

GLOBAL POSITIONING SYSTEM MEASUREMENT OF CRUSTAL
DEFORMATION IN CENTRAL CALIFORNIA

by

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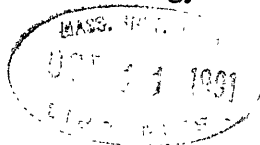
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ABSTRACT

In Chapter 2, we develop a conventional terrestrial reference frame—designated SV6—for the analysis of Global Positioning System (GPS) observations. The reference frame adopts the geocentric origin and scale defined by eleven years of satellite laser ranging (SLR) observations. The precise relative locations of 80 sites that primarily realize the SV6 frame are derived from very-long-baseline interferometry (VLBI) observations. The orientation is consistent with the International Earth Rotation Service terrestrial reference system. The coordinates of sixteen sites with well-determined local vector ties between collocated VLBI and SLR reference points are used to make the VLBI coordinate system commensurate with the SLR system; the sites are globally distributed, including nine in North America, three in Europe, and one each in China, Australia, Hawaii, and Kwajalein. A scales of the VLBI and SLR systems differ by 4 ± 1 parts in 10^9 . SV6 includes estimates of the temporal evolution and uncertainties of the coordinates from VLBI and geophysical observations and we develop a methodology for systematically combining temporally heterogeneous space-geodetic observations with geophysical and geological information, such as global plate tectonic motion models.

In Chapter 3, we present estimates of crustal deformation in central California from thirteen GPS campaigns, conducted primarily by a consortium of four universities from December 1986 to February 1991. Each major campaign occupied a network of seven to twenty sites in California repeatedly for four to five days, with additional fiduciary control provided by sites distributed across North America, and in Hawaii and Europe. The precision of the estimated baseline vectors, based on the day-to-day scatter over each campaign, is 3–5 mm in the horizontal components with $1-3 \times 10^{-8}$ dependence on baseline length for well-designed experiments. The precision in the vertical is 15–25 mm. Precision is most strongly affected by changes to the fiducial network and ionospheric conditions. Systematic constant error sources may be responsible for a slight increase in long-term scatter over short-term scatter.

We then estimate relative motion between twelve sites in California that have observations spanning 1.5–4.2 years, with uncertainties at the 1–1.5 mm/yr level. Confidence in some of the estimated rates is limited by apparent inconsistencies between experiments with poor fiducial control. The estimated relative motions of Owens Valley and Palos Verdes with respect to Vandenberg are consistent with VLBI-derived rates. We estimate north-south convergence across the eastern Santa Barbara channel at 5.2 mm/yr, and our results are consistent with a change to left-lateral shear in the central Santa Barbara channel. The estimated deformation across the Santa Maria fold and thrust belt has less convergence and more right-lateral shear than had been previously estimated from triangulation and trilateration measurements. The relative motion of six sites along the western margin in the vicinity of Vandenberg are consistent with 2–4 mm/yr motion on the Hosgri fault. Relative motion east of the San Andreas suggests that an additional 2–4 mm/yr motion may be accommodated within a shear zone located in Owens Valley and the Mojave Desert.

Thesis supervisor: Thomas H. Jordan, Professor of Geophysics

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CHAPTER 1. INTRODUCTION

De longitudine ac latitudine haec sunt quae digna memoratu putem. universum autem circuitum Eratosthenes (in omnium quidem litterarum subtilitate set in hac utique praeter ceteros solers, quem cunctis probari video) CCLII milium stadiorum prodidit, ...improbum ausum, verum ita subtili argumentatione comprehensum ut pudeat non credere.¹

Pliny, Natural History, II, 247

TERRESTRIAL GEODESY AND THE SAN ANDREAS FAULT

To know the ‘length and breadth’ of the Earth has long captured the imagination of humankind. It is believed that Pythagoras, in sixth century BC, was the first to consider the Earth spherical, based on the aesthetics of geometric purity and on the common-sense observation that the Sun and Moon appear to be so. The next natural question, How big is it?, was nearly inconceivable to answer using the techniques for measuring distances on the ground that were available to the classical world. Eratosthenes, chief librarian at the Alexandrian Museum in the third century BC, arrived at his celebrated answer using an extraterrestrial source of reference: the rays of the Sun. He knew that the midday Sun on the summer solstice cast no shadow in Syrene (near present-day Aswân), and found that at the same time an angle of 1/50 of a circle was cast in Alexandria. Assuming that the rays from the Sun are parallel at the Earth’s surface, the distance between those two cities, which he estimated from the number of days of travel required by camel, should be 1/50 the circumference of the Earth. His estimate of 252,000 stades (approximately 46,620 km)—only 10% too large—was truly, in the words of Pliny, an ‘audacious

¹ These are the facts that I consider worth recording in regard to the Earth’s length and breadth. Its total circumference was given by Eratosthenes (an expert in every refinement of learning, but on this point assuredly an outstanding authority) as 252,000 stades, ...an audacious venture, but achieved by such subtle reasoning that one is ashamed to be skeptical.

venture’.

The major source of error in Eratosthenes’ calculations was the terrestrially measured distance between the two cities. In 1615, the Dutch mathematician Willebrord van Roijan Snell, who is also noted for his work on refraction, improved upon the methodology of Eratosthenes by developing the geodetic technique of triangulation to obtain a more accurate measure of the distance between his astronomical observations. He noted the horizontal angles between his house and the church spires in his hometown of Leiden and then extended his observations to the church spires in nearby towns. Using the geometric constraints imposed by the 33 triangles in his network together with the known 5 kilometer distance from Leiden to Zoeterwoude, he found the distance from Alkmaar to Bergen-op-Zoom, which lie on nearly the same meridian, to be 130 kilometers. His inferred estimate of the Earth’s circumference was only 3.4% too small.

The work of Snell and others, such as the French clergyman Picard and the French expeditions that established the basis of the meter, showed that terrestrial measurements of angles and lengths could provide accurate relative locations over great distances. During the next three centuries, triangulation survey techniques were used extensively to map the Earth’s surface. In California, for example, triangulation surveys were conducted along the coast beginning in 1851 by the Coast and Geodetic Survey. Lacking convenient church spires, such surveys could be arduous and time-consuming; coordinating teams of surveyors usually made nighttime observations of beacons from mountain top to mountain top in order to achieve the greatest accuracy over the longest possible intervisible distances. In such a fashion, a network of primary geodetic stations was established along the coast by 1885 with relative position accuracy on the order of 10–20 parts per million (ppm) proportional to baseline length.

Then on April 18, 1906, a major earthquake ruptured a 450-km segment of the San Andreas fault producing relative displacements of 2–6 m along the fault. Together with the subsequent fire, the earthquake destroyed much of the city of San Francisco and

became the most extensively studied seismological event in history. Many of the field observations were included in the Report of the State Earthquake Investigation Commission, including those by the Coast and Geodetic Survey conducted by *Hayford and Baldwin* [1907]. They realized that such large displacements might seriously degrade the previous triangulation measurements as a means of control for accurate surveys and set about to correct the problem: “The new triangulation to repair the damage was completed in July, 1907. In addition to serving this practical purpose, it has shown the character of the earth movements of 1906, which were found to extend back many miles on each side of the fault. These are very interesting results from a purely scientific point of view.” These results formed the basis for Henry Fielding Reid’s classic exposition on the elastic rebound theory of earthquakes, which was included in the second volume of the Report published several years later [*Reid*, 1910].

Thus, the geodetic study of crustal deformation in California was inaugurated. Arthur L. Day, director of the Geophysical Laboratory of the Carnegie Institution in Washington, recognized the importance of geodetic work to seismologists and their efforts to predict where earthquakes might occur. At his request, William Bowie, chief of the Division of Geodesy in the U.S. Coast and Geodetic Survey, conducted an extensive reoccupation of triangulation sites in California during 1922–5, including those sites observed in 1906–7, plus many more sites along the coast to south of Los Angeles and east to the Sierra Nevada, in an effort to determine the nature of the deformation away from the San Andreas fault, and motions of the coast with respect to eastern California. He found that relative motions due to the earthquake were generally confined to within 20 miles of the fault, but he was unable to detect any change in position between sites in the Sierra Nevada and sites west of the San Andreas in southern California [*Bowie*, 1928].

Bowie also assisted Albert Abraham Michelson in his efforts to determine the velocity of light in an experiment which would be the forerunner of the next major advance in long-distance measurement techniques. Michelson, who was noted for the

famous null result he obtained in his ether-drag experiment conducted with Morley in 1887, long recognized the metrological uses of light (one of his final papers was entitled: “Light-waves as measuring rods for the sounding of the infinite and the infinitesimal”). In 1892–3, he determined that the Paris meter bar was 1553163.5 wavelengths of the red cadmium spectrum line, establishing the current methods for defining length. Towards the end of his life he returned to the problem of determining the velocity of light. At his behest, Bowie organized probably the most accurate triangulation survey ever attempted to determine the 35-km baseline between Mt. Wilson and Mt. San Antonio in the San Gabriel Mountains in California; Bowie claimed the line was accurate to 1:6,800,000, or 0.15 ppm [Michelson, 1927]. From optical measurements over this baseline, Michelson estimated the velocity of light at 299796 ± 4 km/s (comparable with the current estimate of 299792.458 km/s). Ironically, Michelson’s subsequent efforts to determine the velocity of light using a 1.6-km evacuated tube were upset by the occurrence of an earthquake that apparently changed the length of the tube. This event cast doubt on the previous survey because it was then pointed out that on June 25, 1925—after the geodetic determination of the Mt. Wilson–Mt. San Antonio baseline but before the optical measurements—a large earthquake occurred near Santa Barbara, 150 km to the west. Clearly this was an example of the geophysicist’s signal being the metrologist’s noise!

The angles measured in triangulation surveys can establish networks of triangles that extended over great distances; however, the length of at least one side of a triangle has to be known in order to define the *scale* of the network. Eratosthenes relied on camels and Snell used the estimated distance to a nearby town; both measurements were relatively inaccurate. To provide a more uniform and precise standard, the English astronomer Edmund Gunter, who also first noted but failed to correctly interpret the secular variation of the Earth’s magnetic field, introduced the surveyor’s chains in the seventeenth century. This in turn was replaced by the use of invar (an alloy of iron with a low coefficient of thermal expansion) tapes and wires in 1900, which could attain accuracies of 1–2 ppm.

Achieving these accuracies was time-consuming: typical field procedures required suspending 24-m wires under tension in catenary between suitable tripods over fairly flat, obstacle free ground on windless days. Three to four such baselines, typically covering a total length between 7–15 km, could be measured during a few month interval, before the wires became metrologically unstable. Consequently, relatively few such baselines were incorporated into survey networks.

Despite his poor choice of location, Michelson's work implied that rapid, high-precision electromagnetic measurements of length were feasible, if the velocity of light in the atmosphere was known. Work on electromagnetic distance measurement (EDM) was accelerated during the 1930's by the development of radar and its successful use in World War II. In 1947, the Swedish geodesist Erik Bergstrand built the first EDM instrument utilizing white light. This instrument was the prototype for the first Geodimeter which was capable of 1–2 ppm accuracy at ranges up to 40 km at night. The first laser EDM instruments were introduced in 1966, and eventually achieved 1 ppm accuracy at ranges up to 80 km during night or day. The major source of error for these instruments is introduced by atmospheric refraction effects controlled by temperature, barometric pressure, and water vapor pressure. EDM measurements with precisions on the order of 0.2 ppm are feasible if direct measurements of these conditions are taken, for example, by flying along the line of sight between the EDM and the target reflector [e.g., *Savage and Prescott*, 1973]. Two-color laser EDM, introduced in the early 1970's, takes advantage of the dispersive properties of air to nearly eliminate the effects of temperature and pressure [e.g., *Slater and Huggett*, 1976]; these instruments have demonstrated precisions on the order of 0.3 mm + 0.12 ppm over baselines between 1 and 10 km without the need for flying the line [*Langbein et al.*, 1987]. By the early 1970's EDM trilateration had virtually replaced triangulation as the standard surveying technique when high-precision (1 ppm) geodetic surveys were desired over moderate distances.

In California, the triangulation networks were reoccupied approximately once a decade and, at Bowie's suggestion, were supplemented by 50-km arcs extending across the fault in an effort to observe the continuous movements responsible for generating earthquakes as predicted by elastic rebound theory. Displacements due to the 1940 Imperial Valley and 1952 Kern County earthquakes were derived from the existing networks, and the 'slow drift' (about 50 mm/yr) of the crust west of the San Andreas with respect to the east was detected [e.g., *Whitten*, 1955]. In 1957, in response to the reports of Karl V. Steinbrugge of the Pacific Fire Rating Bureau on slippage along the San Andreas fault at the W. A. Taylor Winery south of Hollister, the Coast and Geodetic Survey installed a small network around the winery and began to measure continuous creep along the fault at about 10–12 mm/yr [*Whitten and Claire*, 1960]. By 1963 there were indications that the San Andreas could be divided into at least three zones of deformation: (1) a northern section from Cape Mendocino through San Francisco to Hollister which displayed a relative displacements ranging from 17 to possibly 60 mm/yr across the fault, depending on the width of the zone; (2) a central section running from about Cholame to Riverside which did not display any significant displacements; and (3) a southern section, including the Imperial Valley, with displacements similar to the northern section [*Meade*, 1965].

However, no geological process seemed capable of maintaining relative horizontal motions over long periods of time. Mountain building episodes involving several hundred kilometer horizontal shortening due to thermal contraction of the crust were proposed, but horizontal displacements across faults like the San Andreas, so amply observed following the 1906 earthquake, were considered geologically short-lived phenomena. The suggestion by *Hill and Dibblee* [1953] that similar geological units were displaced by up to 560 km across the San Andreas fault was greeted with skepticism: How could hundreds of kilometers of strike-slip motion diminish to zero at the ends of the faults? Even more controversial was the suggestion of *Carey* [1958] that the San

Andreas was part of a much larger megashear that included Tertiary deformation in Oregon, Washington, Idaho, and the Basin and Range province. The notion of large-scale horizontal displacements was not new—Francis Bacon had noted the similarity of the shores on either side of the Atlantic in 1620, and the nineteenth century Austrian geologist Eduard Suess had proposed the breakup of a super-continent Gondwanaland to form the present-day continents in the southern hemisphere—but no reasonable mechanism could be offered. The theory of continental drift—that the continents freely plow across weak ocean floor—was advocated by the German meteorologist and polar explorer Alfred Wegener from 1911 until his death in 1930; Wegener believed that geodesy provided the strongest support for his proposals. Measurements of astronomic longitude dating back to 1823 in Greenland indicated that the distance from Greenland to Europe was increasing by as much as 36 meters (!) per year [Wegener, 1929]. However, later high-precision geodetic measurements did not confirm these rates, and his proposed driving forces were shown to be several orders of magnitude too small.

Thus, by the early 1960's terrestrial geodetic measurements, although inadequate to fully characterize the broad scales of deformation in California, supported the notion that the San Andreas fault accommodated continuous horizontal displacements that geophysicists and geologists were unable to fully explain. However, these difficulties were soon largely overcome both by the development of a new paradigm for the Earth and new measurement techniques to validate it. These changes were heralded by two unfamiliar melodies: one from the heavens and one from the oceans.

SATELLITE GEODESY AND PLATE TECTONICS

The radio signal of Sputnik 1, launched into orbit on October 4, 1957, announced the dawning of the Space Age. Besides touching off a scientific and technological explosion which led in less than a dozen years to the presence of humans on the Moon, the first

artificial satellites almost immediately improved knowledge about the Earth itself. Soon after the launching Sputnik 2 in November 1957, analysis of its motion, most sensitive to the low-order spherical harmonics of the Earth's gravitational potential, provided a value for the flattening of the Earth ($1/298.3$) that was an improvement over the best terrestrially-derived estimate [Buchar, 1958; Merson and Hale, 1958]. The motions of many of the early satellites with different orbital characteristics provided enough information to estimate other spherical harmonics of the Earth's gravitational potential, such as the coefficients of the low-order tesseral harmonics [Izsak, 1961; Kaula, 1961], and later complete models up to degree and order eight [Guier and Newton, 1965; Lundquist and Veis, 1966]. One of the most recent models, GEM-T1 derived at the NASA Goddard Space Flight Center, includes data from 17 satellites, accounts for solid Earth and ocean tides, and is complete to degree and order 36 [Marsh *et al.*, 1988]; others, such as GEM-10B, include surface gravity and satellite altimeter data as well [Lerch *et al.*, 1981].

The second geodetic advantage artificial satellites provide is to serve as reference points between widely-separated sites on the Earth's surface, greatly expanding the intervisibility limitations that restrict terrestrial techniques. Several methods have been used to observe the location of satellites relative to ground sites. One of the earliest and most simple methods used optical techniques to track the motion of satellites equipped with beacons or reflecting sunlight against the background stars. The azimuth between two ground sites can be derived from the direction cosines of two simultaneous observations of a satellite. These astro-triangulation techniques culminated in 1966–1970 with the establishment by the National Ocean Survey (formerly the Coast and Geodetic Survey) of a world-wide network of 45 stations whose relative site locations were determined to about 5-m accuracy using Wild BC-4 camera observations of the balloon satellite PAGEOS [Schmid, 1974].

Soon after the launch of Sputnik 1, it was realized that the satellite orbit could be estimated from an analysis of the satellite's Doppler-shifted radio transmissions [Guier and Weiffenbach, 1958]. Doppler positioning uses the ratio of the observed Doppler shift to the known transmission frequency to estimate the rate of change of range between the satellite and the ground site. Comparison of this range rate with the assumed satellite velocity provides an estimate of the angle between the instantaneous velocity vector and the direction to the ground site. Positions can then be derived from the intersection of the cones defined by these angles from several orbital passes. The U.S. Navy began launching TRANSIT navigation satellites in 1962; in 1967 TRANSIT was declassified and made available for civilian use, and small, portable receivers were soon available to take advantage of the system. Observing 20–30 passes could provide accuracies for geocentric positions on the order of 2–4 m and for relative positions on the order of 1 m or 1 ppm (whichever was larger) [e.g., Anderle, 1974].

Finally, the ability to make direct measurements of the distance from the ground site to the satellite using radar, microwaves, and lasers led to the development of astro-trilateration methods. The U. S. Army operated the 37-station SECOR network, which used microwave transponders to observe seven SECOR and GEOS 1 and 2 satellites with range accuracies on the order of 3–10 m; the relative accuracy of the coordinates for sites in a network located at low latitudes about the equator was on the order of 25 m [e.g., Henriksen, 1977]. Laser sources and corner-cube reflectors, however, offered several advantages over radar and microwave techniques: concentration of power into very short pulses (0.2–20 ns) providing more precise range measurements, and little sensitivity to refraction in the ionosphere. They have the disadvantage that no measurements can be obtained on cloudy days. The measurement of true ranges to satellites also provide constraints on scale and additional strength to networks determined using Doppler and astro-triangulation methods. For a discussion of early accuracies and uses, see Cook *et al.* [1978].

Thus, by the late 1960's space-geodetic techniques had greatly improved measurement of the 'length and breadth' of the Earth. The low degree and order harmonics of the gravitational potential resolved an ellipticity of the equator and a pear shape to the Earth (as Christopher Columbus had 'predicted' 450 years earlier!). Global networks with relative position accuracies of 1 ppm over several thousand kilometer baselines were tying together the continent-sized networks (datums) and remote islands that had previously been tenuously related by triangulation using flares or radar or the Moon or not at all. And accurate geocentric station locations combined with accurate satellite ephemerides were beginning to establish satellite inertial reference frames capable of studying the motions of the Earth itself, such as polar motion [e.g., *Anderle and Beuglass*, 1970]. But these accuracies would not be enough to satisfy geophysicists who were busy exploring their own brave, new world, a world in motion.

William Menard entitled his personal recollection of the revolution in Earth sciences *The Ocean of Truth*, and it was the curious music of the ocean floors that provided the crucial evidence for sea-floor spreading and unifying theory of plate tectonics. During World War II, Harry H. Hess, who was a navigator and then commander of the U.S.S. *Cape Johnson*, used his troop vessel to conducted marine geological research in the Pacific. Using echo-sounding recorders, which provide a bathymetric profile of the ocean floor, he discovered guyots (flat-topped submarine volcanoes), which he interpreted as having once been islands whose tops had eroded from the actions of wind and wave and then were submerged to great depths [*Hess*, 1946], in much the same fashion that Charles Darwin had explained the origin of atolls in his *The Structure and Distribution of Coral Reefs* in 1842. That interpretation suggested that the ocean floors were mobile (at least, vertically) rather than static, as had been generally assumed. The post-war decade was a time of intense research at sea using many newly developed techniques to study the bathymetry, gravity, heat flow, and crustal structure of the ocean floors. Also, by 1958 ship-towed magnetometers sensitive enough to measure the Earth's magnetic field to 10

ppm were beginning to detect puzzling anomalies in the magnetic field that apparently had their origins in the oceanic crust. For example, extensive surveys in the northeastern Pacific revealed a pattern of linear anomalies that were oriented roughly north-south with steep gradients between the highs and lows and with large offsets in the anomaly patterns that seemed to be associated with the fracture zones recently revealed in the bathymetric profiles [e.g., *Mason*, 1958; *Vacquier et al.*, 1961; *Mason and Raff*, 1961]. The geophysicists were unable to explain the origin of these anomalies and those associated with the mid-ocean ridges, where strong positive anomalies were ubiquitous despite the presence, in some cases, of deep median valleys that should have produced negative anomalies, due to the lack of crustal material.

Arthur Holmes suggested in 1928 that thermal convection within the Earth's mantle was a possible driving force for continental drift [*Holmes*, 1928], and he later proposed that continents passively rode on mobile oceanic crust which was created by diffuse infusion of basaltic magma [*Holmes*, 1929], but the theory of continental drift had little support following the failure of Danish geodesists to confirm Wegener's predicted rates. Interest in continental drift was revived in the late 1950's by evidence of relative motion between Europe and North America from the polar wander paths derived from paleomagnetic data [*Irving*, 1956; *Runcorn*, 1956]. Then in 1962, Hess proposed a model for the evolution of the ocean basins, coined 'sea-floor spreading' by *Dietz* [1961] who had formulated essentially the same ideas simultaneously. Like Holmes, Hess suggested that the continents and oceanic crust are carried passively on a convecting mantle, but he localized the formation of new oceanic crust to the mid-ocean ridges where mantle material rose and then moved laterally away until being swept downward on a descending limb of the convecting cell. Most oceanic crust is therefore relatively young (less than 400 m.y.) as the comparatively shallow sedimentation covering them implied [*Hess*, 1962]. At the same time, recent radioactive dating of magnetized terrestrial rocks began to indicate that the Earth's magnetic field had undergone several polarity reversals during

the most recent 4 m.y. [e.g., *Cox et al.*, 1963; *McDougall and Tarling*, 1963]. Combining these results with the ideas for sea-floor spreading, *Vine and Matthews* [1963] suggested a hypothesis for the origins of the linear marine magnetic anomalies: 'If the main crustal layer (seismic layer 3) of the oceanic crust is formed over a convective up-current in the mantle at the centre of an oceanic ridge, it will be magnetized in the current direction of the Earth's field.... Thus, if spreading of the ocean floor occurs, blocks of alternately normal and reversely magnetized material would drift away from the centre of the ridge and parallel to the crest of it.' Acceptance of the Vine-Matthews hypothesis was not immediate, but by 1966 improvements in the geomagnetic-reversal time scale and in observations of marine magnetic anomalies, such as the *Eltanin*-19 profile in the south Pacific [*Pitman and Heirtzler*, 1966], showed stunning symmetries in the lineations about ridge crests which almost exactly fit the predictions of the time scales assuming a constant rate of sea-floor spreading [e.g., *Vine and Wilson*, 1965; *Vine*, 1966]. The evidence for sea-floor spreading became compelling.

The spreading of the sea floor at mid-ocean ridges implies that the oceanic crust, and the continents as well, are in relative motion. The theory of plate tectonics, offering quantitative descriptions of these motions, soon appeared. *Wilson* [1965] described the Earth's crust as being divided into several large rigid plates and discussed the types of boundaries between these plates, depending on the sense of the relative motion. In addition to extensional and compressive boundaries, he proposed the existence of *transform faults*, across which crust is displaced horizontally and is conserved. Relative motion across transform faults is opposite of that required to displace a ridge, and thus they are fundamentally different than transcurrent faults. He identified the San Andreas to be a right-lateral transform fault offsetting the Juan de Fuca ridge to the north and the East Pacific Rise to the south, and hence a boundary between the Pacific and North America plates. The predicted direction of motion across transform faults was soon confirmed by seismology. *Sykes* [1967], benefitting from the recently installed World-

Wide Standardized Seismograph Network (WWSSN) originally created to help monitor nuclear explosions for the Limited Nuclear Test Ban Treaty, analyzed the seismic radiation patterns from seventeen earthquakes located on mid-ocean ridges, and found that the sense of motion in every case agreed with that predicted by Wilson for transform and ridge-crest earthquakes. Buoyed by this confirmation, *McKenzie and Parker* [1967] extended Wilson's concepts by following the suggestion of *Bullard et al.* [1965] to use Euler's theorem to describe relative motion between two plates on a sphere as a rotation about an axis (Euler pole). It follows that the sum of the angular velocities vectors from a closed circuit of plates (e.g., about a triple junction of plate boundaries) must be zero (closure) if the plates are perfectly rigid and the radius of the Earth does not change. Transform faults, since they conserve crust, describe small circles about the pole of rotation. They also pointed out that earthquake slip vectors, as well, can constrain the direction of relative motion and that marine magnetic anomalies provide information about the rates of motion which vary with distance from the pole. Using the strike of the San Andreas Fault between Parkfield and San Francisco and the average slip vector of all the aftershocks in the Kodiak Island region of the 1964 Alaskan earthquake, they made the first determination of the North America–Pacific Euler pole location (50°N, 85°W).

Thus, the theory of plate tectonics provides a framework for making quantitative tests regarding relative plate motions. It assumes that the plates are rigid and do not deform internally. This assumption clearly breaks down in certain continental regions, like the Mediterranean and Asia. However, the increasingly precise locations of earthquakes from WWSSN data demonstrated that the assumption of rigidity was valid for most parts of the Earth: deformation is typically confined to very narrow zones (on the order of kilometers) defining the plate boundaries, especially in oceanic regions [e.g., *Isacks et al.*, 1968]. Using earthquake slip vector, transform fault azimuth, and marine magnetic anomaly data, *Morgan* [1968] attempted the first quantitative global reconstruction of tectonic motions. He divided the Earth's surface into twenty plates and then investigated

the Africa–North America–Pacific–Antarctica circuit using closure constraints; however, lacking sufficient rate data between the North America and Pacific plates, he used the terrestrial geodetic data summarized in *Hamilton and Myers* [1966] which gave an estimate of 60 mm/yr relative motion across the San Andreas fault. *Le Pichon* [1968] used only transform fault azimuth and magnetic anomaly data along five boundaries to characterize a six-plate Earth; he used an estimate of the opening of the Gulf of California (300–450 km in 10 m.y., implying an average rate of rotation of 0.35–0.52 deg/m.y.) to help constrain the North America–Pacific relative rate (although he chose an average rate of 0.6 deg/m.y. which yields 50 mm/yr relative motion across the San Andreas fault).

The theory of plate tectonics not only describes the present-day relative plate motions, but also indicates their evolution. *McKenzie and Morgan* [1969] discussed the stability of the point where three plates meet (triple junctions), which depends on the type of boundaries between the plates and their relative velocities. As an example, they examined the evolution of the San Andreas Fault, which is delimited by the Mendocino fault-fault-trench triple junction on the north and the Rivera ridge-trench-fault triple junction on the south. They demonstrated that each of these types of triple junctions is stable only if the azimuth of the trench and the transform fault replacing it are collinear, which generally has been the case. They roughly predicted the offset of the triple junctions (2300 km) based on the assumption that the relative motions of the Pacific, North America, and Farallon plates have been constant since their formation in the Oligocene, and noted that the subsequent displacements across the San Andreas fault totaling 350 km [*Hill and Dibblee*, 1953] fell far short of the predicted displacements of about 2000 km, suggesting that other faults, possibly offshore, accommodated relative motion between the plates.

SPACE GEODESY AND THE SAN ANDREAS DISCREPANCY

By the early 1970's, the theory of plate tectonics had gained general acceptance, and geophysicists were reexamining their observations in light of the new paradigm. *Atwater* [1970] provided the first comprehensive discussion of the evolution of western North America based on magnetic anomalies observed offshore. She rejected the notion that the San Andreas is a simple fault separating two rigid plates, suggesting instead that the margin is a 'wide, soft boundary' accommodating deformation on the San Andreas and other active faults (as indicated by the diffuse seismicity), including compression across the Transverse and Coastal Ranges and extension within the Basin and Range province. According to her interpretation, in the late Mesozoic two plates bordered western North America: the Farallon plate subducting along the southern margin, and the Kula plate moving northward accommodating significant strike-slip motion across a proto-San Andreas fault. The triple junction between these three plates migrated north so that by 30 Ma the Farallon plate was obliquely subducting under the majority of the western North America margin. At that time the ridge system between the Pacific and Farallon plate began to collide with the trench and a young, hot continent-ocean boundary began to lengthen. By 24 Ma the margin had cooled sufficiently that Pacific-North America deformation was present along two boundaries: rifting in the Basin and Range province inland, and strike-slip along the San Andreas transform fault on the coast. Between 20 and 10 Ma, the San Andreas lengthened to the north, passing through the location of the present-day Transverse Ranges while the Basin and Range system extended to Baja California where rifting was occurring offshore. This configuration lasted until about 5 Ma when the southern margin became sufficiently strong for rifting to shift inland, commencing the opening of the Gulf of California. This reconfiguration forced the San Andreas to break its way inland to connect to the Gulf, forming a bend in the fault which began to accommodate the compression responsible for the formation of the Transverse

Ranges. These broad interpretations have served as a framework for all subsequent tectonic studies of California and the western Cordillera.

As more geophysical data accumulated, new global reconstructions of plate tectonic motions appeared; instead of simply estimating relative angular velocity poles from pairs of plates independently, closure constraints were utilized to derive self-consistent models from simultaneous inversions of all the observations. *Morgan* [1972] proposed that narrow plumes of deep mantle material, arising from convective processes, impinge on the overlying plates; when these plumes successfully reach the surface they form 'hot spots' which then record the relative motion of the plate with respect to the mantle in the form of volcanic island chains (e.g., Hawaiian-Emperor) and ridges. A comparison between the relative plate motions and the hot-spot traces indicated that the hot spot locations are relatively fixed, and could be used to establish an absolute motion reference frame. *Chase* [1972] used 176 directions and 59 rates of relative motion to derive an eight-plate global model; he noted that the rate of creation of crust was approximately equal to the rate of its consumption. *Minster et al.* [1974] used 62 fracture zone azimuths, 106 earthquake slip vectors, and 68 spreading rates to obtain an eleven-plate global model, denoted RM1, which resolved relative motion between North and South America and hypothesized the existence of a Bering plate. Because of the deformation in the Basin and Range province, they did not include any data from the San Andreas system, but a single magnetic anomaly from the Gulf of California, albeit poorly determined, did provide an independent constraint on the North America–Pacific rate in addition to the constraints imposed by all the other observations by closure conditions.

RM1 predicted 55 mm/yr of relative motion averaged over 5 m.y. (magnetic anomaly three) between the North America and Pacific plates in central California. Geodesists and geologists studying the San Andreas fault in central California, however, were determining significantly smaller rates averaged over various time intervals. *Savage and Burford* [1973] analyzed triangulation, trilateration, and fault creep data collected during

1907–1971 spanning the San Andreas, Calaveras, and Hayward faults from San Francisco to Parkfield, found that an estimate of 32 ± 5 mm/yr of average relative right-lateral motion. Since then, more extensive terrestrial observations have slightly refined the estimate to 34 ± 3 mm/yr [e.g., *Thatcher*, 1979; *Prescott et al*, 1981; *Lisowski and Prescott*, 1981]. Paleoseismic studies of Holocene stream deposit offsets at Wallace Creek give similar rates of 33.9 ± 2.9 mm/yr for the past 3700 yr and $35.8 +5.4/-4.1$ mm/yr for the past 13,250 yr [*Sieh and Jahns*, 1984]. Geologists, building on the earlier work of *Hill and Dibblee* [1953] in central California and *Crowell* [1962] in southern California, studied even longer time intervals contained in the Neogene-Quaternary offsets of geologic formations. For example, *Huffman* [1972] suggested that the rates of offset in central California had changed during the last 22 m.y., and that the 10–m.y. average rate was 25 ± 5 mm/yr.

Minster et al. [1974] suggested that the difference in relative motion between the RM1 estimate and that observed geodetically and geologically, was being accommodated in the Basin and Range province or along faults parallel to the San Andreas. The next generation global plate tectonic model, RM2, included more observations (78 transform fault azimuths, 142 earthquake slip vectors, 110 magnetic anomaly 2 and 2' rates averaged over a 3-m.y. interval) and changed the geometry of its eleven plates to provide a considerably improved model [*Minster and Jordan*, 1978]. The estimated relative motion in central California of 56 ± 3 mm/yr at $N35 \pm 2^\circ W$ was essentially the same as before, but they now noted that the direction of relative motion was significantly different from the $N41 \pm 2^\circ W$ fault azimuth observed along the central San Andreas. *Minster and Jordan* [1978] noted that the difference between the predicted relative plate motion and relative motion across the San Andreas fault in central California, the so-called *San Andreas discrepancy*, could not be fully explained by geological estimates of extension in the Basin and Range province, and suggested that the missing motion might be accommodated on fault systems west of the San Andreas.

Assuming that global plate tectonic models, such as RM2, adequately describe the relative vector motion of regions that are not actively deforming, they provide boundary constraints on crustal kinematics within deformation zones. *Minster and Jordan* [1984] rigorously formulated this vector-boundary constraint method by integrating velocity gradient tensor fields along paths crossing deformation zones. Under this formulation, the San Andreas discrepancy vector must equal the sum of relative velocity vectors describing motions east and west of the San Andreas. Their estimate of the Basin and Range extension, based on a variety of geological and seismic observations, was $\alpha(10.1 \pm 0.7)$ mm/yr, $N63 \pm 5^\circ W$, where α can vary between 0.1 and 1.2. This result, combined with the previously quoted RM2 North America–Pacific relative plate motion vector and the San Andreas motion derived from geodetic and geologic observations, suggested that 6–25 mm/yr of right-lateral motion and 4–13 mm/yr compression might be accommodated on faults west of the San Andreas. Such potentially high rates of relative motion on faults with little seismic activity, such as the San Gregorio-Hosgri system [e.g., *Hall*, 1975; *Weber and Lajoie*, 1977; *Silver and Normack*, 1978], would have serious implications for seismic risk assessment. Similar analysis of Quaternary slip rates along faults in southern California had produced kinematic models for crustal deformation associated with the Transverse Ranges, the Garlock fault, and the southern San Andreas fault system [*Bird and Rosenstock*, 1984; *Weldon and Humphreys*, 1986; *Sedlock and Hamilton*, 1991].

The calculation of vector constraints that combine different measurement techniques, such as plate tectonic models and geodetic surveys, assumes that motions averaged over different time intervals can be treated uniformly. The San Andreas discrepancy was estimated under the assumption that the 3-m.y. average North America–Pacific plate relative motion is a valid boundary constraint for geodetic observations averaged over the last 100 years. While the geophysicists were exploring the implications of plate tectonics, the geodesists were sufficiently improving the accuracy of space geodetic

techniques to make direct estimates of current relative motions. Although some attempt was made to analyze changes of station positions derived from Doppler observations [Anderle and Malyevac, 1983] and from laser ranging to satellites such as the Beacon Explorer C [Smith *et al.*, 1979], the most accurate estimates have been derived from laser ranging to LAGEOS and from a technique that began its development in the late 1960's: very-long-baseline radio interferometry (VLBI).

The Laser Geodynamics Satellite (LAGEOS) was launched in May 1976. Unlike previous satellites, LAGEOS was designed to provide an extremely stable orbit that would be suitable for studies of Earth rotation and crustal motion [e.g., Cohen and Smith, 1985]. The satellite is a totally passive, dense sphere 60 cm in diameter with a mass of 411 kg and is covered with 426 corner cube retroreflectors. These characteristics help to minimize the effect of nonconservative forces like solar radiation pressure and Earth albedo radiation. LAGEOS was placed into a nearly circular orbit at high altitude (about 5900 km) to minimize orbit estimation errors due to atmospheric drag, short-wavelength gravity and tidal effects, while ensuring strong signal returns for the laser systems. By 1985, analysis of laser ranges to LAGEOS had improved Earth gravity models [e.g., Lerch *et al.*, 1985], estimates of GM [e.g., Smith *et al.*, 1985; Tapley *et al.*, 1985], and had observed the rate of change of the second-degree, zeroth-order harmonic of the gravitational field ($\dot{J}_2$) due to postglacial rebound [Rubincam, 1984]. The laser systems had improved ranging precision to 5–10 cm and yielded coordinate precisions of 3–5 cm based on simultaneous estimation of positions, orbit and Earth rotation parameters. These precisions were sufficient to estimate relative motions between the most accurate laser systems which in general showed good agreement with the RM2-predicted motions [e.g., Christodoulidis *et al.*, 1985; Tapley *et al.*, 1985]. NASA's San Andreas Fault Experiment (SAFE) measured the baseline from Quincy on the North America plate to Monument Peak on the Pacific, the length of which was estimated to be changing by -61 ± 25 mm/yr compared with the RM2 rate of -53 mm/yr [Christodoulidis *et al.*, 1985].

Thus, while SLR techniques had matured enough to be able to detect plate tectonic motion, they were not yet of sufficient accuracy to provide significant constraints for geophysical problems.

The other high-precision space-geodetic technique capable of detecting plate tectonic motions by the mid 1980's was based on more distant reference points: extragalactic radio sources discovered by K. Jansky in the 1930's. By the late 1960's, stable hydrogen maser clocks enabled radio signals (typically from quasars) received at widely-separated antennas to be cross-correlated for the study of source structure [e.g., *Broton et al.*, 1967; *Bare et al.*, 1967; *Moran et al.*, 1967]. The geodetic and geophysical possibilities were quickly recognized [e.g., *Gold*, 1967; *MacDonald*, 1967; *Shapiro and Knight*, 1970]. One of the basic measurements produced by very-long-baseline radio interferometry (VLBI) is the group delay in propagation time of the signal arriving at one antenna with respect to a second antenna. Ignoring atmospheric and relativistic effects, the group delay is directly proportional to the dot product of the baseline vector between the sites and the unit vector in the direction to the source. Observation of a sufficient number of sources enables the baseline vector, source positions, and Earth orientation parameters to be determined with great precision [e.g., *Shapiro*, 1976]. Bandwidth-synthesis techniques for estimating group delays and other observables from a combination of S-band (~2.2 GHz) and X-band (~8.4 GHz) signals were developed throughout the 1970's and early 1980's [e.g., *Rogers*, 1970; *Hinteregger et al.*, 1972; *Whitney et al.*, 1976; *Rogers et al.*, 1983; *Clark et al.*, 1985].

The simple geometry inherent to VLBI, with sources at essentially infinite distances defining a nearly perfect inertial reference frame, provides a powerful technique for monitoring Earth rotation and determining baseline vectors. Improvement in data collection techniques and methods for estimating these geodetic parameters and other effects, such as clocks and propagation delays due to the atmosphere [see, e.g., *Shapiro*, 1983; *Ma*, 1978; *Herring*, 1983; *Herring et al.*, 1990], has enabled baselines to be

determined with 2–4 parts per billion precision [e.g., *Clark et al.*, 1987; *Kroger et al.*, 1987; *Davis et al.*, 1991]. High-precision measurements at large, fixed antennas beginning in the early 1980's soon led to the detection of relative motion between North America and Europe that was consistent with RM2 [*Herring et al.*, 1986]. The development of mobile VLBI systems [e.g., *Davidson and Trask*, 1985] somewhat alleviated need to use large, fixed antennas to achieve sufficient signal-to-noise ratios to detect interferometric fringes in the radio signals from quasars and permitted observations to be conducted at many sites in the western United States. For example, the motion of a mobile system located at the Jet Propulsion Laboratory (JPL) near Pasadena, California with respect to fixed antennas at the Owens Valley Radio Observatory (OVRO) and in the Mojave in eastern California was determined to be 25 ± 4 mm/yr, $N40 \pm 7^\circ W$ from observations collected during 1980–1984 [*Lyzenga and Golombek*, 1986]. This rate is significantly slower than the RM2-predicted motion 55 ± 3 mm/yr, $N38^\circ W$ (assuming that JPL is on Pacific plate and OVRO and Mojave are on North America), but is roughly consistent with terrestrial geodetic and geologic estimates of relative motion across the San Andreas. One explanation for these results is that the VLBI baselines do not span the entire zone of deformation associated with the North America–Pacific plate boundary in California and that JPL does not represent stable Pacific plate motion and/or OVRO and Mojave, both located west of the Basin and Range province, do not represent stable North America plate motion.

Thus, by the late 1980's, space-geodetic techniques, especially VLBI, had begun to provide useful constraints on relative tectonic motions and deformations. However, a new space-geodetic technique, combining the high-precision capabilities of VLBI with easy portability and inexpensive operating costs, was rapidly maturing and sparking even greater interest in crustal deformation studies.

THE GLOBAL POSITIONING SYSTEM AND DEFORMATION IN CALIFORNIA

In the late 1960's the U.S. Department of Defense (DoD) began development of the Navigation Satellite Timing and Ranging (NAVSTAR) Global Positioning System (GPS) to provide real-time navigation and time signal control for both civilian and military use [e.g., *Parkinson*, 1979]. The first of the prototype (Block I) satellites was launched in February 1978 and the tenth and last of these was launched in October 1985. The deployment of the next generation of satellites (Block II), delayed by the Challenger disaster, began in February 1989 and currently includes about 10 satellites. Eventually, a total of 21 Block II satellites and 3 spares will be placed into 6 nearly-circular orbital planes inclined 55 degrees to the equator at an altitude of approximately 20,000 kilometers. This constellation ensures that at least 4 satellites will be visible from any point on the Earth's surface at all times, providing sufficient geometric redundancy to do real-time, three-dimensional positioning using the radio signals broadcast by the satellites. Two code-modulated radio signals are transmitted by the satellites: a "precise" code that provides 10-meter positioning accuracy, and a "coarse acquisition" code providing 100-meter accuracy. DoD can encrypt the precise code to restrict non-authorized (i.e., most civilian) users access to only the coarse acquisition signal.

