DIVIDEND VARIABILITY AND VARIANCE BOUNDS TESTS FOR THE RATIONALITY OF STOCK MARKET PRICES

Terry A. Marsh
Robert C. Merton

Sloan School of Management
Massachusetts Institute of Technology

Working Paper #1584-84 August 1984

DIVIDEND VARIABILITY AND VARIANCE BOUNDS TESTS FOR THE RATIONALITY OF STOCK MARKET PRICES†

Terry A. Marsh
Robert C. Merton

Sloan School of Management
Massachusetts Institute of Technology

†Dedicated to the scientific contributions and the memory of John V. Lintner, Jr.

I. Introduction*

Perhaps for as long as there has been a stock market, economists have debated whether or not stock prices rationally reflect the "intrinsic" or fundamental values of the underlying companies. At one extreme on this issue is the view expressed in well-known and colorful passages by Keynes that speculative markets are no more than casinos for transferring wealth between the lucky and unlucky. At the other is the Samuelson-Fama Efficient Market Hypothesis that stock prices fully reflect available information and are, therefore, the best estimates of intrinsic values. Robert Shiller has recently entered the debate with a series of empirical studies which claim to show that the volatility of the stock market is too large to be consistent with rationally-determined stock prices. In this paper, we analyze the variance-bound methodology used by Shiller and conclude that this approach cannot be used to test the hypothesis of stock market rationality.

Resolution of the debate over stock market rationality is essentially an empirical matter. Theory may suggest the correct null hypothesis--in this case, that stock market prices are rational--but it cannot tell us whether or not real-world speculative prices as seen on Wall Street or LaSalle Street are indeed rational. As Samuelson (1965, p. 42) wrote in his seminal paper on efficient markets:

> You never get something for nothing. From a nonempirical base of axioms, you never get empirical results. Deductive analysis cannot determine whether the empirical properties of the stochastic model I posit come close to resembling the empirical determinants of today's real-world markets.

On this count, the overwhelming majority of empirical studies report results which are consistent with stock market rationality.[1] There is, for example, considerable evidence that, on average, individual stock prices respond rationally to surprise announcements concerning firm fundamentals, such as dividend and earnings changes, and that prices do not respond to "non-economic" events such as cosmetic changes in accounting techniques. Stock prices are, however, also known to be considerably more volatile than either dividends or accounting earnings. This fact, perhaps more than any other, has led many, both academic economists and practitioners, to the belief that prices must be moved by waves of "speculative" optimism and pessimism beyond what is reasonably justified by the fundamentals.[2]

Until recently, the belief that stock prices exhibit irrationally-high volatility had not been formally tested. In a series of papers (1981a, 1981b, and 1982), Robert Shiller uses seemingly powerful variance bounds tests to show that variations in aggregate stock market prices are much too large to be justified by the variation in subsequent dividend payments.[3] Under the assumption that the expected real return on the market remains essentially constant over time, he concludes that the excess variation in stock prices identified in his tests provides strong evidence against the Efficient Market Hypothesis. Even if the expected real return on the market does change over time, Shiller further concludes that the amount of variation in that rate necessary to "save" the Efficient Market Hypothesis is so large that the measured excess variation in stock prices cannot reasonably be attributed to this source.

We need hardly mention the significance of such a conclusion. If Shiller's rejection of market efficiency is sustained, then serious doubt is cast on the validity of the most important cornerstone of modern financial economic theory. Although often discussed in the context of profit opportunities for the agile and informed investor, the issue of stock market rationality has implications far beyond the narrow one of whether or not some investors can beat the market. As Keynes (1936, p. 151) noted long ago, and as is evident from the modern Q-theory of investment, changes in stock prices--whether rationally determined or not--can have a significant impact on real investment by firms.[4] To reject the Efficient Market Hypothesis for the whole stock market and at the level suggested by Shiller's analysis implies broadly that production decisions based on stock prices will lead to inefficient capital allocations. More generally, if the application of rational expectations theory to the virtually "ideal" conditions provided by the stock market fails, then what confidence can economists have in its application to other areas of economics where there is not a large central market with continuously-quoted prices; where entry to its use is not free; and where shortsales are not feasible transactions?

The strength of Shiller's conclusions is derived from three elements: (i) the apparent robustness of the variance bound methodology; (ii) the length of the data sets used in the tests--one set has over one hundred years of dividend and stock price data; and (iii) the large magnitude of the empirical violation of his upper bound for the volatility of rational stock prices. Shiller in essence relies upon elements (ii) and (iii) to argue that his rejection of the efficient market model cannot be explained away by "mere" sampling error alone,[5] although Flavin (1983) and Kleidon (1983a,b) have shown that such sampling error can have a nontrivial effect on the variance bound test statistics.

In this paper, we focus exclusively on element (i) and conclude that Shiller's variance bound methodology is wholely unreliable for the purpose of testing stock market rationality. Thus, even if his estimates contained no sampling error at all, his findings do not constitute a rejection of the efficient market model. To support our claim, we present an alternative variance bound test which has the feature that observed prices will, <u>of necessity</u>, be judged rational if they fail the Shiller test. That is, if observed stock prices were to satisfy Shiller's variance bound test, then they would be deemed irrational by our test. It would seem, therefore, that for any set of stock market price data, the hypothesis of market rationality can be rejected by some variance bound test.

This seeming paradox arises from differences in assumptions about the underlying stochastic processes used to describe the evolution of dividends and rational stock prices. Affirmative empirical evidence in support of the class of aggregate dividend processes postulated in our variance bound test is presented in Marsh and Merton (1983). The specific model derived and tested in that paper significantly outperforms the univariate autoregressive model associated with the Shiller analysis.

The Shiller variance bound test and our alternative test share in common the null hypothesis that stock prices are rational, but differ as to the assumed stochastic process for dividends. Since Shiller's data sets strongly reject the joint hypothesis of his test and sustain our's, we conclude that his variance bound test results might better be interpreted as an impressive rejection of his model of the dividend process than as a rejection of stock market rationality.

II. On the Reliability of the Dividend Variance Bound Test of Stock Market Rationality

In his June 1981 American Economic Review article, Robert Shiller (p. 434) concludes that:

> measures of stock price volatility over the past century appear to be far too high--five to thirteen times too high--to be attributed to new information about future real dividends if uncertainty about future dividends is measured by the sample standard deviation of real dividends around their long-run exponential path.

In reaching this conclusion, he relies upon a variance bound test--hereafter called the "p* test"--which establishes an upper bound on the variance of the level of detrended real stock prices in terms of the variance of a constructed "ex post rational" detrended and real price series.[6] In this section, we begin with a brief review of the development of his test and then present an alternative variance bound test which actually reverses the direction of the inequality established in the p* test. That is, the upper bound on the variance of rationally-determined stock prices in the Shiller test is shown to be the lower bound on that same variance in the alternative test.

The key assumptions underlying the p* test can be summarized as follows:

(S.1) Stock prices reflect investor beliefs which are rational expectations of future dividends.

(S.2) The "real" (or inflation-adjusted) expected rate of return on the stock market, r, is constant over time.

(S.3) Aggregate real dividends on the stock market, $\{D(t)\}$, can be described by a finite-variance stationary stochastic process with a deterministic exponential trend (or growth rate) which is denoted by g.

