

THE VALUE PRIORITY HYPOTHESES FOR PURCHASES  
OF CONSUMER DURABLE GOODS

by

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ABSTRACT

Based on the behavioral sciences and mathematical programming, we hypothesize that consumers rank durables by a value (or net value) priority approximated by utility per dollar (or utility minus price) and choose items in that order up to a budget cutoff. This paper derives these behavioral hypotheses and develops a convergent linear programming procedure to estimate utility. Using primary field data on reservation prices, purchase probabilities, lottery orders, and combination prizes we estimate utilities and compare the hypotheses to 215 actual budget plans. LISREL V analysis provides further support for the hypotheses.

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## 1. PERSPECTIVE

Purchases of durable goods such as automobiles, home computers, and video cassette recorders account for substantial budget outlays by consumers. Such purchases have a major impact on national economic conditions and represent a challenging research issue.

Scientific interest is strong because durable goods purchases depend upon inter-category comparisons (e.g., auto versus home computers) and upon the impact of limited consumer budgets. Managerial interest is strong because understanding the effects of relative price and competitive entry along with recession, inflation, and tax policy on families' purchases is critical for established products. In developing new durable goods, managerial and research attention is high because new product development costs are large, (e.g., in automobiles such costs can exceed one billion dollars), and because key strategic decisions must be made prior to new product launch.

This paper reviews the value priority hypothesis for durable purchases, introduces a rival variation, the net value priority hypothesis, and discusses their interrelationships based on economics, marketing, and management science. We describe data collected to test the hypotheses and linear programming procedures to estimate the underlying model from the data. We test the hypotheses by comparing their predictions to actual consumer budgets and we provide convergent tests with an alternative estimation procedure, LISREL V. We close with a discussion of some managerial implications.

## 2. VALUE PRIORITY HYPOTHESIS

For exposition we begin with the single period consumer model. Appendix 1 discusses extensions to multiple periods which include borrowing, savings, depreciation, operating costs, trade-ins, and interproduct complementarity.



We now show that such a prioritized buying order is consistent with a modified MP1.

Suppose that the consumer can assign to each good a marginal utility,  $u_j$ , that represents the amount of utility he gets from possessing that durable good.<sup>1</sup> (We assume  $u_j$  can be ratio scaled.) If the consumer considers more than one unit of the durable good, we assign values  $u_{j1}$ ,  $u_{j2}$ , ..., etc. to the first, second, etc. units of good  $j$  with the usual assumption that  $u_{j1} > u_{j2}$ , etc. However, to simplify exposition we temporarily assume  $g_j$  is at most one item. This is not a restriction in the theory.<sup>2</sup> MP1 now becomes MP2.

$$\begin{aligned} &\text{maximize} && u_1 g_1 + u_2 g_2 + \dots + u_n g_n + u_y(y) \\ & && \text{(MP2)} \\ &\text{subject to} && p_1 g_1 + p_2 g_2 + \dots + p_n g_n + y \leq B \end{aligned}$$

where  $u_y(y)$  is the marginal utility of allocating  $y$  dollars to non-durables.

MP2 is now a mathematical programming problem called the "knapsack" problem. If the  $g_j$  were not restricted to be discrete, that is, if you could buy a fractional automobile, its solution, called the "greedy" algorithm, is well-known (e.g., Gass 1969 p. 204): Allocate the budget to goods in order of  $u_j/p_j$  as long as  $u_j/p_j$  is greater than the budget cutoff,  $\lambda = \partial u_y(y)/\partial y$ , evaluated at the budget constraint.<sup>3</sup> Even when purchases are restricted to be

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<sup>1</sup>Technically this is an assumption of separability (Blackorby, Primont and Russell, 1975). Appendix 1 relaxes this assumption.

<sup>2</sup>If we allow  $g_j$  to be any integer, MP2 becomes  $\max \sum_j \sum_k u_{jk} \delta_{jk} + u_y(y)$ , s.t.  $\sum_j \sum_k p_j \delta_{jk} + y \leq B$  where  $\delta_{jk} = 1$  iff  $g_j \geq k$ . Alternatively, we can redefine goods such that the  $k + 1^{\text{st}}$  item of good  $j$  has a different index than the  $k^{\text{th}}$  item. See also the appendix.

<sup>3</sup>Mathematically,  $\lambda$  is a complex function of all the variables of the problem. For our purposes, we need not evaluate it, we need only that it exists. For a given set of utilities it is quite easy to construct an algorithm that finds  $\lambda$  by iteratively allocating the budget between durables and non-durables according to maximum  $u_j/p_j$  or  $\partial u_y(y)/\partial y$ . Alternatively we can scale all utilities relative to  $\lambda$ .

discrete, "greedy" algorithms are excellent heuristics (Cornuejols, Fisher and Nemhauser 1977 and Fisher 1980).

The "greedy" algorithm is simple yet it provides an excellent approximation to the optimization of MP2 across a variety of situations. We posit that this heuristic provides a reasonable approximation to describe consumer purchasing behavior.

There is a simple behavioral interpretation of the mathematical result; the criterion,  $u_j/p_j$ , of utility per dollar is a measure of "value". We therefore call our proposition the value priority hypothesis. In words,

Value Priority Hypothesis. The consumer purchases durable goods in order of value as long as their value is above some cutoff,  $\lambda$ , which represents the value of spending an additional dollar on non-durable goods. Furthermore, value is measured by utility per dollar.

For example, suppose a consumer is considering a microwave oven, a video cassette recorder, an automobile, a personal computer, a snow blower, and home improvements. He would consider the pleasure and usefulness, i.e. utility, he would get from owning the best choice from each category, consider the price of the best choice, and rank them according to value as shown below.

Microwave Oven  
Video Cassette Recorder  
Automobile  
← Budget Constraint ( $\lambda$ )  
Personal Computer  
Snow Blower  
Home Improvements

He would purchase first the microwave oven (and some non-durables up to  $u_{\text{microwave}}/p_{\text{microwave}}$ ), then the video cassette recorder, then the automobile. At this point he would find that the three durables (plus corresponding non-durables) exhaust his budget. If he were to borrow, or otherwise obtain additional funds, the next durable he would purchase would be a personal computer.

Of course, actual durable purchasing behavior is more complex depending upon unexpected events as well as planning (e.g., Dickson and Wilkie 1978), but we

feel the value priority hypothesis is a good, first-order explanation. It has roots in the econometric (Paroush 1965) and management science (Keon 1980) literatures and is consistent with small sample, exploratory focus group semantics (Bertan and Hauser 1982) such as "you get what you pay for," "I want my money's worth," "good value for the money spent," "I want the most car for my money," "when you buy a car you shop value," etc..

Appendix 1 shows the value priority hypothesis extends beyond the simple single period model. For example, in a multiperiod problem with borrowing (saving) and depreciation, the "value" becomes the depreciated time stream of utility divided by the price in current dollars. Operating costs become an addition to price, discounted overtime; replacement (trade-ins) are incorporated by computing net utility gain and net price; and complementarity is approximated by first order dependence. The budget constraints for each period are related by interest rates.

### 3. ANOTHER VIEWPOINT: NET VALUE PRIORITY

There are two components to the value priority hypothesis. The ordering by value and the means by which value is computed.

In section 2 we treated value as utility per dollar, but in brand choice price is often treated as an attribute. For example, models using conjoint analysis (Green and Srinivasan 1979), perceptual mapping (Hauser and Koppelman 1979), and logit analysis (McFadden 1980) have all included price as another (linear) explanatory variable. Srinivasan (1982) argues that this is a good representation if we recognize that the criterion,  $u_j - \lambda p_j$ , is the Lagrangian solution to MP2 when the problem is one of brand choice where one and only one good is chosen. He then argues that  $u_j - \lambda p_j$  may be a more robust representation when price itself carries utility such as in conspicuous consumption or when perceptions of quality are based on price.

Thus, a variation of the value priority hypothesis is that consumers order durable goods by net value where net value is the surplus of utility over price, that is  $u_j - \lambda p_j$ .

The net value priority hypothesis can be derived by examining the dual program to the mathematical program, MP2. (For those readers unfamiliar with dual linear programs see Gass (1969) or appendix 2.) Let  $\lambda$  continue to be the dual variable of the budget constraint and let  $\gamma_j$  be the dual variables associated with the implicit constraints of  $g_j \leq 1.0$ .<sup>4</sup> Then the dual program is (Gass 1969, p. 90):

$$\begin{aligned} \text{minimize} \quad & B\lambda + \gamma_1 + \gamma_2 + \dots + \gamma_n && \text{(MP3)} \\ \text{subject to} \quad & \lambda p_j + \gamma_j \geq u_j && \text{for all } j \\ & \lambda = \partial u_y(y) / \partial y && \text{at optimum} \end{aligned}$$

We now use mathematical programming theory to provide a behavioral interpretation of MP3.

By the duality theorem (Gass 1969, p. 90) of linear programming, the solution of MP3 equals the solution of MP2 when purchases of fractional goods are allowed. Rearranging the constraints of MP3 we get  $\gamma_j \geq u_j - \lambda p_j$ . If fractional goods are allowed, the complementary slackness theorem (Gass 1969, p. 99) applies. By this theorem,  $\gamma_j = u_j - \lambda p_j$  is greater than zero in the solution to MP3 if and only if  $g_j = 1$  in the solution to MP2. Also,  $\gamma_j$  equals zero if and only if  $g_j < 1$ .

We obtain a behavioral interpretation of MP3 by recognizing that the simplex multiplier,  $\gamma_j$ , is the shadow price of the constraint  $g_j \leq 1$ . In words,  $\gamma_j$  is the value at the margin to the consumer of buying more of durable  $j$ ; that is, the value of relaxing the constraint that durables are discrete. Complementary slackness says that net value,  $u_j - \lambda p_j$ , is greater than zero if and only if

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<sup>4</sup>We can extend this to any integer value for  $g_j$  by the techniques of footnote 2.

a good is purchased. Net value is less than (or equal to) zero if less than one unit of a good is purchased.

