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Design of an Aluminum Differential Housing and Driveline
Components for High Performance Applications

by

Richard A James

Submitted to the Department of Mechanical Engineering in Partial
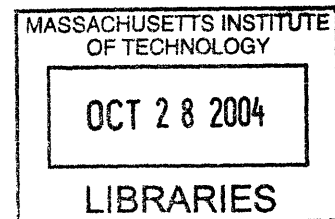Fulfillment of the Requirements for the Degree of
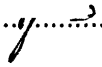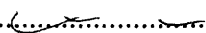
Bachelor of Science

at the

Massachusetts Institute of Technology

June 2004

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Design of an Aluminum Differential Housing and Driveline Components for High Performance Applications Abstract

by

Richard A. James

Submitted to the Department of Mechanical Engineering on May 7th, 2004 in Partial Fulfillment of the Requirements for the Degree of Bachelor of Science in Mechanical Engineering

ABSTRACT

The purpose of the study was to design a lightweight aluminum differential housing to replace the cast-iron housing used in the Torsen® T-1. The redesigned housing was destined for use in the 2004 MIT Formula SAE vehicle, a small high-performance formula-style car.

FEA analysis was used to create a two-piece design that was significantly lighter than the original Torsen housing. Bearing mounts used to attach the unit to the FSAE chassis as well as inboard driveline joints were also designed and fabricated. The remainder of the driveline system was outlined and sourced.

Additionally, a second design incorporating the original cast-iron housing was created for the FSAE vehicle to use in competition while the original design undergoes testing.

TORSEN® is a registered trademark of Toyoda-Koki Automotive Torsen North America Inc.

Thesis Supervisor: Alex Slocum

Title: Professor of Mechanical Engineering

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1.0 Introduction

1.1 Differentials

Most automobiles are propelled by two or more drive wheels. The torque applied by the drive wheels is transferred from the engine through the vehicle's powertrain system. That system includes the engine, transmission, drive shafts, and differentials.

The differential distributes torque to an automobile's drive wheels. The differential earns its name because it allows a vehicle's drive wheels to spin at different speeds (differentiate). This functionality is critical in most automotive applications. The reason behind this can be observed in Figure 1-1.
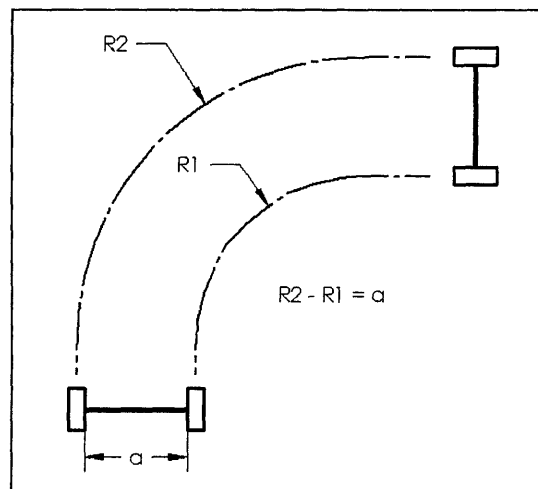
Figure 1-1: Inside & Outside wheel Turning Paths.

In straight-line travel, (assuming two-wheel drive) a vehicle's drive wheels spin at the same velocity. During a turning maneuver, however, the inside and outside wheels must spin at different velocities. The radius of the arc traced out by a vehicle's inside wheels is smaller than that traveled by the outside wheels. Since the outside wheels must travel a greater distance in the same amount of time, in order to complete the turn, they must turn at a faster rate than the inside wheels. This is not a problem for non-drive wheels since they spin independently. A vehicle's drive wheels however are linked together so a single engine and transmission can power them. Without a differential, tire slip would occur during turns, leading to tire wear, increased noise and decreased traction.

1.1.1 Open Differential

There are traditionally two main categories of automotive differentials: open differentials and limited-slip differentials (LSD). Figure 1-2 shows an open differential.

Input Shaft

Outside Housing

Spider Gear

Right Drive Gear

Left output shaft

Right output shaft

Left Drive Gear

spider gear axle attached to outside housing

Outside ring gear attached to housing

■ Blue (or dark) shading indicates bevelled gear teeth.
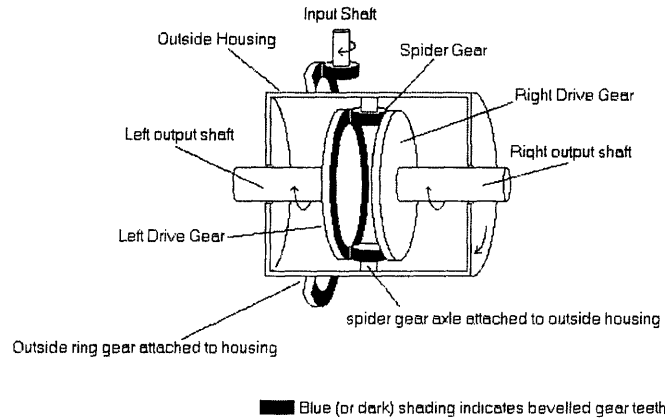
Figure 1-2: Schematic of an open differential showing power flow.[1]

An input shaft transfers power to the differential housing through the ring gear. Spider gears rotate about the output shaft axis with the housing. In straight line driving, the spider gears do not spin, but their movement about the differential housing's axis spins the output shafts at the same velocity. In a turning situation, the right and left output shafts spin at different velocities due to rotation of the spider gears about their own axes. This setup accommodates any velocity difference (slip) between the left and right output shafts.

The open differential however, has the disadvantage of transferring an equal amount of torque between both output shafts. The output shafts drive the vehicle's wheels, therefore the maximum torque that can be transferred is limited by the traction capabilities of the tires. Since torque is distributed equally between both wheels, the maximum total torque transferred to the ground is twice the maximum torque available at the tire with the least traction. This is a major disadvantage for low traction situations such as on off-road terrain, or in cases of large weight transfer like that occurring during racing.

During tight cornering weight is transferred from the vehicle's inside wheels to the outside ones. In reality, the car's mass distribution remains the same, it is the normal force loading at the tires that changes. As the normal force on the outside tires increases, that on the inside tires decreases by the same amount. Since the tractive force produced by the tires is dependant upon the normal force loading, the available traction at the inside wheels decreases. If the vehicle operator attempts to accelerate out of a corner, he will be limited by traction. The open differential will lead to slipping of the inside wheel thus decreasing the rate of acceleration out of a corner.

1.1.2 Limited-Slip Differentials

The solution to these problems is a limited-slip differential (LSD). As the name implies, these devices reduce slip between output shafts. The difference in angular velocities between the output shafts is limited by linking the output gears together. Thus when one output shaft slips too much more than the other (due to tire spin) the force reaction transfers torque to the opposite shaft. LSDs are classified according to the means the transfer is accomplished by.

Some LSDs function on the basis of a mechanical clutch. A clutch between the outer output gear faces and the differential housing prevents excess differentiation between output shafts. Others create a torque reaction through the shearing of a viscous fluid. Figures 1-3 and 1-4 show clutch-type and viscous-coupled (VC) LSDs.
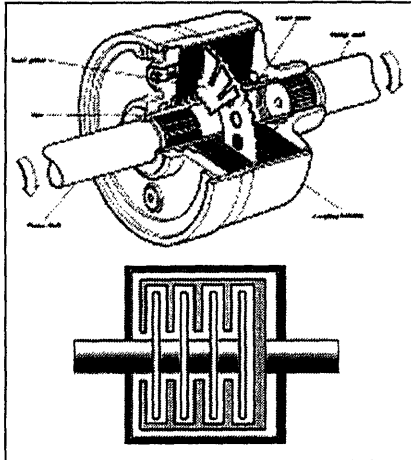
Figure 1-4: Schematic of viscous-coupled plates in Volkswagen VC LSD.[1]
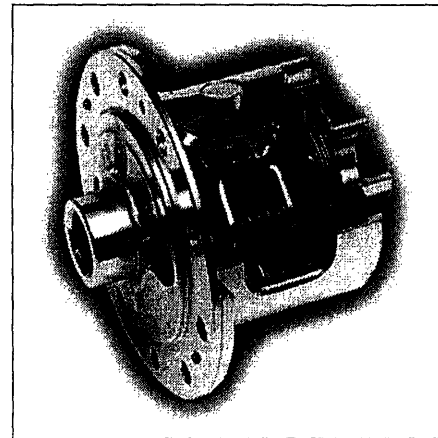
Figure 1-3: Eaton Clutch-type LSD.[2]

The disadvantage of these types of LSDs is that they have a limited useful life. Clutches can wear out and the viscous fluids can breakdown under high operating temperatures.

1.1.3 Torsen® Differential[1]

The Torsen is a purely mechanical differential which functions as a limted-slip device. Unlike clutch-type and viscous coupled differentials, slip limiting is accomplished by means of worm-like gearing and friction. Figure 1-5 presents a sketch of a Torsen T-1 type differential.

The Torsen (**Torque Sen**sing) differential consists of helical side gears and combination spur and helical element (Invex) gearing. These replace the spider gears and bevel side gears of the conventional open differential. Two helical side gears mesh tangentially with the helical surfaces of element gear pairs mounted around the periphery of the side gears. The element gear pairs mesh through their spur gear surfaces.

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[1] TORSEN® is a registered trademark of Toyoda-Koki Automotive Torsen North America Inc.

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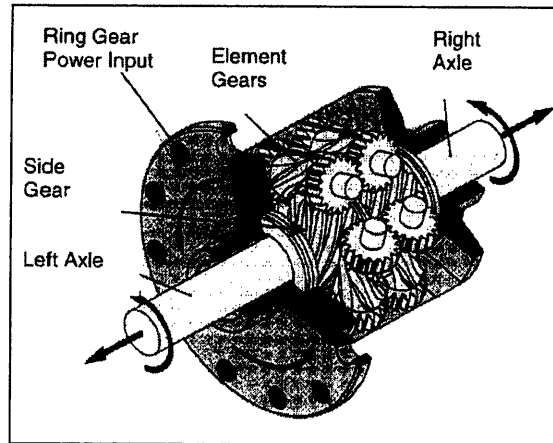
Figure 1-5: Torsen T-1 Differential.[3]

The Torsen differential has the ability to transfer torque from the low-traction axle to the high-traction axle. Under straight-line equal traction conditions, the Torsen's element gear pairs do not rotate about their own axes, similar to the spider gears in a conventional differential. When the left and right drive axles must turn at different velocities (as in turning) or under low-traction conditions ("split-mu") the element gear pairs transfer torque from one side gear to the other (hence from one axle to the other). [3]

The ratio of torque that can be transferred between drive axles is known as the torque bias ratio (TBR). The ratio is a numerical measure of the differential's locking effect. The greater the TBR the less slip is allowed. The TBR in the Torsen is controlled by the friction forces inside the housing. These forces arise from the helical geometry of the Torsen's gear train. As torque is transferred through the side gears axial forces act on the side gears. Both side gears are pushed to one end of the housing generating large frictional forces between the side gear and thrust washer surfaces. These frictional forces serve the same purpose as the clutch and fluid in the differentials previously mentioned.

The Torsen T-1, traditionally found in the center gearbox of second generation Audi A4 Quattros, is a cast-iron open unit. The main reason for its heft is to survive the rigors that an all-wheel drive street car endures. The Torsen differential, more specifically its housing design, will be the main subject of this thesis paper. The cast iron housing will be re-engineered into a lighter design more suitable for small race vehicles like the MIT Formula SAE car.

1.2 MIT Formula SAE

The purpose of the Formula SAE (FSAE) competition is for teams of students to conceive, design, fabricate, and compete with small formula-style racing cars. A typical FSAE car weighs approximately 220 kg, has a wheelbase of 1750 cm and a track width of 1270 cm. These small lightweight cars are powered by 600cc motorcycle engines capable of producing in excess of 50 kW. FSAE cars are designed specifically for low-speed tight courses. High corner exit speeds and low-end acceleration are critical in attaining low lap times. "Putting the

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power down" efficiently however, is difficult given the large power-to-weight ratios and numerous tight corners seen by FSAE vehicles. These requirements necessitate the use of a high-performance, lightweight, limited-slip differential.

