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AN INVESTIGATION OF THE USE OF ELECTRONIC JINKING TO ENHANCE AIRCRAFT SURVIVABILITY

by

Mark E. Connelly

ABSTRACT

Moving the apparent aircraft position from wingtip to wingtip by electronic means is given the name "electronic jinking." Properly timed, this technique can be employed to exploit the intrinsic instability of a missile at close ranges and cause a substantial miss. Physical evasive maneuvers by the aircraft are not nearly as effective.

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TABLE OF CONTENTS

LIST OF FIGURES

1. INTRODUCTION

The U.S. Air Force currently places great emphasis on mission oriented design, a concept in which all the functions a system is expected to perform in carrying out its basic mission are considered from the start in the system design. Instead of optimizing performance for any one task or optimizing specific components, the best overall compromise for the entire sequence of tasks associated with a mission is sought. The new approach integrates functions at the system level, subordinating the component designs and the performance in specific phases of a mission to the overall functional requirements of the mission as a whole. As a practical matter, the design process still attempts to achieve the best possible performance in each individual phase of the mission. These task optimums, however, will inevitably be, to some degree, mutually contradictory, and numerous tradeoffs and compromises will have to be made before a satisfactory mission oriented design is produced.

The implication of the new design philosophy is that the process must start with the requirements imposed by the basic tasks that the system is expected to carry out, not with a hardware concept based largely on the capabilities of current technology. One of the major impediments to more effective use of the USAF approach is the lack of a systematic and rational design methodology. At this stage of its development, the process is still empirical, utilizing trial and error and numerous design iterations, hence costly and time consuming. Refinement of the process itself would be a worthwhile objective of research.

Applying this design approach to command and control systems has led us to a consideration of a fundamental issue that arises in all combat, how to inflict damage on the enemy without suffering losses oneself. Specifically, we have addressed the question of the tradeoffs involved in the close air support or interdiction task, where success in destroying enemy assets and the survival of the aircraft carrying out the attack are almost always conflicting objectives. The methods employed for enhancing aircraft survival (jinking, armor, low level approaches and terrain masking, curvilinear attack trajectories, use of decoys and chaff, ECM, and prior destruction of enemy AA and SAM sites by special weapons systems), without exception, interfere with or degrade the ability to inflict damage on the enemy. Elaborate preliminary measures, like chaff or ECM, eliminate the element of surprise. Evasive, low level maneuvers make it more difficult to acquire the target and to establish an accurate fire control solution. Increased speed penalizes endurance. Utilizing protective armor increases aircraft weight and reduces its speed and payload. Thus, even for the seemingly straightforward mission of attacking ground targets, the tradeoff matrix becomes quite complex. As a consequence, we have concentrated in this preliminary study on a single basic issue, i.e., the optimum tactical tradeoff between the ability to hit a ground target and the ability to survive a surface-to-air missile attack from a site defending the target. Specifically, we have attempted to devise a low level attack trajectory that represents the best compromise between accomplishing the mission successfully and surviving. With this solution in hand, one can then study how it reflects on the requirements imposed upon the command and control system. This is the so-called "from the bottom up" approach to system design. The alternative, "from the top down," starts with an overall concept and, by stages, defines the detailed application of that concept to specific subordinate tasks.

2. SURFACE-TO-AIR MISSILE MODEL

A set of typical SAM trajectories from Reference 1 is shown in Figure 1. The simulated missile trajectories are projected on a vertical plane passing through the launching site and parallel to the target aircraft's path. The missile, in this case, is the U.S. Army SAM-D (Patriot) and the simulated aircraft targets are flying at constant altitude directly over the SAM site, except in Case 2, where the path is offset by 21.5 km. The missile flight time varies from 10 seconds for Case 6 to 50 seconds for Cases 0, 3, and 5. Aside from Case 1 and Case 4, which are delayed intercepts, note that the missile approaches the target from above. Cases 5 and 7 are of particular interest, since they involve a low altitude target. Patriot, with a maximum speed Mach 3.9 and a launch weight of about 2200 lbs, is roughly equivalent to the Russian SA-6, which was used so successfully by Egypt in the Yom Kippur War of 1973.

Since we are assuming a low altitude approach, utilizing terrain masking where possible, it is also reasonable to postulate that the SAM radars will not be able to detect, acquire, and track our tactical aircraft at maximum ranges.

