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## Rail Infrastructure Manager Problem: Analyzing Capacity Pricing and Allocation in Shared Railway System

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# Rail Infrastructure Manager Problem 

# Analyzing Capacity Pricing and Allocation in Shared Railway System 

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#### Abstract

This paper proposes a train timetabling model for shared railway systems. The model is formulated as a mixed integer linear programming problem and solved both using commercial software and a novel algorithm based on approximate dynamic programming. The results of the train timetabling model can be used to simulate and evaluate the behavior of the infrastructure manager in shared railway systems under different capacity pricing and allocation mechanisms. This would allow regulators and decision makers to identify the implications of these mechanisms for different stakeholders considering the specific characteristics of the system.


Keywords—shared railway corridors; capacity pricing; capacity allocation; train timetabling problem for multiple operators

## I. Introduction

Infrastructures supporting basic essential service should be put in place to enable economic activity, economic growth, and development [1], [2]. However, investment in new infrastructure is extremely expensive and not always feasible, especially in densely populated areas with the most critical infrastructure needs.

This research analyzes this complex socio-technical problem in an attempt to reduce infrastructure needs by more efficient use of existing infrastructure. In this paper we look at this problem from the prospective of rail infrastructure, which is especially interesting for two reasons: 1) rail is an infrastructure-intensive industry: infrastructure represents around $40-60 \%$ of the final transportation cost, and 2) there are high interdependencies between the infrastructure capacity and its operation [3].

In the last 15 years, different countries have promoted the use of shared railway systems that allow independent operators to access the infrastructure. This enables higher levels of infrastructure utilization [4]. However, shared railway corridors require coordination between the infrastructure manager typically the owner of the infrastructure- and multiple train operators [3]. This coordination involves determining which trains can access the infrastructure at each time (capacity allocation) and the access price they need to pay (capacity pricing).

According to [5] and [6], ideal capacity pricing and allocation mechanisms should be reproducible and transparent, easy to understand, and non-discriminatory (especially when the

[^0]operators compete in the same market). Simple capacity pricing and allocation rules are used in other network industries (such as electric power or telecommunication). However, in the railway industry, the characteristics of the network and operations critically affect the available capacity and safety. As a result, the implications for the system of the capacity pricing and allocation mechanism even when simple mechanisms used in other applications are applied remain unclear. Rail capacity is endogenous: it depends on the capacity allocation and has to be calculated through the train timetable.

This paper presents a train timetabling model for shared railway systems that would allow regulators and decision makers to simulate the behavior of the infrastructure manager. This model will explicitly consider network effects to analyze the interdependencies between operation and infrastructure and to determine the optimal use of capacity (train timetable) under different capacity pricing and allocation mechanisms.

There are several papers that propose mixed integer linear programming (MILP) approaches to solve the train timetabling problem. References [7], [8], [9], [10] present formulations to compute the train arrival and departure times with different objectives and constraints. Traditionally, these models have been called multi-mode resource constrained project scheduling models.

There is another body of literature also based on MILP approaches that formulates the train timetabling problem as a multi-commodity flow problem [11], [12], [13], [14], [15], representing the final timetable as a collection of nodes and arcs that represent possible train arrival and departure times at stations. Infrastructure and operational constraints are imposed determining subsets of compatible and incompatible arcs.

With the exception of [14], these models assume that there is a single operator trying to schedule trains on the infrastructure. Most of the papers also assume that all trains follow the same path. This paper proposes a multi-mode resource constrained project schedule formulation considering different operators using the same infrastructure for different types of services (commuter, intercity and freight services) with different routes. These additional complexities result in 1) the need to specify safety constraints depending on the path of each train, and 2) the need for operators to specify the desired timetable with flexibility margins to ensure that their trains get scheduled with small adjustments in case of a conflict with other operator's train that make the problem harder to solve.

From a computational standpoint, the size of models increases exponentially with the number of stations and trains to schedule. To partially reduce this "curse of dimensionality", we also propose an alternative class of solution algorithms using approximate dynamic programming techniques [16], [17], [18], [19], [20]. It allows us to decompose and solve the problem faster while still ensuring convergence to the optimal solution within an optimality gap.

The rest of the paper is structured as follows: Section 2 describes and formulates the train timetabling problem for shared railway systems; Section 3 details the implementation of the model and shows the main results obtained. Section 4 discusses some implication of these results for the infrastructure manager, for different train operators, for the users, and for the regulators under different traffic level scenarios. Section 5 presents concluding remarks and identifies lines of future research.

