Long Term Behavior of Cable Stayed Bridges

by

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Laurea, Structural Engineering,
Politecnico di Milano, 2003

SUBMITTED TO THE DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF ENGINEERING IN CIVIL AND ENVIRONMENTAL ENGINEERING
AT THE
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

JUNE 2004

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ABSTRACT

In the design of a structure, one of the most difficult parameter to assess is how the initial condition of a structure will change with time. During its life span, a structure can be subjected to different loads, changes in geometric configuration and even changes in its mechanical properties. Among all factors that can affect the geometrical reconfiguration of a structure (i.e. settlements and overloads) creep is one of the most important given its inevitability and because of its persistent effects (normally estimated as ten thousand days). Due to the effect of creep, a structure tends to amplify its deformation under a given load condition over time, and the final deformation can even reach values five or six time grater than the initial ones.

During the design, the fact that deformations grow with time can be a difficult condition especially for highly indeterminate structures like cable stayed bridges where the stresses are related to the geometric configuration of the structure itself.
In concrete cable stayed bridges, in fact, the increase in the deformation of the deck and the pylons over time leads to a decrease in the initial tension in the stays with an obvious difficulty in the design phase of the structure.

The first chapter of this thesis illustrates and explains one approximate method used to estimate the effect of creep on a concrete structure. The method proposed in this thesis is the "Age-Adjusted Effective Modulus Method". It was chosen among others because it is one of the most commonly used, and because it is highly accessible.
In the second chapter, the Age-Adjusted Effective Modulus Method will be used in conjunction with the force method to study non homogeneous, indeterminate structure under the effect of creep. In this chapter a procedure will be introduced that enables the calculation of an initial value of the prestressing force in the stays that elides the effect of creep on tension.
In the last chapter, the theory developed in the previous section will be used to study the change of tension in the stays in a cable stayed bridge. The bridge chosen for the application of this theory is one of the proposals for the renewal of the Waldo Hancock

Bridge in Maine, USA (M.Eng Project. Alexander Otenti, Andrea Scotti, Richard Unruh III, 2004)
The theory exposed in this thesis is a very powerful procedure that permits to simplification of the problem of creep in cable-stayed bridges, with easy calculations and with an iterative procedure.

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1 CREEP BEHAVIOUR OF A CONCRETE STRUCTURE

In this chapter the theory behind the simple equations of the age adjusted effective modulus method is explained. While the mathematics involved in this chapter are difficult, a deep understanding of all the equations is the only way to fully understand the meaning of all the terms involved in the method.

The chapter will be organized in order to introduce the equations of creep for the simplest cases (constant stress on a homogeneous determinate structure), followed by more complex cases (variable stresses on a non homogeneous indeterminate structure) where the mathematics involved are more complicated and less commonly used. The chapter will conclude with the final version of the equations of the age adjusted effective modulus method with a particular emphasis on the hypotheses at the base of the method.

1.1 INTRODUCTION

Stresses and strains in a reinforced concrete structure or in a prestressed concrete structure are subjected to changes over time due to the creep and shrinkage of the material itself.

It is possible to notice this phenomenon with a simple experiment: If one imposes a constant load on a concrete specimen at time t_0, one observes that the stress-strain relationship is not linear and the strain continues growing until a certain value that may be from five to six times greater than the initial one is achieved.

In Figure 1, the strains under a unitary stress are plotted as function of t, and as explained above the stress-strain relationship it's no linear as in the elastic case.

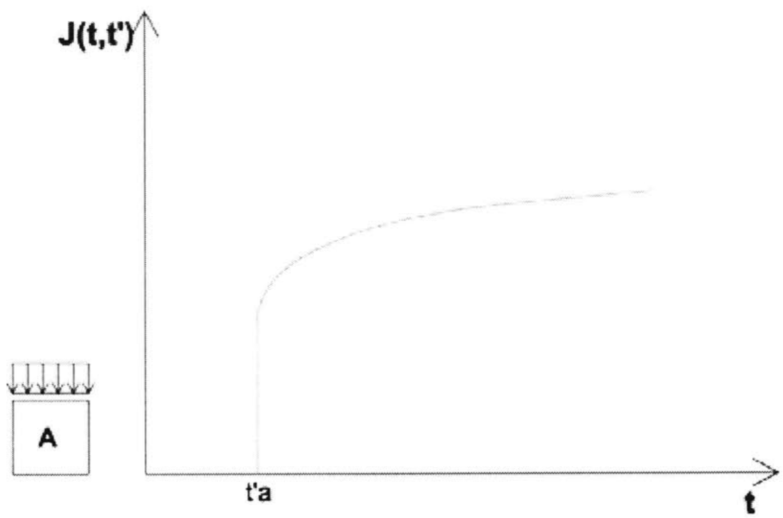

Figure 1: J as function of t

It has been observed that the strain behaviour of the specimen is dependent not only on the duration of the applied stresses, but also on the timing of the application.

If we try to apply the same load to a second specimen but later in time, we can observe that the strains will be lower. This is due to the fact that concrete increases its modulus of elasticity with age and therefore, in the second specimen, the elastic deformations will be less, as explained in Figure 2.

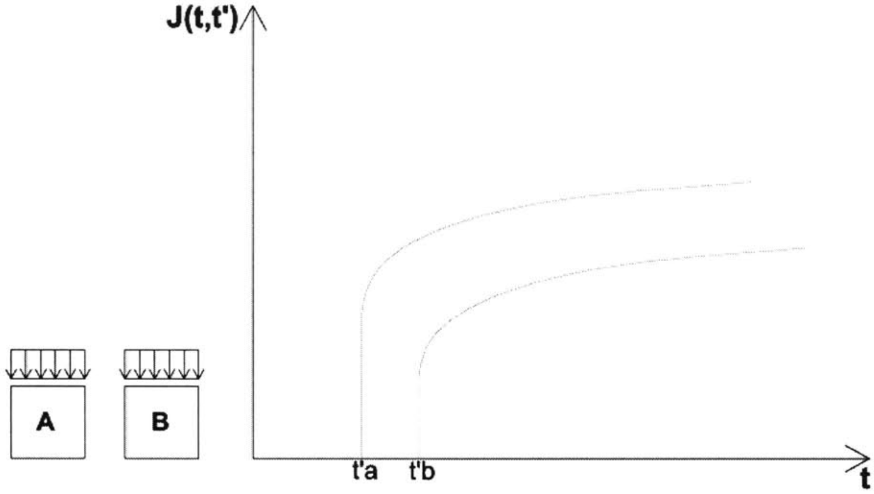

Figure 2:J as function of t, t'

As a result, if we want to study the behavior of a concrete structure during a long period of time, we should find an expression for the strains as a function of the stresses applied to the structure, their duration, and the age at the application of the loads:

$$\varepsilon = \varepsilon\left(t, t_0, \sigma\right) \tag{1.1}$$

where:
t is the duration of the load,

t_0 is the time at which the load is applied

σ is the stress in the structure.

Once ε has been described as function of time, the goal is to express the deformations at time t with a formulation similar to the σ–ε relation in the elastic theory such as:

$$\varepsilon\left(t, t_0, \sigma\right) = f(t, t_0) \cdot \sigma \tag{1.2}$$

Experimentally, it has been shown that a relationship with this form can be written when σ is on the order of σ=0.45-$0.5f'_c$; which is a reasonable value for concrete structures subjected to their self-weight.

Instead of the generic function $f(t,t_0)$, one can use the creep function $J(t,t_0)$ which relates the strains at a certain age t with the stresses applied to the structure.

Based simply on observation it is possible to say that this function should increase with time until it reaches a constant value at approximately ten thousand days when the effects of creep become irrelevant.

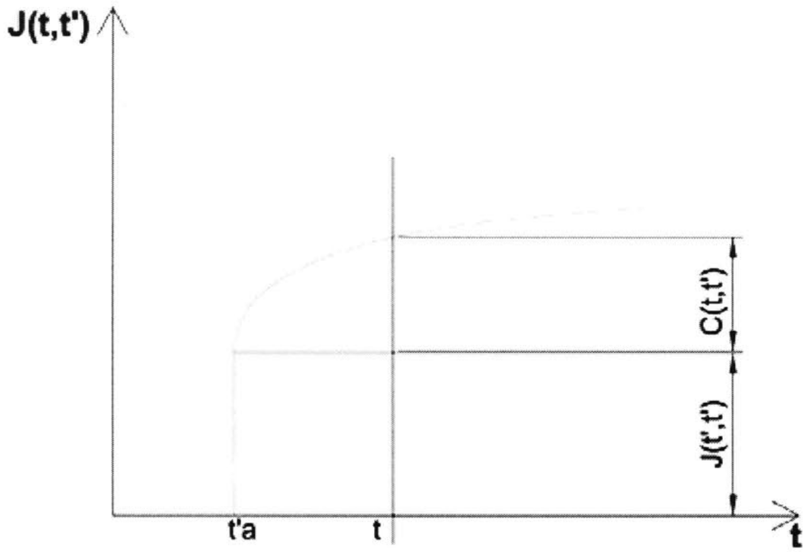

Figure 3: Definition of C(t,t')

As Figure 3 explains, the strains of a structure under an applied load at time *t'* rise instantaneously from zero to a certain value (elastic deformation) and then increase with time due to the effect of creep. It is therefore possible to divide the function $J(t,t')$ as:

$$J(t,t') = J(t',t') + C(t,t') \tag{1.3}$$

where $J(t,t')$ is the strain that the structure experiences at time *t'* under a unitary stress applied at the same time. Therefore:

$$J(t',t') = \frac{1}{E(t')} \tag{1.4}$$

where *E(t')* is the instantaneous deformation for σ=1
and *C(t,t')* is the deformation at time *t* due to σ=1 applied at time *t'*.
Now, using equations (1.2), (1.3)and (1.4) it is possible to rearrange ε as:

$$\varepsilon(t,t',\sigma) = J(t,t')\cdot\sigma = \sigma\left[J(t',t') + C(t,t')\right] = \frac{\sigma}{E(t')}\left[1 + \frac{C(t,t')}{\frac{1}{E(t')}}\right] \tag{1.5}$$

which can be simplified as:

$$\varepsilon(t,t',\sigma) = \frac{\sigma}{E(t')}\left[1 + \varphi(t,t')\right] \tag{1.6}$$

Where φ (t,t')=$\dfrac{C(t,t')}{\dfrac{1}{E(t')}} = C(t,t') \cdot E(t')$ is a dimensionless coefficient function of the age at loading t' and the age t at which the strain is calculated.

"The coefficient φ represents the ratio of strain due to creep to the instantaneous strain; its value increases with the decrease of the time at loading t' and the increase of the length of the period *(t-t')* during which the stress is sustained" [A.Ghali,R.Favre, 2002].

In the coefficient φ are included all the material properties and all the dependencies of the material properties on the surrounding environment. Its determination involves the knowledge of some empirical data about the material used (i.e. the modulus of elasticity at 28 days) and about the environment in which the structure will be placed.[1]

A typical graph of the changing of φ during time is shown in Figure 4.

This graph has been created using the parameters for normal hardening concrete with f_{ck}=20MPa in an environment with a relative humidity of 50%

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[1] The equations used to determine the coefficient φ are the following:

$$\varphi(t,t_0) = \varphi_0 \times \beta_c(t-t_0) \times \beta(t_0) \frac{E_C(t_0)}{E_C(28)}$$

$$\varphi_0 = \varphi_{RH} \times \beta(f_{cm}) \times \beta(t_0)$$

$$\varphi_{RH} = 1 + \frac{1-(RH/100)}{0.46 \times (h_0 / h_{ref})^{1/3}} \ldots\ldots h_{ref} = 100mm$$

$$h_0 = \frac{2A_C}{u}$$

$$\beta(f_{cm}) = \frac{5.3}{\sqrt{f_{cm}/f_{cm_0}}} \ldots\ldots f_{cm_0} = 10MPa$$

$$f_{cm} = f_{ck} + 8MPa$$

$$\beta(t_0) = \frac{1}{0.1 + t_0^{0.2}}$$

$$\beta_c(t-t_0) = \left(\frac{t-t_0}{\beta_H + t - t_0} \right)^{0.3}$$

$$\beta_H = \frac{150h_0}{h_{ref}} \left[1 + (0.012RH)^{18} \right] + 250 \le 1500mm$$

where:
RH is the relative humidity,
At is the area of concrete exposed directly to the environment,
U is the perimeter of the area exposed

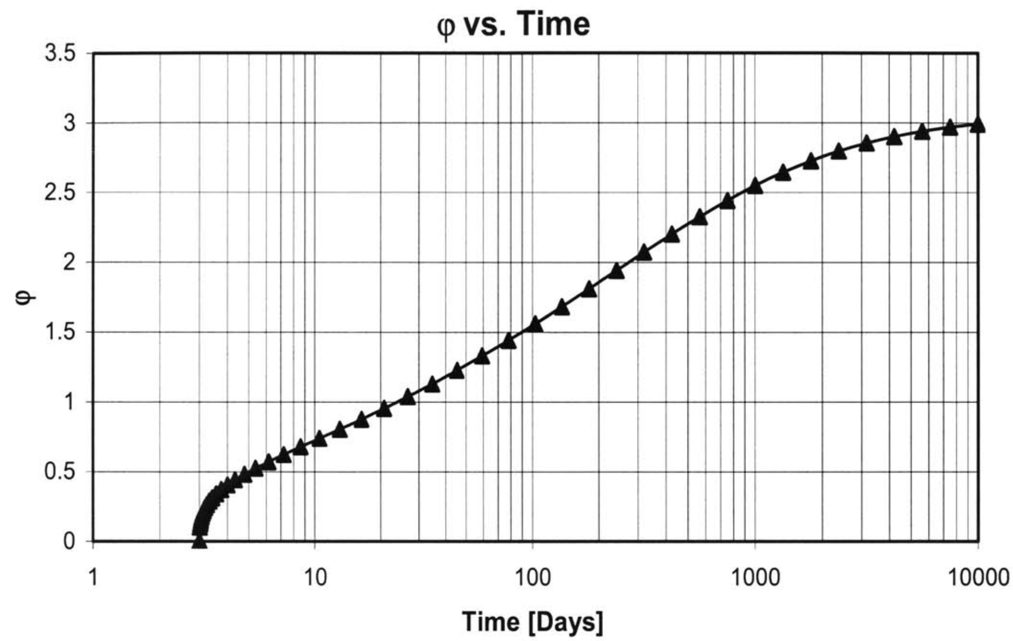


Figure 4:Function phi

It is interesting to observe how the vale of φ increases with time and, by applying this observation to Equation(1.6) how E decreases with time. Therefore, the key factor of this method is to simulate the effect of creep on a structure simply by decreasing its modulus of elasticity during time. From the graph above and using Equation (1.6) it is possible to see that after ten thousand days the structure can be idealized as an elastic structure but with a modulus of elasticity reduced by a factor of 4.

