Statistical Simulation of FM Interference to the Instrument Landing System

by

William W. Zhou

Submitted to the Department of Electrical Engineering and Computer Science

in Partial Fulfillment of the Requirements for the Degrees of

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ABSTRACT

Contrary to the popular views, air travel has been the safest compared to other surface transportation (automobile & railroad) in terms of casualty of passengers. This is mainly due to the vigilant pursuit of safety margin in the design of Air Traffic Control and Navigation (ATCN) system. However, the rising demand of air travels requires us to review the safety factor affecting all phases of aircraft operation. In order to determine the operation benefit of any aircraft instrument landing system (ILS), we need to take into consideration of all factor that could jeopardize the successful landing of any aircraft. The concept of "risk" probability, i.e. the probability of an unsuccessful instrument guided aircraft landing, needs to be applied to ILS. This thesis concentrates on the specifics of such uncertainties of a successful landing. We will be mainly concerned with unsuccessful landings due to unwanted electromagnetic interference from nearby Frequency Modulated radio stations.

To begin with, we formulate the exact path of electromagnetic wave propagation between the FM transmission towers and the receivers located on the landing aircraft. After obtaining the relationship that guides the course of interference, we then translate each parameter into a random variable with known distribution in order to more accurately model the statistical nature of events. To keep the results consistent, normal algebraic operation were replaced with corresponding statistical operation on the relevant distributions. All calculations were minimized before being implemented for real time analysis in the simulation program. The simulation program, unlike many other programs before that only give a deterministic answer of the interference level compared with a pre-set threshold, yields instead a measure of the probability of any interference occurring. Running the simulation program with the same inputs, we could get a feel of how the level of interference is related to the probability risk in landing.

Thesis Supervisor: Dr. Eric Yang

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- **BACKGROUND:**

One of the many factors that could threaten the safe landing of an aircraft is the electromagnetic interference to on-board electronic guidance equipment. Throughout the years, many techniques have been proposed and put into practice to ensure Electromagnetic Compatibility (EMC) between aeronautical radio services in frequency band 108-137 MHz, and the FM broadcast stations located from 88 to 108 MHz. The most sophisticated method up-to-date is Federal Aviation Administration's (FAA) Airspace Analysis Mathematical Model (AAMM) program, which predicts deterministically the level of electromagnetic interference based on the sampling of existing equipment performance.

In practice however, the receiver operational environment is random in nature, and the equipment performance properties are themselves a random variable due to the population of receivers/antennas. This variability "randomizes" the event of the interference characterized by the crossing of a given threshold--which itself may be somewhat random. As a result, a probabilistic model is necessary, and the risk of interference will be used as a reference to match the ground equipment. The classification of the FM interference as a random event is further vindicated by the fact that, though it has known effects on the ILS, those effects are not often obvious to the ILS users. Therefore, this thesis strives to demonstrate a viable method for determining the risk associated with FM broadcast signals interfering with the ILS localizer operation.

- **INTRODUCTION:**

Instrument Landing System (ILS) is an electronic system designed to help airline pilots align their planes with the center of a landing strip during final approach under conditions of poor visibility. In this thesis, we will be solely focusing on the interference caused by the intermodulation products of FM signals on the aircraft guiding comparison channels of ILS. The ILS performs its "centering" function through the comparison of the relative amplitudes of two signals: one (f_{ils1}) at the carrier frequency + 90 Hz, the other (f_{ils2}) at the carrier frequency + 150 Hz. By using separate antennas for transmission of each signal and through careful design of those antennas, the ILS signal-in-space is such that on runway centerline

$$\|f_{ils1}\| = \|f_{ils2}\|$$

Intermodulation occurs as a result of the airborne receiver being driven into non-linearity by a high-power broadcast signal outside the aeronautical band. This distortion could serve to erroneously increase the magnitude of either f_{ils1} or f_{ils2}. If these increases are small, they can degrade signal integrity and cause false guidance being provided to the pilot/autopilot. If large, these increases can disrupt operation by causing the ILS receiver to alarm and stop providing service all together. Figure 1 shows the critical landing area contour, on which desired probability of interference (P_{int}) is indicated. Namely, along the outer boundaries of the service volume (worst case), the P_{int} should be less than 2.4×10^{-4}, while in the critical landing phase (inside the CAT I decision point) a $P_{int} \leq 2 \times 10^{-6}$ is desired.

The statistical approach to this simulation model will allow for quantification of the probability of interference on the ILS from the FM transmission. Our approach is to assume that all of the parameters are either Gaussian random variables or uniform distributions whose means and standard deviations could be derived from measured data on operational equipments. Due to the time constraint, we will restrict our scope to the statistical modeling of interference due to three-frequency third-order intermodulation where the following relationship holds:

$$F_{ils} = F_{fm1} + F_{fm2} - F_{fm3}$$

where F_{ils} is the frequency of the desired ILS localizer channel, and F_{fmi}s are the respective FM station frequencies.

- **FORMULATION:**

Before implementing the simulation, we need to formulate the exact relationship between the FM signals transmitted from station tower and its eventual adverse effect on the receivers located on the landing aircraft. The path of signal propagation must be known before proceeding with the calculations. The free-space transmission formula is:

$$P_D = \frac{P_T * G_T}{4 * \pi * d^2}$$

where

P_D = Power density
P_T = Power transmitted
G_T = Gain of transmitting antenna relative to isotropic
d = Distance from transmitter

The effective power-gathering area A_R of the receiving antenna is given by:

$$A_R = \frac{G_R * \lambda^2}{4 * \pi}$$

So the power received by the receiving antenna can be expressed as:

$$P_R = P_D * A_R$$

or

$$P_R = \frac{P_T * G_T * G_R * \lambda^2}{(4 * \pi)^2 * d^2}$$

where

P_R = Power received
P_T = Power transmitted
G_T = Gain of transmitting antenna relative to isotropic
G_R = Gain of receiving antenna relative to isotropic
λ = Free-space wavelength
d = Distance between antennas

Assuming for the moment that the transmitting and receiving antennas are both isotropic, then expressing the above equation in decibels gives:

$$P_R = [10*\log(P_T)] + [10*\log(\frac{\lambda^2}{(4*\pi*d)^2})]$$

Since the numerical value obtained from the broadcast station database is Effective Radiated Power (ERP) in kilowatts, we need to convert this to a equivalent isotropically radiated power (EIRP) that truly represents P_T:

$$EIRP_{kw} = ERP_{kw}*1.643$$

In order to interface with other equations in AAMM, the received power needs to be expressed in dBm as follows:

$$EIRP_{dBm} = 10*\log(ERP_{kw}*10^6*1.643) = 10*\log(ERP_{kw}) + 62.2$$

The free space loss factor, P_L, is defined as

$$P_L = \frac{\lambda^2}{(4*\pi*d)^2}$$

Since

$$\lambda = \frac{c}{f}$$

where

c = Velocity of light
f = Frequency (in Hz)

then

$$P_L = 10*\log[\frac{c^2}{(4*\pi*d)^2*f^2}]$$

Because the AAMM uses nautical miles (nmi) for distance units and MHz for frequency units, so we need to convert the above equation into:

$$P_L = 10*\log[\frac{161880^2}{(4*\pi*d)^2*(10^6*F)^2}]$$

where

161880 = Velocity of light in nmi/sec
F = Frequency in MHz

After substituting in all the constants, the free-space loss factor can be simplified down to:

$$P_L = -20 * \log(d * F) - 37.8$$

Substituting this in to the original P_R equation, we have:

$$P_R = 10 * \log(ERP_{kw}) + 24.4 - 20 * \log(d * F)$$

Since the receiving antenna is not isotropic, the value of P_R must be adjusted for losses in the receiving antenna, L_R. The value of L_R has been determined empirically by measuring the loss of numerous receiving antennas on a calibrated range. Other corrections to the P_R equation include losses due to the vertical and horizontal radiation patterns of the transmitting antenna. Therefore the final expression of P_R is

$$P_R = 10 * \log(ERP_{kw}) + 24.4 - 20 * \log(d * F) - V_T - H_T - L_R$$

where

V_T = Loss due to vertical radiation pattern of the trans. antenna (dB)
H_T = Loss due to horizontal radiation pattern of the trans. antenna (dB)
L_R = Loss in the receiving antenna (dB)

Knowing the power levels of individual FM station signals present at the receiving terminal, the magnitude of the signal resulting from the three-frequency intermodulation is proportional to the product of the magnitudes of the individual FM signals at the point of intermodulation, so

$$(A_1)(A_2)(A_3) = k(A_0)$$

where

A_1 = the magnitude of the first FM signal
A_2 = the magnitude of the second FM signal
A_3 = the magnitude of the third FM signal
k = a constant that describes the non-linearity of a given receiver
A_0 = the magnitude of the intermodulation product on the ILS localizer

Again, in order to correlate with the database of receiver test results, the relation can be re-written in units of dBm:

$$P_{R1} + P_{R2} + P_{R3} = P_{R\,\text{int}}$$

where

$P_{R1} = A_1$ converted to dBm
$P_{R2} = A_2$ converted to dBm
$P_{R3} = A_3$ converted to dBm
$P_{Rint} = k(A_0)$ converted to dBm

and the value of P_{Rint} is what we want to use to determine the probability of interference.

Having traced the path of FM signal level from the transmitting antenna to the receiving antenna and obtained the precise equation for the received power, the next step is to determine a reasonable receiver threshold above which the received FM signals will distort the normal operation of the receiver. Such threshold levels could be determined using curve fitting technique from a collection of experimental data. In calculating such a threshold, a couple of factors need to be considered. First, the desired signal strength at the receiver terminal, referring to the power level of the un-interfered signal that the receiver is meant to receive information from. The higher the desired signal level, the higher the interference threshold will be. The other factor is the so-called ΔF product, which is calculated by

$$\Delta F = (F_{ILS} - F_{FM1})*(F_{ILS} - F_{FM2})*(F_{ILS} - F_{FM3})$$

The parameter ΔF represents a measure of how far apart the interfering FM frequencies are from the operating frequency at the navigation receiver. Again, the farther apart they are, which in turn gives us a bigger ΔF, the higher the interference threshold will be.

Based on bench measurements, the interference threshold equations for desired signal levels of -86 dBm and -49 dBm can be respectively expressed as

$$TH_{86} = -120.4902 + (6.5931*\log \Delta F) + (4.7004*\log^2 \Delta F)$$

$$TH_{49} = -56.788 + (3.2823*\log \Delta F) + (3.6645*\log^2 \Delta F)$$

Given these regression curves, the interference threshold for any desired signal level can be interpolated by

$$TH = ((-86 - NAV_{dBm}) / 37) * (TH_{49} - TH_{86})$$

where

NAV_{dBm} = desired signal level (in dBm) at the receiver terminal

In the deterministic model, the final step is to compare the intermodulation level at the receiving terminal and the receiver threshold level given the specific desired signal level. And such comparison would be made on a point to point basis in the landing area of interest (see Figure 1).

- **STATISTICAL MODEL EXTENSION:**

Now that we have carefully delineated the deterministic approach to calculating the interference level. It requires a few modifications to turn this model into a statistical one. The probability of interference is a function of four components: the transmitted signal in space, the receiver antenna type factor, the receiver antenna directivity, and the receiver interference threshold. We will examine them in detail one by one in the following paragraphs.

