

Deployable Structures Inspired by the Origami Art

by
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Bachelor of Arts, University of Pennsylvania, 1998

Submitted to the Department of Architecture in partial fulfillment of the requirements for the degree of
Master of Architecture
at the Massachusetts Institute of Technology

February 2004

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signature of author: _____

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Department of Architecture
January 16, 2004

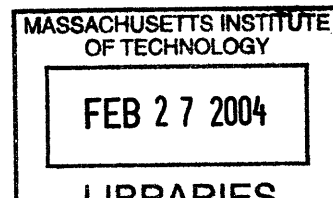
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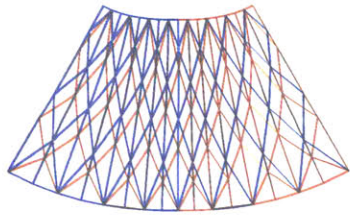
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Deployable Structures Inspired by the Origami Art

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Submitted to the Department of Architecture in partial fulfillment of the requirements for the degree of Master of Architecture at the Massachusetts Institute of Technology
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abstract



My thesis is an exploration of design methods and tools using origami as a vehicle to test their usefulness and coming to terms with their limitations. I have taken my fascination with a particular development in origami and put my belief in its potential for architectural application to the test by way of various investigations: materials and structural analysis, mathematical reasoning, manipulating space and form, parametric modeling, fabrication, and finite element testing.

Parting from conventional, figural forms, mathematicians developed open-surface forms together with theorems that governed the ability of these folded forms to fold flat. I selected a particular form, the Kao-fold, for its simplicity, beauty, and structural properties and imagined many exciting possibilities, specifically for its application in designing a deployable structure.

I analyzed its crease pattern, exploring variations and their corresponding folded forms.

Simultaneously, different material ideas for larger-scale structures were tested and a particular configuration was assessed for internal stresses and its structural stability. Its transformation from a flat sheet to a folded state was scrutinized under the lens of mathematical reasoning, namely trigonometry, by linking the acute angle of its crease pattern and the dihedral angle in its folded state to its final folded configuration.

The rigidity of this investigation was offset by the freedom afforded in manipulating paper models. As such, different spatial qualities and forms were explored while addressing the issue of scale and potential applications. The transformational characteristics discovered were digitally simulated via the construction of parametric models, which was a more controlled manipulation of the form in a virtual space. In order to go beyond the realm of representation and address real-life building issues, a temporary open-air shelter was designed and constructed in detail. The goal was to tackle the complexity of assigning materials, designing components and fabricated them. As a final endeavor, the model's construction was tested for its structural stability using a finite element software.

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acknowledgement

to my advisor, John Ochsendorf, for the consistent enthusiasm he brought to every discussion. It has helped me tremendously to stay guided and motivated

to Carol, my co-advisor, for addressing my assumptions and my shortcomings

to my readers, Shun Kanda and Martin Demaine

to my unofficial reader, Erik Demaine. Your interview in the New Scientist led me to pursue applications of folding in a seminar which paved the way to my thesis

to professor Kenneth Kao who gave the seminar and for teaching us to think critically

to Alex for generously giving me so much of his time and energy toward realizing the "sphere model"

to Susan and Victor for showing me the ropes to Catia

to other members of the "quiet studio" - Mike, Karim, Chris, Aiki and Tracy for their moral support

to Mrs. Marvin E Goody for support of my research and work.

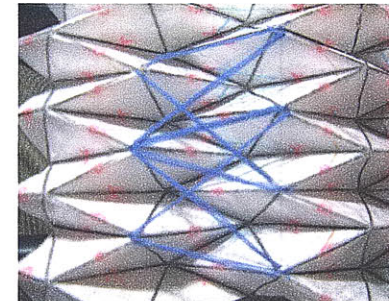
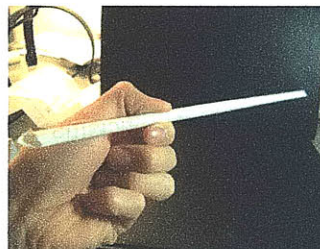
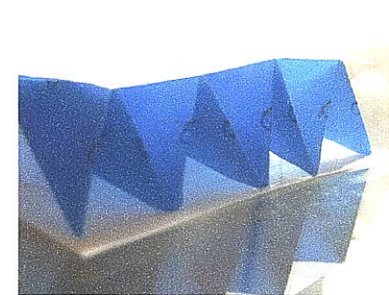
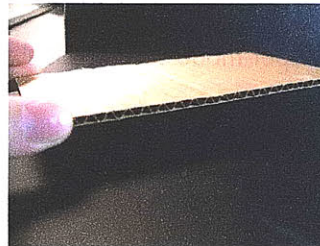
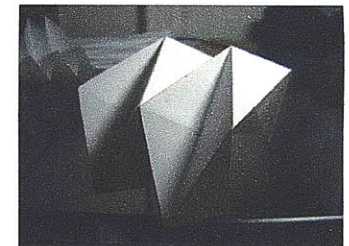
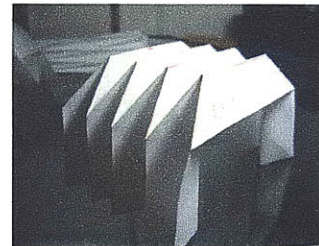
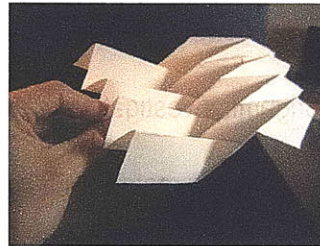
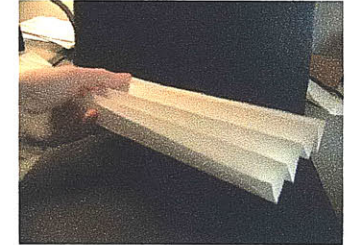
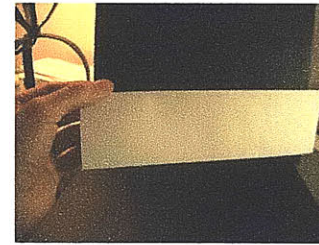
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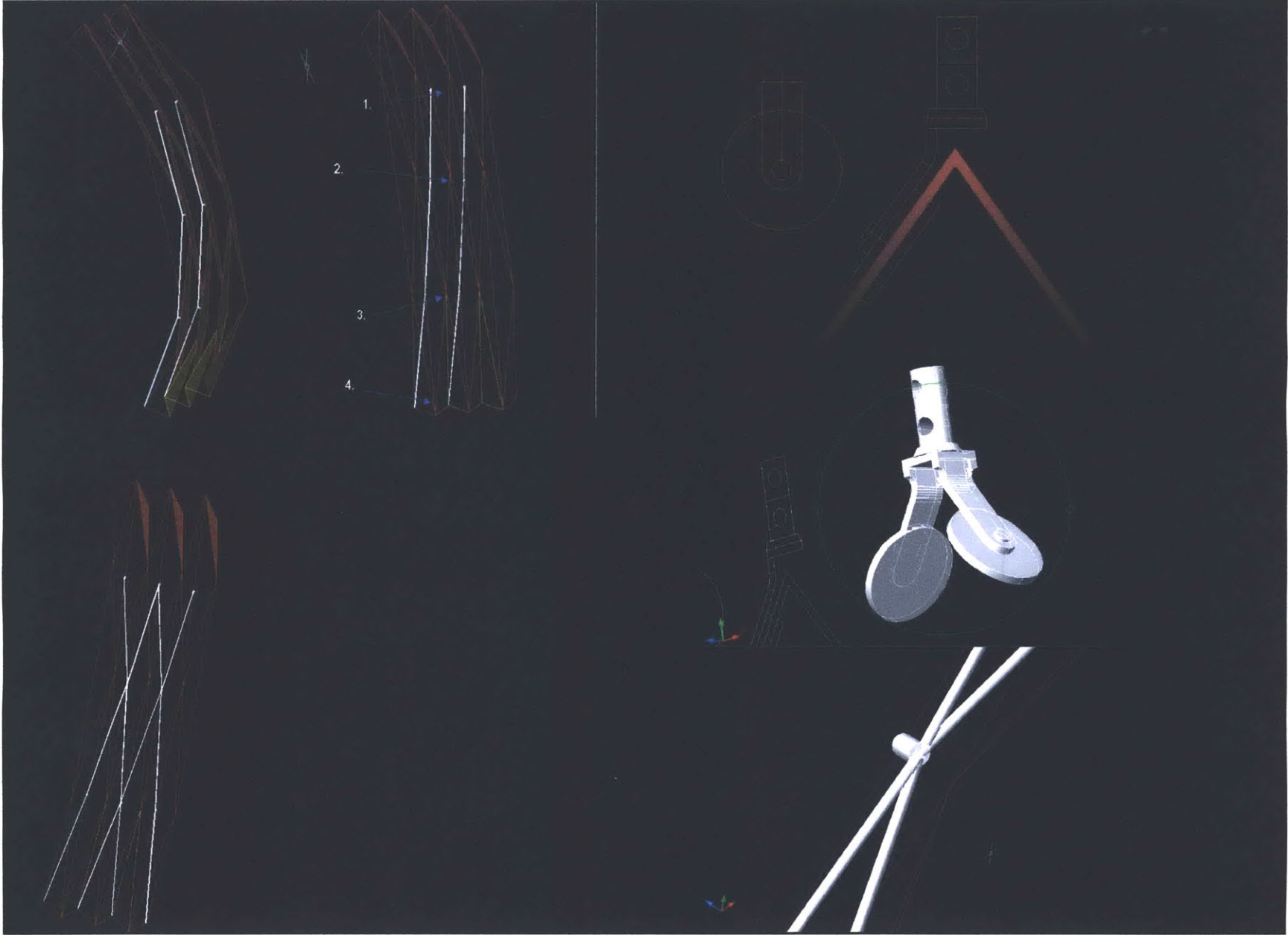
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motivation for research

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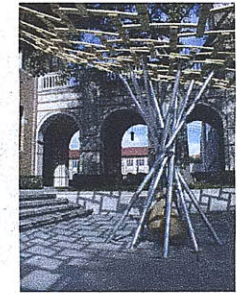
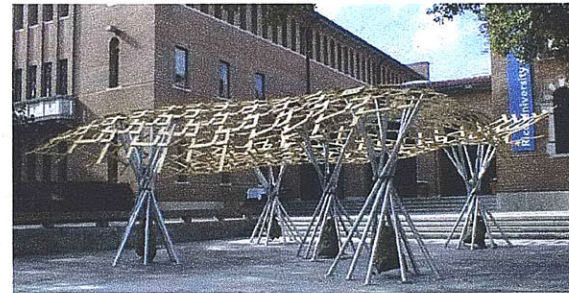
I developed my interest in deployable structures and origami in a seminar taken in the semester preceding my thesis at the Graduate School of Design in Harvard taught by professor Kenneth Kao titled GSD 6410 Concept to Construction: Building Components & Assemblies. The course examined and discussed various case studies of instances in architecture where truly innovative thinking in the engineering of a structural component enabled unique design solutions to emerge. The work of engineers/designers such as Jean Prouvet, Buckminster Fuller, and Peter Rice were examined for their inventiveness. In this context, students were taught to think critically about current-day design and building technologies and try to bring new insight into their use as a starting point for innovation. Eventually, this led to the final assignment for which I researched collapsible furniture and structures and in parallel explored basic folding patterns in origami. The research was guided by the belief in origami to be able to inform the design of deployable structures and focused on the design possibilities of a particular pleated form (which I eventually named the Kao-fold as no inventor could be identified).



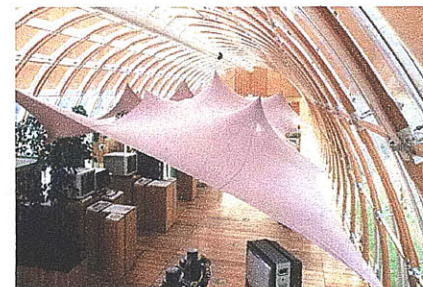
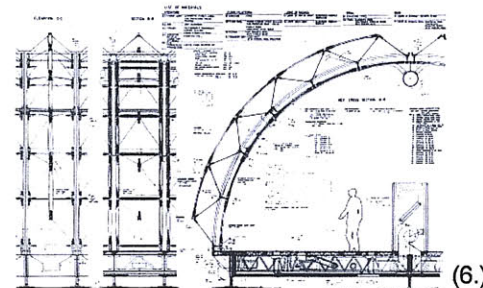


precedent studies

The ideas and thoughts surrounding some of the projects discussed in the course as well as those researched for my final assignment have carried over into the development of my thesis. Certain precedent studies that have been more inspiring may be presented:



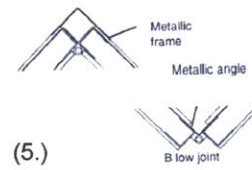
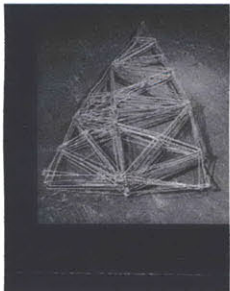
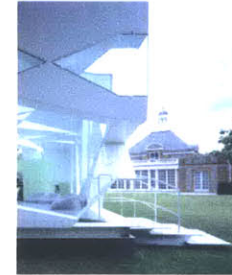
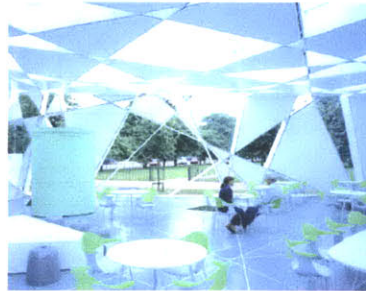
A geodesic dome uses a pattern of self-bracing triangles in a pattern that gives maximum structural advantage, thus theoretically using the least material possible. (A "geodesic" line on a sphere is the shortest distance between any two points.)



In Renzo Piano's IBM pavilion, a series of polycarbonate pyramids are reinforced with cast aluminum connections spliced and glued into laminated beechwood.



Commissioned by Rice University Art Gallery, Bamboo Roof is an outdoor site-specific installation created by Japanese Architect Shigeru Ban. The work features an expansive open-weave canopy of bamboo boards that spans the Gallery Plaza.



(5.)

joint details for an accordion roof



Water-tight hinges drive the design in a motor-driven aluminum sheet roof.



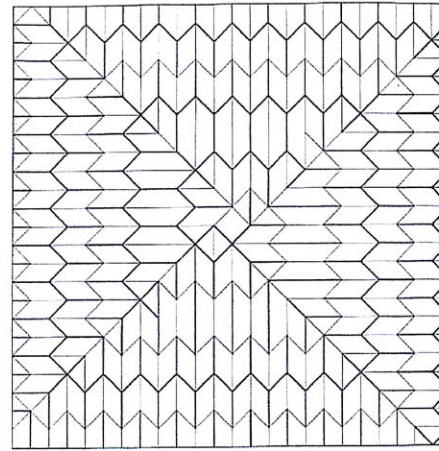
(7.)

origami: investigating open-surface forms

Starting in the 1930's, the art of origami began to extend beyond traditional, figural works with which we most strongly associate with origami. Liapi, 386 Enthusiasts' developed an interest in the mathematical properties found within the crease patterns. The crease pattern is literally the pattern of creases seen in the paper once the origami form is unfolded into a flat sheet. (see examples shown to the right)

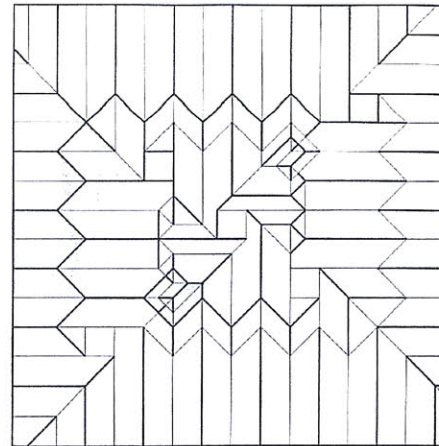
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Thus, working backwards, one is able to determine the final form for a particular design once the crease pattern is known. The only key required to unlocking this DNA which determines a certain form is to know whether each individual crease within the pattern results in a mountain or a valley fold. If either side of the crease is folded towards you, that is a valley fold and if away, you get a mountain fold. (see image on bottom-right)



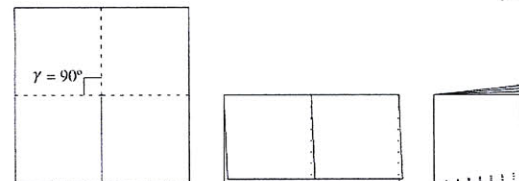
(8.)

Biaxial Flasher Supreme by Ushio Ikegami. The thicker lines represent mountain folds.



(8a.)

Cubic Flasher by Ushio Ikegami. Extremely difficult to fold, the cubic flasher does not expand and contract smoothly like a basic flasher.



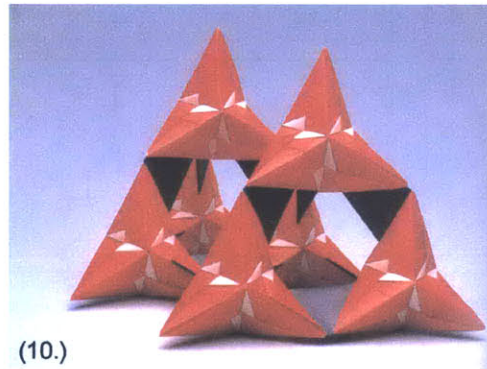
(9.)

In the crease pattern shown, dividing a square into four equal parts, all the creases represent valley folds.

One category of origami, which developed out of this newfound interest in origami's mathematical properties is modular or unit origami. Unit origami, as its name describes, is formed from a number of repeating units which link to form various closed shapes, which in turn, may be linked to one another with certain folded units. (see image)

A second development was open-surface forms. These open-surface forms had captured my interest, as they seemed to carry great potential for being applied as architectural elements, be it an outer skin of a building, an interior surface or an entire habitable structure. Together with the development of theorems which governed the ability of these open-surface forms to be folded flat, one may imagine exciting possibilities for origami's application in designing deployable structures - structures that could be collapsed, transported, and efficiently deployed. These theorems were derived by mathematicians through a careful understanding for how the crease pattern of the pleated form could be manipulated to give specific resulting forms. They determine the underlying mathematical properties of folded forms which allowed them to fold flat. Liapi, 387

These topological conditions that any pattern of creases must fulfill are:



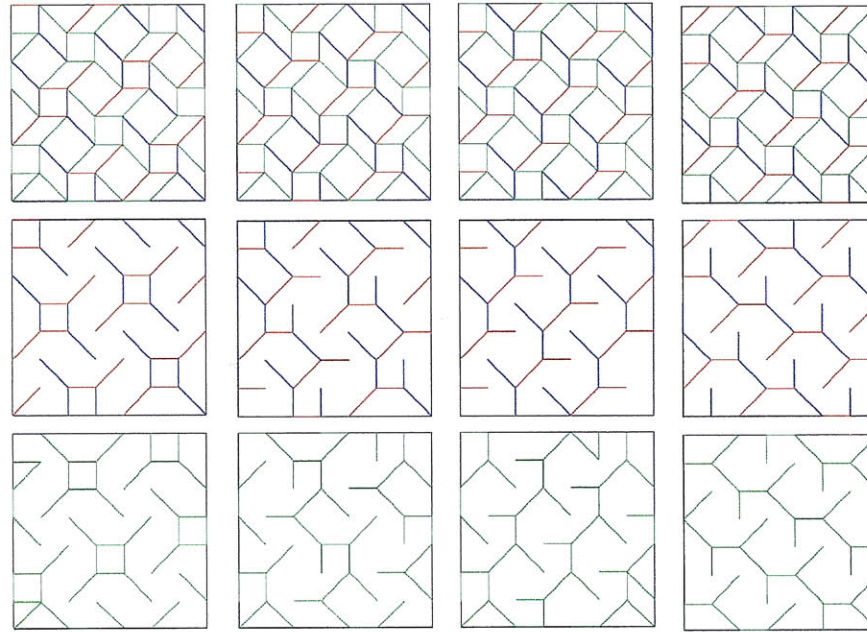
The 6-unit wedge is formed by a series of modular belt-units.

