Distributed Consensus Control of DFIGs with Storage for Wind Farm Power Output Regulation

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Stefanos Baros, *Member, IEEE,* and Marija D. Ilić, *Fellow, IEEE*

***Abstract*—Today the state-of-the-art (SoA) wind generators (WGs) are the double-fed induction (DFIGs) with integrated storage devices. In the future, these WGs are expected to be one of the largest producers of renewable energy worldwide. In this paper, we propose a distributed control methodology for solving the problem of coordinating and controlling a group of SoA WGs to attain fast wind farm (WF) power output regulation with each storage device providing the same amount of power, i.e with equal sharing among the storage devices. Our proposed methodology introduces a consensus protocol for coordinating the grid-side converters (GSCs), whose dynamical equations constitute their closed-loop dynamics, and a particular closed-loop form for the interfacing capacitor dynamics. We establish stability of these closed-loop dynamics by leveraging singular perturbation and Lyapunov theories, proving that with these closed-loop dynamics DFIGs accomplish their assigned control objectives. Finally, we analytically construct a distributed and a Control Lyapunov Function (CLF) -based control law for the GSC and the DC-DC converter respectively, which jointly lead to the desired closed-loop dynamics. We demonstrate the effectiveness of our methodology through simulations on the IEEE 24-bus reliability test system (RTS).**

***Index Terms*—DFIGs with storage devices, wind farm power output regulation, consensus control, distributed control.**

I. Introduction

According to a recent U.S. DOE study [1], by 2020, 10% of the annual U.S. electricity demand is expected to be produced by WGs, offshore and onshore. Projecting more into the future, the study envisions that by 2030, 20% of the U.S. electricity demand could be produced from WGs, and by 2050, this percentage could even reach 35%. This study highlights a trend for integrating large amounts of wind power into power systems today. However, high levels of wind power integration can challenge power systems' stability, reliability and robustness. By realizing that, the current regulations for the operation of WGs mandate that WGs progressively provide multiple ancillary services to the grid through proper design of their controllers. Some of these are frequency regulation, inertial response, power output smoothing, Low Voltage Ride-Through (LVRT) and voltage control [**?**]. Albeit all of these services contribute to the secure operation of power systems, the most crucial is indisputably power output regulation.

Wind DFIGs with integrated storage devices are considered to be the gold standard for WG technology [2]. The proposed control methods for attaining power output regulation of a WF comprised of a group of SoA WGs are centralized. In such control schemes, local wind speed conditions prevalent at the location of the WGs as well as information about their stored energy, are first communicated to a centralized controller. Then, the centralized controller exploits this information to compute the available total wind power, and the total storage power that is required to meet a particular total power reference. Eventually, the centralized controller computes and communicates power set-points to the WGs which employ their local wind turbine and storage controllers to meet them.

In general, centralized control approaches carry several drawbacks. Among others, they are slow responding, they demand high computational effort and extensive communication network [3]. In the case of WGs, the inabilityo f these control schemes to respond fast can be a hurdle compromising a timely dispatch and regulation of their power outputs when these have to be performed rapidly and under highly dynamic conditions, e.g in low-inertia microgrids.

In dispatching and controlling SoA WGs, the real challenge lies into enabling them to compute the power set-points for the wind turbine and storage device in a fast, robust and computationally efficient manner. In particular, it is very important for SoA WGs to be able to retrieve their power set-points fast since that, combined with the fact that the storage devices are able to respond fast, can allow them to attain fast total power output regulation. In this case, the WFs can provide a broader range of services to the grid. In addition, the power set-points of the WGs have to be retrieved in a robust and efficient fashion, so that the WF total power output regulation is reliable and requires minimal computational effort.

In this paper, we recognize that the above challenges can be effectively addressed through distributed control methods and propose a particular design toward this goal. Our proposed control design solves the problem of WF power output regulation via dynamic dispatch and regulation of the power outputs of a group of storage devices.

Related Work. The power output regulation problem for a WF comprised of SoA WGs has been only studied in [4] under a centralized control scheme. In particular, a two-layer centralized constant power control system is proposed where, at the high-layer, a wind farm supervisory controller combines information about the total WF power reference and available wind power to generate the power set-points for both the wind turbine and the storage device of each WG. In the low-layer, proportional integral (PI) controllers for the rotor-side converters (RSCs) and the DC-DC converters meet these power set-points. To the best of our knowledge, distributed

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control methods for dealing with the above problem have not been proposed in the literature. On the other hand, centralized approaches as the one in [4] inherit the discussed weaknesses.

In our work, we develop a distributed control scheme that first allows, the GSCs to coordinatively and dynamically regulate their power outputs using communication so that WF power output regulation is attained and second, the DC-DC converters to dynamically self-organize and control the storage devices to ultimately supply the power demanded by the GSCs. In contrast with centralized approaches, our method allows dynamic and distributed fast WF power output regulation.

Contributions. We introduce a distributed control methodology for the GSC and DC-DC converter that enables SoA WGs to regulate their total power to a reference by controlling their storage devices in an equal sharing fashion. In our context, equal sharing refers to the storage devices continuously adjusting their power outputs so that they remain equal to each other while the total power is tracking the given reference. Our main contributions can be highlighted as:

First contribution. We introduce a leader-follower consensus protocol and a desired closed-loop form for the interfacing capacitor dynamics of SoA WGs that lead to accomplishment of the control objectives described above.

Second contribution. We rigorously establish stability of the proposed closed-loop dynamics of the DFIG system.

Third contribution. We analytically develop a CLF-based controller for the DC-DC converter and a distributed GSC controller that together attain WF power output regulation with fair load sharing among the storage devices.

The rest of the paper is organized in the following way. In Section II, the relevant models are presented and in Section III, the main problem is formulated. In Section IV, our proposed methodology is outlined. Sections V, VI, VII, VIII, IX embody the main results of the paper. In particular, in Section V, the main problem is reformulated into two control subproblems. In Section VI, the consensus protocol and the desired capacitor dynamics are stated. In Section VII and VIII, time-scale separation and stability analyses are conducted. In Section IX, the control design is presented. Finally, in Section X, the proposed methodology is evaluated and in Section XI, the paper is concluded with some remarks.

