

LONG-TERM OIL WARRANTS:  
AN APPLICATION TO  
VENEZUELAN DEBT RELIEF

by

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A.B., Harvard College (1982)

Submitted to the Alfred P. Sloan School of Management  
in Partial Fulfillment of  
the Requirements for the Degree of

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**ABSTRACT**

Long-term warrants have been proposed as part of packages for restructuring the debt of highly-indebted countries (HICs), such as Venezuela. The proposed warrants are essentially call options on a commodity produced by the country (e.g., oil). Valuation of these warrants is problematic, since widely-used methods for derivative-asset valuation, such as the Black-Scholes formula, become less precise as time to maturity increases.

A ten-year strip of one-year warrants is used, with Venezuelan crude oil as the underlying asset. Results from a standard Black-Scholes valuation for the warrant strip are compared with results from a Jacoby-Laughton/Fama-Cox-Ross derivative-asset valuation model using the same set of assumptions about the economic environment (inflation rate, real interest rates, etc.) and asset-related factors (e.g., initial oil price, volatility of oil prices). The theoretical foundations of the Jacoby-Laughton model come from the work of Fama (which led to the Capital Asset Pricing Model, or CAPM), and the Cox-Ross options-pricing model. The Jacoby-Laughton/Fama-Cox-Ross (JL/FCR) model produced the same results as Black-Scholes when given the same parameters. The JL/FCR model permits correction for the Black-Scholes tendency to overvalue long-term options, as well.

A method of measuring the political risk of the warrants was developed, and the amount of debt relief gained from a warrant-based restructuring was calculated. More accurate valuation of such long-term options may lead to greater flexibility in restructuring HIC long-term debt.

Thesis Supervisor: Prof. Henry D. Jacoby  
Title: Professor of Management

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## **1 Introduction**

Large sums of money were borrowed from commercial banks by countries in the developing world in the 1970s to fund natural-resource development projects, such as oil production. These projects were often conceived during times of high market prices for the commodity, and expected returns on the borrowed money clearly exceeded the cost of borrowing (Randall 1987; Teichman 1988). In the 1980s, oil prices fell to around \$14 U.S. per barrel, resulting in severe cash-flow problems for oil-producing countries such as Mexico and Venezuela. Oil production was the main revenue source for these countries, yet oil revenue was insufficient to meet the countries' debt payments.

Both Mexico and Venezuela have sought debt relief from their lenders through renegotiation of existing debt. The lenders seek to maximize the amount repaid while minimizing their exposure to risk of non-payment. The countries want to minimize the impact of debt service while retaining access to loans. Lenders and countries both want to continue a valuable relationship, as recent debt-restructuring agreements in both Mexico and Venezuela clearly show.

Current debt-relief efforts have been heavily influenced by the Brady plan, a framework for HIC debt restructuring proposed and supported by the United States. The Brady plan (McNamee et al. 1990) emphasizes negotiations between the debtor governments and their creditor banks to reduce the amount of existing debt and/or the size of debt payments, with potential supplementary lending from supranational organizations such

as the World Bank and the IMF. Brady plan-style agreements do not match the amount of the government's debt payments to the ability of the government to pay as a function of time. This is a significant flaw in this type of solution to debt relief.

This thesis will concentrate on the financial aspects of proposed solutions to the Venezuelan debt-reduction problem, with special focus on a 1989 proposal based on a bond/warrant combination. A valuation method, based on the Jacoby-Laughton derivative asset valuation method, will be developed for the warrants described in the 1989 proposal. Finally, a measure of the warrants' political risk will be derived from the valuation method.

## **2 Valuation Issues in LDC Debt Reduction**

Determination of the real value of debt, of debt reduction and of risk is one of the major problems seen in debt-reduction negotiations. Debtors and lenders have opposing goals in valuation; debtors seek to gain the maximum credit at the lowest cost, while lenders seek to maximize return on funds lent while minimizing risk. This section will review the recent history of debt-reduction efforts in Mexico and Venezuela as a framework for understanding some of the political and financial issues in valuation of LDC debt relief.

### **2.1 History of Debt-Reduction Efforts**

During the late 1970s and early 1980s, developing countries saw decreases in GNP and government revenue, as world prices for several important commodities declined. Oil was probably the most important of these commodities for fast-growing LDCs, which were either oil exporters (Mexico, Venezuela) or oil importers (Brazil). Heavy government borrowing for projects in the 1960s and 1970s was based on the expectation of higher commodity prices in the 1980s than actually occurred. This brought many 'debt crises' to LDC governments, as they were less able (or unable) to meet their debt payments. As the risk of default increased, negotiations were initiated to restructure foreign debts in countries ranging from Mexico to the Philippines.

The current approaches to the debt-relief problem in Mexico and Venezuela are similar, and the following sections will describe the historical context behind these countries'

methods in greater detail, with emphasis on recent debt-reduction negotiations.

### 2.1.1 Mexico

The deterioration of Mexico's economy during the 1960s and early 1970s decreased government revenue and provoked increasing dissatisfaction with the Mexican government's economic policy. The 'shared development' strategy, which shifted government investment away from the industrial sector and towards agriculture and social-welfare programs, was a political solution to the government's dilemma. Mexico's substantial petroleum reserves were increasingly seen as the source of the financial solution to the government's problem, but exploitation of these reserves for export ran counter to deeply-held beliefs, dating to the Mexican revolution, about the proper use of Mexican natural resources.

Higher levels of oil production required an increase in investment in oil-production capacity. Mexican production capability in the early 1970s was insufficient to meet Mexican demand, so substantial investment was needed to support an export-oriented strategy.

This change in oil-development strategy laid the foundations for the 1982 'debt crisis'. Increased dependence on oil exports required the government to continually increase its capital investments for exploration and production. Increased need for capital investment further accelerated Mexico's acquisition of foreign debt, as Mexico was not

able to finance both increased oil production and increased social spending out of its current income.

The combination of falling prices for crude oil in the early 1980s and increasing short-term world interest rates led to the 1982 debt crisis. As crude prices dropped, Mexico was increasingly forced to seek short-term bank loans to make payments on its huge foreign debt (about \$84 billion in 1982). Foreign banks became less willing to make short-term loans to Mexico as its oil revenues declined, and loan terms were made more restrictive (shorter loan periods, higher interest rates).

#### **2.1.1.1 The 1989 Agreement (exit bond)**

The 1982 debt crisis was resolved through a debt-restructuring agreement which allowed lenders to wind down their lending to Mexico through acceptance of one of two types of "exit bond", or by refinancing the bank's loan through a combination of new debt, or "new money", and existing debt ("old money"). "New debt" loans were limited to an additional amount equal to 25 percent of the existing debt owed to a particular bank. When existing debt was swapped for "exit bonds", Mexico's liability to the lenders was reduced by either refinancing the same amount of debt at a lower interest rate ("interest-reduction bond"), or by reducing the principal amount of the debt ("principal-reduction bond") while keeping the old, higher interest rate. The "exit bond" option allowed lenders pessimistic about Mexico's future prospects to avoid a total loss from their Mexican debt exposure, while providing some relief to the Mexican government by

reducing its payments to the lenders. Refinancing with "new money" was chosen by lenders more optimistic about Mexico's long-term ability to repay both principal and interest on its debt. The agreement allowed the banks to structure their Mexican debt portfolio, using these three basic building blocks in whatever proportion made sense to the bank. No consistent assessment appears to exist of the riskiness of the restructured debt; some lenders are writing down the principal-reduction bonds and the interest-reduction bonds, (e.g., J.P. Morgan), while others (e.g. Manufacturers Hanover) are writing down only the principal-reduction bonds (American Banker 1990).

