AN EXAMINATION OF THE USE OF PROBABILITY MODELING FOR THE ANALYSIS OF INTERFUEL SUBSTITUTION IN RESIDENTIAL FUEL DEMAND

by
Raymond S. Hartman
and
Mark R. Hollyer
(Arthur D. Little, Inc.)

Energy Laboratory Working Paper No. MIT-EL 77-018WP
July, 1977

CONTENTS

ACKNOWLEDGMENTS

Research reported herein was supported by the Energy Research and Development Administration (ERDA) as part of a contract with Arthur D. Little, Inc. Raymond Hartman has subsequently extended the research for analyses for ERDA and EPRI as a staff member of the M.I.T. Energy Laboratory and faculty member at Boston University. The final stages of preparation of this report were supported by the M.I.T. Energy Laboratory.

INTRODUCTION AND SUMMARY

In this world of OPEC, natural gas shut-offs, and nuclear power difficulties, the need is crucial for policy tools to assist the planning of U.S. energy demands and supplies. In the face of that need, the use of probability models for the analysis of fuel demand and interfuel substitution has grown considerably in the recent past. A brief perusal of the bibliography will attest to that fact. Such probability models have provided a powerful set of tools for analyzing the demand for alternative fuels and the capital equipment utilized to burn those fuels for particular residential, commercial or industrial needs.

However, in spite of the varied efforts at probability modeling, a number of difficulties still persist. In the first place, inadequate effort has been directed at quantifying the capital costs of utilizing alternative fuels and the effect of those capital costs for fuel choice. Secondly, the actual model of demand underlying the regression probability formulations has, in many cases, not been explicitly developed. Furthermore, the implicit model of individual choice is not well suited to the data usually utilized for estimation. For example, the probability models purport to explain the choice of an individual consumer facing alternative techniques for providing a given service, say, home heating. The demand model relates the probability of choosing a particular fuel to the relative prices and capital costs of alternative fuels and their related equipment, to economic and other demographic variables. Many empirical applications relate the fuel shares of a given stock of equipment in year t to the independent variables in year t. Such an approach may be be defensible for less-durable durable goods; however, it is difficult to defend it for the durable equipment utilized in home heating unless one is to assume costless and frictionless retrofitting. Given the durability of the housing stock and the difficulty of some types of retrofitting (gas and/or oil to electricity, for example), the fuel shares used in home heating for a given inventory of homes is, in reality, related to a relatively long lagged series of prices and capital costs for alternative fuel and fuel equipment. Probability modeling efforts which don't attempt to deal with the fact will be poorly specified. Alternative formulations could relate the fuel shares in changes in the housing stock to current alternative prices and capital costs.

In the third place, there is some confusion about the relevance of the "assumption" of "independence of irrelevant alternatives." Several authors have utilized it in the analysis of fuel demand and have constrained their estimation procedure as a result. Furthermore, the presence of the "independence of irrelevant alternatives" can substantially ease the computational and analytic difficulties of evaluating new alternatives.[1] The "independecne of irrelevant alternatives" finds its basis in the combination of formal utility/demand analysis and the stochastic assumptions underlying the analysis. The "independence" need not be imposed by general conditional logit formulations.[2] We feel that the "assumption" of the "independence of irrelevant alternatives" can lead to misspecification, and discuss the difficulties in the text and in Appendix A. The econometric analysis in the text compares specifications that impose the "independence of irrelevant alternatives" with those that do not.

---

[1]At the cost, of course, of constant binary odds ratios. See Domencich and McFadden [11], Chaps. 3-5.

[2]See R. Hartman, "A Generalized Logit Formulation of Discrete Consumer Choice", forthcoming.

In light of the potential importance of probability modeling for dealing with energy planning and in light of the aforementioned difficulties found in some of the applications of probability modeling, this paper examines several topics which we summarize here.

Section A summarizes the standard form of the conditional logit model utilizied in many regression applications. We do not examine likelihood formulations (e.g., [13]) here. The underlying theory of individual choice is not pursued; however, references to the source material addressing those issues are provided. The difficulties found in the standard model that have been introduced above are discussed in greater detail. Finally, the standard model is estimated utilizing fuel shares in housing stock data. These results are found to be consistent with most previous analyses: own-price elasticities are usually elastic while cross-price elasticities are usually inelastic. The share eleasticities with respect to gas availability, climatic conditions, and income, accord with expectations. Furthermore, these standard results are compared with results utilizing two alternatives sets of data: 1) fuel shares of the housing stock related to current and past economic and demographic variables as embodied in past fuel shares, and 2) fuel shares in changes in the housing stock related to current economic and demographic variables. The model of individual choice is best approximated by the latter data; the former set of data utilizes lagged fuel shares because the choice model is not adequately approximated utilizing housing stock data.

The elasticity estimates resulting from the analysis of these data sets are similar but generally less elastic that those found in the standard modeling efforts. Finally, Section A utilizes the standard model results (fuel shares in housing inventory) and an alternative model (fuel shares in changes in the housing inventory) to simulate fuel shares 1975 through 2025. The results are found to be similar.

Section B refines the standard model by including the amortized capital costs of alternative fuel equipment. The fuel share elasticities with respect to prices, availability, income and climatic conditions, are found to be relatively stable to the inclusion of capital costs. The capital costs elasticities are extremely elastic. As was found in the standard model, such high elasticities may be due to the fact that fuel shares of the housing stock are analyzed for a cross-section of states; an analysis of the effects of capital costs upon fuel shares of the change in the housing stock is merited.

Finally, Section C presents suggestions for further research that can be pursued to improve and extend this discussion.

The Appendices present supporting information. Appendix A deals explicitly with the criticism that the "assumption" of the "independence of irrelevant alternatives" renders conditional logit useless. We, of course, disagree with that criticism for reasons stated therein. Appendix B presents some detailed simulation results on a regional basis, utilizing the refined model formulations discussed in Section B.

A. THE STANDARD MODEL

Overview

There exists a wide array of regression and likelihood methodologies that have been developed for the analysis of models of qualitative choice. They include linear probability models, probit models, logit models and others.[1] For the purposes of the regression analysis of demand for alternative fuels, the conditional logit formulation has been utilized extensively.[2]

The general logit formulation can be expressed as follows:

$$P_i = F(\alpha + \beta X_i) = F(Z_i) = \frac{1}{1+e^{-Z_i}} = \frac{1}{1+e^{-(\alpha + \beta X_i)}} \qquad (1)$$

where P_i is the probability that an individual makes a particular choice, i, given Z_i, the characteristics of that choice and of the individual; and F is the logistic cumulative distribution function.[3]

---

[1]For a general discussion, see R. Pincyck and D. Rubinfeld, Econometric Models and Economic Forecasts, Chapter 8. For a more detailed discussion of the logit formulation, see H. Theil, "On the Estimation of Relationships Involving Qualitative Variables," American Journal of Sociology, Vol. 76, pp. 103-154, 1970. For an example of analysis for interfuel substitution, see P. Joskow and M. Baughman, "Interfuel Substitution in the Consumption of Energy in the United States, Part I: The Residential and Commercial Sectors," MIT Energy Laboratory Report, #MIT-EL74-002. For a discussion of other forms (.e.g, the Gompit model) and the use of joint estimation, see A. Zellner and T. Lee, "Joint Estimation of Relationships Involving Discrete Random Variables," Econometricia, Vol. 33, April, 1965.

[2]See M. Baughman and P. Joskow, Interfuel Substitution in the Consumption of Energy in the United States, Report #MIT-EL74-002, May 25, 1974; John W. Wilson "Residential Demand for Electricity," Quarterly Review of Economics and Business, 1(1), Spring, 1971; Kent P. Anderson, Residential Energy Use: An Econometric Analysis, Rand Corporation Report R-1297-NSF, October, 1973; William Lin, Eric Hirst and Steve Cohn, Fuel Choices in the Household Sector, Oak Ridge National Laboratory, October, 1976.

[3]Of course, the use of an alternative cumulative distribution function would have required a different model. For example, the cumulative normal distribution would have required the use of the probit model.

This specification can be expressed in a more useful form as follows:

$$
\begin{aligned}
P_i(1+e^{-Z_i}) &= 1 \\
e^{-Z_i} &= \frac{1-P_i}{P_i} \\
e^{Z_i} &= \frac{P_i}{1-P_i} \\
\ln\left(\frac{P_i}{1-P_i}\right) &= Z = \alpha + \beta X_i
\end{aligned}
\qquad (2)
$$

Thus, equation (1) can be reduced in the binary case to relate the log of the odds (or probabilities) of an individual making a particular choice i to the economic variables hypothesized to determine that choice -- the X_i in equations (1) and (2). The X_i's relevant to the fuel choices are developed below.[1]

The conditional logit formulation in equation (2) can be extended to the case where an individual has more than two choices.[2] To extend the binary choice model we write:

$$
\begin{aligned}
\log\left(\frac{P_2}{P_1}\right) &= \alpha_{21} + \beta_{21}X \qquad &(a) \\
\log\left(\frac{P_3}{P_1}\right) &= \alpha_{31} + \beta_{31}X \qquad &(b) \\
\log\left(\frac{P_3}{P_2}\right) &= \alpha_{32} + \beta_{32}X \qquad &(c)
\end{aligned}
\qquad (3)
$$

---

[1]We have clearly not developed a formal model of utility/demand/choice such as those of Domencich and McFadden [11] and Hausman and Wise [14].

[2]The underlying choice model is developed in Hartman [12].

where P_j is the probability that an individual chooses situation j, j = 1, 2 or 3; X is the vector of exogeneous factors (characteristics of <u>all</u> choices 1-3 and individual choosing) influencing the choice; β_{ij} is a vector of estimated coefficients for the log odds ratios (P_i/P_j).

The reader should notice that equations (3a) - (3c) differ from standard regression specifications of logit analysis where homogeneity of tastes and a separable utility function lead to a cancellation of variables that are the same across choices. The inclusion of characteristics of all alternatives in X reflects underlying assumptions of a generalized utility and generalized multinomial logit formulation (see [12]).

It is unnecessary to estimate all three equations in equation (3). The reason is that any one of them can be expressed in terms of the other two. For example:

$$\log\left(\frac{P_3}{P_2}\right) = \log\left(\frac{P_3}{P_1}\right) - \log\left(\frac{P_2}{P_1}\right)$$

$$= \alpha_{31} + \beta_{31}X_i - \alpha_{21} - \beta_{21}X_i$$

$$= (\alpha_{31} - \alpha_{21}) + (\beta_{31} - \beta_{21})X_i;$$

therefore, $\alpha_{32} = \alpha_{31} - \alpha_{21}$ and $\beta_{32} = \beta_{31} - \beta_{21}$.

If equations (3a) and (3b) have been specified and estimated, the estimation of the probabilities of an individual choosing situation j = 1, 2 or 3, utilizes the fact that the choices are mutually exclusive and exhaustive; hence, $P_1 + P_2 + P_3 = 1$. Therefore $\frac{P_3}{P_1} + \frac{P_2}{P_1} + 1 = \frac{1}{P_1}$ and $(\frac{\hat{1}}{P_1}) = 1 - (\frac{\hat{P}_3}{P_1}) - (\frac{\hat{P}_2}{P_1})$.

Utilizing equations 3a and 3b and $\hat{\alpha}_{21}$, $\hat{\beta}_{21}$, $\hat{\alpha}_{31}$, $\hat{\beta}_{31}$ yields $(\frac{\hat{P}_3}{P_1})$ and $(\frac{\hat{P}_2}{P_1})$, hence, $(\frac{\hat{1}}{P_1})$. If the estimates are maximum likelihood, then $\hat{P}_1 = 1/(\frac{\hat{1}}{P_1})$, $\hat{P}_3 = (\frac{\hat{P}_3}{P_1})\hat{P}_1$ and $\hat{P}_2 = (\frac{\hat{P}_2}{P_1})\hat{P}_1$ are maximum likelihood estimates of the actual probabilities.

<u>Application of the General Model to the Analysis of Fuel Shares</u>

Sufficient repetitions of the experiment underlying equations (3a) - (3c) for data cells are required to make OLS (or GLS) a proper estimation procedure.[1] If the unit of choice underlying the model (3) is the household and we have for n households n_1 choosing option #1, n_2 choosing option #2, and n_3 choosing option #3, we can approximate the P_i's as $\hat{P}_1 = \frac{n_1}{n}$, $\hat{P}_2 = \frac{n_2}{n}$, and $\hat{P}_3 = \frac{n_3}{n}$. We may then estimate the logit model in the following form:

$$\log\left(\frac{n_{3/n}}{n_{1/n}}\right) = \alpha_{31} + \beta_{31}X = \log\left(\frac{S_3}{S_1}\right) \text{ and } \log\left(\frac{n_{2/n}}{n_{1/n}}\right) = \alpha_{21} + \beta_{21}X = \log\left(\frac{S_2}{S_1}\right)$$

where S_i is merely the share of the total households choosing option i. X is the single vector of exogenous factors determining the choice. Households are assured homogeneous in this case.

For the specific purposes at hand, S_i is share of households in a given state choosing fuel i for a particular use. X is a vector summarizing fuel prices, income and fuel availability factors in a given state. Individual variables are explained more fully below. Since only three fuels are examined

---

[1]See Pindyck and Rubinfeld, [16], pp. 256-260 and Domencich and McFadden [11]. However, most data used in these analyses are continuous. While Bergson [9] has dealt with the difficulties generated from using continuous data, most authors have ignored them.

(oil, gas and electricity), we specify the gas, oil, and electricity shares in a given state as Sg, So, and Se, respectively. Thus, if we are estimating the effect of prices, income and availability upon the probability of choosing alternative fuels (for home heating or cooking, water heating, etc.), we estimated:

$$\log\left(\frac{So}{Se}\right) = \alpha X \qquad (a)$$

$$\log\left(\frac{Sg}{Se}\right) = \beta X \qquad (b) \qquad (4)$$

where So, Se and Sg are the shares of oil, electricity and gas used for a particular use in a given state, and X is the vector of exogenous price, income and availability factors.

Just as individual probability estimates were obtained above, fuel shares can be estimated given $\hat{\alpha}$, $\hat{\beta}$, X and the fact that $\hat{So} + \hat{Se} + \hat{Sg} = 1$. Thus, for maximum likelihood $\hat{\alpha}, \hat{\beta}$ (dropping the hats (^) for notational ease):

$$\left(\frac{1}{Se}\right) = 1 - \text{antilog}\left(\log\frac{So}{Se}\right) - \text{antilog}\left(\log\frac{Sg}{Se}\right)$$

$$= 1 - \left(\frac{So}{Se}\right) - \left(\frac{Sg}{Se}\right)$$

therefore, $Se = 1/\left(\frac{1}{Se}\right)$; $Sg = Se\left(\frac{Sg}{Se}\right)$; $So = Se\left(\frac{So}{Se}\right)$.

Furthermore, share elasticities are estimated as follows, using the example $e_{SgXi} = \frac{\partial Sg/Sg}{\partial Xi/Xi}$, where Xi is an element of X. We use the facts that:

$$Sg + So + Se = 1 \qquad (a)$$

$$\frac{So}{Se} = e^{\alpha X} \qquad (b) \qquad (5)$$

$$\frac{Sg}{Se} = e^{\beta X} \qquad (c)$$

Using (5a), we have:

$$\frac{1}{Sg} = \frac{Se}{Sg} + \frac{So}{Sg} + 1 \qquad \text{and } Sg = (1 + \frac{Se}{Sg} + \frac{So}{Sg})^{-1}$$

Using the fact that $\frac{Se}{Sg} = e^{-\beta X}$ and $\frac{So}{Sg} = (\frac{So}{Se})/(\frac{Sg}{Se}) = \frac{e^{\alpha X}}{e^{\beta X}}$, we have therefore that:

$$Sg = (1 + \frac{Se}{Sg} + \frac{So}{Sg})^{-1} = (1 + e^{-\beta X} + \frac{e^{\alpha X}}{e^{\beta X}})^{-1}.$$

Since $e_{SgXi} = \frac{\partial Sg}{\partial X_i} \cdot \frac{X_i}{S_g}$, we require $\partial Sg/\partial X_i$.