II. Mathematical Modeling

In this paper, we perform control design for the storage devices of a fleet of SoA DFIGs,k denoted by the set $\mathcal{G}$. Each WG is indexed by i such that $i \in \mathcal{G}$ and is depicted in Fig. 1. The parts of the SoA WG involved in the storage control design are the GSC, the interfacing capacitor between the RSC and the GSC, and the supercapacitor, whose corresponding models are presented next.

A. Grid Side Converter Model

The GSC model represents the dynamics of its current output expressed in a $d-q$ coordinate system as [5]:

$$\frac{dI_{dg,i}}{dt} = -\omega_s\Big(\frac{R_{g,i}}{L_{g,i}}\Big)I_{dg,i}+\omega_s I_{qg,i}+\omega_s\Big(\frac{V_{dg,i}-V_{s,i}}{L_{g,i}}\Big) \quad (1a)$$

$$\frac{dI_{qg,i}}{dt} = -\omega_s\Big(\frac{R_{g,i}}{L_{g,i}}\Big)I_{qg,i}-\omega_s I_{dg,i}+\omega_s\Big(\frac{V_{qg,i}}{L_{g,i}}\Big) \quad (1b)$$

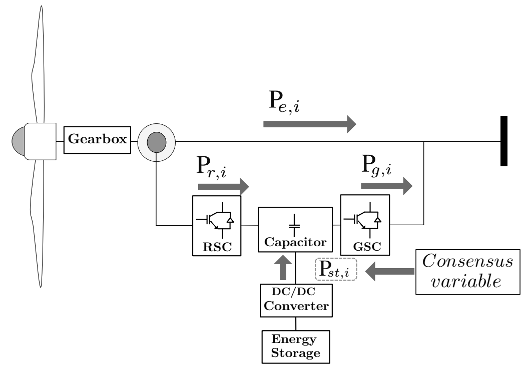


Fig. 1: Wind DFIG with storage

where $i \in \mathcal{G}$. The constants $R_{g,i}, L_{g,i} \in \mathbb{R}_+$ denote the resistance and inductance of the GSC, $V_{s,i} \in \mathbb{R}$ the terminal voltage and $V_{dg,i}, V_{qg,i} \in \mathbb{R}$ the GSC's controllable voltage.

B. Interfacing Capacitor Model

The model of the interfacing capacitor can be stated as:

$$(C_{dc,i}V_{dc,i})\frac{dV_{dc,i}}{dt} = (P_{r,i}+P_{st,i}-P_{g,i}), \qquad \forall i \in \mathcal{G} \quad (2)$$

where $C_{dc,i}, V_{dc,i} \in \mathbb{R}$ are the capacitance and the DC voltage of the capacitor, respectively. Further, $P_{g,i}$, $P_{r,i}$ and $P_{st,i}$ are the electric power outputs of the GSC, the RSC and the storage respectively.

C. Supercapacitor Energy Storage Model

The particular type of storage devices considered here are supercapacitors due to their high efficiency and rapid response. Their model can be stated as:

$$(C_{sc,i}V_{sc,i})\frac{dV_{sc,i}}{dt} = V_{sc,i}\frac{(u_{sc,i}-V_{sc,i})}{R_{sc,i}}, \quad \forall i \in \mathcal{G} \quad (3)$$

where the variable $u_{sc,i} \in \mathbb{R}$ denotes the voltage controlled by the DC-DC converter, whereas $C_{sc,i}, V_{sc,i}, R_{sc,i} \in \mathbb{R}_+$ denote the supercapacitor's capacitance, DC voltage and resistance, respectively. Its storage power output is given by:

$$P_{st,i} = (V_{sc,i}/R_{sc,i})(V_{sc,i}-u_{sc,i}), \qquad \forall i \in \mathcal{G} \quad (4)$$

and can be regulated by the DC-DC converter through $u_{sc,i}$.

III. Problem Formulation

A. WF Power Output Regulation with Fair Utilization of DFIGs' Storage Devices

Consider a WF with n SoA WGs incorporating supercapacitor energy storage devices that receives a power reference P_d from a system operator (SO). This power reference corresponds to the WF's committed power output toward the SO and is the outcome of a wind forecasting method and an economic dispatch (ED) process. The main problem that we seek to solve can be formulated around a particular goal for the SoA WGs. This goal is to coordinate their storage devices in order to dynamically track the total WF power reference P_d while the storage devices contribute equally to the power mismatch required to meet P_d, i.e they are deployed under a fair load sharing regime. In the forthcoming analysis the specific conditions that the storage devices have to meet are analytically derived. First, consider the particular scenario where the storage devices do not generate any power so that at the equilibrium of equation (2) we have:

$$P_{g,i} = P_{r,i}, \quad i \in \mathcal{G} \tag{5}$$

i.e the GSC power output equals the RSC power output. In this case, the total WF power output is approximately equal to the total mechanical power available from the wind, i.e:

$$\sum_{i\in\mathcal{G}} (P_{e,i}+P_{r,i}) \approx \sum_{i\in\mathcal{G}} P_{m,i} \tag{6}$$

This power is highly volatile because it depends on the wind speed conditions. Accordingly, although the WF's total electrical power output is required to match a particular power reference P_d, it is also highly volatile. Thus, it is likely that:

$$\sum_{i\in\mathcal{G}} (P_{e,i}+P_{r,i}) < P_d \tag{7}$$

i.e the WF may not be able to meet SO's request. That can be supported by the following fact. In general, there exists a significant time-delay between the moment the SO issues the scheduled power reference P_d until the moment that it is implemented by the WF [2]. This delay together with the wind speed minute to minute variability, might lead to the WF being unable to meet SO's request. On the other hand, the WF can meet SO's request even with the available wind power being inadequate, when its WGs incorporate storage devices into their systems. Specifically, when the storage devices have sufficient stored energy it might hold that:

$$\sum_{i\in\mathcal{G}} (P_{e,i}+P_{r,i}+P_{st,i}) = P_d \tag{8}$$

In addition, when each storage device generates power, at the equilibrium of (2) it holds that:

$$P_{st,i} = (P_{g,i}-P_{r,i}), \quad i \in \mathcal{G} \tag{9}$$

Hence, the storage devices can provide or draw power so that the WF's power output is regulated to P_d. Mathematically, this is characterized by the following condition.

