Research Into Building Vibrations

by

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This is dedicated to Jo Ann B. Floresca

May 7, 2003

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ABSTRACT

Underground and surface arteries for vehicle or railway traffic can create vibrations that travel to nearby buildings. These vibrations can cause structural damage or human discomfort. Displacement time histories collected from buildings abutting the central surface artery were used to drive mathematical models so that asphaltic and polymeric bearings could be studied as possible passive mitigators of such vibrations. Neither material attenuated vibrations to below threshold levels for human annoyance, but they could dampen levels to resist structural damage if enough material was used to bring the apparent natural frequency away from the range characteristic of traffic vibration. In addition, for resonant cases, the materials did not create enough damping force to counter the inertia of heavy structures, because the materials were too stiff and the displacements and velocities too small. For new construction, it is suggested that these vibrations should be prevented from entering the foundation area by surrounding the foundation with a concrete wall or absorbent foam blocks. For retrofits, polymeric or asphaltic pads could be used and would be relatively easy to install.

Thesis Supervisor: Jerome J. Connor

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1.0 Introduction

With the successful completion of Boston's subsurface Central Artery/Tunnel (CA/T) Project, many other cities around the United States may want to begin similar projects to help alleviate traffic congestion. Underground and surface arteries for vehicle or railway traffic can create vibrations that travel to nearby buildings. These vibrations can cause structural damage or human discomfort. Studies have shown that humans are annoyed by vibration levels right at the threshold to where they can just perceive them, especially if the source is unknown. Such vibrations can disrupt sleep and the work environment.

In addition, these vibrations are undesirable for high performance chip manufacturing plants or genetic laboratories, whose vibration tolerances are significantly more stringent than standard purpose commercial buildings. For these types of structures, the building itself accounts for approximately 25 percent of the total facility cost, and the building is constructed to remain "state of the art" for up to 10 years. Because fabrication plants produce products at microscopic dimensions, the slightest vibration could shift chip-making equipment calibrations.

For these reasons, it is necessary to study the ways in which these types of vibrations can be effectively, easily, and cheaply mitigated.

2.0 Vibration

Vibration is an oscillatory movement around a state of equilibrium. Vibration can be described by particle position (or displacement), particle velocity (change in displacement over time), or particle acceleration (change in velocity over time).

There are five main types of deterministic vibrations. Deterministic means that the vibration motion can be modeled with some mathematical function, in order to predict with some probability the characteristics of the vibration.

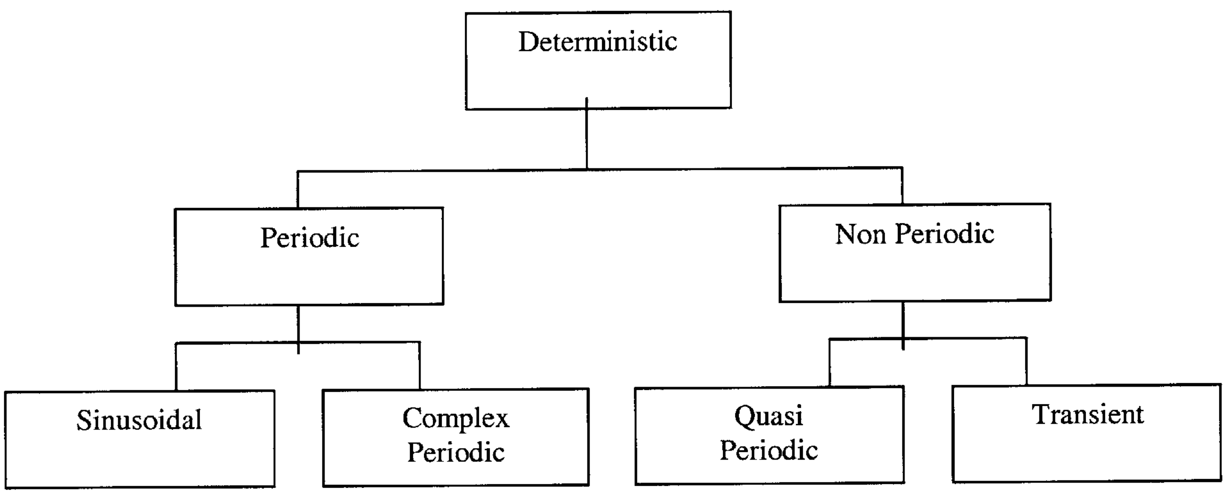

Figure 2.1 Organization of Types of Vibrations

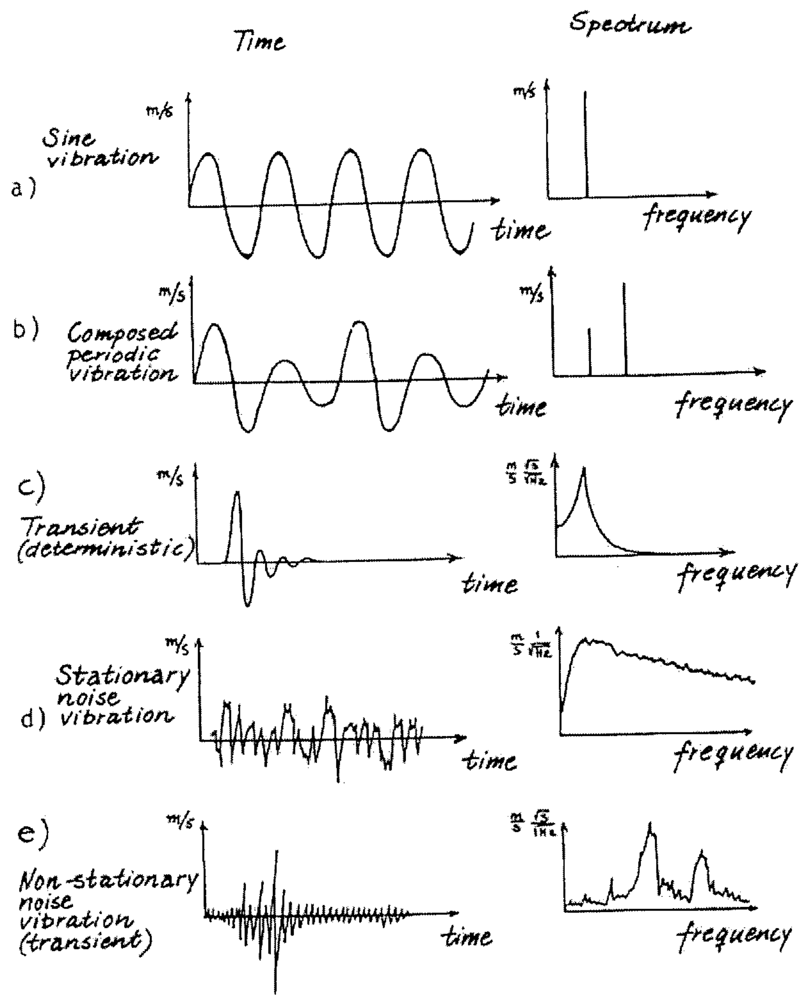

Figure 2.2 Illustrations of Different Types of Vibrations (Holmberg 1984)

2.1 Traffic Vibrations

When a vehicle rolls on a road, vibrations are created due to depressions made by the weight of the vehicle, with the pressure rapidly changing as the wheel rolls. The energy is transmitted from the vehicle to the ground continuously. If the roadway is bumpy, shocks are induced and the energy propagates as waves to the environment. These waves are mostly Rayleigh-type surface waves, but there are also vertically polarized shear waves. The Figure below shows the various parameters by which the particle velocity may be affected.

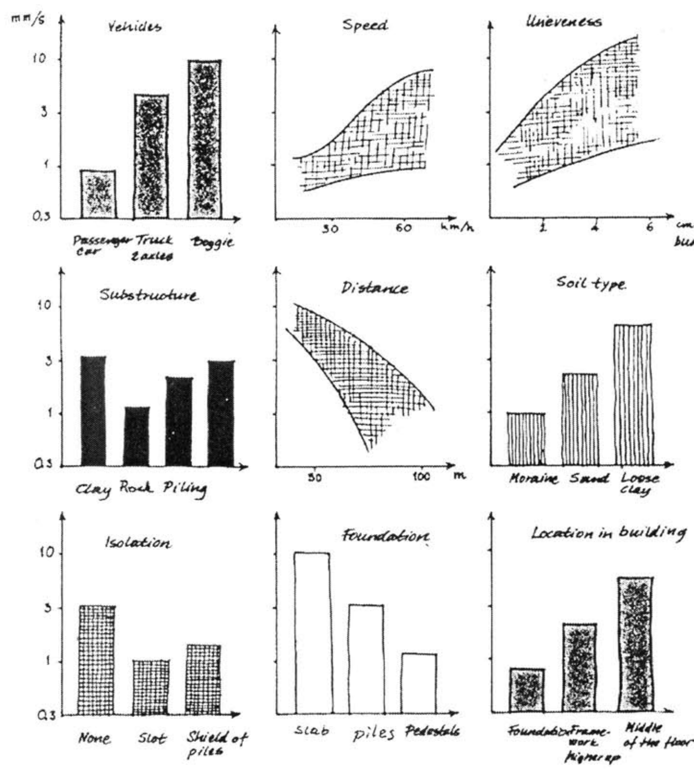

Figure 2.3 Parameters Affection Traffic Vibration Level (Holmberg 1984)

Because the propagation of the surface waves takes place in horizontal ground layers, these waves are more slowly attenuated than the volume waves. Changes in depth and composition of the underground also have large effects on the surface waves. As heavy vehicles are often long and all the wheels function as vibration sources, the vibrations have a long period. They also have rather low frequencies and can therefore cause vibrations of both buildings and separate building elements.

Measurements taken in Alvangen, Sweden found that the frequency of vibrations from railroad traffic nearby varied around 4 Hz. Grootenhuis sites studies indicating wheel-rail interaction from underground railway traffic causing vibrations to nearby buildings to be on the order of 15-30 Hz, depending on the speed of the train.

2.2 Characteristic Wave Motion

Most significant in our discussion of microtremors due to traffic vibration is the Rayleigh-wave (R-wave). The wave particles describe a retrograde elliptical motion, where the vertical component has its maximum just below surface, but diminishes thereafter relatively rapidly with the depth. R-waves have the lowest velocity, compared to shock waves (P-waves) and shear waves (S-waves). The wave velocity is determined by the elastic properties of the material.

2.3 Microtremors

Vibrations induced by vehicular and railway traffic can be characterized as microtremors, which are low amplitude, low frequency ground motions, and are caused by a number of other sources including earthquake aftershocks and construction. It is important to study microtremors because they can cause damage to nearby structures and bring annoyance and discomfort to humans.

Noise vibrations, like microtremors, are said to be random, although governed by natural law. Because of this, they are unlike deterministic vibrations because they cannot be described in mathematical expressions but rather with statistical means, which in principle make it very difficult to predict their behavior.

The most common terms used to describe noise vibrations are r.m.s. (root mean square) (standard deviation), power density spectrum, autocorrelation function, and probability density function. Traffic noise is non stationary, in that these parameters are not independent of absolute time (during a long enough, but limited time space) but rather vary with time and have average values. In many cases, however, the vibration can approximately be regarded as being deterministic, for the sake of design.