## II. Formulation of Train Timetabling Problem In Shared Railway Systems

The train timetabling problem consists of determining the arrival and departure time of all trains scheduled at every station in the path of the train. Figure 1 shows the topology of the railway system considered throughout the paper. It consists of a double-track corridor with 12 stations. The system presented captures all the elements required to represent a corridor such as the Northeast Corridor in the U.S., where the Federal Railroad Administration is now trying to develop a new capacity pricing and allocation mechanism to foster rail efficiency.

Stations 1, 2 and 12 represents main stations in the same metropolitan area (for example, Boston), station 3, 4, 5, 9, 10 and 11 are all in another metropolitan area (New York), and stations 6, 7 and 8 are all also in another different metropolitan area (Washington D.C.). Five types of services are considered: Boston commuter trains travelling around Boston metropolitan area (stations 1, 2, and 12); New York commuter trains; D.C. commuter trains; and intercity and freight trains travelling between Boston and Washington D.C. Intercity and freight trains may not stop at every station. Freight trains will travel the line at speeds much lower than commuter and intercity trains. Intercity trains travel at higher speeds than commuter trains.


Fig. 1. Detailed corridor infrastructure
Today, around 2,000 commuter trains, 150 intercity trains and 70 freight trains travel around the Northeast Corridor every day. In practice, most of the conflicts to schedule trains occur around peak hours; where the infrastructure manager would have to control for conflicts of sets of around 100-250 trains to make changes in the timetable.

There are two main mechanisms to design capacity pricing and allocation for infrastructure capacity markets: mechanisms that determine the price at which capacity will be offered, and let operators decide whether they are willing to access the infrastructure or not (price-based, cost-allocation mechanisms);
and mechanisms that determine the amount of capacity that will be offered, and let the operators reveal the price that they are willing to pay to use that capacity (quantity-based, auction mechanisms) [21]. Cost-allocation mechanisms are typically complemented with priority rules that allow the infrastructure manager to decide which train to schedule when there are conflicts (multiple operators willing to pay the pre-determined access charges).

The model described in this section represents the behavior of the infrastructure manager when an auction mechanism is implemented. Under an auction, every few months (with a predetermined frequency), the operators will have the opportunity to submit bids: list of the trains they want to schedule on the infrastructure, the desired timetable for each train, and the access charges they are willing to pay to schedule each of them. The infrastructure manager will then determine the set of trains that can be finally scheduled, their timetable, and the access charges that the operators will pay with the objective of maximizing its revenue and considering other infrastructure constraints (safety, infrastructure maintenance plans).

We also discuss how to change the model to simulate the behavior of the infrastructure manager under cost-allocation mechanisms. The differences between the infrastructure manager models for each mechanism affect mainly the definition of the parameters and the choice of the objective function. The constraints however are related to the physical operation of the trains and remain unchanged for different mechanisms.

The train timetabling problem sets, parameters, variables, objective function and constrains are defined below.
A. Sets
$i, i \in\{0,1, \ldots, I\}$ trains proposed by the operators in the bidding process

## $j, j \in\{1,2, \ldots, J\}$ railway system stations

$k, k \in\{1,2, \ldots, K\}$ possible types of train services (such as intercity, freight, or commuter)

Each operator has to provide the following information about each train $i$ in the bidding process:
type $_{i k}$, a subset that indicates if train $i$ is of type $k$
$i n i_{i j}$, a subset that indicates the initial station $j$ from which train $i$ departs
fin $_{i j}$, a subset that indicates the final destination (station $j$ ) of train $i$

The information about the topology of the line and the type of service is used to determine the following two subsets:
$s t a t_{i j}$, a subset that indicates whether train $i$ travels through station $j$ or not
next ${ }_{i j j^{\prime}}$, a subset that indicates for each train $i$ the station $j^{\prime}$ that train $i$ will visit immediately after station $j$. Train $i$ may not stop at station $j^{\prime}$.