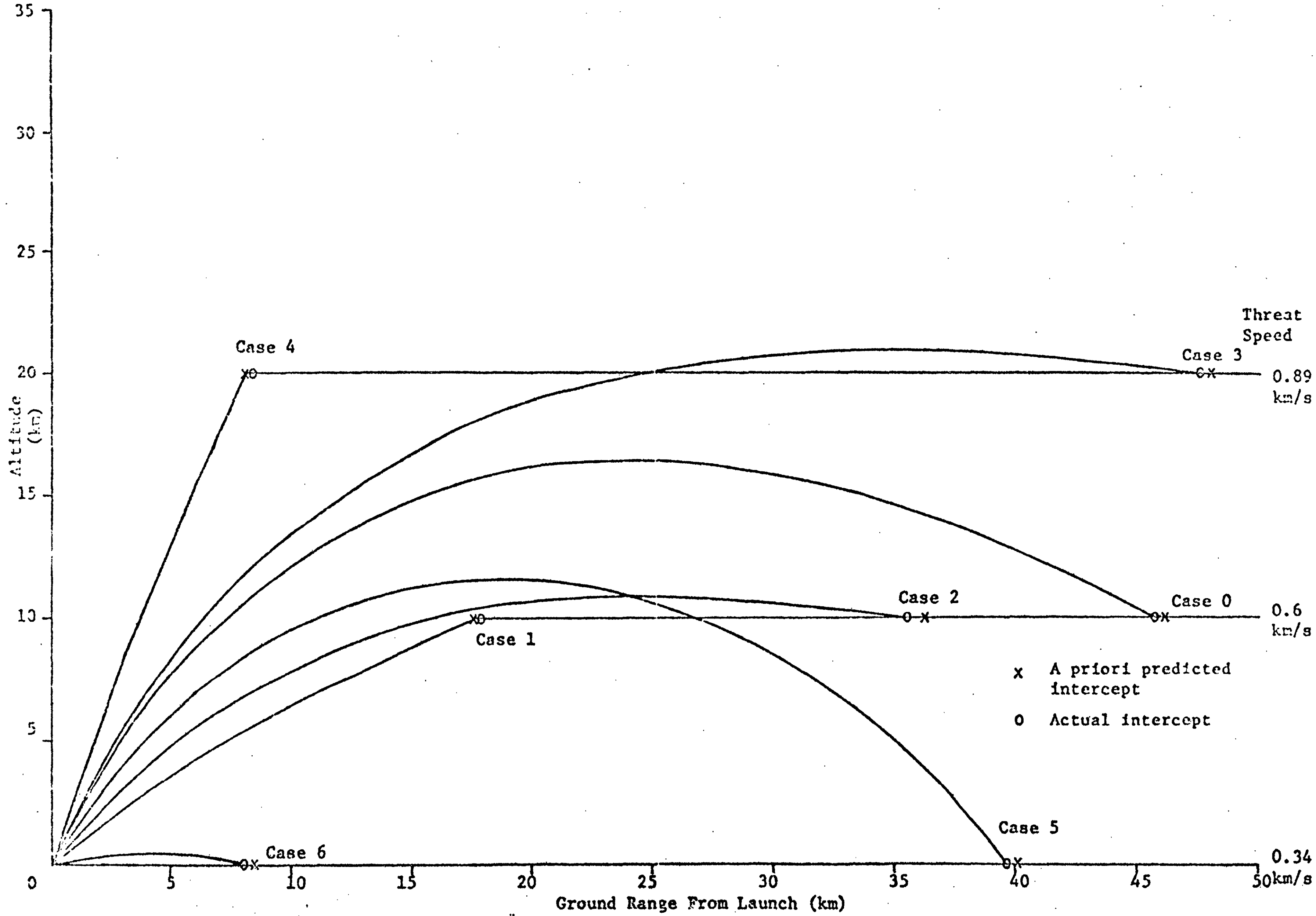


Figure 1. Vertical Profile of Seven SAM Intercepts (Simulated Trajectories)

This implies that the engagement will resemble Case 6 in Figure 1 rather than Case 5. At this relatively short range, we can further assume that both the missile and the aircraft will maneuver approximately in a horizontal plane at constant velocities. Under these conditions, the interception geometry for proportional navigation is shown in Figure 2 (adapted from Reference 2).

The triangle $M_o T_o$ I represents a perfect interception, with impact at point I and with the target position always at a constant bearing ψ_o with respect to the missile velocity vector V_M. It is assumed that deviations from the ideal straightline trajectories, Z_m for the missile and Z_t for the target, are small, so the use of small angle approximation is permissible. These perturbations are perpendicular to the ideal sightline which remains parallel to initial sightline $M_o T_o$. The correct initial flight path angle must satisfy the relation

$$V_m \sin \psi_o = V_t \sin \phi_o$$

The sightline angular perturbation is

$$\tan \theta = \frac{Z_t - Z_M}{r} = \frac{Z_t - Z_M}{V_r \tau} \simeq \theta$$

where the relative velocity V_r is given by

$$V_r = V_M \cos \psi_o - V_t \cos \phi_o$$

and τ is the time-to-go, which is zero when $r = 0$. The miss distance for a non-ideal encounter will be taken as $(Z_t - Z_M)$ at $r = 0$.

In proportional navigation homing, the rate of change of the missile flight path angle $\dot{\psi}$ is proportional to the rate of change of the sightline angle, i.e.,

$$\dot{\psi} = k \, \dot{\theta}$$

Since a constant missile velocity V_M is assumed, this angular rate of change is equivalent to a lateral acceleration

V_R = relative velocity
τ = time-to-go
V_M = Missile velocity
V_t = target velocity

$M(t)$ = Missile position at time t
$T(t)$ = target position at time t
θ = deviation of Sight Line from Ideal
z_M = deviation of Missile Position from Ideal
z_T = deviation of Target Position from Ideal

y_M = Missile lateral displacement
y_T = target lateral displacement

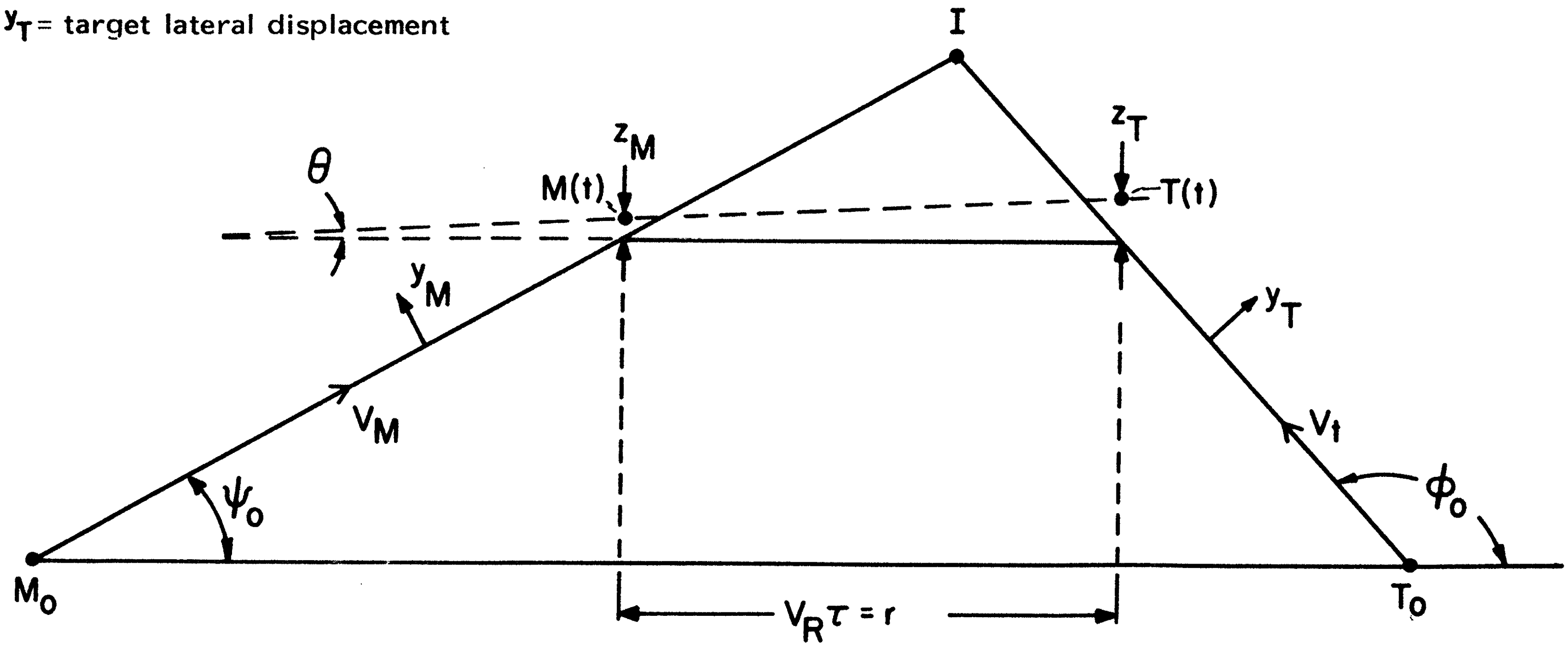


Figure 2. Intercept Geometry in Horizontal Plane Using Proportional Navigation.

$$\ddot{y}_M = \dot{\psi}\, V_M$$

The missile lateral displacement is, therefore

$$y_M = \iint \ddot{y}_M \, dt\, dt = V_M \iint \dot{\psi}\, dt\, dt$$

and the displacement (Z_M) is

$$Z_M = y_M \cos \psi_o$$

Similarly, if we assume a small perturbation (y_τ) of the target perpendicular to the ideal trajectory T_o I, then the target displacement parallel to Z_M is

$$Z_\tau = -y_\tau \cos \phi_o$$

There are two additional dynamic elements in the homing model, the radar seeker, which tracks the target and measures the sightline, and the autopilot, which moves aerodynamic surfaces to produce a lateral acceleration proportional to the rate of change of the sightline angle. Both may be modeled as second order lags with undamped natural frequencies and damping ratios of (ω_H, ρ_H) and (ω_A, ρ_A) respectively. The complete closed loop homing system is shown in Figure 3.