The MIT Formula SAE Team fielded their first FSAE entry in 2003. The Zexel Torsen T-1 was chosen due to its size, performance, low cost and reliability. It has been used by many other teams for several years. Figure 1-6 shows the 2003 MIT Formula SAE vehicle.
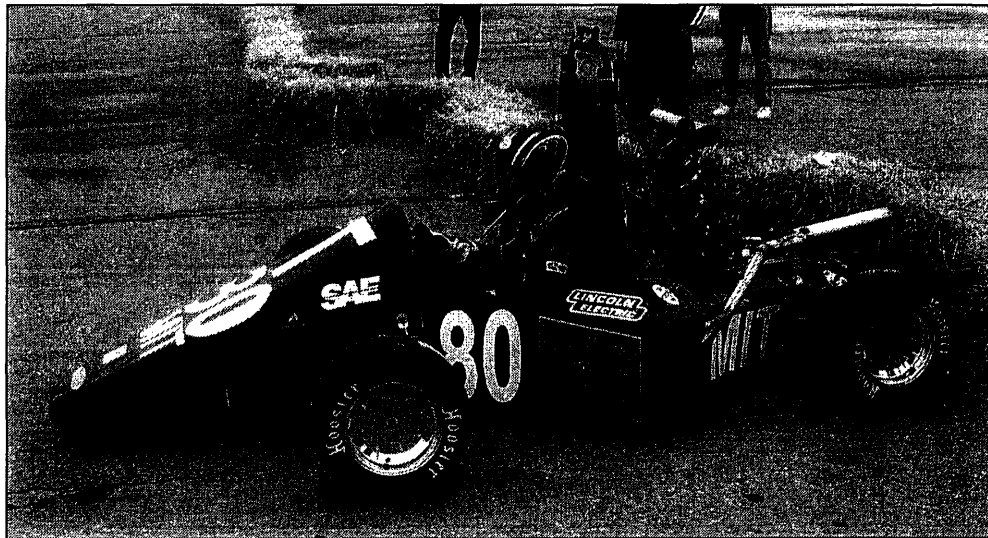
Figure 1-6: MIT Formula SAE vehicle 2003 FSAE competition in Pontiac, MI.

In order to improve upon the mass, integration, reliability and serviceability of the 2003 MIT FSAE driveline system the differential housing and associated driveline components was redesigned for the 2004 MIT vehicle.

2.0 Problem and Objectives

2.1 Functional Requirements and Performance Objectives

There are several functional requirements and objectives that motivate this study. In the fast-paced realm of motorsport competition is fierce and unrelenting. In order to produce a competitive race vehicle, the struggle to lighten yet strengthen must be constantly fought. Significantly reducing the overall mass of the 2004 MIT FSAE driveline system is paramount. Reduction of the polar moment of inertia of the rotating assembly is also desired in order to improve dynamic performance.

Race vehicles are notorious for reliability problems making service and maintenance regularly scheduled activities. In many cases, repairs are performed at the track, a far cry from the organized environment of the machine shop. Ease of serviceability and reliability are crucial in ensuring such a vehicle spends as much time as possible on the track and little time in the repair bay.

The 2003 MIT FSAE drive train met all of its functional requirements however the design and implementation left much to be desired. Figure 2-1 shows a picture of the rear of the 2003 MIT vehicle.
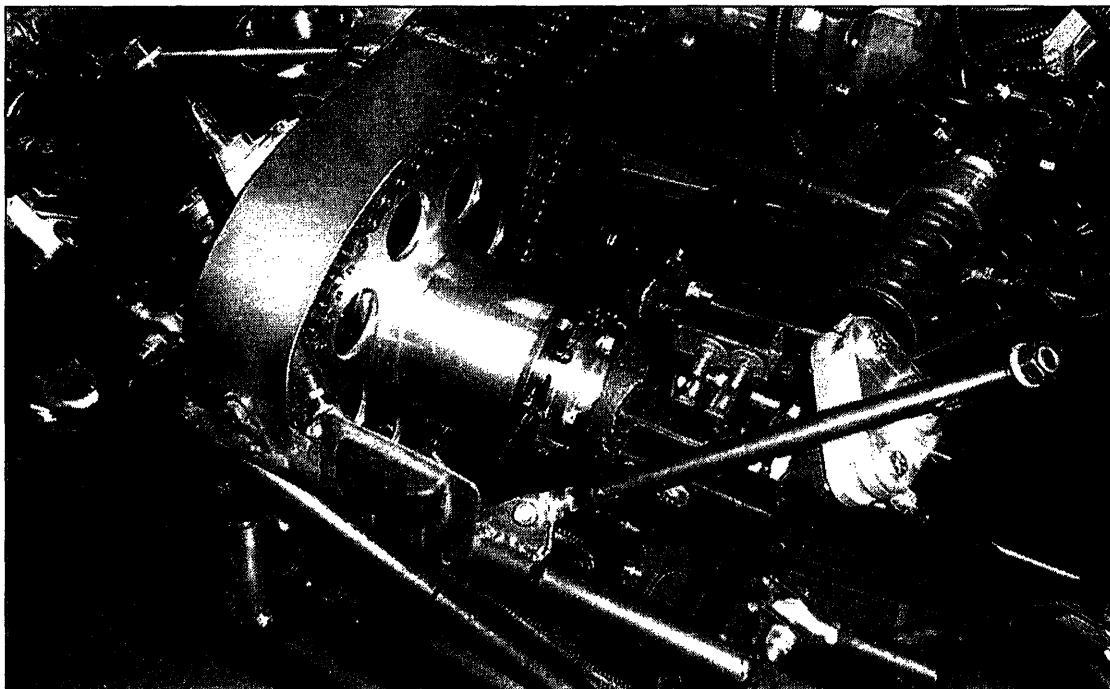
Figure 2-1: 2003 MIT FSAE Drive train system.

The drive train assembly consisted of a modified Zexel Torsen "University Special" (Figure 8 below) with a sprocket and brake rotor solidly mounted to the cast iron housing by means of an aluminum casing. The housing stubs were machined off and holes were tapped into the side of the housing for attaching the
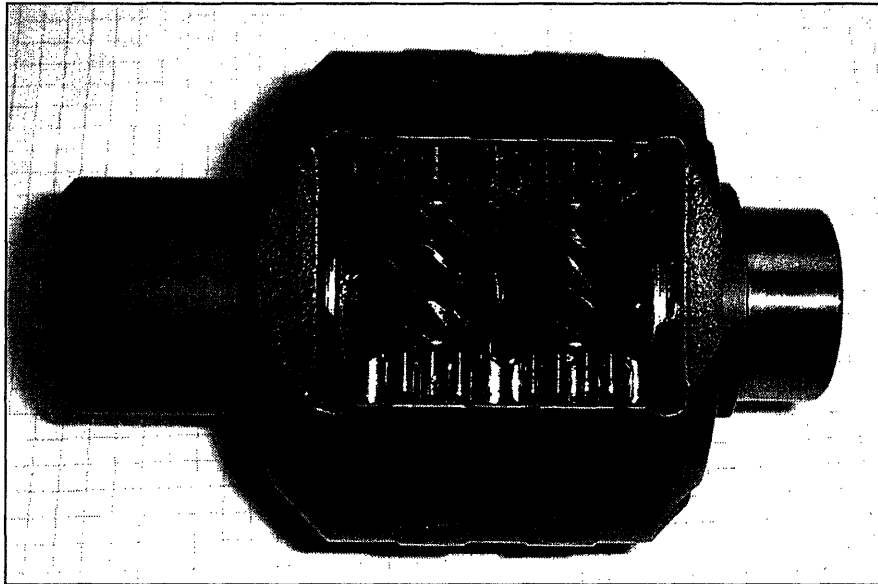
Figure 2-2: Torsen University Special supplied to FSAE teams.

sprocket and rotor. In order to keep the unit lubricated, it was sealed in an aluminum casing. The differential assembly itself was supported by two tapered roller bearings solidly mounted to the vehicle chassis. The design was extremely heavy due to the fabricated steel bearing mounts. Assembly of the system was time consuming and tedious, requiring up to 6 hours to mount the drive train in the vehicle due to awkward fastener placement. The aluminum casing design proved inadequate at properly sealing gear oil within it. Unfortunately this allowed lubricant to leak out of the casing and onto the rear brake rotor hampering braking performance. These experiences led to the reprioritizing the goals of the 2004 drive train systems in order to allow serviceability and reliability issues.

The functional requirements and design goals for the 2004 drivetrain system are as follows:

- Allow for relatively trouble free operation of driveline and rear brake systems for an adequate lifetime. (Avoid mechanical failure and lubricant leakage.)
- Improve accessibility of the drive train to facilitate maintenance and repair operations. (Replacing rotor, sprocket, adjusting chain tension, bleeding hydraulics, filling/draining lubricant and assembly/disassembly of system)
- Significantly reduce the mass and rotating inertia of the drive train.
- Provide an elegant yet simple packaging and mounting solution.

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2.2 Formula SAE Constraints

The integration of the driveline system into the 2004 MIT FSAE vehicle presents another set of constraints and limitations.

According to SAE regulations, all FSAE vehicles must be equipped with a brake system capable of acting on both the front and rear wheels. The vehicle's front wheels will be equipped with individual disc brakes. The dynamics of weight transfer under braking maneuvers means that most of the braking effort is provided by the front tires allowing for a smaller system in the rear. The rear brake system will consist of only one disk brake acting directly on the drive train system. This leads to both a weight and cost savings since only one brake caliper and rotor are required. The design of the rear wheel uprights is also simplified. This means the drive train must be capable of supporting a brake disc and caliper which will act directly on the differential and hence the rear tires.

The engine used in the 2004 vehicle is a Honda CBR600F4i taken from a high-performance motorcycle. The engine has an integral six-speed transmission contained within the block outputting power to a chain sprocket. Modifications to the engine's transmission are both complex and costly so it was decided to retain the stock chain drive. Thus the driveline must be able to accommodate a driven sprocket to transfer power to the differential as well as allow for clearance and adjustment of the chain.

The MIT vehicle will be constructed from steel 4130 aircraft tubing. The shape of the frame is mainly driven by chassis suspension requirements. Thus the drive train must be able to fit inside the rear box section of the frame without interfering with the operation of the rear suspension system. Additionally, drive axles and joints must be chosen so that power can safely be distributed to the rear wheels at all times no matter what the position of the vehicle's suspension is. In other words the differential must be positioned such that shaft misalignment angles do not exceed the rating of the joints employed.

The Formula SAE student competition judges not only the design and performance of the entry vehicles but their cost and manufacturability is evaluated as well. Hence, the drive train system must not be too expensive to manufacture under production conditions nor should it require manufacturing facilities outside the reach of the MIT FSAE team.

Obviously, any drive train system must survive a minimum lifetime. It must endure the harsh conditions encountered during both testing and at the FSAE competition. Additionally, the system must endure sporadic testing use for years to come.

Having designated both functional requirements and design goals the next step is to analyze the physical scenarios the drivetrain must be able to perform in. The following section will present an analysis of the loads and stresses encountered during would-be typical driving maneuvers.

3.0 Load Requirements

In this section the forces and stresses endured by the drive train system will be calculated. Analysis began with determining input torques and forces the system must withstand. Next, internal gear forces were calculated which later used to help determine appropriate system strengths and stiffness.

3.1 Input Forces

The drivetrain system serves to connect engine rotation with that of the tires. Due to its intermediary function the drivetrain sees forces and moments resulting from both engine power and the road surface. The rear brake system will act on the differential casing thus it must be able to withstand braking torques. Reaction forces are also generated at the bearings which support the differential to the chassis.

Engine torque is applied to the differential via a chain drive acting on a sprocket solidly mounted to the differential housing. This force, F_s, acting at the sprocket pitch diameter supplies the differential input torque, T_{in}. The force, F_s, creates reaction forces, F_{R1} and F_{R2}, at the differential support points. The housing is mounted on ball bearings so no torque is applied to the vehicle chassis from these points. As the differential distributes input torque between the left and right drive wheels through the drive axles, the differential sees the corresponding reaction torques, T_L and T_R.

3.1.1 Engine Torque

In stock form, the CBR600F4i engine is capable of producing a peak crankshaft torque of 65Nm. The engine will be fitted with a custom intake manifold and controlled by a stand-alone engine control unit (ECU) so peak power and torquw will vary from stock. SAE regulations, however, require the use of a 20mm restrictor in the engine intake in order to limit power. It is safe to assume the engine will not be able to produce more than 65Nm under race conditions. The torque seen by the differential will be greatest when the transmission is in 1st gear. Allowing for primary, secondary and transmission drive reductions the differential will see a maximum peak torque of **1,405.8Nm** from the engine.

3.1.2 Torque due to Tires

It is important to remember that engine torque is not the only moment seen by the drivetrain. The differential also feels the reaction torques coming from the drive axles. Under most conditions power flows from the engine through the drivetrain and to the vehicle tires. In these circumstances, the maximum torque that can flow into the differential is the previously calculated engine input torque. In some cases however, power actually flows from the vehicle's tires to the engine. This can occur, for example when the vehicle is coasting with the transmission in gear. This is called engine braking as the compression of air in the engine acts as a brake. These forces are much lower than the torque the engine is capable of producing at full throttle. A much more extreme example of engine braking however, can occur during improper transmission gear down-

shifting. When selecting a lower drive gear when the vehicle is at a given speed, engine RPM must rise to match the rotation of the downstream driveline and transmission. If such a downshift is done incorrectly (i.e. engine speed is not sufficiently increased to match driveline speeds) engine compressive braking will generate huge forces sufficient in causing the rotation speed of the driven tires to suddenly fall below those required for the vehicle's longitudinal velocity. In other words the tires will slow so suddenly that they slip on the road surface (the tires appear to lock up). The tires used on the FSAE vehicle are high-performance racing tires capable of generating extremely large frictional forces. The torque generated can be as high if not higher that the generated engine torque.