2.4 Parameter to Describe Vibrations

In describing vibrations, the displacement, velocity, or acceleration time histories can be characterized by several important parameters.

- Peak Value: $\hat{x}$, is the maximum absolute value of the vibration during a time interval.

- R.M.S.-value: rms is defined as

$$rms = \sqrt{\frac{\int_{t_1}^{t_2} x^2 dt}{t_2 - t_1}}$$

For a pure sine wave

$$rms = \frac{\hat{x}}{\sqrt{2}}$$

- The mean value is often defined to be 0.

- Standard Deviation:

$$\sigma = \sqrt{\frac{\int_{t_1}^{t_2} (x - \bar{x})^2 dt}{t_2 - t_1}}$$

If the mean value is zero, then the standard deviation equals the rms.

3.0 Threshold Levels

3.1 Human Annoyance

Man interprets vibrations as warning signals, although it is generally obvious as a soon as the source is identified that no risk exists. The long-term effects from disturbances depend as much on the acceptance of the source as the threshold value of the disturbance. It is widely accepted that threshold levels for human annoyance are right above the level at which they are possible to perceive. In the ISO 2631/DAD 1 (1980) standard, values of 0.14 and 0.2 mm/s are indicated for frequencies exceeding 8 Hz for night and daytime, respectively. The threshold for perception is at about 0.1 mm/s for corresponding frequencies.

3.2 Damage to Buildings

Damage due to vibrations depends on so many factors, that it is very difficult to assess the risk. The method of construction of the building, the vibration's character (intensity, frequency range, wave length), static stress of the building, and how much the original values have been increased by settlements, dampness and temperature variations all contribute to the damage of a building.

There are few generally accepted standard values resulting from the fact that the number of well-documented damage cases is very limited. Vibrations from traffic seldom reach levels where they can cause direct vibration damage. Generally, accelerated aging results. A reasonable starting point then is to relate the intensity of stress caused by the vibrations from an external source to the inevitable stresses that a building withstands due to the indoors environment and climatological factors.

3.3 Damage Criteria

The table below shows a compilation of recommended limit values used when judging the risk for damage by ground vibrations in normal residential areas. Normal residential areas indicate houses with foundations and joists of concrete, outer walls of brick and intermediate partitions of plastered, compact light concrete.

	Sand, Gravel,Clay	Moraine, Slate Stone, Soft lime-stone	Granite gneiss, firm limestone, quartzite sandstone, diabase	Results in normal residential area
Vertical particle velocity v [mm/s]	18	35	70	No noticeable cracking
	30	55	110	Fine cracks, and fall of plaster (threshold value)
	40	80	160	Cracking
	60	115	230	Serious Cracking

Table 3.1 Limit Values for Damage in Buildings. Langefors and Kihlstrom (1967).

Below is the graphical representation of the limiting values shown in the table above. For frequencies exceeding 40 Hz, the particle velocity is the criterion, but at lower frequencies the displacement represents the criterion.

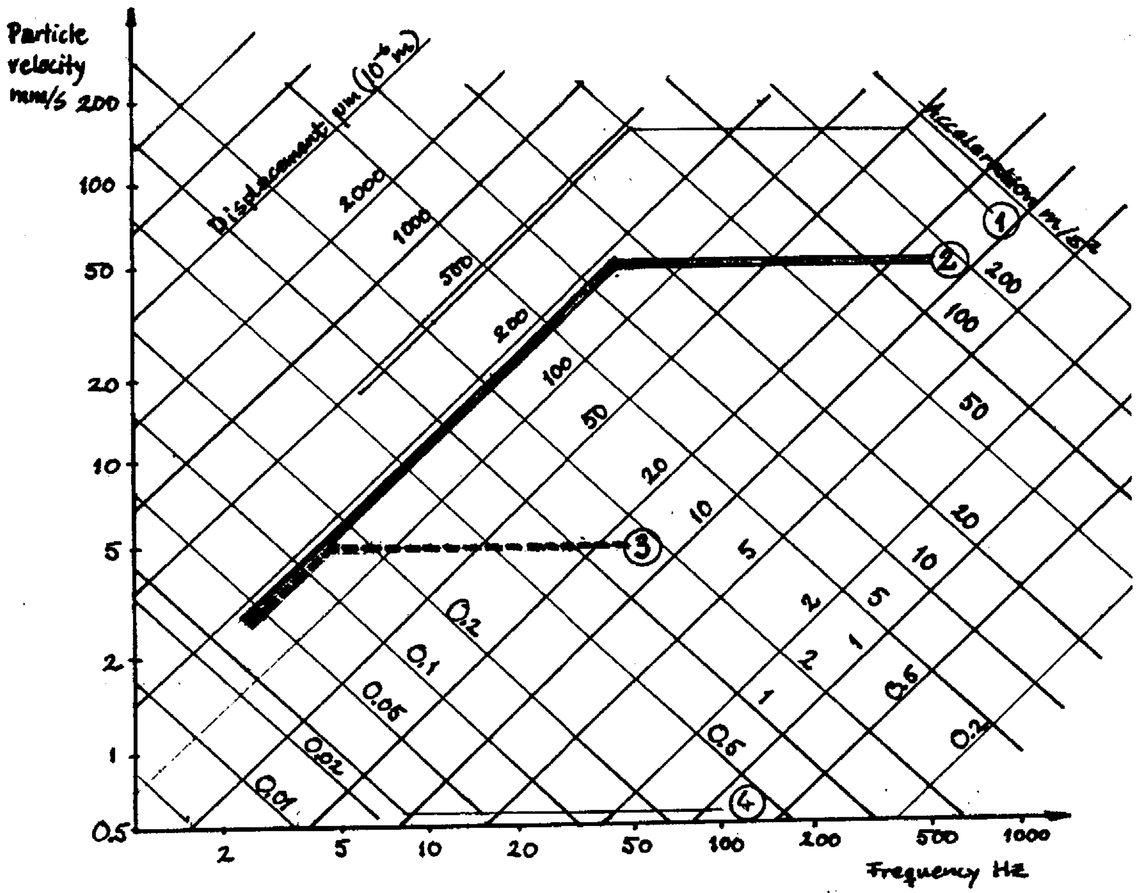

Figure 3.1 Illustration of Threshold Values (Holmberg 1984)

Criteria For Damage and Recommendations

1.) Direct damage from vibrations on buildings at blasting

2.) Recommended upper limit for blasting

3.) Recommended upper limit for piling, sheet piling, vibratory compactors, dynamic deep compaction and traffic

4.) Disturbing vibrations to human beings.

Studer and Suesstrunk (1981) have compiled the results of vibration measurements in Switzerland, carried out later than 1960. There are for nominated criteria for four classes of structures and the associated standards for vibrations caused by various construction procedures and traffic.

Building Class	Frequency range where the standard value is applicable	Maximum resultant v_r [mm/s] of the particle velocities	Estimated maximum vertical particle velocity v_z [mm/s]
I. Industrial buildings of reinforced concrete, steel constructions	10 - 30 30 - 60	12 12 - 18	7.2 - 12 7.2 - 18
II. Buildings on concrete foundation. Concrete walls or bricked walls	10 - 30 30 - 60	8 8 - 12	4.8 - 8 4.8 - 12
III. Buildings with bricked cellar walls. Upper apartment floors on wooden beams	10 - 30 30 - 60	5 5 – 8	3.0 – 5 3.0 - 8
IV. Especially sensitive buildings and historical buildings	10 - 30 30 - 60	3 3 – 5	1.8 – 3 1.8 - 5

Table 3.2 Threshold Levels of Building Damage

Bonde, Rundqvist and others (1981) propose limiting values for vibrations caused by vehicles (traffic). The buildings are assumed to be founded on clay or loosely layered sand. The recommendations refer to the case when the number of vehicles with a total weight of 10 tons exceeds 50 vehicles per 24 hours. If the number is lower, the limiting values can be increased by 50%. A summary of the results are shown below.

Type of building and foundation	Recommended limit value [mm/s]
Especially sensitive buildings and buildings of cultural and historical value	1
Newly-built buildings and/or foundations on a foot plate	2
Buildings on cohesion piles	3
Buildings on bearing piles or friction piles	5

Table 3.3 Threshold Levels for Traffic Vibrations

4.0 Theory and Methodology

4.1 Damping

Damping refers to the dissipation of vibrational energy. All physical systems have some inherent damping, but the level of damping can be augmented to increase energy dissipation in particular vibration modes. In this way, the response of a structure driven at a resonant frequency can be greatly decreased. This in turn can significantly reduce overall motion or acceleration of the structure. The damping process dissipates or absorbs the energy input from external excitations by transferring it to other mechanical forms such as heat, sound, or strain energy.

Viscoelastic damping is a form of passive damping where the input energy is dissipated by a transfer to heat and strain energy to the viscoelastic material. Since for viscous materials the damping force is a function of the rate of deformation, these forces are out of phase with the elastic forces in the system and do not add to the total force at the maximum displacement.

4.2 Viscoelastic Theory

In simple elastic mechanics, the relationship between stress and strain is linear and based on Young's Modulus for simple uniaxial strain.

$$\sigma = E\varepsilon \tag{4.1}$$

And for simple shear strain, the relationship is:

$\tau = G_e\gamma$, where G_e is the Elastic Shear Modulus. (4.2)

This shows that since the ratio of stress to strain is constant, all strain energy created will be stored as potential energy and released when the elastic material decompresses. In reality, there are no materials that are 100% elastic.

Viscoelastic materials behave such that the rate of energy dissipation becomes significant, and the modulus can be adjusted to take into account the energy dissipative nature of the material.

$$E^* = E_1(1+\eta i) \tag{4.3}$$

where $E_1$is the storage modulus and $\eta$is the loss factor, creating an imaginary term.

For 100% viscous materials, the relationship is:

$$\sigma = E_e \dot{\varepsilon} \tag{4.4}$$

where the strain can be modeled as a sinusoidal wave form, resulting in:

$$\varepsilon = \hat{\varepsilon}\sin(\Omega t) \tag{4.5}$$

or in complex form:

$$\varepsilon = \hat{\varepsilon}\, e^{i\omega t} \tag{4.6}$$

Since viscosity produces a time lag in the stress-strain relationship, as revealed through the derivative of the sinusoidal strain, the relationship becomes:

$$\sigma = E_1\, \hat{\varepsilon}\, e^{i\omega t} + E_1 \eta i e^{i\omega t} \tag{4.7}$$

To simplify, let

$$\eta = \tan(\delta) \tag{4.8}$$

such that

$$\sigma = \hat{E}\,\hat{\varepsilon}\sin(\omega t + \delta) \tag{4.9}$$

where

$$\hat{E} = E_1\sqrt{1+\eta^2} \tag{4.10}$$

Below is a graphical representation of the stress strain deformation relations.