3. INTRINSIC MISSILE INSTABILITY AT CLOSE RANGE

One notes immediately that the open loop gain goes to infinity as the range r approaches zero, hence the loop will be unstable for a short period prior to impact. Furthermore, with similar seeker and autopilot characteristics, all homing systems will have the same behavior at a given time-to-go (τ) if

$$\frac{K\, V_M}{V_R} \cos \psi_o = \text{constant} = \alpha$$

Clearly the parameter α, called the kinematic gain, is the main determinant of the dynamic performance. Presuming that there is some fixed optimum value for this parameter, then it follows that the navigation constant (k) must vary

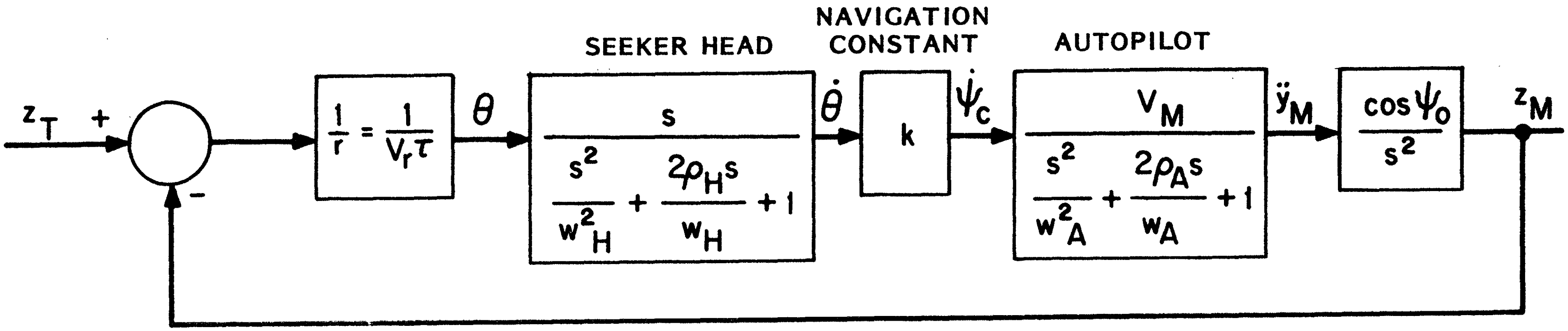


Figure 3. Complete Block Diagram of Proportional Navigation Homing

so that

$$k = \frac{\alpha \, V_R}{V_M \cos \psi_o}$$

Heuristically, one can see that this is sensible, since in a tail chase where the relative velocity (V_R) is low, the navigation constant will be low. Conversely, in a head-on encounter, the high V_R will force k to be high, which is desirable. In most missile systems, α lies in the range 3 to 4. Furthermore, the seeker head is generally limited to ±45° travel, hence ψ_o must be less than 45° and is likely to be much less than this in a typical encounter, i.e., ($.707 < \cos \psi_o < 1$).

Our objective in this analysis is to try to identify functional deficiencies in the SAM performance that the attacking aircraft can exploit to enhance its survivability. In view of the obvious stability problems that the missile design must overcome, the first thought is that the missile's dominant response frequency might be excited by the proper choice of aircraft jinking maneuvers. The principal parameters to be investigated are the amplitude and timing of the jinking action.

4. MODEL SIMPLIFICATION

Before proceeding, we will make one more simplifying assumption in the missile model to the effect that the autopilot bandwidth is much greater than the seeker bandwidth and that the autopilot transfer function can, therefore, be removed from the block diagram shown in Figure 4. The following typical values will be used for the missile:

$\alpha = 3.5$
$\omega_H = 4$ rad/sec
$\rho_H = 0.5$
Limit on lateral acceleration 25g
$V_M = 3348$ ft/sec (Mach 3 at sea level)
$V_R = 893$ ft/sec (Mach 0.8 at sea level)

The speed of sound at sea level is 760.9 mph (1116 ft/sec). With the above parameters, the missile block diagram reduces to the one shown in Figure 4.

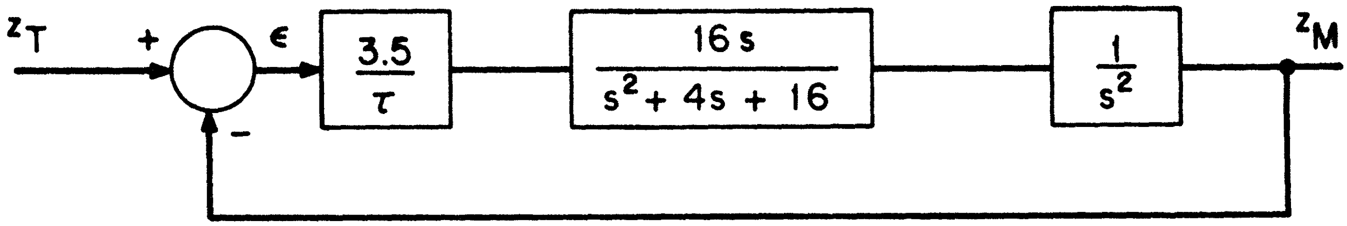


Figure 4. Simplified Block Diagram of Proportional Navigation Homing

The closed loop transfer function is:

$$\frac{Z_M}{Z_T} = \frac{56}{\tau s^3 + 4\tau s^2 + 16\tau s + 56}$$

where τ = time-to-go = T - t, $0 \leq t \leq T$, and

$$T = \frac{r_o}{V_M \cos \psi_o - V_t \cos \phi_o} \qquad \text{(engagement duration)}$$

By the law of sines

$$\frac{V_T\, T}{\sin \psi_o} = \frac{V_M\, T}{\sin (\pi - \phi_o)}$$

$$\sin \psi_o = \frac{V_T}{V_M} \sin (\pi - \phi_o) = \frac{V_\tau}{V_M} \sin \phi_o$$

hence $\cos \psi_o = \sqrt{1 - \sin^2 \psi_o} = \sqrt{1 - \left(\frac{V_T}{V_M}\right)^2 \sin^2 \phi_o}$

so

$$T = \frac{r_o}{V_M \cos \psi_o - V_t \cos \phi_o} = \frac{r_o}{\sqrt{V_M^2 - V_t^2 \sin^2 \phi_o} - V_t \cos \phi_o}$$

The engagement time (T) is completely specified by the initial range (r_o), the angle of approach (ϕ_o), and the missile and target velocities.

