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Citation: Hughes, Scott A. "Listening to the Universe with Gravitational-Wave Astronomy." *Annals of Physics* 303, no. 1 (January 2003): 142–178.

As Published: [http://dx.doi.org/10.1016/S0003-4916\(02\)00025-8](http://dx.doi.org/10.1016/S0003-4916(02)00025-8)

Publisher: Elsevier

Persistent URL: <http://hdl.handle.net/1721.1/101191>

Version: Author's final manuscript: final author's manuscript post peer review, without publisher's formatting or copy editing

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Listening to the Universe with Gravitational-Wave Astronomy

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Abstract

The LIGO (Laser Interferometer Gravitational-Wave Observatory) detectors have just completed their first science run, following many years of planning, research, and development. LIGO is a member of what will be a worldwide network of gravitational-wave observatories, with other members in Europe, Japan, and — hopefully — Australia. Plans are rapidly maturing for a low frequency, space-based gravitational-wave observatory: LISA, the Laser Interferometer Space Antenna, to be launched around 2011. The goal of these instruments is to inaugurate the field of *gravitational-wave astronomy*: using gravitational-waves as a means of listening to highly relativistic dynamical processes in astrophysics. This review discusses the promise of this field, outlining why gravitational waves are worth pursuing, and what they are uniquely suited to teach us about astrophysical phenomena. We review the current state of the field, both theoretical and experimental, and then highlight some aspects of gravitational-wave science that are particularly exciting (at least to this author).

1 Motivation

The current state of gravitational-wave science is very similar to the state of neutrino science circa 1950 [1]: we have a mature theoretical framework describing this form of radiation; we have extremely compelling *indirect* evidence of the radiation's existence; but an unambiguous direct detection has not yet happened. Unlike the case of neutrinos, however, it is unlikely that a bright laboratory source of gravitational radiation (analogous to the Savannah

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River nuclear reactor) will be constructed (though see [2] for an alternative view). The only guaranteed sources of gravitational waves bright enough to be measurable will arise from violent astrophysical events. Though perhaps somewhat frustrating on the one hand — we must remain patient while we wait for nature to supply us with a radiation source bright enough for our fledgling detectors — it offers a great opportunity on the other. Gravitational radiation promises to open a unique window onto astrophysical phenomena that may teach us much about “dark” processes in the universe. Once these detectors have met their “physics goal” of directly and unambiguously detecting gravitational waves, they will grow into observatories that — we hope! — will be rich sources of data on violent astrophysical events.

The properties of gravitational radiation and the processes that drive its emission are quite different from the properties and processes relevant to electromagnetic radiation. Consider the following differences:

- Electromagnetic waves are oscillations of electric and magnetic fields that propagate through spacetime. Gravitational waves are oscillations of spacetime itself. Formally, this is an extremely important difference, and historically has been a source of some controversy regarding the validity of certain computation schemes in gravitational-wave theory (with some members of the relativity community worrying that analogies to electromagnetic radiation were used without sufficient justification). This difference can make it difficult to define what exactly a gravitational wave *is*. One must identify an oscillating contribution to the curvature of spacetime that varies on a lengthscale $\lambda/2\pi$ much shorter than the lengthscales over which all other important curvatures vary. In this sense, gravitational waves are more similar to waves propagating over the ocean’s surface (varying on a lengthscale much smaller than the Earth’s radius of curvature) than they are to electromagnetic radiation.
- Astrophysical electromagnetic radiation typically arises from the incoherent superposition of waves produced by many emitters (e.g., electrons in the solar corona, hot plasma in the early universe). This radiation directly probes the thermodynamic state of a system or an environment. Gravitational waves are coherent superpositions arising from the bulk dynamics of a dense source of mass-energy. These waves directly probe the dynamical state of a system.
- Electromagnetic waves interact strongly with matter; gravitational waves do not. This follows directly from the relative strength of the electromagnetic and gravitational interactions. The weak interaction strength of gravitational waves is both blessing and curse: it means that gravitational waves propagate from emission to observers on the Earth with essentially zero absorption, making it possible to probe astrophysics that is hidden or dark — e.g., the coalescence and merger of black holes, the collapse of a stellar core, the dynamics of the early universe. This also means that the waves

interact very weakly with detectors, necessitating a great deal of effort to ensure their detection. Also, because many of the best sources are hidden or dark, they are very poorly understood today — we know very little about what are likely to be some of the most important sources of gravitational waves.

- The direct observable of gravitational radiation is the waveform h , a quantity that falls off with distance as $1/r$. Most electromagnetic observables [3] are some kind of energy flux, and so fall off with a $1/r^2$ law. This means that relatively small improvements in the sensitivity of gravitational-wave detectors can have a large impact on their science: doubling the sensitivity of a detector doubles the distance to which sources can be detected, increasing the volume of the universe to which sources are measurable by a factor of 8. Every factor of two improvement in the sensitivity of a gravitational-wave observatory should increase the number of observable sources by about an order of magnitude.
- Electromagnetic radiation typically has a wavelength smaller than the size of the emitting system, and so can be used to form an image of the source, exemplified by the many beautiful images observatories have provided over the years. By contrast, the wavelength of gravitational radiation is typically comparable to or larger than the size of the radiating source. Gravitational waves *cannot* be used to form an image. Instead, gravitational-waves are best thought of as analogous to sound: the two polarizations carry a stereophonic description of the source’s dynamics. Many researchers in gravitational-wave physics illustrate their work by playing audio encodings of expected gravitational-wave sources and of detector noise. Some source examples from this author’s research can be found at [4]; I leave it to the reader to judge whether they are beautiful or not.
- In most cases, electromagnetic astronomy is based on deep imaging of small fields of view: observers obtain a large amount of information about sources on a small piece of the sky. Gravitational-wave astronomy, by contrast, will be a nearly all-sky affair: gravitational-wave detectors have nearly 4π steradian sensitivity to events over the sky. A consequence of this is that their ability to localize a source on the sky is not good by usual astronomical standards; but, it means that any source on the sky will be detectable, not just sources towards which the detector is “pointed”. The contrast between the all-sky sensitivity but poor angular resolution of gravitational-wave observatories, and the pointed, high angular resolution of telescopes is very similar to the angular resolution contrast of hearing and sight, strengthening the useful analogy of gravitational waves with sound.

These differences show why we believe that gravitational-wave astronomy will open a radically new observational window for astrophysics, and motivate the efforts to construct sensitive gravitational-wave detectors. The last two points in particular explain why we have chosen to describe gravitational-wave astronomy as “listening to the universe”. (Marcia Bartusiak similarly

expanded on this theme in her very engaging book “Einstein’s Unfinished Symphony” [5].) Gravitational-wave astrophysics can be thought of as learning to speak the language of gravitational-wave sources so that we can understand and learn about the sources that the new detectors will measure.

This article surveys the current state of this field. Sections 2 and 3 are review material — Sec. 2 discusses the major background concepts associated with gravitational radiation and gravitational-wave detectors, and Sec. 3 surveys astrophysical sources and detection methods, categorizing them by the frequency band in which they primarily radiate. We then focus on several aspects of gravitational-wave astronomy involving black holes that are of particular interest to this author. Section 4 discusses the importance of binary black hole systems as sources of gravitational waves, and what can be learned from such observations from the standpoint of astrophysics and physics generally. Section 5 discusses in detail a special kind of binary black hole system — extreme mass ratio binaries, in which one black hole in the binary is far more massive than the other. We discuss the particularly powerful and interesting analyses that measurement of these waves can make possible, and then review the challenges that must be overcome to understand the language of these sources.

2 Major concepts of gravitational-wave physics

The idea that radiation of some sort might be associated with the gravitational interaction has a surprisingly long pedigree. As early as 1776, Laplace [6] suggested that an apparent secular acceleration in the Moon’s orbit (deduced by Edmund Halley from a study of medieval solar eclipses recorded by Al-Batanni and of still older eclipses recorded by Ptolemy [7]) could be explained by requiring that the gravitational interaction propagate at finite speed. (The correct explanation of this effect turned out to be tidal transfer of the Earth’s rotational angular momentum to the Moon’s orbit [7].) Poincaré somewhat tentatively resurrected this idea in 1908 in an attempt to explain the anomalous perihelion shift of Mercury [8]. (This effect was eventually explained by the nonlinear “post-Newtonian” effect of relativistic gravity [9].)

Gravitational waves finally and (almost) unambiguously entered the lexicon of physics as a natural consequence of general relativity. Soon after general relativity was introduced, Einstein predicted the existence of gravitational waves in a 1916 paper [10]. This analysis was flawed by a few important algebraic errors, which were corrected in a 1918 paper [11]. Einstein showed that gravitational radiation arises from variations in a source’s quadrupole moment, and derived (with a factor of 2 error) what has come to be called the “quadrupole formula” for the rate at which the radiation carries energy away

from the source. This is what one expects intuitively — gravitational waves arise from the acceleration of masses in a manner similar to the generation of electromagnetic radiation from the acceleration of charges. At lowest order, electromagnetic waves come from the time changing charge dipole moment, and are thus dipole waves; monopole EM radiation would violate charge conservation. We expect (at lowest order) gravitational waves to come from the time changing quadrupolar distribution of mass and energy, since monopole gravitational waves would violate mass-energy conservation, and dipole waves would violate momentum or angular momentum conservation.

The parenthetical “almost” at the beginning of the preceding paragraph refers to a rather lengthy controversy over the formal underpinnings of gravitational radiation calculations. These controversies mostly came to an end in the 1980s, thanks in large part to the careful, rigorous calculations of Thibault Damour and collaborators (cf. Ref. [12] and references therein) and the excellent correspondence to observations of the Hulse-Taylor binary pulsar [13,14]; see Ref. [7] for extended discussion. It is now generally accepted that Einstein’s original quadrupole formula (corrected for the factor of 2 error) properly describes at lowest order the energy flow from a radiating source (even if that source has strong self gravity, a major issue contributing to the aforementioned controversy), and we are likewise confident that theory can go well beyond this lowest order (see, e.g., the review by Blanchet [15] and references therein).

Gravitational waves act tidally, stretching and squeezing any object that they pass through. Their quadrupolar character means that they squeeze along one axis while stretching along the other. When the size of the object that the wave acts upon is small compared to the wavelength (as is the case for LIGO), forces that arise from the two GW polarizations act as in Fig. 1. The polarizations are named “+” (plus) and “ \times ” (cross) because of the orientation of the axes associated with their force lines.

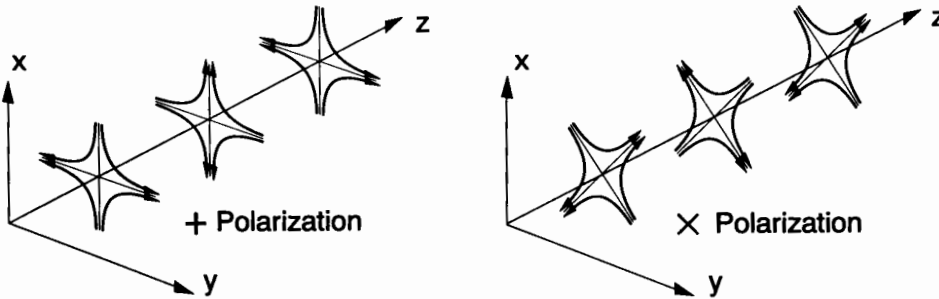


Fig. 1. The lines of force associated with the two polarizations of a gravitational wave (from Ref. [17]).