Condition 1 (Total storage power regulation).

$$\sum_{i\in\mathcal{G}} P_{st,i} = P_d - \sum_{i\in\mathcal{G}} (P_{e,i}+P_{r,i}) \tag{10}$$

In general, the available storage devices are utilized more efficiently when they contribute equally to the total storage power. This can be described by the following condition on the storage power outputs.

Condition 2 (Fair load sharing among storage devices).

$$P_{st,i} = P_{st,j}, \qquad \forall i,j \in \mathcal{G} \tag{11}$$

With the desired conditions for the storage power outputs being defined, our main problem can be stated as follows:

Problem 1. Coordinate and control the energy storage devices of a group of SoA WGs in a distributed way and under a fair load sharing regime to attain WF power output regulation.

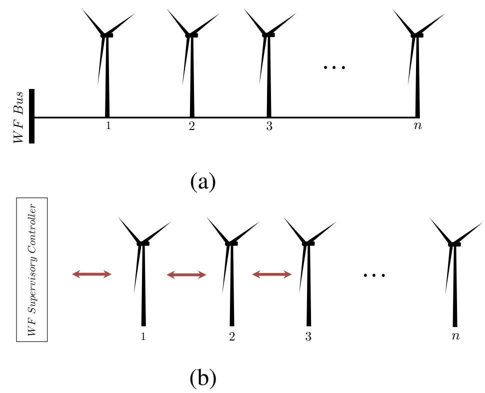


Fig. 2: a) Physical topology b) Communication topology of WF

IV. Distributed Control Methodology

We propose the next methodology for solving Problem 1, partitioned into five main steps.

- **Step 1**: We pose Problem 1 as a twofold control problem, a constrained consensus problem among the GSCs on the variable $z_i := (P_{g,i}-P_{r,i})$, the difference among the GSC and RSC power outputs, and a tracking problem for the storage power outputs so that $\lim_{t\to\infty} P_{st,i} = z_i, \ \forall i \in \mathcal{G}$.
- **Step 2**: We introduce a leader-follower consensus protocol that GSCs can incorporate into their control systems to distributively reach consensus on their $z_i's$ and a desired closed-loop form for the interfacing capacitor dynamics that DC-DC converters can realize through proper control design to ensure storage power output regulation.
- **Step 3**: We perform time-scale separation analysis of the coupled closed-loop consensus protocol's and interfacing capacitor's dynamics and derive conditions on the GSC and DC-DC converter control gains under which these dynamics manifest three time-scales.
- **Step 4**: Given that the GSC's and DC-DC converter's control gains fulfill the Conditions in Step 3, we first employ singular perturbation theory to conduct temporal decomposition of the above closed-loop dynamics and then perform compositional stability analysis to establish asymptotic stability of their equilibrium.
- **Step 5**: We design a distributed controller for the GSC and a cLF-based controller for the DC-DC converter which guarantee respectively that, the closed-loop dynamics of z_i are identical to the consensus protocol dynamics and the closed-loop dynamics of the capacitor have the desired form defined in Step 2.

V. Constrained Consensus and Storage Power Outputs Regulation Problem

In Section III, the problem of WF power output regulation with fair load sharing of the storage devices is formulated as a control problem for the storage devices with objective their power outputs to meet *Conditions* 1 and 2. To ensure that the storage power outputs satisfy *Conditions* 1 and 2, we will control each storage power $P_{st,i}$ so that asymptotically it satisfies $P_{st,i} = (P_{g,i}-P_{r,i})$ and the GSCs such that asymptotically they satisfy the next two conditions for the power difference $z_i := (P_{g,i} - P_{r,i})$.

Condition 3 (**Regulation of the total power z_i's).**

$$\sum_{i\in\mathcal{G}} z_i = \Big(P_d - \sum_{i\in\mathcal{G}} (P_{e,i} + P_{r,i})\Big) \tag{12}$$

Condition 4 (**Fair load sharing among z_i's).**

$$z_i = z_j, \quad \forall i, j \in \mathcal{G} \tag{13}$$

With these conditions, our main problem can be reduced to the two equivalent control subproblems below; the first one is for the GSCs to reach constraint consensus on the variable z_i and the second one for the DC-DC converters to regulate the storage power outputs to their respective $z_i's$.

Problem 2 (**Constrained Consensus on z_i's).** Coordinate and control the GSCs so that the variables z_i asymptotically reach consensus, i.e fulfill Condition 4, while respecting the contraint given in Condition 3.

Problem 3 (**Storage Power Outputs Regulation**)**.** Control the storage devices so that their power outputs $P'_{st,i}s$ are regulated to their respective $z_i's$, i.e $\lim_{t\to\infty} P_{st,i} = z_i, \ \forall i \in \mathcal{G}$.

VI. Consensus Protocol and Closed-loop Capacitor Dynamics

In this Section, a leader-follower consensus protocol that GSCs can adopt into their control design to reach constraint consensus on their $z_i's$ is introduced. Moreover, a desired closed-loop form for the capacitor dynamics that the storage devices can realize through their DC-DC converters to attain regulation of their storage power $P_{st,i}$ to their corresponding z_i.

A. Leader-Follower Consensus Protocol

The proposed leader-follower consensus protocol is stated below where, without loss of generality WG l, with $l := 1$, is the leader and the set of followers is denoted by $\overline{\mathcal{G}} := \{2, ..., n\}$.