Not long after the launch of the first satellite, several groups realized that methods using reconstructed carrier phase signals transmitted in the L-band (~20 cm wavelength), rather than the code-modulated signals, could provide relative positioning capabilities at the centimeter level [*Counselman and Shapiro*, 1979; *MacDoran*, 1979; *Bossler et al.*, 1980]. These techniques, which will be discussed in greater detail in Chapter 3, enable high-precision geodetic measurements to be made with precisions comparable to those obtained using VLBI. Unlike VLBI, GPS relative positioning is sensitive to satellite orbit errors, which may be minimized by positioning relative to a fiducial network of receivers with well-known locations, as discussed in Chapter 2. On the other hand, the signals

transmitted by the GPS satellites are 10^4 to 10^6 times stronger than the quasar signals used by VLBI [MacDoran, 1979]. This advantage means that GPS measurements may be obtained using much smaller antennas, and hence relatively low-cost, easily transportable receivers. This combination of high-precision relative positioning capabilities using inexpensive, highly mobile receivers provides a nearly ideal tool for the study of crustal deformation processes over a wide range of spatial and temporal scales.

By the mid-1980's, single-frequency GPS receivers had been developed with relative positioning capabilities at centimeter-level precisions over baselines of several kilometers [e.g., Counselman *et al.*, 1983; Goad and Remondi, 1984]. These receivers were capable of matching the 1 ppm precisions obtained by terrestrial techniques over networks spanning tens of kilometers [e.g., Bock *et al.*, 1985]. Dual-frequency GPS receivers, which permit the effects of the frequency-dependent contribution of the ionospheric propagation delay to be eliminated, were introduced around 1984. These receivers, located at sites in Massachusetts, Florida, and Texas, were used to independently estimate satellite orbits with a precision of about 0.3 ppm [King *et al.*, 1984; Abbot *et al.*, 1985]. An experiment using these receivers was conducted in March 1985 that estimated a 245-km baseline from Owens Valley to Mojave, California, with a precision of 0.02–0.04 ppm in the horizontal components and 0.08 ppm in the vertical component [Lichten and Border, 1987]. A comparison with independent VLBI and SLR determinations of the same baseline implied that GPS was capable of 0.1–0.2 ppm accuracy [Bock *et al.*, 1986a]. Thus, the GPS space-geodetic technique demonstrated precisions approaching those obtained by VLBI over several hundred kilometer baselines making it highly attractive for studies of broad zones of crustal deformation, such as in California.

The geophysical models of California deformation based on path integral velocity vector constraints suggested that significant relative motion, both right-lateral and compressive, might be occurring along the California margin [Minster and Jordan, 1984; Bird and Rosenstock, 1984; Weldon and Humphreys, 1986]. In 1986, several GPS

experiments were commenced in California to detect this motion. In June 1986, a 4-day, 11-receiver campaign was conducted along the southern margin and included sites on several offshore islands in the Santa Barbara Channel [Dixon *et al.*, 1990]. In December 1986 and January 1987, six 5-day campaigns using 12 receivers were conducted by a consortium of four universities (California Institute of Technology, Massachusetts Institute of Technology, Scripps Institution of Oceanography, and University of California at Los Angeles), the U. S. Geological Survey, and the National Geodetic Survey. In addition to reoccupying the June 1986 sites and sites in central California, these campaigns reoccupied monuments that had been surveyed since the 1880's using triangulation and trilateration. These campaigns were the first in a five-year program dedicated to assessing the long-term precision and accuracy of high-precision GPS measurements and establishing a geodetic network for monitoring crustal deformation in central and southern California [Agnew *et al.*, 1987]. Additional campaigns were conducted in May and September 1987, and in February or March of each year between 1988 and 1991. Analysis of these measurements, both to assess the long-term precision of GPS and to estimate relative motions over the five-year period, will be presented in Chapter 3 of this work.

Analyses of GPS measurements continue to improve the precision and accuracy of their baseline determinations. Errors associated with estimated satellite orbits, which are a rough gauge of baseline precision, are now routinely at the level of 0.01 ppm [e.g., Lichten and Bertiger, 1989; Blewitt, 1989; Dong and Bock, 1989]. Estimates of baseline precision are also provided by day-to-day repeatabilities (weighted root-mean-squared scatters about the weighted mean) of baseline length and vector components. Due to the geometry of the satellite orbits, which have predominantly north-south ground tracks in California, repeatabilities tend to be better in the north rather than east component of baselines. Precision is worst in the vertical component due to atmospheric errors and to the fact that satellite signals cannot be received below the horizon. For the June 1986

campaign which measured baselines between 30 to 600 km in length, for example, repeatabilities were 4 mm and less than 0.001 ppm in the north, 5 mm and 0.022 ppm in the east, and 20–40 mm in the vertical [Dixon *et al.*, 1990]. Comparisons conducted by the U. S. Geological Survey between EDM (Geodimeter) and GPS determinations of baselines ranging from 7 to 50 kilometers, suggest that GPS is consistent with EDM at the 0.1 ppm level [Davis *et al.*, 1989]. For baselines greater than 50 km, comparisons of GPS results with VLBI measurements show sub-centimeter agreement in the horizontal components and 20–40 mm agreement in the vertical [e.g., Dong and Bock, 1989; Davis *et al.*, 1989; Dixon *et al.*, 1990].

The initial results from the GPS campaigns in California have also begun to demonstrate the suitability of GPS to monitor crustal deformation. The first results were obtained from comparisons of GPS measurements with the historical triangulation and trilateration surveys. For example, Feigl *et al.* [1990] determined the direction of maximum compressive strain, NNE with a compressive strain rate of -0.13 ± 0.3 μ strain/yr, in the Santa Maria fold and thrust belt. Assuming that the rotational component of the velocity field is zero, this implies that the rate of deformation integrated across the area is 7 ± 1 mm/yr, N03°E $\pm$ 13° which is consistent with earthquake focal mechanisms, the orientation of the folds and thrusts within the belt, and with the San Andreas discrepancy motion predicted by Minster and Jordan [1984]. Larsen [1990] combined GPS measurements with trilateration measurements from the early 1970's to estimate nearly uniaxial compressive strain across the eastern portion of the Santa Barbara Channel, with equivalent convergence of 6.4 mm/yr, N25°E integrated over the 45-km width channel. Webb [1991] also found significant compression across the Santa Barbara Channel and along the California borderland from a similar study. After several reoccupations of the geodetic network, estimates of relative motion derived solely from GPS measurements were feasible. Larson [1990a] examined measurements collected during the first 2.7 years of the program, and found the estimated rates in good agreement

with VLBI-derived rates and with triangulation and trilateration observations. Repeat GPS observations spanning 1.5 years suggested to *Donnellan et al.* [1990] that 17 ± 8 mm/yr of compression is being accommodated across the northern Ventura Basin. While many of these results, and similar results from surveys conducted around the world, are preliminary, they indicate that GPS measurements obtained at an annual interval over several years are of sufficient precision to detect relative horizontal motions at the 5 mm/yr level for baselines up to several hundred kilometers in length.

The characterization of deformation in California and across the western United States continues to be refined. The fourth generation global plate tectonic model, NUVEL-1, improves upon previous models by altering the plate geometry and adds many high-quality observations obtained over the last decade [*DeMets et al.*, 1990]. Its estimate for North America-Pacific relative motion, benefitting from the addition of several marine magnetic anomaly profiles in the Gulf of California, is 48 mm/yr, N36°W in central California which is significantly slower than the RM2-derived rate [*DeMets et al.*, 1987]. VLBI is adding increasingly more accurate estimates of relative motions [e.g., *Clark et al.*, 1987; *Kroger et al.*, 1987; *Sauber*, 1988; *Caprette et al.*, 1990]. These data have been used to estimate the rate of extension of the Basin and Range at 9 ± 1 mm/yr, N35 ± 3°W [e.g., *Minster and Jordan*, 1987; *Argus and Gordon*, 1991], show the consistency with NUVEL-1 of the relative motion of sites located in the middle of the Pacific plate (and at Vandenberg) with respect to stable North America plate [e.g., *Sauber*, 1988; *Argus and Gordon*, 1990], and to place constraints on the distribution of deformation across the North America-Pacific plate boundary [e.g., *Ward*, 1988; *Ward*, 1990]. Similar estimates of relative motion in the western U. S. have been derived from recent adjustments of satellite laser ranging observations [*Smith et al.*, 1990]. These results decrease the amount of relative motion west of the San Andreas fault necessary to balance the global plate rate with the contributions from the San Andreas fault and regions to the east. If Basin and Range extension is estimated solely from VLBI rates,

the remaining unexplained relative motion is not significantly greater than zero at the 95% confidence level (6 ± 3 mm/yr, N18°W) [*Argus and Gordon, 1991*].

Geodetic techniques and the tectonics of California have had a long and fruitful association. Space-geodetic techniques provide a useful tool for placing kinematic constraints on the distribution of deformation across the 'wide, soft boundary' between the North America and Pacific plates. These techniques also make their measurements with respect to absolute reference frames that enable them to determine block rotations undetectable by terrestrial geodetic techniques. Block rotations, for example, are likely to play an important rôle in the current deformation of the Transverse Ranges [*Jackson and Molnar, 1990*]. GPS, due to its simple and economical operating requirements and (eventual) continuous access to source signals, is ideally suited for continuously monitoring tectonic motions. The National Geodetic Survey has been coordinating the operation of the Cooperative International GPS Network (CIGNET) of orbit tracking receivers collocated with globally-distributed VLBI systems since late 1986 [*Chin et al., 1987*]. Continuously operating networks have been installed in Japan and California to monitor motions in tectonically active areas [e.g., *Bock and Shimada, 1990*]. Such networks might have the potential for detecting seismic precursors [e.g., *Lisowski et al., 1990b; Bilham, 1991*], or could be useful for studies of coseismic and postseismic slip on faults [e.g., *Lisowski et al., 1990a; Clark et al., 1990; Minster et al., 1990; Hager et al., 1991*]. Space-geodetic techniques not only have greatly improved our abilities to know the 'length and breadth' of the Earth and their rate of change, but the variation of their rate of change, as well.

THESIS PLAN

This thesis is divided into two topics related to the analysis of Global Positioning System data: the derivation of a terrestrial reference frame suitable for monitoring crustal

deformation, and the analysis of GPS observations collected between 1986 and 1991 in central and southern California in order to assess the long-term precision and accuracy of GPS and to estimate relative motions of the sites in the central California monitoring network.

In Chapter 2, we present a methodology for deriving a terrestrial reference frame. Several different types of reference frames are used to model space-geodetic observations. The dynamical equations of motion of a satellite are most easily expressed in a quasi-inertial reference frame whose origin translates with the Earth's center of mass, or geocenter, but whose orientation remains fixed with respect to the stars. On the other hand, the relative positions of geodetic sites are most easily expressed in a terrestrial reference frame that is fixed, in some sense, to the surface of the Earth. The relationship between these two reference frames is a function of Earth orientation parameters, such as precession, nutation, polar motion, and UT1. The relative precision of a baseline derived from GPS measurements is strongly related to the relative errors associated with the satellite orbits, which in turn are typically strongly related to the precision of the relative locations between the tracking receivers used to estimate the orbits. Thus, having precise knowledge of the locations of orbit tracking (or fiducial) receivers, such as those in the CIGNET array, improves the precision of the relative locations of other receivers of interest. To determine the precise relative locations of certain receivers, and hence to define a terrestrial reference frame, we use coordinates derived from laser ranging to LAGEOS and from VLBI measurements. SLR coordinates, whose estimation also requires precise modeling of the orbit of LAGEOS, are sensitive to the location of the geocenter. VLBI coordinates lack all sensitivity to the geocenter (since the radio sources are essentially infinitely distant), but provide highly precise relative locations due to the geometrical simplicity of the technique. By combining the coordinates derived from these two space-geodetic techniques, we can derive a terrestrial reference frame with high-precision relative locations and whose origin is located at a dynamically determined

geocenter. We present a methodology for combining space-geodetic coordinates that also takes into account the relative motions of the coordinates due to plate tectonic forces. We apply this methodology to two recent adjustments of SLR and VLBI observations, and derive a terrestrial reference frame, designated SV6 (SV for SLR/VLBI), suitable for monitoring crustal deformation with GPS.

In Chapter 3, we give a detailed description of a five-year program of GPS observations recently conducted along the central and southern California margin. Two of the objectives of this program are to assess the long-term precision and accuracy of GPS measurements and to establish a network for monitoring crustal deformation. In the first part of Chapter 3, we briefly describe the characteristics of GPS measurements, with an emphasis on potential sources of error, such as orbit estimate, resolution of integer phase ambiguities, and atmospheric effects. The California GPS campaigns are then described. A total of 13 campaigns, each conducted over 4–5 days using between 3–18 receivers in California during 1986–1991, are included in our analysis. The measurements from each single-day ‘session’ are adjusted to estimate station positions, satellite state vectors, atmospheric parameters, and integer ambiguities. The day-to-day root-mean-square scatter of baseline components are used to assess the short-term baseline precision for each campaign. The results from the individual campaigns will then be combined to assess the long-term precision of the GPS measurements and to estimate relative motions for 12 sites with sufficient observations. Finally, we examine the uncertainties associated with the relative motions, and discuss some of the tectonic problems they address related to crustal kinematics in central California.

CHAPTER 2. SV6: A TERRESTRIAL REFERENCE FRAME FOR THE GLOBAL POSITIONING SYSTEM

Ὅμοίως δ' ἀνάγκη καὶ περὶ τόπου τὸν φυσικὸν ὥσπερ καὶ περὶ ἀπείρου γνωρίζειν, εἰ ἔστιν ἢ μή, καὶ πῶς ἐστι, καὶ τί ἐστίν.... Ἐχει δὲ πολλὰς ἀπορίας τί ποτ' ἔστιν ὁ τόπος· οὐ γὰρ ταῦτ' οὐ φαίνεται θεωροῦσιν ἐξ ἀπάντων τῶν ὑπαρχόντων.... δηλοῖ δὲ καὶ τὰ μαθηματικά· οὐκ ὄντα γὰρ ἐν τόπτῳ ὁμῶς κατὰ τὴν θέσιν τὴν πρὸς ἡμᾶς ἔχει δεξιὰ καὶ ἀριστερά, ὥστε ἡόνον αὐτῶν νοεῖσθαι τὴν θέσιν, ἀλλὰ μὴ ἔξιν φύσιν τούτων ἕκαστον.

Aristotle, Physics, IV. i. 208a–b

INTRODUCTION

Space-geodetic techniques determine relative locations on the Earth's surface from measurements to extraterrestrial objects. For example, Global Positioning System (GPS) measurements correspond to ranges from satellites orbiting the Earth to receivers near the ground. To first order, a satellite orbiting the Earth is subject to a central force (due to the spherically symmetric component of the Earth's gravitational field), and therefore follows Keplerian motion; that is, in an "inertial" reference frame its orbit describes an ellipse with one focus at the center of mass of the Earth, or geocenter. The Earth (and hence the stations attached to it) rotates in this reference frame, which translates with the geocenter as it orbits about the Sun. Therefore, two

¹ The Natural Philosopher has to ask the same questions about 'place' as about the 'unlimited'; namely, whether such a thing exists at all, and (if so) after what fashion it exists, and how we are to define it....But we encounter many difficulties when we attempt to say what exactly the 'place' of a thing is. For according to the data from which we start we seem to reach different and inconsistent conclusions....The comparison of mathematical figures illustrates the point. For such figures occupy no real places of their own (not being material bodies at all), but nevertheless acquire a right and left with reference to us, thus showing that their positions are merely such as we mentally assign to them and are not intrinsically distinguished by anything in Nature.

natural reference frames need to be considered in order to theoretically describe the observed GPS ranges: a terrestrial reference frame fixed in some fashion to the Earth's surface, and an inertial reference frame defined in some fashion by the motions of the satellites. The relationship between these reference frames can be described in terms of rotational motions of the Earth, such as precession, nutation, polar motion, and universal time. The fact that the Earth's surface is composed of about twenty quasi-rigid plates, whose long-term relative motions are described by global plate tectonic models, complicates the definition of a terrestrial reference frame 'fixed' to the Earth's surface. The goal of this chapter is to develop a terrestrial reference frame that takes into account these motions and is suitable for monitoring crustal deformation with GPS.

In the idealized case where the forces on the satellites are perfectly known, their positions would be perfectly predictable and their ephemerides would provide a well-defined reference frame suitable for relative positioning. In practice, of course, the forces, especially those due to non-gravitational effects such as solar radiation pressure and atmospheric drag, are incompletely known. Take, for example, the LAGEOS satellite whose high-density, perfectly spherical shape was designed to minimize non-gravitational effects. Integrating its orbit using the best available acceleration model over the span of a year would result in kilometer level discrepancies with the observed laser range observations [Reigber, 1989]; the forces on GPS satellites with far more complicated shapes are more poorly understood. High-precision adjustments of LAGEOS and GPS observations typically utilize orbital arcs with shorter intervals ranging from hours to a month that are estimated from ground observations with *a posteriori* residuals at the decimeter level [e.g., Schutz, 1989; Lichten and Bertiger, 1989; Lichten, 1990c].

The precision of these short orbital arcs depends not only on the fidelity of the force model, but also on the precision of the relative locations of the observation sites.

In fact, it is relatively straightforward to demonstrate that the precision of the estimated orbits is roughly proportional to the precision of the relative ground locations, and hence to the terrestrial reference frame. To demonstrate this ‘rule of thumb’, consider the range r_i from station i on the Earth’s surface to a satellite at distance ρ from the geocenter (Figure 2.1):

$$\begin{aligned} r_i^2 &= (\rho - R \cos \theta_i)^2 + (R \sin \theta_i)^2 \\ &= \rho^2 \left[1 + \left(\frac{R}{\rho} \right)^2 - 2 \left(\frac{R}{\rho} \right) \cos \theta_i \right] \end{aligned} \quad (2.1)$$

where R is the radius of the Earth and θ_i is the geocentric angle between the station and the satellite. Given $\rho \cong 26500$ km for GPS satellites and a mean radius of the Earth $R \cong 6371$ km yields $R/\rho \cong 1/4$. Ignoring terms of order $(R/\rho)^2$ for purposes of demonstration:

$$\begin{aligned} r_i &\cong \rho \left(1 - \frac{1}{2} \cos \theta_i \right)^{1/2} \\ &\cong \rho \left(1 - \frac{1}{4} \cos \theta_i - \frac{1}{32} \cos^2 \theta_i - \dots \right) \end{aligned} \quad (2.2)$$

where the second expression makes use of the binomial series expansion.

One of the observations a GPS receiver measures is the phase ϕ of the satellite radio signal (Chapter 3 will provide a more thorough discussion of the GPS radio signal and observables); ignoring error sources such as receiver clock drift, the difference in phase $\Delta\phi$ measured at two stations on the Earth’s surface (Figure 2.1) can be written:

$$\Delta\phi = \frac{\omega}{c} (r_1 - r_2) \quad (2.3)$$

where ω is the frequency of the radio signal and c is the velocity of light. Using (2.2) and ignoring higher order terms:

$$\begin{aligned}\Delta\phi &= \frac{\omega\rho}{4c}(\cos\theta_2 - \cos\theta_1) \\ &= -\frac{\omega\rho}{2c} \sin\left(\frac{\theta_2 + \theta_1}{2}\right) \sin\left(\frac{\theta_2 - \theta_1}{2}\right)\end{aligned}\quad (2.4)$$

Assuming for simplicity that the satellite, geocenter and both stations separated by a distance l are coplanar (the two-dimensional case illustrated in Figure 2.1), it follows that $\sin\left(\frac{\theta_2 - \theta_1}{2}\right) = \frac{l}{2R}$, and letting $\bar{\theta} = \left(\frac{\theta_2 + \theta_1}{2}\right)$ be the mean geocentric angle between the baseline and the satellite:

$$\Delta\phi = -\frac{\omega \sin\bar{\theta}}{4cR} l\rho = kl\rho \quad (2.5)$$

where k is constant.

Assuming that the measured phase difference $\Delta\phi$ is held fixed, an error in orbital radius $d\rho$ is then related to an error in baseline length dl by:

$$d\rho = \frac{\Delta\phi}{k} d\left(\frac{1}{l}\right) = -\frac{\Delta\phi}{k} \frac{1}{l^2} dl \quad (2.6)$$

which upon substituting (2.5) yields the previously stated ‘rule of thumb’:

$$\frac{d\rho}{\rho} = -\frac{dl}{l} \quad (2.7)$$

The preceding demonstration is, of course, a simplification. In particular, the accuracy of baseline estimates can be improved with additional observations of the satellite and other satellites over time. The additional geometric strength gained by using the 7 Block I GPS satellites from recent analysis of GPS orbit determinations indicate that the relationship is $dl/l \approx 0.2 dp/p$ [e.g., *Lichten and Bertiger, 1989; Lichten, 1990b*]. Currently, the ephemerides for the Block I GPS satellites determined and updated weekly by the Department of Defence and broadcast by the satellites provide about 1 part in 10^6 precisions on relative baselines, which is inadequate for monitoring over reasonably short time intervals crustal strains at levels of 1 part in 10^7 or less per year. One method used to obtain higher accuracy GPS satellite orbits, called the *fiducial network concept*, establishes a terrestrial reference frame suitable for estimating improvements to orbits and baselines [e.g., *Davidson et al., 1985*]. In this approach, a number of widely-separated GPS receivers operate at sites whose locations have been determined independently by other high-precision geodetic techniques, such as VLBI and SLR. The accuracy of the satellite ephemerides estimated from observations at these fiducial stations will then be roughly proportional to the accuracy of the relative baselines between the stations, which for continental-scale VLBI-derived baselines are currently estimated with precisions of 2–4 parts in 10^9 [e.g., *Clark et al., 1987; Davis et al., 1988*]. Furthermore, any estimate of baselines between non-fiducial stations utilizing the higher-accuracy satellite orbits will be similarly improved. Thus, the fiducial network concept provides a method for transferring high-precision space-geodetic baseline information via satellite ephemerides to regional-scale baselines, such as those determined in California.

The advantages gained from having fiducial site coordinates with high-precision relative locations provide the motivation for our derivation of a terrestrial reference frame. The remainder of this chapter will be organized as follows. First, we will

review the basic concepts regarding the definition and realization of geodetic reference frames. We will then investigate the particular requirements necessary to define a terrestrial reference frame appropriate for GPS analysis. The principal requirements are that it have a geocentric origin which we shall obtain from SLR observations, that it have high-precision relative positions which we shall obtain primarily from VLBI observations, and that it accounts for relative motions of the coordinates which we shall obtain from a combination of VLBI estimates and geological information. We will then examine recent VLBI and SLR adjustments, and present a methodology to self-consistently combine them that takes into account the full covariance of their observations and the differences in their respective reference frames. The resulting terrestrial reference frame, designated SV6 (for SLR/VLBI), will then be compared with other standard reference frames. Finally, sensitivity of GPS observations to errors in the parameters used to define SV6 will be evaluated.

REFERENCE SYSTEMS AND FRAMES

The development of modern space-geodetic techniques during the last thirty years, with their capability to monitor with high precision the rotation of the Earth and crustal deformations, has led to significant improvements in the accuracy of reference coordinate systems. Many of the general results of these investigations are summarized by *Mueller* [1981, 1989] and by *Kovalevsky et al.* [1989]. It is generally agreed that only two reference coordinate systems are necessary for space-geodetic analysis: a conventional inertial system that might be defined kinematically by extragalactic radio sources or dynamically by the ephemerides of planetary bodies or Earth satellites, and a conventional terrestrial system that is fixed in some fashion to the surface of the Earth. In the following discussion, we will adhere to the terminology defined by *Kovalevsky and Mueller* [1981, 1989] to describe a hierarchy of reference

systems. An *ideal reference system* is a general statement of a theoretical ideal that can not in practice be perfectly realized or constructed. A *conventional reference system* is the set of models and theories that are adopted in an attempt to realize the ideal reference system. A *conventional reference frame* consists of the explicit set of parameters that are used to realize a conventional reference system. A *conventional terrestrial reference frame* is typically specified by an set of site coordinates and velocities that are accessible to geodetic measurement systems; the terrestrial reference system is thus realizable via these sites. The goal of this chapter, therefore, is to derive a set of coordinates that define a terrestrial reference frame that is a realization of a terrestrial reference system. We now examine what terrestrial reference systems are appropriate for the analysis of GPS observations.

Terrestrial Reference Systems for GPS

If the Earth were a non-deforming radially-symmetric sphere, the definition of an ideal terrestrial reference frame would be straightforward. A natural Cartesian coordinate frame would have its origin at the Earth's center of mass whose relative location would not vary with respect to the crust, would orient one axis along the rotation axis whose instantaneous position with respect to the crust would not change, would orient another axis towards an arbitrary origin of the longitudes (e.g., the Greenwich Mean Astronomic Meridian), and would complete a right-handed system with the remaining axis. To first order, this definition is followed for the deforming, non-symmetric Earth, although more elaborate models must be specified to provide precise locations of positions on the surface. Useful discussions on some of the theoretical aspects of defining a terrestrial reference system can be found in *Lambeck* [1988] and *Kovalevsky et al.* [1989].

The current international standard for conventional terrestrial reference frames, defined by the International Earth Rotation Service (IERS), is described in *McCarthy*

[1989] and *Boucher and Altamimi* [1989], and will generally be followed in the derivation of the SV6 terrestrial reference frame, except where noted. The principal characteristics of the standard are:

a) Geocentric origin (including the oceans and the atmosphere)

The natural origin for a terrestrial reference frame to be used with satellite geodetic techniques is the Earth's center of mass, or geocenter, since to first order it occupies one of the foci of the ellipse described by a Keplerian orbit. The relative motions of the geocenter with respect to the crust of the Earth, due to seasonal variations of the oceans and the atmosphere, are probably less than 1 cm [*Stolz*, 1976a,b; *Stolz and Larden*, 1979]. The most useful determination of the geocenter location for GPS is provided by the consistency of the orbital dynamics of other near-Earth satellites. For SV6, the geocenter will be derived from satellite laser ranging to LAGEOS under the assumption that the 6000-km altitude orbit of LAGEOS is sensitive to approximately the same geocenter as the GPS satellites.

b) Scale for local Earth frame, in a general relativistic sense

Space-geodetic techniques are sensitive to the scale of the reference frame in which they observe. For VLBI, scale is principally determined by the speed of light, which defined to be 2.99792458×10^8 m/s by international convention [e.g., *Cohen and Taylor*, 1987]. For SLR, the scale is primarily determined by the speed of light and the value for the geocentric constant of gravitation, GM , if it is not estimated dynamically [*Zielinski*, 1981]. Propagation delay and time models for SLR and VLBI observables have different general relativistic formulations that can affect the scale of the reference frame, depending on whether a geocentric and solar system barycentric convention is adopted. For example, geocentric or barycentric estimates of GM have a scale difference of 1.5×10^{-8} [*Ries et al.*, 1989]. A similar correction applied to VLBI

observations, plus an additional term to account for propagation delays due to the mass of the Earth, lengthened estimated baselines by 6×10^{-9} scale difference [Caprette *et al.*, 1990]. Theoretically, the SLR sensitivity to GM would suggest that the VLBI scale be adopted; however, scale can also be affected by atmospheric propagation delays. The electromagnetic waves at the radio frequencies used by VLBI are strongly dispersive in the ionosphere and experience propagation delays due to both the ‘dry’ gases and water vapor in the troposphere [e.g., Davis, 1985]. The optical-wavelength laser pulses used by SLR experience dry-air refraction delays, but are insensitive to ionospheric effects and are much less sensitive to water vapor [e.g., Abshire and Gardner, 1985]. The GPS technique combines satellite dynamics with radio-frequency observations and, therefore, is sensitive to the speed of light, GM , and radio-frequency atmospheric effects. A recent comparison by Ray *et al.* [1991] using VLBI and SLR coordinates similar to those we shall use to derive SV6 estimated a small difference in scale of $5.4 \pm 0.4 \times 10^{-9}$. They argued that the difference was not statistically significant due to systematic errors that are introduced by the global distribution of the SLR and VLBI sites that were compared, and because the intrinsic accuracy of each system is on the order of $1\text{--}2 \times 10^{-9}$. For the derivation of SV6, we shall adopt the SLR scale and will defer further discussion of scale differences until after we have performed a similar comparison of VLBI and SLR coordinates.

c) BIH orientation at 1984.0

Ideally, the orientation of a reference frame for GPS should be consistent with both the inertial reference frame dynamically defined by the satellite orbits and the Earth rotation parameters (precession, nutation, polar motion, and universal time) that are used to translate between the two frames. The satellites dynamically define a quasi-inertial reference frame whose geocentric origin experiences accelerations about the Earth-Moon and Earth-Sun barycenters, but whose orientation remains fixed with

respect to the celestial sphere. The gravitational masses in the solar system are generally considered small enough that their effects can be added as perturbations to an ideal inertial reference frame formulation for the equations of motion of the satellite. Since the satellites are sensitive to other masses in the solar system besides the Earth, a natural inertial reference frame for GPS would be dynamically defined by planetary ephemerides [e.g., *Williams and Standish*, 1989]. At present, little effort has been made to use a completely self-consistent set of inertial reference frames and Earth rotation parameters. Therefore, we shall adopt the IERS convention for the derivation of SV6. The current IERS terrestrial reference frames (e.g., ITRF89) is derived from the orientation of the Bureau International de L'Heure (BIH) 1984.0 system, and is consistent at 0.003" in orientation [*IERS*, 1990]. The BIH systems were adjusted with respect to the Conventional International Origin (CIO) in 1967; the consistency of the BIH system with the CIO is approximately 0.03". The CIO was defined as the mean position of the instantaneous rotation axis between 1900 and 1905 as observed by five observatories in the International Latitude Service (see, *Moritz and Mueller* [1987] for a discussion regarding the evolution of the CIO into the BIH 1984.0 system).

d) No global net rotation or translation over time of the crust

The surface of the Earth is not static, but deforms due to a number of causes that include plate tectonic motion and deformation, postglacial rebound, and tidal loading [e.g., *Jordan and Minster*, 1988]. The relative motion of the coordinates implies that the definition of the terrestrial reference frame, which is specified by the coordinates, will evolve over time. One possible approach towards minimizing the evolution of the reference frame, would be to align the axes with the principle moments of inertia. However, this definition is not realizable in practice. Another definition, called *Tisserand mean axes*, specifies the velocities such that the linear momentum $\mathbf{p}$ and

angular momentum $\mathbf{h}$ over some volume of the Earth with mass M , such as the lithosphere or crust, sum to zero:

$$\begin{aligned}\mathbf{p} &= \int_M \mathbf{v} \, dM = 0 \\ \mathbf{h} &= \int_M \mathbf{x} \times \mathbf{v} \, dM = 0\end{aligned}\tag{2.8}$$

where $\mathbf{x}$ and $\mathbf{v}$ are the position and velocity of the mass element dM , respectively. This is the standard adopted by IERS; that is, an ideal terrestrial reference system should have no net global rotation or translation with respect to the Earth's crust. This definition, of course, cannot be achieved in practice either. However, maintaining a geocentric origin presumably minimizes the linear momentum of the crust. To minimize the angular momentum of the crust, the no-net-rotation absolute plate tectonic motion model AM0-2 [Minster and Jordan, 1978] is adopted to describe the motions of the coordinates with respect to the reference epoch 1988.0. All other deformations that might be included in the minimization conditions are either periodic (e.g., tidal loading) or not easily specified (e.g., postglacial rebound). The AM0-2 model only requires knowledge of the plate geometry to be derived from the RM2 relative motion model. We shall show how this model and its covariance may be derived in a later section of this chapter.

These are the principal standards used to define the IERS terrestrial reference frame. Two more characteristics are also adopted. The first is that a correction should be applied to the coordinates to account for the permanent solid Earth tidal deformation so that reference coordinates differ from instantaneous coordinates by periodic terms only. The gravitational attractions of the Sun and the Moon induce tidal

deformations on the solid Earth and the mean value of the change in the C_{20} geopotential coefficient is not zero. The permanent deformation due to this is approximately given by:

$$\begin{aligned}\Delta h &= -0.12083 \left(\frac{3}{2} \sin^2 \phi - \frac{1}{2} \right) \\ \Delta n &= -0.05071 \cos \phi \sin \phi\end{aligned}\tag{2.9}$$

where ϕ is the latitude of the site location and Δh and Δn are displacements in meters in the (radial) height and (horizontal) north directions, respectively [McCarthy, 1989]. In practice, most investigators who submit coordinates to IERS do not apply this correction, nor do the IERS terrestrial reference frames seem to include it. We shall not apply this correction either, and we assume that users modeling GPS observations will include the correction in their solid Earth tide calculations. The second characteristic adopted by IERS is to use the GRS 80 reference ellipsoid ($a = 6378137$ m, $1/f = 298.257222$) when converting Cartesian coordinates into a geodetic coordinates [Moritz, 1980].

While we shall retain the above characteristics to be consistent with the IERS terrestrial reference frame standards, we adopt a different methodology to derive SV6 than those used to derive ITRF-0 or its annual updates such as ITRF89 [e.g., Boucher and Altamimi, 1989]. The full covariance of the observations are retained when they are available to provide the correct correlations and relative precision between coordinates. While IERS combines several SLR and VLBI adjustments to estimate coordinates, SV6 is derived from only one SLR and one VLBI adjustment; by choosing only two high-quality coordinate adjustments, we hope to minimize the effect of inconsistent modeling techniques. The most important difference is how we incorporate site velocity information. IERS assumes that all site velocities are exactly described by the no-net-rotation absolute motion model AM0-2. SV6 adopts AM0-2

as the underlying velocity reference frame, but also recognizes that the RM2 relative model is not exact but has uncertainties that are described by a matrix of Euler pole covariances. We derive the AM0-2 Euler pole covariance which is then used to generate an *a priori* site velocity covariance. The assumption of a velocity covariance implies that coordinate uncertainties will grow as the sites are propagated over time with respect to a reference epoch. In recognition of the inhomogeneous observation schedules at VLBI and SLR sites, the mean observation time for a site is adopted as its reference epoch. The *a priori* site velocity covariance is also adjusted to reflect the possibility of deformations not described by global plate tectonic models, such as vertical motions. Finally, independent estimates of site velocity, such as those provided by VLBI observations, are included in the adjustment for SV6.

The development of a methodology to combine uniformly site motion estimates from global plate tectonic models, space-geodetic observations, and other geophysical information follows from the consideration that global plate tectonic models do not always provide an adequate description of relative motions, especially in areas of continental deformation, and is consistent with our goal to derive a terrestrial reference frame with high-precision relative positions.

SPACE-GEODETIC VECTOR POSITION DATA

Three independent sources of data are used to estimate the coordinates and velocities that define the SV6 reference frame. An adjustment of satellite laser ranges to LAGEOS provides the basis for the geocentric origin and scale of SV6. An adjustment of very-long-baseline interferometric group delays provides high-precision relative locations for many sites that are collocated with monuments occupied by GPS. And a set of vector baseline ties between collocated reference points occupied by VLBI and SLR systems makes possible the estimation of a similarity transformation

that, when applied to the VLBI reference frame, makes it consistent with the SLR reference frame so that all coordinates may be combined uniformly. We now describe these three data sets.

Satellite Laser Ranging

The SLR coordinates that define the origin and scale of SV6 were obtained from an adjustment, provided by the Center for Space Research at the University of Texas at Austin, of LAGEOS laser range observations for the interval from May 1976 to February 1989. The coordinates were submitted to International Earth Rotation Service (IERS), which is responsible for defining and maintaining a standard set of conventional terrestrial and celestial reference frames and Earth rotation series based on such submissions, and are described under the designation *CSR 89 L 02* (hereafter CSR8902) in IERS Technical Notes 2 and 5 [Schutz *et al.*, 1989; Eanes *et al.*, 1990]. The adjustment was performed on about 360,000 three-minute normal point ranges from over 37,000 passes collected at more than 100 sites. The geocentric Cartesian position and formal errors for 94 sites were estimated.

In general, the force and measurement models used to calculate the theoretical ranges followed the MERIT standards [Melbourne *et al.*, 1983], which were the basis for the current IERS standards [McCarthy, 1989]. The preliminary gravity field PTGF-3, developed for the TOPEX satellite [Tapley *et al.*, 1987], was used with an adopted value of GM equal to $398600.4404 \text{ km}^3/\text{s}^2$ which was dynamically estimated from the orbits of 14 near-Earth satellites in a general relativistically-defined geocentric reference frame [Reis *et al.*, 1989]. Some ocean tide parameters were adjusted in the solution. The sites were assigned to tectonic plates and were assumed to have absolute motions described by the hot spot model AM1-2 [Minster and Jordan, 1978]. The permanent tide correction was not applied.

The adjustment was performed by the UTOPIA orbit and geodetic parameter estimation system [Tapley *et al.*, 1985]. The adjustment was performed in two steps. First, adjustments were estimated to the initial satellite state vector, along-track acceleration parameters, and ocean tide parameters using the entire 12.8 year data set while station positions and velocities and Earth orientation parameters were fixed at *a priori* values. The weighted root-mean-square (rms) residual of the laser ranges for this long-arc solution was 280 mm. Then, adjustments were estimated to the orbit, Earth orientation parameters, and station positions from shorter arcs whose interval varied from 15 days during 1976–1979, to 6 days during 1980–1983, to 3 days during 1983–1989. The long-arc estimates of the node of the satellite orbit and ocean tide parameters, as well as the station motions given by AM1-2 were held fixed in these short-arc solutions. Because errors in the node are strongly correlated with UT1, the UT1 values were filtered so that at periods greater than 80 days the values correspond to the *a priori* series provided by IERS, while adjustments to UT1 are estimated at shorter periods. The weighted rms residuals of the laser ranges for the shorter-arc solution was 85 mm, although during the latter two intervals 1980–1989, it was 40–50 mm. The geocenter was unconstrained in the adjustment and subsequent analysis of 60-day short-arc solutions indicate that the geocentric determination is stable at the 50 mm level [R. J. Eanes, personal communication, 1990]; note that a 50 mm offset of the geocenter is roughly equivalent to 2×10^{-9} change in scale for GPS orbital errors following the rule-of-thumb given in Equation 2.7.

Several characteristics of the CSR8902 adjustment have a bearing on the development of the SV6 terrestrial reference frame. The adjustment supplied by the Center for Space Research only gives the geocentric Cartesian coordinate positions and their formal errors. No correlations, either between different sites or between the components of a site position, were provided. This lack of correlation means that the local horizontal and vertical components of the formal errors for a site, which from

geometrical considerations ought to show greater uncertainty in the local vertical, cannot be fully determined. Also, the precision on relative baselines between sites cannot be correctly calculated. Incorporating correlations typically increases formal errors; however, the opposite is generally true if intersite correlations are included when deriving relative VLBI baselines from absolute positions. Because relative baselines are the most robustly estimated quantities using VLBI, the incorporation of intersite correlations tends to decrease the formal errors on relative baseline vector components and length. Relative coordinates are also probably more robustly estimated than absolute coordinates using SLR observations, although magnitude of this effect may be lessened by more global nature of the SLR adjustments imposed by the use of satellite dynamics. If relative SLR coordinates are more precise than absolute coordinates, the lack of intersite correlations will allow the SLR coordinates to adjust more easily. This will tend to give greater weight to the relative precision of VLBI baselines, which are derived from an adjustment that includes the full position covariance.

The CSR8902 adjustment did not estimate site motions over the 12.8 year interval. Instead the hot spot absolute motion model AM1-2 was used to propagate the sites and all site positions were estimated at the 1983.0 reference epoch. While SV6 adopts the no-net-rotation absolute motion model AM0-2 to describe the underlying temporal evolution of the reference frame, the difference between the absolute motion models is not significant because the *relative* motions of both AM1-2 and AM0-2 are the same and are given by RM2. The difference between their *absolute* motions is simply a rigid-body rotation given by the vector subtraction between the Euler poles for an arbitrary plate in each model. This implies that the orientation of the frames will not evolve in the same manner. However, the orientation of the CSR8902 reference frame is already somewhat arbitrary in all three components [R. J. Eanes, personal communication, 1990]. The measurements are not sensitive to the origin of the

longitudes and, rather than fix the longitude of a single site, *a priori* constraints were applied to all sites. The UT1 series is fixed at the IRIS VLBI values at periods greater than 80 days while *a priori* constraints were applied to the shorter periods. Both of the longitude and UT1 constraints make the the orientation of CSR8902 poorly-defined. Because GPS adjustments that utilize fiducial networks are not sensitive to the orientation of the terrestrial reference frame to first order, we have decided to apply a rigid-body rotation to CSR8902 so that SV6 will be consistent with an international standard provided by the IERS terrestrial reference frame (e.g., ITRF89) [IERS, 1990; Boucher and Altamimi, 1991].

Assuming AM0-2 (or, equivalently, AM1-2) site motions will degrade the accuracy of baselines between sites whose true relative motion is poorly represented by RM2 as the coordinates are propagated over time. Since site motions were not estimated, the formal errors of the site coordinates also remain constant over the 12.8 yr interval of observations. Obviously, however, the measurements were not collected uniformly over that interval as different laser ranging systems were used at different times and new sites were introduced. To minimize some of the possible biases that might be introduced by this heterogeneous sampling, we have propagated the site coordinates, assuming AM0-2 motion, from the CSR8902 1983.0 reference epoch to the mean epoch of observation for each site provided by the Center for Space Research [R. J. Eanes, personal communication, 1990], and which varied from 1980.9 to 1990.0 with a mean value of 1987.5 (Table 2.1). The formal uncertainty of the site coordinates were associated with their mean observation epochs; these uncertainties will then be increased according to the uncertainties of the estimated site velocities as the coordinates are propagated to other epoches. The mean epoch coordinates of 94 sites with their formal errors from the SLR observations used to derive SV6, and their approximate locations are shown in Figure 2.2.

