

**ON THE GENERATION OF VARIABLE STRUCTURE
DISTRIBUTED ARCHITECTURES FOR C³ SYSTEMS**

**Jacques J. Demaël
Alexander H. Levis**

Laboratory for Information and Decision Systems
Massachusetts Institute of Technology
Cambridge, MA 02139

ABSTRACT

The design of a C³ system architecture for a submarine is described to illustrate a new quantitative approach for generating variable structure architectures. In these architectures, the interactions between the components of the system can change, depending on the task being processed. Colored Petri Nets are used as the appropriate mathematical formalism to represent variable architectures within a well-defined framework. The specifications of design requirements are expressed in that framework. An algorithm to compute the variable structures that satisfy design requirements as well as some generic constraints is outlined, and the solutions are interpreted.

* This work was carried out at the MIT Laboratory for Information and Decision Systems with support provided by the Office of Naval Research under contract no. N00014 - 84 - K- 0519. Paper to appear in Proc. 1989 Symposium on C2 Research, National Defense University, Washington, DC, June 1989.

INTRODUCTION

This paper illustrates a methodology (Demaël, 1989) that has been developed to model and generate variable structure distributed architectures for C³ systems. A hypothetical design of a simplified C³ system that formulates tactical responses in submarines is presented. The submarine is supposed to be in a sensitive tactical situation, and faces two major external threats, torpedoes and depth charges. The goal is to design a Command and Control system that adapts its structure of interaction to the tactical parameters. The methodology has been developed to meet ever increasing requirements of reliability and reconfigurability from users by studying variable structure systems. In these systems, the interactions between components can change depending on the task, while the same task can be performed with different combinations of resources. As some patterns of interactions may be more suitable for the processing of a given input than others, a properly designed variable structure system can be expected to achieve a higher overall performance, provided that it adapts its structure to the most appropriate interactions for each type of input.

In an operational context such as a C³ system, teams of decision makers (Grevet and Levis, 1988) are organized to overcome the individual cognitive limits. Through specialization, individuals acquire the capacity to apply relatively complex cognitive strategies to narrowly defined tasks environments. Through division of labor, substantial cognitive resources can be simultaneously brought to bear on many tasks or information sources at a time. The most important difficulty encountered with the design of those systems is the coordination problem (Crecine and Salomone, 1988), the need for the pattern of activities carried out by individuals in various subunits of the system to fit together in a coherent fashion. Demaël (1989) describes and solves the coordination problem for a well defined class of C³ systems, those that process deterministically a set of simultaneous observations. An appropriate mathematical framework has been defined, which is based on the theory of Colored Petri Nets (Jensen, 1987), an extension of the Petri Net theory (Reisig, 1985). In this approach, the tokens have an identity that represents the information content of the object being modeled. The variable interactions between the elements in the system are described using the language of Linear Algebra.

In the next section, the elements from which a system may be built are identified. The design requirements of a C³ system for a submarine are presented in the third section. The constraints that must be satisfied by a solution to make physical sense are developed in the fourth section. Finally, the fifth section outlines the search for solutions and interprets the results.

MATHEMATICAL MODEL

A C³ system has a *variable structure* if the interactions between the objects that belong to

the system can vary. Reciprocally, a system for which the interactions between objects cannot vary has a *fixed structure*. This paper is restricted to the systems whose variability is triggered by parameters that are *external* to the system, can be *sensed* by some processes, and can be *communicated* to the objects. Furthermore, the emphasis has been placed on describing how interactions in the system are influenced by the external inputs, as opposed to a description of the information content of the interactions. In the submarine example, the focus is thus on studying the interactions that are triggered within the Command and Control system by tactical parameters. These parameters are assessed by sensors (active and passive sonar), which detect the presence of vessels on the surface or in the deep sea environment. The influence of other inputs such as engine failure or emergency have not been incorporated; they have been treated in Weingaertner (1989).

The methodology addresses the coordination problem of the functional entities, the *roles*. A role is a prescribed pattern of behavior, a series of repetitive but uncertain tasks, imposed by the requirements of the mission. Each role is performed by a human being or by an intelligent computerized node. Three roles have been identified in the submarine example: The Anti Submarine Warfare role (ASW), the Anti Surface Warfare role (ASUW), and the Officer of the Deck (OOD). The OOD is at the top of the hierarchical chain, and has the responsibility for integrating all aspects of the ship's mission. However, OOD does not have immediate access to the sonar's observations, which are the responsibilities of ASW and ASUW. The area of competence of ASW is anti submarine warfare. ASW monitors the deep sea environment through different acoustical devices. ASW has been trained to recognize the characteristic noise of foes and friendly ships, as well as their tactical submarine procedures. Similarly, ASUW is the expert in anti-surface warfare. ASUW can recognize the noise of ships, the type of missions to which ships can be assigned, and all relevant aspects of surface warfare. The C³ system has to develop a tactical response that is transmitted to the effectors, the rest of the crew, which includes the ship control party and the combat elements. The goal of the methodology is to create a Colored Petri Net model of dataflow from the sources to the roles, the exchange of information between roles, and the communication of messages to the effectors. A Colored Petri Net can then be used to assess the effectiveness of a structure, using the System Effectiveness Analysis methodology as described in Monguillet (1988).

The C³ systems that are being modeled are *temporally consistent* and *deterministic*.

- 1) They process observations that refer to the same temporal origin, i.e., to an event with a specific time of occurrence (Grevet, 1988);
- 2) The processing of one set of simultaneous observations is achieved while involving a unique set of interactions.

The C³ system of a submarine monitors indeed "on line" the state of the environment, so as to react appropriately. The deterministic assumption indicates that the activities of OOD, ASW, and ASUW have been coordinated so as to produce one single response, that of the system OOD,

ASW, and ASUW process data transmitted by sensors, which can be modeled by two sources of information. Each sensor outputs one signal or symbol, also called a color, from its associated set of possible signals, its output alphabet.

- Sensor 1 describes the deep sea environment. Its output alphabet is
$$X1 = \{\text{NoSubmarine, Submarine, Torpedo}\},$$
NoSubmarine indicates that no submarine is detected by acoustical devices, Submarine indicates that the noise of a submarine vessel has been heard, Torpedo indicates the detection of a torpedo. To make the notation more compact, X1 is abbreviated by $X1 = \{\text{NS, S, T}\}$.
- Sensor 2 describes the surface environment. Its output alphabet is
$$X2 = \{\text{NoSurface, Surface, Depth Charge}\},$$
NoSurface indicates that no particular noise coming from the sea surface hints at the presence of a surface ship or a plane, Surface indicates that some characteristic noise has been heard, and Depth Charge models the detection of an offensive weapon. X2 is abbreviated as $X2 = \{\text{NSU, SU, dC}\}$.
- As the observations are temporally consistent, the inputs to the system from the two sensors are modeled by the cross product, $X1 \times X2$, of the output alphabets. X contains 9 inputs lexicographically ordered:
$$X = \{ \langle \text{NS, dC} \rangle, \langle \text{NS, NSU} \rangle, \langle \text{NS, SU} \rangle, \langle \text{S, dC} \rangle, \langle \text{S, NSU} \rangle, \langle \text{S, SU} \rangle, \langle \text{T, dC} \rangle, \langle \text{T, NSU} \rangle, \langle \text{T, SU} \rangle \}.$$

In the Colored Petri Net model, Sensor i is modeled by a place annotated by X_i , the output alphabet. A transition models the communication of its output. The temporal consistency is modeled by the fact that all sensors are the output of a single process. This process has a single input place p_0 , which is called the external place.