Interferometric gravitational-wave detectors measure this tidal field by observing their action upon a widely-separated set of test masses. In ground-based

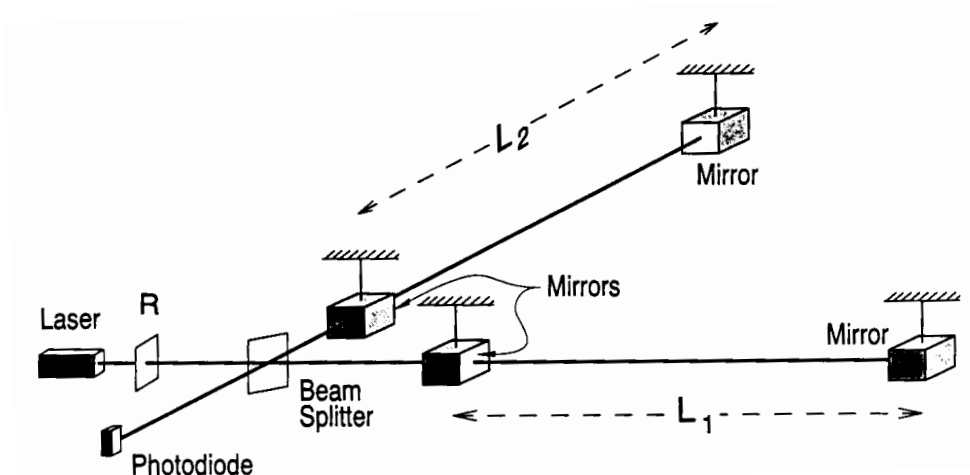


Fig. 2. Layout of an interferometer for detecting gravitational waves (from Ref. [17]).

interferometers, these masses are arranged as in Fig. 2. The space-based detector LISA arranges its test masses in a large equilateral triangle that orbits the sun, illustrated in Fig. 3. On the ground, each mass is suspended with a sophisticated pendular isolation system to eliminate the effect of local ground noise. Above the resonant frequency of the pendulum (typically of order 1 Hz), the mass moves freely. (In space, the masses are actually free floating.) In the absence of a gravitational wave, the sides L_1 and L_2 shown in Fig. 2 are about the same length L .

Suppose the interferometer in Fig. 2 is arranged such that its arms lie along the x and y axes of Fig. 1. Suppose further that a wave impinges on the detector down the z axis, and the axes of the $+$ polarization are aligned with the detector. The tidal force of this wave will stretch one arm while squeezing the other; each arm oscillates between stretch and squeeze as the wave itself oscillates. The wave is thus detectable by measuring the separation between the test masses in each arm and watching for this oscillation. In particular, since one arm is always stretched while the other is squeezed, we can monitor the difference in length of the two arms:

$$\delta L(t) \equiv L_1(t) - L_2(t) . \quad (1)$$

For the case discussed above, this change in length turns out to be the length of the arm times the $+$ polarization amplitude:

$$\delta L(t) = h_+(t)L . \quad (2)$$

The gravitational wave acts as a strain in the detector; h is often referred to as the “wave strain”. Note that it is a dimensionless quantity. Equation (2) is easily derived by applying the equation of geodesic deviation to the separation

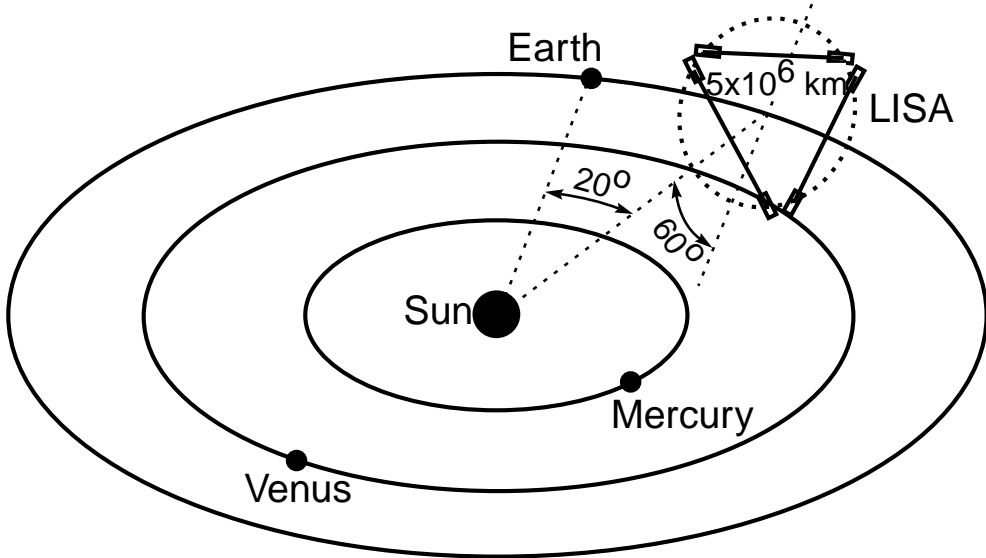


Fig. 3. Orbital configuration of the LISA antenna.

of the test masses and using a gravitational-wave tensor on a flat background spacetime to develop the curvature tensor; see Ref. [18], Sec. 9.2.2 for details.

We obviously do not expect astrophysical gravitational-wave sources to align themselves in as convenient a manner as described above. Generally, both polarizations of the wave influence the test masses:

$$\frac{\delta L(t)}{L} = F^+ h_+(t) + F^\times h_\times(t) \equiv h(t) . \quad (3)$$

The antenna response functions F^+ and F^\times weight the two polarizations in a quadrupolar manner as a function of a source's position and orientation relative to the detector; see [18], Eqs. (104a,b) and associated text.

The energy flux carried by gravitational waves scales as \dot{h}^2 (where the over-dot denotes a time derivative). In order for the energy flowing through large spheres to be conserved, h must fall off with distance as $1/r$. As discussed above, the lowest order contribution to the waves arises from changes in a source's quadrupole moment. To order of magnitude, this moment is given by $Q \sim (\text{source mass})(\text{source size})^2$. By dimensional analysis, we then know that the wave strain must have the form

$$h \sim \frac{G \ddot{Q}}{c^4 r} . \quad (4)$$

The second time derivative of the quadrupole moment is given approximately by $\ddot{Q} \simeq 2Mv^2 \simeq 4E_{\text{kin}}^{\text{ns}}$; v is the source's internal velocity, and $E_{\text{kin}}^{\text{ns}}$ is the nonspherical part of its internal kinetic energy. Strong sources of gravitational

radiation are sources that have strong non-spherical dynamics — for example, compact binaries (containing white dwarfs, neutron stars, and black holes), mass motions in neutron stars and collapsing stellar cores, the dynamics of the early universe.

Violent events that are likely to be interesting gravitational-wave sources are very rare — for example, supernovae from the collapse of massive stellar cores appear to occur in our galaxy once every few centuries. For our detectors to have a realistic chance of measuring observable events, they must be sensitive to sources at rather large distances. For example, to have an interesting shot at measuring the coalescence of binary neutron star systems, we need to reach out to several hundred megaparsecs (*i.e.*, a substantial fraction of 10^9 light years) [19,20,21]. For such coalescences, $E_{\text{kin}}^{\text{ns}}/c^2 \sim 1$ solar mass ($\equiv 1 M_{\odot}$). Plugging into Eq. (4) gives the estimate

$$h \sim 10^{-21} - 10^{-22} . \quad (5)$$

This sets the sensitivity required to measure gravitational waves. Combining this scale with Eq. (3) tells us that for every kilometer of baseline L we need to be able to measure a distance shift δL of better than 10^{-16} centimeters.

This is usually the point at which people decide that gravitational-wave scientists aren't playing with a full deck. How can we possibly hope to measure an effect that is $\sim 10^{12}$ times smaller than the wavelength of visible light? For that matter, how is it possible that thermal motions do not wash out such a tiny effect?

That such measurement is possible with laser interferometry was analyzed thoroughly and published by Rainer Weiss in 1972 [22]. (It should be noted that the possibility of detecting gravitational waves with laser interferometers has an even longer history, reaching back to Pirani in 1956 [23], and has been independently invented by Gertsenshtein and Pustovoit in 1962 [24] and Weber in the 1960s (unpublished), prior to Weiss's detailed analysis. See Sec. 9.5.3 of Ref. [18] for further discussion.) Examine first how a laser with a wavelength of 1 micron can measure a 10^{-16} cm displacement. In a laser interferometer like LIGO, the basic optical layout is as sketched in Fig. 2. A carefully prepared laser state is split at the beamsplitter and sent into the Fabry-Perot arm cavities of the detector. The reflectivities of the mirrors in these cavities are chosen such that the light bounces roughly 100 times before exiting the arm cavity (that is, the finesse \mathcal{F} of the cavity is roughly 100). This corresponds to about half a cycle of a 100 Hz gravitational wave. The phase shift acquired by the light during those 100 round trips is

$$\Delta\Phi_{\text{GW}} \sim 100 \times 2 \times \Delta L \times 2\pi/\lambda \sim 10^{-9} . \quad (6)$$

This phase shift can be measured provided that the shot noise at the photodiode, $\Delta\Phi_{\text{shot}} \sim 1/\sqrt{N}$, is less than $\Delta\Phi_{\text{GW}}$. N is the number of photons accumulated over the measurement; $1/\sqrt{N}$ is the phase fluctuation in a quantum mechanical coherent state that describes a laser. We therefore must accumulate $\sim 10^{18}$ photons over the roughly 0.01 second measurement, translating to a laser power of about 100 watts. In fact, as was pointed out by Ronald Drever [25], one can use a much less powerful laser: even in the presence of a gravitational wave, only a tiny portion of the light that comes out of the interferometer’s arms goes to the photodiode. The vast majority of the laser power is sent back to the laser. An appropriately placed mirror bounces this light back into the arms, *recycling* the light. The recycling mirror is shown in Fig. 2, labeled “R”. With it, a laser of ~ 10 watts drives several hundred watts of input to the interferometer’s arms.

Thermal excitations are overcome by averaging over many many vibrations. For example, the atoms on the surface of the interferometers’ test mass mirrors oscillate with an amplitude

$$\delta l_{\text{atom}} = \sqrt{\frac{kT}{m\omega^2}} \sim 10^{-10} \text{ cm} \quad (7)$$

at room temperature T , with m the atomic mass, and with a vibrational frequency $\omega \sim 10^{14} \text{ s}^{-1}$. This amplitude is huge relative to the effect of gravitational radiation — how can we possibly hope to measure the wave? The answer is that atomic vibrations are random and incoherent. The ~ 7 cm wide laser beam averages over about 10^{17} atoms and at least 10^{11} vibrations per atom in a typical measurement. The effect is thus suppressed by a factor $\sim \sqrt{10^{28}}$ — atomic vibrations are *completely* irrelevant compared to the coherent effect of a gravitational wave. Other thermal vibrations, however, are not irrelevant and in fact dominate LIGO’s noise in certain frequency bands. For example, the test masses’ normal modes are thermally excited. The typical frequency of these modes is $\omega \sim 10^5 \text{ s}^{-1}$ and they have mass $m \sim 10$ kg, so $\delta l_{\text{mass}} \sim 10^{-14} \text{ cm}$. This, again, is much larger than the effect we wish to observe. However, the modes are very high frequency, and so can be averaged away provided the test mass is made from material with a very high quality factor Q — the mode’s energy is confined to frequencies near ω and doesn’t leak into the band we want to use for measurements. Understanding the physical nature of noise in gravitational-wave detectors is an active field of current research; see Refs. [26,27,28,29,30,31,32,33] and references therein for a glimpse of recent work. In all cases, the fundamental fact to keep in mind is that a gravitational wave acts *coherently*, whereas noise acts *incoherently*, and thus can be beaten provided one is able to average away the incoherent noise sources.

3 Gravitational-wave frequency bands and measurement

It is useful to categorize gravitational-wave sources (and the methods for detecting their waves) by the frequency band in which they radiate. Broadly speaking, we may break the gravitational-wave spectrum into four rather different bands: the *ultra low frequency* band, $10^{-18} \text{ Hz} \lesssim f \lesssim 10^{-13} \text{ Hz}$; the *very low frequency* band, $10^{-9} \text{ Hz} \lesssim f \lesssim 10^{-7} \text{ Hz}$; the *low frequency* band, $10^{-5} \text{ Hz} \lesssim f \lesssim 1 \text{ Hz}$; and the *high frequency* band, $1 \text{ Hz} \lesssim f \lesssim 10^4 \text{ Hz}$.