Very-long-baseline Interferometry

The VLBI coordinates that add high-precision relative baseline constraints to SV6 were obtained from an adjustment, provided by the Goddard Space Flight Center, of group delay observations derived from extragalactic quasar radio signals that were collected during the interval from 1979 to 1989. The coordinates and a detailed description of the adjustment are described by *Caprette et al.* [1990]. The adjustment, designated GLB659, includes 472691 group delay measurements from 76 radio sources collected during 1051 observation sessions at 80 fixed and mobile VLBI sites. The group delay and delay rate measurements were obtained using Mark III instrumentation described by *Rogers et al.* [1983] and *Clark et al.* [1985]. In addition to the geocentric Cartesian positions for the 80 sites, the observations from 51 sites were of sufficient quantity and duration to enable the velocity of the site to be estimated. The approximate locations of the 80 sites are shown in Figure 2.3.

In general, the measurement models used to calculate the theoretical group delays followed the IERS standards [McCarthy, 1989]. As in the SLR adjustment, the permanent tide correction was not applied. Daily polar motion and UT1 values, and daily offset corrections to the IAU 1980 nutation model were estimated. General relativistic delay and delay rate models were calculated from the Hellings model [Hellings, 1986] with corrections to make the reference frame geocentric in a general relativistic sense, consistent with the models used in the SLR adjustment, and to take into account propagation delays due to the mass of the Earth. Inverse frequency-squared propagation delay effects caused by the ionosphere and extragalactic plasma were eliminated by taking linear combinations of the X-band (~8.4 GHz) and S-band (~2.2 GHz) observations. Propagation delays due to the hydrostatic troposphere were calibrated with the Saastamoinen model for zenith delay [Saastamoinen, 1972] and mapped to other elevations by the CfA 2.2 model [Davis et al., 1985]. The uncalibrated (i.e., ‘wet’) tropospheric delay at each site was modeled by a continuous,

piecewise-linear function of the residual tropospheric zenith path delay with about 60-minute interval linear segments. No water vapor radiometer observations were used.

The adjustment was performed by the SOLVE/GLOBL geodetic parameter estimation system [e.g., *Caprette et al.*, 1990; *Ma et al.*, 1990]. The adjustment was performed by combining single observation session “arc” parameters, such as UT1, polar motion, daily nutation corrections, clock and atmospheric parameters sequentially with “global” parameters that are affected by more than observation session, such as quasar source positions, site positions and selected site velocities. A series of adjustments were performed to establish preliminary celestial reference frames and Earth orientation parameters from the fixed and some mobile VLBI sites and to translate the geocenter-insensitive VLBI frame to the origin of the IERS terrestrial reference frame ITRF89 [*IERS*, 1990; *Boucher and Altamimi*, 1991]. The final source positions and Earth orientation parameters (415 global and 158593 arc parameters) were determined from 370531 group delays in 848 observing sessions at the fixed and some mobile sites with a weighted rms of 45.6 ps. The final adjustment, denoted GLB659, incorporated all the remaining mobile VLBI observations while applying *a priori* constraints to the previously estimated global parameters. A total of 635 global and 46655 arc parameters were estimated from 102160 group delay measurements with a weighted rms of 42.6 ps. Although the reduced χ^2 was 1.01, indicating that the model and data covariance assumptions are consistent with the *a posteriori* residuals, we have multiplied the position and velocity covariance matrix by 2.0 to more accurately reflect true uncertainties [J. Ray, personal communication, 1990]; this scaling is consistent with the long-term scatter of VLBI estimated positions about their linear trends as described in *Caprette et al.* [1990]. We will use the GLB659 site positions and velocities to estimate SV6 terrestrial reference frame coordinates.

Several characteristics of the GLB659 adjustment have a bearing on the development of the SV6 terrestrial reference frame. Correlations between all the position and estimated velocity parameters were provided. Thus, errors in site coordinates can be correctly decomposed into local horizontal and vertical components, and the precision of relative baselines can be correctly calculated. As noted previously, the formal uncertainties of relative positions, which are more robustly estimated than absolute positions using VLBI, tend to decrease when intersite correlations are incorporated. The no-net-rotation absolute motion model AM0-2 was used to derive the *a priori* motions for each site, and 52 sites had sufficient observations to independently estimate the velocity of the site. AM0-2 velocities were assumed for remaining 28 sites. As with the SLR coordinates, we have propagated the site coordinates from the GLB659 1988.0 reference epoch to the mean epoch of observation for each site. For sites with insufficient observations to estimate their relative motions the mean observation epoch was estimated from the occupation history provided in *Caprette et al.* [1990]. For sites with velocity estimates, we used the information contained in the full position-covariance matrix by assuming that the mean epochs of observation correspond to the times when the position and velocity for each Cartesian component are uncorrelated (at other times, the correlation between position and velocity increases). The mean epochs of observation are given in Table 2.1.

Since both positions and velocities were estimated, constraints must be placed on both the origin and evolution of the origin in order to maintain a well-defined reference frame for the GLB659 adjustment. The origin constraints were applied by fixing the *a priori* coordinates of Westford. The constraints imposed on the evolution of the origin were decomposed into horizontal and vertical components. The horizontal constraint was defined by requiring the direction from Westford to Richmond to change according to the AM0-2 model. The vertical constraint was defined by fixing the vertical rate of

Kauai to zero. These constraints imply that the full position-velocity covariance is not positive definite and has a rank 6 deficiency. Since the origin and its evolution of SV6 will be derived from the SLR adjustment, we shall apply a transformation to the GLB659 covariance to allow its origin and evolution to have a large uncertainty and thus able to adjust without constraints with respect to the SLR reference frame.

Collocation Vector Ties

The SLR and VLBI coordinates are derived in different reference frames. In order to combine the two data sets together uniformly, we determine a seven-parameter similarity transformation to apply to the VLBI coordinates that will make them commensurate with the SLR coordinates. The similarity transformation will be estimated by comparing reference points that are common to both systems. Here a reference point is considered to be either the location in a geodetic measuring system to which all observations are referred, such as the intersection of axes in a VLBI antenna, or to a ground monument whose offset (eccentricity) with respect to the measurement system reference point is known. The CSR8902 and GLB659 adjustments have six reference points that were occupied by both SLR and VLBI systems: Goddard 7102, Haleakala 7120, Monument Peak 7274, Mojave 7288, Owens Valley 7853, and Yuma 7894. The four-digit numbers are the monument numbers assigned by the NASA Crustal Dynamics Project (CDP) and can be found in *CDP Catalogue of Site Information* [CDDIS, 1989]. All of these reference points were occupied by mobile VLBI and SLR systems and all, with the exception of Yuma, are in the vicinity of a fixed SLR or VLBI system with much longer occupation histories. In general, a fixed system can be located with greater precision than a mobile system due to both the increased number of observations and the often superior precision of the fixed measurement system. Local geodetic surveys can provide well-determined (millimeter level) vector baseline ties between collocated systems that allow us to

take advantage of the superior systems and to increase the number of common reference points. Approximately 18 sites have such vector baseline ties between two or more collocated reference points (Figure 2.4).

To provide a useful constraint to the estimate of the similarity transformation parameters, collocation ties should satisfy several conditions. Both the VLBI and SLR reference points should be well determined. In some cases, particularly for occupations by early measurement systems, the formal uncertainties on a site position may not accurately reflect the true uncertainties. For example, the coordinates for both Pasadena 7896 and Flagstaff 7891 were derived from mobile SLR measurements during the early 1980's, and comparisons with collocated VLBI reference points show significant discrepancies. Neither of these ties were used to estimate the similarity transformation. Some site occupations have technical problems that degrade their accuracy. For example, the laser system located at Wettzell 7834 is known to have had a timing problem that was not completely eliminated in the CSR8902 adjustment and may be responsible for a bias in the local vertical location of the reference point [R. Eanes, personal communication, 1990].

The local survey tie should also be well determined. Adjustments of high-precision terrestrial geodetic measurements (triangulation, trilateration, and spirit leveling) are frequently capable of determining baselines shorter than a kilometer between ground monuments to millimeter precision. We determined some of the survey ties by performing adjustments using the HAVAGO algorithm [Vincenty, 1979] from terrestrial observations provided by V. Nelson of the Bendix Field Engineering Corporation. Many of the ties can be found in the *CDP Catalogue of Site Information* [CDDIS, 1989]. In one case, a tie was estimated from GPS surveys conducted in the vicinity of Tidbinbilla 1642 and Orroral 7843. The scatter of the GPS estimates for this 26 km baseline tie are several centimeters in all components [T.

Morrison, personal communication, 1990]. The ties and their sources are given in Table 2.2.

Caution must be applied, however, when a local geodetic survey is used to determine the relative location of the internal reference point of a fixed SLR or VLBI system. Since internal reference points usually cannot be directly observed, such surveys often conduct observations to exterior locations on the measurement system and then use engineering drawings to determine the offset between the assumed location of the reference point and the exterior location. This inability to directly observe the reference point can lead to gross blunders due to the lack of redundancy in the adjustment and has prompted the development of several other methods to estimate its location. VLBI antennas that are capable of pointing to radio sources over a large range of azimuths and elevations typically have mounts with two orthogonal rotation axes that may or may not intersect. The reference point for the antenna is defined to be located at the intersection between the fixed axis and the plane perpendicular to the fixed axis containing the moving axis. The geometry of the rotation axes constrain the possible motions of any point located on the antenna. If a geodetic target (e.g., retroreflector) is attached to the antenna, the locus of the possible target positions defines a sphere if the axes intersect or a torus if the axes are offset. Terrestrial observations to the target combined with the geometrical constraints on its motion, can therefore be used to determine the antenna reference point and the offset between the axes. Appendix A describes the application of this technique to correct a 30 centimeter vertical mislocation of the Kauai 1311 reference point.

The second method that has been used to determine the relative location of fixed VLBI reference points is to obtain simultaneous measurements with a mobile VLBI system occupying a nearby ground monument. Analysis of the phase or group delays gives a direct measurement from the fixed VLBI reference point to the mobile VLBI

reference point and hence to the ground monument via a presumably well-determined eccentricity offset. During 1987–1988, the Crustal Dynamics Project conducted a number of such surveys. These included a combined survey with mobile systems occupying Owens Valley 7853 and Mojave 7288, a survey with a mobile system successively occupying Ft. Davis 7900 and McDonald 7850, and an occupation on Haleakala 7120 to provide a tie between the SLR systems on Haleakala and the VLBI system on Kauai 1311. The preliminary results of these surveys, provided by J. Ray of Interferometrics, Inc., were adjusted using a different model than that used for GLB659. In particular, the relativistic corrections discussed in the previous section were not applied leading to a 6×10^{-9} change in scale between the systems. Since these surveys are the only mobile VLBI occupations of these monuments, and all the measurements (or at least the group delays) are included in the global adjustment GLB659, we have estimated the baselines from GLB659, after propagating the coordinates to the time of the survey to avoid possible discrepancies between estimated and assumed velocity models.

The different methods used to determine the local survey ties listed in Table 2.2 all have measurement uncertainties associated with them. We have tried to retain as much information as possible regarding these uncertainties. In some cases, the survey data was provided by V. Nelson of Bendix Field Engineering Corporation so that we could perform the HAVAGO adjustment to determine the full baseline covariance. In most cases, only the Cartesian formal errors are available. In some cases, no information is available so an error was assumed, depending on the type of survey that was conducted. The distribution of the collocated sites is not geographically uniform, with the largest concentration in North America, followed by Europe and the Pacific (Figure 2.4). The estimated transformation parameters are critically sensitive to the global distribution of sites so that widely separated sites, such as those in Hawaii and Australia, will contribute more information to the

solution. We have explored different methods for giving equal weight to different geographical regions and for minimizing the influence of sites believed to have poorer data, such as at Wettzell 7834 due to timing problems [e.g., *Murray et al.*, 1990], but we have not implemented these methods in the derivation of SV6.

UNIFORM ESTIMATION OF TERRESTRIAL REFERENCE FRAMES

Introduction

The goal of this section is to present a method to estimate the coordinate positions and velocities that comprise the conventional definition of a time-varying terrestrial reference frame. The conventional definition is one realization of a theoretical time-varying terrestrial reference frame that is specified by its scale, origin, orientation, and temporal evolution. As stated before, the conventional positions and velocities will be derived from adjustments of satellite laser ranging (SLR) and very-long-baseline interferometry (VLBI) observations, local vector ties between collocated monuments that have been occupied by SLR and VLBI measurement systems, and geophysical information, such as global plate tectonic models. The methodology takes into account the full covariance of these observations and *a priori* models. All positions, velocities, and other parameters necessary to realize the conventional definition are estimated simultaneously.

Adjustments of SLR and VLBI observations are generally performed in different reference frames as defined by the origin, scale, and orientation of their axes. To make these frames commensurate, we estimate a seven-parameter similarity transformation between the them. Since SLR is sensitive to the Earth's center of mass, we will assume the SLR reference frame defines the origin of our frame. Since the orientation of the axes is somewhat arbitrary, we will initially assume the SLR reference frame and then rotate the estimated reference frame to be consistent with the International

Earth Rotation Service (IERS) terrestrial reference frame. The scale difference between the SLR and VLBI systems is small but significant. Since no satisfactory explanation has been found for this difference, and neither system is demonstrably superior, we will arbitrarily assume the SLR scale. Finally, we shall adopt the AM0-2 no-net-rotation plate tectonic model, consistent with IERS standards, as the underlying description of the time evolution of the reference frame.

High-precision observations have been made using SLR and VLBI measurement systems over the last decade, and for many of these sites the observations are sufficient to provide estimates of relative site motions [e.g., *Clark et al.*, 1987; *Smith et al.*, 1990]. For adjustments of SLR and VLBI data that span several years, the precision of the measurements now necessitates an assumption of an underlying velocity model in order to minimize biases introduced by the relative motions. Usually, the AM0-2 no-net-rotation model has been chosen, although the SLR community sometimes uses the related AM1-2 hotspot frame. Generally, these absolute motions are adopted for sites that do not have sufficient observations to estimate velocities with respect to the underlying velocity frame. However, the assumption of an incorrect velocity, such as for sites located in deformation zones, will progressively degrade the accuracy of a site location as it is propagated with respect to its mean epoch of observation, which varies from site to site. In an effort to more accurately assess the location uncertainty for such a site, we shall assume that the adopted AM0-2 velocity has an uncertainty proportional to one derived from the covariance of the AM0-2 Euler pole for the tectonic plate appropriate for the site. The position uncertainty therefore will increase as the site is propagated with respect to its mean epoch of observation.

Adjustments of SLR and VLBI observations must make assumptions regarding the definition of the reference frame. In addition to assuming an absolute motion model, they also must assume an origin. This can be done either by fixing the location

of a site, or by placing a rank three constraint on several sites. These assumptions tend to result in singularities in the site position covariance matrix that must be accommodated in order to perform least square estimation procedures. In the present example, the SLR adjustment fixed the average location of all the sites and only included the formal errors on the Cartesian positions without any correlations. Lacking the correlations, the SLR position covariance appears to be non-singular. On the other hand, the VLBI adjustment assumed that the position of the Westford VLBI reference point was fixed and did included the full correlation matrix. The resulting covariance matrix is singular, not only due to the fixing the origin, but also due to several other imposed constraints on the site velocities. To overcome these singularities, an origin and absolute motion uncertainty will be assumed and covariance matrix was inflated by quadratic propagation of these uncertainties. These operations do not affect the relative precision of the baselines. Assuming an origin uncertainty is appropriate for the VLBI observations, since VLBI is not sensitive to the origin, and an origin translation with respect to the SLR origin will be estimated.

Site velocities for the adjustments may be partitioned into two sets. The first set corresponds to those sites with insufficient data to adequately estimate relative motion. The *a priori* velocity model is generally adopted for these sites. The AM0-2 no-net-rotation model assumes no radial motions. The SLR adjustment did not estimate relative site motions, and thus AM0-2 is adopted for all sites. The second set corresponds to those sites for which relative motions can be estimated. The VLBI adjustment estimated the relative motions of 52 out of a total of 80 sites (the remaining 28 sites adopt AM0-2 velocities); both horizontal and radial velocity components were estimated. In most cases, the VLBI observations at a site were obtained at more than three epochs and span a sufficiently long interval that the accuracy of the resulting relative motion estimate is comparable or superior to the geological rates. In some cases, however, only two epochs are available and the

uncertainty on the relative motion estimate is large compared with the *a priori* model, especially in the radial component. In an effort to minimize the impact of poorly-resolved estimates, we shall incorporate a *a priori* model and covariance stochastically. Such an assumption should not constrain the robust estimates of relative motion, but should damp the effects of the weaker estimates.

We begin with an examination of the relationship between nearly-equivalent reference frames that lays the groundwork for estimating the transformation necessary to make the VLBI coordinates consistent with the SLR reference frame.

The Bursa-Wolf Similarity Transformation

In general, a linear transformation between two coordinate systems $\mathbf{x}'$ and $\mathbf{x}$ is called an *affine transformation* and can be expressed by:

$$\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{t} \quad (2.10)$$

where $\mathbf{A}$ is an arbitrary 3x3 matrix and $\mathbf{t}$ is an arbitrary 3-dimensional vector. The twelve parameters of $\mathbf{A}$ and $\mathbf{t}$ describing the affine transformation can be divided into two classes: six parameters define an orthogonality transformation (three translation and three rotation parameters), and six parameters define a scaling transformation (three scale parameters along principal axes defined by three orientation parameters) [e.g., *Modenov and Parkhomenko*, 1961]. In geodetic literature, an affine transformation that is defined by an orthogonality transformation combined with a homothetic transformation (one uniform scale parameter defined with respect to the origin) is called a *seven-parameter similarity transformation* (i.e., three translation, three rotation, and one scale parameter). Since there is little reason to believe that the scale is not isotropic for space-geodetic references frames, the similarity transformation is the most commonly adopted model.

Leick and van Gelder [1975] compare several models for definition of the origin and rotation axes in similarity transformation models and favor the use of the *Bursa-Wolf* model [*Bursa*, 1962; *Wolf*, 1963] for global scale networks due to its simplicity and the geometrical significance of t , which describes a translation of the origin. In the Bursa-Wolf model, A is decomposed into a scale parameter s times a rotation matrix R :

$$x' = sRx + t \quad (2.11)$$

A general rotation between coordinate systems can be described by three successive rotations about orthogonal coordinate axes; the magnitudes of the rotations are called *Euler angles* [e.g., *Goldstein*, 1980]. The choice of rotation angles is somewhat arbitrary (the restriction being that no two successive rotations can be about the same axis); for example, one convention frequently used in engineering applications is to take the order of the rotations about the z , y and x axes respectively:

$$R = R_x(\epsilon) R_y(\psi) R_z(\omega) \quad (2.12)$$

where R_x , R_y and R_z are the fundamental orthogonal rotation matrices:

$$R_x(\epsilon) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \epsilon & \sin \epsilon \\ 0 & -\sin \epsilon & \cos \epsilon \end{pmatrix}$$

$$R_y(\psi) = \begin{pmatrix} \cos \psi & 0 & -\sin \psi \\ 0 & 1 & 0 \\ \sin \psi & 0 & \cos \psi \end{pmatrix} \quad (2.13)$$

$$\mathbf{R}_z(\omega) = \begin{pmatrix} \cos \omega & \sin \omega & 0 \\ -\sin \omega & \cos \omega & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The Euler angles ϵ , ψ and ω are defined to be positive for counterclockwise rotation as viewed from the positive end of the rotation axis towards the origin in a right-handed coordinate system (Figure 2.5). The total rotation therefore becomes:

$$\mathbf{R} = \begin{pmatrix} \cos \psi \cos \omega & \cos \psi \sin \omega & -\sin \psi \\ \sin \epsilon \sin \psi \cos \omega - \cos \epsilon \sin \omega & \sin \epsilon \sin \psi \sin \omega + \cos \epsilon \cos \omega & \cos \psi \sin \epsilon \\ \cos \epsilon \sin \psi \cos \omega + \sin \epsilon \sin \omega & \cos \epsilon \sin \psi \sin \omega - \sin \epsilon \cos \omega & \cos \psi \cos \epsilon \end{pmatrix} \quad (2.14)$$

Equations (2.11) and (2.14) represent a uniformly valid Bursa-Wolf model for arbitrary scale changes and finite rotations; however, the VLBI and SLR coordinate systems are nearly equivalent with scale differences on the order of 5×10^{-9} and orientation differences on the order of $0.01''$ (5×10^{-8}). With these small differences, the assumption of infinitesimal rotations and scale changes is valid. Letting $s \equiv 1 + \Delta s$, we assume that $\Delta s \ll 1$, $\epsilon \ll 1$, $\psi \ll 1$ and $\omega \ll 1$ (where the angles are expressed in radians), and it follows that $\sin \epsilon \approx \epsilon$, $\cos \epsilon \approx 1$, etc. Under these assumptions, (2.11) becomes

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = (1 + \Delta s) \begin{pmatrix} 1 & \omega & -\psi \\ -\omega & 1 & \epsilon \\ \psi & -\epsilon & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix} \quad (2.15)$$

The relationship between the coordinates $\mathbf{x}$ and $\mathbf{x}'$, and the parameters describing the similarity transformation in the Bursa-Wolf model (2.15) is non-linear. Letting the estimated model parameters be given by $\mathbf{m} = (t_x, t_y, t_z, \epsilon, \psi, \omega, \Delta s)^T$ and observed data for a single collocation station i be given by $\mathbf{d}_i = (x, y, z, x', y', z')_i^T$, we find that the data and model parameters cannot be explicitly separated (i.e., $\mathbf{f}(\mathbf{m}) - \mathbf{d} = 0$), but

rather a fully implicit, non-linear model (i.e., $f(\mathbf{m}, \mathbf{d}) = 0$) is required. Linearizing an implicit, non-linear model requires partial derivatives of the function f with respect to both the model and data parameters. That is, a Taylor series expansion of f about $\mathbf{m}_k, \mathbf{d}_k$, ignoring second order terms, yields:

$$f(\mathbf{m}_{k+1}, \mathbf{d}_{k+1}) \cong f(\mathbf{m}_k, \mathbf{d}_k) + \frac{\partial f}{\partial \mathbf{m}_k} (\mathbf{m}_{k+1} - \mathbf{m}_k) + \frac{\partial f}{\partial \mathbf{d}_k} (\mathbf{d}_{k+1} - \mathbf{d}_k) = 0 \quad (2.16)$$

where the k refers to an iterated solution. Letting $\mathbf{A} = \partial f / \partial \mathbf{m}_k$ and $\mathbf{B} = \partial f / \partial \mathbf{d}_k$ be the partial derivative matrices evaluated at $\mathbf{m}_k, \mathbf{d}_k$, the system is then related to the *a priori* estimate of the model $\mathbf{m}_0$ and the observed data $\mathbf{d}_0$ by:

$$f(\mathbf{m}_k, \mathbf{d}_k) + \mathbf{A} (\mathbf{m}_{k+1} - \mathbf{m}_0) + \mathbf{B} (\mathbf{d}_{k+1} - \mathbf{d}_0) - \mathbf{A} (\mathbf{m}_k - \mathbf{m}_0) - \mathbf{B} (\mathbf{d}_k - \mathbf{d}_0) = 0 \quad (2.17)$$

or, more simply:

$$\mathbf{w} + \mathbf{A} \delta \mathbf{m} + \mathbf{B} \delta \mathbf{d} = 0 \quad (2.18)$$

where $\delta \mathbf{m} = \mathbf{m}_{k+1} - \mathbf{m}_0$, $\delta \mathbf{d} = \mathbf{d}_{k+1} - \mathbf{d}_0$ and $\mathbf{w} = f(\mathbf{m}_k, \mathbf{d}_k) - \mathbf{A}(\mathbf{m}_k - \mathbf{m}_0) - \mathbf{B}(\mathbf{d}_k - \mathbf{d}_0)$.

Equation (2.18) can be solved by normal least squares techniques [e.g., *Vaníček and Krakiwsky*, 1986]. Letting $\mathbf{C}_d$ be the covariance of the observations, and assuming no *a priori* information on the model parameters (i.e., $\mathbf{C}_m \rightarrow \infty$), the least-square estimate for the $(k+1)$ th perturbations to the model parameters and observation is:

$$\begin{aligned}
\begin{pmatrix} \tilde{\mathbf{m}}_{k+1} \\ \tilde{\mathbf{d}}_{k+1} \end{pmatrix} &= \begin{pmatrix} \mathbf{m}_0 \\ \mathbf{d}_0 \end{pmatrix} - \begin{pmatrix} (\mathbf{A}^T \mathbf{M} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{M} \\ \mathbf{C}_d \mathbf{B}^T \mathbf{M} [\mathbf{I} - \mathbf{A} (\mathbf{A}^T \mathbf{M} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{M}] \end{pmatrix} \{f(\mathbf{m}_k, \mathbf{d}_k) - \mathbf{A}(\mathbf{m}_k - \mathbf{m}_0) - \mathbf{B}(\mathbf{d}_k - \mathbf{d}_0)\} \\
&= \begin{pmatrix} \mathbf{m}_k \\ \mathbf{d}_0 \end{pmatrix} - \begin{pmatrix} (\mathbf{A}^T \mathbf{M} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{M} \\ \mathbf{C}_d \mathbf{B}^T \mathbf{M} [\mathbf{I} - \mathbf{A} (\mathbf{A}^T \mathbf{M} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{M}] \end{pmatrix} \{f(\mathbf{m}_k, \mathbf{d}_k) - \mathbf{B}(\mathbf{d}_k - \mathbf{d}_0)\}
\end{aligned} \tag{2.19}$$

Neglecting second order terms, the partial derivative matrices $\mathbf{A}_i$ (for site i) and $\mathbf{B}$ are given by:

$$\mathbf{A}_i = \begin{pmatrix} 1 & 0 & 0 & 0 & -(1 + \Delta s)z & (1 + \Delta s)y & x + \omega y - \psi z \\ 0 & 1 & 0 & (1 + \Delta s)z & 0 & -(1 + \Delta s)x & y - \omega x + \epsilon z \\ 0 & 0 & 1 & -(1 + \Delta s)y & (1 + \Delta s)x & 0 & z + \psi x - \epsilon y \end{pmatrix} \tag{2.20}$$

$$\mathbf{B} = \begin{pmatrix} 1 + \Delta s & \omega & -\psi & -1 & 0 & 0 \\ -\omega & 1 + \Delta s & \epsilon & 0 & -1 & 0 \\ \psi & -\epsilon & 1 + \Delta s & 0 & 0 & -1 \end{pmatrix} \tag{2.21}$$

where the data partial derivative matrix $\mathbf{B}$ is independent of station.

The above derivation of the implicit, non-linear estimation procedure has been included for completeness. In practice, we have found that perturbations to the coordinates are sub-millimeter after the first iteration due to the near-equivalence of the reference frames and the validity of the infinitesimal rotation and scale difference assumptions. By neglecting the need to iterate (i.e., setting $k = 0$), Equation (2.19) reduces to the normal least squares estimate, and the data partials in Equation (2.21) do not need to be calculated.

Temporal Evolution of the Bursa-Wolf Similarity Transformation

The coordinates $\mathbf{x}$ and $\mathbf{x}'$ used to estimate the similarity transformation are not constant, but vary over time due to relative site motions. We now explore the implications of these motions on the estimate of the similarity transformation

parameters. We assume that a site velocity $\mathbf{v}$ is constant and that the mean epoch of observations at the site is t_0 . In each coordinate system, propagation to a reference epoch t_r is given by

$$\begin{aligned}\mathbf{x}(t_r) &= \mathbf{x}(t_0) + \mathbf{v}(t_r - t_0) \\ \mathbf{x}'(t_r) &= \mathbf{x}'(t_0') + \mathbf{v}'(t_r - t_0')\end{aligned}\tag{2.22}$$

where neither the velocities nor mean observation epochs for a site in the two frames are assumed to be equal. This situation might arise, for example, at a site that has an independent estimate of velocity derived from VLBI observations, but whose SLR-derived coordinates assume AM0-2 velocities.

We now assume that $\mathbf{x}$ and $\mathbf{x}'$ exactly satisfy the Bursa-Wolf model (2.11) at a reference epoch

$$\mathbf{x}'(t_r) = \mathbf{sR} \mathbf{x}(t_r) + \mathbf{t}\tag{2.23}$$

and examine under what conditions the Bursa-Wolf model using the same transformation parameters remains valid at other times. Substituting Equation (2.22)

$$\mathbf{x}'(t_0') + \mathbf{v}'(t_r - t_0') = \mathbf{sR} \mathbf{x}(t_0) + \mathbf{sR} \mathbf{v}(t_r - t_0) + \mathbf{t}\tag{2.24}$$

If we make the assumption that

$$\mathbf{v}' = \mathbf{sR} \mathbf{v}\tag{2.25}$$

Equation (2.24) can be simplified to

$$\mathbf{x}'(t_o') + \mathbf{v}'(t_o - t_o') \equiv \mathbf{x}'(t_o) = \mathbf{sR} \mathbf{x}(t_o) + \mathbf{t} \quad (2.26)$$

indicating that the same Bursa-Wolf model remains valid at t_o , and by implication at all times as well.

Equation (2.25), or $\mathbf{v}' = \mathbf{v}$ since the difference in velocities will be on the order $(1 \times 10^{-8}) \times (50 \text{ mm/yr})$, represents the physically reasonable assumption that velocities at collocation sites do not differ significantly. If this assumption is not made, the estimates obtained for the similarity transformation parameters will vary over time due to velocity discrepancy biases. For this reason, we have included the assumption of homogeneous velocities at collocation sites in the formulation of the methodology used to derive the SV6 terrestrial reference frame.

Methodology for Estimating Terrestrial Reference Frames

We begin by constructing an observation vector that includes the coordinates for site i at its mean epoch of observation t_i

$$\mathbf{d} = \begin{pmatrix} \mathbf{x}^{(v)}(t_i) \\ \mathbf{v}^{(v)} \\ \mathbf{x}^{(s)}(t_i) \\ \Delta \mathbf{x} \\ \Delta \mathbf{v} \end{pmatrix} \quad (2.27)$$

where $\mathbf{x}$ are coordinates, $\mathbf{v}$ are independently estimated velocities (with the v superscript referring to VLBI sites, the s superscript referring to SLR sites), $\Delta \mathbf{x}$ are local vector ties between collocated monuments, and $\Delta \mathbf{v}$ are velocity differences between collocated monuments.

The covariance of the observation vector is given by

$$\mathbf{C_d} = \begin{pmatrix} \mathbf{C_{xx}^{(v)}} & \mathbf{C_{xv}^{(v)}} & 0 & 0 & 0 \\ \mathbf{C_{vx}^{(v)}} & \mathbf{C_{vv}^{(v)}} & 0 & 0 & 0 \\ 0 & 0 & \mathbf{C_x^{(s)}} & 0 & 0 \\ 0 & 0 & 0 & \mathbf{C_{\Delta x}} & 0 \\ 0 & 0 & 0 & 0 & \mathbf{C_{\Delta v}} \end{pmatrix} \quad (2.28)$$

where the non-zero off-diagonal elements correspond to the full covariance of the VLBI position and velocity estimates. The four different sets of observations are assumed to be uncorrelated as indicated by the zero off-diagonal elements.

We define the desired model parameter matrix that contains the positions, velocities, and similarity transformation parameters that define the terrestrial reference frames to be

$$\mathbf{m} = \begin{pmatrix} \mathbf{x} \\ \mathbf{v} \\ \mathbf{s} \end{pmatrix} \quad (2.29)$$

where the coordinates $\mathbf{x}$ refer to the reference epoch t_r , the velocity vector $\mathbf{v}$ contains an estimate for every site rather than just the sites with independent velocities, and $\mathbf{s}$ are the seven similarity transformation parameters (translation, scale, and orientation). The total number of model parameters for n sites is $6n + 7$.

We adopt the following functional relationship between the observations and the model parameters

$$\begin{pmatrix} \mathbf{J}_1^{(v)} & \Delta \mathbf{t}^{(v)} & \mathbf{T}^{(v)} \\ 0 & \mathbf{J}_2^{(v)} & 0 \\ \mathbf{J}_3^{(s)} & \Delta \mathbf{t}^{(s)} & 0 \\ \Delta^{(sv)} & 0 & 0 \\ 0 & \Delta^{(sv)} & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \mathbf{v} \\ \mathbf{s} \end{pmatrix} = \begin{pmatrix} \mathbf{x}^{(v)}(t_i) \\ \mathbf{v}^{(v)} \\ \mathbf{x}^{(s)}(t_i) \\ \Delta \mathbf{x} \\ \Delta \mathbf{v} \end{pmatrix}$$

$\mathbf{A} \qquad \mathbf{m} = \qquad \mathbf{d} \qquad (2.30)$

where the $\mathbf{J}$ matrices are composed of 0's and 1's that select the appropriate observations for the corresponding positions and velocities (i.e., sites that have both VLBI and SLR observations will have two 1's in the same column). $\mathbf{J}_2$ does not equal $\mathbf{J}_1$ since not all VLBI sites have corresponding velocity estimates. The $\Delta \mathbf{t}$ matrices are identical to their associated $\mathbf{J}$ matrices, except that the non-zero elements are the time differences between the mean observation epoch and the reference epoch ($\Delta t_i = t_i - t_r$). The $\mathbf{T}$ matrix represents the partials with respect to the similarity transformation parameters for the VLBI coordinates that were described in the Bursa-Wolf model description section. Finally, the $\Delta^{(sv)}$ matrix, composed of 0's, -1's, and 1's, differences positions and velocities at the collocation sites.

Several aspects regarding the model represented by Equation (2.30) should be noted. The site coordinates are propagated using the estimated reference velocities $\mathbf{v}$, via the $\Delta \mathbf{t}$ matrices, rather than by the *a priori* velocities derived from VLBI or global plate tectonic model estimates that might be inconsistent at collocated sites. Also, we have neglected the effect of the similarity transformation parameters on velocities given in Equation (2.25).

As posed, every site has a reference coordinate $\mathbf{x}$ and velocity $\mathbf{v}$. However, the only space-geodetic constraints on site velocity estimates are the independent VLBI velocity estimates $\mathbf{v}^{(v)}$, which for SV6 are available for 51 out of a total 165 sites. To overcome this deficiency, we incorporate additional information from geophysical

models of global plate tectonic motion and deformation. This information, in the form of an *a priori* model vector $\mathbf{m}_o$ and covariance $\mathbf{C}_m$, can be stochastically combined with the space-geodetic observations to yield the least-squares estimate [e.g., *Franklin, 1970; Jackson, 1979; Tarantola and Valette, 1982*]

$$\tilde{\mathbf{m}} = \mathbf{m}_o + (\mathbf{A}^T \mathbf{C}_d^{-1} \mathbf{A} + \mathbf{C}_m^{-1})^{-1} \mathbf{A}^T \mathbf{C}_d^{-1} (\mathbf{d} - \mathbf{A} \mathbf{m}_o) \quad (2.31)$$

Using this formulation, sites that do not have independent velocity estimates will adopt the *a priori* model, whereas the velocity estimates for the other sites will be a combination of the *a priori* information and the VLBI estimates. Adopting *a priori* information must be done judiciously to maintain a reasonable balance between the weight of the *a priori* information and the weight of the new observations. For the derivation of SV6, we followed a conservative course: the *a priori* velocity estimates were weak enough to give preference to robust VLBI estimates while providing constraints to more poorly-resolved VLBI estimates.

An *a priori* model for the velocities can be derived from the AM0-2 absolute motion model. Let the site i be assigned to plate p whose motion is described by the three-component Euler pole Ω_p vector. The site velocity is given by:

$$\mathbf{v}_i = \Omega_p \times \mathbf{x}_{ip} \quad (2.32)$$

Rearranging the cross product allows this to be written

$$\mathbf{v}_i = \mathbf{X}_{ip} \Omega_p \quad (2.33)$$

where

$$\mathbf{X}_{ip} = \begin{pmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{pmatrix}_{ip} \quad (2.34)$$

Combining the Euler poles for all the plates into a single vector Ω , an *a priori* velocity for all the sites can be derived from

$$\mathbf{v}_0 = \mathbf{X} \Omega \quad (2.35)$$

where $\mathbf{X}$ is composed of 0's and the appropriate $\mathbf{X}_{ip}$ matrices.

The *a priori* velocity covariance determines the weight of the *a priori* velocities with respect to the VLBI estimates. In the next section, we will describe how the covariance $\mathbf{C}_\Omega$ matrix of the no-net-rotation Euler poles Ω may be derived and we will discuss some of the inadequacies of this covariance, principally due to the breakdown of the assumption of rigid plate motion in areas of distributed crustal deformation. To compensate for these inadequacies we will also derive in the next section a covariance matrix $\mathbf{D}$ to accommodate deformation in the form of uncorrelated site motions. The *a priori* velocity covariance $\mathbf{C}_v$ will then take the form

$$\mathbf{C}_v = \mathbf{X} \mathbf{C}_\Omega \mathbf{X}^T + \mathbf{D} \quad (2.36)$$

Finally, the *a priori* model vector $\mathbf{m}_0$ and covariance $\mathbf{C}_m$ can be given by

$$\mathbf{m}_0 = \begin{pmatrix} 0 \\ \mathbf{v}_0 \\ 0 \end{pmatrix}; \quad \mathbf{C}_m = \begin{pmatrix} \infty & 0 & 0 \\ 0 & \mathbf{C}_v & 0 \\ 0 & 0 & \infty \end{pmatrix} \quad (2.37)$$

where the infinity signs indicate that no *a priori* constraints will be applied to the site positions or the similarity transformations.

A Priori Absolute Motion Models

The global plate motion model RM2 is specified by Euler pole vector differences and their covariances representing relative positions in angular velocity space. The data used to estimate these models—marine magnetic anomalies, transform fault azimuths, and earthquake slip vectors—describe purely relative plate motions. The models are usually tabulated by listing Euler poles with respect to an arbitrary reference plate such as the Pacific, and the Euler poles describing relative motion with respect to other plates are derived by vector subtraction. In other words, the origin of the angular velocity space is unconstrained by the relative motion data. The choice of an arbitrary, fixed plate establishes an origin and defines a so-called *absolute* plate motion frame. Several methods have been proposed to define other physically meaningful absolute motion frames. For example, the Wilson-Morgan hypothesis that hot spots are fixed to the mantle implies that the relative motion of the lithosphere with respect to the mantle can be estimated from azimuth and rate data derived from hot spot traces (e.g, the Hawaii and Marquesas island chains). *Minster and Jordan* [1978] use nine hot spot traces to estimate the AM1-2 absolute motion model, which is specified by the Euler pole for the Pacific plate with respect to the mean mesospheric frame (61.66°S, 97.19°E, 0.967 degrees/m.y.); such a model depends on the choice of hot spot data. Another proposed absolute motion frame requires that the lithosphere as a whole possesses no net rotation (e.g., *Minster et al.*, 1974; *Solomon and Sleep*, 1974; *Solomon et al.*, 1975); to the extent that the plate boundaries are specified, a no-net-rotation model can be uniquely defined. *Minster and Jordan* [1978] estimated the AM0-2 absolute motion model by performing a vector spherical harmonic expansion of the RM2 surface velocity field from which the coefficients of the toroidal harmonic of degree one provide the amount of net rotation; the AM0-2 Pacific Euler pole is given by (62.93°S, 111.50°E, 0.736 degrees/m.y.).

The AM0-2 no-net-rotation model was adopted as the standard station motion model by the International Earth Rotation Service (IERS) for its terrestrial reference frames [McCarthy, 1989]. It is typically used as the underlying velocity reference frame for estimating station positions, velocities, and polar motion from VLBI and lunar laser ranging observations. For example, the Goddard VLBI solution GLB659, which will be used to define relative positions for the SV6 reference frame, assumes the *a priori* station velocities are given by AM0-2 and estimates deviations from AM0-2 for those sites with sufficiently robust observations spanning an adequate interval. The AM1-2 hot spot model is used by some SLR analysts who argue that dynamic orbit models incorporating the Earth's gravitational field are more sensitive to the mantle; the Center for Space Research solution CSR8902 based on LAGEOS observations, is derived from estimates of station position at a single epoch assuming AM1-2 station velocities over the 13-year observation interval. We have adopted the AM0-2 no-net-rotation model as the underlying absolute angular velocity frame for SV6.

The use of RM2-derived velocities at sites whose motions differ significantly from RM2, such as those in areas of continental deformation, implies that the estimated relative site locations will become systematically less accurate as they are propagated from the mean observation epoch. In an effort to minimize the biases introduced by assuming incorrect station velocities, the RM2 uncertainties on these motions are incorporated into the propagation of coordinates to epochs other than the mean observation epoch, thus increasing the site coordinate uncertainties. The RM2 Euler pole uncertainties are specified by a 30 by 30 full covariance matrix that is available in the PMOTION software package developed by Bernard Minster and submitted to the Crustal Dynamics Project Data Information System (B. Minster, personal communication, 1986); the AM0-2 Euler pole uncertainties were not derived. We therefore present a general method for deriving no-net-rotation Euler poles and

their covariances and apply it to the RM2 model in order to estimate the AM0-2 uncertainties that will underlie the SV6 reference frame.

We derive a no-net-rotation Euler pole covariance matrix by first determining the linear transformation used to estimate the no-net-rotation Euler poles as a function of the relative motion Euler poles. The covariance is then formed by standard error propagation methods in which the transformation is applied to the relative Euler pole covariance quadratically. *Minster and Jordan* [1978] estimated the AM0-2 no-net-rotation Euler poles from a vector spherical harmonic expansion of the RM2 surface velocity field; this method requires that an arbitrarily dense distribution of the velocity vectors be specified *a priori* and does not readily yield a simple linear transformation between the relative and no-net-rotation Euler poles. We have used an alternative method, suggested by *Solomon and Sleep* [1974] and *Solomon et al.* [1975], that minimizes the total angular momentum, as in Equation (2.8). By assuming that the plates are infinitesimally thin and have uniform surface density, the condition of no-net-angular momentum is equivalent to no-net rotation. This approach can be completely specified by the definition of the plate boundaries and provides the simple linear transformation we require.