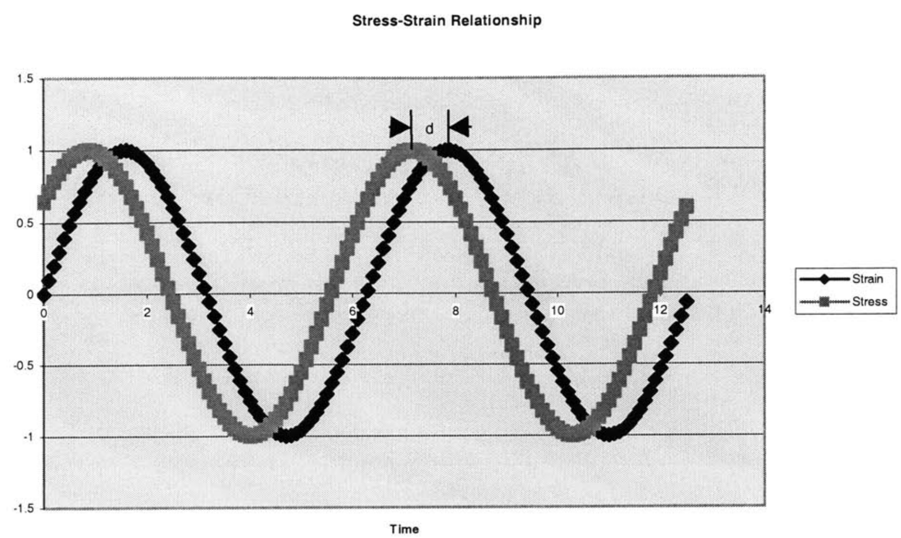

Figure 4.1 Stress Strain Relationship

5.0 Materials

5.1 Asphalt

Asphalt is considered for use as a damping material because of its use in road pavement. The loading of pavement structures is due to mobile forces, and the contact with the surface unevenness of the pavement causes a transient state of stress. Systematic experimental testing of real highway pavements and asphalt have been carried out, which have correlated well with theoretical models of pavement structures and their dynamic behavior.

Asphalt primarily consists of aggregate and binder materials. Aggregate sizes range from 3.35 mm to 60 microns, and may be natural crushed rock, gravel, or sand, or artificial, as in slag or calcined bauxite. Today, tar has been replaced by bitumen for the binder material. Bitumen is a hydrocarbon, coming off at the heavy end of the distillation of crude oil.

The bituminous materials in asphalt have distinct features of viscoelastic behavior. Below are selected data gathered from experiments conducted over several years by G. Martincek and his team at the Institute of Civil Engineering and Architecture at the Slovak Academy of Sciences, in Bratislava, Slovak Republic. Measurements were performed using mechanical impedance, which is the complex ratio of the exciting force to the velocity of motion at this point. The results of measurements at temperature T =

10, 20, and 40° C, in the form of isochrones and damping parameter variations are shown below for a dense-cover asphalt layer. The values of the damping parameter Delta are average values because it does not present a regular and distinct change with frequency. However, the dependence of Delta on temperature T is very strong. In addition, the complex modulus is very dependent on T and vibration frequency.

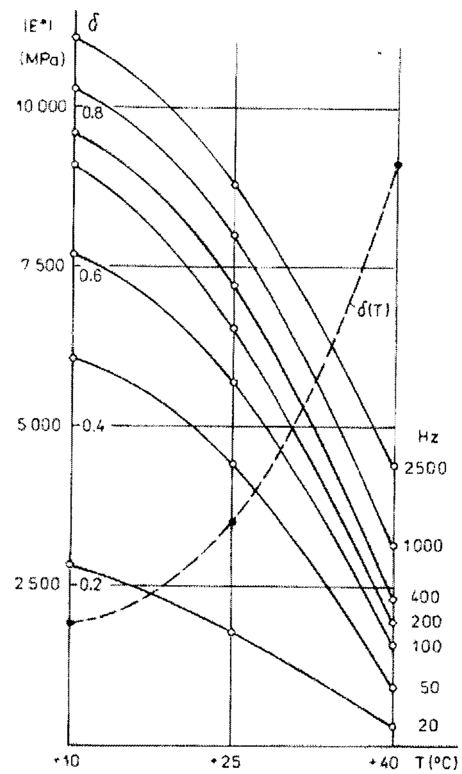

Figure 5.1 Isochrones for Typical Flexible System Asphalt

5.2 Viscoelastic Polymers

Natural rubber bearings have been used as damping devices in bridges for almost 40 years. Plastics are materials made from long molecules (Polymers). Generally Plastics contain numerous chemical additives to refine their mechanical properties. Plastics are widely used throughout industry because they can be formed into complicated shapes at a very low cost using mass production methods. The range of properties available from plastics materials has made them the prime choice for many applications. Plastics are light, and durable. They generally are not able to withstand high temperatures and they are not as strong as metals.

A rubber/elastomer is a polymeric material with long flexible molecular chains and the ability to deform elastically when vulcanized. During the vulcanization process, rubber molecules are linked with adjacent molecules at intervals along their lengths, usually by sulfur to form a cross-linked elastic material that is stable over a wide range of temperatures.

Another unique characteristic of rubber is that its modulus of elasticity is a complex quantity, having both a real and imaginary component. Furthermore, this complex modulus varies as a function of many parameters, most important of which are the temperature and frequency of a given application. It is therefore necessary to establish an accurate and thorough understanding of these parameters in order to design effective damping treatments.

Below is the nomograph for a typical damping polymer, made by 3M.

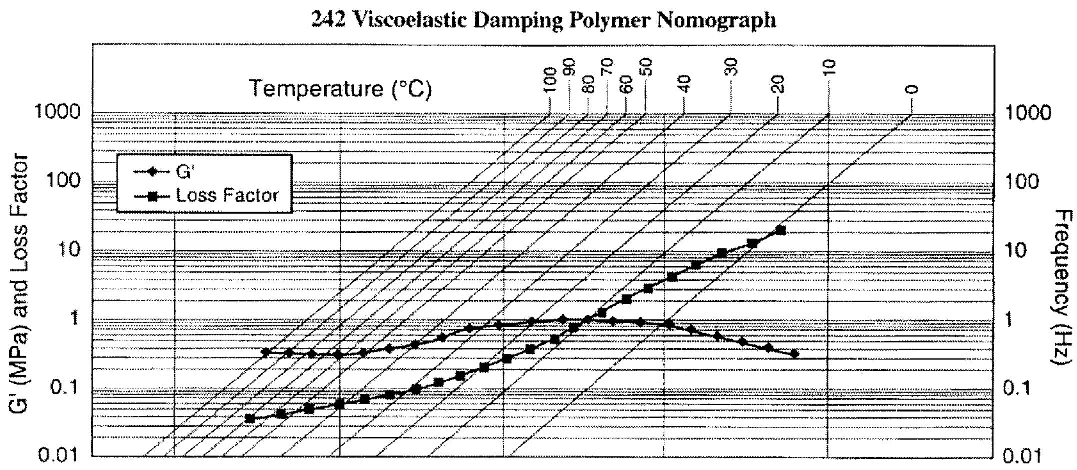

Figure 5.2 Nomograph for 3M 242 Damping Polymer

6.0 Mathematical Model

6.1 Single Degree of Freedom System

For simplicity, a building will be modeled as a single degree of freedom system subject do ground motion, as shown below.

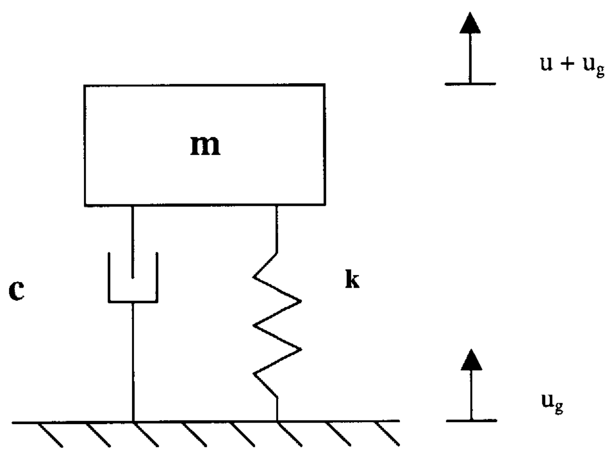

Figure 6.1 Schematic of Single Degree of Freedom System

The equation of motion is

$$m\ddot{u}(t) + c\dot{u}(t) + k\,u(t) = -m\ddot{u}_g(t) \tag{6.1}$$

For design purposes, let the maximum relative motion of the mass to the ground be a typical value of

$$u^* = 0.1\text{m} \tag{6.2}$$

Assume that the building, idealized as an SDOF system, behaves like a continuous-time linear time-invariant filter described by the ordinary differential equation (6.1) and generalized below

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \ldots\ldots + a_1 \frac{d^1 y}{dt^1} + a_0 y = b_m \frac{d^m u}{dt^m} + b_{m-1} \frac{d^{m-1} u}{dt^{m-1}} + \ldots\ldots. + b_1 \frac{d^1 u}{dt^1} + b_0 u$$

(6. 3)

where y(t) denotes the system output subject to input u(t).

Since the Laplace Transform of the derivative of a function is given by

$$L\left\{\frac{d^n f}{dt^n}\right\} = s^n F(s) \tag{6.4}$$

then the system output in the frequency domain is given by

$$Y(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots\ldots. + b_1 s^1 + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots\ldots\ldots + a_1 s^1 + a_0} U(s) \tag{6.5}$$

$$Y(s)=H(s)U(s) \tag{6.6}$$

$$H(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots\ldots. + b_1 s^1 + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots\ldots\ldots + a_1 s^1 + a_0} \tag{6.7}$$

where *H(s)* is known as the transfer function for the system.

For the ODE specified in equation (6.1), where the input function is $u_g(t)$ and the output function is u(t), the transfer function is

$$H(s) = \frac{-ms^2}{ms^2 + cs + k} \qquad (6.8)$$

Dividing the numerator and the denominator by m yields

$$H(s) = \frac{s^2}{s^2 + 2\zeta\omega_n + \omega_n^2} \qquad (6.9)$$

$$c = 2\zeta\omega_n m$$

$$\omega_n = \sqrt{\frac{k}{m}}$$ [1]

Since the Laplace transform cannot be solved in closed form for the input motion $u_g(t)$, the system equation must be put back into the time domain for processing the output. If for the frequency domain, the output is found by equation (6.6), in the time domain the output is found by

$$y(t) = h(t) \otimes u(t) \qquad (6.10)$$

or the convolution integral

$$y(t) = \int_{-\infty}^{t} h(\tau)u(t-\tau)d\tau \qquad (6.11)$$

Therefore, the time domain impulse response for the transfer function must be found, using the inverse Laplace transform.

[1] It is noted that the actual natural frequency of the system is w = wn*(1-xi²)^(1/2)

$$h(t) = \frac{1}{2\pi j}\int_{\sigma - j\infty}^{\sigma + j\infty} H(s)e^{st}ds = \frac{1}{2\pi j}\int_{\sigma - j\infty}^{\sigma + j\infty} \frac{-ms^2}{ms^2 + cs + k}e^{st}ds \qquad (6.12)$$

Once system variables such as damping ratio and natural frequency are specified, one can solve for the inverse Laplace transform by the use of look-up tables, Matlab functions, or partial fractions in closed form.

6.2 Damping Device

Assuming a design shown in Figure (9.2), where sheets of damping material could be placed between the column and footing or underneath the base floor slab of a house, the behavior could be modeled using the formulations in the previous section. The inertia force due to gravity will be neglected in this study because the nature of the vibrations in question (microtremors) do not cause significant displacements.

