OPTIMAL EXPLORATION AND PRODUCTION OF A NONRENEWABLE RESOURCE*

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ABSTRACT

Earlier studies of exhaustible resource production and pricing usually assume that there is a fixed reserve base that can be exploited over time. In reality there is no "fixed" reserve base (in an economically meaningful sense), since as price rises, additional proved and potential reserves become economical. Here we view a resource like oil as being "nonrenewable" rather than "exhaustible." There is a proved reserve base which is the basis for production, and exploratory activity is the means of increasing or maintaining this proved reserve base. "Potential reserves" are unlimited, but as depletion ensues, given amounts of exploratory activity result in ever-smaller discoveries. Thus resource producers must determine simultaneously their optimal rate of exploratory activity and their optimal rate of production. Optimal trajectories for exploratory activity and production are determined for both competitive and monopolistic producers, and are applied to a simple model of oil production in the Permian region of Texas.

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1. Introduction

The optimal pricing and production of an exhaustible resource in different market settings has by now been fairly well analyzed. Hotelling [10] first demonstrated that with constant marginal extraction costs, price minus marginal cost should rise at the rate of discount in a competitive market, and rents (marginal revenue minus marginal cost) should rise at the rate of discount in a monopolistic market.[1] The monopoly price will initially be higher (and later will be lower) than the competitive price, but the extent to which the two prices will differ depends on the level of production cost and the particular way in which demand elasticities change as the resource is depleted.[2] If extraction costs rise as the resource is depleted, both the monopolist and competitor will be more "conservationist," i.e. they will set prices that are initially higher but that grow less rapidly relative to the case of constant extraction cost.[3]

More recent work has extended the basic Hotelling model in a number of directions.[4] There has been particular concern about the effects of uncertainty (over the resource reserve base, the appearance of substitutes for the resource, and changes in demand) on the rate of extraction. As one would expect, a resource should be extracted more slowly (by a monopolist or a competitor) when the reserve base is not known with certainty. The characteristics of extraction paths under reserve uncertainty have been examined by Gilbert [4] and Loury [12]. Dasgupta and Stiglitz [2] and Hoel [9] studied optimal extraction paths when a substitute for the resource

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[1] For other derivations and interpretations of Hotelling's results, see Herfindahl [8] and Gordon [6]. For further discussion see Solow [15].

[2] This is examined by Stiglitz [17] and Sweeney [18]. Stiglitz shows that if extraction costs are zero and the demand elasticity is constant, the monopoly and competitive price trajectories will be the same.

[3] The case of rising extraction costs has been examined by Heal [7], Levhari and Leviatan [11], and Solow and Wan [16]. Price trajectories for several empirical examples have been calculated by Pindyck [14].

[4] For a general development and presentation of most of the the recent results in the economics of exhaustible resources, see Dasgupta and Heal [1]. For a survey, see Peterson and Fisher [13].

may be introduced at some uncertain future time, under alternative market structures for both the resource and the substitute. Gilbert [4,5] examined the use and value of exploratory activity to obtain a better estimate of the size of the reserve base.

In this paper we also examine exploratory activity. Here, however, the result of exploration is not better information about the reserve base (proved reserves are assumed known with certainty, and "potential reserves" are assumed unlimited), but rather <u>additions</u> to the reserve base. This allows us to deal with the problem that in reality there is no "fixed" reserve base (in an economically meaningful sense) for any resource. If the price of oil were to rise to $200 per barrel (and the demand for oil did not drop to zero), oil would probably be found in some rather strange places. It makes more sense to think of a resource like oil as being "nonrenewable," rather than "exhaustible."

In our model exploratory activity is the means of accumulating or maintaining a level of reserves, and we treat depletion by assuming that reserve additions ("discoveries") resulting from exploratory activity fall as <u>cumulative</u> discoveries increase. The desired level of reserves depends in part on the behavior of production costs. If production costs were independent of reserves (and if there were no uncertainty about the discoveries resulting from exploratory activity), producers would postpone much of their exploratory activity (thereby discounting its cost) and maintain <u>no</u> reserves. In fact, production costs rise as reserves decline (although the exact relationship between the two may be complex).[5] Thus producers must simul-

[5] For resources like oil and gas, at the level of individual pools and fields lower reserves means higher extraction costs as the rate of physical output per unit of capital equipment declines, and eventually as secondary and tertiary recovery techniques are needed. Even at the aggregate level, however, reserve depletion will be accompanied by higher average extraction costs since lower cost deposits are usually produced first, and of those individual deposits with similar cost characteristics, reserves per deposit will on average be lower when aggregate reserves are lower. For many mineral resources extraction costs will similarly increase as higher cost deposits are tapped and as deeper mines must be utilized for individual deposits.

taneously determine optimal levels of exploratory activity and production - resulting in an optimal reserve level - that balance revenues with exploration costs, production costs, and the "user cost" of depletion.

The design of an optimal exploration strategy to accumulate reserves has already been examined by Uhler [19], who calculated an optimal rate of exploratory effort assuming a fixed price for the resource. The price (and rate of production) of the resource, however, will change over time, and the optimal production rate and exploration rate are interrelated. Here we examine exploration and production simultaneously, and study the joint dynamics of the two.

2. Exploration and Production under Competition and Monopoly

We consider first competitive producers of a nonrenewable resource.[6] Producers take the price p as given, and choose a rate of production q from a proved reserve base R. The average cost of production $C_1(R)$ increases as the proved reserve base in depleted. Additions to the proved reserve base occur in response to the level of exploratory effort w.[7] The rate of flow of additions to proved reserves depends on both w and cumulative reserve additions x, i.e. $\dot{x} = f(w,x)$, with $f_w > 0$ and $f_x < 0$. Thus as exploration and discovery proceed over time, it becomes more and more difficult to make new discoveries. The cost of exploratory effort $C_2(w)$ increases with w. We assume that $C_2''(w) \geq 0$, and that the marginal discovery cost, $C_2'(w)/f_w$, increases as w increases.[8] The producer's problem, then, is as follows:

$$\max_{q,w} W = \int_0^\infty [qp - C_1(R)q - C_2(w)]e^{-\delta t}dt \tag{1}$$

subject to

$$\dot{R} = \dot{x} - q \tag{2}$$

$$\dot{x} = f(w,x) \tag{3}$$

and

$$R \geq 0,\ q \geq 0,\ w \geq 0,\ x \geq 0 \tag{4}$$

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[6] We are ignoring the problem of common access. In effect we are assuming here that there are a large number of identical firms that all ignore each other, or, equivalently but more realistically, that a state-owned company has sole exploration and production rights, and sets a competitive price.

[7] w might represent the number of exploratory wells drilled, or it might be an index of drilling footage adjusted for depth.

[8] Note that $C_2'(w)$ and f_w are, respectively, the additional cost and the additional discoveries associated with one more unit of exploratory effort.

The solution of this optimization problem is straightforward. The Hamiltonian is

$$H = qpe^{-\delta t} - C_1(R)qe^{-\delta t} - C_2(w)e^{-\delta t} + \lambda_1(f(w,x)-q) + \lambda_2 f(w,x) \quad (5)$$

Note that H is a linear function of q but in general a nonlinear function of w. Differentiating H with respect to R and x gives the dynamic equations for λ_1 and λ_2:

$$\dot{\lambda}_1 = C_1'(R)qe^{-\delta t} \quad (6)$$

$$\dot{\lambda}_2 = -(\lambda_1 + \lambda_2)\, f_x \quad (7)$$

From (5) we see that each producer should produce either nothing or at some maximum capacity level, depending on whether $pe^{-\delta t} - C_1(R)e^{-\delta t} - \lambda_1$ is negative or positive. Since this expression depends on the price p, market clearing will ensure that

$$pe^{-\delta t} - C_1(R)e^{-\delta t} - \lambda_1 = 0 \quad (8)$$

Note that λ_1 is the marginal profit-to-go resulting from an additional unit of reserves. λ_1 is always positive, but $\dot{\lambda}_1$ is negative, since $C_1'(R)$ is negative by assumption, so that λ_1 approaches zero as depletion ensues. We can see immediately then that at some point production will cease (generally before proved reserves become zero), even though further exploration could yield more reserves.

Differentiating (8) with respect to time, substituting (2) for $\dot{R}$, and equating with (6) gives us the equation describing the dynamics of the price path:

$$\dot{p} = \delta p - \delta C_1(R) + C_1'(R)f(w,x) \quad (9)$$

Observe that price rises more slowly than in the case of production without exploration.[9] Note also that if $C_1'(R)$ is zero, i.e. if production costs do not depend on reserves, the rate of change in the price path is unaffected by exploration and is identical with that in the standard constant-cost Hotelling problem. The <u>level</u> of the price path, however, will be affected by exploration; since "planned" reserves (i.e. the total amount of the resource available for production, including what will ultimately be discovered) are greater than initial reserves, our producer can set the <u>initial</u> price at a lower level. Price trajectories with and without exploration are shown for zero extraction costs in Figure 1.