The velocity of point $\mathbf{x}$ assumed to be located on plate p is given by:

$$\mathbf{v} = \Omega_{pm} \times \mathbf{x} \quad (2.38)$$

where Ω_{pm} is the Euler pole for plate p with respect to the absolute reference frame m . The condition of no-net-angular momentum is then:

$$\sum_p \int_p \mathbf{x} \times (\Omega_{pm} \times \mathbf{x}) dA = 0 \quad (2.39)$$

where the integrations are carried out over the surface of each infinitesimally thin plate. Recalling the vector identity $\mathbf{x} \times (\boldsymbol{\Omega} \times \mathbf{x}) = (\mathbf{x} \cdot \mathbf{x}) \boldsymbol{\Omega} - (\mathbf{x} \cdot \boldsymbol{\Omega}) \mathbf{x}$, and defining the radius of the Earth to equal one so that $\mathbf{x} \cdot \mathbf{x} = 1$, (2.39) can be written:

$$\sum_p (A_p \mathbf{I} - \mathbf{S}_p) \boldsymbol{\Omega}_{pm} = 0 \quad (2.40)$$

where $\mathbf{I}$ is an identity matrix, A_p is the total area of plate p , and $\mathbf{S}_p$ is a symmetric moment matrix defined by:

$$S_{p,ij} = \iint_p d\phi d\theta x_i x_j \sin\theta \quad (2.41)$$

where ϕ is the longitude, θ is the colatitude, and $\mathbf{x} = (\sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta)$. It is straightforward to show that:

$$\sum_p (A_p \mathbf{I} - \mathbf{S}_p) = \frac{8\pi}{3} \mathbf{I} \quad (2.42)$$

Defining

$$\mathbf{Q}_p = \frac{3}{8\pi} (A_p \mathbf{I} - \mathbf{S}_p) \quad (2.43)$$

the no-net-rotation condition (2.40) becomes:

$$\sum_p \mathbf{Q}_p \boldsymbol{\Omega}_{pm} = 0 \quad (2.44)$$

Let Ω_{om} be the Euler pole of an arbitrary reference plate (e.g., the Pacific) with respect to an absolute angular velocity frame, such as the mantle. It follows that the Euler poles for the remaining plates can be determined by $\Omega_{pm} = \Omega_{po} + \Omega_{om}$, where Ω_{po} represent the relative Euler poles with respect to the reference plate. An absolute angular velocity frame can therefore be defined simply by the Euler pole of the arbitrary reference plate Ω_{om} . From (2.44), it follows:

$$\sum_p (Q_p \Omega_{po}) + \left(\sum_p Q_p \right) \Omega_{om} = 0 \quad (2.45)$$

Recalling (2.42) and (2.43), the bracketed expression in the second term is the identity matrix, and thus the desired Euler pole of the arbitrary reference plate is

$$\begin{aligned} \Omega_{om} &= - \sum_p Q_p \Omega_{po} \\ &= (-Q_1 - Q_2 \dots) \begin{pmatrix} \Omega_{1o} \\ \Omega_{2o} \\ \vdots \end{pmatrix} \end{aligned} \quad (2.46)$$

To obtain the full no-net-rotation covariance matrix, we extend the above result to simultaneously derive all the no-net-rotation Euler poles

$$\begin{pmatrix} \Omega_{om} \\ \Omega_{1m} \\ \Omega_{2m} \\ \vdots \end{pmatrix} = \begin{pmatrix} -Q_1 & -Q_2 & \dots \\ I - Q_1 & -Q_2 & \dots \\ -Q_1 & I - Q_2 & \dots \\ \dots & \dots & \ddots \end{pmatrix} \begin{pmatrix} \Omega_{1o} \\ \Omega_{2o} \\ \vdots \end{pmatrix} \quad (2.47)$$

Equation (2.47) represents the desired linear transformation of a relative motion model specified with respect to an arbitrary plate Ω_{po} to a no-net-rotation motion

model with respect to the mantle Ω_{pm} . Quadratic application of this transformation to the covariance of the Euler poles for the relative motion model yields the desired no-net-rotation covariance. The no-net-rotation covariance is a matrix of order $(3n \times 3n)$, whereas the relative motion model covariance is a matrix of order $(3(n-1) \times 3(n-1))$, where n is the total number of plates in the model. Since no additional information has been added in the construction of the no-net-rotation covariance, it follows that it is a non-negative matrix with a rank 3 deficiency.

Construction of the transformation matrix in (2.47) requires that the areal moment matrices S_p in (2.40), as well as the area of each plate, be estimated. We performed the double integrals by evaluating the longitude ϕ analytically, and the co-latitude θ numerically at each point on the plate boundaries, following the method of *Solomon et al.* [1975]. For example, for the integral for the area of plate p :

$$\begin{aligned}
 A_p &= \iint_p d\phi d\theta \sin\theta \\
 &= \sum_i \phi_i \sin\theta \Delta\theta_i
 \end{aligned}
 \tag{2.48}$$

where (ϕ_i, θ_i) are points on the boundary, and the line integral using $\Delta\theta_i = \theta_i - \theta_{i-1}$ is performed clockwise about the boundary. Due to the discontinuity in longitude ϕ at 0 and 2π , a branch cut contribution must be added when ϕ goes from 2π to 0 and subtracted when ϕ goes from 0 to 2π for plates on the Greenwich meridian. For the area integral, for example, this term is:

$$\begin{aligned}
A'_p &= \int_0^{2\pi} d\phi \int_0^\alpha d\theta \sin\theta \\
&= 2\pi (1 - \cos \alpha)
\end{aligned}
\tag{2.49}$$

when $\theta = \alpha$.

The integrations were performed using a digitized plate boundary data set, specified by 2008 points, corresponding to the NUVEL-1 plate configuration (C. DeMets, pers. comm., 1988) with the exception of the India-Australia plate boundary zone which was not included; a nominal boundary was constructed to bisect the proposed diffuse deformation zone [DeMets *et al.*, 1990](Figure 2.6). For purposes of integration, the Easter Island microplate was incorporated into the Nazca plate. The estimated areas and coefficients of the areal moment matrices are given in Table 2.3. The total estimated area of 12.563 is slightly smaller than the true surface area of a sphere with radius equal to 1 ($4\pi \equiv 12.566$); this difference and the other minor differences between the estimated coefficients and the theoretical values are probably due to inaccuracies introduced by evaluating the integrations numerically, and thus provide a measure of the precision of the technique.

The values listed in Table 2.3 were used to construct transformation matrix (Equation 2.47) to estimate the no-net-rotation frame for RM2 relative motion Euler poles. The Philippine plate, which was not estimated in either global plate motion model, was combined with the Pacific plate. The India and Australia plates were combined in order to estimate the AM0-2 Euler poles. The estimated Pacific plate angular velocity vectors with respect to the no-net-rotation frame (63.125°S , 110.887°E , $0.7162^\circ/\text{m.y.}$) differs from the *Minster and Jordan* [1978] estimate (62.93°S , 111.50°E , $0.736^\circ/\text{m.y.}$) at about the one-sigma level; presumably, this difference can be attributed to minor changes in the plate boundaries and to the different methodologies used to estimate the no-net-rotation Euler poles. Integration

of the areal moment matrices to minimize rotation has the advantage that it is only a function of the geometry of the plate boundaries, whereas the vector spherical harmonic expansion technique must also specify an arbitrary distribution of velocity vectors within each plate. Since the goal of the present work is to use the AM0-2 covariance to minimize the biases introduced in the definition of the SV6 reference frame by potential velocity errors, the errors in the covariance matrix resulting from using the transformation based on the coefficients in Table 2.3 rather than the true AM0-2 values are negligible. However, to maintain internal consistency, the areal moments for the Pacific plate were adjusted so that the numerical sums equal the theoretical values. The *a priori* model and covariance was constructed using the true AM0-2 velocities with the derived uncertainties; the resulting velocities at some of the SV6 sites are shown in Figure 2.7.

Relaxing the Rigid Plate Assumption

An absolute motion frame is established for the SV6 reference frame by describing the temporal evolution of the sites with respect to the AM0-2 no-net-rotation model. As mentioned earlier, adopting AM0-2 as the underlying frame adheres to international convention and merely represents the arbitrary definition of an origin in angular velocity space. Choosing a plate or minimizing hot-spot absolute motions represent alternate, but equally arbitrary, methods for fixing the origin (ignoring arguments that hot spot motions are fixed with respect to a mantle reference frame that may be more appropriate for techniques sensitive to the gravitational field of the Earth). However, SV6 also uses the AM0-2 model to derive velocities for sites lacking independent space-geodetic estimates, either because the measurements are insufficient in number or duration, or because the motions of the sites are fixed at AM0-2 (or AM1-2) rates to estimate their coordinates. The assumption that AM0-2 or, equivalently, RM2 rates describe the relative motion between sites is inadequate

in many cases, and raises the possibility that the relative precision of the site coordinates, and hence the SV6 terrestrial reference frame, will become progressively degraded as the sites are propagated by these erroneous velocities with respect to their mean observation epochs. There are several reasons why RM2-derived rates may poorly represent the motions of space-geodetic sites.

Global plate tectonic models, such RM2 and NUVEL-1 [*Minster and Jordan, 1978; DeMets et al., 1990*], assume that tectonic plates are rigid spherical caps that do not deform internally and that their relative motions on the surface of the sphere may be described by an angular velocity vector, or Euler pole, that represents a rotation about the pole location. In this ideal world, a transform fault on a plate boundary lies on a small circle centered at the Euler pole describing the relative motion between the two plates, and a spreading ridge lies on a great circle that passes through the Euler pole. In reality, these assumptions break down in many areas, especially along continental plate boundaries where deformation of the lithosphere is prevalent. The western United States is a notable example, from the extension occurring in the Basin and Range province in the interior of the North America plate to the complexities of the San Andreas transform fault system along the Pacific-North America plate boundary. These discrepancies have long been noted, and observations from these areas are typically excluded when global plate tectonic models are derived. It is also possible that the 2-m.y. average rates from global tectonic models are not consistent with rates derived from space-geodetic observations over the last decade due to changes in the global mean rate of motions. Space-geodetic measurements show, however, good agreement with global mean rates [e.g., *Smith et al., 1990; Argus and Gordon, 1990*].

In addition to shortcomings regarding the assumptions of global plate tectonic models with respect to the more instantaneous space-geodetic motions, the global models themselves may be in error. For example, relative motions between certain plate pairs for the RM2 model are known to be biased toward high rates due to errors

in published marine magnetic anomaly figures [DeMets *et al.*, 1990]. These biases have been corrected in the NUVEL-1 model. One example is the North America–Pacific plate relative Euler pole: AM0-2 predicts 56 mm/yr relative motion across the San Andreas fault in Central California, whereas NUVEL-1 predicts 48 mm/yr. However, until the NUVEL-1 model has been accepted as the international standard and is used in adjustments of SLR and VLBI observations, we will continue to use the AM0-2 model both to establish the absolute motion reference frame for SV6 and to provide velocity estimates for sites that have insufficient space-geodetic observations to estimate their velocities independently.

Assuming an inaccurate velocity for a site derived from a global plate tectonic model degrades the accuracy of the predicted site location as it is propagated with respect to the mean observation epoch of the site. Several approaches might be considered to minimize the effect on site coordinates of poor velocity model predictions. If space-geodetic observations are insufficient to provide a robust estimate, site velocities based on other geophysical and geologic information, such as seismic moment tensors [e.g., *Ekström and England*, 1989] or neotectonic measurements [e.g., *Minster and Jordan*, 1984], might be used. However, these methods generally have large uncertainties compared to global plate tectonic models and are not globally available. Instead, we shall take the conservative approach of deriving an estimate of uncertainty in a site’s location that incorporates uncertainties in its relative motion, including those arising from plate deformation.

Space-Geodetic Constraints on Non-Rigid Plate Motion

We derived the AM0-2 Euler pole covariance in a previous section by using areal moment functions to linearly transform the RM2 covariance. The Euler pole uncertainties imply approximately 2–3 mm/yr uncertainty in rates. For example, if the Pinon Flat site is assumed to lie on the Pacific plate, its AM0-2 estimated rate is 27.7

± 1.2 mm/yr north, -47.1 ± 0.9 mm/yr east. Space-geodetic measurements offer an independent estimate of the motions for certain sites. The VLBI measurements included in the GLB659 adjustment over the last decade are sufficiently numerous to provide independent velocity estimates for about 50 sites; the GLB659 estimate for the velocity of Pinon Flat 7256 is 4.2 ± 2.4 mm/yr north, -31.2 ± 1.8 mm/yr east. Clearly, the VLBI and the global plate tectonic estimates are significantly different. Pinon Flat is located in a tectonically complex area approximately 15 km west of the San Andreas fault and 30 km east of the San Jacinto fault. If Pinon Flat is assumed to lie on the North America plate, its AM0-2 estimated rate is -14.6 ± 1.2 mm/yr north, -13.0 ± 0.9 mm/yr east, which is also significantly different than the VLBI estimate. On the other hand, the mean of the two AM0-2 estimated rates, 6.6 mm/yr north, -30.1 mm/yr east, is consistent with the VLBI estimate, suggesting that the site is at the midpoint of the North America–Pacific plate boundary.

Figure 2.8 shows the discrepancies of the horizontal GLB659 velocities with respect to AM0-2 for sites in North America, where the ellipses represent 95% confidence levels derived from the VLBI velocity estimates. The most significant discrepancies occur at sites located in the vicinity of the Pacific–North America plate boundary, both along the San Andreas transform fault system and the Aleutian subduction zone. In California, the largest discrepancies are associated with the sites nearest the San Andreas fault, Pearblossom and Pinon Flat. These sites have comparable VLBI estimates but for purposes of this figure, Pearblossom is assumed to lie on the Pacific plate while Pinon Flat is assigned to the North America plate. The resulting large discrepancies indicate the inadequacy of the AM0-2 motion for either plate to explain the site motions. The sites in eastern California (Hat Creek, Owens Valley, and Mojave) show discrepancies that are consistent with predicted motions resulting from the extension of the Basin and Range province with respect to the North America plate [Minster and Jordan, 1987; Argus and Gordon, 1991]. The

discrepancy at Vandenberg (the southernmost coastal site shown) is consistent with the known velocity bias in the Pacific–North America relative motion Euler pole [e.g., *DeMets et al.*, 1987; *Sauber*, 1988; *Argus and Gordon*, 1990], although this bias is not evident for the site on Kauai that should be located on the most stable portion of the Pacific plate. The largest discrepancy in the Aleutian subduction zone is at the Yakataga site. *Ma et al.* [1990] have proposed that this anomalous motion may be due elastic straining of the overriding plate due to a locked main thrust fault zone; they did not include VLBI observations following the 1987 and 1988 Gulf of Alaska earthquakes in the vicinity of Yakataga in their analysis and found a rate fairly consistent with NUVEL-1. On the other hand, GLB659 did include the post-earthquake VLBI observations, resulting in an estimate inconsistent with AM0-2 or NUVEL-1 [*Caprette et al.*, 1990]. Most of the sites in the presumably stable interior of the North America plate do not show significant discrepancies in the horizontal components.

To assess the overall validity of the AM0-2 absolute motion model and its covariance, we have calculated the statistical significance of the discrepancy of the available VLBI-derived velocity estimates with respect to AM0-2. For site i , let the VLBI-derived velocity estimate and covariance be given by $\mathbf{v}_{v,i}$ and $\mathbf{C}_{v,i}$, respectively, and let the corresponding global plate tectonic model estimate be given by $\mathbf{v}_{o,i}$ and $\mathbf{C}_{o,i}$, respectively. The difference between the two estimates and its covariance are determined straightforwardly by

$$\begin{aligned}\Delta \mathbf{v}_i &= (\mathbf{I} \ -\mathbf{I}) \begin{pmatrix} \mathbf{v}_{v,i} \\ \mathbf{v}_{o,i} \end{pmatrix} = \mathbf{v}_{v,i} - \mathbf{v}_{o,i} \\ \mathbf{C}_{\Delta,i} &= (\mathbf{I} \ -\mathbf{I}) \begin{pmatrix} \mathbf{C}_{v,i} & 0 \\ 0 & \mathbf{C}_{o,i} \end{pmatrix} \begin{pmatrix} \mathbf{I} \\ -\mathbf{I} \end{pmatrix} = \mathbf{C}_{v,i} + \mathbf{C}_{o,i}\end{aligned}\tag{2.50}$$

The estimated motions will be considered significantly different if $\Delta \mathbf{v}_i$ is statistically different from zero at the 95% confidence level. This hypothesis may be tested by forming a quadratic misclosure statistic

$$\phi_{\Delta,i} = \Delta \mathbf{v}_i^T \mathbf{C}_{\Delta,i}^{-1} \Delta \mathbf{v}_i \sim \chi_f^2 \quad (2.51)$$

where the notation on the right hand side of the equation indicates that the statistic is chi-square (χ^2) distributed with f degrees of freedom. For the case of a single site ($f = 3$), $\Delta \mathbf{v}_i$ is significantly different from zero if $\phi_{\Delta,i}$ is greater than 7.81 at the 95% confidence level. The overall consistency of the velocity estimates is determined by calculating a statistic from simultaneously differencing all the observations $\Delta \mathbf{v}$ and including their full covariance. For a total of 50 sites ($f = 150$), $\Delta \mathbf{v}$ is significantly different from zero if ϕ_{Δ} is greater than 179.58 at the 95% confidence level.

The GLB659 adjustment includes 51 sites with independent estimates for their horizontal and vertical components of velocity. The constraints imposed on the vertical velocity for Kauai 1311 and the horizontal velocity for Richmond 7219 in the adjustment render the total velocity covariance singular. Therefore, we omitted these components and evaluated the total χ^2 -misclosure statistics based on 50 sites. AM0-2 assumes that all motions are horizontal (normal to the local vertical) so that any vertical motions that are significantly greater than zero will add a contribution to the χ^2 -misclosure statistic. To assess these effects, we determined the horizontal and vertical contributions to the total misclosure statistics.

We performed two series of tests. The first used only the VLBI covariance to calculate the statistics (i.e., $\mathbf{C}_{\Delta,i} = \mathbf{C}_{v,i}$), to provide a measure of the VLBI velocity discrepancies with respect to AM0-2. Table 2.4 gives the χ^2 -misclosure statistics for the vertical, horizontal, and total velocity difference at each site; these cases have 1, 2, and 3 degrees of freedom, respectively. The same tests were then applied to all the

sites simultaneously, with 50, 100, and 150 degrees of freedom. The total χ^2 -misclosure statistic, equal to about 12400, is significantly greater than the 95% confidence level for zero velocity differences of 180. This high value is due primarily to horizontal component differences, although the vertical component differences are also significant at the 95% confidence level. We then added the formal AM0-2 covariance to the VLBI covariance, as given in Equation (2.50); the statistics for the vertical components are not affected, since AM0-2 has no vertical covariance. While the total χ^2 -misclosure statistic for the horizontal components drops from about 12000 to 3800 when the AM0-2 covariance is added, the difference remains significantly greater than zero. These results indicate that the AM0-2 model and covariance do not provide an adequate description of present-day deformation. We therefore consider an addition to the absolute motion velocity covariance designed to take into account deformation that is uncorrelated between sites.

Uncorrelated Site Motion

The AM0-2 model assumes that the Earth's surface is divided into 11 rigid plates that are in relative motion. The horizontal (two-component) motion of any site is purely a function of its location with respect to the three-component Euler pole of the plate associated with the site. If all sites on a plate are known to have motions exactly described by an Euler pole, it follows that knowledge of the motions of two sites completely determines the Euler pole (with one degree of freedom) and hence the motions of all other sites on the plate. In practice, the site velocities derived purely from the AM0-2 covariance are highly correlated, and often difficult to invert due to numerical instabilities.

The preceding discussion indicated the extent to which the Euler pole representation fails to completely describe observed relative motions and gave some possible sources for these discrepancies. To take into account possible plate

deformations, we therefore add to the *a priori* site velocities additional covariance that is uncorrelated between sites. If the Earth were composed of a single plate, a covariance with isotropic components might be added to all sites (e.g., 10 mm/yr uncertainty in all directions), or perhaps a deformation covariance decomposed into horizontal and vertical uncertainties (e.g., 10 and 5 mm/yr, respectively) might be more appropriate. However, Figure 2.8 suggests that horizontal velocity discrepancies are highly correlated with the presence of plate boundaries. To test this hypothesis, we show in Figure 2.9 the magnitude of the horizontal velocity difference between rates derived from VLBI and AMO-2 plotted as a function of distance to the nearest plate boundary. The nearest plate boundary distances were determined from the digitized NUVEL-1 plate boundary set and are provided in Table 2.1. The plate boundary distances are approximate; note, for example that Pinon Flat 7256, which is the site closest to a boundary (about 15 km from the San Andreas fault), is estimated to be 4 km distant. In general, the velocity discrepancies are between 8–30 mm/yr at sites within 200 km of a plate boundary and less than 5 mm/yr at greater distances. The high scatter in the 500–1000 km range is due to a number of sites with poorly-resolved velocities (Hartbeetoeek 7232, Canberra 1642, Penticton 7283, and Madrid 1665). The largest discrepancy and greatest outlier is due to Yakataga 7277 followed by two sites near the San Andreas fault, Pearblossom 7254 and Pinon Flat 7256.

The general trend of the horizontal discrepancies was estimated from a weighted least-square estimate fit to an empirically-defined power-law relationship. The main considerations in defining this relationship were that it should approach small positive values at large distances from plate boundaries, and large values in the vicinity of boundaries. The inverse-square-root of distance relationship

$$|\Delta v| = 1.0 + 78.2 L^{-1/2} \quad (2.52)$$

where $|\Delta v|$ is the magnitude of the horizontal velocity differences in mm/yr, and L is the distance to nearest plate boundary in kilometers, was found to satisfy these criteria. The scatter of the observations about this empirically-derived relationship for uncorrelated horizontal deformation is approximately 5 mm/yr. Since the range of site distances for the entire SV6 data set varies from 4–3588 km, corresponding to 40–4 mm/yr in deformation, we can safely neglect the singularity at zero distance. Such singularities could be avoided by using a more physically realistic relationship based on theoretical models of strain and deformation [e.g., *Savage and Burford, 1973*], but are beyond the scope of this work.

The magnitudes of the vertical velocity differences of VLBI with respect to AM0-2 are shown in Figure 2.10. With exception of several sites with poorly-resolved velocity estimates, the majority of the discrepancies are less than 10 mm/yr and do not display a strong dependence on the distance to the nearest plate boundary. Based on this comparison, we have assumed a constant vertical velocity uncertainty at all sites.

To determine the magnitude the vertical uncertainty and to properly scale the horizontal uncertainties, we apply the chi-square misclosure statistics defined in Equation (2.47), except that we now add additional horizontal and vertical components of deformation to the AM0-2 covariance:

$$C_{\Delta,i} = C_{v,i} + C_{o,i} + \alpha^2 D_h + \beta^2 D_v \quad (2.53)$$

The horizontal covariance matrix D_h is block diagonal (no correlations between sites) with elements composed of north and east variances equal to $|\Delta v|^2/2$, where $|\Delta v|$ is given by Equation (2.48) and the factor of two is included so that the total magnitude of the uncertainty is $|\Delta v|$. The vertical covariance matrix D_v is block diagonal with elements composed of variances equal to a nominal $(1 \text{ mm/yr})^2$ value. The α and the β are the scaling parameters that make the total chi-square misclosure statistics (50

sites simultaneously) consistent for horizontal and vertical components at the 95% confidence level. We estimated the scaling parameters by a series of trial determinations of the misclosure statistics; the results are given in Table 2.5. Setting the horizontal scalar $\alpha = 1.0$ yields a chi-square statistic $\phi_{\Delta} = 122.60$ that agrees with the theoretical statistic with 100 degrees of freedom at the 95% confidence level; $\alpha = 1.0$ implies that no scaling is required to the empirical relationship given by Equation 2.48. Setting the vertical scalar $\beta = 1.5$ yields a chi-square statistic $\phi_{\Delta} = 64.31$ that agrees with the theoretical statistic with 50 degrees of freedom at the 95% confidence level; this value of β implies that a uniform 1.5 mm/yr vertical uncertainty is appropriate.

The sum of the last three terms in Equation (2.53) constitutes the *a priori* velocity model covariance, which is shown for a subset of sites in Figure 2.11. It incorporates global plate tectonic information, including inter- and intra-plate correlations between sites, and is consistent with space-geodetic information provided by independent VLBI velocity estimates. This covariance matrix will help to constrain poorly-resolved VLBI velocities, such as those for Yellowknife 7285 and Penticton 7283, and will be applied to all sites that do not have independent velocity constraints.

ESTIMATION OF SV6

Model Assumptions

The SV6 terrestrial reference frame is realized by the geocentric positions and velocities for 165 reference points contained in the GLB659 and CSR8902 adjustments. The reference epoch is 1988.0 (January 1, MJD = 47161), near the average of the mean epochs of observations (1987.5) given in Table 2.1. It is also the reference epoch adopted by the IERS for their ITRF-0 and ITRF89 terrestrial reference frames [Boucher and Altamimi, 1989, 1991; IERS, 1990].

The data matrix was constructed by determining the mean-epoch coordinates for the SLR and VLBI adjustments from the information provided in the data description section. The mean epochs of observation are given in Table 2.1. The independently-determined VLBI velocities were taken directly from the GLB659 adjustment. The collocation sites for both position and velocity differences are listed in Table 2.1, and the local survey ties are given in Table 2.2. The velocity differences for the collocated sites were calculated according to differences in AM0-2 velocities at the two positions, assuming the site is assigned to the plate listed in Table 2.2. For most of the sites, of course, the differential velocity Δv is essentially zero. The largest magnitude difference, across the 386 km Hawaiian baseline, is approximately 5 mm/yr.

We made several assumptions in order to construct a well-conditioned data covariance matrix (i.e., numerically invertible). The local survey tie covariance $C_{\Delta x}$ is block diagonal and is constructed from the sigmas and correlations provided in Table 2.2; for ties that have unknown uncertainties (i.e., the ties with zero sigmas, such as at Goddard and Kwajalein) a uniform 10 mm uncertainty on all components was assumed. The uncertainty on the differential velocities was assumed to be 0.1 mm/yr in all components to ensure little relative motion between the collocated sites. The full GLB659 position and velocity covariance has several singularities due to the constraints that were applied to fix the origin and velocity frames. We assumed an origin uncertainty equal to 1 mm which was propagated to all sites. The uncertainty of the radial velocity at Kauai 1311 was set to 100 mm/yr. The horizontal velocity constraint applied to Richmond 7219 was more problematic due to the correlations between Richmond and the other sites. We were unable to satisfactorily add an uncertainty to the Westford-Richmond azimuthal variation and attribute this problem to instabilities stemming from the lack of numerical precision on the intersite correlations, since *Caprette et al.* [1990] provided correlations with only three significant digits for the GLB659 adjustment. Similar problems were encountered with

the velocities for three other sites whose velocities were poorly-resolved: Penticton 7283, Noto 7547, and Fort Ord 7241. We therefore eliminated the estimated velocities for Richmond and these sites from the observations. The CSR8902 position covariance $C_x^{(s)}$ is a non-singular diagonal matrix.

Results

The observations used to estimate the 165 site position and velocity parameters that realize the SV6 terrestrial reference frame include 80 VLBI site positions, 47 VLBI velocity estimates, 91 SLR site positions, and 16 position and velocity differences. Including the similarity transformation parameters, 997 parameters were estimated ($165 \times 6 + 7 = 997$) from 750 observations ($((80+47+91+16+16) \times 3 = 750)$). Normally, this would be an under-determined system; however, we have added constraints to the estimates of the 165 site velocities. Because these constraints completely determine the 118 velocities of the sites lacking space-geodetic information, we will assume that $997 - 118 \times 3 = 643$ parameters are effectively estimated with a resulting $750 - 643 = 107$ degrees of freedom.

We define the weighted norm of the estimated residuals e^2 , from Equation 2.31, to be

$$e^2 = (d - A \tilde{m})^T C_d^{-1} (d - A \tilde{m}) \quad (2.54)$$

and expect this to be χ^2 -distributed with $f = 107$ degrees of freedom. The sampling variance, or variance of unit weight, σ_o^2 is defined to be

$$\sigma_o^2 = e^2 / f \quad (2.55)$$

and has an expected value of one. Significant departures of the sampling variance from one indicate either that the *a priori* estimates of the observation uncertainties are incorrect, the model is incorrectly formulated, or that certain observations are significantly different from the model, perhaps due to gross blunders (outliers).

The weighted norm of the residuals from the adjustment of the 750 observations was 206.61, implying that the sampling variance is 1.931. This is a significant departure from the expected value of 1.0. To check for outliers, we define a weighted residual r_i for an observation d_i to be the estimated residual e_i divided by the formal uncertainty of d_i . Note that this is not the standardized residual typically used in statistical testing for outlier detection in which the residual is divided by its uncertainty and not the observation uncertainty [e.g., *Vaníček and Krakiwsky, 1986*]; however, the weighted residual r_i is a close approximation that does not require the calculation of the residual covariance matrix. The weighted residuals for 534 observations are shown in a histogram with bin widths of one in Figure 2.12. The residuals corresponding to the velocities for 72 sites without independent velocity constraints have been removed from the residual set as they were identically equal to zero. Most of the weighted residuals are less than 10; however, the residuals for five sites are greater than 10: Madrid 1665, Yellowknife 7285, Algonquin 7282, Mammoth Lakes 7259, and Yakataga 7277. These sites all have VLBI estimated velocities with vertical components greater than 9 mm/yr and the highest uncertainties associated with their vertical component amongst all the VLBI estimated velocities. The large weighted residuals appear to be derived from the constraints placed on these rates by the *a priori* velocity model, which assumed 0.0 ± 1.5 mm/yr vertical motion.

A second estimate of the consistency of the model, data, and covariance used to derive the SV6 terrestrial reference frame may be obtained from a comparison with another terrestrial reference frame. The adjustment describe above transformed the VLBI coordinates to make them consistent with the SLR coordinate system. We then

estimated a seven-parameter similarity transformation of the SLR coordinate system with respect to the ITRF89 terrestrial reference frame [*IERS*, 1990; *Boucher and Altamimi*, 1991] in order to make the orientation of SV6 consistent with IERS. The ITRF89 reference frame was derived in a manner similar to the method we have described to derive SV6. In particular, it uses the CSR8902 adjustment to establish its origin and scale. ITRF89 combines 18 VLBI, SLR, and lunar laser ranging adjustments to realize its site coordinates, rather than a one VLBI and one SLR adjustment that we have chosen to use to minimize potential modeling differences. The reference epoch for both terrestrial reference frames is 1988.0. ITRF89 adopts velocities derived from the AM0-2 model for all sites. ITRF89 includes all reference points in SV6 except for Canberra 1642, Madrid 1665, Owens Valley 7084, and Goddard 7100.

To perform the similarity transformation estimate, the Cartesian position and formal uncertainty without correlations of the SV6 sites were used. An additional five sites (Easter Island 7061, Hobart 7242, Metsuhova 7805, Riga 1884, and Goldstone 7085) were deleted from the estimate due to large differential coordinate residuals indicating possible blunders. For example, the coordinates for Easter Island 7061 differed by 3 meters in the vertical, suggesting that the definition of the reference monument might not be consistent between the two reference frames. The results of the similarity transformation are given in Table 2.6. The estimate had 461 degrees of freedom and the weighted norm of the estimated residuals was 748.2 giving a sampling variance of 1.623. Again, this is significantly greater than one. The range of the residual coordinate differences after the transformation was applied to the SV6 coordinate frame are indicated in a histogram shown in Figure 2.13. The majority of coordinate differences are less than 40 mm. The coordinate differences were rotated into the site local geodetic coordinate frame, and most of the differences greater than

60 mm are associated with vertical components, although the greatest difference, -277 mm, is in the east component of Santiago de Cuba 1953.

The estimated sampling variances of the VLBI to SLR transformation and the SLR to IERS transformation are significantly greater than one. The VLBI to SLR sampling variance, however, does not formally take into account the *a priori* constraints placed on the velocity parameters that change both the degrees of freedom (we attempted an ad hoc compensation for this by excluding unconstrained velocity parameters) and the calculation of the weighted norm of the residuals (it should include a model norm, as well). Additional outliers appear to be partly responsible for the large SLR to IERS sampling variance, but we did not investigate these discrepancies more fully since trial exclusions of the observations did not significantly change the estimated transformation parameters. Due to these considerations, we did not scale the uncertainties of the SV6 coordinates and velocities by the *a posteriori* scatter represented by the sampling variance.

The seven similarity transformation parameters estimated to make the VLBI frame consistent with the SLR coordinate system and the seven similarity transformation parameters estimated to make the SLR frame consistent with the IERS frame are given in Table 2.6. The three orientation parameters (ϵ , Ψ , ω) were used to rotate the orientation of the SV6 coordinates into the IERS coordinate system. The resulting locations of the 165 sites derived from VLBI and SLR observations constitute the definition of the SV6 terrestrial reference frame. The SV6 positions in a Cartesian coordinate system and the formal uncertainties in local geodetic coordinate systems (with up defined by the spherical radius) are given in Table 2.7. The site velocities and their formal uncertainties, both given in local geodetic systems, are listed in Table 2.8. The horizontal position uncertainties and horizontal rates and uncertainties for a subset of the sites that have had GPS occupations on collocated monuments are shown in Figures 2.14 and 2.15, respectively.

DISCUSSION

SV6 Positions and Velocities

The 165 coordinates that define the SV6 terrestrial reference frame are given in Table 2.7. The sites are nominally assigned to seven tectonic plates, with the greatest concentration of sites on the North America and Eurasia plates, followed by the Pacific, South America, Australia, Africa, and the Nazca (Easter Island) plates. The formal (one-sigma) uncertainties of their horizontal components vary from about 4 to 266 mm, with uncertainties less than 7 mm for the majority of the sites with VLBI observations and less than 40 mm for the sites with SLR observations. The vertical uncertainties are slightly greater, ranging from 7 to 300 mm, with VLBI sites generally having uncertainties less than 10 mm and SLR sites less than 40 mm. Again, we should point out that the lack of a full covariance matrix for the SLR positions hinders our estimates of the formal uncertainties in a local geodetic coordinate system. The horizontal uncertainties for a subset of 30 sites are shown in Figure 2.14. The error ellipses with two degrees of freedom represent 95% confidence levels. The majority of these sites have been occupied predominantly by VLBI systems and show high relative precisions, especially in North America. The site in South America, Arequipa 7907, has one of the longest SLR occupations, but using one of the oldest laser systems with a mean observation epoch in 1982. In contrast, a site in Europe, Zimmerwald 7810, has more recent high-quality laser observations and consequently smaller formal uncertainties.

To assess how much the SV6 coordinates adjusted with respect to their original SLR and VLBI locations, the SLR and VLBI adjustments were propagated from their mean epochs to 1988.0 using their *a priori* velocity estimates (either according to the AM0-2 model or according to a VLBI estimated rate). The post-fit residuals obtained

after a seven-parameter similarity transformation for each set of coordinates was estimated with respect to the SV6 coordinates were used to assess the magnitude of the position adjustments. The VLBI coordinates, by virtue of their high relative precision, in general adjusted less than 4 mm in the horizontal components and less than 6 mm in the vertical. Exceptions to this were generally for those sites mentioned previously with poorly-resolved vertical velocities. Yellowknife, for example, with a mean epoch in 1985 and an *a priori* vertical velocity estimate of 55 ± 57 mm/yr (based on two occupations during 1984–1985) adjusted 144 mm in the vertical. The SLR coordinates, having higher uncertainties and lacking intersite correlations, tended to adjust 10–30 mm in all components at sites collocated with VLBI systems., and for both the SLR and VLBI coordinate data sets. The SLR and VLBI coordinates were propagated by

The estimated velocities and their uncertainties that define the temporal evolution of the SV6 terrestrial reference frame are listed in Table 2.8, which also includes horizontal rates derived from the underlying absolute motion model AM0-2. The horizontal velocities for the 30 reference points collocated with GPS receiver occupations are shown in Figure 2.15 with their 95% confidence level error ellipses. In general, sites that have robust independent VLBI velocity constraints have horizontal component uncertainties on the order of 2 mm/yr or less, while sites lacking VLBI constraints have horizontal component uncertainties ranging from 2–19 mm/yr. Vertical rates are all less than 4 mm/yr with uncertainties less than or equal to the *a priori* model-defined 1.5 mm/yr. The only vertical velocities that are significant at the 95% confidence level are at McDonald 7086/Ft. Davis 7216, Wettzell 7224 and 7834, and Gilcreek 7225.

To assess the velocity estimates, we compare with their *a priori* values. Figure 2.16 shows the differences between the estimated velocities with respect to the *a priori* velocities at the sites with independent VLBI estimates. Most adjustments are

less than 2 mm/yr. The largest adjustments occur at Hartesbeetoe 7232 and Madrid 1665 and are not significant at the 95% confidence level of the original velocity uncertainties; the adjusted values are consistent with AM0-2. Figure 2.17 shows the differences between the estimated velocities with respect to the *a priori* velocities at the sites without independent VLBI velocity estimates. The largest adjustments occur at four sites in California: Quincy 7109, Monument Peak 7110, Mojave 7288, and Owens Valley 7853. The velocities for all of these sites were constrained to be equal to those of collocated monuments with independent VLBI velocity estimates to ensure uniform estimation of the similarity transformation. The other sites shown in Figure 2.17 tend to show systematic adjustments that are correlated with tectonic plates. For example, the North American sites all move approximately 2–3 mm/yr faster in the north component and approximately 1 mm/yr faster in the west component. These systematic variations represent adjustments to the Euler pole that was used to derive their *a priori* motions due to correlations between these sites and sites that have independent velocity estimates. They therefore represent an estimate based on VLBI velocities of some of the systematic biases inherent to the AM0-2 model.

In general, the adjustments to the positions and velocities are consistent with their *a priori* uncertainties. The largest adjustments are associated with vertical components and are usually a result of the *a priori* assumption of 1.5 mm/yr vertical velocity uncertainty. More investigation is required to assess whether 1.5 mm/yr is an appropriate level of uncertainty.

Comparison of SLR and VLBI

The 16 local survey ties shown in Figure 2.4 and listed in Table 2.2, plus the implicit ties between SLR and VLBI systems that occupied the same monument, provide the necessary constraints on the estimated similarity transformation parameters that make the VLBI coordinate frame consistent with the SLR coordinate

frame (see Table 2.6). The adjustments to these ties are all less than 1 mm, except at 7 sites: Canberra (~50 mm), Hawaii (~15 mm), Grasse (~10 mm), McDonald (5 mm), Kwajalein (~5 mm), Mojave (~4 mm), and Goddard (~2 mm). The adjustments are all consistent with their *a priori* uncertainties, except for Hawaii and Canberra. The estimated transformation parameters show a significant 30 ± 8 mm offset in the origin and significant scale 4 ± 1 ppb difference. It is interesting to note that, except for the scale change, the VLBI to SLR transformation is almost exactly the opposite of the SLR to IERS transformation, indicating that the GLB659 VLBI coordinate frame is nearly consistent with the IERS both in orientation and origin, since GLB659 includes an estimated offset to the IERS origin.

The estimate of similarity transformation parameters is strongly dependent on the global distribution of collocated VLBI and SLR reference monuments (Figure 2.4). The current distribution of collocation sites is dominated by sites in North America, with only three sites in Europe, two sites in the Pacific and one site each in Australia and China. Quality of collocation—that is, the combined quality of the SLR and VLBI measurement systems and the local survey ties—differs from site to site and is especially variable for sites outside of North America. For example, the SLR occupation at Medicina is weak, and the SLR system at Wettzell has a timing problem associated with its early observations that may have biased its vertical location. No direct tie has been conducted at Kwajalein, but rather two independent Doppler-based estimates were differenced. The 26-km tie at Canberra between the Tidbinbilla VLBI antenna and the Orroral SLR system is the result of a combination of terrestrial and GPS observations that have several centimeter scatter, and is probably of low quality. It is, however, the only site in the southern hemisphere and contributes strong weight to the estimate despite the relatively high uncertainty assigned to its tie. It may, for example, be responsible for the large residuals noted at Hawaii.

Several methods have been adopted to assess geometric effects. *Murray et al.* [1990], in the derivation of the SV5 terrestrial reference frame, explored applying weights to the sites to incorporate some of the other considerations mentioned above that might affect the true quality of the collocation. Under this methodology, which used the same SLR and VLBI adjustments but only the collocation sites given in Table 2.2 plus Yuma 7894 and propagated the sites to 1988.0 via their *a priori* velocity estimates, Grasse was given the strongest weight in Europe while the vertical component of Wettzell was given less weight, and the weights of Canberra and Kwajalein were less with respect to Hawaii. The estimated similarity transformation parameters are similar in orientation, but the scale difference is 7 ± 1 ppb and the z-origin offset is not significant while the x-origin offset is -19 ± 10 mm. The weighted rms of the coordinate differences was 21 mm, although the difference in vertical at Wettzell was 82 mm and the east and vertical components at Canberra were 70 mm. The change in the SV5 and SV6 estimated scale differences may be attributed to the introduction of vertical uncertainties in the *a priori* motion model which propagates greater uncertainty into the SLR vertical coordinates. Changes to the weights on the sites appears to affect the estimate of the origin at the several centimeter level.

An alternative approach was taken by *Ray et al.* [1991] who used nearly the same sites and local survey offsets, and used a similar VLBI adjustment, but used an SLR adjustment that only included data over an 18 month period beginning in November 1987. The short SLR data span was chosen to eliminate the possible biases that might be introduced by assuming incorrect site velocities, since no velocities were estimated in the SLR adjustment. Their results were similar to the ones discussed above in orientation and scale (5.4 ± 0.4 ppb), but the origin offset was most significant in the y-component (-24 ± 4 mm). They estimated systematic biases that might be introduced by the geometry of the site distribution by eliminating sites from the estimate and also found the origin to be stable at the 20 mm level.