To develop the p* test from these assumptions, Shiller defines an ex post rational detrended price per share in the market portfolio at time t :

$$p^*(t) \equiv \sum_{k=0}^{\infty} \eta^{k+1} d(t+k) \quad , \tag{1}$$

where $d(s) \equiv D(s)/(1+g)^{s+1}$ is the detrended dividend paid at the end of period s and $\eta \equiv (1+g)/(1+r)$. $p^*(t)$ is called an ex post (detrended) rational price because it is the present value of actual subsequent (to time t) detrended dividends. If as posited in (S.1), actual stock prices, $\{P(t)\}$, are ex ante rational prices, then it follows from (1) that

$$p(t) = \varepsilon_t[p^*(t)] \quad , \tag{2}$$

for each t where $p(t) \equiv P(t)/(1+g)^t$ is the detrended real stock price per share of the market portfolio at the beginning of period t and ε_t is the expectation operator conditional on all information available to the market as of time t.

If, as Shiller (1981a, p. 422) points out, $p(t)$ is an ex ante rational price, then it is also an optimal forecast of $p^*(t)$. If $p(t)$ is such an optimal forecast, then the forecast error, $u(t) \equiv p^*(t) - p(t)$, should be uncorrelated with $p(t)$. It follows therefore that under this hypothesis, $\mathrm{Var}[p^*(t)] = \mathrm{Var}[p(t)] + \mathrm{Var}[u(t)] > \mathrm{Var}[p(t)]$. That is, in a set of repeated experiments where a forecast $p(t)$ and a sequence of subsequent dividends, $d(t+k)$, $k = 0,1,\ldots,$ are "drawn," it should turn out that the sample variance of $p^*(t)$ exceeds the sample variance of the forecast $p(t)$.

If (detrended) dividends follow a regular stationary process, then rationally-determined (detrended) stock prices must also. Hence, from assumption (S.3), it follows by the Ergodic Theorem that time series ensembles of $\{p(t)\}$ and $\{p^*(t)\}$ can be used to test the "cross-sectional" proposition that $Var[p^*(t)] > Var[p(t)]$.[7]

To compute an estimate of $p^*(t)$ with a finite sample time period, it is, of course, necessary to truncate the summation in (1). If, as Shiller (1981a, p. 425) notes, the time series sample is "long enough," then a reasonable estimate of the variance of $p^*(t)$ can be obtained from that truncated summation. At the point of truncation, a "terminal" value, $p^*(T)$, is assigned which is the average of the detrended stock prices over the sample period. That is,

$$p^*(T) = [\sum_{t=0}^{T-1} p(t)]/T \quad , \tag{3}$$

where T is the number of years in the sample period.

Under the posited conditions (S.1)-(S.3), the null hypothesis of the p^* test for rational stock prices can be written as

$$Var[p^*] \geq Var[p] \quad , \tag{4}$$

where from (1) and (3), the constructed $p^*(t)$ series used to test the hypothesis is given by

$$p^*(t) = \sum_{k=0}^{T-t-1} \eta^{k+1} d(t + k) + \eta^{T-t} p^*(T) \; , \quad t = 0,\ldots,T - 1 \quad . \tag{5}$$

As summarized by Shiller in the paragraph cited at the outset of this section, the results reported in his Table 2 (1981a, p. 431) show that the variance bound in (4) is grossly violated by both his Standard and Poor's 1871-1979 data set and his modified Dow Industrial 1928-1979 data set.

Although widely interpreted as a rejection of stock market rationality (S.1),[8] these findings are more precisely a rejection of the joint hypothesis of (S.1), (S.2), and (S.3). As noted in our Introduction, Shiller

(1981, pp. 430-433) argues that a relaxation of (S.2) to permit a time-varying real discount rate would not produce sufficient additional variation in prices to "explain" the large magnitude of the violation of the derived variance bound. However, even if (S.2) were known to be true, this violation of the bound is not a valid rejection of stock market rationality unless (S.3) is also known to be true. Nevertheless, to some, (S.3) may appear to encompass such a broad class of stochastic processes that any plausible real-world time series of dividends can be well-approximated by some process within its domain.[9] If this were so, then, of course, the p* test, viewed as a test of stock market rationality, would be robust. In fact, however, this test is very sensitive to the posited dividend process. We show this by deriving a variance bound test of rational stock prices that reverses the key inequality (4). While maintaining assumptions (S.1) and (S.2) of the p* test, this alternative test replaces (S.3) with the assumption of a different, but equally-broad, class of dividend processes. As background for the selection of this alternative class, we turn now to discuss some of the issues surrounding dividend policy and the sense in which rational stock prices are a reflection of expected future dividends, this to be followed by the derivation of our test.

If the required expected real rate of return on the firm is constant, then its intrinsic value per share at time t, V(t), is defined to be the present value of the expected future real cash flows of the firm that will be available for distribution to each of the shares currently outstanding. From the well-known accounting identity,[10] it follows that the firm's dividend policy must satisfy the constraint:

$$V(t) = \varepsilon_t[\sum_{k=0}^{\infty} D(t + k)/(1 + r)^{k+1}] \quad . \tag{6}$$

Although management can influence the intrinsic value of its firm by its investment decisions, management has little, if any, control over the stochastic or unanticipated changes in $V(t)$. In sharp contrast, management has sole responsibility for, and control over, the dividends paid by the firm. There are, moreover, no important legal or accounting constraints on dividend policy. Hence, subject only to the constraint given in (6), managers have almost complete discretion and control over the choice of dividend policy.

This constraint on dividend choice is very much like the intertemporal budget constraint on rational consumption choice in the basic lifetime consumption decision problem for an individual. In this analogy, the intrinsic value of the firm, $V(t)$, corresponds to the capitalized permanent income or wealth of the individual, and the dividend policy of the firm corresponds to the consumption policy of the individual. Just as there are an uncountable number of rational consumption plans which satisfy the consumer's budget constraint for a given amount of wealth, so there are an uncountable number of distinct dividend policies which satisfy (6) for a given intrinsic value of the firm. Hence, like rational consumers in selecting their plans, rational managers have a great deal of latitude in their choice of dividend policy.[11]

If stock prices are rationally determined, then

$$P(t) = V(t) \quad \text{for all} \quad t \quad . \tag{7}$$

Hence, the only reason for a change in rational stock price is a change in intrinsic value. Since a manager can choose any number of different dividend policies which are consistent with a particular intrinsic value of the firm, the statement that "rational stock prices reflect expected future dividends" needs careful interpretation. It follows from (6) and (7) that rational stock prices will satisfy:

$$(8) \qquad P(t) = \varepsilon_t[\sum_{k=0}^{\infty} D(t+k)/(1+r)^{k+1}] \qquad .$$

Thus, rational stock price reflects expected future dividends through (8) in the same sense that an individual's current wealth reflects his expected future consumption through the budget constraint. Pursuing the analogy further: if because of an exogeneous event (for example, a change in preferences), a consumer changes his planned pattern of consumption, then it surely does not follow from the budget constraint that this change in the expected future time path of his consumption will cause his current wealth to change. Just so, it does not follow from (8) that a change in dividend policy by managers will cause a change in the current rationally-determined prices of their shares.[12] For a fixed discount rate, r, it does however follow from (8) that an unanticipated change in a rationally-determined stock price must necessarily cause a change in expected future dividends, and this is so for the same feasibility reason that with a constant discount rate, an unanticipated change in a consumer's wealth must necessarily cause a change in his planned future consumption. In short, (8) is a constraint on future dividends and not on current rational stock price.

Since management's choice of dividend policy clearly affects the time series variation in observed dividends, the development of the relation between the volatility of dividends and rational stock prices requires analysis of the linkage between the largely-controllable dividend process and the largely-uncontrollable process for intrinsic value.