In deriving the stiffness, k, and the equivalent viscous damping coefficient, c, of the mat, let

$$u(t) = \varepsilon(t)L = \frac{\sigma(t)L}{E^*} = \frac{P(t)L}{E^*A} \qquad (6.13)$$

such that

$$k = \frac{E_1 A}{L} \qquad \text{where L is the thickness of pad of damping material.} \qquad (6.14)$$

For equivalent viscous damping, the stress is proportional to the rate of strain as in equation (4). An expansion of the above equations yields:

$$F = \frac{AE_1}{L}\hat{u}_g[\sin\Omega t + \eta\cos\Omega t] \qquad (6.15)$$

Such that the energy dissipated per cycle is

$$W_{viscoelastic} = \pi\eta\frac{AE_1}{L}\hat{u}_g^2 \qquad (6.16)$$

Comparing this with the energy dissipation for a 100% viscous damper

$$W_{viscous} = c\pi\Omega\hat{u}^2 \qquad (6.17)$$

yields

$$c_{equivalent} = c = \frac{AE_1\eta}{L\Omega} \qquad (6.18)$$

This is reasonable because the viscosity of the material, and thus the damping effect, should vary inversely with the rate of loading.

6.3 Preliminary Analysis: Pure Sinusoidal Excitation

For design purposes, let us assume that the excitation is deterministic and can be expressed as a sinusoid or a sum of sinusoids, with zero phase for simplicity.

$$u_g(t) = \hat{u}_g \sin(\Omega t) \tag{6.20}$$

The input motion in the frequency domain is merely the inverse Laplace transform of the ground motion.

$$U_g(s) = L^{-1}\{u_g(t)\} = L^{-1}\left\{\hat{u}_g \sin(\Omega t)\right\} = \frac{\hat{u}_g \Omega}{s^2 + \Omega^2} \tag{6.21}$$

Since it holds that the system output is equal to the system transfer function multiplied by the input, in the frequency domain, the SDOF resultant relative motion response is

$$U(s) = H(s)U_g(s) = \frac{s^2}{s^2 + 2\zeta\omega_n + \omega_n^2} \cdot \frac{\hat{u}_g \Omega}{s^2 + \Omega^2} \tag{6.22}$$

Solving for the response in the time domain requires applying the inverse Laplace transform to this fourth order rational function. The solution yields:

$$u(t) = \frac{\hat{u}_g \sin(\Omega t - \delta)}{\sqrt{(\omega_n^2 - \Omega^2)^2 + (2\zeta\omega_n\Omega)^2}} \tag{6.23}$$

$$\delta = \arctan\left(\frac{-2\zeta\omega_n\Omega}{\omega_n^2 - \Omega^2}\right) \tag{6.24}$$

$$\dot{u}(t) = v(t) = \frac{\hat{u}_g \Omega \cos(\Omega t - \delta)}{\sqrt{(\omega_n^2 - \Omega^2)^2 + (2\zeta\omega_n\Omega)^2}} \tag{6.25}$$

7.0 Simulation Procedure

7.1 Model Parameters

Although it is very difficult to predict the exact frequency of micretremor excitation due to traffic (as explained in section 6.2), for this analysis, we will assume a frequency ranging from 5 Hz to 30 Hz for automobile and railway traffic. This is consistent with observations made by the Swedish Council for Building Research and the Imperial College of Science and Technology, London.

In addition, for ease of analysis, we will consider two cases for building type, which will only affect the natural frequency for our simplified SDOF system. The first case is a moderate size home, 2500-3000 sq-ft, with wood framing and possibly a few concrete columns. The participating mass for this structure would be about 200,000 lbs. or 100,000 kg. The second case is a moderate size, short and stiff, brick or concrete office building with a participating mass of 1,500,000 kg.

We will consider two materials for damping: asphalt and viscoelastic polymer. For asphalt, Figure 5.1 reveals that the magnitude of the complex Elastic Modulus is between 1800 and 4000 MPa at 25° C for the loading frequency range 5 Hz to 30 Hz respectively, with a loss factor of 0.7.

For viscoelastic polymers, values for compressive (normal) storage moduli could not be obtained. However, for the frequency range and temperature of interest, this will be approximated by the shear storage modulus and loss factor. For 3M Ultra-pure Viscoelastic Damping Polymer (242F01/242F02/242F04), using Figure 5.2, the storage modulus is 1 MPa and loss factor is 1 for 5 Hz and 1 MPa and 3 for 30 Hz. The parameters for the test matrix are summarized below.

Material and Property Frequency	Case 1: House M = 100,000 kg	Case 2: Office Building M = 1,500,000 kg
Asphalt: 5 Hz E1 = 1579 MPa, n = 0.7	$\omega_n =$ 5, 40 Hz	$\omega_n =$ 5, 40 Hz
Asphalt: 30 Hz E1 = 3508 MPa, n = 0.7	$\omega_n =$ 5, 40 Hz	$\omega_n =$ 5, 40 Hz
Polymer: 5 Hz E1 = 1 MPa, n = 1	$\omega_n =$ 5, 40 Hz	$\omega_n =$ 5, 40 Hz
Polymer: 30 Hz E1 = 1 MPa, n = 3	$\omega_n =$ 5, 40 Hz	$\omega_n =$ 5, 40 Hz

Table 7.1: Simulation Test Matrix

7.2 Controlling Velocity

The problem concerns controlling the building velocity to under the threshold levels for human annoyance, since it is the more stringent requirement compared to building damage levels.

$$\hat{v}_t = 1\frac{\text{mm}}{\text{s}}$$

Substituting in mass values of 100,000 kg and 1.5 Mkg and excitation values of 5 Hz and 30 Hz, and max ground velocity in equation (6.25) and relating the material properties to the natural frequency and damping ratio, the relationship between the normal Elastic storage modulus and the loss factor can be expressed graphically to show the necessary values to hold the peak total velocity to under 1 mm/s.

Below are graphs showing the relationship between the response velocity and the natural frequency of the damped structure, considering excitations of 5 Hz and 30 Hz (approximately 30 rad/s and 200 rad/s), with varying damping ratios. It is clear that if the natural frequency of the structure in the vertical direction falls in this range, it is susceptible to resonant phenomena and will need heavy damping to limit the relative velocity response to below 1 mm/s. For the figures below, this translates to a ratio of response magnitude to excitation magnitude, or amplification, of 1 [mm/s]/ 10 [mm/s], which equals 0.01.

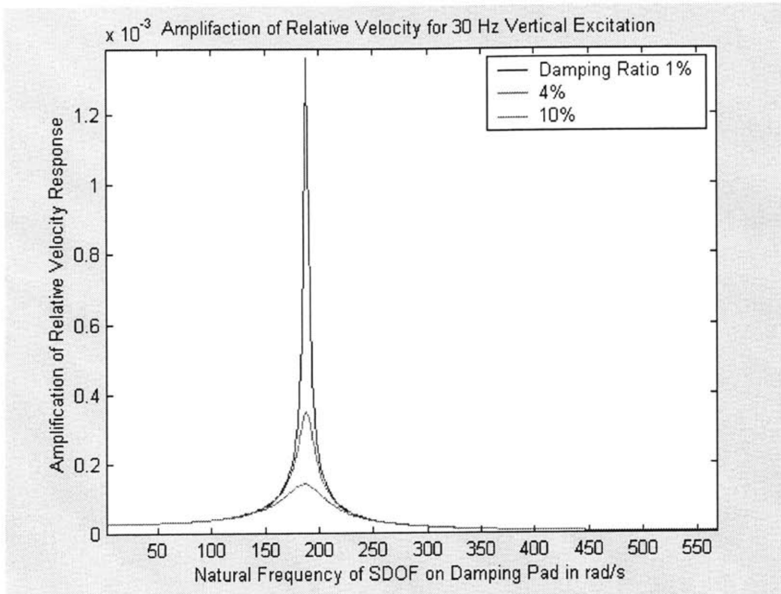

Figure 7.1 Amplification of Velocity for Natural Frequency 30 Hz

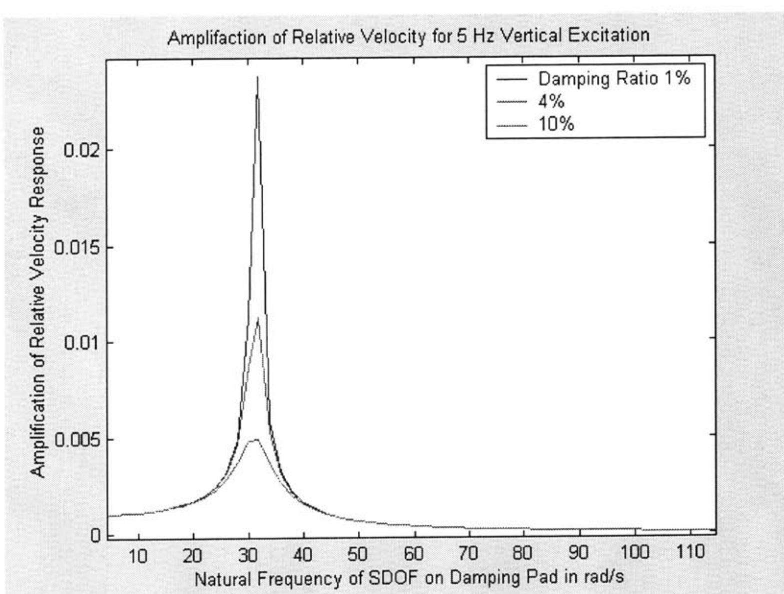

Figure 7.2 Amplification of Velocity for Natural Frequency 5 Hz

Actually, for the case of 30 Hz excitation, the amplification of relative velocity is so small that the values already fall below threshold values. So the main concern is to keep the natural frequency away from 5 Hz or 30 rad/s, or to make sure the material provides sufficient damping. To find the optimum storage modulus, we make the following calculations.

$$20\frac{rad}{s} > \omega_n > 40\frac{rad}{s}$$

$$400\left(\frac{rad}{s}\right)^2 > \omega^2{}_n > 1600\left(\frac{rad}{s}\right)^2$$

$$400\left(\frac{rad}{s}\right)^2 > \frac{E_1 A}{Lm} > 1600\left(\frac{rad}{s}\right)^2$$

$$4000000N > E_1 A\big|_{house} > 16000000N \qquad 60000000N > E_1 A\big|_{office} > 240000000N$$

where the height of the pads have been assumed to be 0.1 m. Considering the order of magnitude for the necessary stiffness, it seems that polymer dampers would work best for the case of a house and asphalt would work best for an office building. Using values for the storage modulus of the individual materials, with respect to an excitation frequency close to 5 Hz, the required areas are:

$$4m^2 > A\big|_{polymer,house} > 16m^2 \qquad 0.0375m^2 > A\big|_{asphalt,office} > 0.15m^2$$

7.3 Input Vibration Data

Under the supervisory of the Central Artery and Tunnel Project of Boston, Massachusetts, engineers from Bechtel/Parsons Brinckerhoff collected vibration data from traffic passing over the elevated surface artery. These time histories will be used as a starting point for the evaluation of damping materials to mitigate such traffic vibrations from affecting nearby buildings.

The available data comes from instruments placed on grade, 15-20' away from a bent column of the elevated highway, in the North End of Boston. The seismographs were sensitive to 100 mV/g accelerations and were broad-band, collecting data from 5 Hz – 20,000 Hz. The instruments sampled vibrations at 1 sample per second. Because the data is not band-limited and the sampling frequency is well below the Nyquist sampling rate, no frequency domain processing could be done because aliasing would otherwise occur.