For compact sources (mass/energy configurations that are of compact support), the band in which gravitational waves are generated is typically related to the source’s size R and mass M . R is meant to set the scale over which the source’s dynamics vary; for example, it could be the actual size of a particular body, or the separation of members of a binary. The “natural” gravitational-wave frequency of such a source is $f_{\text{GW}} \sim (1/2\pi)\sqrt{GM/R^3}$. Because $R \lesssim 2GM/c^2$ (the Schwarzschild radius of a mass M), we can estimate an upper bound for the frequency of a compact source:

$$f_{\text{GW}}(M) < \frac{1}{4\sqrt{2}\pi} \frac{c^3}{GM} \simeq 10^4 \text{ Hz} \left(\frac{M_{\odot}}{M} \right). \quad (8)$$

This is a rather hard upper limit, since many interesting sources are quite a bit larger than $2GM/c^2$, or else evolve through a range of sizes before terminating their emission at $R \sim 2GM/c^2$. Nonetheless, this frequency gives some sense of the types of compact sources that are likely to be important in each band — high frequency compact sources are of stellar mass (several solar masses); low frequency compact sources are of thousands to millions of solar masses, or else contain widely separated stellar mass bodies; etc. Other interesting sources of waves, particularly in the lower frequency bands, are not well-described by these compact body rules; we will discuss them separately in greater depth below.

3.1 High frequency

The high frequency band, $1 \text{ Hz} \lesssim f \lesssim 10^4 \text{ Hz}$, is the band targeted by the new generation of ground-based laser interferometric detectors, such as LIGO. (It also corresponds roughly to the audio band of the human ear: when converted to sound, LIGO sources are human audible without any frequency scaling.) The low frequency end of this band is set by the fact that it is extremely difficult to isolate against ground vibrations at low frequencies, and probably impossible to isolate against gravitational coupling to ground vibrations, human activity, and atmospheric motions [31,32,33]. The high end of the band

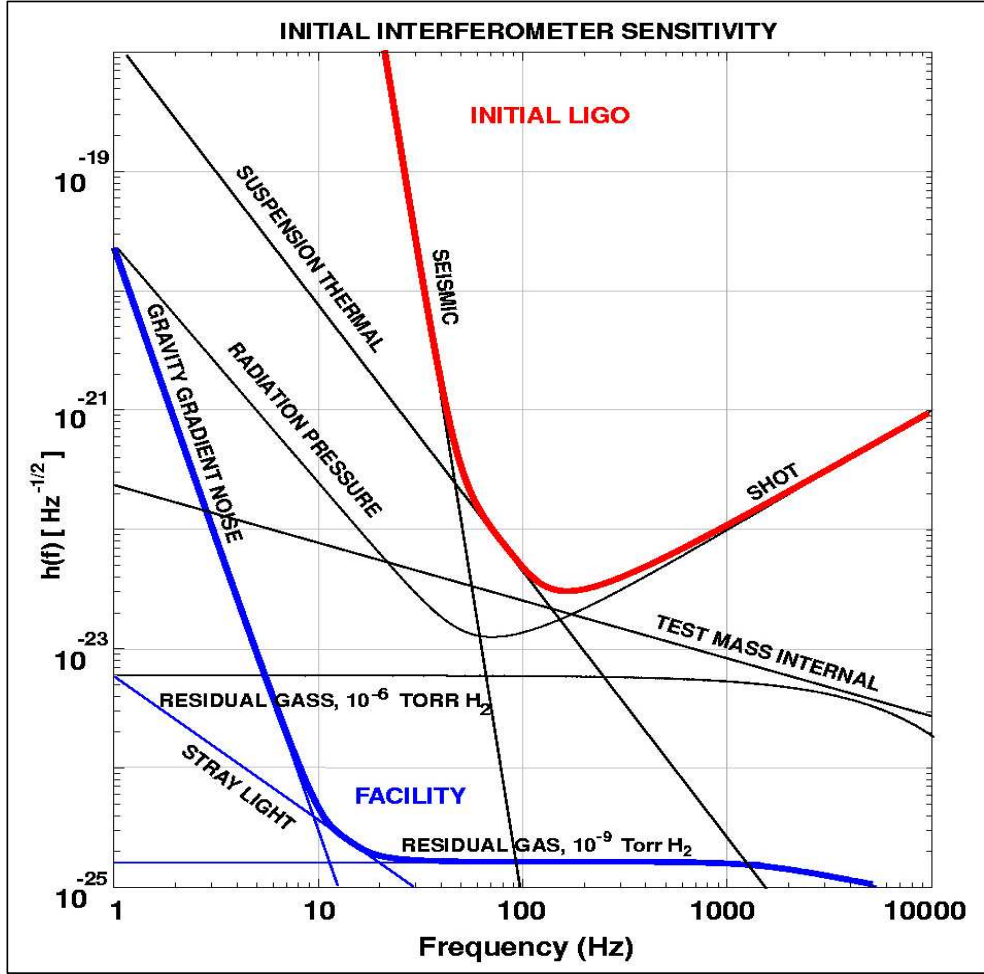


Fig. 4. Sensitivity goals of the initial LIGO interferometers, and facility limits on the LIGO sensitivity (taken from Ref. [16]).

is set by the fact that it is unlikely any interesting gravitational-wave source radiates at frequencies higher than a few kilohertz — from the arguments sketched above, such a source would have to be relatively low mass but extremely compact.

The operating principles of a ground-based laser interferometric detector have already been sketched in Sec. 2 [cf. the text following Eq. (5)]. The curve describing the sensitivity of such detectors typically takes a shape similar to that shown in Fig. 4. At high frequencies, the detectors' sensitivities rapidly degrade because of photon shot noise — fluctuations in the number of photons used in the measurement process. Making a measurement at a frequency f essentially means averaging for a timescale $T = 1/f$. As the time T becomes shorter, a smaller number of photons are gathered in the course of the measurement, and hence the typical fluctuation in the number of photons is relatively more important. At intermediate frequencies, thermally excited normal modes in the test mass mirrors (at the ends of the arms in Fig. 2) and

in the mirrors' suspensions dominate the noise budget. The resonant frequencies of these modes are carefully chosen to be rather far above the band of greatest interest for gravitational-wave observation; and, the Q of the masses and suspensions are made as large as is practical so that the modes' energy bleeds into the gravitational-wave band as little as possible. Some contamination is of course inevitable. At very low frequencies, seismic motions dominate the detectors' noise. The test masses are carefully suspended on multi-level pendular systems to isolate them from local ground motions. This makes the masses effectively free falling above the resonant frequency of the pendulum; below that frequency, however, the noise due to ground motion dominates the motion spectrum of the masses.

Several interferometric gravitational-wave observatories are either operating or being completed in the United States, Europe, Japan, and Australia. Multiple observatories widely scattered over the globe are extremely important, both as checks on one another for assured detection and to aid in the interpretation of measurements. For example, position determination and thence measurement of the distance to a source follows from triangulation of time-of-flight differences between separated detectors. The major interferometer projects are:

- **LIGO.** The Laser Interferometer Gravitational-Wave Observatory currently consists of three operating interferometers: a single four kilometer interferometer in Livingston, Louisiana, as well as a pair of interferometers (four kilometers and two kilometers) in the LIGO facilities at Hanford, Washington. The sites are separated by 3000 kilometers, and are situated to support coincidence analysis of events.
- **Virgo.** Virgo is a three kilometer French-Italian detector under construction near Pisa, Italy [34]. In most respects, Virgo is quite similar to LIGO. A major difference is that Virgo employs a very sophisticated seismic isolation system that promises extremely good low frequency sensitivity.
- **GEO600.** GEO600 is a six hundred meter interferometer constructed by a German-English collaboration near Hannover, Germany [35]. Despite its shorter arms, GEO600 is expected to achieve sensitivity comparable to the multi-kilometer instruments by incorporating advanced interferometry techniques from the beginning. This will make it an invaluable testbed for technology to be used in later generations of the larger instruments, as well as enabling it to make astrophysically interesting measurements.
- **TAMA300.** TAMA300 is a three hundred meter interferometer operating near Tokyo. It has been in operation for several years now [36]; the most recent run achieved a displacement sensitivity 10^{-16} cm/ $\sqrt{\text{Hz}}$ [37] at frequencies near 1000 Hz. The TAMA team is currently designing a three kilometer interferometer [38], building on their experiences with the three hundred meter instrument.
- **ACIGA.** The Australian Consortium for Interferometric Gravitational-

Wave Astronomy is currently constructing an eighty meter research interferometer near Perth, Australia [39], hoping that it will be possible to extend it to multi-kilometer scale in the future. Such a detector would likely be a particularly valuable addition to the worldwide stable of detectors, since all the Northern Hemisphere detectors lie very nearly on a common plane. An Australian detector would be far outside this plane, allowing it to play an important role in determining the location of sources on the sky.

All of these detectors have or will have sensitivities similar to that illustrated in Fig. 4 (which shows, in particular, the sensitivity goal of the first generation of LIGO interferometers). This figure also shows the “facility limits” — the lowest noise levels that can be achieved even in principle within an interferometer facility. The low level facility limits come from *gravity-gradient noise*: noise arising from gravitational coupling to fluctuations in the local mass distribution (such as from seismic motions in the earth near the test masses [31], human activity near the detector [32], and density fluctuations in the atmosphere [33]). At higher frequencies, the facility limit arises from residual gas (mostly hydrogen) in the interferometer vacuum system. Stray molecules of gas effectively cause stochastic fluctuations in the index of refraction, a source of noise as we try to make ever more precise measurements.

There’s a great deal of room for improvement between the sensitivity goals of the first detectors and the facility limits. Much active research and development work is geared towards developing improved interferometers which will have greater astrophysical reach than the first generation of detectors. The first detectors have been designed somewhat conservatively, ensuring that they can be operated for several years without requiring too much technology development. Upgraded detectors will have the seismic “wall” pushed down to lower frequencies and will have noise curves that are moderately “tunable”, shaping the detector response to chase down signals that are particularly interesting or important [29,30,40,41]. We should emphasize that, at present, much effort is being put into reaching the initial sensitivity goals. The LIGO detectors have made enormous strides in improving their sensitivity recently (gaining several orders of magnitude over the course of 2002), but are still some distance from the design goals. Seismic noise in particular has proven to be a greater problem than was anticipated (largely because of increased human activity near the two LIGO sites), so improvements to the test masses’ isolation systems will be implemented quite quickly.

In the remainder of this subsection, we take a quick tour of some of the more well-understood possible sources of measurable gravitational waves in the high-frequency band. We emphasize at this point that such a listing of sources can in no way be considered comprehensive: we are hopeful that some gravitational-wave sources may surprise us, as has been the case whenever we have studied the universe with a new type of radiation. If we regard gravitational-wave

astrophysics as learning to speak the language of gravitational-wave sources, then surprise sources will be somewhat akin to discovering a lost language written in an unknown script — interpreting and understanding their message will be quite difficult.

3.1.1 Compact binaries

Compact binaries — binary star systems in which each member is a collapsed, compact stellar corpse (neutron star or black hole) — are currently the best understood sources of gravitational waves. Double neutron stars have been studied observationally since the mid 1970s; three such systems [20] tight enough to merge within a few 10^8 or 10^9 years have been identified in the galaxy (two in the galactic field, one in a globular cluster). Detailed studies of these systems currently provide our best data on gravitational-wave generation [42,43,44], and led to the 1993 Nobel Prize for Joseph Taylor and Russell Hulse. Extrapolation from these observed binaries in the Milky Way to the universe at large [19,20,21] indicates that gravitational-wave detectors should measure at least several and at most several hundred binary neutron star mergers each year (following detector upgrades; the rates for initial detectors suggest that detection is plausible but not very probable — the expected rate is of order one per decade). Population synthesis (modeled evolution of stellar populations) indicates that the measured rate of binaries containing black holes should likewise be interestingly large (perhaps even for initial detectors) [45,46,47,48,49,50,51]. The uncertainties of population synthesis calculations are rather large, however, due to poorly understood aspects of stellar evolution and compact binary formation; data from gravitational-wave detectors is likely to have a large impact on this field.