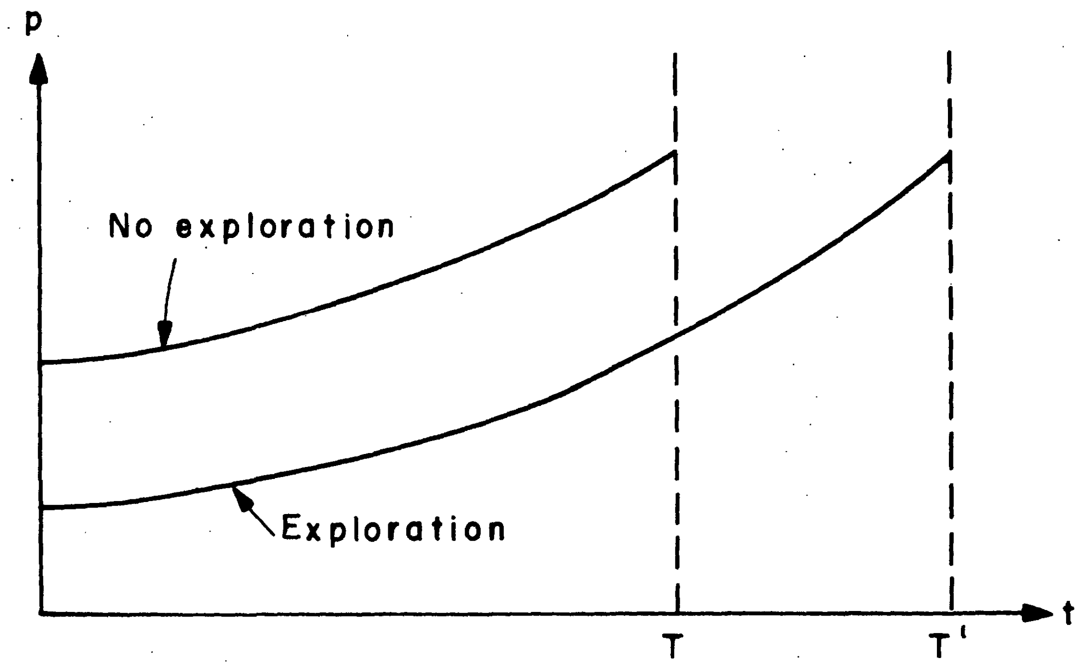

Figure 1: Price Paths for Zero Extraction Costs

[9] We showed in an earlier paper [14] that if extraction costs rise as reserves fall, but there is no exploration, price follows the equation $\dot{p} = \delta p - \delta C_1(R)$. Note however, that the introduction of exploration does not make our producer more conservationist. Given any <u>initial</u> reserve level R_o, total production will be larger if there were no exploration, so that price can begin at a <u>lower</u> level and rise more slowly over a longer period of time.

We can now determine the optimal rate of exploration by setting $\partial H/\partial w$ equal to 0, and substituting in equation (8) for λ_1. This yields the following equation for λ_2:

$$\lambda_2 = \frac{C_2'(w)}{f_w} e^{-\delta t} - pe^{-\delta t} + C_1(R)e^{-\delta t} \tag{10}$$

Using equations (8) and (10), we can rewrite equation (7) as:

$$\dot{\lambda}_2 = -\frac{f_x}{f_w} C_2'(w)e^{-\delta t} \tag{11}$$

Differentiating equation (10) with respect to time and substituting (2), (3), and (9) for $\dot{R}$, $\dot{x}$, and $\dot{p}$ yields:

$$\dot{\lambda}_2 = -C_2'(w)\frac{f_{wx} \cdot f}{(f_w)^2} e^{-\delta t} + \frac{f_w C_2''(w) - C_2'(w) f_{ww}}{(f_w)^2} \dot{w}e^{-\delta t}$$
$$- \delta \frac{C_2'(w)}{f_w} e^{-\delta t} - C_1'(R)qe^{-\delta t} \tag{12}$$

Equating this with (11) and rearranging gives us an equation that describes the dynamics of exploratory effort:

$$\dot{w} = \frac{C_2'(w)\left[\frac{f_{wx}}{f_w} \cdot f - f_x + \delta\right] + C_1'(R)qf_w}{C_2''(w) - C_2'(w)\frac{f_{ww}}{f_w}} \tag{13}$$

Since the Hamiltonian evaluated at the terminal time T (when production ceases) must be zero, we know that at the terminal time exploratory effort must be zero, and this provides a boundary condition for w. A second boundary condition can be obtained from the transversality condition. Since there is no terminal cost associated with cumulative discoveries x, we know that $\lambda_2(T) = 0$. Then from equation (10) and the fact that $w_T = 0$, we have that $p_T = C_1(R_T)$, i.e. price rises and reserves fall (raising extraction costs) until the profit resulting from the extraction of the last bit of the resource is just zero. Given particular functional forms for f, C_1, and C_2, and a demand function relating p and q, equations (9) and (13) can be solved together with the boundary conditions described above to yield paths for price (and hence production) and exploratory effort.

The particular pattern of exploratory effort, price, and production depends critically on the initial value of reserves. The intertermporal trade-off in exploration involves balancing the gain from postponing exploration (so that its cost can be discounted) with the loss from higher current production costs resulting from a lower reserve base. If initial reserves are large so that $C_1(R)$ is small, most exploration can be postponed to the future, whereas if initial reserves are small, exploration must occur early on so as to increase the inventory of proved reserves. In this latter case production will increase initially (as price falls), and later reserves and production will fall as exploratory effort diminishes. We will examine the behavior of price and exploratory effort in more detail in Section 4 of the paper.

Let us now turn to the case of a monopolistic producer. The monopolist also chooses q and w to maximize the sum of discounted profits in equation (1), but faces a demand function p(q), with $p'(q) < 0$. Equations (6) and (7) still apply, but maximizing H with respect to q yields

$$\lambda_1 = MR_t e^{-\delta t} - C_1(R)e^{-\delta t} \qquad (14)$$

with $MR = p + q(dp/dq)$. Differentiating (14) with respect to time and equating with (6) gives us the equation describing the dynamics of marginal revenue:

$$\dot{MR} = \delta MR - \delta C_1(R) + C_1'(R)f(w,x) \qquad (15)$$

Again note that if extraction costs do not depend on the reserve level, marginal revenue follows the same differential equation as in the standard Hotelling problem, i.e. marginal revenue net of extraction cost rises at the rate of discount. Given any initial reserve level, however, exploration permits the initial price (and marginal revenue) to be lower since the total quantity that can be extracted will be greater.

Maximizing H with respect to w and substituting (14) for λ_1 gives us an expression for λ_2:

$$\lambda_2 = \frac{C_2'(w)}{f_w} e^{-\delta t} - MR_t e^{-\delta t} + C_1(R) e^{-\delta t} \tag{16}$$

Differentiating this with respect to time, and equating with (7) yields the differential equation for w:

$$\dot{w} = \frac{C_2'(w)\left[\frac{f_{wx}}{f_w} \cdot f - f_x + \delta\right] + C_1'(R) q f_w}{C_2''(w) - C_2'(w)\frac{f_{ww}}{f_w}} \tag{17}$$

This is identical to equation (13), but this does not mean that the pattern of exploratory effort is the same in the monopoly and competitive cases. Initially q is lower for the monopolist, and since $C_1'(R)$ is negative, $\dot{w}$ is larger. Whether initial proved reserves are small or large, the monopolist will initially undertake less exploratory activity than the competitor, but later he will undertake more.[10]

3. Measuring Resource Scarcity

In the United States, policy makers often use estimated "potential reserves" of oil, natural gas, and various minerals as a measure of resource scarcity. This,

[10] Unless extraction costs are zero and the elasticity of demand is constant, in which case both price and exploratory activity will be the same for the monopolist and the competitor. Stiglitz demonstrated [17], for the case of production without exploration, that these special conditions result in monopolistic and competitive price trajectories that are identical. When extraction costs are zero the differential equations for price (in the competitive case) and for marginal revenue (in the monopoly case) do not depend on reserves or exploratory activity, so that price (and quantity) trajectories are again identical. Since equations (13) and (17) are identical, the trajectories for exploratory effort will also be the same.

of course, implies viewing these resources as exhaustible, which as we have argued makes little economic sense. But even if such resources were exhaustible, the volume of potential reserves does not provide a useful measure of scarcity, since it does not reflect the difficulty of actually obtaining these reserves. As Fisher points out [3], an appropriate scarcity measure "should summarize the sacrifices required to obtain a unit of the resource," and if by a resource we mean the raw material in the ground, the "rent" or "user cost" component of price (i.e. the components of price other than extraction cost) is a better measure of scarcity. Extraction costs may rise or fall independently of how much of the resource is left in the ground, but rent (i.e. the difference between price and marginal extraction cost in a competitive market) represents the opportunity cost of resource extraction, which better reflects resource scarcity.

In this paper we have argued that most mineral resources can be best thought of as nonrenewable but inexhaustible, so that "potential reserves" has little meaning as a scarcity measure. On the other hand, "rent" provides a scarcity measure that is particularly appropriate. To see this, rearrange equation (8) for price in the competitive case:

$$p = C_1(R) + \lambda_1 e^{\delta t} \tag{18}$$

The second term on the right-hand side of this equation is undiscounted rent, and by setting $\partial H/\partial w$ equal to 0, we see that it has two components:

$$\lambda_1 e^{\delta t} = \frac{C_2'(w)}{f_w} - \lambda_2 e^{\delta t} \tag{19}$$

The second term on the right-hand side of (19) is the shadow price of an additional unit of cumulative discoveries, and measures the impact of this additional unit on future marginal discovery costs. We would usually expect λ_2 to be negative, since discoveries today result in an increase in the amount of exploratory effort that will be needed to obtain future discoveries.[11]

One might ask why both the marginal discovery cost and the opportunity cost of additional cumulative discoveries should be included in a measure of scarcity, rather than simply lumping discovery cost together with extraction cost and using only the last term in (19) to measure scarcity. Note from equation (11) that (assuming λ_2 is negative) $\dot{\lambda}_2$ is positive, so that the discounted value of this opportunity cost becomes smaller in magnitude over time - as the <u>actual</u> value of marginal discovery cost grows. The reason is that once marginal discovery cost has become very large - and the resource is very scarce - resource use decreases as potential future profits become small, so that the opportunity cost of additional discoveries is small. For example, it might be that 30 years from now the marginal discovery cost of oil will exceed \$100 per barrel, at which time oil will be extremely scarce, even though the opportunity cost of additional discoveries will be small. Thus the full rent of equation (19) should be used to measure scarcity.[12]

4. The Behavior of Optimal Exploration and Production

In the solution of the typical exhaustible resource problem for a competitive market, price rises slowly over time as reserves are depleted, so that demand is choked off just as the last unit is extracted (if extraction costs are constant), or just as the profit on the last unit extracted becomes zero (if extraction costs rise as reserves decline). In our model of a nonrenewable resource, price

[11] As Fisher [3] and Uhler [19,20] point out, additional cumulative discoveries might initially result in a <u>decrease</u> in the amount of exploratory effort needed to obtain future discoveries by providing geological information. In this case λ_2 would be positive initially, and would later become negative as the effects of depletion offset the informational gains from cumulative discoveries. Uranium is a resource for which λ_2 might conceivably be positive today, but for most other resources of policy interest (and particularly oil and gas), λ_2 is negative.