Since the traction a tire is capable of generating is proportional to the normal force experienced by the tire, one must take into account weight transfer effects that will change the weight supported by each tire. The equations below demonstrate the derivation. The coefficient of static friction of race tires is assumed to be 1.2.
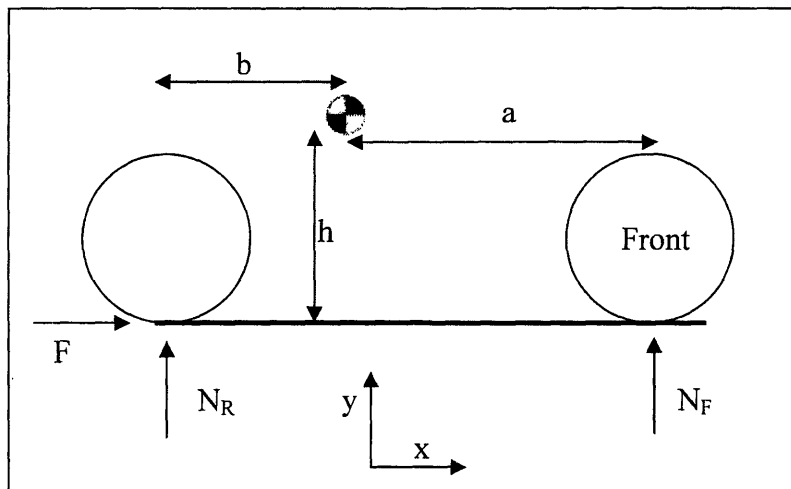

Figure 3-1: FBD of traction forces under acceleration.

$$\sum F_y = 0 \qquad\qquad \sum F_x = m\ddot{x} \qquad F = \mu N_R$$

$$N_R + N_F = mg \qquad\qquad F = m\ddot{x}$$

$$N_F = mg - N_R$$

$$\sum M_z = 0$$

$$Fh - N_R b + N_F a = 0$$

$$Fh - N_R b + (mg - N_R)a = 0$$

$$Fh - \frac{Fb}{\mu} + \left(mg - \frac{F}{\mu}\right)a = 0$$

$$F = \frac{mga}{\dfrac{a+b}{\mu} - h}$$

The above equation predicts the maximum possible friction force the combined rear tires can generate in an acceleration maneuver for a given car weight distribution. Under a deceleration, weight is unloaded from the rear wheels and the sign of the h term becomes negative. Therefore a 228kg vehicle with a 50-50% weight distribution can produce about 2700N and 1760N of friction force at the rear tires under acceleration and deceleration respectively. Since the limits of a vehicle's performance are in fact set by the tires these are the maximum longitudinal forces the rear of the vehicle can generate.

The diameter of the rear tires of the FSAE vehicle will be 533mm (21"). Therefore, the resulting maximum rear wheel torque will be **720N/m**. This implies that under steady-state conditions the vehicle is only able to deliver that much torque at the tire contact patches. Thus under steady-state conditions the drive train will only ever see 720N/m. However, brake clamping force was ignored in the previous derivation.

3.1.3 Brake Torque

As mentioned previously, the rear brake system acts directly on the differential housing. The brake system works by means of a hydraulically actuated caliper which squeezes the differential mounted brake disc. Friction between the caliper pads and the disk provides a torque to slow the rotation of the driveline assembly. Although the vehicle is only capable of applying a limited engine torque at the ground; it is possible for the brake system to provide the difference in reaction torque between the traction capabilities of the tires and the torque produced by the engine. This could occur, for example, when both the brake and full engine power are applied simultaneously which may occur due to driver error or advanced driving techniques.

Therefore the design input torque will in fact be the maximum engine torque of 1,406N/m.

3.2 Helical Gearing

Under normal differential operation, as torque is distributed between drive axles, internal forces are generated which attempt to pull the differential housing apart. These forces are generated from the helical gear geometry used in the Torsen. The differential housing must be designed to withstand loads resulting from both input torques as well as generated internal forces. Figure 3-2 shows an example of a typical helical gear, where the teeth are cut at an angle to the gear's axis. This angle is known as the helical angle, ψ.
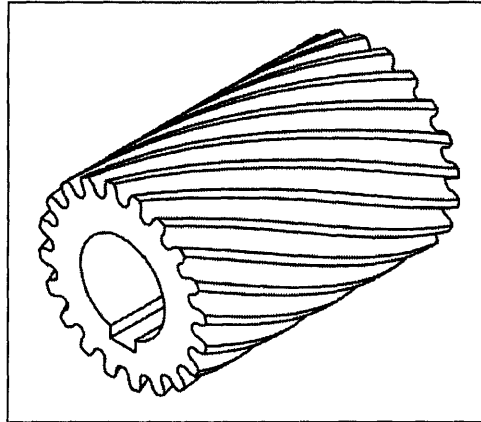

Figure 3-2: Diagram of a typical helical gear [4]

When helical gears mesh both radial and thrust loads are generated. Figure 3-3 summarizes the forces components acting on a meshed helical gear.
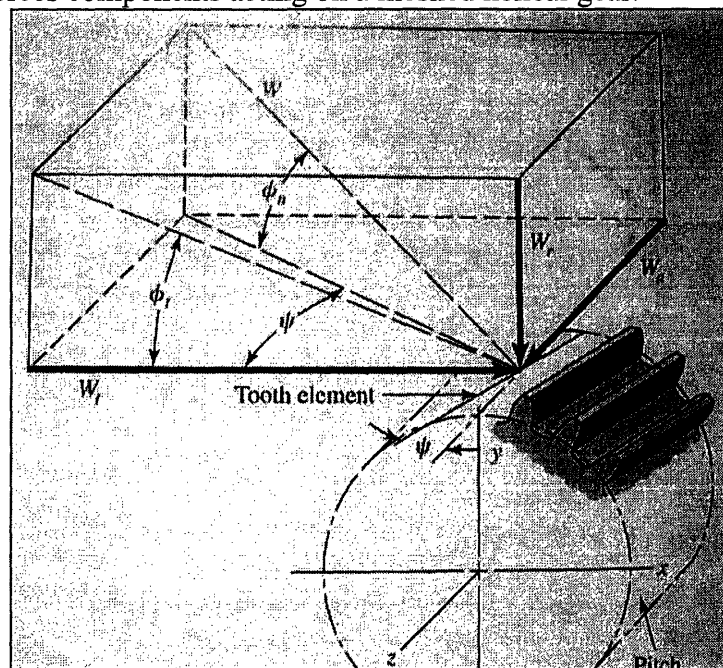
Figure 3-3: Forces acting on a meshed helical gear [5]

The three components of the normal tooth force, W, are:[5]

$$W_r = W \sin\phi_n \qquad (3.1)$$

$$W_t = W \cos\phi_n \cos\psi \qquad (3.2)$$

$$W_a = W \cos\phi_n \sin\psi \qquad (3.3)$$

where W is the total force, W_r is the radial component, W_t is the tangential component, W_a is the axial component, and ϕ_n is the normal pressure angle.[5]

W_t is the transmitted load which can determined from the gear pitch radius and the torque being generated, W_a is the thrust load. The forces W_r and W_a act directly on the differential housing and are the forces must be designed for.

Figure 3-4 shows the Torsen gear train. The element gears mesh with the side gears helically while they are interconnected via spur tooth engagement.

Figure 3-4: Torsen Invex gear train[3]

In order for crossed-axis helical gears to mesh they must be of the same hand and their helix angles must be complementary to the angle between the gears' axes. All the Torsen gears are right handed and have a 45° helix angle. Since the gears are all right-handed, both side gears, under normal driving conditions, generate thrust loads in the same direction. Figure 3-5 shows a diagram of the Torsen differential for reference.

Figure 3-5: Schematic of Torsen T1 Differential [3]

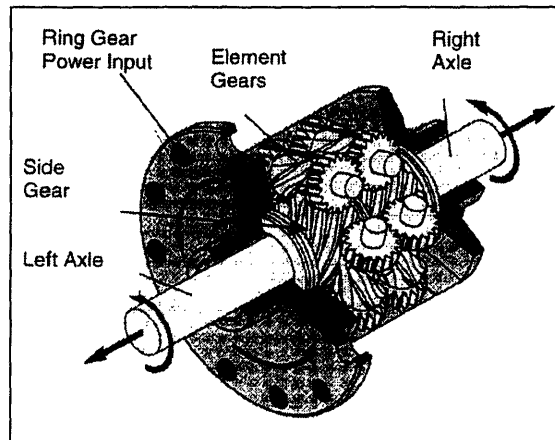Looking at the forces generated by the side gears one can see that the transmitted load becomes the torque applied to the drive axle. The thrust loads will be applied directly to the differential casing at the interface between the housing face and side gear thrust washers. Since both axles will always be turning in the same direction the thrust load generated by both side gears acts will act in the same direction. That direction is dependant on whether torque is being applied from the casing (from the sprocket or brake) or whether torque is coming from the axles (i.e. under engine braking). The side gears in the Torsen are not mounted on a shaft. Instead they are free to float in the housing. The three pairs of element gears share the radial loads generated by the side gears thus fixing them in the housing.

The element gears' radial and tangential loads are supported by the journal pins they are mounted on. These loads have a component parallel to the main differential axis. The axial loads generated by the element gears interestingly enough must equal to the equivalent input torque acting at that radius. This makes sense because looking at the sketch of the Torsen housing the only way that torque can be transferred (neglecting friction at the thrust washers) to the side gears is through the element gears. The element gears are free to spin on journal pins thus the only way to apply a torque from the casing is friction at either end.

The simplest way to calculate the gear forces within the Torsen gear train is to work backwards from wheel torque.

3.3 Calculating Internal Forces

From section 3.1.3 the maximum torque at the differential was calculated to be 1,406Nm. Assuming initially that both axles receive the same amount of torque, each side gear sees a torque of 703Nm. The side gears have a pitch radius of 18.4mm meaning the transmitted load at each side gear is 38,175.4N. Since the helix angle of the side gears is 45° according to Eqs 3.2 and 3.3 the transmitted and thrust loads are equal. Thus each

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side gear generates a 38,175.4N thrust load. **Both side gears generate loads in the same direction due to their handedness, thus the total side gear thrust load is 76,350.8N.**

The total tooth force, W, seen by each side gear is can be calculated to be 57,453N. Each element gear will see one third of this force (since each side gear has 3 element gears). Therefore the tangential and radial forces on each element gear are 12725.1N and 6550N respectively. Predictably, summing the tangential forces over the six element gears yields the total side gear load of 76,350.8N. The axial force at each element gear is 12725.1N. The torque created by the axial forces at the six element gears is equal to the input torque of 1,406Nm.

Figure 3-6 summarizes the internal forces acting on the Torsen housing.
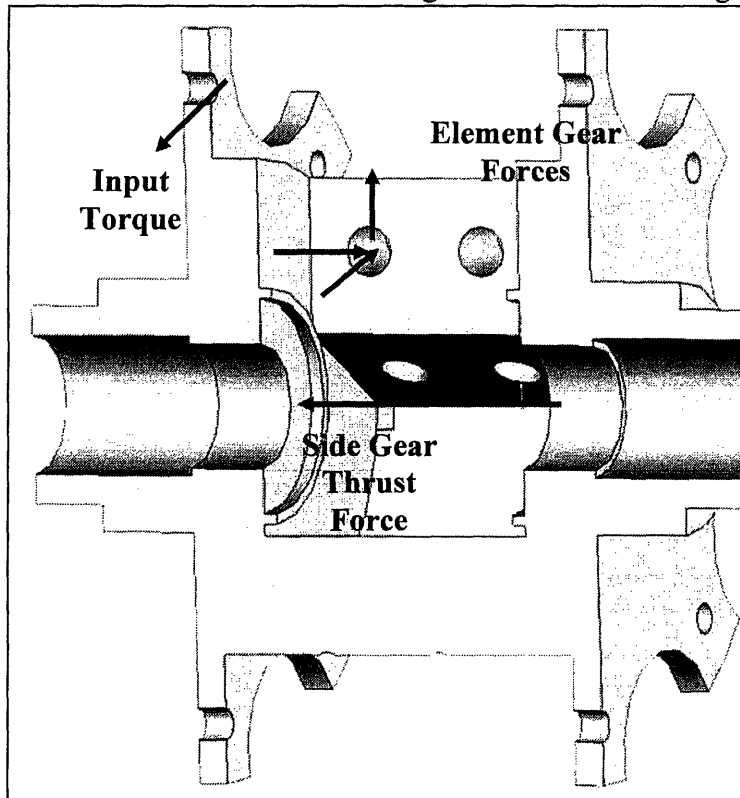
Figure 3-6: Diagram of Internal Forces acting on Differential Housing
(one set of element gear reactions forces shown)

It was assumed in the previous calculation that both axles received the same amount of torque. The Torsen is capable of transferring up to three times as much torque to one axle over the other. This does not affect the sum of the magnitudes of the forces calculated above. However, it does mean that the loading experienced by one side gear and its three element gears will be three times that of the opposing gears.