This is determined as follows:

$$\frac{\partial Sg}{\partial X_i} = \frac{\partial}{\partial X_i}(1 + e^{-\beta X} + \frac{e^{\alpha X}}{e^{\beta X}})^{-1} = -1(1 + e^{-\beta X} + \frac{e^{\alpha X}}{e^{\beta X}})^{-2}[e^{-\beta X}(-\beta_i) + \frac{e^{\alpha X}}{e^{\beta X}}\alpha_i + \frac{e^{\alpha X}}{e^{\beta X}}(-\beta_i)]$$

Therefore, for e_{SgXi}, we have:

$$e_{SgXi} = \frac{\partial Sg}{\partial X_i}\frac{X_i}{Sg} = -1(1 + e^{-\beta X} + \frac{e^{\alpha X}}{e^{\beta X}})^{-2}[e^{-\beta X}(-\beta_i) + \frac{e^{\alpha X}}{e^{\beta X}}(\alpha_i) + \frac{e^{\alpha X}}{e^{\beta X}}(-\beta_i)]\frac{X_i}{(1+e^{-\beta X} + \frac{e^{\alpha X}}{e^{\beta X}})^{-1}}$$

$$= -1(1 + e^{-\beta X} + \frac{e^{\alpha X}}{e^{\beta X}})^{-1}[\frac{e^{\alpha X}}{e^{\beta X}}(\alpha_i X_i) - \beta_i X_i(e^{-\beta X} + \frac{e^{\alpha X}}{e^{\beta X}})]$$

The elasticity estimates for the other two fuel shares with respect to an independent variable X_i are estimated in similar fashion.

Hypothesized Factors Affecting Fuel Shares

The principal focus here is the factors affecting the fuel shares. The logit specification in equation (1) relates the cumulative probability (F) of choosing a particular option to a set of determining variables X.[1] In equations (4a) and (4b), this basic relationship has been extended to the log of the ratio of fuel shares for particular residential uses. The residential uses usually analyzed include home heating, water heating, clothes drying and cooking. For all of these uses, wood, solar and coal presently contribute insignificantly; gas, oil, and electricity are the major fuels. Furthermore, for some uses, such as cooking and clothes drying, only gas and electricity serve as substitute fuels. For the remainder of this paper we analyze the fuel shares utilized utilized in home heating.

The vector X includes relative price terms, income, demographic factors and supply factors. The full list of exogenous variables to be considered is as follows:

Pe: User cost of electricity in $/(10^6 BTU) for a given state, calculated from the average cost of the first 250 kWh/mo consumed. Typical Electric Bills, 1960, 1970; Federal Power Commission, Washington, D.C.

Pg: User cost of natural gas in $/(10^6 BTU) for a given state, averaged consumer cost. Gas Facts, 1961, 1971; American Gas Association, Arlington, VA.

Po: User cost of oil in $/(10^6 BTU) for a given state, derived from American Petroleum Institutes, Petroleum Facts and Figures, 1971 edition; wholesale prices multiplied by retail markup of 54% in 1960 and 78% in 1970. Markups are the difference between the average Bureau of Labor Statistics price and API's.

---

[1]More extended discussions of the implications of this relationship are found in the sources listed in footnote #1, page 1.

PCI: State per capita incomes from U.S. Department of Commerce, Bureau of Economic Analysis, Survey of Current Business, August 1976, Volume 56, No. 8.

TEMP: A variable proxying the severity of climatic conditions in each state. It is annual heating degree days for each state, averaged over 1931-1960. State weighted average degree days were calculated on an SMSA basis by percent of a state's population residing in SMSA's for which heating degree day data were available. Similarly, the state averages were aggregated into regions using data on percent of households in each region by state. The data on heating degree days by region come from the ASHRAE 1973 Systems Handbook in Chapter 43 Energy Estimating Methods. The data were provided for United States cities from a publication of the United States Weather Bureau, Monthly Normals of Temperature, Precipitation, and Heating Degree Days, 1962, and are the period 1931 to 1960, inclusive. These data also include information from the 1963 Revisions to this publication, where available. The yearly totals are based on 65°F.

AV: Availability index computed as the number of distribution main miles per state resident, multiplied by 100. Source: Gas Facts, American Gas Association, 1961 and 1971 Editions; U.S. Census of Population, 1960, 1970.

RU: Rural-urban dummy variable assuming a value of 1 for the rural segment of the sample.

REG1: Dummy variable equal to 1 for Northeast region (according to Census Bureau definition).

REG2: Dummy variable equal to 1 for North Central Region.

REG3: Dummy variable equal to 1 for the South region.

REG4: Dummy variable equal to 1 for the Western region.

Observations of these variables for both urban and regional areas in all 50 states and Washington, D.C have been gathered for 1950, 1960, and 1970. Hence, a given year's data base provides 102 observations.

Notice that alternative costs of the relevant equipment have not been included here. They are introduced in Section B.

Utilizing these exogenous variables, equations (4a) and (4b) become:

$$\log\left(\frac{So}{Se}\right) = \alpha_1 Pe + \alpha_2 Pg + \alpha_3 Po + \alpha_4 PCI + \alpha_5 TEMP + \alpha_6 AV + \alpha_7 RU + \alpha_8 REG1 + \alpha_9 REG2 + \alpha_{10} REG3 + \alpha_{11} REG4 \quad (6a)$$

$$\log\left(\frac{Sg}{Se}\right) = \beta_1 Pe + \beta_2 Pg + \beta_3 Po + \beta_4 PCI + \beta_5 TEMP + \beta_6 AV + \beta_7 RU + \beta_8 REG1 + \beta_9 REG2 + \beta_{10} REG3 + \beta_{11} REG4 \quad (6b)$$

Our priors on the price variables are $\alpha_1>0$, $\alpha_3<0$, $\beta_1>0$, and $\beta_2<0$. Our prior for gas availability in (6b) is $\beta_6>0$. The priors for the cross-price coefficient of the price of the fuel not included in the dependent variable (e.g., Pg in (6a), and Po in (6b)) are unclear; the price effect will depend upon the relative effect upon both other fuels. The priors for PCI are $\alpha_4>0$ and $\beta_4>0$. The relative impact of temperature, rural/urban location and region are discussed with the results below.

Equations (6a) and (6b) are the basic forms applied to the explanation of fuel shares for the residential sector for home heating. In some cases, a third equation for $\left(\frac{Sg}{So}\right)$ is also estimated.

Difficulties with the Standard Model

Three difficulties arise in utilizing equations (6a) and (6b) for specifying and estimating the determinants of fuel shares. In the first place, these equations are better specifications when the share data (So, Sg, Se) refer to changes in the stock of consumer durables or the housing stock, rather than the stock itself. This is particularly true for the stock of home heating equipment. The fuel shares of a housing stock that include a range of vintages from very new to very old are a crude representation of the underlying model of consumer choice. These shares are determined by current fuel prices and availabilities only to a small extent unless rational expectations are operating to a superlative degree. Generally, additions to the housing stock in a given year amount to 1-2% of the stock. As a result, the effect of patterns of past fuel prices and availabilities should be built into the explanation if observed fuel shares of the existing housing stock are utilized to measure consumer choice. For a region like New England, the current fuel shares of the existing housing stock are a lagged function of past prices, incomes and availabilities. This lag could be anywhere from 10-100 years, depending on the age of the housing stock and the expense of retrofitting (i.e., the capital costs incurred to switch from one fuel to another for a given structure). Many analyses do not deal with this difficulty.[1] Our analysis does not examine this difficulty in the level of detail that would be desirable. However, some lagged specification of prices, income and availability have been tried; furthermore, several specifications utilizing changes in the housing stock have been estimated and reported. These specifications and estimations are discussed below.

A second difficulty can arise if the "assumption" of the "independence of irrelevant alternatives" underlying the logit specification[2] is valid.

---

[1]For example, Baughman and Joskow [4], Wilson [19], and Anderson [1, 2 and 3] do not. It must be admitted that later versions of the Baughman and Joskow analysis attempt to deal with this shortcoming through a partial adjustment formulation.

[2]See H. Theil, "A Multinomial Estimation of the Linear Logit Model," [17], and J. Hausman, "Project Independence Reports: An Appraisal U.S. Energy Needs up to 1985" [13]. It is shown in Hartman [12] that this assumption is a product of two factors: usual assumptions regarding utility in addition to the assumed stochastic structure of the problem.

It can be demonstrated[1] that if this assumption is valid, then equations (4a-b) can be written:

$$\log\left(\frac{So}{Se}\right) = f\left(\frac{Po}{Pe}, X\right) \qquad (7a)$$

$$\text{and } \log\left(\frac{Sg}{Se}\right) = g\left(\frac{Pg}{Pe}, X\right) \qquad (7b)$$

or, more generally,

$$\log\left(\frac{Si}{Sn}\right) = Fi\left(\frac{Pi}{Pn}, X\right) \qquad (7c)$$

where X includes nonprice exogenous factors.[2] Given equations of the form (7c), it can be demonstrated[3] that "the cross-elasticities between the i^{th} quantity and the j^{th} price is constant for all $j \neq i, n$: that is,

$$\frac{\partial \log Qk}{\partial \log Pj} = \frac{\partial \log Qm}{\partial \log Pj}$$

This implies "for instance, the cross elasticity of coal with respect to fuel price is identical to the cross elasticity of natural gas with respect to fuel oil price."[3]

However, this difficulty is inherent in the combination of assumed utility and stochastic structure.[4] Certainly, if equation (7c) is the proper specification, then the mathematics require that:

$$\frac{\partial \log Qk}{\partial \log Pj} = \frac{\partial \log Qm}{\partial \log Pj},$$

as stated above.

---

[1]Jerry Hausman, [13].

[2]Formally, X should not include any variables common to the choice of i over n. See [11], [12] and [14].

[3][13], p. 649.

[4]See Hartman [12].

However, this becomes a problem only if there exist 2 or more other fuels j where $j \neq i$ or n in equation (7c). Since we are dealing with only 3 fuels, then for fuel k, we have

$$\frac{\partial \log Qk}{\partial \log Pj} = \frac{\partial \log Qk}{\partial \log Pj},$$

hardly something to worry about.

Furthermore, this criticism is relevant to the combined models of utility/demand and stochastic structure rather than the logit specification. We have specified the logit formulation[1] so that all price terms affect the fuel shares. Hence, rather than (7c), our specification is:

$$\log \left(\frac{Si}{Sn}\right) = F(Pi,\ Pn,\ Pj,\ X) \tag{7d}$$

where Pj includes all other fuel choice prices.

The assumption is, of course, that $\frac{\partial Sj}{\partial Pj} \neq 0$ and since $\Sigma S_k = 1$, $\frac{\partial Si}{\partial Pj} \neq 0 \neq \frac{\partial Sn}{\partial Pj}$. Therefore, Pj can affect both Si and Sn, hence $\log(\frac{Si}{Sn})$. The inclusion of Pj in equation (7c) to generate (7d) renders the criticisms unnecessary. This point is discussed more fully in Appendix A.

The third difficulty is that equations (6a) and (6b) ignore capital costs which will also have an important effect upon fuel choice. Such capital costs of alternative home heating equipment have been left out of most probability model analyses of alternative fuel demand. Those studies which have included capital costs have utilized a single capital cost across all fuel-burning equipment.[2] Clearly, what is required is the inclusion of

---

[1]Such a generalized logit formulation is discussed in [12].

[2]See W. Lin, E. Hirst and S. Cohn, "Fuel Choice in the Household Sector," [15]. They use the capital cost of oil-burning equipment.

capital costs for alternative fuel equipment. These capital costs are included in Section B, when the model is refined.

Model Estimation and Comparison with Other Efforts

The introduction and the preceding discussion have indicated particular difficulties with the usual probability modeling of fuel demand. As a result, the basic formulations found in equations (6a) and (6b) are extended to three sets of data here. We classify the analysis into three areas:

Category 1

Explanation and prediciton of fuel shares in the existing inventory of housing units as a function of current and lagged prices, incomes, availability, temperature and other demographic characteristics. This category is most consistent with the received literature.[1]

Category 2

Explanation and prediction of fuel shares in the existing inventory of housing units as a function of current and lagged prices, incomes, availability, etc., in addition to past fuel shares. The inclusion of past fuel shares will indicate the effect of past fuel choices upon the current fuel mix in the housing inventory.[2]

Category 3

Explanation and prediction of fuel shares in the net additions to the housing stock as a function of current and lagged prices, incomes,

---

[1]Baughman and Joskow, [4], Wilson, [19], Anderson, [1, 2 and 3], Lin, Hirst and Cohn [15].

[2]Experimentation with lagged shares is found in [5] and [7].

availability, temperature and other demographic characteristics. This category is most consistent with analyzing the underlying model of consumer choice.

The results of all the equational specification and estimation are not reported here.[1] (They are available to the interested reader if desired.) The best equations from each category are discussed. They are best in the sense of statistical and predictive performance.

The sets of equations are discussed here for each category.[2] They are:

Category 1

Ai) $$\text{Log}\left(\frac{Sg}{So}\right) = \underset{(5.07)}{.409548}\ Pe + \underset{(1.62)}{.73698}\ Po - \underset{(-7.39)}{2.56219}\ Pg - \underset{(-1.49)}{.000268565}\ PCI - \underset{(-7.40)}{.000344065}\ TEMP + \underset{(8.86)}{.00958623}\ AV - \underset{(-11.38)}{2.01589}\ RU$$

R^2 = .89
D.W. = 1.20
S.e. = .88

Aii) $$\text{Log}\left(\frac{Sg}{Se}\right) = \underset{(5.55)}{.336974}\ Pe - \underset{(-3.94)}{1.35637}\ Po - \underset{(-4.24)}{1.10485}\ Pg + \underset{(2.36)}{.000319227}\ PCI + \underset{(4.15)}{.000144928}\ TEMP + \underset{(6.77)}{.0055975}\ AV - \underset{(-10.93)}{1.45322}\ RU$$

R^2 = .80
D.W. = 1.20
S.e. = .66

B) Same as A) but with the coefficient of PCI set to zero.

---

[1]OLS was used throughout.

[2]We have not restricted ourselves to reporting only equations of the form (6a) and (6b). Notice log (Sg/So) is also utilized.

Category 2

Ai)
$$\text{Log}\left(\frac{Sg}{So}\right) = \underset{(1.05)}{.086728\ Pe} - \underset{(-3.65)}{1.04641\ Pg} - \underset{(-.08)}{.0787107\ Po} + \underset{(1.88)}{3.72053\ Sg(-10)}$$
$$+ \underset{(.21)}{.442757\ Se(-10)} - \underset{(-1.19)}{2.47104\ So(-10)} + \underset{(1.79)}{.000288014\ PCI}$$

R^2 = .91
D.W. = 1.56
S.e. = .79

Aii)
$$\text{Log}\left(\frac{Sg}{Se}\right) = \underset{(1.43)}{.117855\ Pe} - \underset{(-3.26)}{.926836\ Pg} - \underset{(-2.26)}{2.23137\ Po} + \underset{(2.46)}{4.81915\ Sg(-10)}$$
$$- \underset{(-1.86)}{3.92376\ Se(-10)} + \underset{(1.43)}{2.93912\ So(-10)} + \underset{(4.41)}{.000705372\ PCI}$$

R^2 = .73
D.W. = 1.09
S.e. = .78

Bi)
$$\text{Log}\left(\frac{Sg}{So}\right) = \underset{(2.04)}{.165084\ Pe} - \underset{(-3.05)}{.951740\ Pg} + \underset{(.42)}{.412226\ Po} + \underset{(.76)}{1.62931\ Sg(-10)}$$
$$- \underset{(-.73)}{1.57422\ Se(-10)} - \underset{(-1.75)}{3.8085\ So(-10)} + \underset{(1.49)}{.000232622\ PCI}$$
$$- \underset{(-2.61)}{.000113779\ TEMP} + \underset{(2.30)}{.00263748\ AV}$$

R^2 = .92
D.W. = 1.37
S.e. = .74

Bii)
$$\text{Log}\left(\frac{Sg}{Se}\right) = \underset{(.46)}{.0348794\ Pe} - \underset{(1.54)}{.459603\ Pg} - \underset{(-1.92)}{1.78528\ Po} + \underset{(1.62)}{3.31976\ Sg(-10)}$$
$$- \underset{(-2.16)}{4.44449\ Se(-10)} + \underset{(.41)}{.856141\ So(-10)} + \underset{(4.24)}{.000630998\ PCI}$$
$$+ \underset{(4.70)}{.000195757\ TEMP} + \underset{(.99)}{.00108248\ AV}$$

R^2 = .78
D.W. = 1.13
S.e. = .71

Category 3

Ai)

$$\log\left(\frac{So}{Se}\right) = .466072\ Pe - 7.40945\ Po + 2.28009\ Pg - .000213175\ PCI$$

(1.66) (-4.77) (2.75) (-.23)

R^2 = .18
D.W. = 1.69
S.e. = 3.70

Aii)

$$\log\left(\frac{Sg}{So}\right) = -.202231\ Pe + 6.52106\ Po - 3.853\ Pg + .00105059\ PCI$$

(-.63) (3.65) (-4.04) (.97)

R^2 = .16
D.W. = 1.61
S.e. = 4.33

Bi)

$$\log\left(\frac{So}{Se}\right) = .556738\ Pe - 5.33321\ Po - .00271306\ Pg + .000448728\ PCI$$

(1.69) (-3.32) (-.22) (.44)

$$+ .000093886\ TEMP - .0159395\ AV$$

(.41) (-2.58)

R^2 = .23
D.W. = 1.77
S.e. = 3.59

Bii)

$$\log\left(\frac{Sg}{So}\right) = -.462149\ Pe + 4.45602\ Po - 1.08877\ Pg + .0000281986\ PCI$$

(-1.20) (2.37) (-.75) (.24)

$$+ .00011884\ TEMP + .0185927\ AV$$

(.44) (2.57)

R^2 = .23
D.W. = 1.63
S.e. = 4.20

The equations detailed above merit some comment. The R^2, Durbin Watson (D.W.) statistics, standard error (S.e.) of regression, and t statistics (Ho: parameter coefficient = zero) are reported for all equations. The first set of equations for Category 1 relate fuel shares of the housing stock in a given year to the fuel prices (Pe, Po, Pg), per capita income (PCI), temperature (TEMP), gas availability (AV) and a variable correcting for urban or rural location (RU) for that year. The t statistics of most coefficients are quite good. The coefficients for Po, Pg, AV in Ai, are the expected signs. The rural dummy indicates oil is more popular in rural areas. This perhaps captures the greater availability of oil in rural areas where gas pipelines may be less prevalent given the small consuming base.