Consensus Protocol $\mathcal{P}_1$

Leader

$$\frac{d\xi_h}{dt} = \Big(P_d - \sum_{i\in\mathcal{G}} (P_{e,i} + P_{r,i} + z_i)\Big) \quad \xi_h \in \mathbb{R} \tag{14a}$$

$$\frac{dz_l}{dt} = -k_{\alpha,l}(z_l - \xi_h)\ , \qquad z_l \in \mathbb{R},\ z_l := z_1 \tag{14b}$$

Followers

$$\frac{dz_i}{dt} = -k_{\alpha,i}(z_i - z_{i-1})\ , \qquad z_i \in \mathbb{R},\ \forall i \in \overline{\mathcal{G}} \tag{14c}$$

The state variables are $z_i := (P_{g,i} - P_{r,i})$, and an auxiliary variable of the leader ξ_h. The variable ξ_h is the one that drives the protocol dynamics and guarantees regulation of the total WF power to the reference P_d. Notice that, every WG is allowed to communicate with its two neighbors.

The protocol $\mathcal{P}_1$ can be executed in the following manner. The leader WG obtains information about the total power reference P_d from the WF supervisory controller. Combining this with the information $\sum_{i\in\mathcal{G}} (P_{e,i} + P_{g,i})$, the leader WG controls the dynamics of the state-variables z_1 and ξ_h and continuously communicates z_1 to its neighbors. Synchronized with the leader, all followers control the dynamics of their $z_i's$ by exploiting information from their respective neighbors (z_{i-1}) while they communicate to them their $z_i's$.

B. Closed-loop Interfacing Capacitor Dynamics

We now introduce the next desired closed-loop form for the interfacing capacitor's dynamics given with respect to the variable $\Delta E_{dc,i} := (E_{dc,i} - E_{dc0,i})$, that denotes the deviation of energy around the equilibrium.

$$\frac{d(\Delta E_{dc,i})}{dt} = -k_{2,i}(\Delta E_{dc,i}), \qquad \forall i \in \mathcal{G} \tag{15}$$

The DC-DC converters can shape their capacitors' closed-loop dynamics to obtain this form and by doing that they will guarantee that their storage power outputs $P_{st,i}$ track the variables z_i. The open-loop dynamics of the energy variable are:

$$\frac{d(\Delta E_{dc,i})}{dt} = (P_{r,i} + P_{st,i} - P_{g,i}), \qquad \forall i \in \mathcal{G} \tag{16}$$

When the closed-loop dynamics of the capacitor are identical to (15), the dynamical equation for the storage power $x_i := P_{st,i}$ can be obtained, by first differentiating (15) and (16) and letting them be equal, and then substituting dz_i/dt from (14c):

Storage power dynamics

$$\frac{dx_i}{dt} = -k_{\alpha,i}(z_i - z_{i-1}) - k_{2,i}(x_i - z_i), \qquad \forall i \in \mathcal{G} \tag{17}$$

The consensus protocol dynamics in (14c) represent the desired closed-loop dynamics of the variable z_i and can be realized by the GSC while, the dynamics in (17), that of the storage power $P_{st,i}$, that can be realized by the DC-DC converter. Altogether, the model comprised of the equations (14a)-(14c) and (17) describes our main dynamical system.

VII. Time-scale Separation Analysis

The dynamics of the variable z_i and the storage power x_i can be expressed more compactly in vector form as:

$$\frac{d\xi_h}{dt} = \Big(P_d - \sum_{i\in\mathcal{G}} (P_{e,i} + P_{r,i} + z_i)\Big) \tag{18a}$$

$$\frac{d\mathbf{z}}{dt} = \mathbf{g}_z \tag{18b}$$

$$\frac{d\mathbf{x}}{dt} = \mathbf{g}_x \tag{18c}$$

where:

$$\mathbf{z} := [z_1, ..., z_n]^\top \in \mathbb{R}^n,\ \mathbf{x} := [x_1, ..., x_n]^\top \in \mathbb{R}^n$$
$$\mathbf{g}_z := [-k_{\alpha,1}(z_1 - \xi_h), ..., -k_{\alpha,n}(z_n - z_{n-1})]^\top \in \mathbb{R}^n$$
$$\mathbf{g}_x := \mathbf{g}_z - [k_{2,1}(x_1 - z_1), ..., -k_{2,n}(x_n - z_n)]^\top \in \mathbb{R}^n$$

This dynamical system is characterized by three distinct time-scales when the gains $k_{\alpha,i},\ k_{2,i}$ respect the conditions stated in the next Lemma.

Lemma 1. *The dynamical system* (18a)-(18c) *manifests three distinct time-scales when* $k_{\alpha,i} \gg 1,\ \ k_{2,i} \gg\ k_{\alpha,i},\ \forall i \in \mathcal{G}$.

Proof. Without loss of generality we define:

$$k_{\alpha,1} = ... = k_{\alpha,i} = ... = k_{\alpha,n} = (1/\varepsilon_1) \tag{19}$$

$$k_{2,1} = ... = k_{2,i} = ... = k_{2,n} = (1/\varepsilon_2) \tag{20}$$

Further, we express equations (18b), (18c) in scalar form and divide them by $k_{\alpha,i}$ and $k_{2,i}$, respectively. These, yield:

$$\varepsilon_1 \frac{dz_i}{dt} = -(z_i - z_{i-1}) \ , \ z_0 = \xi_h, \qquad \forall i \in \mathcal{G} \tag{21}$$

$$\varepsilon_2 \frac{dx_i}{dt} = -\frac{\varepsilon_2}{\varepsilon_1}(z_i - z_{i-1}) - (x_i - z_i) \ , \qquad \forall i \in \mathcal{G} \tag{22}$$

Altogether, we finally obtain the system:

$$\frac{d\xi_h}{dt} = \Big(P_d - \sum_{i \in \mathcal{G}} (P_{e,i} + P_{r,i} + z_i)\Big) \tag{23a}$$

$$\varepsilon_1 \frac{d\mathbf{z}}{dt} = \bar{\mathbf{g}}_z \tag{23b}$$

$$\varepsilon_2 \frac{d\mathbf{x}}{dt} = \bar{\mathbf{g}}_x \tag{23c}$$

where the new vector fields are:

$$\bar{\mathbf{g}}_z = \mathbf{g}_z \varepsilon_1, \ \ \bar{\mathbf{g}}_x = \mathbf{g}_x \varepsilon_2 \tag{24}$$

When $\varepsilon_1 \ll 1$, $\varepsilon_2 \ll \varepsilon_1$, this system obtains the *standard singularly perturbed* form with three distinct time-scales t, $\tau = t/\varepsilon_1$, $\tilde{\tau} = t/\varepsilon_2$. Correspondingly, ξ_h is the slow state-variable, $\mathbf{z}$ are the fast and $\mathbf{x}$ the very fast state-variables. ■

Lemma 1 is practically useful since the conditions that it involves can guide the choice of appropriate control gains for the GSCs ($k_{\alpha,i}$) and the DC-DC converters ($k_{2,i}$) that will grant three time-scales in the dynamics of the system (18a) - (18c).