Both VLBI and SLR are sensitive to the scale of the reference frame, and the adjustments used to derive SV6 nominally assumed relativistic models that are consistent with a geocentric frame. The estimated differences in scales, which range from 4 to 7 ± 1 ppb, are significant and indicate that the models used to estimate the VLBI and SLR coordinates are not complete. Several explanations are possible. A known bias exists in the CSR8902 reference frame due to an incorrect assumption for the satellite center of mass correction for LAGEOS. The MERIT and IERS standards specify a 240 mm correction, but it was recently realized the correction should be 251 mm [Ray *et al.*, 1991]. Consequently, Ray *et al.* [1991], adopted a value of $GM = 398600.4415 \text{ km}^3/\text{s}^2$ for the SLR adjustment. This 2 ppb change in scale is consistent with the 11 mm range bias, but it *increases* the SLR scale and thus increases the scale discrepancy with respect to VLBI. The most likely effect that might account for the scale difference is modeling of propagation delays due to the ionosphere and troposphere to which VLBI radio waves are more sensitive than optical wavelengths used by the laser systems. No satisfactory explanation has been found for the scale difference at this time, so we shall continue to adopt the SLR scale.

GPS Reference Points

The SV6 terrestrial reference frame consists of 165 reference points derived from either VLBI or SLR measurement systems. Since late 1986, the National Geodetic Survey has coordinated the operation of the Cooperative International GPS Network (CIGNET) of continuously tracking GPS receiver that are collocated near VLBI reference points included in SV6 [Chin, 1987]. Since that time more GPS receivers have been similarly collocated with SLR and VLBI reference points [e.g., Bock and Shimada, 1990], and numerous investigators have made temporary occupations using monuments included in SV6 (for example, Chapter 3 will discuss the analysis of observations from California sites that include Ft. Ord, Vandenberg, Palos Verdes,

Owens Valley and Mojave). Figure 2.14 shows a representative sample of GPS-occupied sites near SV6 reference points. Provided the offset between the GPS antenna reference point and a collocated SV6 reference point is known, the SV6 terrestrial reference frame can provide useful fiducial constraints for GPS orbit and baseline determination.

The offset between a GPS antenna reference point and an SLR or VLBI reference point generally, but not necessarily, consists of three different parts. The SLR or VLBI reference point may refer to either a location internal to the measurement system, such as the intersection of the VLBI antenna mount rotation axes, or it may refer to a ground monument, as in the case of mobile occupations. The first part of the offset establishes the relationship between a monument whose relative location with respect to the VLBI or SLR reference point and the GPS antenna is known. For example, the GPS antenna may be located over another ground monument whose relative location with respect to the first ground monument is known from previous terrestrial measurements, or a direct survey using GPS receivers to determine the relative location can be conducted (e.g., the location of SIO1 near San Diego depends on the location of Pinon Flat 7256). Table 2.9 gives offsets from SV6 reference points to GPS reference points for the sites depicted in Figure 2.14, where the GPS reference point refers either to a ground monument or to the L1 phase center of the antenna.

The second part of the offset establishes the relationship between an accessible point on the GPS antenna and the ground monument. In the case where a direct GPS survey was conducted to establish the antenna phase center offset with respect to the ground monument, this offset is not necessary. Typically, however, the offset consists of a measurement of the height of the antenna above a ground monument. Care must be taken to adequately specify the physical point on the antenna that was used as one of the endpoints of the measurement. Typically, such a measurement is conducted either vertically or in a slant from the ground monument to a point on the base of the

antenna. Table 2.10 provides the antenna height information for some of the CIGNET sites found in Figure 2.14. Note that in some cases, such as Tromso, the antenna height changed several times over the occupation history, while in other cases, such as Kokee, a different antenna was installed over the same ground monument.

Finally, antenna-dependent offsets specify the location of the GPS L1 and L2 phase centers with respect to the accessible point on the antenna. The assumed vertical phase center offsets for some antennas are given in Table 2.11. Some documentation for these offsets exists. For example the Texas Instrument TI-4100 conical spiral antenna is described in *Sims* [1985], and a correction for the location of the L2 phase center for later series TI4100 antennas is given by *Prescott et al.* [1989]. Most of the other antennas phase center offsets were obtained from unpublished engineering tests.

By combining the information in Tables 2.9, 2.10, and 2.11, the relative location of the GPS phase centers with respect to SV6 reference points may be determined. Since the uncertainties in these ties is usually one millimeter or less, the relative precision of the GPS phase center locations should be determined from their corresponding SV6 reference point uncertainties (Table 2.7).

CONCLUSIONS

We have presented a methodology for deriving a terrestrial reference frame suitable for crustal deformation monitoring with the Global Positioning System. The SV6 terrestrial reference frame is derived from SLR, VLBI and terrestrial observations, and utilizes VLBI and geologic estimates of velocity to define the temporal evolution of the sites. We conclude with a few general remarks regarding future work.

The continuing development of space-geodetic techniques and the rapid deployment of GPS receivers around the globe provide a challenge for the timely development of terrestrial reference frames. SV6 is likely to be superseded in a very short time by the introduction of several improvements. One of the major limitations of SV6 is the absence of position correlations or estimate sites velocities in the SLR adjustment . The SLR technique has now reached a level of maturity that allows robust estimates of site velocities for many of the sites [e.g., *Smith et al.*, 1990]. The techniques developed in this chapter can be easily modified to accommodate this information. Another limitation is the weak global distribution of sites that limits the accuracy of the similarity transformation. This will improve with the addition of more sites and the improvement of long ties using GPS, such as at Canberra, China, Japan, South America, and Antarctica.

The need to incorporate geophysical models for site velocities will probably not disappear, due to the presence of many sites that simply will not have enough data to robustly estimate a velocity. However, the *a priori* model is now out of date with the introduction of the NUVEL-1 model [*DeMets et al.*, 1990] that substantially improves upon the RM2 model used to derive AM0-2. Efforts to derive hot spot [*Gripp and Gordon*, 1990] and no-net-rotation absolute motion frames (D. Argus, personal communication, 1991) will soon allow NUVEL-1 to become international standard. The velocity constraints described above also could be extended to include a more regional definition of correlated velocities, rather than just constraining two collocated sites to have the same velocity.

Several large, globally distributed GPS experiments have recently been conducted, including the 24-receiver Global Orbit Tracking Experiment (GOTEX) Campaign coordinated by NGS in November 1989, and the 100-receiver IERS/GIG'91 Campaign coordinated by JPL in January 1991. These campaigns are beginning to extend the accuracy of the VLBI/SLR derived reference frames to areas that are impractical to

occupy with VLBI and SLR systems. Because these global experiments include sites used by the U.S. Department of Defense to operationally define the World Geodetic System 1984 (WGS84), analysis of the data will enable us to determine the relationship between the WGS84 reference frame and those such as SV6 based on SLR and VLBI observations. When the full constellation of GPS satellites is available and is tracked from a global distribution of sites, GPS-derived terrestrial reference frames may possibly rival the accuracy and precision of terrestrial reference frames derived from VLBI and SLR observations.

Table 2.1. SV6 Site Information

Mon.	Site	Mean Epoch		Vel Est	Lat., °N	Lon., °E	Plate	km to Bound.	Tie
		SLR	VLBI						
1181	POTSDM	1986.3918			52.380	13.065	aura	1425.1	
1311	KAUAI		1988.6739	Y*	22.126	-159.665	pcfc	3398.4	7210
1513	GOLDVENU		1985.8232	Y	35.248	-116.795	noam	138.8	
1561	ROBLED32		1983.4000		40.429	-4.249	aura	483.6	
1642	DSS45		1989.3084	Y	-35.398	148.978	aust	1883.6	7843
1665	DSS65		1989.0691	Y	40.427	-4.251	aura	483.3	
1856	KASHIMA		1987.2437	Y	35.954	140.663	noam	76.5	
1884	RIGA	1988.8805			56.949	24.059	aura	1899.5	
1953	SANCUB	1988.5785			20.012	-75.762	noam	56.7	
4968	KWAJAL26		1986.5399	Y	9.399	167.482	pcfc	2292.6	7092
7035	OTAY2	1988.6620			32.601	-116.841	pcfc	95.8	
7038	MILES MON		1988.4000		46.397	-105.861	noam	1614.7	
7051	QUINCY	1980.9383			39.973	-120.940	noam	214.9	
7061	EASTER	1987.9462			-27.148	-109.384	nazc	372.3	
7062	OTAY	1978.9999			32.601	-116.841	pcfc	95.8	
7063	STALAS	1979.8537			39.020	-76.828	noam	2139.5	
7067	BERMDA	1978.4827			32.354	-64.656	noam	1388.4	
7069	RAMLAS	1979.7283			28.228	-80.606	noam	1066.5	
7080	MCDON4	1988.6456			30.680	-104.015	noam	791.2	
7082	BERLAK	1979.3499			41.934	-111.421	noam	1011.0	
7084	OWENSV	1978.1977			37.232	-118.296	noam	225.5	
7085	GOLDST	1978.2168			35.424	-116.886	noam	154.1	
7086	MCDON	1985.1906			30.677	-104.016	noam	791.0	7216
7090	YARAG	1984.0088			-29.047	115.347	aust	2102.8	
7091	HAYSTK	1988.7247			42.623	-71.488	noam	2495.1	7209
7092	KWAJLN	1980.5665			9.394	167.476	pcfc	2292.2	4968
7096	SAMOA	1980.3485			-14.335	-170.725	pcfc	96.1	
7097	EASTR2	1988.6390			-27.148	-109.384	nazc	372.3	
7100	GRF100	1977.8112			39.021	-76.820	noam	2139.5	
7101	GRF101	1979.6998			39.021	-76.821	noam	2139.5	
7102	GORF7102	1981.3113	1989.7000		39.021	-76.828	noam	2139.6	7105
7103	GRF103	1981.9005			39.021	-76.828	noam	2139.6	
7104	GRF104	1979.2793			39.021	-76.823	noam	2139.5	
7105	GRF105	1985.6141			39.021	-76.828	noam	2139.6	7102
7109	QUINC2	1985.7277			39.975	-120.945	noam	214.7	7221
7110	MNPEAK	1985.8668			32.892	-116.423	pcfc	45.4	7274
7112	PLATVL	1983.5246			-0.183	-104.726	noam	1269.1	7258
7114	OWNSV2	1980.6133			37.233	-118.294	noam	225.8	
7115	GLDST2	1980.4394			35.248	-116.792	noam	139.0	
7120	HALEAKAL	1981.3335	1988.6000		20.708	-156.256	pcfc	3588.5	
7121	HUAHIN	1984.8101			-16.734	-151.041	pcfc	2134.3	
7122	MAZTLN	1986.1238			23.343	-106.459	noam	135.4	
7123	HUAHI2	1988.0695			-16.734	-151.041	pcfc	2134.3	
7125	GRF125	1985.4580			39.020	-76.827	noam	2139.5	
7130	GRF130	1986.9243			39.021	-76.827	noam	2139.6	
7203	EFLSBERG		1981.4587	Y	50.525	6.884	aura	1480.0	
7204	NRAO-140		1984.2680	Y	38.438	-79.836	noam	2132.4	
7205	HAYSTACK		1984.0268	Y	42.623	-71.488	noam	2495.1	
7207	OVRO-130		1985.5337	Y	37.231	-118.283	noam	226.4	7853
7209	WESTFORD		1988.0040		42.613	-71.494	noam	2493.9	7091

7210	HOLLAS	1985.0485			20.707 -156.256	pcfc	3588.6	7120
7213	ONSALA60		1987.3523	Y	57.396 11.926	aura	1718.3	
7214	NRAO85-3		1989.5000		38.430 -79.843	noam	2131.7	
7215	CHLBOLTN		1980.9000		51.145 -1.438	aura	1628.1	
7216	HRAS-085		1987.4521	Y	30.637 -103.947	noam	793.9	7086
7217	MARPOINT		1986.9920	Y	38.374 -77.231	noam	2075.0	
7218	HATCREEK		1987.7333	Y	40.817 -121.471	noam	219.5	
7219	RICHMOND		1987.4096	Y*	25.614 -80.385	noam	780.5	7295
7220	MONPK2	1984.7088			32.892 -116.423	pcfc	45.4	
7221	QUINCY		1987.8741	Y	39.975 -120.944	noam	214.8	7109
7222	MOJAVE12		1988.1073	Y	35.332 -116.888	noam	144.4	7288
7223	VNDNBERG		1987.7525	Y	34.556 -120.616	pcfc	128.3	
7224	WETZELL		1987.4354	Y	49.145 12.877	aura	1101.6	7834
7225	GILCREEK		1987.6227	Y	64.978 -147.498	noam	643.4	
7226	SHANGHAI		1986.5000		31.190 121.430	aura	863.5	
7227	SESHAN25		1988.8602	Y	31.099 121.200	aura	877.8	7837
7228	CARROLGA		1987.6000		33.573 -85.110	noam	1760.2	
7229	SEATTLE1		1986.7000		47.686 -122.249	noam	437.8	
7230	MEDICINA		1988.6946	Y	44.520 11.647	aura	761.0	7546
7231	DSS15		1988.8000		35.422 -116.887	noam	153.9	
7232	HARTRAO		1987.7994	Y	-25.890 27.685	afrc	2157.2	
7234	PIETOWN		1988.9000		34.301 -108.119	noam	666.9	
7241	FORTORDS		1989.1059	Y*	36.589 -121.772	pcfc	43.8	
7242	HOBART26		1990.0000		-42.804 147.441	aust	1140.6	
7244	NOBEY-6M		1990.0000		35.941 138.472	noam	128.2	
7251	PT-REYES		1987.9098	Y	38.104 -122.937	pcfc	29.9	
7252	PRESIDIO		1987.7532	Y	37.805 -122.455	pcfc	11.5	
7254	PBLOSSOM		1986.3499	Y	34.512 -117.922	pcfc	34.8	
7255	SANPAULA		1987.9396	Y	34.388 -118.999	pcfc	39.0	
7256	PINFLATS		1986.9720	Y	33.609 -116.459	noam	4.1	
7258	PLATTVIL		1987.2599	Y	40.183 -104.726	noam	1269.1	7112
7259	MAMMOTH		1985.8215	Y	37.642 -118.945	noam	223.3	
7261	FLAGSTAF		1986.9173	Y	35.215 -111.635	noam	453.2	
7263	JPL-MV1		1986.6394	Y	34.205 -118.173	pcfc	10.9	
7265	MOJAVE	1984.0953			35.331 -116.891	noam	144.2	
7266	FORT-ORD		1987.0473	Y	36.670 -121.773	pcfc	39.6	
7267	DEADMANL		1987.6783	Y	34.255 -116.279	noam	69.6	
7268	PVERDES		1987.9756	Y	33.744 -118.404	pcfc	62.7	
7269	BLKBUTTE		1987.1986	Y	33.664 -115.720	noam	61.4	
7270	OCOTILLO		1984.8000		32.790 -115.796	noam	8.9	
7271	AUSTINTX		1987.6000		30.339 -97.696	noam	1261.4	
7274	MON-PEAK	1985.4778	1987.1609	Y	32.892 -116.423	pcfc	45.4	7110
7277	YAKATAGA		1986.8785	Y	60.081 -142.486	noam	98.2	
7278	KODIAK		1988.1404	Y	57.740 -152.497	noam	129.9	
7279	NOME		1985.7047	Y	64.563 -165.371	noam	1141.6	
7280	SNDPOINT		1988.5729	Y	55.352 -160.475	noam	118.4	
7281	SOURDOGH		1986.2050	Y	62.664 -145.484	noam	375.2	
7282	ALGOPARK		1985.2641	Y	45.955 -78.073	noam	2915.9	
7283	PENTICTN		1985.4495	Y*	49.323 -119.620	noam	620.8	
7284	WHTHORSE		1988.5368	Y	60.711 -135.077	noam	363.0	
7285	YELLOWKN		1985.2635	Y	62.479 -114.472	noam	1428.7	
7286	ELY		1988.1088	Y	39.293 -114.843	noam	603.1	
7288	MOJ-7288	1988.4843	1987.9000		35.331 -116.892	noam	144.2	7222
7290	VERNAL		1988.2039	Y	40.327 -109.571	noam	964.6	
7291	BLOOMIND		1987.7000		39.179 -86.498	noam	2387.1	
7292	LEONRDOK		1987.7000		35.909 -95.795	noam	1746.3	
7294	BERMUDA		1987.7000		32.361 -64.669	noam	1389.0	
7295	RICHMO	1988.3855			25.613 -80.384	noam	780.4	7219

7400	SNTAGO	1984.2867			-33.150	-70.669	soam	198.5	
7401	CERTOL	1984.4222			-30.172	-70.800	soam	125.3	
7510	ASKITS	1987.1895			40.928	25.566	aura	535.2	
7512	RHODES	1986.9843			35.952	27.781	aura	133.2	
7515	DIONS2	1987.3915			38.079	23.932	aura	314.6	
7517	ROUMEL	1987.3611			35.404	24.694	aura	133.7	
7520	KARITS	1986.2858			39.734	20.665	aura	108.8	
7525	XRISOK	1987.4832			36.791	21.878	aura	93.2	
7530	BARGIY	1987.3324			31.722	35.089	afrc	42.7	
7540	MAT540	1986.1136			40.649	16.704	aura	165.2	
7541	MAT541	1986.0767			40.649	16.704	aura	165.2	
7544	LAMPED	1987.8038			35.518	12.567	afrc	112.9	
7545	CAGLIA	1986.8279			39.135	8.973	aura	208.7	
7546	MEDICI	1988.4047			44.520	11.646	aura	761.0	7230
7547	NOTO		1989.5421	Y*	36.876	14.989	aura	27.8	
7550	BASOVI	1986.3409			45.643	13.876	aura	730.0	
7575	DIYARB	1987.3387			37.920	40.195	aura	105.7	
7580	MELENG	1987.3198			37.378	33.191	afrc	264.5	
7585	YOZGAT	1987.4531			39.801	34.813	aura	336.5	
7587	YIGILC	1987.4583			40.937	31.439	aura	578.4	
7596	WET596	1985.6585			49.144	12.879	aura	1101.4	
7599	WET599	1984.8936			49.145	12.878	aura	1101.5	
7600	HOHENFRG		1989.6000		53.051	10.476	aura	1564.6	
7601	METSHOVI		1989.6000		60.242	24.384	aura	1626.3	
7602	TROMSONO		1989.7000		69.663	18.922	aura	572.6	
7603	CARNUSTY		1989.7000		56.478	-2.783	aura	1356.7	
7604	BREST		1989.7000		48.408	-4.504	aura	1335.5	
7605	GRASSE		1989.8000		43.755	6.921	aura	738.6	7835
7805	METFIN	1983.5366			60.217	24.395	aura	1629.1	
7810	ZIMMER	1987.7250			46.877	7.465	aura	1076.4	
7831	HELWAN	1988.6103			29.859	31.343	afrc	339.0	
7833	KOOTWK	1981.3121			52.178	5.810	aura	1665.6	
7834	WETZEL	1985.1309			49.145	12.878	aura	1101.5	7224
7835	GRASSE	1987.2313			43.755	6.921	aura	738.6	7605
7837	SHAHAI	1986.6136			31.098	121.192	aura	878.4	7227
7838	SHO	1985.9722			33.578	135.937	aura	59.1	
7839	GRAZ	1986.6705			47.067	15.493	aura	810.5	
7840	RGO	1986.6398			50.867	0.336	aura	1570.3	
7843	ORLLR	1986.6801			-35.636	148.939	aust	1867.9	1642
7850	MCD-7850		1988.8000		30.680	-104.015	noam	791.2	
7853	OVR-7853	1988.8183	1987.9000		37.233	-118.294	noam	225.8	7207
7882	CABO	1988.0203			22.918	-109.864	pcfc	172.1	
7885	MCDON3	1982.6464			30.677	-104.016	noam	791.0	
7886	QUINC3	1984.5769			39.975	-120.945	noam	214.7	
7888	MTHOP2	1982.1780			31.685	-110.878	noam	313.8	
7891	FLGSTF	1981.5112			35.215	-111.635	noam	453.2	
7892	VERNAL	1981.8531			40.327	-109.571	noam	964.6	
7894	YUMA	1983.3811	1987.2873	Y	32.939	-114.203	noam	110.1	
7896	PASDNA	1980.9027			34.206	-118.172	pcfc	10.8	
7899	GRF899	1980.7143			39.021	-76.820	noam	2139.5	
7900	FTD-7900		1988.9000		30.636	-103.947	noam	793.8	
7907	ARELAS	1982.5005			-16.466	-71.493	soam	248.5	
7921	HOPLAS	1977.6434			31.684	-110.878	noam	313.8	
7929	NATLAS	1978.3973			-5.928	-35.165	soam	898.6	
7939	MATERA	1986.2398			40.649	16.705	aura	165.2	
7943	ORRLAS	1979.1062			-35.625	148.955	aust	1867.7	
8833	KOTWK2	1987.1210			52.178	5.810	aura	1665.6	

* Velocity not included in observations due to singularities or numerical instabilities in covariance

Table 2.2. Local Survey Ties from VLBI to SLR Reference Points

Site	Ref. Point		Cartesian Offset, mm			Sigma, mm			Correlation		
	SLR	VLBI	x	y	z	x	y	z	xy	xz	yz
GRAS	7835	7605	-6015.0	33735.0	8102.0	0.0	0.0	0.0	0.0	0.0	0.0
HAWA	7210	1311	77839492.7	-349863760.8	-145626353.7	14.0	6.9	6.3	0.923	-0.852	-0.820
MCDO	7086	7216	-5914395.7	3496514.5	4031857.9	3.6	12.1	7.0	0.725	-0.717	-0.890
GSFC	7105	7102	33112.0	2452.0	-4398.0	0.0	0.0	0.0	0.0	0.0	0.0
RICH	7295	7219	60877.0	-0931.0	-44212.0	3.0	2.0	2.0	0.0	0.0	0.0
WEST	7091	7209	247005.5	851714.0	800434.5	1.0	1.4	1.4	-0.27	0.25	-0.55
MEDI	7546	7230	29535.1	-29903.9	-48551.8	0.5	1.1	4.9	0.0	0.0	0.0
SHAN	7837	7227	598875.0	469603.0	-154995.0	10.0	10.0	10.0	0.0	0.0	0.0
MOJA	7288	7222	-323131.5	148209.9	-43973.2	2.1	2.2	2.2	0.03	-0.03	-0.05
MNPK	7110	7274	11394.7	-7802.1	-2522.5	4.6	2.4	1.3	-0.87	0.16	0.08
OVRO	7853	7207	-820483.1	549137.4	87110.0	2.4	3.5	3.6	0.75	-0.08	-0.25
QUIN	7109	7221	-3889.8	38986.6	38409.4	1.2	1.0	1.0	0.02	0.05	-0.22
PLAT	7112	7258	29894.4	-9035.5	-1025.3	0.5	0.7	0.5	0.21	-0.08	-0.13
KWAJ	7092	4968	89163.4	702928.4	-544305.0	0.0	0.0	0.0	0.0	0.0	0.0
CANB	7843	1642	14458347.6	-4638683.3	-21870027.7	19.9	25.7	14.3	0.250	0.808	-0.121
WETT	7834	7224	-10003.0	46149.0	-11072.0	0.4	0.4	0.4	0.0	0.0	0.0

Table 2.3 Plate Areas and Areal Moment Coefficients

Plate	Area*	S_{11}	S_{22}	S_{33}	S_{12}	S_{13}	S_{23}
afrc	1.911	1.256	0.375	0.280	0.250	-0.087	-0.109
anta	1.467	0.110	0.262	1.095	0.047	-0.106	-0.065
arab	0.124	0.047	0.055	0.021	0.050	0.034	0.032
aust	1.227	0.371	0.591	0.266	-0.217	0.277	-0.295
carb	0.087	0.007	0.075	0.005	-0.021	0.006	-0.019
coco	0.076	0.002	0.073	0.002	0.006	-0.001	-0.011
aura	1.692	0.280	0.600	0.812	-0.096	0.299	0.413
indi	0.282	0.020	0.241	0.022	0.055	0.015	0.061
nazc	0.406	0.011	0.334	0.060	0.013	0.004	0.116
noam	1.420	0.152	0.433	0.835	-0.074	-0.100	-0.387
pcfc	2.661	1.460	0.637	0.564	0.398	-0.060	0.056
phil	0.141	0.062	0.064	0.016	-0.061	-0.032	0.029
soam	1.069	0.409	0.448	0.212	-0.352	-0.249	0.179
Total**	12.563	4.187	4.188	4.190	-0.002	0.000	0.000

*Earth's radius equals 1

**Theoretical totals: $\text{area} = 4\pi \cong 12.566$; $S_{ii} = 4\pi/3 \cong 4.189$; $S_{ij} = 0$, when $i \neq j$.

Table 2.4. Chi-Square Misclosure Statistics for Scaling Deformation Uncertainty

site	monu	VLBI only			VLBI + AM0-2		VLBI + AM0-2 +Deformation		
		ϕ_{vert}	ϕ_{horiz}	ϕ_{total}	ϕ_{horiz}	ϕ_{total}	ϕ_{vert}	ϕ_{horiz}	ϕ_{total}
HAYSTACK	7205	3.04	3.94	10.62	0.67	4.21	1.45	0.18	1.68
NRAO-140	7204	0.06	4.92	5.11	0.46	0.53	0.04	0.19	0.23
OVRO-130	7207	0.69	338.40	388.92	21.79	21.80	0.43	4.30	4.64
EFLSBERG	7203	0.08	0.93	0.93	0.88	0.88	0.08	0.63	0.63
HRAS-085	7216	10.22	13.87	49.07	0.37	11.68	4.75	0.14	4.92
ONSALA60	7213	0.05	7.68	35.03	3.55	4.59	0.03	1.35	1.38
GOLDVENU	1513	0.00	129.29	129.29	13.01	13.01	0.00	2.08	2.08
MARPOINT	7217	0.82	6.23	7.03	1.68	2.50	0.71	0.46	1.17
HATCREEK	7218	0.02	201.65	243.57	21.14	22.69	0.01	3.40	3.44
PLATTVIL	7258	0.47	2.62	2.98	1.10	1.50	0.44	0.24	0.67
MOJAVE12	7222	1.09	375.72	376.84	17.12	18.48	0.58	2.65	3.20
WETTZELL	7224	5.75	4.86	147.92	2.31	22.56	3.90	0.81	6.48
RICHMOND	7219	6.36	0.00	6.36	0.00	6.36	0.01	0.00	0.01
KASHIMA	1856	2.13	37.98	47.58	22.82	38.68	1.79	4.00	6.47
KAUAI	1311	0.00	6.74	6.74	4.99	4.99	0.00	3.25	3.25
VNDNBERG	7223	1.04	236.65	243.15	39.34	40.25	0.69	2.43	3.15
GILCREEK	7225	6.12	0.48	12.44	0.13	8.46	4.17	0.03	4.32
KWAJAL26	4968	1.09	0.90	1.75	0.83	1.67	1.05	0.68	1.51
ALGOPARK	7282	0.19	0.07	0.38	0.07	0.38	0.19	0.06	0.35
YELLOWKN	7285	0.47	0.44	0.80	0.43	0.80	0.47	0.40	0.77
PENTICTN	7283	0.35	6.62	6.95	6.48	6.79	0.35	5.89	6.22
HARTRAO	7232	0.97	2.11	2.11	2.09	2.09	0.95	1.98	1.99
MEDICINA	7230	0.05	5.58	7.01	5.00	6.20	0.05	3.47	4.13
SESHAN25	7227	1.91	0.31	2.64	0.24	2.48	1.89	0.21	2.38
DSS45	1642	0.00	1.69	1.89	1.66	1.85	0.00	1.61	1.78
DSS65	1665	5.71	5.77	8.84	5.63	8.76	5.68	4.56	8.07
NOTO	7547	0.02	0.03	0.04	0.03	0.04	0.02	0.02	0.03
KODIAK	7278	1.13	51.11	55.73	18.85	20.99	1.11	4.18	5.49
SNDPOINT	7280	0.64	14.81	16.40	7.81	8.73	0.63	1.26	1.96
WHTHORSE	7284	0.20	17.74	18.12	13.94	14.29	0.20	2.77	2.99
JPL-MV1	7263	0.22	486.28	491.88	195.06	195.44	0.22	1.78	1.98
MON-PEAK	7274	0.47	471.64	480.31	123.84	124.94	0.43	3.24	3.69
QUINCY	7221	0.00	128.96	132.52	29.44	29.58	0.00	4.49	4.50
PBLOSSOM	7254	0.91	374.87	395.79	191.25	199.46	0.90	8.43	9.49
MAMMOTH	7259	1.03	37.05	37.56	24.65	25.43	1.03	8.13	8.94
FORT-ORD	7266	0.30	32.83	32.83	19.28	19.34	0.29	0.86	1.14
PRESIDIO	7252	0.18	344.87	351.36	154.10	155.75	0.18	1.82	2.01
PT-REYES	7251	0.01	85.08	85.25	42.35	42.45	0.00	1.49	1.49
SANPAULA	7255	0.43	109.99	112.42	74.79	75.48	0.43	4.11	4.44
PINFLATS	7256	0.09	350.53	363.66	112.59	113.46	0.09	0.87	0.95
YUMA	7894	1.00	2.27	3.64	0.49	1.57	0.97	0.07	1.05
BLKBUTTE	7269	0.72	28.21	28.24	12.38	12.80	0.71	0.89	1.55
PVERDES	7268	0.82	76.58	80.85	49.06	52.26	0.81	4.48	5.45
DEADMANL	7267	4.00	3.75	9.29	3.12	8.29	3.99	0.84	5.28
FLAGSTAF	7261	2.29	0.82	3.59	0.41	2.95	2.27	0.19	2.58
ELY	7286	0.06	11.46	11.46	5.76	5.82	0.06	1.18	1.22
VERNAL	7290	0.63	1.66	2.73	1.36	2.35	0.63	0.57	1.34
FORTORDS	7241	1.44	2.00	2.81	1.99	2.80	1.44	1.38	2.34
NOME	7279	0.12	0.52	0.73	0.45	0.66	0.12	0.31	0.48
SOURDOGH	7281	0.36	3.82	4.30	2.27	2.71	0.35	0.67	1.04
YAKATAGA	7277	5.79	292.13	301.45	143.31	150.78	5.76	29.26	35.34
95% Confidence		3.84	5.99	7.81	5.99	7.81	3.84	5.99	7.81
Simultaneous		134.61	11857.79	12366.94	3822.27	4326.48	64.31	122.60	187.87
95% Confidence		67.50	124.70	179.58	124.70	179.58	67.50	124.70	179.58

Table 2.5. Tests for Horizontal α and Vertical β Deformation Scalars

α	β	ϕ_{horiz}	ϕ_{vert}	ϕ_{total}
0.0	0.0	3822.27	134.61	4326.48
0.5	0.5	313.72	110.19	432.72
1.0	1.0	122.60	81.65	206.22
0.9	1.4	142.17	67.04	210.70
1.0	1.5	122.60	64.31	187.87
1.1	1.6	107.14	61.85	169.58
1.5	1.5	68.87	64.31	133.43
2.0	2.0	45.47	54.25	99.48
2.5	2.5	32.74	48.05	80.42
95% Confidence		124.70	67.50	179.58

Table 2.6. Similarity Transformation Parameters

		VLBI to SLR		SLR to IERS	
t_x	(mm)	8.91	± 8.05	-10.02	± 2.74
t_y	(mm)	-4.21	± 8.90	8.17	± 2.81
t_z	(mm)	30.06	± 7.61	-20.48	± 2.60
s	(ppb)	4.42	± 1.09	2.71	± 0.37
ϵ	(mas)	5.84	± 0.33	-4.34	± 0.11
ψ	(mas)	-0.84	± 0.27	2.14	± 0.10
ω	(mas)	13.22	± 0.26	-12.91	± 0.08
f		107		461	
σ_o^2		1.931		1.623	

Table 2.7. SV6 Site Positions and Uncertainties at 1988.0

mon	site	Cartesian position, mm			Local Geodetic Uncertainties, m.m		
		x	y	z	σ_n	σ_e	σ_u
1181	POTSDM	3800621301.3	882005350.5	5028859575.1	31.9	35.2	28.7
1311	KAUAI	-5543845956.5	-2054564169.1	2387813802.0	6.9	6.7	7.6
1513	GOLDVENU	-2351129027.0	-4655477076.0	3660956908.9	5.0	5.1	8.9
1561	ROBLED32	4849245383.6	-360278257.3	4114884426.4	20.6	23.0	87.4
1642	DSS45	-4460935061.9	2682765783.3	-3674381651.3	12.8	14.6	14.4
1665	DSS65	4849336821.8	-360488909.4	4114748666.7	9.4	10.1	10.1
1856	KASHIMA	-3997892244.7	3276581301.2	3724118325.2	10.3	9.4	10.6
1884	RIGA	3183895735.7	1421496372.2	5322803976.5	227.2	264.8	204.7
1953	SANCUB	1474548701.5	-5811242391.8	2168945010.9	97.4	98.6	76.7
4968	KWAJAL26	-6143536463.7	1363997227.9	1034707400.4	10.3	9.7	12.4
7035	OTAY2	-2428826566.8	-4799754336.6	3417267044.8	18.2	21.0	18.8
7038	MILESMON	-1204438897.9	-4239211087.0	4596266061.6	10.4	8.0	59.0
7051	QUINCY	-2516893105.2	-4198845158.2	4076411378.9	37.0	38.4	21.9
7061	EASTER	-1884984692.7	-5357609392.8	-2892854117.5	33.7	44.0	35.6
7062	OTAY	-2428825853.7	-4799750540.4	3417272730.7	60.8	60.9	22.5
7063	STALAS	1130714785.7	-4831367738.8	3994087359.5	21.2	24.0	18.3
7067	BERMDA	2308538568.1	-4874079970.4	3393628569.2	71.0	69.8	60.4
7069	RAMLAS	917958960.3	-5548370217.1	2998776605.2	91.2	118.7	75.5
7080	MCDON4	-1330020920.7	-5328401888.7	3236480822.4	13.3	15.8	11.5
7082	BERLAK	-1735997126.3	-4425048139.4	4241430385.8	32.2	36.6	27.5
7084	OWENSV	-2410590042.3	-4477745321.2	3838648383.6	85.4	107.6	84.5
7085	GOLDST	-2353393828.0	-4641529303.1	3676898883.6	92.7	103.9	89.4
7086	MCDON	-1330125243.4	-5328526617.5	3236150235.0	5.3	5.6	8.4
7090	YARAG	-2389006528.6	5043329265.1	-3078525370.1	15.2	15.7	12.4
7091	HAYSTK	1492453793.1	-4457278762.3	4296815910.8	5.7	6.0	6.0
7092	KWAJLN	-6143447294.5	1364700150.7	1034163090.7	13.3	12.9	13.3
7096	SAMOA	-6100045853.6	-996203169.9	-1568976340.3	53.8	54.5	18.9
7097	EASTR2	-1884984335.4	-5357608143.1	-2892853421.7	19.9	22.7	18.9
7100	GRF100	1131355748.7	-4831169076.4	3994131940.6	142.1	161.8	139.1
7101	GRF101	1131240480.5	-4831179834.7	3994148426.7	71.7	79.6	66.0
7102	GORF7102	1130686698.9	-4831353013.1	3994110884.5	5.6	6.0	9.7
7103	GRF103	1130685661.6	-4831348717.2	3994116329.3	23.5	27.7	23.1
7104	GRF104	1131095874.1	-4831196919.4	3994170694.1	54.1	62.9	53.0
7105	GRF105	1130719807.3	-4831350558.5	3994106484.7	8.2	9.3	9.3
7109	QUINC2	-2517234678.7	-4198556229.1	4076569684.0	4.9	5.0	8.2
7110	MNPEAK	-2386277906.3	-4802354365.3	3444881439.0	4.9	6.5	7.2
7112	PLATVL	-1240678129.6	-4720463382.2	4094480623.8	4.8	5.1	8.1
7114	OWNSV2	-2410422366.3	-4477802727.1	3838686662.1	35.8	36.6	18.2
7115	GLDST2	-2350861591.5	-4655546341.9	3660997863.3	43.7	44.2	18.6
7120	HALEAKAL	-5465998413.9	-2404408550.9	2242228432.0	7.5	7.1	12.9
7121	HUAHIN	-5345865249.1	-2958246664.4	-1824623800.1	17.9	16.4	13.7
7122	MAZTLN	-1660089269.4	-5619100396.7	2511637858.1	16.8	17.8	10.2
7123	HUAHI2	-5345865222.6	-2958246638.8	-1824623847.9	22.7	21.9	19.6
7125	GRF125	1130745853.9	-4831368043.5	3994077055.5	96.4	131.0	101.5
7130	GRF130	1130735348.3	-4831324623.4	3994132312.0	96.8	127.1	101.6
7203	EFLSBERG	4033947655.0	486990419.4	4900430677.3	13.5	15.9	18.7
7204	NRAO-140	882880071.2	-4924482286.1	3944130641.7	5.9	6.1	10.2
7205	HAYSTACK	1492404932.0	-4457266482.5	4296881706.5	6.1	6.3	8.0
7207	OVRO-130	-2409600644.1	-4478349540.1	3838603210.7	4.6	4.9	6.2
7209	WESTFORD	1492206787.7	-4458130476.5	4296015476.6	5.6	6.0	5.8
7210	HOLLAS	-5466006465.9	-2404427933.8	2242187452.2	7.2	7.0	8.6
7213	ONSALA60	3370606205.1	711917412.8	5349830618.3	8.6	8.9	8.6

7214	NRAO85-3	882325752.8	-4925137942.9	3943397608.6	6.9	6.8	18.5
7215	CHLBOLTN	4008310230.7	-100650851.9	4943794660.8	18.0	18.5	21.1
7216	HRAS-085	-1324210845.3	-5332023125.0	3232118371.7	4.8	5.2	5.7
7217	MARPOINT	1106629487.6	-4882907169.4	3938086927.8	6.0	6.1	11.9
7218	HATCREEK	-2523969832.8	-4123506348.8	4147752592.2	4.6	4.8	5.9
7219	RICHMOND	961258190.1	-5674090015.8	2740533733.4	5.9	6.3	6.6
7220	MONPK2	-2386292228.9	-4802347508.3	3444880911.9	35.2	36.0	19.9
7221	QUINCY	-2517230788.8	-4198595215.7	4076531274.7	4.8	4.9	8.2
7222	MOJAVE12	-2356170883.7	-4646755884.3	3668470604.0	4.6	4.9	5.4
7223	VNDNBERG	-2678094640.6	-4525450821.1	3597410132.2	4.7	4.9	5.8
7224	WETTZELL	4075540082.8	931735172.7	4801629251.5	9.0	9.6	8.9
7225	GILCREEK	-2281547068.8	-1453645014.0	5756993219.9	5.9	5.8	6.5
7226	SHANGHAI	-2847698143.9	4659872949.7	3283958876.8	32.3	80.0	256.2
7227	SESHAN25	-2831686628.8	4675733850.6	3275327801.2	12.1	11.1	14.1
7228	CARROLGA	453520738.7	-5300506748.7	3507207409.1	12.2	13.2	58.0
7229	SEATTLE1	-2295347899.3	-3638029477.4	4693408731.4	10.7	9.7	62.2
7230	MEDICINA	4461370192.6	919596714.4	4449559063.2	9.4	10.1	10.1
7231	DSS15	-2353538655.3	-4641649522.6	3676670008.9	7.5	6.8	17.0
7232	HARTRAO	5085442818.1	2668263399.0	-2768697253.3	16.5	18.6	16.9
7234	PIETOWN	-1640953535.0	-5014815976.3	3575411897.8	5.5	5.7	7.6
7241	FORTORDS	-2699840090.3	-4359126998.9	3781050886.7	17.7	12.1	111.6
7242	HOBART26	-3950236379.8	2522347569.0	-4311562966.3	15.6	16.3	20.0
7244	NOBEY-6M	-3871168188.2	3428273971.5	3723697670.1	22.4	21.7	83.9
7251	PT-REYES	-2732332998.7	-4217634889.9	3914491046.7	5.1	5.1	13.1
7252	PRESIDIO	-2707704770.6	-4257609601.2	3888374179.9	5.0	5.0	12.2
7254	PBLOSSOM	-2464070824.1	-4649425640.1	3593905704.8	6.2	5.6	19.3
7255	SANPAULA	-2554476570.5	-4608627384.0	3582138294.2	5.5	5.4	23.3
7256	PINFLATS	-2369635844.7	-4761324916.4	3511116139.5	5.4	5.2	12.7
7258	PLATTVIL	-1240708024.0	-4720454346.6	4094481649.1	4.8	5.1	8.1
7259	MAMMOTHL	-2448246667.5	-4426738316.0	3875435886.9	7.9	6.5	29.5
7261	FLAGSTAF	-1923992566.6	-4850854511.9	3658589295.3	6.1	5.7	20.5
7263	JPL-MV1	-2493305885.8	-4655197591.2	3565519327.7	5.2	5.1	13.9
7265	MOJAVE	-2356475637.7	-4646618322.9	3668424740.8	29.9	31.8	23.5
7266	FORT-ORD	-2697026665.2	-4354393308.2	3788077599.3	5.5	5.2	16.3
7267	DEADMANL	-2336819552.4	-4732586958.4	3570329977.1	6.7	6.0	23.5
7268	PVERDES	-2525452693.4	-4670035670.6	3522886753.1	5.6	5.4	22.7
7269	BLKBUTTE	-2306306851.7	-4787914428.8	3515736418.5	5.3	5.2	16.2
7270	OCOTILLO	-2335601015.9	-4832244220.3	3434392523.1	62.1	62.0	47.7
7271	AUSTINTX	-737793694.5	-5459892259.4	3202990467.7	7.2	6.5	26.5
7274	MON-PEAK	-2386289300.4	-4802346563.5	3444883961.7	4.8	5.0	7.2
7277	YAKATAGA	-2529744111.2	-1942091279.0	5505027945.0	6.5	6.1	20.4
7278	KODIAK	-3026940081.2	-1575911770.8	5370362501.5	6.2	5.9	15.8
7279	NOME	-2658150332.1	-693821858.8	5737236619.4	8.4	8.1	24.6
7280	SNDPOINT	-3425461823.7	-1214669111.6	5223858283.9	6.7	6.3	18.3
7281	SOURDOGH	-2419993385.5	-1664228725.3	5643538270.2	6.5	6.3	17.2
7282	ALGOPARK	918034940.7	-4346132204.5	4561971113.8	7.5	7.6	12.8
7283	PENTICTN	-2058840397.4	-3621286401.5	4814420844.6	28.6	24.0	303.9
7284	WHHORSE	-2215213507.9	-2209261576.9	5540292515.9	5.7	5.7	16.8
7285	YELLOWKN	-1224124406.8	-2689530587.6	5633555287.6	9.5	8.7	41.2
7286	ELY	-2077236204.0	-4486712705.2	4018753744.7	5.0	5.0	13.5
7288	MOJ-7288	-2356494010.5	-4646607678.8	3668426628.2	4.9	5.1	5.8
7290	VERNAL	-1631473190.9	-4589128911.9	4106759877.2	5.2	5.2	15.0
7291	BLOOMIND	302384562.8	-4941699019.1	4007908458.4	13.7	15.9	62.7
7292	LEONRDOK	-522231471.1	-5145676870.0	3720152342.6	8.3	6.6	31.5
7294	BERMUDA	2307209604.6	-4874215801.3	3394317849.4	7.5	7.4	18.4
7295	RICHMO	961319067.1	-5674090946.9	2740489521.5	6.2	6.9	6.8
7400	SNTAGO	1769699799.7	-5044612937.3	-3468260056.4	31.9	34.9	24.8
7401	CERTOL	1815517158.9	-5213464884.0	-3187999400.9	28.9	30.4	18.8