In equation Aii, the coefficients of Pe, Pg and AV are again the correct signs. The coefficient of PCI suggests that there exists a positive income effect in the choice of gas over electricity. The coefficient of TEMP, the number of heating degree days is the correct sign, it indicates that as the heating degree days rise in colder climates (i.e., there are more heating days required), gas is favored over electricity. This apparently captures the fact that in colder climates operating costs seem to dominate fuel choice and the operating costs associated with gas are favorable. In warmer climates where the overall heating cost is lower, the capital costs will dominate a fuel choice decision, making electricity more desirable. The rural areas are less inclined to use gas than electricity, given the sign of RU. This also makes sense, given the economics of small scale gas distribution in rural areas, and the presence of a rural electrification program in this country.

The second set (B) is the same as A except the coefficient of PCI is set to zero. The coefficient of PCI is not significantly different from zero in equation A ; furthermore, the sign is different from expectations. As a result, we felt the coefficient of PCI might affect fuel share projections in ways we did not believe during simulation. Hence, it was dropped in equations B for comparison with the simulation results using equations A.

The equations in Category 1 utilize the same type of data used by most of the literature; to wit, fuel shares in the current inventory of houses are related to current exogenous variables in the conditional logit formulation. The problems of this approach have been documented above. Let us examine the sensitivity of the results of the use of such data by examining the other 2 categories of equations.

The equations in Category 2 relate the share variables for housing stock, Sg/So and Sg/Se, to current fuel prices (Pe, Pg and Po) per capita income (PCI), temperature (TEMP), the availability of gas (AV), and the fuel shares that characterized the housing stock ten years before (Sg(-10), So(-10), and Se(-10)). The purpose is, of course, to incorporate the effects of past prices and other variables upon past fuel decisions and the effect of those decisions upon the composition of the current housing stock.

As expected in the equations for Sg/So, Sg(-10) has a positive effect and So(-10) has a negative effect. Likewise, in the Sg/Se equations, Sg(-10) has a positive sign, while Se has a negative effect. (The actual elasticities are presented below.) In equations B of Category 2, the coefficients of Pg, Po, Sg, So, PCI, TEMP and AV have the correct signs in Bi. In Bii, the coefficients of Pg, Sg, Se, TEMP and Av have the correct signs; Pe has the wrong sign but is insignificantly different from zero.

The equations in Category 3 relate the fuel shares So/Se and Sg/So of net changes in the housing stocks (i.e., additions minus deductions) to fuel prices (Pe, Po, and Pg) and income (PCI) in the first set and to these variables plus temperature (TEMP), and gas availability (AV) in the second set. The statistical performance of the second set is not good; of the 12 coefficients estimated, only four are statistically different from zero. This is disappointing because it is the data relating the fuel shares of changes in the housing stock that most nearly captures the individual choice model underlying the logit formulation. However, since net changes are being explained (for data availability reasons), there is an aggregation problem in that the share of alternative fuels represented in the net additions will be different from that of the gross additions. It is gross additions that most closely capture the individual choice model. Further effort is required in developing that data. In any case, in equation Ai of Category 3, the coefficients of Pe, Po and PCI are the expected sign. In Aii, the coefficients of Po, Pg, and PCI are the correct sign. In Bi, the signs of the coefficients of Po, Pg, PCI, TEMP and AV are of the correct sign.

Table 1 summarizes the elasticity estimates that are derived from the Category 1, 2 and 3 equations. Furthermore, these results are compared with the price elasticities of four other analyses.[1] On the whole, the elasticity estimates look good; where the signs of the elasticities are incorrect, the elasticity estimates are usally not significantly different from zero. All own-price elasticity estimates are negative except for the own-price elasticity for electricity in equation B of Category 2. However, that estimate is numerically close to zero. The cross-elasticities are usually

---

[1]Table 1D comes directly from Lin, Hurst and Cohn [15], Table 1.

positive; again, when they are not, the estimates are usually not statistically different from zero. The lagged share elasticities are the correct signs. The gas availability elasticity is the correct sign for all equations and significant. The elasticities of income in Table 1A are suspect given the priors mentioned above (p. 13); likewise, in Tables 1B and 1C, the income elasticity of oil is always positive, electricity always negative, and gas sometimes positive and sometimes negative. We expected a positive income effect away from oil and toward gas and electricity. Furthermore, these results differ from the empirical work of Lin, Hirst and Cohn [15], and Baughman and Joskow [4]. We are not sure why our results differ. We do feel that the statistical estimates of the income elasticity of the oil share are spurious, reflecting the fact that states with high per capita incomes have been generally industrialized and such industrialization has as a concommitent, an infrastructure of oil supply for industrial use.[1] Furthermore, many of these states, particularly in the Northeast, have been characterized by a lack of availability of gas. As a result, oil has been used for home heating given its availability. However, we do not feel that the causal link is from demand, particularly for single family units. We also feel that the income elasticity for electricity is positive, and will continue to be so. As a result, the interpretation of the income elasticities must be cautious.

---

[1]Baughman and Joskow faced a similar problem in the price elasticity for total industrial energy demand which they solved with ease by breaking out demand into regional components. See [5]. Since the analysis performed here is cross-sectional, we feel a pooled time-series cross-sectional analysis would eliminate this unexpected income elasticity. Such pooling occurs in Baughman and Joskow [5].

Table 1A) ELASTICITY ESTIMATES FOR EQUATIONS IN CATEGORY 1

Share Elasticities for Equations A: Elasticities with PCI Included

	Pe	Pg	Po	PCI	TEMP	AV	RU
Se	-2.250	0.910	2.385	-1.090	-.762	-1.142	.553
Sg	.296	-.250	.107	-.081	-.140	.224	-.084
So	-2.504	2.148	-1.040	.714	1.259	-1.916	.713

Share Elasticities for Equations B: Elasticities excluding PCI

	Pe	Pg	Po	TEMP	AV	RU
Se	-1.745	.410	2.975	-1.214	-.736	.496
Sg	1.101	-.898	.407	-.468	.758	-.309
So	-1.644	1.438	-.773	.878	-1.273	.470

Table 1B) ELASTICITY ESTIMATES FOR EQUATIONS IN CATEGORY 2

Share Elasticities for Equations A:

	Pe	Pg	Po	Se(-10)	Sg(-10)	So(-10)	PCI
Se	-.706	.670	3.807	.177	-1.465	-1.573	-2.049
Sg	.316	-.439	-.594	-.019	.648	-.817	.604
So	-.456(.129)	.337	-.669	-.060	-.29	.968	8.121

Share Elasticities for Equations B:

	Pe	Pg	Po	Se(-10)	Sg(-10)	So(-10)	PCI	TEMP	AV
Se	.081	.193	3.270	..175	-1.11	-.832	-1.83	-1.002	-.176
Sg	.393	-.341	.308	-.0478	.367	-.411	.518	-.0161	.0217
So	-1.040	.767	-1.061	.032	-.267	1.473	.136	.558	-.051

Table 1C) ELASTICITY ESTIMATES FOR EQUATIONS IN CATEGORY 3

Share Elasticities for Equations A:

	Pe	Pg	Po	PCI
Se	-1.567	1.286	.245	-1.203(0)*
Sg	.730	-.611	-.437(0)*	.568(0)*
So	2.491	4.037	-10.829	-1.654(0)*

*Elasticity value if parameters not significantly different from zero are set to zero.

Table 1C) (Continued)

Share Elasticities for Equations B:

	Pe	Pg	Po	PCI	TEMP	AV
Se	-.561	.886	.952	-.679	-.722	-.356
Sg	-.274	-.430	-.465	.328	.350	.180
So	4.285	.883	-7.543	.269	-.249	-3.596

Table 1D) FUEL PRICE SATURATION ELASTICITIES FROM ALTERNATIVE STUDIES

	a) Lin, Hirst & Cohn			a) Lin, Hirst & Cohn		
	Pe	Pg	Po	Pe	Pg	Po
	Space Heating					
Electricity	-2.63	0.44	1.37	-3.19	0.38	1.09
Gas	0.39	1.57	0.03	0.57	-1.33	0.03
Oil	0.03	3.51	-1.09	-0.18	2.95	01.01

	b) Wilson		
	Pe	Pg	Po
	Space Heating		
Electricity	-4.88	1.20	
Gas			
Oil			

	c) Anderson			d) Baughman & Joskow		
	Pe	Pg	Po	Pe	Pg	Po
	Space Heating					
Electricity	-2.04	2.21	0.55	-2.08	2.12	3.30
Gas	0.17	-1.80	0.55	0.23	-1.48	3.30
Oil	0.17	2.21	-1.58	0.23	2.12	-7.21

Sources for 1D):
a) Lin, Hirst & Cohn [15]
b) Wilson [19]
c) Anderson [1, 2, 3]
d) Baughman & Joskow [4].

The own-price elasticities are elastic for the price of electricity and oil in Table 1A equations A. The cross-price elasticities are all the correct sign except for the elasticity of So with respect to Pe. Lin, Hurst and Cohn found the same difficulty with their estimations using the semilog formulation (Table 1D). The fact that Anderson and Baughman and Joskow constrained their cross-elasticities to be equal may have avoided this sign difficulty. The sizes of the price elasticities in Table 1A accord generally with the others summarized in Table 1D. e_{SoPg} and e_{SePo} are elastic in all studies except Anderson's. e_{SgPe} and e_{SgPo} are inelastic in most analyses. In Table 1A we also find negative income elasticities for Se and Sg while So has a positive income elasticity. As mentioned above, we do not believe these elasticities. The elasticities of Se and Sg with respect to TEMP are negative, while that of So is positive. This last elasticity reflects the fact that increasing TEMP reflects greater heating needs and the greater heating needs favors the fuel providing the cheapest operating cost compared with its capital cost. Oil is such a fuel when compared to electricity. But so is gas; hence, e_{SgTEMP} should be > 0. Of course, the Sg elasticity with respect to gas availability is positive, while the Se and So elasticities are < 0. The elasticity with respect to rural or urban locations indicates the prevalence of electricity and oil in rural areas.[1]

The analyses in Table 1A and 1D utilize stock data in year t and exogenous variables in year t. In light of the difficulties of that approach discussed above, Table 1B utilizes the alternative specifications discussed as

---

[1]When the income term is left out of the equation in B of Table 1A, the TEMP, AV and RU elasticities are unchanged in sign.

Category 2. As seen in Table 1B, the effect of including lagged fuel shares is to generally diminish the price sensitivity of current shares to current fuel prices, particularly own-price.

However, the elasticities of the other factors retain the same signs and relative magnitudes. Such a result makes sense. If share data for new homes were available, one would expect fuel choice to be sensitive to current prices. However, when shares of the current inventory are utilizied, only the incremental additions to that housing stock and new retrofits are sensitive to current variables; much of the determination of fuel shares of current housing inventory rests in past prices and other economic/demographic variables that are embodied into the lagged shares. The inclusion of the lagged shares lowers the price sensitivity as indicated by the price elasticity estimates. The signs of the lagged share elasticities are as expected: own lagged share elasticity is positive while the cross-elasticities are negative. The sizes of the lagged share elasticities are also instructive. For example, the cross-elasticities of Se with respect to Sg(-10) and So(-10) are much greater than any other lagged-share cross-elasticities in Table 1B. This indicates the technical difficulties of switching from oil or gas to electricity; in other words, although current prices may indicate the desirability of electricity in home heating, retrofitting from oil or gas equipment and ductwork to electrical equipment can be economically prohibitive. The switch from oil to gas or gas to oil is much easier, and the cross elasticities of Sg with respect to So(-10) and So with respect to Sg(-10) are substantially less negative.

Table 1C reports the elasticity results from data which comes the closest to reflecting the model of individual choice underlying a logit formulation of heating fuel/equipment demand. The price elasticities are generally less than those estimated in Table 1A, except for e_{SoPo}. However, that estimate is spurious; it is due to the fact that the share data is for net additions to the housing stock (i.e., additions - withdrawals). Over the sample period, most of the houses withdrawn from the housing stock have utilized oil. As a result, even though there have been oil burning homes in the gross additions (chosen by the buyer/contractor in response to current prices and expectations), the withdrawals have been greater and usually forced oil's share in net additions to zero (or negative).

Some Simulation Results

Table 1A and 1D examined implied elasticities for fuel shares, given the traditional fuel share modeling. Tables 1B and 1C presented the elasticity results utilizing alternative data sets. The equations underlying the elasticity estimates can be utilized to examine the effects of alternative prices and availability scenarios for the U.S. as a whole and on a regional basis. Let us examine the simulation performance of Category 1 equations and Category 3 equations. We report some results here for two scenarios.[1] These scenarios are summarized in Table 2 in terms of the exogenous variables. All price and income variables are expressed in 1970 dollars. PCI, TEMP and AV are assumed to be the same under both scenarios. The two scenarios project identical Pe through 2025. However, Pg and Po are clearly different. Pg and Po for 1975 in scenario one come from the sources discussed in Section A above. The projections of these variables assume the equality of the

[1]The scenarios were developed in an analysis for Energy Research and Development Agency (ERDA) by Arthur D. Little, Inc. (ADL). The scenarios used here are purely suggestive. They do not embody current ERDA or ADL projections.

prices per 10^6 BTU in 2000. Scenario two projections, on the other hand, foresee a relative price advantage for gas versus oil throughout the period. Furthermore, both gas and oil have a relative price advantage versus electricity through 2025 under scenario two that does not exist under scenario one.

The results of these assumptions are readily seen in the predicted shares of net additions to the U.S. housing stock in Table 3. Under scenario one, the net new units are initially about 4 to 1 in favor of gas over electricity. However, by 2025, 22% of the new units are gas and 78% are electricity.[1] This reflects the changing relative price of gas and electricity of the period: $(Pe/Pg)_{1975} = 6.35$, while $(Pe/Pg)_{2025} = 4.17$. As electricity becomes relatively less expensive, given everything else affecting the fuel choice, more new households are built using electricity. Furthermore, oil is not chosen at all over the period. By 2025, under price scenario one, 77 million housing units will use gas, 17 million oil (the same as 1975), and 35 million will use electricity.

Compared with scenario one, under price scenario two, we see that both gas and oil have a relative price[2] advantage to electricity and that gas has a relative price advantage over oil. The results of such assumptions are predictable: the share of gas in net new additions to the housing stock over the entire period averages about 90%. By 2024, 96% of new houses are gas heated. While electricity accounts for about 10-15% of new houses through 2000, the relative price advantage of gas becomes strong enough by 2025 ($Pe/Pg_{1970} = 7.89$, while $Pe/Pg_{2025} = 8.41$) that only 4% of new additions are electrical.

---

[1]The fact that no oil is added reflects the aggregation bias caused by using <u>net</u> additions rather than gross additions.

[2]These price advantages refer entirely to operating costs.