## B. Parameteres

The information that the train operators provide in the bidding process for every train $i$ is:
accessprice $_{i}$, the maximum access price that the operator is willing to pay if train $i$ is scheduled. For price-based mechanisms the access price will be pre-determined (using for example a cost-allocation model) and fixed by the infrastructure manager depending on the characteristics of the service. It is important to note that the operator will only operate a train if that price is lower or equal than his willingness to pay.
$\operatorname{arrtime}_{i j}$, deptime $_{i j}$, the desired arrival and departure time of train $i$ at every station $j$ in the path of train $i$
traslation $_{i}$, penaltytr $r_{i}$, maximum acceptable translation (see figure 2) of train $i$ and per-unit penalty imposed by the operator if the infrastructure manager translates the train over the desired timetable
traveltime $_{i}$, penaltytt ${ }_{i}$, maximum acceptable change in train $i$ total travel time (see figure 2 ) and per-unit penalty imposed by the operator if the infrastructure manager increases the travel time of train $i$ at any station over the desired timetable


Fig. 2. Possible changes with respect to desired timetable
In addition, the topology of the track and the signaling system will determine the minimum safety headway (time elapsed) between consecutive maneuvers at every station:
headwaydep ${ }_{j}$, headwayarr $r_{j}$ minimum headway between consecutive departures/arrivals from/to station $j$

In some cases the minimum safety headway depends also on the characteristics of the rolling stock. If that is the case, the former parameters will have different values for different train pairs.

## C. Variables

The decision variables of this problem are:
$S C H E D U L E_{i}$ binary variable that indicates whether train $i$ is scheduled

ARRTIME $E_{i j}$, DEPTIME $_{i j}$ final arrival and departure time (timetable) of every train $i$ scheduled at every station $j$ in the path of the train

TRASLATION $_{i}, \triangle$ TRAVELTIME $_{i j}$ final train $i$ translation and increment of travel time per station $j$. Note that these variables can be determined knowing

ARRTIME $_{i j}$, DEPTIME $_{i j}$ and vice versa. This research assumes $\triangle T R A V E L T I M E_{i j} \geq 0$ to ensure that the resulting train timetable is feasible. TRASLATION $N_{i}$ can either be positive or negative; so the positive variable TRASLATIO $N_{i}^{+}$is defined to compute the portion of the access charge that each operator receives from the infrastructure manager in case that train $i$ is re-scheduled.

## D. Objective Function

As discussed before, the objective of the problem is to determine which trains should be scheduled and when to maximize the infrastructure manager's revenue:

$$
\begin{aligned}
& \max \sum_{i} \text { accessprice }_{i} \text { SCHEDULE }_{i} \\
&+ \text { penaltytr }_{i} \text { TRASLATION }
\end{aligned}
$$

Similar objective functions can be defined for different mechanisms. For instance the functions:

$$
\begin{gathered}
\max \sum_{i} S C H E D U L E_{i} \\
\max \sum_{i} \text { priority }_{i} S C H E D U L E_{i}
\end{gathered}
$$

could be used to maximize the number of trains scheduled or the number of priority trains scheduled respectively under a costallocation and priority rules mechanisms. In this case priority $i_{i}$ would be a parameter that indicates the priority level of each train $i$.

## E. Constraints

The first sets of constraints establish the relation between the desired timetable and the final timetable of every train scheduled:

The departure time at the first station can be determined as:

$$
\text { DEPTIME }_{i j}=\text { deptime }_{i j}+\text { TRASLATIO N }_{i}, \quad \forall i, j: i n i_{i j}
$$

The travel time at intermediate stations can be determined as:

$$
\begin{aligned}
& \text { DEPTIME }_{i j}-\text { DEPTIME }_{i j^{\prime}} \\
&=\text { deptime }_{i j}-\text { deptime }_{i j^{\prime}} \\
&+\Delta \text { TRAVELTIME }_{i j^{\prime}}, \\
& \forall i, j, j^{\prime}: \text { next }_{i j^{\prime} j} \text { and }_{i j}+\text { fin }_{i j}=0
\end{aligned}
$$

At the final station, the travel time can be determined using:

$$
\begin{aligned}
& \operatorname{ARRTIME}_{i j}-\text { DEPTIME }_{i j^{\prime}} \\
&=\operatorname{arrtime}_{i j}-\text { deptime }_{i j^{\prime}} \\
&+\Delta \text { TRAVELTIME }_{i j^{\prime}}, \\
& \forall i, j, j^{\prime}: \text { next }_{i j^{\prime} j} \text { and }^{\prime} \text { fin }_{i j}
\end{aligned}
$$

Note that the arrival time to the initial station is not defined in the timetable; neither the departure time from the last station.