5. COMPUTER SIMULATION OF A HEAD-ON ENCOUNTER

The target displacement (Z_T) is, in effect, a forcing function applied to the third order differential equation governing the missile displacement (Z_M), i.e.,

$$Z_T = -y_T \cos \phi_o = \frac{\tau}{56} \frac{d^3 Z_M}{d t^3} + \frac{4\tau}{56} \frac{d^2 Z_M}{d t^2} + \frac{16\tau}{56} \frac{d Z_M}{d t} + Z_M$$

Let us examine the missile performance in response to various target actions for the specific case of a <u>head-on encounter</u>. In this situation

$$\phi_o = 180° \qquad \cos \phi_o = -1$$

$$\psi_o = 0° \qquad \cos \psi_o = 1$$

$$T = \frac{r_o}{V_m + V_t}$$

With an initial range (r_o) of 15 miles, for example, the total encounter time T is 18.7 seconds, so

$$\tau = \text{time-to-go} = 18.7 - t \qquad 0 \leq t \leq 18.7$$

It is instructive to look at the root locus plot for the missile, which is shown in Figure 5. At the start of the engagement, the time-to-go is 18.7 seconds, hence the loop gain at this point is ($56/\tau = 56/18.7 = 3$). The damping factor is satisfactory ($\rho = 0.5$) and the negative real root dominates the response. However, as the intercept proceeds, τ decreases to zero and the loop gain goes to infinity. The negative real root moves far to the left and the complex root locus crosses the imaginary axis at a gain of 64, i.e., ($56/\tau = 64$), hence ($\tau = 56/64 = 0.875$ seconds). Thus, 0.875 seconds before impact, the missile actually becomes <u>unstable</u>, and, just prior to this time, it is <u>underdamped</u> and vulnerable to resonant excitation.

Employing a modified Euler predictor-corrector algorithm with a variable time step (0.01 seconds $\leq \Delta t \leq$ 0.05 seconds) to solve the third order differential equation governing the encounter, we will now evaluate the relative worth

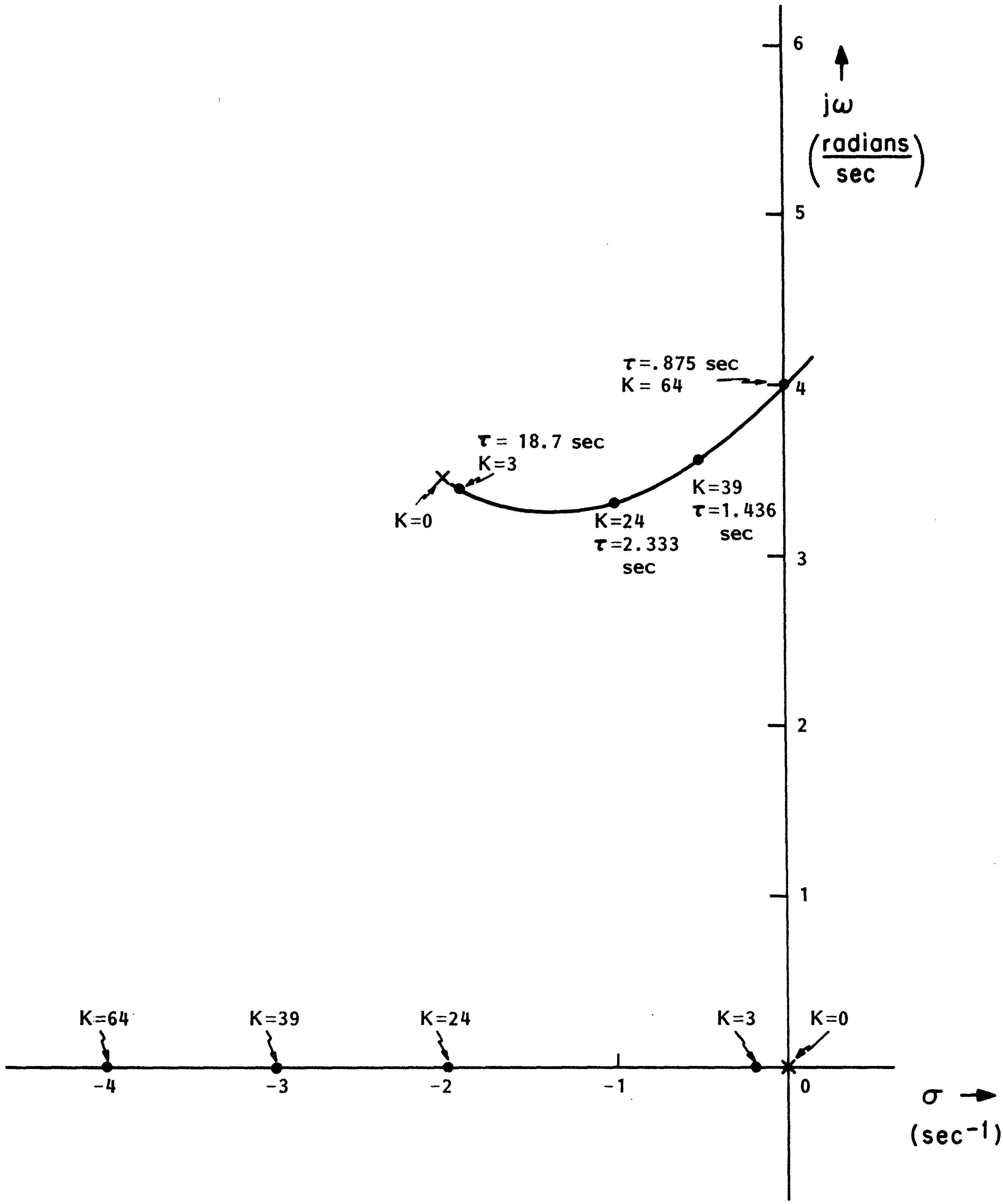


Figure 5. Root Locus for Typical Surface-to-Air Missile

of various target evasive tactics. In the first case, the target is assumed to be offset 20 feet from a straight-line trajectory passing over the missile site. A plot of the missile offset (Z_M) versus time is shown in Figure 6. The initial range is 15 miles and the total encounter time is 18.7 seconds. Quite clearly, a non-maneuvering target has very little chance of surviving an attack by our idealized missile model. With the exception of the second order lag in the seeker head response, the missile model is assumed to execute its proportional nagivation task perfectly. In subsequent encounters, however, we will also place a 25 g acceleration limit on the missile. Numerous other sources of error such as ground clutter, multipath reflections at low altitude, additional dynamic lags, radome refraction, tracking noise, lobbing structure of the target-image interference pattern, speed loss due to drag, aeroelastic effects, imperfect antenna stabilization, and scintillation noise are neglected (Referece 3). In short, the performance of an actual missile will not be as good as the performance of our simplified model. Hence, if we can develop evasive tactics that are effective against a nearly ideal missile, these same tactics should be highly successful against operational missiles.