As under scenario one, oil is miniscule in the additions to new houses. It is interesting to note that although gas availability is constrained to be constant 1990-2025 (AV = 350 in Table 2), the price advantage of gas still indicates a radical shift to gas.

The simulations in Table 3 utilized share data for net additions to the housing stock, data closest to the underlying model of individual choice. How do these simulations compare to those utilizing the usual share data of the housing inventory? To examine that question, we have assessed price secnarios one and two utilizing the equations in Category 1.[1]

On the whole, the simulations in Table 4 agree with those in Table 3. As a matter of fact, they are elaborated. The form of the equations in Table 3 did not permit an analysis of the composition of houses coming out of the housing stocks. The form of equations in Table 4 permits such an analysis. As a result, we see in Table 4A that under scenario one, the number or units using oil actually declines. The shift to electricity is even stronger than seen in Table 3. Likewise, under scenario two, the shift to gas is even stronger than that found in Table 3B; both gas and oil consuming units decline in number. However, in general, the larger price sensitivity exhibited in the elasticities in Table 1A generates projected shifts in fuel using components in the housing inventory greater than the total changes predicted exogenously.

For example, utilizing the relative prices for the year 1980 and the price sensitivity embodied in Category 1 equations, predicted fuel shares implied additions to the group of houses using electricity that was greater than the exogenously predicted increase in the <u>entire housing stock</u> over

---

[1]We have also used those in Category 2. They have produced the same results.

1975-1980. Basically, the model is predicting a large amount of retrofitting to electricity, the costs of which are quite severe and have not been built into the analysis. As a result, the additions to the housing stock over 1975-1980 using electricity is constrained to be 9,300,000 units (Table 4A column B for 1980). 9,300,000 units is the total increase in the housing stock predicted for 1975-1980.

Table 2: EXOGENOUS VARIABLES FOR ALTERNATIVE SCENARIOS

A) Exogenous Variables for Scenario One*

Year	Pe	Po	Pg	PCI	TEMP	AV
1975	9.08	2.03	1.43	4,258.00	5,150.00	300.00
1980	10.27	2.33	1.76	4,630.00	5,150.00	325.00
1985	11.62	2.67	2.16	5,343.00	5,150.00	340.00
1990	13.15	3.07	2.67	6,045.00	5,150.00	350.00
1995	14.87	3.52	3.28	6,839.00	5,150.00	350.00
2000	16.83	4.04	4.04	7,738.00	5,150.00	350.00
2005	19.04	4.57	4.57	8,755.00	5,150.00	350.00
2010	21.54	5.17	5.17	9,981.00	5,150.00	350.00
2015	24.37	5.85	5.85	11,207.00	5,150.00	350.00
2020	27.58	6.62	6.62	12,776.00	5,150.00	350.00
2025	31.20	7.49	7.49	14,346.00	5,150.00	350.00

B) Exogenous Variables for Scenario Two*

Year	Pe	Po	Pg	PCI	TEMP	AV
1975	9.08	1.94	1.15	4,258.00	5,150.00	300.00
1980	10.27	2.01	1.52	4,630.00	5,150.00	325.00
1985	11.62	2.09	2.01	5,343.00	5,150.00	340.00
1990	13.15	2.27	2.16	6,045.00	5,150.00	350.00
1995	14.87	2.47	2.34	6,839.00	5,150.00	350.00
2000	16.83	2.69	2.53	7,738.00	5,150.00	350.00
2005	19.04	2.93	2.73	8,755.00	5,150.00	350.00
2010	21.54	3.19	2.94	9,981.00	5,150.00	350.00
2015	24.37	3.47	3.18	11,207.00	5,150.00	350.00
2020	27.58	3.78	3.43	12,776.000	5,150.00	350.00
2025	31.20	4.12	3.71	14,346.00	5,150.00	350.00

NOTES:

Pe is the user cost of electricity in $\$/(10^6 BTU)$.
Po is the user cost of oil in $\$/(10^6 BTU)$.
Pg is the user cost of gas in $\$/(10^6 BTU)$.
PCI is average per capita income.
TEMP is annual average degree days.
AV is the simple average availability index acrsss all 50 states and Washington DC computed as the average number of distribution miles of main per capita multiplied by 100.
All dollar figures in 1970$.

*Prices for the years 2000-2025 have been extrapolated at the 1985-2000 growth rates.

Table 3: SIMULATION OF FUEL SHARES IN INCREMENTAL ADDITIONS TO HOUSING STOCK IN THE RESIDENTIAL SECTOR, USING CATEGORY 3 EQUATIONS, VERSION B.

A) Price Scenario One

	a) Proportion of Increment Going to			b) Five Year Incremental Increase			c) Stock of Houses Using		
	Gas	Oil	Electricity	Gas	Oil	Electricity	Gas	Oil	Electricity
1975	--	--	--	--	--	--	39,481	16,827	8,412
1980	.82435	.00013	.17552	6,389	1.0	1,360	45,870	16,828	9,772
1985	.79554	.00005	.20441	7,740	0.5	1,989	53,610	16,828	11,761
1990	.76761	.00003	.23236	4,976	0.2	1,506	58,587	16,829	13,268
1995	.70993	.00001	.29006	4,602	0.1	1,880	63,189	16,829	14,148
2000	.62225	.00001	.37774	4,034	0.1	2,449	67,223	16,829	17,597
2005	.49674	0	.50325	2,687	--	2,273	69,911	16,829	20,320
2010	.43059	0	.56941	2,329	--	3,081	72,240	16,829	23,400
2015	.36493	0	.63507	1,974	--	3,436	72,214	16,829	26,836
2020	.28396	0	.71604	1,536	--	3,874	75,751	16,829	30,710
2025	.21776	0	.78224	1,178	--	4,232	76,929	16,829	34,941

NOTES:
Price scenario one found in Table 2.
Columns B = Columns A + Incremental increase in housing stocks entries in Columns C for Year t = Housing stock (T-1) + incremental additions (t-1 to t) in columns B.
Columns B and C in 1000 housing units.

B) Price Scenario Two

	a) Proportion of Increment Going to			b) Five Year Incremental Increase			c) Stock of Houses Using		
	Gas	Oil	Electricity	Gas	Oil	Electricity	Gas	Oil	Electricity
1975	--	--	--	--	--	--	30,481	16,827	8,412
1980	.86284	.00014	.13702	6,687	1.1	1,062	46,168	16,828	9,474
1985	.85468	.00014	.14518	8,316	1.4	1,413	54,484	16,829	10,887
1990	.84640	.00019	.15341	5,487	1.2	995	59,971	16,831	11,881
1995	.84655	.00017	.13528	5,605	1.1	877	65,576	16,832	12,758
2000	.87896	.00018	.12086	5,698	1.2	784	71,274	16,833	13,542
2005	.89522	.00020	.10458	4,843	1.1	566	76,118	16,834	14,107
2010	.91268	.00023	.08709	4,938	1.2	471	81,055	16,835	14,579
2015	.93233	.00026	.06740	5,044	1.5	365	86,099	16,837	14,943
2020	.94702	.00034	.05263	5,123	1.8	285	91,222	16,839	15,228
2025	.93619	.00045	.03636	5,211	2.4	197	96,433	16,841	15,425

NOTES:
Price scenario two found in Table 2.
Columns B = Columns A + Incremental increase in housing stocks entries in Columns C for Year t = Housing stock (t-1) + incremental additions (t-1 to t) in Columns B.
Columns B and C in 1000 housing units.

Table 4) SIMULATION OF FUEL SHARES IN THE RESIDENTIAL SECTOR USING CATEGORY 1 EQUATIONS, VERSION B.

A) Price Scenario One*

	a) Proportion of Housing Units Using			b) Incremental Increase or Decrease in Period in Units Using			c) Stock of Housing Units Using		
Year	Gas	Oil	Electricity	Gas	Oil	Electricity	Gas	Oil	Electricity
1975	61.0	26.0	13.0	--	--	--	39,481	16,827	8,412
1980	54.2	21.3	24.4	-180	-1,369	9,300	30,300	15,603	17,712
1985	47.5	19.0	33.5	-274	145	9,860	39,026	15,603	27,572
2000	32.5	16.1	51.4	-6,000	733	24,716	33,026	16,336	52,288
2025	19.8	9.8	70.4	-7,570	-3,680	38,300	25,456	12,656	90,588

B) Price Scenario Two*

Year	Gas	Oil	Electricity	Gas	Oil	Electricity	Gas	Oil	Electricity
1975	61.0	26.0	13.0	--	--	--	39,481	16,827	8,412
1980	61.2	21.1	17.7	4,865	-1,550	4,435	44,346	15,277	12,847
1985	58.6	22.5	19.0	3,798	3,194	2,739	48,144	18,471	15,586
2000	70.9	13.7	15.4	23,905	-4,539	84	72,049	13,932	15,670
2025	85.7	9.1	5.2	38,301	-2,247	-9,003	110,350	11,685	6,667

NOTES:

Price scenarios one and two for 1975, 1980, 1985, 2000 and 2025 are found in Table 2.

Columns B and C in 1000 housing units.

*Prices for years 2000-2025 have been extrapolated at the 1985-2000 growth rates.

B. MODEL REFINEMENT

Given the criticisms above concerning the lack of capital costs in the probability model analysis, this section refines the basic equations (6a) and (6b) by including amortized capital costs for electrical, oil and gas fueled heating equipment. The derivation of these capital cost estimates is discussed in the notes to Table 5. Category 1 equations (using housing stock) are utilized in the refinement, given the availability of data. Regression results are presented in Table 5 utilizing 1970 data (1970 shares and 1970 independent variables), pooled 1960 and 1970/current \$ data (i.e., 1960 and 1970 shares related to current \$ 1960 and 1970 independent variables). Furthermore, some more detailed regression analysis is presented for the data in Table 5C.

For the 1970 equations (Table 5A) all coefficient signs are as hypothesized, except for equation 3, $\partial(\text{Log } \frac{So}{Se})/\partial Pe$. However, the coefficient is insignificantly different from zero. The equations utilizing pooled 1960/1970 current \$ data increase estimate efficiency in Table 5B. All parameter estimates for those parameters for which priors exist have the hypothesized sign.

Likewise, the use of pooled 1960/1970 constant 1970 \$ data increases estimation efficiency in Table 5C over 5A. Equations C1-C3 of Table 5C duplicate the results for the equations in 5A and 5B for this data. Equations C4-C6 demonstrate the effect of regional factors on these equations by including regional dummies. Equation C7 corrects for rural/urban differences. The signs of almost all the coefficients in equations C1, C2 and C3 for which priors exist are correct. The price and capital cost coefficient are, for the most part, significantly different from zero, particularly in C1 and

C2. The important price and capital cost coefficient estimates are quite stable across the three equations, as availability (AV), income (PCI) and heating degree days (TEMP) are included.

It is interesting to observe the effect of regional characteristics upon the coefficient estimates (comparing C4 with C1, C5 with C2 and C6 with C3). In most cases, the size of the coefficient estimates does not change much; the signs are relatively stable.

This is not true, however, for Po, the price of oil. In most cases, the inclusion of regional dummies reverses the coefficient sign of Po. In many cases, the estimated coefficient is significantly different from zero (e.g., with one sign in C1 and the opposite sign in C4). The signs of the coefficient estimates in C4-C6 are usally contrary to our priors. The reason seems to invariably be the presence of the dummy: when its coefficient is positive (negative), the coefficient for Po becomes negative (positive). We cannot, at the moment, explain the reason for this; nor can we fully explain the disturbing significance of the regional dummies in C4-C6. The significance certainly suggests some variables are being excluded. One possibility is that the dummy variables suggest that the expectation of log Sg/Se and log So/Se are too high for all regions based only upon prices (Pe, Po, Pg), capital costs (CAPe, CAPo, CAPg), availability (AV), income (PCI), and weather (TEMP). This may be caused by the large scale non-price-related promotional effort to expand electrical use over the 1960's on the part of construction contractors and utilities.

Finally, the inclusion of a rural/urban dummy in equations C7 does not affect parameter estimates (compare with C3). The signs of the estimated parameters are as expected.

The elasticities for the equations found in Table 5A and found in 6A, 5B and 6B, and those for equations C1-C3 of 5C are found in Table 6C. The elasticities estimated utilizing only (Pe, Po, Pg, CAPe, CAPo, and CAPg) are almost always greater than those estimated for the full set of variables. We feel that the elasticity results utilizing 1960/1970 constant 1970 $ are most reliable. All own-price and own-capital cost elasticity estimates are negative. All cross-price and cross-capital cost elasticities are positive except for e_{SeCAPg} and e_{SgCAPe}. The income elasticities for gas and electricity are positive while that of oil is negative. Thus the inclusion of capital costs eliminate the unexpected income coefficient signs found in Section A. The elasticity of the gas share with respect availability (AV) is positive while that of oil is negative. The elasticity of the electricity share with respect to availability of gas (AV) is positive but almost zero. This contradicts common sense and the results in Tables 6A and 6B. The temperature elasticity of oil demand is positive reflecting its operating cost desirability in colder areas. The temperature (TEMP) elasticities of electricity and gas are negative reflecting the fact for electricity that as less heat is required, the relative capital cost advantage of electricity becomes a factor. However, the gas elasticity with respect to temperature should also be greater than zero.

While the signs of the capital cost elasticities are, for the most part, correct, they seem, in general, to be quite large. They are certainly larger than those found by Lin, Hurst and Cohn.[1] However, it is unclear how their elasticity estimates were obtained since all their equations for space heating include only the equipment price of oil.[2]

---

[1]Lin, Hirst and Cohn [15], Table 6.

[2]Ibid. [15], Table 3 and Appendix C.

Detailed simulation results utilizing the equations estimated in equations C3 Table 5C are presented in detail in Appendix B. The simulations are performed for four regions: the Northeast, the North Central States, the South, and the West. Since these simulations are rather involved, discussion is left to Appendix B. However, for purposes of comparison with the simulations found in Section A, we repeat those national simulations including capital costs. Those results are presented in Table 7. The results in Table 7 are to be compared with those in Table 4.

The basic trends found in Table 4 are repeated in Table 7; however, the results are not identical. It should be noticed that the alternative capital costs are assumed constant over the forecast period; greater variation based upon deeper analysis is, of course, possible. In Table 7, we see that for scenario one, the final fuel shares in 2025 are not as strongly in favor of electricity over gas as they were when capital costs were ignored (Table 4). Such a result is surprising since the lower capital cost of electricity would favor even greater shifts toward electricity than those based upon operating costs alone. However, the inclusion of the capital costs have changed other estimated parameters, and we believe that the elimination of the misspecification due to excluded capital cost variables improves these other parameter estimates. For example, when the capital costs of alternative equipment are included, the own-price elasticity estimate for electricity rises 72% from -2.25 (Table 1A) to -3.86 (Table 5C). Thus, while the capital cost desirability would increase the penetration of electricity, we feel the parameter estimates are consistent and the increased own-price elasticity for electricity would limit the switch to electicity, given the significant electricity price rises documented in Table 7. Thus, in Table 7, electricity still acounts for a major portion of the housing stock in

2025 (57%) under price scenario one; it is still less than that found in Table 4 (70.4%) when the forecasting equations suffer from probable misspecification bias/inconsistency.

Under scenario two, the results in Table 7 are very similar to those found in Table 4. However, the switch to gas is even more pronounced in Table 7 (88% versus 85.7%). Although the difference is not extreme, it does reflect several things: 1) the own- and cross-capital cost elasticities (Table 6C) make gas even more desirable, 2) the (more) consistent parameter estimates in Tables 5C/6C indicate greater cross-elasticities, in particular e_{SgPe}, and greater own-elasticity (e_{SgPg}); as a result, when the relative cost of electricity versus gas rises in this scenario (as compared to scenario one), these elasticities increase the shift to gas (as compared with Table 4 and its underlying equations). The increased shift to gas in Table 7 under scenario two occurs over the entire period and almost entirely at the expense of electricity. The predicted oil shares are very similar in Table 4 and in Table 7 under scenario two (and under one, for that matter). The projected incremental changes in the housing stock and the actual housing stock estimates found in Table 7 are derived directly from the share estimates and the exogenously projected total housing stock (TOT) and increments (INCRE) to and withdrawals from (DECRE) that stock.

Table 5: SELECTED REGRESSION RESULTS FOR FUEL SHARES OF EXISTING INVENTORY OF HOUSING UNITS (CATEGORY 1) WITH CAPITAL COSTS INCLUDED.