Each element in $X = X1 \times X2$ represents a combination of tactical parameters. The deterministic assumption implies that there is one and only one fixed structure architecture that is associated with any element of X, i.e., that to each combination of tactical parameters corresponds one and only one fixed pattern of interactions between roles. Of course, several elements in X might correspond to the same fixed global pattern of interactions, if similar tactical parameters require the same set of interactions between OOD, ASW, and ASUW, while radically different tactical situations might correspond to very different fixed patterns of interactions.

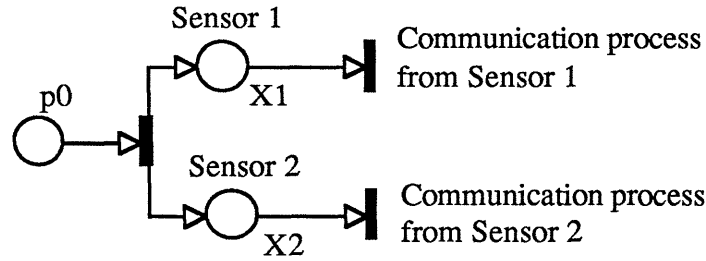


Fig. 1 Sensors

In the Colored Petri Net model, each interaction is represented by a link between two transitions, as depicted in Fig. 2. The arcs of the link are annotated by a 9×9 *diagonal* matrix L .

- $L_{ii} = 1$ if the i -th input in the lexicographic ordering of X activates the link.
- $L_{ii} = 0$ if the i -th input in the lexicographic ordering of X does not activate the link.

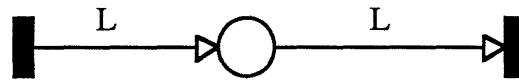


Fig. 2 Link

Consider the matrix $L1$ for example. This interaction is activated if and only if the input to the system is $\langle S, NSU \rangle$ or $\langle T, NSU \rangle$. The diagonal elements that correspond to these two inputs are unity.

$$L1 = \text{diag} \begin{bmatrix} \langle NS, dC \rangle & \langle NS, NSU \rangle & \langle NS, SU \rangle & \langle S, dC \rangle & \langle S, NSU \rangle & \langle S, SU \rangle & \langle T, dC \rangle & \langle T, NSU \rangle & \langle T, SU \rangle \\ 0, & 0, & 0, & 0, & 1, & 0, & 0, & 1, & 0 \end{bmatrix}.$$

There are three basic types of interactions in a variable structure:

- The *inadmissible* links. These are the links for which L is the $|X| \times |X|$ null matrix. No input requires this interaction to be processed. These links are not depicted on a Colored Petri Net model of the C3 system.
- The *permanent* links. These are the links for which L is the $|X| \times |X|$ identity matrix. Every input requires this interaction to be processed. By convention, these links are depicted without annotation on a CPN model.
- The *variable* links. These are the links for which L has 0s and 1s on the diagonal. Some inputs require this interaction in order to be processed, while some do not.

Inadmissible links and permanent links are of little interest as far as the coordination problem between roles is concerned. If a link is inadmissible or permanent, the existence of the interaction

is not subordinated to the information content of the input. This cannot be the case for variable links. In those cases, the decision to interact or not is based on the information content of the input, and must be simultaneously recognized by the roles that interact. The major difficulty is that different roles might not have access to the same sources of information, and cannot be meaningfully coordinated. Two notions have been identified to deal rigorously with the coordination of variable interactions. A source of information Sensor i (an output alphabet X_i) is said to be *accessible* at some process t if the process can receive information generated by the output of Sensor i . A source of information Sensor i (an alphabet X_i) is said to be *effective*, if the decision whether or not to interact is based, partially or in whole, on the output of Sensor i .

Each role in a fixed structure has been modeled by a subnet with four transitions and three internal places, presented in Fig. 3 (Levis, 1984 and 1988). The four stage decision making process consists of four algorithms SA, IF, CI, and RS. In Figure 3, x represents an input signal from an external source of information or from the rest of the organization, i.e., from another role. The *Situation Assessment (SA)* algorithm processes the incoming signal to obtain an assessment of the situation. The assessed situation z may be transmitted to other decision making processes. Concurrently, the role may incorporate one or several signals z'' from other parts of the system. The signals z and z'' are fused together in the *Information Fusion (IF)* to produce the final situation assessment z' . The next algorithm, the *Command Interpretation (CI)* receives and interprets possible commands (v') from other roles, which restrict the set of responses that can be generated. The CI stage outputs a command v which is used in the *Response Selection* algorithm (*RS*) to produce the response of the role, the output y . This output can be sent to the effectors and/or to other roles in the system. The input stage of a role may be SA, IF or CI; all stages that accept external inputs. The final output stage, however, must be RS, the stage at which the role selects its response.

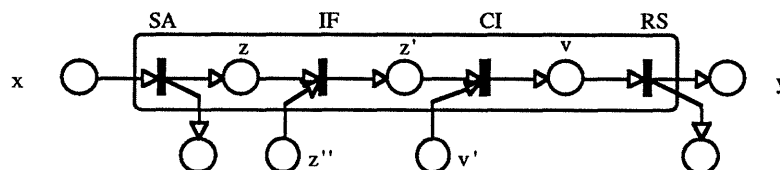


Fig. 3 Four Stage Model

Every component of the system might not have access to all the sensors' observations. It might base its processing on a restricted number of observations. In the submarine, the communication of the observations from the two sensors are modeled as shown Fig. 4, where S_{ij} , for $i = 1, 2, j = 1, 2, 3$, models the communication of the output of i -th sensor to the SA stage of Role j .

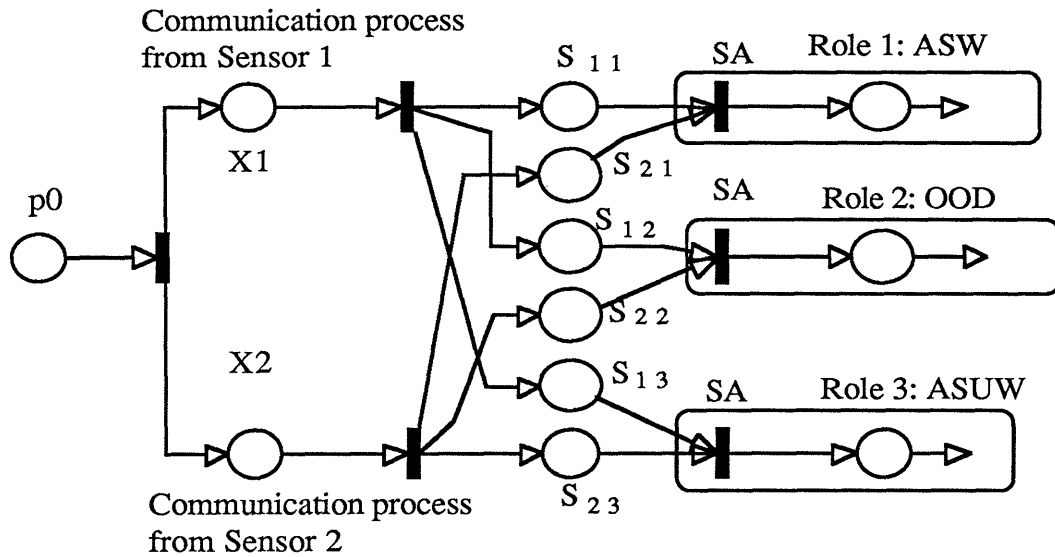


Fig. 4 Interactions Roles-Sensors

Only certain types of interactions make sense within the model (Remy and Levis, 1988). They are depicted on Fig. 5. For the sake of clarity, only the links from the i -th role to the j -th role have been represented. The symmetrical links from i to j are valid interactions as well.