6. ENCOUNTER WITH EVASIVE TURNS PRIOR TO IMPACT

The most obvious evasive tactic is a sharp turn just prior to missile impact. If constant g is assumed, the forces acting on the turning aircraft are indicated in Figure 7. To prevent change of altitude, the vertical forces must cancel:

$$mg = L \cos \phi$$

In a 2g turn, the lift (L) is twice the aircraft weight (mg), so ($\cos \phi = 0.5$) and the bank angle ϕ is 60°. In a 4g turn, ($\cos \phi = 0.25$) and the bank angle ϕ is 75.5°. For a 6g turn, the bank angle is 80.4°, a somewhat precarious attitude flying close to the ground.

The acceleration in the horizontal place (a_H) is determined by the net horizontal force

$$a_H = \frac{V_T^2}{r} = \frac{L \sin \phi}{m}$$

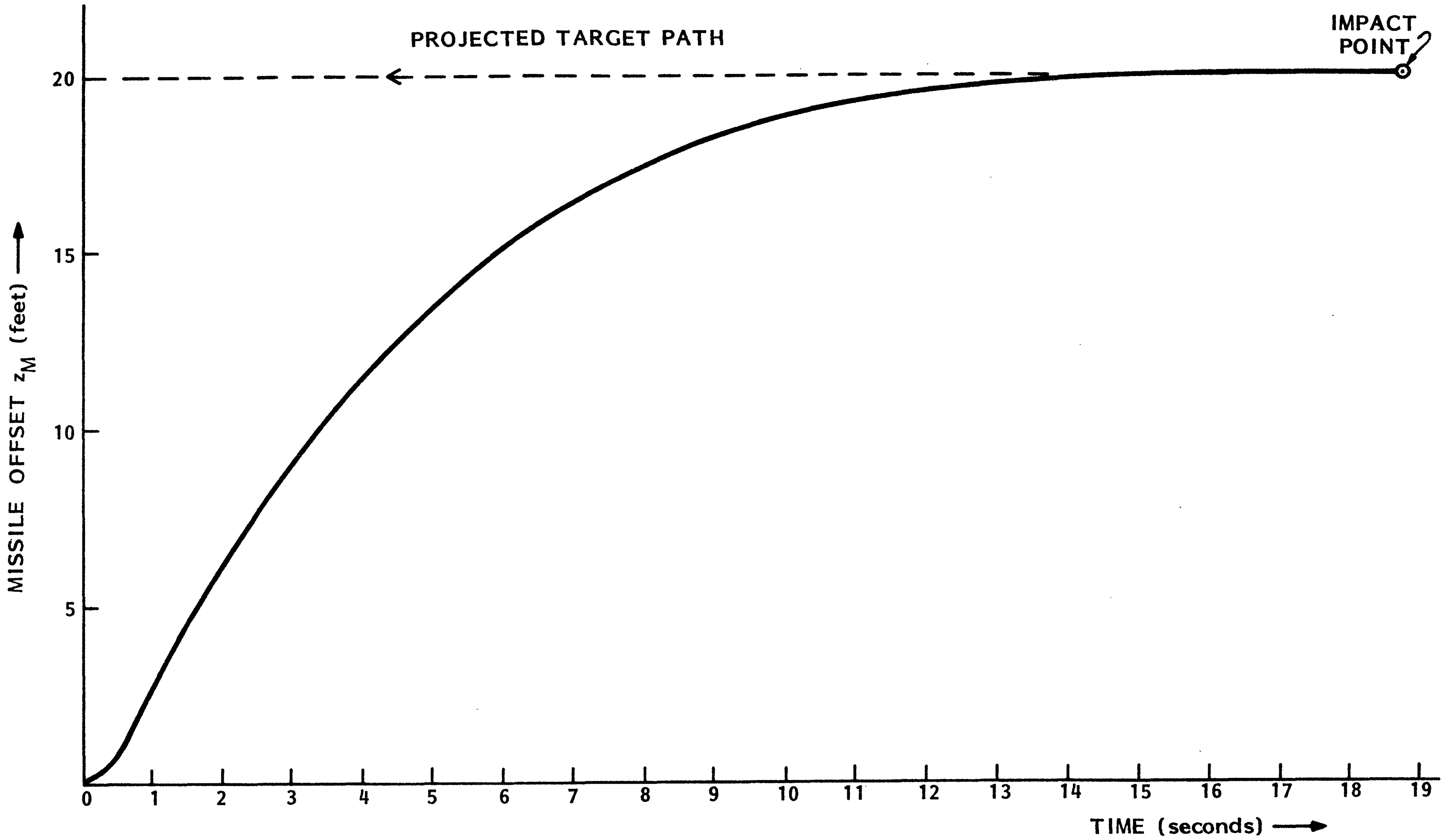


Figure 6. Missile Trajectory for 20 Foot Target Offset

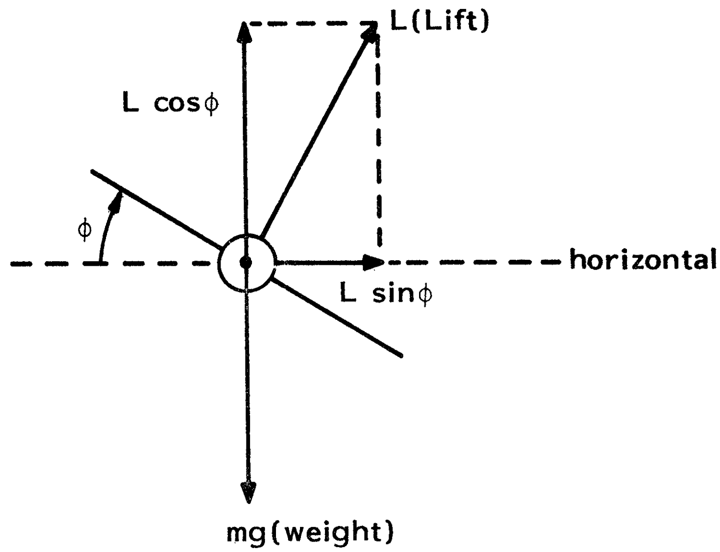
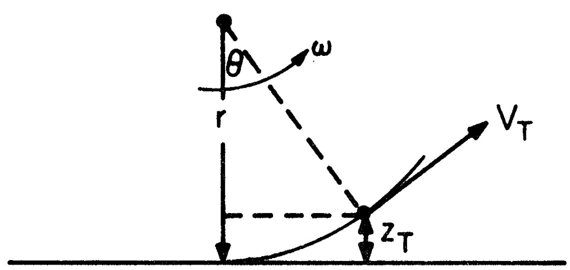


Figure 7. Forces Acting on Turning Aircraft

from which the radius of curvature (r) can be found.

$$r = \frac{m V_T^2}{L \sin \phi} = \frac{V_T^2}{g \tan \phi}$$

The horizontal displacement of an aircraft in a constant g turn for t seconds, therefore, can be derived from the diagram in Figure 8.



$$\theta = \omega t$$

$$Z_T = r\ (1 - \cos \omega t)$$

$$\text{where } \omega = \frac{V_T}{r}$$

Figure 8. Horizontal Displacement of Aircraft

For the assumed target aircraft at Mach 0.8 (V_T = 893 ft/sec at sea level), the following table summarizes these results for a 2g, 4g, and 6g evasive turn.