For four data sets taken on different days, the sample averages for the peak particle vertical velocity and mean vertical velocity are 4.5 mm/s and 0.3 mm/s, respectively. These are consistent with the values shown in Figure 2.3 for the variation of parameters such as vehicle speed and soil type.

Because the samples were not taken often enough to accurately describe the microtremors, these samples of the random variable, or noise, will be combined with deterministic sinusoidal excitations of similar magnitude (10 mm/s), with frequencies between 5 Hz and 30 Hz, encompassing the zone of possibility for vibrations due to traffic. This is done, as opposed to merely applying a pure sinusoidal ground motion, in order to study the effects of the random or nondeterministic characteristic of the excitation.

A sample time history is shown below. The first being the original data set, and the second being the interpolated data set (interpolated to a sufficient sampling rate), using Matlab's INTERP, which resamples the data at a higher rate using a symmetric lowpass interpolation, so that the mean square error between the points and their ideal values is minimized.

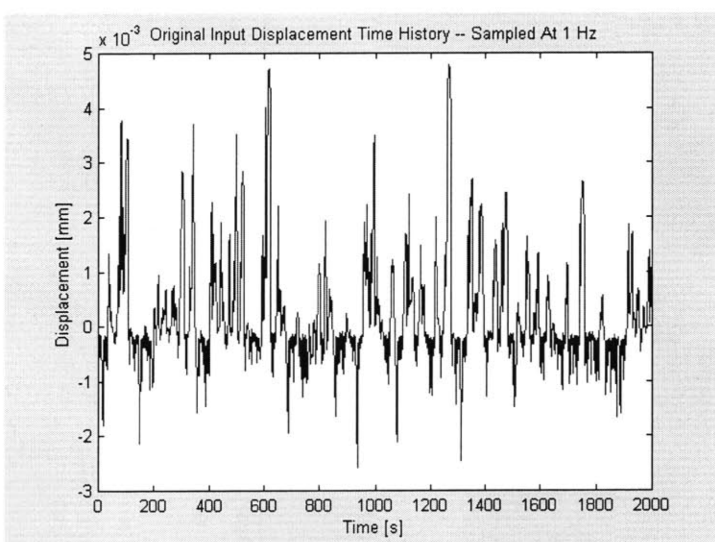

Figure 7.3 Sample of Original Recorded Displacement Time History, Sampled at 1 Hz

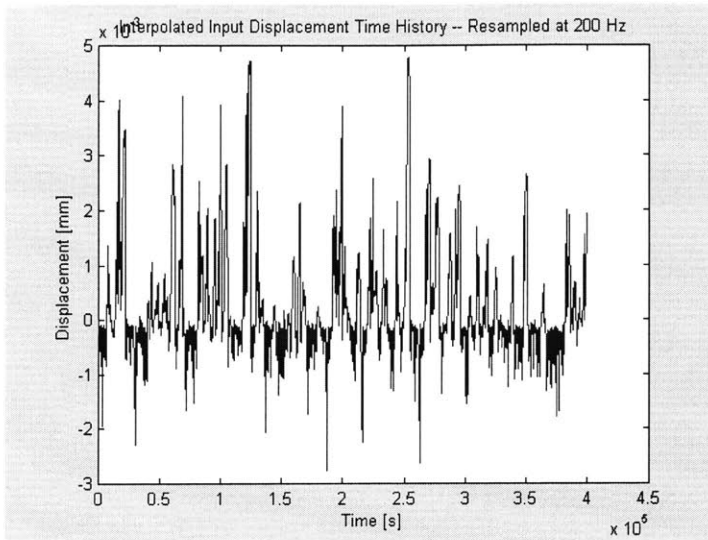

Figure 7.4 Interpolated Displacement Time History, Sampling Rate 200 Hz

7.4 Model and Test Procedure

SDOF models based on the transfer function expressed in equations (6.8) and (6.9) were formed using the material properties shown in Table 7.1. They were loaded with vertical excitations created by adding deterministic sinusoidal functions to the random data sets of displacements collected from the Central Artery/Tunnel Project. The models based on material properties assuming 5 Hz excitation frequency were loaded with a 5 Hz frequency, and the same for 30 Hz. In addition, the models were also loaded with an excitation that consisted of a sum of sinusoids with frequencies 5, 10, 20, and 30 Hz all together. To study the total affect of the damping material, natural frequencies for the

SDOF systems were chosen at 5 Hz and 40 Hz; the former to check damping, latter to check for isolation. The Matlab script used to run the simulation is included in the Appendix.

8.0 Results

As a simplified, first pass approach, the simulation concluded that a polymer would outperform asphalt for damping the microtremors studied here. This is not a surprise because rubber has a lower stiffness and higher loss factor. However, in choosing a damping device, there are other considerations, including ease of installation and cost. Asphalt would obviously be much cheaper than a rubber bearing, and perhaps easier to install for structures such as moderate sized homes, where the asphalt could be sandwiched between the soil subgrade and the concrete foundation.

In addition, neither material could attenuate ground motion sufficiently to below threshold values for human perception or annoyance across the board. However, for the case where the thickness of the material was sufficient to result in a natural frequency away from that of traffic vibration (5 – 30 Hz), levels were maintained at 1 mm/s. When the excitation frequency was not driving at resonance, the materials were able to attenuate the velocities enough to within recommended levels for Type I, II, and III buildings.

From this, we can also conclude that although damping ratios were very high, at resonance, where damping was needed the most, the materials could not sufficiently damp the structural vibrations. In the worst case, the velocity was amplified to six times the input magnitude. This is not obvious when looking at the material properties and the impulse response of the system. An example is shown below.

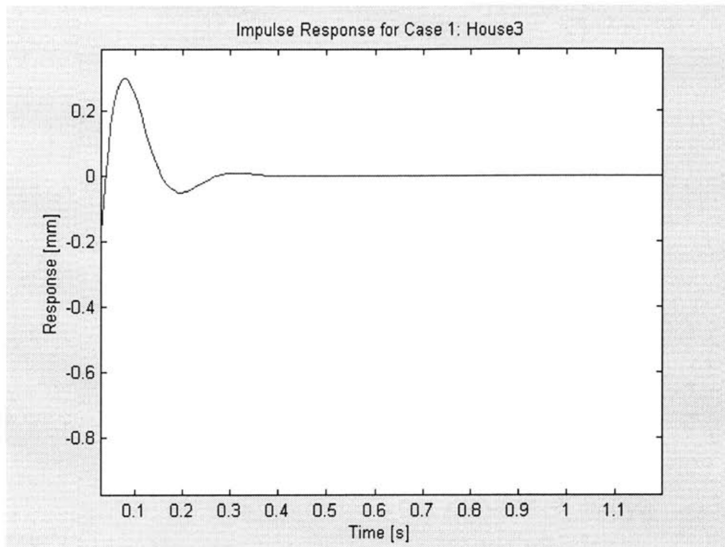

Figure 8.1 Sample of System Impulse Response

One of the problems is that in order for damping to be effective, the rate of displacement, or velocity of the structure, must be large enough to counter the large inertia coming from the mass of the structure. However, for microtremors, the displacements and velocities are too small to effectively dissipate energy for this scale. This is discussed further in the Hysteresis Loops section of the report. Below are the power spectral densities showing the frequencies apparent in the resulting displacements and the levels. Because the resulting frequencies are almost exactly at the natural frequencies, even when driven by a multitude of frequencies, it is clear that damping was not achieved.

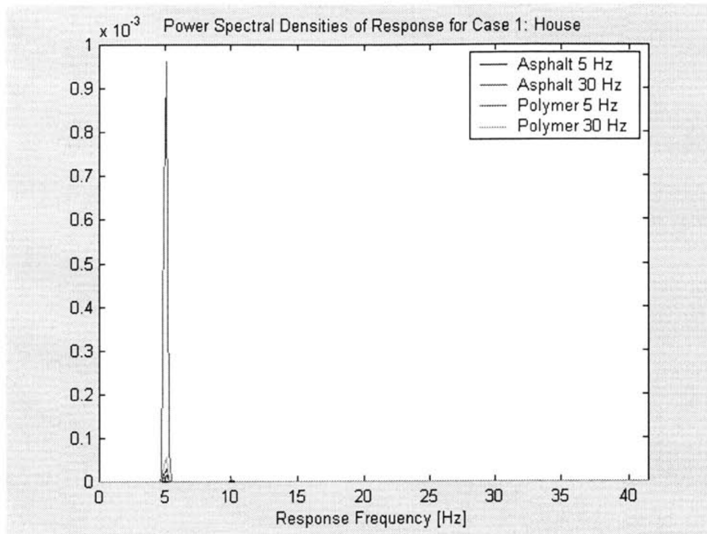

Figure 8.2 Case for Structural Natural Frequency is 5 Hz, Driving Frequencies: 5, 10, 20, 30 Hz

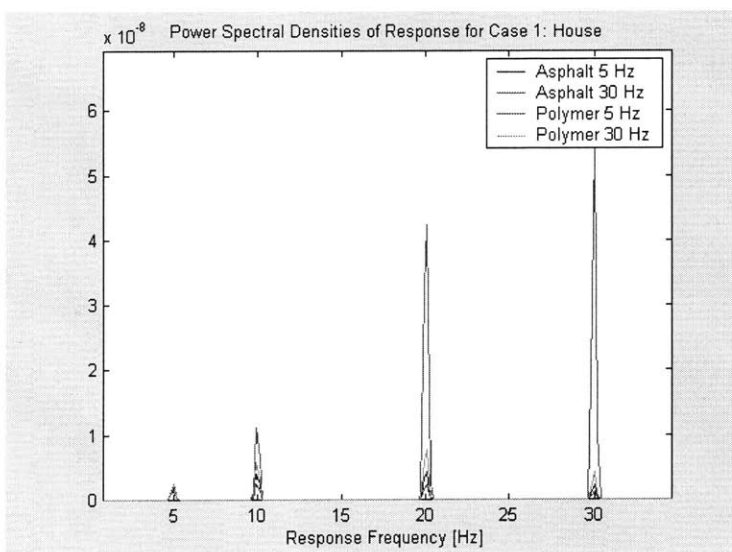

Figure 8.3 Case for Structural Natural Frequency is 5 Hz, Driving Frequencies: 5, 10, 20, 30 Hz

Practically, it is difficult to achieve high levels of damping for these viscoelastic materials responding to microtremors. The damping energy is converted to deformation and heat, and the materials exhibiting highly viscous behavior tend to become more viscous with increasing temperature and also stiffer with increasing number of cycles. Therefore, the dampers lose effectiveness as the duration increases, unless a large volume of material is used.

8.1 Hysteresis Loops

In the study of vibration, hysteresis loops as measures of energy dissipation are very often applied. A hysteresis loop can be obtained by recording the magnitude of a force versus the displacement brought about by its action. If there exists any damping in the system then the graphical representation of this recording resembles a loop. The area enclosed by the loop is proportional to the amount of energy dissipated in the system within one period. There are different possible interpretations of hysteresis loops according to the kind of force recorded during measurements.

a.) External loop – reflecting the relationship between externally applied loads and the displacements measured in the system.

b.) Internal – if displacements are related to internal forces in the system.

c.) Damping – when damping forces are taken into consideration

The materials did exhibit damping, as shown by sample polymer hysteresis loops below. However, as discussed earlier, it was not sufficient to attenuate the vibrations to below threshold levels for human annoyance.