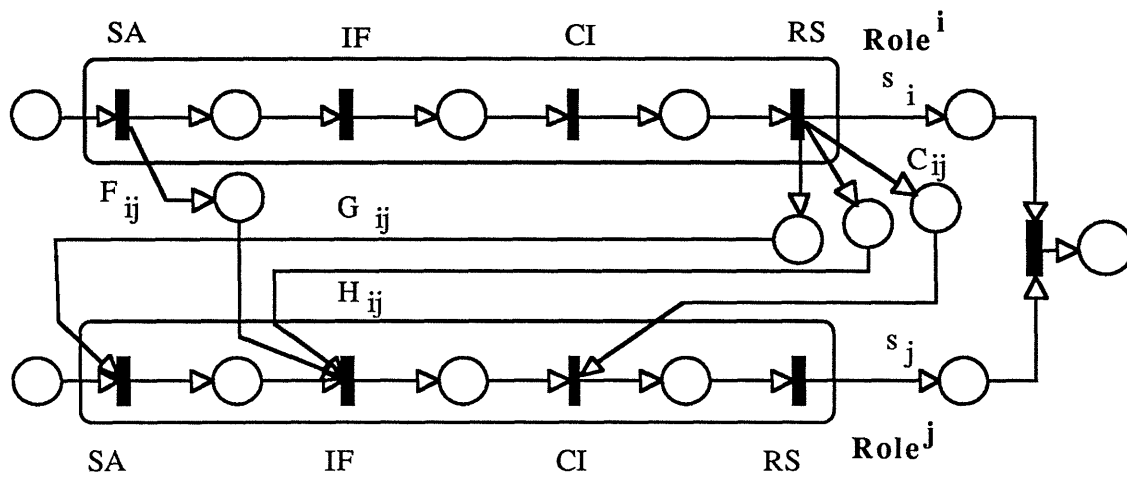


Fig. 5 Allowable Interactions between two Roles

The parameter s_i models the case in which the i -th role communicates the response it has selected to the external environment through the effectors. If Role i sends its response to the effectors, then there exists a link between the RS stage of Role i and the output transition. This output transition has a unique output place, which is called the sink. F_{ij} is the interaction that occurs when the situation assessment which is produced as an output of the SA stage is sent to the j -th role to be

fused with the assessment of the j -th role, and/or assessments from other roles. G_{ij} depicts the case where the response selected by the i -th role is the input of the j -th role. H_{ij} shows the sharing of a result. The i -th role informs the j -th role of its final decision. The j -th role may or may not take this information into account. Finally, C_{ij} has been introduced to model hierarchies between roles. It describes the possibility of role i sending a command to role j .

A variable structure for the submarine system is completely determined by the six-tuple

$$\Pi = (S, s, F, G, H, C) .$$

- S is a 2×3 block array.
 - s is a 1×2 block array.
 - F, G, H, C are four 3×3 block arrays.
- F_{ij} models the link from the SA stage of Role i to the IF stage of Role j .
 G_{ij} models the link from the RS stage of Role i to the SA stage of Role j .
 H_{ij} models the link from the RS stage of Role i to the IF stage of Role j .
 C_{ij} models the link from the RS stage of Role i to the CI stage of Role j .
- Every block in $\Pi = (S, s, F, G, H, C)$ is a 9×9 diagonal matrix L .
 $L_{ii} = 1$ if the i -th input in the lexicographic ordering activates the link.
 $L_{ii} = 0$ if the i -th input in the lexicographic ordering does not activate the link.

The set of all variable structures is called V . Similarly, a fixed variable structure can be represented as a six-tuple

$$\Sigma = (S', s', F', G', H', C')$$

- S' is an 2×3 array
 - s' is a 1×2 array,
 - F', G', H', C' are four 3×3 arrays.
- Their entries are in $\{0, 1\}$: 1 if the interaction is present; 0 if the interaction is not present.

The set of fixed structures is called W . The results of the methodology are based on the following properties of the sets V and W .

Property 1: Each element of V can be equivalently described in matrix form by Π or by a Colored Petri Net.

There exists a one to one relationship between the representation of a variable structure in a matrix form and a Colored Petri Net model of the structure. One can thus work with the language that is most appropriate for one's needs.

Property 2: Each element of W can be equivalently described in matrix form by Σ or by an Ordinary Petri Net.

There exists a one to one relationship between the representation of a fixed structure in a matrix form and an Ordinary Petri Net model of the structure.

Property 3: Any variable structure corresponds to a mapping

$$\begin{aligned} \Pi: \quad X &\rightarrow W \\ x &\rightarrow \Pi(x), \end{aligned}$$

which associates with each input in X one and only one fixed structure.

These properties relate the results that have been developed for fixed structure systems to properties of variable structure systems. Finally, some relations that ease the investigation of the set of variable structures, V , and the set of fixed structures, W , must be introduced. These relations are partial orderings that allows one to sort the elements of V and W .

- The set V of variable structures is ordered by the binary relation ACT, where

$$\Pi \text{ ACT } \Pi'$$

is equivalent to the statement that every input that activates an interaction in Π activates the same interaction in Π' , i.e., Π has fewer than or equal number of interactions as Π' .

The elements of V can be ordered using ACT from the ones with the least activation to the ones with the most activation.

- The set W is ordered by the binary relation SUB, where

$$\Sigma \text{ SUB } \Sigma'$$

means that every interaction in Σ is present in Σ' , i.e., the Ordinary Petri Net that represents Σ is a subnet of the Ordinary Petri Net that represents Σ' .

The elements of W can be ordered using SUB from the least connected ones to the maximally connected ones.

DESIGN REQUIREMENTS

The requirements for the C^3 system can be translated into requirements on the matrix forms of

the structures. The designer may rule in or rule out some links, or force the variability of some other links. If a link must be ruled in, the 9×9 Identity matrix I must be put in the appropriate entry of Π . If a link has to be ruled out, the designer must put the 9×9 null matrix in the appropriate entry of Π . Finally, a variable link can be defined by putting a matrix L that describes the variable interaction in the appropriate entry of Π . At the end of the specifications, some links may remain unspecified, and constitute the degrees of freedom of the design. An unspecified link is annotated by the symbol "#".