Even if this approach is theoretically incorrect, because the modulus of elasticity of concrete increases with time, the results obtained are in good accordance with the observed behavior.

From Equations (1.5) and (1.6), it is clear that under the condition σ=0.45-0.5f'_c it is possible to idealize the behavior of the structure exactly like an elastic structure but with the effective modulus $J(t,t')$ instead of $1/E$. It is therefore important to study the properties of the function $J(t,t')$ to understand the behavior of the structure during time.

1. J monotonically increases with t for a fixed t':

$$\frac{\partial J(t,t')}{\partial t} > 0 \qquad \text{For fixed } t'.$$

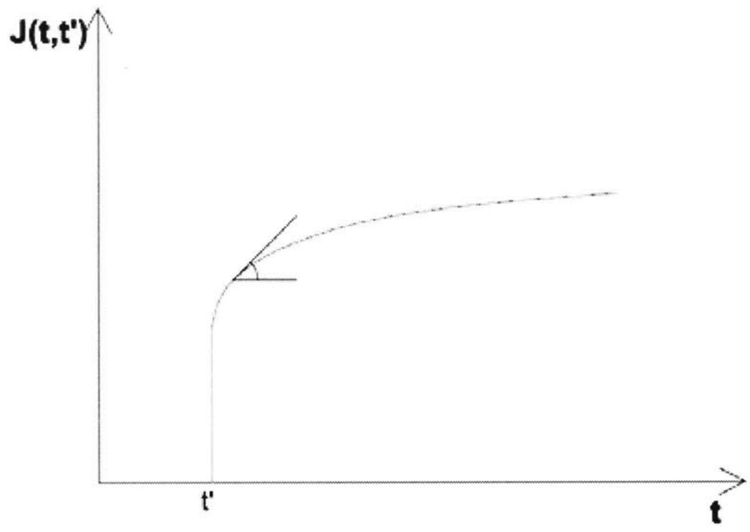


Figure 5: Properties of J

2. *J* decreases with *t'*:

$$\frac{\partial J(t,t')}{\partial t'} < 0$$

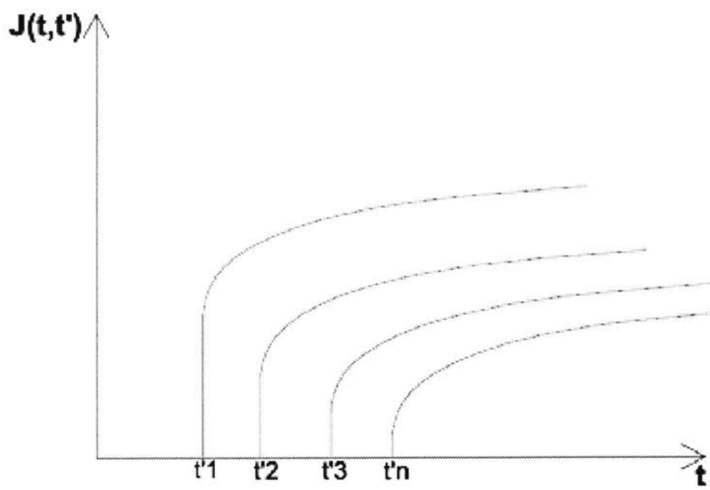


Figure 6: Properties of J

1.2 STRESSES VARYING WITH TIME

Usually structures are not subjected to a constant stress during their life but to a combination of stresses due to different loads applied at different times.

For the case in which all the loads are applied at the same time t' it is obvious that it is possible to use the superposition principle to study the simultaneous effect of all the loads. The problem arises when the loads and the consequent stresses are applied at different times in the life of the structure. In this case it is not completely accurate to use the superposition principle because the response of a previously loaded structure is different from the response of a structure that is not loaded.

From experimental data [Mola et al.1983] it has been proven that superposition can be used if:

- σ is less than $0.5 f'_c$ or less than $0.9 f_{ct}$
- There are no sudden variations of ε.

If we suppose that those two limitations are verified and that the variations of stresses are discrete as in Figure 7:

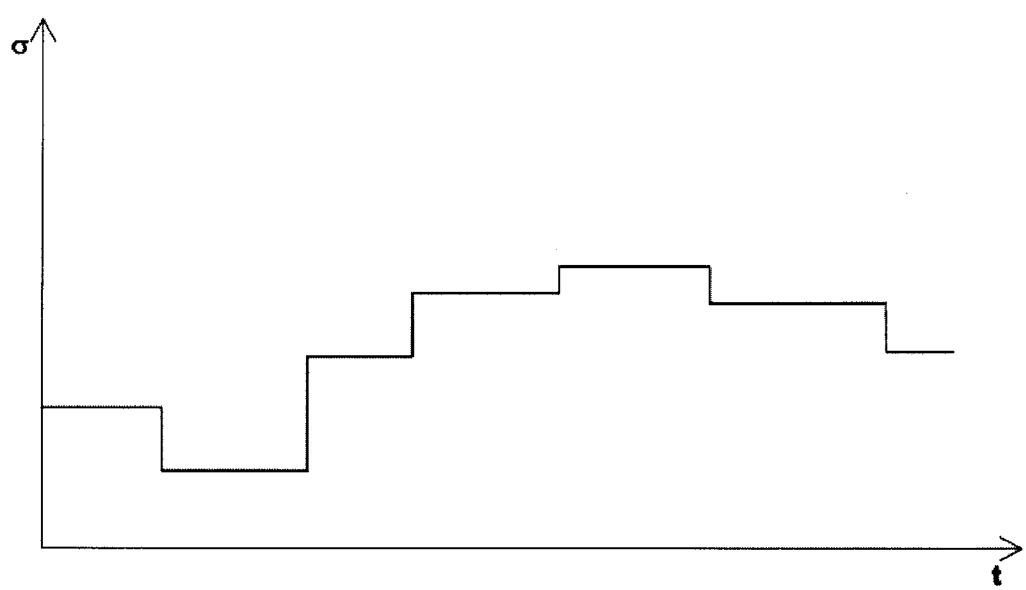


Figure 7: Sigma discrete

It is possible to write an expression for strains in the form:

$$\varepsilon(t) = \sigma(t_0) \cdot J(t,t_0) + \sum_{i=1}^{n} \Delta\sigma_i \cdot J(t,t_i) \tag{1.7}$$

For the case in which stresses are varying continuously, the strains are given by the equation:

$$\varepsilon(t) = \int_{t'}^{t} J(t,t')d\sigma \tag{1.8}$$

The problem is that the function that describes the stresses in a structure during time is not generally continuous but almost continuous as explained in Figure 8.

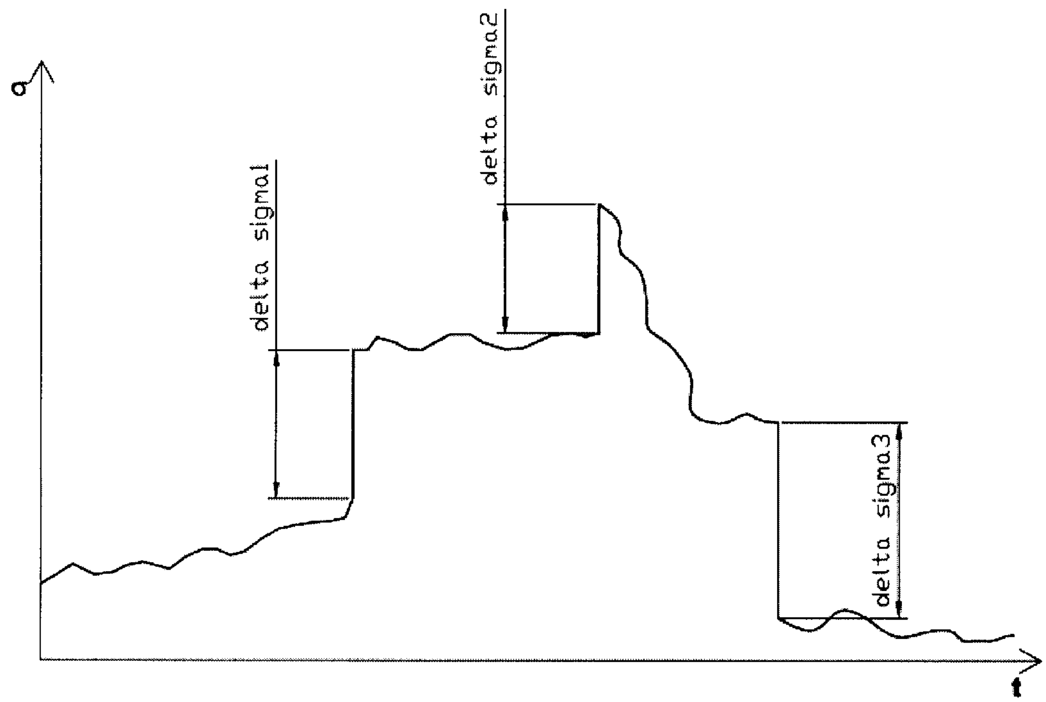


Figure 8: Sigma almost continuous

For this reason it is necessary to rewrite Equation (1.8) in order to account for the discontinuities in the stress function

$$\varepsilon(t) = \int_{t_0}^{t_1} J(t,t')d\sigma + \int_{t_1}^{t_{21}} J(t,t')d\sigma + \int_{t_2}^{t_3} J(t,t')d\sigma + \Delta\sigma_0 \cdot J(t,t_0) + \Delta\sigma_1 \cdot J(t,t_1) + \Delta\sigma_2 \cdot J(t,t_2) \tag{1.9}$$

In order to summarize equation (1.9) it is useful to use the Stiltjes-Lebesge integrals, which are a different kind of integral used for non-continuous or quasi-continuous functions.

With the Stieltjes-Lebesge integrals Equation (1.9) becomes:

$$\varepsilon(t) = \int_0^t J(t,t')d\sigma \tag{1.10}$$

The only problem with this formulation is that we're taking into account only the deformations due to a change of stress and not considering the non-tensional deformations (i.e. shrinkage, temperature).

In order to have a more complete formulation and to include those non-tensional deformations it is useful to use the Mc Henry formulation for viscous-elastic constitutive law:

$$\boxed{\varepsilon(t) = \int_0^t J(t,t')d\sigma + \varepsilon_{sh}(t) + \varepsilon_t(t) = \int_0^t J(t,t')d\sigma + \bar{\varepsilon}(t)} \tag{1.11}$$

Where:

$\varepsilon_{sh}(t)$ is the strain due to shrinkage

$\varepsilon_t(t)$ is the strain due to temperature

This equation is a very powerful law because it makes it possible to find the deformation of a structure knowing the stress history of the structure itself.

1.3 STATICALLY DETERMINATE STRUCTURES

It is useful at this moment to propose an example that can explain the use of this equation for a statically determinate structure. In this case the stresses acting on the structure can be determined easily by using only equilibrium equations.

The equation to determine the strains can be applied directly in the form proposed by Mc Henry because, by knowing sigma from equilibrium and J from material properties, the only unknown in Equation (1.11) remains ε, which can be determined easily.

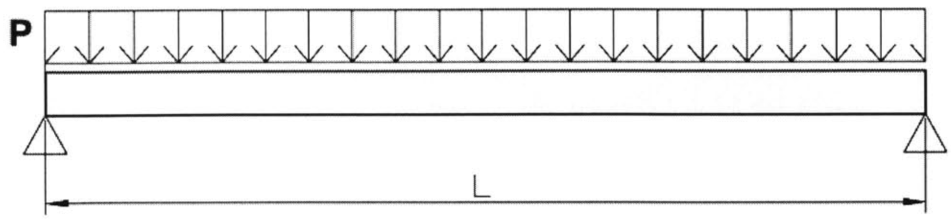


Figure 9: Statically determinate structure

If we consider the beam in Figure 9 it is possible to express the tension σ in the lowest fiber of the beam as: $\sigma = \frac{M}{I}\bar{y}$ where $\bar{y}$ is the distance of the lowest fiber of the section from the center of mass.