Unlike the theory of consumer choice, there is no generally accepted theory of optimal dividend policy.[13] Empirical researchers have, therefore, relied on positive theories of dividend policy to specify their models. The prototype for these models is the Lintner model (1956) based on stylized facts first established by him in a classic set of interviews of managers about their dividend policies. Briefly, these facts are: (L.1) Managers believe that their firms should have some long-term target payout ratio; (L.2) In setting dividends, they focus on the change in existing payouts and not on the level; (L.3) A major unanticipated and nontransitory change in earnings would be an important reason to change dividends; (L.4) Most managers try to avoid making changes in dividends which stand a good chance of having to be reversed within the near future. In summary, managers set the dividends that their firms pay to have a target payout ratio as a long-run objective, and they choose policies which smooth the time path of the changes in dividends required to meet that objective.

As most textbook discussions seem to agree, these target payout ratios are measured in terms of long-run sustainable ("permanent") earnings rather than current earnings. In the special case where the firm's cost of capital r is constant in real terms, real permanent earnings at time t, $E(t)$, are related to the firm's intrinsic value by $E(t) = rV(t)$.

With this as background, we now develop a model of the dividend process as an alternative to the p^* test's (S.3) process. A class of dividend policies which captures the behavior described in the Lintner interviews is given by the rule:

$$\Delta D(t) = gD(t) + \sum_{k=0}^{N} \gamma_k[\Delta E(t-k) - gE(t-k)] \quad , \tag{9}$$

where Δ is the usual forward difference operator and it is assumed that

$\gamma_k \geq 0$ for all $k = 0,1,\ldots,N$. In words, managers set dividends to grow at rate g, but deviate from this long-run growth path in response to changes in permanent earnings that deviate from their long-run growth path. Describing the policies in terms of the change in dividends rather than the levels, and having these changes depend on changes in permanent earnings, is motivated by Lintner's stylized facts (L.2) and (L.3). His behavioral fact (L.4) is met in (9) by specifying the change in dividends as a moving average of current and past changes in permanent earnings over the previous N periods.

Equation (9) can be rewritten in terms of detrended real dividends and permanent earnings as:

$$(10) \qquad \Delta d(t) = \sum_{k=0}^{\infty} \lambda_k \, \Delta e(t-k) \quad .$$

where $e(s) \equiv E(s)/(1+g)^s$ and $\lambda_k \equiv \gamma_k/(1+g)^{k+1}$. By integrating (10),[14] we can express the level of detrended dividends at time t in terms of current and past detrended permanent earnings as:

$$(11) \qquad d(t) = \sum_{k=0}^{N} \lambda_k e(t-k) \quad .$$

By inspection of (11), the dividend policies in (9) satisfy Lintner's (L.1) condition of a long-run target payout ratio where this ratio is given by $\rho \equiv \sum_{0}^{N} \lambda_k$.

Consider an economy in which the p^* test assumptions (S.1) and (S.2) <u>are known</u> to hold, but instead of (S.3), assume that (9) describes the stochastic process for aggregate real dividends on the market portfolio. From the assumption of a constant discount rate (S.2) and the definition of permanent earnings, we have from (11) that detrended real dividends at time t can be written as:

$$(12)\qquad d(t) = r \sum_{k=0}^{N} \lambda_k v(t - k)$$

$$= r\delta \sum_{k=0}^{N} \theta_k v(t - k)$$

where $v(s) \equiv V(s)/(1 + g)^s$ is the detrended real intrinsic value of the firm at time s and $\theta_k \equiv \lambda_k/\delta \geq 0$ with $\sum_{0}^{N} \theta_k = 1$.

From (S.1), stock prices are known to be rationally-determined, and therefore, it follows from (7) that $p(t) = v(t)$ for all t. Hence, from (12), current detrended dividends can be expressed as a function of current and past detrended stock prices: Namely,:

$$(13)\qquad d(t) = \rho \sum_{k=0}^{N} \theta_k p(t - k) \qquad ,$$

where $\rho \equiv r\delta$ is the long-run or steady-state dividend-to-price ratio on the market portfolio.[15] Thus, from (S.1), (S.2), and (9), detrended aggregate real dividends are a moving average of current and past detrended real stock prices. Moreover, under these posited conditions, the ex-post rational price series constructed for the sample period $[0,T]$ can be expressed as a convex combination of the observed detrended stock prices, $p(t)$, $t = -N,\ldots,0,1,\ldots,T - 1$. That is, from (3) and (13), (5) can be rewritten as:

$$(14)\qquad p^*(t) = \sum_{k=-N}^{T-1} w_{tk} p(k) , \qquad t = 0,1,\ldots,T - 1 \qquad ,$$

where as is shown in the Appendix, the derived weights satisfy

$$\sum_{k=-N}^{T-1} w_{tk} = 1; \quad \sum_{t=0}^{T-1} w_{tk}^2 \leq 1 + (\sum_{t=0}^{T-1} w_{tk})^2/T; \quad \text{and} \quad w_{tk} \geq 0 \quad \text{with}$$

$w_{tk} = 0$ for $k < t - N$.

<u>Theorem I</u>: If, for each t, $p^*(t) = \sum_{k=0}^{T-1} \pi_{tk} p(k)$ where

$$\sum_{k=0}^{T-1} \pi_{tk} = 1; \quad \sum_{t=0}^{T-1} \pi_{tk}^2 \leq 1 + (\sum_{t=0}^{T-1} \pi_{tk})^2/T; \quad \text{and} \quad \pi_{tk} \geq 0,$$

then <u>for each and every sample path</u> of stock price realizations, $\mathrm{Var}(p^*) \leq \mathrm{Var}(p)$, with equality holding if and only if all realized prices are identical in the sample $t = 0,\ldots,T - 1$.

The formal proof is in the Appendix. However, a brief intuitive explanation of the theorem is as follows: For each t, $t = 0,\ldots,T - 1$, $p^*(t)$ is formally similar to a conditional expectation of a random variable p with possible outcomes $p(0),\ldots,p(T - 1)$ where the $\{\pi_{tk}\}$ are interpreted as conditional probabilities. $\mathrm{Var}(p^*)$ is, therefore, similar to the variance of the conditional expectations of p which is always strictly less than the variance of p itself (unless, of course, $\mathrm{Var}(p) = 0$).

The variance inequality in Theorem I is the exact opposite of inequality (4) which holds that $\mathrm{Var}(p^*) \geq \mathrm{Var}(p)$. That is, if the ex post rational price series satisfies the hypothesized conditions of Theorem I, then the p^* test inequality will be violated whether or not actual stock prices are ex ante rational. Because Theorem I applies to each and every time path of prices, its derived inequality $\mathrm{Var}(p^*) \leq \mathrm{Var}(p)$ holds <u>in-sample</u>. A fortiori, it will obtain for any distribution of prices. Thus, even for a "bad draw," $\mathrm{Var}(p^*)$ will not exceed $\mathrm{Var}(p)$.