To ensure that the timetable is feasible, the final stopping and travel time at each station have to be greater than the stopping
and travel time in the desired timetable:

$$
\begin{array}{r}
\text { DEPTIME }_{i j}-\text { ARRTIME }_{i j} \geq \text { deptime }_{i j}-\text { arrtime }_{i j} \\
\forall i, j: \text { stat }_{i j} \text { and ini }_{i j}+\text { fin }_{i j}=0 \\
\text { ARRTIME }_{i j^{\prime}}-\text { DEPTIME }_{i j} \geq \operatorname{arrtime~}_{i j^{\prime}}-\text { deptime }_{i j}, \\
\forall i, j: \text { next }_{i j^{\prime} j} \text { and ini }_{i j}+\text { fin }_{i j}=0
\end{array}
$$

Then, the maximum translation and increment of travel time are established:

The maximum translation of a train is bounded by the maximum translation defined by the operator:

- traslation $_{i} \leq$ TRASLATIO $_{i} \leq$ traslation $_{i}, \quad \forall i$

In addition, the absolute value of the translation is determined using the following constraints:

$$
- \text { TRASLATIO }_{i} \leq \text { TRASLATION }_{i}^{+} \geq \text {TRASLATIO }_{i}, \quad \forall i
$$

The maximum change on travel time is bounded by the maximum increment on travel time defined by the operator:

$$
\sum_{j: \text { stat }_{i j}} \Delta \text { TRAVELTIME }_{i j} \leq \Delta \text { traveltime }_{i}, \quad \forall i
$$

The operator may impose additional conditions over the acceptable changes with respect to the desired timetable. That happens when the operator is not interested in operating the train if the departure from or the arrival at one major station is changed. In this case, additional constraints are included to ensure that the timetable includes the operator requests if the train is finally scheduled.

The final set of constraints ensures that the timetable proposed by the infrastructure manager can be accommodated in the existing infrastructure.

Essentially, the difference between the departure times of every pair of trains scheduled has to be greater or equal than the minimum safety headway, so at least one of the following equations must hold:

$$
\begin{aligned}
& \text { DEPTIME }_{i j}-\text { DEPTIME }_{i^{\prime} j} \geq \text { headwaydep }_{j} \\
& \text { DEPTIME }_{i^{\prime} j}-\text { DEPTIME }_{i j} \geq \text { headwaydep }
\end{aligned}
$$

The binary disjunctive variable $O R D_{i i^{\prime} j}$ is used to automatically activate only one of the constraints depending on the value of the other variables. $O R D_{i i^{\prime} j}$ has value 1 if train $i$ departs before train $i^{\prime}$ at station $j$. Although the desired timetable can be used to determine the value of some of these variables; in general this problem will have on the order of $\mathrm{O}\left(\mathrm{I}^{2} J\right)$ binary variables and will be very difficult to solve for large I.

Very similar equations are formulated for inter-arrival times too, ensuring also that the order of the trains is preserved in the inter-stations.

Note that these constraints are valid for any shared railway system. The information about the topology of the infrastructure, the route of the trains, the safety headways imposed by the signaling system, etc. is introduced in the model parameterization.

## III. ImPLEMENTATION AND RESULTS

This section discusses how the train timetabling problems proposed has been implemented and solved, and shows the main results obtained.

## A. Execution

The problem has been written in GAMS 24.1.2 and solved by CPLEX 12.5 on a PC at $2.40 \mathrm{GHz}, 4 \mathrm{~GB}$ running with Microsoft Windows 764 bits. Different solver options were used to speed the execution.

In addition, a novel approximate dynamic programming algorithm was used to solve the problem. The algorithm proposed (see [22] for further details) decomposed the problem in different decision stages analyzing the benefits of scheduling one train at a time using an adaptive relaxed linear programming version of a Q-factor Bellman equation (QARLP). Figure 3 shows a comparison of the execution time and the iterations required to convergence (within 5\% integrality gap) of the MIP approach and the QARLP algorithm. Note that QARLP execution time grows polinomically with the size of the problem (number of stations and number of trains to schedule), allowing for solving larger problems than the MIP approach.


Fig. 3. Comparison of the execution time and the iterations required to convergence (within 5\% integrality gap) of the MIP approach and the QARLP algorithm.

## B. Results

Table 1 shows the number of equations, variables and discrete variables of a problem with different number of commuter and intercity trains.