The first constraint on the design is that OOD does not monitor the sonar. Because of the hierarchical relationships, it is assumed that ASW cannot issue a command to ASUW, and conversely that ASUW cannot issue a command to ASW. It is further assumed that the design should recognize structures that optimize the existing expertise in the system. For that purpose, ASW should be the role that formulates the tactical response if the submarine is only threatened in the deep sea environment. Similarly, if a threat is detected in the surface environment only, the ASUW should be the role that issues commands to the effectors. Finally, if no threat has been detected, or if threats are detected in both environments, the OOD should formulate the tactics to be followed. These constraints yield:

- The Constraints on S are given by:

$$S = \begin{array}{ccc|l} & \text{ASW} & \text{OOD} & \text{ASUW} \\ \hline & \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & I \end{bmatrix} & & \begin{array}{l} \text{Sensor 1} \\ \text{Sensor 2} \end{array} \end{array}$$

The matrix S is completely specified. S indicates that the output of Sensor 1, the state of the deep sea environment, can only be monitored by ASW, and that the output of Sensor 2, the state of the surface environment, can only be accessed by ASUW.

- The constraints on s are determined by the fact that the objective of the design is to obtain variable structures that recognize the expertise of the different roles. Thus ASW must select a response to be sent to the effectors if and only if the inputs to the system are $\langle S, NSU \rangle$, $\langle T, NSU \rangle$. These inputs describe the situations in which there is only a submarine threat.
- ASUW must select a response to be sent if and only if the inputs to the system are $\langle NS, dC \rangle$, $\langle NS, SU \rangle$. These inputs describe the situations in which there is only a surface threat.
- OOD must select the response in all other cases, which are $\langle NS, NSU \rangle$, $\langle S, dC \rangle$, $\langle S, SU \rangle$, $\langle T, dC \rangle$, $\langle T, SU \rangle$. These inputs describe the situations in which there are either no threats, or threats in both the submarine and the surface environments.

$$s = \begin{array}{ccc} \text{ASW} & \text{OOD} & \text{ASUW} \\ \hline [L1, & L3, & L2] \end{array}$$

$$L1 = \text{diag} \begin{bmatrix} \langle \text{NS}, \text{dC} \rangle & \langle \text{NS}, \text{NSU} \rangle & \langle \text{NS}, \text{SU} \rangle & \langle \text{S}, \text{dC} \rangle & \langle \text{S}, \text{NSU} \rangle & \langle \text{S}, \text{SU} \rangle & \langle \text{T}, \text{dC} \rangle & \langle \text{T}, \text{NSU} \rangle & \langle \text{T}, \text{SU} \rangle \\ 0, & 0, & 0, & 0, & 1, & 0, & 0, & 1, & 0 \end{bmatrix}.$$

$$L2 = \text{diag} \begin{bmatrix} \langle \text{NS}, \text{dC} \rangle & \langle \text{NS}, \text{NSU} \rangle & \langle \text{NS}, \text{SU} \rangle & \langle \text{S}, \text{dC} \rangle & \langle \text{S}, \text{NSU} \rangle & \langle \text{S}, \text{SU} \rangle & \langle \text{T}, \text{dC} \rangle & \langle \text{T}, \text{NSU} \rangle & \langle \text{T}, \text{SU} \rangle \\ 1, & 0, & 1, & 0, & 0, & 0, & 0, & 0, & 0 \end{bmatrix}.$$

$$L3 = \text{diag} \begin{bmatrix} \langle \text{NS}, \text{dC} \rangle & \langle \text{NS}, \text{NSU} \rangle & \langle \text{NS}, \text{SU} \rangle & \langle \text{S}, \text{dC} \rangle & \langle \text{S}, \text{NSU} \rangle & \langle \text{S}, \text{SU} \rangle & \langle \text{T}, \text{dC} \rangle & \langle \text{T}, \text{NSU} \rangle & \langle \text{T}, \text{SU} \rangle \\ 0, & 1, & 0, & 1, & 0, & 1, & 1, & 0, & 1 \end{bmatrix}.$$

- Constraints related to F: Both ASW and ASUW must inform the OOD of their situation assessment. The role OOD does not receive information at its SA stage, therefore it cannot send any situation assessment. No other constraints have been stated on the sharing of information. Thus F is

$$F = \begin{array}{c} \text{ASW OOD ASUW} \\ \begin{bmatrix} 0 & I & \# \\ 0 & 0 & 0 \\ \# & I & 0 \end{bmatrix} \begin{array}{l} \text{ASW} \\ \text{OOD} \\ \text{ASUW} \end{array} \end{array}$$

- Constraints on G: In this example, the input of one role must be either the observation from a sonar (ASW and ASUW) or a situation assessment (OOD), thus

$$G = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- Constraints on H: No constraint has been expressed on the sharing of responses

$$H = \begin{bmatrix} 0 & \# & \# \\ \# & 0 & \# \\ \# & \# & 0 \end{bmatrix}$$

- Constraints on C: The constraint on the hierarchical structure is that both ASW and ASUW cannot issue commands or advisories regarding the tactical response to each other. However, they can issue advisories to OOD, or can receive commands or advisories from it.

$$C = \begin{array}{c} \text{ASW OD ASUW} \\ \begin{bmatrix} 0 & \# & 0 \\ \# & 0 & \# \\ 0 & \# & 0 \end{bmatrix} \begin{array}{l} \text{ASW} \\ \text{OD} \\ \text{ASUW} \end{array} \end{array}$$

The requirements are translated into requirements on the CPN model of the variable structures on Fig. 6. To ease the readability of the net, a link that has been ruled out has not been represented, a link that is permanent is drawn with a bold line without annotations, and a link whose variability is imposed has been drawn in a bold line and annotated by the appropriate matrix.

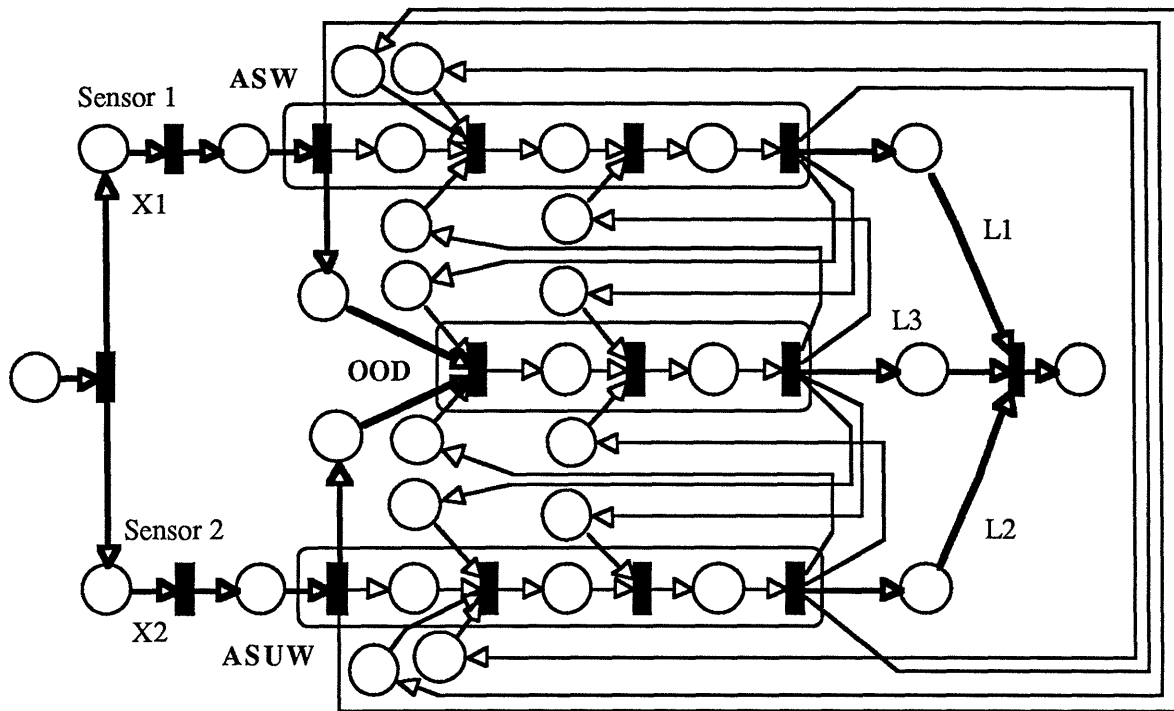


Fig. 6 Colored Petri Net model of Design Requirements.