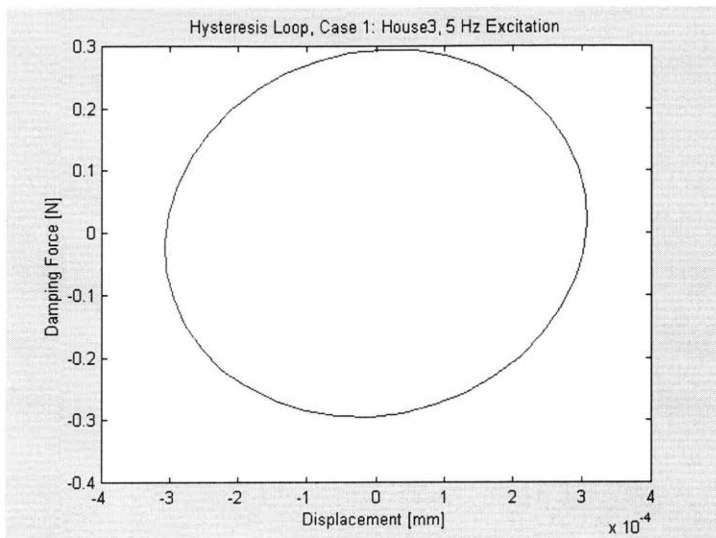

Figure 8.4 Sample of Hysteresis Loop for Polymer Bearing with 5 Hz Excitation

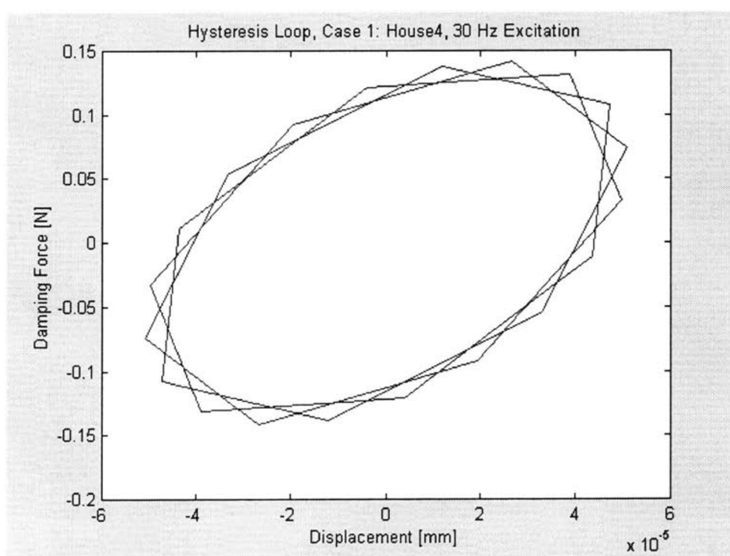

Figure 8.5 Sample of Hysteresis Loop for Polymer Bearing with 30 Hz Excitation

9.0 Feasibility

9.1 Vertical Load Capacity

For the application studied here, the compressive strength of a polymeric bearing depends on the type of rubber, the bearing plan size, the bearing shape factor, and the thickness. If elastomeric rubber is used, like that used for high damping rubber isolators, then the compressive strength is sufficient and assisted by steel plates that prevent the rubber from bulging, so that the bearing can support higher vertical static loads with only small deformations.

For asphalts, the figures below show the relationship between the approximate thickness and allowable bearing strength. For the thickness of 0.1m assumed in this study, the maximum vertical is well within the limits of asphalt.

Design of the Flexible System 167

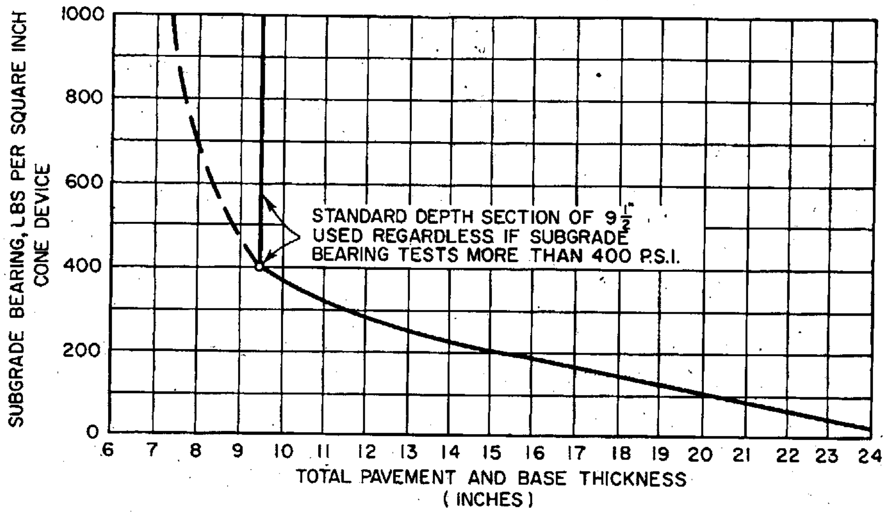

FIG. 9-16. Flexible-pavement design curve.

Figure 9.1 Compressive Strength of Typical Asphalt Layer (Rogers and Wallace 1958)

9.2 Possible Design of Isolation System

To isolate the building from microtremors like those caused by vehicular or railway traffic, pads could be inserted as resilient bearings on top of the pile caps or on the footings. A sketch of such an installation is shown below. The bearings, about 100mm thick when unloaded, are bonded with a self-levelling epoxy grout to the pile cap or to a ground beam. A beam is placed on top to distribute the loads. A failsafe upstand can be placed in between some of the pads to take the load in the event of a complete failure.

Where a large load has to be supported on a single pile cap, a cluster of 10-30 rubber pads could be used, with failsafe strips in between.

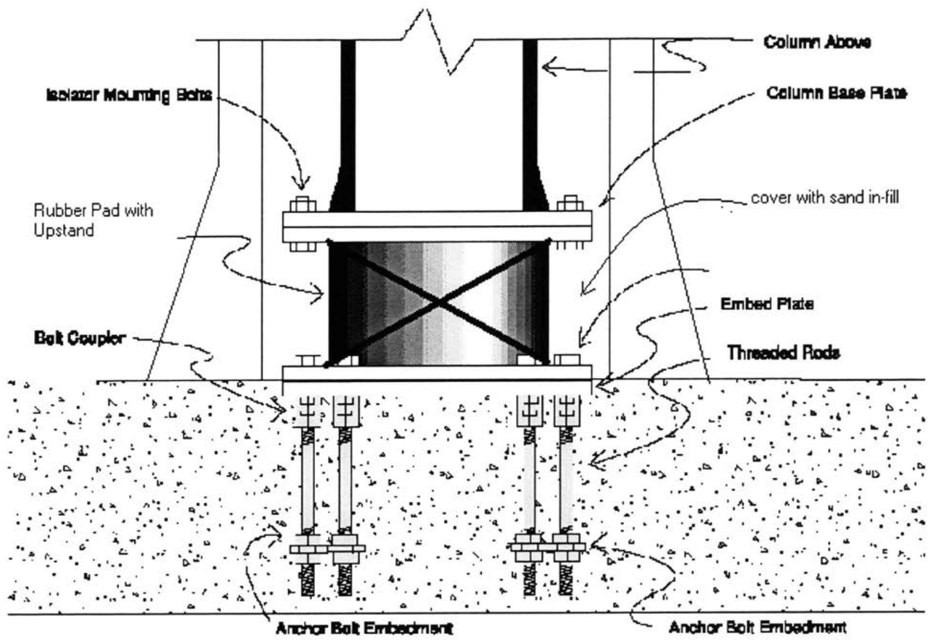

Figure 9.2 Design Schematic for Possible Column/Footing and Damping Material Connection

The ratio of dynamic to static stiffness for natural rubber at these stress levels and at a frequency of say 10Hz is between 1.7 and 2. The ratio for synthetic rubber is somewhat higher. There is a marked effect of the shape factor on the natural frequency of a pad design; pads with the same thickness and with a smaller plan area will normally have a lower natural frequency for the same stress. The shape factor is defined as the ratio of the loaded area to the area free to bulge. A better isolation can therefore be obtained by installing a large cluster of small pads than only one or two very large ones. There is, of course, a limit to this idea because the stability of the pads must be maintained. For a typical damping material, TICO CV/CA, with thickness 25 mm, tests have indicated a loss factor of 0.12. This corresponds to a damping ratio of 6%.

If there was any moment capacity considered in the column/footing connection in the original design, then moment connections for the rubber or asphalt damper, would be needed, as in most connections for rubber bearings used for earthquake shear base-isolation. This detail would be more easily applied in new construction; retrofits would be more difficult. For a steel column connected to a pile cap or footing, the design would mimic that for shear base-isolators. For concrete columns, the column would be spliced, with the bearing placed in between. If it is a retrofit, the load would have to be transferred away from the column using standard methods. For small structures, like houses, which normally sit on a concrete slab, the bearings could be placed underneath the slab, directly on to the soil foundation.

9.3 Other Materials: Foam

Expanded polystyrene (EPS), or Geofoam, is a very common product that is widely used for packaging and in building construction. Geofoam is used as infill, as opposed to earth materials which are heavy and can cause undesirable settlement or instability. Manufacturing of EPS blocks begins with expandable polystyrene resin beads that are generally less than 3 mm in diameter and contain microscopic cells filled with a blowing agent. Because of its mechanical structure, Geofoam has proven to be an excellent absorber of small ground vibrations. Unfortunately, it is not viable as a pad to place underneath a structure because the comperessive strength is very low. At 1% strain, the compressive resistance is only 75 kPa, which is not sufficient to take the load of a supporting column.

Conclusion

From this study, it is clear that to attenuate ground born microtremors due to vehicle or railway traffic, the ideal situation is to block the vibrations from getting into the building. This can be done by surrounding the footings or piles by a concrete wall, or filling the nearby area with Geofoam.

However, if this is not an option, a building could be retrofitted with polymeric bearings to sufficiently attenuate vertical vibrations, as long as the bearings were large enough to bring the apparent vertical natural frequency away from the 5 – 30 Hz resonant range. This would in effect be more of an isolation approach, as opposed to damping. Otherwise, the material cannot sufficiently dampen the structure. In addition, if cost is a constraint, asphaltic dampers could be installed, that could sufficiently maintain levels to resist structural damage, but not human annoyance.

