

Performance Analysis of Metamaterials With Two-dimensional Isotropy

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Abstract—A two-dimensional isotropic metamaterials formed by crossed split-ring resonators (CSRRs) are studied in this paper. The effective characteristic parameters of this media are determined by quasi-static Lorentz theory. The induced current distributions of a single CSRR at the resonant frequency are presented. Moreover, the dependence of the resonant frequency on the dimensions of single CSRR and the spaces of the array are also discussed.

Index Terms—crossed split-ring resonators (CSRRs), effective characteristic parameters, current distribution, resonant frequency.

I. INTRODUCTION

IN 1968, V. G. Veselago theoretically demonstrated the properties of substances with negative permittivity ϵ and permeability μ simultaneously [1]. He named it as left-handed material (LHM) since the electric field \mathbf{E} , magnetic field \mathbf{H} and the wave vector \mathbf{k} obey a left-handed law in the media. The LHM possesses some surprising properties, such as negative refractive index, backward-wave propagation. But this kind of media does not exist in the nature especially the negative μ . After over thirty years, the research works to a composite material, which has negative ϵ and μ , done by John Pendry and his colleagues [2, 3] spur a new investigative upsurge. In succession, David Smith experimentally realized a LHM for the first time [4]. In his experiment, the composite media consists of two-dimensional periodic arrays of copper split-ring resonators (SRRs) and wires. The array of SRRs is built up for negative magnetic permeability. They called this artificial media metamaterials to show their supernatural properties beyond a classical media.

Thus far, the inclusions of metamaterials having the magnetic response are circle [3], square [5] SRRs, and their modified shapes such as cross SRRs [6]. Others are distributed L-C network [7] or embedded-circuit transmission line model [8]. Most of inclusions are highly anisotropic. They must be arranged by some way [3] in

order to obtain an isotropic magnetic media. Whereas the three-dimensional magnetic resonator structure CSRR presented by Martin are highly 2-D isotropy. In his paper, several different configurations of CSRRs have been studied to observe if they are isotropic by using the scattering cross section (SCS) [9]. In this paper, we mainly study a CSRR, which is shown in Fig. 1, and analyze in detail its more properties from the media characteristic parameters, induced current distributions and resonance.

II. STRUCTURE MODEL

The inclusion that we consider is depicted in Fig. 1. It is composed of two perpendicularly intersecting SRRs. Each SRR is made of two aluminum strips [6]. The structure of CSRR is defined by following parameters: the gap width $g = 1mm$, the strip width $w = 0.5mm$, the inner radius $r_{in} = 2mm$, and the outer radius $r_{out} = 3mm$. The spaces of array composed by CSRRs, denoted by d_x , d_y and d_z , are same and set as $8mm$. In the numerical simulation, we omit the thickness of the aluminum strip and consider it as perfecting conductor.

In order to obtain a magnetic response, we consider the polarization of incident electric field is in parallel with the axis of CSRR, that is z axis. The propagation direction and the polarization of magnetic field are in the xoy plane. In this paper, the electric field integral equation (EFIE) and method of moment (MoM) with roof-top basis function and line matching are used in the numerical simulation. Then the induced currents densities on the surface of CSRR are obtained. Utilizing the currents, the scattered electromagnetic field at any point in the propagation space and the scattering cross section (SCS) can be evaluated.

III. THE PERFORMANCES OF SINGLE CSRR AND ARRAY

A. Effective permittivity and permeability

In generally, a bianisotropic media can be expressed by a

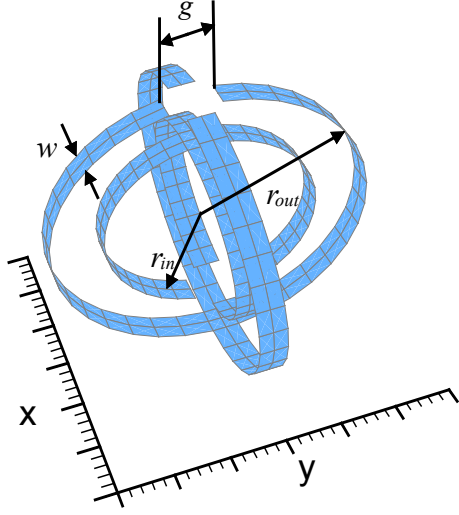


Fig. 1. The structure of the single CSRR.