Although the inequality in Theorem I is an analytic result, it does not strictly hold for all possible sample paths of the $p^*(t)$ series generated by the dividend process (9) and rational stock prices. By inspection of (3) and (14), for each t, $N \leq t \leq T$, $p^*(t)$ is a convex combination of the sample stock prices $\{p(0),\ldots,p(T-1)\}$ that satisfies the hypothesized conditions of Theorem I. However, for $0 \leq t \leq N-1$, $p^*(t)$ will depend upon both the sample period's stock prices and one or more "out-of-sample" stock prices $\{p(-N),\ldots,p(-1)\}$. Hence, with the exception of one member of the class of processes given by (9),[16] $Var(p^*) \leq Var(p)$ need not obtain for each and every sample path of prices.[17] The problem created here by out-of-sample prices is similar to the general "start-up" problem in using a finite sample to estimate a moving average or distributed lag process. Because only the first N of the T sample elements in the p^* series depend on out-of-sample prices, the influence of these prices on the sample variance of p^* becomes progressively smaller as the length of the sample period is increased. Indeed, as proved in the Appendix, we have that:

> Theorem II: If (S.1) and (S.2) hold and if the process for aggregate real dividends is given by (9), then in the limit as $T/N \to \infty$, $Var(p^*)/Var(p) \leq 1$ will hold almost certainly.

As noted in the Introduction, the Shiller variance bound theorem has been widely interpreted as a test of stock market rationality. However, as with Theorem I, Theorem II concludes that $Var(p^*)$ is a lower bound on $Var(p)$ whereas, the corresponding Shiller theorem concludes that $Var(p^*)$ is an upper bound on $Var(p)$. Both Theorem II and the Shiller theorem are mathematically correct and both share in common the hypothesis (S.1) that stock prices are rationally determined. Therefore, if these variance bound theorems are interpreted as tests of stock market rationality, then we have the empirical paradox that this hypothesis can always be rejected. That is, if observed stock prices were to satisfy the p* test of stock market rationality, then this same sample of prices must fail our test, and conversely. This finding alone casts considerable doubt on the reliability of such variance bound theorems as tests of stock market rationality.

The apparent empirical paradox is, of course, resolved by recognizing that each of the variance bound theorems provides a test of a different joint hypothesis. In addition to (S.1), both theorems also assume that the real discount rate is constant. Hence, neither (S.1) nor (S.2) of the respective joint hypotheses is the source of each theorem's contradictory conclusion to the other.[18] It therefore follows necessarily, that the class of aggregate dividend processes (9) postulated in Theorem II is incompatible with the Shiller theorem assumption (S.3) of a regular stationary process for detrended aggregate dividends.[19] That is, given that (S.1) and (S.2) hold, nonstationarity of the dividend process is a necessary condition for the validity of Theorem II[20] whereas stationarity of the dividend process is a sufficient condition for the validity of the p* test inequality (4). Thus, the diametrically-opposite conclusions of these variance bound theorems follow directly from the differences in their posited dividend processes.

In this light, it seems to us that if the p* test is to be interpreted as a test of any single element of its joint hypothesis, (S.1), (S.2) and (S.3), then it is more appropriately viewed as a test of (S.3) than of (S.1). Viewed in this way, the previously-cited empirical findings of a large violation of inequality (4) would appear to provide a rather impressive rejection of the hypothesis that aggregate real dividends follow a stationary stochastic process with a trend. As noted, Shiller has argued extensively that his results are empirically robust with respect to assumption (S.2). In a parallel fashion, we would argue that they are also robust with respect to (S.1). That is, even if stock prices were irrationally volatile, the amount of irrationality required to "save" the stationarity hypothesis (S.3) is so large that the measured five-to-thirteen times excess variation in stock prices cannot reasonably be attributed to this source.

Perhaps the p* test might still be saved as a test of stock market rationality if there were compelling a priori economic reasons or empirical evidence to support a strong prior belief that aggregate dividends follow a stationary process with a trend. We are, however, unaware of any strong theoretical or empirical foundation for this belief. Indeed, the standard models in the theoretical and empirical literature of both financial economics and accounting assume that stock prices, earnings, and dividends are described by nonstationary processes.[21] In his analyses of the Shiller and other variance bounds tests, Kleidon (1983a,b) uses regression and other time series methods to show that the hypothesis of stationarity for the aggregate Standard and Poor's 500 stock price, earnings, and dividend series can be rejected.

Marsh and Merton (1983) develop and test an aggregate dividend model based on the same Lintner stylized facts used to motivate (9) here. In this model, the dividend-to-price ratio follows a stationary process, but both the dividend and stock price processes are themselves nonstationary. This model is shown to significantly outperform empirically the univariate autoregressive model (with a trend) normally associated with a stationary process. These results not only cast further doubt on the stationarity assumption, but also provide affirmative evidence in support of the class of dividend processes hypothesized in Theorems I and II.

The Marsh and Merton model can also be used to reinterpret other related empirical findings which purport to show that stock prices are too volatile. For example, to provide a more-visual (if less-quantitatively precise) representation of the "excess volatility" of stock prices, Shiller (1981a, p. 422) plots the time series of the levels of actual detrended stock prices and the constructed ex post rational prices, $p^*(t)$. By inspection of these plots, it is readily apparent that $p(t)$ is more volatile than $p^*(t)$. Instead of implying "too much" stock price volatility, these plots can be interpreted as implying that the p^* series has "too little" volatility to be consistent with a dividend process which is not smoothed. They are however, entirely consistent with rational and nonstationary stock prices and dividend policies like (9) which smooth the dividend process.

It also appears in these plots that the levels of actual prices "revert" toward the p^* trend line. In the context of (14), this apparent correspondence in trend should not be surprising since $p^*(t)$ is in effect a weighted sum of <u>future</u> actual prices which were, of course, not known to investors at time t. The ex post "mistakes" in forecasts of these future prices by the market at time t are, thus, "corrected" when the subsequent "right" prices (which were already contained in $p^*(t)$) are revealed.[22]

In his latest published remarks on the plots of these time series, Shiller (1983, p. 237) concludes:

> The near-total lack of correspondence, except for trend, between the aggregate stock price and its <u>ex post</u> rational counterpart (as shown in Figure 1 of my 1981a paper) means that essentially no observed movements in aggregate dividends were ever correctly forecast by movements in aggregate stock prices!

This conclusion does not, however, appear to conform to the empirical facts. As shown in Marsh and Merton (1983), the single variable which provides, by far, the most significant and robust forecasting power of the subsequent year's change in aggregate dividends is the previous year's unanticipated change in aggregate stock price.[23]

Shiller (1981a, pp. 425-427) presents a second variance bound test of rational stock prices which uses the time series of "price innovations" which he denotes by $\delta p(t) \equiv p(t) - p(t-1) + d(t-1) - \rho p(t-1)$. Under the assumption that detrended dividends have a stationary distribution, he derives as a condition for rational stock prices that

$$\text{(15)} \qquad \mathrm{Var}(d) \geq \mathrm{Var}(\delta p)[(1+\rho)^2 - 1] \quad ,$$

where $\mathrm{Var}(d)$ and $\mathrm{Var}(\delta p)$ denote the sample variances of the level of detrended dividends and the innovations of price changes, respectively. As reported in Shiller's cited Table 2, the null hypothesis of rational stock prices seems, once again, to be grossly violated by both his data sets.

If, however, dividends are generated by a process like (9) and rational stock prices follow a nonstationary process, then the inequality (15) is no longer valid. If, for example, $D(t) = \rho P(t)$ and stock prices follow a geometric Brownian motion with a variance rate given by σ^2, then

$$\text{(16)} \qquad \varepsilon_0[\mathrm{Var}(d)] \leq \varepsilon_0[\mathrm{Var}(\delta p)]\rho^2/\sigma^2 \quad .$$

Although inequalities (15) and (16) are not mutually inconsistent for all parameter values, using the $\hat{\sigma} = 0.176$ and $\hat{\rho} = 0.048$ reported by Shiller for his 1871-1979 S&P data set, we have that (15) implies that $\mathrm{Var}(d) \geq .0983\ \mathrm{Var}(\delta p)$ whereas (16) implies that $\mathrm{Var}(d) \leq .0744\ \mathrm{Var}(\delta p)$. Moreover, (16) was derived under the polar case of (9) where $N = 0$. If, as empirically seems to be the case, managers select "averaging" dividend policies with $N > 0$, then the effect of such dividend smoothing is to reduce Var(d), but not to reduce Var(δp). Thus, an empirical finding that Var(d) << .0983 Var(δp) --while inconsistent with the stationarity assumption (S.3)--is entirely consistent with rational stock prices and the aggregate dividend process (9).