Appendix A: Results from Simulation

A.1 Structural Natural Frequency 5 Hz, Excitation Amplitude 10 mm/s at 5 Hz

Structure and Material Property	Max Velocity [mm/s]	RMS Velocity [mm/s]	Min Velocity [mm/s]
House *Asphalt 5 Hz*	0.0138	0.0097	-.0300
Office *Asphalt 5 Hz*	0.0138	0.0097	-0.0300
House *Polymer 5 Hz*	0.0096	0.0068	-0.0293
Office *Polymer 5 Hz*	0.0096	0.0068	-0.0293

A.2 Structural Natural Frequency 5 Hz, Excitation Amplitude 10 mm/s at 30 Hz

Structure and Material Property	Max Velocity [mm/s]	RMS Velocity [mm/s]	Min Velocity [mm/s]
House *Asphalt 30 Hz*	0.0094	0.0066	-0.0752
Office *Asphalt 30 Hz*	0.0094	0.0066	-0.0752
House *Polymer 30 Hz*	0.0093	0.066	-0.0304
Office *Polymer 30 Hz*	0.0093	0.066	-0.0304

A.3 Structural Natural Frequency 5 Hz, Excitation Amplitude 10 mm/s at 5, 10, 20, and 30 Hz

Structure and Material Property	Max Velocity [mm/s]	RMS Velocity [mm/s]	Min Velocity [mm/s]
House *Asphalt 5 Hz*	0.0224	0.0147	-0.030
Office *Asphalt 5 Hz*	0.0224	0.0147	-0.030
House *Polymer 5 Hz*	0.0208	0.0125	-0.0293
Office *Polymer 5 Hz*	0.0208	0.0126	-0.0293
House *Asphalt 30 Hz*	0.0880	0.0581	-0.0752
Office *Asphalt 30 Hz*	0.0880	0.0581	-0.0752
House *Polymer 30 Hz*	0.0269	0.0177	-0.0304
Office *Polymer 30 Hz*	0.0269	0.0177	-0.0304

A.4 Structural Natural Frequency 40 Hz, Excitation Amplitude 10 mm/s at 5 Hz

Structure and Material Property	Max Velocity [mm/s]	RMS Velocity [mm/s]	Min Velocity [mm/s]
House *Asphalt 5 Hz*	0.00026	0.00008	-.0011
Office *Asphalt 5 Hz*	0.00026	0.00008	-.0011
House *Polymer 5 Hz*	0.00014	0.00007	-0.0008
Office *Polymer 5 Hz*	0.00014	0.00007	-0.0008

A.5 Structural Natural Frequency 40 Hz, Excitation Amplitude 10 mm/s at 30 Hz

Structure and Material Property	Max Velocity [mm/s]	RMS Velocity [mm/s]	Min Velocity [mm/s]
House *Asphalt 30 Hz*	0.0055	0.0039	-0.0056
Office *Asphalt 30 Hz*	0.0055	0.0039	-0.0056
House *Polymer 30 Hz*	0.0015	0.0011	-0.0015
Office *Polymer 30 Hz*	0.0015	0.0011	-0.0015

A.6 Structural Natural Frequency 40 Hz, Excitation Amplitude 10 mm/s at 5, 10, 20, and 30 Hz

Structure and Material Property	Max Velocity [mm/s]	RMS Velocity [mm/s]	Min Velocity [mm/s]
House *Asphalt 5 Hz*	0.0019	0.0007	-0.0039
Office *Asphalt 5 Hz*	0.0019	0.0007	-0.0039
House *Polymer 5 Hz*	0.0013	0.0005	-0.0028
Office *Polymer 5 Hz*	0.0013	0.0005	-0.0028
House *Asphalt 30 Hz*	0.0116	0.0028	-0.0114
Office *Asphalt 30 Hz*	0.0116	0.0028	-0.0114
House *Polymer 30 Hz*	0.0028	0.0009	-0.0052
Office *Polymer 30 Hz*	0.0028	0.0009	-0.0052