We will revisit and discuss in greater depth this class of sources in Sec. 4.

3.1.2 Stellar core collapse

Core collapse in massive stars (the engine of Type II supernova explosions) has long been regarded as likely to be an important source of gravitational waves; see, for example, Ref. [52] for an early review. Stellar collapse certainly exhibits all of the *necessary* conditions for strong gravitational-wave generation: large amounts of mass ($1-100 M_\odot$) flow in a compact region (hundreds to thousands of kilometers) at relativistic speeds ($v/c \sim 1/5$). However, these conditions are not *sufficient* to guarantee strong emission. In particular, the degree of asymmetry in collapse is not particularly well understood [cf. the text following Eq. (4), arguing that non-spherical dynamics drives gravitational-wave emission]. If stellar cores are rapidly rotating, instabilities can develop that are certain to drive strong gravitational-wave emission. An example of such an instability

is the development of a rapidly rotating bar-like mode in the dense material of the stellar core [53,54,55]. Such an instability has a rapidly varying quadrupole moment and potentially generates copious amounts of gravitational waves.

Fryer, Holz, and Hughes [56] recently surveyed the status of core-collapse simulations with an eye to understanding whether such collapses are likely to produce interesting and measurable waves. They find that stellar cores in fact are quite likely to have enough angular momentum to be susceptible to secular or dynamical instabilities such as the bar mode. The detectability of the waves from these modes will depend quite strongly on the coherence of the emission mechanism: detectable waves arise from modes that hold together long enough to radiate several tens of gravitational-wave cycles without changing their peak frequency too strongly. Even in this case, observers will need to wait for upgrades before such detection is likely to become commonplace (unless we get lucky and a star collapses relatively close by). Future theoretical progress in this field will come from detailed three-dimensional simulations of core collapse processes. We note that significant progress has been made on this problem recently [57], and are confident that we will have a grasp of core collapse wave emission robust enough to enable the design of useful detection algorithms and astrophysical studies by the time that the upgraded detectors are likely to be operating.

3.1.3 *Periodic emitters*

Periodic sources of gravitational waves radiate at constant or nearly constant frequency, like radio pulsars. In fact, the prototypical source of continuous gravitational waves is a rotating neutron star, or gravitational-wave pulsar. A non-axisymmetric neutron star (caused, for example, by a crust that is somewhat oblate and misaligned with the star’s spin axis) will radiate gravitational waves with characteristic amplitude

$$h_c \sim \frac{G I f^2 \epsilon}{c^4 r}, \quad (9)$$

where I is the star’s moment of inertia, f is the wave frequency, and r is the distance to the source. The crucial parameter ϵ characterizes the degree to which the star is distorted; it is rather poorly understood. Various mechanisms have been proposed to explain how a neutron star can be distorted to give a value of ϵ interesting as a gravitational-wave source; see [58,59] for further discussion. Examples of some interesting mechanisms include misalignment of a star’s internal magnetic field with the rotation axis [60] and distortion by accreting material from a companion star [61,62].

Whatever the mechanism generating the distortion, it is clear that ϵ will be relatively small, so that $h_c \sim 10^{-24}$ or smaller — rather weak. (Note that if these sources were not weak emitters, the backreaction of gravitational-wave emission would make their frequencies change more quickly — they would not be periodic emitters.) Measuring these waves will require coherently tracking their signal for a large number of wave cycles — coherently tracking N cycles boosts the signal strength by a factor $\sim \sqrt{N}$. This is actually fairly difficult, since the signal is strongly modulated by the Earth’s rotation and orbital motion, “smearing” the waves’ power across multiple frequency bands. Searching for periodic gravitational waves means demodulating the motion of the detector, a computationally intensive problem since the modulation is different for every sky position. Unless one knows in advance the position of the source, one needs to search over a huge number of sky position “error boxes”, perhaps as many as 10^{14} . One rapidly becomes computationally limited. (Note that radio pulsar searches face this same problem, with the additional complication that radio pulses are dispersed by the interstellar medium. However, in this case, it is known in advance which sky position is being examined, so the computational cost is usually not as great.) For further discussion, see [68]; for ideas about doing hierarchical searches that require less computer power, see [69].

Finally, we note that the r-mode instability (a source of waves from a current instability in rotating neutron stars) would generate waves that are nearly periodic [63,64,65,66,67]. Although the physics of this source is rather different from the physics of bumpy neutron stars, the character of the waves is quite similar, at least as far as detection goes. We note, though, that recent results [70,71,72,73] indicate that the r-mode is suppressed rather more robustly than previously appreciated. Conventional wisdom currently suggests that r-mode waves are unlikely to be important sources from isolated neutron stars, though r-modes driven by accretion from a companion may turn out to be quite important [74]. See [75] for further discussion.

3.1.4 *Stochastic backgrounds*

Stochastic backgrounds are “random” gravitational waves, arising from a large number of independent, uncorrelated sources that are not individually resolvable. A particularly interesting source of backgrounds is the dynamics of the early universe — an all-sky gravitational-wave background, similar to the cosmic microwave background. Backgrounds can arise from amplification of primordial fluctuations in the universe’s geometry, phase transitions as previously unified interactions separated, or the condensation of a brane from a higher dimensional space. These waves can actually spread over a wide range of frequency bands; waves from inflation in particular span all bands, from ultra low frequency to high frequency. We will discuss such inflationary waves in

greater detail in Sec. 3.3; here, we briefly discuss how these backgrounds are characterized at higher frequencies, and the sensitivity to them that LIGO should achieve.

Stochastic backgrounds are described by their contribution to the universe’s energy density, ρ_{gw} . In particular, one is interested in the energy density as a fraction of that needed to close the universe, over some frequency band:

$$\Omega_{\text{gw}}(f) = \frac{1}{\rho_{\text{crit}}} \frac{d\rho_{\text{gw}}}{d \ln f} , \quad (10)$$

where $\rho_{\text{crit}} = 3H_0^2/8\pi G$ is the critical density needed to close the universe. (H_0 is the value of the Hubble constant today.) Different cosmological sources produce different levels of $\Omega_{\text{gw}}(f)$, centered in different bands. In the high frequency band, waves produced by inflation are likely to be rather weak: estimates suggest that the spectrum will be flat across LIGO’s band, with magnitude $\Omega_{\text{gw}} \sim 10^{-15}$ at best [76]. Waves from phase transitions can be significantly stronger, but are typically peaked around a frequency that depends on the temperature T of the phase transition [77,78]:

$$f_{\text{peak}} \sim 100 \text{ Hz} \left(\frac{T}{10^5 \text{ TeV}} \right) . \quad (11)$$

The temperature required to enter the LISA band, $f \sim 10^{-4} - 10^{-2}$ Hz, is $T \sim 100 - 1000$ GeV, nicely corresponding to the electroweak phase transition. Waves arising from extradimensional dynamics should peak at a frequency given by the scale b of the extra dimensions [79,80]:

$$f_{\text{peak}} \sim 10^{-4} \text{ Hz} \left(\frac{1 \text{ mm}}{b} \right)^{1/2} . \quad (12)$$

For the waves to be in LIGO’s band, the extra dimensions must be rather small, $b \sim 10^{-15}$ meters. LISA’s band is accessible for a scale similar to those discussed in modern brane-world work [81,82]. It’s worth noting that extradimensional models which attempt to explain the acceleration of the universe typically predict relic spectra of gravitational waves that are rather large, and thus may be falsified by gravitational-wave observations [83].

Because of their random nature, stochastic gravitational waves look just like noise. Ground-based detectors will measure stochastic backgrounds by comparing data at multiple sites and looking for “noise” that is correlated [84,85]. For comparing to a detector’s noise, one should construct the characteristic stochastic wave strain,

$$h_c \propto f^{-3/2} \sqrt{\Omega_{\text{gw}}(f) \Delta f} . \quad (13)$$

(For further discussion and the proportionality constants, see [84].) Note that this strain level grows sharply with decreasing frequency. As we will discuss in Sec. 3.4, observations in the very low frequency band are likely to provide the best constraints on stochastic waves in the near future.

Early LIGO detectors will have fairly poor sensitivity to the background, constraining it to a level $\Omega_{\text{gw}} \sim 5 \times 10^{-6}$ in a band from about 100 Hz to 1000 Hz. This is barely more sensitive than known limits from cosmic nucleosynthesis [77]. Later upgrades will be significantly more sensitive, able to detect waves with $\Omega_{\text{gw}} \sim 10^{-10}$, which is good enough to place interesting limits on cosmological backgrounds.

3.2 Low frequency

There is no hope of measuring gravitational waves in the low frequency band, $10^{-5} \text{ Hz} \lesssim f \lesssim 1 \text{ Hz}$, using a ground-based instrument: even if it were possible to completely isolate one's instrument from local ground motions, gravitational coupling to fluctuations in the local mass distribution ultimately limits the sensitivity to frequencies $f \gtrsim 1 \text{ Hz}$. As we shall discuss below, however, many extremely interesting gravitational-wave sources radiate in this band. The only way to measure these waves is to build a gravitational-wave observatory in the quiet environment of space, far removed from low-frequency noise sources.

Such an instrument is currently being designed jointly by NASA in the United States and ESA, the European Space Agency: LISA, the Laser Interferometer Space Antenna. If all goes well, LISA would be launched into orbit in or near 2011. Like LIGO, LISA will be a laser interferometer — changes in the distance between widely separated test masses will be monitored to find variations consistent with the action of gravitational waves. However, the scale of LISA is vastly different from that of LIGO, and so details of its operations are quite different. In particular, LISA has armlengths $L \simeq 5 \times 10^6 \text{ km}$, vastly larger than LIGO and all other ground-based detectors. The three spacecraft which delineate the ends of LISA's arms are placed into orbits such that LISA forms a triangular constellation orbiting the sun, inclined 60° with respect to the plane of the ecliptic and following the Earth with a 20° lag. This configuration is sketched in Fig. 3. Since it essentially shares Earth's orbit, the constellation orbits the sun once per year, “rolling” as it does so. This orbital motion plays an important role in pinpointing the position of gravitational-wave sources by modulating the measured waveform — the modulation encodes source location and makes position determination possible.

Each spacecraft contains two optical assemblies, each of which houses a 1 watt laser and a 30 centimeter telescope. Because of the extreme lengths of

the interferometer’s arms, Fabry-Perot interferometry as in LIGO is not at all possible: diffraction spreads the laser beam over a diameter of about 20 km as it propagates the 5×10^6 km from one spacecraft to the other. With this much spread, multiple bounces in LISA’s arms obviously aren’t feasible. Instead, a portion of that 20 km wavefront is sampled with the telescope. That light is then interfered with a sample of light from the on-board laser. Each spacecraft thus generates two interference data streams; six signals are generated by the full LISA constellation. From these six signals, we can construct the time variations of LISA’s armlengths and then build both gravitational-wave polarizations. More information and details can be found in Refs. [86,87,88,89].

It is worth noting at this point that the LISA armlengths are *not* constant — as the constellation orbits, the distances between the various spacecraft vary by about 1% (including effects such as planetary perturbations). This is *far* larger than the effect produced by gravitational waves, which is of order picometers. However, these variations occur over timescales of order months, and are extremely smooth and well modeled. It will not be difficult to fit out these very low frequency variations, leaving clean data in the interesting low-frequency gravitational-wave band. Note also that these picometer scale variations are not too difficult to measure in this frequency band: measuring in this band entails gathering photons for a time $10 \text{ sec} \lesssim T \lesssim 1 \text{ day}$. Even though the bulk of the laser’s emitted power is lost due to diffraction, enough photons are gathered on this timescale that the phase shift due to the gravitational-wave can be determined [cf. the argument outlined in and near Eq. (6)].