[12] This is still an imperfect measure of scarcity in that it does not reflect external costs (such as environmental damage resulting from resource exploration, discovery, and production).

will also rise (more slowly than before) and reserves will steadily decline, but only if reserves are very large to begin with (so that extraction costs are low). As we will see, if reserves are initially very low, price will start high, fall as reserves increase (as a result of exploratory activity), and then rise slowly as reserves decline.

If reserves are initially very large, $C_1(R)$ and $C_1'(R)$ will be small, so that $\dot{p}$ will be positive - in fact the rate of growth of p will be just slightly below the discount rate. If reserves are large, $\dot{w}$ will also be positive. To see this, observe that the denominator of the right-hand side of (13) is always positive, while the first term in the numerator is positive, and the second term is very small.[13] Thus w will begin growing from some very low level (when reserves are large, new discoveries are not needed initially, so that the cost of exploration can be postponed and thus discounted). Since initially there are almost no discoveries, reserves will fall. Reserves will fall more and more slowly, however, as exploration increases. At some point after reserves have become small enough, $\dot{w}$ will become negative (as $C_1'(R)$ becomes large), and exploration will decline towards zero as most of the reserves are used up. Price will increase until demand is choked off just as profit on the last unit of the resource is zero, and just as exploratory activity becomes zero. At this point the resource has not been "exhausted," but it no longer pays to explore for new reserves. This pattern of exploratory activity and reserves is shown in Figure 2.

Suppose extraction costs are small relative to price and to the costs of exploration. Then there is no value in holding a large stock of reserves, and most exploratory activity will be postponed until near the end of the planning horizon. This is illustrated by the dotted lines in Figure 2.

[12]By assumption, $d/dw[C_2'(w)/f_w] > 0$. Then, since $f_w > 0$, $[C_2'' - C_2' f_{ww}/f_w] > 0$. Since w is small initially, $(f_{wx}/f_w)f - f_x < \delta$, and since R is large, $C_1'(R)$ is small.

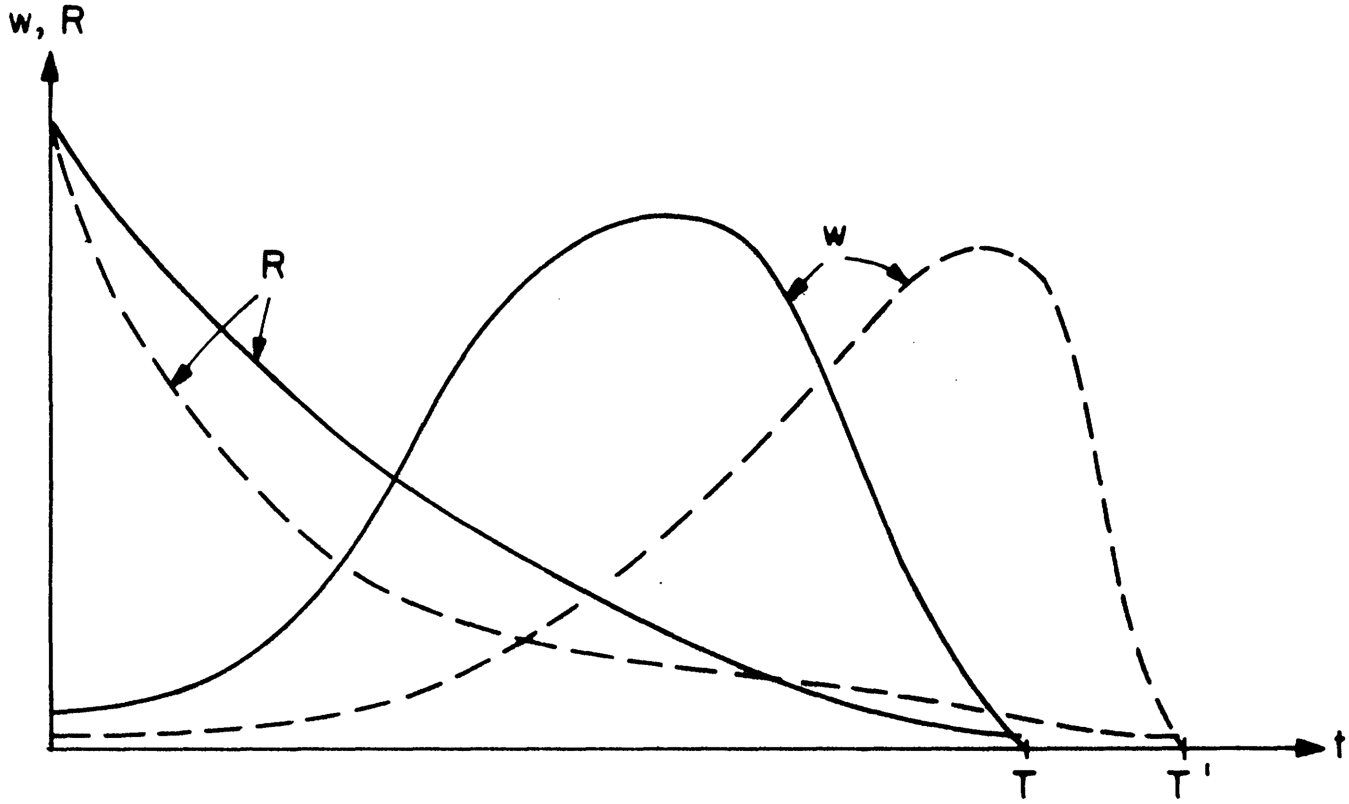

Figure 2: Exploratory Activity and Proved Reserves - Initial Reserves Large

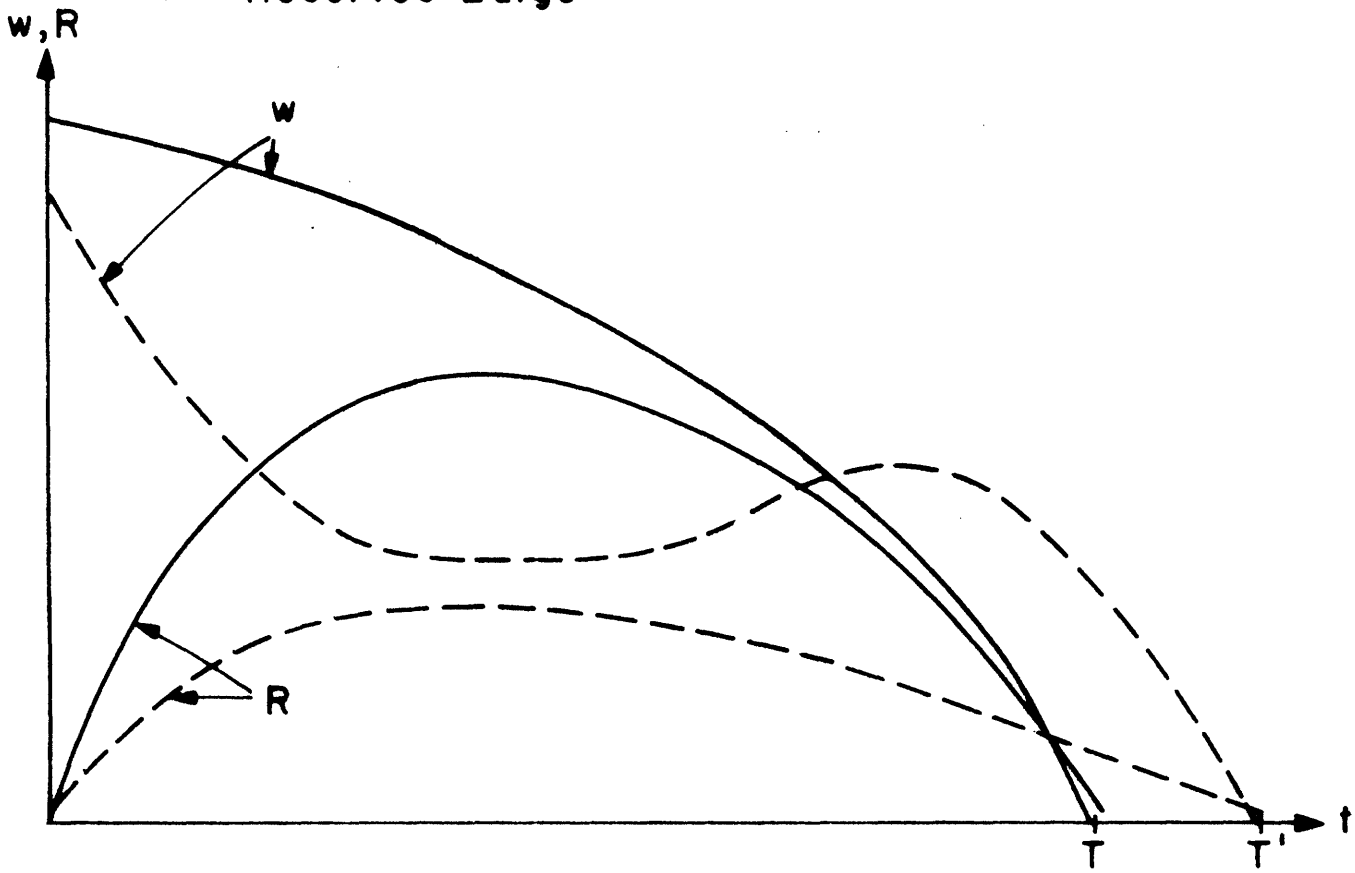

Figure 3: Exploratory Activity and Proved Reserves - Initial Reserves Small

If reserves are initially very small, price will begin declining from a high level (since $C_1(R)$ and $C_1'(R)$ are large in magnitude). Exploration will also begin declining from some high level (again because $C_1'(R)$ is a large negative number). Reserves will at first increase in response to exploration, but later will decrease as exploration diminishes and the average product of exploration increases. As reserves decrease price will increase, until demand becomes zero as exploratory activity becomes zero and the profit on the last extracted unit of the resource becomes zero. This is illustrated in Figure 3.

If extraction costs are small, exploration can decline more rapidly since there is no need to build up as large a stock of reserves. Later as production increases, $\dot{w}$ can become positive; exploration then increases so that the stock of reserves does not fall to zero too quickly. Finally, as the returns from exploration diminish, $C_1'(R)$ will dominate the numerator of (13), $\dot{w}$ will become negative, and exploration will fall to zero. This is illustrated by the dotted lines in Figure 3.