matrix of 36 components, that is $\begin{bmatrix} \bar{\boldsymbol{\epsilon}} & \bar{\boldsymbol{\xi}} \\ \bar{\boldsymbol{\zeta}} & \bar{\boldsymbol{\mu}} \end{bmatrix}$, where $\bar{\boldsymbol{\epsilon}}$ and $\bar{\boldsymbol{\mu}}$ represent the permittivity and permeability tensors, while the magnetoelectric crosscoupling tensors are denoted by $\bar{\boldsymbol{\xi}}$ and $\bar{\boldsymbol{\zeta}}$, respectively. Using the quasi-static Lorentz theory and numerical simulation to single CSRR, all the characteristic parameters of a three-dimensional array, which is built up by CSRRs, can be evaluated [10]. In the rectangular coordinates, the effective permittivity components of $\epsilon_{xx} = \epsilon_{xxr} - i\epsilon_{xxi}$, $\epsilon_{yy} = \epsilon_{yyr} - i\epsilon_{yyi}$, and $\epsilon_{zz} = \epsilon_{zzr} - i\epsilon_{zzi}$ are shown in Fig. 2. And the effective permeability components of $\mu_{xx} = \mu_{xxr} - i\mu_{xxi}$, $\mu_{yy} = \mu_{yyr} - i\mu_{yyi}$, and $\mu_{zz} = \mu_{zzr} - i\mu_{zzi}$ are shown in Fig. 3. The other components are negligibly small according to the numerical results. In these figure, the legends of xx, r and xx, i denote the real and image part of the component ϵ_{xx} or μ_{xx} , respectively. The similar to yy, r , yy, i , zz, r and zz, i .

The real parts of the permittivity and permeability components ϵ_{xx} , ϵ_{yy} , μ_{xx} and μ_{yy} become negative simultaneously about from 6.9GHz ~ 7.2GHz. ϵ_{zz} and μ_{zz} are nearly constant with frequency variety. Especially, we can observe that the value of ϵ_{xx} accords with that of ϵ_{yy} , and the same case to μ_{xx} and μ_{yy} . This is enough to testify its 2-D isotropy in the xoy plane. The slight discrepancy is due to a little coarse step of frequency.

B. Currents distribution of single CSRR and array

In this section, we use the SCS to seek the resonant frequency point first. For the dimension of Fig. 1, the SCS

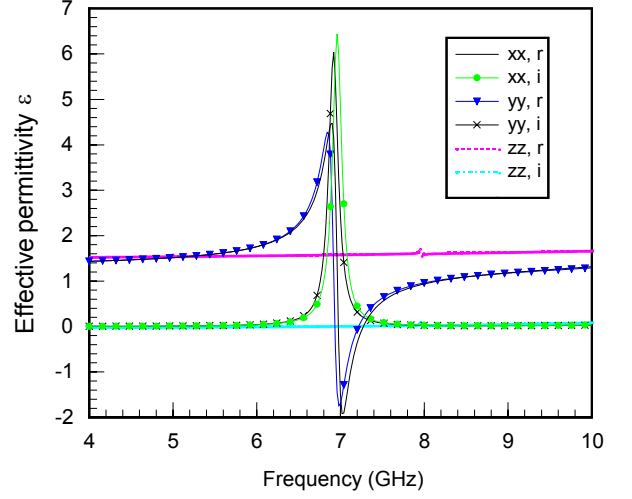


Fig. 2. Variation of ϵ_{xx} , ϵ_{yy} , and ϵ_{zz} of three-dimensional

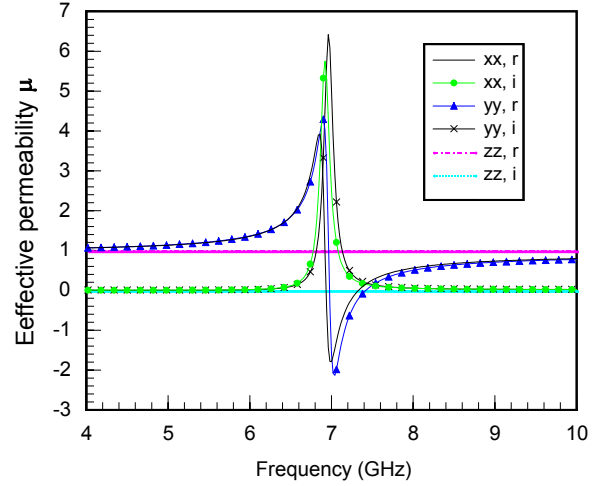


Fig. 3. Variation of μ_{xx} , μ_{yy} , and μ_{zz} of three-dimensional.