The transmitted signal in space is a measure of the interfering signal strength, which is a function of the transmitter power, the transmitter antenna gain, and the signal propagation loss. Notice that the transmitted signals have a distribution per FM signal. Since we are concerned about the three-frequency intermodulation, there are going to be three Gaussian distributions with mean μ_i and standard deviation σ_i. Because the parameters are expressed in unit of dBm, the multiplicative operation needed to obtain the intermodulation product was simplified to the addition of the three random variables. From what we know about the properties of Gaussian distribution, it is obvious that the sum of Gaussian distributions (after performing necessary convolution) is still a Gaussian distribution with mean μ and standard deviation σ given by:

$$\mu_{fm} = \mu_{fm1} + \mu_{fm2} + \mu_{fm3}$$

$$\sigma_{fm} = \sqrt{\sigma_{fm1}^2 + \sigma_{fm2}^2 + \sigma_{fm3}^2}$$

The new mean μ_{fm} corresponds to the intermodulation power present at the receiver, P_{Rint}, which we have already learned to calculate in the deterministic model. The value for σ_{fmi} can be based on the data presented in Table I of Canadian Document JIWP 8-10/1 CAN 2, July 25, 1988, which says received undesired signal strength at an aircraft has a standard deviation of 4.2 dBm with respect to predicted values. Therefore, the value for the new standard deviation σ is

$$\sigma_{fm} = \sqrt{4.2^2 + 4.2^2 + 4.2^2} = 7.2746$$

The antenna type factor is simply another word for the antenna rejection loss L_R that we have taken into consideration in the before-mentioned expression on P_{Rint}. For the statistical model, the mean and standard deviation of the antenna rejection distribution can be derived from composite antenna tests conducted by Transport-Canada. As a result, we have

$$\mu_{LR} = 2717.9 + (-84.6073 * F) + (0.85708 * F^2) + (-0.00284696 * F^3)$$

$$\sigma_{LR} = 5.0$$

where

F = Frequency in MHz

Like the signal in space distribution, the antenna rejection random variable has a distribution per FM signal. Performing the convolution will give us the resulting Gaussian distribution with:

$$\mu_{LR} = \mu_{LR1} + \mu_{LR2} + \mu_{LR3}$$

$$\sigma_{LR} = \sqrt{\sigma_{LR1}^2 + \sigma_{LR2}^2 + \sigma_{LR3}^2} = 8.66$$

The third component, receiver antenna directivity, can be characterized by a uniform distribution with maximum deviation of ±3 dB. The Transport-Canada data states that "it can be seen from this data that the gain of the antenna relative to frequency varies by about 18 dB over the FM broadcast band with an additional 2-3 dB variation at each frequency due to aircraft orientation." In order to worst case this analysis, the larger (3 dB) value was assumed. Again, a single distribution is required for each FM frequency. Applying the properties of uniform distribution, we have:

$$\sigma = \sqrt{\frac{(b-a)^2}{12}} = \sqrt{\frac{[3-(-3)]^2}{12}} = \sqrt{\frac{36}{12}} = \sqrt{3}$$

and

$$\sigma_{uni} = \sqrt{3 * \sigma^2} = \sqrt{3 \times 3} = 3$$

The last component, the receiver interference threshold, is addressed as a Gaussian random variable, with mean equals to threshold value we calculated in the deterministic case using the function of *log(ΔF)* and a standard deviation equals to the average standard deviation of the data for each unique log frequency product. The average standard deviation was calculated to be σ_{th}=6.58 dB.

After deciding on the distribution pattern for all four components, we can draft up a table of parameter distributions as shown in Table 1. The answer we are looking for is:

$$(P_{fm1}+P_{fm2}+P_{fm3})-(L_{R1}+L_{R2}+L_{R3})-(D_1+D_2+D_3)-T$$

The arithmetic operations on these random variables naturally translate into convolutions on the respective distributions to yield the composite distribution:

$$A*B*C*D*E*F*G*H*I*J$$

with the combined deviation of:

$$\sigma=\sqrt{\sigma_{fm}{}^2+\sigma_{LR}{}^2+\sigma_{uni}{}^2+\sigma_{th}{}^2}\approx 13.42$$

In order to minimize complication, we will let all the convolving distributions be zero-mean distributions and perform the proper arithmetic operations on their means separately. In the end, the distribution offset that we are interested in is found by

$$[(\mu_{fm1}+\mu_{fm2}+\mu_{fm3})-(\mu_{LR1}+\mu_{LR2}+\mu_{LR3})]-TH$$

The distribution offset is then mapped onto a point on the zero-mean composite distribution and the area under the curve to the left of this point gives us the probability of interference. This scheme makes sense if one looks at the distribution offset as a simple difference between the receiver threshold level and the intermodulation signal level at the receiver. If the two values are equal, the offset point will be zero, and we have 0.5 as our probability of interference. If the intermodulation level is above the threshold, the offset point will be on the positive side of the distribution, and the probability will fall between 0.5 and 1. On the other hand, if the intermodulation level is less than the threshold, the offset point will be on the negative side of the zero-mean distribution, and the probability will fall between 0 and 0.5.

Since the present AAMM simulation program has already performed most of the analysis needed to yield such a distribution offset, the main task remaining is to find an accurate description of the resulting composite zero-mean distribution. All the Gaussian curves could easily be grouped together since the result of convolving zero-mean Gaussian distribution will still be a zero-mean Gaussian distribution, except with a different standard deviation. The challenging part is to convolve this composite Gaussian distribution with the three uniform distributions left.

- **CONVOLUTION USING FOURIER TRANSFORMS:**

Instead of performing the straight convolution, a better approach is to transform the distributions into their respective characteristic functions. The relationship between any probability distribution and its characteristic function resembles those of a Fourier Transform pair:

$$\varphi(t) = \int_{-\infty}^{+\infty} e^{ixt} f(x)dx$$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-ixt} \varphi(t)dt$$

Thus, convolution between the probability distributions can be equivalently accomplished by multiplication between their corresponding characteristic functions. The main motivation for mapping the operation into the characteristic domain is to simplify the calculation of a cumulative probability distribution. To see that, let there be Fourier Transform Pair

$$f(t) \Leftrightarrow F(\omega) = R(\omega) + jX(\omega)$$

We shall evaluate the transform *G(w)* of the real integral

$$g(t) = \int_{-\infty}^{t} f(\tau)d\tau$$

in terms of the transform *F(w)* of the integrand *f(t)*. The above integral is, obviously, a convolution of *f(t)* with the unit step *U(t)*,

$$g(t) = f(t) * U(t)$$

Apply the convolution theorem, we get:

$$G(\omega) = F(\omega)\cdot[\pi\cdot\delta(\omega) + \frac{1}{j\omega}] = \pi\cdot R(0)\cdot\delta(\omega) + \frac{X(\omega)}{\omega} - j\cdot\frac{R(\omega)}{\omega}$$

because $F(\omega)\cdot\delta(\omega) = F(0)\cdot\delta(\omega) = R(0)\cdot\delta(\omega)$. Using the inverse transform formula, we can write *g(t)* in terms of *F(w)*:

$$g(t)=\frac{R(0)}{2}+\frac{1}{\pi}\int_0^{\infty}[\frac{X(\omega)}{\omega}\cos\omega t+\frac{R(\omega)}{\omega}\sin\omega t]d\omega$$

which allows us to arrive at the bounded area under the composite distribution curve with minimum calculation.

In our case, since the fundamental function that we are interested in is of the following form:

$$f(t)=\frac{1}{\sqrt{2\pi}}e^{-t^2/2}*\frac{1}{2}[U(t+1)-U(t-1)]*\frac{1}{2}[U(t+1)-U(t-1)]$$
$$*\frac{1}{2}[U(t+1)-U(t-1)]$$

the corresponding $F(w)$ is an all real function:

$$F(\omega)=e^{-\omega^2/2}[\frac{\sin(\omega)}{\omega}]^3$$

Thus, we arrive at:

$$G(\omega)=\frac{\pi}{2}\cdot\delta(\omega)-j\cdot e^{-\omega^2/2}\cdot[\frac{\sin(\omega)}{\omega}]^3\cdot\frac{1}{\omega}$$

$$g(t)=\frac{1}{2}+\frac{1}{\pi}\int_0^{\infty}e^{-\omega^2/2}[\frac{\sin(\omega)}{\omega}]^3[\frac{\sin(\omega t)}{\omega}]d\omega$$

In order to further simplify the calculation, we can temporarily ignore the $(\pi/2)\delta(w)$ term and add $1/2$ to the result of inverse Fourier transform afterward. Now we have instead a new Fourier pair:

$$G'(\omega)=-j\cdot e^{-\omega^2/2}\cdot[\frac{\sin(\omega)}{\omega}]^3\cdot\frac{1}{\omega}$$
$$g'(t)=\frac{1}{\pi}\int_0^{\infty}e^{-\omega^2/2}[\frac{\sin(\omega)}{\omega}]^3[\frac{\sin(\omega t)}{\omega}]d\omega$$

Please refer to Figure 2 and Figure 3 for the plots of $G'(w)$ and $g'(t)$.

- **IMPLEMENTATION:**

Having devised the methodology, the next step is the implementation of the procedures and the incorporation of the new codes into the existing AAMM source files. Appendix 1 lists the independent Fortran module that computes the cumulative probability function *[g'(t)+0.5]* of the desired composite distribution. Parameter *t* and *w* represent respectively the time and the frequency domain variables. Variable *dw* is the width of minimum integration interval. Since the number of integration 'slices' are set by $n=2^{nu}$, the value of *dw* is found by dividing the range, *wrange*, which is set to be 15, by the number *n*.

Having determined the *dw*, we iterate through each interval to carry out the integration. Exploiting the fact that the frequency domain function *G'(w)* was odd and imaginary, we know from the symmetry property of Fourier transform that the corresponding time domain function *g'(t)* will be odd and real. This fact helps us restrict our computation to the manipulation of read numbers only. So, instead of typing:

```
sum=sum+cmplx(cos(t*z), sin(t*z))*y*dw+
            cmplx(cos(t*(-z), sin(t*(-z))*(-y)*dw
```

we simply type:

```
sum = sum + 2*sin(t*z)*y*dw
```

In order to eliminate the singularities of *G'(w)* around the origin caused by *1/w*, we first subtracted from it a function *H(w)* that has the same order of singularities around the origin. This operation is then compensated in time domain by adding the result onto *h'(t)*, the known analytic inverse Fourier transform of *H'(w)*, to obtain *g'(t)*:

$$G'(\omega)-H'(\omega)=\{-j\cdot e^{-\omega^2/2}\cdot[\frac{\sin(\omega)}{\omega}]^3\cdot\frac{1}{\omega}\}-(-j\cdot\frac{1}{1+4\omega^2}\cdot\frac{1}{\omega})$$

$$\Rightarrow g'(t) = F^{-1}[G'(\omega) - H'(\omega)] + h'(t) = F^{-1}[G'(\omega) - H'(\omega)] + \frac{1}{2}\cdot(1 - e^{-\frac{1}{2}t})$$

This explains the following codes in the appendix:

```
      do 10 t=0.05,5.0,0.05
      do 5 i=1,n/2
            w=i*dw
            y=(exp(-w**2/2)*(sin(w)/w)**3/w)-((0.25/(0.25+w**2))/w)
5        sum=sum+2*sin(t*w)*y*dw
      indx=int(t/0.05+0.5)
10    x(indx)=sum/(2*pi)+0.5*(1-exp(-0.5*t))
```

Therefore, the result we get is an array of 100 elements representing the 100 time-axis values from 0 to 5 with 0.05 unit separation. Since we know that the resulting function *g'(t)* is an odd function, we only need to cover one side of the *t* axis because *g'(-t) = - g'(t)*. This way, given an input variable called *sigdiff*, which is the difference between interfering signal strength (*sig3*) and the interference threshold level at the receiver (*th3*), we can calculate the probability of interference at a particular point by the method of interpolation, as reflected in the codes listed in Appendix 2.