The behavior of exploration and production under different initial conditions and different extraction costs can be summarized by the phase diagram in Figure 4. From equations (2) and (3) we see that the $\dot{R}=0$ isokine will be nearly vertical for large values of R, but as R becomes small, q will become small, so that this isokine will bend in towards the origin. From equation (13) it is clear that the $\dot{w}=0$ isokine will be downward sloping, since increased R and increased w both make $\dot{w}$ larger. Note, however, that this isokine will shift to the left if q decreases, or if cumulative discoveries x increases. Also, this isokine will be closer to the origin if extraction costs are relatively low. In the figure, the isokine $[\dot{w}=0]_1$ corresponds to large extraction costs. The isokines $[\dot{w}=0]_2$ and $[\dot{w}=0]_3$ correspond to relatively low extraction costs, with q small and/or x large for $[\dot{w}=0]_2$, and the opposite for $[\dot{w}=0]_3$.

If reserves are initially large, the optimal trajectory is given by curve A in Figure 4. Note that reserves always decrease, with exploration increasing and then decreasing. If reserves are initially small, the optimal trajectory could be given by curves B or C, depending on extraction costs. If extraction costs are large, exploration will be at a higher level and will continually decrease, as in B. If extraction costs are small, exploration can decrease, increase, and decrease again as in C. Here the trajectory crosses the $\dot{w}=0$ isokine so that $\dot{w}$ becomes positive, the isokine shifts to the right as q increases so that $\dot{w}$ becomes negative again,

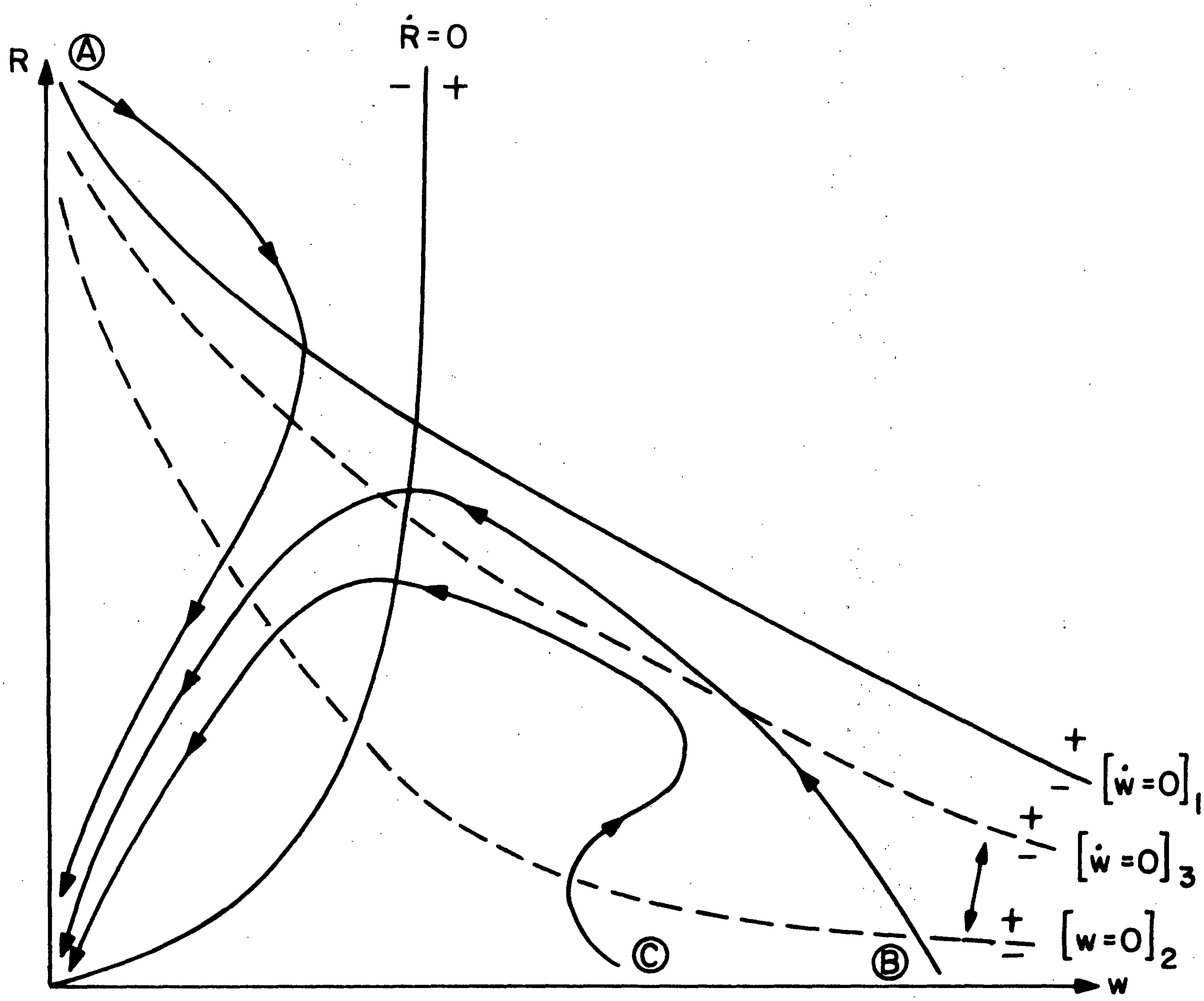

Figure 4: Phase Diagram and Optimal Trajectories

and reserves keep falling as the isokine moves back to the left as a result of decreasing q and increasing x.

5. The Case of No Depletion

If the returns from exploration do not decline as cumulative discoveries increases, i.e. if $f_x = 0$, production can go on indefinitely. In this case there will be an initial transient period during which reserves approach some long-run steady-state level $\bar{R}$, and after which steady-state exploration $\bar{w}$ results in discoveries just equal to steady-state production $\bar{q}$. This can be seen from the phase diagram in Figure 5. Since $f_x = 0$, increases in cumulative discoveries will not result in a shift of the $\dot{w}=0$ isokine. Trajectories A and B (large initial reserves and small initial reserves, respectively) lead to a long-run equilibrium of constant reserves and production. Any other trajectory leads to reserves and a level of exploration that grow large without limit, or else to a decline in reserves and cessation of production.

We can examine the characteristics of this steady-state by setting f_x and $\dot{w}$ equal to 0 in equation (13). From this we obtain

$$\frac{C_2'(\bar{w})}{f_w} = \frac{C_1'(\bar{R})\bar{q}}{\delta} \tag{20}$$

The right-hand side of (20) is the present discounted value of the annual flow of extraction cost savings resulting from one extra unit of reserves. If this quantity is less than the marginal discovery cost incurred in maintaining that extra unit of reserves (the left-hand side of (20)), profits would be greater with a level of exploration below the steady-state level, and indeed, we will have $\dot{w} > 0$, $w < \bar{w}$, and $R > \bar{R}$. Similarly, if this quantity is greater than the marginal discovery cost, we will have $\dot{w} < 0$, $w > \bar{w}$, and $R < \bar{R}$. In the first case the initial reserve level is larger than necessary, and in the second case it is too small.

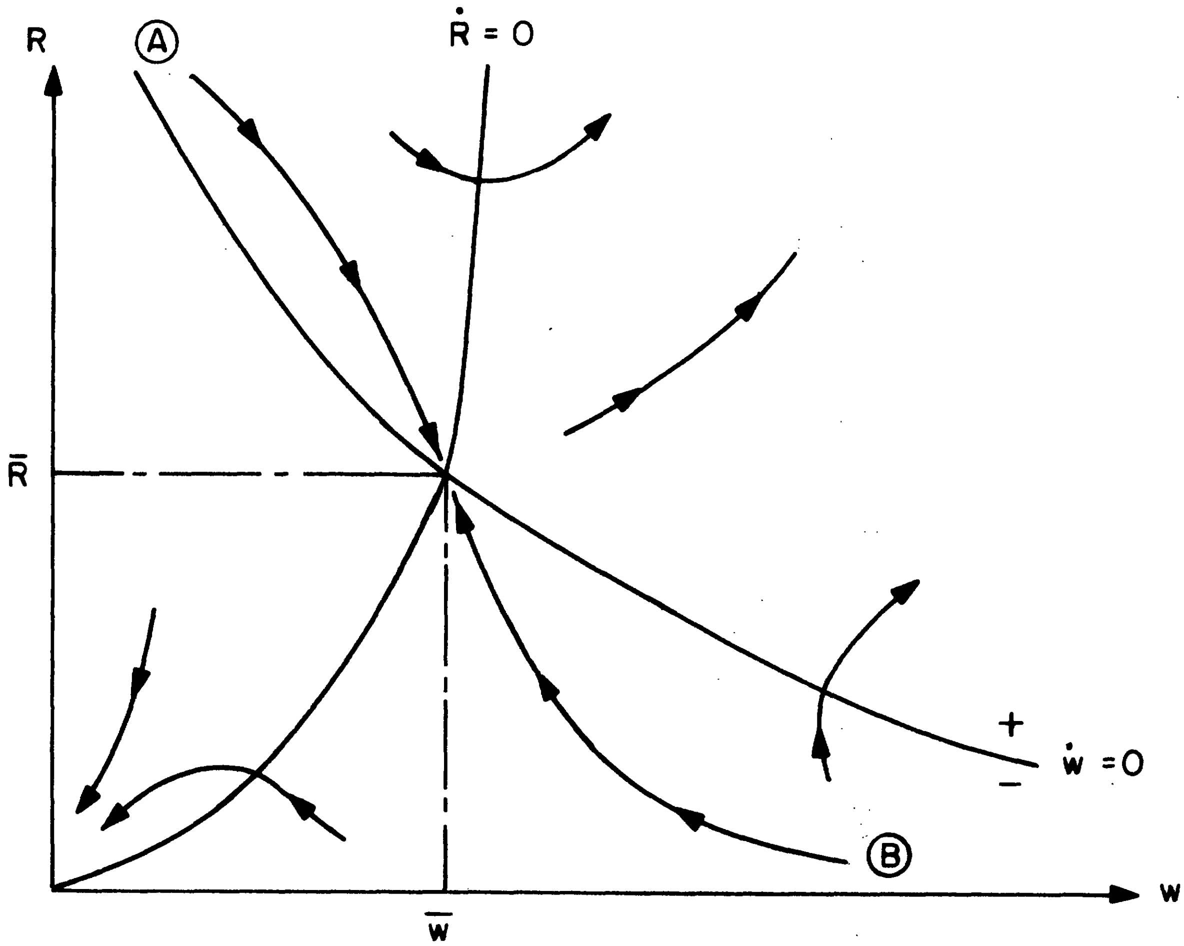

Figure 5: Phase Diagram for Case of No Depletion

We can also see that the optimum steady-state $\bar{w}$, $\bar{R}$, and $\bar{q}$ is independent of initial reserves. Since $\dot{R} = 0$ in the steady-state, $\bar{q} = f(\bar{w})$. The under competition p is taken as given, and $\bar{w}$ is chosen to maximize profit:

$$\max_{\bar{w}} \Pi = pf(\bar{w}) - C_1(\bar{R})f(\bar{w}) - C_2(\bar{w}) \tag{21}$$