The gravitational-wave signals are actually read out by monitoring the position of the so-called “gravitational sensor” on each optical assembly; in particular, the position of a “proof mass” which floats freely and constitutes the test mass for the LISA antenna is monitored. Because it is freely floating, the proof mass responds solely to gravitational forces (or, in relativistic language, follows a geodesic of the spacetime). Micronewton thrusters keep the bulk spacecraft centered on these proof masses, forcing the craft to follow the average trajectory of the two proof masses. In this way, LISA is isolated from low frequency noises that could impact the ability to measure gravitational waves (e.g., variations in solar radiation pressure). This is called a drag-free system, since such systems were first used to reduce the effect of Earth’s atmospheric drag on low altitude satellites.

The sensitivity of LISA to gravitational waves is shown in Fig. 5. At high frequencies, the noise budget is dominated by the accuracy with which laser interferometry can determine variations in the 5×10^6 km distance between proof masses on distant spacecraft, which is largely limited by photon shot noise. Wiggles in the sensitivity curve at this point arise because, in this band, the gravitational wavelength is shorter than LISA’s armlength; see [90] for further discussion. At lower frequencies, the instrumental noise is domi-

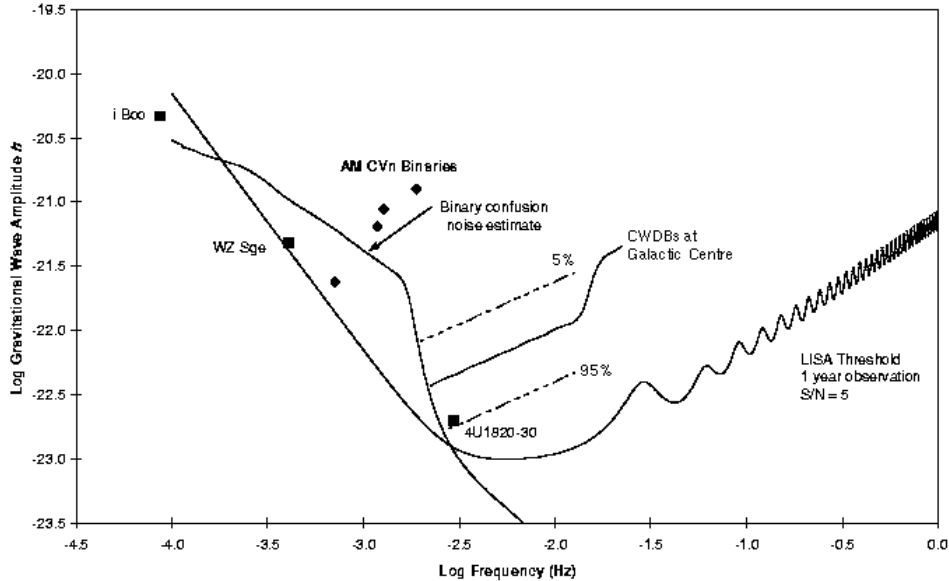


Fig. 5. LISA sensitivity, including a few interesting known sources, taken from Ref. [16]. Points are the expected signal amplitude of certain known monochromatic binary stars. “CWDB” stands for close white dwarf binary.

nated by spurious accelerations on the proof mass. LISA requires that these accelerations be kept at a level below $3 \times 10^{-15} \text{ m/sec}^2 \text{ Hz}^{-1/2}$ in this band. This subsystem will be tested by SMART-2 (Small Mission for Advanced Research and Technology), to be launched in 2006 by ESA with participation from NASA.

Note in Fig. 5 the curve labeled “Binary confusion estimate” over the band $10^{-4} \text{ Hz} \lesssim f \lesssim 3 \times 10^{-3} \text{ Hz}$. In this band, LISA’s “noise” actually comes not from the instrument itself but from a confused stochastic background of gravitational waves! It is expected that so many binary star systems (primarily double white dwarf binaries) in the galaxy will be radiating in this band that we will not have sufficient information to resolve them — $10^2 - 10^4$ binaries may contribute to the waves measured in a single frequency bin of width $\delta f \sim 10^{-7} \text{ Hz}$ [91]. This confused background of waves is “noise” from the point of view of observers wishing to measure other sources in this band (though of course it is extremely interesting “signal” to an astrophysicist interested in close binary populations).

This aspect of LISA’s “noise” budget points to an important difference in sources in the high-frequency and low-frequency bands: whereas many (though certainly not all) high-frequency sources are short-lived and comparatively rare (e.g., binary coalescence and stellar collapse), most low-frequency sources are quite long-lived and may not be so rare. As in Sec. 3.1, we now take a quick

tour through some interesting LISA sources.

3.2.1 *Periodic emitters*

For LIGO, the source of most periodic gravitational waves is expected to be isolated neutron stars, essentially gravitational-wave pulsars. LISA’s periodic sources will come primarily from binary star systems in the Milky Way. These systems do not generate waves strong enough to backreact significantly on the system, so that their frequencies typically change very little or not at all over the course of LISA observations. Certain systems are well-known in advance to be sources of periodic waves for the LISA band; cf. the points in Fig. 5. These sources are understood well enough that they may be regarded as “calibrators” — LISA had better detect them, or else something is wrong!

Aside from these sources that are known in advance, it is expected that LISA will discover a good number of binary systems that are too faint to detect with telescopes. Joint observations by LISA and other astronomical instruments are likely to be quite fruitful, helping to understand these systems much better than can be done with a single instrument alone. For example, it is typically difficult for telescopes to determine the inclination of a binary to the line of sight (a factor needed to help pin down the mass of the binary’s members). Gravitational waves measure the inclination angle almost automatically, since this angle determines the relative magnitude of the polarizations h_+ and h_\times .

3.2.2 *Coalescing binary black holes*

Coalescing binary black hole systems will be measurable by LISA to extremely large distances; even if such events are very rare, the observed volume is enormous, so that an interesting measured rate seems quite likely. One class of such binaries consists of systems in which the member holes are of roughly equal mass. These binaries can form following the merger of galaxies (or pregalactic structures) containing a black hole in their core. Depending on the mass of the binary, the waves from these coalescences will be detectable to fairly large redshifts ($z \sim 5 - 10$), possibly probing an early epoch in the formation of the universe’s structure. (The optimal system mass is near $10^5 - 10^6 M_\odot$ — the waves from smaller systems aren’t so loud, and so can’t be measured quite as well; the waves from larger systems come out at low frequencies where noise is strong.) The rate at which such events are likely to occur, however, is extremely uncertain. It seems clear that, following the merger of their host structures, the black holes will form a bound binary. It is not clear, however, whether this hole becomes bound tightly enough that gravitational-wave emission importantly impacts its dynamics: some simulations show that the binary “stalls” well before gravitational waves become important [92]. It is possible

that a later mechanism drives the holes closer together (see, for example, Ref. [93]); some observations hint that this in fact may be happening [94]. If black hole mergers are “efficient” (there is roughly one binary black hole merger for every merger of host structures), then the rate at which LISA measures these events could be several per year [95].

The other major class of binary black hole systems consists of relatively small bodies (black holes with mass $\sim 10 M_\odot$, neutron stars, or white dwarfs) that are captured by larger black holes ($M \sim 10^5 - 10^7 M_\odot$) such as are found at the cores of many galaxies. These extreme mass ratio binaries are created when the smaller body is captured onto an extremely strong field, highly relativistic orbit, generating strong gravitational waves. Such systems are measurable to a distance of a few gigaparsecs if the inspiraling body is a $10 M_\odot$ black hole, and to a distance of a few hundred megaparsecs if the body is a neutron star or white dwarf. LISA will measure the waves that come from the last year or so of the smaller body’s inspiral, probing the nature of the larger black hole’s gravitational field from deep within the hole’s potential. The rates for such events are, again, not so well understood, depending in some detail on the dynamical nature of the cores of galaxies. Extremely conservative estimates typically find that the rate of measurable events for LISA should be at least several per year [96,97]. Recent thinking suggests that these rates are likely to be rather underestimated — black holes (which are measurable to much greater distances) are likely to dominate the measured rate, perhaps increasing the rate to several dozen or several hundred per year.

Both of these types of black hole binaries will be discussed in greater depth in Sec. 4 and 5.

3.2.3 Stochastic backgrounds

As discussed in Sec. 3.1, ground-based detectors can measure a stochastic background by correlating the data streams of widely separated detectors. LISA obviously cannot do this, since it consists of a single antenna. However, it can take advantage of a different trick: by combining its six data streams in an appropriate way, it can construct an observable that is completely *insensitive* to gravitational waves, measuring noise only [98]. This makes it possible to distinguish between a noise-like stochastic background and true instrumental noise, and thereby to learn about the characteristics of the background [99].

The sensitivity of LISA will not be good enough to set interesting limits on an inflationary gravitational-wave background: LISA will only reach $\Omega_{\text{gw}} \sim 10^{-11}$, about four orders of magnitude too large to begin to say something about inflation [76]. However, as was discussed in Sec. 3.1, LISA’s band is well placed for other possible sources of cosmological backgrounds. In particular, waves

generated by the electroweak phase transition at temperature $T \sim 100 - 1000$ GeV would generate waves in LISA's band; they are likely to be detectable if the phase transition is strongly first order (a scenario that does not occur in the standard model, but is conceivable in extensions to the standard model [78]). Likewise, LISA is well-positioned to measure waves that may arise from extradimensional dynamics in the early universe (depending rather strongly on the scale of the extra dimensions [79,80]).

3.3 Ultra low frequency

The ultra low frequency band, $10^{-18} \text{ Hz} \lesssim f \lesssim 10^{-13} \text{ Hz}$, is better described by converting from frequency to wavelength: for these waves, $10^{-5} H_0^{-1} \lesssim \lambda \lesssim H_0^{-1}$, where $H_0^{-1} \sim 10^{10}$ light years is the Hubble length today. Waves in this band oscillate on scales comparable to the size of the universe. They are most likely to be generated during inflation: quantum fluctuations in the spacetime metric are parametrically amplified during inflation to relatively high amplitude. The rms amplitude to which the waves are amplified depends upon the energy scale of inflation:

$$h_{\text{rms}} \propto \left(\frac{E_{\text{infl}}}{m_{\text{Planck}}} \right)^2 . \quad (14)$$

Measuring these inflationary gravitational waves would be a direct probe of inflationary physics. Detection of these waves has been described as the “smoking gun” signature of inflation [100].

During inflation, quantum fluctuations impact both the scalar field which drives inflation itself (the inflaton ϕ) and the metric of spacetime. These scalar and tensor perturbations, $\delta\phi(\vec{r}, t)$ and $h_{ab}(\vec{r}, t)$, each satisfy a massless Klein-Gordon equation. The Fourier modes of each perturbation, $\delta\tilde{\phi}(\vec{k}, t)$ and $\tilde{h}_{ab}(\vec{k}, t)$, are thus describable as harmonic oscillators in the expanding Universe [101]. Each mode undergoes zero-point oscillations in the harmonic potential. However, the potential itself is evolving due to the expansion of the universe. The evolution of this potential parametrically amplifies these zero-point oscillations, creating quanta of the field [77]. During inflation, the scale factor grows faster than the Hubble length H^{-1} , and so each mode's wavelength likewise grows faster than the Hubble length. Amplification of each mode occurs while its wavelength is smaller than H^{-1} ; when the scale factor has grown such that $\lambda \gtrsim H^{-1}$, the crests and troughs of each mode are no longer in causal contact and the fluctuation ceases to grow, becoming frozen at its amplified magnitude [101]. Fluctuations in the inflaton seed density fluctuations, $\delta\rho(\vec{r}) = \delta\phi(\vec{r})(\partial V/\partial\phi)$ [where $V(\phi)$ is the potential that drives the inflaton field]. Fluctuations in the spacetime metric are gravitational waves.

Both density fluctuations and gravitational waves imprint the cosmic microwave background (CMB). First, each contributes to the CMB temperature anisotropy. However, even a perfectly measured map of temperature anisotropy cannot really determine the contribution of gravitational waves very well because of *cosmic variance*: since we only have one universe to use as our laboratory experiment, we are sharply limited in the number of statistically independent influences upon the CMB that we can measure. Large angular scales are obviously most strongly affected by this variance, and these scales are the ones on which gravitational waves most importantly impact the CMB [102].