CONSTRAINTS

There are many degrees of freedom and, consequently, the set of variable structure systems which fulfill the designer's constraints is quite large. However, all the variable structures that belong to this set might not be solutions that make physical sense. Some structural constraints must be defined to restrict the class of solutions to those that are acceptable. The structural constraints on V can be divided into two classes. The class of constraints relates the properties of variable structures to the properties of fixed structures, as described in Remy (1988). The second class of constraints is specific to variable structures (Demaël, 1989).

Let Π be a variable structure. For any x in X , the fixed structure $\Pi(x)$ must satisfy:

- (R1) (a) The Ordinary Petri Net that corresponds to $\Pi(x)$ should be connected, i.e., there should be at least one (undirected) path between any two nodes in the Net (b) A directed path should exist from the external place to every node of the PN and from every node to the sink.
- (R2) The Ordinary Petri Net that corresponds to $\Pi(x)$ should have no loops, i.e., the structure must be acyclic.
- (R3) In the Ordinary Petri Net that corresponds to $\Pi(x)$, there can be at most one link from the RS stage of a role i to another role j , i.e., for each i and j , only one element of the triplet

- $\{G(x)_{ij}, H(x)_{ij}, C(x)_{ij}\}$ can be non-zero.
- (R4) Information fusion can take place only at the IF and CI stages. Consequently, the SA stage of a role can either receive observations from sensors, or receive one and only one response sent by some other role.
 - (R5) There cannot be one link from the SA stage of role i to the IF stage of role j and a link from the RS stage of role i to the SA stage of role j .

Constraint R1(a) eliminates a data flow structure that does not represent a single structure. Constraint R1(b) insures that the flow of information is continuous within the organization, that there are no other sources of information than Sensors 1 and 2. Constraint R2 allows acyclical fixed dataflow structures only. This restriction is imposed to avoid deadlocks and infinite circulation of messages within the organization. Constraint R3 indicates that it does not make sense to send the same output to the same role at several stages. It is assumed that once the output has been received by a role, this output is stored in its internal memory and can be accessed at later stages. Constraints R4 and R5 ensures that the IF stage is indeed a stage at which items of information coming from different sources are fused.

- (R6) If the first stage of a role is SA, then each input link in S and G is permanent
- (R7) If the first stage of a role i is IF, then each input link F_{ji}, H_{ji} for j in $[1..R]$ is permanent.
- (R8) If the first stage of a role i is CI, then each input link C_{ji} for j in $[1..R]$ is permanent
- (R9) Let L be a variable link between two stages $t1$ and $t2$, and let us suppose that X_i is an effective alphabet of the variable interaction. If $t1$ is a SA stage and $t2$ is a IF stage, then X_i must be accessible at $t1$ and at the SA stage of the role that contains $t2$. If $t1$ is a RS stage and $t2$ is a IF stage, then X_i must be accessible at t and at the SA stage of the role that contains $t2$. If $t1$ is a RS stage and $t2$ is CI stage, then X_i must be accessible at $t1$ and at the IF stage of the role that contains $t2$.

Constraints R6, R7, and R8 proceed from a common rationale. They state that a role at its input stage does not have any knowledge about the input to the system, and cannot exhibit a variable interaction. Thus, at the SA stage, any link between the sensors and the roles must be fixed. Similarly, if a role receives the response from another role, the latter must always communicate its response (R6). Constraints R7 and R8 incorporate the fact that the input stage of a role can be the Information Fusion or Command Interpretation stages. Constraint R9 states that a variable interaction between two stages $t1$ and $t2$ must be based on sources of information that are accessed jointly by the roles that interact. The stage $t1$ must determine, based on some information it has accessed, whether or not it has to send a message to $t2$. Similarly, the role that contains $t2$ must infer from some of the information it has already received, whether or not it must wait for a message from $t1$

before initiating process t2. Observe that the role containing t2 has received this information at the internal stage preceding t2.

COMPUTATION OF SOLUTIONS

The characterization of the set of solutions to the design problem is closely related to the determination of the maximal and the minimal solutions. A solution Π_0 is *minimal* in V if it is not possible to have a variable structure Π , with $\Pi \text{ ACT } \Pi_0$, without violating one of the constraints R1 to R9. A solution Π^0 is *maximal* in V if it is not possible to have a variable structure Π , with $\Pi^0 \text{ ACT } \Pi$, without violating one of the constraints R1 to R9. Minimal and maximal solutions in W are defined in a similar manner.

The computation of the minimal and maximal solution in V proceeds as follows. First, an analysis of the constraints of the designer's requirements is performed so as to determine the number of different tactical situations relevant to the problem. Each tactical situation is then assigned to one and only one fixed pattern of interaction between OOD, ASW, ASUW. Three tactical situations are present in the submarine example.

- The first tactical situation happens if there is a threat in the submarine environment only, i.e., when the input to the system belongs to $EX_1 = \{ \langle S, NSU \rangle, \langle T, NSU \rangle \}$. ASW formulates the tactical response in that case. Each input in EX_1 must be processed according to the same fixed structure Σ^1 . The constraints on Σ^1 are that Σ^1 must contain a fixed link from the RS stage of ASW to the effectors, and that there shall not be fixed links from the RS stages of OOD and ASUW to the effectors. Let W^1 be the set of fixed structures that verify these constraints.
- The second tactical situation corresponds to a threat in the surface environment only, i.e., when the input to the system belongs to $EX_2 = \{ \langle NS, dC \rangle, \langle NS, SU \rangle \}$. ASUW formulates the tactical response in that case. Each input in EX_2 must be processed according to the same fixed structure Σ^2 . The constraints on Σ^2 are that Σ^2 must contain a fixed link from the RS stage of ASUW to the effectors, and that there shall not be fixed links from the RS stages of OOD and ASW to the effectors. Let W^2 be the set of fixed structures that verify these constraints.
- The third tactical situation corresponds to the case in which OOD has to formulate the tactical response, i.e., when the input belongs to $EX_3 = \{ \langle NS, NSU \rangle, \langle S, dC \rangle, \langle S, SU \rangle, \langle T, dC \rangle, \langle T, SU \rangle \}$. Each input in EX_3 must be processed according to the same fixed structure Σ^3 . The constraints on Σ^3 are that Σ^3 must contain a fixed link from the RS stage of OOD to the effectors, and that there shall not be fixed links from the RS stages of ASUW and ASW to

the effectors. Let W^3 be the set of fixed structures that verify these constraints.

Then, maximal and minimal candidate solutions are computed based on the properties exposed hereafter. A candidate solution is a variable structure that satisfies all constraints of the design but R9, which must be checked independently.