It is now possible to use the Mc Henry equation for viscous-elastic material:

$\varepsilon(t) = \int_0^t J(t,t')d\sigma$ and, by referring to the Riemann Integrals:

$$\varepsilon(t) = \sigma(t_0) \cdot J(t,t_0) + \int_{t_0}^{t} J(t,t')d\sigma$$

Given that the tension is constant with time, the integral is equal to zero. Therefore:

$$\varepsilon(x,t) = \frac{M(x)}{I}\bar{y} \cdot J(t,t_0) = \frac{M(x) \cdot \bar{y}}{I}[1+\varphi(t,t')]$$

To find the curvature (χ(x)) of this beam, it is only necessary to remember that the curvature of the beam, at each section is the inclination of the strain diagram acting on the section itself, therefore:

$$\chi(x,t) = \frac{\varepsilon(x,t)}{\bar{y}} = \frac{M(x)}{I}[1+\varphi(t,t')]$$

Now, by dividing and multiplying this expression by E(t_0) and remembering that

$$v''(x,t_0) = \chi(x,t_0) = \frac{M(x)}{E(t_0) \cdot I}$$

it is possible to rearrange the equation as:

$$\chi(x,t) = \chi(x,t_0)[1+\varphi(t,t_0)]$$

To find the deflection of the beam at mid-span, we can use the equations for a simply supported beam:

$$v_{max} = \frac{5}{384} \cdot \frac{p \cdot L^4}{E \cdot I} \text{ and,}$$

$$M_{max} = \frac{p \cdot L^2}{8} \text{ so:}$$

$$v_{max}(t) = \frac{5}{48} \cdot \frac{M_{max}(t)}{E \cdot I} \cdot L^2 = \frac{5}{48} \cdot \chi_{max}(t) \cdot L^2 = \frac{5}{48} \cdot \chi_{max}(t_0) \cdot [1 + \varphi(t,t_0)] \cdot L^2 = v_{max} \cdot [1 + \varphi(t,t_0)]$$

Therefore, the displacement at mid-span of this beam is the sum of the elastic displacement that the beam experiences at the time in which the load is applied plus the creep effect. For conventional values of E and C, φ reaches value between 4 and 5, and the displacement reaches values between 5 and 6 times the initial strain.

1.4 STATICALLY INDETERMINATE STRUCTURES

If we consider an indeterminate structure like the one in the figure below it is clear that in this case the problem is more complex:

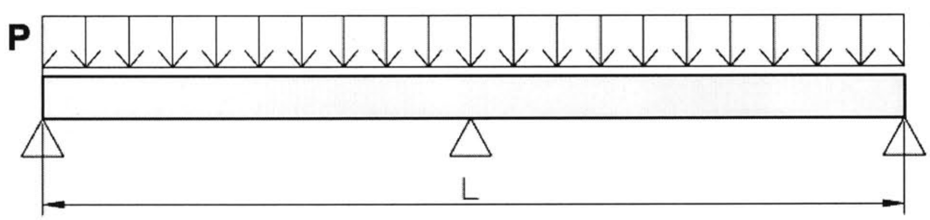


Figure 10: Statically indeterminate structure

In this case, the only known quantities are the deformation of the bean at certain fixed points (bearings). It is impossible to determine the stresses in the beam by just using equilibrium.

The difficulty of this case is in the fact that now, in equation (1.11), the unknown variable (σ) is within the integral and the known variable (ε) is free.

If now we consider equation (1.11) as a Volterra integral equation with:

$\varepsilon(t)$ = Known term,

$d\sigma$ = unknown function,

$J(t,t')$ = integral's kernel

it is possible to find an equation identical in form to equation (1.11) but that expresses stresses in terms of applied deformation and not deformations in term of applied stresses. The equation found using the Volterra integral equation is:

$$\sigma = \int_0^t d(\varepsilon - \bar{\varepsilon}) \cdot R(t,t') \qquad (1.12)$$

Where the function $R(t,t')$ is the relaxation function and it defined as:

$$1 = R(t_0,t_0) \cdot J(t,t_0) + \int_{t_0}^{t} \frac{\partial R(t',t_0)}{\partial t'} \cdot J(t,t')dt' \qquad (1.13)$$

so, for $t = t_0$:

$$R(t_0,t_0) = \frac{1}{J(t,t_0)} = E(t_0) \qquad (1.14)$$

and, when $t \neq t_0$:

$$-\varphi(t,t_0) = \int_{t_0}^{t} \frac{\partial R(t',t_0)}{\partial t'} \cdot J(t,t')dt' \qquad (1.15)$$

Using equations (1.15) and (1.14) it is possible to study the behavior of the function $R(t,t_0)$ in function of the time t.

If we consider the graphs below it is clear that:

- $\frac{\varphi(t,t_0)}{E(t_0)} = C(t,t_0)$ is monotonically increasing with time therefore the term $-\varphi$ in equation (1.15) is negative with its absolute value increasing with time.
- $J(t,t')$ is a positive quantity always increasing.

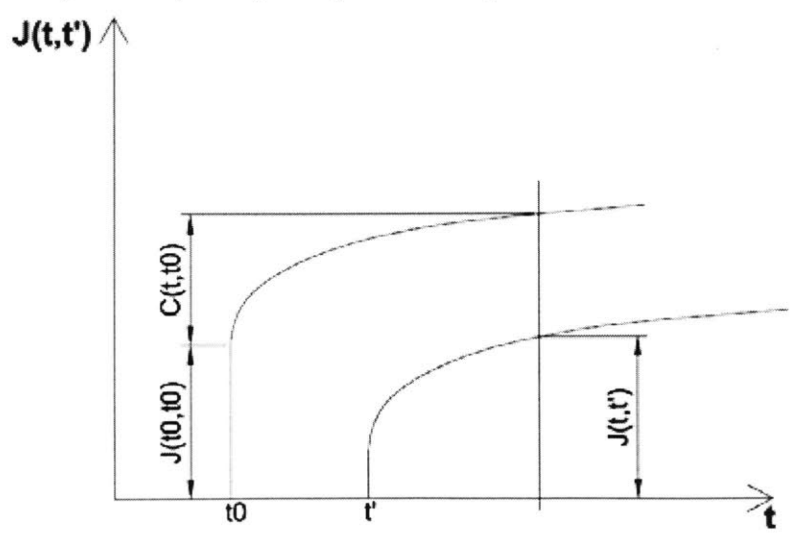


Figure 11: Study of the behavior of the function R

Therefore, to validate Equation(1.15), the following condition must be true:

$$\frac{\partial R(t',t_0)}{\partial t'} > 0 \tag{1.16}$$

From equations (1.16) and (1.14) it is possible to have an idea of the behavior of the function R during time:

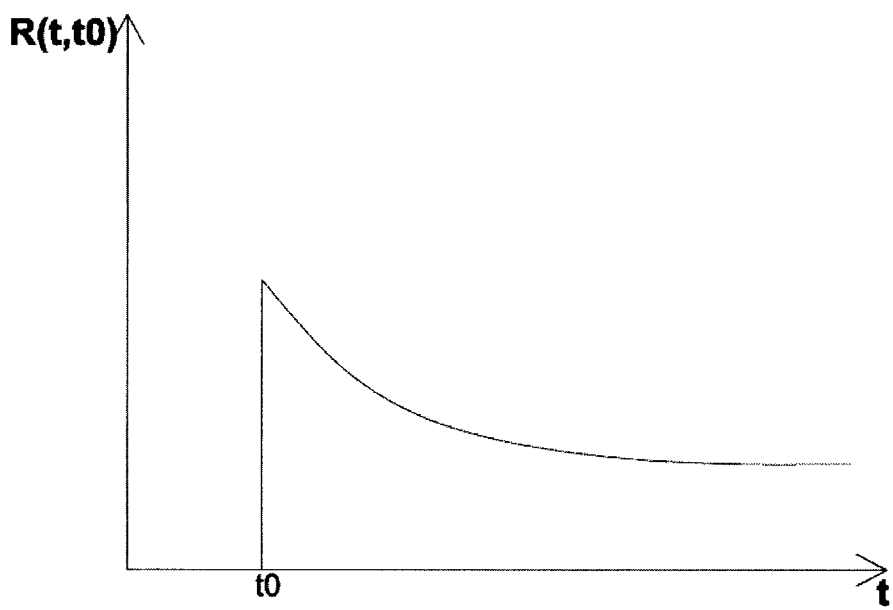


Figure 12: Function R(t,t')

It is interesting to notice how this function decreases during time, starting with the value of the initial elasticity modulus of the structure. This means that if a deformation is impressed to the structure at a certain time t_0, the stresses in the structure due to that deformation will decrease during time in the same way in which R decreases in time.

1.5 NON HOMOGENEOUS STRUCTURES

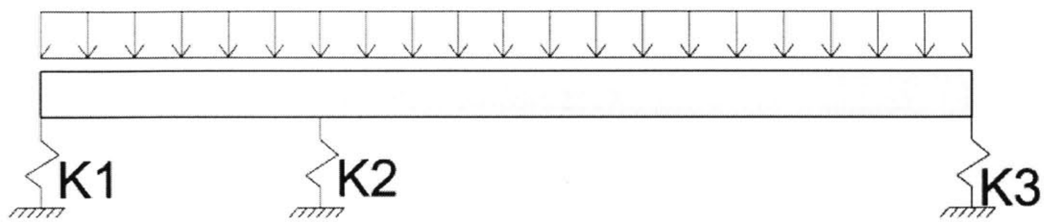


Figure 13: Non homogeneous structure

Figure 13 gives an example of a non-homogeneous indeterminate structure.

In this kind of structure the problem is that the displacements are functions of the stiffness of the non-concrete elements, and this leads to an indeterminacy of the problem.

If we consider equation (1.11) without non-tensional strains:

$$\varepsilon(t) = \int_0^t J(t,t')d\sigma = \sigma(t_0)\cdot J(t,t') + \int_{t_0}^t J(t,t')d\sigma \tag{1.17}$$

it is possible to observe that, for these kind of problems (indeterminate, non homogeneous) the first addendum in the right hand side of the equation is known, but not the second.

With the theory developed in the previous paragraphs, in fact, it is impossible to forecast how the stresses will change during time in a non-homogeneous structure.

In order to solve this problem we should provide a method to obtain the solution (or an approximation of it) to the integral in equation (1.17) without actually solving it.

One approach is to solve that integral by using an approximation given by Lagrange's theorem for continuous functions: If we consider a function $y=f(x)$ continuous between x and x_0, Lagrange's theorem asserts that:

$$\int_{x_0}^{x} y\cdot d\xi = \int_{x_0}^{x} f(\xi)\cdot d\xi = (x-x_0)\cdot\lambda\cdot f(x_0)$$

So, in the same way it is possible to simplify the integral in equation (1.17) as:

$$\int_{t_0}^{t} J(t,t')d\sigma = \left[\sigma(t)-\sigma(t_0)\right]\cdot\lambda\cdot J(t,t_0) \tag{1.18}$$

The only remaining problem is the determination of the parameter λ.

$$\lambda = \frac{\int_{t_0}^{t} J(t,t')\cdot d\sigma(t')}{\left[\sigma(t)-\sigma(t_0)\right]} \tag{1.19}$$

Using Lagrange's theorem, it is possible to be sure about the existence of λ, but it is impossible to find its expression.

The idea is to relate the parameter λ to a particular history of deformation and then to find its approximate value. The value of λ found will be an approximation if the history of deformation is different from the assumed one, but it will be the correct value when strains are exactly the assumed ones.

The assumption for the history of the strains is:

$$\varepsilon = a + b\cdot\varphi(t,t_0) \tag{1.20}$$

The closer ε is to this approximation, the more accurate the solution will be.