Together MP2 and MP3 suggest the behavioral interpretation that net value is the appropriate criterion if the consumer focuses on the marginal benefit of purchasing more of a given durable; value is the appropriate criterion if the consumer focuses on his overall budget allocation problem. Both are reasonable theoretical descriptive hypotheses. Empirical data will shed further light on both hypotheses.

In summary, value priority and net value priority are two reasonable hypotheses about consumer durable purchasing. Both are derived by assuming the consumer uses an heuristic decision rule to maximize utility subject to a budget constraint and implicit constraints that fractional items are not available. Both imply that a consumer will order his durable purchasing (within his budget) according to a simple criterion based on value. The two hypotheses differ in how this criterion is computed. Value priority focuses on the budget constraint and postulates a criterion of utility per dollar. Net value priority focuses on the marginal value of the next purchase and postulates a criterion of net value gained. We now examine data on both hypotheses.

#### 4. DATA

The value priority and net value priority hypotheses are formulated for decision making units such as individuals or families. To test their implications we require data on budget allocations of individual decision making units.

In March 1983 we were given the opportunity to apply and test our hypotheses. An American automobile manufacturer planned to introduce a new automobile in Spring 1984 and, among other things, wanted to know with which durable products the automobile would compete. The new automobile was a luxury model for upscale consumers, hence competition from vacations, second homes,

pools, boats, and college tuition was a management concern.

The data collection was part of an ongoing project. The questioning procedure described below is based on focus groups in Boston, MA and Troy, MI in June 1982, automobile show interviews in Boston, MA in November 1981, 1982, and 1983, a pretest (30 consumers) in Troy, MI in June 1982, a mini-test (40 consumers) in Phoenix AZ in January 1983, and a series of informal tests throughout the period.

The consumer tasks were administered with trained and experienced professional interviewers. The consumer tasks took approximately 50 minutes and were the opening part of a larger, two-hour interview in which respondents were paid \$25 for their time. (See Hauser, Roberts and Urban 1983 for details on the full interview.) The 174 respondents were chosen at random from the Cincinnati, OH area, but in proportion to previous purchases of automobiles similar to the automobile of interest. For 12 percent of the interviews, both husbands and wives participated in making joint budget allocations.

Since our hypotheses and the data are at the level of the individual consumer, this data should be sufficient for an initial test of the value priority hypotheses. However, the specific durables and the magnitude of the budgets are not generalizable to the U.S. population because our sample was weighted towards potential luxury car buyers. Furthermore, our analyses are limited to any extent that luxury car buyers are different in their budgeting processes.

#### Budget Task

To obtain budget information, we gave consumers a deck of cards in which each card represented a potential purchase. For example, these cards included college tuition, vacations, home improvements, major clothing purchases, landscaping, cameras and accessories, furniture, home fuel savings devices, dishwashers, color televisions, stereo systems, jewelry, etc. After an extensive pretest, we were able to identify 52 items that accounted for most

purchases. (Consumers were given blank cards for additional purchases.)

Consumers first sorted these cards according to whether they (A) now owned the durable, (B) would consider purchasing it in the next three years, or (C) would not consider purchasing it in the next three years. Consumers next considered pile A, "currently own", and removed those items they would either replace or supplement by buying an additional unit. Finally, they selected from pile B, "would consider," and from the replacement/additional pile, those items for which they would specifically budget and plan. These items are now their budgetable durable goods.

Consumers then allocated these items to the years 1983, 1984, and 1985 and ordered the items according to priority within each year. This rank order of items becomes our measure of their budget allocation. We estimate utilities with other data, described below, and attempt to forecast the measured rank order buying priorities.

#### Explanatory Measures

Obtaining utility measures that can be used to infer value among product categories is a difficult task. Almost every utility measurement procedure of which we are familiar, including conjoint analysis, preference regression, logit analysis, expectancy values, and von Neumann-Morgenstern assessment measures utility within a product category. In a series of pre-test measurements in 1981, 1982, and 1983, we tried over a dozen different methods including directly scaled (0 - 100 scale) points on "utility" and on "value", constant sum paired comparisons among items, and constant sum allocations among all items. We found four measures that appeared feasible and provided meaningful tasks to the consumer. These four measures were included in our interviews.

None of the four measures were explicit measures of utility. However, for each consumer measure, we use the value priority hypotheses to infer relationships among utilities. Details are given in the estimation section

below. The measures were:

Reservation Price. The consumer was asked to specify the minimum price at which he, she, or they would no longer purchase the durable.

Purchase Probability. The consumer was asked to estimate the probability that he, she, or they would actually purchase the durable in the period of interest (0 to 10 "Juster" scale). See Juster (1966).

Lottery Order. The consumer was asked to imagine that he, she, or they had won a lottery and would be allowed to select a prize. They were then to rank the durables allocated to each year in the order corresponding to the order in which he, she, or they would choose a prize in the lottery. Note that this ordering will usually be different than the budget allocation ordering because price is not to be considered in this task.

Combination Lottery Prizes. The consumer was again told that he, she, or they had won a lottery, but this time the task was to choose among two pairs of prizes. For example, the consumer(s) might be asked to choose among receiving either (a) the first and fourth ranked prize, or (b) the second and third ranked prize. Consumers were asked up to eight such pairs or combinations for each budget year.

#### Example Respondent

Table 1 lists the actual data obtained from one respondent. This respondent, a 30 year old, married woman with three children and a \$35,000 per year family income, has six durable goods in her 1985 budget. For example, she expects to purchase a \$5,000 automobile with probability .70. This durable good is ranked first in the lottery prize question and has a reservation price of \$10,000. If price were not an issue she would rather have the automobile plus a freezer than paid tuition plus a vacation.

For each respondent, there are three tables such as Table 1, one for each year.

TABLE 1

## DATA FROM EXAMPLE RESPONDENT

<u>DURABLE</u>	<u>PRICE</u>	<u>RESERVATION PRICE</u>	<u>PURCHASE PROB.</u>	<u>LOTTERY ORDER</u>
Automobile	\$5,000	\$10,000	.70	1
Furniture	2,000	4,000	.60	2
Tuition	2,000	5,000	.99	3
Movie Camera	500	1,000	.60	4
Vacation	1,000	1,500	.70	5
Freezer	300	500	.50	6

COMBINATION LOTTERY PRIZES

(1) Automobile, Freezer	>	Tuition, Vacation
(2) Automotile, Vacation	>	Tuition, Camera
(3) Tuition, Vacation	>	Furniture, Freezer
(4) Tuition, Freezer	>	Camera, Vacation
(5) Freezer, Vacation	>	Camera
(6) Tuition	>	Camera, Freezer
(7) Tuition, Freezer	>	Furniture

## 5. ESTIMATION: CONVERGENT LINEAR PROGRAMMING

Each of the measures in Table 1 provides information about utility values, but none is a direct measure of utility. For example, the purchase probability might be a non-linear function of utility and of  $\lambda$ , while the lottery order and combination lottery prizes provide only rank order information about utility.

Because two data types, lottery orders and combination lottery prizes, are rank order relationships and because the other data types are continuous and non-linear, traditional methods based on continuous, linear relationships may not be appropriate or, at least, must be modified. We present in this section a modified linear programming (LP) procedure which can incorporate rank order and continuous data types in a single convergent estimation procedure. Section 7 presents an alternative estimation procedure which uses covariance analysis (LISREL V). In that section the relative predictive capabilities of the two procedures are examined and the convergent indications about our hypotheses are discussed.

## The Basic Idea

The idea behind convergent LP estimation is quite simple. Each datum implies a relationship either among various utility values or among a utility value and the datum. The relationship varies by data type. Our goal is to select utility values such that all relationships are satisfied. However, in the presence of measurement error and approximation error, it is unlikely that we will be able to satisfy all relationships simultaneously. Thus, for each datum, say a lottery prize answer, we may be able only to satisfy approximately the relationship. The amount by which we cannot satisfy the relationship we call "error". Thus, we choose utility values to minimize a weighted sum of errors where the weights (chosen by the analyst) allow us to put different emphasis on different data types.

This minimization of errors can be accomplished with a linear program. The objective function is the weighted sum of errors and the constraints are the relationships implied by each datum. In general terms this is (LP1):<sup>5</sup>

$$\begin{aligned} \text{minimize} \quad & W_1 * (\text{errors based on reservation price answers}) \\ & + W_2 * (\text{errors based on purchase probability answers}) \\ & + W_3 * (\text{errors based on lottery order answers}) \\ & + W_4 * (\text{errors based on combination lottery prize answers}) \end{aligned}$$

subject to relationships implied by the value priority (or net value priority) model. We now illustrate the specific mathematical relationships.

### Reservation Price Relationships

The reservation price is the price at which the durable good leaves the budget. Thus, if  $r_j$  and  $u_j$  are the reservation price and utility of the  $j^{\text{th}}$  item, then the value priority hypothesis implies:

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<sup>5</sup>It is useful to distinguish between the mathematical programs, MP1, MP2, and MP3, which are the consumers' budget problems, and the linear program, LP1, which is the analyst's estimation problem.

$$u_j/r_j = \lambda \quad (1)$$

because at the reservation price, the  $j^{\text{th}}$  item falls just below the budget cutoff,  $\lambda$ .