Setting $\partial\Pi/\partial\bar{w} = 0$ gives us a relationship between $\bar{w}$, $\bar{R}$, and $\bar{p}$:

$$\bar{w} = g(\bar{R},\bar{p}) \tag{22}$$

Since $\dot{w} = 0$, we have from equation (3)

$$\delta C_2'(\bar{w}) + C_1'(\bar{R})f(\bar{w})f'(\bar{w}) = 0 \tag{23}$$

Finally, we have

$$f(\bar{w}) = \bar{q} \tag{24}$$

and

$$\bar{p} = p(\bar{q}) \tag{25}$$

Thus equations (22), (23), (24) and (25) provide a unique solution for $\bar{w}$, $\bar{R}$, $\bar{q}$, and $\bar{p}$ that is independent of the initial conditions. This can be thought of as a "Golden Rule" of reserve accumulation; whatever "endowed" initial reserves are, they will be increased (or, if they are very large, allowed to decline) until a profit-maximizing steady state level is reached.

6. A Numerical Example

It is useful to examine the characteristics of the competitive and monopoly solutions for exploration and production in the context of a specific numerical example. We have therefore specified functional forms for $f(w,x)$, $C_1(R)$ and $C_2(w)$, and fit these to data for oil exploration, discovery, and production in the Permian region of Texas. We do not pretend that this example provides a realistic representation of the real world; the functions themselves are over-simplified, and we ignore important aspects of market structure.[14] On the other hand, by using these functions to compute optimal competitive and monopoly solutions for exploration and production (and comparing these solutions to actual data over the past decade), we can examine the implications of our results in an empirical context.

[14] We are describing the highly complex process of exploration and discovery by a simple deterministic function, the actual market may not be perfectly competitive, we are ignoring problems of common access, and perhaps most important, we are ignoring the effects of government controls. We can only hope to have captured enough of the real world to tell an interesting story!

Although the characteristics of average production cost may be complex, our aim here is only to capture the fact that this cost increases as reserves decrease. We therefore assume for convenience that average extraction costs increase hyperbolically as the proved reserve base goes to zero, i.e.[15]

$$C_1(R) = m/R \tag{26}$$

In 1966 extraction costs were \$1.25 per barrel, and Permian reserves were 7170 million barrels, so we set m = 8960.

We represent the level of exploratory activity by the number of exploratory and development wells drilled each year. Over the years, the cost per well has been lower when the number of wells drilled has been higher, suggesting mild economies of scale. We therefore choose the following cost function:

$$C_2(w) = a_1 w + a_2 \tag{27}$$

Measuring C_2 in millions of 1966 dollars, and w in number of wells, we obtain the following estimated equation using data for 1966-1974 (t-statistics in parentheses):

$$\frac{C_2(w)}{w} = \underset{(5.09)}{0.0670} + \underset{(2.43)}{103.2/w} \tag{28}$$

$R^2 = .458$ S.E. = .0039 $F(1/7) = 5.90$

We assume that the discoveries function is of the form:[16]

$$f(w,x) = Aw^{\alpha}e^{-\beta x}, \quad \alpha,\beta > 0 \tag{29}$$

---

[15] One might argue that aggregate average production cost will rise very slowly over a broad range of reserve levels, and will increase sharply only when reserves become very small. This would suggest the function $C_1(R) = m/R^2$. We use equation (26) since it more closely represents the behavior of production cost at the level of individual pools.

[16] Uhler [19] finds that the following discovery function provides a fairly close fit for oil and gas producing regions in Alberta:

$$f(w,x) = Aw^{\alpha}e^{-\gamma(x-k)^2 - \beta x}$$

Equation (29) is more tractable, and provides a reasonably close approximation to this function if exploration has gone on for some time, i.e. if x is not small.

Actual crude oil reserve additions consist of three components - new discoveries, extensions (discoveries in the vicinity of an existing reservoir, and often part of the same pool), and revisions (changes in the estimates of existing reserves that often result from new information that becomes available after production begins). Although new discoveries and extensions can be seen to have a strong dependence on well drilling and cumulative reserve additions, revisions usually show no such dependence, but indeed behave like a random process with a mean value several times (6.0 in the Permian region) larger than the mean value of discoveries plus extensions.[17] Since we wish to account for reserve additions, and not simply discoveries and extensions, we multiply our data on discoveries plus extensions by the ratio of the mean value of reserve additions to the mean value of discoveries plus extensions. It is this constructed series that we use as "discoveries" in our model, and to which we fit equation (29):

$$\log DISC = \underset{(0.77)}{2.389} + \underset{(1.53)}{0.599} \log w - \underset{(-5.86)}{.0002258}x \qquad (30)$$

$$R^2 = .837 \qquad S.E. = 0.172 \qquad F(2/7) = 17.93$$

Here both DISC and x are measured in millions of barrels.

Finally, we need a market demand function to complete our specification. We use a linear demand function with a price elasticity of -0.1 at a price of \$3.00 and production of 600 million barrels (roughly the average price and production level during the 1965-1974 period):[18]

$$q = 660 - 20p \qquad (31)$$

[17] Which is why a major limitation of this paper is its failure to deal with uncertainty.

[18] The reflects oil demand elasticity estimates for the 1960's, a period during which real oil prices were roughly constant at about \$3. Elasticity estimates for today's higher prices are in the range of -0.2 to -0.5; equation (31) implies an elasticity of -0.45 at a price of \$10. Equation (31) is also consistent with a "backstop" price of \$33; at this price demand becomes zero as oil is replaced with alternative energy sources.

To obtain numerical solutions for this example, we write difference equation approximations to our differential equations for w and p, and substitute in our estimated functions. In the competitive case:

$$p_t = 1.05p_{t-1} - \frac{450}{R_{t-1}} - \frac{9.81x10^4}{R^2} w_{t-1}^{.6} e^{-.000226x_{t-1}} \tag{32}$$

$$w_t = 1.125w_{t-1} - 2.196x10^6 \frac{q_{t-1}}{R_{t-1}^2} w_{t-1}^{.6} e^{-.000226x_{t-1}} \tag{33}$$

To these equations we add the identities

$$x_t = x_{t-1} + 10.9w_t^{.6} e^{-.000226x_t} \tag{34}$$

and

$$R_t = R_{t-1} - q_t + x_t - x_{t-1} \tag{35}$$

To obtain an optimal solution, we repeatedly simulate this model, varying the initial conditions for p_o and w_o until the terminal condition that w, q, and average profit all become zero simultaneously is satisfied. (To obtain a solution to the monopoly case, we replace p_t in equation (32) with marginal revenue, and then obtain an expression for marginal revenue from equation (31)).

Solutions for the competitive case are given in Table 1, and for the monopoly case in Table 2. These solutions are also shown graphically in Figures 6, 7, and 8.

Note that as expected, the competitive price is initially lower, but later higher, than the monopoly price. Since production is initially lower in the monopoly case, less discoveries are needed to maintain the reserve base, so that exploratory effort is smaller. In the competitive case exploration and production cease after about 55 years, but since average production over this period is smaller in the monopoly case, monopoly exploration and production continues for an additional 37 years - although at the points of termination, cumulative discoveries are about the same for the two cases.

Table 1: Solutions to Competitive Case

Year	Production (10^6 barrels/year)	Price ($/barrel)	Rent* ($/barrel)	W-Explor. Activity	Reserves (10^6 barrels)	Cum. Disc. (10^6 barrels)	Profits (10^6 $/year)
1965	552.0	5.400	4.150	9353	7170	0.0	1556
1966	557.0	5.146	4.177	4779	9243	2630	1901
1967	554.9	5.254	4.284	4120	9648	3590	2019
1968	551.9	5.402	4.488	3794	9801	4295	2118
1969	548.5	5.573	4.661	3612	9822	4865	2209
1970	544.7	5.760	4.842	3511	9763	5350	2298
1971	540.7	5.962	5.033	3462	9649	5777	2385
1972	536.4	6.176	5.232	3450	9496	6161	2471
1973	531.9	6.403	5.441	3464	9315	6511	2557
1974	527.1	6.641	5.658	3500	9112	6835	2643
1975	522.1	6.891	5.883	3554	8892	7138	2729
1976	516.9	7.152	6.117	3623	8660	7423	2815
1977	511.4	7.425	6.361	3705	8419	7693	2900
1978	505.7	7.711	6.614	3798	8170	7950	2985
1979	499.8	8.008	6.876	3902	7917	8196	3070
1980	493.6	8.317	7.147	4014	7659	8433	3154
1985	458.8	10.05	7.99	4681	4361	9500	3550
1990	417.0	12.14	10.39	5411	5117	10428	3867
1995	368.1	14.59	12.35	5978	3994	11247	4039
2000	312.4	17.37	14.41	6012	3031	11960	3996
2005	251.0	20.44	16.45	5062	2243	12552	3684
2010	185.4	23.72	18.20	2968	1623	12993	3071
2015	113.6	27.31	19.58	669.8	1159	13244	2075
2020	22.04	31.89	22.12	3.074	917.1	13308	383.7
2021	0.180	32.99	23.23	3.415	917.9	13309	-99.03

*Marginal discovery cost & opportunity cost of additional cumulative discoveries.