With this assumption, and knowing that $\varphi(t,t_0) = E(t_0)\cdot J(t,t_0) - 1$, it is possible to find:

$$\sigma = (a-b)\cdot R(t,t_0) + E(t_0)\cdot b \tag{1.21}$$

Combining Equation(1.21) with Equation(1.19):

$$\lambda = \frac{\int_{t_0}^{t} \frac{\partial R(t',t)}{\partial t'}\cdot J(t,t')\cdot dt'}{\left[R(t,t_0)-E(t_0)\right]\cdot J(t,t_0)} \tag{1.22}$$

If we try again to use Volterra's integral equation applied to equation (1.22), it is possible to find an easier form for the parameter λ as:

$$\lambda = \frac{1-E(t_0)\cdot J(t,t_0)}{\left[R(t,t_0)-E(t_0)\right]\cdot J(t,t_0)} = \frac{\varphi(t,t_0)}{\left[R(t,t_0)-E(t_0)\right]\cdot J(t,t_0)} \tag{1.23}$$

Using this expression of λ, equation(1.19) and equation(1.17), it is possible to find:

$$\varepsilon(t) = \sigma(t_0)\cdot J(t_0,t_0) + \left[\sigma(t)-\sigma(t_0)\right]\cdot J(t,t_0)\cdot\lambda(t,t_0) \tag{1.24}$$

By the substitution: $\left[\sigma(t)-\sigma(t_0)\right]\cdot J(t,t_0)\cdot\lambda(t,t_0) = \left[\sigma(t)-\sigma(t_0)\right]\cdot\frac{(1+\chi\cdot\varphi)}{E(t_0)}$, it is finally possible to find the final form of the age effective adjusted modulus method as:

$$\varepsilon(t) = \frac{\sigma(t_0)}{E(t_0)} \cdot (1+\varphi) + \frac{[\sigma(t) - \sigma(t_0)]}{E(t_0)} \cdot (1 + \chi \cdot \varphi) \tag{1.25}$$

where χ is defined as:

$$\chi = \frac{1}{1 - \frac{R(t,t_0)}{E(t_0)}} - \frac{1}{\varphi} \tag{1.26}$$

It can be to observed that equation (1.25) is a key expression because it allows the reduction of the non-linear creep theory into a simple elastic formulation. Both terms in the equation are, in fact, simple elastic terms with a modified elasticity modulus.

Given the difficulties related to the determination of the parameter χ, a lot of different theories have been proposed to approximate this function.(Appendix A)

In this thesis, the non approximate value for the function χ will always be used.

These values have been found using a FORTRAN program sold with the book: "Concrete Structures, Stresses and Deformations. A.Ghali, R. Favre, M.Elbadry"(third edition).

The program uses equation (1.26) coupled with the procedure to compute the function φ explained in the footnote of page 5. The results obtained for t'=3 days, RH =50% and normal hardening concrete are shown in the following graph.

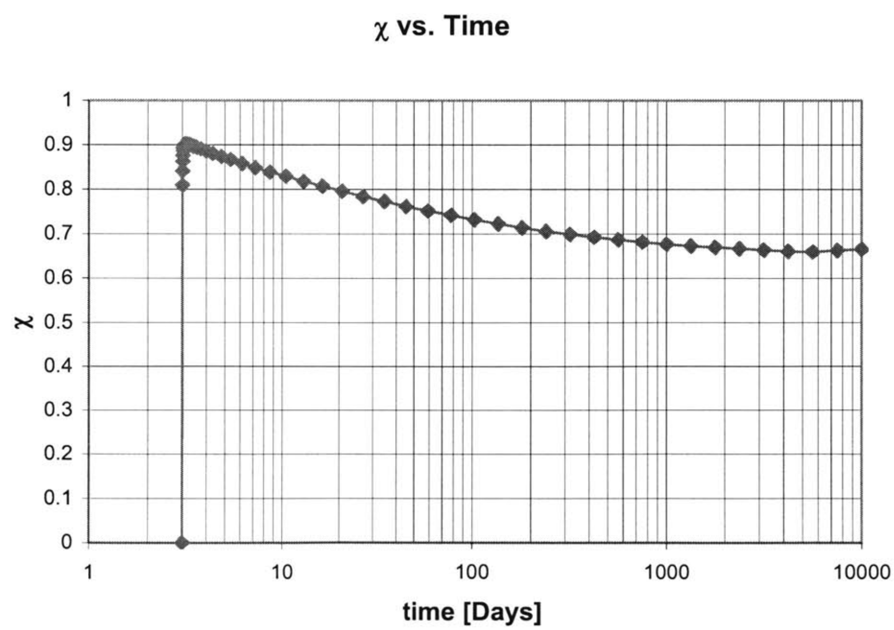


Figure 14: Chi as a function of time

1.6 AGE ADJUSTED EFFECTIVE MODULUS METHOD.

In the previous paragraphs we proved that, under certain hypotheses and assumptions, it is possible to study the long term behavior of a concrete structure simply by using a pseudo-elastic theory with an "adjusted" elasticity modulus.

$$\boxed{\begin{array}{l} hp: \varepsilon = a + b\cdot\varphi(t,t_0) \\ \begin{cases} \varepsilon(t) = \dfrac{\sigma(t_0)}{E(t_0)}\cdot(1+\varphi) + \dfrac{[\sigma(t)-\sigma(t_0)]}{E(t_0)}\cdot(1+\chi\cdot\varphi) \\ \chi = \dfrac{1}{1-\dfrac{R(t,t_0)}{E(t_0)}} - \dfrac{1}{\varphi} \end{cases} \end{array}} \qquad (1.27)$$

This method can be used for any kind of concrete structure and its only problem is the accuracy of the parameters φ and χ because of their dependency on the environment.

It is important to explain again the importance of the assumption at the base of the method: In order to find a convenient form for the parameter λ, we assumed the strains of the structure to be linearly dependent on the functional φ. This assumption implies that the strains are constrained to be only monotonically increasing, decreasing or constant. In the case in which the strains are not following this behavior, the only way to solve the creep problem is to solve the integral in equation (1.17) numerically. In all the other cases, the formulation of the age adjusted effective modulus method provides results in good accordance with observations.

2 CREEP EFFECT ON CABLE STAYED BRIDGES

Cable Stayed bridges can be classified as indeterminate-non-homogeneous structures, and for this reason the effect of creep can be one of the most important parameters to assess during the design of these kinds of structures.

During time, the effect of creep acting on the deck can affect the geometry and the tension of all the cables with a consequent discrepancy with the initial design condition.

The ideal solution would then be to try to find the optimal level of the initial prestress applied to the stays in order to minimize the effect of creep on the tension of the cables.

To find a solution to this problem, the idea used in this thesis is to use the force method in conjunction with the age effective adjusted modulus method in order to find the initial optimal prestressing tension in the stays.

2.1 MATHEMATICAL PROCEDURE

The mathematical procedure used in this section was introduced by F.Martynez Y Cabrera, P.G.Malerba, F.Bontempi and F.Biondini at the seventh international conference on computing in civil and building engineering (Seoul, Korea 1997). The goal of this procedure is to find the initial value of the prestressing force in order to completely avoid the effects of creep on the tension of the stays.

Consider, for example, the following cable stayed bridge:

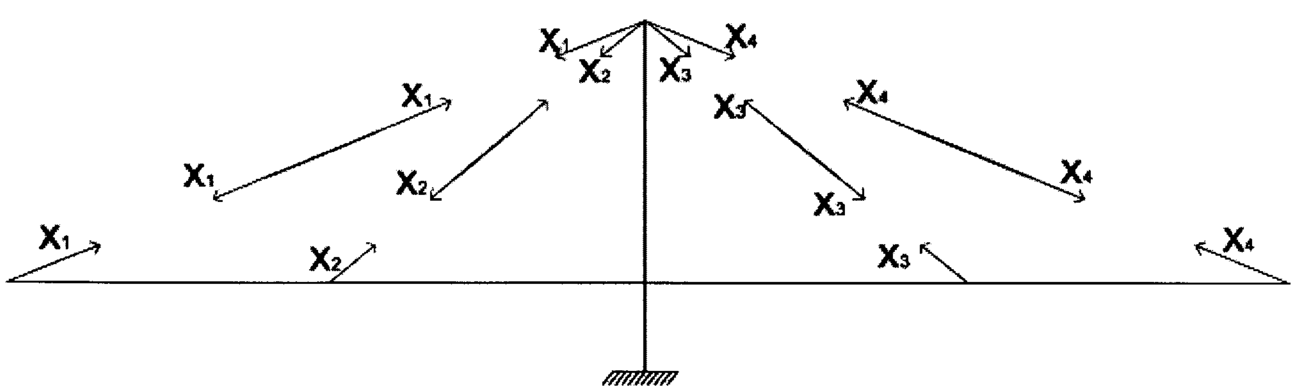


Figure 15

It is possible to study the system with the force method:

If we consider as unknown the n redundant forces, denoted as X_i, applied from the stays into the system (so the n forces will be in the direction of the cables) it is possible to express the internal forces (axial, shear and moment) $\underline{f} = |N \quad V \quad M|^T$ in a generic section of the bridge as the sum of the internal forces given by the tensions of the cables plus the internal forces given by the applied loads on the released structure.

So, by calling $X(t)$ the vector composed by all the components X_i, it is possible to write the internal forces due to the cable tension as $f_c = S_x \cdot X(t)$ and the internal forces given by the applied load as $\overline{f}$ Therefore the vector f can be expressed as:

$$\underline{f} = |N \quad V \quad M|^T = S_x \cdot X(t) + \overline{f} \tag{2.28}$$

Where S_x is a (3xn) matrix that expresses the internal forces in the deck due to a unitary force applied in the direction of each cable.

So, for example, at time t=0, the internal forces in a generic section of the bridge will be: $\underline{f}_0 = S_x \cdot X(0) + \overline{f}_0$.

If we consider the deck of the bridge to behave entirely as an elastic material with the constitutive law expressed by equation (1.22):

$$\varepsilon(t,t') = \frac{\sigma(t)\cdot(1+\chi\varphi)}{E(t_0)} + \frac{\sigma(t_0)}{E(t_0)}\cdot\varphi\cdot(1-\chi)$$

it is possible to write the strains over time with the expression:

$$\varepsilon = \left|\varepsilon_a \quad \gamma_a \quad -\frac{1}{\rho}\right|^T = (1+\chi\cdot\varphi)\cdot V\cdot f + \varphi\cdot(1-\chi)\cdot V\cdot f_0 \tag{2.29}$$

where T is the matrix that transforms internal forces into stresses:

$$T = \frac{1}{E(t_0)}\cdot\begin{vmatrix} \frac{1}{A} & 0 & 0 \\ 0 & \frac{1}{A_S} & 0 \\ 0 & 0 & \frac{1}{I} \end{vmatrix} \tag{2.30}$$

and:

A=Area of the section,

As=Shear area=$A/2(1+\chi)\chi_s$ where χ_s is the shear factor

I=Moment of inertia of the section.

If we now consider the prestress of the cable as an impressed displacement Λ between the head of the cables and the deck it is possible to equate the internal work with the external work in the system as:

$$\int_l \left[(\partial X(t)^T \cdot S_x^T) \cdot (1+\chi \cdot \varphi) \cdot T \cdot f \right] \cdot dl + \int_l \left[(\partial X(t)^T \cdot S_x^T) \cdot \varphi \cdot (1-\chi) \cdot T \cdot f_0 \right] \cdot dl = \partial X(t)^T \cdot \Lambda \quad (2.31)$$

Considering that we are studying a cable stayed bridge it is now possible to divide the integrals contained in equation (2.31) into two integrals:

The first one refers to concrete, where φ is different from zero.

The second one refers to steel, where it is possible to assume that φ is equal to zero since steel doesn't creep.

With this distinction, and by using the expression of f in Equation(2.28), Equation (2.31) becomes:

$$\left[(1+\chi \cdot \varphi) \cdot \int_{l_c} S_x^T \cdot T \cdot S_x .dl + \int_{l_s} S_x^T \cdot T \cdot S_x .dl \right] \cdot X(t) + \left[(1+\chi \cdot \varphi) \cdot \int_{l_c} S_x^T \cdot T \cdot \bar{f} .dl + \int_{l_s} S_x^T \cdot T \cdot \bar{f} .dl \right] +$$
$$+ \varphi \cdot (1-\chi) \cdot \left[\left(\int_{l_c} S_x^T \cdot T \cdot S_x .dl \right) \cdot X(t_0) + \int_{l_s} S_x^T \cdot T \cdot \bar{f}_0 .dl \right] = \Lambda$$

(2.32)

If we now consider the bridge to be loaded only by its self weight, it is possible to assume that the internal forces induced by the self load are constant in time ($\bar{f}(t) = \bar{f}_0$). Therefore, equation (2.32) can be rearranged as:

$$\left[(1+\chi \cdot \varphi) \cdot \int_{l_c} S_x^T \cdot T \cdot S_x .dl + \int_{l_s} S_x^T \cdot T \cdot S_x .dl \right] \cdot X(t) + \varphi \cdot (1-\chi) \cdot \left(\int_{l_c} S_x^T \cdot T \cdot S_x .dl \right) \cdot X(t_0) +$$
$$+ (1+\varphi) \cdot \int_{l_c} S_x^T \cdot T \cdot \bar{f}_0 .dl + \int_{l_s} S_x^T \cdot T \cdot \bar{f}_0 .dl = \Lambda \quad (2.33)$$

In order to obtain a procedure less complex it is possible to consider the following definitions:

$$A_c = \int_{l_c} S_x^T \cdot T \cdot S_x .dl$$
$$A_s = \int_{ls} S_x^T \cdot T \cdot S_x .dl$$
$$Bc = \int_{l_c} S_x^T \cdot T \cdot \overline{f}_0 .dl \tag{2.34}$$
$$B_s = \int_{ls} S_x^T \cdot T \cdot \overline{f}_0 .dl = 0$$

where:

- The elements $a^c{}_{ij}$ of the matrix A_c are the displacements of the deck in the direction of the force X_j due to a unit load applied in the direction of the force X_i ,
- The elements $a^s{}_{ij}$ of the matrix A_s are the displacements of the cables in the direction of the force X_j due to a unit load applied in the direction of the force X_i ,
- The elements $b^s{}_i$ of the vector B_c are the displacements of the deck in the direction of the force X_i due to the loads applied to the structure (i.e. self weight).