To include equation (1) as a relationship in an LP, we define "errors based on reservation price answers" as the absolute value of the difference between  $u_j/r_j$  and  $\lambda$ , that is,  $|u_j/r_j - \lambda|$ . In linear programming mathematics, this becomes

$$\text{errors based on reservation price answers} = e_{rj}^+ + e_{rj}^- \quad (2)$$

To assure consistent scale of errors across data types in LP1 we multiply through by  $r_j$ . The constraint relationships become,

$$u_j - e_{rj}^+ + e_{rj}^- = \lambda r_j \quad (3)$$

$$u_j, e_{rj}^+, e_{rj}^- \geq 0.$$

Equations (2) and (3) are the standard LP formulation for minimizing absolute error, e.g., Gass (1969, pp. 320). If values for  $u_j$  and  $\lambda$  are estimated and  $u_j/r_j$  exceeds  $\lambda$ , only  $e_{rj}^+$  will take on a positive value because minimization of equation (2) in LP1 forces  $e_{rj}^-$  to zero. If  $\lambda$  exceeds  $u_j/r_j$ , only  $e_{rj}^-$  will be positive.

Since the LP seeks to minimize  $e_{rj}^+ + e_{rj}^-$  and since it can simultaneously set  $u_j$  and  $\lambda$ , one trivial solution is to set all variables equal to zero. We avoid this problem by recognizing that utility, and hence  $\lambda$ , are ratio scales and thus unique to a positive constant. Thus, we can set one utility value, or  $\lambda$ , arbitrarily. In our formulations we set  $\lambda = 1$ , thus scaling everything relative to  $\lambda$ .

When  $\lambda = 1$ , the net value priority hypothesis implies  $u_j - \lambda r_j = u_j - r_j = 0$  which implies the same constraint as (3) above. This is consistent with the complementary slackness theorem and the interpretation of

MP2 and MP3. The duality theorem implies that at optimum, for a given B, the items in the budget, as implied by the optimal  $\lambda^*$  and  $u_j$ 's, are the same. However, the priority order predicted by value and net value may be quite different. See discussion in section 8.

#### Purchase Probability Relationships

The purchase probability is the consumer's estimate of the probability that the durable good will actually be purchased in the budget period. It is based on the utility and price of the durable good but also upon unobserved events that make the purchase more or less favorable. If these unobserved events represent observation error, then, according to the value priority model, the probability of purchasing good  $j$  is given by:

$$L_j = \text{Prob} \{u_j/p_j + \text{error} \geq \lambda\} \quad (4)$$

That is, the likelihood of purchase ( $L_j$ ) is the probability that the value ( $u_j/p_j$ ) is greater than the budget constraint ( $\lambda$ ) after adjusting for error.

If we multiply through by  $p_j$  to assure consistent scaling in LP1, and assume that the resulting observation error is distributed with a double exponential probability distribution, then equation (4) becomes the logit model shown in equation (5) where  $\beta$  is a parameter to be estimated.

$$L_j = \frac{\exp\{\beta u_j - \lambda \beta p_j\}}{\exp\{\beta u_j - \lambda \beta p_j\} + 1} \quad (5)$$

For derivation, see McFadden (1974). Equation (5) can be linearized by dividing through by  $(1-L_j)$  and taking logarithms.

Finally, we again use the standard LP formulation for minimizing absolute error to obtain the objective function and constraint relationships for purchase probability. For the criterion function in LP1:

$$\text{errors based on purchase probability answers} = e_{\ell j}^+ + e_{\ell j}^- \quad (6)$$

and the associated constraint is

$$u_j - (\beta^{-1})\{\log[L_j/(1-L_j)]\} - e_{\ell j}^+ + e_{\ell j}^- = \lambda p_j \quad (7)$$

$$u_j, \beta^{-1}, e_{\ell j}^+, e_{\ell j}^- \geq 0$$

In these equations,  $L_j$  and  $p_j$  are observed and  $u_j$ ,  $\beta$ ,  $e_{\ell j}^+$  and  $e_{\ell j}^-$  are variables. As before, we establish the scale by setting  $\lambda = 1$ , and, as before, constraint (7) also estimates utilities for the net value priority hypothesis.

### Lottery Orders

The lottery order is a rank order of the durable goods according to their usefulness or desireability to the consumer. As such, they imply rank orders on the magnitude of the utilities. For example, if  $u_1$  is the utility of the first ranked durable,  $u_2$  is the utility of the second ranked durable, etc., then the lottery orders imply:

$$u_1 > u_2$$

$$u_2 > u_3 \quad (8)$$

etc.

The reader will notice that this data and the constraints implied by equations (8) are similar to the LP conjoint analysis algorithm LINMAP as proposed by Srinivasan and Shocker (1973). The only difference is that we are interested in the utilities of alternative durable goods whereas Srinivasan and Shocker were interested in the utilities of factorial combinations of product characteristics.

Following similar methods, we count errors only when the inequality relationships are violated. That is,

$$\text{lottery order error} = (1-\delta_{jk})e_{ojk}^+ + (\delta_{jk})e_{ojk}^- \quad (9)$$

$$u_j - u_k - e_{ojk}^+ + e_{ojk}^- = 0 \quad (10)$$

$$u_j, u_k, e_{ojk}^+, e_{ojk}^- \geq 0$$

where

$$\delta_{jk} = \begin{cases} 1 & \text{if } j \text{ is preferred to } k \\ 0 & \text{if } k \text{ is preferred to } j \end{cases}$$

In equations (9) and (10), the (0,1) variable,  $\delta_{jk}$ , is the "answer" to the lottery order question which tells us which product is preferred as a prize in the lottery. Because the relationship is specified directly in terms of utility, equations (9) and (10) apply for both the value priority and net value priority hypotheses. Unlike Srinivasan and Shocker (1973), we need not worry about the scaling of the utilities because the scaling is already established by the constraints associated with the reservation price and/or purchase probability data<sup>6</sup>.

#### Combination Lottery Prizes

The combination lottery prize questions imply rank order relationships among pairs of utilities. For example, if the combination of goods 1 and 4 are preferred to the combination of goods 2 and 3, then

$$u_1 + u_4 \geq u_2 + u_3 \quad (11)$$

Objective functions for the paired comparison lottery error,

$$(1-\delta_m)e_{cm}^+ + (\delta_m)e_{cm}^- \quad (12)$$

and constraints similar to (9) and (10) can be established for each combination lottery question, m. For ease of exposition, we do not repeat them here.

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<sup>6</sup>This implies that either the weight associated with reservation price ( $W_1$  in LP1) and/or with purchase probabilities ( $W_2$  in LP1) must be non-zero to establish scaling in terms of  $\lambda$ .

## Summary

The estimation LP is now to minimize the weighted sum of errors, given by equations (LP1), (2), (6), (9), and (12) subject to the constraints of (3), (7), (10), and the mathematical formulation of (11). For example, for the six durable goods in Table 1, there are six reservation price relationships, six probability relationships, five lottery order relationships, and seven combination lottery prize relationships totalling 24 constraints and 24 independent errors in the objective function. Because of the complementary slackness and duality theorems, LP1 applies for both the value priority and net value priority hypotheses.

## 6. PREDICTIVE TESTS

The data on reservation prices, purchase probabilities, lottery orders, and combination lottery prizes give us the ability to estimate the utilities of the durable goods in an individual's or a family's budget. If the value priority hypothesis and/or the net value priority hypothesis is a reasonable descriptive representation of consumer purchasing behavior, then the rank order of "value" ("net value"), that is, estimated utility divided by price (minus price), should provide an estimate of the consumer's rank order buying priorities. We formulate a predictive test by comparing the estimated utilities (divided by or minus price) to the consumer's budget priorities.

We illustrate the predictive tests with an example.

### Example Predictive Test

Consider the data in Table 1 and suppose we place equal weight on each data type, that is  $W_1 = W_2 = W_3 = W_4$ . Applying convergent LP estimation provides the estimates of utility shown in the second column of Table 2. Dividing by price (third column) gives the estimates in the fourth column of Table 2. Notice that the estimated utilities would predict that this consumer

would rank 'tuition' as her first budget priority (value = 5.1), a movie camera as her second budget priority (value = 2.5),..., and a freezer as her last budget priority (value = 1.0).

We now compare the budget priority predicted by the estimated utilities to that actually observed. Remember, the observed budget priorities were not used in the estimation, thus, the comparison in Table 2 is a test of predictive ability, rather than of data fitting ability. Comparing rank orders implied by the data in the fifth column of Table 2 to the sixth column we see that the predictions are reasonable but not perfect.

TABLE 2  
EXAMPLE PREDICTIVE TEST

<u>DURABLE</u>	<u>ESTIMATED UTILITY</u>	<u>PRICE (000's)</u>	<u>UTILITY ÷ PRICE (000's)</u>	<u>VALUE PRIORITY ORDER</u>	<u>ACTUAL BUDGET PRIORITY ORDER</u>
Automobile	10.00	5.0	2.0	3	4
Furniture	4.00	2.0	2.0	3	3
Tuition	10.27	2.0	5.1	1	2
Movie Camera	1.22	0.5	2.5	2	1
Vacation	1.50	1.0	1.5	5	6
Freezer	0.30	0.3	1.0	6	5

CORRELATION OF ESTIMATE WITH BUDGET PRIORITY

Spearman  $\rho = .87$

Kendall  $\tau = .69$

Tuition and the movie camera are predicted and observed to be the top two items, but estimated "value" predicts tuition as the top priority while the consumer feels that the movie camera is her top priority. Overall, the Spearman rank order correlation of the predicted rank from utility per dollar (column 5) and the actual rank (column 6) is .87, while the Kendall rank order correlation is .69.

However, equally weighting of the data types is not the only choice. For example, Table 3 indicates the results we obtained by using each data source separately.<sup>7</sup> For this consumer, it appears that the purchase probabilities, lottery orders, and paired lottery prizes each, alone, provide reasonable estimates of budget priorities; however, in this case, reservation prices do not appear to be as good data as the other measures. In fact, if we drop reservation prices and use equal weights on the other three data sources, we get a higher rank order correlation, .93, than if we use all four data sources.