A couple of concerns need to be addressed here. First of all, the reason we restrict the region of interest to from -5 unit to +5 unit is because beyond this bounded region the value of *g'(t)* approaches almost exactly -0.5 or 0.5, i.e. the probability of interference, *g(t) = g'(t) + 0.5*, converges to either 0 or 1. Since what we want to determine are the in-between values, this assumption does no harm. Also, until now, we have not specifically defined what the unit of the horizontal axis really is. It should be the standard deviation of the composite distribution σ. However, in this case, σ is not 1 because of the choice of the convolving distributions. Let's calculate what the standard deviation for this distribution really is:

$$f_1(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2} \longrightarrow \sigma_1 = 1$$

$$f_2(t) = \frac{1}{2}[U(t+1) - U(t-1)] \longrightarrow \sigma_2 = \sqrt{1/3}$$

$$f_3(t) = \frac{1}{2}[U(t+1) - U(t-1)] \longrightarrow \sigma_3 = \sqrt{1/3}$$

$$f_4(t) = \frac{1}{2}[U(t+1) - U(t-1)] \longrightarrow \sigma_4 = \sqrt{1/3}$$

Therefore, the composite distribution deviation is found to be:

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_3^2} = \sqrt{1 + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = \sqrt{2}$$

With this piece of information, we can now refer to the region of interest as from $-5/\sqrt{2}$ standard deviations to $5/\sqrt{2}$ standard deviations. The reason we decided to pick the particular *f(t)*s is because that they are simple in forms and easy to work with. But in real life, as we have calculated in the STATISTICAL MODEL EXTENSION section, the actual standard deviation of the composite distribution is $\sigma \approx 13.42$. Therefore, this explains the (sqrt(2)/13.42) scaling factor in the calculation of index variable *indx*. In order to further convert the horizontal axis values into the corresponding index values of the array, we need to multiply the result by 20 because the mapping was defined such that:

$$0.05 \longrightarrow x(1),\ 0.10 \longrightarrow x(2)\ \cdots\cdots\ 4.95 \longrightarrow x(99),\ 5.00 \longrightarrow x(100)$$

Combining all steps, we find the value of the index variable *indx* to be:

*indx = int(abs(sigdiff)*sqrt(2)*20/13.42+0.5)*

Again, we restrict our region of interest by letting all points beyond the region assume the values at the boundary, i.e. *x(100)*. Having located the closest *indx* value, the Fortran codes then performs the interpolation according to the decimal digits of *preindx*. Then depending on whether *sigdiff* is positive or negative, the proper addition or subtraction is carried out to yield the final value of *intprob*, the probability of interference given the interfering signal strength (*sig3*) and the receiver threshold level at the frequency (*th3*).

The implementation stage could be considered complete by this point. Before we begin incorporating, we have to understand how the existing AAMM codes executes to render the simulation. To quickly summarize, the program first prompts the user for a geographic location anywhere on the surface of earth, and search the FM broadcast station database for all possible proponent FM stations within three-mile radius of the specified latitude/longitude. When the proponent location and transmitting frequency have been determined, the AAMM automatically searches the navaid database for all navaids within 60 nmi of the proponent FM broadcast station. Finally, another search yields a list of all FM stations within 30 nmi radius of the selected navaid. Then simulation is run to check for interference and the results are written into files to be printed and plotted.

The stage of operation which we are interested in will be the simulation procedure for detecting and recording the three-frequency intermodulation interferences. This helps us narrow down our target to *b1_3freq.exe* file and the Fortran code associated with it, *b1_3freq.for*. Again, it is necessary to first examine the working sequence of the source codes before making any modifications. The *b1_3freq.for* program includes all the calculations needed to compute the interfering signal level at a particular location and receiver interference threshold at a particular frequency. Then comparisons are made to see if the interfering signal level indeed exceeds the receiver threshold. The program first performs such comparisons over a coarse grid, where the area of interest were divided up into fairly large blocks. If definite interference is detected, the program switches to the fine grid and records into a *.plt* file how much the interfering signal exceeded the threshold level at selective points in the area of interest. Afterward another module of the AAMM program will be able to display the data in these *.plt* files to give us an interference level plot.

Since we are concerned with the probabilities of interference rather than the magnitudes of interference, we are interested in those interfering signal levels that do not exceed the interference threshold in addition to those interfering signal levels that actually do. In such case, instead of claiming interferences do not exist, the statistical model will tell us that there is a smaller probability of interference whereas if the interfering signal levels do exceed the interference threshold, the statistical analysis will record a bigger probability of interference. So a couple of conditional statements need to be changed. For example:

if (sig3 .ge. th3) then

However, this does not mean we no longer need these conditional statements. Otherwise we will be wasting computation time on those points where the probabilities of interference are extremely low. Therefore we need to establish a boarder line probability of interference as a cutoff. A good choice will be the FAA recommended value of 2.4×10^{-4}. So this corresponds to about three standard deviation to the left of the zero mean point on the composite distribution curve, which in turn corresponds to a *sigdiff* value of -40.26 dB. So the new conditional statement will be:

if (sig3 .ge. (th3-40.26)) then

Thus, we will be able to embrace those points that were not considered before because they did not exceed the threshold, and also to avoid including those points that will produce insignificant results. That will take care of the coarse grid section of the code. Then in the fine grid section of the code where file operation took place, we need to delete the line

ilev = int(sig3 - th3 +0.5)

and replace it with our own comparison codes. As one can see, in the AAMM program, the variable *ilev* represents an integer value of the interference level. When the resulting interference files are displayed, different characters are used on the plot to indicate different levels of interference. To avoid complication, we want to establish a mapping between our *intprob* variable and the existing *ilev* variable so that the range of values that *ilev* would assume remains the same. This way, the new interference files would be displayed in the same manner except now the different characters that used to represent different levels of interference now indicates different ranges of interference probability values. To do that, we incorporate the section of codes shown in Appendix 3.

Before we recompile the modified program, there is one more change we need to make. The AAMM program uses the worst case data for the receiver threshold calculation because there is no variance involved. In our statistical version, we need to use the average/mean threshold value. This can be accomplished by rewriting the equation for the variable *th3*:

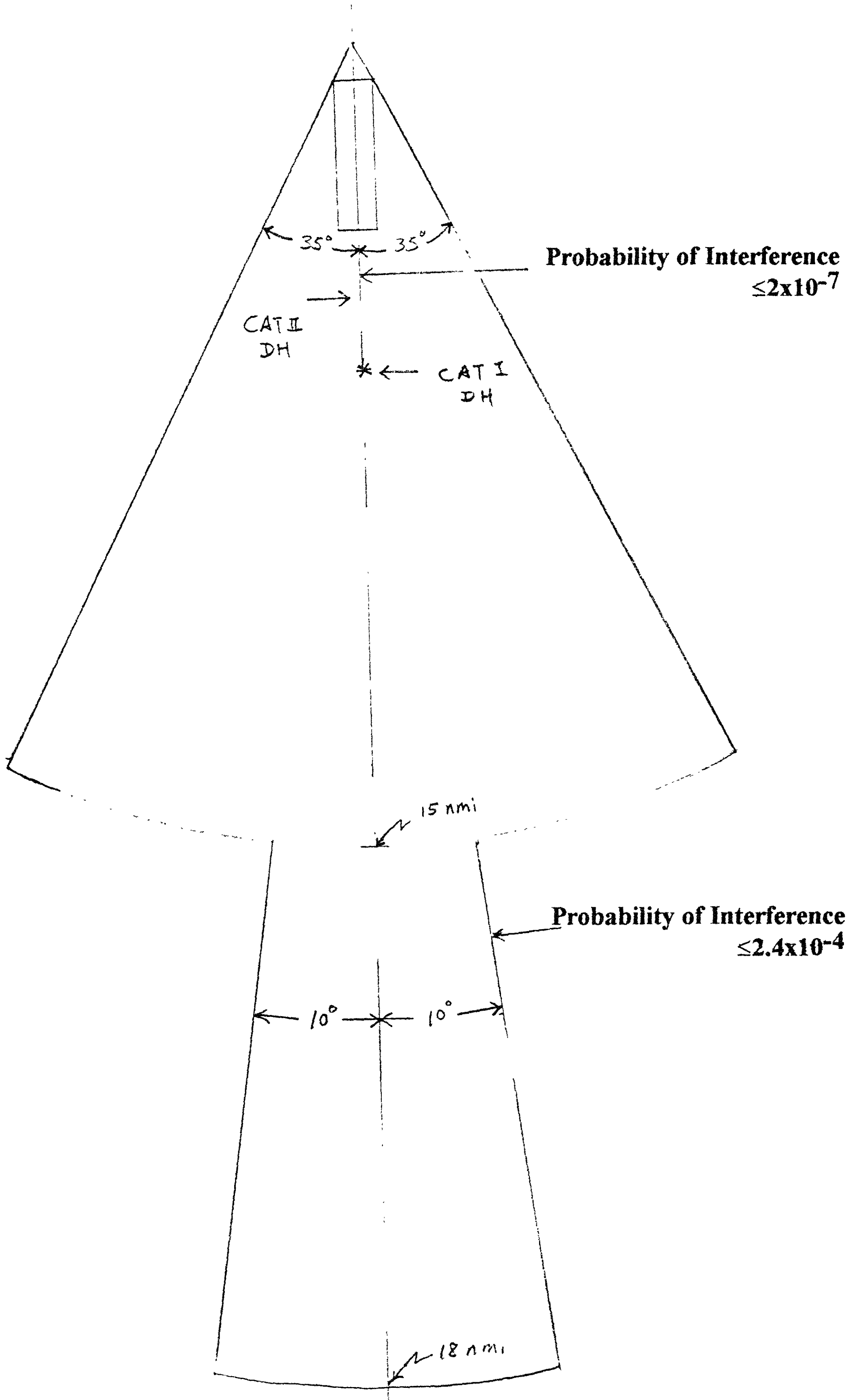
*th3=21*th3t - 99.0*

where the variable *th3t* is the ΔF product mentioned before.

After all changes has been implemented and source codes recompiled and run, the new statistical model yield a series of plots that make up Figure 4. To facilitate the comparison between the two models, the corresponding deterministic model plots were also arranged in the same order to make up Figure 5. Please refer to the bottom of Figure 4 and Figure 5 for relavant information on the proponent navaid and the interfering FM stations.

- **CONCLUSION & ANALYSIS:**

As one can see, whereas the deterministical model plots out interference level in a restricted area in the landing 'arrow,' the statistical model covers more area due to the inclusion of all those points where the intermodulation signal levels are smaller than the receiver threshold levels. The probability of interference in the statistical plots were also approximately represented by different numbers as noted in the legend. The bigger the number is, the more likely the interference will occur at that point. Overall, the two plots follow each other fairly well in terms of their density patterns. Nonetheless, the statistical plots reveal useful insights about interference patterns that were not available through the deterministic analysis.