Table 1. Trajectory Parameters for Various Evasive Turns

Evasive turn	Bank angle ϕ	r Radius of curvature (feet)	Z_T Target displacement	Z_T (feet)		
				1 sec	2 sec	3 sec
2g	60.0°	14,298	14,298 (1 - cos 0.06246t)	27.9	111.4	250.3
4g	75.5°	6,405	6,405 (1 - cos 0.1394t)	62.1	247.3	552.0
6g	80.4°	4,189	4,189 (1 - cos 0.2132t)	94.8	375.1	828.0

Computing the missile trajectories for this set of evasive maneuvers produces the results shown in the table below.

Table 2. Simulation Results for Evasive Turns

Evasive turn	Duration (seconds)	Target Z_T (feet)	Missile Z_M (feet)	Miss distance $(Z_T - Z_M)$ (feet)	Maximum missile g
2g	1	27.9	24.3	3.6	25.0 limit
	2	111.4	113.9	- 2.5	5.9
	3	250.3	250.3	0.0	4.0
4g	1	62.1	52.8	9.3	25.0 limit
	2	247.3	253.0	- 5.7	12.3
	3	552.0	552.0	0.0	8.6
6g	1	94.8	75.2	19.6	25.0 limit
	2	375.1	384.2	- 9.1	18.5
	3	828.0	829.0	- 1.0	12.8

A typical encounter (4g turn for one second) is plotted in Figure 9.

7. EVALUATION OF ELECTRONIC JINKING

It is clear that abrupt evasive turns are <u>not</u> very effective against the near-perfect missile model assumed in this analysis. Even when the evasive

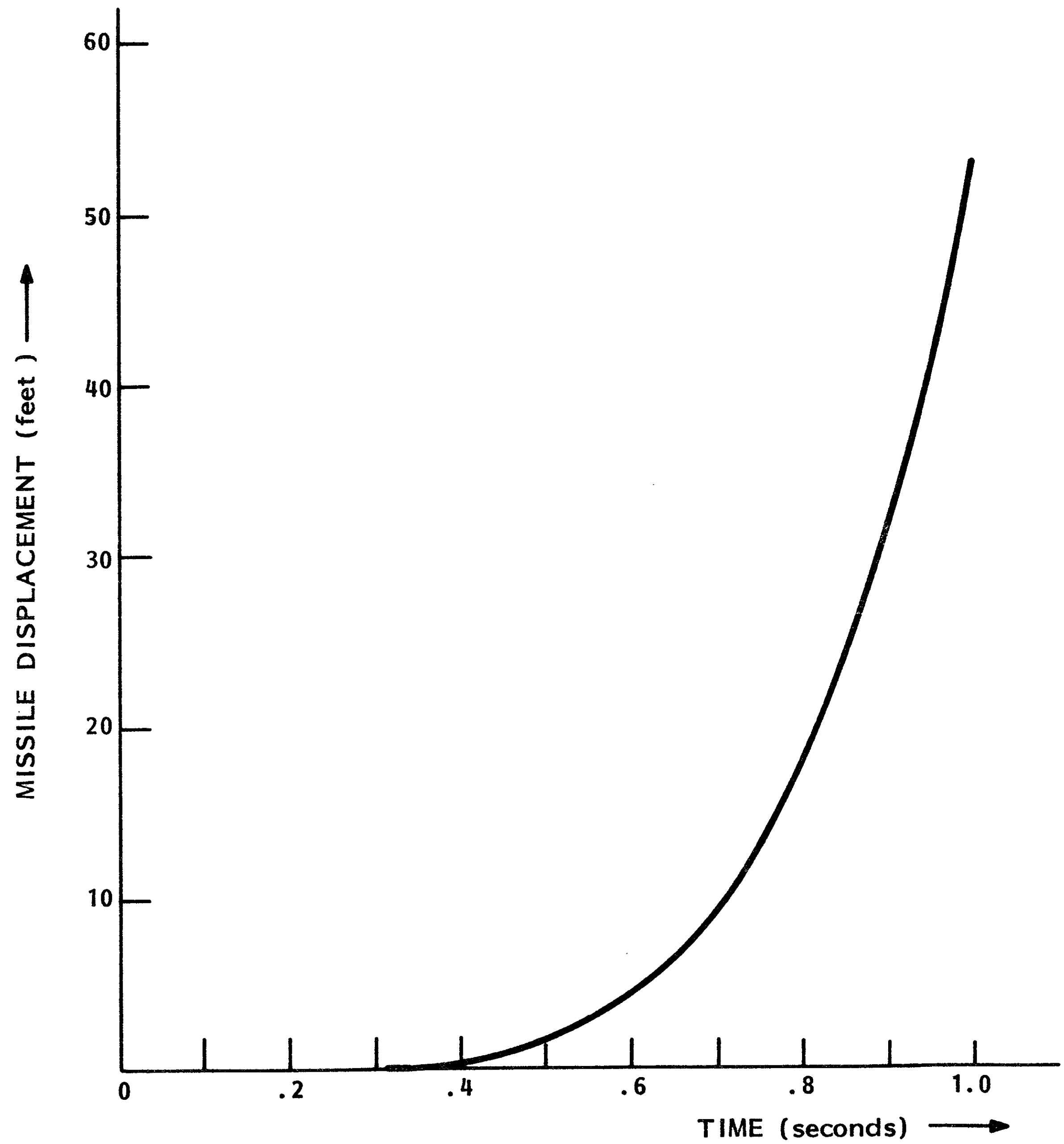


Figure 9. Missile Response to a 4g Evasive Turn Initiated One Second Before Impact

action is optimally initiated about one second prior to impact, the miss distance is typically less than the dimensions of the aircraft. It the turn commences two seconds or more before impact, the missile has sufficient time to alter course and achieve a hit without exceedings its 25g acceleration limit. The large missile advantage in speed and lateral acceleration appears to make the aircraft extremely vulnerable, whether it employs evasive maneuvers or not.

To circumvent the instrinsic limitations on aircraft lateral acceleration, one might consider electronic jinking, i.e., moving the apparent aircraft position from wingtip to wingtip by electronic means. Simply retransmitting the missile radar signal from one wingtip or the other would achieve this effect. Although the amplitude of the jink would be limited to the maximum dimensions of the aircraft, higher frequencies could be achieved and the timing relative to the impact time could be controlled very precisely.

To investigate the potential value of electronic jinking, a series of simulated encounters were run employing the missile model against a square wave target motion. The amplitude of the square wave was fixed at ±20 feet, but the period and timing relative to the impact time were varied. Typical missile responses in these tests are shown in Figures 10, 11, and 12.