		Pe	Po	Pg	CAPe	CAPo	CAPg	AV	PCI	TEMP	R^2	D.W.	S.e.
A) 1970 Equations													
1)	$\log(\frac{Sg}{So})$	.03304 (1.36)	.644859 (.52)	-4.31613 (-9.26)	-.113858 (-2.51)	.993173 (3.67)	-1.00306 (-3.10)				.62	.93	1.616
	$\log(\frac{So}{Se})$	.162627 (1.46)	-2.01492 (-2.19)	2.25694 (6.52)	.149904 (4.46)	-1.09028 (-5.43)	1.11071 (4.63)				.65	1.38	1.20
2)	$\log(\frac{Sg}{So})$	.268038 (2.16)	1.40999 (1.37)	-1.88118 (-3.55)	-.0469285 (-1.19)	.660095 (2.88)	-.754053 (-2.80)	.0126801 (6.61)			.75	.59	1.33
	$\log(\frac{So}{Se})$	.120322 (1.238)	-2.51495 (-3.13)	.665635 (1.60)	.105837 (3.49)	-.872609 (-4.87)	.947975 (4.50)	-.00828679 (-5.57)			.74	1.61	1.04
3)	$\log(\frac{Sg}{So})$	.354705 (2.72)	1.27232 (1.26)	-2.19868 (-4.14)	-.0134811 (-.34)	.583366 (2.28)	-.692852 (-2.30)	.0112111 (5.76)	.000241376 (.80)	-.0002035 (-2.45)	.76	.56	1.30
	$\log(\frac{So}{Se})$	-.0316777 (-.33)	-2.513 (-3.42)	1.06678 (2.77)	.0671061 (2.32)	-.619706 (-3.34)	.682712 (3.12)	-.00623865 (-4.41)	.00008956 (.41)	.00028548 (4.74)	.79	1.32	.94
B) Pooled Equations/Current$													
1)	$\log(\frac{Sg}{So})$	.149652 (1.69)	.311482 (.39)	-3.85064 (-12.19)	-.136335 (-4.12)	.952671 (4.72)	-.933038 (-3.97)				.61	1.08	1.67
	$\log(\frac{So}{Se})$	.571508 (6.51)	-2.03918 (-2.59)	2.00597 (6.41)	.192703 (5.88)	-.630647 (-3.16)	.527704 (2.27)				.54	1.24	1.66
2)	$\log(\frac{Sg}{So})$	.112886 (1.506)	.641069 (.96)	-2.23427 (-6.91)	-.0906272 (-3.19)	.590473 (3.37)	-.608497 (-3.02)	.0121991 (8.85)			.72	.79	1.41
	$\log(\frac{So}{Se})$	.58977 (6.95)	-2.20289 (-2.90)	1.20308 (3.29)	.169989 (5.29)	-.458735 (-2.27)	.366497 (1.60)	-.00605936 (-3.88)			.57	1.29	1.60
3)	$\log(\frac{Sg}{So})$	.291194 (3.34)	.477163 (.72)	-2.58623 (-7.88)	-.0559047 (-1.90)	.501366 (2.59)	-.540511 (-2.44)	.0103885 (7.15)	.000300969 (1.75)	-.000205647 (-3.01)	.74	.77	1.37
	$\log(\frac{So}{Se})$	.306206 (3.26)	-1.65265 (-2.31)	1.73801 (4.92)	.122475 (3.87)	-.511409 (-2.46)	.49206 (2.06)	-.00378103 (-2.42)	-.000834184 (-4.50)	.00024608 (3.41)	.64	1.38	1.47
C) Alternate Pooled Equations													
	$\log(\frac{Sg}{Se})$	.72615 (8.71)	-1.66564 (-2.23)	-1.76485 (-5.94)	.0618586 (1.99)	.310762 (1.64)	-.39975 (-1.81)				.36	1.23	1.57
	$\log(\frac{Sg}{Se})$	.706586 (8.87)	-1.49027 (-2.09)	-.904776 (-2.63)	.08619 (2.85)	.118036 (.63)	-.227062 (-1.06)	.00648112 (4.43)			.41	1.06	1.50
	$\log(\frac{Sg}{Se})$	.614268	-1.10433	-.748327	.0762232	-.0462438	-.0193038	.00680388	-.000525492	.0000222973	.44	1.14	1.47

Statistics for (H_o: Parameter = 0) in Parentheses.

C) Pooled Equations/Constant 1970$

		Pe	Po	Pg	CAPe	CAPo	CAPg	AV	PCI	TEMP	REG1	REG2	REG3	REG4	RU	R^2	D.W.	S.e.
1)	$\log(\frac{Sg}{So})$	.115874 (1.48)	.216795 (.32)	-3.14341 (-11.41)	-.118631 (-3.99)	.826259 (4.55)	-.88891 (-3.83)				13.3807	15.1394	13.0263			.58	1.05	1.72
	$\log(\frac{Sg}{Se})$	.508523 (7.52)	-1.29428 (-2.18)	-1.42792 (-6.00)	.0605533 (2.36)	.322229 (2.05)	-.406703 (-2.23)									.42	1.27	1.49
	$\log(\frac{So}{Se})$	.387915 (5.36)	-1.56543 (-2.46)	1.64391 (6.46)	.174052 (6.35)	-.490902 (-2.92)	.393962 (2.20)									.58	1.29	1.59
2)	$\log(\frac{Sg}{So})$	.216221 (3.39)	.216842 (.39)	-1.85968 (-7.29)	-.084959 (-3.52)	.526151 (3.52)	-.543772 (-3.16)	.0125695 (10.22)								.73	.80	1.39
	$\log(\frac{Sg}{Se})$	.542063 (8.12)	-1.29427 (-2.23)	-.998852 (-3.75)	.0718078 (2.85)	.221921 (1.42)	-.318082 (-1.77)	.00420123 (3.27)								.45	1.13	1.45
	$\log(\frac{So}{Se})$	.317776 (4.85)	-1.56547 (-2.75)	.746636 (2.85)	.150517 (6.07)	-.281138 (-1.83)	.20864 (1.18)	-.00878558 (-6.97)								.66	1.53	1.43
3)	$\log(\frac{Sg}{So})$	.277244 (3.99)	.324225 (.60)	-2.05567 (-7.56)	-.9566947 (-2.20)	.390905 (2.35)	-.412427 (-2.17)	.0112509 (8.32)	.000107042 (.66)	-.000184964 (-2.88)						.74	.76	
	$\log(\frac{Sg}{Se})$	.567269 (7.20)	-1.02007 (-1.73)	-.925096 (-3.23)	.0818419 (3.02)	.0442556 (.25)	-.11941 (-.60)	.00442554 (3.10)	-.000265459 (-1.56)	-.0000748543 (-1.10)						.47	1.21	1.44
	$\log(\frac{So}{Se})$	.238414) (3.34)	-1.41039 (-2.45)	1.02728 (3.67)	.130335 (4.92)	-.311844 (-1.82)	.264114 (1.35)	-.00716432 (-5.14)	-.000374331 (-2.25)	.000123048 (1.80)						.67	1.50	
4)	$\log(\frac{Sg}{So})$	.172017 (2.24)	-4.38729 (-3.43)	-2.78228 (-8.45)	-0.255438 (-3.60)	.79793 (2.96)	-.696575 (-2.44)				13.3807 (4.09)	15.1394 (4.29)	13.0263 (4.08)	13.8436 (4.16)	-1.74387	.62	1.19	1.66
	$\log(\frac{Sg}{Se})$	.479668 (7.13)	.93635 (.83)	-1.61199 (-5.59)	-.00250965 (-.04)	.513319 (2.17)	-.542885 (-2.16)				-6.5871 (-2.30)	-6.53641 (-2.12)	-7.18358 (-2.57)	-7.72711 (-2.65)		.46	1.26	1.45
	$\log(\frac{So}{Se})$	.297054 (4.71)	5.64091 (5.37)	1.11036 (4.11)	.256589 (4.41)	-.242167 (-1.09)	.107785 (.46)				-21.1633 (-7.88)	-22.9159 (-7.92)	-21.3623 (-8.16)	-22.7037 (-8.31)		.70	1.59	1.36
5)	$\log(\frac{Sg}{So})$	.213355 (3.30)	.0480577 (.04)	-1.65607 (-5.46)	-.167426 (-2.78)	.680371 (3.00)	-.641648 (-2.67)	.0127376 (8.95)			-.485541 (-.15)	.489515 (.14)	-.692663 (.22)	-.574841 (.18)		.74	.78	1.39
	$\log(\frac{Sg}{Se})$	.505729 (8.13)	3.73249 (3.28)	.673472 (-3.09)	.222446 (.19)	-.196562 (2.01)	.0864774 (-2.20)	.00494134 (5.87)			-15.3287 (-5.05)	-15.772 (-4.84)	-15.8324 (-5.34)	-16.8168 (-5.42)		.54	1.12	1.34
	$\log(\frac{So}{Se})$	.281017 (4.59)	3.92028 (3.50)	.673472 (2.34)	.222446 (3.89)	-.196562 (-.92)	.0864774 (.38)	-.00494134 (-3.67)			-15.7841 (-5.28)	-17.2327 (-5.38)	-16.0402 (-5.49)	-17.1103 (-5.60)		.72	1.53	1.32
6)	$\log(\frac{Sg}{So})$	.291878 (4.16)	-.615641 (0.51)	-1.98197 (-6.27)	-.157268 (-2.65)	.648894 (2.65)	-.625452 (-2.37)	.0102703 (6.39)	.000122672 (.69)	-.000230841 (-3.09)	3.35712 (.91)	4.47295 (1.15)	2.58537 (.73)	3.36778 (.90)		.75	.73	1.37
	$\log(\frac{Sg}{Se})$	.479417 (6.95)	3.8905 (3.25)	-.79925 (-2.57)	.0493793 (.85)	.43353 (1.80)	-.495597 (-1.91)	.00879067 (5.56)	-.0000631 (-.36)	.000069432 (.94)	-16.62689 (-4.49)	-16.7591 (-4.40)	-16.6088 (-4.77)	-17.7923 (-4.84)		.55	1.09	1.34
	$\log(\frac{So}{Se})$	.173688 (2.70)	4.87922 (4.37)	1.12412 (3.87)	.208717 (3.83)	-.140499 (-.62)	.0498017 (.21)	-.00151566 (1.03)	-.000149999 (-.92)	.000321892 (4.69)	-21.3157 (-6.31)	-22.957 (-6.45)	-20.7798 (-6.39)	-22.7771 (-6.64)		.75	1.39	1.25
7)	$\log(\frac{Sg}{So})$	.182183 (2.94)	.351264 (.72)	-2.01891 (-8.51)	-.0553977 (-2.47)	.520255 (3.37)	-.563313 (-3.38)	.0121312 (10.24)	.00045384 (3.07)	-.000139551 (-2.47)					-1.74387 (-7.77)	.80	1.11	1.19
	$\log(\frac{Sg}{Se})$	.433809 (6.52)	-.993485 (-1.89)	-.888957 (-3.48)	.0831171 (3.44)	.171426 (1.09)	-.267754 (-1.49)	.00529109 (4.15)	.0000754961 (.47)	-.0000302068 (.50)					-1.71449 (-7.09)	.58	1.32	1.28
	$\log(\frac{So}{Se})$	.238439 (3.26)	-1.4104 (-2.44)	1.02727 (3.66)	.130335 (4.91)	-.311875 (-1.81)	.264154 (1.34)	-.00716455 (-5.11)	-.000374422 (-2.14)	.000123036 (1.84)					.00045828 (.002)	.67	1.50	1.41

TABLE 5: (Continued)

where

CAP_e is the annual amortization and maintenance costs of an average electric heating system, not including heat pump but including direct electric and electric furnace system[1] in the given region of the United States.

CAP_o is the annual amortization and maintenance costs of an average oil heating system[1] in the given region of the United States.

CAP_g is the annual amortization and maintenance costs of an average gas heating system[1] in the given region of the United States.

and where Sg, So, Pe, Po, Pg, AV, PCI and TEMP are defined above.

---

[1] For a full discussion of the assumptions and parameters underlying the amortization see J. G. Delene, "A Regional Comparison of Energy Resource Use and Cost to Consumers of Alternate Residential Heating Systems," Oak Ridge National Laboratory, ORNL-TM-4689, November 1974, Table 13. [10]

TABLE 6: FUEL SHARE ELASTICITY ESTIMATES

A) 1970 EQUATIONS

	Share Electricity	Share Oil	Share Gas	Share Electricity	Share Oil	Share Gas	Share Electricity	Share Oil	Share Gas
Pe	-2.35	-0.940	0.818	-2.88	-1.84	0.478	-1.69	-1.96	1.10
Po	2.71	-1.25	0.170E-01	2.40	-2.55	0.227	2.80	-2.14	0.364
Pg	0.838	3.51	-1.60	0.281	1.07	-0.183	0.520	1.78	-0.821
CAPe	-6.86	9.92	-2.83	-6.93	4.91	-0.282	-5.70	1.80	0.296
CAPo	48.2	-105.	34.5	39.9	-82.8	10.0	25.6	-61.6	20.5
CAPg	-43.8	93.5	-30.5	-34.3	82.8	-10.4	-20.9	63.5	-22.1
AV				-0.721	-3.06	0.519	-0.432	-2.19	0.972
PCI							-0.864	-0.529	0.372
TEMP							-0.643	0.827	-0.221

B) POOLED EQUATIONS/CURRENT $

	Share Electricity	Share Oil	Share Gas	Share Electricity	Share Oil	Share Gas	Share Electricity	Share Oil	Share Gas
Pe	-5.42	-0.476	0.820	-5.39	-0.287	0.690	-3.96	-1.31	1.23
Po	3.11	-0.513	0.410E-01	3.05	-0.867	0.273	2.30	-0.641	0.207
Pg	0.366	2.74	-1.82	0.175	1.60	-1.05	-0.195	1.87	-1.28
CAPe	-10.4	8.87	-4.79	-10.9	6.14	-2.94	-8.41	3.86	-1.74
CAPo	6.40	-73.0	47.0	10.9	-45.9	28.5	24.8	-39.6	23.6
CAPg	3.91	62.3	-40.9	0.376	40.9	-26.4	-17.3	37.1	-22.3
AV				-0.314	-1.79	1.18	-0.578	-1.50	1.0
PCI							1.80	-0.631	0.24
TEMP							-0.594	0.674	-0.38

TABLE 6: FUEL SHARE ELASTICITY ESTIMATES

(Continued)

C) POOLED EQUATIONS/CONSTANT (1970$)

	Share Electricity	Share Oil	Share Gas	Share Electricity	Share Oil	Share Gas	Share Electricity	Share Oil	Share Gas
Pe	-4.35	-0.465	0.695	-4.26	-1.08	1.08	-3.86	-1.47	1.31
Po	2.76	-0.410	0.289E-01	2.76	-0.412	0.269E-01	2.34	-0.521	0.136
Pg	0.315	2.57	-1.74	0.490	1.51	-1.04	0.278	1.69	-1.13
CAPe	-11.1	8.70	-4.82	-10.7	6.42	-3.26	-10.4	4.45	-2.01
CAPo	-0.662	-71.0	47.4	-4.99	-45.3	30.1	10.3	-34.4	21.7
CAPg	10.9	60.5	-41.3	14.3	40.6	-27.9	-1.82	31.4	-20.5
AV				0.276	-1.87	1.20	0.865E-01	-1.66	1.08
PCI							0.954	-0.261	0.867E-01
TEMP							-0.558E-01	0.578	-0.375

SOURCE: Equations in Table 5.
Elasticities estimated at sample means.