- a. At any vertex, the sum of the angles is 360 degrees
- b. The number of creases originating at a vertex must always be even.
- c. (the Hushimi theorem) When four creases meet at a common point, the difference between two adjacent angles has to be equal to the difference between the remaining two angles.
- d. (the Kawasaki theorem) The angles $a_1, a_2, a_3, \dots, a_{2n}$, surrounding a single vertex in a flat origami crease pattern must satisfy the following requirement:
 $a_1 + a_3 + a_5 + \dots + a_{2n-1} = 180$ degrees
 and
 $a_2 + a_4 + a_6 + \dots + a_{2n} = 180$ degrees

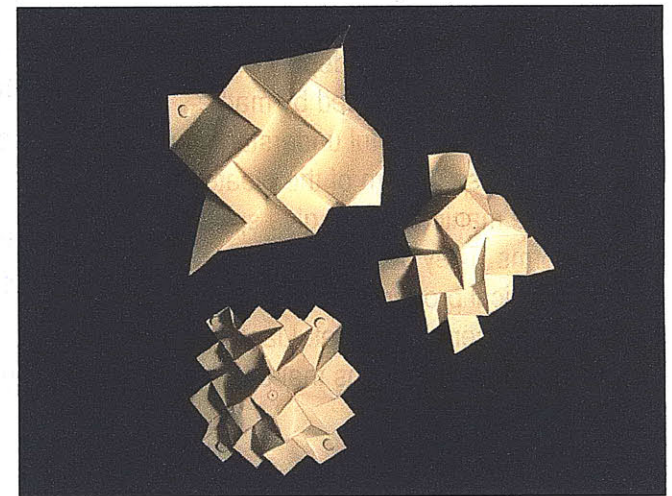
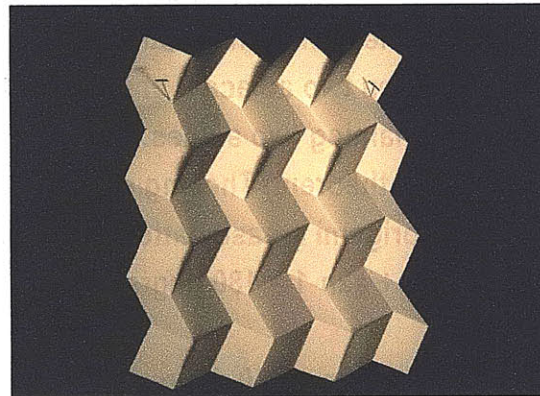
One such open-surface form that was of particular interest was the Mars fold, patented by Taborda Barreto. The Mars fold is composed of squares and rhombuses, lying adjacent to the squares. What was so fascinating about the Mars fold was how its simple crease pattern could yield such a variety of forms depending on the direction of the folds.

Yet, the problem with these forms was that they often required large amounts of bending and twisting both to develop the form and to keep it in its folded state. If the structure were to be constructed on a larger scale, a material other than paper would not be as forgiving.

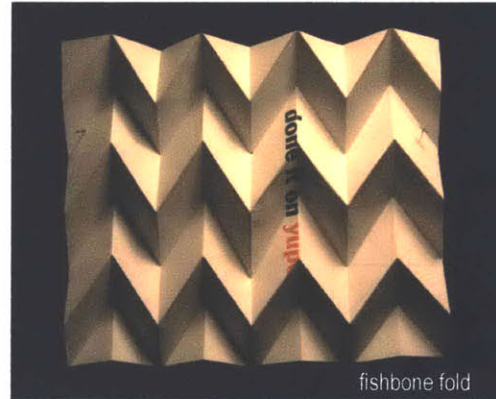
While the Mars fold and other open-surface forms could be shaped without any distortion within the surface it was also vital that one could give the pleated surface a degree of curvature.



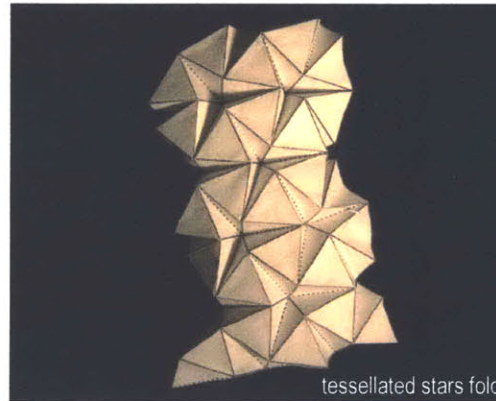
The red and blue lines represent mountain folds and the green, valley folds. Variations in the Mars fold are based on two possible positions for the rhombuses. While all patterns fold flat, the only restriction is that the lines forming each obtuse angle of the rhombs be folded in the same direction.



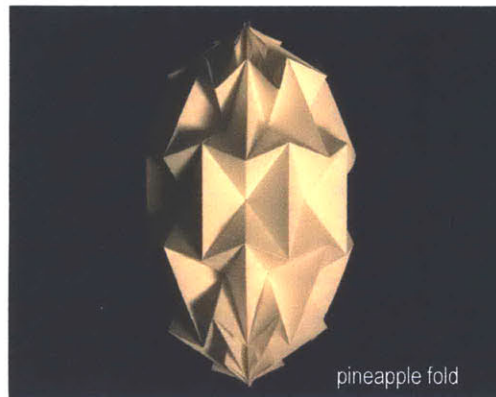
This second characteristic (which can not be found in the fishbone or tessellated stars fold) was important in being able to contain space and offer a sense of enclosure. This curvature could be implemented into the pineapple fold, however, it would not translate into an efficient structure as there is a significant amount of overlapping between the panels. Ultimately, I returned to the Kao-fold for its versatility, beauty, but most importantly, its inherent structural value. Among other open-surface forms, it has a clearly identifiable load path and a considerable depth to its section. It reminds one of a truss, whereby the greater the depth of the section, the stronger it becomes and the greater the distance it may span. However, as the form is not composed of linear elements, differing from the truss, the panels offer coverage and allow one to imagine a variety of uses.



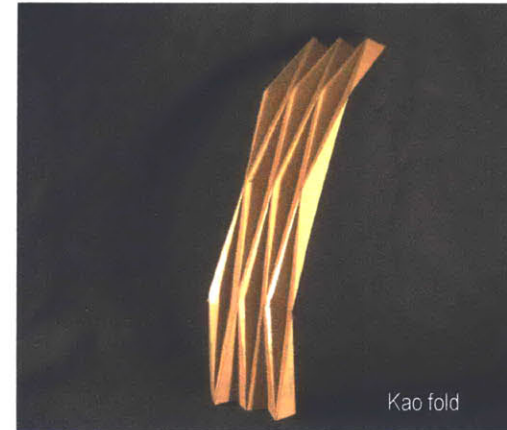
fishbone fold



tessellated stars fold



pineapple fold



Kao fold

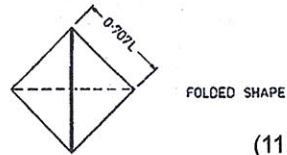


Kao fold - variations

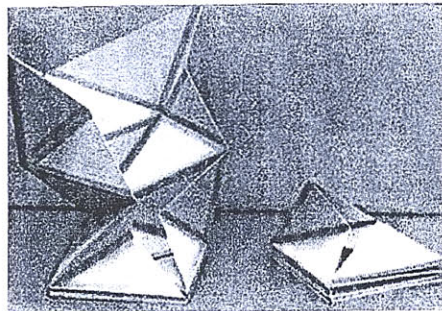
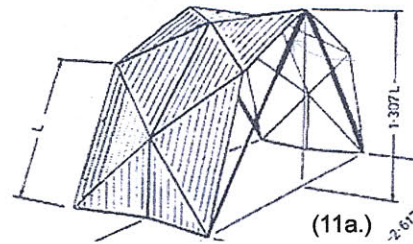
addressing materials: a preliminary structural analysis

After playing with different configurations of the Kao fold in paper, I was eager to work with materials besides paper that would begin to suggest the difficulties of working on a larger scale. Different material/assembly ideas were tried with the goal of keeping the paper model's ability to expand and collapse freely. In looking at different material possibilities, it soon became apparent that paper could not be substituted with a single sheet material but that it would be necessary to make a composite where the material along the crease lines of the paper model would be flexible differing from the rest of the form that could be rigid. For this flexible material along the fold-lines, drywall and vinyl tape were tested. In the panels, chipboard, acrylic, and thin plywood were tried. The elastic vinyl tape was used for, in order to fold the structure, the material would need to be stretched at the nodes. As can be seen in the chipboard and drywall tape model, another solution was to simply extract material at the nodes thereby disconnecting one seam from the next.

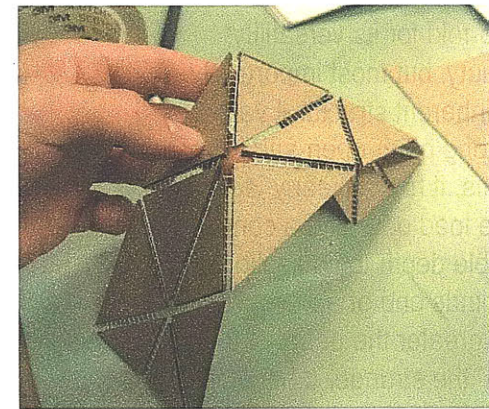
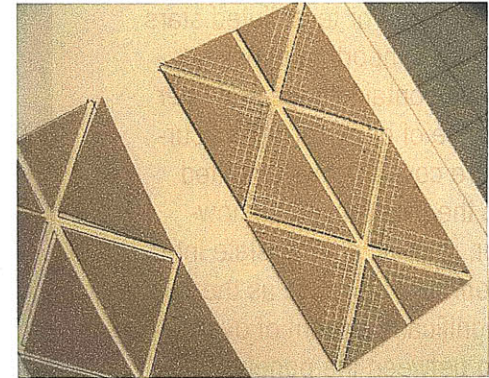
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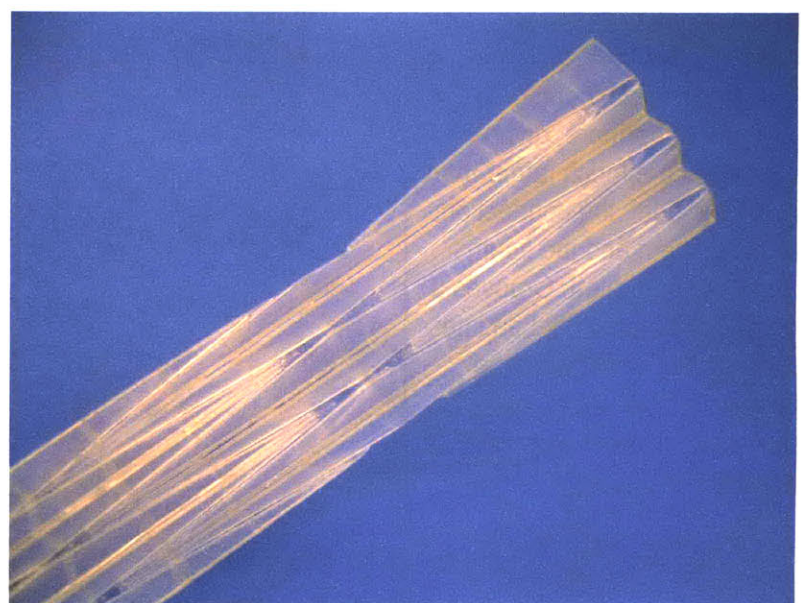
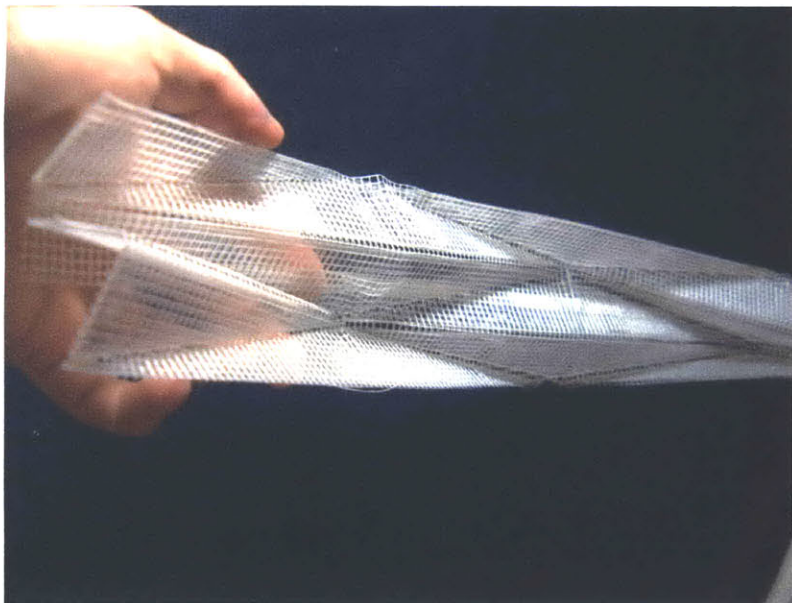
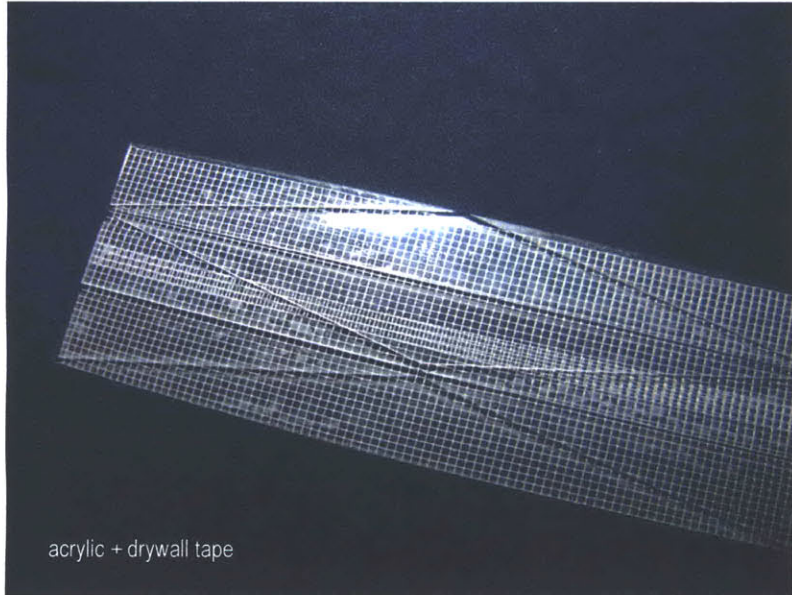
(11.)



(11b.)



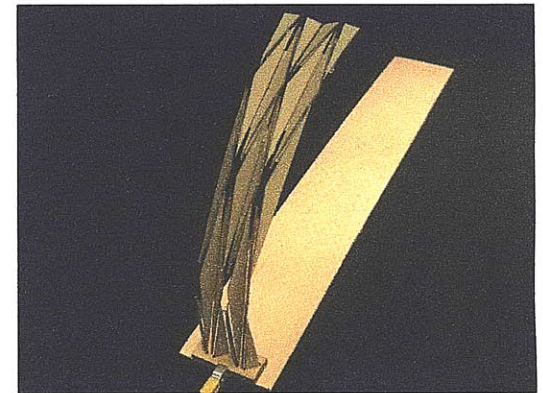
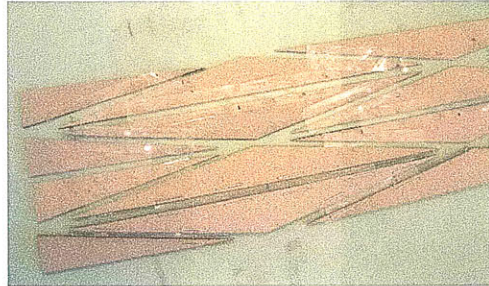
The chip model formed by two 90 degree modules joined at their ends was part of a study by C.G. Foster with S. Krishnakumar in 1986 of a family of foldable, portable structures based on triangular panels.



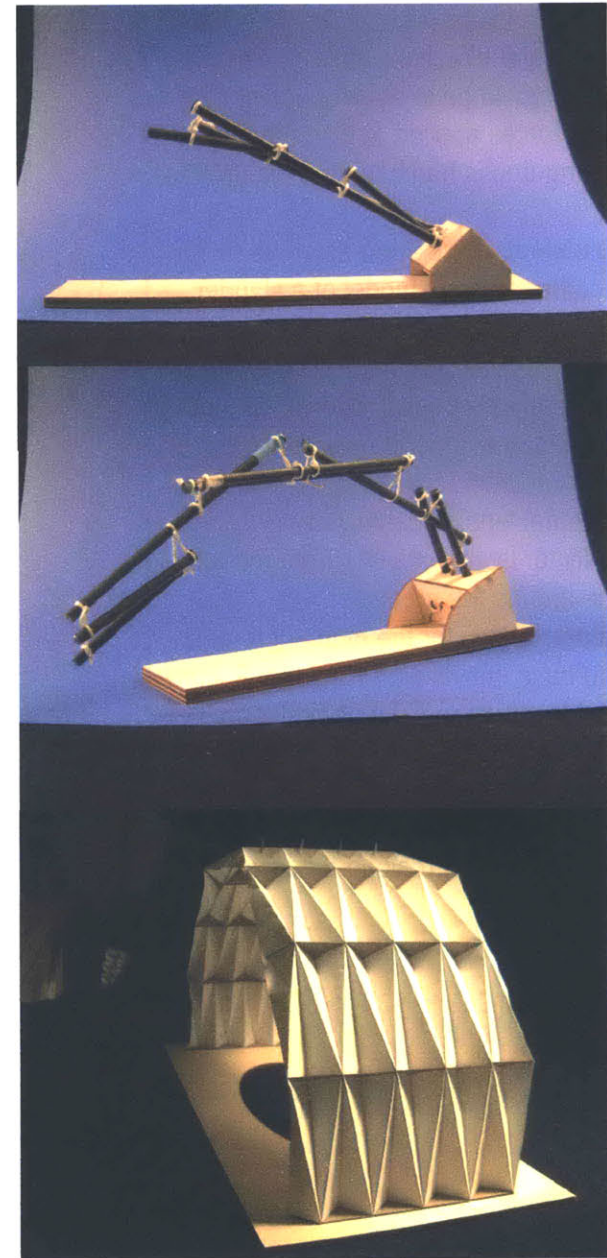
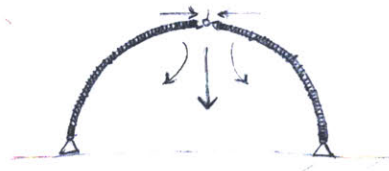
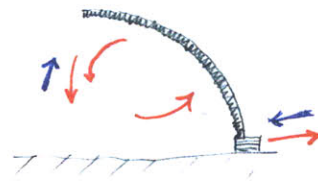
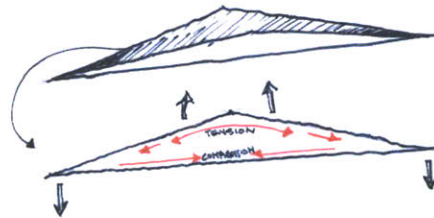
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I assigned the Kao fold the form of a half-arch as a starting point to understand how the stresses from its own dead weight are carried through the structure to the ground. The making of a half-arch chip model using zip-lock bags (polyethylene) along the seams was particularly instructive in being able to see how the stresses caused by the materials own dead weight were carried through from one panel to the next, that is, along the seams.

A heavier plywood model was made to increase the stresses on the seams and thereby clarify exactly how the seams were resisting the dead load of the panels. Furthermore, given that the panels are much larger than the chip model, the connections between them and the seams could be elaborated. Grooves were routed out so that an interstitial strip of wood could be sandwiched between two layers and hold the sheet material along the entire edge.



The transfer of weight from the top of the arch to the base could be abstracted using short steel rods and rope. From this abstraction, it becomes clearer as to how the internal stresses inside the panels are operating, that is, in compression along the fold and in tension on the other two sides. (see sketch) The steel models, moreover, are instructive in identifying the Kao-fold's weaknesses as a structure. In the shape of a half-arch, a tremendous moment force exists at the base. Furthermore, one can observe that, laterally, it is very unstable (as it would sway quite freely with a slight force to the side!) A first reaction to increase lateral stability would be to complete the arch, thereby transmitting the gravitational forces from one end to the other. The model was constructed using two half-arches connected by a pin joint at the ends.