Testing the net value priority hypothesis proceeds similarly. The only difference is we subtract price (in \$000's) from the estimated utility rather than divide by price. For example, for the automobile the net value criterion is  $10.00 - 5.00 = 5.00$  which turns out to be ranked second. For equal W's for this respondent the net value priority hypothesis produces a Spearman rank order correlation of .54. Thus, for this respondent (with equal W's), the value priority hypothesis appears to predict better than the net value priority hypothesis. Unfortunately, because the tests are of different hypotheses rather than nested hypotheses, we can not be rigorous and state whether this difference is significant statistically.

#### Predictive Tests Across Individuals for the Value Priority Hypothesis

Our sample yields 522 potential budgets (174 families times 3 years). Sixty budgets (11.5%) had one or more values missing for either an explanatory or predictive measure. These were spread across measures and demographics and did not appear to represent a systematic bias in measurement. Of the remaining 462 budgets, 247 had 0, 1, or 2 durables

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<sup>7</sup>We report only the Spearman correlation for ease of exposition. Results are similar with Kendall's  $\tau$ . This applies for the remainder of the paper.

TABLE 3

## VARYING WEIGHTS ON TYPES OF INPUT DATA FOR EXAMPLE RESPONDENT

	<u>SPEARMAN CORRELATION</u>
EQUAL WEIGHTS TO ALL FOUR TYPES	.87
RESERVATION PRICE WEIGHTED HEAVILY*	.31
PURCHASE PROBABILITY WEIGHTED HEAVILY	.82
LOTTERY ORDER WEIGHTED HEAVILY	.82
PAIRED LOTTERY PRIZES WEIGHTED HEAVILY	.87

\*"Weighted heavily" means the relevant weight is 100 times more than others. Weights are not set equal to zero to maintain scaling as discussed earlier.

planned. Although the value priority (or net value priority) hypothesis applies to such small budgets, predictions would be perfect by definition for 0 or 1 items and perfect by chance 50% of the time for 2 items in a budget. We felt this would bias our results upward artificially, so we restricted ourselves to the more difficult task of predicting the 215 budgets which contained at least 3 durables.<sup>8</sup>

We applied the predictive tests as illustrated in Tables 1, 2, and 3 to each individual's (or family's) budgets in the resulting sample. To investigate the relative effectiveness of various measures we estimated utilities for each individual (or family) for equal weights ( $W_1 = W_2 = W_3 = W_4$ ), for weighting heavily each data source (as per table 3), and for weighting heavily combinations of data sources (e.g., reservation prices and purchase probabilities, ...). Even with today's mainframe computers and efficient LP software, it was not feasible computationally to search all possible combinations of W's.

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<sup>8</sup>Of these annual budgets, 84 had three items, 54 had four items, 35 had five items and 22 had six items. The remainder had seven or more items up to a high of twelve items. We detected no systematic bias based on the number of items in a budget.

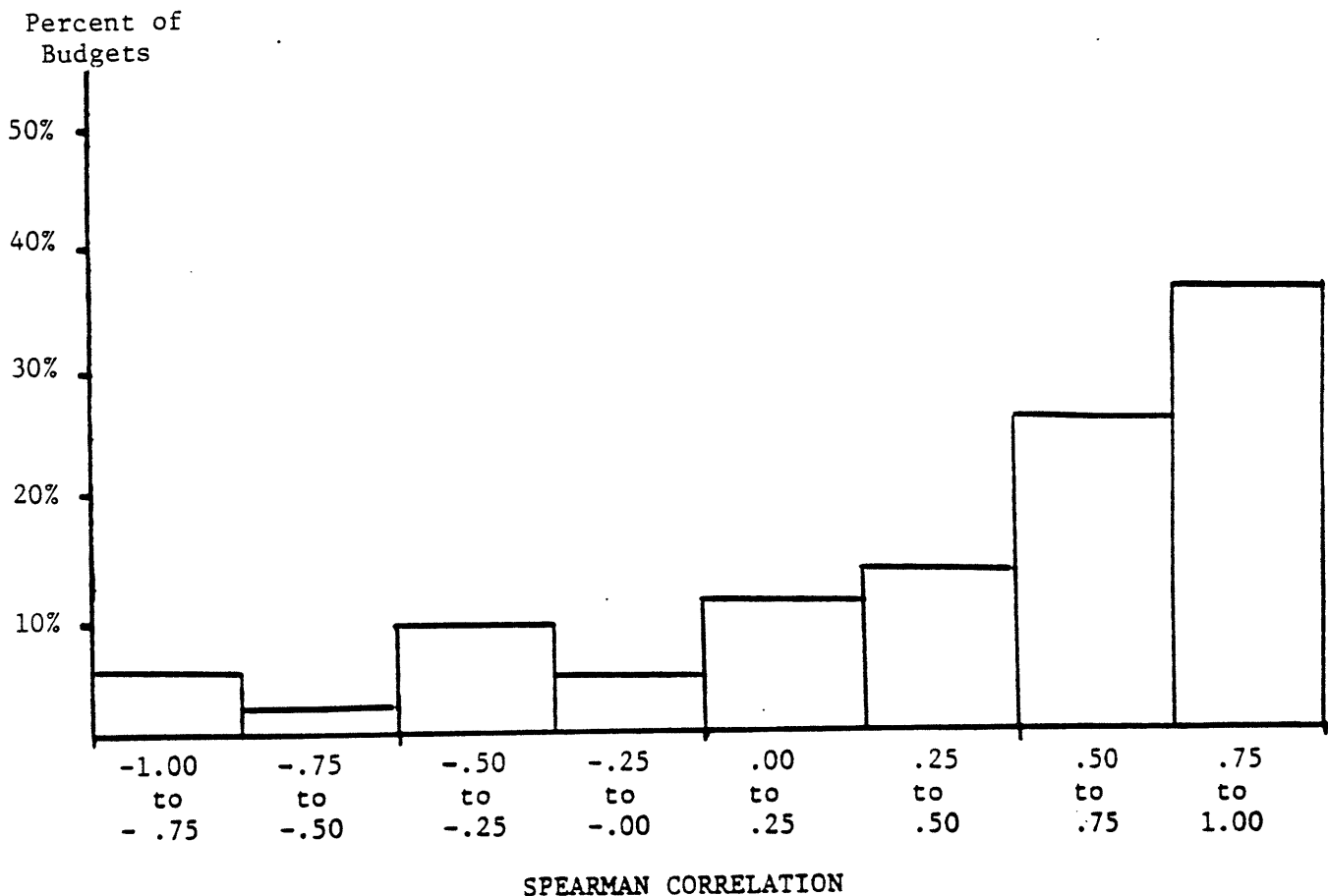
We summarize the data in two ways. To examine the value priority and net value priority hypotheses we report predictions based on the best set of W's (from our limited search as described above) for each individual or family. Then, to examine the relative merits of each data source, we keep the W's the same for all individuals and families. Other means of summarizing the data provide the same qualitative implications and are noted as appropriate.

We begin with figure 1 which reports the Spearman correlations of the predicted and actual budget priorities for the value priority hypothesis. Figure 1 is based on the best W's for each individual (or family) but we use the same weights for all his, her, or their budgets.

Overall, the value priority hypothesis seems reasonable. Roughly 83% of the budgets have positive correlations, 60% have correlations of .50 or better, and