By using the definition of these new matrices it is possible to reduce expression (2.33) to:

$$\left[(1+\chi\cdot\varphi)\cdot A_c + A_s\right]\cdot X(t) + \varphi\cdot(1-\chi)\cdot A_c\cdot X(t_0) + (1+\varphi)\cdot B_c = \Lambda \tag{2.35}$$

If we now try to express Equation(2.35), not in terms of the actual tension in the stays, but in terms of the variation of the tension in the stays, it is sufficient to use the expression:

$$X(t) = X(t) - X(t_0) + X(t_0) = \Delta X(t) + X(t_0).$$

With this expression, equation (2.35) can be rearranged as:

$$\left[(1+\chi\cdot\varphi)\cdot A_c + A_s\right]\cdot\Delta X(t) + \left[(1+\chi\cdot\varphi)\cdot A_c + A_s\right]\cdot X(t_0) + \varphi\cdot(1-\chi)\cdot A_c\cdot X(t_0) + (1+\varphi)\cdot B_c = \Lambda \tag{2.36}$$

and if the value of $X(t_0)$ is:

$$X(t_0) = (A_c + A_s)^{-1}\cdot(\Lambda - B_c) \tag{2.37}$$

expression (2.36) becomes:

$$\left[(1+\chi\cdot\varphi)\cdot A_c + A_s\right]\cdot\Delta X(t) + \varphi\cdot\left[A_c\cdot X(t_0) + B_c\right] = 0 \tag{2.38}$$

If we look at equation (2.38), it is clear that if $X(t_0) = -(A_c)^{-1}\cdot B_c = X^{opt}(t_0)$ then $\Delta X(t)$ should be equal to zero for every value of t and therefore X(t) will be constant over time.

If we substitute the value of $X^{opt}(t_0)$ into equation (2.37) it is possible to find the optimal value of the initial prestressing (Λ) in the cable that will provide a constant tension during time.

$$X(t_0) = X^{opt}(t_0) = -(A_c)^{-1} \cdot B_c = (A_c + A_s)^{-1} \cdot (\Lambda - B_c) \tag{2.39}$$

Therefore, the optimal value of Λ will be:

$$\boxed{\Lambda = \Lambda^{opt} = -A_s \cdot A_c^{-1} \cdot B_c} \tag{2.40}$$

2.2 EXAMPLE 1

In order to have a clearer view of the theory an easy application is introduced: An example of a cantilever beam with a cable attached at the end to reduce the deflections is proposed.

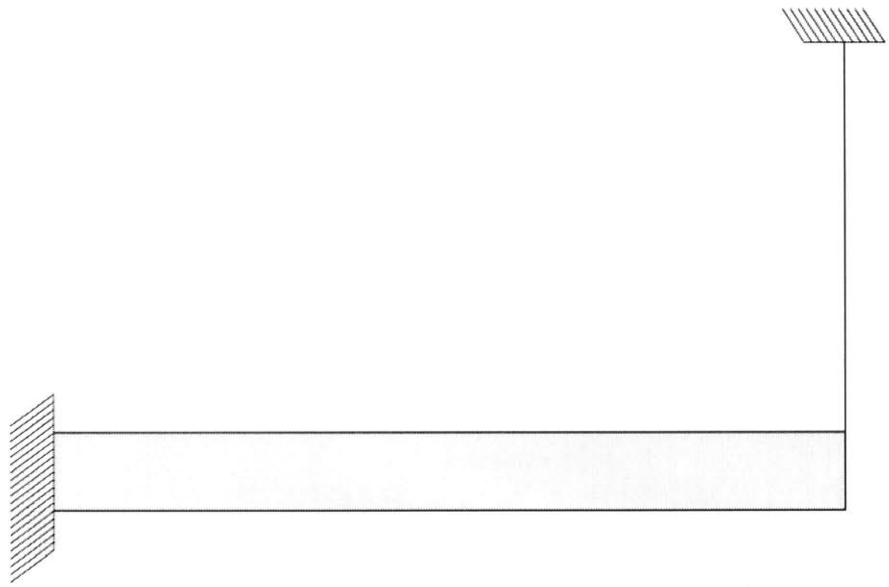


Figure 16

The beam considered in this example is a 20 meters long concrete beam supported at the end by a steel cable with an area of 6.3 cm2.

The dimensions, expressed in meters, of the beam are the following:

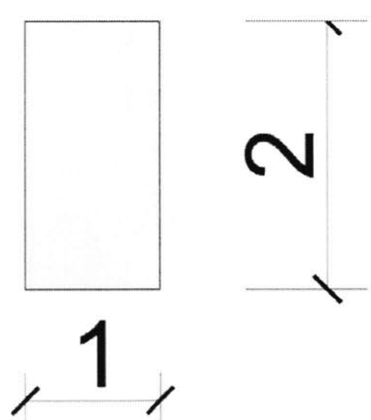


Figure 17

The load applied on the beam is just the dead load. In this case, for a 2x1 beam the dead load is approximately equal to $\rho \cdot b \cdot h = 25000\left[\frac{N}{m^3}\right] \cdot 1[m] \cdot 2[m] = 50000\left[\frac{N}{m}\right]$ where ρ is the specific weight of reinforced concrete.

If we want to use the approach explained in the previous paragraph it is necessary to release the structure and to study the following case:

Figure 18

It is now possible to find A_c, A_s and B_c defined by equations(2.34):

- A_c is the displacement of the beam in the direction of the cable due to X=1.

In this case, given that we are studying a simple cantilever beam, the expression of Ac will be: $A_c = \frac{L^3_{beam}}{3 \cdot E_{concrete} \cdot I_{beam}} = 0.0002129\left[\frac{mm}{N}\right]$

- A_s is the displacement in the direction of the cable due to X=1 applied on the cable itself:

In this case: $A_s = \frac{L_{cable}}{E_{steel} \cdot A_{cable}} = 0.000079\left[\frac{mm}{N}\right]$

- B_c is the displacement of the beam in the direction of the cable due to self weight of the beam.

In this case, it is simply the displacement of a cantilever beam subjected to a uniform load P and its value is equal to: $B_c = \frac{-P \cdot L^4_{beam}}{8 \cdot E_{concrete} \cdot I_{beam}} = -50[mm]$ where the minus sign indicate that this displacement is in the opposite direction of A_c

.

From the values of A_s, A_c and B_c, the optimal prestressing value Λ_{opt}, can be found by applying equation(2.40): $\Lambda^{opt} = -A_s \cdot A_c^{-1} \cdot B_c = -\frac{0.00079}{0.0002129} \cdot (-50) = 29.841[mm]$

Once Λ_{opt} has been calculated it is possible to find the value of the initial tension in the cable by applying equation(2.39) : $X^{opt}(t_0) = -(A_c + A_s)^{-1} \cdot (\Lambda - B_c) = -\frac{29.841+50}{0.0002129+0.000079} = 375000[N]$.

The values of X_{opt} and Λ_{opt} are the solution of the force method for the elastic case. To study the behavior of the structure during time it is now necessary to use equation(2.35) for the released structure.

The system can be studied, now, by using the superposition method.

The response of the system is in fact the sum of the response of the same system under the following separate loading conditions:

- Dead Load
- Prestressing of the cables.

DEAD LOAD.

Under only the action of the dead load (Bs≠0; Λ=0), equation(2.35) can be rearranged as:

$$[(1+\chi\cdot\varphi)\cdot A_c + A_s]\cdot X(t) + \varphi\cdot(1-\chi)\cdot A_c \cdot X(t_0) + (1+\varphi)\cdot B_c = 0 \quad (2.41)$$

And therefore, the displacement induced in the structure due to the dead load at time t is:

$$D_{Dead}(t) = -[\varphi\cdot(1-\chi)\cdot A_c \cdot X(t_0) + (1+\varphi)\cdot B_c],$$

Then, from equation (2.41), it is possible to find the tension in the cable at time t due to only the effect of the dead load as:

$$X(t)_{Dead} = \frac{D_{Dead}}{[(1+\chi\cdot\varphi)\cdot A_c + A_s]} = -\frac{[\varphi\cdot(1-\chi)\cdot A_c \cdot X(t_0) + (1+\varphi)\cdot B_c]}{[(1+\chi\cdot\varphi)\cdot A_c + A_s]} \quad (2.42)$$

CABLE PRESTRESSING

Using the same procedure used for the dead load, equation(2.35) can be rearranged (Bs=0; Λ≠0) as:

$$\left[(1+\chi\cdot\varphi)\cdot A_c + A_s\right]\cdot X(t)+\varphi\cdot(1-\chi)\cdot A_c\cdot X(t_0)=\Lambda$$

That will allow writing the displacements experienced by the beam at time *t* under the action of the cable prestressing as:

$$D_{Prestressing}(t)=-\varphi\cdot(1-\chi)\cdot A_c\cdot X(t_0)+\Lambda$$

and the tension in the stay at time t due to only the prestressing action as:

$$X(t)_{Prestressing}=\frac{D_{Prestressing}}{\left[(1+\chi\cdot\varphi)\cdot A_c + A_s\right]}=\frac{-\varphi\cdot(1-\chi)\cdot A_c\cdot X(t_0)+\Lambda}{\left[(1+\chi\cdot\varphi)\cdot A_c + A_s\right]} \quad (2.43).$$

DEAD LOAD+PRESTRESSING.

Combining the expression of the displacement experienced by the beam under the separated actions of dead loads and prestressing, it is finally possible to find the real displacement of the beam at time t as:

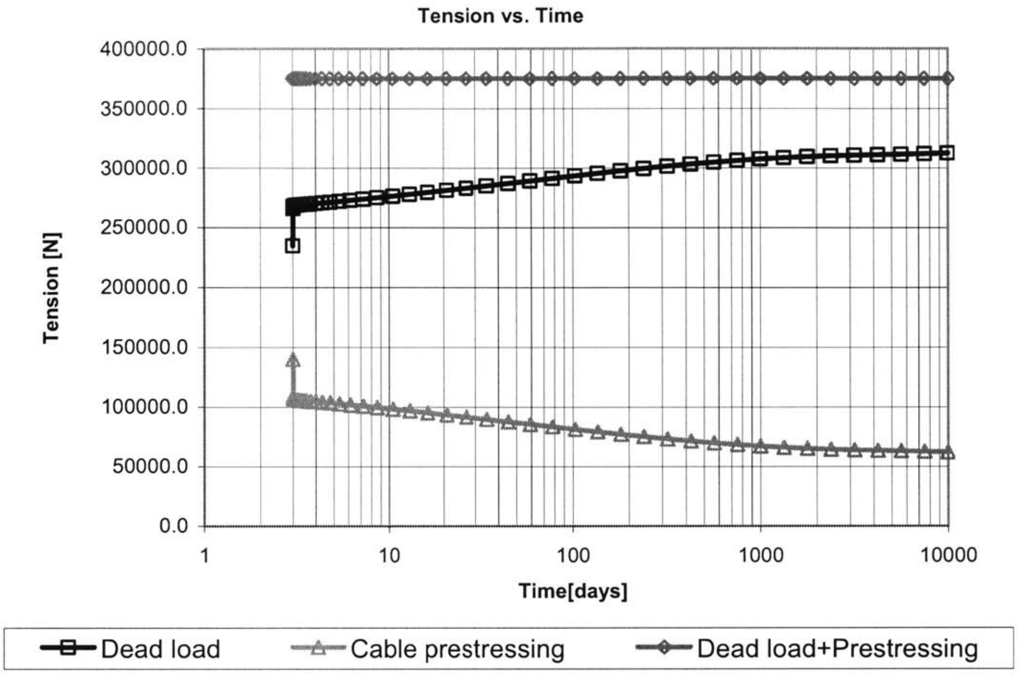
$$D(t)=-\varphi\cdot(1-\chi)\cdot A_c\cdot X(t_0)-(1+\varphi)\cdot B_c+\Lambda \quad (2.44)$$

And the total tension in the stays as:

$$X(t)=\frac{D}{\left[(1+\chi\cdot\varphi)\cdot A_c + A_s\right]}=\frac{-\varphi\cdot(1-\chi)\cdot A_c\cdot X(t_0)-(1+\varphi)\cdot B_c+\Lambda}{\left[(1+\chi\cdot\varphi)\cdot A_c + A_s\right]} \quad (2.45).$$

By simply using expressions (2.42),(2.43) and (2.45), it is possible to create an Excel spreadsheet that computes the tension of the stays for the different values of φ and χ at different times (APPENDIX B).

Using the expression of φ and χ explained in the previous chapter, it is possible to plot the tension in the stays over time as follows:



From the graph above it is possible to observe that, due to the effect of creep:

- The tension in the stay increases during time due to the effect of the dead load. This fact can be explained simply by considering that under the effect of the dead load, the beam tends to amplify its deflection due to creep and pull the cable.