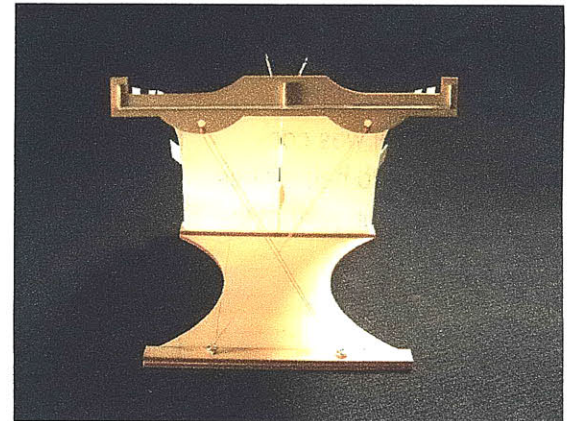
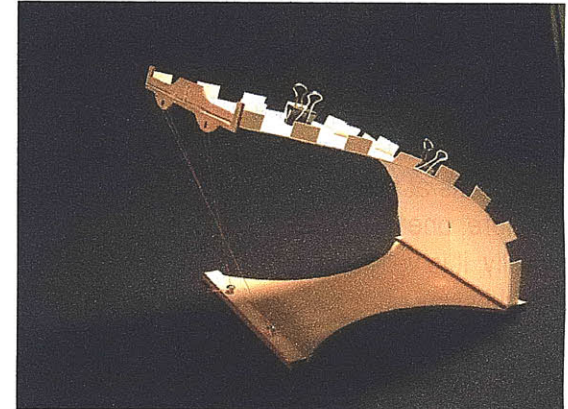
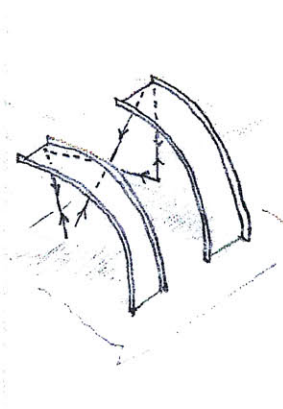
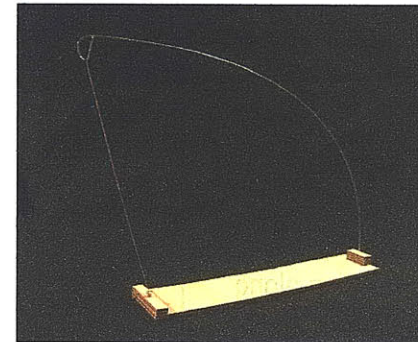


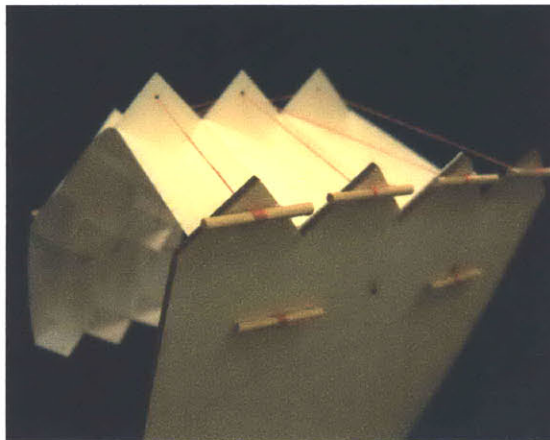
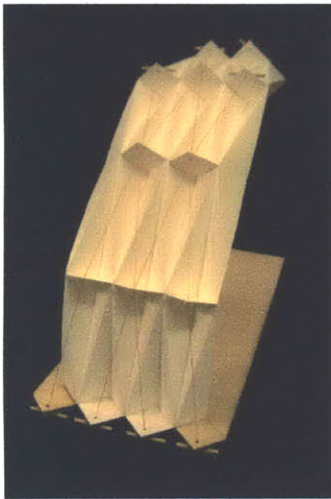
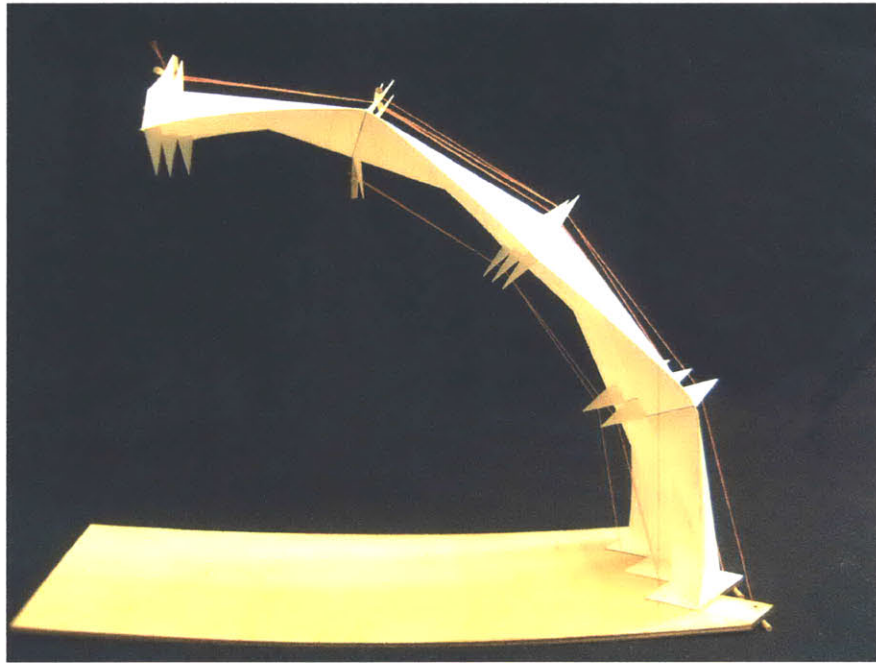
One means to stabilize the structure in the lateral direction with external supports would be to have opposing tension forces applied to it, in the way a young tree is harnessed to the ground.

In observing the internal forces of another abstract model of a slender steel rod held in tension by a string, another consideration was to use embedded pre-tensioned cables which would force the structure to erect itself while counteracting that force with external tension cables running diagonally for lateral stability, from the top of the arch to the ground.

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In accounting for the moment force at the base, however, it would be a more stable structure if the embedded tension cables were brought on the outside, again running diagonally, and, for sake of elegance, the counteracting cables be placed along the inside surface.



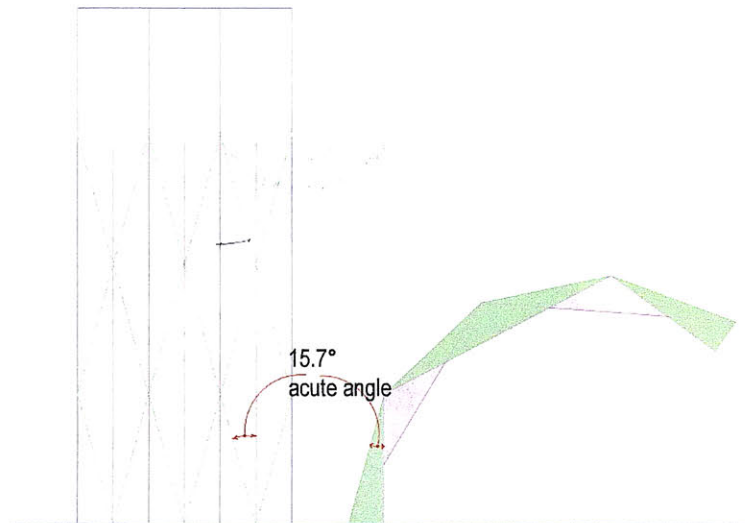
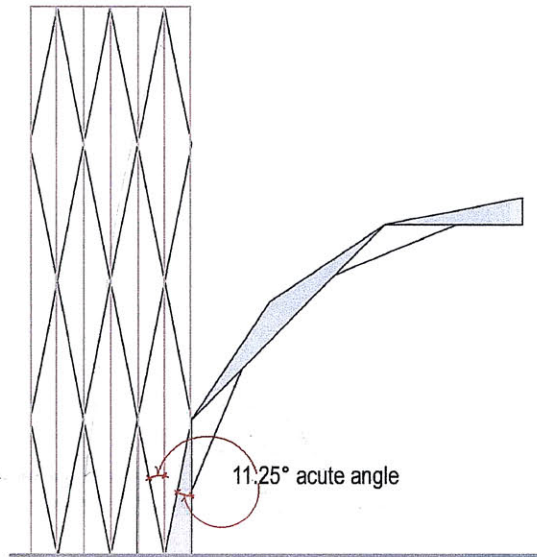


A Mathematical Investigation: comprehending the transformation by trigonometry

20

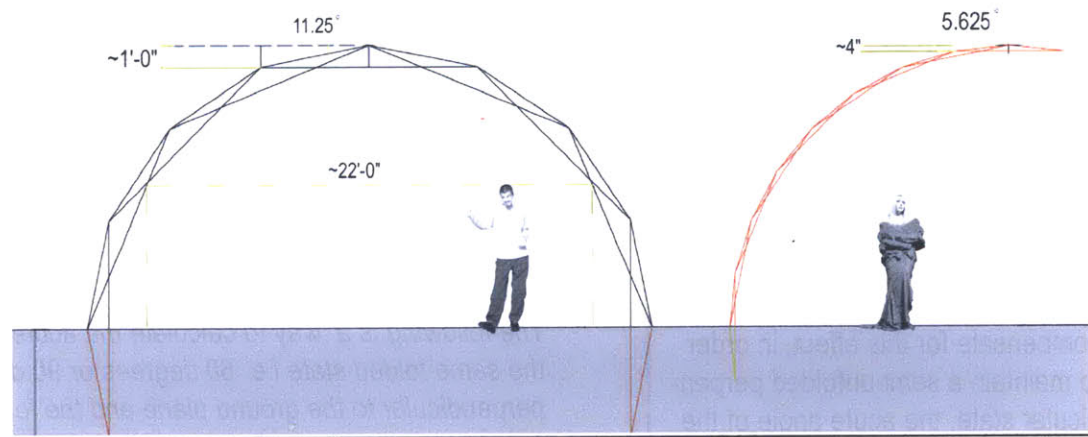
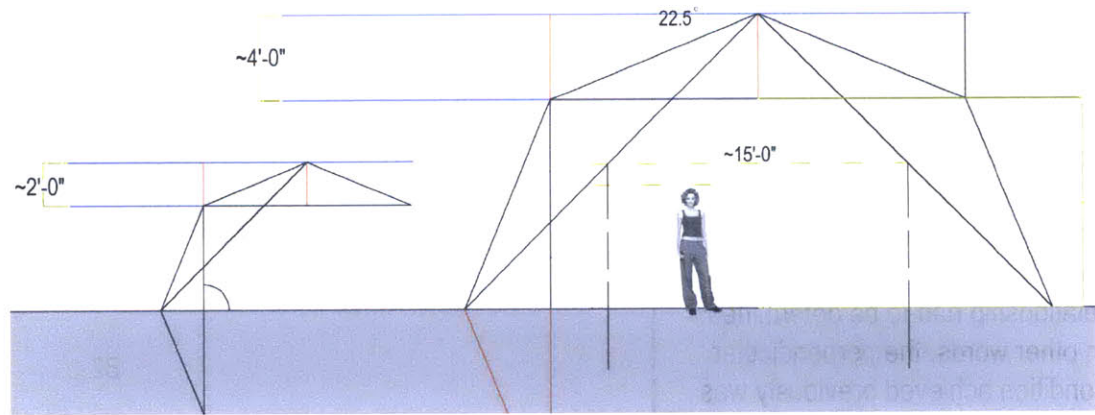
Simultaneous to thinking about materials and structural stability, I wanted to come to terms with the mathematical principles behind the transformation of the Kao fold. That is, to determine the relationship between the geometry of the crease pattern and its corresponding folded form.

After playing with paper models it became apparent that the size of the acute angle in the crease pattern controlled the degree of curvature in the pleated arch. The smaller the acute angle, the "tighter" the curl of arch would be.



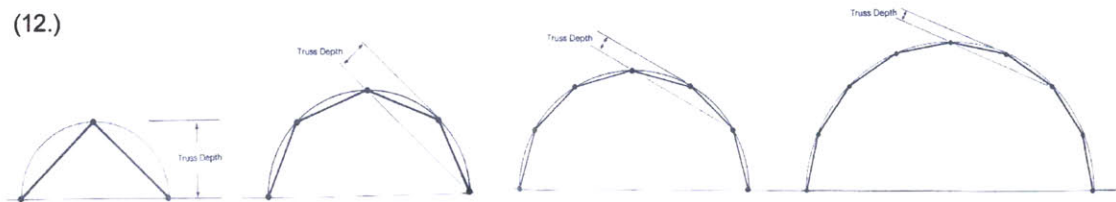
Furthermore, in looking at the section, it may be deduced that in order for a half-arch to stand perpendicular to the ground plane (In this state, the bottom segment of the arch is perpendicular to the ground plane and the top segment parallel to it. Also the vertices of the half-arch are tangent to a circle), the acute angles would need to add up to 90 degrees.

As the image shows, the smaller the acute angle is, the greater are the number of folds. As a completed arch, structurally, the Kao-fold behaves much like a truss in a half-circle formation. (see image 12) Just as “increasing the modular incrementation of the truss decreases the ability of the vertices to resist concentrated loads”, the more faceted the surface of the Kao-fold, it loses depth compromising its strength. Pearce, p.27 Thus based on an intuitive assessment of the stability of the structure in accordance with its depth, it was determined that the 11.25-degree model would be developed.

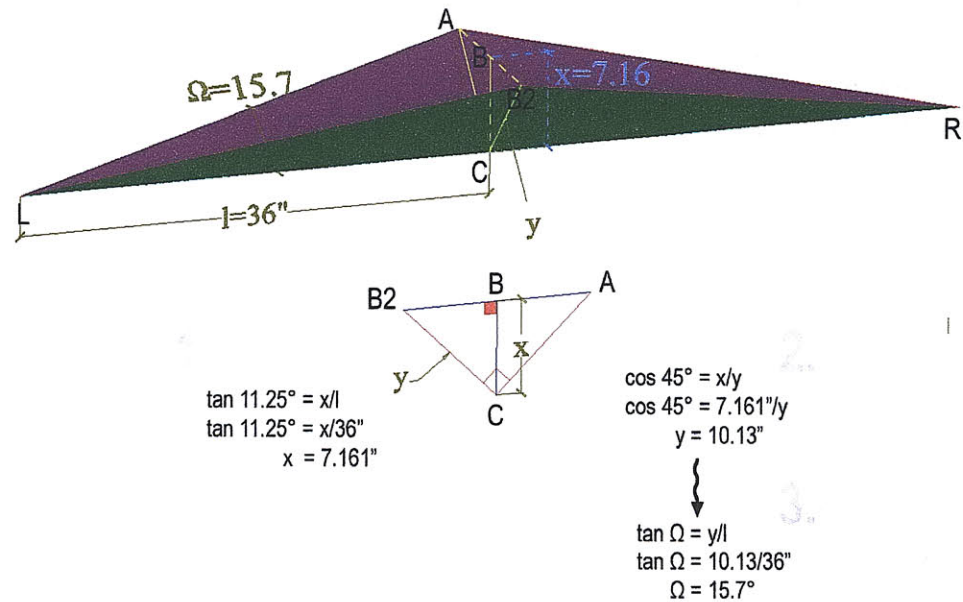


*The size of the arched structure is dependent on the length of the seams. Each arch shown in the image was sized to house 2-3 people.

(12.)



It is important to note that the study only assesses the impact of the acute angles on the contained space seen in section. That is, there is no consideration for the impact of the dihedral angle. As the dihedral angle would affect the curvature of the arch as well, its mathematical relationship had to be determined. In other words, the perpendicular condition achieved previously was based on the assumption that the form was folded flat and, as such, should the arch be deployed in a semi-unfolded state, the effect of the dihedral angle would need to be accounted for. By expanding and collapsing a paper model, we learn that as the arch folds flat, it acquires a tighter curvature and vice-versa. Intuitively, one may presume that to compensate for this affect, in order to maintain a semi-unfolded perpendicular state, the acute angle of the crease pattern must be increased from 11.25 degrees to some greater number. Through some basic trigonometry we find that, should the dihedral angle be set at 90 degrees, this new value for the acute angle is 15.7 degrees. (see page 20)

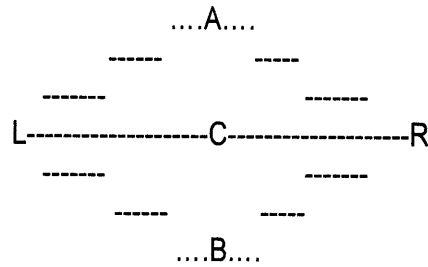


The following is a way to calculate the acute angle, given that all units in the folded piece are in the same folded state i.e. 60 degrees or 90 degrees, so that the bottom segment of the arch is perpendicular to the ground plane and the top segment parallel to it.

It has been previously deduced that the acute angle must be a factor of 90 (22.5, 11.25, etc.) to allow for this condition. However, these calculations depend on the arch being in a fully folded/closed state.

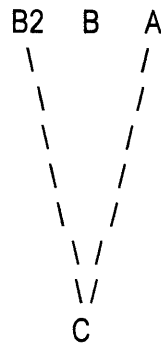
The less folded the arch, the greater its tendency to be "less arch like" and become a straight, flat form. In other words, the arch "rises", losing its curvature.

Therefore, in order to compensate for this effect, the acute angle would need to be slightly greater than say...22.5 Through some basic trigonometry the problem may be solved allowing for the design of arches with various curvatures while controlling the degree of folding.



For example, let's say, in the above folded pair of triangles, the acute angle RLB is 11.25 degrees when the triangles are completely folded and the length of the folded edge is determined. In its closed state, we draw a line from point B to the midpoint of LR or C which gives us a right triangle LCB .

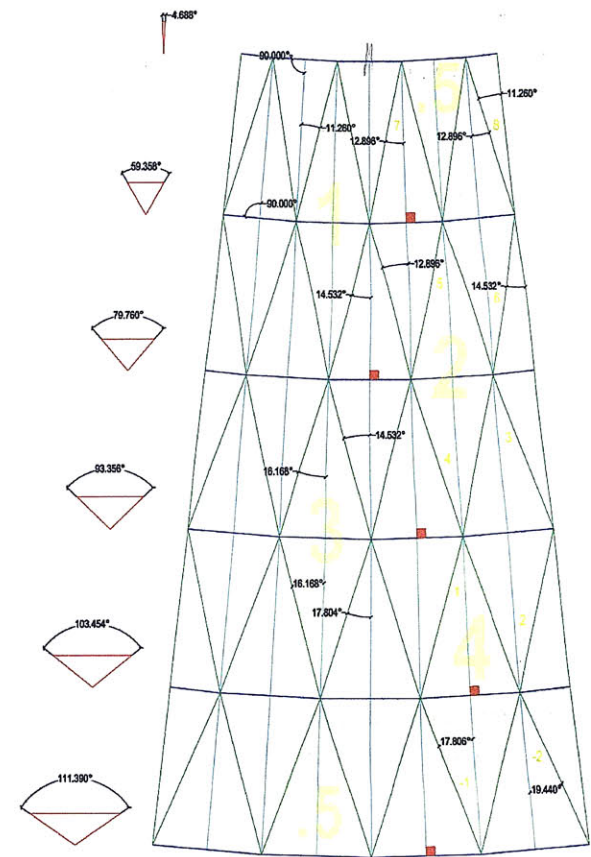
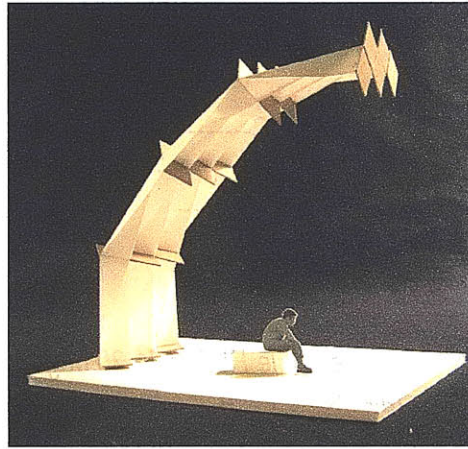
Next we find the length of BC . (In its closed state, we can draw a line through B and C which is perpendicular to LR .) Then we open the fold by a determined angle, say 60 degrees. The diagram below shows the fold in section.