Section 4 will illustrate the redesign of the Torsen housing according the maximum internal forces that have just been calculated.

4.0 Housing Design

In order to design a functional replacement for the original Torsen housing, the geometric and stiffness requirements must be established.

4.1 Geometric Requirements

The redesigned housing must be able to properly support the Invex gear train and allow it to spin freely. The first step in creating the new housing design was to take stock of the dimensional requirements of the helical gears. A solid model of the original Torsen unit was created from the manufacturing drawings (Appendix A).
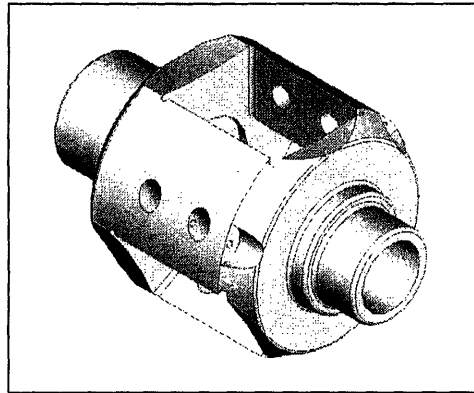
Figure 4-1: Solid model of original Torsen housing
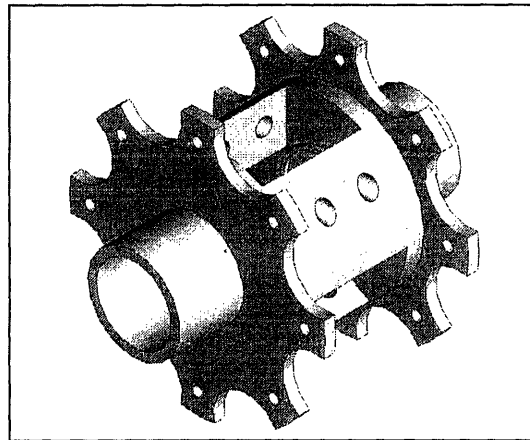
Figure 4-2: Solid model of original Torsen housing

One of the main objectives of the new unit is to better integrate the rear driveline. This meant mounting the brake rotor and sprocket directly to the differential housing. Flanges were added to on either end of the unit to serve as mounting surfaces. The gear train must be submerged in lubricant while operating. The unit's internals must be carefully sealed to prevent fluid loss and leakage on the attached brake rotor. The original Torsen unit is a one-piece cast iron housing. Due to the manufacturing complexity of the geometry it was decided to redesign the housing as two-piece machined aluminum unit.

4.2 Finite Element Modeling

In order to design a robust housing, given the functional requirements and loading previously established, the stresses and deflections of the various housing features was calculated. The complexity of the given geometry does not lend itself to solving by analytical methods. Instead a numerical computer analysis was undertaken. The method used was Finite Element Analysis (FEA).

Computerized FEA allows a user to create model geometry, impose certain loads and boundary conditions with the aim of numerically calculating the predicted stresses and deflections of that geometry. The software package used was Algor FEA® in conjunction with Solidworks 2003. The FEA software allowed for easy importing and analysis of generated solid models. The complexities of creating a two-piece FEA model forced the author to begin with the analysis of a one-piece design. The design could later be easily converted into two-separate modules.

An in-depth discussion of FEA is outside the scope of this paper thus the steps taken in modeling will appear in a summarized form.

The geometry was meshed upon importing the solid model into Algor. This important step sets the size, number and shape of the elements used in calculating the various system properties. After meshing the next important step was setting the boundary and loading conditions.

The boundary conditions were set so to simulate the conditions in the real world as closely as possible. First the right-side face of the differential was fixed. This was done at the interior surfaces of the brake disc mount holes. Ideally, the holes would be modeled as rotational constraints while the rear bearing boss constrained translation. However, the software did not allow constraining of rotation over about a point over several features. This is not expected to drastically affect the results of the FEA model. The next boundary condition was pinning the bearing boss surface so that it would constrain the model in all directions yet permit rotation about the differential's main axis.

The loading conditions were established after setting the boundary conditions. The loads calculated in the previous sections were used in loading various parts of the model. The first loading entered was sprocket input torque. This was modeled as a linear force acting tangent to the sprocket mounting holes. The side gear thrust loading was set to act on the surface were the thrust washers would bear on. Since thrust loading would only occur in one direction it was chosen set to act towards the sprocket side of the differential. The element gear forces were modeled as acting at the journal pin-hole surfaces. The axial forces were distributed around a circle centered at the journal pinhole of a size approximating the face of the element gear.

Figure 4-3 shows a capture screen of the FEA model as the loading and boundary conditions were established. Once the model was fully meshed the FEA simulation could be run. The program allows displaying several stress tensors, Von Mises stresses, trains, displacement, etc directly on a color-coded model. Figure 4-3 shows the Von Mises

stresses inn the model. In the first iteration of the analysis, the highest stress seen by the unit was 310 MPa occurring at the side gear interface area.

Aluminum 7075 has yield strength of 503 MPa. This corresponds to a safety factor of 1.6. This safety factor was considered too low for a final design. Yield failure is not the only failure condition that must be taken into account. The Torsen housing is a stiff unit designed to keep the gear train in proper contact at all times. Thus deflections of the aluminum housing must be kept low to prevent high gear teeth stresses and possible timing errors in the gear train. The first iteration saw maximum deflections of upwards of 0.15mm. The manufacturing tolerances only allow for ±0.1mm. Such high deflections could harm the gear train. Additionally, fatigue would become a serious factor with such large strains. Figures 4-4 and 4-5 show the FEA results for both Von Mises stress and displacement of the first design iteration.

From the results of the initial FEA analysis it was decided to add material to strengthen several critical locations. As expected, the regions seeing the highest stresses where the side gear thrust face, the torque input locations, and the bosses supporting the element gears.
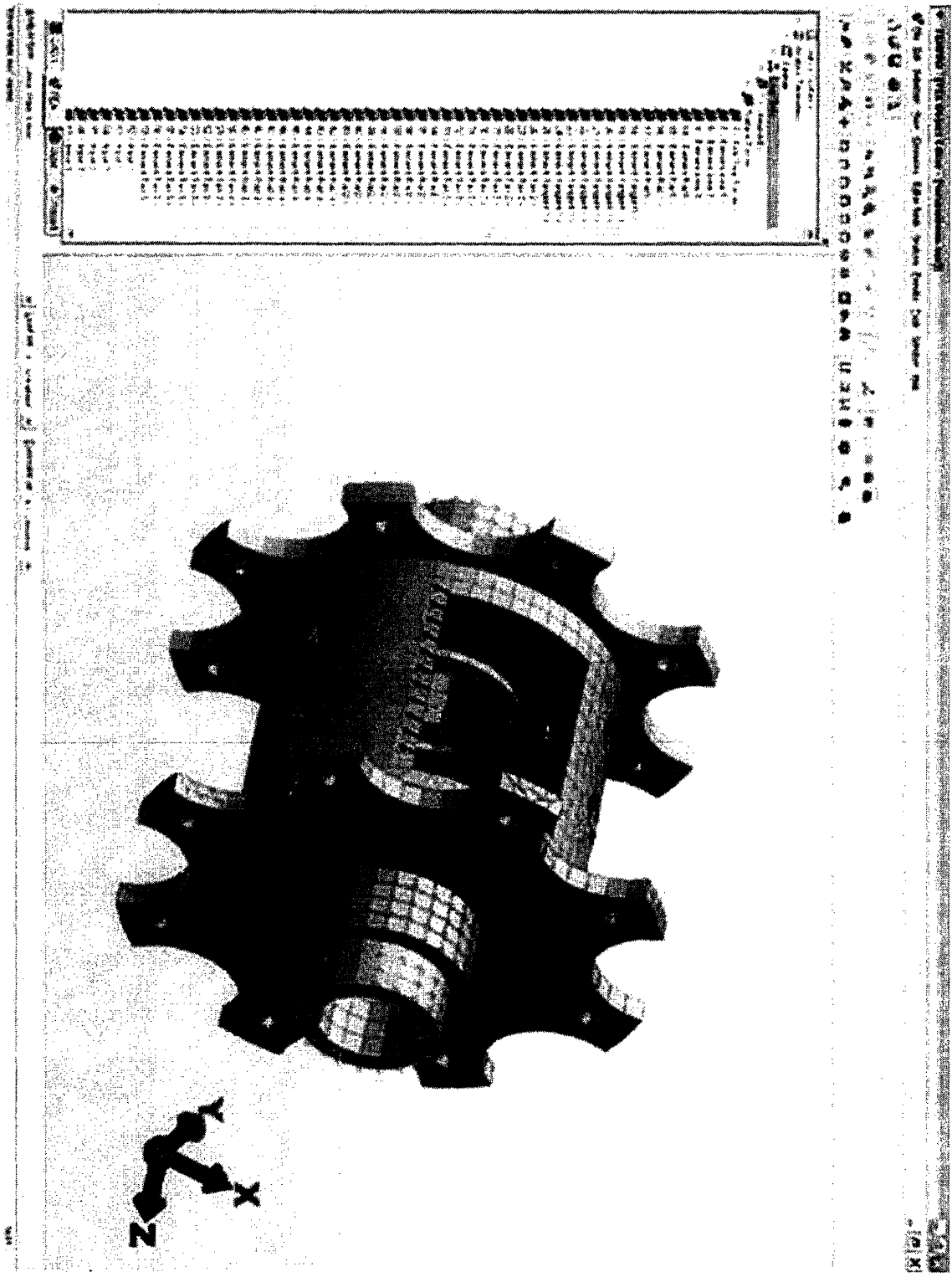
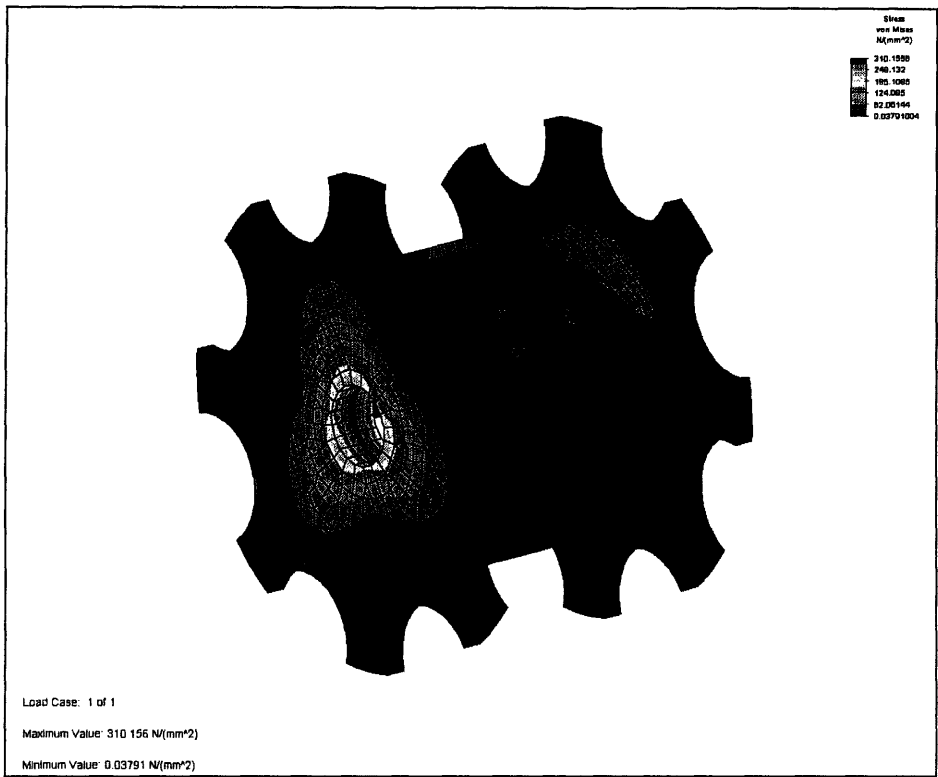**Figure 4-3: Screen capture from Algor; setting FEA boundary conditions**