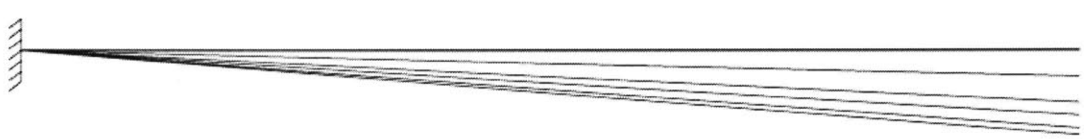


Figure 19: Dead load effect

- The tension in the stay decreases during time due to the effect of the prestress. In this case it is also possible to understand this phenomenon simply by considering that under the pulling action of the prestressing force the beam will increase its deflection due to creep pushing the cable

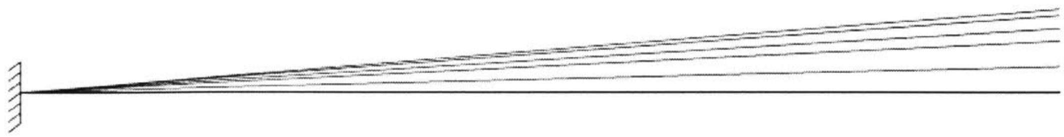


Figure 20:Prestress effect

- Considering the combined effect of dead load and prestressing, the tension in the stays remains constant during time.

2.3 EXAMPLE 2

The second example proposed is similar to the first one but it involves two cables instead of only one.

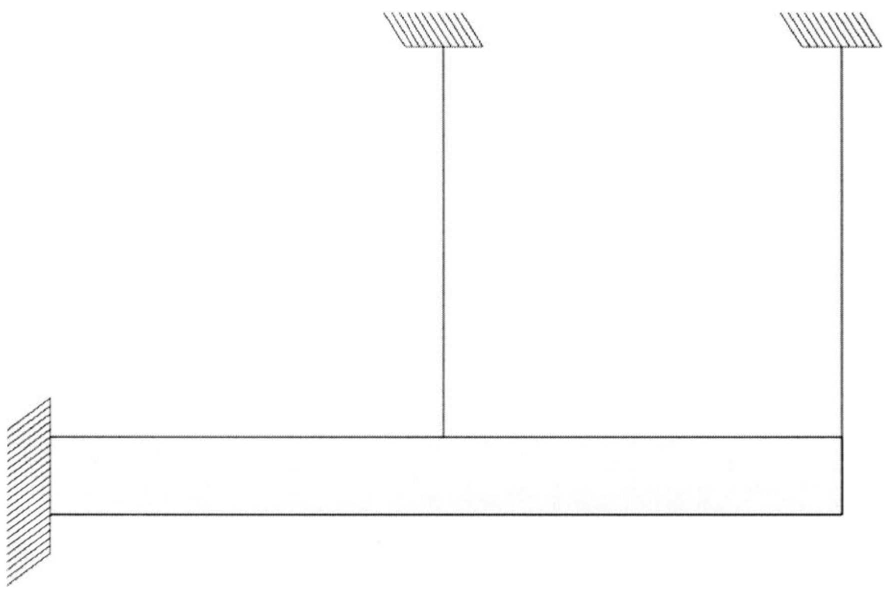


Figure 21

For this example it is also necessary to release the structure and to use the force method:

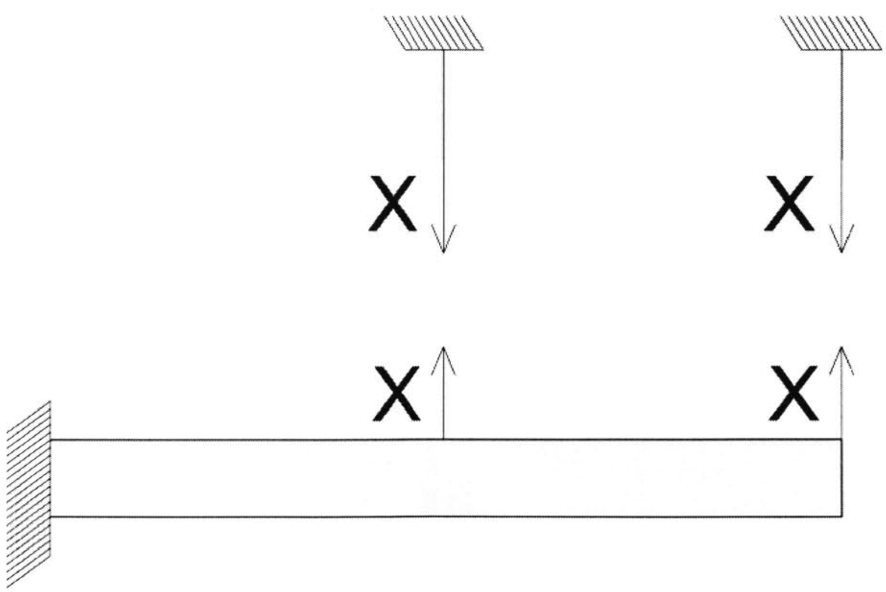


Figure 22

The difference between the previous example and this one is that, in this case, it is necessary to use a matrix expression for all the terms involved.

Considering the beam to have the same geometrical properties of the beam in the previous example the resulting parameters are:

$$A_c = \begin{vmatrix} \dfrac{L_{beam}^{\ 3}}{3 \cdot E_{concrete} \cdot I_{beam}} & \dfrac{5}{48} \cdot \dfrac{L_{beam}^{\ 3}}{E_{concrete} \cdot I_{beam}} \\ \dfrac{5}{48} \cdot \dfrac{L_{beam}^{\ 3}}{E_{concrete} \cdot I_{beam}} & \dfrac{L_{beam}^{\ 3}}{24 \cdot E_{concrete} \cdot I_{beam}} \end{vmatrix}$$

$$A_s = \begin{vmatrix} \dfrac{L_{cable}}{E_{steel} \cdot A_{cable}} & 0 \\ 0 & \dfrac{L_{cable}}{E_{steel} \cdot A_{cable}} \end{vmatrix}$$

$$B_c = \begin{vmatrix} -\dfrac{1}{8} \cdot \dfrac{P \cdot L_{beam}^{\ 4}}{E_{concrete} \cdot I_{beam}} \\ -\dfrac{17}{384} \cdot \dfrac{P \cdot L_{beam}^{\ 4}}{E_{concrete} \cdot I_{beam}} \end{vmatrix}$$

where:

- The elements of the matrix A_c are the displacement of the beam in direction j due to a unitary force applied in direction i.
- The elements of the matrix A_s are the displacements of the cable i due to a unitary load applied in direction j and,
- The elements of the vector B_c are the displacements in the direction of the unknowns due to the self weight.

With the expression of A_c, A_s and B_c it is possible to find the optimal value of the initial prestressing simply by again applying equation (2.40):

$$\Lambda^{opt} = -A_s \cdot A_c^{-1} \cdot B_c = \begin{vmatrix} 0.015633563 \\ 0.045465565 \end{vmatrix} [m]$$

Using the same procedure used for the previous example it is possible to find the values of the tension in the stays due to the dead load and the prestressing action and their combination at different ages. Thee are plotted as follow:

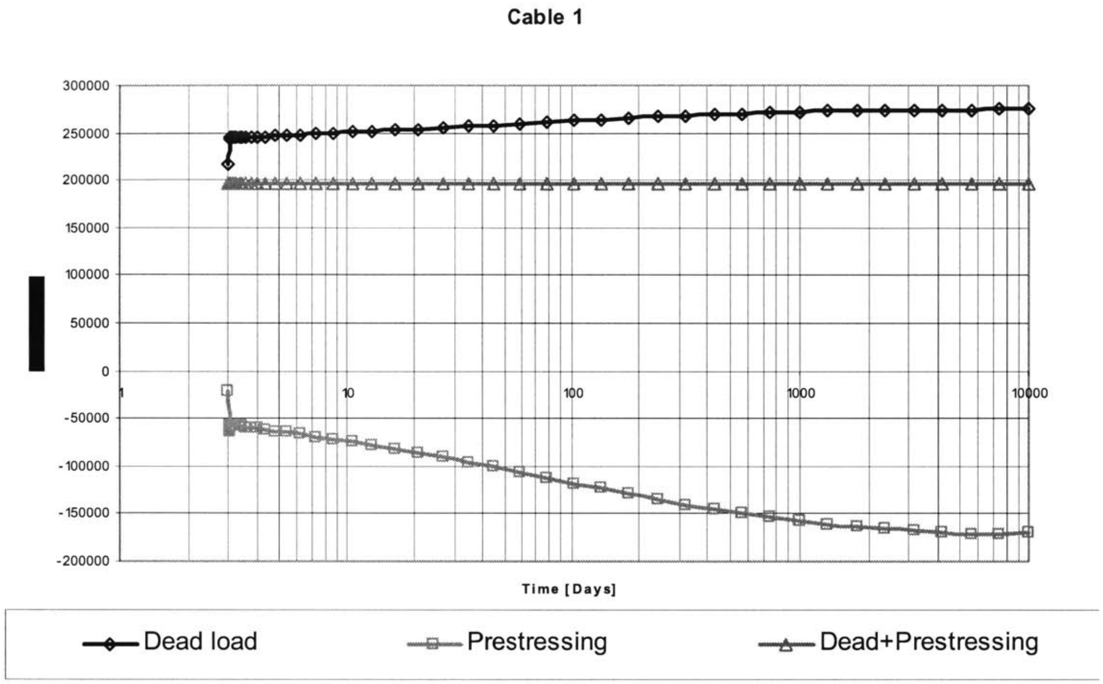


Figure 23

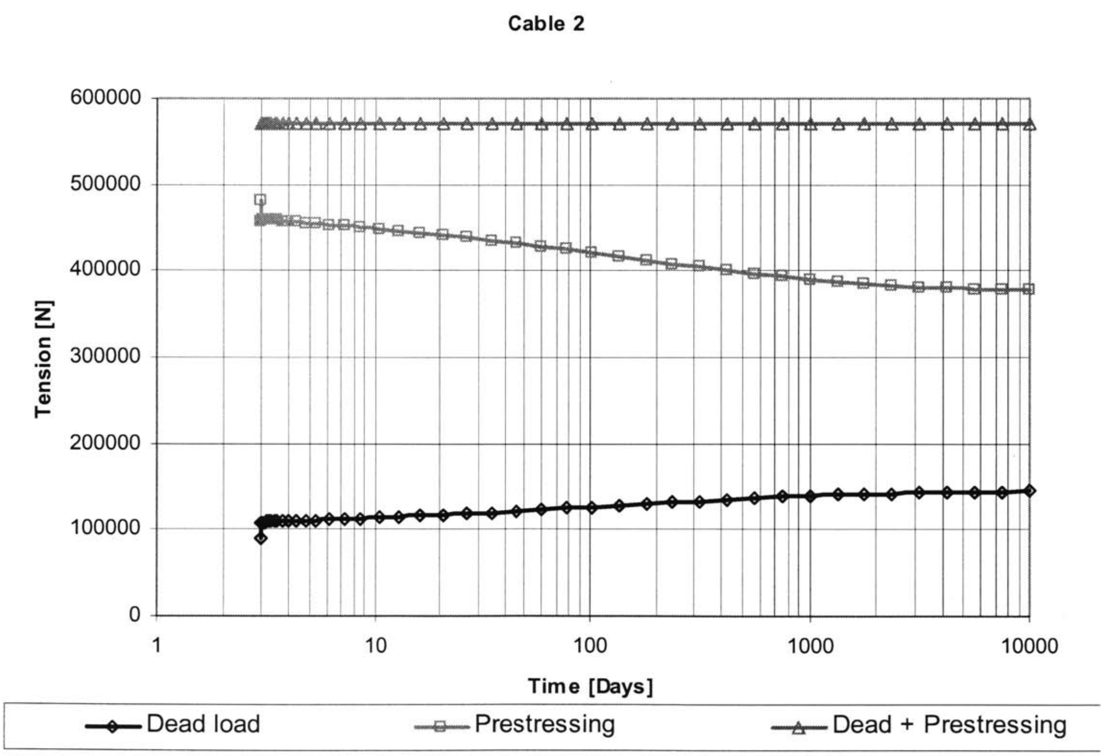


Figure 24

In this case, with the action of two cables acting on the beam, the resultant tension remains constant during time for the first ten thousands days.

As the two previous examples showed, this method provides a simple way to find the initial value of the prestressing force applied to the stays that allows the negation of the effects of creep n the tension of the cables.

It will be observed in the next chapter that this procedure works perfectly in the case in which the response of the beam is not controlled only by flexure (as in the previous examples) but also by axial and shear components. In cable stayed bridges, one of the parameters that can affect the tension in the stays is also the creep behavior of the deck under the compression given by the horizontal projection of the force in the cables.

3 CASE STUDY

In this chapter the procedure to find the optimal prestressing action in the stays is used on one of the proposals for the redesign of the Waldo-Hancock (Maine, USA).
This bridge was chosen because of the complexity of its geometry. As it is possible to see in the picture below, the geometry of the bridge is developed in three directions due to the curvature of the deck and the inclination of the arch. Compared to the examples illustrated in the previous chapter, the effects of creep in this bridge affect the deformations of the bridge in all directions and not only for the vertical deflection. Due to the self weight of the bridge there exists a tendency to either deflect vertically or horizontally. Additionally, (due to the action of the cables) the deck tends to shrink during time.