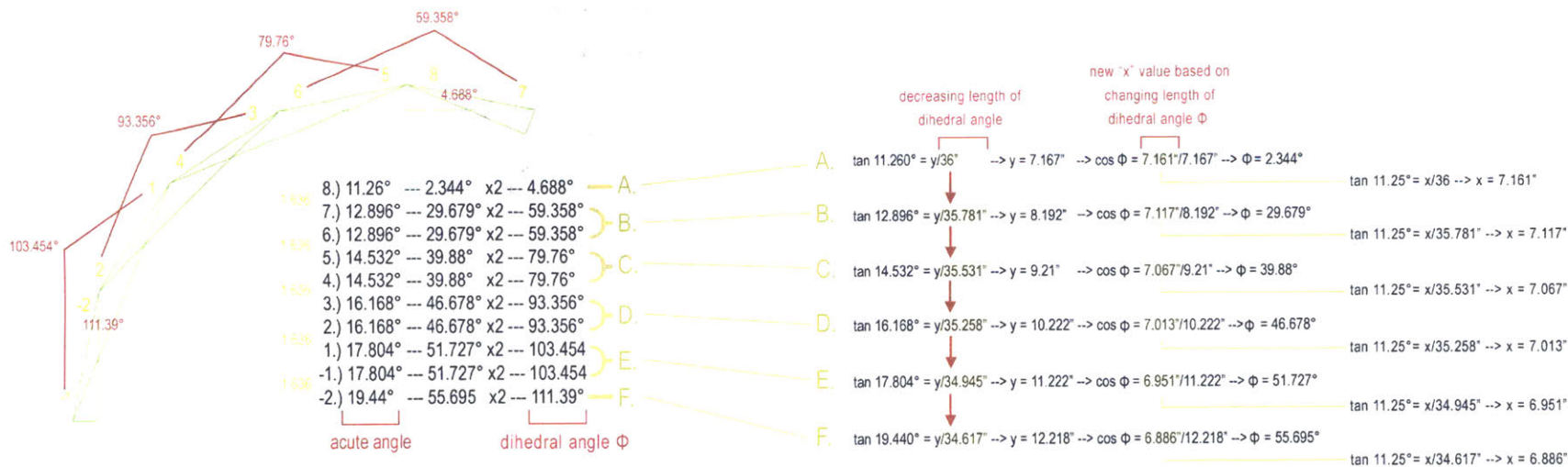
Using the value of BC , we can calculate $B2C$ the "new" length of BC once the fold is opened 60 degrees. Then we work backwards and calculate the acute angle given our "new" length of BC or $B2C$.

The illustration on p.20 shows the two corresponding crease patterns with the acute angles of 11.25 and 15.7 degrees as well as their completely folded state. When completely folded we can see the impact of the acute angle. That is, the higher the value, the tighter the curvature, and thus, if we were to unfold and expand the arch by setting the dihedral angle at 90, the arch opens up and rises to the aforementioned perpendicular condition. This condition was modeled by inserting square pieces of chip into the center of every folded unit demonstrating the logic and clarity of mathematical thinking.

To make things more interesting, I collapsed one end of the half-arch configuration. The result is a form that resembles an orange peel as the first curvature along the length of the arch is supplemented by a second curvature along the arch's width. In this model, essentially, what is happening is that the dihedral angles are diminishing from one end of the arch to the other. Using the same thought process and trigonometric calculations as before, one may determine the value for each dihedral angle in a semi-unfolded perpendicular state.

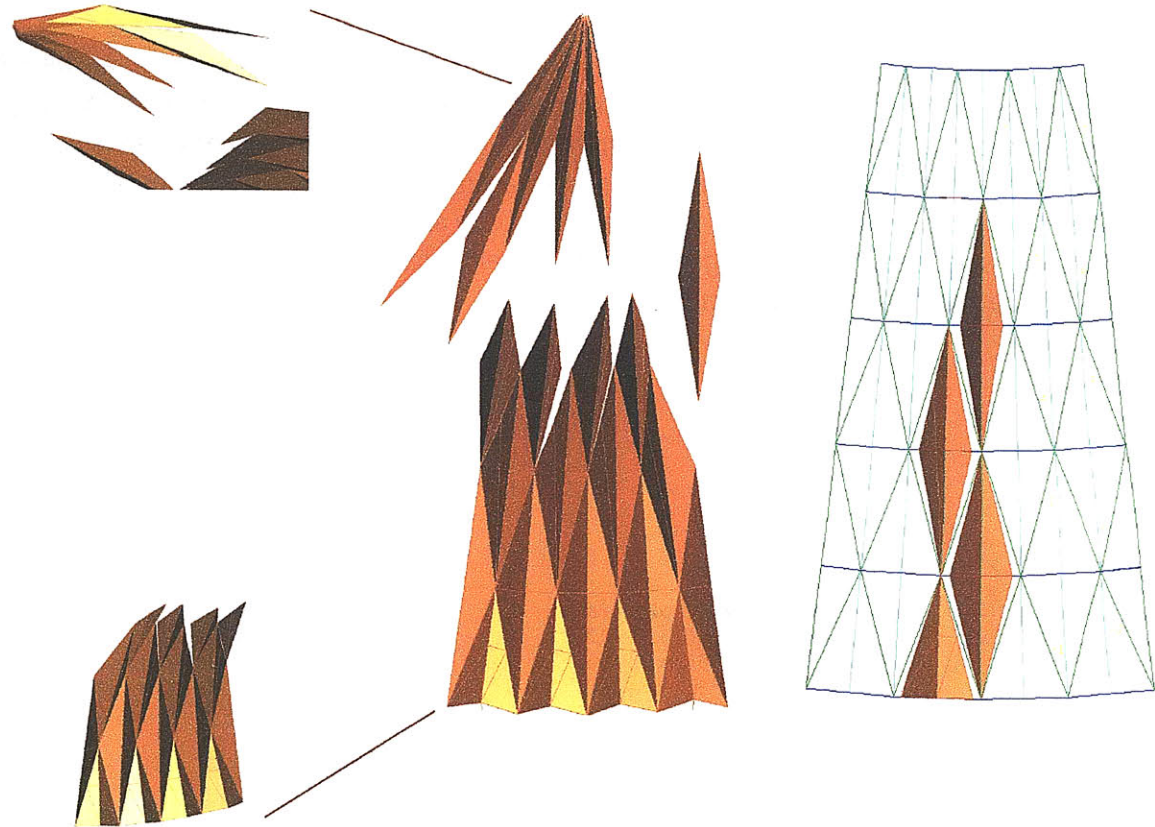


The 11.25-degree, perpendicular condition dictates that in a half arch, there will be three full folded panels and two, half folded panels. In the image for the crease pattern, the arch extends an additional 25%, giving four full folded panels. Once the smallest dihedral angle is determined, the difference between its corresponding acute angle and the next pair is established. (If one wants to work with the 11.25-degree, perpendicular condition, the smallest acute angle cannot be less than 11.25 degrees, as the planes can not close in on themselves!) In this design, I chose 1.636 degrees. Once all the acute angles are determined, their corresponding dihedral angle is calculated using the same, right-angle principles as before but going backwards. That is why it is important that the acute angles are paired, that is, to be able to use the same method as before.



While this investigation allowed me to establish the relationship between the 2-dimensional crease pattern and its corresponding folded form through mathematical reasoning, the precision required was limiting and far removed from the freedom of working in paper models.

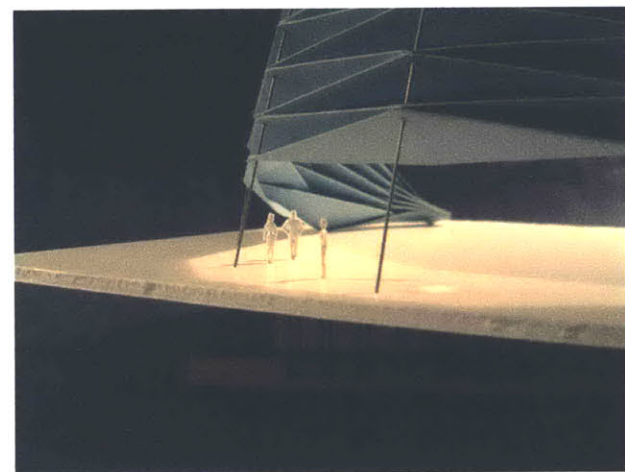
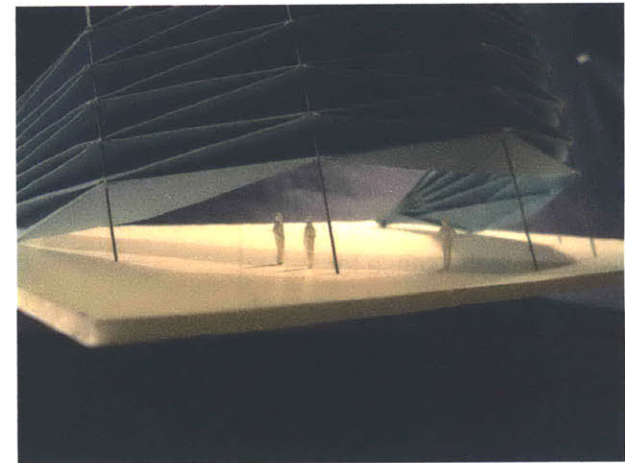
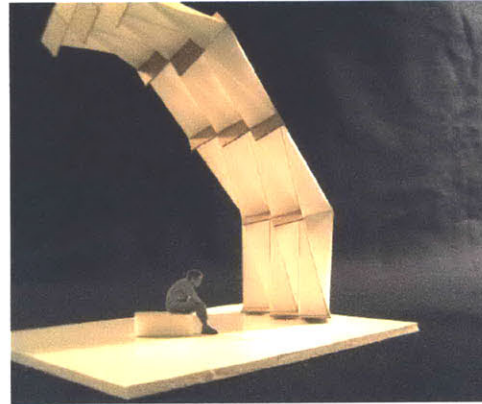
26



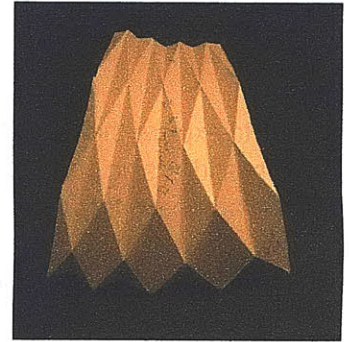
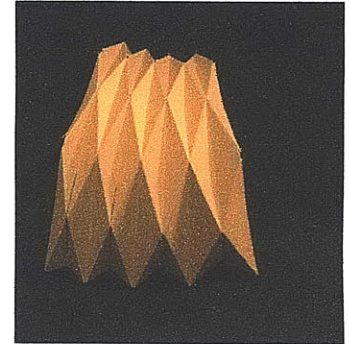
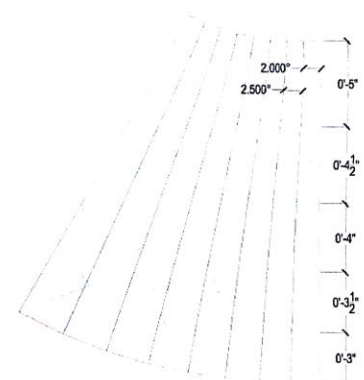
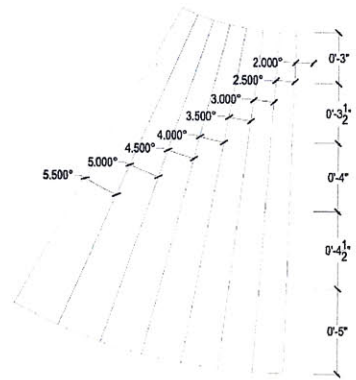
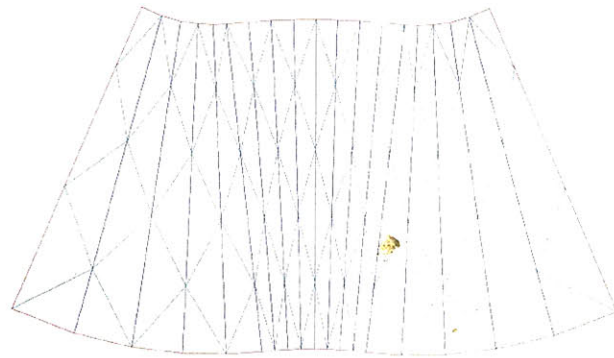
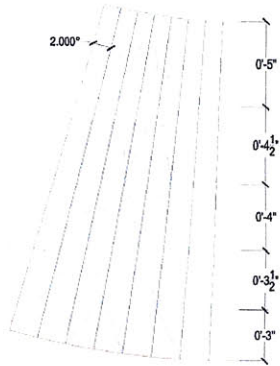
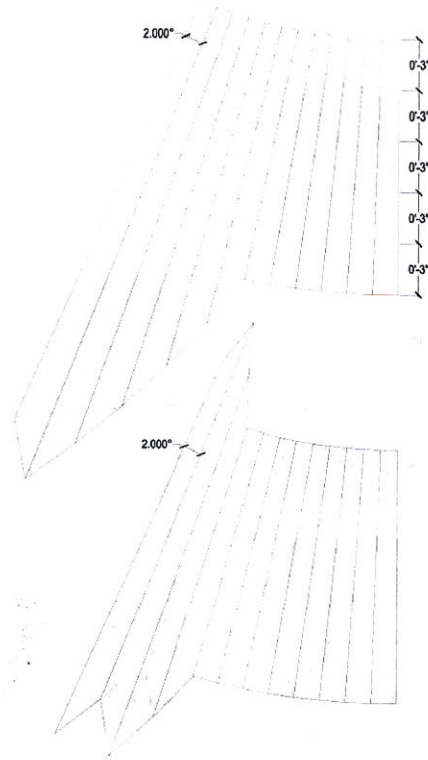
I had attempted to apply the math to the making of a 3d, digital model. Unfortunately, while in theory, the mathematical reasoning is sound, due to rounding-off the values within the calculations, we see a tight fit between panels at the base and as we work our way towards the top of the arch, the marginal errors become accumulate and are progressively transparent.

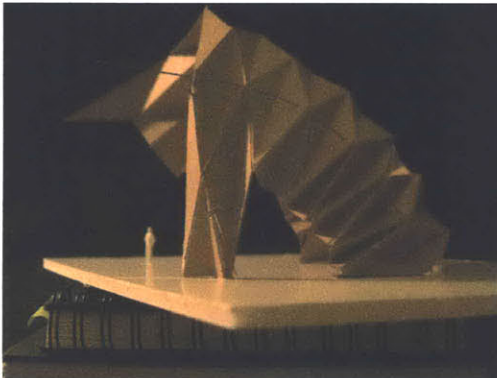
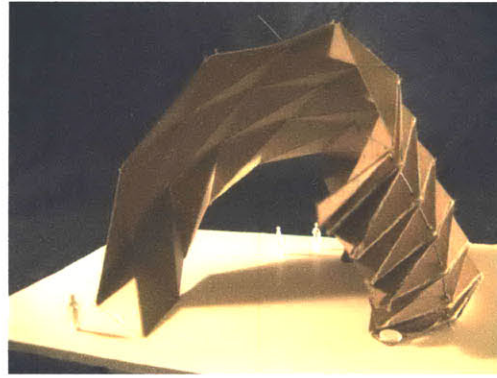
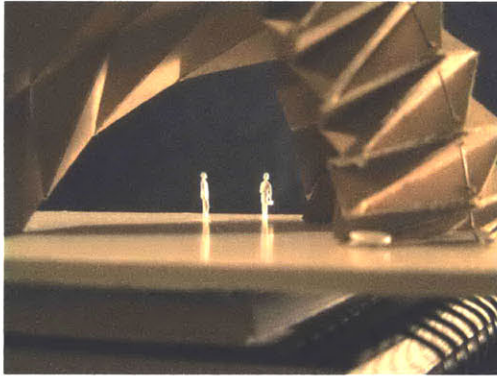
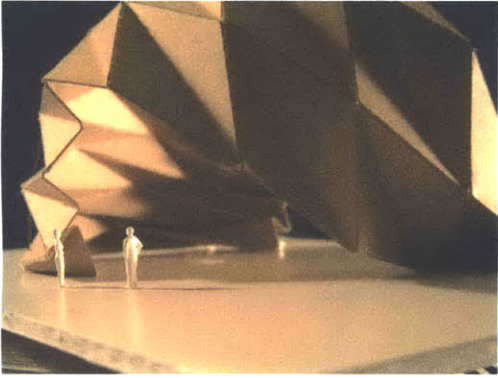
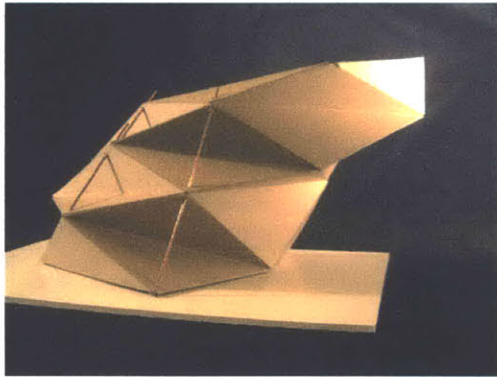
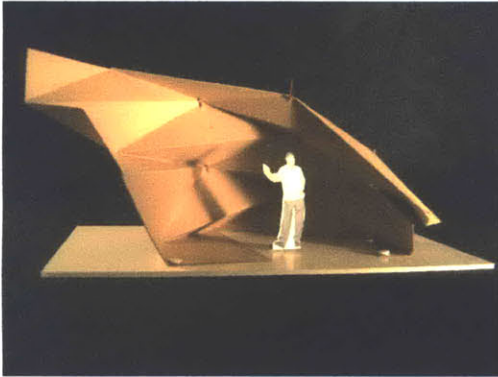
paper models: a physical manipulation of space & form

In order to explore the habitational qualities, addressing scale and potential applications, I returned to the freedom found in working with paper models. In exploring forms, I looked first for design ideas using the basic crease pattern of the Kao-fold. The first gesture to suggest occupation would be to simply insert a scale figure. And so I took the half-arch and orange-peel configurations and modeled them as such. Another idea would be to merge the qualities of the full-arch configuration and the orange-peel and collapse the full arch at the ends and link the vertices with linear members.

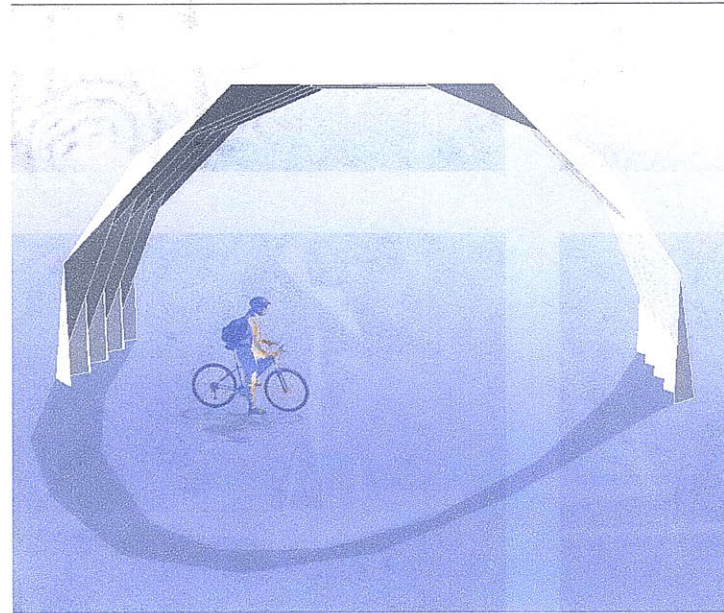


Subsequently, to go further, I identified specific characteristics of the 2-dimensional crease pattern, namely that of the axis of the dihedral angle and length of unit. (By axis, what is referred to are the lines along which every folded unit lies, collinear with the vertices of the dihedral angles.) The following diagrams show variations of these two characteristics and, for some, their corresponding folded forms. The use of 1/8" scale figures immediately gave the sense for the habitational/spatial quality of these forms. (see opposite page)



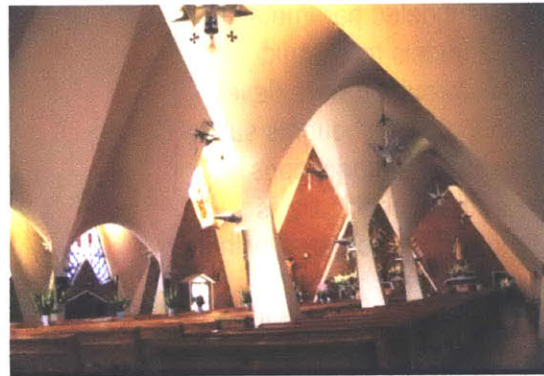


To address a certain scale of construction and different applications for the Kao-fold, collages were made as well. The half-arch construction could be used for booths in a carnival or craft fair, the full arch could be a monumental entry-way. If the Kao-fold was constructed out of steel, it could serve a more permanent function much like Calder's out-door sculptures, or if cast-in-place concrete, we may think of Felix Candela's entryway to the Pabellon Music School, or the interior of his church in Migrosa

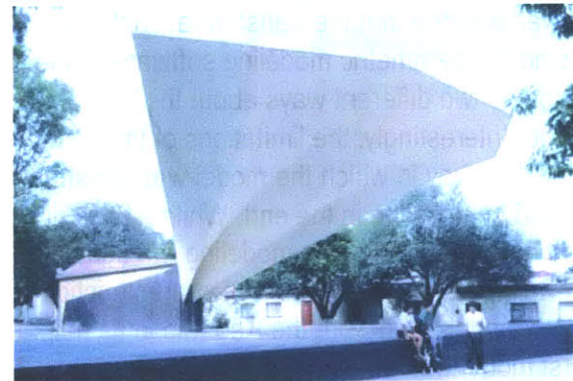




(13)



(14)



(14a)

parametric modeling: digital manipulation through defining parameters

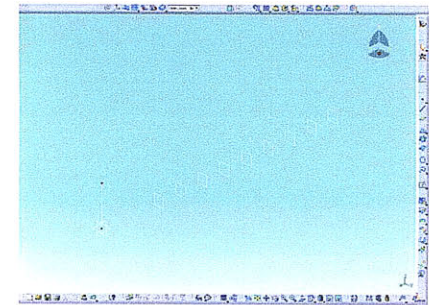
Parallel to my exploration of space and form by using the hands to manipulate paper models, a different quality of freedom was found in the digital manipulation of form. While playing in paper, a certain insight on the transformational qualities was gained, that is, to understand exactly how the individual folded units move in relation to each other as they contract and expand. As such, it must be stressed that both investigations took place at the same time. The relationship was a two-way road as the thinking applied to defining the Kao-fold in the virtual space of the computer also helped to identify ways in which to develop the various physical models.