FIGURE 1

VALUE PRIORITY HYPOTHESIS - PREDICTED VS. ACTUAL

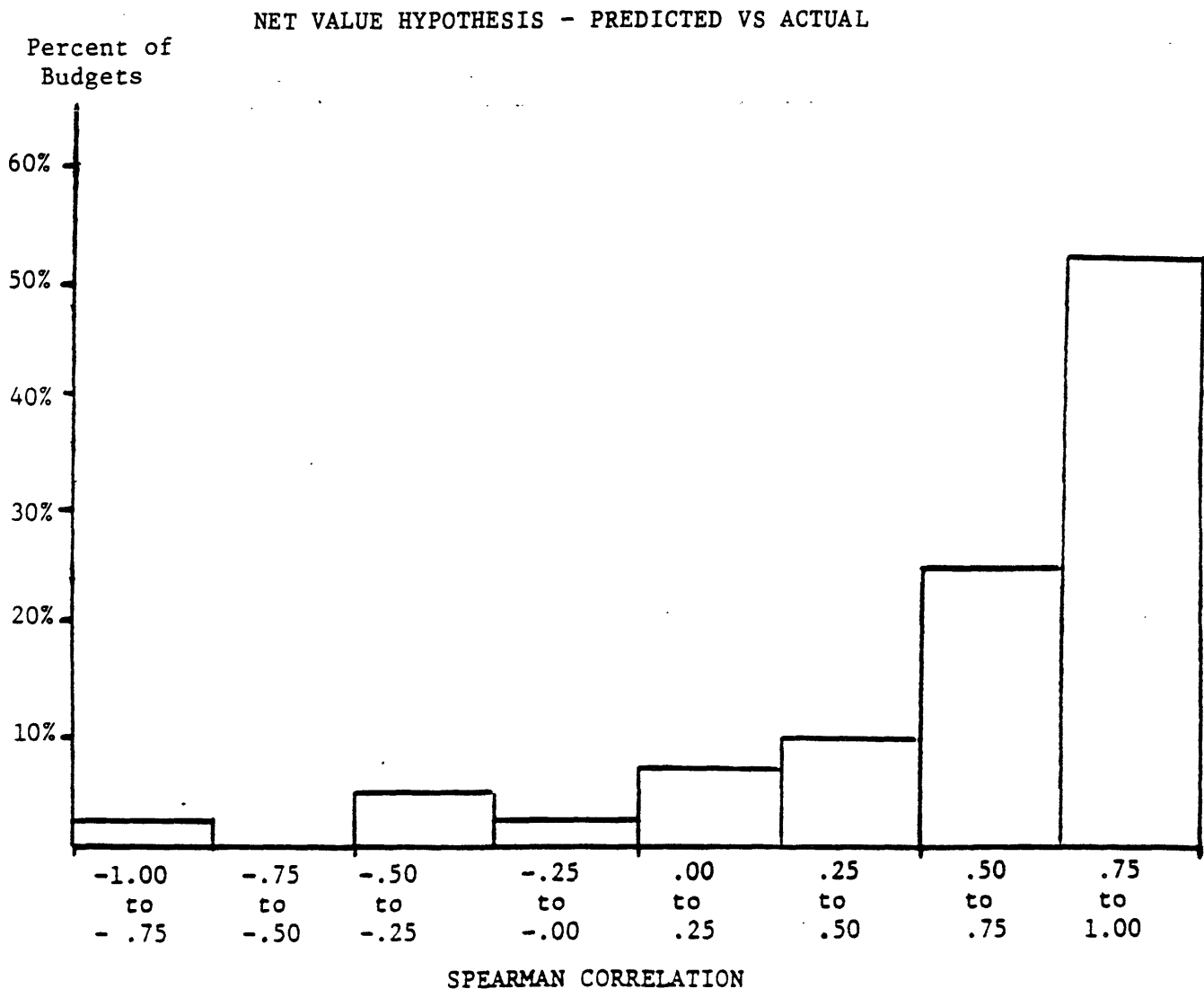


35% have correlations of .75 or better. Significance levels are complex (because many ties are possible), vary by budget (the number of items in each budget varies), and do not apply between hypotheses. There is no single overall critical value that can be applied to Figure 1.

Comparison of the Value Priority and Net Value Priority Hypotheses

Figure 2 reports the Spearman correlations of predicted and actual budget priorities for the net value hypothesis. The net value priority hypothesis also appears to be a reasonable description of consumer purchasing behavior. Roughly 91% of the budgets have positive correlations, 84% have correlations of .50 or better, and 51% have correlations .75 or better. The net value hypothesis appears to do somewhat better than the value hypothesis.

FIGURE 2



Examining the budgets consumer by consumer, 122 budgets (57%) were predicted better by net value priority, 56 budgets (26%) were predicted better by value priority, and 37 budgets (17%) were predicted equally well by both. As Table 4 suggests, we found no significant demographic differences that suggest when one hypothesis predicts better than the other.

TABLE 4

COMPARISON OF VALUE PRIORITY AND NET VALUE PRIORITY HYPOTHESES

	<u>Value Priority Predicts Best</u>	<u>Net Value Priority Predicts Best</u>
No. of Budgets	56	122
Ave. No. of Products/Budget	4.57	4.61
Ave. No. of Autos/Budget	0.48	0.36
Ave. Price of Products in Budget	\$3278	\$3635
Ave. Reservation Price of Product in Budget	\$4232	\$4349
Ave. Age	43.7	44.1
Ave. Income	\$36,200	\$36,300

In summary, both hypotheses do well; neither are rejected; and both are retained for future empirical testing. Although net value priority does better in our tests, the issue is very complex because of the theoretical interrelationships among the hypotheses (through the duality theorem). We return in section 8 to interpret our results in light of these interrelationships.

We now study further the empirical evidence by examining variation in predictive ability across data sources and estimation techniques.

Variation Across Alternative Weightings of Data Types

We report results for emphasizing each of the data sources. For example, if we make  $W_1$  much larger than  $W_2$ ,  $W_3$ , and  $W_4$ , we emphasize reservation prices as the primary data source. The results are shown in Figure 3a.<sup>9</sup>

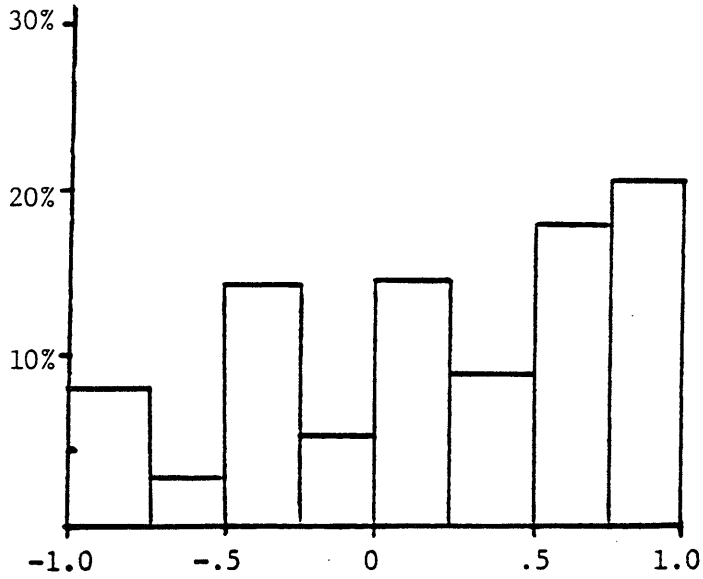
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<sup>9</sup>We report the results for net value priority here. Results for value priority are qualitatively the same and in about the same relationship as summarized by figures 1 and 2.

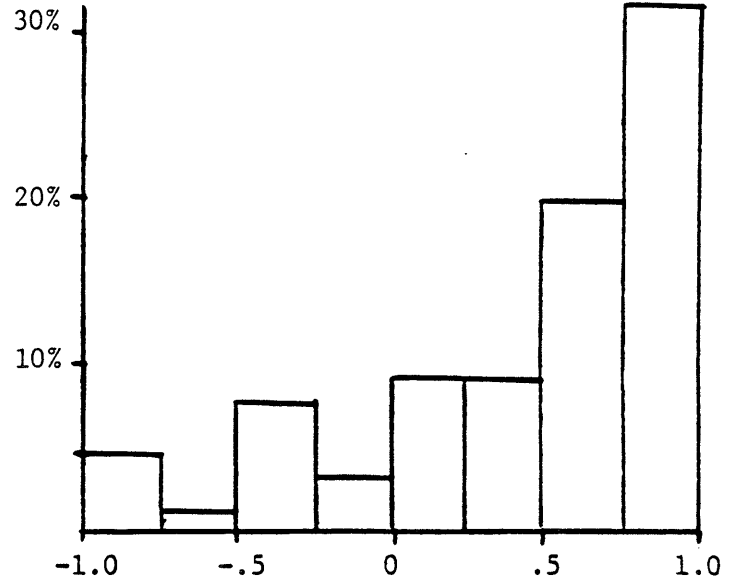
FIGURE 3

PREDICTIVE RESULTS EMPHASIZING DIFFERENT DATA SOURCES

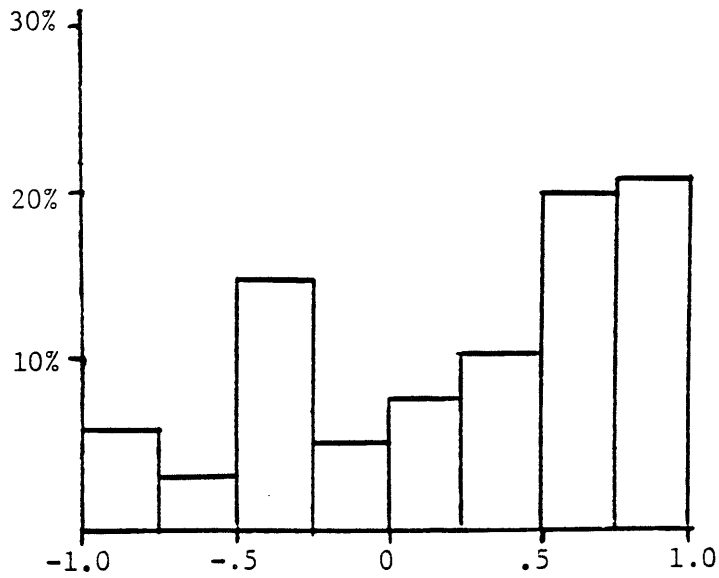
(Percent of budgets with indicated Spearman correlation.)



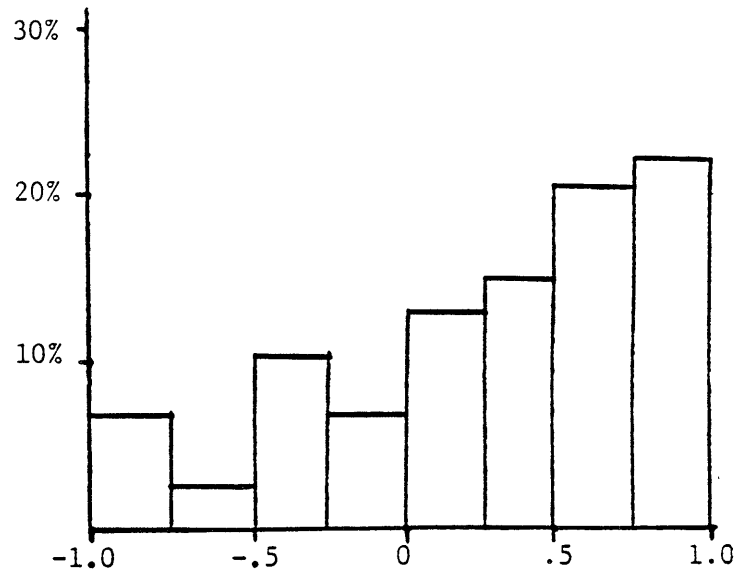
(a) Reservation Price



(b) Purchase Probabilities



(c) Lottery Orders



(d) Combination Lottery Prizes

(Figure 3 report results where the weights do not vary across individual families.) Overall, reservation prices do better than random (67% are positive), but not nearly as well as the results in figure 2. This is not surprising because "reservation price" is a complex concept for many consumers causing the quality of data to vary across consumers. Our example respondent appears to understand the concept, but other respondents clearly did not. For example, some consumers gave a reservation price of \$2001 for an item with an expected price of \$2000.