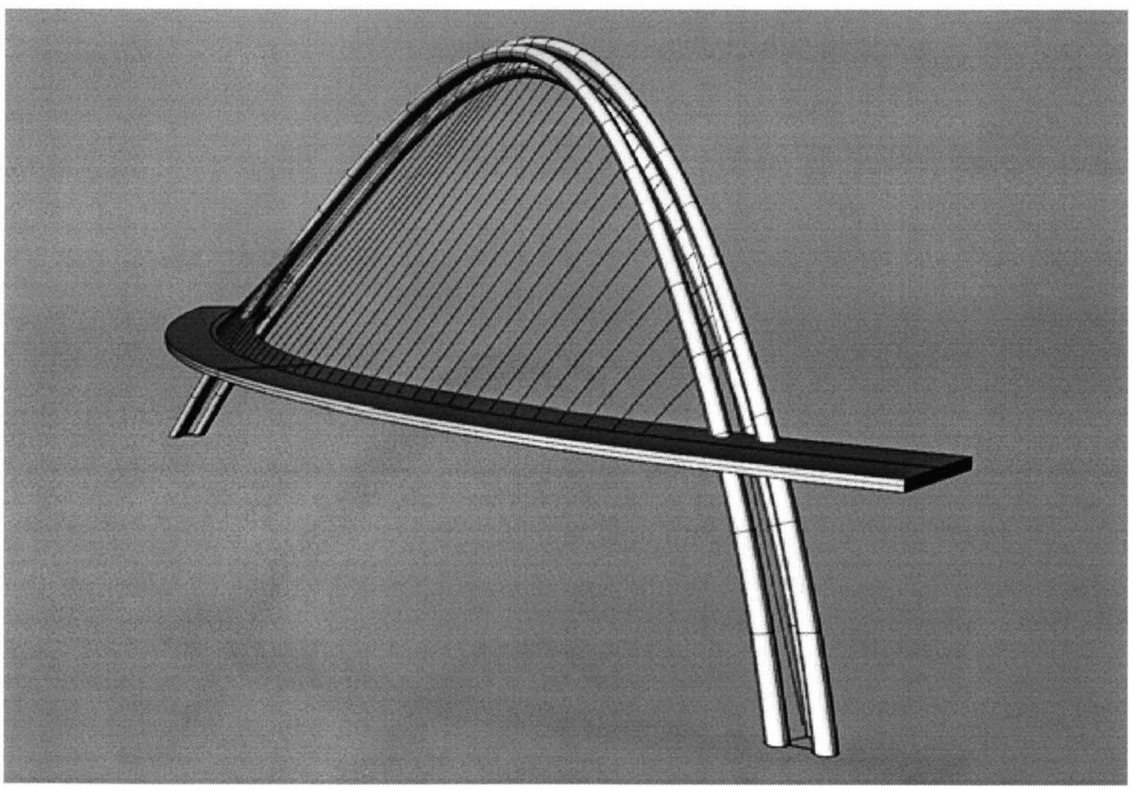


Figure 25:Proposal for the Redesign of the Waldo-Hancock Bridge

3.1 BRIDGE DESCRIPTION

The bridge chosen for the case study is a 273 m long curved arch bridge.

Its geometry is characterized by the presence of an inclined arch that, through the cables, counterbalances the weight of the deck. The arch is 375 meters long and its height above the water level is 120 meters. The arch height above the deck is only 80 meters.

The deck, made entirely of concrete, is curved with a constant curvature of radius 330 m. Its width, of 25 meters, is also constant.

The cables are spaced every six meters in order to simulate a constant action on the deck and on the arch. Their angle from the deck varies from 40° for the first cables to 60° for the central cables. Also, the length of the cables varies from the borders of the bridge to the center, but their area was chosen to be constant in order to facilitate the construction phase of the structure.

Due to the curvature of the deck and to the inclination of the arches, the cables are inclined with different angles with respect to each other. Thus, the actions that they transfer onto the deck and to the arch are different for each cable.

3.2 DETERMINATION OF "Ac", "As" AND "Bc"

The determination of the matrices A_c, A_s and of the vector B_c is more complex in this case, due to the complex geometry of the system. The procedure used to find the values of these matrices is defined as follow:

-A model of the released structure of the bridge was created in the program SAP 2000

-The program was run 43 times; each time with a unitary force applied in a different location of the unknowns

-From the program it was possible to obtain the values of the displacements of the points under each different load.

-By summing the values of the displacements of the arch and the deck and by projecting them in the direction of the stays it was possible to build the matrices A_c,A_s,B_c.

A_c

To find matrix A_c with the specifications of the previous chapter it is necessary to apply a unitary force to each cable and than to study the deflections in the directions of the cables due to those forces. The first vector of the matrix A_c will be the vector of the displacements of the deck in the directions of the cables due to a unit force applied to the first cable.

A_s

As in the previous examples, the matrix *As* is the matrix of the displacements of the cables due to An applied unit load. Again, the first column vector of this matrix is the vector of the displacements of the cables due to a unit load applied to the first stay. The vector will be such that the first element is non zero and all the others are zeros

B_c

This vector is the vector which includes the deflection of the system due to the effect of the dead load. Its first component is the deflection of the system at the location of the first cable, in the direction of the first cable, due to the external load.

3.3 RESULTS

It is difficult to report all of the procedure because, due to the presence of 43 cables in the system, the matrices used are all [43x43] and the vectors are [43x1]. Nonetheless, the procedure used is exactly the same as for the second example in the previous chapter. From A_c,A_s and B_c it is possible to find the optimal value of the initial prestressing Λ_{opt} and also the initial tension in the stays X_{opt}.

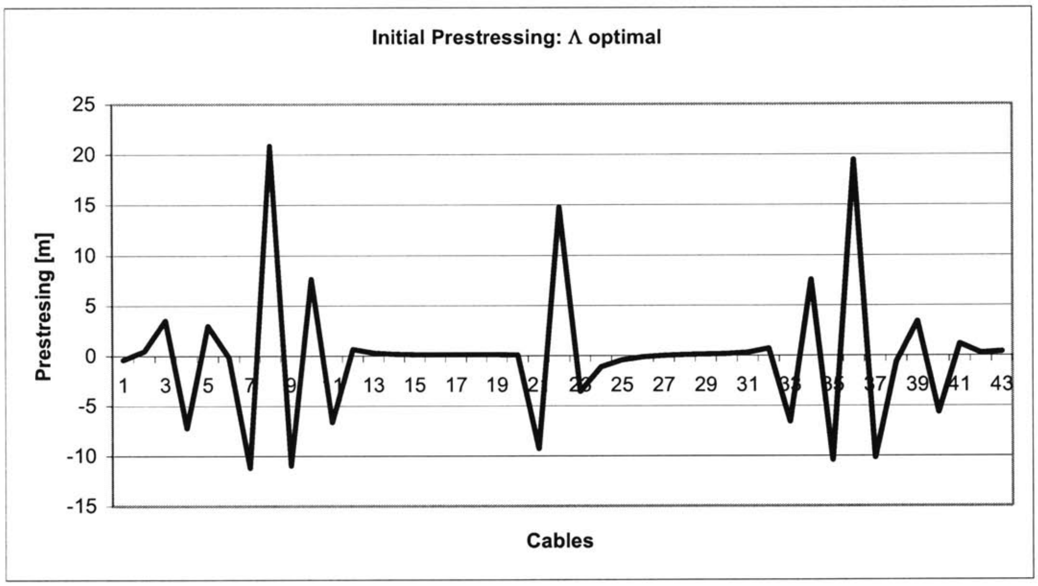


Figure 26 Prestressing of the Stays

From Figure 26 one can observe that the values of prestressing are not reasonable. To obtain the solution desired it is necessary to have some cables in compression and some other cables prestressed with a force equivalent to 20 meters of deformation.

From the graph above it is clear that this unfeasible situation is due to the large number of cables used for the bridge. When the first cable is tensioned it pulls the deck up, leading the second cable to be unnecessary or even in compression. Even if this solution is wrong the results obtained with this analysis behave in the expected way. If we try to plot the tension in the stays over time, the solution found is in line with what is expected.

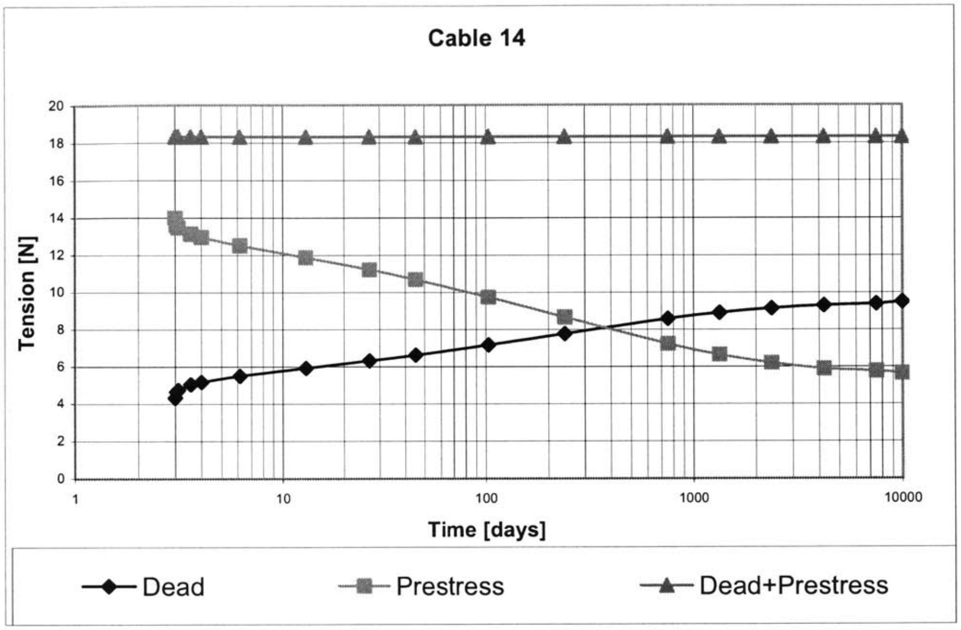


Figure 27: Tension in cable 14

Given the unfeasibility of the results obtained, a sensitivity analysis was performed on the bridge in order to find the correct positioning of the stays.

In the next table is reported the prestress force, expressed in meters, that was calculated for the same bridge but with half of the cables.

The original design of the bridge provided one cable every six meters and the solution proposed in this sensitivity analysis uses one cable every twelve meters.

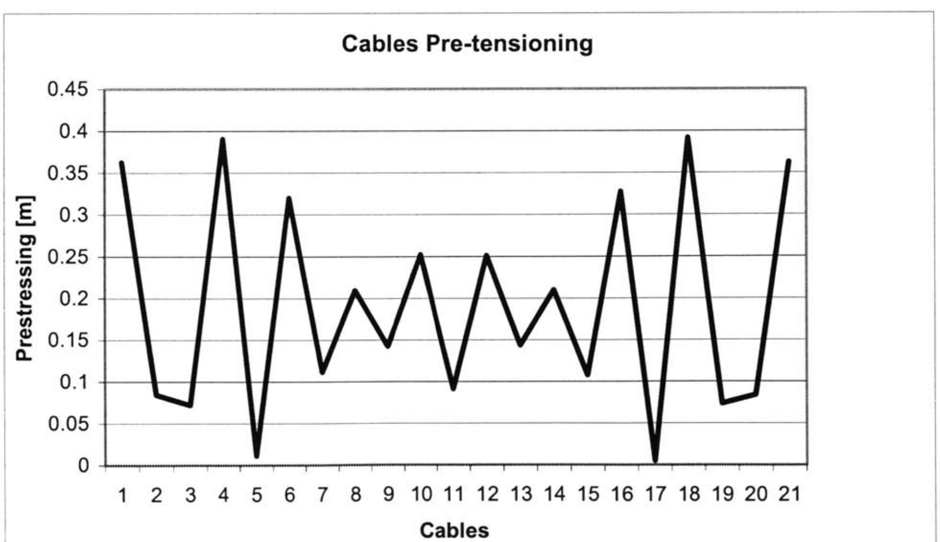


Figure 28:Sensitivity Analysis: Prestressing in the Stays.

Using the new values of the prestressing force in the cables, the same procedure used for the examples in the previous chapter was employed (Appendix C).

The results for two of the cables are reported in the following charts; all others are found in Appendix C.