Table 2: Solutions to Monopoly Case

Year	Production (10^6 barrels/yr)	Price ($/barrel)	Rent ($/barrel)	W-Explor. Activity	Reserves (10^6 barrels)	Cum. Disc. (10^6 barrels)	Profits (10^6 $/yr.)
1965	303.0	17.85	1.450	3618	7170	0.0	4681
1966	304.8	17.75	1.427	2293	8353	1488	4828
1967	305.2	17.73	1.448	1871	8851	2291	4876
1968	305.1	17.74	1.499	1641	9137	2883	4901
1969	304.9	17.75	1.538	1495	9310	3360	4916
1970	304.5	17.77	1.588	1396	9409	3764	4925
1971	304.1	17.79	1.633	1326	9459	4117	4930
1972	303.6	17.81	1.674	1275	9471	4433	4933
1973	303.0	17.84	1.732	1239	9454	4720	4934
1974	302.4	17.87	1.788	1214	9416	4985	4934
1975	301.8	17.90	1.843	1197	9360	5230	4932
1976	301.1	17.94	1.915	1188	9289	5460	4929
1977	300.3	17.98	1.987	1184	9206	5677	4925
1978	299.6	18.01	2.037	1185	9113	5884	4921
1979	298.8	18.05	2.106	1190	9010	6080	4915
1980	297.9	18.10	2.194	1200	8901	6269	4909
1985	293.1	18.34	2.597	1295	8276	7120	4868
1990	287.1	18.64	3.097	1460	7576	7868	4811
1995	280.0	18.99	3.671	1690	6847	8554	4736
2000	271.5	19.42	4.375	1983	6116	9198	4639
2005	261.4	19.92	5.182	2338	5403	9813	4515
2010	249.5	20.52	6.142	2746	4720	10403	4359
2015	235.7	21.21	7.224	3180	4080	10969	4165
2020	219.8	22.00	8.432	3592	3489	11511	3928
2025	201.7	22.91	9.788	3901	2955	12022	3644
2030	181.5	23.92	11.226	3994	2479	12495	3313
2035	159.1	25.04	12.735	3742	2062	12920	2937
2040	134.8	26.25	14.226	3050	1699	13281	2519
2045	108.0	27.59	15.715	1960	1386	13563	2046
2050	77.60	29.11	17.241	801.6	1123	13751	1481
2055	40.05	30.99	19.396	145.7	934.9	13842	743.2
2056	31.29	31.43	20.042	99.19	912.6	13850	565.5
2057	22.05	31.89	20.798	73.65	897.6	13858	374.5
2058	12.33	32.38	21.706	64.55	891.2	13864	167.6

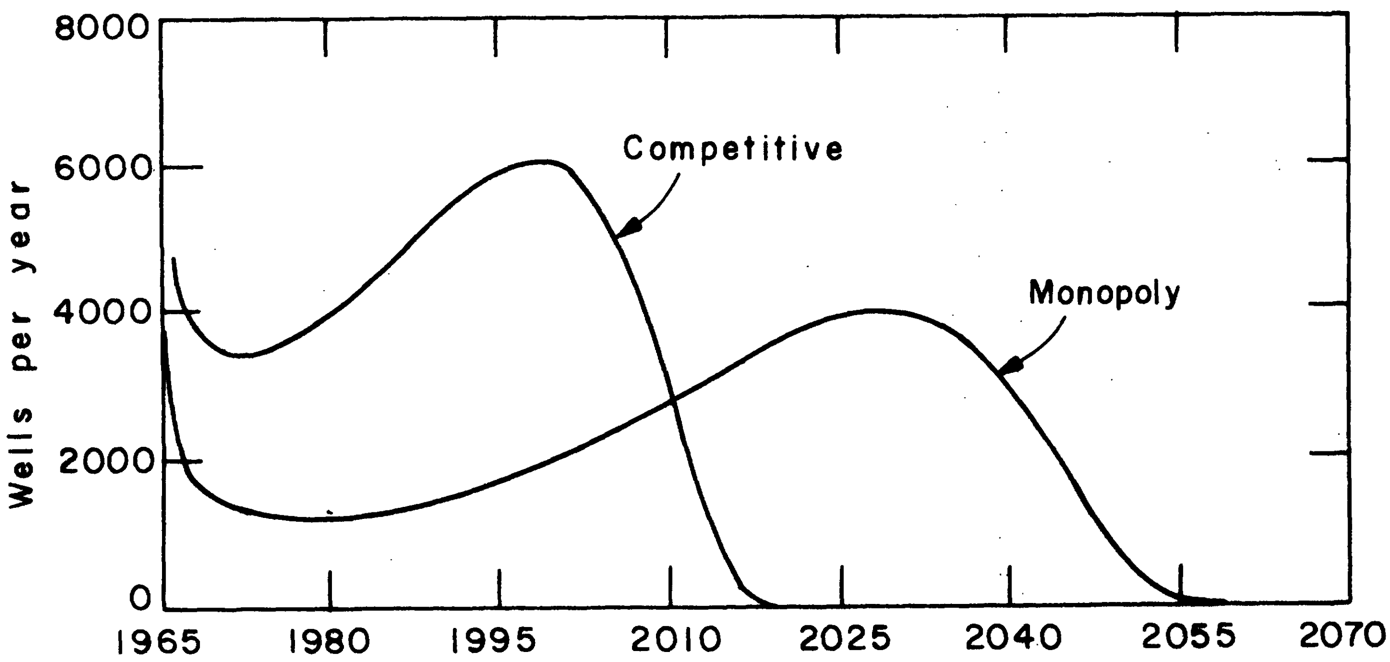

Figure 6: Well Drilling, Competitive and Monopoly Cases

Both the competitive and monopoly cases could be characterized by curve C in the phase diagram of Figure 4. The initial reserve base is too small, and therefore well drilling begins at a high level (so that reserves are quickly increased), falls (to a level sufficient to maintain these reserves for some years), slowly rises over a long period (as depletion reduces the discovery rate per well), and then, over the last 15 or 20 years of the horizon, falls to zero (as production decreases to zero, and proved reserves falls to the level at which extraction cost approaches the cut-off price of $33). Note that discovery rates (the slopes of the cumulative discovery curves in Figure 8) are high only during the first decade or two; discovery rates are lower after this period first because of reduced exploration, and later because of depletion. Thus after reaching a maximum at the end of 5 or 10 years, reserves steadily decline.