Figure 3 also reports the results for emphasizing data on purchase probabilities, lottery orders, and combination lottery prize answers. Of the data sources, purchase probabilities are clearly the best (81% positive and 60% with correlations of .50 or better). Lottery orders and combination lottery prizes (67% and 69% positive respectively) do about as well as reservation prices. Results for combinations of two, three, and four data sources tend to be in the range of those in figure 3. Those results also suggest that of the four data sources, purchase probabilities tend to predict best.

Although purchase probability measures appear to be the best indicators of budget priorities, figure 2 suggests that consumers do vary in their abilities to answer any given question format. We recommend a convergent estimation approach that utilizes all four data sources. Convergent linear programming is one such approach. Section 7 illustrates another.

#### Summary of Predictive Tests

Based on convergent linear programming estimation with all four data sources, we are able to estimate utility values for durable products which, with price, forecast well consumers' budget orders. We feel that this is reasonable preliminary evidence that the hypotheses are good first approximations to consumers' purchasing of durable goods.

The comparison of value priority and net value priority shows both criteria do well. Net value priority (focusing on the marginal increase in net utility) does better than value priority (focusing on the budget constraint), but the results do vary by individuals and/or families. We found no systematic reason for the variations, but further research may suggest some hypotheses.

Finally, consumers do vary in their ability to respond to complex utility questions suggesting that utility is best measured with multiple questions and with at least one form of convergent estimation.

#### 7. RESULTS FOR AN ALTERNATIVE ESTIMATION PROCEDURE: LISREL V

Convergent linear programming is one procedure to incorporate multiple data sources. Its strengths are that it can readily accommodate both ordinal and cardinal measures and that the theoretical relationships suggested by the value priority hypotheses can be represented exactly within the structure. Furthermore, it is readily applied on a consumer by consumer basis to identify potential heterogeneities in response to question format and/or behavior. Its disadvantages are that: (1) the cost of searching all combinations of weights ( $W_1, W_2, W_3$ , and  $W_4$ ) to find a single best fit is prohibitive and (2) statistical properties of linear programming estimation are not well known.

There are other estimation procedures which use multiple data sources. Each has its relative strengths and weaknesses. We select one such estimation procedure to demonstrate that our basic result -- the reasonableness of the value priority hypotheses -- is robust with respect to the estimation procedure. The procedure we choose is covariance analysis as implemented by Jöreskog and Sörbom's (1981) LISREL V. (See Heise (1975), Duncan (1975), and Bentler and Bonett (1980) for more details on covariance analysis.)

The advantages of LISREL V is that a best set of weights (in the maximum likelihood sense) can be found for the proper emphasis among data sources and

that the statistical properties are well known when normality conditions hold. The disadvantages are that (1) because LISREL is very sensitive to departures from normality (see Jöreskog and Sörbom 1981 page I.39) it may not do well with our ordinal data or with our transformed probabilities and (2) the estimation is infeasible for small sample sizes as would be the case with individual by individual analyses. See Bentler and Bonett (1980, p. 591).

Since the relative advantages and disadvantages of LISREL V compensate those of convergent linear programming estimation, the resulting estimation provides valuable insight on the value priority hypotheses.

#### Basic Estimation Model

The LISREL V analysis corresponding to convergent LP estimation is the measurement model shown in figure 4. The data sources (boxes in Figure 4) are indicators of the unobservable utility values (circle in Figure 4), thus each measurement, say a reservation price, can be thought of as resulting from the unobserved utility value and a measurement error ( $\delta$ 's in Figure 4). The goal of LISREL V is to estimate the correlations (known as factor loadings) relating the observables to the unobserved utility and then to use the structure to estimate a value (known as a factor score) for the unobserved utility. We use the theoretical relationships as implied by the value priority hypotheses to specify the appropriate transformations of the raw measurements.

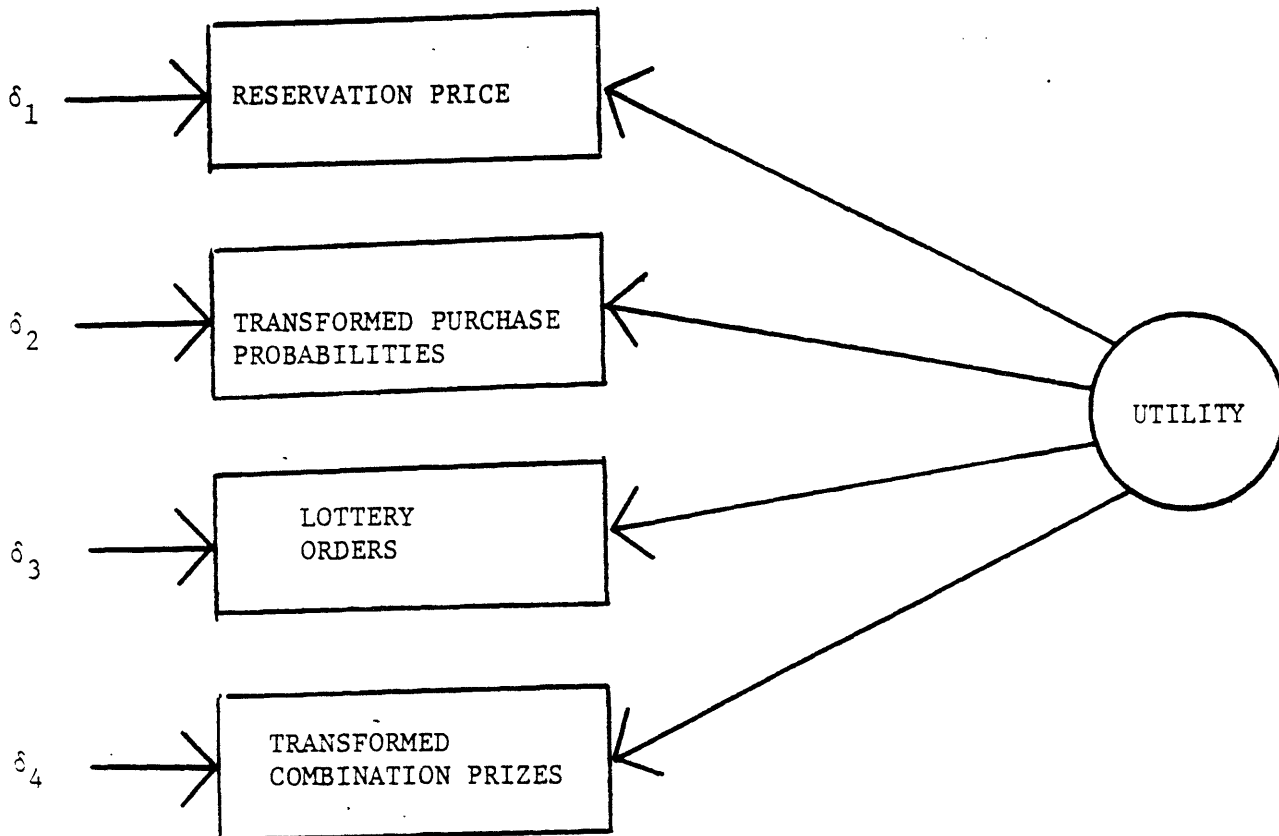
#### Measures

Based on the theory derived in section 5, the appropriate measures are:

- (1) Reservation prices as implied by equation 3 with  $\lambda = 1$ .
- (2) Logit transformed probabilities as implied by equation 7 with  $\lambda = 1$ . We allow the estimation to determine the scaling constant  $\beta^{-1}$ .

- (3) Lottery orders. These orderings are rank order measures and may violate strict normality assumptions, but they are monotonic in utility.
- (4) Combination lottery prizes. The rank order relationships implied by equations 11 and 12 are complex, dependent on each individual, and interrelated with lottery orders. They are not readily handled by the linear equations of LISREL V. We use as a surrogate the number of times a durable is chosen from the set of combinations. This measure is clearly monotonic in utility. Again, normality is a concern.

FIGURE 4  
MEASUREMENT MODEL FOR LISREL V



## Estimation Results

The maximum likelihood estimation results are shown in Table 5. The estimation is based on 932 observations corresponding to the total number of budgeted items in the 215 budgets. Overall, the measurement model does remarkably well. The goodness of fit index (which "is independent of sample size and relatively robust against departures from normality" - Jörgskog and Sörbom 1981 page I.41) suggests that 99.9% of relative covariance is accounted for by the model. Even adjusted for degrees of freedom this measure is 98.9%. The coefficient of determination for the overall model is 92.4% suggesting high overall reliability of the measurement model. The chi-squared value is low, 0.87, indicating that no addition of free parameters would improve the model significantly.

TABLE 5

### ESTIMATION RESULTS FOR LISREL V MEASUREMENT MODEL

<u>Variable</u>	<u>Estimated Factor Loading</u>	<u>Asymptotic t-statistic</u>	<u>Squared Multiple Correlation (Reliability)</u>
Reservation Price	.954	23.0	.911
Trans. Probabilities	.812	20.7	.660
Lottery Orders	.166	8.6	.086
Combination Prizes	.293	4.9	.027
<hr/>			
Goodness of Fit Index		.999	
Adjusted Goodness of Fit Index		.989	
Coefficient of Determination		.924	
Root Mean Square Residual		.005	
Chi-Squared (1 degree of Freedom)		0.87	

For the specific measures, reservation prices have the highest reliability followed by transformed probabilities. Both have excellent asymptotic t-statistics. The rank order measures fair less well with low reliabilities but

good t-statistics. Normalized residuals (not shown) are reasonable for the cardinal measures, but do depart somewhat for the rank order measures. Since the latter is to be expected, since the t-statistics are acceptable, and since we desire a comparison with the convergent LP estimation, we retain all measures in the model.