Property 4: Consider the fixed structure of Fig. 7, called the Universal Net, which contains all the interactions that have not been ruled out at the design specifications stage. Then, Σ^1 , Σ^2 , and Σ^3 must be subnets of the Universal Net.

Σ^1 , Σ^2 , Σ^3 are characterized by *simple information flow paths*. A simple information flow path is a directed path without loops from the external place of the Universal Net to the sink.

Proposition 5: Σ is a fixed structure in W^i , $i = 1, 2$, or 3 , if and only if Σ lies between a maximal solution and a minimal solution in W^i :

$$\exists \Sigma_1 \text{ and } \Sigma_2 \text{ such that } \Sigma_1 \text{ SUB } \Sigma \text{ SUB } \Sigma_2$$

Σ is a union of simple information flow paths of ΣU , the Universal Net.

Finally, a candidate solution to the design verifies Proposition 6.

Proposition 6: A variable structure verifies all constraints of the design in the submarine example but R9 if and only if Π lies between a maximal candidate solution and a minimal candidate solution:

$$\exists \Pi_1 \text{ and } \Pi_2 \text{ such that } \Pi_1 \text{ ACT } \Pi \text{ ACT } \Pi_2$$

Π_1 is such that

$$EX_1 \text{ --> } \Sigma^1, \text{ with } \Sigma^1 \text{ minimal solution in } W^1$$

$$EX_2 \text{ --> } \Sigma^2, \text{ with } \Sigma^2 \text{ minimal solution in } W^2$$

$$EX_3 \text{ --> } \Sigma^3, \text{ with } \Sigma^3 \text{ minimal solution in } W^3$$

Π_2 is such that

$$EX_1 \text{ --> } \Sigma^1, \text{ with } \Sigma^1 \text{ maximal solution in } W^1$$

$$EX_2 \text{ --> } \Sigma^2, \text{ with } \Sigma^2 \text{ maximal solution in } W^2$$

$$EX_3 \text{ --> } \Sigma^3, \text{ with } \Sigma^3 \text{ maximal solution in } W^3$$

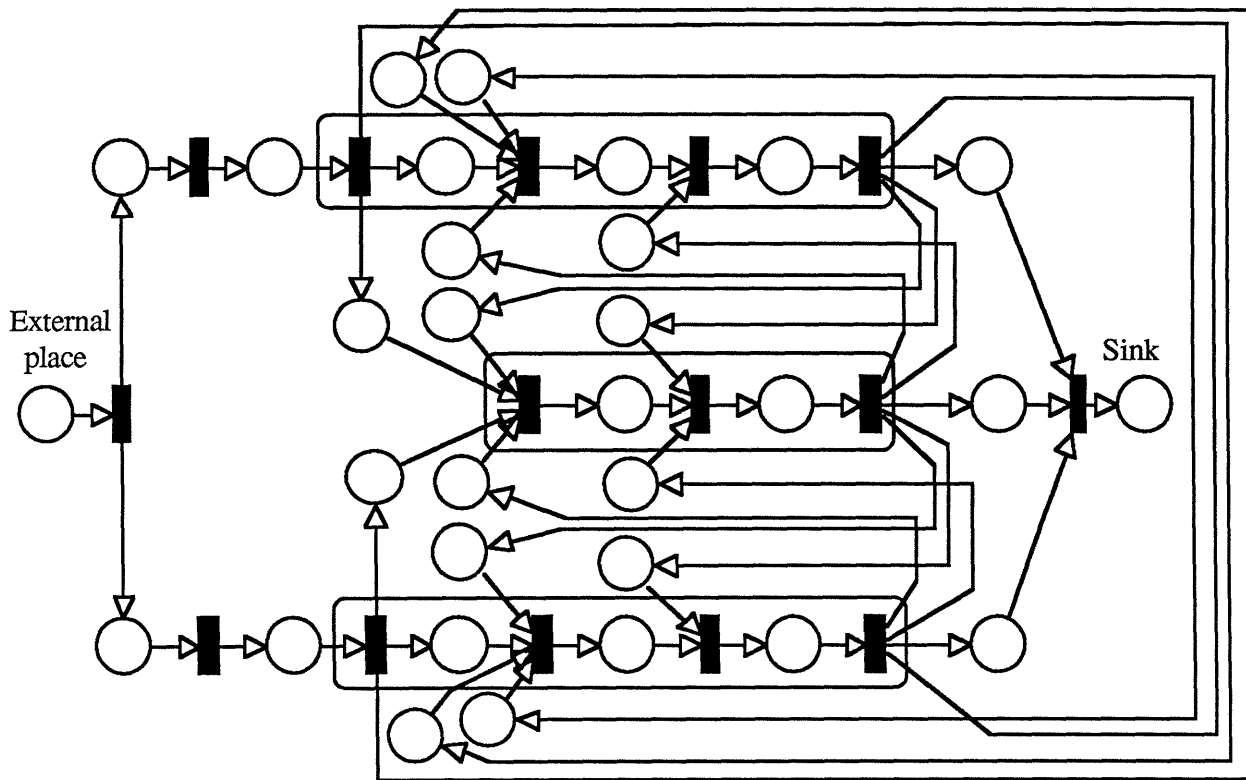


Fig. 7 Universal Net

Candidate maximal and minimal solutions are computed using the Lattice algorithm (Remy, 1988). This algorithm determines the minimal and maximal solutions in W^1 , W^2 , and W^3 . Then, using the characterization of a variable structure as a mapping from X into V , the minimal mappings Π_1 and the maximal mappings Π_2 can be translated into Colored Petri Nets. The output of the computation in the submarine case yields 512 candidate minimal solutions and 512 candidate maximal solutions.

Unfortunately, all variable structures between minimal candidate solutions and maximal candidate solutions do not fulfill constraint R9. The reason is that many variable structures between the candidate maximal and the minimal solutions exhibit variable links in which the effective alphabets are not accessible. However, the set of solutions can be completely determined once minimal and maximal solutions are computed. The last step of the computation takes as input candidate minimal and maximal solutions, and checks constraint R9 to find minimal and maximal solutions. R9 is checked 1) by determining for each variable link its effective alphabets; 2) If X_i $i = 1$ or 2 is an effective alphabet, by checking if each fixed structure Σ^1 , Σ^1 , and Σ^1 contains a simple information flow path which includes the link and the place Source i (X_i is effective). If one variable link does not fulfill R9, the structure is rejected.

Minimal solutions are computed by checking first all the candidate minimal solutions. If some

candidate minimal solutions verify R9, then the computation stops. Otherwise, the search continues inductively on the variable structures that are immediately above, until a solution is found. Symmetrically, the maximal solutions are computed by scanning first all the candidate maximal solutions. If some such structures fulfill R9, the search is stops. Otherwise, it continues inductively by checking the set of candidate solutions that are immediately below the ones just scanned.