We are not alone in questioning the specification of the dividend process in the Shiller model. In addition to the cited Kleidon analysis, Copeland (1983) has commented on the assumption of a deterministic trend. In his reply to Copeland, Shiller (1983, p. 236) had this to say on the specification issue:[24] "Of course, we do not literally believe with certainty all the assumptions in the model which are the basis of testing. I did not intend to assert in the paper that I know dividends were indeed stationary around the historical trend." We have shown, however, that variance bound inequality (4) is critically sensitive to the assumption of a stationary process for aggregate dividends. If aggregate dividend policy is described by a smoothing or averaging of intrinsic values which follow a nonstationary process, then the misspecification of stationarity in the dividend process does not just weaken the power of this bound as a test of stock market rationality--it destroys it--because in that case the fundamental inequality is exactly reversed.

In summary, the story that dividends follow a stationary process with a trend leads to the empirical conclusion that aggregate stock prices are grossly irrational. It has, therefore, the deep and wide-ranging implications for economic theory and policy that follow from this conclusion. The vast majority of empirical tests of the efficient market theory do not, however, concur with this finding. Hence, to accept this dividend story, we must further conclude that the methodologies of these tests were sufficiently flawed that they failed to reject this hypothesis in spite of the implied substantial irrationality in stock prices. Similar flaws must also be ascribed to the extensive studies in finance and accounting that claim to show earnings, dividends, and stock prices follow nonstationary processes. If, however, this dividend story is rejected, then the empirical violation of inequality (4) implies nothing at all about stock market rationality. In the spirit of Leamer's (1983) discussion of hypothesis testing, we therefore conclude that the Shiller variance bound theorem is a wholely unreliable test of stock market rationality because "...there are assumptions within the set under consideration that lead to radically different conclusions." (p. 38).

III. Overview and Conclusion

In the previously-cited reply to Copeland, Shiller (1983, p. 237) proclaims:

> The challenge for advocates of the efficient markets model is to tell a convincing story which is consistent both with observed trendiness of dividends for a century and with the high volatility of stock prices. They can certainly tell a story which is within the realm of possibility, but it is hard to see how they could come up with inspiring evidence for the model.

We believe that the theoretical and empirical analysis presented here provides such "inspiring evidence."

Economists have long known that fluctuations in stock prices are considerably larger than the fluctuations in aggregate consumption, national income, the money supply, and many other similar variables whose expected future values presumably play a part in the rational determination of stock prices. Indeed, as noted in the Introduction, we suspect that the sympathetic view held by some economists toward the proposition of excess stock market volatility can largely be traced to this long-established observation. Those who make this inference implicitly assume that the level of variability observed in these economic variables provides the appropriate frame of reference from which to judge the rationality of observed stock price volatility. Although quantitatively more precise, the Shiller analysis adopts this same perspective when it asks: "If stock prices are rational, then why are they so volatile (relative to dividends)?" The apparent answer is that stock prices are not rational.

Our analysis turns this perspective "on its head" by asking: "If stock prices are rational, then why do dividends exhibit so little volatility (relative to stock prices)?" Our answer is simply that managers choose dividend policies so as to smooth the effect of changes in intrinsic values (and hence, rational stock prices) on the change in dividends. The a priori economic arguments and empirical support presented for this conclusion surely need no repeating. We would note, however, that this explanation is likely to also apply to the time series of other economic flow variables. There are, for example, good economic reasons for believing that aggregate accounting earnings, investment, and consumption have in common with dividends that their changes are smoothed either by the behavior of the economic agents that control them or by the statistical methods which are used to measure them. An initial examination of the data appears to support this belief. If a thorough empirical evaluation confirms this finding, then our analysis casts doubt in general over the use of volatility comparisons between stock prices and economic variables which are not also speculative prices, as a methodology to test stock market rationality.

In summary of our view of the current state of the debate over the efficient market theory, Paul Samuelson said it well when he addressed the practicing investment managers of the financial community a decade ago (1974):

> Indeed, to reveal my bias, the ball is in the court of the practical men: it is the turn of the Mountain to take a first step toward the theoretical Mohammed ... If you oversimplify the debate, it can be put in the form of the question,
>
> > Resolved, that the best of money managers cannot be demonstrated to be able to deliver the goods of superior portfolio-selection performance.
>
> Any jury that reviews the evidence, and there is a great deal of relevant evidence, must at least come out with the Scottish verdict:
>
> > Superior investment performance is unproved.

Just so, our evidence does not prove that the market is efficient, but it does at least warrant the Scottish verdict:

Excess stock price volatility is unproved.

The ball is once again in the court of those who doubt the Efficient Market Hypothesis.

FOOTNOTES

*This paper is a substantial revision of a part of Marsh and Merton (1983) which was presented in seminars at Yale and Harvard. We thank the participants for their helpful comments. We also thank G. Gennotte, S. Myers, and R. Ruback; and J. Hausman for his advice on the econometric issues. We are especially grateful to F. Black, both for his initial suggestion to explore this topic, and for sharing with us his deep insights into the problem. Without either, the paper would never have been written. We are pleased to acknowledge financial support from the First Atlanta Corporation for computer services.

1 To be sure, of the hundreds of tests of efficient markets, there have been a few which appear to reject market efficiency [cf. "Symposium on Some Anomalous Evidence on Capital Market Efficiency," Journal of Financial Economics (June-September 1978)]. For the most part, however, these studies are joint tests of both market efficiency and a particular equilibrium model of differential expected returns across stocks such as the Capital Asset Pricing Model, and therefore, rejection of the joint hypothesis may not imply a rejection of market efficiency. Even in their strongest interpretation, such studies have at most rejected market efficiency for select segments of the market.

2 For example, in discussing the problems of Tobin's Q theory in explaining investment, Bosworth (1975, p. 286) writes "Nor does it seem reasonable to believe that the present value of expected corporate income actually fell in 1973-1974 by the magnitude implied by the stock-market decline of that period, when q declined by 50 percent. ... As long as management is concerned about long-run market value and believes that this value reflects 'fundamentals,' it would not scrap investment plans in response to the highly volatile short-run changes in stock prices."

3 Using the variance bound methodology, LeRoy and Porter (1981) claim to show that stock prices are "too volatile" relative to accounting earnings. For a similar discussion of their analysis, see Marsh and Merton (1984).

4 For a recent discussion of the "causal" effect of stock price changes on investment, see Fischer and Merton (1984).

5 Shiller (1981a, p. 434) notes on this general point: "The lower bound of a 95 percent one-sided χ^2 confidence interval for the standard deviation of annual changes in real stock prices is over five times higher than the upper bound allowed by our measure of the observed variability of real dividends. The failure of the efficient markets model is thus so dramatic that it would seem impossible to attribute the failure to such things as data errors, price index problems, or changes in tax laws."

6 Shiller also develops a second variance bound test which establishes an upper bound on the variance of unanticipated changes in detrended real stock prices in terms of the variance of detrended real dividends. A brief analysis of this "innovations test" is presented in Section II.