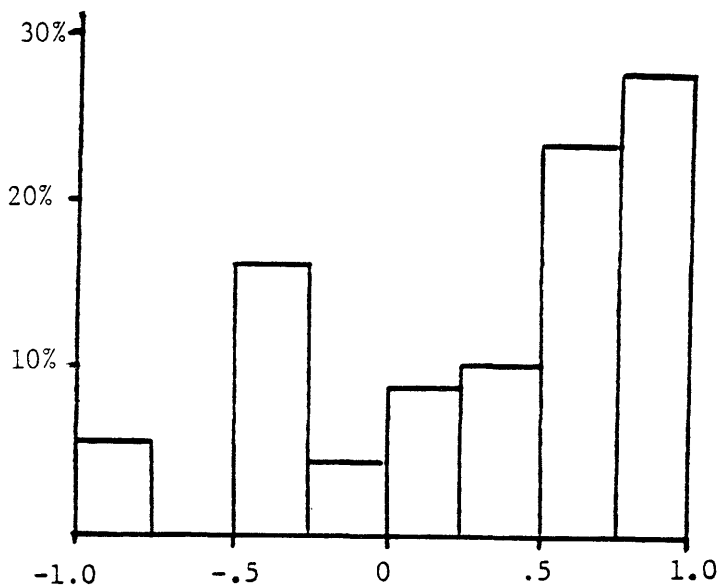
Predictive Tests

Based on the measurement model in Table 5, we use the LISREL V factor score regressions<sup>10</sup> to estimate utility for each durable in each budget. We then divide by price to forecast value priorities or subtract price to forecast net value priorities. As per section 6 we compare the LISREL V forecasts to actual consumer budget orders. See Figure 5.

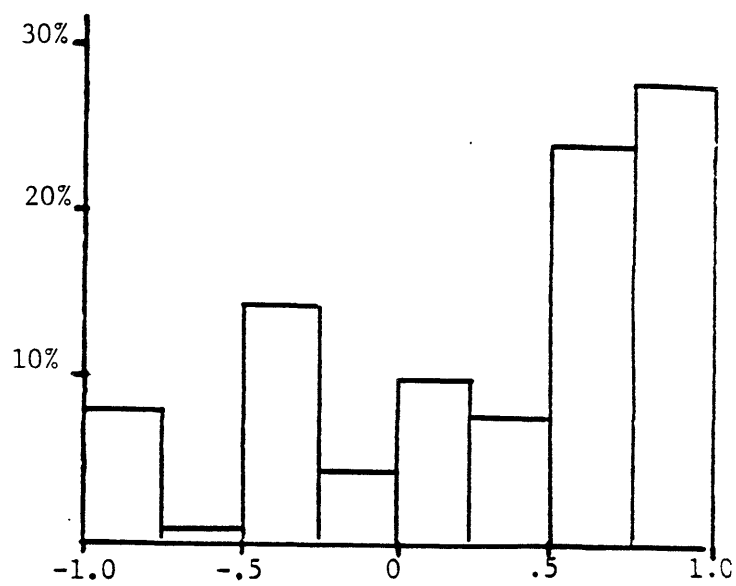
FIGURE 5

PREDICTIVE RESULTS WITH LISREL V ESTIMATES

(Percent of budgets with indicated Spearman correlation.)



(a) Value Priority Hypothesis



(b) Net Value Priority Hypothesis

<sup>10</sup>The factor score coefficients are .805, .180, .026, and .015, respectively, for the four measures.

The LISREL V estimates appear comparable to the convergent LP estimates which do not vary by family, 73% of the correlations are positive and 55% are .50 or better for the value priority hypothesis while 71% are positive and 54% equal or above .50 for the net value priority hypothesis.<sup>11</sup> These are better than those obtained in figure 3 for reservation prices, lottery orders, or combination prizes as single measures and almost as good as for purchase probabilities. Of course, LISREL V does not do as well as the family by family estimates in Figures 1 and 2.

Based on the similarity of LISREL analysis to the convergent LP analysis we have more confidence in our proposition that at least one of the value priority hypotheses is a reasonable first order model of durable purchasing behavior.<sup>12</sup>

## 8. DISCUSSION

The value priority and net value priority hypotheses are models of how consumers allocate their budgets to durable goods. Both hypotheses are derived from the standard economic model of maximizing utility subject to a budget constraint. However, both recognize the evidence in a variety of scientific disciplines that suggests that behavior as observed may differ from behavior as prescribed. Both hypotheses imply that the consumer (or family) uses a simple

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<sup>11</sup>The value priority hypothesis does slightly better with LISREL V than does the net value priority hypothesis, but this result is probably not significant. The best comparison among the hypotheses remains the family by family analysis. Review Table 4.

<sup>12</sup>Curiously, LISREL V selects reservation price as the most reliable measure but the predictive tests (a measure of validity) suggest that purchase probabilities may predict better. Further research might estimate a more complex structural model including the dependent variable in the estimation. We did not do this because we felt it more appropriate to have an independent test of predictive ability that did not use the budget orders in the estimation and because the dependent measure was at best an ordinal measure which clearly causes problems with LISREL V. Furthermore our goal is to test the value priority hypotheses not compare the relative merits of linear programming and LISREL V. Finally, a full comparison of technique would best be done with multiple measures of the dependent variable in a variety of contexts.

heuristic that leads to near optimal behavior under a wide variety of conditions. This heuristic is to rank order durables according to value (or net value) and purchase items in that order up to and including a budget cutoff. The two hypotheses differ only in their derivation of the numeraire by which durables are ranked.

The empirical evidence in sections 6 and 7 suggests that both hypotheses are reasonable. We are comfortable with proposing both for further testing.

Figures 1 and 2 also suggest that the net value priority hypothesis may be the better predictor. However, before embracing the net value hypothesis, there are a number of cautions worth considering. These include the complex interrelationship among the two hypotheses, the distinction among descriptive and prescriptive theories, and the empirical observation of high rank order correlation among  $u_j/p_j$  and  $u_j - p_j$ . We discuss each in turn.

Interrelationships Among the Hypotheses

If durables were not discrete, then the duality and complementary slackness theorems would imply that the optimal solutions to MP2 (value priority) and MP3 (net value priority) are the same. This means that for a given budget, B, and budget cutoff,  $\lambda^*$ , the two criteria,  $u_j/p_j \geq \lambda^*$  and  $u_j - \lambda^*p_j \geq 0$  would yield the same overall budget. (This can also be seen by dividing through by  $p_j$  in the net value criterion.) This does not mean that the order of purchasing, the rank order of  $u_j/p_j$  and  $u_j - \lambda^*p_j$ , will be the same.

Consider three items, a home improvement, landscaping, and a food processor, that are part of the budget of one of our respondents, a married 37 year old male with two children and a \$65,000 family income. Following the estimation procedure described in section 5, we get the following utilities scaled such that  $\lambda = 1$ .

	<u>Utility</u>	<u>Price (\$000's)</u>	<u><math>u_j/p_j</math></u>	<u>Value Priority</u>	<u><math>u_j - p_j</math></u>	<u>Net Value Priority</u>
Home Improvement	.994	0.60	1.7	3	.394	1
Landscaping	.657	0.30	2.2	2	.357	2
Food Processor	.328	0.08	4.1	1	.248	3

Net value priority predicts the order as shown, home improvement, then landscaping, then the food processor. Value priority predicts the reverse order. For these three items the respondent actually chose home improvement, then landscaping, then the food processor. For this consumer, net value priority appears to be a better descriptive model. (Review table 4 for more general results.)

#### Assumption of Stable $\lambda$

The example above does not indicate what would happen if the budget, B, the utilities,  $u_j$ , or the availability of products changed. The value priority criterion,  $u_j/p_j$ , would not change. On the other hand, the net value criterion would remain unchanged only if  $\lambda^*$  did not change.

However,  $\lambda^*$ , which equals  $\partial u_j(y^*)/\partial y$  at the optimum solution to MP3, may change if B or the  $u_j$ 's change.<sup>13</sup> If the change were sufficiently dramatic, the net value ordering could change. Thus, the net value priority hypothesis assumes that  $\lambda^*$ , or at least the consumer's perceived  $\lambda$ , changes slowly. This assumption is worth testing.

#### Descriptive vs. Prescriptive Hypotheses

We stated both the value priority and net value priority hypotheses as descriptive hypotheses. They may or may not lead the consumer to the "best" decisions.

For example, consider the consumer discussed above and suppose there were another durable, say a tabletop convection oven, with the same utility and price as the food processor. Then, for \$600 of budget allocations the two hypotheses recommend:

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<sup>13</sup>A change in  $\lambda^*$  would affect our scaling convention of  $\lambda = 1$ . The utilities are a function of  $\lambda^*$  and may themselves change if we change the budget problem yet restrict  $\lambda$  to be 1.0. For the analyses of sections 6 and 7 the budget problem does not change hence the restriction,  $\lambda = 1$ , is not critical for our predictive tests. It could become critical in other situations.

<u>Value Priority Utility</u>		<u>Net Value Priority Utility</u>	
Food Processor	.328 for \$ 80	Home Improvements	.994 for \$600
Convection Oven	.328 for \$ 80		
Landscaping	.657 for \$300		
Total	1.313 for \$460	Total	.994 for \$600

Prescriptively, (if our utilities are accurate) the consumer would have been better off (more utility and less money) using the value priority than using net value priority.

Such examples are easy to create. Indeed, if there were no integer constraints on durable purchases, the value priority algorithm is, prescriptively, the best algorithm. Even with integer constraints, it does not do badly and has reasonable worst case properties<sup>14</sup> (Cornuejols, Fisher and Nemhauser 1977 and Fisher 1980). However, we can also create examples to favor net value priority over value priority, so, again, we must interpret all prescriptive results with caution.