35°
35°
Probability of Interference
≤2x10-7
CAT II
DH
CAT I
DH
15 nmi
Probability of Interference
≤2.4x10-4
10°
10°
18 nmi

Figure 1

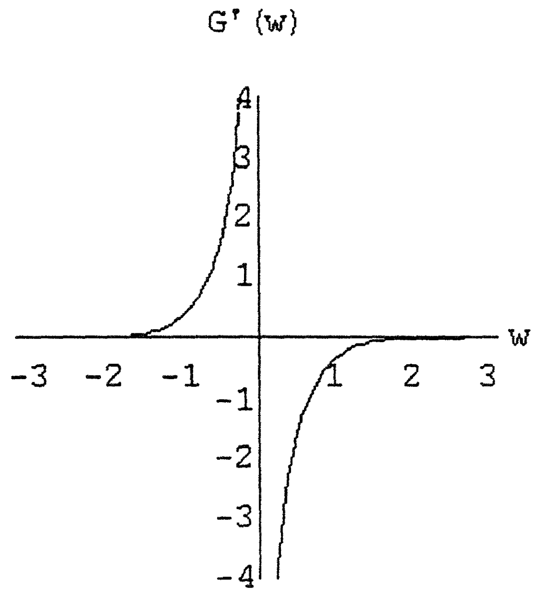


Figure 2

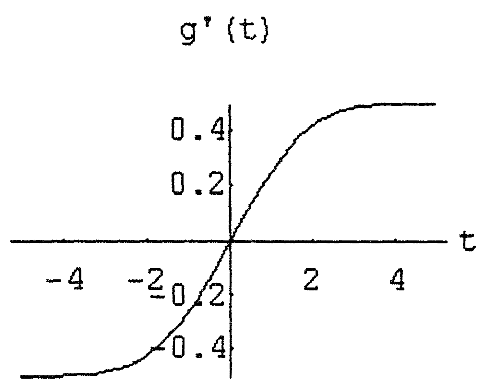
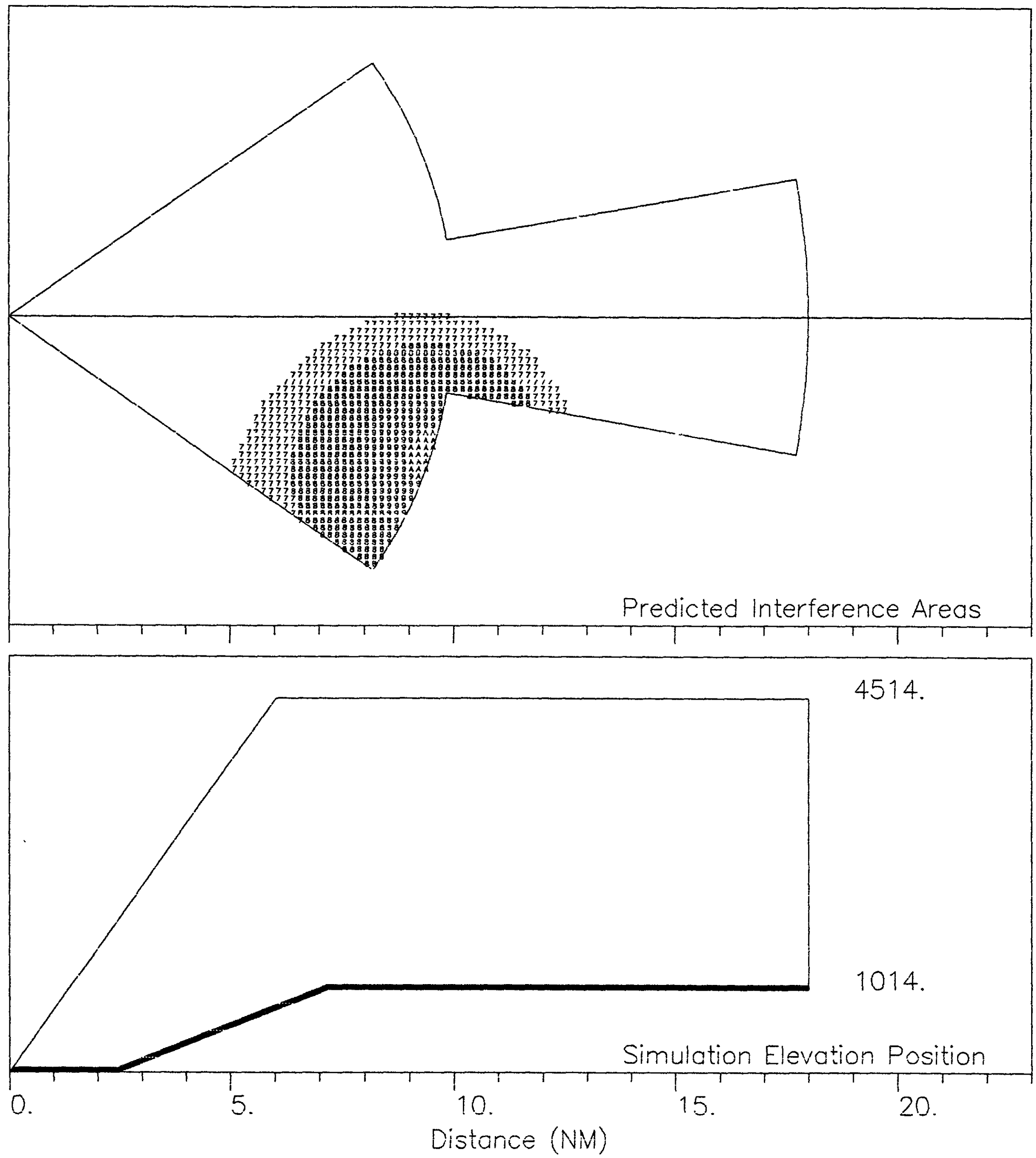
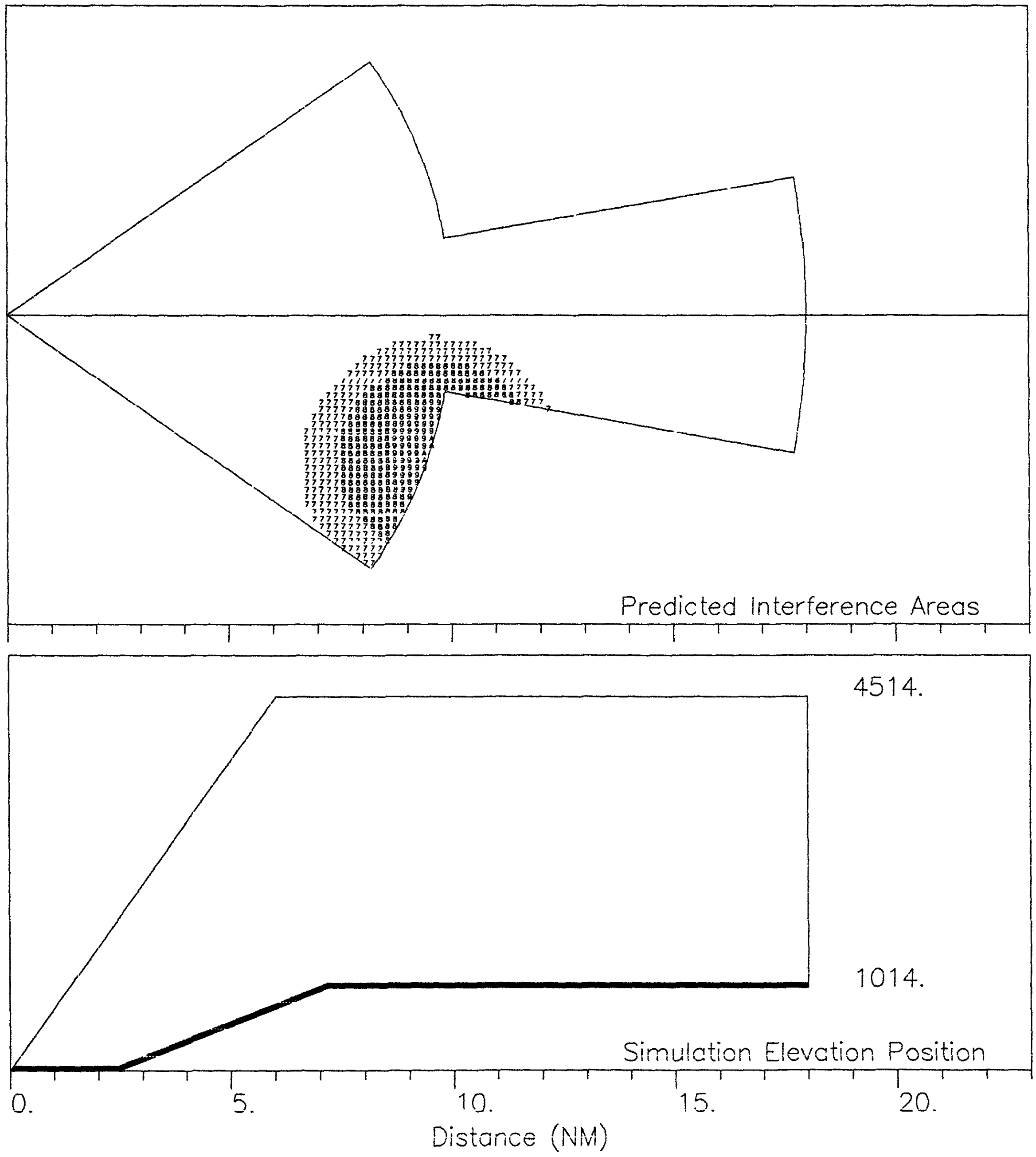


Figure 3



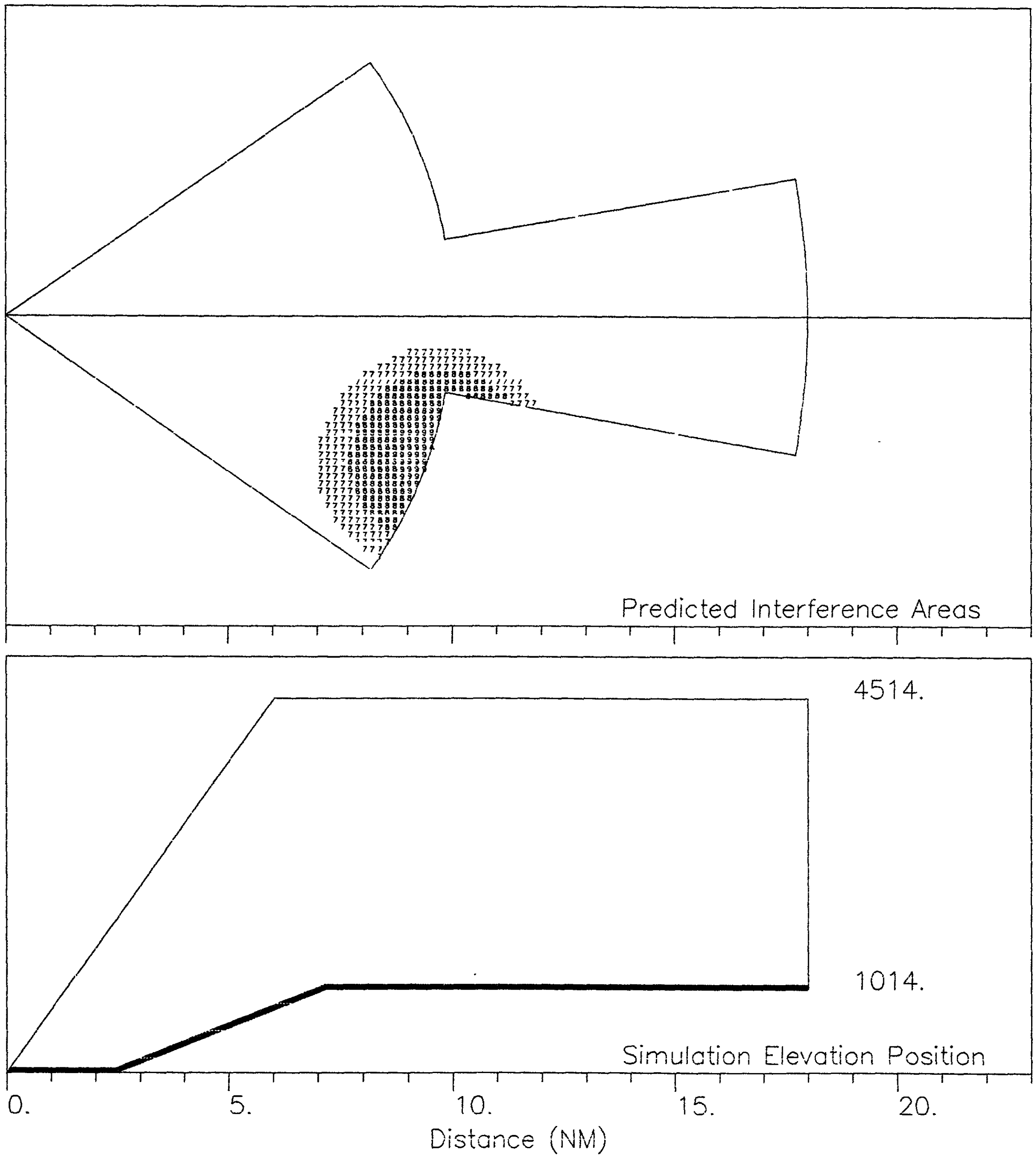
Airspace case #: 2 Site:
Date: Plot filename: 2E_21_1U.plt Service Volume Bottom
Intermod (B1) plot: WKDN (86), WMCK (73), & WEAZ (66)
Frequencies: WKDN = 106.90 MHz WMGK = 102.90 MHz WEAZ = 101.10 MHz
Navaid: MYY Frequency: 108.75 MHz Elevation (Ft. MSL): 14.
Runway heading: 159.0