Figure 4-4: FEA output from first iteration, Von Mises stress
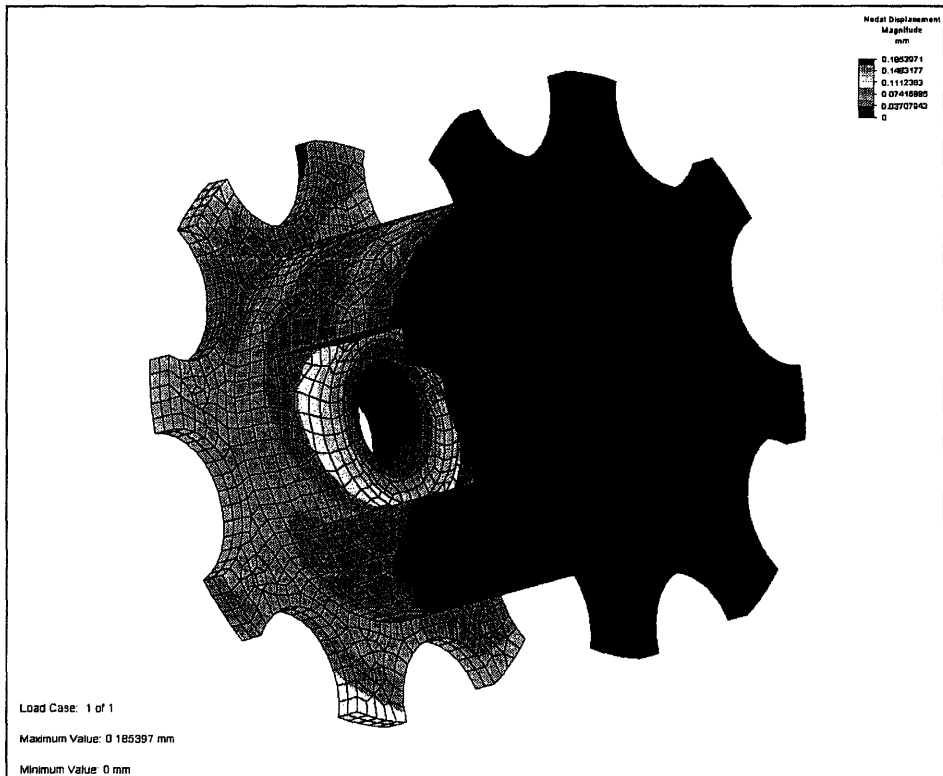

Figure 4-5: FEA output from first iteration, displacement

In order to reduce stresses at the side gear face, the axle shaft opening was made smaller and the wall thickness at that location was increased. The axle shaft opening was made just larger than the diameter needed to clear the drive axles. Wall thickness was increased by approximately 20%. If that face were thought of as a beam in bending, it was made shorter and stouter to decrease displacement. The cross-section of the element gear supports was increased by enlarging the outer diameter of the housing. It was decided not to increase the thickness of the sprocket and brake flanges since the stresses seen were at reasonable levels and the displacement at those points is not a large concern. Additionally, the first FEA model saw the input torque delivered at a single bolt hole whereas the load was spread out over all 8 holes in subsequent modeling.

Figures 4-6 and 4-7 indicate the areas where material was added to increase strength.
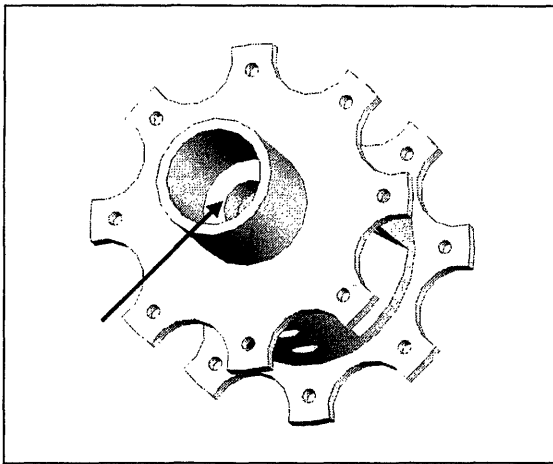
Figure 4-6: Material was added behind the side gear thrust face
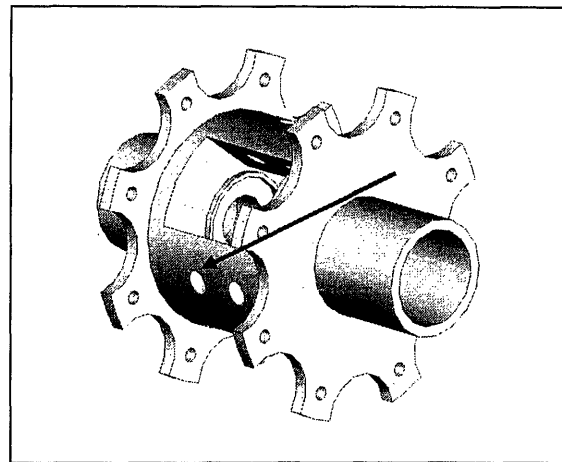
Figure 4-7: OD of the gear support bosses was increased for added stiffness

After completing several design iterations the final housing dimensions were finalized. Maximum Von Mises stress was found to be 117 MPa resulting in an acceptable safety factor of 4.3. Maximum displacements were significantly reduced to 0.049mm. In fact the largest displacement seen by any critical gear supporting surface was only 0.04mm. It was decided that this performance was acceptable ensuring adequate stiffness and strength in the gear housing. At this juncture the housing's mass was 1.42kg, where as the original iron housing's mass was 2.13kg. Since the original housing also requires ancillary components such as end caps and extra fasteners the weight savings were in fact greater.

Figures 4-8 and 4-9 show the screen output from the FEA model of the final housing design.
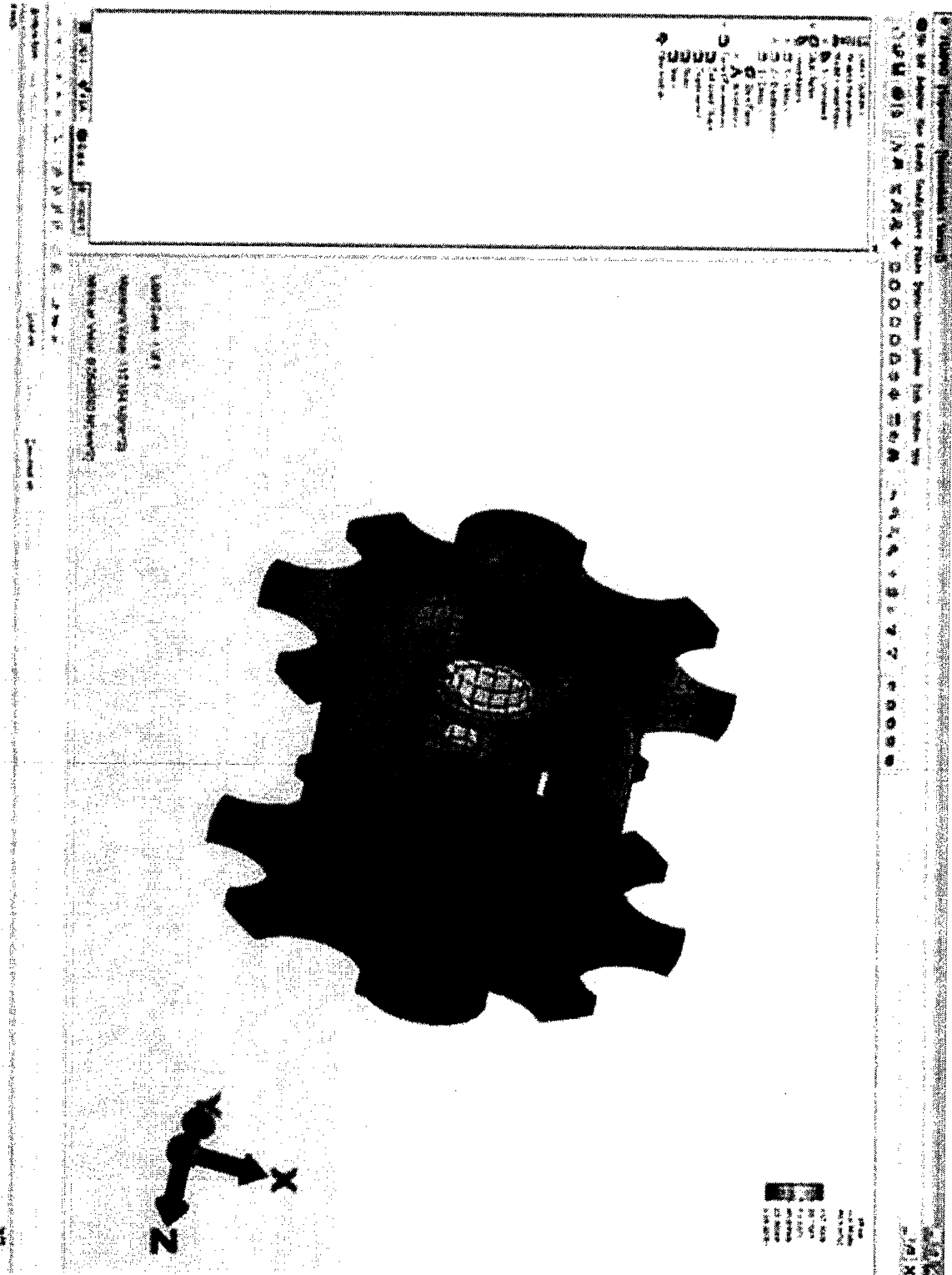
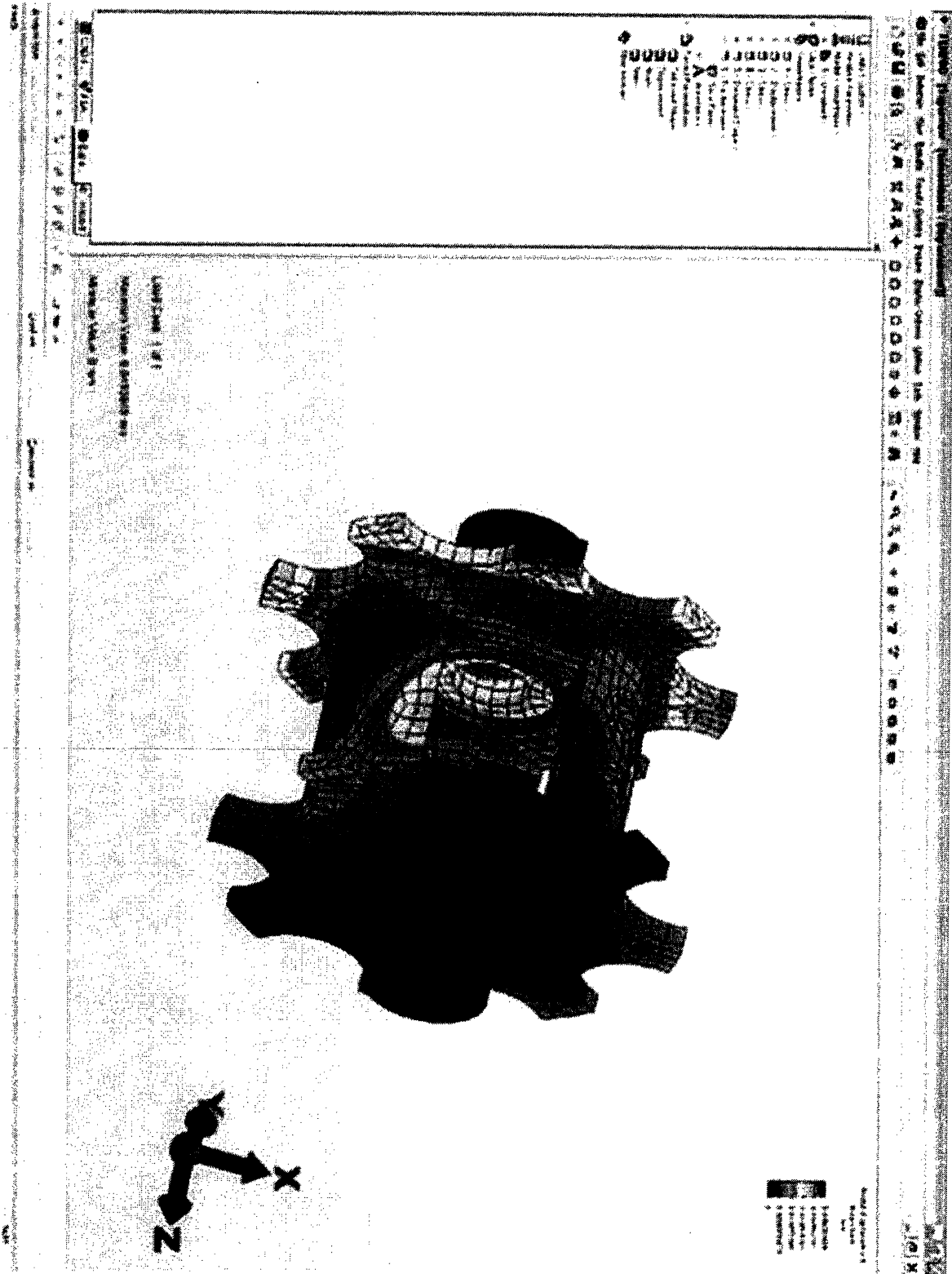**Figure 4-8: FEA output of final design, Von Mises stress**

Figure 4-9: FEA output of final design, displacement

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4.3 Joint Design

The new differential housing was intended to be a two-piece design in order to simplify manufacture and allow for easier assembly and maintenance. The housing would be split at the plane where the brake rotor flange begins. Since the unit must hold strict tolerances to protect the gear train, the choice of attachment and alignment methods is crucial.