7510	ASKITS	4353444941.7	2082666331.0	4156506650.2	16.6	19.5	16.5
7512	RHODES	4573400053.7	2409322255.1	3723881776.5	18.9	21.8	18.2
7515	DIONS2	4595216458.8	2039435372.4	3912629514.0	16.1	18.6	15.9
7517	ROUMEL	4728694702.5	2174373423.7	3674572970.1	16.0	17.7	15.2
7520	KARITS	4596042635.9	1733476864.9	4055720842.9	23.3	26.8	21.8
7525	XRISOK	4745949559.1	1905705997.2	3799169056.3	19.5	21.9	19.3
7530	BARGIY	4443968445.4	3121946911.5	3334695467.3	24.0	25.1	23.5
7540	MAT540	4641984609.2	1393057762.3	4133233448.6	27.9	31.5	27.4
7541	MAT541	4641990326.2	1393042328.2	4133231839.6	20.9	24.8	19.2
7544	LAMPED	5072832021.5	1130886292.5	3684836964.5	21.5	27.8	21.9
7545	CAGLIA	4893398089.5	772673366.1	4004140975.4	19.5	25.4	18.6
7546	MEDICI	4461399727.7	919566810.5	4449510511.4	10.0	10.1	10.6
7547	NOTO	4934563284.5	1321201169.9	3806484277.8	22.1	24.1	82.5
7550	BASOVI	4336738610.6	1071271526.8	4537911026.4	31.4	42.7	31.0
7575	DIYARB	3848636021.6	3251760952.1	3898909226.7	32.5	37.2	34.2
7580	MELENG	4247620539.9	2778639020.1	3851607511.6	23.4	30.5	25.3
7585	YOZGAT	4029730709.6	2802093344.1	4062068003.8	35.5	49.7	39.0
7587	YIGILC	4117362032.2	2517076864.5	4157679035.4	24.3	27.2	25.5
7596	WET596	4075585304.5	931837765.9	4801559243.5	25.5	34.2	24.2
7599	WET599	4075517439.5	931754102.9	4801630300.6	53.0	64.3	49.9
7600	HOHENFRG	3778215037.1	698644685.6	5074053491.1	9.7	10.0	14.3
7601	METSHOVI	2890652928.3	1310295259.2	5513958605.1	10.0	10.3	21.2
7602	TROMSONO	2102904253.9	721602470.1	5958201250.4	10.2	10.5	19.5
7603	CARNUSTY	3526416527.9	-171421193.9	5294098746.3	10.2	10.5	25.1
7604	BREST	4228877256.9	-333104282.0	4747180876.7	9.9	10.2	19.4
7605	GRASSE	4581697822.8	556125653.0	4389351315.8	9.4	10.1	11.0
7805	METFIN	2892595126.9	1311808346.3	5512609972.1	123.6	122.0	95.6
7810	ZIMMER	4331283631.5	567549571.7	4633139934.1	14.7	19.0	14.5
7831	HELWAN	4728283541.8	2879670736.7	3156894263.0	147.9	174.1	179.4
7833	KOOTWK	3899224109.9	396742881.3	5015073866.5	34.3	38.8	29.9
7834	WETZEL	4075530079.8	931781321.7	4801618179.5	9.0	9.6	8.9
7835	GRASSE	4581691817.8	556159397.3	4389359401.0	9.4	11.1	10.1
7837	SHAHAI	-2831087754.0	4676203453.4	3275172806.9	15.4	14.8	17.1
7838	SHO	-3822388384.0	3699363444.2	3507573079.8	20.7	20.9	13.4
7839	GRAZ	4194426727.9	1162693927.1	4647246529.5	12.3	15.2	11.8
7840	RGO	4033463872.1	23662368.9	4924305026.9	11.6	15.3	11.0
7843	ORRLR	-4446476737.0	2678127023.2	-3696251676.4	12.1	13.2	12.3
7850	MCD-7850	-1330008037.7	-5328391547.1	3236502711.3	6.3	6.1	17.6
7853	OVR-7853	-2410421125.9	-4477800403.5	3838690320.2	5.0	5.0	7.0
7882	CABO	-1997241718.1	-5528041257.0	2468355241.4	20.5	21.5	18.2
7885	MCDON3	-1330125312.7	-5328526564.7	3236150236.3	32.2	36.1	32.6
7886	QUINC3	-2517242288.0	-4198550980.4	4076570301.6	22.2	25.3	17.3
7888	MTHOP2	-1936744266.5	-5077639076.6	3331993876.0	38.2	40.8	30.0
7891	FLGSTF	-1923976615.4	-4850871705.8	3658574712.0	36.4	40.3	32.7
7892	VERNAL	-1631484649.3	-4589133836.1	4106749515.7	27.6	31.2	25.7
7894	YUMA	-2196777818.6	-4887337049.9	3448425229.1	4.9	5.1	10.2
7896	PASDNA	-2493211911.7	-4655229484.7	3565574622.2	127.1	127.6	27.4
7899	GRF899	1131364755.4	-4831168760.3	3994128969.6	37.6	41.5	37.2
7900	FTD-7900	-1324227829.2	-5332063025.4	3232023029.7	7.0	6.5	21.5
7907	ARELAS	1942791952.5	-5804077722.9	-1796919309.7	28.3	28.2	12.1
7921	HOPLAS	-1936760158.6	-5077707156.5	3331922446.5	45.0	45.5	24.3
7929	NATLAS	5186467850.6	-3653856008.0	-654321749.0	61.4	56.3	46.6
7939	MATERA	4641965119.2	1393069976.3	4133262234.5	15.1	17.2	12.4
7943	ORRLAS	-4447548707.1	2677134003.1	-3694996648.8	25.8	24.0	19.1
8833	KOTWK2	3899237900.4	396769217.5	5015055219.6	20.4	26.0	19.1

Table 2.8. SV6 Local Geodetic Coordinate Site Velocities and Uncertainties

mon	site	AM0-2 Rate, mm/yr		SV6 Rate, mm/yr			SV6 Rate Uncertainty, mm/yr		
		n	e	n	e	u	σ_n	σ_e	σ_u
1181	POTSDM	16.40	19.12	16.9	17.8	0.0	2.3	2.3	1.5
1311	KAUAI	37.27	-67.91	35.2	-64.4	-0.5	0.8	0.8	1.0
1513	GOLDVENU	-14.72	-13.37	-6.9	-17.8	0.2	0.7	0.6	1.3
1561	ROBLED32	17.84	17.99	17.8	16.8	0.0	3.3	3.3	1.5
1642	DSS45	51.56	20.87	51.7	19.6	0.2	1.9	1.6	1.1
1665	DSS65	17.84	17.99	20.3	17.6	0.2	2.9	2.8	1.5
1856	KASHIMA	-17.31	9.78	-7.5	-1.5	-1.5	1.2	1.2	1.3
1884	RIGA	14.70	20.54	15.5	19.3	0.0	2.2	2.1	1.5
1953	SANCUB	2.69	-10.18	4.0	-12.3	0.0	8.1	8.1	1.5
4968	KWAJAL26	30.90	-75.37	31.2	-73.1	0.4	1.3	1.3	1.0
7035	OTAY2	27.83	-48.25	25.7	-46.2	0.0	6.4	6.4	1.5
7038	MILESMON	-10.45	-17.99	-7.9	-19.0	0.0	2.2	2.1	1.5
7051	QUINCY	-16.20	-13.94	-13.3	-15.8	0.0	4.5	4.5	1.5
7061	EASTER	-10.14	89.41	-9.7	92.0	0.0	4.6	4.5	1.5
7062	OTAY	27.83	-48.25	25.7	-46.2	0.0	6.4	6.4	1.5
7063	STALAS	2.21	-17.25	3.6	-18.0	0.0	2.0	2.0	1.5
7067	BERMDA	7.55	-14.40	8.3	-15.5	0.0	2.3	2.3	1.5
7069	RAMLAS	0.52	-13.47	2.1	-15.1	0.0	2.5	2.5	1.5
7080	MCDON4	-9.71	-13.43	-7.2	-15.3	0.0	2.7	2.7	1.5
7082	BERLAK	-12.68	-16.02	-10.0	-17.4	0.0	2.5	2.5	1.5
7084	OWENSV	-15.26	-13.69	-12.4	-15.6	0.0	4.4	4.4	1.5
7085	GOLDST	-14.75	-13.40	-11.9	-15.3	0.0	5.2	5.2	1.5
7086	MCDON	-9.71	-13.43	-8.1	-13.8	-2.6	0.3	0.3	0.7
7090	YARAG	60.54	43.03	60.0	42.4	0.0	2.2	2.1	1.5
7091	HAYSTK	4.58	-18.19	5.6	-18.6	-0.1	1.4	1.4	1.1
7092	KWAJLN	30.90	-75.37	31.2	-73.1	0.4	1.3	1.3	1.0
7096	SAMOA	36.43	-68.76	35.7	-67.3	0.0	6.4	6.4	1.5
7097	EASTR2	-10.14	89.41	-9.7	92.0	0.0	4.6	4.5	1.5
7100	GRF100	2.22	-17.25	3.6	-18.0	0.0	2.0	2.0	1.5
7101	GRF101	2.21	-17.25	3.6	-18.0	0.0	2.0	2.0	1.5
7102	GORF7102	2.21	-17.25	3.7	-16.6	-0.4	1.1	1.2	1.0
7103	GRF103	2.21	-17.25	3.6	-18.0	0.0	2.0	2.0	1.5
7104	GRF104	2.21	-17.25	3.6	-18.0	0.0	2.0	2.0	1.5
7105	GRF105	2.21	-17.25	3.7	-16.6	-0.4	1.1	1.2	1.0
7109	QUINC2	-16.20	-13.94	-9.7	-21.9	0.2	0.9	0.7	1.0
7110	MNPEAK	27.66	-47.84	12.3	-44.2	0.3	0.7	0.5	1.0
7112	PLATVL	-10.00	-16.39	-8.8	-17.3	-0.1	0.8	0.7	1.0
7114	OWNSV2	-15.26	-13.69	-12.4	-15.6	0.0	4.4	4.4	1.5
7115	GLDST2	-14.72	-13.37	-11.9	-15.3	0.0	5.4	5.4	1.5
7120	HALEAKAL	37.26	-67.79	35.8	-66.0	0.5	1.5	1.5	1.4
7121	HUAHIN	36.96	-71.27	35.6	-69.8	0.0	2.0	2.1	1.5
7122	MAZTLN	-10.72	-10.65	-8.1	-12.9	0.0	5.5	5.5	1.5
7123	HUAHI2	36.96	-71.27	35.6	-69.8	0.0	2.0	2.1	1.5
7125	GRF125	2.21	-17.25	3.6	-18.0	0.0	2.0	2.0	1.5
7130	GRF130	2.21	-17.25	3.6	-18.0	0.0	2.0	2.0	1.5
7203	EFLSBERG	17.09	18.14	16.8	16.1	0.0	1.7	1.8	1.5
7204	NRAO-140	0.87	-17.11	2.0	-17.4	-0.1	0.5	0.4	1.2
7205	HAYSTACK	4.58	-18.19	4.0	-17.5	1.7	0.5	0.4	1.0
7207	OVRO-130	-15.26	-13.69	-6.9	-19.8	-0.3	0.4	0.3	0.8
7209	WESTFORD	4.58	-18.19	5.6	-18.6	-0.1	1.4	1.4	1.1
7210	HOLLAS	37.26	-67.79	35.1	-64.3	-0.2	0.8	0.8	1.0
7213	ONSALA60	16.53	17.65	17.1	14.4	0.6	0.7	0.6	0.9
7214	NRAO85-3	0.87	-17.11	2.4	-18.0	0.0	2.0	2.0	1.5

7215	CHLBOLTN	17.71	16.02	17.8	14.6	0.0	2.2	2.2	1.5
7216	HRAS-085	-9.68	-13.42	-8.1	-13.8	-2.6	0.3	0.2	0.7
7217	MARPOINT	2.03	-17.05	1.5	-18.3	-0.5	0.7	0.5	1.4
7218	HATCREEK	-16.38	-14.05	-9.8	-20.5	0.4	0.5	0.4	1.1
7219	RICHMOND	0.62	-12.46	2.2	-14.2	0.0	2.0	2.0	1.1
7220	MONPK2	27.66	-47.84	25.6	-45.8	0.0	9.0	9.0	1.5
7221	QUINCY	-16.20	-13.94	-9.7	-21.9	0.2	0.9	0.7	1.0
7222	MOJAVE12	-14.75	-13.38	-5.9	-17.3	-0.8	0.4	0.3	0.7
7223	VNDNBERG	29.40	-47.24	21.8	-43.0	-0.9	0.5	0.4	1.1
7224	WETTZELL	16.43	19.78	17.8	17.2	-3.9	0.8	0.6	0.8
7225	GILCREEK	-23.34	-10.20	-23.2	-10.1	3.1	0.6	0.5	1.0
7226	SHANGHAI	-12.12	25.53	-10.8	25.8	0.0	2.8	3.1	1.5
7227	SESHAN25	-12.06	25.55	-10.9	26.2	-0.1	1.9	2.2	1.1
7228	CARROLGA	-1.49	-15.42	0.3	-16.7	0.0	2.1	2.1	1.5
7229	SEATTLE1	-16.64	-15.44	-13.7	-16.9	0.0	3.4	3.4	1.5
7230	MEDICINA	16.58	20.41	18.3	17.7	-0.1	1.6	1.7	1.1
7231	DSS15	-14.75	-13.40	-11.9	-15.3	0.0	5.2	5.2	1.5
7232	HARTRAO	21.89	22.17	20.6	21.6	0.0	1.9	1.9	1.5
7234	PIETOWN	-11.39	-14.21	-8.8	-16.0	0.0	2.9	2.9	1.5
7241	FORTORDS	29.85	-45.43	27.8	-43.3	0.0	9.1	9.1	1.5
7242	HOBART26	52.37	14.73	52.2	14.5	0.0	2.7	2.5	1.5
7244	NOBEY-6M	-16.57	10.15	-16.1	5.7	0.0	5.6	5.7	1.5
7251	PT-REYES	30.29	-44.17	19.7	-36.4	0.0	1.5	1.1	1.5
7252	PRESIDIO	30.11	-44.34	10.5	-32.8	-0.1	1.2	0.9	1.5
7254	PBLOSSOM	28.29	-46.52	4.0	-30.0	-0.2	1.6	1.1	1.5
7255	SANPAULA	28.74	-46.95	10.9	-40.9	0.1	1.8	1.4	1.5
7256	PINFLATS	-14.60	-12.95	4.5	-30.9	0.0	1.6	1.1	1.5
7258	PLATTVIL	-10.00	-16.39	-8.8	-17.3	-0.1	0.8	0.7	1.0
7259	MAMMOTHL	-15.50	-13.69	-8.0	-23.5	-0.1	2.5	1.9	1.5
7261	FLAGSTAF	-12.77	-14.07	-10.0	-15.0	0.2	2.0	1.5	1.5
7263	JPL-MV1	28.40	-46.92	6.8	-39.2	0.2	1.0	0.7	1.5
7265	MOJAVE	-14.75	-13.38	-11.9	-15.3	0.0	5.4	5.3	1.5
7266	FORT-ORD	29.85	-45.35	22.1	-41.9	0.1	1.5	1.1	1.5
7267	DEADMANL	-14.53	-13.16	-8.1	-18.5	0.1	4.3	3.5	1.5
7268	PVERDES	28.49	-47.46	14.1	-40.5	-0.1	2.0	1.5	1.5
7269	BLKBUTTE	-14.32	-13.06	-10.3	-19.3	-0.1	1.7	1.2	1.5
7270	OCOTILLO	-14.35	-12.80	-11.5	-14.8	0.0	19.3	19.3	1.5
7271	AUSTINTX	-7.04	-13.77	-4.8	-15.5	0.0	2.3	2.3	1.5
7274	MON-PEAK	27.66	-47.84	12.3	-44.2	0.3	0.7	0.5	1.0
7277	YAKATAGA	-22.34	-11.64	9.1	-28.5	0.3	2.0	1.8	1.5
7278	KODIAK	-24.18	-7.99	-14.0	-16.3	0.2	1.5	1.4	1.5
7279	NOME	-25.45	-3.26	-22.2	-5.0	0.0	2.0	1.9	1.5
7280	SNDPOINT	-25.12	-5.03	-22.9	-12.2	0.1	1.6	1.5	1.5
7281	SOURDOGH	-22.96	-10.80	-25.0	-11.2	0.1	1.5	1.4	1.5
7282	ALGOPARK	1.66	-19.43	2.9	-19.4	0.1	1.7	1.6	1.5
7283	PENTICTN	-15.72	-16.34	-12.8	-17.6	0.0	3.0	3.0	1.5
7284	WHTHORSE	-20.54	-14.11	-20.5	-20.0	-0.1	1.6	1.4	1.5
7285	YELLOWKN	-13.83	-19.82	-11.1	-19.8	0.0	2.2	2.1	1.5
7286	ELY	-13.99	-14.79	-13.0	-18.1	-0.1	1.3	0.9	1.5
7288	MOJ-7288	-14.75	-13.38	-5.9	-17.3	-0.8	0.4	0.3	0.7
7290	VERNAL	-11.96	-15.84	-9.6	-14.6	0.1	1.7	1.4	1.5
7291	BLOOMIND	-2.11	-17.31	-0.3	-18.2	0.0	1.9	1.9	1.5
7292	LEONRDOK	-6.21	-15.82	-4.0	-17.2	0.0	2.1	2.1	1.5
7294	BERMUDA	7.55	-14.40	8.3	-15.5	0.0	2.3	2.3	1.5
7295	RICHMO	0.62	-12.46	2.2	-14.2	0.0	2.0	2.0	1.1
7400	SNTAGO	9.81	-3.09	9.9	-4.2	0.0	4.8	4.8	1.5
7401	CERTOL	9.79	-3.74	9.9	-4.8	0.0	5.8	5.8	1.5
7510	ASKITS	14.44	23.49	15.2	22.4	0.0	3.3	3.2	1.5

7512	RHODES	14.02	24.26	14.9	23.3	0.0	5.6	5.6	1.5
7515	DIONS2	14.74	23.51	15.5	22.5	0.0	4.0	3.9	1.5
7517	ROUMEL	14.60	23.85	15.4	22.9	0.0	5.6	5.5	1.5
7520	KARITS	15.30	22.80	16.0	21.7	0.0	6.1	6.0	1.5
7525	XRISOK	15.10	23.31	15.8	22.3	0.0	6.5	6.5	1.5
7530	BARGIY	21.14	26.51	20.4	25.6	0.0	9.2	9.2	1.5
7540	MAT540	15.91	21.99	16.5	20.9	0.0	5.1	5.1	1.5
7541	MAT541	15.91	21.99	16.5	20.9	0.0	5.1	5.1	1.5
7544	LAMPED	22.26	20.93	21.6	20.3	0.0	5.9	5.9	1.5
7545	CAGLIA	16.89	20.80	17.3	19.6	0.0	4.6	4.6	1.5
7546	MEDICI	16.58	20.41	18.3	17.7	-0.1	1.6	1.7	1.1
7547	NOTO	16.16	22.18	16.7	21.1	0.0	11.2	11.2	1.5
7550	BASOVI	16.30	20.66	16.8	19.4	0.0	2.9	2.8	1.5
7575	DIYARB	11.27	25.80	12.4	24.9	0.0	6.2	6.1	1.5
7580	MELENG	21.36	25.18	20.6	24.4	0.0	4.2	4.1	1.5
7585	YOZGAT	12.53	25.00	13.5	24.0	0.0	3.9	3.8	1.5
7587	YIGILC	13.27	24.42	14.2	23.4	0.0	3.2	3.1	1.5
7596	WET596	16.42	19.78	16.9	18.5	0.0	2.5	2.5	1.5
7599	WET599	16.42	19.78	16.9	18.5	0.0	2.5	2.5	1.5
7600	HOHENFRG	16.71	18.38	17.1	17.0	0.0	2.3	2.2	1.5
7601	METSHOVI	14.64	19.85	15.4	18.5	0.0	2.3	2.2	1.5
7602	TROMSONO	15.55	15.98	16.2	14.5	0.0	3.2	3.1	1.5
7603	CARNUSTY	17.76	14.20	17.8	12.8	0.0	2.3	2.3	1.5
7604	BREST	17.84	16.02	17.8	14.7	0.0	2.3	2.3	1.5
7605	GRASSE	17.10	19.59	19.3	17.6	0.1	2.0	2.0	1.1
7805	METFIN	14.64	19.86	15.4	18.5	0.0	2.3	2.2	1.5
7810	ZIMMER	17.04	19.08	17.4	17.8	0.0	2.5	2.5	1.5
7831	HELWAN	21.56	26.06	20.8	25.2	0.0	3.8	3.8	1.5
7833	KOOTWK	17.19	17.49	17.5	16.1	0.0	2.2	2.2	1.5
7834	WETZEL	16.42	19.78	17.8	17.2	-3.8	0.8	0.6	0.8
7835	GRASSE	17.10	19.59	19.3	17.6	0.1	2.0	2.0	1.1
7837	SHAHAI	-12.06	25.55	-10.9	26.2	-0.1	1.9	2.2	1.1
7838	SHO	-15.05	23.60	-14.0	24.1	0.0	7.9	8.1	1.5
7839	GRAZ	16.08	20.73	16.6	19.5	0.0	2.8	2.7	1.5
7840	RGO	17.61	16.52	17.7	15.1	0.0	2.2	2.2	1.5
7843	ORRLLR	51.59	20.67	51.7	19.6	-0.1	1.9	1.6	1.1
7850	MCD-7850	-9.71	-13.43	-7.2	-15.3	0.0	2.7	2.7	1.5
7853	OVR-7853	-15.26	-13.69	-6.9	-19.8	-0.3	0.4	0.3	0.8
7882	CABO	24.63	-56.40	22.5	-54.4	0.0	5.0	5.0	1.5
7885	MCDON3	-9.71	-13.43	-7.2	-15.3	0.0	2.7	2.7	1.5
7886	QUINC3	-16.20	-13.94	-13.3	-15.8	0.0	4.5	4.5	1.5
7888	MTHOP2	-12.48	-13.07	-9.8	-15.0	0.0	3.9	3.9	1.5
7891	FLGSTF	-12.77	-14.07	-10.1	-15.9	0.0	3.4	3.4	1.5
7892	VERNAL	-11.96	-15.84	-9.3	-17.3	0.0	2.6	2.5	1.5
7894	YUMA	-13.75	-13.05	-11.7	-13.9	0.1	1.2	0.8	1.5
7896	PASDNA	28.40	-46.92	26.3	-44.8	0.0	17.6	17.6	1.5
7899	GRF899	2.22	-17.25	3.6	-18.0	0.0	2.0	2.0	1.5
7900	FTD-7900	-9.68	-13.42	-7.2	-15.3	0.0	2.7	2.7	1.5
7907	ARELAS	9.69	-6.62	9.8	-7.8	0.0	4.4	4.4	1.5
7921	HOPLAS	-12.48	-13.07	-9.8	-15.0	0.0	3.9	3.9	1.5
7929	NATLAS	13.30	-9.39	13.3	-10.6	0.0	2.7	2.9	1.5
7939	MATERA	15.91	21.99	16.5	20.9	0.0	5.1	5.1	1.5
7943	ORRLAS	51.57	20.67	51.4	20.3	0.0	2.4	2.2	1.5
8833	KOTWK2	17.19	17.49	17.5	16.1	0.0	2.2	2.2	1.5

Table 2.9. Local Offset From SV6 Reference Points to GPS Reference Points

Site	To GPS	From SV6	Offset, meters		
			x	y	z
ALGO	ALGORMA	ALGO7282	92.7590	70.2790	13.0650
AROT	ARO_TI41	ALGO7282	94.7956	60.8626	6.8292
CANB	CANBN194	CANB1642	-80.1339	-45.9138	82.6301
DIOC	DIONRM_C	DION7515	1.8930	2.1520	-3.3140
FTDV	FTDV4	FTDV7216	19.1246	-36.6944	-75.0238
FAIR	FAIRRM_1	FAIR7225	-31.6890	-25.8340	-33.5480
HATC	HATCNCMB	HATC7218	97.9760	-73.5820	-33.4940
HAYS	HAYSOCP3	WEST7209	203.3090	840.1750	803.6850
HOB	HOBAMMAC	HOB7242	52.4530	17.0090	-25.8460
JPL1	JPLIARI1	JPL17263	176.0880	-134.5530	-21.0070
JPLM	JPLM7272	JPL17263	1.9870	-18.0630	-22.0520
KOKR	KOKESNR8	KAUA1311	7.7929	-23.6717	-4.3426
KOK0	KOKECIGN	KAUA1311	28.0809	-18.9910	44.3985
KOKW	KOKEWVR	KAUA1311	28.8716	-24.2070	40.1038
MOJA	MOJATI41	MOJA7222	-43.6686	21.8614	-10.1945
MOJF	MOJAFRPA	MOJA7222	-43.6906	21.8550	-10.1832
MOJM	MOJAMMAC	MOJA7222	-44.8717	19.3895	-14.3794
MOJ1	MOJANCM1	MOJA7222	-253.4726	142.2684	-8.3585
ONS0	ONSAO31	ONSA7213	52.6310	-40.4640	-43.8650
ONS1	ONSA_401	ONSA7213	96.3520	-112.2820	-66.6150
OVRO	OVRO7114	OVRO7207	-821.7057	546.8763	83.4928
BPA3	OVROBPA3	OVRO7207	-42.4073	80.9875	35.2678
PENT	PENTGPS	PENT7283	-324.1780	178.0540	11.6380
PFO1	PFO1	PINY7256	125.5880	117.6550	279.9100
RICH	RICHFRPA	RICH7219	60.2712	31.9715	31.4952
RICM	RICHMMAC	RICH7219	51.3171	14.3615	5.1987
RICT	RICHTIMR	RICH7219	59.7220	35.2130	29.9190
SIO1	SIO1	PINY7256	-85885.6603	-5888.4740	-69461.3565
TROM	TROMGPSM	TROM7602	36.2870	-33.1150	-9.2190
TRO2	TROMTII	TROM7602	47.1860	64.1010	-21.4500
TSUK	TSUKMMAC	KASH1856	40698.5322	33609.9630	13614.9284
VNDN	VNDNRM1	VNDN7223	23.0948	-0.9667	17.2418
WSFD	WESTTI41	WEST7209	26.2945	38.8295	30.4057
WSFF	WESTFRPA	WEST7209	26.2995	38.8146	30.4202
WSFM	WESTMMAC	WEST7209	26.3384	42.1712	32.5385
WETT	WETTTI41	WETT7224	6.1990	90.5260	-29.8470
WETM	WETTMMAC	WETT7224	12.4490	90.5330	-40.1900
YKNF	YKNFTI41	YKNF7285	59.9968	-302.5595	-122.8432
YKN1	YKNFGACP	YKNF7285	-327.8480	314.5990	82.8820
ZIMM	ZIMMTI41	ZIMM7810	13.8319	6.0322	-6.0773

Table 2.10. Antenna Height Information

Site	Start date		Height, m	Antenna	Ref. point	Description
	yr	day				
HOB	90	021	0.000	MIN6AT	L1PHC	Hobart Mini-Mac
JPLM	90	081	0.093	ROGSNR	DHBCR	JPL MESA Rogue
KOK0	88	092	0.2445	FRPA-2	L1PHC	Kokee FRPA-2
KOK0	90	183	0.3810	TRMSST	DHPAB	Kokee Trimble
MOJA	86	314	0.000	TI4100	L1PHC	Mojave TI4100
MOJF	88	198	0.000	FRPA-2	L1PHC	Mojave FRPA-2
MOJM	89	036	0.000	MIN6AT	L1PHC	Mojave Mini-Mac
ONS0	87	352	1.6605	TI4100	L1PHC	Onsala TI4100
ONS1	91	027	1.580	ASHXII	L1PHC	Onsala Ashtech
PFO1	90	165	1.8431	ROGSNR	DHBCR	PFO1 Rogue
RICH	88	040	0.000	FRPA-2	L1PHC	Richmond FRPA-2
RICM	89	036	0.000	MIN6AT	L1PHC	Richmond Mini-Mac
TROM	87	353	1.824	TI4100	DHPAB	Tromso TSS GPSM
TROM	88	301	2.356	TI4100	DHPAB	Tromso TSS GPSM
TROM	87	311	2.624	TI4100	DHPAB	Tromso TSS GPSM
TROM	89	263	2.6165	TI4100	DHPAB	Tromso TSS GPSM
TROM	90	199	2.4734	ROGSNR	DHBCR	Tromso TSS GPSM
TRO2	90	148	1.623	ROGSNR	DHBCR	Tromso TSS III
TRO2	90	181	1.546	ROGSNR	DHBCR	Tromso TSS III
TRO2	91	018	2.543	ROGSNR	DHTCR	Tromso TSS III
TSUK	88	191	0.000	MIN6AT	L1PHC	Tsukuba Mini-Mac
WELL	90	049	0.000	TRMSST	L1PHC	Wellington
WSFD	86	273	0.000	TI4100	L1PHC	Westford TI
WSFF	88	217	0.000	FRPA-2	L1PHC	Westford FRPA-2
WSFM	89	029	0.000	MIN6AT	L1PHC	Westford Mini-Mac
WETT	87	315	0.000	TI4000	L1PHC	Wettzell TI4100
WETM	89	143	0.000	MIN6AT	L1PHC	Wettzell Mini-Mac
YKNF	87	001	0.000	TI4000	L1PHC	Yellowknife 869211
YKN1	89	188	0.000	TI4000	DHPAB	Yellowknife 889201
YKN1	91	022	0.111	ROGSNR	DHBCR	Yellowknife 889201

Antenna types:

TI4100 = TI4100 (100 series assumed)
 TI4000 = TI4100 (4000 series)
 FRPA-2 = FRPA-2 microstrip
 TRMSST = Trimble 4000 SST microstrip
 ROGSNR = Rogue SNR-8
 MIN6AT = Mini-Mac 2816 AT
 ASHXII = Ashtech L-XII

Reference point:

L1PHC = L1 phase center
 DHPAB = direct height to center of pre-amp base
 DHBCR = direct height to bottom of choke ring

Table 2.11. Antenna L1 and L2 Phase Center Vertical Offsets

Antenna	Reference Point	L1, meters	L2, meters
ASHXII	L1PHC	0.000	0.000
FRPA-2	L1PHC	0.000	0.000
MIN6AT	L1PHC	0.000	-0.0015
ROGSNR	DHBCR	0.0779	0.0964
TI4100	DHPAB	0.227	0.202
TI4100	L1PHC	0.000	-0.025
TI4000	DHPAB	0.227	0.212
TI4000	L1PHC	0.000	-0.015
TRMSST	DHPAB	0.0692	0.0677

Antenna types:

TI4100 = TI4100 (100 series assumed)
 TI4000 = TI4100 (4000 series)
 FRPA-2 = FRPA-2 microstrip
 TRMSST = Trimble 4000 SST microstrip
 ROGSNR = Rogue SNR-8
 MIN6AT = Mini-Mac 2816 AT
 ASHXII = Ashtech L-XII

Reference point:

L1PHC = L1 phase center
 DHPAB = direct height to center of pre-amp base
 DHBCR = direct height to bottom of choke ring

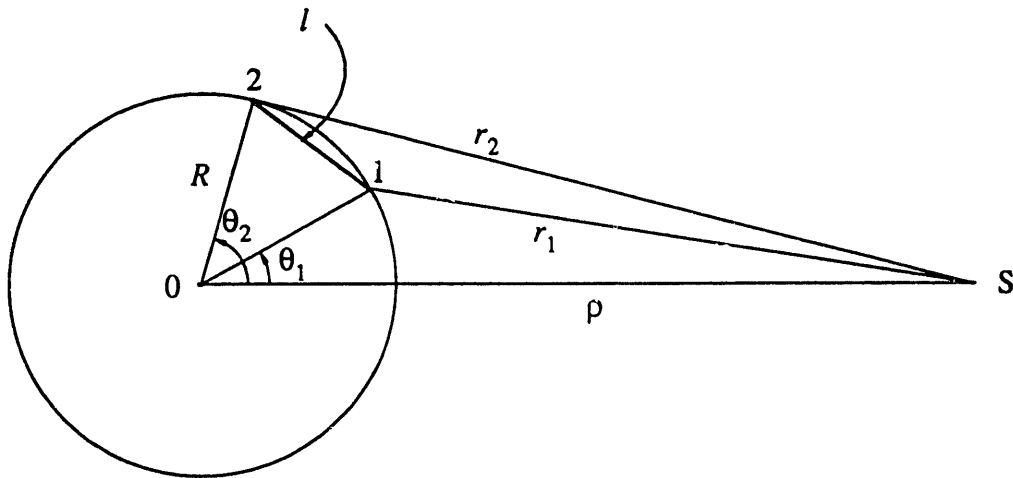


Figure 2.1. The baseline-to-satellite geometry. The geocentric vector to the satellite S , with length ρ , and the baseline between stations 1 and 2, with ranges r_i to the satellite, are assumed to be co-planar. Given the Earth's radius R , and the hour angles between the satellites and the stations θ_i , the length of the baseline between the stations is $l = 2R\sin(\theta_2 - \theta_1)$.

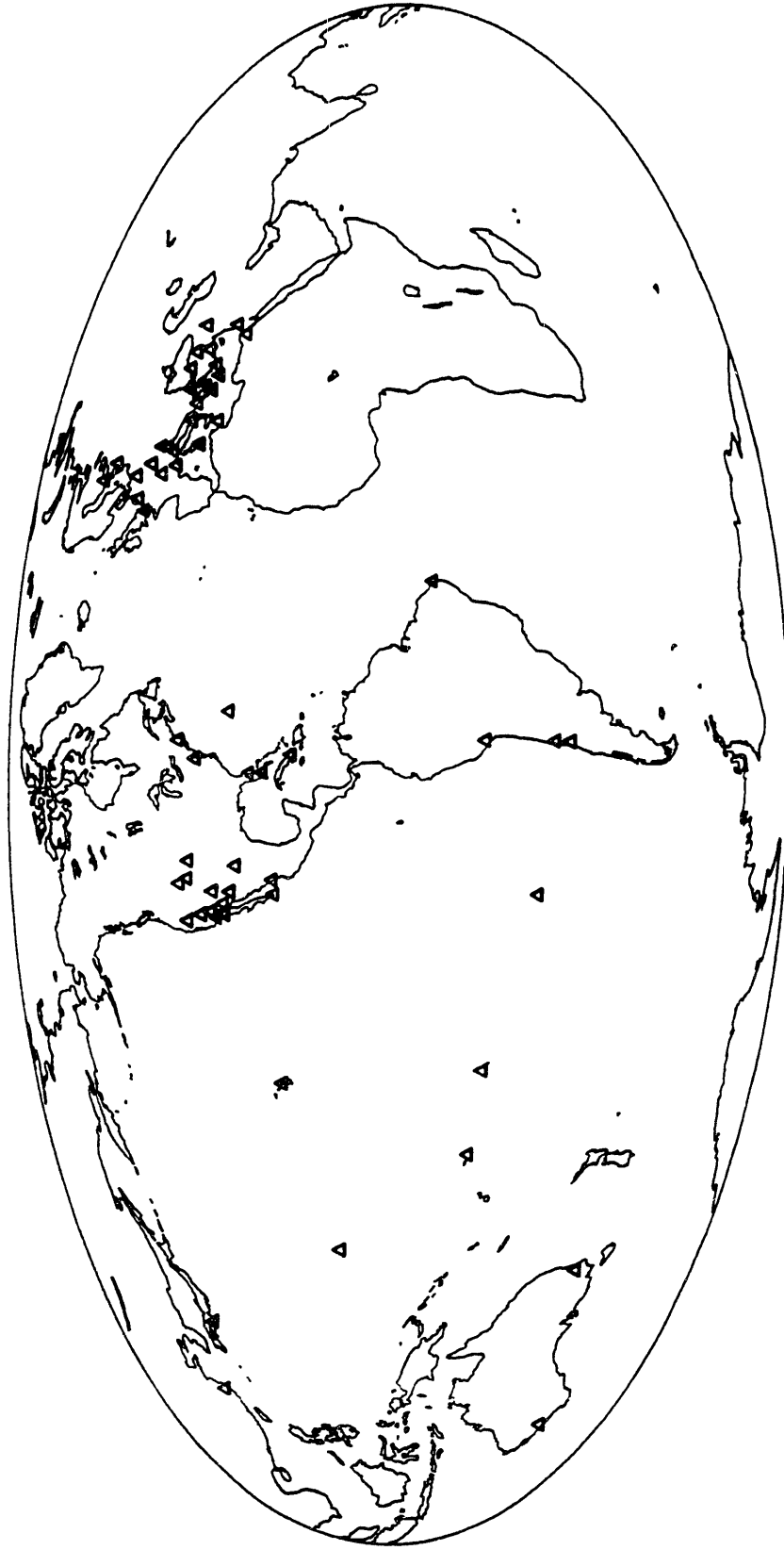


Figure 2.2. Approximate location of the 91 monuments in the CSR8902 adjustment of satellite laser ranging observations on a Mollweide projection.

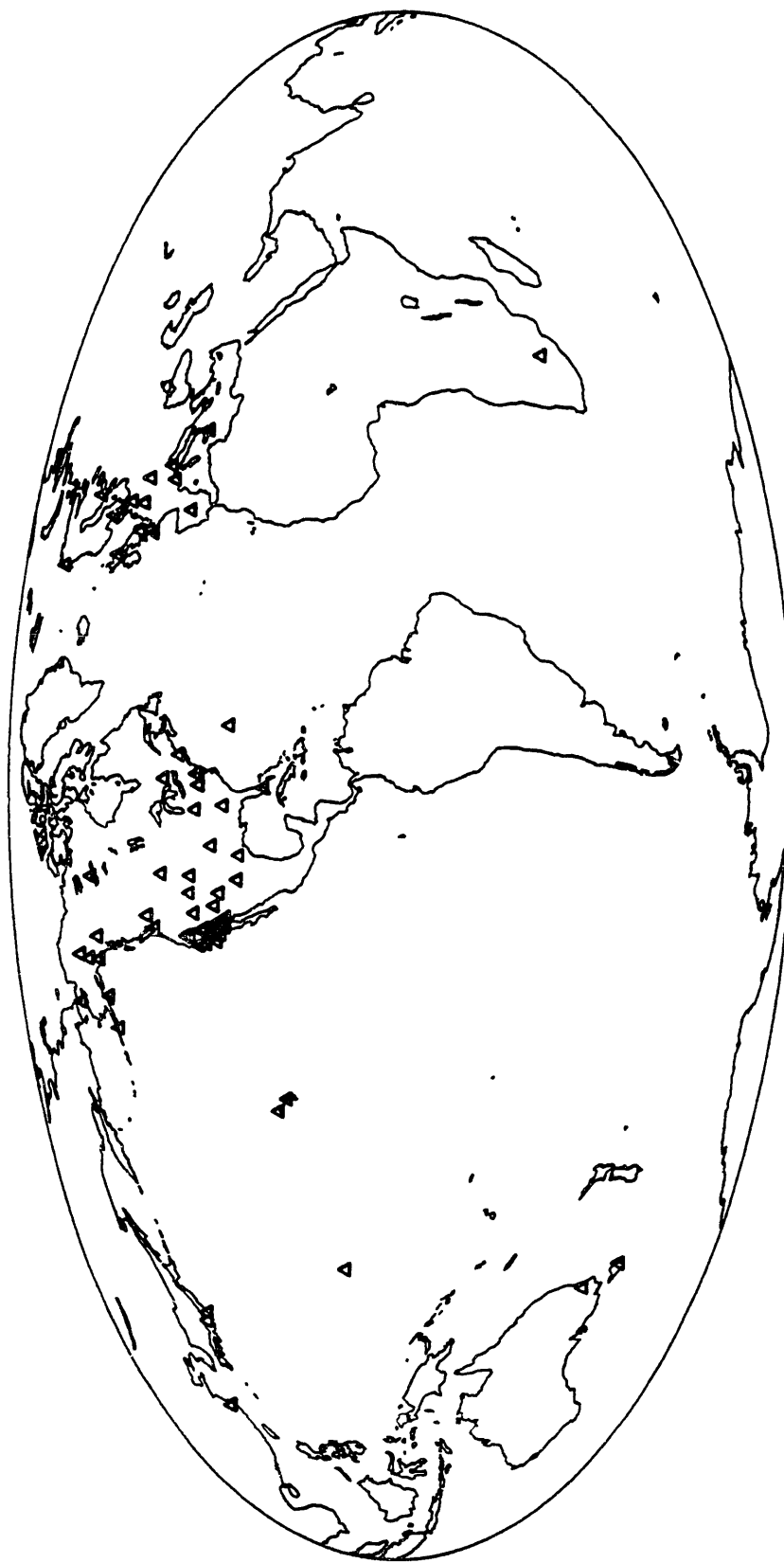


Figure 2.3. Approximate location of the 80 reference points in the GLB659 adjustment of very-long-baseline interferometry observations on a Mollweide projection.