Appendix B: Matlab script used to run simulation

```
%%                                                                 %%
%%                                                                 %%
%%    Diane Floresca                                               %%
%%    Massachusetts Institute of Technology                        %%
%%    Department of Civil and Environmental Engineering            %%
%%    Masters of Engineering Thesis 2003                           %%
%%    Research Into Building Vibrations                            %%
%%    Using Aphalt or Polymers to Dampen Traffic Vibrations in     %%
%%    Buildings                                                    %%
%%                                                                 %%
%%    The following script was written to simulate 2 SDOF systems %%
%%    which model a typical house structure and a small office     %%
%%    building, with asphalt and polymer pads placed between the   %%
%%    building and the foundation.                                 %%
%%                                                                 %%
%%                                                                 %%
%%                                                                 %%
```

```
E = [1579000000 3508000000 1000000 1000000];
L = 0.1; %100 mm
n = [0.7 0.7 1 3];
m = [100000 1500000];
omega = 2*pi*[5 30 5 30];

%Set target vertical period to specified frequency of study
%Frequencies studied: 31, 50, and 250 rad/s
A_house = (E.^(-1)).*((31^2)*m(1)*L);
A_office = (E.^(-1)).*((31^2)*m(2)*L);


for i=1:4

   k_house(i) = E(i)*A_house(i)/L;
   c_house(i) = A_house(i)*E(i)*n(i)/(L*omega(i));
   wn_house(i) = (k_house(i)/m(1))^(0.5);
   xi_house(i) = c_house(i)/(2*wn_house(i)*m(1));

   k_office(i) = E(i)*A_office(i)./L;
   c_office(i) = A_office(i)*E(i)*n(i)/(L*omega(i));
   wn_office(i) = (k_office(i)/m(2))^(0.5);
   xi_office(i) = c_office(i)/(2*wn_office(i)*m(2));

end

%Graphing the theoretical amplification factor for velocity response
w = 0:2:200*2*pi;
xi = [0.01 0.04 0.1];
```

```
vamp_house1_x1 = ((((w.^2)-
(omega(1)^2)).^2)+((w.*2*xi(1)*omega(1)).^2)).^(-1/2);
vamp_house1_x4 = ((((w.^2)-
(omega(1)^2)).^2)+((w.*2*xi(2)*omega(1)).^2)).^(-1/2);
vamp_house1_x10 = ((((w.^2)-
(omega(1)^2)).^2)+((w.*2*xi(3)*omega(1)).^2)).^(-1/2);
figure
plot(w,vamp_house1_x1,w,vamp_house1_x4,w,vamp_house1_x10);
xlabel('Natural Frequency of SDOF on Damping Pad in rad/s');
ylabel('Amplification of Relative Velocity Response');
title('Amplifaction of Relative Velocity for 5 Hz Vertical
Excitation');
legend('Damping Ratio 1%','4%','10%');

vamp_house2_x1 = ((((w.^2)-
(omega(2)^2)).^2)+((w.*2*xi(1)*omega(2)).^2)).^(-1/2);
vamp_house2_x4 = ((((w.^2)-
(omega(2)^2)).^2)+((w.*2*xi(2)*omega(2)).^2)).^(-1/2);
vamp_house2_x10 = ((((w.^2)-
(omega(2)^2)).^2)+((w.*2*xi(3)*omega(2)).^2)).^(-1/2);
figure
plot(w,vamp_house2_x1,w,vamp_house2_x4,w,vamp_house2_x10);
xlabel('Natural Frequency of SDOF on Damping Pad in rad/s');
ylabel('Amplification of Relative Velocity Response');
title('Amplifaction of Relative Velocity for 30 Hz Vertical
Excitation');
legend('Damping Ratio 1%','4%','10%');

plot(w,vamp_house1_x1,w,vamp_house1_x4,w,vamp_house1_x10,w,vamp_house2_
x1,w,vamp_house2_x4,w,vamp_house2_x10);
xlabel('Natural Frequency of SDOF on Damping Pad in rad/s');
ylabel('Amplification of Relative Velocity Response');
title('Amplifaction of Relative Velocity for 5 Hz and 30 Hz Vertical
Excitation');
legend('Damping Ratio 1% - 5 Hz','4% - 5 Hz','10% - 5 Hz','Damping
Ratio 1% - 30 Hz','4% - 30 Hz','10% - 30 Hz');



load d:\vibetest1.txt
%sampled at 1 Hz (uips)


signal_velocity = ((10.^(vibetest1./10)).^(1/2))*0.0254*0.000001;
%[m/s]
%zero out the velocity
signal_velocity = detrend(signal_velocity);

%just taking 2000 points from input, somewhere in the middle
signal_disp = indefintegral(signal_velocity(1999:3999));


%Check statistics
rms_velocity =
(trapz(signal_velocity.^2)/(length(signal_velocity)))^(0.5);
rms_disp = (trapz(signal_disp.^2)/(length(signal_disp)))^(0.5);
over_threshold=length(find(signal_velocity>0.001));
```

```
mean1 = mean(signal_velocity);
std1 = std(signal_velocity);
max1 = max(abs(signal_velocity));
rms1= (trapz(signal_velocity.^2)/length(signal_velocity))^(1/2);

%Assuming highest frequency content to be 80 Hz or 160pi rad/s
%Nyquist = 160 Hz --> 200 Hz
ws = 2*pi*200;
fs = 200;
ts = 1/200;


%Since data sampled at 1 Hz, we're going to insert points to make fs =
200 Hz
%we will assume at this point that the input is band limited to 1 Hz
input1 = interp(signal_disp,200);
l=400000;
t1 = 0:0.005:length(signal_disp)-1;
t1 = t1(1:l);
input5 = 0.0003*(input1(1:l) + sin(2*pi*5*t1(1:l)));
input30 = 0.00005*(input1(1:l) + sin(2*pi*30*t1(1:l)));
inputall = 0.0003*input1(1:l) + 0.0003*sin(2*pi*5*t1(1:l)) +
0.00015*sin(2*pi*10*t1(1:l))+
0.00007*sin(2*pi*20*t1(1:l))+0.00003*sin(2*pi*30*t1(1:l));


%Creating Object Transfer Functions
s = tf('s');
H_house1 = (-m(1)*(s^2))/((m(1)*s^2)+(c_house(1)*s)+k_house(1));
H_house2 = (-m(1)*(s^2))/((m(1)*s^2)+(c_house(2)*s)+k_house(2));
H_house3 = (-m(1)*(s^2))/((m(1)*s^2)+(c_house(3)*s)+k_house(3));
H_house4 = (-m(1)*(s^2))/((m(1)*s^2)+(c_house(4)*s)+k_house(4));

H_office1 = (-m(2)*(s^2))/((m(2)*s^2)+(c_office(1)*s)+k_office(1));
H_office2 = (-m(2)*(s^2))/((m(2)*s^2)+(c_office(2)*s)+k_office(2));
H_office3 = (-m(2)*(s^2))/((m(2)*s^2)+(c_office(3)*s)+k_office(3));
H_office4 = (-m(2)*(s^2))/((m(2)*s^2)+(c_office(4)*s)+k_office(4));

%Checking impulse response
impulseinput = 1+zeros(1,1000);
timpulse = 0:0.005:5;
timpulse = timpulse(1:1000);
[Y_house3_5_impulse,timpulse] = lsim(H_house3,impulseinput,timpulse);
title('Impulse Response for Case 1: House3');
xlabel('Time [s]');
ylabel('Response [mm]');

[Y_office1_5_impulse,timpulse] = lsim(H_office1,impulseinput,timpulse);
title('Impulse Response for Case 2: Office1');
xlabel('Time [s]');
ylabel('Response [mm]');


%Creating Response Time Histories for Inputs of Various Frequencies
[Y_house1_5,t]=lsim(H_house1,input5,t1);
[Y_house3_5,t]=lsim(H_house3,input5,t1);
[Y_house2_30,t]=lsim(H_house2,input30,t1);
```

```
[Y_house4_30,t]=lsim(H_house4,input30,t1);
[Y_office1_5,t]=lsim(H_office1,input5,t1);
[Y_office3_5,t]=lsim(H_office3,input5,t1);
[Y_office2_30,t]=lsim(H_office2,input30,t1);
[Y_office4_30,t]=lsim(H_office4,input30,t1);

[Y_house1_all,t]=lsim(H_house1,inputall,t1);
[Y_house3_all,t]=lsim(H_house3,inputall,t1);
[Y_house2_all,t]=lsim(H_house2,inputall,t1);
[Y_house4_all,t]=lsim(H_house4,inputall,t1);
[Y_office1_all,t]=lsim(H_office1,inputall,t1);
[Y_office3_all,t]=lsim(H_office3,inputall,t1);
[Y_office2_all,t]=lsim(H_office2,inputall,t1);
[Y_office4_all,t]=lsim(H_office4,inputall,t1);

[V_house1_5] = diff(Y_house1_5)/ts;
[V_house3_5] = diff(Y_house3_5)/ts;
[V_office1_5] = diff(Y_office1_5)/ts;
[V_office3_5] = diff(Y_office3_5)/ts;
[V_house2_30] = diff(Y_house2_30)/ts;
[V_house4_30] = diff(Y_house4_30)/ts;
[V_office2_30] = diff(Y_office2_30)/ts;
[V_office4_30] = diff(Y_office4_30)/ts;

[V_house1_all] = diff(Y_house1_all)/ts;
[V_house2_all] = diff(Y_house2_all)/ts;
[V_house3_all] = diff(Y_house3_all)/ts;
[V_house4_all] = diff(Y_house4_all)/ts;
[V_office1_all] = diff(Y_office1_all)/ts;
[V_office2_all] = diff(Y_office2_all)/ts;
[V_office3_all] = diff(Y_office3_all)/ts;
[V_office4_all] = diff(Y_office4_all)/ts;

rmsV_house1_5 = rms(V_house1_5)
rmsV_house2_30 = rms(V_house2_30)
rmsV_house3_5 = rms(V_house3_5)
rmsV_house4_30 = rms(V_house4_30)

rmsV_office1_5 = rms(V_office1_5)
rmsV_office2_30 = rms(V_office2_30)
rmsV_office3_5 = rms(V_office3_5)
rmsV_office4_30 = rms(V_office4_30)

rmsV_house1_all = rms(V_house1_all)
rmsV_house2_all = rms(V_house2_all)
rmsV_house3_all = rms(V_house3_all)
rmsV_house4_all = rms(V_house4_all)

rmsV_office1_all = rms(V_office1_all)
rmsV_office2_all = rms(V_office2_all)
rmsV_office3_all = rms(V_office3_all)
rmsV_office4_all = rms(V_office4_all)



maxV_house1_5 = max(V_house1_5)
maxV_house2_30 = max(V_house2_30)
```

```
maxV_house3_5 = max(V_house3_5)
maxV_house4_30 = max(V_house4_30)

maxV_office1_5 = max(V_office1_5)
maxV_office2_30 = max(V_office2_30)
maxV_office3_5 = max(V_office3_5)
maxV_office4_30 = max(V_office4_30)

maxV_house1_all = max(V_house1_all)
maxV_house2_all = max(V_house2_all)
maxV_house3_all = max(V_house3_all)
maxV_house4_all = max(V_house4_all)

maxV_office1_all = max(V_office1_all)
maxV_office2_all = max(V_office2_all)
maxV_office3_all = max(V_office3_all)
maxV_office4_all = max(V_office4_all)


minV_house1_5 = min(V_house1_5)
minV_house2_30 = min(V_house2_30)
minV_house3_5 = min(V_house3_5)
minV_house4_30 = min(V_house4_30)

minV_office1_5 = min(V_office1_5)
minV_office2_30 = min(V_office2_30)
minV_office3_5 = min(V_office3_5)
minV_office4_30 = min(V_office4_30)

minV_house1_all = min(V_house1_all)
minV_house2_all = min(V_house2_all)
minV_house3_all = min(V_house3_all)
minV_house4_all = min(V_house4_all)

minV_office1_all = min(V_office1_all)
minV_office2_all = min(V_office2_all)
minV_office3_all = min(V_office3_all)
minV_office4_all = min(V_office4_all)

[p_house1_all,f_house1_all]=psd(Y_house1_all,1026,200);
[p_house2_all,f_house2_all]=psd(Y_house2_all,1026,200);
[p_house3_all,f_house3_all]=psd(Y_house3_all,1026,200);
[p_house4_all,f_house4_all]=psd(Y_house4_all,1026,200);
[p_office1_all,f_office1_all]=psd(Y_office1_all,1026,200);
[p_office2_all,f_office2_all]=psd(Y_office2_all,1026,200);
[p_office3_all,f_office3_all]=psd(Y_office3_all,1026,200);
[p_office4_all,f_office4_all]=psd(Y_office4_all,1026,200);


figure
plot(f_house1_all,p_house1_all,f_house2_all,p_house2_all,f_house3_all,p
_house3_all,f_house4_all,p_house4_all);
title('Power Spectral Densities of Response for Case 1: House');
xlabel('Response Frequency [Hz]');
legend('Asphalt 5 Hz', 'Asphalt 30 Hz', 'Polymer 5 Hz', 'Polymer 30
Hz');
```

```
figure
plot(f_office1_all,p_office1_all,f_office2_all,p_office2_all,f_office3_
all,p_office3_all,f_office4_all,p_office4_all);
title('Power Spectral Densities of Response for Case 1: office');
xlabel('Response Frequency [Hz]');
legend('Asphalt 5 Hz', 'Asphalt 30 Hz', 'Polymer 5 Hz', 'Polymer 30
Hz');

figure
plot(0:1:length(signal_velocity)-1,signal_velocity);
title('Original Input Velocity Time History -- Sampled At 1 Hz');
xlabel('Time [s]');
ylabel('Displacement [mm/s]');

figure
plot(0:1:length(signal_disp)-1,signal_disp);
title('Original Input Displacement Time History -- Sampled At 1 Hz');
xlabel('Time [s]');
ylabel('Displacement [mm]');

figure
plot(0:1:length(input1)-1,input1);
title('Interpolated Input Displacement Time History -- Resampled at 200
Hz');
xlabel('Time [s]');
%ylabel('Displacement [mm]');


%Hysteresis Loops
h_house = xi_house.*wn_house;
Fd_house1_5 = -2*h_house(1)*V_house1_5;
Fd_house3_5 = -2*h_house(3)*V_house3_5;
Fd_house2_30 = -2*h_house(2)*V_house2_30;
Fd_house4_30 = -2*h_house(4)*V_house4_30;

h_office = xi_office.*wn_office;
Fd_office1_5 = -2*h_office(1)*V_office1_5;
Fd_office3_5 = -2*h_office(3)*V_office3_5;
Fd_office2_30 = -2*h_office(2)*V_office2_30;
Fd_office4_30 = -2*h_office(4)*V_office4_30;


Fd_house1_all = -2*h_house(1)*V_house1_all;
Fd_house2_all = -2*h_house(2)*V_house2_all;
Fd_house3_all = -2*h_house(3)*V_house3_all;
Fd_house4_all = -2*h_house(4)*V_house4_all;
Fd_office1_all = -2*h_office(1)*V_office1_all;
Fd_office2_all = -2*h_office(2)*V_office2_all;
Fd_office3_all = -2*h_office(3)*V_office3_all;
Fd_office4_all = -2*h_office(4)*V_office4_all;



figure
plot(Y_house1_5(1000:5000),Fd_house1_5(1000:5000));
title('Hysteresis Loop, Case 1: House1, 5 Hz Excitation');
xlabel('Displacement [mm]');
```

```
ylabel('Damping Force [N]');
figure
plot(Y_house3_5(1000:5000),Fd_house3_5(1000:5000));
title('Hysteresis Loop, Case 1: House3, 5 Hz Excitation');
xlabel('Displacement [mm]');
ylabel('Damping Force [N]');
figure
plot(Y_house2_30(1000:5000),Fd_house2_30(1000:5000));
title('Hysteresis Loop, Case 1: House2, 30 Hz Excitation');
xlabel('Displacement [mm]');
ylabel('Damping Force [N]');
figure
plot(Y_house4_30(1000:5000),Fd_house4_30(1000:5000));
title('Hysteresis Loop, Case 1: House4, 30 Hz Excitation');
xlabel('Displacement [mm]');
ylabel('Damping Force [N]');



figure
plot(Y_office1_5(1000:5000),Fd_office1_5(1000:5000));
title('Hysteresis Loop, Case 2: office1, 5 Hz Excitation');
xlabel('Displacement [mm]');
ylabel('Damping Force [N]');
figure
plot(Y_office3_5(1000:5000),Fd_office3_5(1000:5000));
title('Hysteresis Loop, Case 2: office3, 5 Hz Excitation');
xlabel('Displacement [mm]');
ylabel('Damping Force [N]');
figure
plot(Y_office2_30(1000:5000),Fd_office2_30(1000:5000));
title('Hysteresis Loop, Case 2: office2, 30 Hz Excitation');
xlabel('Displacement [mm]');
ylabel('Damping Force [N]');
figure
plot(Y_office4_30(1000:5000),Fd_office4_30(1000:5000));
title('Hysteresis Loop, Case 2: office4, 30 Hz Excitation');
xlabel('Displacement [mm]');
ylabel('Damping Force [N]');


figure
plot(Y_house1_all(1000:399999),Fd_house1_all(1000:399999));
title('Hysteresis Loop, Case 1: House1, 5,10,20,30 Hz Excitation');
xlabel('Displacement [mm]');
ylabel('Damping Force [N]');
figure
plot(Y_house2_all(1000:399999),Fd_house2_all(1000:399999));
title('Hysteresis Loop, Case 1: House2, 5,10,20,30 Hz Excitation');
xlabel('Displacement [mm]');
ylabel('Damping Force [N]');
figure
plot(Y_house3_all(1000:399999),Fd_house3_all(1000:399999));
title('Hysteresis Loop, Case 1: House3, 5,10,20,30 Hz Excitation');
xlabel('Displacement [mm]');
ylabel('Damping Force [N]');
figure
```

```
plot(Y_house4_all(1000:399999),Fd_house4_all(1000:399999));
title('Hysteresis Loop, Case 1: House4, 5,10,20,30 Hz Excitation');
xlabel('Displacement [mm]');
ylabel('Damping Force [N]');


figure
plot(Y_office1_all(1000:399999),Fd_office1_all(1000:399999));
title('Hysteresis Loop, Case 2: office1, 5,10,20,30 Hz Excitation');
xlabel('Displacement [mm]');
ylabel('Damping Force [N]');
figure
plot(Y_office2_all(1000:399999),Fd_office2_all(1000:399999));
title('Hysteresis Loop, Case 2: office2, 5,10,20,30 Hz Excitation');
xlabel('Displacement [mm]');
ylabel('Damping Force [N]');
figure
plot(Y_office3_all(1000:399999),Fd_office3_all(1000:399999));
title('Hysteresis Loop, Case 2: office3, 5,10,20,30 Hz Excitation');
xlabel('Displacement [mm]');
ylabel('Damping Force [N]');
figure
plot(Y_office4_all(1000:399999),Fd_office4_all(1000:399999));
title('Hysteresis Loop, Case 2: office4, 5,10,20,30 Hz Excitation');
xlabel('Displacement [mm]');
ylabel('Damping Force [N]');
```

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