Fortunately, the scalar and tensor contributions also impact the *polarization* of the CMB. These two contributions can be detangled from one another in a model-independent fashion. This detangling uses the fact that the polarization tensor $P_{ab}(\hat{\mathbf{n}})$ on the celestial sphere can be decomposed into tensor harmonics. These harmonics come in two flavors, distinguished by their parity properties: the “gradient-type” harmonics $Y_{(lm)ab}^G(\hat{\mathbf{n}})$ [which pick up a factor $(-1)^l$ under $\hat{\mathbf{n}} \rightarrow -\hat{\mathbf{n}}$], and the “curl-type” harmonics $Y_{(lm)ab}^C(\hat{\mathbf{n}})$ [which pick up a factor $(-1)^{l+1}$ under $\hat{\mathbf{n}} \rightarrow -\hat{\mathbf{n}}$]. These harmonics are constructed by taking covariant derivatives on the sphere of the “ordinary” spherical harmonics $Y_{lm}(\hat{\mathbf{n}})$; see [103] for details. (An alternative, but equivalent, formulation labels the gradient-type harmonics “E-modes” and the curl-type harmonics “B-modes” [111]; the analogy to electric and magnetic fields is obvious. Interestingly, the various multipole formalisms used to describe polarization maps are identical to those used to expand gravitational radiation fields, as in Ref. [112]; see Ref. [103] for further discussion.) Because scalar perturbations have no handedness, they *only* induce gradient-type polarization. Gravitational waves induce both gradient- and curl-type polarization. Thus, an unambiguous detection of the curl-type polarization would confirm production of gravitational waves by inflation.

The gradient-type polarization has recently been measured for the first time [104]. These modes are reduced relative to the CMB temperature anisotropy by an order of magnitude; the curl component should be smaller by an additional order of magnitude [105]. Detecting the gravitational-wave component of CMB polarization will be quite a challenge — aside from the instrumental sensitivity needed to measure this effect [106], astrophysical foregrounds can cause important complications [107,108,109], such as conversion of gradient modes to curl modes [110]. But this is likely to be the only direct probe of physical processes in the inflationary era.

3.4 Very low frequency

The very low frequency band, $10^{-9} \text{ Hz} \lesssim f \lesssim 10^{-7} \text{ Hz}$, corresponds to waves with periods ranging from a few months to a few decades. Our best limits on waves in this band come from observations of millisecond pulsars. First suggested by Sazhin [113] and then carefully analyzed and formulated by Deweiler [114], gravitational waves can drive oscillations in the arrival times of pulses from a distant pulsar. The range encompassed by the very low frequency band is set by the properties of these radio pulsar measurements: the high end of the frequency band comes from the need to integrate the radio pulsar data for at least several months; the low end comes from the fact that we have only been observing millisecond pulsars for a few decades. One cannot observe a periodicity shorter than the span of one’s dataset!

Millisecond pulsars are very good “detectors” for measurements in this band because they are exquisitely precise clocks. Andrea Lommen has recently [115] performed a rather massive analysis of the data from several millisecond pulsars that are widely spaced on the sky. Her analysis extends the data used for a previous analysis [116] so that nearly 17 years of observations are represented. A detailed description of Lommen’s methodology is given in Ref. [115]; her punchline is the following limit on the density of stochastic gravitational waves:

$$\Omega_{\text{GW}} h_{100}^2 < 2 \times 10^{-9} \tag{15}$$

(where h_{100} is the Hubble constant in units of $100 \text{ km sec}^{-1} \text{ megaparsec}^{-1}$). This is the best observed limit on gravitational waves that has been achieved to date. Though it is not quite at the level where it can constrain sources of stochastic gravitational-wave backgrounds, it is extremely close; with further observations and the inclusion of additional pulsars in the datasets, it is likely to become interesting quite soon. It is expected that the background in this band will be dominated by many unresolved coalescing massive binary black holes [117] — binaries that are either too massive to radiate in the LISA band, or else are inspiraling towards the LISA band en route to a final merger several centuries or millenia hence. Constraints from pulsar observations in this band will remain an extremely important source of data on stochastic waves in the future — the limits they can set on Ω_{GW} are likely to be better than can be set by *any* of the laser interferometric detectors.

4 Binary black holes

As has been mentioned already in Secs. 3.1 and 3.2, one of the most important sources of gravitational radiation in the high- and low-frequency bands is the coalescence of compact objects. One of the reasons for this importance is that this source is amenable, at least to some degree, to fairly detailed theoretical analysis: for the most part, the only tools needed to understand the evolution of these systems are the nature of gravitational-wave emission and the manner in which it drives these binaries to coalesce.

Analysis of binaries becomes considerably more complicated when its members come close together. Then, the nature of these members can become extremely important — their finite size and the material of which they are made importantly influences the binary’s evolution and the character of the waves that it generates. For binaries that contain neutron stars, the late stages of the “inspiral” (when the members of the binary are well separated and evolve primarily due to gravitational-wave backreaction) and the final “merger” (when the bodies come into contact and fuse into some kind of remnant) will depend in detail on the nature of neutron star matter [118,119,120,121,122].

The problem remains “clean”, at least in principle, if both members of the binary are black holes. There is then no matter to complicate the problem — black holes are vacuum solutions to the Einstein field equations, and so a binary black hole system is likewise just a vacuum solution. The dynamics of binary black holes can be stated quite concisely: they are given by the family of dynamical spacetimes, $g_{ab}(t)$, which: (a) satisfy the vacuum Einstein field equations $G_{ab} = 0$; (b) consist of a pair of widely separated black holes in the asymptotic past; (c) consist of a single rotating black hole in the asymptotic future; (d) allow only outgoing radiation to reach distant observers (who are located at “outgoing null infinity”); and (e) allow only ingoing radiation to propagate down event horizons. [For careful definitions of the Einstein tensor G_{ab} and outgoing null infinity, see, e.g. [123]. Note that the time parameter t introduced in the metric $g_{ab}(t)$ is intended to be any future-directed label that parameterizes the evolution of the system. For the purposes of gravitational-wave astronomy, a convenient such label is time measured by very distant observers — i.e., us.]

As is often the case in mathematics, the ease with which the problem can be stated belies the difficulty one has in solving it. The field equation $G_{ab} = 0$ is shorthand for ten coupled nonlinear partial differential equations. The location of event horizons (upon which one might naively want to place the “ingoing radiation only” boundary condition) is not known in advance, and as a matter of principle *cannot* be known until the full spacetime is built. And, in general relativity one has a great deal of freedom to specify coordinates. It is not often

clear, for the purposes of a calculation, what particular choice will turn out to be “good”. Despite their “clean” character, binary black hole systems are not at all easy to describe.

A useful (albeit very crude) characterization of binary systems breaks their evolution into three broad epochs. The characterization that we will use here is based on that presented in Ref. [124]; as we will discuss further below, there is a fair amount of arbitrariness associated with this characterization. The first two epochs have already been mentioned: the *inspiral* describes the binary when its members are separated, discrete objects, evolving primarily due to the backreaction of gravitational-wave emission. The *merger* which follows describes the violent dynamics of the two bodies merging into a single body. For binary black hole systems, this remnant will itself be a black hole. (The remnant most likely will contain a black hole for binaries with neutron stars as well.) This remnant hole must “settle down” to the Kerr solution [125] which describes all rotating black holes — the “no hair” theorem of general relativity [126] guarantees that the Kerr solution describes the final state, no matter what conditions describe the binary which produced it. This “settling down” process has been named the *ringdown* since the waves generated in this epoch take the form of damped sinusoids, similar to the sound of a struck bell. In fact, the quality factor Q of black holes is quite low ($Q_{\text{BH}} \sim 20$ or so, compared to $Q_{\text{bell}} \sim 10^3 - 10^5$); when translated into sound, one finds that black holes don’t ring so much as thud [132]. Ringdown waves “shave” the remnant, ensuring that all of the “hairiness” characterizing the system right after the merger is lost, and what remains is a perfectly hairless Kerr black hole [130,131].

Breaking the coalescence process into three broad epochs likewise divides its gravitational waves into three broad frequency bands. (This is one reason that this characterization is useful, despite its crudeness — it illustrates what source dynamics are “audible” to the observatories.) Roughly speaking, for inspiral waves we have [124]

$$f \lesssim 400 \text{ Hz} \left[\frac{10 M_{\odot}}{(1+z)M} \right], \quad (16)$$

where z is the cosmological redshift and M is the total system mass. The ringdown waves come out at frequency

$$\begin{aligned} f &\sim \frac{c^3}{2\pi G(1+z)M} \left[1 + 0.63(1 - a/M)^{0.3} \right] \\ &\sim (1200 - 3200) \text{ Hz} \left[\frac{10 M_{\odot}}{(1+z)M} \right]. \end{aligned} \quad (17)$$

The parameter a describes the spin of the merged remnant: it is related to the vectorial black hole spin \vec{S} by $a \equiv G|\vec{S}|/Mc$, and is in the range $0 \leq a \leq M$. The span in frequency given in Eq. (17) reflects this range. These ringdown waves are generated by a bar-like perturbation to the black hole that rotates in the same sense as the hole’s spin. The “merger” then consists of all waves that come out between these two frequencies.

This division into three bands, particularly our definition of the “merger”, is rather crude and ad hoc. The notion of “inspiral” is wholly defensible when the holes which comprise our binary are widely separated. The binary’s dynamics are then well described using the post-Newtonian approximation to general relativity [15]: the lowest order dynamics are described by Newtonian gravity, and corrections to this motion are given in terms of a power series in $x \sim (GM/rc^2)^{1/2}$, where r is orbital separation. The parameter x is roughly orbital speed over c . This expansion works well when x is small. Late in the inspiral, when $x \sim 0.2 - 0.4$, the convergence of this power series is not so good. The frequency given in Eq. (16) corresponds roughly to this x . (Further discussion and caveats can be found in Sec. III of Ref. [124].) Likewise, the notion of “ringdown” is quite rigorous and defensible as a means of describing the last waves that flutter out of the merged system — the remnant of the binary can be treated as a Kerr black hole plus some distortion; perturbation theory accurately describes the waves generated in this state [133,134]. This is in fact how Eq. (17) was found [134,135].

Difficulties come in the middle: what we have called “merger” sweeps together all of the poorly understood physics associated with the end of the inspiral and the complex gravitational dynamics describing the transition of our binary into a single black hole. Note that, for binaries of several tens of solar masses, the frequencies associated with these poorly understood waves lie very near the most sensitive frequencies of ground-based gravitational wave detectors. These waves, which we currently understand least well, may be perfectly suited for gravitational-wave observatories to measure!

This is the vanguard of current research in binary systems in general relativity, motivated quite a bit by the likely observational importance of the late inspiral and merger waves. Much of the community’s efforts to understand strong-field binary black hole dynamics use *numerical relativity*: direct solution of the Einstein field equations by large scale computations. In principle, numerical relativity should be able to provide, in detail, a description of the binary’s dynamics as a function of the two black holes’ masses and spins, and thus the gravitational waveforms produced by these dynamics. These waveforms should depend uniquely on these masses and spins since they are the only parameters that can describe the binary’s holes. Comparison of the numerically generated waveform with those measured by gravitational-wave observatories is arguably the most stringent test of general relativity imaginable, probing

what are probably the strongest and most violently varying gravitational fields produced by nature since the big bang.

Numerical solution of the two black hole problem has proven to be quite difficult. Unanticipated problems have slowed the rate of progress in this field to the point that astrophysically relevant binary solutions are just beginning to be produced today. Some idea of how unanticipated these problems were can be inferred from the following statement by Kip Thorne:

...numerical relativity is likely to give us, in the next five years or so, a detailed and highly reliable picture of the final coalescence and the wave forms it produces, including the dependence on the holes' masses and angular momenta.