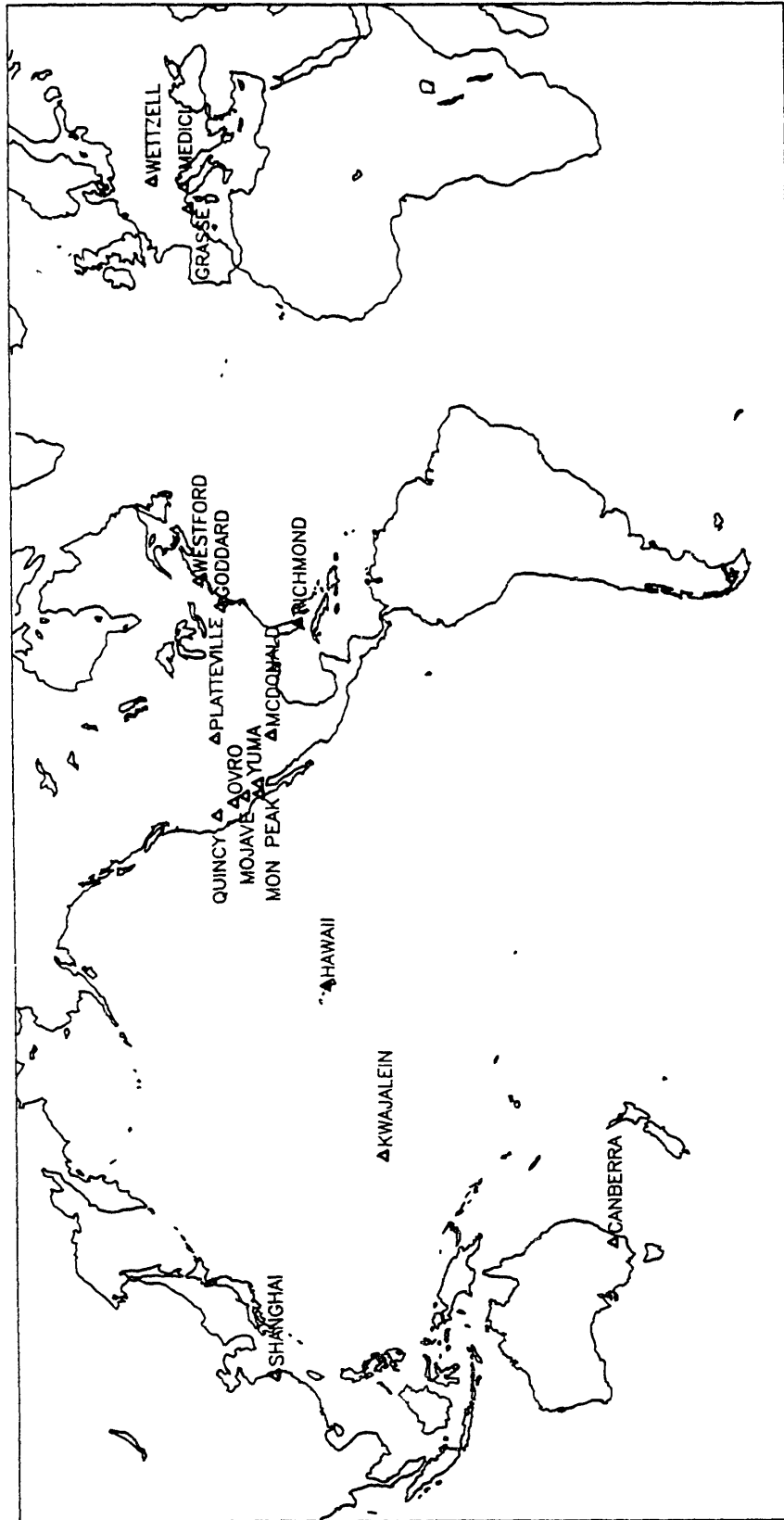


Figure 2.4. Approximate location of the 17 sites with collocated SLR and VLBI reference points on a Mercator projection. These sites are used to constrain the estimate of a similarity transformation between the SLR and VLBI reference frames.

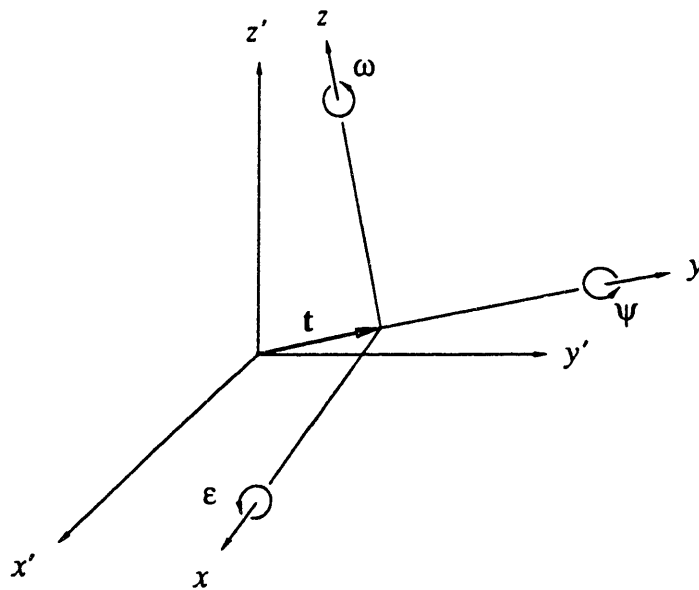


Figure 2.5. Geometry of the Bursa-Wolfe similarity transformation from the (xyz) to the $(x'yz')$ Cartesian coordinate systems.

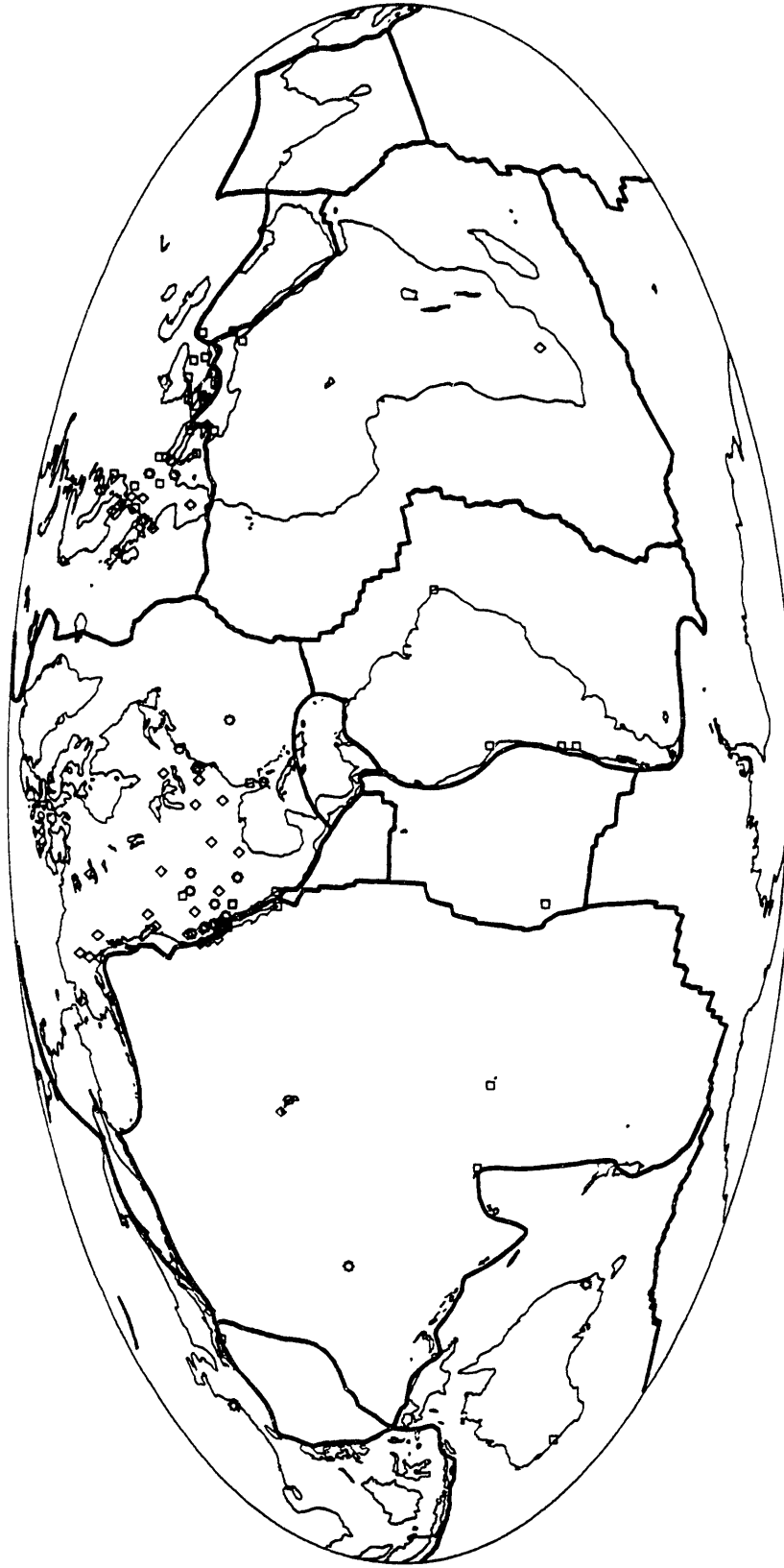


Fig. 2.6 Geometry of the tectonic plate boundary data set used to estimate the RM2 and NUVEL-1 no net rotation reference frames. The twelve plates and their four letter abbreviations are: Africa (afr), Antarctica (anta), Arabia (arab), Australia (aust), Caribbean (carb), Cocos (coco), Eurasia (eura), India (indi), Nazca (nazc), North America (noam), Pacific (pafc), Philippines (phil), and South America (soam). The squares and diamonds represent the satellite laser ranging and very-long-baseline interferometry sites included in the SV6 data sets, respectively. The eight pointed stars thus represent the potential intercomparison sites.

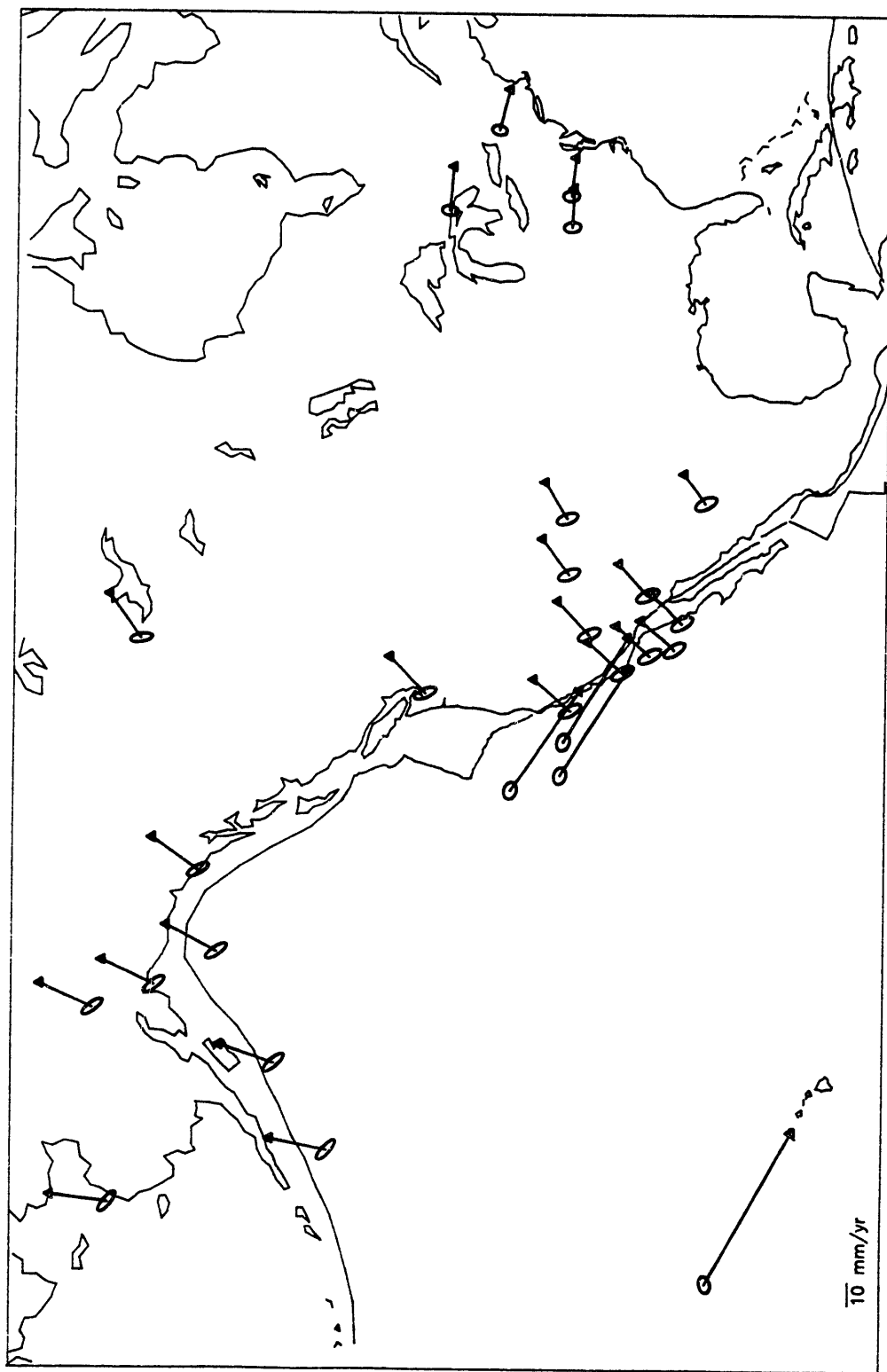


Figure 2.7. Horizontal component site velocities estimated from the AM0-2 absolute motion model. The error ellipses were derived from the RM2 covariance linearly transformed via plate areal moments. The error ellipses represent 95% confidence regions. Many sites in California have been excluded for purposes of clarity.

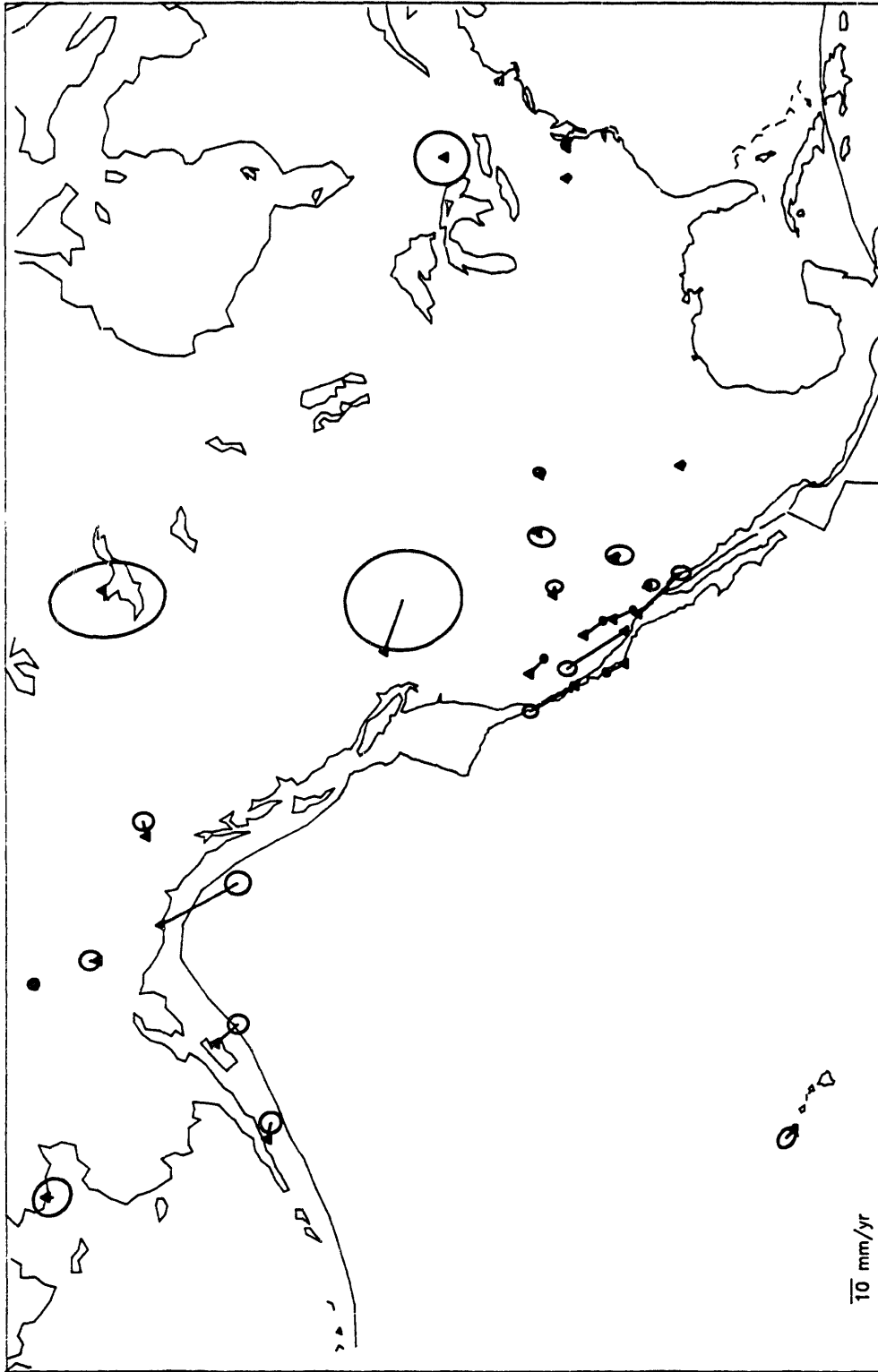


Figure 2.8 Horizontal component site velocities of the VLBI estimates with respect to AM0-2. The error ellipses are derived solely from the VLBI covariance and represent 95% confidence regions.

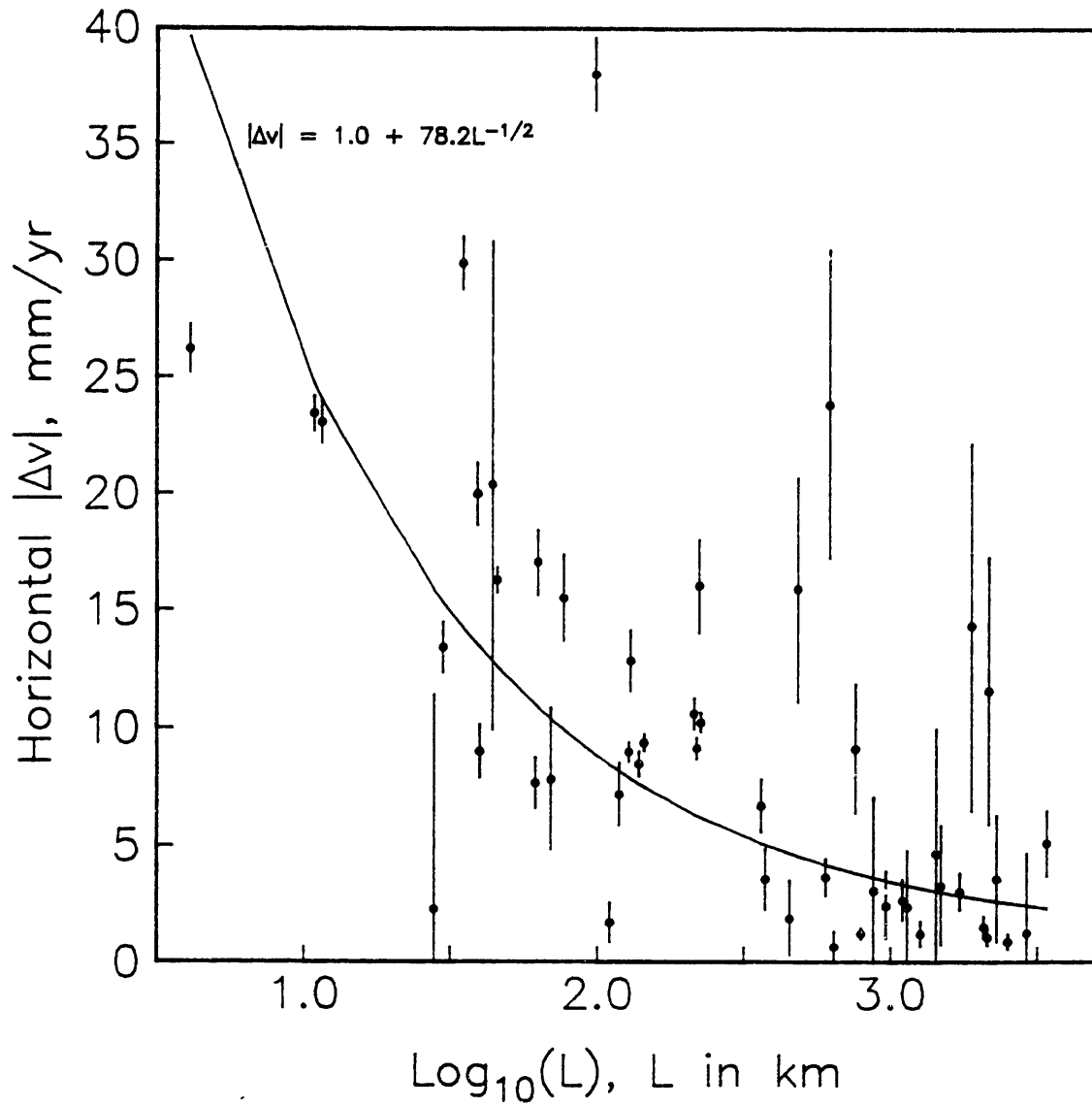


Figure 2.9. Magnitude of the horizontal velocity vector difference, $|\Delta v|$ in millimeters per year, of VLBI estimates with respect to AM0-2 plotted as a function of distance to the nearest plate boundary, L in kilometers. The VLBI estimates for 50 sites were derived from the GLB659 adjustment and the one-sigma formal uncertainties are based on the VLBI covariance only. The line is the weighted least square fit to the observations of an empirically-defined power-law relationship given by $|\Delta v| = 1.0 + 78.2 L^{-1/2}$.

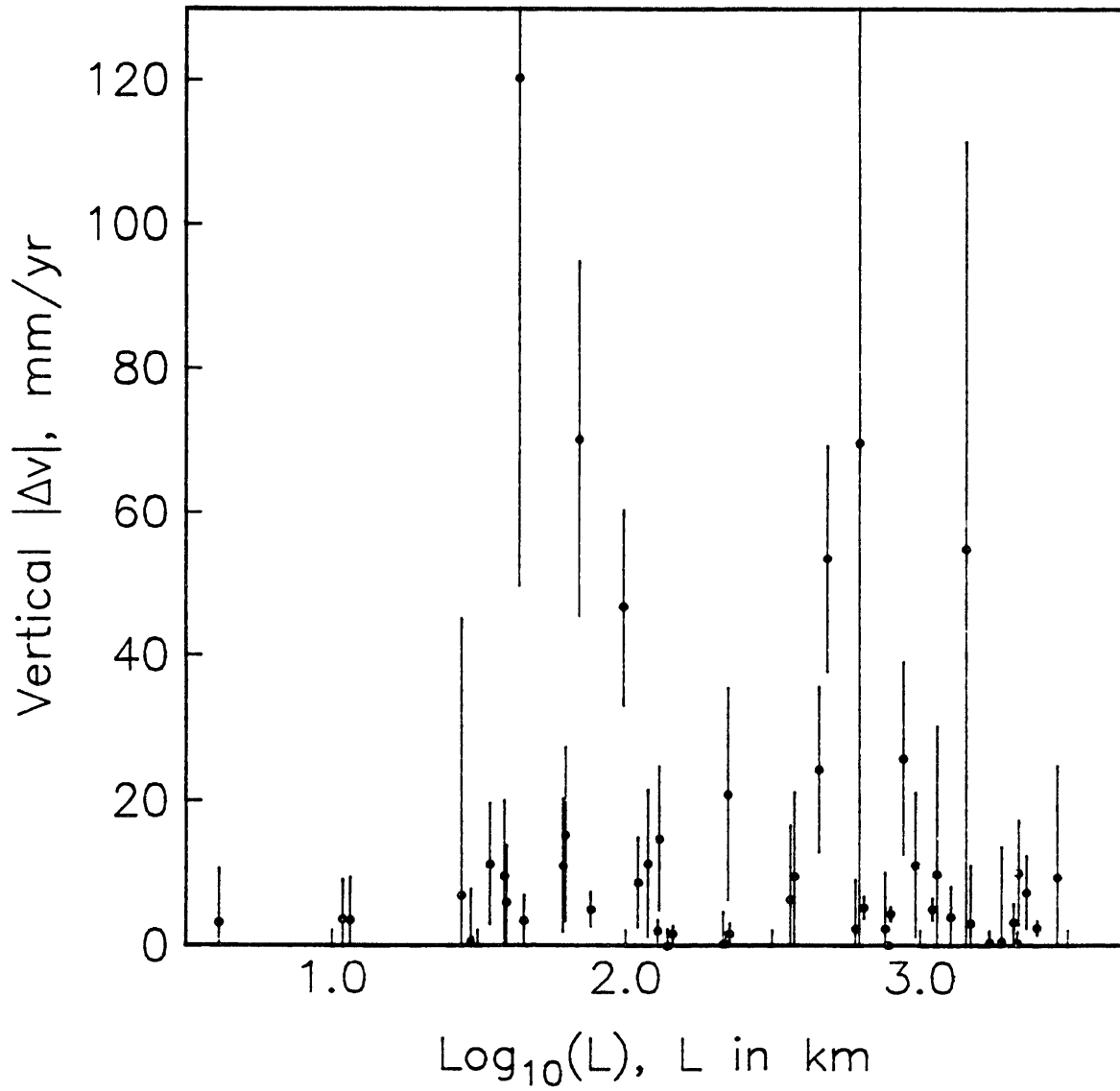


Figure 2.10. Magnitude of the vertical velocity vector difference, $|\Delta v|$ in millimeters per year, of the VLBI estimates with respect to AM0-2 plotted as a function of distance to the nearest plate boundary, L in kilometers. The VLBI estimates for 50 sites were derived from the GLB659 adjustment and the one-sigma formal uncertainties are based on the VLBI covariance only.

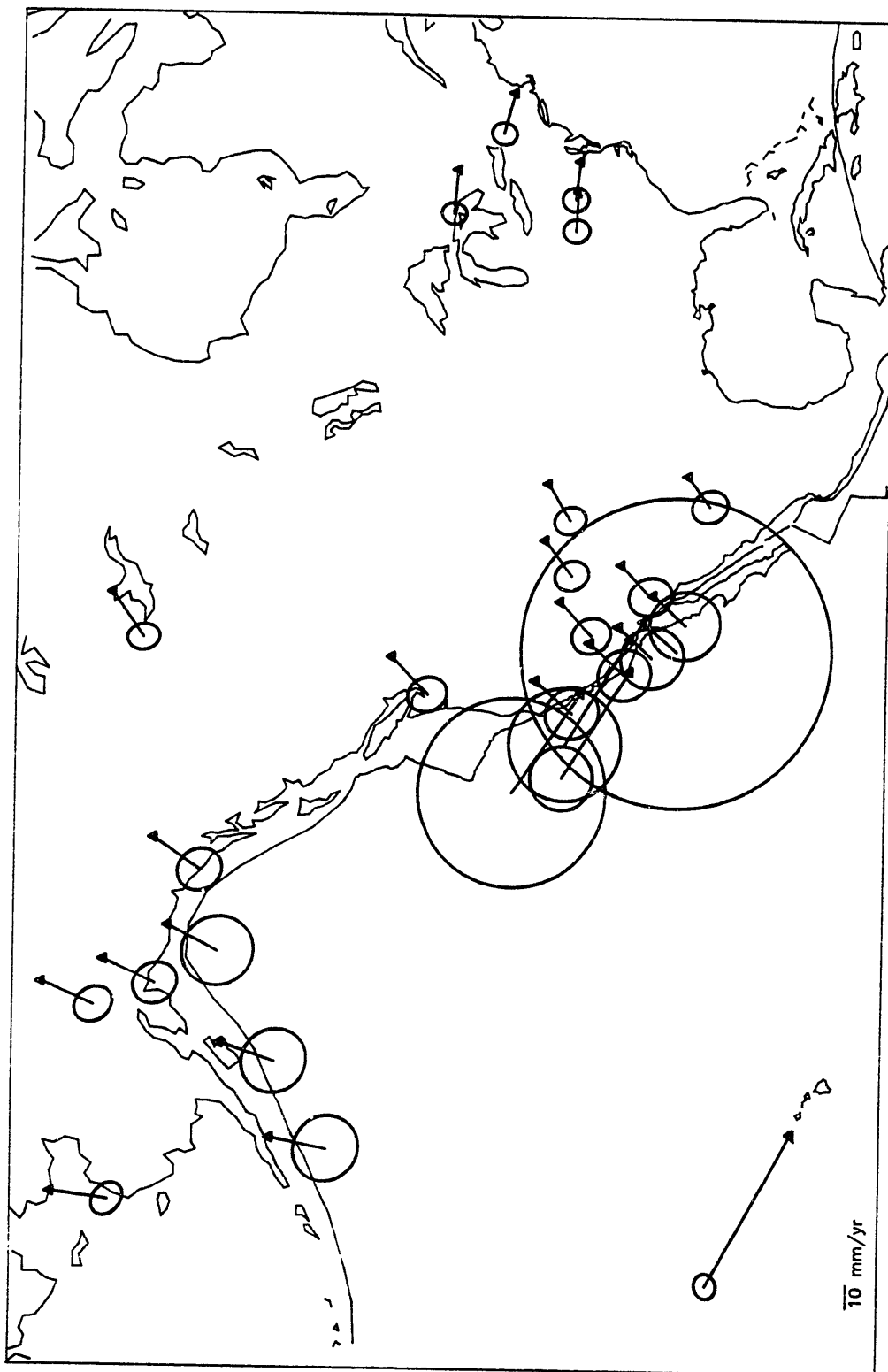


Figure 2.11 Horizontal components of the estimated a priori site velocities. The vectors are AM0-2 rates. The error ellipses are a combination of AM0-2 covariance and scaled uncorrelated site deformation and represent 95% confidence regions.

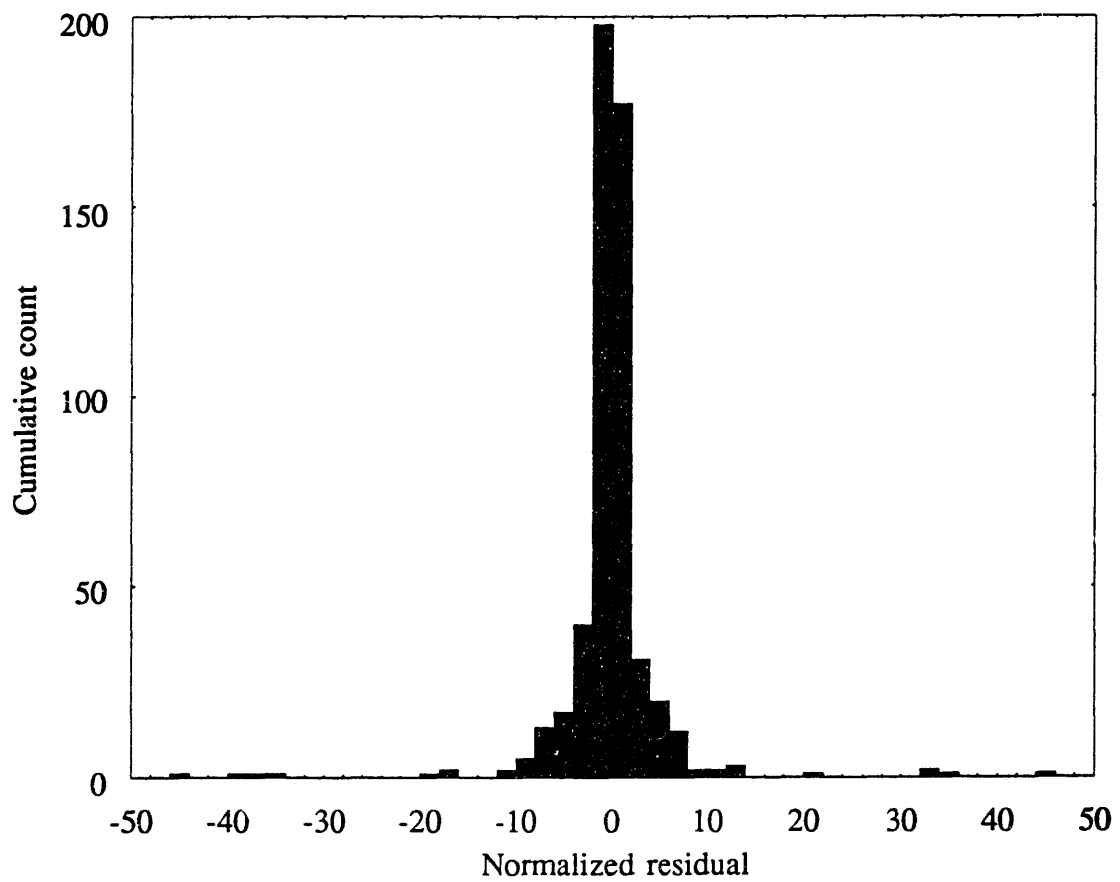


Figure 2.12. Normalized residuals of 534 position, velocity, and position and velocity differences. Zero residuals for sites lacking independent velocity estimates have been deleted. Bin width is one.

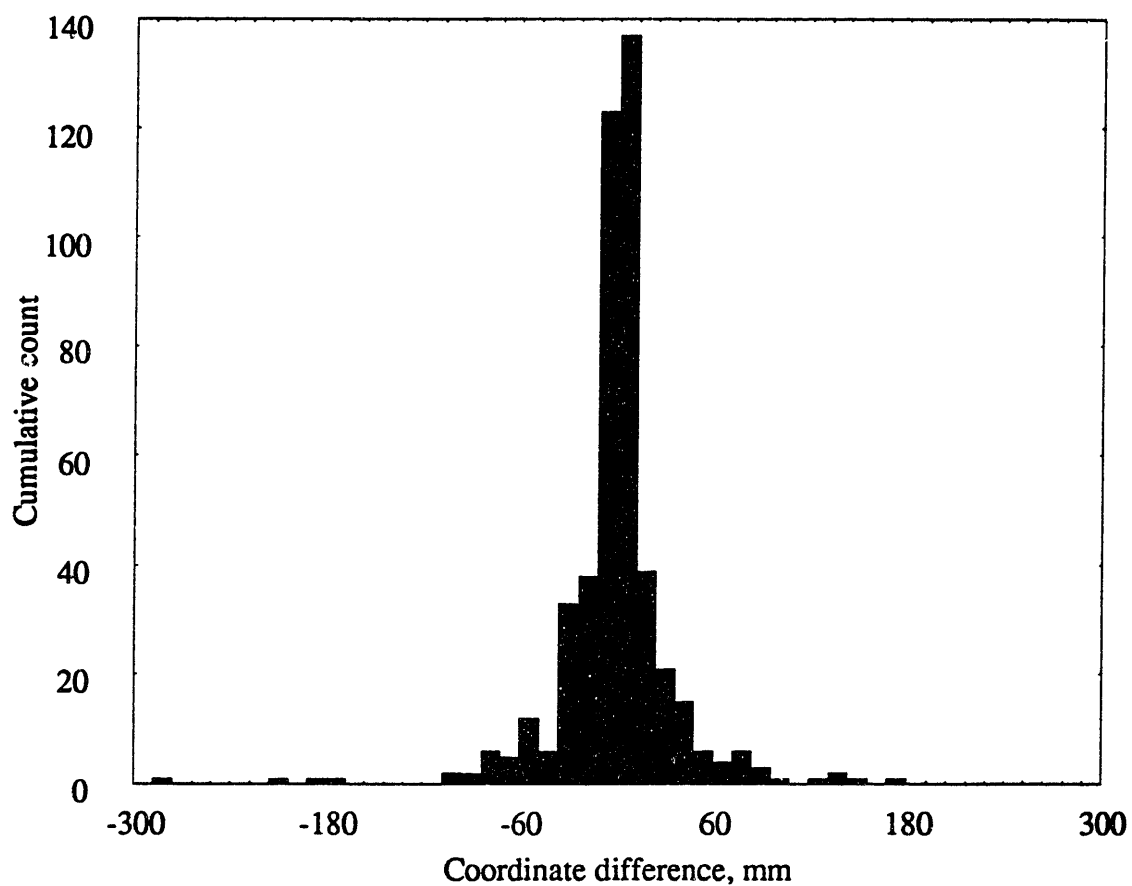


Figure 2.13. Residual coordinate differences between 156 sites in common between the ITRF-89 and SV6 terrestrial references. A 7-parameter similarity transformation was applied to the SV6 terrestrial reference frame to make it commensurate with ITRF-89. The residuals were rotated into the corresponding local geodetic frame of the site. Bin width is 12 mm.

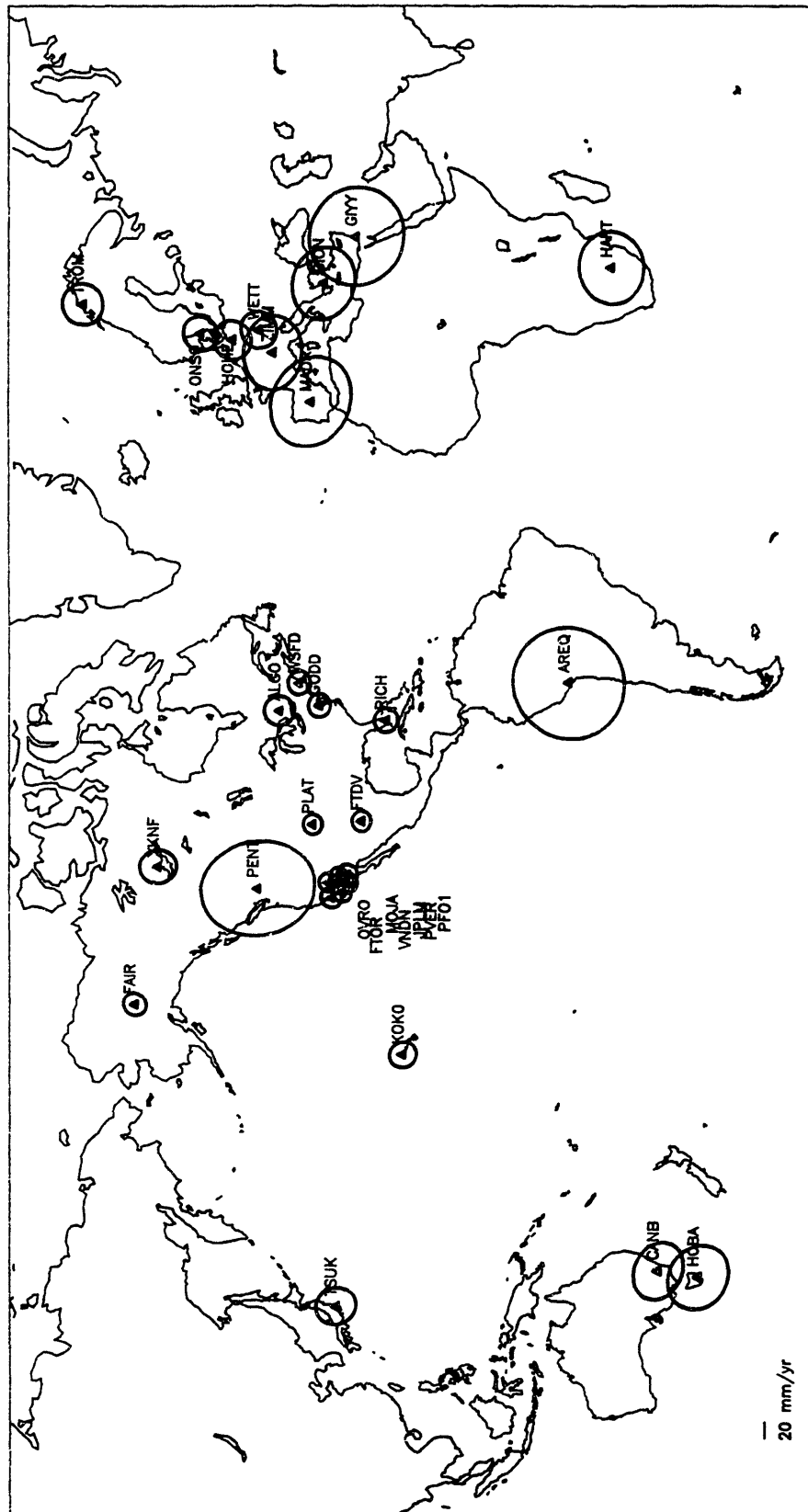


Figure 2.14. Positions and uncertainties for 30 SV6 reference monuments that are collocated with monuments that have been occupied by GPS systems. The scale of the uncertainties is given in the lower left corner, and the error ellipses represent 95% confidence regions with two degrees of freedom.