The flexibility of the Mexican restructuring plan makes Mexican-style restructurings attractive to both lenders and governments, and the plan has served as a model for refinancing proposals in other countries. The following section will discuss the historical context of the recent Venezuelan debt-relief negotiations and two recent restructuring proposals (one based on a bond/warrant combination and the other a Mexican-style proposal).

### **2.1.2 Venezuela**

Revenue from oil production is very important to Venezuela. Oil export revenue is a very large percentage (over 90 percent through the 1970s) of all Venezuelan revenue received from exports. This great dependence on one source of income from a non-renewable resource has been of great concern to recent Venezuelan governments, which have sought to direct this income towards investments in non-petroleum renewable resources and general infrastructure improvements. Investments in the petroleum sector

were taken to assure future income flows from oil sufficient to fund Venezuela's drive towards greater economic independence. This strategy, similar to the Mexican development strategy, was originally referred to as *sembrar el petróleo* [sowing the oil]. (Tugwell 1975, 33)

Lower oil-price levels in the 1980s severely reduced Venezuelan government revenue and the government's ability to meet payments on its debt. By 1989, Venezuela and its bank lenders were locked in debt-relief negotiations. Several debt-relief proposals were considered during the 1989-90 debt crisis; the next two sections will describe the basics of both the November 1989 warrant-based proposal, and the early 1990 Mexico-style proposal (which was accepted by both sides in March 1990).

#### **2.1.2.1 The 1989 Bond/Warrant Proposal**

The November 1989 debt-restructuring proposal divided Venezuelan external debt into two parts. Part of the debt (e.g., one-half) would be refinanced through a deeply discounted bond; the rest would be restructured as a ten-year (120-month) strip of one-month warrants on Venezuelan oil (Lysy 1989). Each warrant is an option to purchase some portion of future production of Venezuelan oil (e.g., 1000 barrels) at some specified price (e.g., \$25 U.S.) at the end of a one-month period. The purchase price specified in the warrant is called the "exercise price"; the date when the warrant comes due is called the "maturity date".

In finance-theory terms, the warrants are European call options, since the warrant-holder has bought the right to purchase Venezuelan oil, and cannot exercise that right until the end of the one-month period (the "maturity date") covered by the warrant. Obviously, the warrant-holder will not choose to purchase the oil if the oil price at the maturity date is less than the exercise price (this would be like paying \$25 for an asset worth \$20).

Initial proceeds from the sale of the warrants would go to the Venezuelan government (the "writer" of the warrants). The purchasers, who initially would be the banks holding Venezuelan debt, would get the proceeds (if any) as the warrants mature. The warrants would have a cap limiting the maximum payoff from the Government to the banks (Lysy 1989). The warrants potentially would be tradeable on international exchanges like any other international option. The bond/warrant approach could serve as the basis for a total restructuring of Venezuela's external debt, depending on relevant factors, such as Venezuela's desired rate of oil production and export, projected world interest rates, or Venezuelan government need for working capital.

#### **2.1.2.2 The 1990 Venezuelan Agreement**

On March 20, 1990, an debt-restructuring agreement was reached between the Venezuelan government and its principal creditor banks. The agreement gives lenders five options, including: debt buyback by the Venezuelan government, interest or principal reduction on the bank's portion of Venezuela's \$20.6 billion (U.S.) reschedulable foreign debt (American Banker, 1990), and temporary interest-rate

reductions which require Venezuela to pay market interest rates if oil prices rise above a certain level. Bank participation in the refinancing is voluntary, and, as in the Mexican restructuring, the bank can choose the combination of options which appears to be in its own best interest "without fearing either peer pressure from other banks or indirect retaliation from [Venezuelan] bank regulators." (American Banker 1990) The restructuring also included \$2 billion (U.S.) in additional World Bank and IMF funding.

The 1989 Mexican and 1990 Venezuelan packages give lenders such a broad range of options because no firm agreement could be reached on the fair value of the countries' debt. Accurate valuation requires accurate estimation of the riskiness of the debt and the value of the (implicit) options to hold or to sell the debt. Since there is substantial disagreement about the future prospects of these countries, valuations can vary just as widely. The following section will explore the valuation problem in greater detail.

## **2.2 Valuation Problems**

As implied above, valuation of existing debt and of new debt can be reduced to two problems: first, what discount rate fully reflects the riskiness of the debt? That is, what is the minimum price (expressed as an interest rate) that lenders will accept from borrowers for the use of the lenders' money? The work of Eugene Fama on asset pricing (which led to the widespread Capital Asset Pricing Model, or CAPM) provides a powerful tool for determining answers to these questions. Secondly, what is the value of the options implicit in holding the debt? The theory of options pricing suggests

several approaches to this problem, which is the crucial issue for debt reduction. Should lenders continue to lend, and at what terms? What is the true cost of borrowing to the borrower? The Fama model, combined with options pricing, can be used to rigorously develop answers to these questions.

Political risk assessment is part of the process of understanding the true cost of borrowing. Both lenders and borrowing countries need to understand the effects of borrowing on their future prospects. High levels of debt may constrain governments' financial flexibility and restrict their ability to respond to politically-sensitive situations. The political dimension of financial risk is the subject of the next section.

### **2.3 Political Risk**

Any corporation which does business across national borders may be affected by political risk. Political risk (sometimes called "country risk") may be defined as:

"exposure to either an outright loss or to an unanticipated lower earnings stream in cross-border business, caused by economic, financial or sociopolitical events or conditions in a particular country that are not under the control of a private enterprise or individual .... [Country risk] refers to uncertainty about future conditions within a country or those current conditions that cause inherent instability in or uncertainty about the future." (Business International Corp. 1981)

Dan Haendel, in Foreign Investments and the Management of Political Risk, notes that

[a]nyone considering a foreign investment should separate risk (the probability that a loss will occur) from uncertainty (the subjective doubt regarding the occurrence of such a loss). At least potentially, risk is measurable, insurable, and avoidable; uncertainty is not. Yet information bridges the two; the same information that is useful in accurately calculating risk also serves to reduce uncertainty. It is crucial, therefore,

that the investor always search for the most complete information available. (Haendel 1979, 84)

Haendel further notes that the uncertainty that is part of political risk may offer the opportunity for both loss and gain (Haendel 1979). For example, Dow Chemical was able to return to Chile in 1973 after the Allende government was overthrown (Kennedy 1987). Political-risk analysis which does not allow for the possibility of a pleasant surprise from a political-risk event will miss these opportunities.