The interior face of the brake rotor flange piece locates one of the side gears both axially and radially. Radial location is extremely important since it determines the concentricity of the side gears.

One of the methods surveyed was the use of a kinematic coupling (KC). A KC is an accurate method of constraining two parts in all six degrees of freedom. The KC relies on the point contact between three balls and three v-grooves. In this case, a KC could be formed by machining a v-groove on each element gear support and creating three corresponding balls on the brake rotor flange. In order to attach the two pieces together a quasi-kinematic coupling could be used (QKC). A QKC emulates the function of a normal KC but instead relies on the arc contact formed between axi-symmetric balls and grooves.[6] Figure 4-10 shows a illustrates a typical QKC. The QKC balls and grooves would allow a bolt to be passed through their centers which would then thread into the

Ball

Axi symmetric groove
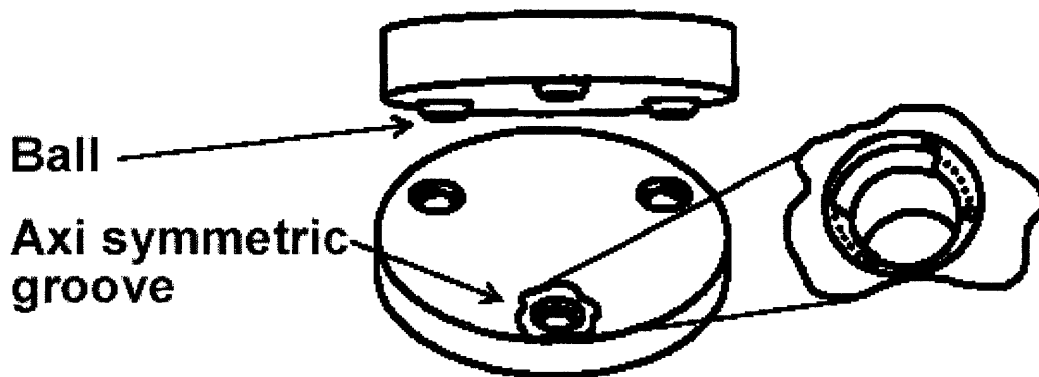
Figure 4-10: Schematic of typical QKC[6]

main housing piece. The QKC appeared to be an elegant way of ensuring accurate assembly of the housing pieces. Conversations with Professor Martin Culpepper, an expert on QKCs, however led the author to look at other means of attachment since the repeatability required was not stringent enough to warrant the use of a QKC.

An alternative would take advantage of the Degree of Freedom constraints already provided by the existing part geometry. The interface between the face of the brake rotor flange and the face of the element gear supports would be sufficient in providing translational constraint in the Z direction as well as provide rotational constraint about the X and Y (Φ_X and Φ_Y) axes. The remaining constraints in the X,Y, and Φ_Z axes could be accomplished by creating special geometry on the faces of the two pieces. An L-shaped

boss would locate the brake rotor flange translationally while a straight boss on another element gear support would constrain radially. These bosses would form line contact with two hollow dowel pins featuring concentric bolts. Figure 4-11 demonstrates this concept.
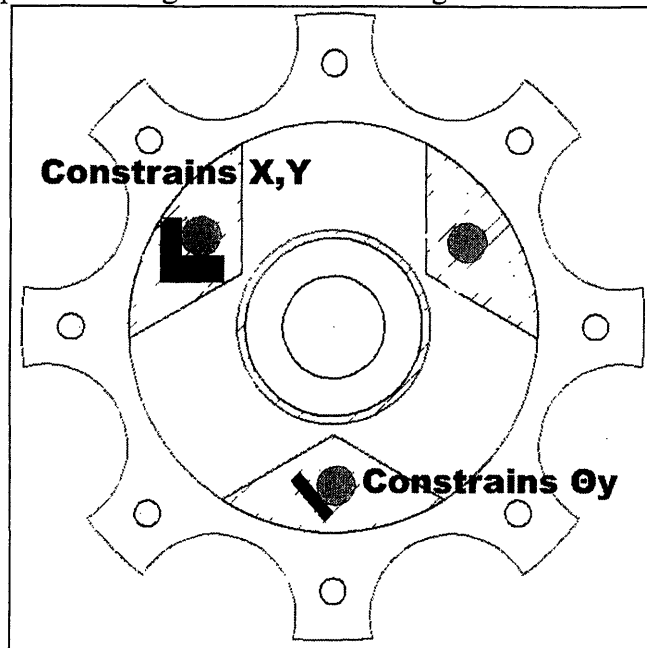

Figure 4-11: Line contact alignment concept

Using 3/8" diameter AN fasteners would provide the needed tensile and shear strength to keep the unit halves together and transfer torque between them.

4.4 Unit Sealing

Proper sealing was critical functional requirement since lubricant leakage can lead both to gear train damage as well as diminished performance from the rear braking system. Additionally, the unit must be sealed to prevent foreign object from entering the gear train.

To prevent leakage through the gaps between the element gear supports, a concentric aluminum sleeve was used. The sleeve will be held in place between the inner faces of the sprocket and rotor flanges. RTV sealant will be used to close any small gaps present. This also allows the sleeve length to be slightly smaller than the distance between the flange pieces which would otherwise prevent the rotor flange from fully seating with the element gear support faces. Additionally shaft seals will be inserted inside the interior of the bearing support bosses so that lubricant cannot leak past the axle stub shafts.

The following section will describe how the unit will be mounted within the FSAE vehicle as well as outlining the remainder of the driveline system.

5.0 Differential Mounting and Associated Driveline Components

Obviously, the differential must be properly supported within the FSAE vehicle in order to maintain chain sprocket alignment and allow the reaction forces created by the chain input torque and braking torque to be fed into the chassis.

This section will also outline the components that will allow torque from the differential to reach the rear drive wheels.

5.1 Differential Mounting

Since the housing must be free to spin within the chassis it must be mounted on roller bearings. The bearings will be pressed into machined aluminum plates which are rigidly mounted the vehicle's tube frame.

These supports must be able to withstand the reaction forces created by the drive chain as well as those created from the rear brake caliper. A design for lightweight bearing supports was created with the help of further FEA. The plates were machined by the author from ¾" 6061 aluminum on a CNC mill. Pockets were machined into both sides in order to reduce their mass. Both left and right bearing supports are identical in design except for the added material needed to mount the brake caliper on the right-hand bracket. The outer bearing races are constrained axially by a machined shoulder and an internal retaining ring. The bearing supports were designed so that the entire differential unit could be easily lifted from the vehicle in one piece. The supports are attached to the vehicle chassis by bolting into steel tabs welded to the rear frame members. The bearing supports have a mass of approximately 0.6kg and can withstand support forces in excess of the 9200N generated by chain tension. Figure 5-1 illustrates the bearing support brackets.
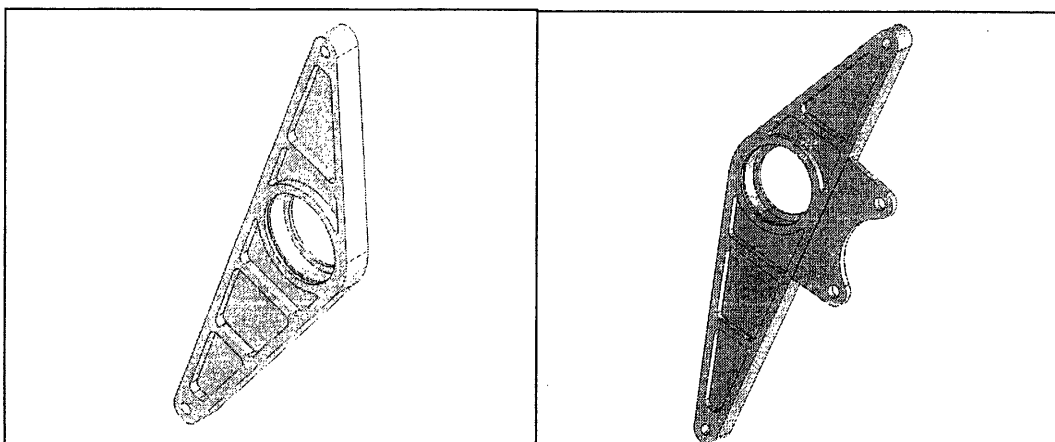

Figure 5-1: Solid model of differential bearing supports

5.2 Driveline Components

The torque from the differential side gears must reach the rear drive wheels in an efficient manner. The vehicle's rear suspension allows for 2" of vertical wheel travel. The drive wheels must receive power under all possible suspension conditions. This requires the use a constant velocity (CV) joint. Unlike u-joints typically used in drive shafts, CV joints allow for a constant rate of rotation regardless of the misalignment angle between the connected shafts. In this case tripod-type joints were specified due to their lightweight and compactness.

A tripod joint uses a central hub, or tripod with three trunnions fitted with spherical rollers on needle bearings.[7] The spherical rollers ride in the tulip-shaped cavity of an outer housing. Tripod joints allow for high misalignment angles and allow the drive axles plunge in and out with suspension movement. In order to keep stub shafts straight at both the differential and the wheel hub two joints, inboard and outboard, must be used. Figure 5-2 shows a schematic of a typical tripod joint and the actual tripod housing used in the MIT FSAE vehicle. The MIT FSAE vehicle uses tripod joints, housings, axles and
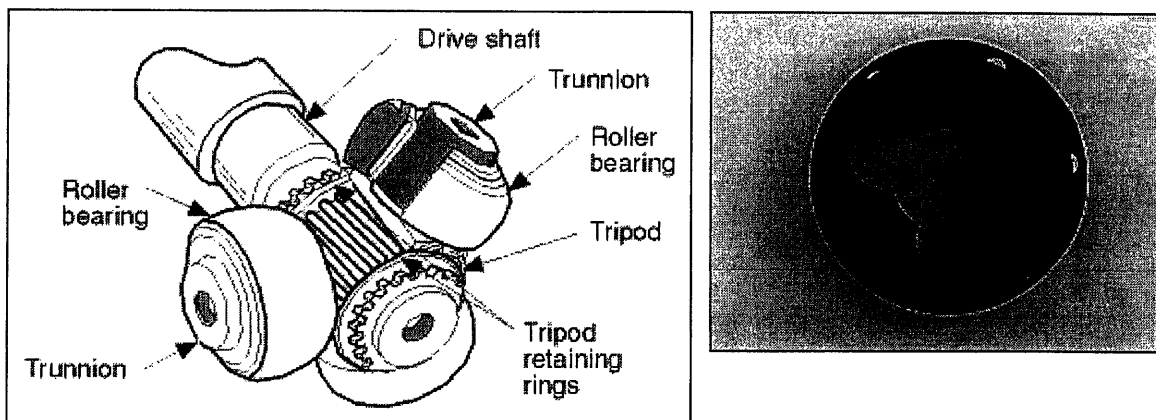

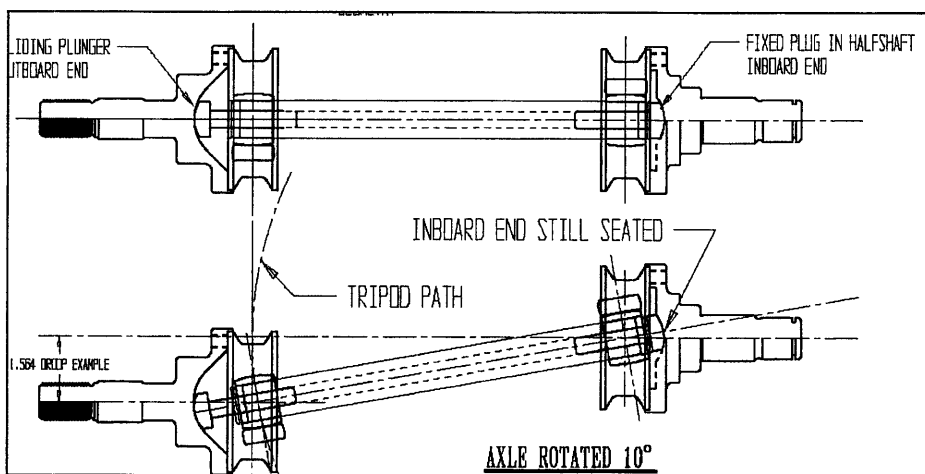Figure 5-2: Example of a typical tripod joint and the tripod housing used in the MIT FSAE vehicle.