It turns out that there is one and only one solution in the submarine example. This example shows therefore that constraint R9 can be quite stringent, and restricts dramatically the number of solutions to the design problem, as compared to the size of the set of variable structures that satisfy all constraints but R9. One feels immediately that this reduction in size can have both positive and negative effects. On one hand, this reduction facilitates the computational problem and any implementation of the methodology. Another positive effect is that it may provide the designer with a small set of solutions, which may be analyzed more thoroughly than if the set of solutions were large. On the other hand, this reduction in size may interfere with the probability that a solution exists, because the more drastic the reduction, the less likely the existence of a solution. Finally, note that it is unlikely that this reduction in size is just an artifact of the model; there are many practical and theoretical reasons to believe that coordinating the activity of distributed entities is very difficult. A CPN model of the solution is depicted in Fig. 8, which corresponds to a mapping:

EX1 $\rightarrow \Sigma^1$, with Σ^1 represented in Fig. 9

EX₂ $\rightarrow \Sigma^2$, with Σ^2 depicted on Fig. 10

EX₃ $\rightarrow \Sigma^3$, with Σ^3 shown on Fig. 12.

Σ^1 represents the exchange of information between roles when there is a threat in the submarine environment only. In this pattern of interaction, the situation assessments of ASW and ASUW are sent to every role in the system. ASUW chooses a response which is communicated to OOD. OOD incorporates the situation assessments of ASW and ASUW to the response selected by ASUW, and selects a command to be sent to ASW. Finally, ASW interprets this command and chooses an appropriate course of action, which is communicated to the effectors.

Σ^2 represents the case of a surface threat. The interactions are symmetrical to those in Σ^1 . The situation assessments are sent to every role in the system. ASW communicates its response to OOD. OOD incorporates the situation assessments and the response of ASW so as to select a command to be sent to ASUW. Finally, ASUW interprets this command and chooses an appropriate course of action which is communicated to the effectors.