Analysis of political risk is especially relevant for companies which hold large amounts of illiquid assets (e.g., large amounts of debt, or large holdings of concessions to produce oil in a foreign country), since it would be difficult at best for the company to move these assets out of the foreign country.

Companies subject to substantial political risks, such as multinational companies engaged in large-scale manufacturing, natural-resource extraction or financial efforts, have developed various methods for measuring and predicting risk probabilities. These methods are generally either **qualitative** or **quantitative** in nature. Qualitative analysis methods attempt to measure or predict levels of political risk through assessment of qualitative data. The data come from the subjective impressions of people who are knowledgeable about political, economic and other relevant conditions in the country, such as political analysts specializing in that country or company managers based in the country. Quantitative methods approach the same goal through analysis of "hard" macroeconomic data, such as the country's money supply or gross national product

(GNP), or through deduction and refinement of theories about the country's behavior through empirical testing. The strongest analysis methods are built around a consistent, proven model of the country's behavior. Inputs to these models may be based on either qualitative or quantitative data. (Haendel 1979; Business International Corp. 1981; Kennedy 1987; Snider 1988)

Lenders to LDCs have used political-risk models, with varying success, to predict the politically-related likelihood of unfavorable outcomes, such as debt nonpayment or repudiation since the 1970s. Painful losses incurred during the 1982 debt crisis have forced bankers to more carefully consider political-risk factors when lending to LDCs (Kennedy 1987). The following section gives a more detailed listing of generally-used methods for assessing political risk.

### **2.3.1 Political Risk Methodologies**

As noted above, methods for measuring and predicting political risk fall into two general categories: **qualitative** and **quantitative**. A third, hybrid category, covers integrative methods which combine both subjective and objective data (**integrated methods**).

The qualitative methods in wide use are (Haendel 1979; Business International Corp. 1981; Kennedy 1987):

- 1) **Checklist.** Current events are compared to a list of items believed to be related to political risk. The greater the number of items checked off, the

higher the assumed political risk. Flaws of the method include: items may not have the same importance across countries, or at different times in the same country.

- 2) **Component.** These methods, such as Haendel's Political System Stability Index (PSSI), break down the political system's behavior into a set of variables (ultimately derived from subjective assessments) which are taken as quantitative inputs to a model of political risk. These methods potentially can have both flawed data and flawed models, compounding the systems' error rate. System performance is highly dependent on which variables are incorporated into the model.
- 3) **Old hands.** Consult the opinions of people who have great experience with or expertise in the country. These experts may be internal or external to the organization performing the analysis. If the country's current situation is unlike its past history, or if new, relatively unknown leaders take charge, this method becomes less reliable.
- 4) **Delphi.** A structured method for eliciting opinions from experts, the Delphi technique is subject to many of the same flaws that reduce the reliability of the 'old hands' method.
- 5) **Grand tour.** Company managers travel to the country, meet influential people and subjectively assess the country's status. The power of this method is limited by the managers' ability to evaluate the country's situation and by the willingness of the country to allow the executives to

seek out information potentially damaging to the country.

Popular quantitative approaches include:

- 1) **Regression-based** models, such as discriminant analysis and probit/logit analysis. A regression is run on data from quantitative factors believed to be related to political risk (e.g., percentage of exports as commodities, or rate of capital flight). The result is an equation for the relationship between the factors.
- 2) **Decision-tree** models. Construct a probability tree for major events affecting a decision faced by the company. The tree starts with the decision, then events the decision depends upon, then events those events depend upon, and so on. The power of this model is a direct function of the relationship of the subjective model probabilities to the real probability of the event. Low-probability events excluded from the tree may also prove to be more important than others included in the model.

Under the integrated approach, the problem is decomposed into parts and the proper qualitative or quantitative method for that part is used. To arrive at a conclusion, the results of the separate analyses are integrated according to an overall model of political risk for the country (Kennedy 1987). Like all other approaches which integrate various sources of data, determination of the proper relationship and weighting of the subordinate analyses is the key problem.

The common flaw of all the political-risk-assessment methods discussed above is that each method's predictive power is limited by the quality of available information inputs. Valuation of debt, and of debt-reduction options greatly depends on assumptions (information) about the risk of lending to the borrowing country, and on the method chosen for valuing the options implicit in lending.

In the following sections, two valuation methods for derivative assets (such as the oil warrants described above) will be described and the behavior of the warrants tested in a valuation experiment. The results of the valuation experiment will be used to estimate the amount of debt-reduction provided by the oil-warrant approach. Finally, a measure of the oil warrants' political risk (from the Venezuelan government's point of view) will be derived from the warrant valuations.

### 3 Valuation Methods

Several major valuation methods exist for determining the value of options, including the **Black-Scholes** method, which is solved analytically, and the approximation developed by Cox and Ross, where risk-adjusted payoffs are calculated (by Monte-Carlo simulation in this application) and discounted at the risk-free rate (Cox and Rubenstein 1985). Both methods can be used to determine the value of an asset (the **derivative asset**) which depends solely on the value of another asset (the **underlying asset**). This section will describe the basic Black-Scholes method and contrast it with the integrated Fama-Cox-Ross method used later in this thesis.

#### 3.1 Notation

Notation used to describe the warrants, the stochastic behavior of their underlying assets and the measures of political risk developed in this thesis will be derived from standard usage in finance theory and from the work of Jacoby and Laughton. Stochastic process variables, i.e. process variables that may randomly take any value from a range of possible values will be indicated with a tilde '~'. The expected magnitude and median of a variable will be indicated as  $E(\text{variable})$  and  $M(\text{variable})$ , respectively. Any variable may be subscripted to indicate relative position in a sequence, e.g., if PO is a stochastic process variable that changes as a function of time (subscript variable  $t$ ), then it would be represented as  $E_t(P\tilde{O})$ .

### **3.2 Black-Scholes Method**

The work of Fischer Black, Myron Scholes and Robert Merton on options valuation is the foundation of most current options-pricing models in use by practitioners today. The Black-Scholes method is independent of investors' risk preferences and is relatively easy to implement. The assumptions underlying the Black-Scholes model are (Hull 1989):

- 1) Stock prices are distributed lognormally
- 2) Short selling of stock is allowed
- 3) No transaction costs or taxes
- 4) No dividends during the life of the derivative asset
- 5) No riskless arbitrage
- 6) Trading is continuous
- 7) The risk-free rate, stock-price volatility ( $\sigma$ ), and the expected return on the stock are the same (constant) for all maturities. The Black-Scholes model is still correct if the risk-free rate is the instantaneous risk-free rate and is a known function of the stock price  $S$  and of time  $t$ .

Many of these assumptions have been relaxed in later extensions of Black-Scholes. For example, the version of Black-Scholes used in this thesis is a modification for stocks paying a constant dividend yield (Hull 1989).