VIII. Stability Analysis

Presuming that the control gains of the GSCs and DC-DC converters respect the conditions stated in Lemma 1, singular perturbation theory is now employed to perform compositional stability analysis of the above system.

A. Equilibrium and Desired Properties

The equilibrium of the coupled consensus protocol and storage power system (18a)-(18c) is:

$$\xi_{h0} = \Big(P_d - \sum_{i \in \mathcal{G}} (P_{e,i} + P_{r,i})\Big)/n \tag{25a}$$

$$\mathbf{z}_0 = (\xi_{h0} \cdot \mathbf{1}_n) \tag{25b}$$

$$\mathbf{x}_0 = (\xi_{h0} \cdot \mathbf{1}_n) \tag{25c}$$

We define the state-vector of the full system as:

$$\boldsymbol{\phi} = [\xi_h, \ \mathbf{z}, \ \mathbf{x}]^\top \in \mathbb{R}^{2n+1} \tag{26}$$

and a *consensus subspace* as:

$$\mathcal{S} := \{\boldsymbol{\phi} \in \mathbb{R}^{2n+1} \mid \boldsymbol{\phi} = \beta \cdot \mathbf{1}_{2n+1} \ , \ \ \beta \in \mathbb{R}\} \tag{27}$$

WF power output regulation with fair load-sharing among the storage devices is guaranteed when the equilibrium $\boldsymbol{\phi}_0$ of the full system (18a)-(18c) possesses the following properties.

Property 1. $\boldsymbol{\phi}_0 \in \mathcal{S}$.
Property 2. $\boldsymbol{\phi}_0$ *is asymptotically stable.*

The system's equilibrium readily has property 1, since $\boldsymbol{\phi}_0 = (\xi_{h0} \cdot \mathbf{1}_{2n+1})$. We now have to show that it possesses property 2 as well. By defining the new shifted state-variables:

$$\psi_h := (\xi - \xi_{h0}) \tag{28}$$

$$\mathbf{y} := (\mathbf{z} - \xi_h \cdot \mathbf{1}_n), \qquad \mathbf{y} := [y_1, ..., y_n]^\top \in \mathbb{R}^n \tag{29}$$

$$\boldsymbol{\eta} := (\mathbf{x} - \mathbf{z}), \qquad \boldsymbol{\eta} := [\eta_1, ..., \eta_n]^\top \in \mathbb{R}^n \tag{30}$$

the system (18a)-(18c) can be transformed to:

$$\frac{d\psi_h}{dt} = -n\psi_h - \mathbf{1}_n^\top \mathbf{y} \tag{31a}$$

$$\varepsilon_1 \frac{d\mathbf{y}}{dt} = \mathbf{g}_y \tag{31b}$$

$$\varepsilon_2 \frac{d\boldsymbol{\eta}}{dt} = \mathbf{g}_\eta \tag{31c}$$

where the vector fields are:

$$\mathbf{g}_y := [-y_1, ..., -(y_n - y_{n-1})]^\top - \varepsilon_1 \frac{d\psi_h}{dt} \cdot \mathbf{1}_n \tag{32}$$

$$\mathbf{g}_\eta := -\boldsymbol{\eta} \tag{33}$$

We define the state-vector of (31a)-(31c) as:

$$\bar{\boldsymbol{\phi}} = [\psi_h, \ \mathbf{y}, \ \boldsymbol{\eta}]^\top \in \mathbb{R}^{2n+1} \tag{34}$$

that has equilibrium $\bar{\boldsymbol{\phi}}_0 = \mathbf{0}_{2n+1}$ i.e the origin. The aim of the forthcoming analysis is to derive conditions under which $\bar{\boldsymbol{\phi}}_0$ is asymptotically stable. But first, realize that, in the transformed system, the consensus protocol dynamics (31a), (31b) are decoupled from the storage power dynamics (31c). This facilitates stability analysis of these dynamics since by merely establishing stability of (31a), (31b) and independently of (31c), is sufficient to infer stability of the full system (31a)-(31c). In other words, if:

$$\tilde{\mathbf{y}}_0 = [\psi_{h0}, \ \mathbf{y}_0]^\top = \mathbf{0}_{n+1}, \ \ \boldsymbol{\eta}_0 = \mathbf{0}_n \tag{35}$$

are asymptotically stable equilibria of (31a), (31b) and (31c) respectively, then $\bar{\boldsymbol{\phi}}_0$ will be of (31a)-(31c).

B. Stability of the Consensus Protocol Dynamics

We first study stability of the consensus protocol dynamics (31a)-(31b) that have a standard singular perturbation form with two distinct time-scales, the slow one t and the fast one τ. The state-variable ψ_h is the slow one while the state-variables $\mathbf{y}$ the fast ones. Stability of this system is established in the following way. We perform temporal decomposition to obtain fast and slow decoupled subsystems and establish their asymptotic stability.

a) **Stability of Fast-boundary Layer Subsystem**

First, we establish asymptotic stability of the equilibrium of the protocol's decoupled fast subsystem, $\mathbf{y}_0 = \mathbf{0}_n$. This system can be obtained by approximating the slow state-variable ψ_h in equation (31b) as constant, i.e $d\psi_h/d\tau = 0$:

Fast-boundary Layer Subsystem

$$\frac{dy_i}{d\tau} = -(y_i - y_{i-1}), \ \ \forall i \in \mathcal{G} \tag{36}$$

where $\tau = t/\varepsilon_1$. Stability of this system is established through the following lemma.