### Correlation

The value priority and net value priority hypotheses have quite different behavioral interpretations. However, they may be difficult to distinguish from observations of behavior because the two criteria,  $u_j/p_j$  and  $u_j - p_j$ , have high rank order correlations. To illustrate this we drew 10,000 random values of  $u_j$  and  $p_j$  from a uniform distribution, each of five times. The resulting linear correlation<sup>15</sup> of  $\log(u_j/p_j)$ , which is monotonic in

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<sup>14</sup>The theoretical worst case is a factor of two. For example, with a budget of \$1000 and two products of utility 5.02 and 5.00 that cost \$501 and \$500 respectively, the optimum is two units of the second product rather than one unit of the first product. But if the budget can be relaxed or non-durables purchased this worst case result is mitigated.

<sup>15</sup>We seek to demonstrate the rank order correlation of  $u_j/p_j$  and  $u_j - p_j$ . Thus, we seek monotonic transformations of either or both variables such that the linear correlation is maximized. The logarithmic transformation is effective providing a reasonable lower bound on the maximum rank correlation. Technically, some transformation is necessary because the mean and variance of  $(u_j/p_j)$  are both infinite for  $u_j$  and  $p_j$  i.i.d. uniform. In addition, the logarithmic transformation gives the intuitive interpretation that  $\log(u_j/p_j) = \log u_j - \log p_j$  which we expect to be related to  $u_j - p_j$ .

$u_j/p_j$ , and  $u_j - p_j$  was quite high, 0.87. This suggests an even higher rank order correlation.

Because of this rank order correlation, we must interpret with caution any empirical comparisons of observed behavior. This does not mean the hypotheses are indistinguishable. For example, verbal protocols or process tracing technology may be able to distinguish among the hypotheses.

In summary the evidence favoring at least one of the hypotheses as a description of durable purchasing is favorable. However, comparisons among the hypotheses must be made with caution and subject to further testing.

## 9. SUMMARY AND SOME MANAGERIAL IMPLICATIONS

### Summary

Based on our data, estimation, and predictive tests we feel:

- the ranking of durables according to a budget priority appears to be a reasonable descriptor/predictor of planned durable purchases;
- both value and net value provide reasonable approximations to the numeraire by which durables are ranked;
- it is feasible to measure "utility" across categories if multiple convergent measures are used; and
- convergent LP estimation is feasible, provides reasonable estimates of "utility," and appears consistent with LISREL V.

For these postulates the empirical evidence is strong.

In addition, our analyses suggest the following postulates, subject to future research:

- consumers vary in the heuristic numeraire, value or net value, they use for ranking; and
- consumers vary in their ability to answer specific question types.

The former postulate is based on the empirical evidence of section 6 but must be interpreted relative to the discussion of section 8.

The second postulate is no surprise to the behavioral researcher who faces often the difficult task of estimating unobserved constructs. It does provide a caution to the market researcher or management analyst faced with limited measurement budgets who wants to forecast durable purchases. Convergent measurement is probably necessary.

### Managerial Implications

We close on a practical note. The value priority (or net value priority) hypotheses can and are useful in forecasting sales in existing durable product classes. Once utilities are estimated for a sample of consumers we can forecast the implications of new products, improved products, or changes in prices or economic conditions. New or improved products change utilities and economic conditions change the budgets. For each consumer we compute the value criterion (or net value criterion if  $\lambda$  does not change) and recompute the buying order. For example, a megabyte personal computer, a digital stereo/VCR, or a mini van may have high enough value (or net value) to enter the budget of some consumers. The percent of consumers who now budget for the new product form is a forecast of its category sales.

The measurement system described in this paper is in use at General Motors and has provided valuable managerial insight into which durable goods compete most with luxury automobiles. For example, Table 6 lists those goods which were ranked above automobiles when both were in the budget. Management has reacted to this input and recent automobile design and marketing campaigns have been based on budget priority analyses that consider product improvements or advertised image (less maintenance, improved comfort, etc.). Such changes are designed to increase the utility of an auto purchase and thereby move it up in the buying priority. The value priority model is also used to determine if the introduction of a new automobile will improve the position of an automobile purchase in the ordering. After having consumers drive the new automobile in a

clinic environment, the new automobile's utility is measured relative to the respondent's current first choice automobile to determine if it is higher in the respondent's priority ordering than the previous automobile.

Application work is continuing to ascertain the managerial usefulness of the value priority hypotheses in managing new and established durable consumer goods.

TABLE 6

DURABLE GOODS COMPETING WITH AUTOMOBILES

(Percent of Time Ranked Above Automobile When Both are in Budget)

<u>DURABLE</u>	<u>PERCENT</u>	<u>DURABLE</u>	<u>PERCENT</u>
1. School Tuition 1983	96.4	13. Dishwasher	63.2
2. Vacation 1983	92.8	14. Color Television	59.1
3. Home Improvement (Minor)	84.0	15. Stereo System	57.9
4. Major Clothing	78.8	16. Jewelry	55.6
5. Landscaping	77.8	17. House	53.3
6. School Tuition 1984	76.7	18. Oven	50.0
7. Gifts/Donations	76.0	19. Movie/Video Camera	50.0
8. Cameras and Accessories	70.6	20. Video Tape Recorder	46.9
9. Furniture	68.0	21. Refrigerator/Freezer	46.2
10. Home Fuel Savings Device	67.7	22. School Tuition 1985	45.0
11. Home Improvement (Major)	67.3	23. Home Computer	44.7
12. Vacation 1985	64.2	24. Vacation 1985	37.0

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APPENDIX 1

Extension of the Value Priority Hypotheses to Multiple Periods with Borrowing (Savings), Depreciation, Operating and Maintenance Costs, Price Expectations, Trade-ins, and Pairwise Complementarity.

The value priority hypotheses of sections 2 and 3 are readily extendable. We state here the equations as derived in Hauser and Urban (1982).

Let  $u_{ji}$  be the utility of the  $i$ th item of the  $j$ th good,  $p_{jt}$  be the expected price of that good at time  $t$ ,  $\delta_{jit}$  be a zero-one indicator of whether the  $i$ th item of good  $j$  is purchased. Note  $\delta_{jit} > 0$  only if  $\delta_{j,i-1,t} = 1$ . Let  $u_y(y_t)$  be the utility of spending  $y_t$  on non-durables in time  $t$ . Let  $B_t$  be the consumer's budget constraint in time  $t$ ,  $D_t$  be his debt in time  $t$ , and  $b_t$  be the amount borrowed (saved) in that period. Let  $d_j$  be the depreciation rate for good  $j$  and  $r$  be the interest rate. Let  $c_{jn}$  be the operating and maintenance cost of durable  $j$ ,  $n$  periods after purchase.

The consumers' problem (MP4) is:

$$\text{maximize } \sum_{t=1}^T \sum_j \sum_i \left[ \sum_{q=0}^{T-t} d_j^q u_{ji} \delta_{jit} \right] + \sum_{t=1}^T u_y(y_t)$$

$$\text{subject to } \sum_j p_{jt} \left( \sum_i \delta_{jit} \right) + \sum_{n=1}^{T-t} c_{jn} \delta'_{j\&lt;t} + y_t - b_t \leq B_t$$

$$D_t = D_{t-1}(1+r) + b_t, \quad D_T = 0 \quad \text{for all } t$$

$$\delta_{jit} = 0,1; \quad \delta'_{jis} = 1 \quad \text{iff} \quad \delta_{jit} = 1 \quad \text{and} \quad s > t$$

The value priority criterion for the LP relaxation of the integer constraints now becomes:

$$u_{ji} \left[ \sum_{q=0}^{T-t} d_j^q / (1+r)^{T-t} \right] / \left[ p_{jt} + \sum_{n=1}^{T-t} c_{jn} \right] \quad (A1)$$

Finally, trade-ins are handled by computing net depreciated utility gain divided by net price and pairwise complementarity is added with hierarchical dependence of  $u_{ji}$  on another good  $k$ . For the net value hypothesis, the criterion is (numerator of A1) -  $\mu$  (denominator of A1) where  $\mu$  is the simplex multiplier of the debt constraint,  $D_T = 0$ .

## APPENDIX 2

### Dual Linear Programs - Non-Technical Summary

An important concept in linear programming is that for every linear program, there is a related dual linear program. The variables of the dual are known as simplex multipliers, or shadow prices. Each variable of the dual corresponds to a constraint in the original linear program and represents the "sensitivity" of relaxing that constraint, i.e., the amount by which the objective function would change if that constraint were relaxed.

If the original linear program is a maximization problem, then the dual program has as its objective the minimization of a weighted sum of the dual variables. The weights are the constants in the constraints of the original linear program. The constraints of the dual are based on the constraints and objective function of the original linear program. For example.

<u>Original</u>	<u>Dual</u>
max $c_1x_1 + c_2x_2$	min $b_1u_1 + b_2u_2$
subject to: $a_{11}x_1 + a_{12}x_2 \leq b_1$	s.t. $a_{11}u_1 + a_{21}u_2 \geq c_1$
$a_{21}x_1 + a_{22}x_2 \leq b_2$	$a_{12}u_1 + a_{22}u_2 \geq c_2$
$x_1, x_2 \geq 0$	$u_1, u_2 \geq 0$

Note that  $u_1$  corresponds to the first constraint in the original program and represents the value of relaxing that constraint.

The duality theorem states the amazing result that the optimal values of the object functions of the two linear programs are identical. Complementary slackness states that if a dual variable has a non-zero value in the optimal solution to the dual, then the corresponding constraint in the original program must be binding, and visa versa. For a more complete and technical exposition see Gass (1969) or any linear programming text.

Note that the dual of the dual is the original linear program.