The following parameters are used in this version of Black-Scholes:

- 1) initial stock price (S)
- 2) exercise (strike) price (X)
- 3) underlying asset-price volatility ( $\sigma$ )
- 4) risk-free interest rate ( $r_f$ )
- 5) stock dividend percentage, or convenience yield ( $\delta$ )
- 6) time to maturity ( $\tau$ )
- 7) warrant cap (K)

The equation for constant-dividend-yield Black-Scholes is:

$$S e^{-q\tau} * N(d_1) - X e^{-r\tau} * N(d_2) \quad (1)$$

where

$\tau$  = (maturity time - information time)

$\delta$  = convenience yield

$r_f$  = risk-free interest rate

and

$$d_1 = \frac{\ln(S/X) + (r - q + \sigma^2/2)\tau}{\sigma \tau}$$

$$d_2 = \frac{\ln(S/X) + (r - q - \sigma^2/2)\tau}{\sigma \tau}$$

This equation gives the Black-Scholes value of one warrant in the strip.

The present value of an entire warrant strip is the sum of the present values of all the warrants in the strip:

$$\sum Se^{-qt} * N(d_1) - Xe^{-rt} * N(d_2) \quad (2)$$

### 3.3 Fama-Cox-Ross Method

The derivative-asset-valuation (DAV) method used in this thesis is based on the work of Jacoby and Laughton, which itself is based on asset-pricing principles established by Fama, and Cox and Ross' work in options pricing. The Fama-Cox-Ross (hereafter FCR) method gives the price of a derivative asset in terms of both the expected value and the riskiness level of the underlying asset. The Fama method, similar to the Capital Asset Pricing Model (CAPM) later derived from it, is used to derive a measure of risk for the underlying asset. The Jacoby-Laughton method used here integrates FCR with a model of uncertainty as information flow over time.

The Jacoby-Laughton method distinguishes between two distinct classes of time: **information time** (subscript variable  $s$ ), which relates the information available to observers as they pass through time, and **variable definition time** (subscript variable  $u$ ), which relates events which define variables to the variables as time passes. **Maturity**

time (subscript variable  $t$ ) is a special subclass of variable definition time. It denotes the variable definition time for a discrete amount of cash flow in a series of cash flows (Jacoby and Laughton 1990).

The Jacoby-Laughton basic information model describes a process by which expectations about future prices change as a function of time. The basic information model will be augmented in a later section to allow for non-constant underlying-asset volatility over time (the half-life effect).

The basic information model may be expressed as (Jacoby and Laughton 1990)

$$E_{s+\Delta s}(\tilde{P}_u) - E_s(\tilde{P}_u) = E_s(\tilde{P}_u) \sigma_{P_u, s} \Delta \tilde{z}_s, \quad s < u. \quad (3)$$

where

$s$  and  $u$  (as subscripts) indicate information and variable definition time respectively,

where

$\Delta s$  = one information-time period (the incremental change in information time),

$\Delta \tilde{z}_s$  = a normalized representation of the information affecting oil prices (i.e.,

the distribution of information at any (information) time is assumed to be normal, with an expected value of zero and variance of  $\Delta s$ ),

$\sigma_{P_u, s}$  = the information proportionality constant for oil prices, which controls how the expectation of the oil price at (variable definition) time  $u$  changes at (information) times after time  $s$ . This is assumed to be known with certainty at all (information) times (Jacoby and Laughton 1990).

A simulation based on this model starts with an initial set of expected prices of oil and takes a random value from a normal distribution at each (information) time  $s$ . When multiplied by the proportionality constant, the random sample simulates the expected change in information affecting oil prices for that (information) time period. A time-series of realized oil prices can then be generated, where the price at any given (information) time depends only on the price in the prior period and the expected change in price during that period. For valuation purposes, the same process is applied to risk-adjusted (certainty-equivalent) oil prices (Jacoby and Laughton 1990).

In Fama-Cox-Ross valuation, the set of expected oil prices generated from the information model, and the risk-price information derived from the Fama equation, are parameters to the option-pricing calculator. An explanation of how the Fama equation is used to determine the discount for the underlying asset follows.

The Fama equation used in Jacoby and Laughton (1990) is:

$$R_{P_{u^s}} = R_f + b_s * \sigma_{P_{u^s}} \quad (4)$$

where the term  $R_{P_{u^s}}$  is the expected rate of return from holding oil,  $R_s$  is the real risk-free discount rate at (information) time  $s$ ,  $b_s$  is the oil price of risk and  $\sigma_{P_{u^s}}$  is the standard deviation of the expected price of oil at (variable definition) time  $u$  as expected at (information) time  $s$ .

The expected rate of return for holding oil may also be expressed as

$$R_{P_{u^s}} = \frac{\Delta P}{P} + \delta \quad , \quad (5)$$

where  $\frac{\Delta P}{P}$  is the expected rate of capital gain from holding oil (assumed constant over time) and  $\delta$  is the convenience yield (also assumed constant).

In this thesis, the Jacoby and Laughton (1990) estimate of  $b_s$  is used:

$$b_s = PRSK * \rho_{m, P_u} \quad (6)$$

Stated in terms of  $R_{P_{w,s}}$ ,  $r_f$  and  $\sigma_{P_{w,s}}$ ,  $b_s$  is:

$$b_s = \frac{R_{P_{w,s}} - r_f}{\sigma_{P_{w,s}}} \quad (7)$$

Jacoby and Laughton (1990) assume that the the market price of risk (PRSK), is 0.5 (Brealey and Myers 1988), and that the correlation factor ( $\rho$ ) between the price of oil and the value of the market portfolio, is 0.8 when  $\sigma = .10$ , resulting in  $b_s = .4$ . In this thesis,  $b_s$  is held at 0.4 even though  $\sigma$  is raised to 0.15; thus (holding the other parameters equal) the implied PRSK is 0.533.

These relations will later be used to calculate the proper oil-price correlation factor for other combinations of parameters to the FCR model.

The FCR valuation model used here is based on a Monte-Carlo simulation process of the price of oil. In this Monte-Carlo process, multiple runs of the derivative-asset cash-flow model are performed. The accuracy of this type of simulation is a direct function of the number of runs (the simulated value converges toward the theoretical value as the number of runs increases). However, as the number of runs increases to very high numbers (>10,000), the convergence rate slows considerably.

The valuation experiment conducted in the next section will show that the FCR method

produces the same results as Black-Scholes, given an appropriate transformation of parameters.

#### **4 Setup for the Valuation Experiment**

One goal of this experiment is to show that a restricted subset of the Fama-Cox-Ross valuation method is functionally equivalent to the Black-Scholes options-pricing method when the parameters are correctly related. Environmental assumptions applicable to both models will be established, and the valuations for a simple warrant under both models will be calculated and compared. The experiment will first approximate the monthly warrants proposed in the 1989 World Bank paper with a 10-year strip of annual warrants. The calculation of results for shorter time periods (quarterly, monthly) is beyond the scope of this thesis, but the required corrections are given below.

The original Black-Scholes equation, still in wide use by most practitioners, makes no provision for assets with convenience yields. The effect of convenience yield on Black-Scholes will be explored and contrasted with original or "straight" Black-Scholes, and FCR.