results are shown in Fig. 4. The resonant frequency is around 7.3GHz. To observe the resonance of single CSRR, we present induced current distributions at the resonant frequency. The current distributions of a single inclusion for different incident angle are shown in Fig. 5. When $\phi = 0^\circ$ as Fig. 5a, magnetic fields \mathbf{H} completely penetrate through the split rings in yoz plane, whereas none of \mathbf{H} to those in the xoz plane. The induced current densities on their surface are much stronger than those in the xoz plane. The currents vanish at the proximity of the gaps. The strength of the magnetic coupling only depends on the magnetic flux in the SRRs, and therefore we can observe the current densities on SRRs in xoz plane are very weaker. For $\phi = 90^\circ$ as Fig. 5c, the situation is just opposite to that of $\phi = 0^\circ$. For $\phi = 45^\circ$ as Fig. 5b, we

just choose this angle because we can decompose \mathbf{H} into two components, H_x and H_y , in the xoy plane, with the same amplitudes. This can make us better understand some phenomena. It is well known that these two magnetic field components penetrate through the split rings in yoZ and xoz plane, respectively. In consequence, the same current distributions are generated on them. Obviously, total of magnetic fields can completely penetrate through the crossed split-ring resonator, no matter what incident angle. It confirms that CSRR is isotropic in the xoy plane once more. In these figures, the current densities at the middle of the opposite side to the gap, which is named by neutral point in [6], are the maximum at the resonant frequency for any incident angle. The charges with opposite sign accumulate on both sides of gaps respectively and strong electrical fields are produced.

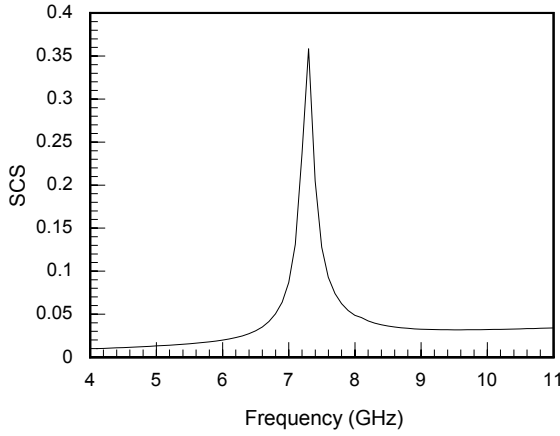
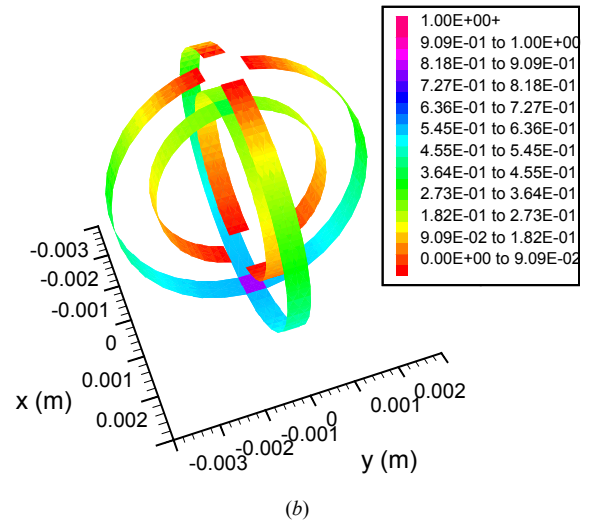


Fig. 4. Scattering cross section (SCS) versus frequency for a single CSRR.

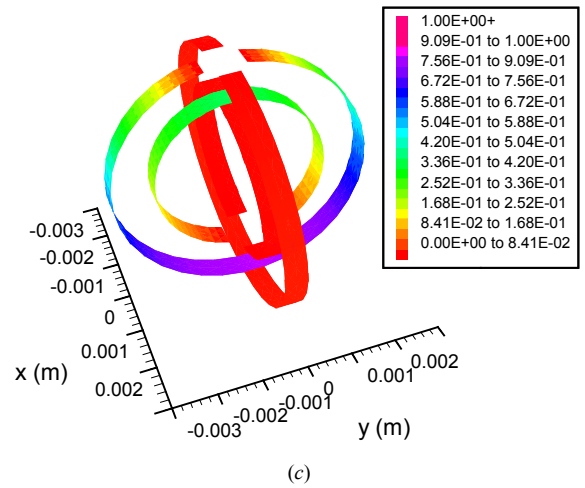
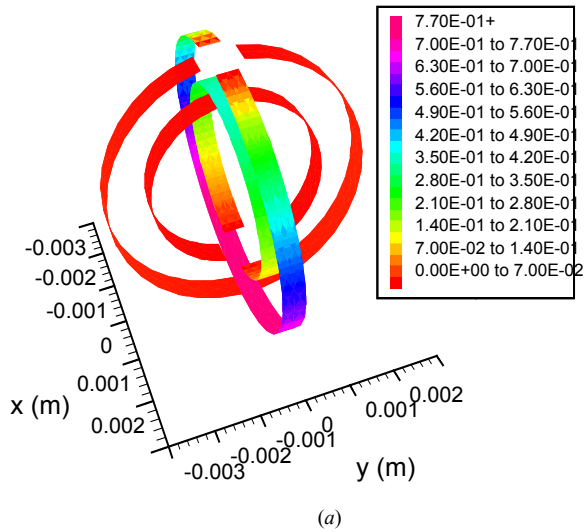


Fig. 5. The induced current density distributions under different incident angles, (a) $\phi = 0^\circ$, (b) $\phi = 45^\circ$, (c) $\phi = 90^\circ$.