TABLE I. TRAIN TIMETABLING PROBLEM SIZE

| Number <br> of <br> Trains | Equations | Variables | Discrete <br> Variables |
| :---: | :---: | :---: | :---: |
| $\mathbf{1 5}$ | 970 | 510 | 91 |
| $\mathbf{3 0}$ | 3,715 | 1,607 | 292 |
| $\mathbf{6 0}$ | 14,533 | 5,565 | 919 |
| $\mathbf{1 2 0}$ | 57,481 | 20,537 | 3,145 |

Figures 4, 5 and 6 show time-space diagrams with the timetable designed by the infrastructure manager model for different capacity use demand scenarios. The y-axes represents distance in miles from station 1 and the x -axes represents time in minutes at which different trains are scheduled to pass through each point of the line (vs. desired scheduled in dashed line). The horizontal segments represents stopping time at stations. There is no interaction between trains travelling in different directions.

Figure 4 shows the timetable for a scenario with demand to schedule an intercity train in the system when commuter trains around the three urban areas operate every 30 minutes. Figure 5 shows a scenario where two competing intercity operators try to schedule intercity trains at the same time in the system with the same commuter demand. Figure 6 shows a scenario with demand to schedule a freight train in the system when commuter trains around the three urban area operate every 1 hour. The conflicts in the desired timetable are adjusted with the model (infrastructure manager) as a trade-off between eliminating trains and readjusting the desired schedules.


Fig. 4. Timetable proposed by infrastructure manager to schedule an intercity train in a system with commuter trains operating every 30 minutes.

Assuming that each commuter operator pays 1 unit to schedule a commuter service and gets a $5 \%$ discount over the original access fee for every minute their train are re-scheduled; the timetable presented in figure 4 involves a 2.1 units of total discount for commuter operators (the intercity service will initially conflict with 14 commuter trains). That means that the infrastructure manager would only schedule the intercity train if it represents more than 2.1 units of revenue.

If the frequency of commuter trains increases, for example to one commuter service every 15 minutes instead of 30 minutes, the intercity train will initially conflict with 22 commuter trains and will only be scheduled if it represents more than 3.6 units of revenue for the infrastructure manager.

Conversely, if the frequency of commuter trains decreases to one train every 60 minutes, the intercity train will be scheduled if it represents at least 0.3 units of revenue for the infrastructure manager. The model can be used to quantify the trade-off between commuter and intercity trains for any other frequency of service.


Fig. 5. Timetable proposed by infrastructure manager to schedule two intercity trains in a system with commuter trains operating every 30 minutes.

Scheduling two intercity trains in Figure 5, assuming the same commuter willingness to pay to schedule trains, involves a 4.0 units of total discount for commuter operators (the intercity service will initially conflict with 14 commuter trains). That means that the infrastructure manager would only schedule the two intercity trains if they represent more than 4.0 units of revenue. If the revenue of scheduling the intercity trains represent from 2.1 to 4.0 units, at most one of the intercity trains would be scheduled.

Furthermore, note that although both trains would like to depart station 1 at minute 0 , one of them will depart at minute 3 and the other one at minute 8 . In some cases, none of the operators may be interested in operating a second intercity service just 5 minutes after another one. In this example the figures show that intercity operators may avoid getting their train scheduled just after other intercity by controlling 1) how flexible their schedule is, 2 ) how much discount in the access fee they obtain if the schedule of the train is changed, and 3) how much they are willing to pay to access the infrastructure.


Fig. 6. Timetable proposed by infrastructure manager to schedule a freight train in a system with commuter trains operating every 60 minutes.

Figure 6 assumes that the willingness to pay to access the infrastructure for a freight operator is lower than the willingness to pay of other operators. We also assumed that the freight operator's flexibility (total allowed translation and increment of travel time) is higher than other operators' flexibility.

For the same commuter frequency, a freight train will initially conflict with more commuter trains than an intercity train since it travels at a lower speed. As a result, independently of how much each commuter pays, the freight train will be scheduled if there is capacity as long as the net access fee paid to the infrastructure manager is positive. The minimum access fee that a freight operator should pay when the line is more congested will depend on how many services have to be rescheduled to find a feasible timetable. If the commuter operator wants to increase the frequency of commuter from one train per hour to one train every 30 minutes, the freight train will only be scheduled if the net access fee that the freight operator is willing to pay represents more than 3 units of revenue for the infrastructure manager (since three commuter services could not be operated).

Note that the relative speed difference between different services has a major impact on the capacity of the system.

## IV. Implications For Different Stakeholders

This section summarizes the results of the timetabling model for different operator's infrastructure access demand scenarios. The main implications for the operators, the infrastructure manager, the users, and the regulators are presented:
A. The capacity pricing and allocation mechanism determines how different operators are able to compete to access capacity
This effect is important when there are different types of operators (such as commuters and long-distance operators) competing to access the infrastructure.