This statement was written in a well-known review article from 1987 (Ref. [18], p. 379); clearly, Thorne's estimate of the timescale needed to get out interesting information was optimistic.

Many of the most important problems are beginning to be understood — progress in numerical relativity has been quite impressive recently. We will just summarize some of the recent highlights; the interested reader will find more details in the review by Lehner, Ref. [136]. One of the fundamental difficulties has been casting Einstein's equations into a form that behaves well under numerical integration. Some formulations which behaved quite well on earlier testbed problems with high degrees of symmetry have been found to perform extremely badly in general [137]: they allow unphysical modes (which are seeded by very small scale numerical errors) to grow exponentially and destroy the physical content of a calculation. Understanding this behavior will hopefully make controlling it possible, so that we will be able to construct evolution schemes that are not susceptible to unphysical mode growth [138,139].

Despite the fact that codes currently cannot model the full binary black hole merger right now, success has been achieved by taking present codes as far as they can go and then using perturbation theory to carry the evolution still further. This very pragmatic approach takes the point of view that the “full” codes should only be used for a limited section of the merger process [140]. Dubbed “The Lazarus Project” (since it works by resurrecting a fallen code), this direction makes it possible to get some insight into the properties of the waves generated late in the merger process [141].

Even with good evolution equations and perfect codes, it is necessary to match the strong-field portion of the coalescence which has been numerically modeled to the earlier inspiral — the initial data with which one starts the numerical evolution must latch onto what came before. It now seems likely that such data will be well-developed fairly soon. A way to approximate an evolution is to

consider it to be a sequence of initial data snapshots. This works well provided that the evolution of the system is not too rapid — the binary can be treated as in *quasi-equilibrium*. Such techniques were originally developed to study binary neutron star systems [142,143,144,145,146]. Recently an extension to this technique has been developed which goes beyond the “slices of initial data” view, endowing the spacetime with a helical timelike Killing vector which describes with good accuracy the circular motion of binary black holes [147,148]. With these tools, it should not be too difficult to go from the earlier inspiral regime into the very strong field merger, covering the full range of binary black hole coalescence.

In parallel to the recent progress in numerical relativity, techniques have been developed by Thibault Damour and colleagues [149,150,151,152] that promise to greatly improve our *analytical* understanding of strong-field binary systems. This work is based on combining “resummation methods” to improve the post-Newtonian description of the binary with a novel recasting of the binary’s dynamics in terms of the motion of a single body in an “effective one-body metric” (usefully regarded as a deformed black hole). The resummation techniques are, essentially, Padé approximants that improve the behavior of the poorly convergent Taylor series form of the post-Newtonian expansion. The one-body remapping is based on tools that were originally developed to describe two-body problems in quantum electrodynamics; further discussion can be found in Ref. [149]. Good agreement has been found between important invariant dynamical quantities describing strong-field binary orbits using this effective one-body technique and numerical relativity [153].

These rapidly maturing approaches to strong field dynamics gives us hope that theory will be able to play an important role aiding and interpreting gravitational-wave observations of black hole binaries. As has already been mentioned above, comparing measured binary black hole waves to those predicted by theory is about the most stringent test of general relativity imaginable. In addition to this “physics measurement”, the waves will provide a wealth of astrophysical information. As discussed in Secs. 3.1 and 3.2, we currently know very little about the rate at which these mergers are likely to take place. Any information about the rate will provide a great deal of information: observations in the high-frequency band by LIGO-type instruments can strongly constrain the various scenarios (e.g., Refs. [45,46,47,48,49,50,51]) by which stellar mass binaries can form; observations with LISA may be able to directly observe the consequences of early hierarchical mergers that were the building blocks of galaxies [95].

Detailed information about the binary that generates a particular signal will be measurable in cases in which we can fit the data to a model waveform — such fits provide (with varying degrees of accuracy) certain combinations of the black holes’ masses, information about their spins, the source’s position on

the sky, and the distance to the source (cf. Refs. [91,135,155,156,157,158] for further discussion). This information greatly increases the astrophysical value of gravitational-wave measurements. For example, using LISA it should be possible to survey the evolution of black hole masses as a function of redshift [158], tracing the development of black holes and the structures that host them over the evolution of the universe. If an electromagnetic counterpart can be associated with the gravitational-wave event, the measurement could provide a standard candle with extraordinarily low intrinsic error [159].

Though much is unknown about binary black holes in the universe, it is clear they are exquisite gravitational-wave sources — they are intrinsically “loud” radiators, they are incredible labs for testing gravity under extreme conditions, and they are powerful probes of astrophysical processes.

5 Bothrodesy

One subset of binary black holes comprises a LISA source with particularly wonderful characteristics. These are the extreme mass ratio binaries mentioned in Sec. 3.2 — binaries formed by the capture of stellar mass compact objects onto highly relativistic orbits of massive black holes. (As described in Sec. 3.2, the captured object can be a neutron star or a white dwarf as well as a black hole. Since black holes are likely to dominate the measured rate, we will consider this source to be a special case of binary black holes.)

In the general case, the spacetime of a binary black hole is a violently dynamical entity, varying in a manner that is extremely difficult to model (cf. the discussion in Sec. 4). The character of extreme mass ratio binaries is quite different. Because the captured body is so much less massive than the large black hole, the binary’s spacetime is largely that of the black hole plus a perturbation. The major effect of this perturbation is to create gravitational radiation. The motion of the small body is essentially an orbit that evolves due to this radiation. The properties of this evolving orbit — and thus of the waves that it generates — depend almost entirely on just the large black hole’s spacetime. These waves provide an extremely clean probe of the black hole’s spacetime.

Einstein’s theory of gravitation predicts that black holes are objects with event horizons, and whose structure is completely described by two numbers, the mass M and spin parameter a (ignoring the astrophysically uninteresting possibility of a charged black hole — macroscopic charged objects are rapidly neutralized in astrophysical environments by interstellar plasma). Extreme mass ratio inspirals provide a way to test this: the gravitational waves generated as the compact body spirals through the strong field of the black hole depend upon, and thus encode, the structure of the hole’s spacetime metric.

The waves that LISA will measure come from the captured body spiraling through the very strong field of the large black hole — the orbital radius is a few times the Schwarzschild radius of the hole, so that the captured body is near the hole’s event horizon. The small body executes many orbits as gravitational-wave backreaction drives it to spiral inwards — it orbits about $10^5 - 10^6$ times before it reaches a dynamical instability and then plunges into the hole. These orbits happen over a period of several months to years. By tracking the gravitational wave’s phase evolution over this time, we will be able to follow the evolution of the smaller body’s orbital frequencies with high precision.

It is these frequencies, or rather the sequence of frequencies that the small body follows, which encode such information about the black hole spacetime. Consider for a moment an eccentric, inclined orbit about a spherical body with mass M . The concept of “inclination” is of course rather artificial in this case — the field will be spherically symmetric, so the orbits had better not depend on that inclination. Ignoring this common sense for a moment, we can define three orbital timescales: T_r is the time it takes to move through the full range of motion in the radial coordinate; T_θ is the time it takes to move through the full range of latitudinal angle; and T_ϕ is the time it takes to move through 2π radians of azimuth.

For spheres in Newtonian gravity, these three timescales are of course identical: $T_r = T_\theta = T_\phi \equiv T = 2\pi\sqrt{R^3/M}$ — Newtonian orbits are closed ellipses with semi-major axis R . That $T_\theta = T_\phi$ follows from the spherical symmetry of the gravitational field. That T_r is equal as well is something of a miracle that follows from the $1/r$ form of Newton’s gravitational potential [160]. Now imagine adding some multipolar structure to the sphere. This changes the character of the potential, and thus the character of the frequencies. For example, if we add a quadrupolar distortion to our sphere, the gravitational potential picks up a bit that goes as $1/r^3$ and that has an angular dependence:

$$V_{\text{grav}} = -\frac{GM}{r} + \frac{\mathcal{Q}Y_{20}}{r^3}. \quad (18)$$

(\mathcal{Q} heuristically represents the quadrupolar distortion of the central body; Y_{20} is a spherical harmonic.) This extra piece changes all of the timescales — we no longer have $T_\phi = T_\theta$ for example, because the potential is no longer spherical.

Measuring the orbital frequencies thus maps the shape of a body’s gravitational field, which in turn maps the body’s structure. Using satellite orbits, we have measured with high precision quite a few of the multipolar distortions that characterize the Earth; NASA’s recently launched GRACE mission [161] promises to improve these measurements quite a bit (see [162] for further discussion). The science of performing these measurements is known as

geodesy.

In a very similar way, by tracking the evolution of the orbital frequencies that describe black hole orbits through the gravitational waves that they generate, we can map the shape of a black hole’s spacetime metric. In analogy to geodesy, this science has been given the name *bothrodesy*. This name comes from the Greek word “bothros” ($\beta\theta\theta\rho\varsigma$), meaning (roughly) “garbage pit”. (In archeology, “bothros” refers to a sacrificial pit — an appropriate connotation since a black hole is Nature’s ultimate sacrificial pit!)

Bothrodesy is particularly powerful because black holes have a unique multipolar structure. As we have already stated, the “no-hair” theorem [126] tells us that the spacetime of a black hole can only depend on its mass M and spin a . On the other hand, it is well understood that the spacetime of a compact object can be built from a multipolar description of that object [112]. The object is fully described by a family of mass moments M_{lm} (similar to electric multipole moments) and current moments S_{lm} (analogous to magnetic multipole moments) given roughly by

$$M_{lm} \simeq \int dV r^l Y_{lm}(\theta, \phi) \rho(r, \theta, \phi) , \quad (19)$$

$$S_{lm} \simeq \int dV r^l Y_{lm}(\theta, \phi) \rho(r, \theta, \phi) v(r, \theta, \phi) ; \quad (20)$$

ρ is the mass density at the coordinate (r, θ, ϕ) , and ρv is the current density. Although a black hole has no matter, its spacetime is also generated by multipole moments of this form. The moments of a black hole are

$$M_{l0} + iS_{l0} = M(ia)^l , \quad (21)$$

$$M_{lm} = S_{lm} = 0 \quad \text{for } m \neq 0 . \quad (22)$$

Condition (22) simply enforces the fact that rotating black holes are axisymmetric. Condition (21) is far more interesting: it enforces the no-hair theorem! For $l = 0$, it tells us $M_{00} = M$ — the zeroth mass moment is the mass, no great surprise. For $l = 1$ we find $S_{10} = aM$. This is the magnitude of the hole’s spin $|\vec{S}| = S$ (in units with $G = 1 = c$). *All higher multipoles are completely determined by these first two moments.*

This is a remarkably powerful statement. It tells us that measuring three multipole moments is sufficient to falsify whether an object is a black hole. For example, many galaxies are known to contain extremely massive, compact gravitating objects in their centers. It is most plausible that these objects are black holes, but it is possible they could be something even more bizarre, such as a gravitational condensation of bosonic cold dark matter [163,164,165,166,167]. If we measure gravitational waves from inspiral into one of these massive ob-

jects and find that the moment M_{20} is not consistent with the measured values of M and S , then that object is *not* in fact a black hole, but is indeed something even more bizarre. Conversely, if we can measure a good sized set of multipoles and find that they are all consistent with Eq. (21) then we have extremely compelling evidence that the “black hole” is in fact a black hole exactly as described by general relativity.

How well can LISA perform this kind of measurement in practice? Laying the foundations to answer this question is an area of very active research right now. Some guidance can be found from calculations performed by Fintan Ryan [168]. Ryan examined how well one can measure the moment structure of a large body with gravitational waves in the context of a toy calculation. In his setup, the inspiraling body is confined to orbits that lie in the large body’s equatorial plane and are of zero eccentricity. These restricted orbits throw away a lot of useful information about the multipolar structure which would be encoded in the precessional motion of an orbit that is inclined and eccentric. Ryan’s calculation instead “weighs” the different multipoles by the fact that each impacts the orbital frequency with a different radial dependence, and so affects the waveform phasing at different rates as the small body spirals in. Even in the context of this excessively simplified problem, Ryan finds that at least three and in some cases five multipoles will be measurable by LISA. We are certain that, due to his restricted orbit families, Ryan’s calculation underestimates how well LISA will be able to measure these moments.