C. Resonant frequency

In order to know the influence of structure dimensions of CSRR on resonant frequency, we first change the inner radius r_{in} to obtain the different spaces t between the inner rings and the outer rings. The resonant frequencies versus different spaces t are shown in Fig. 6. From this figure, we can find the resonant frequency increases fast when t becomes larger.

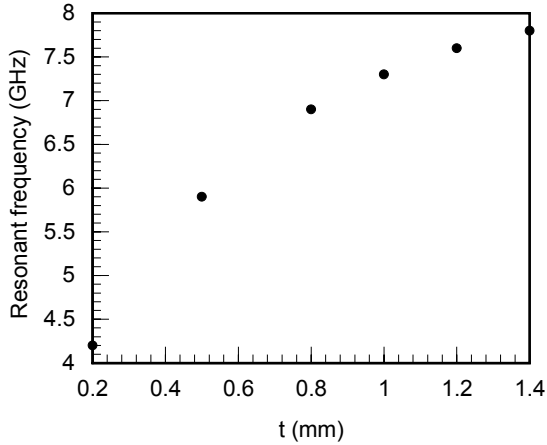


Fig. 6. The resonant frequency versus the space t .

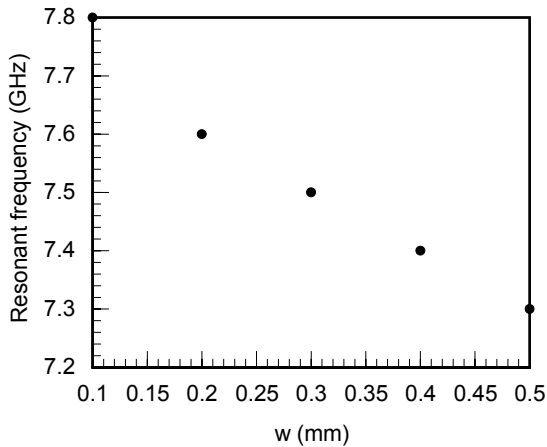


Fig. 7. The resonant frequency versus the strip width w .

The other dimensions, such as the gap width g , the strip width w are also analysed. We find that the change of g affects the resonant frequency slightly. And the narrowing of strip width w can make the resonant frequency increase slowly. We can observe the phenomenon from Fig. 7. These results are consistent with the formulas presented by J. B. Pendry [3, referred to (42) and (45)]. The capacitance of the gaps little affects the total capacitance and has been omitted in that paper. The increase of t or decrease of w makes the capacitance between the two elements of the split ring decreased and the resonant frequency is enhanced.

Finally, the spaces of array denoted by dx , dy and d_z are considered. We change the number of CSRR in \hat{x} , \hat{y} , \hat{z} direction from 1 to 3, respectively. The spaces are changed from 8mm to 40mm. We find that the resonant frequency is slightly changed, yet the SCS is grown more and more stronger and its peak becomes very sharper with the increasing of the spaces.

IV. CONCLUSION

An isotropic magnetic resonator is studied in this paper. We use the electrical field integral equation to calculate the induced currents on the crossed split-ring resonators. For a infinite three-dimensional array of CSRRs, the effective permittivity and permeability components are obtained by utilizing the quasi-static Lorentz theory. We can observe the frequency range in which there are real parts of four components, ϵ_{xx} , ϵ_{yy} , μ_{xx} and μ_{yy} , to appear negative value simultaneously. And besides, $\epsilon_{xx} = \epsilon_{yy}$ and $\mu_{xx} = \mu_{yy}$ show the two-dimensional isotropy of this artificial media. The induced current densities of a single CSRR under different incident angle are shown to more certify and explain its isotropy in xoy plane. Our conclusions agree with those obtained by Martin applying the scattering cross section.

According to our results, the resonant frequency can be tuned by changing the space t between the inner and outer rings, strip width or gap width. Especially the space t can be greatly contributed to the resonant frequency. It can make us find the best design according with the practical requirements before coming to fabricate and make an experiments on it.

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