32

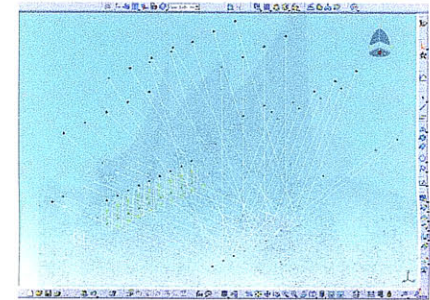
It was decided that the transformational qualities would be best modeled parametrically using the parametric modeling software, Catia. Two parametric models that would address two different ways about thinking of the Kao-fold's transformation were developed. Interestingly, the limitations of the software forced me to focus my thoughts such that the way in which the model was constructed would limit what transformations would be possible in the end. While a limit, one may argue that this, specifically, is the power of parametric modeling - to construct a digital 3d model in accordance with certain parameters or inherent characteristics of that form.

Both models work with the Kao-fold in its half-arch configuration. The drawing of the first model was based on assigning an axis to the dihedral angles and linking them to offset planes as well as to rotating planes. While the dimensions of the folded form would be distorted, the intention of the model would be to play with the angle of the dihedral angle axis and the distance between them to give morphings of the original geometry.

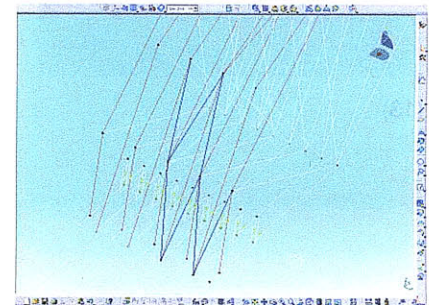
Model was drawn referring to 2-d sketches to give the 3-d coordinates of the corner points, which subsequently allows one to link them giving the seams of the folds. The following story board, illustrates the model's construction:



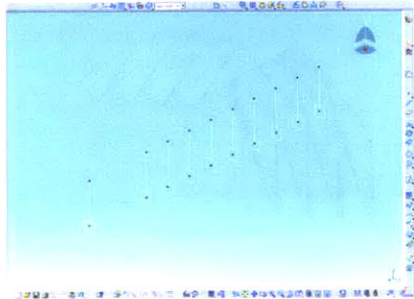
1. Parallel planes offset from each other were constructed.



5. Here we see all sketches in 3d space.



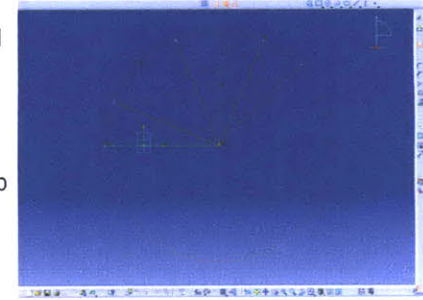
9. All nodes or extremities of the two-dimensional sketch are connected to give the triangulation of the pleated form.



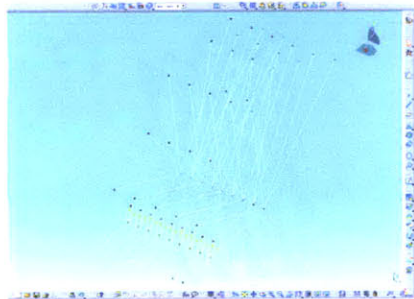
2. Extension lines were projected...



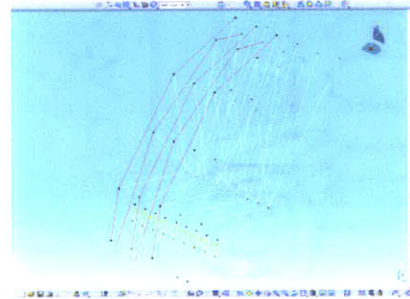
3. ...which would function as a rotational axis for the rotational planes.



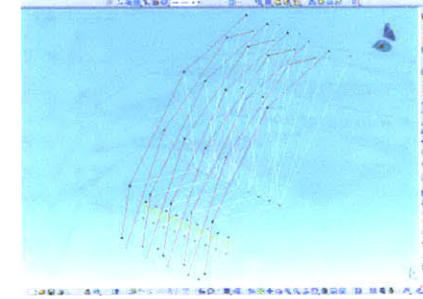
4. Each rotational plane (which is linked to the offset planes) is associated with a two-dimensional sketch. Catia operates between two workbenches - that of the sketch and the 3d model. In the sketch mode, points that would later give the 3-d coordinates for the vertices were drawn.



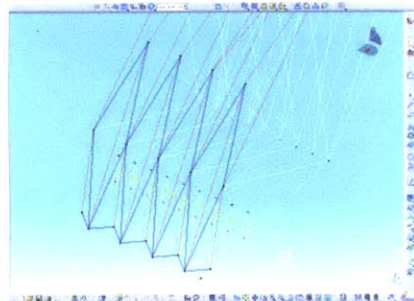
6.



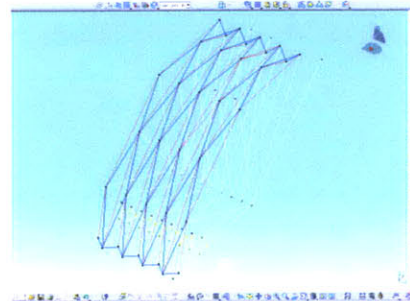
7. Every rotational plane represents an axis for each dihedral angle.



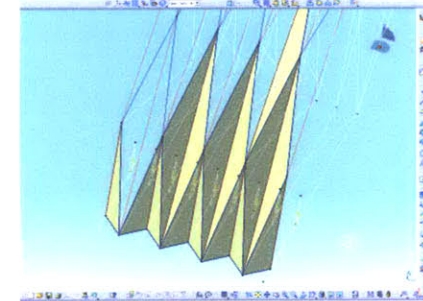
8.



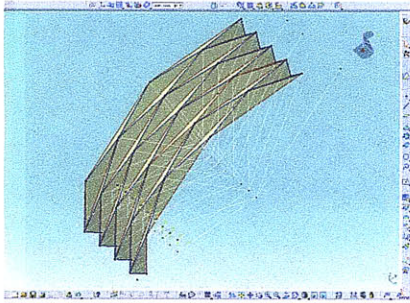
10.



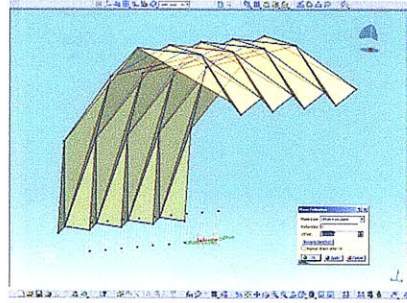
11,



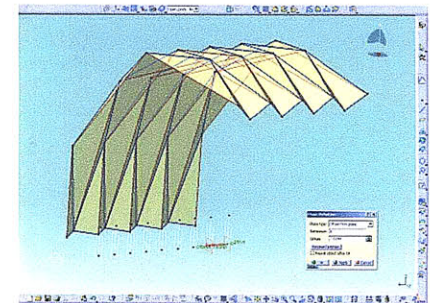
12. The outlines are filled to give panels of zero thickness.



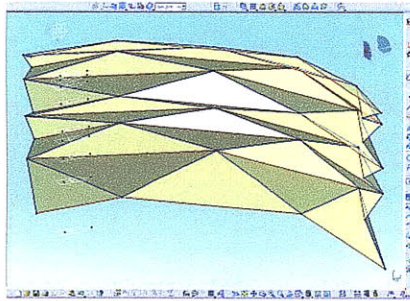
13.



14. The first parameter, the distance between the offset planes, may be adjusted incrementally to expand the width of the half arch.

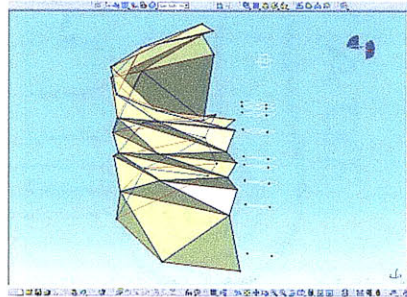


15.

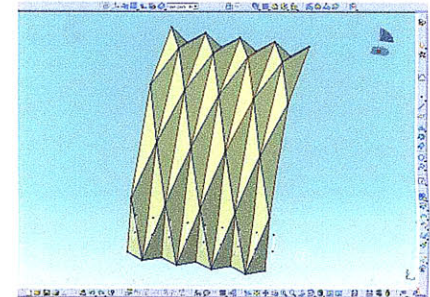


34

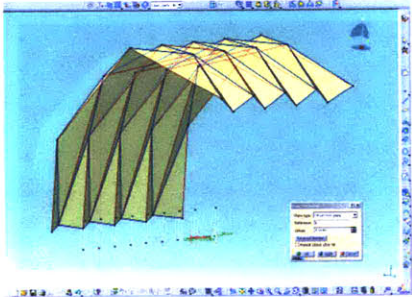
19. ...and the camera angle may be changed to give the impression that the structure is lying on its side.



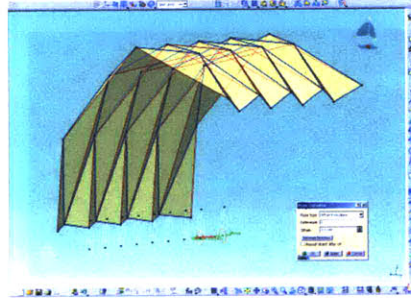
20.



21. Next, the second parameter, the position of the rotational planes may be adjusted.



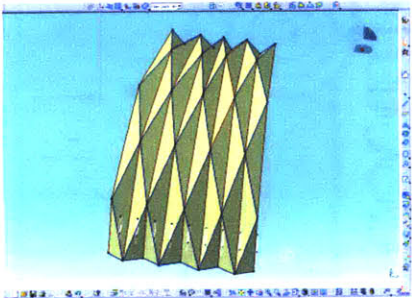
16.



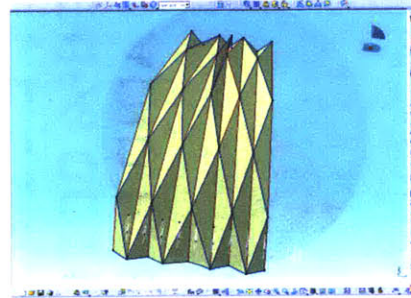
17.



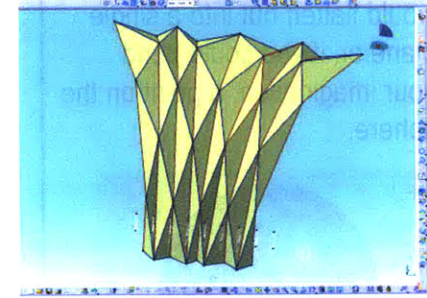
18. ...or the assigned offset values may be randomized...



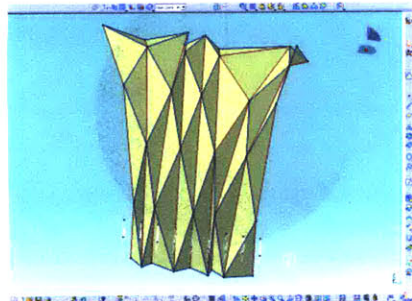
22. As with the offset planes, their assigned value may be controlled and incremental, fist closing in on one end...



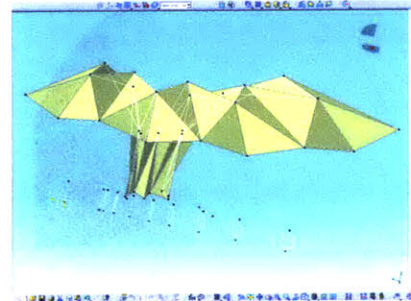
23.



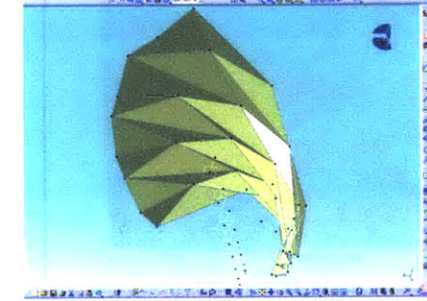
24. ...and then expanding



25. ...or random and chaotic

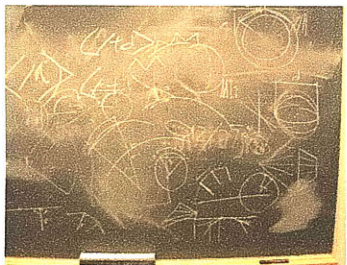
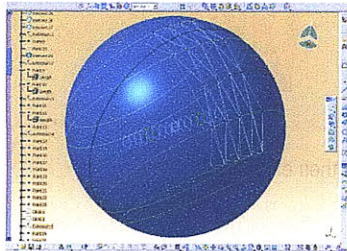


26. If we play with both parameters, more exciting forms can be given.

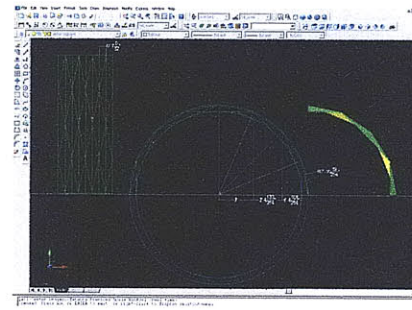


27.

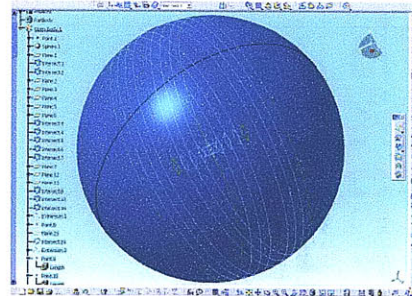
The second model was conceived based on the premise that the outer-most points of the form lay on the surface of a sphere. (see image below) The parametric underpinning of the model would be the function that as the diameter of the sphere increased, so did the dihedral angle of the folded form. And as the diameter approached infinity, the form would flatten out into a single plane or, if you could bend your imagination, a point on the sphere.



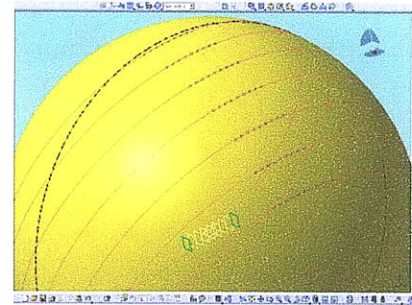
brainstorming with Alex on the chalkboard...



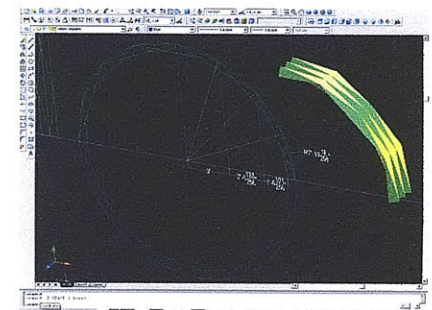
1. I started by using the dimensions from a determined 2-d crease pattern.



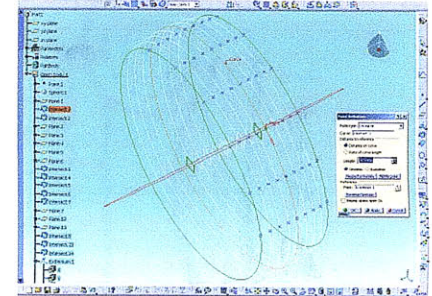
6. ...allowing me to get the intersections between the sphere and the planes.



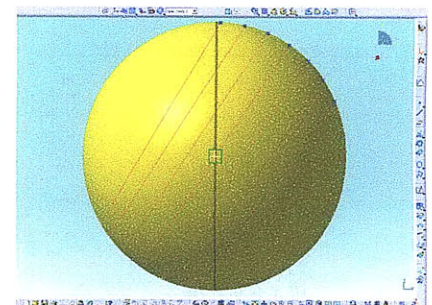
11. Along the x-axis or the width of the form, each "collection" of points lie on a circle...



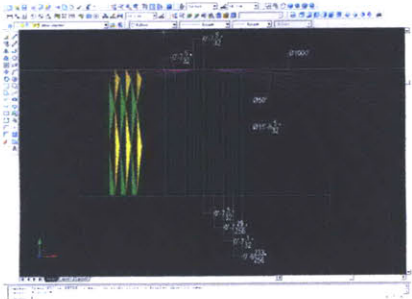
2. What was important about this particular pattern was that its outer "nodes" lying in the same plane also lie on a circle and that its inner nodes lie on a smaller, concentric circle in the same plane.



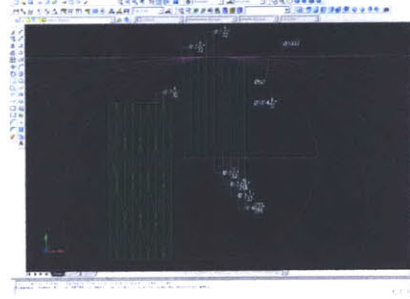
7. Point definitions along the length of the arch were drawn taking the assigned extreme as the origin on each circle.



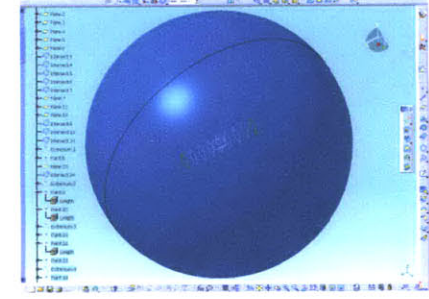
12. ...which taper around an axis running through the center of the sphere.



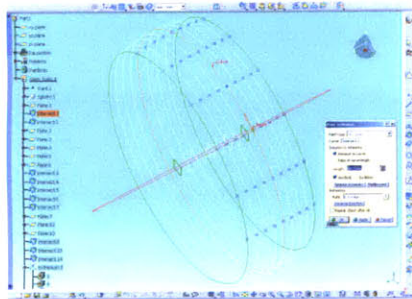
3. Before working in Catia, a new 2-d drawing to determine the distance between the planes on which each and every dihedral angle lies had to be developed.



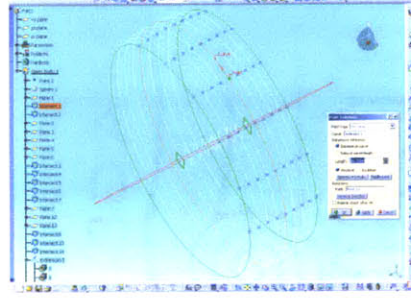
4.