7 That is, the time series estimator $\sum_{0}^{T-1} [p(t) - \bar{p}]^2/T$ can be used to estimate Var[p(t)] and similarly, for p*(t).

8 As a recent example, see Tobin (1984).

9 This belief may perhaps explain why Shiller devotes twenty percent of his paper (1981a) to justify the robustness of his findings with respect to assumption (S.2) and virtually no space to justifying (S.3).

10 The cash flow accounting identity applies only to dividends paid net of any issues or purchases of its outstanding securities. "Gross" dividends are, of course, subject to no constraint. Hence, all references to "dividends" throughout the paper are to "net" dividends paid.

11 The fact that individual firms pursue dividend policies which are vastly different from one another is empirical evidence consistent with this view.

12 By the accounting identity, "net" dividend policy (as described in footnote 9) cannot be changed without changing the firm's investment policy. However, changes in investment policy need not change the current intrinsic value of the firm. Managers can implement virtually any change in net dividends per share (without affecting the firm's intrinsic value) by the purchase or sale of financial assets held by the firm or by marginal changes in the amount of investment in any other "zero net present value" asset held by the firm (e.g., inventories). Such transactions will change the composition of the firm's assets and the time pattern of its future cash flows, but not the present value of the future cash flows. Since these "trivial" changes in investment policy will not affect the intrinsic value of the firm, they will not affect the current level of rationally-determined stock price.

13 Indeed, the classic Miller-Modigliani (1961) theory of dividends holds that dividend policy is irrelevant, and hence, in this case, there is no optimal policy.

14 The constant of integration must be zero since $e(t) = 0$ implies that $V(t) = 0$ which implies that $e(t + s) = 0$ and $d(t + s) = 0$ for all $s \geq 0$.

15 The target payout ratio δ and the long-run growth rate g are related by $g = r - \rho = (1 - \delta)r$.

16 The exception is the polar case of (9) where $N = 0$ and managers choose a dividend policy so as to maintain a target payout ratio in both the short and long runs. In this case, with $d(t) = \rho p(t)$ for all t, we have the stronger analytic proposition that the Shiller variance bound inequality (4) must be violated in all samples if stock prices are rational.

17 For example, if all in-sample prices happened to be the same (i.e., $p(t) = \bar{p}$, $t = 0,\ldots,T - 1$), but the out-of-sample prices were not, then for that particular sample path, $\mathrm{Var}(p^*) > \mathrm{Var}(p) = 0$.

18 Since the two theorems share the assumption (S.2) and for any sample of prices, one must fail, they cannot reliably be used to test this hypothesis either. However, as Fama (1977) and Myers and Turnbull (1977) have shown, we note that a constant discount rate is inconsistent with a stationary process for dividends when investors are risk averse. Hence, the assumptions (S.2) and (S.3) are a priori, mutually inconsistent.

19 If $V(t)$ follows a stationary process and the dividend process is given by (9), then the innovations or unanticipated changes in intrinsic value, $\Delta V(t) + D(t) - rV(t)$, will not form a martingale as is required by (6). If, as is necessary for the validity of (9), the intrinsic value follows a nonstationary process, then from (6) and (7), both dividends and rational stock prices must also be nonstationary.

20 If $p(t)$ and $d(t)$ follow nonstationary processes, the variances of the price and dividend are, of course, not well-defined in the time series sense that they were used in Shiller's variance bound test. However, $\mathrm{Var}(p^*)$ and $\mathrm{Var}(p)$ can be simply treated as sample statistics constructed from the random variables $\{p(t)\}$ and $\{d(t)\}$, and for any finite T, the conditional moments of their distributions will exist. If, moreover, the processes are such that the dividend-to-price ratio coverages to a finite-variance steady-state distribution, then the conditional expectation of the variance bound inequality as expressed in Theorem II, $\varepsilon_0[\mathrm{Var}(p^*)/\mathrm{Var}(p)]$, will exist even in the limit as $T \to \infty$.

21 In financial economics, the standard assumption is that the per period rate of return on stocks is independently and identically distributed over time. Together with limited liability on stock ownership, this implies a geometric Brownian motion model for stock prices which is not, of course, a stationary process. There is a long-standing and almost uniform agreement in the accounting literature that accounting earnings (either real or nominal) can best be described by a nonstationary process (cf. Foster (1978, Chapter 4)).

22 The strength of this apparent reversion to trend is further accentuated by using the ex post or in-sample trend of stock prices to detrend both the actual stock price and the $p^*(t)$ time series.

23 As shown in Fischer and Merton (1984), in addition to predicting dividend changes, aggregate real stock price changes are among the better forecasters of future changes in business cycle variables including GNP, corporate earnings, and business fixed investment. These empirical findings might also be counted in the support of the hypothesis of stock market rationality.

24 We surely echo this view with respect to our own dividend model (9). We do not however, assert that the variance bound condition of Theorem I and II provides a reliable method for testing stock market rationality.

APPENDIX 1

PROOF OF THEOREM I:

Define Π as the T x T matrix of elements π_{tk} in Theorem I, so that $\underline{p}^* = \Pi\underline{p}$. We show that:

$$\text{Var}(\Pi\underline{p}) \leq \text{Var}(\underline{p}) \tag{A.1}$$

if and only if:

$$||[(I - \frac{\underline{1}\,\underline{1}'}{T})\Pi]_k|| \leq 1 \tag{A.2}$$

where $[(I - \frac{\underline{1}\,\underline{1}'}{T})\Pi]_k$ is defined as the k'th column of the T x T matrix $(I - \frac{\underline{1}\,\underline{1}'}{T})\Pi$, and Var(.) is the usual sample variance operator. Note that condition (A.2) may be re-expressed, as in the statement of Theorem I, as:

$$\sum_{t=0}^{T-1} \pi_{tk}^2 \leq 1 + (\sum_{t=0}^{T-1} \pi_{tk})^2/T \quad . \tag{A.2a}$$

Necessity is easily proved. Let $\underline{u}_k$ be a basis vector with zeros everywhere except for a one in the k'th position. Then the k'th column of $[(I - \frac{\underline{1}\,\underline{1}'}{T})\Pi]$ is defined as:

$$\underline{x}_k = (I - \frac{\underline{1}\,\underline{1}'}{T})\Pi\, \underline{u}_k \tag{A.3}$$

$$=> \qquad ||\underline{x}_k||^2 = || (I - \frac{\underline{1}\,\underline{1}'}{T})\Pi\, \underline{u}_k ||^2 \tag{A.4}$$

But, applying (A.1) to the case $\underline{p} = \underline{u}_k$:

$$||(I - \frac{\underline{1}\,\underline{1}'}{T})\Pi\,\underline{u}_k||^2 \leq ||(I - \frac{\underline{1}\,\underline{1}'}{T}\,\underline{u}_k||^2 \leq ||\underline{u}_k||^2 = 1 \qquad (A.5)$$

To prove sufficiency, let $\underline{p}$ be any arbitrary vector which can be decomposed into a mean vector and a vector of deviations around that mean. That is:

$$\underline{p} = \underline{p}_1 + \mu\underline{1} \qquad (A.6)$$

where

$$(\underline{p}_1' + \mu\underline{1}')\Pi'(I - \frac{\underline{1}\,\underline{1}'}{T})\Pi(\underline{p}_1 + \mu\underline{1}) \leq (\underline{p}_1' + \mu\underline{1}')(I - \frac{\underline{1}\,\underline{1}'}{T})(\underline{p}_1 + \mu\underline{1}) \qquad (A.7)$$

which implies:

$$\underline{p}_1'\Pi'(I - \frac{\underline{1}\,\underline{1}'}{T})\Pi\underline{p}_1 \leq \underline{p}_1'(I - \frac{\underline{1}\,\underline{1}'}{T})\underline{p}_1 \qquad (A.8)$$

since $\Pi\underline{1} = \underline{1}$ and $(I - \frac{\underline{1}\,\underline{1}'}{T})\underline{1} = 0$.