The relationship of the Black-Scholes parameters to those used with FCR is relatively simple. The asset price and option strike price are the same in both models, as is the asset volatility. The asset discount rate in Black-Scholes is the real risk-free rate used in both models, and the same convenience yield ( $\delta$ ) will be tested. (See Equation (5) for the definition of  $\delta$ .)

#### **4.1 Environmental Assumptions**

From a market perspective, Venezuelan oil differs from the benchmark crudes widely used as pricing standards, such as West Texas Intermediate and Arabian Light. These differences impart slight variations in the price behavior of Venezuelan oil relative to the benchmark oils. Adjustments for these effects can be made in a more complex model of Venezuelan oil prices; however, for the purposes of this thesis, the price for Venezuelan oil will be assumed to be a "world price". Further, the price distribution at any point in time will be assumed to be lognormal.

The Jacoby-Laughton set of assumptions will be used in this experiment: 1) assets are tradeable at unique prices, and 2) some key variables can be known with certainty, including future risk-free interest rates, future prices of risk, and the proportional covariance between future changes in underlying variables (Jacoby and Laughton 1988).

The following macroeconomic assumptions will be used in the sample calculations below:

$r_f$  (real risk-free interest rate) = 3% per year

Zero inflation rate for the FCR valuation experiment (This makes the effects of changes in other parameters easier to see. The information model can be easily extended to include the effects of oil-price inflation on the price distributions.)

#### **4.2 Oil Price Assumptions**

Expected oil prices and the volatility (sigma) of oil prices are chosen to make the results

easy to interpret, although they are consistent with the past history of world oil prices. In the FCR valuation experiment, the expected initial price of oil used will be \$20, and the expected growth rate of oil prices will be 3% per year. An oil price volatility of  $\sigma = .15$  will be used for both Black-Scholes and FCR.

### **4.3 Valuation of the Underlying Asset**

The assumption of an oil-price volatility of  $\sigma = 0.15$  requires recalculation of the oil price of risk. This parameter is needed for valuation of the expected initial price of oil. The calculations of the price of oil risk  $b_s$  and the oil-price correlation factor  $\rho$  for  $\sigma = 0.15$  use the method established earlier in Equations (6) and (7):

    Holding the expected return on oil (0.07) and the risk-free rate (0.03) constant, use Equation (7) with  $\sigma = 0.15$  to compute the price of oil risk  $b_s$ . Use this  $b_s$ , which is 0.2666, with the market price of risk, PRSK, which is 0.5, in Equation (6) to compute the new correlation factor  $\rho = 0.5333$ .

The correlation factor and oil price of risk are parameters for the Fama-Cox-Ross valuations performed below.

### **4.4 Transformations for Shorter Time Periods**

The following equations provide the conversions for annual variables into their equivalents for time periods less than a year, where  $n$  = the number of the shorter time

periods in a year (e.g., 4 for quarterly variables):

volatility (sigma, or  $\sigma$ ) by

$$\sigma_n = \sqrt{\frac{\sigma_{ann}}{n}}, \quad (8)$$

continuous interest rates by

$$r_{n_{cont}} = \frac{r_{ann_{cont}}}{n}, \quad (9)$$

period interest rates by

$$1 + r_{n_{per}} = \sqrt[4]{1 + r_{ann_{per}}}. \quad (10)$$

The following section applies the assumptions laid out in earlier sections to the Venezuelan warrant-valuation problem.

## 5 Venezuelan Example

As noted above, the Venezuelan government in the fall of 1989 was negotiating with its major bank creditors for debt relief on its \$20.6 billion of foreign debt. One proposed refinancing used a combination of a deeply-discounted bond with strips of warrants on future Venezuelan oil production. This section will compare the valuation of these warrant strips under the Black-Scholes and Fama-Cox-Ross methods. The next section will give calculations for the amount of expected debt relief provided by the warrants under a FCR valuation.

Production from Venezuela's oil reserves provides substantial collateral for debt reduction. As of 1989, the country produced roughly 1.6 million barrels of oil per day (MMBD) (Lysy 1989). The amount of debt-relief available from the oil-warrant approach depends on the amount of this production set aside to pay off warrants as they mature. The debt-reduction calculations performed later in this section will assume that 1 MMBD are available to back the warrants.

The bond/warrant approach's most elegant feature is the close matching of the size of debt payments to the country's ability to pay. However, this approach has political consequences: for example, the government may believe that paying out more than a certain percentage of its income to foreign debtholders would be unacceptable to the populace. The government can avoid this outcome by placing a cap on the warrants, limiting the maximum expected payout at the desired level. The probabilistic nature

(when based on Monte-Carlo simulation) of the FCR method allows this probability to be easily estimated. This approach to measurement of political risk will be developed further in succeeding sections.

The FCR approach, as a method for analyzing political risk, can be best characterized as an **integrated** method, as it is based both on a firm quantitative foundation (the information and options-pricing models) and on the judgement of human experts.

The next section describes the results of analysis of the Venezuelan debt-relief problem using the Black-Scholes method.

### **5.1 Black-Scholes Analysis**

For this analysis, a Black-Scholes calculator was implemented using Lotus 1-2-3. Hull (1989) and Benninga (1989) were the sources used, respectively, for the calculator's Black-Scholes equation and its implementation of the cumulative normal probability density function. The calculator was verified according to the standards given in Benninga (1989). This section will discuss the calculation results for an oil-bond warrant (zero strike price) and both uncapped and capped warrants with a non-zero strike price. The effects of a non-zero convenience yield will also be explored by comparison with a standard (zero-convenience-yield) Black-Scholes valuation. The initial price of oil will be assumed to be \$20 per barrel.

As the strike price of an option approaches zero, the value of that option approaches the underlying-asset price (Cox and Rubinstein, 1985). Therefore, this section will only consider warrants with non-zero strike prices in evaluating the behavior of Black-Scholes with and without convenience yields.

The results for Black-Scholes without and with convenience yields of  $\delta=0.04$  for a ten-year strip of uncapped annual warrants ( $K=\infty$ ) with  $S=20$ ,  $X=15$ ,  $\sigma=0.15$ , and  $r_f=0.03$  are:

| Year        | Present Value, $\delta=0.00$ | Present Value, $\delta=0.04$ |
|-------------|------------------------------|------------------------------|
| 0           | 0.00                         | 0.00                         |
| 1           | 5.45                         | 4.68                         |
| 2           | 5.94                         | 4.49                         |
| 3           | 6.41                         | 4.36                         |
| 4           | 6.87                         | 4.24                         |
| 5           | 7.31                         | 4.13                         |
| 6           | 7.72                         | 4.02                         |
| 7           | 8.12                         | 3.91                         |
| 8           | 8.49                         | 3.80                         |
| 9           | 8.86                         | 3.70                         |
| 10          | 9.21                         | 3.59                         |
| Strip Total | 74.38                        | 40.92                        |

This example shows that Black-Scholes valuation without convenience yields can lead to

large errors. For oil, with a convenience yield of 4 percent, "straight" Black-Scholes overstates the present value of the entire warrant strip by nearly 45 percent. More importantly, the overvaluation gets worse as time to maturity of each warrant increases. Similar valuation errors should be expected whenever no correction for oil convenience yield is made. For example, in Lysy (1989) (which does not account for convenience yield), the value of an uncapped warrant-strip with  $S=16$ ,  $X=\$28$ ,  $\sigma=0.30$ , and  $r_f=0.08$  (nominal), was calculated as \$328. The actual value of this strip is probably \$180 or less.