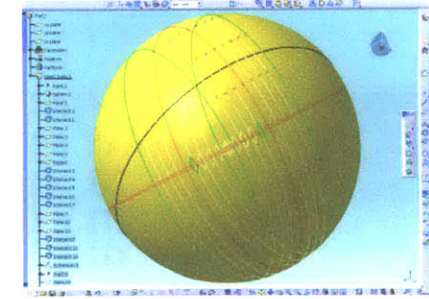
5. These planes were subsequently transferred to Catia...



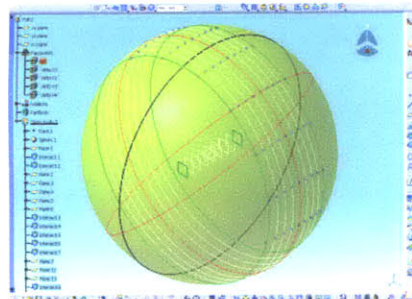
8. Each point would reference the location of the vertices within the crease pattern.



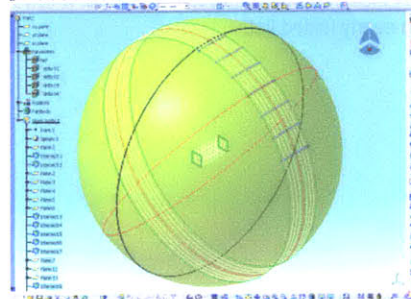
9. Points were defined on either side of the extremum...



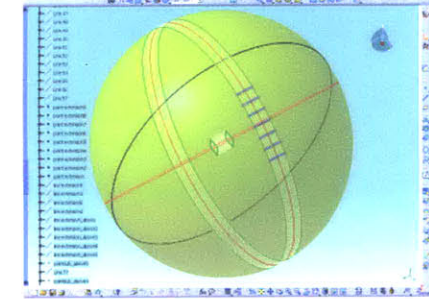
10. ...giving us a "patch" of points.



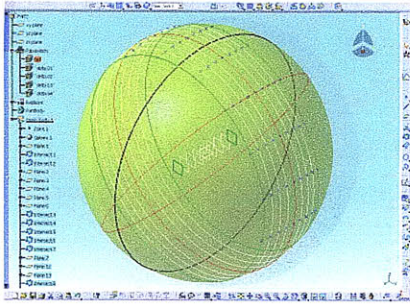
13. The distance between the points along the curve is identical for each pair of points and taken from the determined 2-d crease pattern.



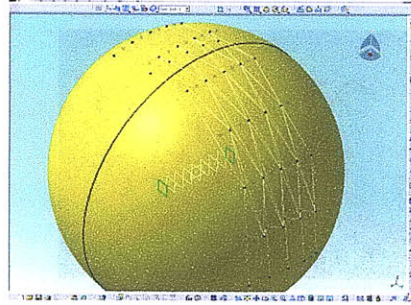
14. It was critical to fix the distance between the points so that the "patch" or area defined by the points on the sphere's surface would stay the same...



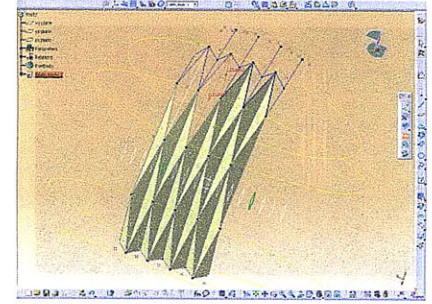
15. ...regardless of the diameter of the sphere.



16. Returning to the original radius...

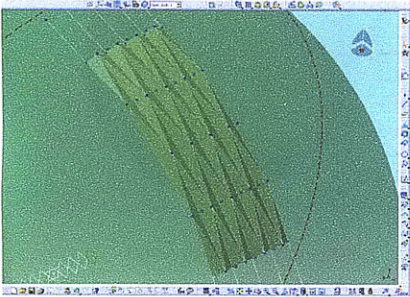


17. ...lines were drawn by connecting the points...

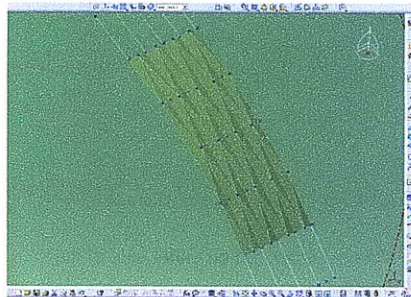


18. ...and filled to create panels.

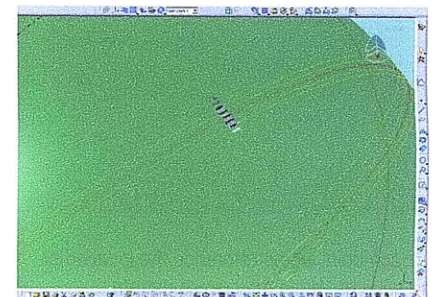
38



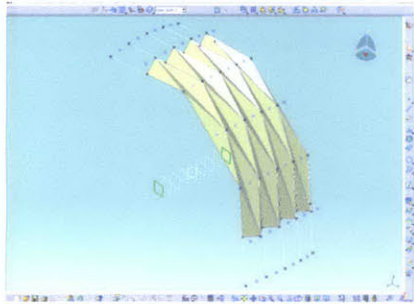
22. We see the arch expand and unfold.



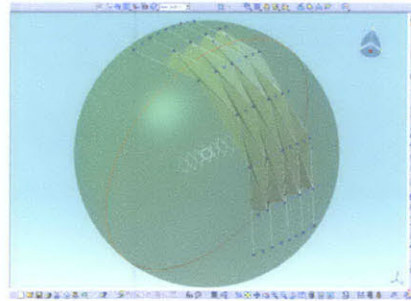
23. Finally, the radius is set to 1000ft, at which point the arch has nearly folded flat.



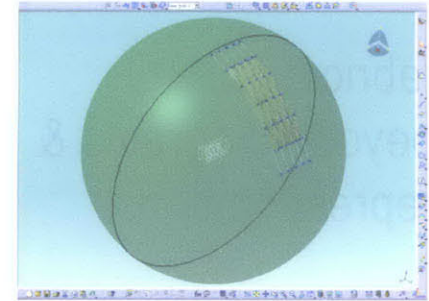
24.



19.



20. The final step would be to assign a parametric function to the radius of the sphere so that as it increased in size, the "patch" or folded form would flatten. The original radius is set at 7'-10".



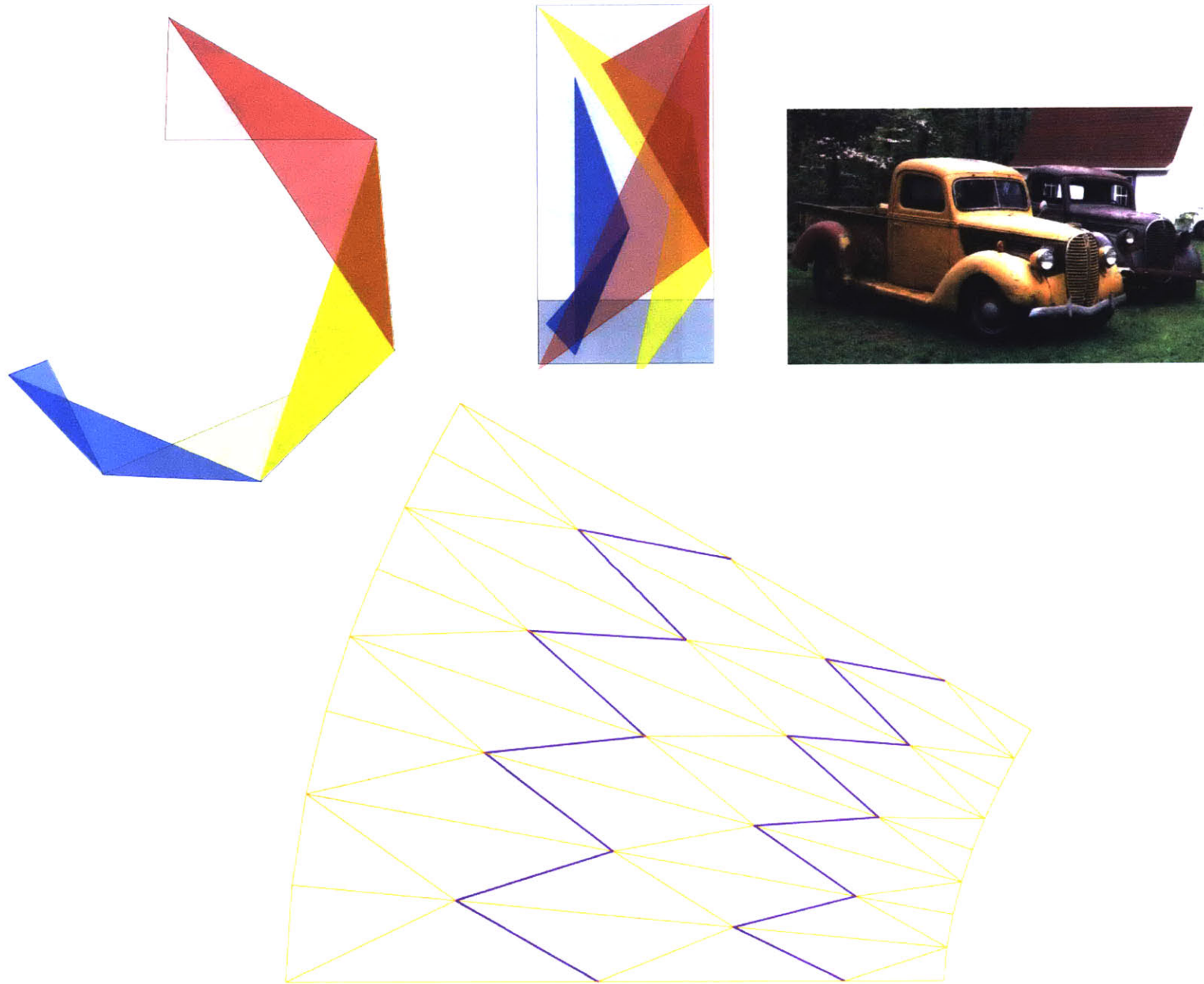
21. Subsequently, the assigned value for the radius is raised to 100ft.

fabrication: beyond diagrams & representations

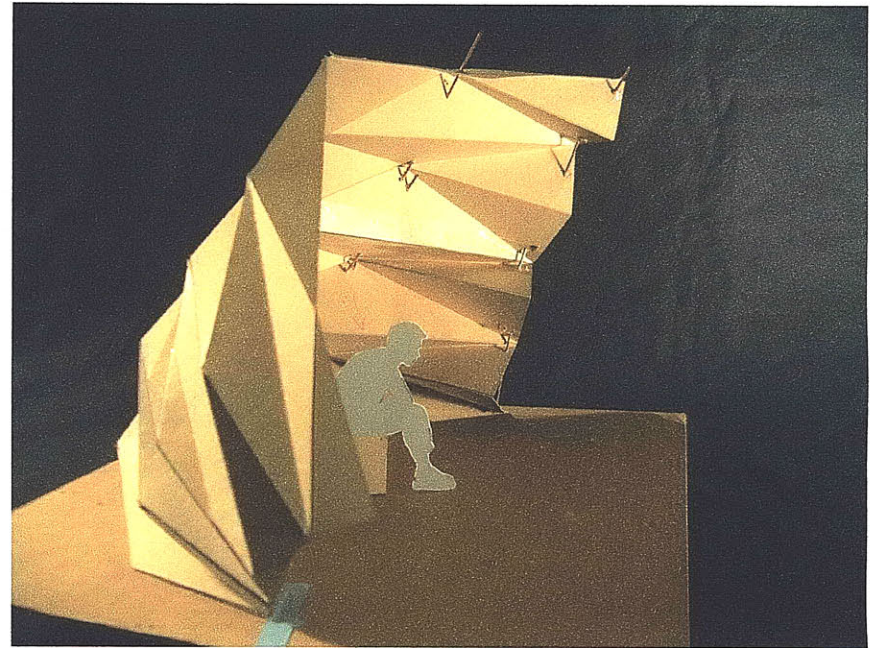
While origami may serve as an extremely rich source of beautiful forms and while mathematical reasoning, paper models, and digital simulations address interesting and useful methods for design, the big question in my mind has always been how to go beyond diagrams, and abstract representations? The challenge would be to tackle the complexity of addressing materials, components, and fabrication, as well as other real-life issues, such as, how does this thing get transported and deployed?

- 40** To begin thinking of building in reality and building a detailed model, the first step would be to determine an application. Working within the anticipated time constraint, it was determined that the structure would be a small-scale, open-air shelter for a temporary out-door event such as an arts & crafts fair. The ensuing criteria are that it be easily transportable and easily deployed. More specifically it: (a.) should be lightweight, (b.) may fold up and fit in the back of a pick-up truck, (c.) would be intuitive and quick to assemble, that is, have a small number of parts to assemble on site and assemble without the use of a crane but the labor of a few people, and, (d.) should use non-precious materials, preferably reconstituted or recycled. As a deployable structure, the form is not manipulated or reconfigured on the site but will be brought there as a prefabricated assembly whereby the form and its corresponding folding pattern would be predetermined after a virtual exploration in the computer. To this end, fabrication would be based directly on files generated from the computer model, all components manufactured for a particular configuration of the folded form.



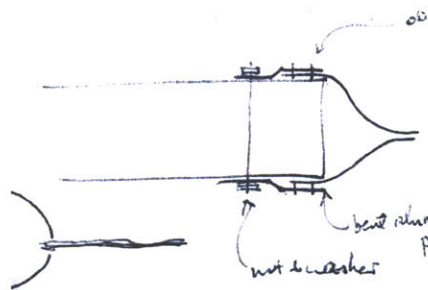


The crease pattern and its form was chosen for its gentle curvature which suggests a structure where the wall and roof are a continuous surface providing an adequate sense of enclosure and separating one booth or function from another, yet to not be too enclosed, resulting in large areas of non-habitable space. (Looking at the previous collage of the half-arch configuration on page 30, we can see how much space is given over to "storage".) I had decided against the arched form as from previous paper models used to study habitation possibilities and from the simple materials experimentation models, I determined that the structure would be most simply erected on site if the seams were not put under stress. One end of the shelter is considerably higher and outstretched over the other for sake of orientation. It's form is simple allowing for easy repetition in a larger, open area such as a park or plaza.



Using the same assembly as the plywood model, the detail model recreates the Kao fold using honeycomb board with canvas at the seams. A mockup construction using trash bags demonstrates the basic assembly of two sheets of fabric pinned to and connecting the panels on either side. (see image to right) Honeycomb board was selected for its recycled paper content and high strength to weight ratio. The honeycomb is such an efficient construction that it is used in train bodies and airplane wings. (see image 15, 15a)

To give the structure lateral stability, a flexible yet relatively stiff membrane would be required at the seams. As such, canvas was chosen as something very affordable and available. To connect the canvas to the honeycomb board, strips of 144# YUPO, a recyclable, tear-resistant, synthetic paper was used. Due to a short supply, 3/2nd" birch plywood was substituted on the exterior. The canvas would be riveted to the YUPO and birch strips, which, in turn, would be secured by a bolt running through the panel. (see sketch)



(15a.)



Modular belts of different thicknesses were folded to test YUPO's flexibility.

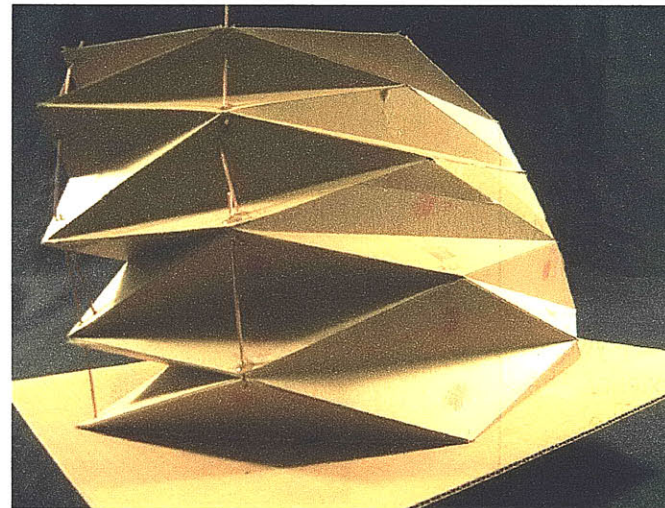
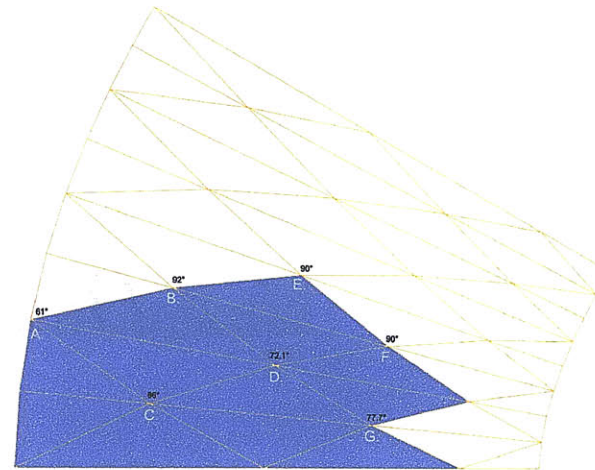


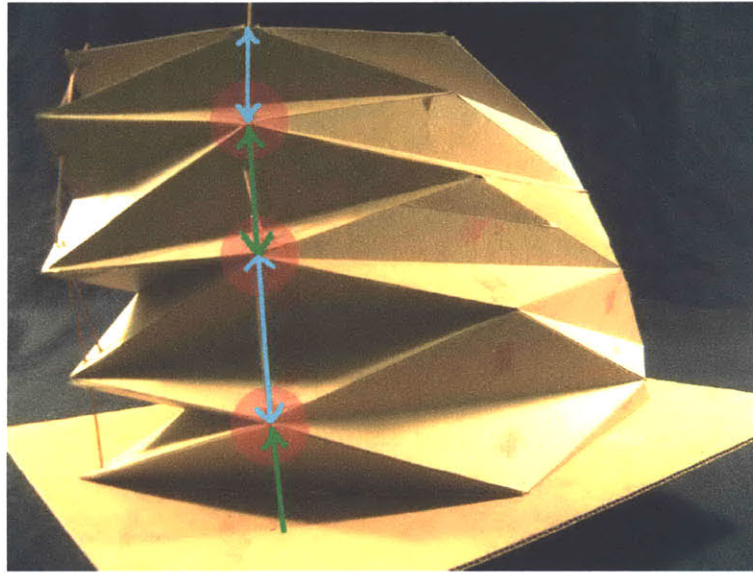
(15.)

Using the dimensions of the honey-comb board sold as a limitation, the detail model was constructed at a scale of 3/8ths of the full scale. Furthermore, given material and time constraints, the model focused on a portion of the entire structure that can be seen highlighted in red on the crease pattern and marked in red on the chip-board model.

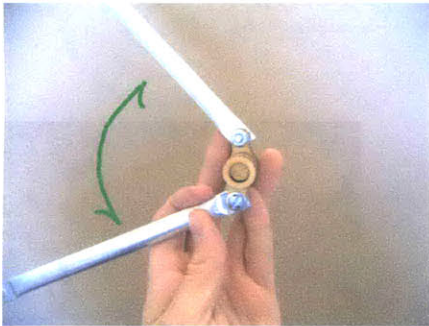
With this particular form, the structure essentially behaves like an accordion requiring a secondary armature to hold the panels in place. This armature would take the form of compressive struts spanning vertically between one vertex and the next, their lengths defining the respective location of the vertices.