Also,

$$\frac{\underline{1}\,\underline{1}'}{T}\underline{p}_1 = \underline{0} \qquad (A.9)$$

because $\underline{p}_1$ has a zero mean. Then, from (A.8) and (A.9):

$$||(I - \frac{\underline{1}\,\underline{1}'}{T})\Pi\underline{p}_1||^2 = \underline{p}_1'\Pi'(I - \frac{\underline{1}\,\underline{1}'}{T})\Pi\underline{p}_1 \leq \underline{p}_1'\underline{p}_1 = ||\underline{p}_1||^2 \quad .$$

Using the definition that $\underline{u}_k$ is a vector with one in the k'th position and zeros elsewhere:

$$\underline{p}_1 = \sum_{k=0}^{T-1} \alpha_k\underline{u}_k \qquad (A.10)$$

$$\Rightarrow \qquad ||\underline{p}_1||^2 = \sum_{k=0}^{T-1} \alpha_k^2 \tag{A.11}$$

Then, by the Minkowski inequality:

$$||(I - \frac{\underline{1}\,\underline{1}'}{T})\Pi\underline{p}_1||^2 \leq \sum_{k=0}^{T-1} ||(I - \frac{\underline{1}\,\underline{1}'}{T})\Pi(\alpha_k\underline{u}_k)||^2$$

$$= \sum_{k=0}^{T-1} \alpha_k^2||(I - \frac{\underline{1}\,\underline{1}'}{T})\Pi\underline{u}_k||^2 \tag{A.12}$$

$$= \sum_{k=0}^{T-1} \alpha_k^2||\underline{p}_k||^2 \quad .$$

Thus, (A.2) is sufficient for:

$$||(I - \frac{\underline{1}\,\underline{1}'}{T})\Pi\underline{p}_1||^2 = \underline{p}_1'\Pi'(I - \frac{\underline{1}\,\underline{1}'}{T})\Pi\underline{p}_1 \leq \sum_{k=0}^{T-1} \alpha_k^2 = \underline{p}_1'\underline{p}_1 = ||\underline{p}_1||^2 \tag{A.13}$$

$$\Rightarrow \qquad T\ Var(\Pi\underline{p}) \leq T\ Var(\underline{p}) \tag{A.14}$$

$$\Rightarrow \qquad Var(\Pi\underline{p}) \leq Var(\underline{p}) \tag{A.15}$$

Q.E.D.

PROOF OF THEOREM II:

Using the definition of the (detrended) ex post rational price, $p^*(t)$, given in (5) in the text, and allowing (detrended) dividends to be a general distributed lag of (detrended) prices as in (13) in the text, ex post rational prices can be expressed in terms of the observed and pre-sample (detrended) prices as:

$$\begin{bmatrix} p^*(T-1) \\ p^*(T-2) \\ p(T-3) \\ \vdots \\ p^*(0) \end{bmatrix} = \frac{1}{T} \begin{bmatrix} a & \dots & a & 0..0 \\ a^2 & \dots & a^2 & 0..0 \\ a^3 & \dots & a^3 & 0..0 \\ \vdots & & \vdots & \\ a^T & \dots & a^T & 0..0 \end{bmatrix} + \rho \begin{bmatrix} a & 0 & 0 & \dots & 0 \\ a^2 & a & 0 & \dots & 0 \\ a^3 & a^2 & a & \dots & 0 \\ \vdots & \vdots & \vdots & & \\ a^T & a^{T-1} & a^{T-2} & \dots & 0 \end{bmatrix} X$$

$$\left[\begin{array}{cccccccc|ccc} \theta_0 & \theta_1 & \cdots\cdots & \theta_N & 0 & \cdots & & 0 & 0 & \cdots\cdots & 0 \\ 0 & \theta_0 & \cdots\cdots & \theta_{N-1} & \theta_N & \cdots & & 0 & 0 & \cdots\cdots & 0 \\ \vdots & & & \vdots & & & & \vdots & \vdots & & \vdots \\ \vdots & & & \vdots & & & & \theta_N & 0 & & 0 \\ \vdots & & & \vdots & & & & \vdots & \vdots & & \vdots \\ \vdots & & & \vdots & & & \theta_0 & \theta_1 & \vdots & \theta_N & 0 \\ 0 & \cdots & \cdots & 0 & \cdots & 0 & & \theta_0 & \theta_1 & \cdots\cdots & \theta_N \end{array}\right] \begin{bmatrix} p(T-1) \\ p(T-2) \\ \vdots \\ p(0) \\ p(-1) \\ \vdots \\ p(-N) \end{bmatrix} \qquad \text{(A.16)}$$

where $a = 1/(1+\rho)$, and the level of dividends is a distributed lag of the level of past prices, as in (13), i.e., $d(t) = \rho \sum_{k=0}^{N} \theta_k p(t-k)$.

(A.16) may be conveniently rewritten as:

$$\underline{p}^* = [A_1 + A_2\Theta]\underline{p} \qquad \text{(A.17)}$$

where A_1 is the first matrix on the RHS of (A.16), (i.e., the matrix which involves multipliction by the scalar $1/T$), A_2 is the next matrix, which involves multiplication by the scalar ρ, and Θ is the matrix which contains the elements $\theta_1,\theta_2,\ldots,\theta_N$. The weights in the matrix A_1 reflect the contribution $1/T$ of each of the observed prices $[p(T-1),\ldots,p(0)]$ to $p^*(T)$ in accordance with (3), together with the weight $[1/(1+\rho)]^{T-t}$ attached to $p^*(T)$ in the determination of $p^*(t)$ in (5). The matrix A_2 contains the discount weights which (5) places on dividends as components of each $p^*(t)$, while Θ contains the distributed lag weights of dividends on past prices, as given in (13). Using these definitions of A_1, A_2, and Θ, (A.17) is equivalent to:

$$\underline{p}^* = W\,\underline{p} \tag{A.18}$$

where $W = [w_{tk}]$, the w_{tk} being those defined in (14) in the text.

The elements of W satisfy the conditions stipulated in the text. By inspection, $(A_1 + A_2)$ satisfies the conditions, and hence, given the structure of Θ, $(A_1 + A_2\Theta)$ will also.