The effects of capping the warrant under Black-Scholes were also explored. The capped-warrant value was calculated by computing the value of an uncapped warrant with a strike price equal to the cap value (this could be called a "cap warrant"). The value of the cap warrant was then subtracted from the original uncapped warrant. Here are the valuation results for a ten-year strip of capped annual warrants with  $S=20$ ,  $\sigma=0.15$ ,  $X=15$ ,  $K=25$ ,  $\delta=0.04$ , and  $r_f=0.03$  compared with the uncapped strip ( $K=1000$ ):

| Year | K = 1000 | K = 25 |
|------|----------|--------|
| 0    | 0.00     | 0.00   |
| 1    | 4.68     | 4.45   |
| 2    | 4.49     | 4.17   |
| 3    | 4.36     | 3.88   |
| 4    | 4.24     | 3.62   |
| 5    | 4.13     | 3.38   |
| 6    | 4.02     | 3.18   |
| 7    | 3.91     | 2.99   |
| 8    | 3.80     | 2.81   |

| Year           | K = 1000 | K = 25 |
|----------------|----------|--------|
| 9              | 3.70     | 2.66   |
| 10             | 3.59     | 2.51   |
| Strip<br>Total | 40.92    | 33.65  |

The cap pulls down the present value of the warrant strip by 17.5 percent. The cap has a stronger effect on the warrants maturing after year 5, with the 10-year warrant value cut by 29 percent. The effect of a cap on a Black-Scholes valuation, then, is biased against cash flows which occur further away in time.

## 5.2 Basic Fama-Cox-Ross Analysis

A Fama-Cox-Ross calculator for the Venezuelan warrants was constructed by modifying a FCR valuation system written in Fortran by Jacoby and Laughton. Lotus 1-2-3 spreadsheets were used to generate the initial set of expected oil prices and to calculate the oil price of risk. These parameters, along with others describing the warrants, were then fed into the Fortran model for each validation run (which was later performed on an IBM mainframe). Results from the mainframe runs were downloaded to a Lotus 1-2-3 spreadsheet for further analysis.

### 5.2.1 FCR Model Validation

The price of oil is assumed to be constant in real terms for the validation runs discussed in this section. The cap is set to 1000 for all cases requiring infinite caps. An infinite

half-life (1000 time-periods) for the oil-price distributions is also assumed.

The results of the Black-Scholes analysis performed above provide a standard for validating results of the Fama-Cox-Ross method. This section will compare the Black-Scholes results with those from FCR valuations of the same warrants under the FCR assumptions given above.

The results for  $\sigma=0.15$ ,  $X=15$ ,  $K=\infty$ ,  $r_f=0.03$  and  $R_s=0.07$  for Fama-Cox-Ross and Black-Scholes valuations are:

| Year        | Black-Scholes | Fama-Cox-Ross |
|-------------|---------------|---------------|
| 0           | 0.00          | 0.00          |
| 1           | 4.68          | 4.69          |
| 2           | 4.49          | 4.50          |
| 3           | 4.36          | 4.38          |
| 4           | 4.24          | 4.28          |
| 5           | 4.13          | 4.16          |
| 6           | 4.02          | 4.06          |
| 7           | 3.91          | 3.95          |
| 8           | 3.80          | 3.86          |
| 9           | 3.70          | 3.73          |
| 10          | 3.59          | 3.60          |
| Strip Total | 40.92         | 41.21         |

FCR valuation results for a ten-year strip of capped annual warrants with  $\sigma=0.15$ ,  $X=15$ ,

$K=25$ ,  $r_f = 0.03$  and  $R_s=0.07$  (implying  $\delta=0.04$ ), compared with the Black-Scholes results, are:

| Year        | Black-Scholes | Fama-Cox-Ross |
|-------------|---------------|---------------|
| 0           | 0.00          | 0.00          |
| 1           | 4.45          | 4.60          |
| 2           | 4.17          | 4.21          |
| 3           | 3.88          | 3.89          |
| 4           | 3.62          | 3.64          |
| 5           | 3.38          | 3.40          |
| 6           | 3.18          | 3.23          |
| 7           | 2.99          | 3.03          |
| 8           | 2.81          | 2.88          |
| 9           | 2.66          | 2.74          |
| 10          | 2.51          | 2.56          |
| Strip Total | 33.65         | 34.18         |

Note the very close match to results given by the Black-Scholes analysis above. The slight differences between the results from the two methods is attributable to computational errors in the numerical methods used in the calculations (e.g., Monte-Carlo convergence towards the Black-Scholes value improves as the number of iterations of the simulation increases. A very small amount of floating-point error may also be present).

### 5.2.2 Debt-Capacity Analysis

Assuming the Venezuelan government has 1 million barrels of oil per day, or 365 million barrels of oil per year, to fund restructuring of its debt, it could potentially write (sell) up to 365,000 contracts of 1000 ten-year strips of warrants. How much debt reduction is provided if warrants are used? Environmental assumptions for this experiment will be the same as those used in the earlier analysis above.

Given Venezuela's 1989 foreign debt of \$20.6 billion (U.S.), and assuming the Venezuelan government applies all proceeds from warrant sales towards debt refinancing, the following table gives the amounts received and the percent of debt reduction given for three possible warrant strips:

| Initial Oil Price | Strike Price | Cap      | Value of Warrant Strip | Amount of Debt Reduction (\$ billions) | Percent Debt Reduction |
|-------------------|--------------|----------|------------------------|--|------------------------|
| 20                | 15           | $\infty$ | \$41.21                | 15.04                                  | 73                     |
| 20                | 15           | 35       | \$39.90                | 14.56                                  | 71                     |
| 20                | 15           | 25       | \$34.18                | 12.48                                  | 61                     |
| 20                | 25           | 35       | \$21.01                | 7.67                                   | 37                     |

Clearly, Venezuela could achieve the 50 percent debt-reduction required in the 1989 proposal with oil warrants if the strike price and cap are properly set. Too high a strike price, combined with too low a cap, will result in insufficient debt relief (less than 50 percent). However, the government must be concerned not only with the value of debt

reduction now; it must also consider whether the expected cash outflows over the life of the warrant strip are politically acceptable. The government's exposure to the possibility of politically-risky large payments on the warrants can be estimated as part of the warrant-valuation process, and the level of the exposure can be managed by proper setting of the warrant strike price and cap. The next section will explore these issues in greater detail.