44 Components that would receive the struts had to be designed. Given the twist in the surface of the overall structure, the individual pairs of panels and their respective struts do not align orthogonally. (see image_ortho) This means the receiving ends for the two struts meeting at a vertex would have slightly differing orientations and can not function as a simple hinge (see image_protoA). To accommodate for this difference, one approach would be to insert a gasket like material to allow for the strut to hinge laterally as well as locally. (see image_protoB) In this model, Tyvek was used at the seams connecting the panels (see mockup assembly on previous page) and the hinge component is attached to an excess portion of fabric at the vertex using grommets. (see image_protoC)

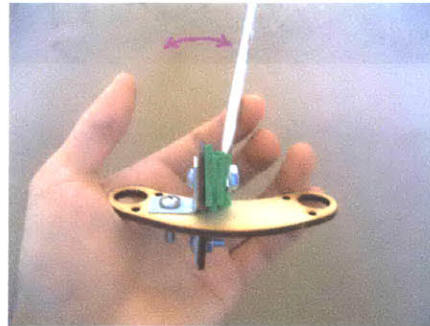




image_ortho



image_protoA

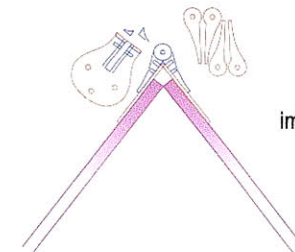
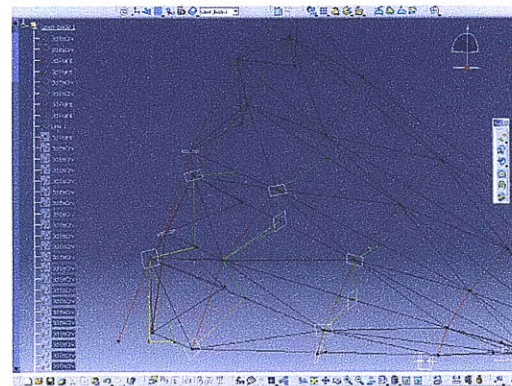
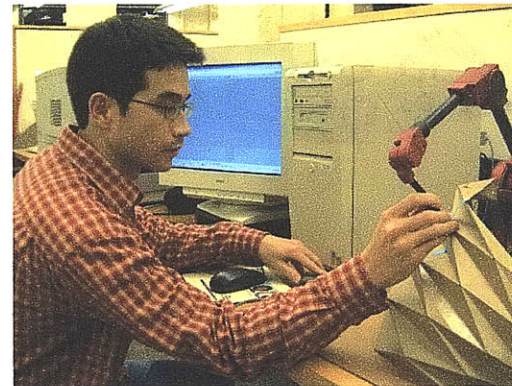
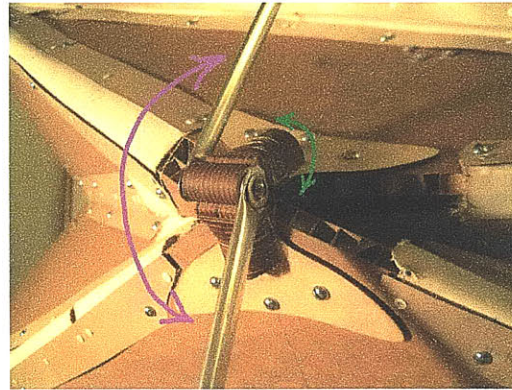


image_protoB

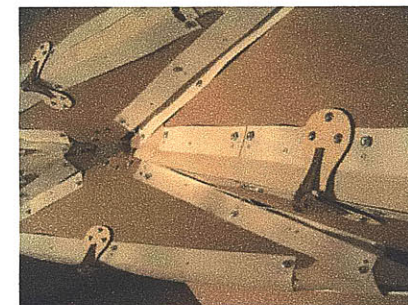


image_protoC

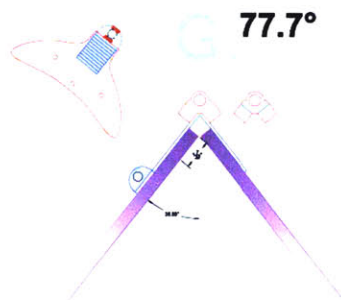
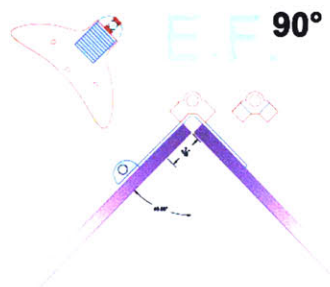
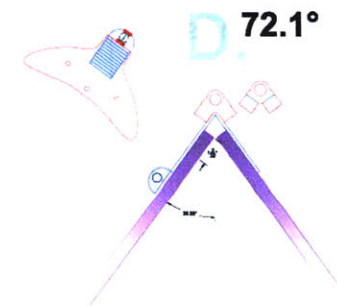
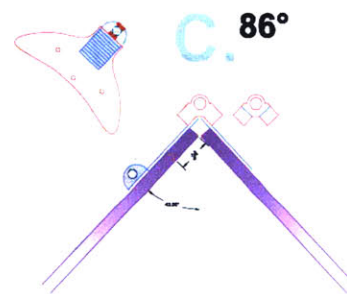
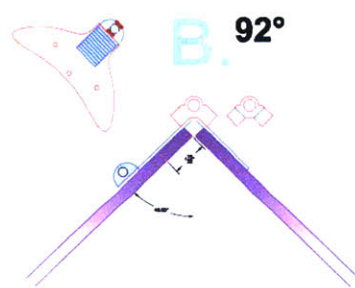
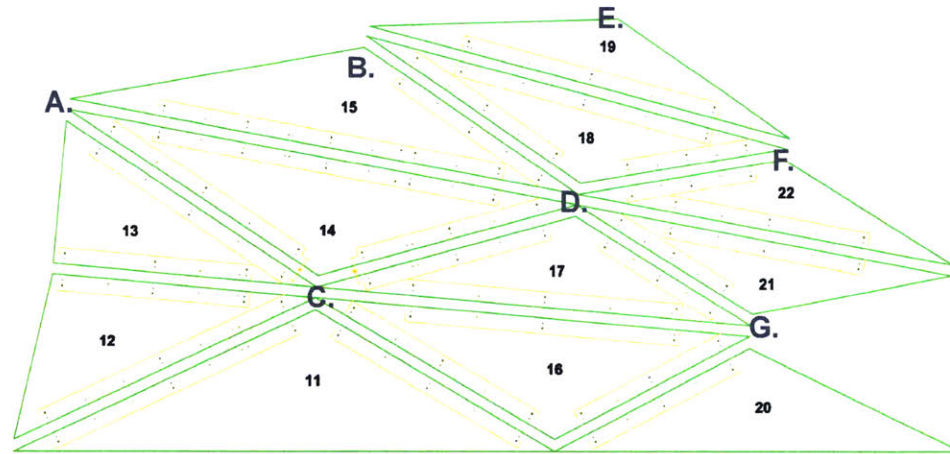
The problem with this method is that a lot of stress is placed on the pin that connects the hinge to the strut such that it may easily fatigue and fail under the structure's own dead weight. A different approach was adopted in which the hinge would connect directly to the panels and allow the area of contact to pivot. (see image to the right) In order to further rigidify the structure, the hinging action would be limited to the struts while the component would hold the two panels to which it was secured at a fixed angle. This meant knowing each different angle for every vertex. Once the panels of the chip-board model were crudely held in place with brass struts and epoxy and its final shape resolved, the model was brought to the 3d-digitizer for a 3d scan giving a digital model in Rhinoceros. Subsequently, the 3d model was imported into Catia and by bisecting a plane through the vertex, their respective angles could be determined. As a third layer of structural reinforcement, the panels forming the dihedral angles would be locked into place using a secondary system of hinges. (see image_hingeA) These hinges are also bolted directly to the panels. They are a simple hinge that may be tightened once all hinges on the vertices, together with the struts, are implemented and assembled.



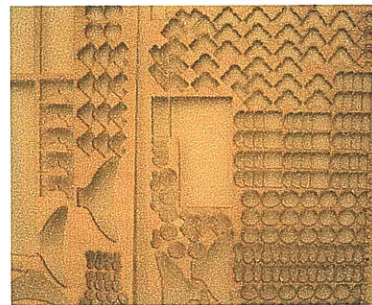
image_hingeA



All components were drawn in AutoCAD and designed to be able to be laser-cut, glued together, and assembled.

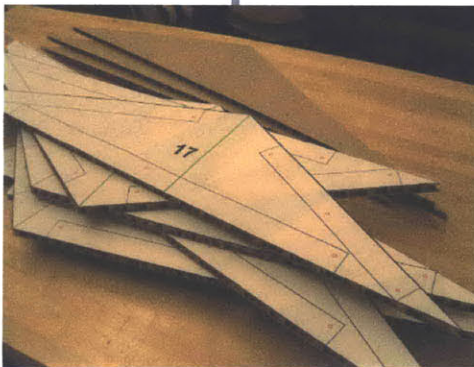


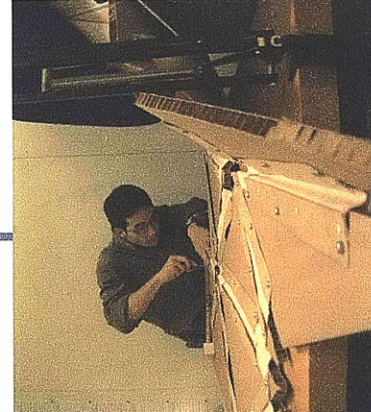
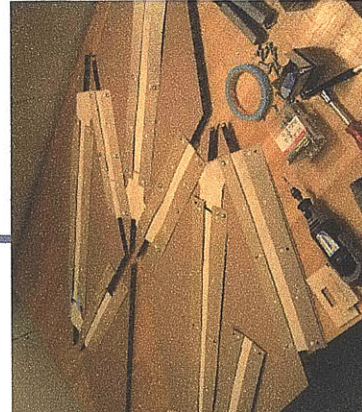
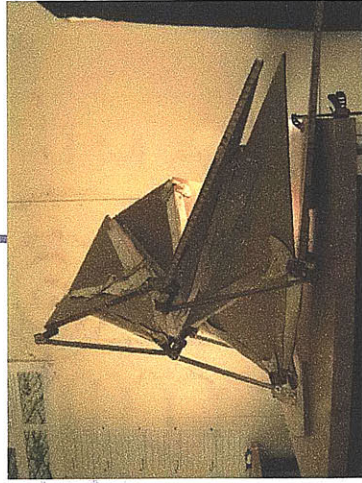
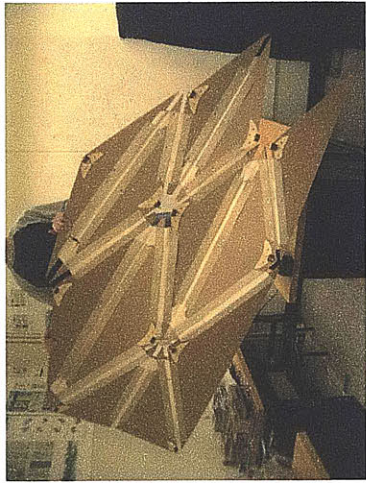
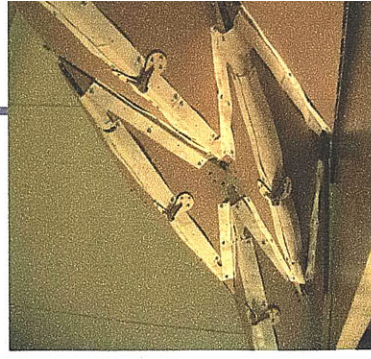
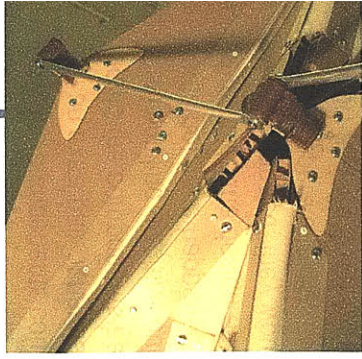
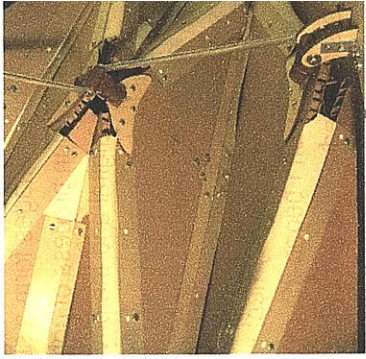
Also the YUPO strips and the canvas joints were drawn-up digitally including holes for all bolts and rivets for precise assembly. (see previous page) Cutting the canvas was particularly challenging as it had to be secured to a layer of chip board in order that it not catch on fire. Furthermore, the dimensions of the laser bed led to splicing separate pieces to create a continuous joint. While speed and precision were certainly advantages in using the laser cutter, it may be pointed out that the constraint imposed by the dimension of the bed led to a large amount of wasted material.



1/8" thick, birch-faced plywood was cut in a single pass. Though cut-lines were prepared for laser cutting by nesting, a lot of material was wasted.

The following thumbnail images form a story board which highlight the design process of going back and forth between the digital and the physical. What I learned was a design process. The model is a prototype, not an answer but the result of a chosen design path with the tools that were given to me. While ease of deployment was a prime concern with which I began the model, this objective was side-lined as the structural response during deployment had not been given sufficient attention. As a consequence of the hinges breaking under too much bending stress and forcing them to delaminate, the components could not be secured alone and an extra pair of hands was often needed. Should the components have been machined by a 3-axis milling machine out of steel, of course, this would not have happened. Yet, the limitations served to guide the design and should the connections be redesigned, they would not be given a fixed angle but operate freely (as do the hinges on the interior) and the 3-dimensional location of each vertex would only be dependent on the length of the struts. This would take off a lot of pressure from the connections and enable one to lock each vertex in place starting at the base and working upwards. Overall, it was a very frustrating process to assemble the structure but what I learned was how often you lose a certain amounts of control along the way due to various inaccuracies and overlooked difficulties and thus, the result is an interpretation of the intentions with which I began.





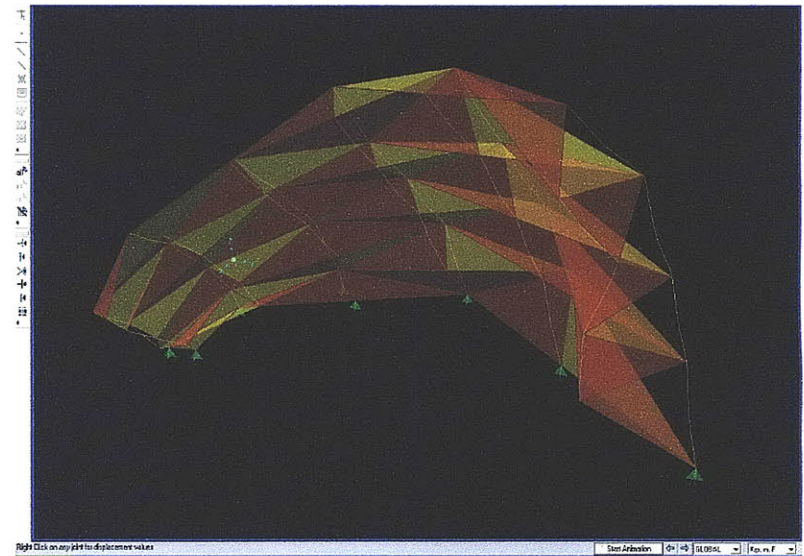
finite element analysis: an assessment of structural stability

After constructing the detailed model, I wanted to assess the dimensions of the components for structural stability as well as test the form for different material assemblies. How would the structure hold up under its own weight if constructed in full scale? Would it be able to withstand lateral wind loads? Numerous simulations were run in the finite element analysis program, SAP 2000. Different materials were tested under their own dead weight and various wind loads. In doing so, the thickness of the plates was varied and the diameter of the struts adjusted.

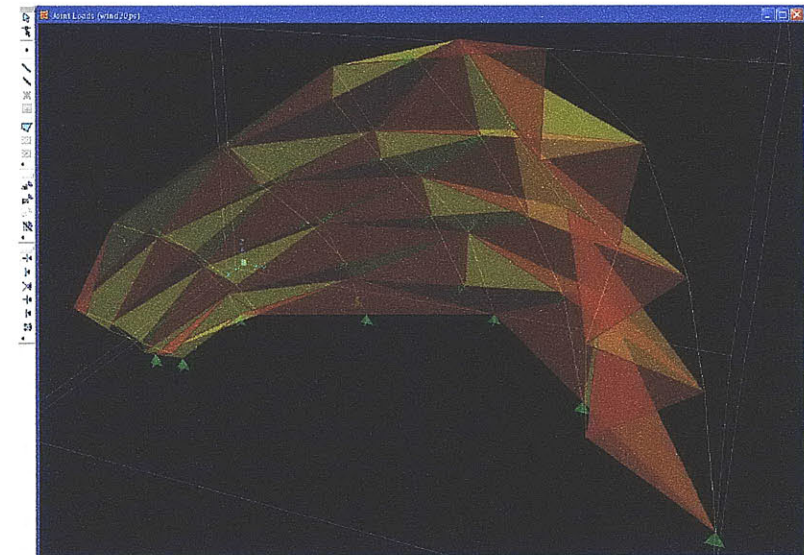
52

Unfortunately, the honeycomb board could not be simulated and so the material that was closest in its properties, aluminum, was used. Moreover, given the time constraint, a flexible seam, as used in the prototype, could not be modeled resulting in a rigid folded-plate structure. Nevertheless, it was believed that testing the model for different load conditions, albeit a different construction, would give legitimacy to its form.

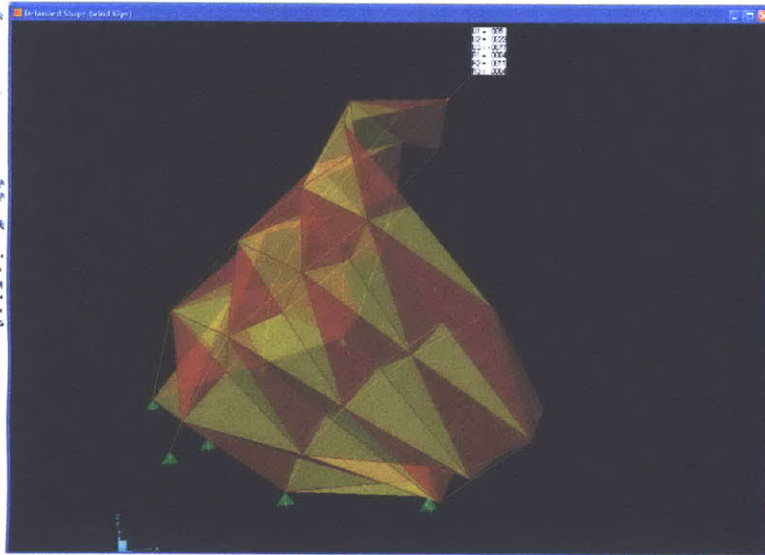
I began by running a simulation for an aluminum structure with 3/16th inch thick plates, and a 1/4"/.063" thick aluminum pipe to serve as the struts. (see: Analysis_1) The displacement, measured at the highest point was negligible. Next, a wind load of 30psf was distributed over its entire 337 sq.ft. area giving a total of 10 kips force. (see: Analysis_2,2a)



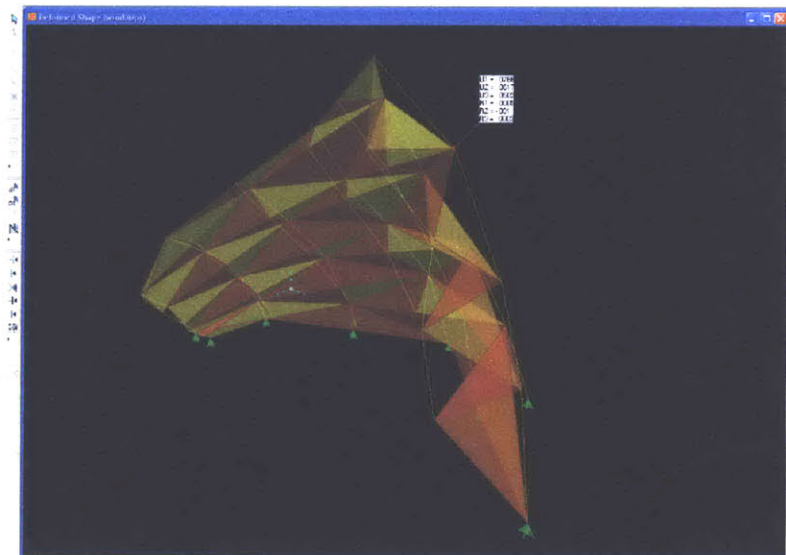
Analysis_1: 3/16" thick alupanel - 1/4" diam/.063"thick alupipe struts



Analysis_2: 30psf wind load over 337 sq. ft. (10 kips) negative x -global



Analysis_2a: 30psf wind load over 337 sq. ft. (10 kips) positive x -global



Applied both to the concave and convex direction of the surface, the displacement along the axis of the wind load measure less than an inch.