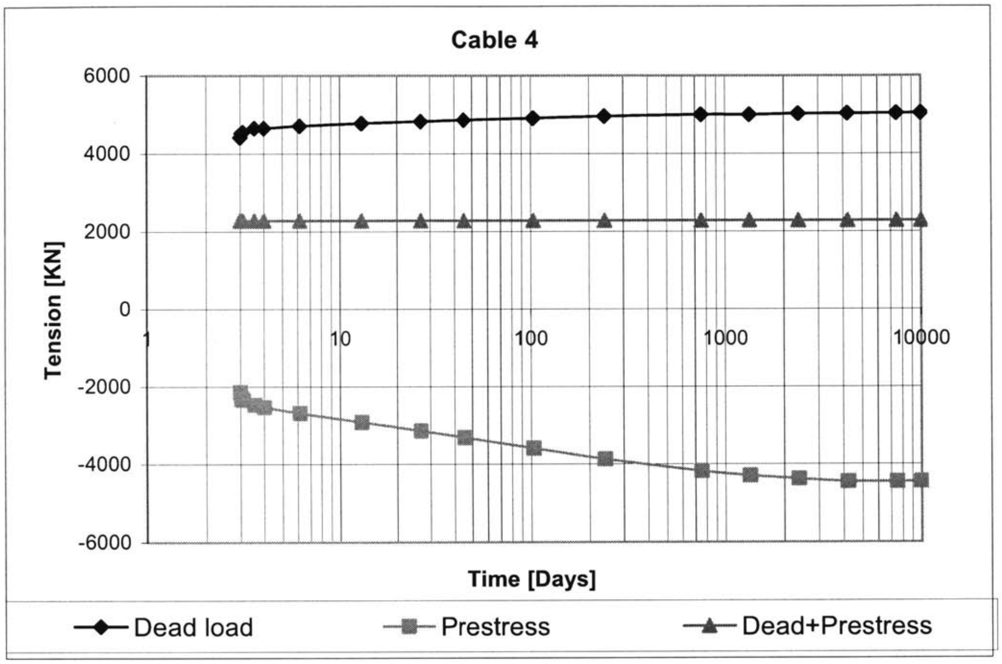


Figure 29:Cable 4

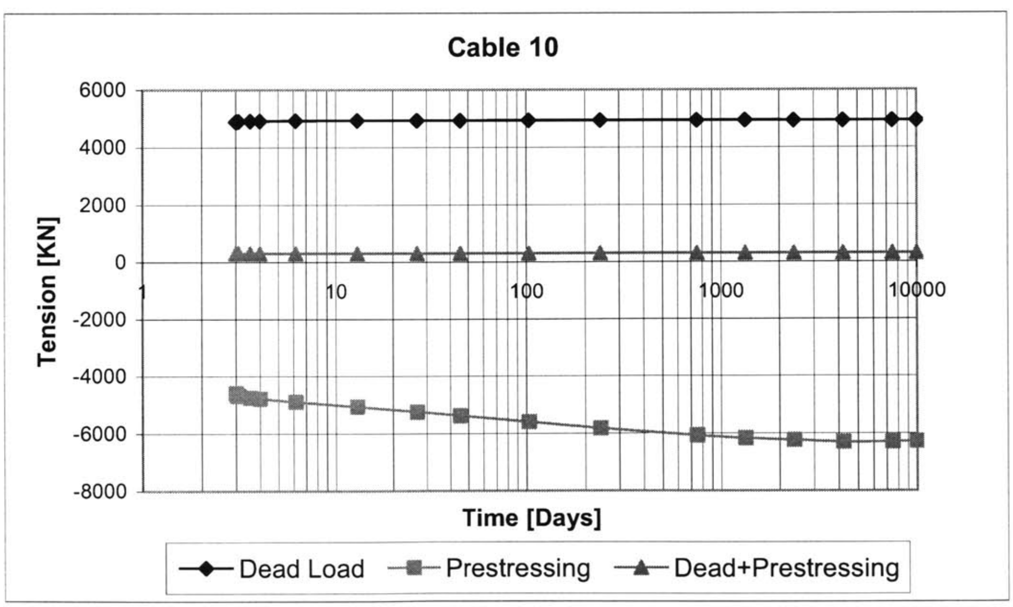


Figure 30:Cable10

From figures 5 and 6 it is possible to observe that the goal of having a constant tension in the cables during time was achieved with a reasonable level of tensioning in the stays.
For cable 4, the final tension is on the order of 2000 [KN], which requires 90 wires. Considering that the biggest cable ever produced contains 400 wires the value seems to be acceptable.

4 CONCLUSIONS

Trough the theory, the example and the case study proposed in this thesis, it has been proven and shown a procedure that can allow neutralizing the effect of creep on the tension of the stays in a cable stayed bridge. This procedure is based on the application of the force method in conjunction with the Age-Adjusted effective Modulus Method in order to keep into account the effects of creep.

The long term behavior of cable stayed bridges was, in fact, simplified a sequence of discrete steps in which the bridge can be studied as a pseudo elastic structure.

Once the response of the bridge over time was found, it has been possible to find the optimal value of the prestressing force in the stays in order to neutralize the effect of creep.

The interesting aspect of this procedure is that it permits to study a complex phenomenon like the effect of creep on an indeterminate, non-homogeneous structure, with an easy and automatic model. As shown in the examples proposed, it is possible to simulate in an accurate way the long term behavior of a complex concrete structure only by using a simple spreadsheet.

Given its simplicity and it general applicability, the theory exposed is probably one of the most efficient way to study the effects of creep in cable stayed bridges. Trough the application of this theory t is also possible to find interesting solutions where the effect of creep is counterbalanced by the prestressing of the stays in order to minimize its effect.

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APPENDIX A

The theories proposed to simplify the expression of c are essentially two

1. **Boltzman Model.**

This model considers the function J(t,t') to be not dependent on t'. That means that the function J doesn't decrease with t' but it remains the same function just translated over the time axe. With this assumption it is possible to find the value of R at t equal to infinity as:

$$R(\infty,t_0)=\frac{E}{[1+\varphi(\infty,t_0)]}$$

This results in a value of c of:

$$\chi=\frac{1}{1-\frac{E}{[1+\varphi(\infty,t_0)]\cdot E}}-\frac{1}{\varphi(\infty,t_0)}=\frac{1+\varphi(\infty,t_0)}{1+\varphi(\infty,t_0)-1}-\frac{1}{\varphi(\infty,t_0)}=1$$

2. **CEB Model.**

This model assumes that at t=t0 the function R has vertical tangent. With this assumption it's possible to know that at t=t0 the function c is always 0.5.

Experimentally has been proven that for t near to infinity the values of the function c are between 0.9 and 0.75. So, under the first assumption of R with vertical tangent at t=t0 it's possible to –do another simplification:

$$0.75\le\chi\le0.9\Rightarrow\chi=0.8$$

APPENDIX B

SYSTEM PROPERTIES		
$E_{concrete}$	3.E+10	[N/mm2]
E_{steel}	2.E+11	[N/m2]
L_{beam}	20	[m]
b_{beam}	1	[m]
h_{beam}	2	[m]
I_{beam}	1	[m4]
L_{cable}	10	[m]
r_{cable}	0.02	[m]
A_{cable}	0.0006283	[m2]
P	50000	[N/m]

t= 3 Days

φ	0.00000000
χ	0.00000000

A_c

1.33333E-07	4.16667E-08
4.16667E-08	1.66667E-08

A_s

7.95775E-08	0
0	7.95775E-08

B_c

-0.05
-0.017708

Λ_{opt}

0.015633563
0.045465565

X_{t0}

196457.1429
571337.1429

---

t= 13 Days

φ	0.87112376
χ	0.81800000

A_c

0.00000013	0.00000004
0.00000004	0.00000002

A

0.00000031	0.00000007
0.00000007	0.00000011

B_1

0.08562896
0.03032635

X_1

251562.12317883
114460.45506774

B_2	0.00770634		X_2	-78393.71135193
	0.04265806			446280.34395995
B_{tot}	0.10126252		X_{tot}	196457.14285714
	0.07579192			571337.14285714

---

t= 4220 Days

φ	2.90267942			
χ	0.66100000			
A_c	0.00000013	0.00000004		
	0.00000004	0.00000002		
A	0.00000047	0.00000012		
	0.00000012	0.00000013		
B_1	0.14593355		X_1	274240.27374260
	0.05168383			142979.47476246
B_2	-0.03356685		X_2	-170241.20616472
	0.02804075			380153.19812181
B_{tot}	0.16156712		X_{tot}	196457.14285714
	0.09714939			571337.14285714

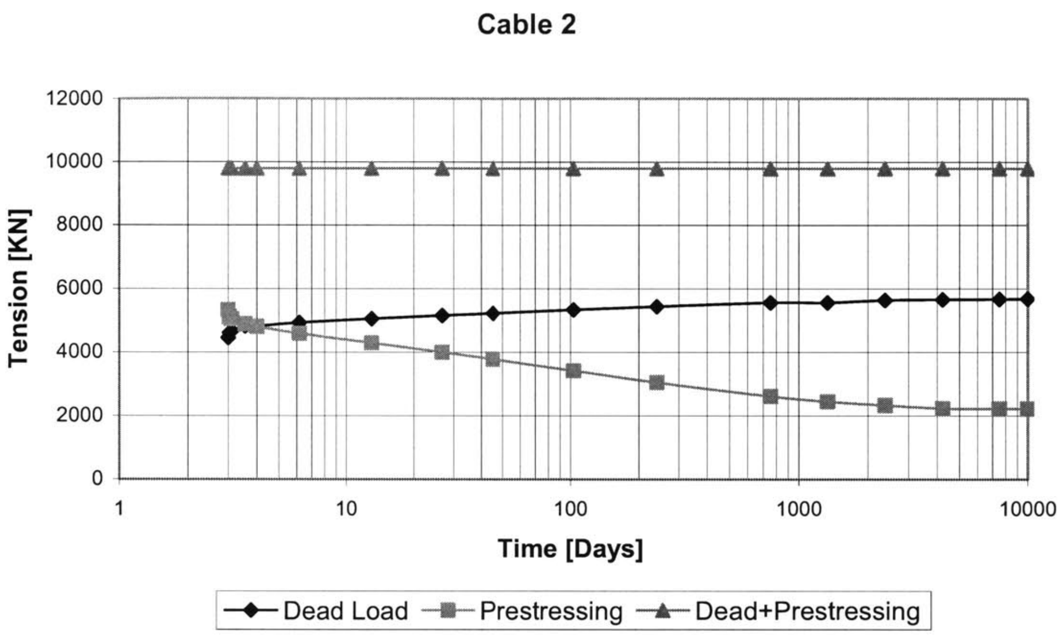
---

t= 7502 Days

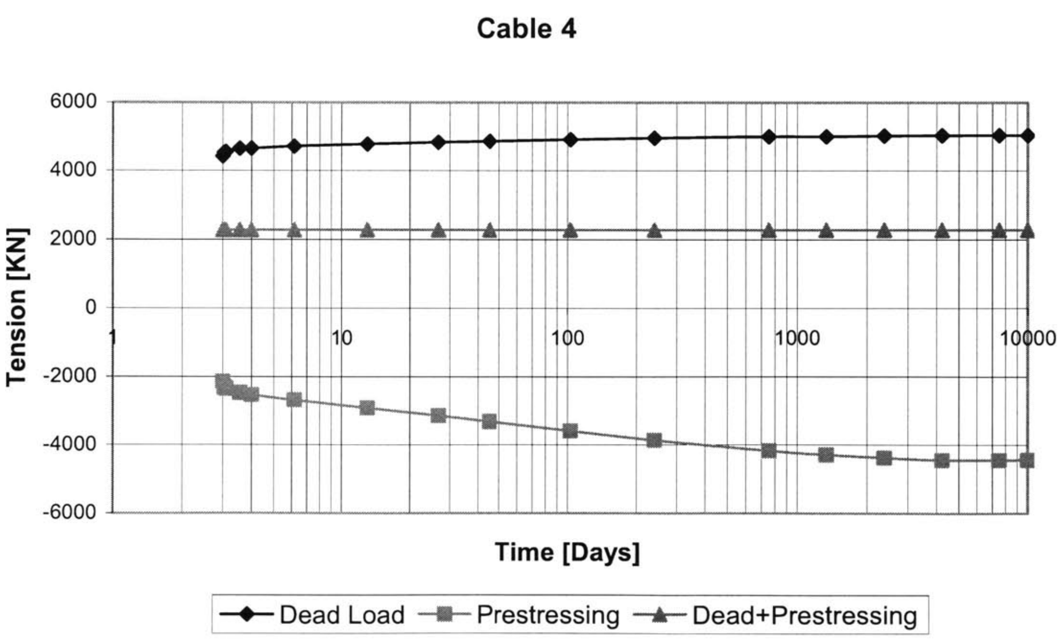
φ	2.96978024			
χ	0.66300000			
A_c	0.00000013	0.00000004		
	0.00000004	0.00000002		
A	0.00000048	0.00000012		
	0.00000012	0.00000013		
B_1	0.14844822		X_1	274767.42960709
	0.05257442			143993.31749684
B_2	-0.03440723		X_2	-170932.35829129
	0.02774312			378804.74132935
B_{tot}	0.16408178		X_{tot}	196457.14285714
	0.09803999			571337.14285714

t=	**10000**	**Days**			
φ	2.99281340				
χ	0.66600000				
A_c	0.00000013	0.00000004			
	0.00000004	0.00000002			
A	0.00000048	0.00000012			
	0.00000012	0.00000013			
B_1	0.14966069			X_1	275014.07563246
	0.05300383				144478.61596172
B_2	-0.03434642			X_2	-170399.34897296
	0.02776465				378609.12312109
B_{tot}	0.16529425			X_{tot}	196457.14285714
	0.09846939				571337.14285714

APPENDIX C



Cable 2
12000
10000
8000
6000
4000
2000
0
Tension [KN]
1
10
100
1000
10000
Time [Days]
Dead Load
Prestressing
Dead+Prestressing



Cable 4
6000
4000
2000
0
-2000
-4000
-6000
Tension [KN]
1
10
100
1000
10000
Time [Days]
Dead Load
Prestressing
Dead+Prestressing