### **5.2.3 Effects of Political Risk**

The results of the last section show that the payoff cap is very important, as it may be imposed to limit cash outflows to warrant-holders, reducing the political risk of the warrants to the government. Note that as the cap value decreases (all other factors held equal), the present value of the warrant strip decreases, as would be expected (the cap limits the payoffs from the government to warrant-holders during times of high oil prices). Thus the government must choose the cap level carefully to avoid the risk of paying too large an amount to warrant-holders at any given time without lowering the present value of the strip too much (and therefore buyers' offer price and the government's income from the sale).

Testing the effects of different cap levels on the present value of a warrant strip and on the expected payoff of each warrant in the strip is quite easy. The following table shows the effect of lowering the cap from \$35 to \$25 on a ten-year strip of annual warrants with  $X=15$ ,  $\sigma=.15$ ,  $r_f = 0.03$  and  $R_s = 0.07$ :

| Year           | K = 35 | K = 25 |
|----------------|--------|--------|
| 0              | 0.00   | 0.00   |
| 1              | 4.69   | 4.60   |
| 2              | 4.48   | 4.21   |
| 3              | 4.32   | 3.91   |
| 4              | 4.20   | 3.64   |
| 5              | 4.02   | 3.40   |
| 6              | 3.91   | 3.21   |
| 7              | 3.78   | 3.02   |
| 8              | 3.67   | 2.85   |
| 9              | 3.52   | 2.69   |
| 10             | 3.31   | 2.53   |
| Strip<br>Total | 39.90  | 34.07  |

Lowering the cap decreases the present value of the warrant strip by nearly 15 percent. Note especially the effect of the cap on the warrant values in years 4 through 7 (a 15 to 20 percent reduction in present value). The effect of the cap is especially marked, given that oil prices in these simulations are rising at 3% per year (zero inflation in real terms).

The political risk of excessive cash outflow during the life of the warrant strip must be managed by the government as well. The Fama-Cox-Ross valuation method used in this thesis can provide a direct measure of the probability that the expected cash outflow in any given year will exceed a particular level. This is the measure of political risk which

will be used here.

As an example of political-risk assessment, the government might want to avoid situations where there is a high (greater than .5) likelihood that the expected warrant cash outflow will exceed \$10 per warrant. Using the assumptions established in section 5.2.2, \$10 per warrant means \$3.65 billion in cash outflows (\$10 per warrant x 365 million warrants). The table below gives the probability in each year of the warrant value exceeding \$10 (these probabilities were computed as part of the FCR valuation process performed earlier in this thesis) for warrants with three different cap levels:

| Probability of Expected Payoff Exceeding \$10 Per Warrant |       |        |        |
|---|-------|--------|--------|
| Year  | K = ∞ | K = 35 | K = 25 |
| 1   | 0.10  | 0.08   | 0.00   |
| 2   | 0.21  | 0.18   | 0.00   |
| 3   | 0.30  | 0.25   | 0.00   |
| 4   | 0.36  | 0.31   | 0.00   |
| 5   | 0.41  | 0.35   | 0.00   |
| 6   | 0.45  | 0.38   | 0.00   |
| 7   | 0.48  | 0.41   | 0.00   |
| 8   | 0.51  | 0.43   | 0.00   |
| 9   | 0.53  | 0.45   | 0.00   |
| 10  | 0.56  | 0.48   | 0.00   |

The government would certainly have to impose a cap to keep the payoff probabilities in the desired range. A cap at \$35 appears to work for this purpose; however, if the

volatility or the rate of growth of oil prices should increase above assumed levels, it is possible that the government might be exposed to the risk it is trying to avoid. Note that if the cap is set at \$25, the government can avoid this risk altogether (all other things held equal).

### 5.3 Alternative Specification of the Information Model

The Jacoby-Laughton information model can be extended to simulate underlying assets which have non-constant price volatilities, non-constant expected prices and other features beyond the scope of the Black-Scholes method used here. The constant-volatility assumption for underlying-asset price distributions leads to mispricing of a warrant strip, as this assumption causes the spread of the asset's price distribution to increase as time to maturity increases.

In Equation (3),  $\sigma_{P_{u,s}}$  was held constant over  $u$ , which implies an assumption that the arrival of new information  $\Delta z_t$  has the same proportional effect on all future oil prices  $\bar{P}_u$  in periods  $u$ . Here we test the assumption that this effect erodes over time, using a half-life model. The year-to-year value of sigma,  $\sigma_s$ , is held constant at  $\sigma_s = 0.15$ , but now

$$\sigma_{P_{u,s}} = \sigma_s e^{-\beta(u-s)} \quad (11)$$

where

$$\beta = \frac{\log 2}{HLPO}$$

and HLPO is the "half-life" parameter symbolizing a judgment of how fast the effect of a surprise change in oil price wears off.

The tests performed in this section will isolate the effects of changes in the half-life by holding all other parameters equal while varying the half-life. As the time to maturity increases, the variance of the oil price distribution also increases. The half-life effect "pulls in" the tails of the price distribution, which means that, as time to maturity increases, the simulated price distribution should better match the probable future price distribution. (Infinite half-life implies no reduction in the variance of the price distributions as time to maturity increases).

The basic FCR valuations performed above were repeated with a half-life setting of 10 time-periods (HLPO = 10). Since the oil price volatility  $\sigma_{p,t}$  for HLPO = 10 is no longer constant over the time period (u - s) between the time oil-price information arrives and the time the price is actually realized, the oil price of risk  $b_s$  must be adjusted accordingly. How can this be done, and which parameters should be held constant?

Assuming the oil-price correlation factor  $\rho$  is held constant at 0.533, there are three

possible choices. The first is to hold constant the price of a pure claim on oil (an "oil bond") over (information) time  $s$ . This implies that the market price of risk (PRSK) is rising over time. The second option holds PRSK constant over time and reduces the volatility ( $\sigma$ ). Unfortunately, this overvalues the oil bond as time to maturity increases. The third choice is to find out which value of PRSK keeps the return on a pure claim to oil at its infinite-half-life value over a critical range of time. This imposes the least bias on the valuation of the oil-bond when the other parameters are held constant, and was the method used to calculate the PRSK for the following valuations. The critical range was taken as years 4 through 7 of the ten-year warrant.

The method used to calculate the initial value of PRSK was as follows:

- i) Compute all  $\sigma_{P,s}$  for times  $(u - s)$  4 through 7.
- ii) Compute the average volatility  $\bar{\sigma}_{P,s}$  for times 4 through 7.
- iii) Compute  $b_s$  by substituting  $\bar{\sigma}_{P,s}$  into Equation (7) while holding all other parameters equal.
- iv) Compute the new PRSK as in Equation (6).

