A Meshfree Weak- Strong-form (MWS) method for solid and fluid mechanics

G. R. LIUa,b and Y. T. Gua

(1) Dept. of Mechanical Engineering, National University of Singapore, Singapore 119260
(2) Singapore-MIT Alliance, 4 Engineering Drive 3, National University of Singapore, Singapore-117576

***Abstract*— Mesh free methods can be largely categorized into two main categories: mesh free methods based on strong forms (e.g. collocation methods) and mesh free methods based on the weak forms (EFG, MLPG, PIM, etc.; see Mesh Free Methods, by G. R. Liu, CRC Press, 2002). The mesh free collocation method is simple to implement and computationally efficient. However, it is often found unstable and less accurate, especially for problems governed by partial differential equations with Neumann (derivative) boundary conditions, such as solid mechanics problems with stress (natural) boundary conditions. On the other hand, the mesh free methods based on the weak form exhibits very good stability and excellent accuracy. However, the numerical integration makes them computational expensive, and the background mesh (global or local) for integration is responsible for not being "truly" mesh free. In this paper, a new idea of combination of both the strong form and the local weak form is proposed to develop truly meshless method for 2-D elasto-statics.**

A novel truly meshfree method, the meshfree weak-strong (MWS) form method, is originated by Liu et al. (2002) based on a combined formulation of both the strong and local weak forms. As shown in Figure 1, the problem domain and boundaries are represented by properly scattered nodes. The key idea of the MWS method is that in establishing the discrete system equations, both the strong-form and the local Petrov-Galerkin weak-form are used for the same problem, but for different nodes.

This paper details the MWS method for solid and fluid mechanics problems. In the MWS method, the problem domain and its boundary is represented by a set of points or nodes. The strong form or collocation method is used for all the internal nodes and the nodes on the essential (Dirichlet) boundaries. The local weak form (Petrov-Galerkin weak form) is used for nodes on the natural (Neumann) boundaries. There is no need for numerical integrations for all the internal nodes and the nodes on the essential boundaries. The local numerical integration is performed only for the nodes on the natural/Neumann boundaries. The natural/Neumann boundary conditions can then be easily imposed to produce stable and accurate solutions. The locally supported radial point interpolation method (RPIM) and moving least squares (MLS) approximation are used to construct the shape functions. The final system matrix will be sparse and banded for computational efficiency.

Numerical examples of solids and fluids are presented to demonstrate the efficiency, stability and accuracy of the proposed meshfree method.

[Full Text Not Available]