Table 7: SIMULATION OF FUEL SHARES IN THE RESIDENTIAL SECTOR USING THE REFINED EQUATIONS (C3, TABLE 5C) WITH ALTERNATIVE CAPITAL COSTS INCLUDED

EXOGENOUS VARIABLES

A) Exogenous Variables for Scenario One

Year	Pe	Po	Pg	CAPe	CAPo	CAPg	AV	PCI	TEMP	TOT	INCRE	DECRE
1975	9.08	2.03	1.43	115.84	145.93	128.30	300.00	4,258.00	5,150.00	70,830.00	0.00	0.00
1980	10.27	2.33	1.76	115.84	145.93	128.30	325.00	4,630.00	5,150.00	78,580.00	9,300.00	-1,550.00
1985	11.62	2.67	2.16	115.84	145.93	128.30	340.00	5,343.00	5,150.00	88,360.00	11,280.00	-1,500.00
2000	16.83	4.04	4.04	115.84	145.93	128.30	350.00	7,738.00	5,150.00	107,793.00	25,433.00	-6,030.00
2025	31.20	7.49	7.49	115.84	145.93	128.30	350.00	14,346.00	5,150.00	134,807.00	32,264.00	-11,250.00

B) Exogenous Variables for Scenario Two

Year	Pe	Po	Pg	CAPe	CAPo	CAPg	AV	PCI	TEMP	TOT	INCRE	DECRE
1975	9.08	1.94	1.15	115.84	145.93	128.30	300.00	4,258.00	5,150.00	70,830.00	0.00	0.00
1980	10.27	2.01	1.52	115.84	145.93	128.30	325.00	4,630.00	5,150.00	78,580.00	9,300.00	-1,550.00
1985	11.62	2.09	2.01	115.84	145.93	128.30	340.00	5,343.00	5,150.00	88,360.00	11,280.00	-1,500.00
2000	16.83	2.69	2.53	115.84	145.93	128.30	350.00	7,738.00	5,150.00	107,793.00	25,433.00	-6,030.00
2025	31.20	4.12	3.71	115.84	145.93	128.30	350.00	14,346.00	5,150.00	134,807.00	38,264.00	-11,250.00

SIMULATION RESULTS

A) Simulations for Secnario One

Year	Proportion of Housing Units Using			Incremental Increase or Decrease in Period in Units Using (1000 Units)			Stock of Housing Units Using (1000 Units)		
	Gas	Oil	Electricity	Gas	Oil	Electricity	Gas	Oil	Electricity
1975	.64	.24	.12	6,853	-112		44,730	16,828	8,412
1980	.66	.21	.13	6,853	-112	1,869	51,583	16,716	10,281
1985	.68	.17	.15	8,195	-1,500	3,085	59,778	15,216	13,366
2000	.52	.12	.36	-4,264	-1,826	25,433	55,575	13,390	38,799
2025	.36	.07	.57	-7,654	-3,596	38,264	47,920	9,794	77,063

B) Simulations for Scenario Two

Year	Proportion of Housing Units Using			Incremental Increase or Decrease in Period in Units Using (1000 Units)			Stock of Housing Units Using (1000 Units)		
	Gas	Oil	Electricity	Gas	Oil	Electricity	Gas	Oil	Electricity
1975	.64	.24	.12				44,730	16,828	8,412
1980	.70	.21	.09	9,300	-280	-1,270	54,030	16,548	7,142
1985	.68	.24	.08	5,277	4,524	-20	59,307	21,072	7,122
2000	.79	.15	.06	25,433	-5,144	-885	84,740	15,927	6,236
2025	.88	.09	.03	33,682	-3,906	-2,762	118,421	12,021	3,474

NOTE: Pe, Po, Pg, CAPe, CAPo, CAPg, AV, PCI and TEMP are defined above. All dollar estimates are in 1970$. TOT, INCRE and DECRE refer to the total housing stock, additions to the housing stock and withdrawals from the housing stock, respectively. They are estimated exogenously.

C. ADDITIONAL RESEARCH EFFORTS

The discussions of Sections A and B have pointed to some difficulties in the standard treatment of probability modelling of alternative fuel demand and have introduced some attempts to eliminate those difficulties. However, much work remains to be done. We feel useful research could be directed at the following:

1. A more explicit examination and formulation of the model of individual choice underlying the demand analysis. We have in mind the efforts of the type Hausman and Wise [14], Domecich and McFadden [11] and Hartman [12]. Such an explicit formulation would identify the data required for empirical work (e.g., fuel shares in changes in the housing stock as opposed to shares in the actual stock). Such a formulation would also settle the relevance of the assumption of the "independence of irrelevant alternatives."

2. The capital cost data utilized in the analysis requires improvement. Furthermore, fuel share data in the gross and net additions to the housing stock need improvement. Such data should be developed and added to the existing data bank in order to provide the foundation for further model specification, estimation and hypothesis testing.

3. More efficient estimation techniques should be utilized to improve the results from current and proposed data including Zellner joint estimation and GLS.[1] Our results for equations utilizing fuel shares in net additions to the housing stock and including capital costs[2] could have been improved with either technique.

4. The ability of this probability modeling to handle new technologies should be examined. If the model of choice can be fully developed and estimates of operating cost and capital cost elasticities

---

[1]These techniques have been used in [15] and [20].

[2]These results are not reported here.

satisfactorily obtained, then new technologies such as the heat pump can be characterized by such capital and operating costs in addition to other qualitative characteristics. As a result, market penetration of such new technologies can be assessed using the estimated probability models.

5. In terms of longer run research, the well specified and estimated fuel share models that would result from the proposed research could be integrated into a more aggregate model of total energy demand and alternative energy supplies.

BIBLIOGRAPHY

1. Anderson, Kent P. The Price Elasticity of Residential Energy Use, The Rand Paper Series, #P-5180 (February, 1974).

2. Anderson, Kent P. Residential Demand for Electricity: Econometric Estimates for California and the United States, Rand Corporation, R-905-NSF (January, 1972).

3. Anderson, Kent P. Residential Energy Use: An Econometric Analysis, Rand Corporation, R-1297-NSF (October, 1973).

4. Baughman, M. and Joskow, P. "Interfuel Substitution in the Consumption of Energy in the United States, Part I: The Residential and Commercial Sectors," MIT Energy Laboratory Report, #MIT-EL74-002.

5. Baughman, M. and Joskow, P. "The Future of the U.S. Nuclear Energy Industry," The Bell Journal of Economics, (Spring, 1976, Vol. 7, #1) pp. 3-32.

6. Baughman, M. and Joskow, P. "A Regionalized Electricity Model," M.I.T. Energy Laboratory Report #75-005, Dec. 1974.

7. Baughman, M. and Joskow, P. "Energy Consumption and Fuel Choice by Residential and Commercial Consumers in the United States," Energy Systems and Policy.

8. Baughman, M. and Zerhoot, F. "Interfuel Substitution in the Consumption of Energy in the United States, Part II: The Industrial Sector," M.I.T. Energy Lab. Report #75-007, April, 1975.

9. Bergson, J. "Maximum likelihood and minimum X^2 estimates of the logistic function," Journal of the American Statistical Association, 50, pp. 130-161.

10. Delene, J. G. A Regional Comparison of Energy Resource Use and Cost to Consumers of Alternate Residential Heating Systems, Oak Ridge National Laboratory, #w-7405-ENG-26 (November, 1974).

11. Domencich T. and McFadden, D. Urban Travel Demand, A Behavioral Analysis, North Holland/American Elsevier, 1976.

12. Hartman, R. "A Generalized Logit Formulation of Discrete Consumer Choice," mimeo.

13. Hausman, J. "Project Independence Report: An Appraisal of U.S. Energy Needs up to 1985," The Bell Journal (Autumn, 1975), pp. 517-551.

14. Hausman, J. and Wise, D. "A Conditional Probit Model for Qualitative Choice: Discrete Heterogeneous Preferences," forthcoming in Econometrica.

15. Lin, W., Hirst, E. and Cohn, S. Fuel Choices in the Household Sector, Oak Ridge National Laboratory, #W-7405-ENG-26 (October, 1976).

16. Pindyck, R. and Rubinfeld, D. Econometric Models and Economic Forecasts, McGraw-Hill; New York (1976).

17. Theil, H. "A Multinomial Estimation of the Linear Logit Model," International Economic Review (Volume 10, #3, October, 1969), pp. 251-259.

18. Theil, H. "On the Estimation of Relationships Involving Qualitative Variables," American Journal of Sociology, (Volume 76, 1970), pp. 103-154.

19. Wilson, J. W. "Residential Demand for Electricity," The Quarterly Review of Economics and Business, 11:1 (Spring, 1971), pp. 7-19.

20. Zellner, A. and Lee, T. H., "Joint Estimation of Relationships Involving Discrete Random Variables," Econometrica (Volume 33, April, 1975), pp. 382-394.

APPENDIX A: Model Criticisms

The use of the logit specification for the estimation of price elasticities in demand models has been criticized for apparently imposing the assumption that all cross price elasticities with respect to a given price change are restricted to be identical.[1] However, as demonstrated here, that criticism must be addressed to the assumed model of demand,[2] not against the use of conditional logit.

The criticism is demonstrated through a model specifying total fuel demand (TOT). As:

$$TOT = G(P_{INDEX}, X_1) \qquad (A1)$$

where P_{INDEX} is the price index (value weighted sum of individual fuel prices) and X_1 is a vector of exogenous macroeconomic variables. The relative share equations are:

$$\frac{S_i}{S_n} = F_i\left(\frac{P_i}{P_n}, X_{2i}\right) \qquad i = 1, \ldots n-1 \qquad (A2)$$

where P_i/P_n is the ratio of relative fuel prices, X_{2i} are exogenous, and of course F_i is the usual exponential formulation. The usual restriction, $\sum_{i=1}^{n} S_i = 1$ is assumed to hold. Using the fact that $F_i = S_i/S_n =$

---

[1]For the full development of this criticism, see J. Hausman, "Project Independence Report: An Appraisal of U.S. Energy Needs up to 1985," The Bell Journal, (Autumn, 1975), pp. 517-551. In turn, Hausman's criticisms are based upon an article by Henri Theil, "A Multinomial Extension of the Linear Logit Model," International Economic Review (Oct. 1969), Vol. 10, #3, pp. 251-259. Both articles are discussed here and the notation of the original authors is used for simplicity and comparability.

[2]In reality, it is a criticism against the implications derived from the combination of usual separable utility and stochastic assumptions. See Hartman [12].

$Q_i/TOT/Q_n/TOT = Q_i/Q_n$ (where Q_i is the amount of fuel i consumed), we have $Q_i = Q_n F_i$ and the cross elasticity demand Q_i with respect to P_j is:

$$e_{Q_i P_j} = \frac{\partial Q_i}{\partial P_j}\frac{P_j}{Q_i} = \frac{P_j}{Q_i}\left(\frac{\partial Q_n}{\partial P_j} F_i + Q_n \frac{F_i}{P_j}\right) \tag{A3}$$

Clearly, given the formulation of (A2), $\frac{\partial F_i}{\partial P_j}\frac{P_j}{F_i} = 0 \ \forall j \neq i,n$

Hence, $\frac{\partial Q_i}{\partial P_k}\frac{P_k}{Q_i} = \frac{\partial Q_n}{\partial P_k}\frac{P_k}{Q_n}$ for $k \neq i,n$; the cross-elasticities of Q_i and Q_n with respect to P_k are always equal.[1]

Therefore, using equation A3 for j=i, and n, we have

$$\begin{aligned} \frac{\partial Q_i}{\partial P_i}\frac{P_i}{Q_i} &= \frac{\partial Q_n}{\partial P_i}\frac{P_i}{Q_n} + \frac{\partial F_i}{\partial P_i}\frac{P_i}{F_i} \\ \frac{\partial Q_i}{\partial P_n}\frac{P_n}{Q_i} &= \frac{\partial Q_n}{\partial P_n}\frac{P_n}{Q_n} + \frac{\partial F_i}{\partial P_n}\frac{P_n}{F_i} \end{aligned} \tag{A4}$$

while for $j \neq i,n$

$$\frac{\partial Q_i}{\partial P_j}\frac{P_j}{Q_i} = \frac{\partial Q_n}{\partial P_j}\frac{P_j}{Q_n} \tag{A5}$$

It should be clear that the estimates of elasticities and the constancy

---

[1]Using equation (A3), Hausman continues the derivation to

$$\frac{\partial Q_i}{\partial P_j}\frac{P_j}{Q_i} = \frac{\partial TOT}{\partial P_j}\frac{P_j}{TOT} - S_j \frac{P_j}{F_j}\frac{\partial F_j}{\partial P_j} + \frac{P_j}{F_i}\frac{\partial F_i}{\partial P_j} \tag{AT}$$

with a more detailed examination of the proposition that all cross-elasticities for a given P_j are equal. However, as in the discussion above, the derivation depends crucially on the form of demand F_i, and the fact that $\partial F_i/\partial P_j = 0$.

of cross elasticities in (A5) depends crucially not upon the use of conditional logit, but upon the critical formulation of demand F_i in (A2). If (A2) were formulated as

$$\frac{S_i}{S_n} = F_i\ (P_1,\ \ldots\ P_i,\ \ldots\ P_n,\ X_{2i}) \qquad \text{(A2')}$$

then (A5) would become

$$\frac{\partial Q_i}{\partial P_j}\frac{P_j}{Q_i} = \frac{\partial Q_n}{\partial P_j}\frac{P_j}{Q_n} + \frac{\partial F_i}{\partial P_j}\frac{P_j}{F_i} \qquad \text{(A5')}$$

where $\frac{\partial F_i}{\partial P_j} \neq 0\ \forall\ j \neq i,n$. Furthermore, equation (A2)' is more realistic. It is equation (A2)' that is used for the logit specification in the main body of this discussion.

The use of (A2)' can be defended upon several grounds. In the first place, since (A2)' is specified and estimated in conjunction with $\frac{S_i}{S_n} = F_i(\qquad)$, i = 1, ... n-1, subject to $\sum^n S_i = 1$ and since $\frac{\partial S_j}{\partial P_j} \neq 0$, then $\frac{\partial S_i}{\partial P_j} \neq \frac{\partial S_n}{\partial P_j} \neq 0$. As a result, there is no reason to constrain $\frac{\partial F_i}{\partial P_j} = 0$ which is effectively what formulation (A2) does.

This can be stated more precisely as follows: Let Y be a polychotimous random variable described by the set of M multinomial probabilities $Pr(Y\text{-}y_i) = P_i$ where $\sum^M P_i = 1$, where $0<P_i<1\ \forall_i$.[1] If we relate the probability of choosing fuel i to a set of exogenous variables X through the functional form F_i

---

[1]Clearly this holds for fuel share data also.

where X includes <u>all fuel prices</u> (own and other), then[1]

$$\Pr(Y=y_i|X) = P_i = \frac{e^{F_i(X)}}{\sum_{r=1}^{M} e^{F_r(X)}}$$

Likewise, $\Pr(Y=y_j|X) = P_j = \frac{e^{F_j(X)}}{\sum_{r=1}^{M} e^{F_r(X)}}$

Using these specifications for P_i and P_j, we get $\frac{P_i}{P_j} = \frac{e^{F_i(X)}}{e^{F_j(X)}}$ and

$$\log \frac{P_i}{P_j} = F_i(X) - F_j(X). \tag{A6}$$

If $F_i(X) = \alpha_i X$ for $\forall_i$, then we have

$$\log \frac{P_i}{P_j} = \alpha_i X - \alpha_j X \tag{A7}$$

$$= (\alpha_{i1} - \alpha_{j1})X_1 + (\alpha_{i2} - \alpha_{j2})X_2 + \ldots (\alpha_{iL} - \alpha_{jL})X_L$$

$$= \beta_1 X_1 + \beta_2 X_2 + \ldots \beta_L X_L$$

where again $X(X_1 \ldots X_L)$ includes <u>all</u> fuel prices. Clearly, if we replace fuel share data for probability data, we obtain equation (A2)', not equation (A2). Only if the price coefficients in (A7) or (A2)' are $\equiv 0$ will the equational form reduce to (A2). There is no reason to believe that is the case.

Finally, another way of indicating this is by utilizing the discussion

---

[1]This is the form of the development in Baughman and Joskow, <u>op. cit</u>.

of Theil.[1] Theil uses the formulation,

$$\frac{P_i}{P_j} = e^{\alpha_{ij}} \prod_{h=i}^{m} \chi_h^{\beta_{hij}} \prod_{k=1}^{n} \left(\frac{Y_{ki}}{Y_{kj}}\right)^{\gamma_{kij}} \qquad (2.1)$$

Given the "circularity" relations,[2] Theil demonstrates that

$$\alpha_{ij} = \alpha_i - \alpha_j$$

$$\beta_{hij} = \beta_{hi} - \beta_{hj}$$

and $\gamma_{kij} = \gamma_k$ for all i and j.

Hence, equation 2.1 becomes equation 2.2.

$$\frac{P_i}{P_j} = e^{\alpha_i - \alpha_j} \prod_{h=1}^{m} \chi_h^{\beta_{hj} - \beta_{hj}} \prod_{k=1}^{n} \left(\frac{Y_{ki}}{Y_{kj}}\right)^{\gamma_k} \qquad (2.2)$$

To estimate a share elasticity for χ_h we have

$$d(\log\frac{P_i}{P_j}) = (\beta_{hi} - \beta_{hj})\ d(\log\chi_h)$$

which Theil develops to show that

$$e_{P_i\chi_h} = \frac{d(\log P_i)}{d(\log\chi_h)} = \beta_{hi} - \sum_{j=1}^{N} P_j\beta_{hj}$$

If P_i is assumed to be a fuel share Q_i/TOT as assumed above, the quantity elasticity is identical:

---

[1]See Footnote 1, p. A1 for source.