The conclusion from this study is that properly timed electronic jinking with a reversal period of about one second can be employed to excite the natural frequencies of the missile and cause it to miss the aircraft by substantial distances. In Figure 10, for example, the missile overshoots the target and is displaced about 75 feet from the aircraft center of gravity at the time of passage. Correct timing of the jinking, however, is essential to achieve a guaranteed miss. To illustrate, a "position" reversal at 6 seconds in Figure 11 (0.4 seconds before impact) does not allow enough time for the overshoot to develop and the missile, consequently, is only a few feet from the aircraft center of gravity at the time of passage. However, if the "position" reversal occurs at 5.6 seconds (0.8 seconds before impact) or at 5.5 seconds (0.9 seconds before impact), then the missile will overshoot and miss the target as indicated in Figure 12. For guaranteed safety, therefore, electronic jinking must have available some measure of the time-to-go before impact. The last "position" reversal should occur no less than 0.8 seconds before impact

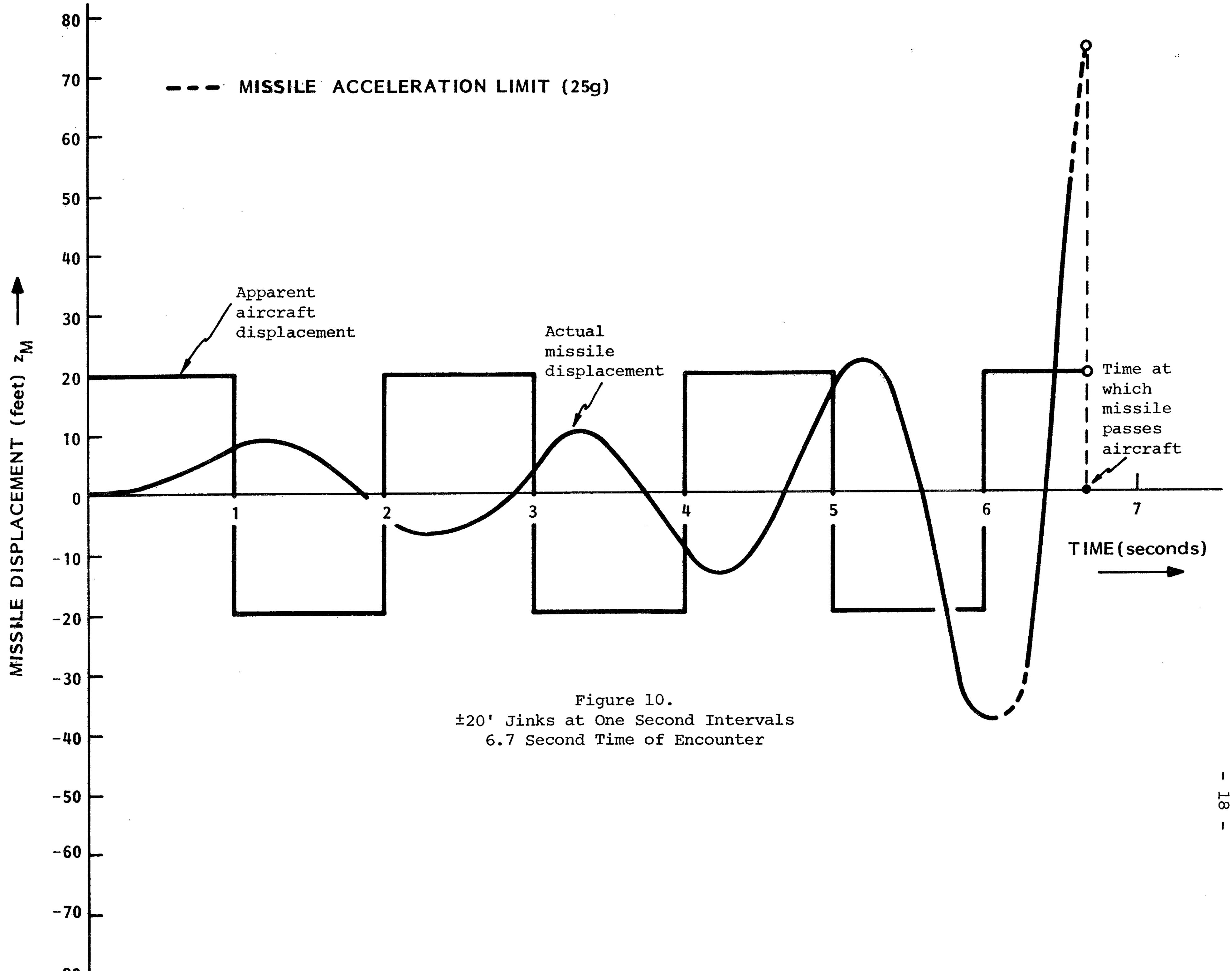


Figure 10.
±20' Jinks at One Second Intervals
6.7 Second Time of Encounter

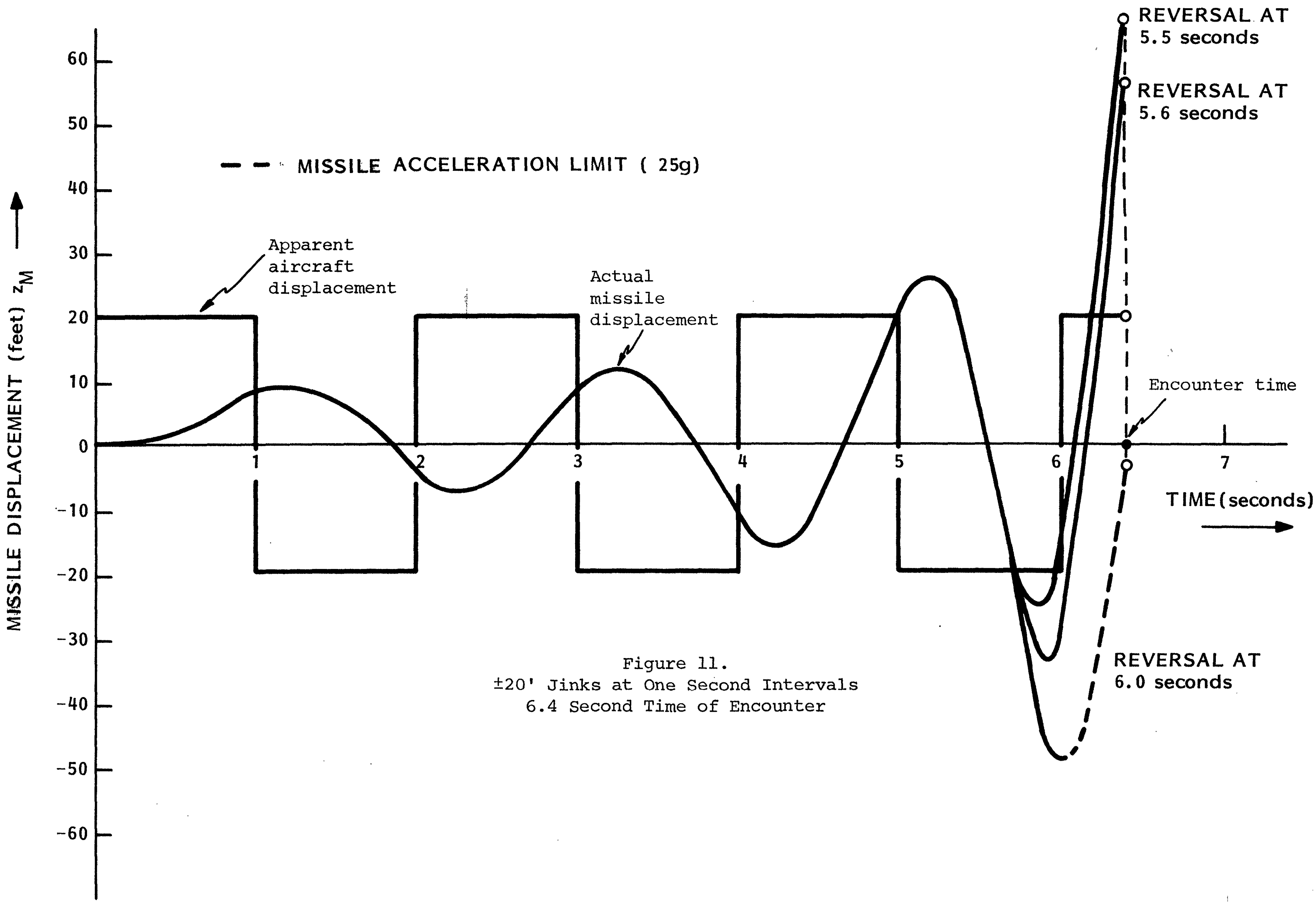


Figure 11.
±20' Jinks at One Second Intervals
6.4 Second Time of Encounter