Figure 4.a



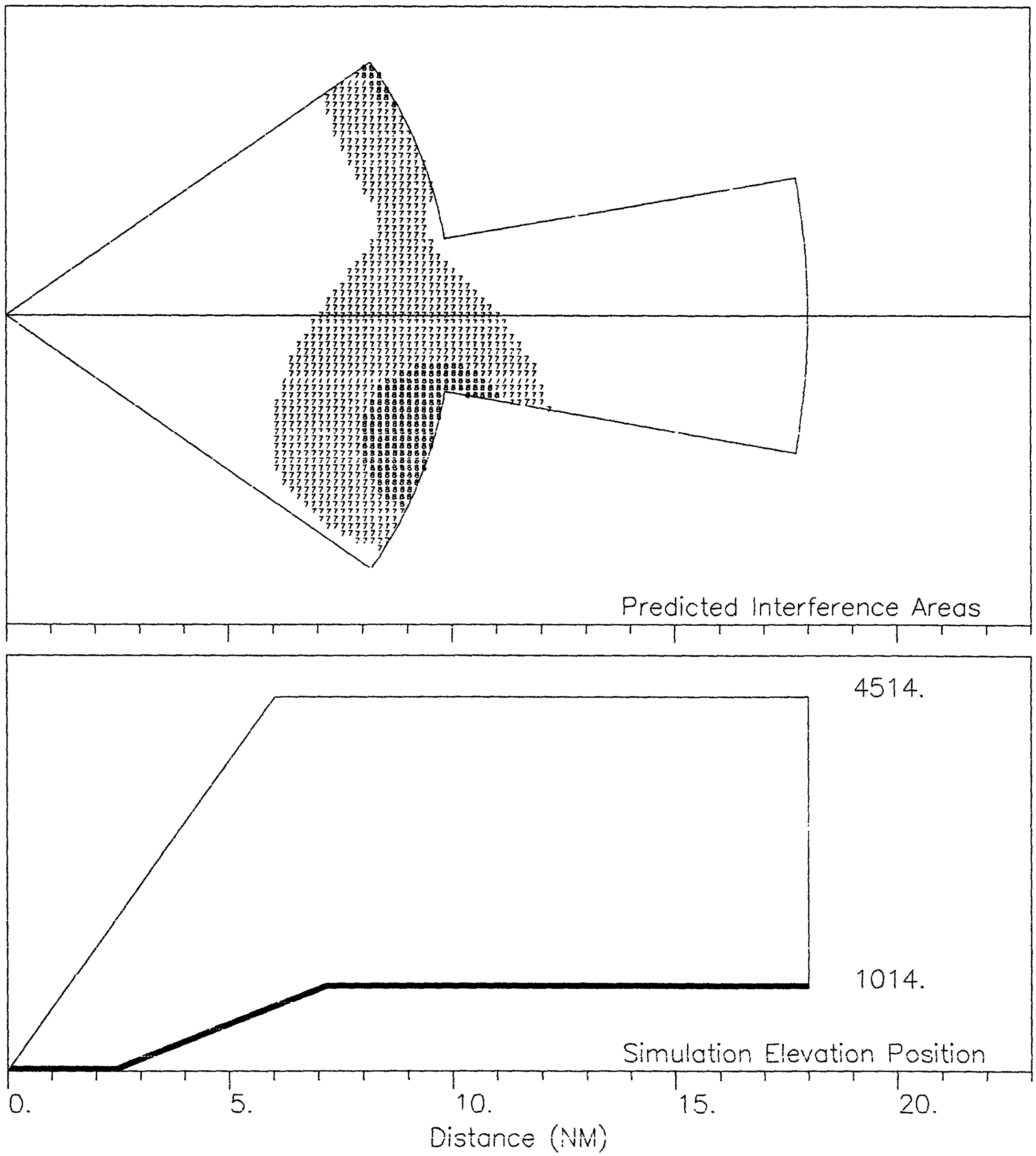
Airspace case #: 2 Site:
Date: Plot filename: 2E_25_1Y.plt Service Volume Bottom
Intermod (B1) plot: WKDN (86), WIBF (77), & NEWx (70)
Frequencies: WKDN = 106.90 MHz WIBF = 103.90 MHz NEWx = 102.10 MHz
Navaid: MYY Frequency: 108.75 MHz Elevation (Ft. MSL): 14.
Runway heading: 159.0

Figure 4.b



Airspace case #: 2 Site:
Date: Plot filename: 2E_25_1Z.plt Service Volume Bottom
Intermod (B1) plot: WKDN (86), WIBF (77), & WIOQ (71)
Frequencies: WKDN = 106.90 MHz WIBF = 103.90 MHz WIOQ = 102.10 MHz
Navaid: MYY Frequency: 108.75 MHz Elevation (Ft. MSL): 14.
Runway heading: 159.0

Figure 4.c



Airspace case #: 2 Site:

Date: Plot filename: 2E_1Y_1Q.plt Service Volume Bottom

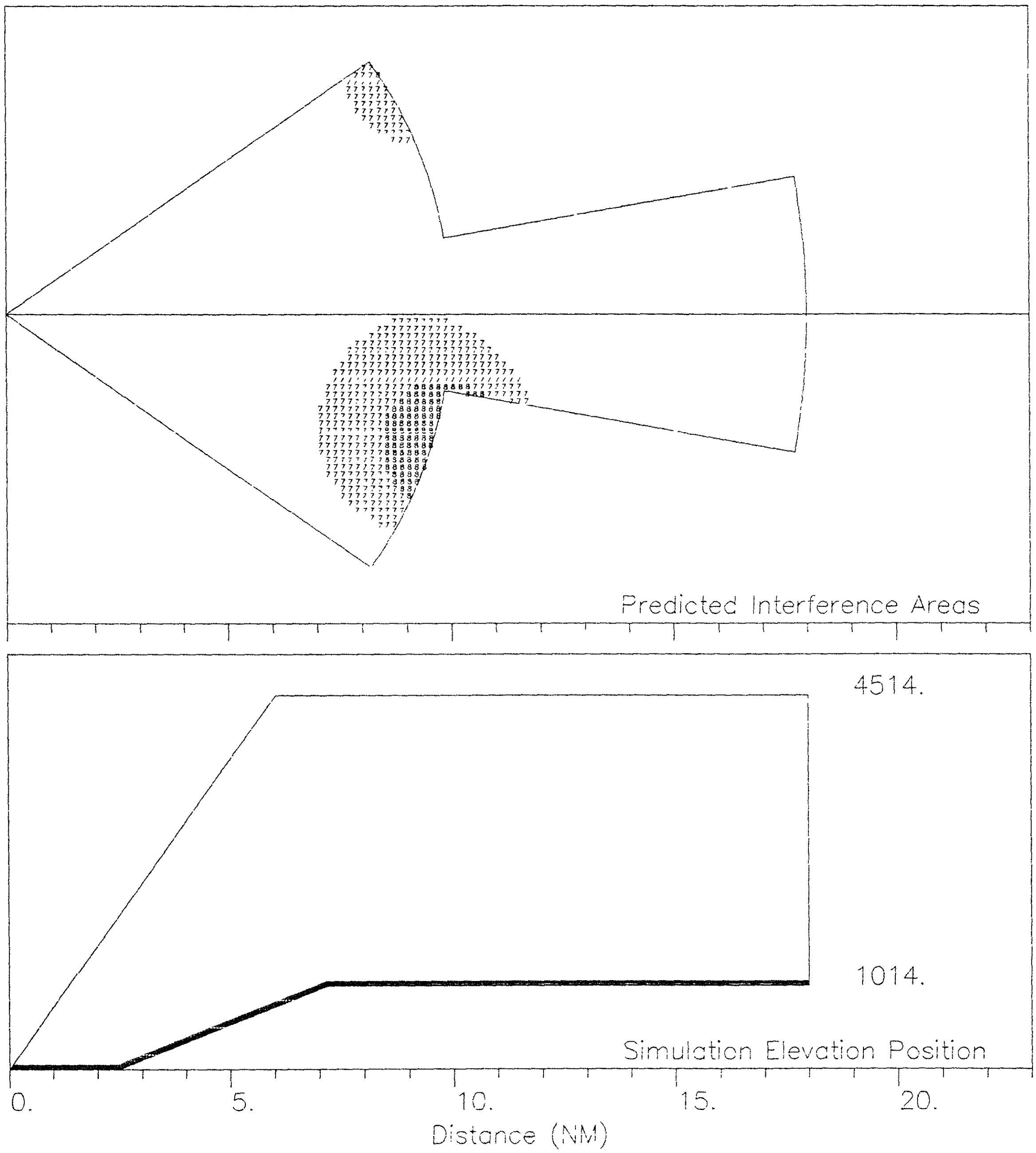
Intermod (B1) plot: WKDN (86), NEWx (70), & WKSZ (62)

Frequencies: WKDN = 106.90 MHz NEWx = 102.10 MHz WKSZ = 100.30 MHz

Navaid: MYY Frequency: 108.75 MHz Elevation (Ft. MSL): 14.