Suppose that the $(T + N)$ vector $\underline{p}$ is decomposed into a vector $\bar{p}\,\underline{1}$ and a vector $\underline{p}_1$, where $\bar{p}$ is the mean of $p(j)$, $j = -N,\ldots,T-1$, and $\underline{p}_1$ contains the deviations about the mean, just as in the proof of Theorem I. Then:

$$\underline{p}^* = [A_1 + A_2\Theta][\bar{p}\,\underline{1} + \underline{p}_1] \tag{A.19}$$

$$= [A_1 + A_2\Theta]\bar{p}\,\underline{1} + [A_1 + A_2\Theta]\underline{p}_1 \tag{A.20}$$

$$= A_1\bar{p}\,\underline{1} + A_2\Theta\bar{p}\,\underline{1} + A_1\underline{p}_1 + A_2\Theta\underline{p}_1 \quad . \tag{A.21}$$

Since the row elements of A_1 are constant, $A_1\underline{p}_1 = \underline{0}$. Also, since the row elements of Θ sum to unity, $A_2\Theta\bar{p}\,\underline{1} = A_2\bar{p}\,\underline{1}$. Thus, (A.21) simplifies to:

$$\underline{p}^* = [A_1 + A_2]\bar{p}\,\underline{1} + A_2\Theta\underline{p}_1 \tag{A.22}$$

$$= \bar{p}\,\underline{1} + A_2\Theta\underline{p}_1 \quad \text{(since the row elements of } (A_1 + A_2) \text{ sum to unity)} \ . \tag{A.23}$$

From (A.23):

$$\mathrm{Var}(p^*) = \frac{1}{T+N}(\underline{p}_1'\Theta'A_2' + \bar{p}\,\underline{1}')\left(I - \frac{\underline{1}\,\underline{1}'}{T}\right)(\bar{p}\,\underline{1} + A_2\Theta\underline{p}_1) \tag{A.24}$$

$$= \frac{1}{T+N}\underline{p}_1'\,\Theta'A_2'\left(I - \frac{\underline{1}\,\underline{1}'}{T}\right)A_2\Theta\underline{p}_1 \quad . \tag{A.25}$$

By inspection of (A.16), the row elements of A_2 are positive, sum to unity or less than unity, and have a norm less than unity. Thus, by using Theorem I twice, we have that:

$$\mathrm{Var}(p^*) \leq \frac{1}{T+N}\underline{p}_1'\Theta'\left(I - \frac{\underline{1}\,\underline{1}'}{T}\right)\Theta\underline{p}_1 \leq \frac{1}{T+N}\underline{p}_1'\left(I - \frac{\underline{1}\,\underline{1}'}{T}\right)\underline{p}_1$$

$$\Rightarrow \qquad \mathrm{Var}(p^*) \leq \frac{1}{T+N}\underline{p}_1'\underline{p}_1 = \mathrm{Var}(p) \tag{A.26}$$

(A.26) can be interpreted as an extension of the inequality in Theorem I to the case where a $(T + N)$ vector of sample and pre-sample prices is mapped into a T vector of an ex post rational prices. However, in the market rationality tests, the variance of the ex post rational prices is compared not to the variance of the $(T + N)$ vector of T observed <u>and</u> N pre-sample prices, but to the variance of only the T observed prices. Partitioning of the $(T + N)$ prices into in-sample and out-of-sample prices, it is straightforward to show that:

$$\frac{\mathrm{Var}(p^*)}{\mathrm{Var}(p_T)} \leq 1 + \frac{T.N}{(N+T)^2}\,\frac{(\bar{p}_T-\bar{p}_N)^2}{\mathrm{Var}(p_T)} - \frac{N}{T}\left[\frac{\mathrm{Var}(p_T)-\mathrm{Var}(p_N)}{\mathrm{Var}(p_T)}\right] \tag{A.27}$$

where:

$$\bar{p}_T = \sum_{t=0}^{T-1} p(t)$$

$$\bar{p}_N = \sum_{t=-N}^{-1} p(t)$$

$$Var(p_T) = \sum_{t=0}^{T-1} [p(t) - \bar{p}_T]^2/T$$

$$Var(p_N) = \sum_{t=-N}^{-1} [p(t) - \bar{p}_N]^2/N$$

If the sum of the last two terms on the RHS of (A.27) is nonpositive, $Var(p^*) \leq Var(p_T)$, and our "variance bound test" will hold analytically for all such sample paths. For given T and N, the start-up effects of out-of-sample prices are less important the more the (unobserved) pre-sample prices "look like" the sample prices (e.g., if $\bar{p}_T = \bar{p}_N$ and $Var(p_T) = Var(p_N)$, the adjustments vanish).

The sum of the last two terms on the RHS of (A.27) can be positive for some sample paths (if $\bar{p}_N$ is very different from $\bar{p}_T$ while at the same time $Var(p_N)$ is bigger than $Var(p_T)$). However, for a given N, and irrespective of whether (detrended) prices are stationary or non-stationary, it is clear that almost surely for every sample path, the start-up adjustment terms in (A.27) converge to zero as $T \to \infty$.

REFERENCES

Bosworth, B., 1975, "The Stock Market and the Economy," Brookings Papers on Economic Activity, 2, 257-300.

Copeland, B.L. Jr., 1983, "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?: Comment," The American Economic Review, 73(1), 234-235.

Fama, E.F., 1977, "Risk-Adjusted Discount Rates and Capital Budgeting Under Uncertainty," Journal of Financial Economics, 5, 3-24.

Fischer, S., and R.C. Merton, 1984, "Macroeconomics and Finance: The Role of the Stock Market," National Bureau of Economic Research Working Paper, No. 1291, March.

Flavin, M.A., 1983, "Excess Volatility in the Financial Markets: A Reassessment of the Empirical Evidence," Journal of Political Economy, 91(6), 929-956.

Foster, G., 1978, Financial Statement Analysis, Prentice-Hall, Englewood Cliffs, NJ.

Keynes, J.M., 1936, The General Theory of Employment Interest and Money, Harcourt, Brace & Co., New York.

Kleidon, A.W., 1983a, "Variance Bounds Tests and Stock Price Valuation Models," Unpublished Working Paper, Graduate School of Business, Stanford University, (January).

____________, 1983b, "Bias in Small Sample Tests of Stock Price Rationality," Unpublished Paper, University of Chicago.

Leamer, E.E., 1983, "Let's Take the Con Out of Econometrics," American Economic Review, 73(1), 31-43.

LeRoy, S.F., and R.D. Porter, 1981, "The Present-Value Relation: Tests Based on Implied Variance Bounds," Econometrica, 49(3), 555-574.

Lintner, J., 1956, "Distribution of Incomes of Corporations Among Dividends, Retained Earnings, and Taxes," The American Economic Review, XLVI(2) (May), 97-113.

Marsh, T.A., and R.C. Merton, 1983, "Aggregate Dividend Behavior and Its Implictions for Tests of Stock Market Rationality," Working Paper 1475-83, Sloan School of Management, September.

__________________________, 1984, "Earnings Variability and Variance Bounds Tests for the Rationality of Stock Market Prices," Working Paper 1559-84, Sloan School of Management, March.

Miller, M.H., and F. Modigliani, 1961, "Dividend Policy, Growth and the Valuation of Shares," Journal of Business, October, 411-433.

Myers, S.C., and S.M. Turnbull, 1977, "Capital Budgeting and The Capital Asset Pricing Model: Good News and Bad News," Journal of Finance, 32(2), 321-333.

Samuelson, P.A., 1965, "Proof That Properly Anticipated Prices Fluctuate Randomly," Industrial Management Review, 6, Spring, 41-49.

______________, 1974, "Challenge to Judgment," Journal of Portfolio Management, 1, pp. 17-19.

______________, 1976, "Optimality of Sluggish Predictors Under Ergodic Probabilities, International Economic Review, 17(1), February, 1-7.

Shiller, Robert J., 1981a, "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?" The American Economic Review, 71(3), June, 421-436.

________________, 1981b, "The Use of Volatility Measures in Assessing Market Efficiency," The Journal of Finance, XXXVI(2), May, 291-311.

________________, 1982, "Consumption, Asset Markets, and Macroeconomic Fluctuations," Carnegie-Rochester Conference Series on Public Policy, 17, 203-238.

________________, 1983, "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?: Reply," The American Economic Review, 73(1), 236-237.

Tobin, J., 1984, "On the Efficiency of the Financial System," Hirsch Memorial Lecture, New York, May 15.