[2]These relationships and the notation will be confusing to the reader unfamiliar with the Theil article. However, the elasticity discussion that follows does not depend upon a full understanding of the circularity.

$$d(\log\frac{P_i}{P_j}) = d(\log\frac{Q_i/TOT}{Q_j/TOT}) = \frac{dQ_i}{Q_i} - \frac{dQ_j}{Q_j} = (\beta_{hi} - \beta_{hj})d(\log\chi_h)$$

Multiplying by Q_j, we have:

$$Q_j\frac{dQ_i}{Q_i} - dQ_j = (Q_j\beta_{hi} - Q_j\beta_{hj})d(\log\chi_h)$$

since TOT = $\sum_j Q_j$ and $\sum_j dQ_j = 0$

As a result,

$$\frac{dQ_i}{Q_i} = (\beta_{hi} - \Sigma\frac{Q_j}{TOT}\beta_{hj})d(\log\chi_h) \text{ and}$$

$$e_{Q_i\chi_h} = \frac{d\log Q_i}{d\log\chi_h} = \beta_{hi} - \Sigma P_j\beta_{hj} = e_{P_i\chi_h}$$

Clearly, for the substitute prices entering into equation (2.1) as χ_{h1}, χ_{h2}, χ_{h3}, ... , etc., $e_{Q_i\chi_{h_2}} \neq e_{Q_i\chi_{h_2}} \neq e_{Q_i\chi_{h_3}}$ ... That is the general manner in which the prices have entered equation (A7 and (A2)'.

APPENDIX B: Regional Model Simulations

Overview

The simulations examined and compared in the main text of this discussion are national in purview. However, regional differences in fuel prices, capital costs, the availability of alternative fuels and in other economic and demographic variables will generate differential fuel share effects. It is the aim of our modelling effort to disaggregate our simulations in order to take account of regional differences in the independent variables. This Appendix examines the regional simulations for 1975, 1980, 1985, 2000 and 2025 for four geographical regions:[1]

- Northeast
- North Central
- South
- West

The simulations are based upon regression results utilizing the 1960/1970 constant 1970 dollars pooled data found in Table C.

---

[1]Northeast comprises:
Maine, New Hampshire, Vermont, Massachusetts, Rhode Island, Connecticut, New Jersey, New York, and Pennsylvania.

North Central comprises-
Ohio, Indiana, Illinois, Michigan, Wisconsin, Minnesota, Iowa, Missouri, North Dakota, South Dakota, Nebraska, and Kansas.

South comprises:
Delaware, Washington, D.C., Maryland, Virginia, West Virginia, North Carolina, South Carolina, Georgia, Florida, Kentucky, Tennessee, Alabama, Mississippi, Arkansas, Louisiana, Oklahoma, and Texas.

West comprises-
Montana, Idaho, Wyoming, Colorado, New Mexico, Arizona, Utah, Nevada, Washington, Oregon, California, Alaska, and Hawaii.

The exogenous variables for each region are documented in Table 1A of this appendix. They have been developed in two steps: 1) specification of a national scenario, and 2) regionalization of the national projections.

National Projections

The exogenous variables which have been projected include the prices of the various fuels, the capital costs of the heating systems which utilize the various fuels, per capita income, gas availability and heating degree days. Heating degree day (temp) was assumed to be constant over time for both the nation and the regions. The derivation of these variables has been discussed in Section A of the main body of this report.

Fuel price projections were developed through discussions with the Energy Research and Development Administration (ERDA). Though these projections are not official ERDA estimates, we feel that they represent reasonable assumptions about the future.[1] Based on these projections, we feel it is plausible to assume that gas and oil prices per BTU would be equal by the year 2025. As a result, the gas price (Pg) projections are based upon the oil price (Po) projections. Fuel oil used for space heating, namely #2, has been assumed to rise at a national rate of roughly 1.5% per annum. The resultant gas price forecasts, generated by the assumption of equivalent prices in 2025, increase at about 2.3% per annum.

The projections of these fossil fuel prices along with forecasts of the prices of coal and nuclear fuels[2] were combined with our assumptions of

[1]We look on these projections not as point estimates but rather as variable time paths helpful in exploring solution space.

[2]Prepared by Arthur D. Little Inc.

improved capacity factors for electric utilities (either through load management, rate structure changes, or more productive generating equipment) to yield projections of roughly 1% per annum average increase in the price of electricity (Pe).

Gas availability (AV) has been projected to increase through 1985 at the historical annual average rate realized between 1960 to 1975. After 1985, gas availability has been forecast to remain at that constant level throughout the forecast period. We feel that even given increases in the level of LNG imports and flow of gas from Alaska that this is an optimistic projection.

Per Capita Personal Income (PCI) has been forecast to rise at the 1960-1975 annual average rate of roughly 2.5% per annum throughout the forecast period.

Capital costs (CAPe, CAPo, CAPg) for the various types of space heating equipment has been assumed to remain constant. This is our most naive assumption; it does not reflect the likely changes in the mixes of equipment. The most notable change is the introduction of heat pumps. Another such innovation is the combination of a solar and conventional systems. The changes could be reflected in the capital costs of the systems as well as the operating costs.

In addition to these exogenous variables, we also forecast the level of the housing stock to increase on average by roughly 1.3% per annum over the forecast period. Housing stock for 1975 is the actual number taken from Bureau of Census statistics.

Regionalization of National Projections

The four regions for which projections have been made are introduced earlier in this appendix. The data sources for each of the regional exogenous variables which are not projected to change over the forecast period have been described in Section A and the supplement to Table 5 of the main body of this paper. Gas availability (AV) and per capita personal income (PCI) have been projected to increase for each region at the national rates.

The regionalized projections of the fuel prices reflect assumptions that regional price disparities will narrow substantially. This assumption yields the forecasts detailed in Table 1A of this appendix.

Housing stock projections (and the projected increments and decrements to the stock) are assumed to change by region (according to historical regional rates and expected economic activity: Thus, areas which have shown more building activity recently (for example, the South) are projected to remain the most active. The housing demand in all regions is assumed to be the greatest in the period from 1975 through 1985.

Form of the Simulation Results

This simulation model utilizes parameters estimated in the 1960/1970 pooled equations and the exogenous variables shown in Table 1A. The estimated regional fuel shares from the simulation model for each region for the years 1975, 1980, 1985, 2000 and 2025, are shown in Tables 1B-E.

Table 1B, 1C, 1D, and 1E present the simulation results for the Northeast, Northcentral, South and West regions respectively. Each of these regional tables contains five charts. The first two charts present the estimated fuel shares over 1975-2025 based upon the exogenous variables

(Table 1A) and the equations in Table 5C of the text. Estimates of shares and numbers of households (i.e. share times total required housing stock) are presented in the first two charts by region. These are the "unconstrained results." If, however, the increments to or decrements from the housing stocks implied by the unconstrained estimates of numbers of households is greater than exogenous estimates of these increments or decrements (Table 1A), the and household estimates are constrained the exogenous estimates. The resulting "constrained estimates" are presented in the third and fourth charts of Tables 1B-1E. Chart 5 in each table indicates the implied incremental changes in the number of households using each fuel, based on Chart 4.

Regional Simulation Results

The following discussion will only deal with the constrained estimates (charts 3, 4 and 5) of the simulation results. The discussion will be limited to the long-run changes over the forecast period.

Electric home heating shows the most dramatic increase in all regions rising by two or three times its original share in each region. This is clearly due to the projected increase in price advantage of electricity. As mentioned above, these optimistic price projections for electricity result from the assumption that more cost effective generation facilities will come on stream throughout the forecast period. Furthermore, the projected increase in the electric share of home heating results from the estimated positive income elasticity of demand for electricity and the clear capital cost advantage projected for electricity in all regions throughout the forecast period.

The share of homes heated by oil falls in every region, except the West, by the end of the forecast period. The two principal reasons for oil's declining share in home heating are the strong negative elasticity with respect to capital costs and the negative income elasticity. Capital costs for oil heating equipment is projected to remain the highest in all regions throughout the forecast period. Also, the price of oil is projected to remain higher than gas in all but the Northeast region, and it contributes to the projected decline in the share of houses heated by oil.

The share of homes heated by gas falls in all regions but the Northeast. However, the change in gas share is the least dramatic of the changes among the three fuels. The declining share is due to the fact that the price of gas rises the most rapidly of the three fuel prices; however, it remains the cheapest fuel ($/BTU) except in the Northeast where oil is projected to be the cheapest only after 1985. The decline in gas share is further ameliorated by the estimated positive income elasticity and the projected capital cost advantage gas holds over oil through the forecast period.

NORTHEAST REGION (TABLE 1B, CHARTS 3-4)

In the Northeast region, the share of gas increases by more than 5 percentage points, from just over 38% to nearly 44% of the total home heating market. In absolute numbers, the households using gas for spaceheating rises from about 6,171,000 to over 10,306,000 between 1975 and 2025. The shift to gas reflects the positive income elasticity and the capital cost advantage over oil equipment. This occurs in spite of oil's price advantage.

The share of homes heated by electricity rises from 4.4% to over 11.6%, in absolute numbers from 709,000 to over 2,750,000. Electricity's increase is due to a strong positive income elasticity, the cheapest projected capital costs and lowest rate of increase in prices.

The oil heat share declines in the Northeast from over 57% to just under 45% of the total heating market, but the absolute number of units rises marginally from about 9,216,000 to slightly more than 10,542,000 homes.

While oil is currently the dominant fuel used in the Northeast (due of course to the historically easy availability of cheap imported fuel), over the forecast period oil's price advantage is projected to decline with respect to electricity. Capital cost for oil furnaces is projected to remain the highest, roughly 25% greater than electricity's over the forecast period. Also oil's income elasticity is negative. As a result, the share of oil used for home spaceheating will decline markedly.

NORTH CENTRAL REGION (TABLE 2C, CHARTS 3-4)

In the North Central region, both the share of gas and oil are projected to decline while electricity is projected to increase. The share of gas used in home heating is projected to decline from 77.6% to almost 72%, while in the number of household units it is projected to increase from about 14,546,000 to over 23,142,000 between 1975 and 2025. The share of homes heated by oil is projected to decline from 16.8% to 10.4%, while the number of households increases marginally from about 3,148,000 to over 3,340,000. Electricity's share of the home heating market rises, showing the most dramatic relative change among the three fuels, from 5.6% to 17.7% and in actual number of units from about 1,041,000 to about 5,695,000.

The reason for the appeal of electricity is the same in this region as in the others. The projected price rise is the least radical of the three fuels, capital costs are projected to remain the lowest among the three types of burning equipment and the income elasticity is estimated to be positive.

The increase in the number of household units using gas for space heating is attributable to the estimated positive income elasticity, its projected operating cost advantage over the other two fuels and its capital cost advantage over oil.

The oil market fairs the worst because of the estimated negative income elasticity, its projected capital cost disadvantage to all other fuels, and the projected price disadvantage with respect to gas.

<u>SOUTH REGION</u> (TABLE 2D, CHARTS 3 and 4)

In the South, the trend toward electricity continues unabated. In this region the share of homes heated with electricity is projected to rise from 21.3% to 47.6% and the number of household units to increase from roughly 4,584,000 to about 23,135,000. As in the other regions, the reason for this projected change is that the price of electricity rises at a slower rate than does the price of oil or gas. Furthermore, electricity shows the strongest income elasticity of demand and is projected to maintain the cheapest capital costs.

The share of homes heated by gas is projected to decrease from 62.2% to just under 48%, though the actual number of houses is expected to rise from about 13,398,000 to just less than 23,330,000. These projections result from reasons similar to those discussed above.

The share and number of homes in the south heated by oil is projected to decline over the forecast period, from 16.6% to 4.4% and from 3,567,000 units to just under 2,152,000. These projections are an indication that the expected capital cost disadvantage, the price disadvantage with respect to gas and the negative income elasticity.

WEST REGION (TABLES 2E-5E)

In the West, by 2025, both oil and electricity are projected to increase in market share and gas to decline.

The share of homes heated by electricity is expected to increase, after a short period of some decline, from 16.5% to about 34.2%. The number of household units is projected to rise from 2,078,000 to over 9,762,000.

The share and number of homes heated by fuel oil is projected to increase from 7.1% to over 8.4% and the number of units is expected to grow from 897,000 to over 2,407,000. Oil's share as in most regions first increases and then decreases. The reasons for this cyclical phenomenon can be attributed to the fact that the price of oil is projected to rise faster in the latter half of the forecast period than in the first half, while the prices of gas and electricity indicate increases at a slower rate in the second half of the forecast period than in the first half.

The share of homes heated by gas is shown in the simulation results to decrease from 76.4% to 57.4%, though the number of units is projected to increase from 9,615,000 to over 16,385,000. The decrease in gas share is principally due to the sharpest price increases projected among the three fuels. The expected market loss is ameliorated somewhat by the expectation that gas furnace capital costs will remain less expensive than oil's and by the estimated positive income elasticity.

CONCLUSION

In this brief discussion, we have focused on what we feel are the three principal factors affecting our fuel share projections: price, capital costs and income. We have also run other simulations which indicate that the results are sensitive to the variation in gas availability projections. In this set of simulations we have been satisfied with the naive projection that gas availability will increase for the first ten years of the forecast period and remain at the same level throughout the rest of the forecast period. Further efforts will be needed to reflect more refined assumptions for all the forecast exogenous variables.

APPENDIX B: REGIONAL MODEL SIMULATIONS

TABLE 1A

EXOGENOUS VARIABLES FOR NORTHEAST REGION

YEAR	PE	PO	PG	CAPE	CAPO	CAPG	AV	PCI	TEMP	TOT	INCRE	DECRE
1975	9.45	1.96	1.74	115.33	143.88	126.66	153.00	4365.87	6272.00	16282.00	0.0	0.0
1980	9.26	1.99	1.98	115.33	143.88	126.66	166.00	4939.55	6272.00	17253.00	1250.00	-279.00
1985	9.07	2.02	2.23	115.33	143.88	126.66	179.00	5588.71	6272.00	18483.00	1500.00	-270.00
2000	10.70	2.61	2.87	115.33	143.88	126.66	179.00	8093.77	6272.00	20760.00	3357.00	-1080.00
2025	14.05	4.07	4.16	115.33	143.88	126.66	179.00	15005.94	6272.00	23785.00	5050.00	-2025.00

EXOGENOUS VARIABLES FOR NORTHCENTRAL REGION

YEAR	PE	PO	PG	CAPE	CAPO	CAPG	AV	PCI	TEMP	TOT	INCRE	DECRE
1975	7.02	2.07	1.12	133.06	159.29	140.31	303.00	4397.90	6665.00	18993.00	0.0	0.0
1980	7.14	2.11	1.56	133.06	159.29	140.31	328.00	4975.62	6665.00	20664.00	2074.00	-403.00
1985	7.27	2.14	1.99	133.06	159.29	140.31	353.00	5629.43	6665.00	22779.00	2505.00	-390.00
2000	8.56	2.76	2.51	133.06	159.29	140.31	353.00	8153.81	6665.00	26865.00	5646.00	-1590.00
2025	11.24	4.30	4.14	133.06	159.29	140.31	353.00	15115.99	6665.00	32435.00	8495.00	-2925.00

EXOGENOUS VARIABLES FOR SOUTH REGION

YEAR	PE	PO	PG	CAPE	CAPO	CAPG	AV	PCI	TEMP	TOT	INCRE	DECRE
1975	5.94	1.96	1.20	102.01	134.09	116.85	414.00	3619.72	3208.00	22488.00	0.0	0.0
1980	6.33	1.99	1.57	102.01	134.09	116.85	449.00	4095.07	3208.00	25726.00	3827.00	-589.00
1985	6.74	2.02	1.94	102.01	134.09	116.85	484.00	4633.35	3208.00	29803.00	4647.00	-570.00
2000	7.93	2.61	2.44	102.01	134.09	116.85	484.00	6710.94	3208.00	38027.00	10504.00	-2280.00
2025	10.43	4.07	3.94	102.01	134.09	116.85	484.00	12441.11	3208.00	49555.00	15803.00	-4275.00

EXOGENOUS VARIABLES FOR WEST REGION

YEAR	PE	PO	PG	CAPE	CAPO	CAPG	AV	PCI	TEMP	TOT	INCRE	DECRE
1975	5.41	2.01	1.09	113.83	147.21	129.95	330.00	4312.68	5245.00	13067.00	0.0	0.0
1980	5.95	2.05	1.51	113.83	147.21	129.95	357.00	4879.76	5245.00	14937.00	2149.00	-279.00
1985	6.49	2.07	1.94	113.83	147.21	129.95	384.00	5520.78	5245.00	17295.00	2628.00	-270.00
2000	7.64	2.68	2.50	113.83	147.21	129.95	384.00	7995.84	5245.00	22141.00	5926.00	-1080.00
2025	10.05	4.18	4.02	113.83	147.21	129.95	384.00	14823.06	5245.00	29032.00	8916.00	-2025.00

TABLE 1B

NORTHEAST REGION

1) UNCONSTRAINED RESULTS

SHARE DATA GIVEN BY YEAR FOR URBAN, RURAL, AND ALL AREAS RESPECTIVELY

****

1975	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.31735	0.45427	0.03044
	0.06604	0.11829	0.01361
	0.38339	0.57256	0.04405

1980	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.07563	0.64423	0.03015
	0.02521	0.21474	0.01005
	0.10084	0.85897	0.04020

1985	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.05972	0.66416	0.02613
	0.01991	0.22139	0.00871
	0.07962	0.88554	0.03483

2000	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.09230	0.61303	0.04467
	0.03077	0.20434	0.01489
	0.12307	0.81737	0.05956

2025	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.30734	0.30858	0.13409
	0.10245	0.10286	0.04470
	0.40978	0.41144	0.17878

TABLE 1B (cont'd.)