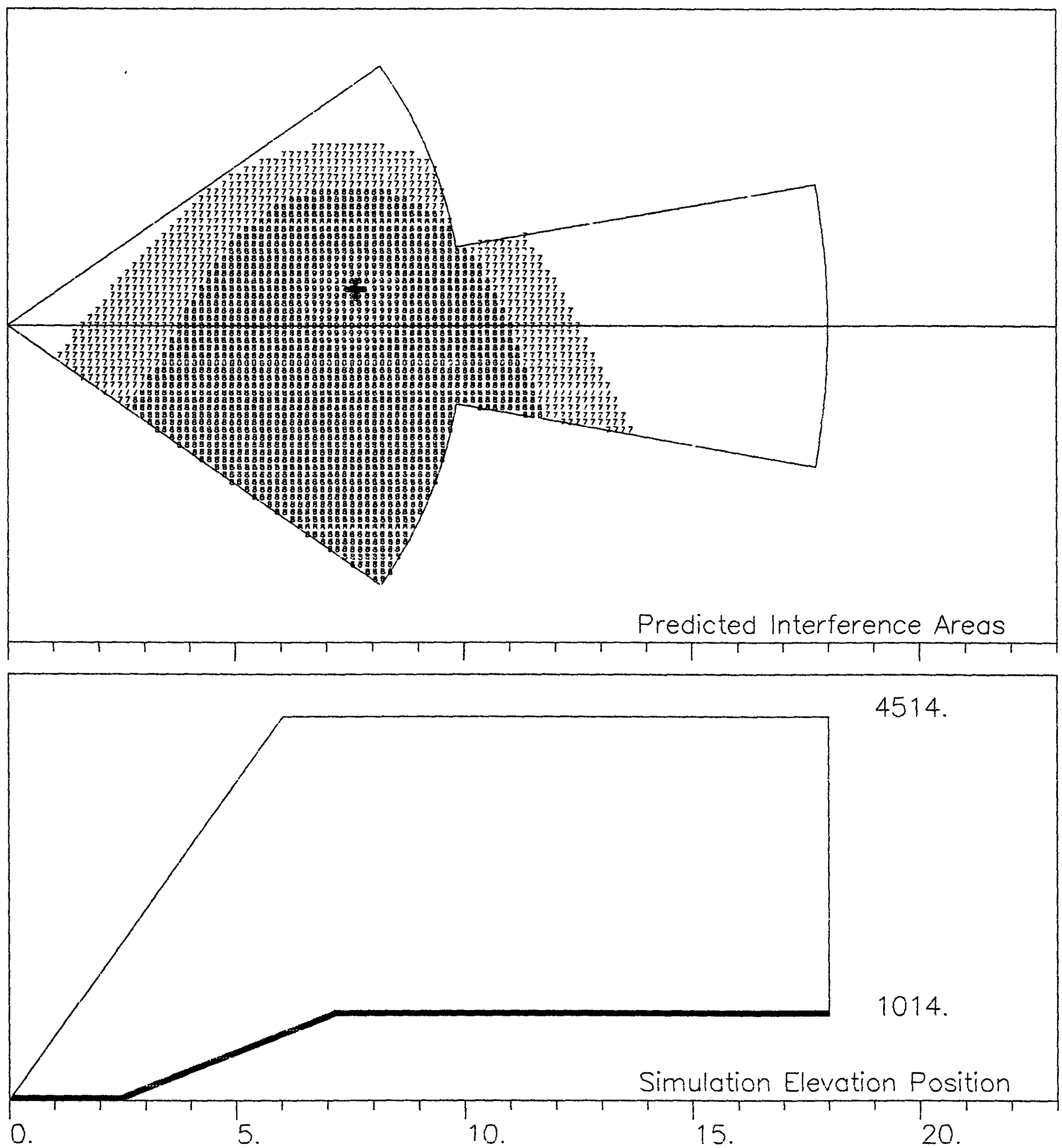
Runway heading: 159.0

Figure 4.d



Airspace case #: 2 Site:
Date: Plot filename: 2E_1Z_1Q.plt Service Volume Bottom
Intermod (B1) plot: WKDN (86), WIOQ (71), & WKSZ (62)
Frequencies: WKDN = 106.90 MHz WIOQ = 102.10 MHz WKSZ = 100.30 MHz
Navaid: MYY Frequency: 108.75 MHz Elevation (Ft. MSL): 14.
Runway heading: 159.0

Figure 4.e



Airspace case #: 2 Site:
Date: Plot filename: 2K_2E_2C.plt Service Volume Bottom
Intermod (B1) plot: WHHS (92), WKDN (86), & WECX (84)
Frequencies: WHHS = 107.90 MHz WKDN = 106.90 MHz WEGX = 106.10 MHz
Navaid: MYY Frequency: 108.75 MHz Elevation (Ft. MSL): 14.
Runway heading: 159.0

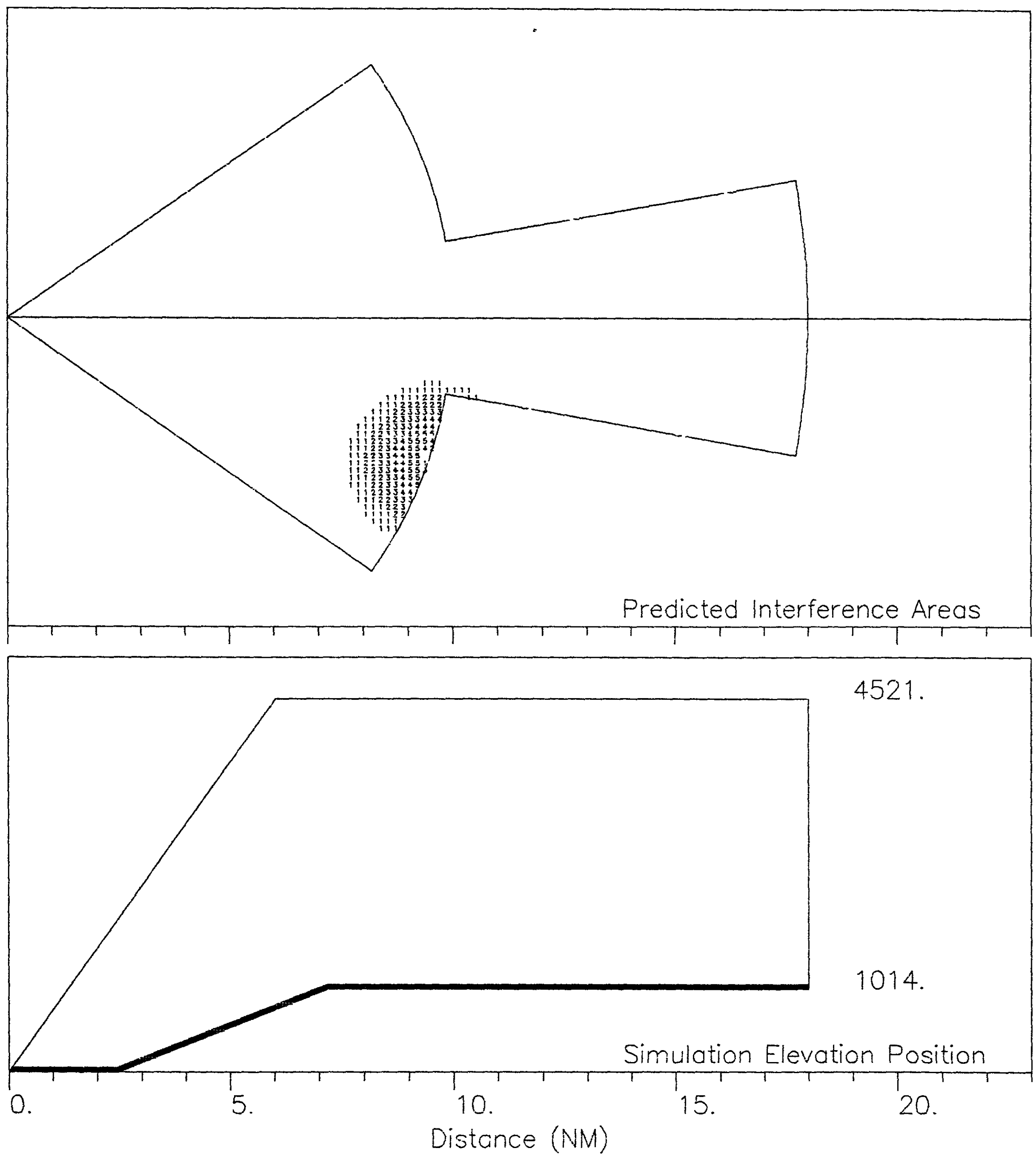
Figure 4.f

Legend for Airspace Analysis Model Plots

111111 111111 111111	$10^{-10} < X \leq 10^{-9}$
222222 222222 222222	$10^{-9} < X \leq 10^{-8}$
333333 333333 333333	$10^{-8} < X \leq 10^{-7}$
444444 444444 444444	$10^{-7} < X \leq 10^{-6}$
555555 555555 555555	$10^{-6} < X \leq 10^{-5}$
666666 666666 666666	$10^{-5} < X \leq 10^{-4}$
777777 777777 777777	$10^{-4} < X \leq 10^{-3}$
888888 888888 888888	$10^{-3} < X \leq 10^{-2}$
999999 999999 999999	$10^{-2} < X \leq 10^{-1}$
AAAA AAAA AAAA	$10^{-1} < X \leq 1$

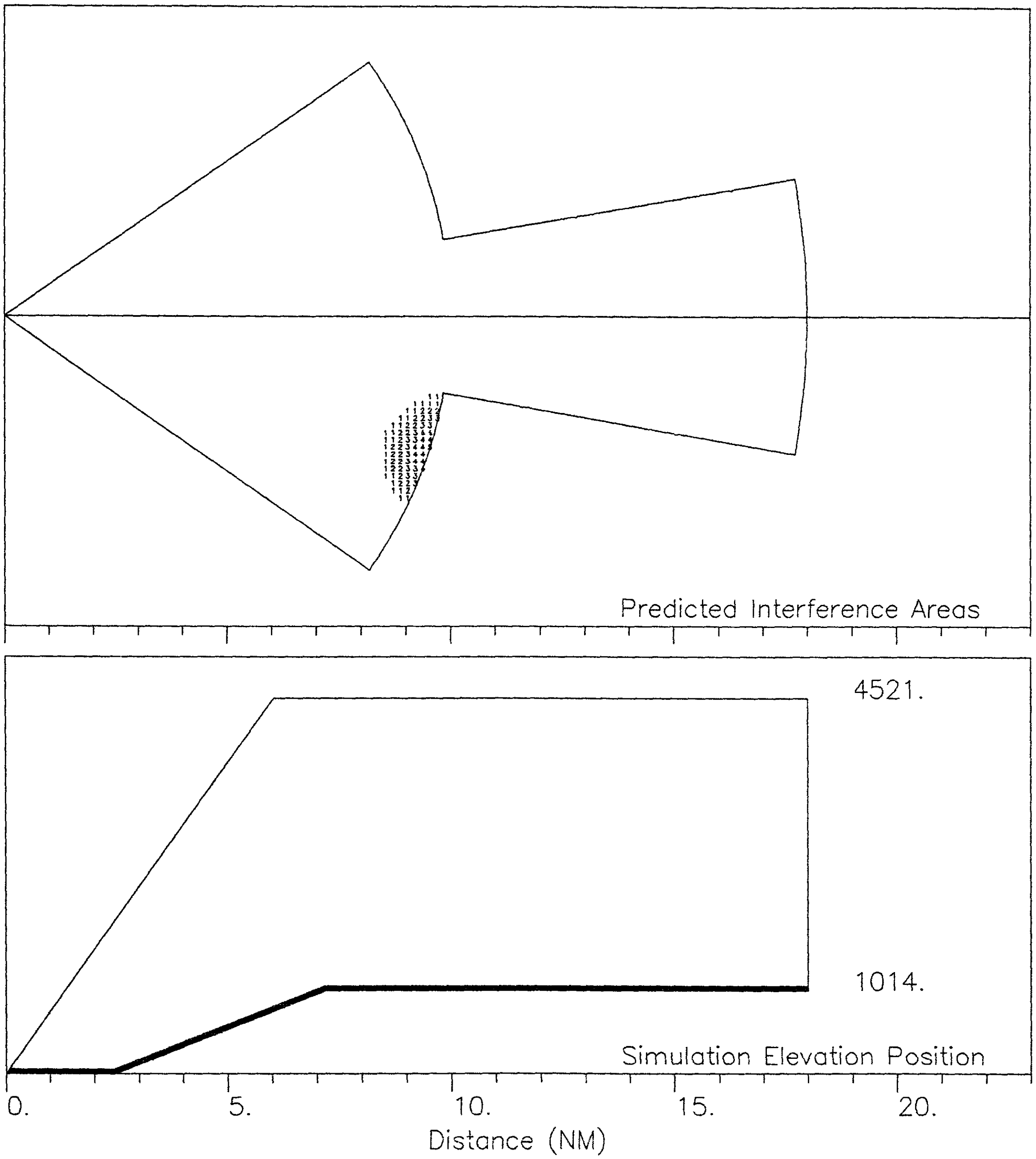
Where X is the Probability of Interference