Figure 5-3: Drawing of a drive axle under suspension droop. Courtesy of Taylor Race Engineering.[8]

outboard stub shafts purchased from Taylor Race Engineering. The outboard stub shafts mate with the rear wheel hubs transferring torque to the rear drive wheels. The inboard stub shafts were machined by the author. The side gear splines of the Torsen unit are unfortunately a non-standard metric module. This meant it was extremely difficult to find the tooling necessary to make them in-house or to find a vendor capable of manufacturing them within a reasonable time frame.

Ideally, the inboard stub shafts would be machined from a single piece of steel 4340. However, due to the difficulty of finding a spline cutter these were fabricated from two separate components. Pre-splined 4340 shafts were purchased. These had to be cut in half, machined on a lathe, and welded to a custom-made flange. The drive flanges were machined from a 4" diameter 4340 round. The bolt pattern on the flange was drilled to match that on the Taylor Race tripod housings. The splined shafts were pressed into the flanges and welded. Because of the high forces involved the diameter of the splined shafts was kept as large as possible at the weld point in order to keep shear stress endured by the weld fillets low. This procedure was similar to the one carried out on the 2003 MIT FSAE inboard stub shafts. However, the previous system manifested cracks at the welds, so the shaft diameter at the weld was increased and a ¼" key was also used to fix the flange and shaft together.

4340 loses a great deal of hardness once welded so it was necessary to heat treat the stub shafts to prevent driveline failure. After being welded the stubshafts were annealed and the bulk of the machining work was completed. The shafts were then heat-treated to a hardness of approximately Rc 50 by oil quenching after heating to 860°C. Figure 5-4 shows a picture of the right and left stub shafts immediately after being removed from oil quenching. The relative angular velocity between the stub shafts and the differential
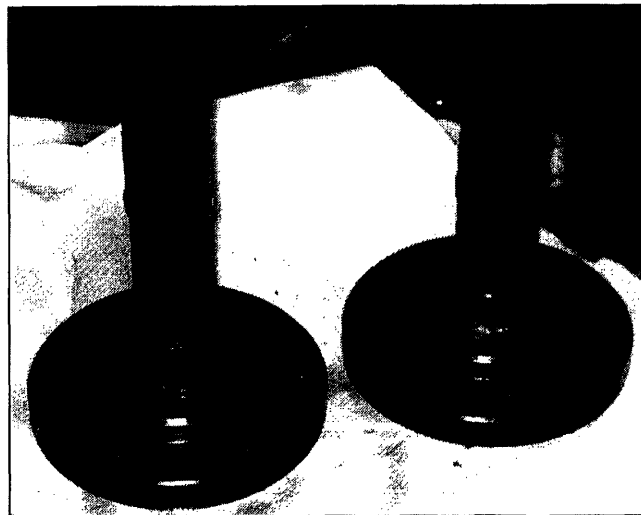
Figure 5-4: Right and left inboard stub shafts after being heat-treated to Rc 50

housing is relatively low. The difference in angular velocity is half the difference in rotational speed of the drive wheels. The short width between drive wheels means this difference remains below 60 RPM under even the tightest of turning. The inboard stub

shafts will be supported by oil-impregnated bronze bushings pressed into the aluminum differential housing.

Once the FSAE vehicle's rear suspension has been assembled, the exact distance between the drive flanges on the inboard and outboard stub shafts can be measured. The drive axles will be cut to size from the extra length supplied by Taylor Race. These axles have been gun-drilled in order to save weight.

Rear wheel hubs were also purchased from Taylor Race due to the difficulty and cost of manufacturing splines in-house. The rear hubs are fastened to the outboard stub shafts as they pass through the rear upright and wheel bearing assembly. Figure 5-5 shows a drawing of the Taylor Race outboard stub shafts and wheel hubs attached to a generic upright. They are also made from hardened 4340.
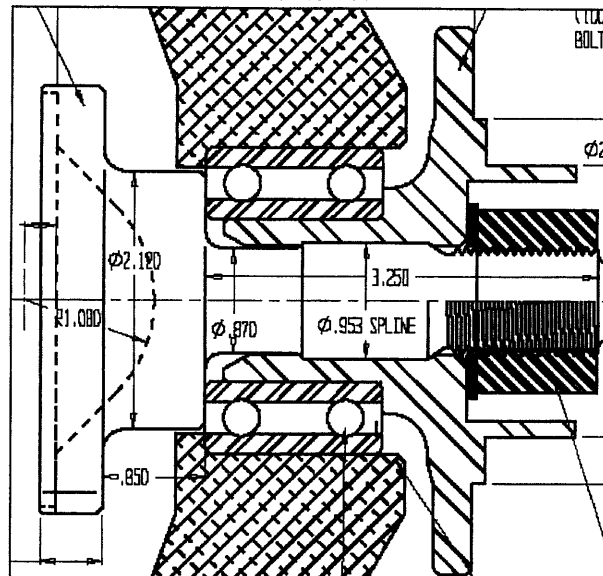

Figure 5-5: Drawing of Taylor Race outboard stub shaft and wheel hub. Courtesy of TRE[9]

In order to prevent the tripod joints from plunging too far into or out of their respective housings (hence losing contact or damaging the rollers) an anti-plunge system will be installed. At either end of each drive shaft, a delrin plunger is inserted into the inner diameter of the shafts. The plunger at the inboard end in pinned directly to the shaft. The outboard plunger is free to move along the axle's axis. A spring and a plastic filler rod are placed in the shaft's bore behind the outboard plunger. The spring helps keep the shaft in the same axial location relative to the housings since the outboard plunger can slide to bear on the curved surface of the outboard stub shafts at all times.

6.0 Alternative Differential System

The aluminum differential housing described in the previous sections represents an attempt to significantly lighten the drive train system of the 2004 MIT Formula SAE vehicle. The MIT FSAE vehicle is destined to compete at the national FSAE competition in Detroit in mid-May 2004. Analysis shows that creating a robust housing out of aluminum is possible. However, such a radical redesign of the drivetrain requires extensive track testing to ensure that it would not fail under race conditions as well ensuring that it could survive such punishment for hundreds of track sessions. Lacking specialized equipment to load the housing the only viable testing option is actual on-track testing of the MIT FSAE vehicle. Unfortunately, at the time of this writing the MIT car is still under construction, hence limiting possible testing to only one or two sessions before competition. The author has decided that insufficient testing time will require the design and manufacture of an alternate yet already proven system for use at the 2004 competition.

This alternate housing is an evolution of the system used in 2003. In this case, the original cast-iron housing is modified to provide the same functionality outlined in section 2. The system will be designed such that it will incorporate the same mounting and drive axle scheme as the aluminum housing. This will allow the housing to be swapped at a later date to allow for testing of the custom aluminum unit.

6.1 Modifying the Torsen Housing

The original Torsen housing is designed to accept input torque through a large hollow splined shaft. This can be seen in figure 6-1. This method cannot easily be used to supply
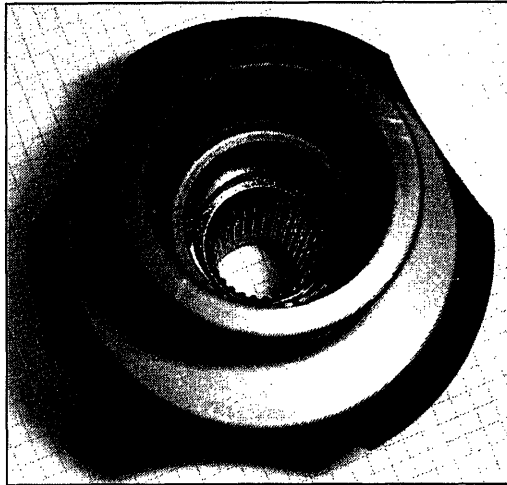

Figure 6-1: View of the Torsen housing's large input spline

input torque to the differential. Instead, end caps will be bolted directly to the housing in order to mount to the sprocket and brake rotor. These end caps will also supply the necessary bearing surfaces to mount the differential in the vehicle. First it was necessary to shorten the external bosses of the original housing by grinding. A 3-hole bolt circle was drilled and tapped into both ends of the housing to serve as the end cap attachment

points. 5/16" high grade fasteners have already proven sufficient in previous years' design to attach these end caps. The external bosses were further machined to a suitable outer diameter to allow their use in aligning the housing to the end caps. (Appendix B)

Bronze bushing, as previously mentioned, were made to fit inside the bore of the housing bosses to support the inboard stub shafts. These bronze bushings are located axially by the protruding side gear faces and by a machined step in the end caps.

A simple FEA was carried out to see if the caps could transfer torque from the sprocket/rotor mount points to the differential housing with a reasonable yield safety factor. The wall thickness behind the end cap bearing surface was dimensioned to prevent chain tension forces from causing a shear failure at that plane. The end caps were designed to use the same bearing supports as the original proposed design so their geometry is very similar to the flanges on the all aluminum design. Indeed many of the design details such as seals and retaining rings have been duplicated in this design. The mounting holes for the sprocket and rotor match those on the aluminum design. The sprocket and rotor end cap are almost identical except for the extra length on the sprocket cap to allow for mounting bolt clearance. Figure 6-2 shows a CAD model of the brake rotor end cap.
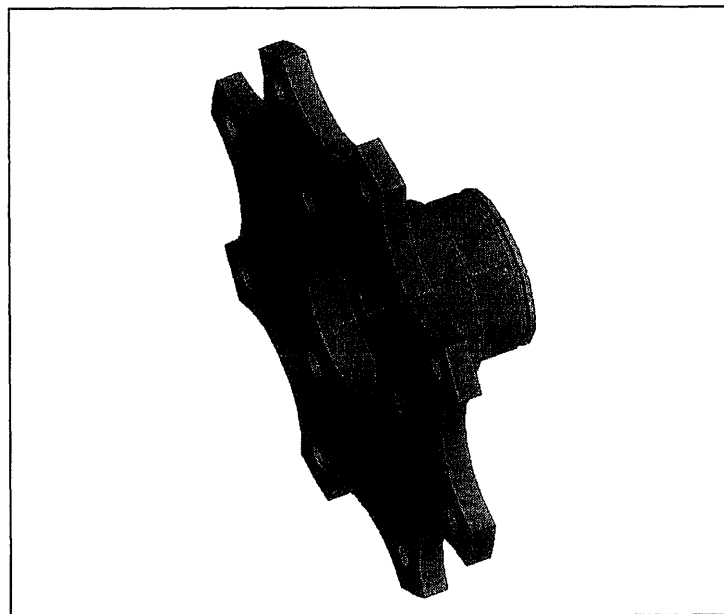
Figure 6-2: CAD model of the rotor end cap for the alternate differential design

Figure 6-3 shows a photograph of the finalized differential components. These components are currently mounted in the 2004 MIT FSAE vehicle. (See Appendix C for models).
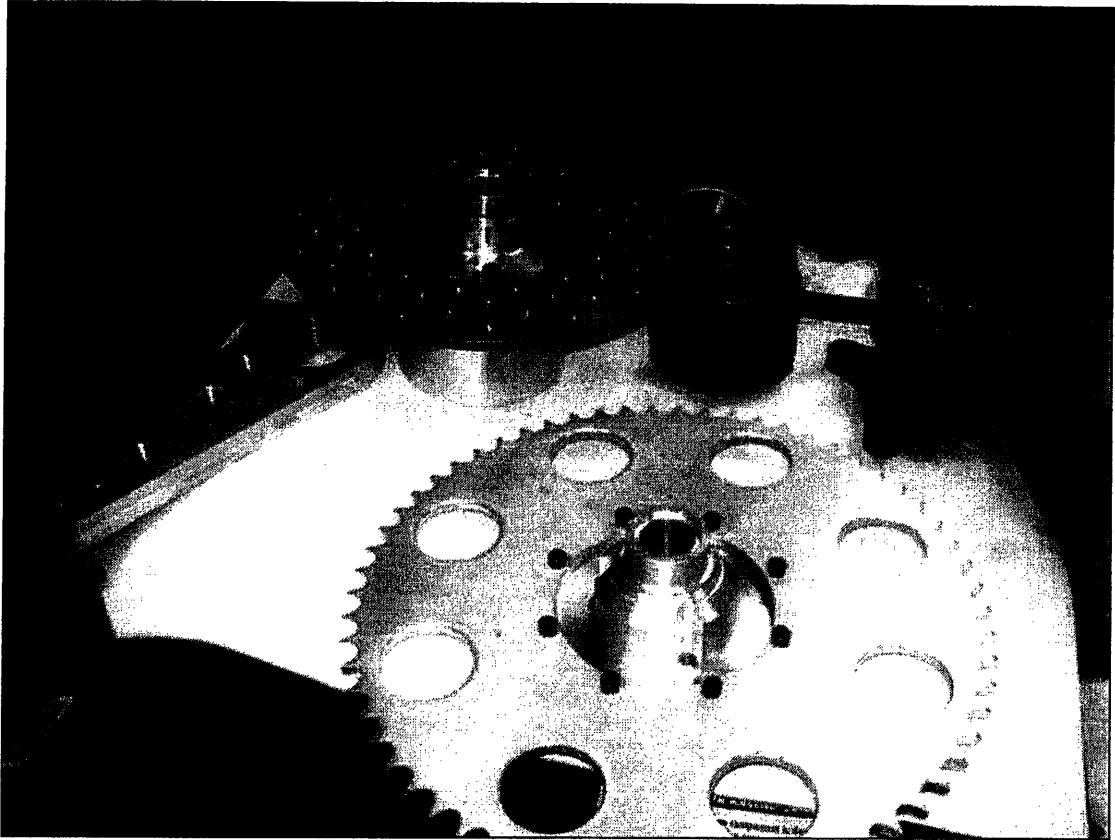
Figure 6-3: Manufactured components for the alternate differential design