2) NORTHEAST REGION

HOUSEHOLD DATA GIVEN BY YEAR FOR URBAN, RURAL, AND ALL AREAS RESPECTIVELY

****

1975	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	5108.00	7312.00	490.00
	1063.00	1904.00	219.00
	6171.00	9216.00	709.00

1980	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	1304.79	11114.82	520.15
	434.93	3704.94	173.38
	1739.71	14819.77	693.53

1985	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	1103.75	12275.61	482.88
	367.92	4091.87	160.96
	1471.67	16367.49	643.85

2000	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	1916.14	12726.53	927.34
	638.71	4242.18	309.11
	2554.85	16968.70	1236.46

2025	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	7309.97	7339.49	3189.28
	2436.66	2446.50	1063.09
	9746.63	9785.99	4252.37

TABLE 1B (cont'd.)

NORTHEAST REGION

3) CONSTRAINED RESULTS

SHARE DATA GIVEN BY YEAR FOR URBAN, RURAL, AND ALL AREAS RESPECTIVELY

****

1975	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.31735	0.45427	0.03044
	0.06604	0.11829	0.01361
	0.38339	0.57256	0.04405

1980	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.28707	0.48336	0.02867
	0.05821	0.12987	0.01282
	0.34528	0.61323	0.04149

1985	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.25844	0.51235	0.02501
	0.05119	0.14164	0.01138
	0.30962	0.65399	0.03639

2000	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.26932	0.47756	0.04384
	0.05868	0.13327	0.01732
	0.32800	0.61083	0.06117

2025	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.34787	0.35199	0.08564
	0.08885	0.09473	0.03091
	0.43673	0.44672	0.11655

TABLE 1B (cont'd.)

4) NORTHEAST REGION

HOUSEHOLD DATA GIVEN BY YEAR FOR URBAN, RURAL, AND ALL AREAS RESPECTIVELY

****

1975	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	5108.00	7312.00	490.00
	1063.00	1904.00	219.00
	6171.00	9216.00	709.00

1980	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	4899.48	8249.50	489.27
	993.49	2216.50	218.76
	5892.97	10466.00	708.03

1985	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	4728.64	9374.49	457.60
	936.55	2591.50	208.20
	5665.19	11965.99	665.81

2000	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	5541.02	9825.40	902.06
	1207.34	2741.80	356.35
	6748.37	12567.21	1258.42

2025	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	8209.48	8306.65	2021.10
	2096.83	2235.55	729.37
	10306.32	10542.21	2750.47

TABLE 1B (cont'd.)

5) NORTHEAST REGION

INCREMENTAL DATA GIVEN BY YEAR FOR URBAN, RURAL, AND ALL AREAS RESPECTIVEL

****

1980	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	-208.52	937.50	-0.73
	-69.51	312.50	-0.24
	-278.03	1250.00	-0.97

1985	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	-170.83	1125.00	-31.67
	-56.94	375.00	-10.56
	-227.78	1500.00	-42.22

2000	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	812.38	450.91	444.46
	270.79	150.30	148.15
	1083.18	601.21	592.61

2025	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	2668.46	-1518.75	1119.04
	889.49	-506.25	373.01
	3557.95	-2025.00	1492.05

TABLE 1C

1) NORTHCENTRAL REGION
UNCONSTRAINED RESULTS
SHARE DATA GIVEN BY YEAR FOR URBAN, RURAL, AND ALL AREAS RESPECTIVELY
****

1975	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.53899	0.09090	0.03235
	0.23742	0.07713	0.02322
	0.77641	0.16803	0.05556

1980	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.22321	0.48516	0.04164
	0.07440	0.16172	0.01388
	0.29761	0.64687	0.05551

1985	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.16358	0.55101	0.03541
	0.05453	0.18367	0.01180
	0.21810	0.73469	0.04721

2000	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.26054	0.42700	0.06247
	0.08685	0.14233	0.02082
	0.34738	0.56933	0.08329

2025	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.37086	0.21915	0.15999
	0.12362	0.07305	0.05333
	0.49448	0.29220	0.21332

TABLE 1C (cont'd.)

2) NORTHCENTRAL REGION

HOUSEHOLD DATA GIVEN BY YEAR FOR URBAN, RURAL, AND ALL AREAS RESPECTIVELY

****

1975	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	10098.00	1703.00	606.00
	4448.00	1445.00	435.00
	14546.00	3148.00	1041.00

1980	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	4612.41	10025.24	860.35
	1537.47	3341.75	286.78
	6149.88	13366.99	1147.14

1985	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	3726.15	12551.55	806.54
	1242.05	4183.85	268.85
	4968.20	16735.41	1075.39

2000	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	6999.33	11471.27	1678.16
	2333.11	3823.76	559.39
	9332.44	15295.02	2237.55

2025	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	12028.80	7108.09	5189.35
	4009.60	2369.36	1729.78
	16038.41	9477.46	6919.13

TABLE 1C (cont'd.)

3) NORTHCENTRAL REGION

HOUSEHOLD DATA GIVEN BY YEAR FOR URBAN, RURAL, AND ALL AREAS RESPECTIVELY

****

1975	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	10098.00	1703.00	606.00
	4448.00	1445.00	435.00
	14546.00	3148.00	1041.00

1980	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	9795.75	3242.51	621.99
	4347.25	1958.17	440.33
	14143.00	5200.68	1062.32

1985	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	9519.99	5121.26	605.25
	4255.33	2584.42	434.75
	13775.32	7705.67	1040.00

2000	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	12793.17	4040.97	1476.86
	5346.39	2224.32	725.29
	18139.56	6265.29	2202.15

2025	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	16545.11	1847.22	4096.17
	6597.03	1493.07	1598.39
	23142.14	3340.29	5694.56

TABLE 1C (cont'd.)

4) NORTHCENTRAL REGION
CONSTRAINED RESULTS
SHARE DATA GIVEN BY YEAR FOR URBAN, RURAL, AND ALL AREAS RESPECTIVELY
****

1975	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.53899	0.09090	0.03235
	0.23742	0.07713	0.02322
	0.77641	0.16803	0.05556

1980	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.48004	0.15890	0.03048
	0.21304	0.09596	0.02158
	0.69308	0.25486	0.05206

1985	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.42272	0.22740	0.02687
	0.18895	0.11476	0.01930
	0.61167	0.34216	0.04618

2000	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.48082	0.15188	0.05551
	0.20094	0.08360	0.02726
	0.68176	0.23548	0.08277

2025	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.51419	0.05741	0.12730
	0.20502	0.04640	0.04967
	0.71921	0.10381	0.17698

TABLE 1C (cont'd.)

5) NORTHCENTRAL REGION

INCREMENTAL DATA GIVEN BY YEAR FOR URBAN, RURAL, AND ALL AREAS RESPECTIVELY

****

1980	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	-302.25	1539.51	15.99
	-100.75	513.17	5.33
	-403.00	2052.68	21.32

1985	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	-275.76	1878.75	-16.74
	-91.92	626.25	-5.58
	-367.68	2505.00	-22.32

2000	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	3273.18	-1080.29	871.61
	1091.06	-360.10	290.54
	4364.24	-1440.38	1162.15

2025	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	3751.94	-2193.75	2619.31
	1250.65	-731.25	873.10
	5002.58	-2925.00	3492.41

TABLE 1D

1) SOUTH REGION

UNCONSTRAINED RESULTS

SHARE DATA GIVEN BY YEAR FOR URBAN, RURAL, AND ALL AREAS RESPECTIVELY

****

1975	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.35691	0.09323	0.13133
	0.26484	0.07230	0.08140
	0.62175	0.16553	0.21272

1980	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.52844	0.03940	0.18216
	0.17615	0.01313	0.06072
	0.70458	0.05254	0.24288

1985	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.52880	0.04414	0.17706
	0.17627	0.01471	0.05902
	0.70506	0.05886	0.23608

2000	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.51141	0.02424	0.21435
	0.17047	0.00808	0.07145
	0.68188	0.03232	0.28580

2025	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.38181	0.00795	0.36024
	0.12727	0.00265	0.12008
	0.50907	0.01060	0.48033

TABLE 1D (cont'd.)

2) SOUTH REGION

HOUSEHOLD DATA GIVEN BY YEAR FOR URBAN, RURAL, AND ALL AREAS RESPECTIVELY

****

1975	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	7691.00	2009.00	2830.00
	5707.00	1558.00	1754.00
	13398.00	3567.00	4584.00

1980	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	13594.59	1013.69	4686.21
	4531.53	337.90	1562.07
	18126.13	1351.59	6248.28

1985	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	15759.72	1315.60	5276.93
	5253.24	438.53	1758.98
	21012.96	1754.13	7035.91

2000	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	19447.28	921.80	8151.16
	6482.43	307.27	2717.05
	25929.71	1229.07	10868.22

2025	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	18920.39	393.93	17851.91
	6306.79	131.31	5950.64
	25227.18	525.24	23802.55

TABLE 1D (cont'd.)

3) SOUTH REGION
CONSTRAINED RESULTS
SHARE DATA GIVEN BY YEAR FOR URBAN, RURAL, AND ALL AREAS RESPECTIVELY
****

1975	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.35691	0.09323	0.13133
	0.26484	0.07230	0.08140
	0.62175	0.16553	0.21272

1980	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.39593	0.06323	0.14432
	0.25879	0.05691	0.08081
	0.65472	0.12014	0.22513

1985	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.41502	0.06476	0.14440
	0.24724	0.05236	0.07622
	0.66226	0.11712	0.22062

2000	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.42242	0.03978	0.18988
	0.22556	0.03721	0.08515
	0.64798	0.07699	0.27503

2025	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.31142	0.01949	0.34439
	0.16846	0.02477	0.13147
	0.47988	0.04426	0.47586

TABLE 1D (cont'd.)

4) SOUTH REGION

HOUSEHOLD DATA GIVEN BY YEAR FOR URBAN, RURAL, AND ALL AREAS RESPECTIVELY

****

1975	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	7691.00	2009.00	2830.00
	5707.00	1558.00	1754.00
	13398.00	3567.00	4584.00

1980	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	9813.97	1567.25	3577.28
	6414.66	1410.75	2003.09
	16228.63	2978.00	5580.37

1985	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	11979.09	1869.16	4168.00
	7136.36	1511.39	2200.00
	19115.46	3380.54	6368.00

2000	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	15666.65	1475.36	7042.23
	8365.55	1380.12	3158.08
	24032.21	2855.48	10200.30

2025	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	15139.75	947.49	16742.98
	8189.91	1204.16	6391.66
	23329.68	2151.66	23134.64

TABLE 1D (cont'd.)

5) SOUTH REGION

INCREMENTAL DATA GIVEN BY YEAR FOR URBAN,RURAL, AND ALL AREAS RESPECTIVEL'

****

1980	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	2122.97	-441.75	747.28
	707.66	-147.25	249.09
	2830.63	-589.00	996.37

1985	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	2165.13	301.91	590.72
	721.71	100.64	196.91
	2886.84	402.54	787.63

2000	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	3687.56	-393.79	2874.23
	1229.19	-131.27	958.08
	4916.74	-525.06	3832.30

2025	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	-526.89	-527.87	9700.75
	-175.63	-175.96	3233.58
	-702.52	-703.83	12934.34

TABLE 1E

1) WEST REGION
UNCONSTRAINED RESULTS
SHARE DATA GIVEN BY YEAR FOR URBAN, RURAL, AND ALL AREAS RESPECTIVELY
****

1975	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.64067	0.04202	0.11533
	0.12303	0.02923	0.04972
	0.76370	0.07125	0.16505

1980	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.28728	0.34900	0.11372
	0.09576	0.11633	0.03791
	0.38304	0.46533	0.15163

1985	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.25442	0.39539	0.10019
	0.08481	0.13180	0.03340
	0.33923	0.52719	0.13358

2000	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.31534	0.28459	0.15007
	0.10511	0.09486	0.05002
	0.42045	0.37945	0.20010

2025	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.35137	0.10945	0.28918
	0.11712	0.03648	0.09639
	0.46850	0.14593	0.38557

TABLE 1E

2) WEST REGION

HOUSEHOLD DATA GIVEN BY YEAR FOR URBAN, RURAL, AND ALL AREAS RESPECTIVELY

Year	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
1975	8066.00	529.00	1452.00
	1549.00	368.00	626.00
	9615.00	897.00	2078.00
1980	4291.06	5212.99	1698.70
	1430.35	1737.66	566.23
	5721.42	6950.65	2264.93
1985	4400.27	6838.25	1732.73
	1466.76	2279.42	577.58
	5867.02	9117.67	2310.31
2000	6981.88	6301.08	3322.78
	2327.29	2100.36	1107.59
	9309.17	8401.44	4430.38
2025	10201.07	3177.58	8395.34
	3400.36	1059.19	2798.45
	13601.43	4236.77	11193.79

TABLE 1E

3) WEST REGION
CONSTRAINED RESULTS
SHARE DATA GIVEN BY YEAR FOR URBAN, RURAL, AND ALL AREAS RESPECTIVELY
****

1975	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.64067	0.04202	0.11533
	0.12303	0.02923	0.04972
	0.76370	0.07125	0.16505

1980	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.54334	0.14471	0.10375
	0.10230	0.06149	0.04440
	0.64564	0.20620	0.14816

1985	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.47366	0.22106	0.09123
	0.09012	0.08508	0.03885
	0.56378	0.30614	0.13008

2000	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.48687	0.14681	0.14422
	0.10968	0.05779	0.05463
	0.59655	0.20460	0.19885

2025	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	0.46030	0.05620	0.25268
	0.11352	0.02611	0.08920
	0.57382	0.08431	0.34188

TABLE 1E

4) WEST REGION

HOUSEHOLD DATA GIVEN BY YEAR FOR URBAN, RURAL, AND ALL AREAS RESPECTIVELY

****

1975	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	8066.00	529.00	1452.00
	1549.00	368.00	626.00
	9615.00	897.00	2078.00

1980	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	7856.75	2092.47	1500.28
	1479.25	889.16	642.09
	9336.00	2981.63	2142.37

1985	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	7965.95	3717.73	1534.31
	1515.65	1430.91	653.44
	9481.61	5148.64	2187.75

2000	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	10547.56	3180.56	3124.36
	2376.19	1251.85	1183.45
	12923.75	4432.42	4307.82

2025	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	13143.72	1661.81	7215.20
	3241.58	745.60	2547.07
	16385.30	2407.42	9762.26

TABLE 1E

5) WEST REGION

INCREMENTAL DATA GIVEN BY YEAR FOR URBAN,RURAL, AND ALL AREAS RESPECTIVEL

****

1980	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	-209.25	1563.47	48.28
	-69.75	521.16	16.09
	-279.00	2084.63	64.37

1985	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	109.20	1625.26	34.03
	36.40	541.75	11.34
	145.61	2167.02	45.38

2000	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	2581.61	-537.17	1590.05
	860.54	-179.06	530.02
	3442.15	-716.23	2120.07

2025	GAS SHARE	OIL SHARE	ELECTRICITY SHARE
	2596.16	-1518.75	4090.83
	865.39	-506.25	1363.61
	3461.55	-2025.00	5454.45