Figure 4.g



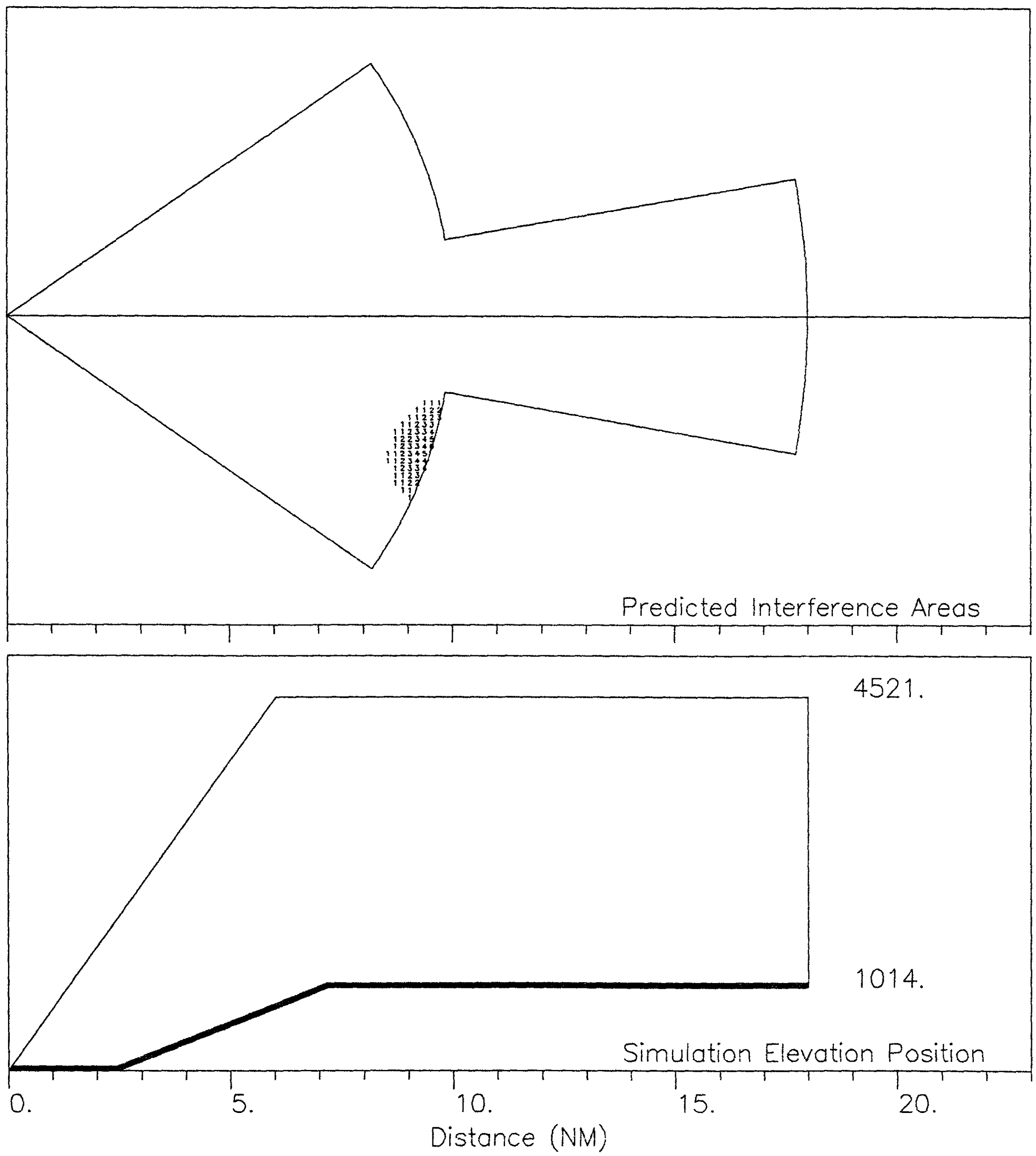
Airspace case #: 2 Site:
Date: Plot filename: 2E_21_1U.plt Service Volume Bottom
Intermod (B1) plot: WKDN (86), WMGK (73), & WEAZ (66)
Frequencies: WKDN = 106.90 MHz WMGK = 102.90 MHz WEAZ = 101.10 MHz
Navaid: MYY Frequency: 108.75 MHz Elevation (Ft. MSL): 14.
Runway heading: 159.0

Figure 5.a



Airspace case #: 2 Site:
Date: Plot filename: 2E_25_1Y.plt Service Volume Bottom
Intermod (B1) plot: WKDN (86), WIBF (77), & NEWx (70)
Frequencies: WKDN = 106.90 MHz WIBF = 103.90 MHz NEWx = 102.10 MHz
Navaid: MYY Frequency: 108.75 MHz Elevation (Ft. MSL): 14.
Runway heading: 159.0

Figure 5.b



Airspace case #: 2 Site:
Date: Plot filename: 2E_25_1Z.plt Service Volume Bottom
Intermod (B1) plot: WKDN (86), WIBF (77), & WIOQ (71)
Frequencies: WKDN = 106.90 MHz WIBF = 103.90 MHz WIOQ = 102.10 MHz
Navaid: MYY Frequency: 108.75 MHz Elevation (Ft. MSL): 14.
Runway heading: 159.0

Figure 5.c

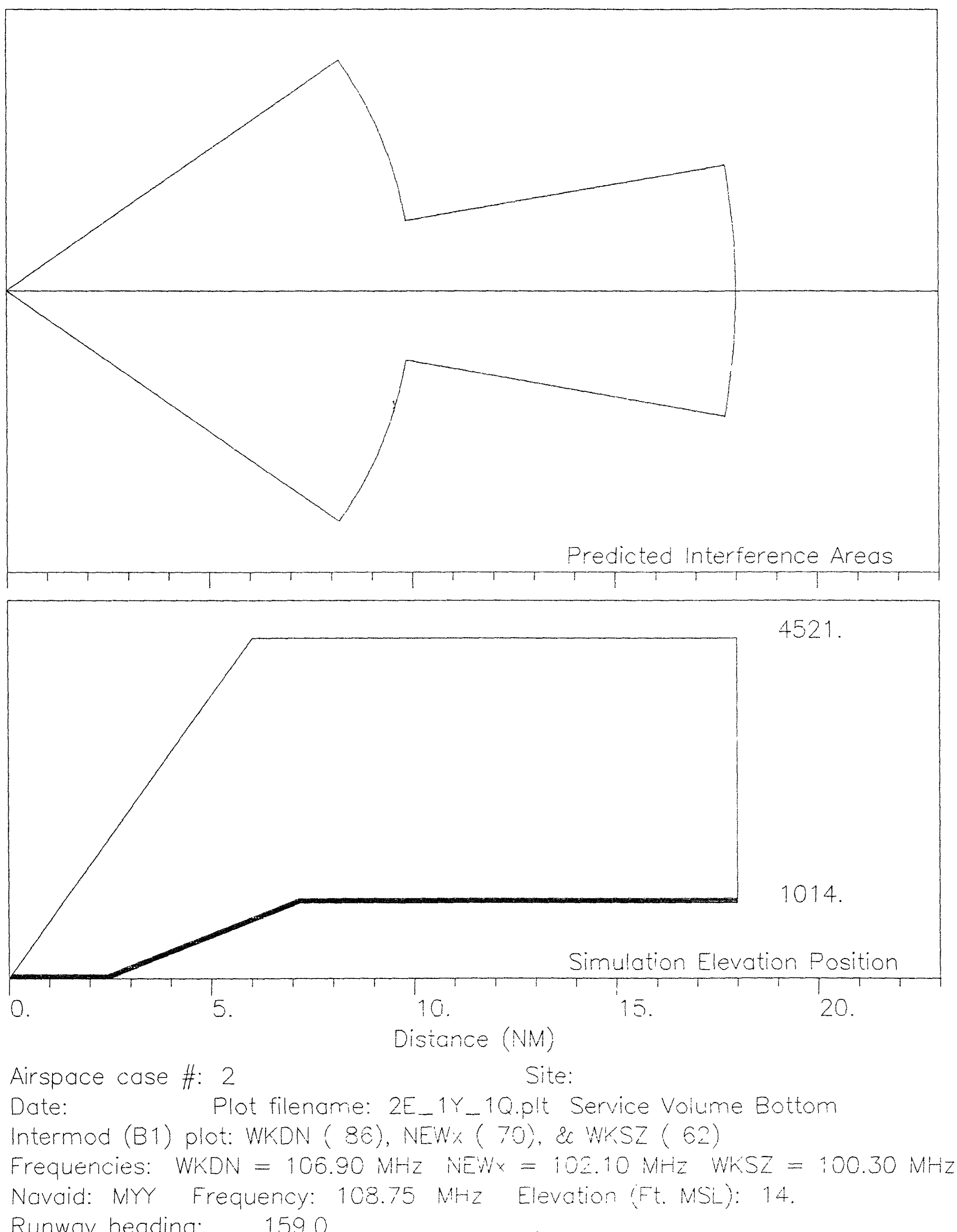
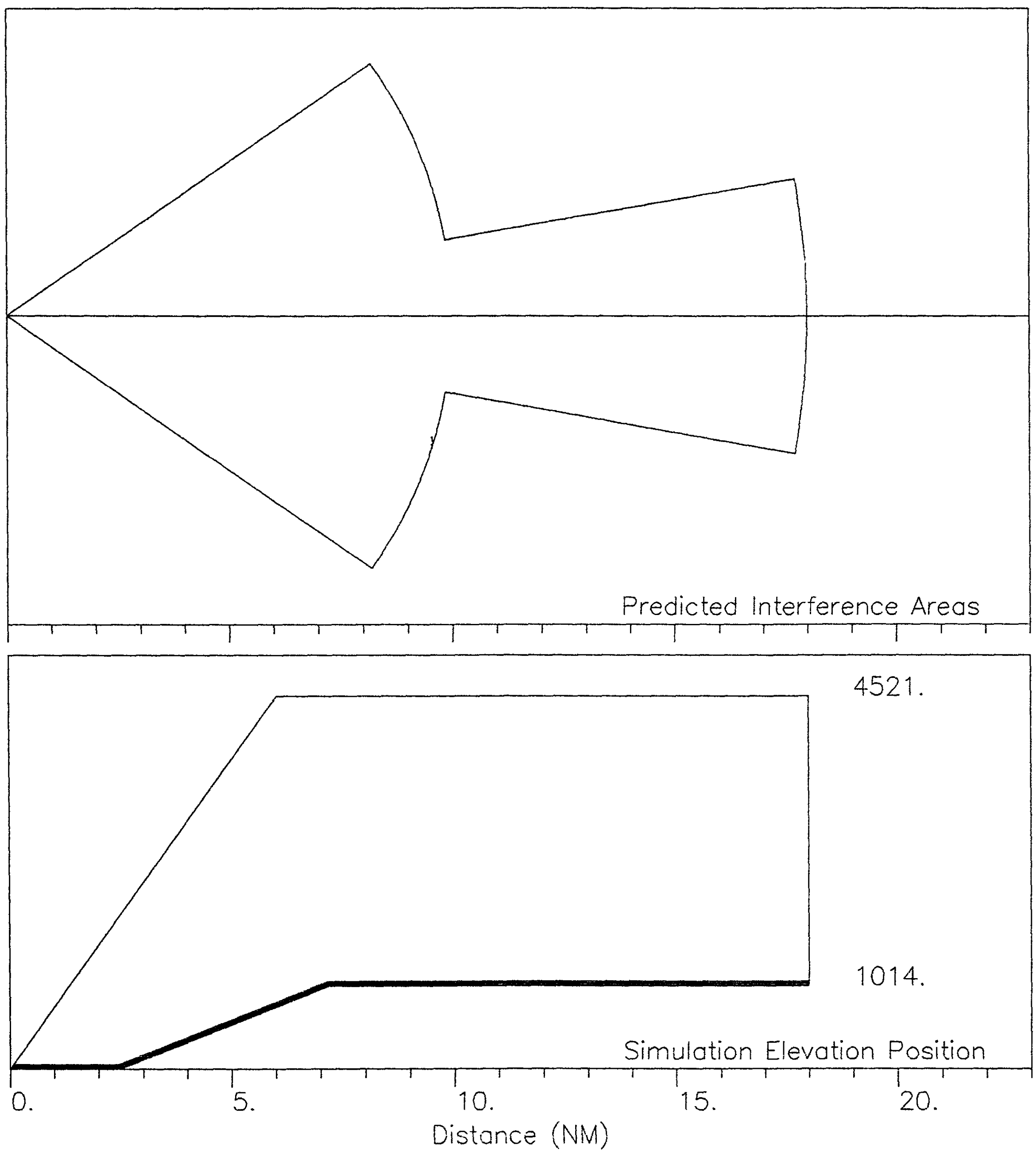
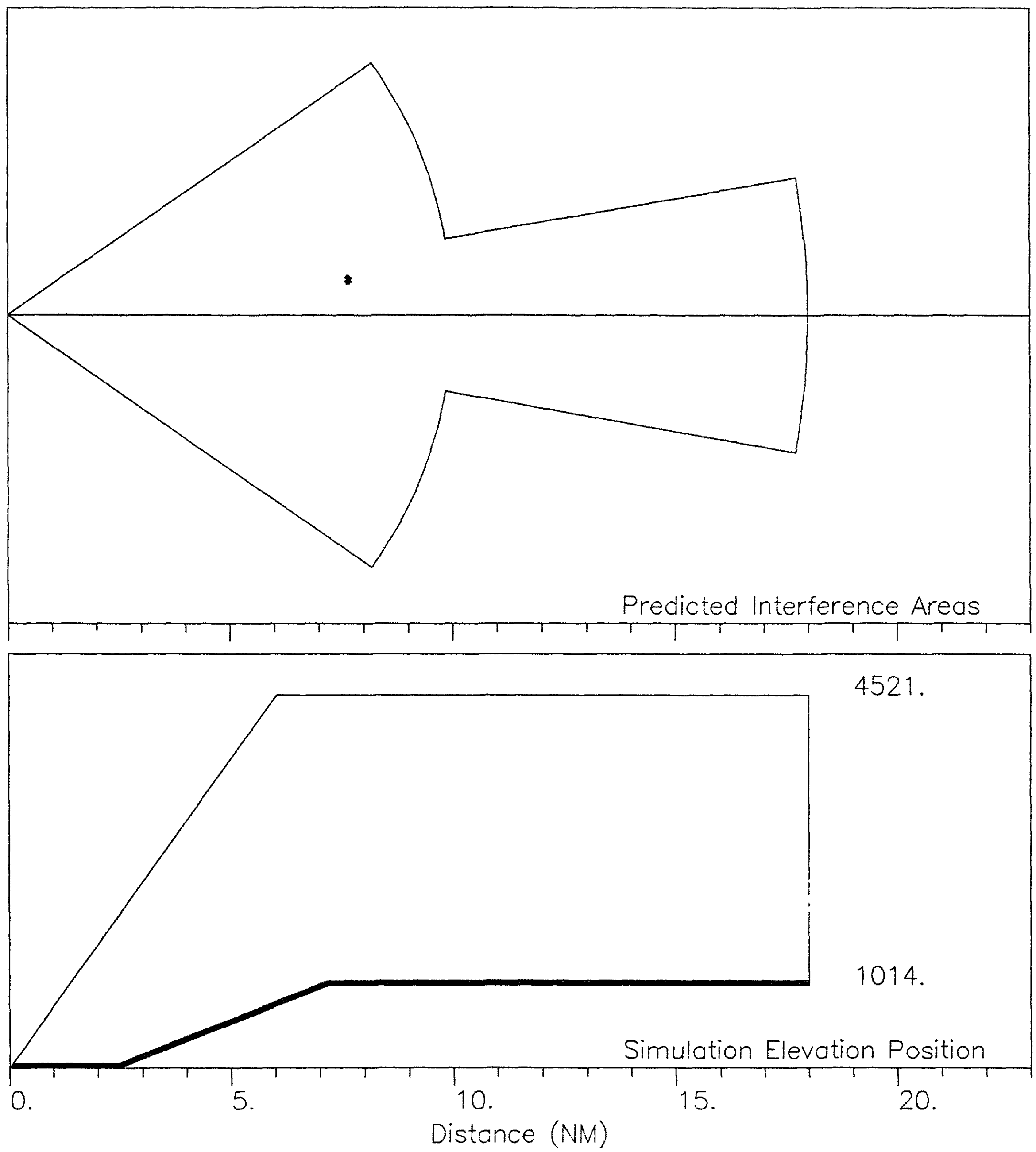