Lemma 2. *The equilibrium* $\mathbf{y}_0 = \mathbf{0}_n$ *of the fast boundary-layer system* (36) *is asymptotically stable.*

Proof. The system (36) in matrix form can be written as:

$$\frac{d\mathbf{y}}{d\tau} = \mathbf{A}_f\mathbf{y}, \quad \mathbf{A}_f \in \mathbb{R}^{n\times n} \tag{37}$$

$$\mathbf{A}_f = \begin{bmatrix} -1 & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & & & \vdots \\ 0 & 0 & \cdots & 1 & -1 \end{bmatrix}$$

We denote the eigenvalues of this matrix by $\boldsymbol{\lambda} = [\lambda_l, ..., \lambda_n]^\top$. The matrix $\mathbf{A}_f$ is lower triangular so it holds that:

$$\boldsymbol{\lambda} = -\mathbf{1}_n \preceq 0 \tag{38}$$

From this, we conclude that $\mathbf{A}_f$ is *Hurwitz*. It also follows from Theorem 4.5 ([6]) that $\mathbf{y}_0$ is asymptotically stable. With this, we complete the proof. ∎

The stability property established above will be useful in proving asymptotic stability of the full system (31a)-(31b). Therefore, a parameterized Lyapunov function that captures this property and serves as a stability certificate of the fast boundary-layer system can be defined as:

$$V_f = \mathbf{y}^\top\mathbf{P}\mathbf{y}, \quad V_f > 0, \ \ \forall \mathbf{y} \in \overline{\mathcal{D}}_y \tag{39}$$

where $\overline{\mathcal{D}}_y = \mathcal{D}_y \backslash \{\mathbf{0}_n\}$, $\mathcal{D}_y \subset \mathbb{R}^n$ and $\mathbf{P} \in \mathbb{R}^{n\times n}$ is a positive definite matrix satisfying the Lyapunov equation:

$$\mathbf{P}\mathbf{A}_f + \mathbf{A}_f^\top\mathbf{P} = -\mathbf{Q} \tag{40}$$

for a particular choice of $\mathbf{Q} \succ 0$.

b) **Stability of Slow Reduced-order Subsystem**

We will now establish asymptotic stability of ψ_{h0}, equilibrium of the protocol's slow reduced-order susbystem. Focusing on the slow time-scale t and approximating the fast state-variables $\mathbf{y}$ with their quasi-steady state values $\mathbf{y} = \mathbf{0}_n$, yields the slow reduced-order subsystem:

Slow Reduced-order Subsystem

$$\frac{d\psi_h}{dt} = -n\psi_h \tag{41}$$

Stability of (41) is established through the following lemma.

Lemma 3. *The equilibrium point* $\psi_{h0} = 0$ *of the slow reduced system* (41) *is asymptotically stable.*

Proof. A candidate Lyapunov function for the system (41) is:

$$V_h = \psi_h^2, \quad V_h > 0, \quad \forall \psi_h \in \overline{D}_{\psi_h} \tag{42}$$

where $\overline{D}_{\psi_h} = D_{\psi_h} \backslash \{0\}$, $D_{\psi_h} \subseteq \mathbb{R}$. The time-derivative of (42) along the trajectories of (41) is:

$$\dot{V}_h = -2n\psi_h^2 < 0, \ \forall \psi_h \in \overline{D}_{\psi_h}, \ \ n > 0 \tag{43}$$

By invoking Lyapunov's stability theorem we conclude that $\psi_{h0} = 0$ is asymptotically stable. ∎

With the stability properties of the decoupled subsystems established, we are now ready to state our main stability result.

c) **Stability of the Full Consensus Protocol System**

The stability properties of the fast boundary-layer and slow reduced-order subsystems just established can be exploited to infer asymptotic stability of the equilibrium $\tilde{\mathbf{y}}_0$ of the full consensus protocol system (31a), (31b). This is carried out through the next theorem.

Theorem 1. $\exists \bar{\varepsilon}_1$ *such that* $\forall \varepsilon_1 < \bar{\varepsilon}_1$ *the equilibrium* $\tilde{\mathbf{y}}_0 = \mathbf{0}_{n+1}$ *is asymptotically stable.*

Proof. Follows from Theorem 11.4 ([6]) and *Lemmas* 2 and 3. ∎

The intuition driving Theorem 1 is the following: since the equilibria of the approximated fast and slow subsystems, $\mathbf{y}_0$ and ψ_{h0} are exponentially stable, the equilibrium $\tilde{\mathbf{y}}_0$ of the full protocol system will also be exponentially stable when the GSC control gains $k_{\alpha,i}$ are large enough so that the small parameter ε_1 respects an upper bound $\bar{\varepsilon}_1$.

C. Stability of the Storage Power Dynamics

We now establish stability of the storage power dynamics:

$$\varepsilon_2\frac{d\boldsymbol{\eta}}{dt} = -\boldsymbol{\eta} \tag{44}$$

Lemma 4. *The equilibrium* $\boldsymbol{\eta}_0 = \mathbf{0}_n$ *of the storage power dynamics* (44) *is asymptotically stable.*

A candidate Lyapunov function for this system is:

$$V_\eta = \|\boldsymbol{\eta}\|_2^2, \ \ V_\eta > 0, \ \ \forall \boldsymbol{\eta} \in \overline{\mathcal{D}}_\eta \tag{45}$$

where $\overline{\mathcal{D}}_\eta = \mathcal{D}_\eta \backslash \mathbf{0}_n$, $\mathcal{D}_\eta \subseteq \mathbb{R}^n$. Along the trajectories of the system (44), the derivative of V_η is:

$$\dot{V}_\eta = -\frac{2}{\varepsilon_2}\|\boldsymbol{\eta}\|_2^2, \ \ \dot{V}_\eta < 0, \ \ \forall \boldsymbol{\eta} \in \overline{\mathcal{D}}_\eta \tag{46}$$

By applying *Lyapunov's stability theorem*, it can be concluded that V_η is a Lyapunov function and $\boldsymbol{\eta}_0 = \mathbf{0}_n$ is an asymptotically stable equilibrium point.