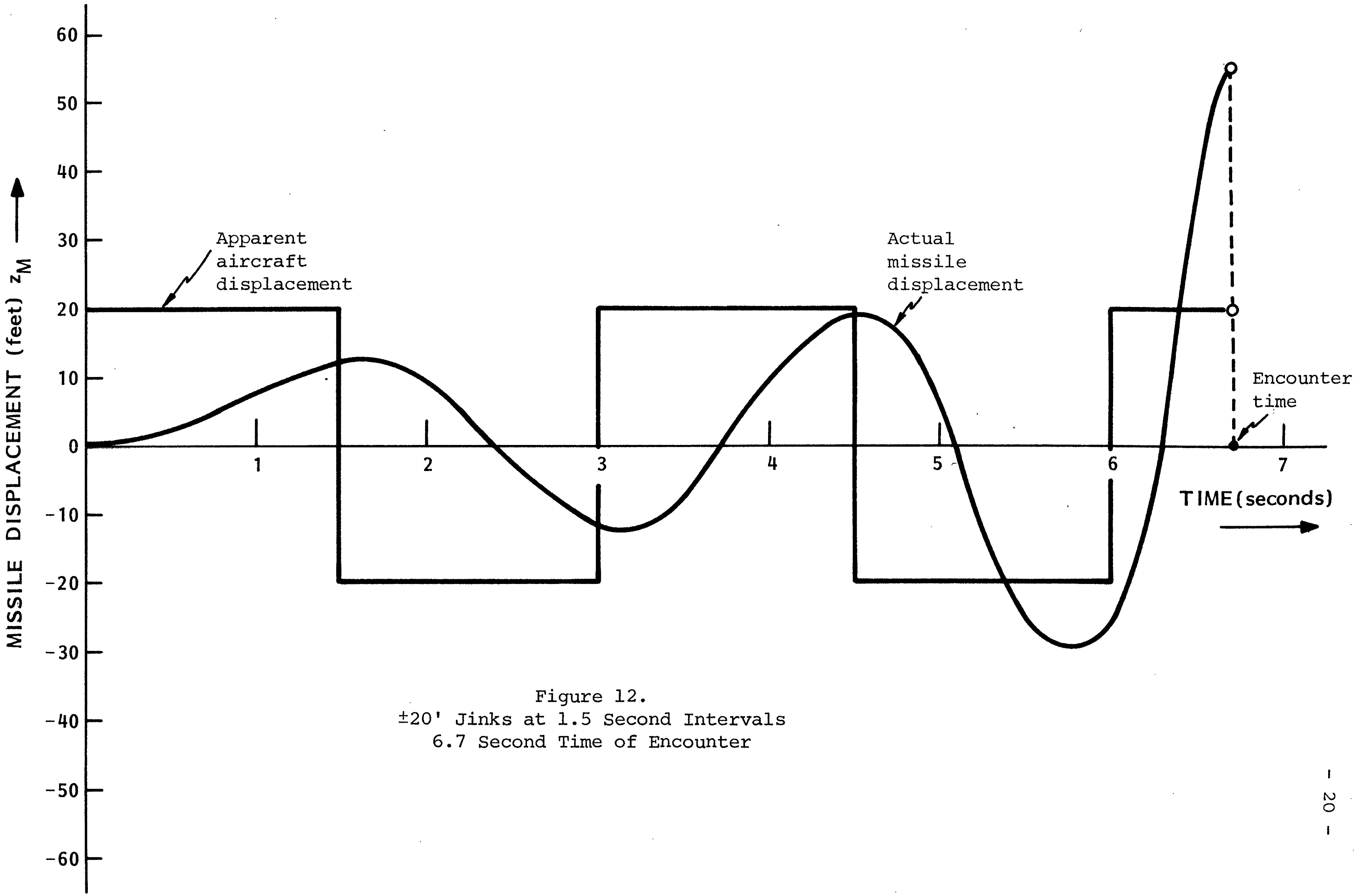


Figure 12.
±20' Jinks at 1.5 Second Intervals
6.7 Second Time of Encounter

to allow enough time for the missile overshoot to develop.

In Figure 12, the reversal period has been increased to 1.5 seconds. This obviously gives the missile more time to adapt to the apparent jink, and reduces the overshoot and the miss distance.

8. ADVANTAGES OF ELECTRONIC JINKING

An important advantage of electronic jinking is that it can be carried out without interfering with the pilot's primary fire control task or the actual trajectory of the aircraft. Conventional jinking, on the other hand, requires high aircraft accelerations and adds to the pilot's workload precisely during the period when he is trying to acquire his target and establish a fire control solution. Electronic jinking, of course, would not be effective against small arms fire from the ground, but the principal reason for flying at low altitudes (where such weapons can be used against aircraft) is to avoid surface-to-air missiles. If electronic jinking proves to be effective against SAM attacks, then aircraft will be able to utilize the higher approach altitudes again where small arms fire is not a great danger. Needless to say, approaching the target at higher altitudes also makes the target acquisition and fire control problem easier.

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