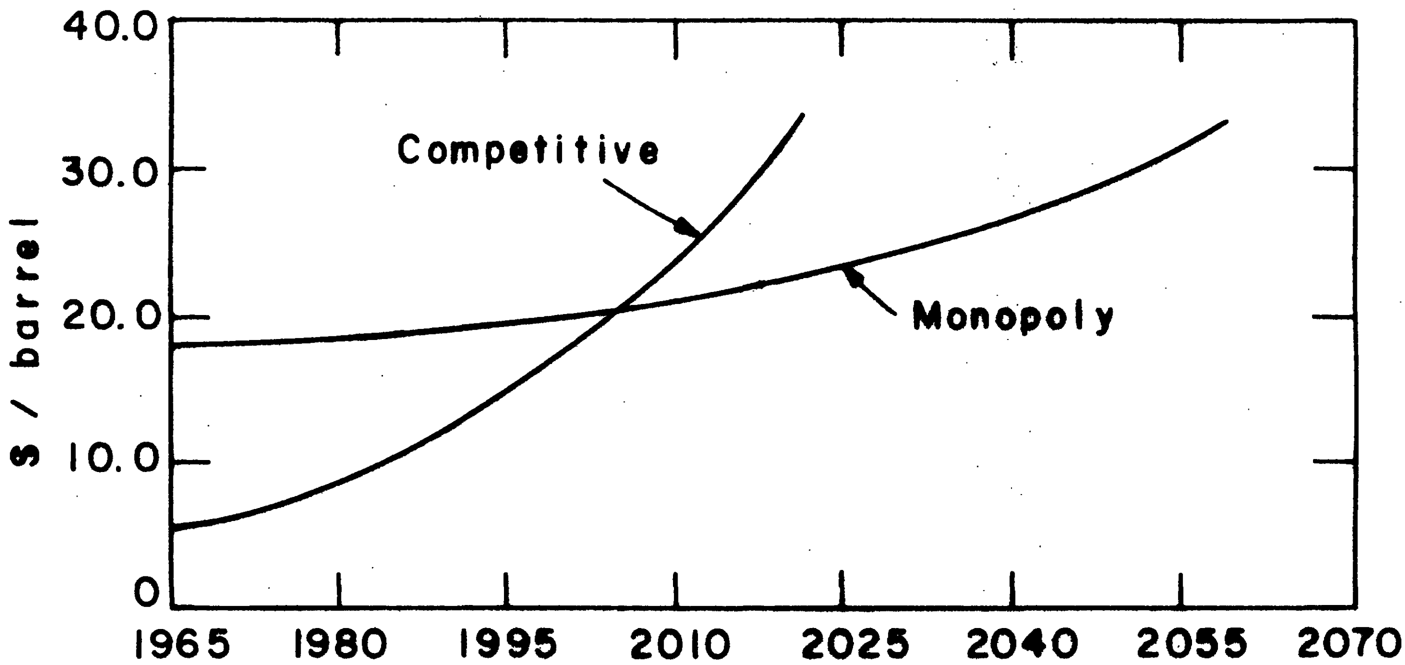

Figure 7: Price Trajectories, Competitive and Monopoly Cases

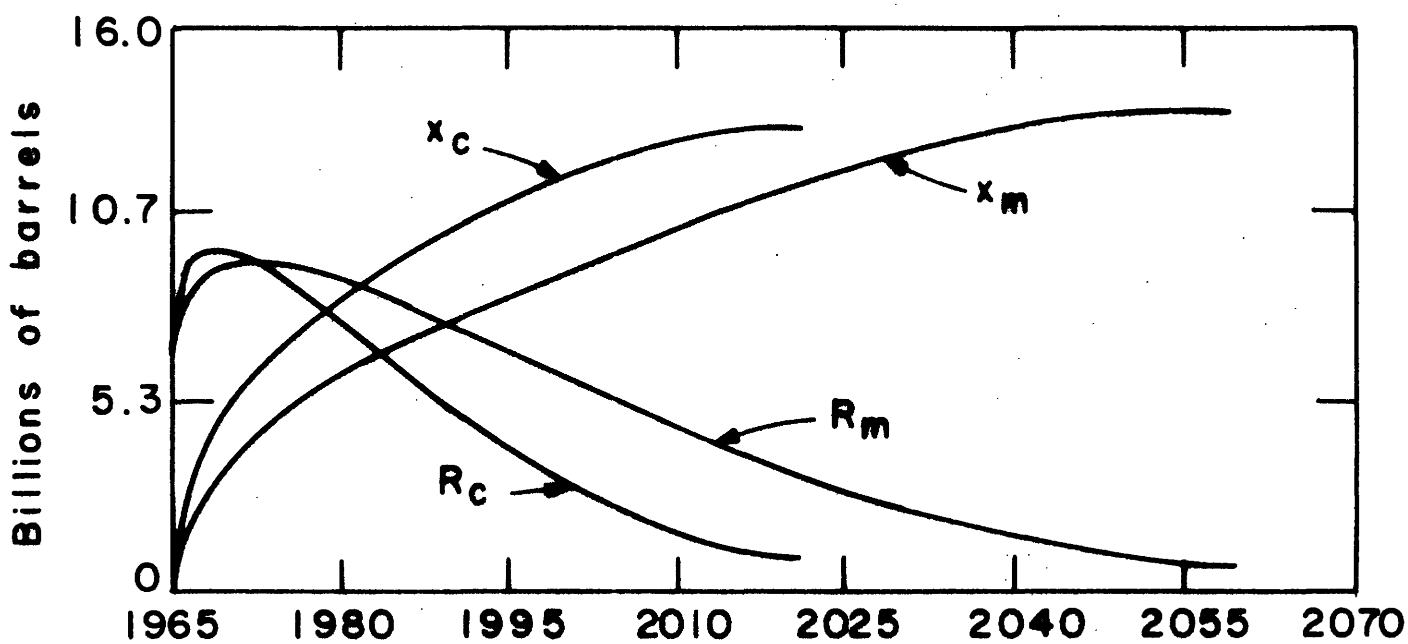

Figure 8: Reserves and Cumulative Discoveries

Suppose that oil in Texas were a non-depletable resource, i.e. that $\beta=0$ in equation (29) so that cumulative discoveries had no effect on the discovery rate per well drilled. In this case, exploratory activity, production, price, and reserves would all approach some steady-state levels. We can determine these levels for our numerical example by applying equations (22), (23), (24), and (25).[19] Doing this, we find that for the competitive case, $\bar{w}$ = 913 wells per year, $\bar{q}$ = 651.4 million barrels per year, $\bar{p}$ = \$0.43 per barrel, and $\bar{R}$ = 54.1 billion barrels. For the monopoly case, $\bar{w}$ = 288, $\bar{q}$ = 326, $\bar{p}$ = \$16.70, and $\bar{R}$ = 43.0. Note that the steady-state prices are always well below the corresponding optimal prices in Tables 1 and 2, and the steady-state reserve levels are much larger than even the highest reserve levels reached when depletion occurs. In the competitive case, for example, no depletion means that well drilling should begin at a high level and then decline towards the steady-state value of 913 wells per year, as reserves are increased to 54 billion barrels. The discoveries resulting from this steady-state well drilling would just be sufficient to replace the steady-state production of 651 million barrels per year. The steady-state price (43¢) will then be somewhat higher than the sum of the marginal extraction cost (16.6¢) and the marginal cost (for an additional barrel of production) of well drilling (15.9¢); the difference between steady-state price and steady-state marginal cost represents the amortized value (per barrel) of the additional well drilling needed initially to raise reserves to the level of 54 billion barrels. The steady-state price is still lower than it would be if depletion occured because extraction costs are lower (a larger reserve base is maintained), marginal discovery costs do not grow over time, and there is no opportunity cost of additional cumulative discoveries. In fact, as can be seen in Tables 1 and 2, when depletion occurs, rent is a large component of price, particularly in later years.

Although we cannot view the simple model used in this example as being very representative of the real world, it is still interesting to compare the optimal

[19] In the monopoly case, equation (22) is obtained bu substituting (25) into (21) before maximizing with respect to w.

values of well drilling, price, reserves, and profits to historical values over the period for which we have data. This is done in Figures 9, 10, 11, and 12. In Figures 9 and 10 we also include the optimal myopic values of price and well drilling, i.e. the prices and amounts of well drilling that would occur if future depletion were ignored but the reserve-production ratio were maintained at its initial level (12.0) from period to period.[20]

We can see that optimal well drilling would have initially been much larger than actual well drilling (so that optimal reserves are larger than actual reserves), but would be close to actual well drilling in later years. In addition, the optimal price is always at least \$2 above the actual price. The higher price, together with slightly lower extraction costs, results in a much greater level of profit.

It might be that oil producers were myopic over the past decade. Note that the myopic price is just below the actual price, and the corresponding myopic pattern of well drilling more closely follows the slow rise in the actual data. Producers might have ignored the future gains from reduced production costs that would have resulted from higher initial well drilling (as in the optimal solution), and might have ignored the opportunity cost component of rent in determining output.[21]

[20] We take the competitive "myopic" price to be the sum of marginal production cost and average well drilling cost. We use average (with respect to output) drilling cost rather than marginal cost because average costs decline with output in our model. Thus this competitive price corresponds to zero profits. Average drilling cost will depend on the amount of exploration needed to maintain the reserve-production ratio, and this will rise over time as depletion ensues. It is easy to show that if the reserve-production ratio is to be constant, the discoveries needed in each period are

$$\Delta x_t = R_{t-1}(q_t/q_{t-1}) + q_{t-1} - R_{t-1}$$

so that necessary well drilling is given by

$$w_t = A^{-1/\alpha} e^{(\beta/\alpha)x_t} [R_{t-1}(q_t/q_{t-1}) + q_{t-1} - R_{t-1}]^{1/\alpha}$$

Since the cost of well drilling is $C_2(w) = a_1w+a_2$, the average cost of exploration is

$$AC_{exp} = (a_1/q_t)A^{-1/\alpha} e^{(\beta/\alpha)x_t} [R_{t-1}(q_t/q_{t-1}) + q_{t-1} - R_{t-1}]^{1/\alpha} + a_2/q_t$$

[21] On the other hand, it is just as likely that the model used in this example simply does not capture enough of the true market structure, costs, etc.