D. Stability of the Full Consensus Protocol - Storage System

The consensus protocol and storage power subsystems appear decoupled in the transformed state-space (31a)-(31c). We leverage that by deploying the already established stability properties for these subsystems to infer stability of the full system comprised of the protocol and storage power dynamics through the next Theorem.

Theorem 2. *Let the closed-loop storage power dynamics have the form* (17). *Then, the equilibrium* $\bar{\boldsymbol{\phi}}_0 = \mathbf{0}_{2n+1}$ *of the full consensus protocol and storage power output system* (31a)-(31c) *is asymptotically stable for* $\varepsilon_1 < \bar{\varepsilon}_1$ *where,* $\bar{\varepsilon}_1$ *is an upper bound as stated in Theorem 1.*

Proof. Let $V_c := (1/2)V_f + (1/2)V_h, \forall \varepsilon_1 < \bar{\varepsilon}_1$, be a Lyapunov function for the consensus dynamics (31a), (31b). Then, $\dot{V}_c = \tilde{\mathbf{y}}^\top\mathbf{G}\tilde{\mathbf{y}} < 0, \ \forall (\tilde{\mathbf{y}}, \varepsilon_1) \in \overline{\mathcal{D}}_{\tilde{\mathbf{y}}} \times \mathcal{E}_1$ where:

$$\overline{\mathcal{D}}_{\tilde{\mathbf{y}}} = \mathcal{D}_{\tilde{\mathbf{y}}} \backslash \mathbf{0}_{n+1}, \ \ \mathcal{D}_{\tilde{\mathbf{y}}} \subseteq \mathbb{R}^{n+1} \text{ and } \mathcal{E}_1 := \{\varepsilon_1 \in \mathbb{R}_+ | \varepsilon_1 < \bar{\varepsilon}_1\}$$

and $\mathbf{G}$ is a matrix for which $\mathbf{G} \prec 0$ when $\varepsilon_1 \in \mathcal{E}_1$. In this case, a candidate Lyapunov function for the full protocol and storage power output system can be defined as:

$$V_{full} = V_c + V_\eta, \ \ V_{full} > 0, \ \ \bar{\boldsymbol{\phi}} \in \overline{\mathcal{D}}_{\bar{\phi}} \tag{47}$$

where $\overline{\mathcal{D}}_{\bar{\phi}} = \mathcal{D}_{\bar{\phi}} \backslash \{\mathbf{0}_{2n+1}\}, \quad \mathcal{D}_{\bar{\phi}} \subseteq \mathbb{R}^{2n+1}$. The time-derivative of V_{full} is:

$$\dot{V}_{full} = \tilde{\mathbf{y}}^\top \mathbf{G} \tilde{\mathbf{y}} - \frac{2}{\varepsilon_2} \|\boldsymbol{\eta}\|_2^2 < 0, \ \ \forall(\bar{\boldsymbol{\phi}}, \varepsilon_1) \in \overline{\mathcal{D}}_{\bar{\phi}} \times \mathcal{E}_1 \quad (48)$$

From Lyapunov's stability theorem, we conclude that $\bar{\boldsymbol{\phi}}_0 = \mathbf{0}_{2n+1}$ is an asymptotically stable equilibrium $\forall \varepsilon_1 < \bar{\varepsilon}_1$. ■

The intuition behind the above results is that stability of the coupled consensus protocol and storage power output system is certified when the GSC gains (consensus protocol's gains) respect the inequality $\varepsilon_1 < \bar{\varepsilon}_1$ (*Theorem* 1). That being the case, provable WF power output regulation and asymptotic consensus on the variables $\mathbf{z}$ will be reached. On the other hand, the role of the DC-DC converters is to shape the closed-loop storage power dynamics such that they are identical to the dynamics in (17). When that holds, the storage power outputs $\mathbf{x}$ will be provably regulated to the consensus state-variables $\mathbf{z}$, i.e $\lim_{t\to\infty} \boldsymbol{\eta} = \mathbf{0}_n \Rightarrow \lim_{t\to\infty} \mathbf{x} = \mathbf{z}$, as long as the control gains $k_{2,i}$ are positive. This has the implication that the variables $\mathbf{x}$ will also reach consensus through tracking of the variables $\mathbf{z}$. Albeit tracking will be attained as long as the gains $k_{2,i}$ are positive, ideally, high enough gains $k_{2,i}$ should be chosen so that the storage power regulation occurs much faster than the consensus on the variables $\mathbf{z}$.

IX. Design of the Controllers

We now proceed to design the controllers for the GSC and the DC-DC converter which respectively realize the closed-loop consensus protocol and storage power output dynamics.

A. Design of the GSC Controller

The control objective of the GSC is to shape the physical closed-loop dynamics of z_i given below so that they are identical to the consensus protocol dynamics $\dot{z}_i$.

$$\dot{z}_i = (\dot{P}_{g,i} - \dot{P}_{r,i}), \quad \forall i \in \mathcal{G} \quad (49)$$

where $P_{g,i}$ is the power output of the GSC:

$$P_{g,i} = I_{dg,i} V_{s,i}, \quad \forall i \in \mathcal{G} \quad (50)$$

Now, let the following assumptions be true.

Assumption 1. $dP_{r,i}/dt = 0, \ dV_{s,i}/dt = 0, \ \ \forall i \in \mathcal{G}$.