Predicted Interference Areas
4521.
1014.
Simulation Elevation Position
0.
5.
10.
15.
20.
Distance (NM)
Airspace case #: 2
Site:
Date:
Plot filename: 2E_1Y_1Q.plt Service Volume Bottom
Intermod (B1) plot: WKDN (86), NEWx (70), & WKSZ (62)
Frequencies: WKDN = 106.90 MHz NEWx = 102.10 MHz WKSZ = 100.30 MHz
Navaid: MYY Frequency: 108.75 MHz Elevation (Ft. MSL): 14.
Runway heading: 159.0

Figure 5.d



Airspace case #: 2 Site:
Date: Plot filename: 2E_1Z_1Q.plt Service Volume Bottom
Intermod (B1) plot: WKDN (86), WICQ (71), & WKSZ (62)
Frequencies: WKDN = 106.90 MHz WIOQ = 102.10 MHz WKSZ = 100.30 MHz
Navaid: MYY Frequency: 108.75 MHz Elevation (Ft. MSL): 14.
Runway heading: 159.0

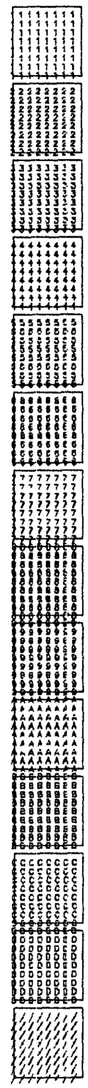
Figure 5.e



Airspace case #: 2 Site:
Date: Plot filename: 2K_2E_2C.plt Service Volume Bottom
Intermod (B1) plot: WHHS (92), WKDN (86), & WEGX (84)
Frequencies: WHHS = 107.90 MHz WKDN = 106.90 MHz WEGX = 106.10 MHz
Navaid: MYY Frequency: 108.75 MHz Elevation (Ft. MSL): 14.
Runway heading: 159.0

Figure 5.f

Legend for Airspace Analysis Model Plots



$.0 < x \leq 3.0$

$3.0 < x \leq 6.0$

$6.0 < x \leq 9.0$

$9.0 < x \leq 12.0$

$12.0 < x \leq 15.0$

$15.0 < x \leq 18.0$

$18.0 < x \leq 21.0$

$21.0 < x \leq 24.0$

$24.0 < x \leq 27.0$

$27.0 < x \leq 30.0$

$30.0 < x \leq 33.0$

$33.0 < x \leq 36.0$

$36.0 < x \leq 39.0$

$39.0 < x$

Where x is the Level of Interference (in dB) above Threshold

Figure 5.g

Table of Parameter Distributions

Dist	*Identifier*	*R.V.*		*Dist Type*	*Mean (dB)*	*Stan. Dev.*
A	*Transmitted Signal*	P_{fm1}	*FM1*	*Gaussian*	μ_{fm1}	σ=4.2 dB
B		P_{fm2}	*FM2*	*Gaussian*	μ_{fm2}	σ=4.2 dB
C		P_{fm3}	*FM3*	*Gaussian*	μ_{fm3}	σ=4.2 dB
D	*Receiving Antenna*	L_{R1}	*FM1*	*Gaussian*	μ_{LR1}	σ=5.0 dB
E	*Factor*	L_{R2}	*FM2*	*Gaussian*	μ_{LR2}	σ=5.0 dB
F		L_{R3}	*FM3*	*Gaussian*	μ_{LR3}	σ=5.0 dB
G	*Antenna Directivity*	D_1	*FM1*	*Uniform*	0	±3 dB
H		D_2	*FM2*	*Uniform*	0	±3 dB
I		D_3	*FM3*	*Uniform*	0	±3 dB
J	*Interference Thresh.*	T		*Gaussian*	*TH*	σ=6.58 dB

Table 1

Fortran Subroutine Calculating The Cumulative Distribution

```
c Fortran module for calculation of the cumulative probability
c distribution of the final composite distribution.

c*************************************************************
      real*8 w,dw,y,x(101),sum
      real t,wrange,sigdiff
      integer indx
      nu=12
      n=2**nu
      pi=3.14159265358979
c**************************************************************
      do 10 t=0.05,5.0,0.05
      sum=0.0
      wrange=15.00
      dw=wrange/n
      do 5 i=1,n
            w=i*dw
            y=(exp(-w**2/2)*(sin(w)/w)**3/w)-((0.25/(0.25+w**2))/w)
5        sum=sum+2*sin(t*w)*y*dw
      indx=int(t/0.05+0.5)
10    x(indx)=sum/(2*pi)+0.5*(1-exp(-0.5*t))
c********************************************************
```

Appendix 1

Fortran Codes Determining The Probability of Interference

```
c Fortran codes performing the interpolation to find the
c probability of interference associated with a particular
c 'sigdiff' value

c****************************************************
      sigdiff = sig3-th3
      preindx=abs(sigdiff)*sqrt(2)*20/13.42
      indx=int(preindx+0.5)
      if (indx .ge. 100) then
            indx = 100
      end if
      intprob=x(indx-1) + (x(indx) - x(indx-1)) *
     &           (mod(int(preindx*100+0.5),100)/100.0)
      if (sigdiff .eq. 0.0) then
            intprob=0.5
      else if (sigdiff .ge. 0.0) then
            intprob=0.5+intprob
      else
            intprob=0.5-intprob
      endif
c****************************************************
```

Appendix 2

Fortran Codes Modifying B1_3FREQ.for

```
c  Fortran module that translates values of probability
c  of interference into proper display scale value, 'ilev.'

c*******************************************
       ilev =  0
       if (intprob .ge. 0.9) then
              ilev = 10
       else if (intprob .ge. 0.8) then
              ilev = 9
       else if (intprob .ge. 0.7) then
              ilev = 8
       else if (intprob .ge. 0.6) then
              ilev = 7
       else if (intprob .ge. 0.5) then
              ilev = 6
       else if (intprob .ge. 0.4) then
              ilev = 5
       else if (intprob .ge. 0.3) then
              ilev = 4
       else if (intprob .ge. 0.2) then
              ilev = 3
       else if (intprob .ge. 0.1) then
              ilev = 2
       else if (intprob .ge. 0.0) then
              ilev = 1
       end if
c*******************************************
```

Appendix 3

Bibliography

1. R. J. Kelly and M. G. Biggs, "FM Interference to the Instrument Landing System--Statistical Model Consideration," Private Memorandum, Bendix Communication division, December 17, 1991.

2. Federal Aviation Administration, "User's Manual and Technical Reference for the Airspace Analysis Mathematical Model, Version 4.1," January, 1992.