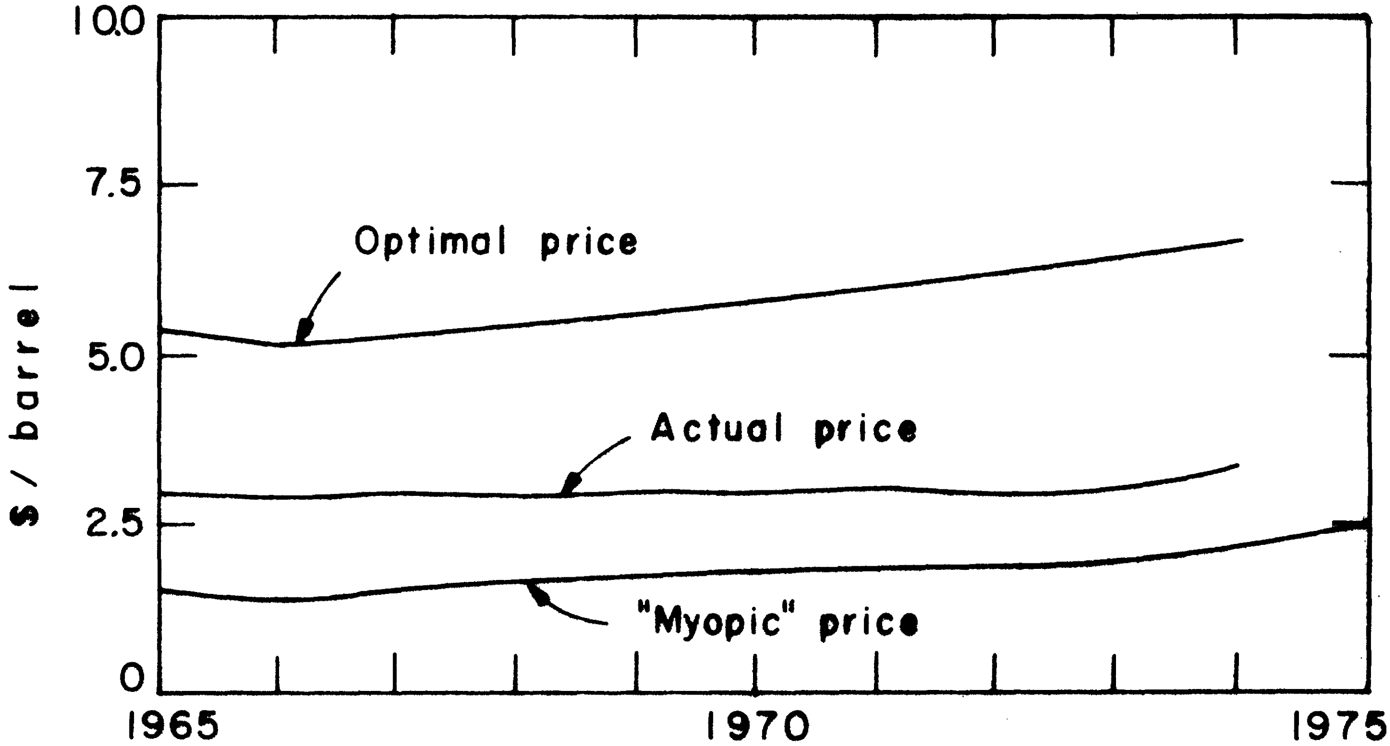

Figure 9: Actual vs. Optimal Competitive Well Drilling

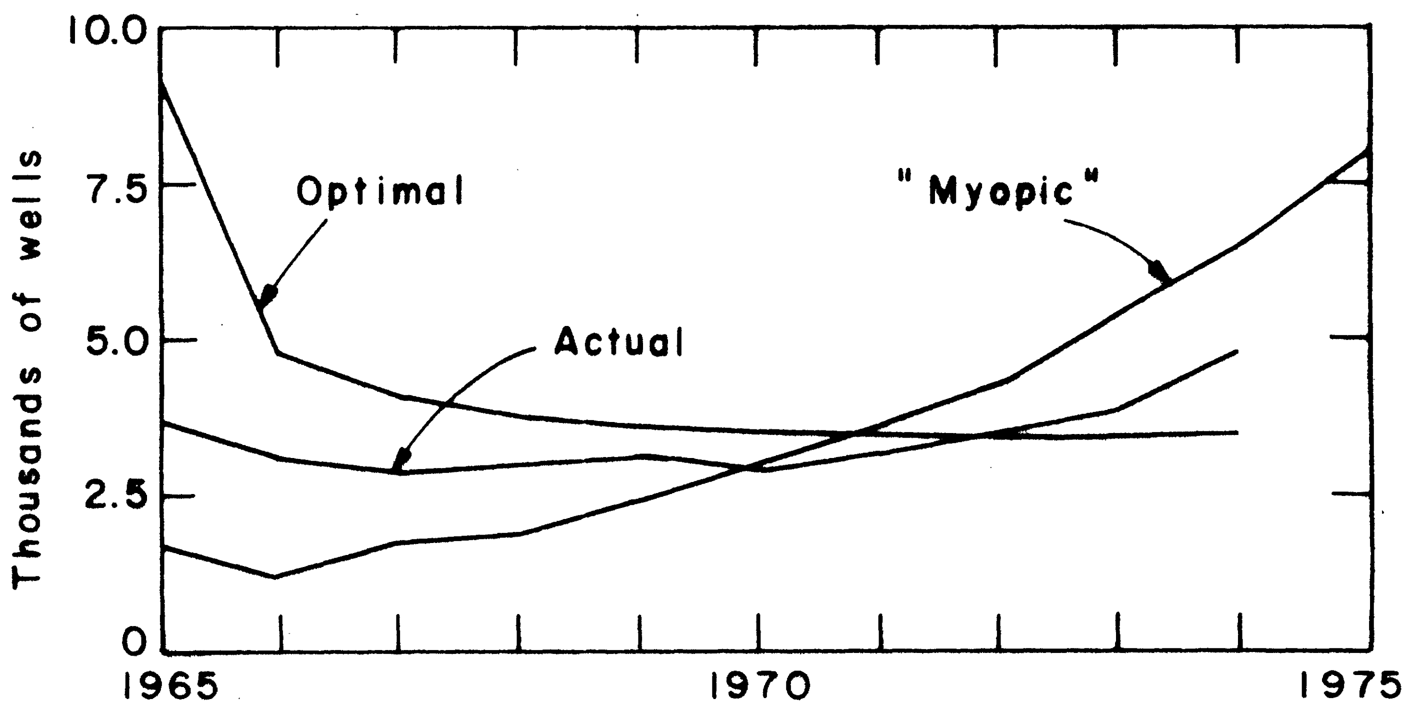

Figure 10: Actual vs. Optimal Competitive Price

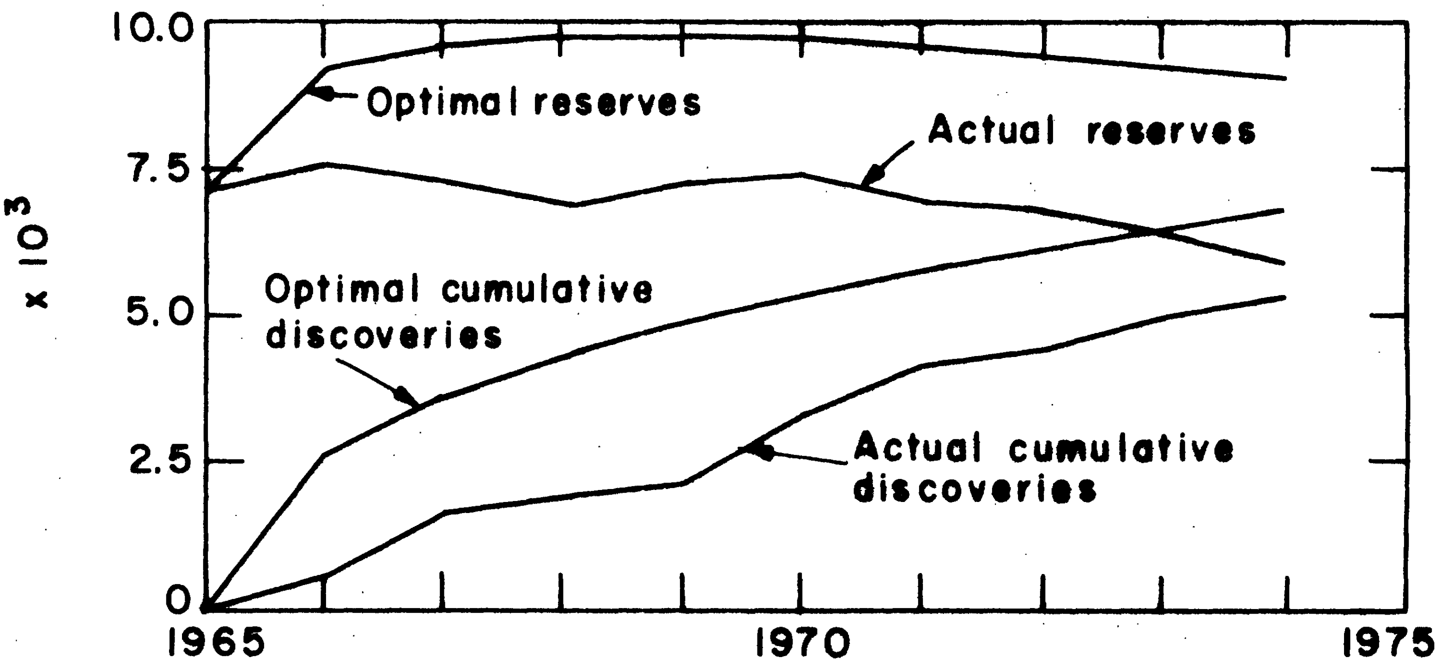

Figure 11: Actual vs. Optimal Competitive Reserves and Cumulative Discoveries

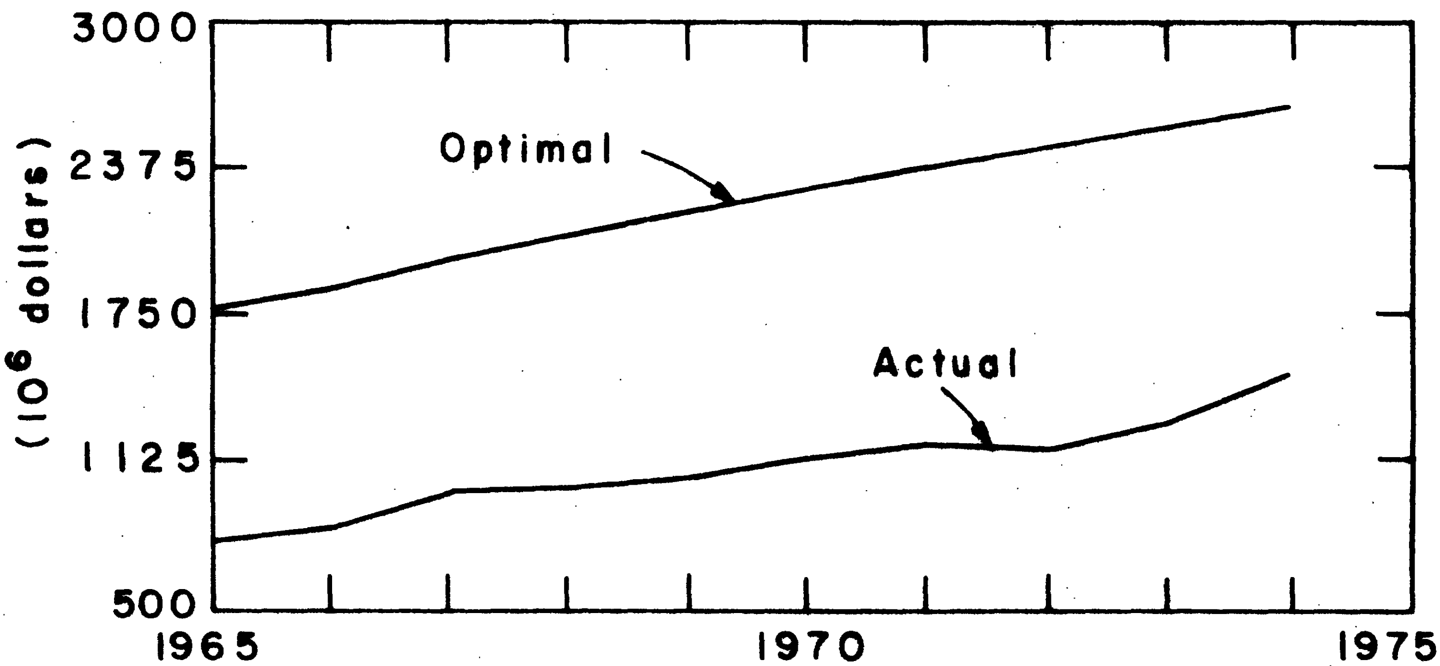

Figure 12: Actual vs. Optimal Competitive Profits

7. Concluding Remarks

We have argued that many "exhaustible" resources could be better thought of as inexhaustible but nonrenewable, and that the optimal rates of exploration and production for these resources are interrelated and must be jointly determined. We saw that exploratory activity has the effect of reducing the rate of increase of price (so that rates of growth of resource rents below market interest rates need not be indicative of monopoly power). We showed that exploratory activity should be chosen to build the reserve base up to an optimal level, and then should be adjusted over time so as to trade off cost savings from postponed exploration with savings from lower extraction costs and revenue gains from greater total production, and therefore the pattern of optimal exploratory activity depends highly on initial reserve levels and on rates of depletion. We suggested the use of "rent" as a scarcity measure, and showed in our simple example how this measure would change to reflect depletion. Finally, we saw that in developing a new resource for which depletion is not significant (but for which exploration and reserve accumulation are necessary), an optimal steady-state reserve level should be reached that is independent of any initial reserve endowment.

Obviously our approach ignored a number of important problems, including the effect of common access, market structures other than monopoly and perfect competition, the effects of government controls, and the effect of uncertainty. This last factor is perhaps the most important deficiency in our approach. Any representation of the response of discoveries to exploratory activity will be an uncertain one, both in terms of specification and estimated parameters, and the presence of uncertainty could significantly alter the "optimal" rates of exploration and production. Despite these shortcomings, we have tried to tell a story that is somewhat more complete than those usually told about nonrenewable resources.

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