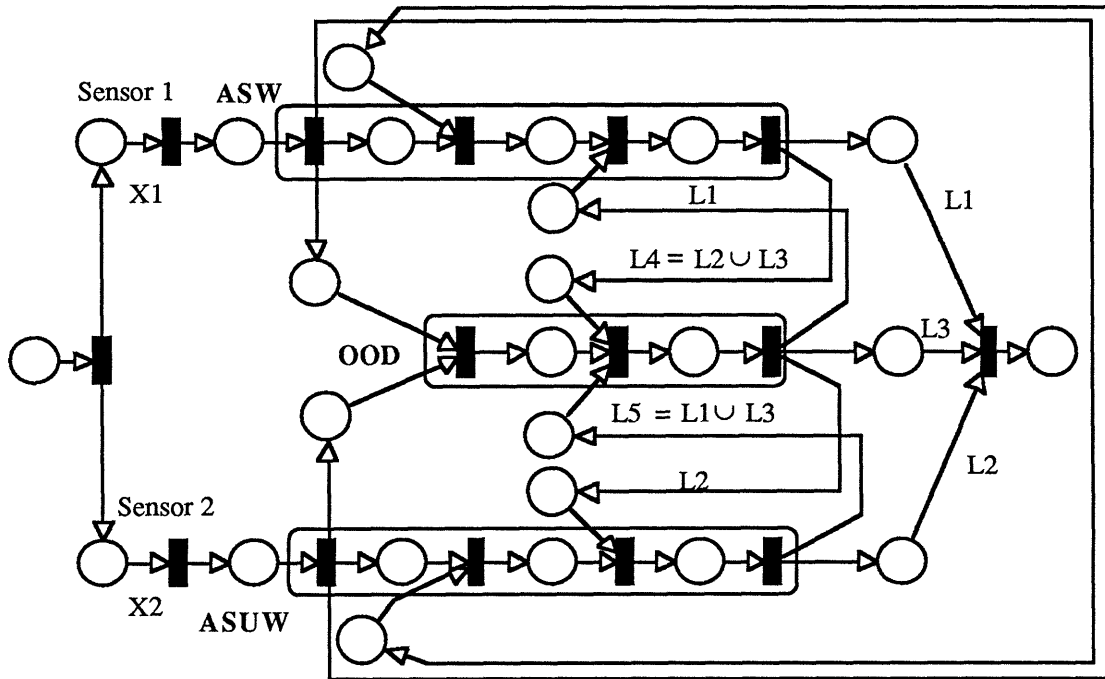


Fig. 8 Solution to the Design Problem

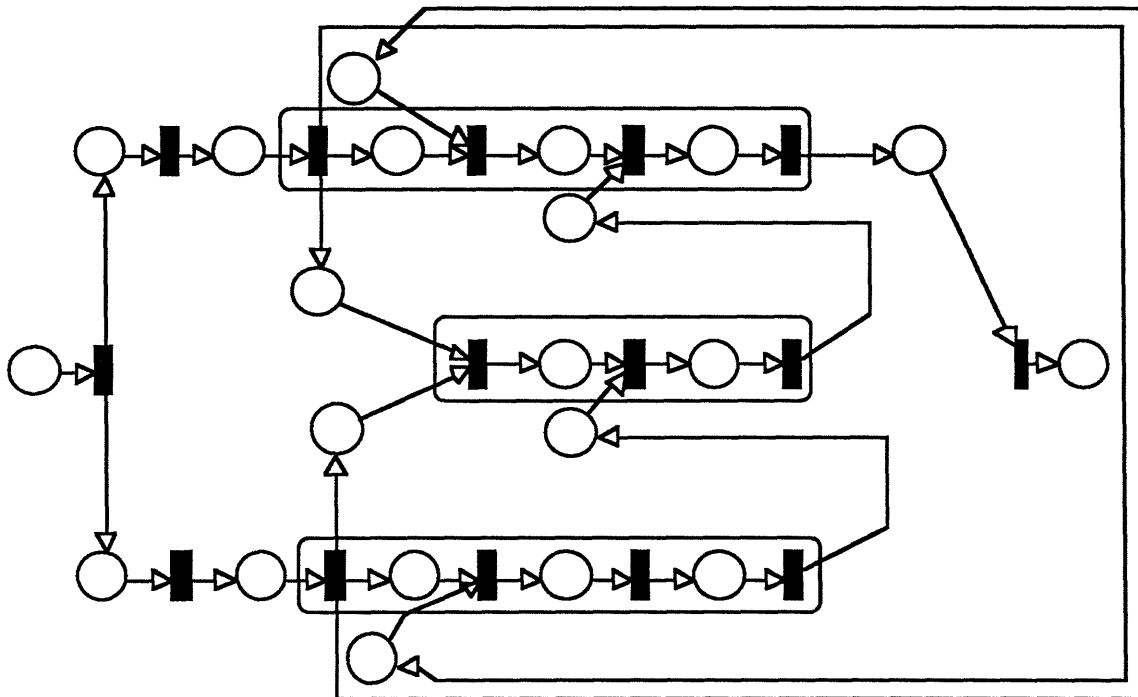


Fig. 9 Fixed Structure Σ^1

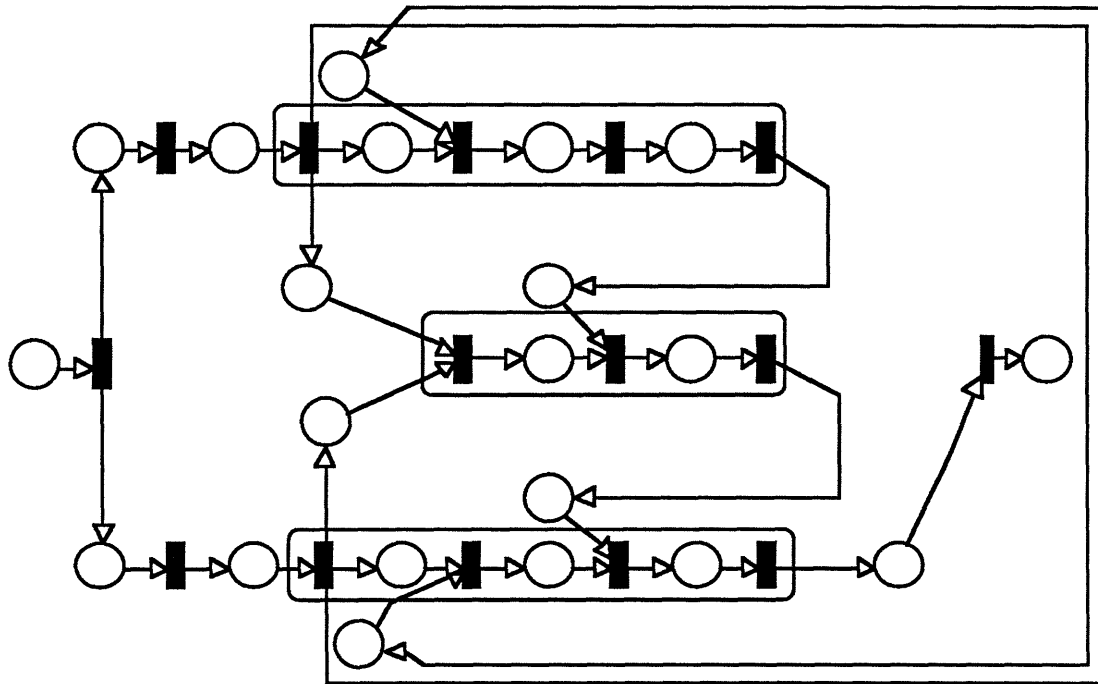


Fig. 10 Fixed Structure Σ^2

Finally, if the tactical response is formulated by the Officer of the Deck, the exchange of information between the roles is depicted by Σ^3 . OOD receives the situation assessments of ASW and ASUW, waits for their expert responses, and interprets their expert advisories with his assessment of the mission, so as to produce the response to be communicated to the effectors.

It must be noted that the variable structure is fairly simple and robust. Each expert subordinate has two modes of interactions overall. Each role always communicates its situation assessment to the other expert subordinate and OOD. Every expert expects to receive the situation assessment of the other role, irrespective of the tactical parameters. Then, if an expert role has to formulate the response, it waits for a command from OOD. Otherwise, it sends its expert advisory to the Officer of the Deck. Similarly, OOD receives the situation assessments of both subordinates, and infers from them the state of the environment. Then, if OOD has to formulate the tactical response, he waits for the expert responses. Otherwise, he integrates the response from the expert who has not detected a threat with its own assessment, and issues a command to the subordinate who issues the tactical response.

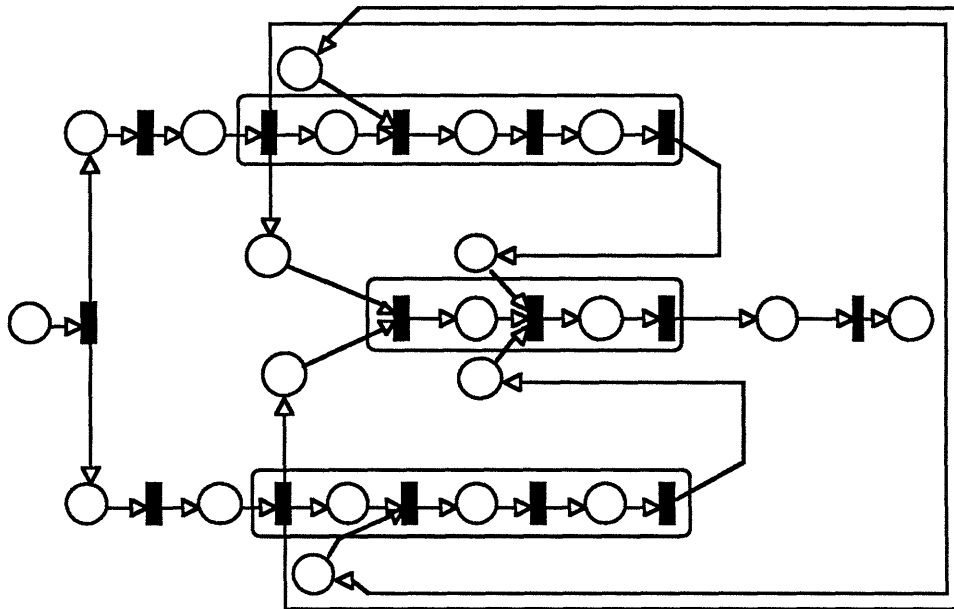


Fig. 11 Fixed Structure Σ^3

CONCLUSION

In this paper a quantitative methodology for modeling variable structures and for solving their coordination problem is presented. The class of systems considered process deterministically a set of simultaneous observations. This methodology models variable structures with Colored Petri Nets, which are used as the basic mathematical framework to generate the set of structures that satisfy design requirements. The designer of a system can describe his degree of knowledge about the requirements in a matrix form that is translated into the language of Colored Petri Nets. Then, an algorithm determines the set of variable structures that satisfy the designer's requirements as well as some generic constraints. This methodology provides a basic step towards the modeling and analysis of more realistic C^3 systems.

REFERENCES

- Crecine, J.P., and Salomone, M.D. (1988). "Organization theory and C^3 ". *Proc. 1988 Symposium on Command and Control Research*. Science Applications International Corporation, McLean, Virginia.
- Demaël, J.J. (1989). "On the Generation of Variable Structure Distributed Architectures". LIDS-TH-1869. Laboratory for Information and Decisions Systems, MIT, Cambridge.

Grevet, J.-L., and Levis, A. H. (1988). "Coordination in Organizations with Decision Support Systems". *Proc. 1988 Symposium on Command and Control Research*. Science Applications International Corporation, McLean, Virginia.

Jensen, K. (1987). " Colored Petri Nets " in *Advanced Course on Petri Nets 1986*, Lecture Notes in Computer Science, Springer-Verlag, Berlin, Germany.

Levis, A.H. (1984). "Information Processing and Decision-Making Organizations: a Mathematical Description." *Large Scale Systems*, No. 7, pp. 151-163.

Levis, A. H. (1988) ."Human Organizations as Distributed Intelligence Systems." *Proc. IFAC Symposium on Distributed Intelligence Systems*. Pergamon Press, Oxford, England.

Monguillet, J.-M., and Levis A. H. (1988). " Modeling and Evaluation of Variable Structure Command and Control Organizations." *Proc. 1988 Symposium on Command and Control Research*. Science Applications International Corporation, McLean, Virginia.

Reisig, W. (1985). *Petri Nets, An Introduction*. Springer-Verlag, Berlin, Germany.

Remy, P. and Levis, A. H. (1988). " On the Generation Of Organizational Architectures Using Petri Nets. " in *Advances in Petri Nets 1988*, Lecture Notes in Computer Science, G.Rozenberg Ed. Springer Verlag, Berlin, Germany.

Weingaertner, S. and Levis A.H (1989). "Evaluation of Decision Aiding in Submarine Emergency Decisionmaking." *Automatica* . Vol. 25, No.3.