After the new value of PRSK was computed, the 10th and 90th percentiles for expected oil prices were calculated for each time period, along with the FCR warrant-strip values (see Figure 1). Compared to the earlier results with half-life = 1000, note the smaller spread of the HLPO = 10 distributions, especially as time to maturity increases:

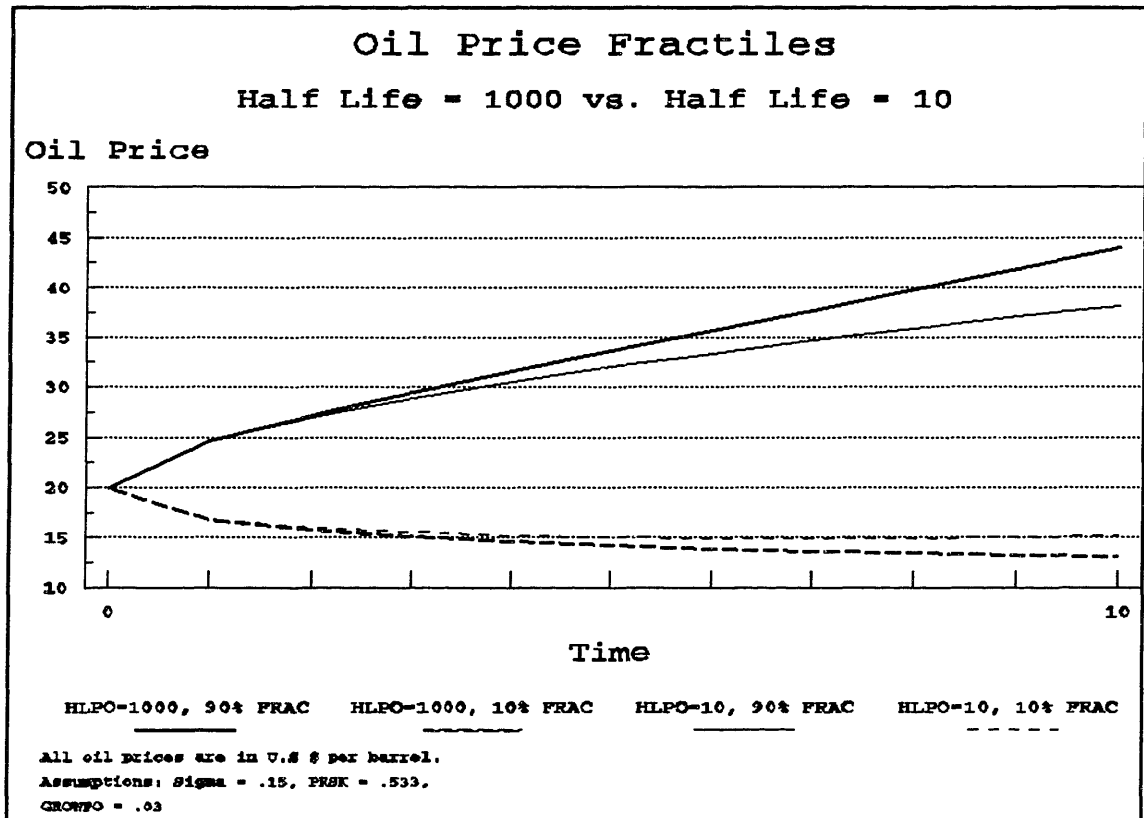


Figure 1

Valuations with HLPO = 1000 and HLPO = 10 for a ten-year strip of annual warrants with  $X=15$ ,  $K=\infty$ ,  $\sigma=.15$ ,  $r_f=0.03$  and  $R_s=0.07$  follow:

| Year        | HLPO = 1000 | HLPO = 10 |
|-------------|-------------|-----------|
| 0           | 0.00        | 0.00      |
| 1           | 4.69        | 4.55      |
| 2           | 4.50        | 4.25      |
| 3           | 4.38        | 4.02      |
| 4           | 4.28        | 3.88      |
| 5           | 4.16        | 3.67      |
| 6           | 4.06        | 3.56      |
| 7           | 3.95        | 3.44      |
| 8           | 3.86        | 3.35      |
| 9           | 3.73        | 3.21      |
| 10          | 3.60        | 3.08      |
| Strip Total | 41.21       | 37.01     |

The expected distribution of oil prices given by a half-life of 10 sharply reduces the calculated value of the warrants (around 10 percent of the total value). Like the Black-Scholes valuation performed above, FCR valuation with an infinite half-life overprices the warrants. Moreover, infinite half-life gives too much weight to early cash flows.

The same warrant strips were valued with a cap of  $K = 25$ :

| Year | HLPO = 1000 | HLPO = 10 |
|------|-------------|-----------|
| 0    | 0.00        | 0.00      |
| 1    | 4.60        | 4.47      |
| 2    | 4.21        | 4.03      |

| Year           | HLPO = 1000 | HLPO = 10 |
|----------------|-------------|-----------|
| 3              | 3.91        | 3.70      |
| 4              | 3.64        | 3.46      |
| 5              | 3.41        | 3.22      |
| 6              | 3.21        | 3.07      |
| 7              | 3.02        | 2.91      |
| 8              | 2.85        | 2.79      |
| 9              | 2.69        | 2.68      |
| 10             | 2.53        | 2.54      |
| Strip<br>Total | 34.08       | 32.87     |

The cap prevents oil prices in the upper tail of the price distribution (as it is narrowed by the half-life effect) from affecting the price of the warrant, reducing the warrants' expected present value the most in the middle years (especially in years 3 through 7).

The half-life effect clearly reduces the impact of high oil prices on the present value of the warrants, as does the cap. However, when decreased half-life is used with capped warrants, these results show that the cap effect is more important than the half-life effect for distant maturity dates.

## **6 Summary**

The Jacoby-Laughton/Fama-Cox-Ross (JL/FCR) approach to derivative asset valuation gives the analyst great flexibility in modelling the price behavior of both underlying assets and derivative assets. The integration of an information-flow model with an asset-valuation model allows analysts to more easily model the financial consequences of unexpected information and events.

### **6.1 Possible Applications**

As seen above, the flexibility of the JL/FCR approach allows it to be used in very different applications. The method has been applied to project evaluation and the valuation of undeveloped oil reserves (Jacoby and Laughton 1988, 1990; Hong 1989) and here, in a debt-restructuring application. More accurate valuation of options could allow a highly-indebted country with substantial illiquid assets, such as Venezuela's oil reserves, to trade equity in those assets while avoiding liquidity problems for purchasers. Many other applications are possible, since this approach can be used wherever a choice must be made between two or more possible actions in a context of significant uncertainty and information flow.

### **6.2 Summary of Results**

The JL/FCR method for avoids the flaws of the Black-Scholes method. As seen above, when given equivalent assumptions, the JL/FCR method produces equivalent results. However, it is more easily extended to accept conditions such as non-constant volatility

of underlying-asset prices. When the Black-Scholes method is modified to use convenience yields (holding all other assumptions equal), it produces superior results compared to "straight" Black-Scholes, and is an adequate substitute for JL/FCR for cases where the Black-Scholes assumptions hold. Otherwise, the greater flexibility of the JL/FCR approach, especially its ability to model the impact of information on underlying-asset prices, leads to greater valuation accuracy.

More accurate methods, such as the JL/FCR approach, for valuing options will ease the task of making implicit options explicit, thus allowing better management of risk through selling and buying of these options. More accurate methods should lead to greater agreement about the value of options and more widespread use of derivative assets.

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