It’s worth noting at this point the accuracy with which some of these moments can be measured. Ryan finds [168] that the mass of the large object is typically measured with an accuracy $\delta M/M \sim 10^{-4} - 10^{-5}$. This is phenomenal precision — the precision with which we measure black hole masses today is no better than $\sim 10\%$ for the Milky Way’s black hole, and usually much larger ($\delta M/M \sim 1$ or larger is not uncommon). Ryan finds that the spin can be measured with an accuracy $\delta S/S \sim 0.01$. This again is extremely precise — presently, we have very little information about black hole spins, other than indications that the spin must be rapid in some cases [169,170]. It’s worth re-emphasizing that his accuracy estimates are likely to be pessimistic owing to his excessively restricted orbit families. Bothrodesy will provide high precision probes of the nature of black holes.

Preparing for these LISA observations requires that we understand the nature of the waves that inspiral into black holes will provide. Because of the extreme mass ratio of inspiral systems, this is a relatively simple task: black hole perturbation theory using the system’s mass ratio as an expansion parameter describes these binaries very well [173,174,175,176,177,178,179,180]. Although there remain issues of principle that are currently being worked out (particularly the issue of rigorously computing the perturbation’s backreaction on the inspiral in full generality [181,182,183,184,185,186,187,188]), this problem

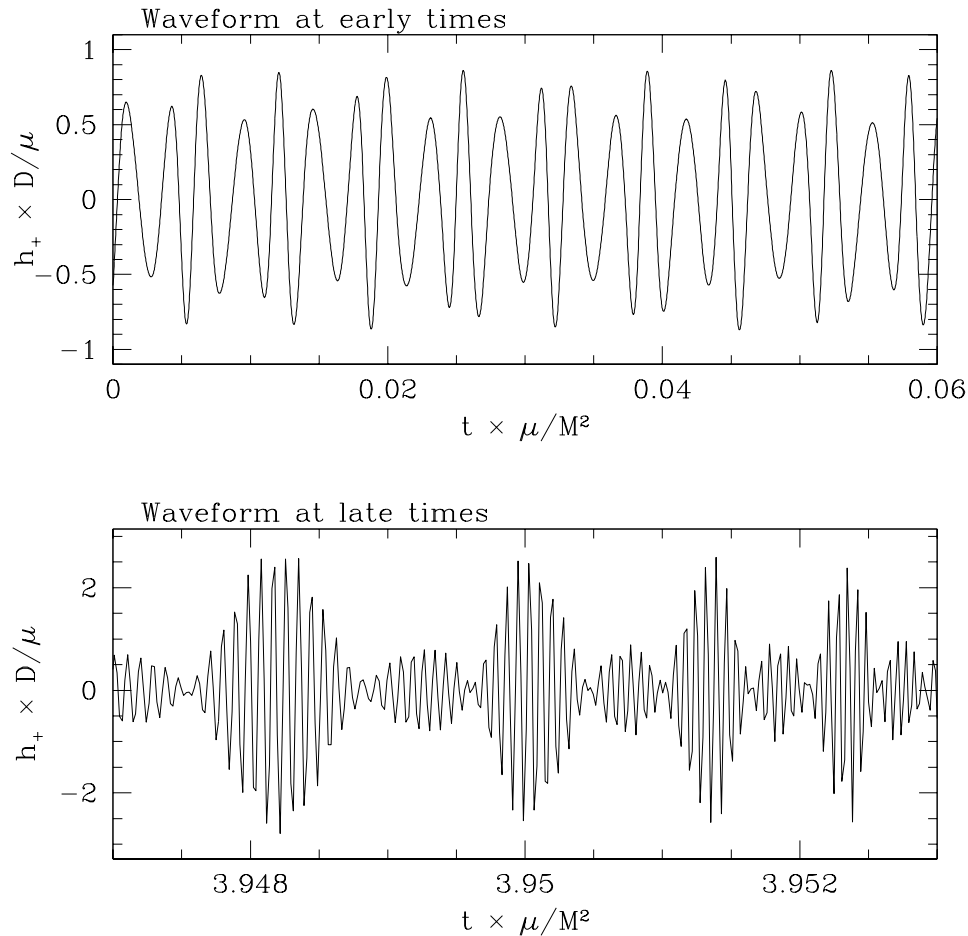


Fig. 6. The waveform generated by “circular” inspiral, from Ref. [179]. Early on, the modulation is small and happens on a short timescale. This is because the frequencies Ω_ϕ and Ω_θ describing circular motion are not very different. The frequencies evolve at different rates, changing the nature of the modulation dramatically as time proceeds. At late times, the modulation is very strong, and there are many more cycles of “carrier” in each cycle of modulation. Note the different timescales in the top and bottom panels — orbital frequencies are much higher late in inspiral. Audio encodings of this waveform can be downloaded from [4].

is not nearly as difficult as that of the general binary black hole evolution. Indeed, there are two special cases in which perturbative codes have already been able to tell us a great deal about the character of these inspirals. These cases correspond to orbits that are “circular” but inclined, and orbits that are eccentric but confined to the hole’s equatorial plane.

Let us look at the circular inspirals first. Circular orbits would be of constant radius if radiative backreaction were not shrinking them. Waveforms generated in this case are influenced by two orbital frequencies, Ω_ϕ (related to the time required for an orbit to move through 2π radians of azimuth) and Ω_θ (related

to the time required to span its full range of latitude). These frequencies differ for rotating black holes, in part because rotation makes black holes oblate [cf. the discussion near Eq. (18)] and in part because of *frame dragging* — the tendency of objects near a spinning source of gravity to be dragged into corotation with that spin. Under the combined influence of these two effects, $\Omega_\phi > \Omega_\theta$. The difference leads to a modulation of the gravitational waveform — essentially, there is beating between these two frequencies.

This modulation is illustrated in Fig. 6 (taken from Ref. [179]). Here we show an example of an inclined, circular inspiral into a rapidly rotating black hole (spin parameter $a = 0.998M$). Segments of the waveform are presented early in the inspiral and again much later (as the inspiraling body approaches the final plunge orbit). Note the evolving character of the waveform’s modulation: the amplitude of the modulation is much stronger at the end, and there are many more cycles of the carrier wave per cycle of the modulation. This is a signature of the black hole’s strong field: near the event horizon, Ω_θ decreases (a redshifting effect due to the proximity of the event horizon), whereas Ω_ϕ grows to a maximum (the body “locks” onto the dragging of inertial frames and is forced to orbit at a rapid rate [192]). In the physical space near the hole, the small body appears to whirl very rapidly near the black hole while slowly moving in its latitude angle. This stamp on the waveform is a clear signature of a black hole’s strong field nature.

Eccentricity introduces yet another layer of complexity, owing to modulations between the inspiraling object’s azimuthal motion and its motion in the radial direction. Strong-field eccentric orbits show what has been named a “zoom-whirl” character [193]. If gravity were purely Newtonian, the inspiraling body would accumulate 2π radians of azimuth while moving through its full range of radius. General relativity tells us that in fact the body moves through an extra bit of azimuth over the orbit. This effect is nothing more than perihelion precession, well-known from studies of Mercury’s orbit in the solar system.

In the case of Mercury, the excess azimuth is rather puny — an extra 43 arcseconds of azimuth accumulate every century due to general relativity, or about 0.1 arcsecond per orbit. In the strong field of a rapidly rotating black hole, the extra azimuth can amount to thousands of degrees per orbit! The inspiraling body appears to “whirl” around the black hole many times when it is near peribothron; it then “zooms” out to apobothron and back, to whirl again on the next cycle. An example of the waveform from such an orbit (taken from Ref. [180]) is shown in Figure 7. Note the multiple high frequency cycles occurring every $t \sim 700$; this is due to the rapid whirling of the inspiraling body at peribothron.

The ornate character of the waves illustrated in Figs. 6 and 7 gives some sense of the information that they encode. These figures don’t really do the

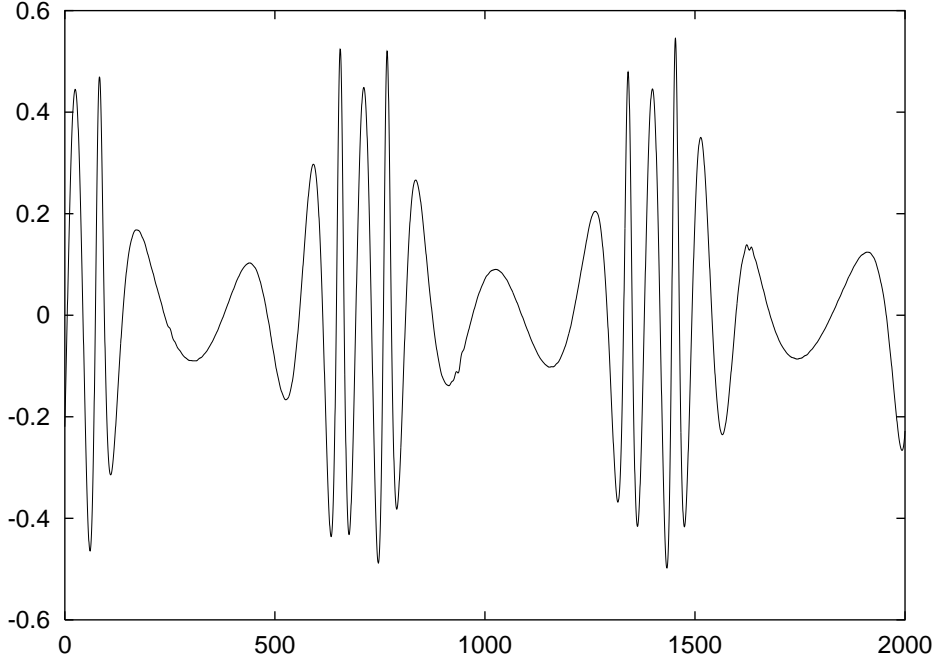


Fig. 7. A “zoom-whirl” waveform, generated by an eccentric, equatorial orbit, from Ref. [180]. The high frequency peaks near $t \sim 0$, $t \sim 700$, and $t \sim 1400$ are due to the whirling motion of the inspiraling body at peribothron. This is a relatively gentle zoom-whirl structure — it is not difficult to find cases that exhibit stronger whirling at peribothron. Audio encodings of waveforms that incorporate this kind of structure can be heard at [4].

waveform justice, though — to really get a sense of their harmonic content, one should listen to an audio encoding of these waves. The reader is invited to listen to such encodings which have been placed on the World Wide Web at the URL given in [4]. The sounds presented there illustrate a variety of extreme mass ratio inspiral signals, and how their features vary as a function of the system’s parameters.

6 Conclusions

In this article, we have taken a brief tour of various ways that the Universe produces gravitational waves, surveying the different bands in which this “voice” operates, and how we can build — or are building — “ears” for listening to what it is saying. Sections 4 and 5 have focused on the waves produced from black hole sources, a particular favorite of this author, outlining the challenges in learning to speak the language of these sources and showing a few snippets of what we have learned so far.

Before too long, we will hopefully begin to hear these voices directly from

Nature, and not just as output from theorists' computations.

Acknowledgments

In presenting an overview of gravitational-wave astrophysics, I have tried to emulate the style by which I learned the subject from Kip Thorne. I also thank Kip for his hospitality, without which I probably would never have finished this article, as well as for permission to reproduce Figs. 1 and 2 here. I thank Kostas Glampedakis and Daniel Kennefick for permission to use Fig. 7 (taken from Ref. [180]). Finally, I thank Daniel Holz for providing useful comments on a careful, critical reading of this paper. Some of the background material presented here was adapted from a previous review, Ref. [16]. This work was supported by NSF Grant PHY-9907949.

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