7.0 Conclusion

The principal intention of this study was to create a design for a lightweight differential housing for use in the 2004 MIT Formula SAE racing vehicle. Specifically, the cast-iron housing of the Torsen T-1 differential was evaluated and redesigned as a two-piece aluminum unit. The impetus behind this study was to create an elegant, yet simple and lightweight driveline system to improve upon the system currently in place in the 2003 MIT FSAE vehicle. The system not only had to function mechanically but had to fit within the constraints of the FSAE vehicle and allow easy service.

After an overview of typical differentials available on the market the function of the Torsen T-1 was investigated in great detail. The Torsen relies on friction forces, generated by axial loading from crossed-axis helical gears, to transfer torque from its low traction end to the higher traction end. The housing must be strong enough to withstand the internal housing forces generated by the Invex gear train and stiff enough to prevent relative gear movement and large gear tooth stresses. The maximum input torque available was calculated from the capabilities of the FSAE vehicle which determined the ultimate magnitude of the internal forces within the housing.

Finite Element Analysis was carried out on CAD models of potential design. Using the FEA output a design was created that met all the stipulated functional requirements and possessed a reasonable factor of safety while still saving significant mass and rotational inertia as compared to the original Torsen housing. Lightweight aluminum bearing mounts were designed and fabricated as well as inboard stub shafts. The remaining axles and joints were sourced from a private vendor.

Due to insufficient available testing time, the newly designed unit never made it into the MIT FSAE vehicle. It was decided instead to rely on an already proven design from years past. An alternated design using the original cast iron housing was designed and fabricated. Although not as lightweight as the all-aluminum design it still provided significant mass savings over the system present in the 2003 vehicle.

There are many possibilities for furthering this study. The geometry of the redesigned housing does not lend itself easily to manufacture. Further work in this area could reap benefits for the MIT FSAE Team who will likely use the design exclusively in the 2005 vehicle. Lastly, the notion of Torque Bias Ratio (TBR) was touched upon briefly in this study. Increasing the TBR could yield significant performance benefits, allowing the differential to "put the power down" under more strenuous cornering conditions. Changing the friction forces present within the housing would accomplish this. 4

8.0 References

8.1 Cited References

1. "Differentials (Open, Locked, Limited Slip, EDL, Quattro)" [Online Document], Available HTTP: http://www.houseofthud.com/_top#_top
2. How Stuff Works "How Differentials Work" [Online Document], Available HTTP: http://howstuffworks.com
3. Chocholek, SE. The Development of a Differential for the Improvement of Traction Control. IMechE 1998, C368/88 p.75-82
4. Isocalc, [Online Document], Available HTTP: http://www.isocalc.com/isocalc3/examples/images/helical-gear.gif
5. Shigley JE, Mischke CR. Mechanical Engineering Design – 6th ed. New York: McGraw-Hill; 2001 pgs 851-874
6. Culpepper ML, Slocum A, Shaikh FZ. Quasi-kinematic Couplings for Low-cost Precision Aligment of High-volume Assemblies. ASME J Mechanical Design: Sept 2002
7. Autosite, "Axle, Differential, CV and U-joints" [Online Document], Available HTTP: http://www.autosite.com/garage/subsys/baaxlejo.asp
8. Taylor Race Engineering, "FSAE Technical Drawings" [Online document], Available HTTP: http://www.taylor-race.com/pdf/chaind.pdf
9. Taylor Race Engineering, "Generic Upright with center-lock stub axle" [Online document], Available HTTP: http://www.taylor-race.com/pdf/fsae_upright_design_centerlock.pdf

8.2 Additional References

1. Crandall SH, Dahl NC, Lardner TJ. An Introduction to The Mechanics of Solids – 2nd ed. New York: McGraw-Hill; 1978
2. Dowling NE. Mechanical Behavior of Materials – 2nd ed. Upper Saddle River, New Jersey: Prentice Hall; 1999
3. Oberg E, Jones FD, Horton HL, Ryffel HH. Machinery's Handbook – 26th ed. New York: Industrial Press; 2000
4. Shih S, Bowerman W. An Evaluation of Torque Bias and Efficieny of Torsen Differential. SAE Technical Paper Series 2002-01-1046
5. Jawad BA, Ziemke M, Young AP. Design of an Aluminum Differential for a Racing Style Car. SAE Technical Paper Series 2000-01-1156
6. Conversation with James Gleason of the Gleason Corporation. Co-inventor of the Torsen differential. 10/17/03
7. Conversations with Prof. Alex Slocum, MIT. 10/03-5/04
8. Conversations with Prof. Martin Culpepper, MIT 1/04-3/04
9. Autozone Technical School, "Different Types of 4WD" 2000 [Online Document], Available HTTP: http://autozine.kyul.net/technical_school/traction/tech_traction_4wd_2.htm

9.0 Appendices
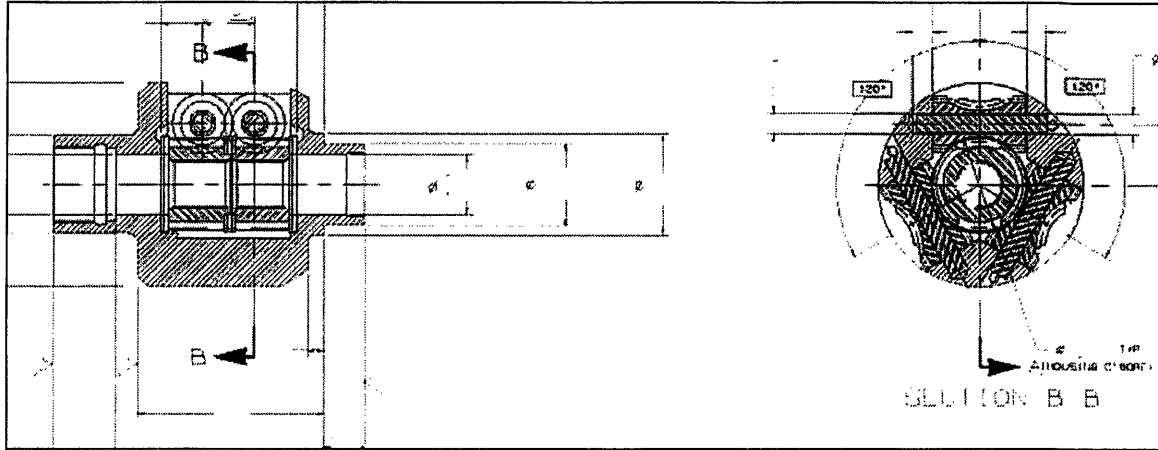
9.1 Appendix A – Torsen T1 Manufacturing print

Figure A-1: Drawing print of Torsen T-1 housing

9.2 Appendix B – Calculating Bolt Shear Stress

The ultimate shear capacity of an AN bolts is 652 MPa. Shear stress, τ, is

$\tau = \dfrac{P}{A}$ where P is load and A is the total cross - sectional area in shear

$P = \dfrac{T}{d}$ where T is torque and d is the effective radial distance the torque acts at.

$T = 1406\text{Nm} \quad d = 0.03175\text{m}$

$\rightarrow$ **P = 44,283N**

the total cross - sectional are of (3) 5/16" bolts is 1.484E - 4m

$\therefore \tau = 298.3\text{MPa}$

This leaves a safety factor of 2.19 under the most extreme input torques
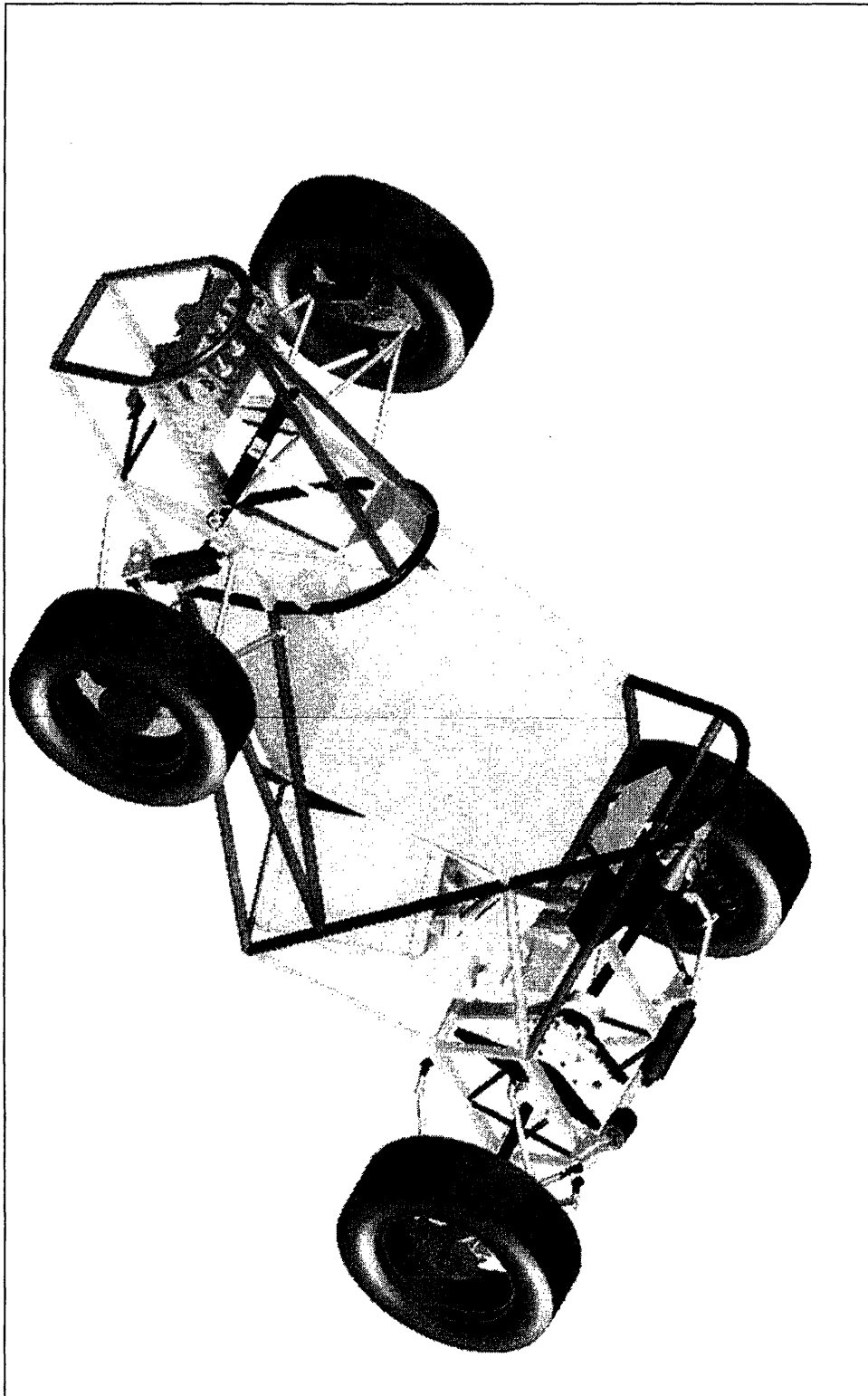
9.3 Appendix C -2004 MIT vehicle model

Figure A-2: CAD model of the 2004 MIT FSAE vehicle.

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