The table lists displacement values only under its own dead load for an increasing plate thickness and a diminishing diameter of the struts. As expected, as the thickness of the plates was increased, the greater the deflection, however, the change was not linear given the greater rigidity obtained from the weld along the seams. Surprisingly, the struts could be reduced to a diameter of 1/8 of an inch and the displacement remains under half an inch.

- .0105" displacement: 3/16" thick alupanel - 1"diam/.063"thick alupipe struts*
- .0217" displacement: 1/2" thick alupanel - 1"diam/.063"thick alupipe struts*
- .0314" displacement: 1" thick alupanel - 1"diam/.063"thick alupipe struts*
- .0332" displacement: 2" thick alupanel - 1"diam/.063"thick alupipe struts*

- .0105" displacement: 3/16" thick alupanel - 1"diam/.063"thick alupipe struts*
- .0416" displacement: 3/16" thick alupanel - 1/2"diam/.063"thick alupipe struts*
- .0481" displacement: 3/16" thick alupanel - 1/4"diam/.063"thick alupipe struts*
- .0525" displacement: 3/16" thick alupanel - 1/8"diam/.05"thick alupipe struts*

- .17" displacement: 3/16" thick alupanel - no struts*
- .1672 displacement: 3/16" thick steel - no struts*

1.8" & -1.8" in x and z axis: 1/16" thick steel - no struts --> analysis with 30psf wind load in x-axis

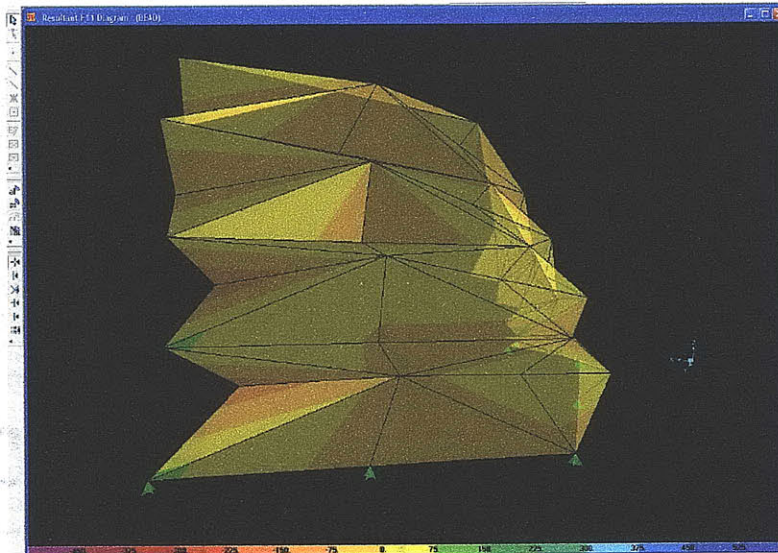
- .0022" displacement: 1/2" concrete shell*
- .0022" displacement: 1/8" concrete shell*
- .0009" displacement: 1/8" steel plate*

In running two separate simulations for aluminum, while the scale factor exaggerates the deflection, we see that the structure would distort more uniformly with struts on the interior as well.

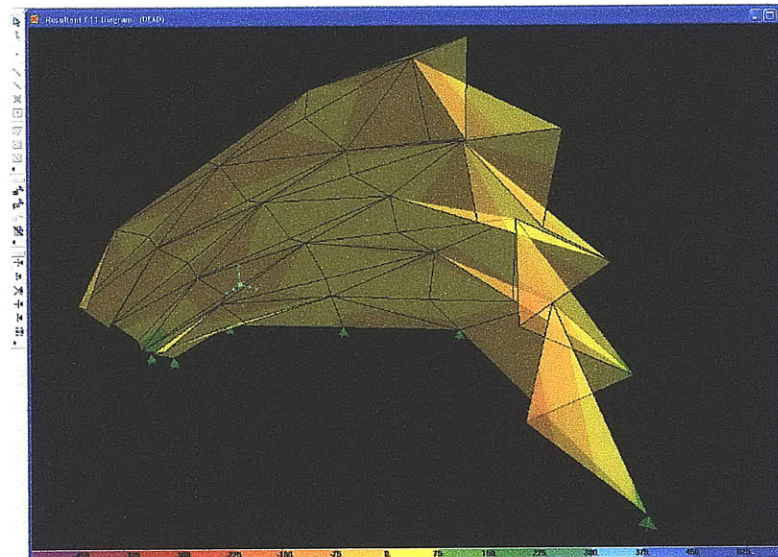
Imagining a more permanent application, stress diagrams were obtained for a model with 3/16th inch thick steel plates showing that it is generally stable but would first show signs of fatigue at the folds in the base where the dead weight accumulates.

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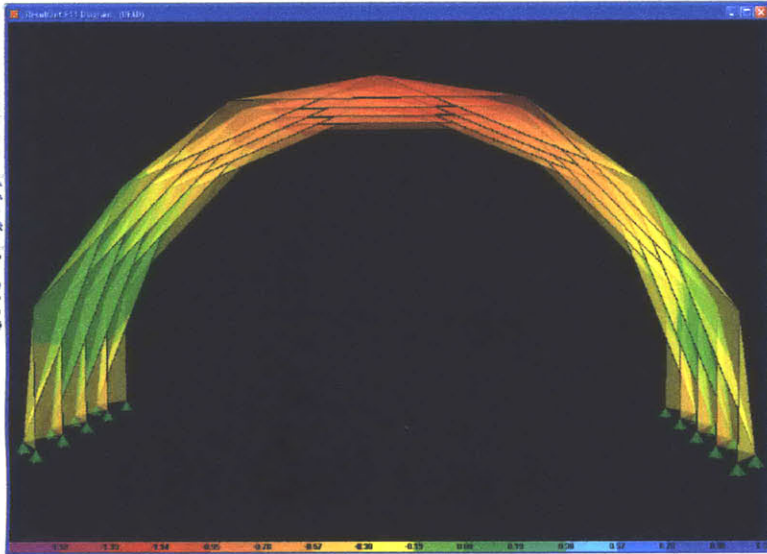
Finally, displacement readings were obtained for a full-arch configuration using both steel and concrete. They demonstrate that, for either material, the Kao-fold is a very stable structure.



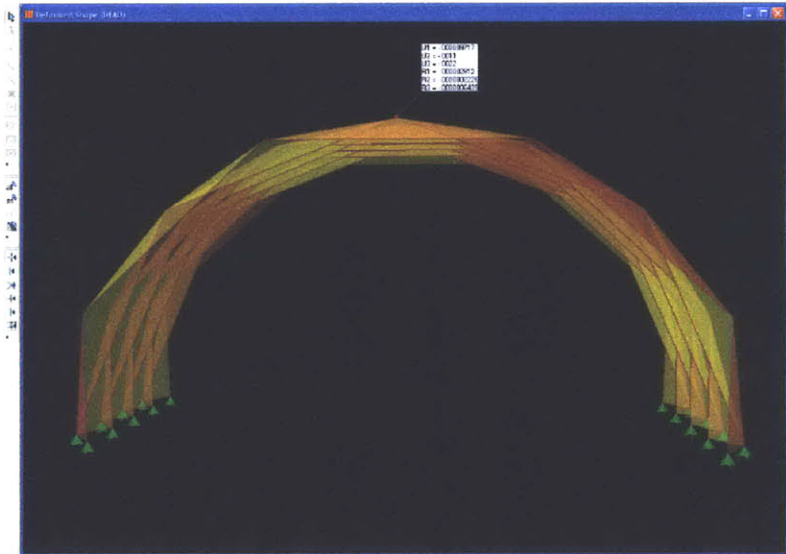
Analysis_3: 3/16th" steel plate - stress diagram



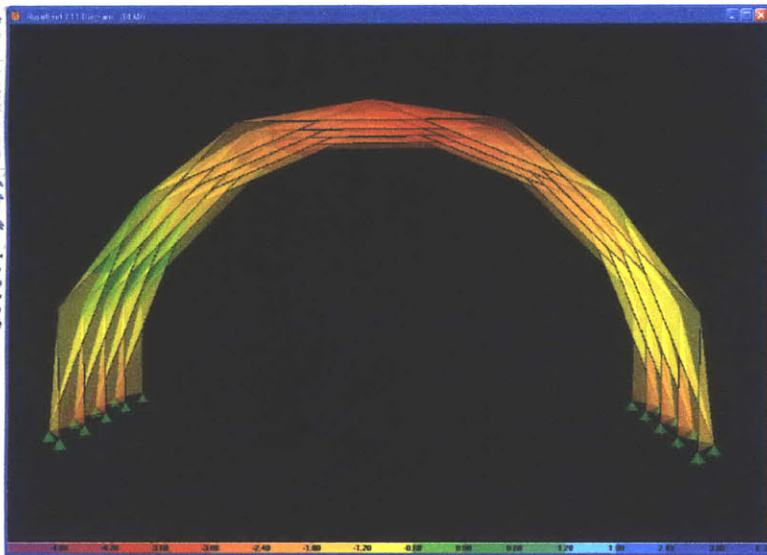
Analysis_3a: 3/16th" steel plate - stress diagram



Analysis_4: 1/8" concrete shell w/ $-0.0022''$ displacement at summit



Analysis 6: 1/2" concrete shell w/ $-0.0022''$ displacement at summit

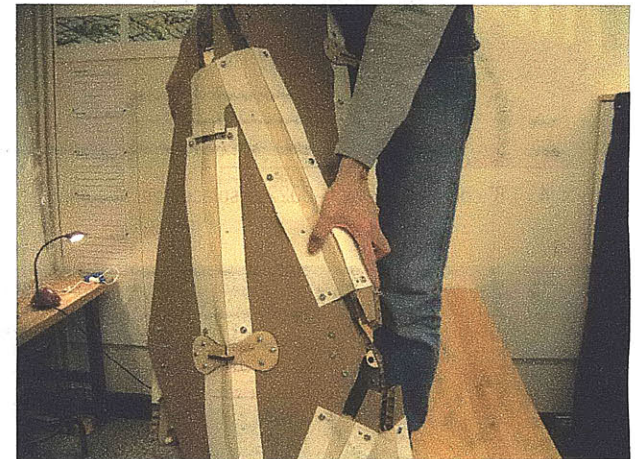


Analysis_5: 1/8" steel shell w/ $-0.0009''$ displacement at summit

conclusion:

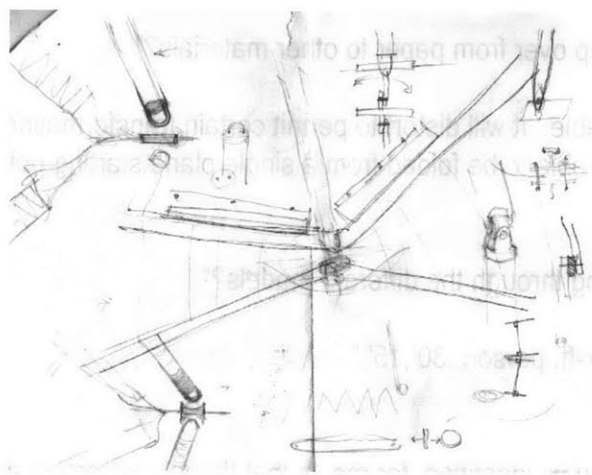
What I learned was a design process; a process of questioning through making. Though, at the onset, certain goals were stated and kept in mind, each step would inform the next allowing the design to evolve as if by a will of its own. Every investigation, every tool had offered certain insights, yet due to various inaccuracies, the outcome was always a transfiguration of the original, or as mentioned, an interpretation of the intentions with which I began. The results address the creative potential of origami as a database of beautiful, sophisticated forms from which design ideas may be conceived. Should the exploration be pursued further, I would like to test different pleated forms for material and assembly ideas with a revised approach to detailing of the connections and of transportation, deployment, and dismantling mechanisms. For the structure's overall optimization, the geometry and mechanism for deployability should be tailored more specifically to its application.

Further future considerations include an evaluation of the form for natural and artificial lighting conditions within and seen from the outside. From the inside, changing natural light addresses the notion of temporality by altering one's perception of the contained space. As such, the forms could be assessed for the nature of their facets and their ability to create a rich and varied distribution of light in response to varying sun angles throughout the day. Should the structure be lit from within, at night, it's presence, much like a glowing lantern, can celebrate a civic space for all to enjoy.



CONCRETE & STEEL

What follows is a partial structural plan of the structure, with a legend by the same name. The plan shows the layout of the structure, including the location of the columns, beams, and slabs. The legend identifies the various components of the structure, such as the columns, beams, and slabs, and provides a key to the symbols used in the plan.



comments & criticisms:

What follows is a partial documentation of the comments and criticisms by my thesis committee and two invited guests, Patricia Patkau of Patkau and Associates in Vancouver and Hugh Stewart from Foster and Partners in London.

Patricia Patkau:

“What was the biggest obstacle in making the jump over from paper to other materials?”

K.G.:

“Paper is extremely thin and therefore very malleable...it will distort to permit certain transformation as well as to permit distortion in the final form...meaning certain folded forms would not be able to be folded from a single plane starting point or at all if the units were made of stiff materials such as plywood or sheet metal.

P.P.:

“Were you following one scale as you were working through the different models?”

K.G.:

“Yes, I determined this was a small space for a six-ft. person..30',15'.”

Hugh Stewart:

“It's a fantastic investigation and what it seems you've identified, for me, is that though origami is an attractive proposition at the scale of paper models, it doesn't translate at the scale of real-world building components...what you end up doing is to rigidify it by introducing these struts...and are not using the strength in the folds, as is the case in the paper models.

If this is your area of interest, what I would say you do is to identify what you are trying to achieve and then harness your analytical abilities to make that happen...because you could go on forever...you need to identify a real task, a real problem and then work backwards as to how you meet the demands imposed by that problem...”

K.G.:

“When working in a smaller scale, felt it was something I physically had a handle on...on a larger scale, I must admit that this was not an efficient deployment of material...I was constantly struggling with the model..needed 3, 4 people to help me hold the nodes in place while I secured the connections...had to go back and forth between modifying what the computer told me was the solution to what reality permitted..”

John Ochsendorf:

"That is one of the key lessons learned - working between the two modes of digital and physical and what hit me was when you said you didn't want to go back to the math...how you work between the two. I applaud mentioning the pile of waste material.

H.S.:

"Do you still have a fascination for origami...do you still think there's an application?
As an inspiration for form - yes, but I'm less convinced about the construction aspects."

H.S.:

"Struts are hard to engineer...you could cover the surface with some splurge and rigidify it like spray-on concrete...the origami could serve as the formwork."

P.P.:

"I appreciate the investigation and how clear you were about your intentions...might recommend you look more carefully at the inhabitants...consider daily living...where do you put the furniture and so on?"

Shun Kanda:

"What will you do in the next three weeks?"

K.G.:

"Document the work and reflect on the process and its outcome...take a break!"

Carol Burns:

"I think there is room for one more wringing out...and one thing I would recommend is this evaluation of tools - one incredible strength of project is that you have gone from your simple fascination of the sophistication of origami, you have bombarded your intuition by way of making with every kind of tool you could put in your hands...and in this school there a lot of very different tools, right? So with your fabulous friends you made these fabulous forms...and your thesis could be about the limits of tools available now or a method of probing with a variety of tools...you talked about the digital and the physical but there is something else that is very strong in your work which is the purely mathematical...which is neither digital nor physical...and I would say that's an area on its own that's worth pursuing.

I'm convinced that you, Ken, could do 8 or 10 more wringing exercises...just to put a capstone on this investigation..."

J.O.:

Just to wrap up you really covered a full range of things...you worked satisfactorily in the digital world...tried to build them and I applaud you for that...taught us a lot about geometry...from this simple form came an astounding richness and complexity of investigation.

bibliography

BOOKS:

Marks, Robert W. "Dymaxion World of Buckminster Fuller", Reinhold Publishing Corporation - NY, 1960 p.178,9 & p.206-8

Fuse, Tomoko "Unit Origami - Multidimensional Transformations", Japan Publications, Inc. - Tokyo 2001

Escrig, F. & Brebbia, C.A. "Mobile and Rapidly Assembled Structures 2", Computational Mechanics Publications, Southampton 1996 p.63-71

Escrig, F. & Brebbia, C.A. "Mobile and Rapidly Assembled Structures 3", WIT Press, Southampton 2000 p.155-61

Gantes, C.J. "Deployable Structures: Analysis and Design", WIT Press, Southampton 2001 p.3-15,52-6,149

Pearce, Peter "Structure in Nature as a Strategy for Design", MIT Press, Cambridge 1979 p.x-xv, 24-8

JOURNALS:

Praxis vol. 1 "Deceit/fabrication office dA" p.10-13, "Interference Screen" p.24,5

A+U Architecture & Urbanism July-Sept. 1984, p.67-72

CONFERENCE PROCEEDINGS:

Barreto, Paulo Taborda 1992 Lines Meeting on a Surface The "Mars" Paperfolding. in Proc. 3rd Int. Meeting of Origami Science, Mathematics, and Education p.342-59

De Focatiis, D.S.A. & Guest, S.D. 2002 Deployable membranes designed from folding tree leaves Proc. R. Soc. Lond., BE/1-10 <http://www-civ.eng.cam.ac.uk/dsl/leaves.pdf>

Hall, Judy 1997 Teaching Origami to Develop Visual/Spatial Perception. in Proc. 2nd Int. Conference on Origami Science & Scientific Origami, p.279-91

Kobayashi, H., Kresling, B. & Vincent, J.F.V. 1998 The geometry of unfolding tree leaves. Proc. R. Soc. Lond., p.147-54

Kresling, Biruta 1996 Folded and unfolded nature. in Proc. 2nd Int. Conference on Origami Science & Scientific Origami, p.93-107

Ikegami, Ushio 2002 Iso-Area Polyhedra. in Proc. 3rd Int. Meeting of Origami Science, Mathematics, and Education , p.20, 141

Sakoda, James Minoru 1992 Hikari-ori: Reflective Folding. in Proc. 2nd Int. Conference on Origami Science & Scientific Origami, p.333-41

THESES:

Liapi, Katherine A. 2002 Transformable Architecture Inspired by the Origami Art: Computer Visualization as a Tool for Form Exploration. University of Texas

WEB SITES:

<http://www.uspto.gov/patft/index.html>

<http://kdg.mit.edu/Matrix/matrix.html>

<http://theory.lcs.mit.edu/~edemaine/hypar/>

<http://www-civ.eng.cam.ac.uk/dsl/leaves.pdf>

<http://www.britishorigami.org.uk/theory/chen.html>

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http://www.greatbuildings.com/buildings/Air_Force_Academy_Chapel.html

http://www.greatbuildings.com/architects/Felix_Candela.html

<http://www.bluffton.edu/~sullivanm/bigisail/bigisail.html>

<http://www.americansteelspan.com/storagesheds.html>

<http://www.yupo.com/>

*Construction Toy Store "Construction Site" in Waltham -

<http://www.constructiontoys.com>

<http://images.google.com/images>

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in this section

- (1) <http://ricegallery.org/view/exhibition/bamboorooft.html>
- (2) http://www.0111.com/lud/pages/architecture/archgallery/ito_serpentine/
- (3), (4) Marks, Robert W. "Dymaxion World of Buckminster Fuller"
- (5) Escrig, F. & Brebbia, C.A. "Mobile and Rapidly Assembled Structures 2"
- (6) A+U Architecture & Urbanism July-Sept. 1984, p.67-72
- (7) Escrig, F. & Brebbia, C.A. "Mobile and Rapidly Assembled Structures 3"
- (8, 8a) Ikegami, Ushio 2002 Iso-Area Polyhedra
- (9) De Focatiis, D.S.A. & Guest, S.D. 2002 Deployable membranes designed from folding tree leaves p.9
- (10) Fuse, Tomoko "Unit Origami - Multidimensional Transformations"
- (11-11b) Gantes, C.J. "Deployable Structures: Analysis and Design" p.54,55
- (12) Pearce, Peter "Structure in Nature as a Strategy for Design" p.26,7
- (13) <http://www.bluffton.edu/~sullivanm/bigisail/bigisail.html>
- (14, 14a) http://www.greatbuildings.com/architects/Felix_Candela.html
- (15, 15a) <http://images.google.com/images>



(16.)