Although the results presented before were calculated assuming that the capacity pricing and allocation mechanism in place is an auction; note that similar effects happen when other mechanisms are implemented. For instance, the results of the model demonstrate that intercity and freight services would have a disadvantage to compete with commuter trains under capacity allocation methods that maximize how many trains are scheduled in the system. This problem could be solved by designing mechanisms that account for the miles or passenger miles, by assigning higher priority to long distance services, or by assigning a societal utility to each service. The exact extent of these measures should be quantified using the model proposed for the specific system.

Competing companies may be concerned about getting their train in the schedule just after another competing train. The results for this problem suggest that when the infrastructure is congested, bundling two services together also represents a high opportunity cost for the infrastructure manager. As a consequence, the companies may be able to avoid having their train scheduled after another competing train by controlling the flexibility of their desired timetable and their willingness to pay to access the infrastructure.
B. The characteristics of the system and the capacity pricing and allocation mechanism affects the ability of the infrastructure manager to recover infrastructure costs
The results of the model also show that depending on the operators’ infrastructure access demand (like the demanded frequency of commuter services) the price that an intercity or freight operator will have to bid to be able to schedule a train varies considerably. This price reflects the congestion rent. In any case, this also implies that the infrastructure manager's ability to recover costs under auctions depends on the level of service (understood as the number of train services offered and the mix between different types of train services) of the line. Therefore in congested infrastructure the costs recovered will be higher.

Although cost-allocation mechanisms are usually presented as capacity pricing and allocation mechanisms that allow the infrastructure manager to recover infrastructure costs, the operators' demand to access the infrastructure will certainly depend on the infrastructure access fee. For instance, the operators would require high average ridership levels to provide a service when they pay high access charges and vice-versa.
C. The characteristics of the system and the capacity pricing and allocation mechanism also condition the level of service experienced by the users of the railway system (passengers and freight shippers)
This implication is a consequence of the previous ones. The capacity pricing and allocation mechanism determines how different operators can compete to get access to the infrastructure and the access charges they will have to pay. This will ultimately affect the operators' infrastructure access demand, which translates into how many services each operator will schedule. This will have a direct impact on the level of service.

Furthermore, the equilibrium between different types of services will also impact the level of service that the users will experience: whether it is possible to have frequent intercity services, whether it is possible to operate higher-speed services in the system, which long-distance services can be operated during peak-hours, etc.
D. Regulator's design of capacity pricing and allocation mechanisms should consider not only overarching regulation goals but also the specific characteristics of the railway system
This paper shows that the implications of the capacity pricing and allocation mechanism for the operators, the infrastructure manager, and the users strongly depend on the characteristics of the system. As a consequence, regulators should consider these characteristics to analyze if overarching regulation goals such as mix of services, level of infrastructure cost recovered, and level of service are met.

## V. Conclusions And Further Research

This paper proposes a train timetabling model for shared railway systems. The model is formulated as a mixed integer linear programming problem and solved both using commercial software and a novel algorithm based on approximate dynamic
programming. The results of the train timetabling model can be used to simulate and evaluate the behavior of the infrastructure manager in shared railway systems under different capacity pricing and allocation mechanisms. The main conclusions of the paper are:

1) The capacity pricing and allocation mechanisms impact the operations on a shared railway system for all the stakeholders: it determines whether the infrastructure manager will be able to recover costs; it determines how operators offering different or competing services will be able to compete to access the infrastructure; and hence it ultimately determines the level of service that will be offered to rail users.
2) The extent of this impact depends on the characteristics of the system and the traffic level.

The use of models as the one presented in this paper would allow regulators and decision makers to understand the implications of different capacity pricing and allocation mechanisms in particular shared railway systems.

This research simulates the optimal behavior of the infrastructure manager under a capacity pricing and allocation mechanism. It also assumes the characteristics of the railway systems and the operators' infrastructure access demand (characterized both as the demand to schedule trains and the revealed willingness to pay to access the infrastructure). However, the operator's infrastructure access demand depends strongly on the capacity pricing and allocation mechanism.

Further research will integrate the infrastructure manager model with an operator model to better quantify the trade-offs between utilization and level of service on the one hand, and infrastructure cost recovered under different capacity pricing and allocation mechanisms. These results will be valuable to design and evaluate appropriate capacity pricing and allocation mechanisms aimed at the particular characteristics of a specific shared railway system.

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