Intuitively, these assumptions can be justified by the fact that the power output of the RSC ($P_{r,i}$) and the terminal voltage ($V_{s,i}$) vary in a much slower time-scale than that of the $\dot{z}_i$ dynamics. By deploying equation (50) and under these assumptions, equation (49) can be expanded as:

$$\dot{z}_i = V_{s,i} \Big[-\omega_s \Big(\frac{R_{g,i}}{L_{g,i}} \Big) I_{dg,i} + \omega_s I_{qg,i} + \omega_s \Big(\frac{V_{dg,i} - V_{s,i}}{L_{g,i}} \Big) \Big] \quad (51)$$

The control input of the GSC is given by the term $V_{dg,i}$. The closed-loop form of the above physical dynamics matches the consensus protocol dynamics (14c) with the distributed GSC control law:

$$V_{dg,i} = \Big(\frac{-k_{\alpha,i}(z_i - z_{i-1})}{V_{s,i}} + \omega_s \Big(\frac{R_{g,i}}{L_{g,i}} \Big) I_{dg,i} - \omega_s I_{qg,i} \Big) \frac{L_{g,i}}{\omega_s} + V_{s,i} \quad (52)$$

B. Design of the DC-DC Converter Controller

The control objective of the DC-DC converter is to shape the physical closed-loop dynamics of the interfacing capacitor to the ones given by equation (15). This can be achieved by enforcing a constraint on the capacitor dynamics of the form:

$$(P_{r,i} + P_{st,i} - P_{g,i}) = -k_{2,i}(\Delta E_{dc,i}) \quad (53)$$

Finally, the control law for the DC-DC converter can be derived from this constraint as:

$$u_{sc,i} = \Big(P_{g,i} - P_{r,i} - k_{2,i} \Delta E_{dc0,i} \Big) (R_{sc,i}/V_{sc,i}) + V_{sc,i} \quad (54)$$

X. Case Study

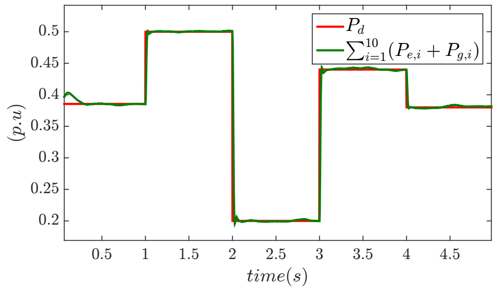
We evaluate the performance of the derived controllers and corresponding consensus protocol and closed-loop interfacing capacitor dynamics on solving the problem of WF power output regulation with fair load-sharing of the storage power outputs. For this purpose, a modified version of the IEEE 24-bus reliability test system is adopted here where, at bus 22, a WF comprised of 10 SoA WGs with supercapacitor energy storage devices is placed. The physical and communication topologies are depicted in Fig. 2a, 2b. The GSCs and DC-DC converters of the WGs are controlled according to the distributed control law (52) and the CLF-based control law (54), respectively. The simulations are conducted under the following scenario.

Scenario 1: The WF power reference P_d varies in a step-wise manner as shown in Fig. 3a.

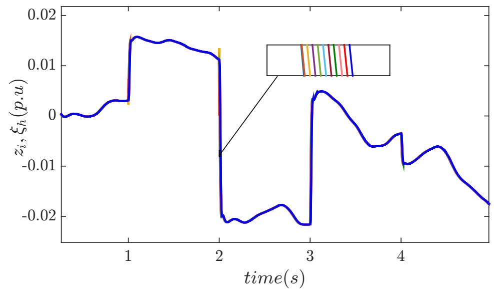
Observe from Fig. 3a that, the distributed controllers for the GSCs and the CLF-based controllers for the storage controllers are able to attain WF power reference tracking with good performance as the total WF power is rapidly and closely tracking the fast-varying reference without exhibiting an overshoot.

Proceeding to Fig. 3b, we realize that the GSCs regulate their power outputs according to the proposed protocol and in response to the reference changes causing in that way their corresponding z_i variables to vary. These variables manifest indistinguishable dynamical responses throughout their trajectories since, at any point on their trajectory (Fig.3b) they rapidly reach consensus and converge to the variable ξ_h which is quasistatic. In the slow time-scale, this variable converges to the equilibrium ξ_{h0} (which depends on P_d) driving the variables $\mathbf{z}$ to the equilibrium ξ_{h0}. Particularly, these slower dynamics are depicted in Fig. 3b where the variables $\mathbf{z}$ and ξ_h are together driven to the quasistatic equilibrium ξ_{h0} while they already reached consensus between each other.

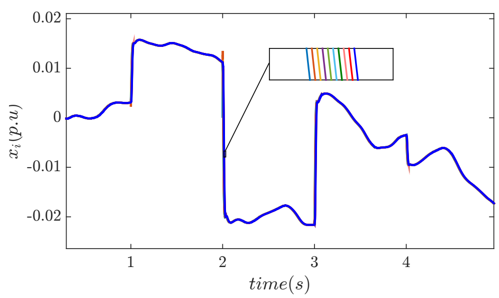
From Fig. 3c, it can be observed that the responses of the variables $\mathbf{x}$ are one-to-one identical to the responses of the variables $\mathbf{z}$ while they are also indistinguishable between them throughout their trajectories. This can be explained as follows. The CLF-based controllers for the DC-DC converters regulate the storage power outputs $\mathbf{x}$ to their corresponding $\mathbf{z}$ in order to continuously meet the power demands of the GSCs, needed to attain WF power output regulation. Through that, the storage power outputs $\mathbf{x}$ eventually reach consensus as well, carrying out in that way the fair load-sharing objective.



(a) Total power output of WF



(b) Power variables (z_i) of all WGs



(c) Storage power outputs (x_i) of all WGs

Fig. 3: Dynamical Response of SoA WGs

In conclusion, the proposed distributed GSC controllers and CLF-based storage controllers effectively attained WF power output regulation with fair utilization of the storage devices.

XI. Concluding Remarks

In this paper, a distributed control methodology for DFIGs with energy storage devices is introduced. These SoA WGs can adopt the proposed methodology to attain WF power output regulation by deploying their storage devices in a fair load-sharing manner. We built our methodology on a consensus protocol and a desired form for the closed-loop capacitor dynamics which are realized by a distributed control law for the GSCs and the DC-DC converters. Further, we performed compositional stability analysis of the closed-loop dynamics by using singular perturbation and Lyapunov theories . Finally, we analytically derived the GSC's and the DC-DC converter's controllers. Their performance as well as the theoretical results are numerically verified via simulations on a modified version of the IEEE 24-bus RTS.

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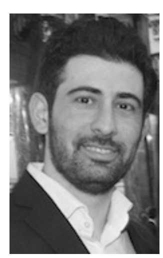

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