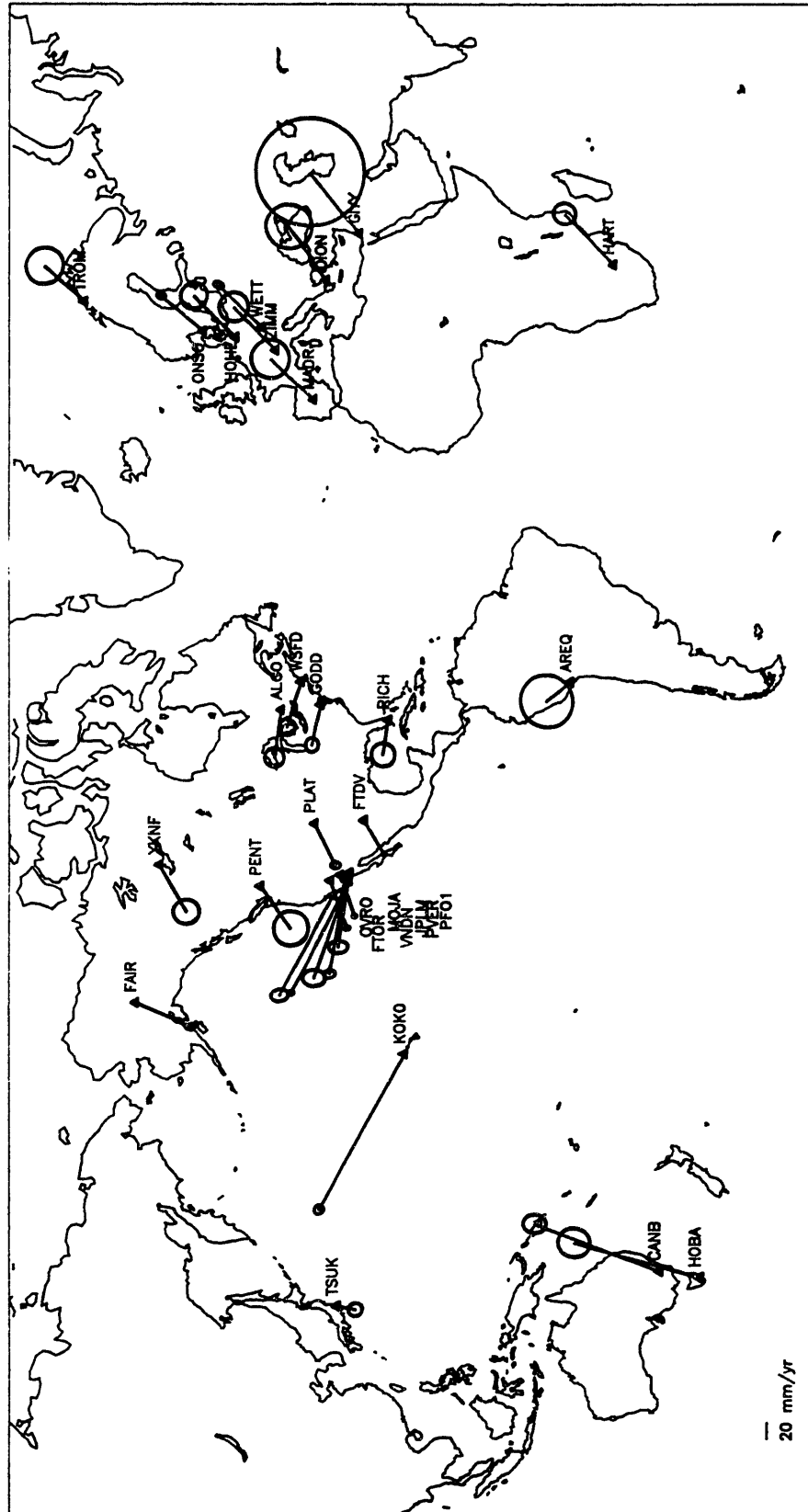


Figure 2.15. Velocities and their uncertainties with respect to AM0-2 for 30 SV6 reference monuments that are collocated with monuments that have been occupied by GPS systems. The scale of the uncertainties is given in the lower left corner, and the error ellipses represent 95% confidence regions with two degrees of freedom.

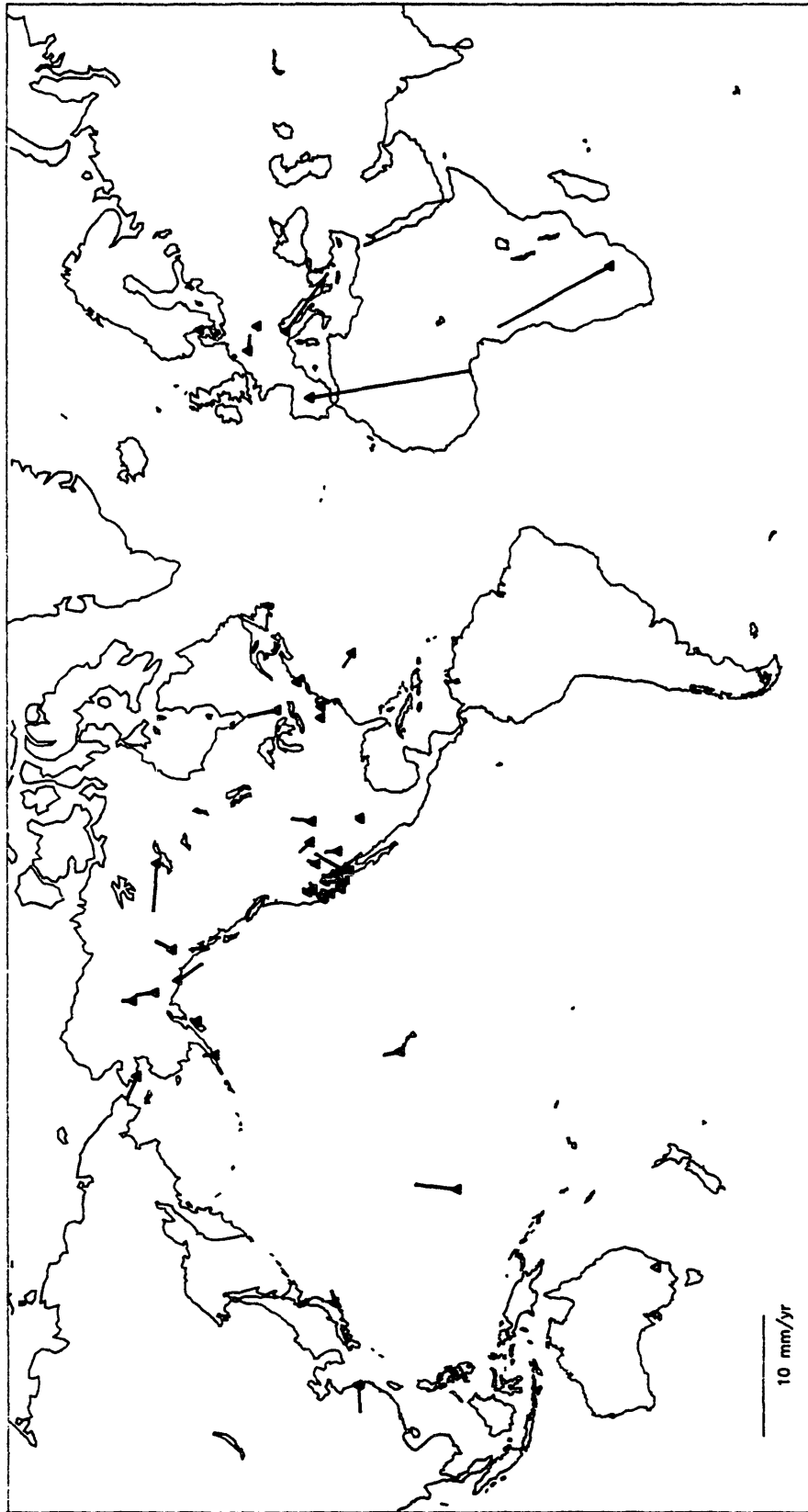


Figure 2.16. Horizontal component velocity differences of the SV6 estimates with respect to the independent VLBI estimates at 46 sites included in the GLB659 adjustment.

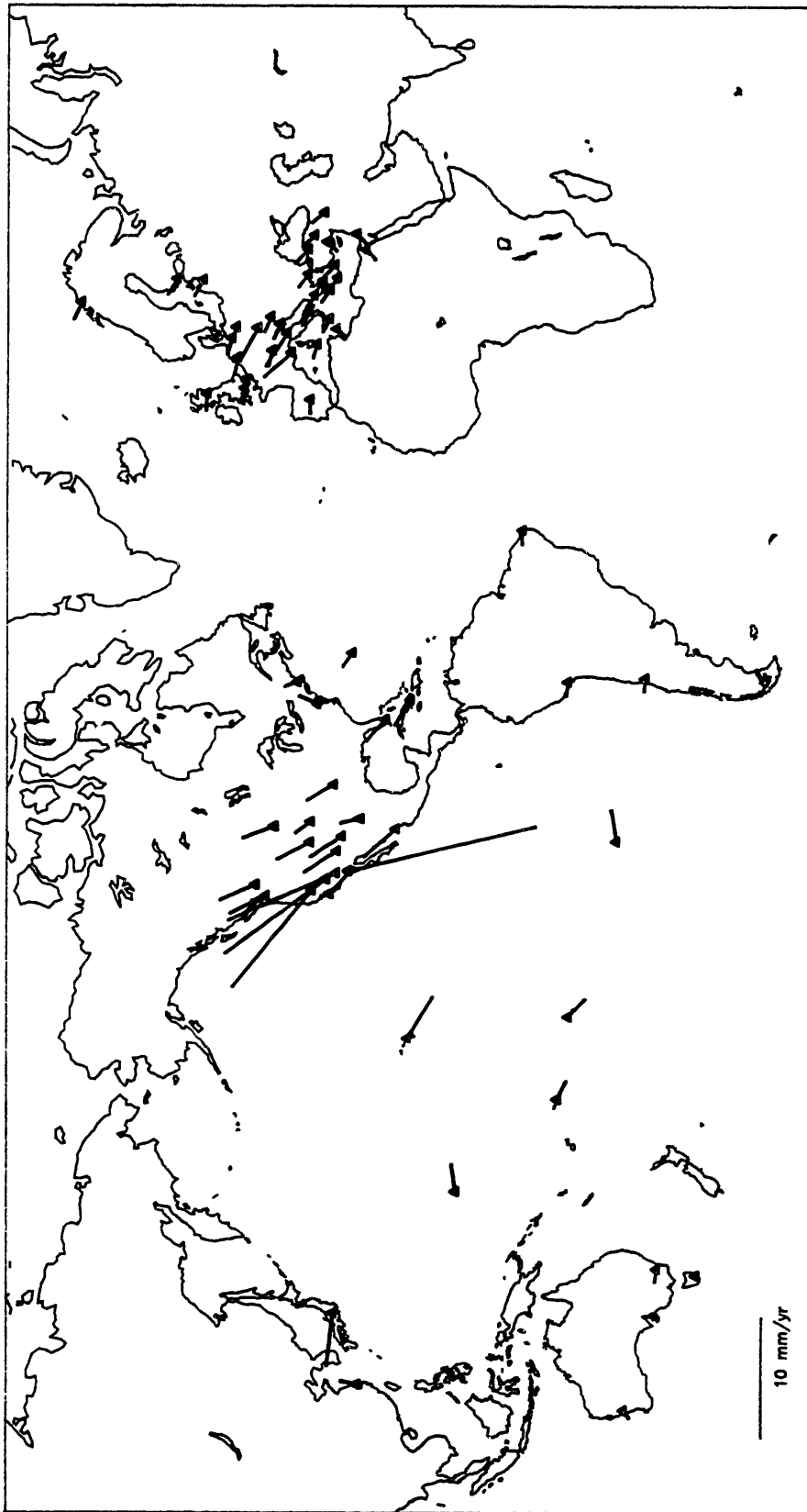


Figure 2.17. Horizontal component velocity differences of the SV6 estimates with respect to AM0-2 model estimates at all sites that do not have independent VLBI velocity estimates.

CHAPTER 3. GLOBAL POSITIONING SYSTEM MEASUREMENTS IN CENTRAL CALIFORNIA

Compared with all other theories of similarly wide scope, [continental] drift theory has the great advantage that it can be tested by accurate astronomical position-finding. If continental displacement was operative for so long a time, it is probable that the process is still continuing, and it is just a question of whether the rate of movement is enough to be revealed by our astronomic measurements in a reasonable period of time.

A. Wegener, The Origin of Continents and Oceans, 1929

INTRODUCTION

In December 1986, we initiated a five-year program of geodetic measurements in central and southern California using Global Positioning System (GPS) techniques. The observation program was the joint effort of a consortium of four universities (California Institute of Technology, Massachusetts Institute of Technology, Scripps Institution of Oceanography, and University of California at Los Angeles) with assistance from the U.S. Geological Survey, National Geodetic Survey, Jet Propulsion Laboratory, and Defense Mapping Agency. We acquired observations during three to six week field ‘campaigns’, each divided into a series of independent experiments lasting four to five days. The first series of experiments, conducted in December 1986 and January 1987, established a geodetic network with about 30 sites that was subsequently reoccupied by major experiments each year around March from 1988 to 1991. Subsets of this network were reoccupied in May and September 1987, and January and September 1990. In addition, many historical triangulation and trilateration monuments, some of which were established in the 1880’s by the U.S. Coast and Geodetic Survey, were occupied. The primary objectives of this recently completed program were threefold: (1) to assess the

long-term precision and accuracy of high-precision GPS measurements for studying crustal deformation; (2) to compare the GPS measurements with existing triangulation and trilateration data; and (3) to establish a network for long-term monitoring of deformations along the California plate margin [Agnew *et al.*, 1987; Agnew *et al.*, 1988].

Analyses of the GPS measurements by members of the consortium have produced preliminary results that address all of these objectives. *Dong and Bock* [1989] discuss methods to improve the precision of baseline estimates suitable for the scale of the California monitoring network and assess the short-term repeatability and accuracy of the GPS observations with an analysis of one of the five-day experiments in January 1987. Several investigators have derived significant estimates of horizontal strain and strain rate from a combination of the GPS observations with historical triangulation and trilateration data. The estimates of strain along the California margin in the central Coastal ranges [Shen, 1991], in the Santa Maria fold and thrust belt [Feigl *et al.*, 1989; Feigl, 1991], and across the Santa Barbara Channel [Webb, 1991; Larsen, 1991] are similar in magnitude and orientation and appear to agree well with other geologic and geophysical indicators of deformation. Finally, preliminary estimates of relative motion in central and southern California based solely on GPS measurements collected over a several-year interval have been obtained. *Donnellan et al.* [1990] used GPS measurements over a 1.5-year span to confirm the high strain rates across the Ventura Basin estimated from historical observations. *Larson* [1990a] examines longer-term precision and accuracy based on measurements spanning 2.7 years and finds that the GPS measurements alone are sufficiently robust to provide estimates of relative motions in central California that are consistent with independent estimates derived from very-long-baseline interferometry (VLBI) and with the motions derived from the historical terrestrial observations.

This chapter will present an analysis of the GPS measurements acquired between December 1986 and February 1991 both to assess the long-term repeatability and accuracy of the GPS observations and to derive estimates of crustal deformation in

central California. This work focuses on the geodetic aspects of the GPS measurements. Based on observations spanning 4.2 years, it represents an extension of the results presented by *Larson* [1990a], and is complementary to the work of *Feigl* [1991] who examines the tectonic implications of some of the estimated rates of relative motion between sites located near the Santa Maria Basin. We will begin by discussing some of the aspects that affect the precision and accuracy of GPS estimates. We will then discuss the GPS measurements collected in California and the analysis that was performed on the data to estimate geodetic parameters, such as relative baseline vectors. We will then assess the precision of the vector baselines over both short-term and long-term intervals. We will estimate rates of relative motion between several of the sites in the geodetic network and assess the precision and accuracy of these motions. Finally, we will examine some of the tectonic implications of the estimated site motions.

GEODETIC PARAMETER ESTIMATION FROM GPS MEASUREMENTS

Relative Positioning with GPS

In this section, we review the methodologies used to estimate geodetic parameters, such as baseline vectors, from GPS measurements. This review is intended to be a brief introduction to the major concepts and potential sources of error that will be considered in the analysis of the California GPS measurements that follows. More detailed discussions of high-precision GPS geodetic techniques are provided, for example, by *King et al.* [1985], *Leick* [1990], and *Dixon* [1991].

GPS positioning is based on a simple trigonometric principle: a position can be uniquely determined from distance measurements to several known locations. GPS satellites transmit radio-frequency signals that can be tracked by receivers with antennas, and the satellites and receivers contain oscillators that provide the time of the transmission and reception of the signals. Ignoring clock errors and effects that delay the

propagation of the signal, the distance, or range, between the satellite and the receiver is the time difference between transmission and reception divided by the speed of light. Provided the relative locations of the satellites are known and assuming the receiver is near the Earth's surface, the ranges to three satellites provide sufficient information to locate the receiver with respect to the satellites; the range to an additional satellite allows the receiver clock offset to be estimated as well. When the complete constellation of 24 GPS satellites is in operation by the mid-1990's, four satellites will be visible at all times anywhere on the Earth's surface.

A GPS satellite transmits signals at two *L*-band carrier frequencies that are integer multiples of its 10.23 MHz atomic oscillator frequency: *L1* (1575.42 MHz) and *L2* (1227.60 MHz). These carrier frequencies are modulated in order to encode timing information. The *L1* carrier modulation has two components: a 10.23 MHz (~30 m wavelength) precision (*P*) code that is essentially pseudorandom noise (PRN) with a long repeat time (267 days) and a 1.023 MHz (~300 m wavelength) clear/acquisition (*C/A*) code. The *L2* carrier is modulated only with the *P* code. The carrier frequencies are also modulated with a 50 Hz code that contains satellite ephemerides and other information. The signal structure was designed so that a receiver with knowledge of these codes could cross-correlate the signals to obtain time differences which are then combined with the ephemeris information to derive pseudorange measurements (ranges that contain clock errors). This system is capable of achieving real-time absolute positioning at approximately a quarter of the wavelengths of the *P* and *C/A* codes. For security reasons, the Department of Defence developed two capabilities for altering the signal structure. The first, called *selective availability* (*SA*), reduces the accuracy of the ephemeris message and the clock epoch, and "dithers" the oscillator frequency by about 1 Hz over several-minute periods. The second, called *antispoofing* (*AS*), encrypts the *P* code so that non-authorized users can track only the less-precise *C/A* code. The first satellites launched ("Block I"), all prior to 1989, did not have *SA* or *AS* capability, but all of the

satellites ("Block II") of the final constellation will. A more detailed description of the GPS signal structure is given in *Spilker* [1978], and a discussion of the implications of SA for geodetic users is given in *Feigl* [1991].

Not long after the launch of the first GPS satellite, it was realized that many of the phase differencing techniques developed for quasar VLBI and spacecraft tracking [e.g., *Counselman et al.*, 1972; *Counselman et al.*, 1973; *King et al.*, 1976] could provide centimeter-level relative-positioning precision with GPS [*Counselman and Shapiro*, 1979; *MacDoran*, 1979; *Bossler et al.*, 1980]. Receivers were soon designed that measured the carrier phase of four or more satellites at both the $L1$ and $L2$ frequencies, even without knowledge of the codes. Carrier-phase ranges to satellite are obtained by multiplying the carrier wavelength by the sum of the observed phase and the integer number of wavelengths between the satellite and receiver. The integer number of wavelengths is initially unknown, but can be estimated for doubly differenced observations. Several methods have been developed for optimally determining the correct integer values, variously called 'bias fixing' or 'ambiguity resolution' [e.g., *Bender and Larden*, 1985; *Blewitt*, 1989; *Counselman and Abbot*, 1989; *Dong and Bock*, 1989].

Geodetic measurements with the highest precision are usually obtained by determining relative positions within a network of receivers. Relative positioning takes advantage of several methods of combining phase and pseudorange observables that eliminate or minimize many of the common errors associated with satellites or receivers, such as clock, orbit, and atmospheric propagation effects. The California GPS campaigns were designed to study relative-positioning techniques appropriate for a network containing baselines ranging from 30–450 kilometers, with a goal of obtaining several millimeter level precision in the horizontal baseline components and several centimeter level precision in the vertical. We now review some of the methods for modeling the phase and pseudorange observables that minimize the effect of the various error sources.

Phase and Pseudorange Observations

While the clocks used by GPS satellites and receivers are based on high-stability oscillators (cesium with rubidium as backup for satellites, and quartz for receivers), variations in the frequency of the oscillators are one of the dominant sources of error for GPS positioning [e.g., *King et al.*, 1985]. Since these variations are receiver and satellite dependent, combining simultaneous observations from different sites can eliminate their effect. For example, if simultaneous observations at two sites from the same satellite are differenced, the satellite clock variations can be cancelled. Similarly, if the simultaneous observations of two satellites by the same receiver are differenced, the receiver clock error can be cancelled. Thus, by 'double differencing' the observations between two receivers and two satellites, clock errors can be nearly eliminated; *King et al.* [1985] discuss circumstances where residual clock errors can be a significant effect. Clock errors can be eliminated by either constructing an operator that forms all the linearly independent, doubly differenced observations at a particular time [e.g., *Bock et al.*, 1986b; *Schaffrin and Bock*, 1988], or by treating clock variations in undifferenced observations stochastically [e.g., *Goad*, 1985].

Double-differencing techniques require that the measurements be obtained simultaneously. However, not all receivers obtain measurements (i.e., sample) at the same time. If the satellite frequency is fairly stable, the differences in sample times can often be neglected. However, significant errors can result if the frequency of the satellite oscillator changes significantly between these transmission times, as is possible under SA conditions. Most current receivers sample on the even second in GPS time (essentially UTC without the additional leap seconds that roughly synchronize UTC with UT1), which effectively minimizes this problem. However, other commonly used receivers sample several milliseconds before or after the even second. *Wu et al.* [1990] and *Feigl et al.* [1991] describe methods of modeling the satellite oscillators that minimize the errors introduced by double differencing non-simultaneous observations.

Estimates of range based on carrier-beat-phase observations require that the integer number of cycles between the satellite to the receiver be well-determined. If the signal is interrupted, due to an obstruction between the satellite and the receiver, for example, the receiver can experience a loss of lock on the phase resulting in a discontinuity in the integer number of cycles ("cycle slip"). Determining the magnitude of the cycle slip requires a new parameter that degrades the estimation of other geodetic parameters. Various methods to eliminate the slips have been developed. Large cycle slips can usually be detected by simple differencing between measurement samples. Slips of only a few cycles can be more difficult to determine, especially in the presence of noise, and require more sophisticated signal processing methods if only phase observations are available (D. N. Dong, personal communication, 1990). Methods for removing slips that take advantage of pseudorange observations are described in *Blewitt* [1990]. Correction of cycle slips is critically important for obtaining high-precision geodetic estimates since a slip of a single cycle in both $L1$ and $L2$ results in approximately a 10 cm change in propagation path length when ionosphere-free linearly combined observations are used (see below). Detection of cycle slips can be complicated by the presence of high ionospheric activity, noise introduced by SA dithering of the satellite's oscillator, and gaps in the data.

GPS signals propagating through the atmosphere experience delays with respect to theoretical travel times in a vacuum due to changes in the index of refraction. Refraction effects are significant at radio frequencies in both the ionosphere, between 50 and 500 km altitude, and the troposphere, between 0 and 10 km altitude. Ionospheric refraction delays are proportional to the integrated free electron content along the signal path caused by solar ultraviolet radiation interactions with the upper atmosphere and depend on time of day, latitude, and solar activity which is greatest at the maximum of the solar sunspot cycle. Relative positioning errors due to ionospheric effects are roughly proportional to baseline length, since they tend to cancel over short baselines where the signals propagate

through highly correlated ionospheric paths. These effects are typically between 0.2–2 parts in 10^6 of baseline length during solar-cycle-minimum conditions [e.g., *Schaffrin and Bock, 1988*].

Ionospheric effects are frequency-dispersive so that first-order effects of the ionosphere can be eliminated by forming an *ionosphere-free* linear combination (*LC*) of the phase measurements obtained at two frequencies. Simplified observation equations (ignoring terms due to clock errors that are removed by double differencing, for example) for carrier-beat-phase observations ϕ for *L1* and *L2*, in terms of cycles, can be written:

$$\begin{aligned}\phi_1 &= \frac{\rho}{c} f_1 - \frac{\kappa}{f_1} + n_1 + e_1 \\ \phi_2 &= \frac{\rho}{c} f_2 - \frac{\kappa}{f_2} + n_2 + e_2\end{aligned}\tag{3.1}$$

where ρ is the geometric range from the satellite to the station, c is the speed of light, f_1 and f_2 are the *L1* and *L2* frequencies, n_1 and n_2 are integer cycle ambiguities, e_1 and e_2 are random measurement errors, and κ is proportional to the integrated free electron content [e.g., *Dong and Bock, 1989*]. An ionosphere-free linear combination can be derived by eliminating κ from Equation 3.1:

$$\begin{aligned}\phi_{LC} &= \frac{f_1}{f_1^2 - f_2^2} (f_1 \phi_1 - f_2 \phi_2) \\ &= \frac{\rho}{c} f_1 + \frac{f_1}{f_1^2 - f_2^2} (f_1 n_1 - f_2 n_2) + e_{LC}\end{aligned}\tag{3.2}$$

Higher-order ionospheric effects are not eliminated using the *LC* observable and can sometimes become significant at sites located near the equator or at high latitudes.

Linearly combining the phase measurement errors e_1 and e_2 increases the variance of the ionosphere-free random measurement error e_{LC} . Let us assume that the random errors of $L1$ and $L2$ phase measurements are independent and have a Gaussian distribution described by: $\langle e_i \rangle = 0$, $\langle e_i^2 \rangle = \alpha^2$ ($i = 1, 2$), where the brackets denote the expected value, and α is a fraction of a cycle. We note from Equation 3.2 that $\phi_{LC} = 2.546\phi_1 - 1.984\phi_2$, which by propagation of errors implies that $\langle e_{LC} \rangle = 0$, $\langle e_{LC}^2 \rangle = 10.418\alpha^2$. However, this tenfold increase in variance is offset by the elimination of first-order ionospheric effects. The fact that ionospheric effects are roughly proportional to baseline length and cancel out over short baselines, suggests that $L1$ and $L2$ observations should be treated independently for the analysis of short baselines while ionosphere-free observations should be used to estimate longer baselines. A rough estimate of this trade-off is provided by *Davis et al.* [1989] who found the scatter in estimated components of baselines with lengths between 6–12 km was approximately the same for both $L1$ and LC observations. *Bock et al.* [1986b] describe a sequential least-square method suitable for the analysis of a network with a wide range of baseline lengths which incorporates weighted constraints on ionospheric effects (see also *Schaffrin and Bock* [1988]). This method reduces to using $L1$ and $L2$ phases independently on zero-length baselines and reduces to using LC phases on long baselines.

Formation of the ionosphere-free observable also linearly combines the $L1$ and $L2$ integer-cycle ambiguities, which cannot be independently resolved from the resulting real-valued LC ambiguity. Ambiguity resolution (i.e., determining the correct integer values) must therefore incorporate an adjustment of $L1$ and $L2$ phases that includes explicit estimates and uncertainties of the integer ambiguities. Methods for determining the correct integer numbers from such estimates are discussed by *Blewitt* [1989] and *Dong and Bock* [1989]. These methods consider the uncertainty of an ambiguity estimate and its nearness to an integer to determine the statistical confidence associated with fixing it to an integer. However, adjustments which use $L1$ and $L2$ phases separately also

include first-order ionospheric effects that increase the uncertainty of the ambiguities on long lines and thus make them more difficult to resolve.

Several techniques have been developed to optimize the resolution of ambiguities in light of these problems. The $n_2 - n_1$ ambiguity difference, which is known as the “wide-lane” ambiguity, is associated with an 86-cm wavelength for code-based receivers and therefore can be resolved more easily in the presence of noise than the “narrow-lane” n_1 ambiguity which has a 19-cm wavelength [Counselman et al., 1980; Hatch and Larson, 1985]. Resolution of wide-lane ambiguities can be further enhanced by applying ionospheric constraints to the phase observations [Bender and Larden, 1985; Bock et al., 1986b], and by forming linear combinations of the phases with the P -code pseudorange measurements at both the $L1$ and $L2$ frequencies [Bender and Larden, 1985; Blewitt, 1989].

The method of Dong and Bock [1989] incorporates these techniques into a sequential least-square “bootstrapping” procedure that first resolves the ambiguities with the highest levels of confidence and then progresses to less certain ambiguities. After the ambiguity that can be most confidently resolved (typically those associated with the shortest baselines) is fixed to its correct value, all the other parameters are reestimated. The resolution of the ambiguity both improves the precision of the orbits and decreases the uncertainties of all the other parameters, and thus improves the chances of resolving ambiguities on longer baselines (see also Counselman and Abbot [1989]). This sequential procedure continues until no more ambiguities can be confidently resolved. Once a wide-lane ambiguity has been resolved, ionosphere-free observables can be used to estimate its corresponding narrow-lane ambiguity. The narrow-lane ambiguities estimated in this fashion have an equivalent wavelength of 10.7 cm, but also have smaller uncertainties due to the elimination of the first-order ionospheric refraction effects. The bootstrapping method using LC observables is then used to resolve as many narrow-lane ambiguities as possible.

Non-dispersive tropospheric effects result from variations in the index of refraction due to the atmosphere at hydrostatic equilibrium and to excess water vapor [e.g., *Davis et al.*, 1985]. Atmospheric propagation delays due to both hydrostatic (dry) and water vapor (wet) components are usually estimated with two models: one to calibrate the zenith propagation delay [e.g., *Saastamoinen*, 1972], and another to map the delay to other elevations [e.g., *Davis et al.*, 1985]. Residuals to these models are typically estimated from the observations either as a single zenith delay residual, or by using stochastic models [e.g., *Tralli et al.*, 1988; *Dixon and Kornreich Wolf*, 1990; *Herring et al.*, 1990].

Another source of error is associated with the relative locations of the satellites. As discussed in Chapter 2, the precision of a baseline estimate divided by its length is roughly proportional to the precision of the satellite orbits divided by their altitude. The orbital information transmitted by the satellites (“broadcast ephemeris”) has a relative precision of approximately 1 part in 10^6 , which is insufficient for high-precision geodetic applications such as monitoring crustal deformation. To improve this precision, orbital parameters associated with the satellite’s initial position and velocity and solar radiation models are usually estimated from GPS measurements. If the GPS measurements are acquired from receivers whose relative locations are well known, the precision of their relative locations can be transferred to the satellite orbits. Establishing high-precision relative “fiducial” site locations is equivalent to defining a terrestrial reference frame in which site and orbit parameters are estimated. An example of such a definition, using observations from other space-geodetic techniques such as very-long-baseline interferometry and satellite laser ranging that provide relative locations with precisions of less than 1 part in 10^8 , was presented in Chapter 2.

DATA

General Description

We have dubbed the five-year program of GPS observations in central and southern California the Transverse Range EXperiments (TREX), although they encompass several other tectonic regions. To facilitate the description of the TREX experiments, which were conducted at discrete intervals using varying numbers of receivers, we define several conventions regarding their temporal and spatial scales. Individual phase and pseudorange measurements were sampled generally near the GPS minute, with exceptions to be noted in the discussion of the receivers. The time between measurements used in the adjustments, the *sampling interval*, is either 30 or 120 seconds; the shorter interval enables cycle slips to be detected in the presence of noise with greater confidence. A single observation period by a receiver, a *session*, typically lasted six to eight hours (i.e., they included around 225 or 900 measurements depending on the sampling interval) and were scheduled to optimize the geometry of the positions of the six to nine satellites observed from the receiver. Usually, the observations from all the receivers operating during a single session are adjusted simultaneously to yield estimates of the relative positions of the receivers and other geodetic parameters. An *experiment* generally consisted of three to five sessions in which the same monuments were repeatedly occupied. Three to five sessions per experiment provide sufficient redundancy to detect blunders, such as incorrectly measuring the antenna height, and the scatter of the estimates provides some measure of their precision. The largest unit of time from an operational point of view is a *campaign* which consisted of one to six experiments that were conducted over the course of a month or so after all the receivers and personnel had converged upon California. We conducted approximately 22 experiments during 10 campaigns in California over a 4.2-yr interval beginning in December 1986.

The experiments may also be distinguished by their geographic distribution and primary objectives, although these categories tend to be less well defined. For the purposes of this work, we distinguish five types of experiments. Two types involved the occupation of the primary crustal monitoring network, which was generally done in halves with some overlap. The northern half spans central California from Ft. Ord to Pasadena, while the southern half spans the area south of Pasadena and includes the sites on most of the Santa Barbara Channel islands. The other types of experiments have concentrated on smaller areas, including a study of the Santa Maria fold and thrust belt [Feigl *et al.*, 1990] and a study of the Ventura Basin [Donnellan *et al.*, 1990]. The final type of experiment is a generic description concerned with occupations of historical triangulation and trilateration monuments which were not part of the primary network. We will refer to sites by their location name except when ambiguities arise due to occupations of several monuments at a particular location when the four-character monument name will be given (see Table 3.1). For example, one of the principal GPS monuments of the network, VNDN (stamped "VLBI STA 7223 RM-1"), which is located near the VLBI reference point 7223, will generally be referred to as Vandenberg, although there have been a few occupations on other monuments that are within several hundred meters of VNDN, including 7223, another of its reference marks which we call VND2, and a historical triangulation monument PARG.

The work presented in this chapter concentrates on the occupations of the primary network in central California (Figure 3.1). This portion of the network was established during three experiments in December 1986 and January 1987. Subsets of the network were occupied in May 1987 and September 1987. Major occupations of the network were then conducted annually in March 1988, March 1989, March 1990, and February 1991. In addition to these seven experiments spanning a little over four years, we will also include observations from the Santa Maria fold and thrust belt which were conducted to assess the extent of crustal deformation within a hundred kilometers of the Vandenberg

VLBI antenna 7223. These observations are described in detail by *Feigl* [1991]. We include them in this work to provide some assessment of shorter baseline lengths and to provide additional information regarding several sites in the primary network in the vicinity of Vandenberg. Some of these observations were conducted September 1987 and April 1989. Longer separate experiments were also conducted by K. Feigl in February 1990 and September 1990. The site occupation history for these experiments is given in Table 3.2. For convenience, each of the TREX experiments has been assigned a number, beginning with TREX 1 in December 1986 and ending with TREX 20 in February 1991; for historical reasons the experiments conducted in the vicinity of Vandenberg in February 1990 and September 1990 are denoted VF 1 and VF 2 (for Vandenberg Footprint), respectively (see Table 3.2).

In addition to the measurements obtained in California, GPS receivers operated simultaneously at other sites in North America, Europe, and Hawaii (Figure 3.2). The majority of these sites are part of the Cooperative International GPS Network (CIGNET) coordinated by the National Geodetic Survey since late 1986 [*Chin*, 1987]. These sites are collocated with VLBI reference points to provide a fiducial orbit tracking network that is well-defined in a terrestrial reference frame, such as SV6 described in the previous chapter. Observations are also available from a few sites that are not part of CIGNET, including Platteville (collocated with VLBI), Austin, and Churchill. For convenience all receivers located outside of California will be called part of a *continental* network, whereas the network in California will be described as *regional*, referring to baseline length scales from 30 to 450 km.

The work in this chapter will therefore concentrate on GPS observations obtained during 13 experiments comprising a total of 71 single sessions spanning 4.2 years. Measurements were obtained during a total of 745 receiver-sessions at 58 sites (Table 3.2). The 42 regional sites are shown in Figure 3.6, with four-character labels shown for the principal sites considered in this work (sites with sufficient observations to estimate

long-term precision and relative motions). Their approximate locations and occupation summaries are given in Table 3.1. The number of occupations at the regional sites varies from 10 sites with one to Vandenberg with 68 session occupations.

GPS Receivers

Our observations were collected using several different types of GPS receivers. The majority of measurements were obtained using Texas Instruments TI4100 receivers and antennas [Henson *et al.*, 1985; Sims, 1985]. The TI4100 is dual-band (i.e., receives both *L1* and *L2* frequencies), code-correlating receiver (i.e., it is able to track the *P* code at both frequencies). The measurements are sampled 0.92 seconds before the GPS minute. The TI4100 antenna is a conical spiral design that is somewhat prone to multipath interference effects at low elevations. At a few of the CIGNET sites, such as Kokee KOKE and Mojave MOJF, the TI4100 antenna was replaced with a FRPA-2 microstrip antenna that was thought to have lower sensitivity to multipathing effects. The TI4100 is able to track only four satellites simultaneously.

In late 1989, the TI4100 receivers at three CIGNET sites (Mojave MOJF, Richmond RICH, and Westford WSFD) were replaced by Mini-Mac 2816AT receivers using Macrometer cross-dipole antennas located over slightly different monuments (MOJM, RICM, and WSFM). The Mini-Mac 2816AT is a dual-band receiver that uses codeless technology to track the *L2* signal, enabling it to operate under *AS* conditions when the *P* code is not available to non-authorized users. It uses the *C/A* code to track the *L1* carrier phase and pseudorange. The *L2* carrier phase is extracted by squaring the signal or mixing upper and lower side bands of the signal, removing the modulation which allows the *L2* carrier to be tracked at twice the original frequency [Counselman, 1987]. The Mini-Mac receiver samples approximately on the GPS minute [see Feigl, 1991]. The Macrometer cross-dipole antenna is mounted on a ground plane and is less susceptible

than the TI4100 and FRPA-2 antennas to multipathing effects. The Mini-Mac can track up to eight satellites simultaneously.

The three experiments considered in this work (February 1990, September 1990 and February 1991) used Trimble 4000 SST receivers at the non-CIGNET regional sites. The Trimble is also a codeless receiver that tracks the *C/A* code and squares the *L2* carrier signal. The Trimbles used in these experiments sample on the even GPS minute. The Trimble antenna uses a microstrip design that helps to minimize multipathing effects. Tests to determine the locations of the *L1* and *L2* phase centers of the Trimble 4000 SST antennas indicate that they might be horizontally offset from the physical center of the antenna by several millimeters [C. Rocken and C. Meertens, personal communication, 1990]. This offset needs to be taken into consideration when combining Trimble observations with those obtained by other receivers, but is not a major effect for the two experiments mentioned above except for baselines between Trimble and CIGNET receivers, such as at Mojave.

The California Campaigns

The California campaigns were designed to study different sources of error, and baseline repeatability and accuracy over a wide range of spatial scales, both regional and continental (30 to 4200 km). Each major campaign consisted of one or more four- or five-day experiments during which different subsets of the network were occupied. Four-day occupations of each site were chosen to provide adequate redundancy to detect blunders and to assess short-term repeatability within each experiment. In general, measurements were obtained from six to nine satellites using similar seven to eight hour schedules of satellites during each observation session.

Since the window of satellite observability advances by two hours each month, the observations during different times of the year obtain measurements at different times of day, thus sampling different ionospheric and tropospheric conditions. The midwinter

observations were entirely at night, the March observations during the evening and early morning, the May observations in the afternoon and evening, and the September observations from sunrise to midafternoon, the time of peak ionospheric gradients. While the first-order effects of the ionosphere can be removed from the observations using the ionosphere-free linear combination of $L1$ and $L2$, their magnitude significantly affects the ability to resolve phase ambiguities and thus to obtain the most accurate estimates of site coordinates [Dong and Bock, 1989]. Over the five year program, the mean level of ionospheric activity has changed as well, due to sunspot activity that reached its eleven year high in spring of 1989. We will now briefly discuss the characteristics of each experiment to be used in the following analysis to highlight some of the measurement differences.

The first experiment (TREX 1) comprises observations from five sessions collected during December 16–20, 1986 (days 350–354 of the year). The experiment used eight receivers in central California and served as a training exercise. Five of the sites occupied (Fort Ord FTOR, Owens Valley OVRO, Palos Verdes PVER, Santa Paula SNPA, and Vandenberg VNDN) were located near VLBI reference monuments. The remaining regional sites were all located near San Luis Obispo and include Black Hill BLHL, Lospe LOSP, and Pozo POZO. The continental network for this experiment is particularly weak, with only four sessions at the VLBI-collocated Platteville site and three occupations at Austin.

The second experiment (TREX 2) comprises observations from five sessions collected between December 29, 1986 and January 2, 1987 (days 363–002), and included a New Year's Eve celebration. This was the first major occupation of the central California network with eleven receivers, plus the CIGNET receiver at Mojave, making simultaneous observations in California. Principal network sites include Black Hill BLHL, Piedras Blancas BLAN, Center (on Santa Cruz Island) CENT, Fort Ord FTOR, Fibre (near Buttonwillow) FIBR, La Cumbre LACU, Lospe LOSP, Madre MADC,

Owens Valley OVRO, and Vandenberg VNDN. Few observations were obtained at Mojave and Lospe due to receiver problems. A 20-cm correction had to be applied to Center observations due to a consistent incorrect measurement of the vertical antenna height during all five sessions (F. Webb and B. Hager, personal communication, 1987). A multipathing experiment was conducted at Piedras Blancas (located near a lighthouse) on January 1 and 2 by placing the antenna directly on the monument; the horizontal centering of the antenna during these two sessions is suspect (R. King, personal communication, 1987). The continental network included measurements at Platteville and Westford WSFD, and two sites in Canada: Algonquin, Ontario ALGO and Churchill, Manitoba CHUR.

The last experiment (TREX 3) we shall examine from the first campaign comprises observations from five sessions collected during January 3–7, 1987 (days 003–007). This experiment included eleven field receivers to complete the establishment of the central California network and also occupied several important historical monuments with baselines crossing the Santa Barbara Channel, including Devil's Peak (on Santa Cruz Island) DEVL, Gaviota GAVI, Soledad (on Santa Rosa Island) SOLE, and Chaffee CHAF and its alternative monument Solimar SOLI. The continental network was also fairly robust, with observations from Algonquin, Churchill, Platteville, and Westford.

The second campaign included only one small experiment (TREX 7), conducted from 24 to 28 May 1987 (days 144–148). A subset of the central California network was occupied for five days at six sites in California, and at Austin, Westford, and Yellowknife, Northwest Territories YKNF. Four of the six California sites were at VLBI collocated sites; the remaining two sites, Black Hill BLHL and Center CENT on Santa Cruz Island, were chosen to provide a variety of baseline lengths and atmospheric environments. One of the goals of this experiment was to assess how the afternoon and evening observations in May compare with the nighttime observations from January.

The third campaign is split into two experiments. The first experiment (TREX 8) was conducted over five sessions during September 22–26, 1987 (days 265–269). Seven field receivers plus Mojave obtained measurements for a subset of the sites in central California, with primary reoccupations at Black Hill, Center, Fort Ord, Owens Valley, Palos Verdes, and Vandenberg. The continental network includes observations from Churchill, Platteville, Westford and Yellowknife.

The second part of the third campaign (TREX 9) comprised occupations of twenty-eight historical triangulation and trilateration sites for one to three days, including several in the vicinity of the October 1987 Whittier Narrows earthquake that occurred during the experiment. For the purposes of this work, we shall only use measurements from two sessions from TREX 9 on days 274 and 275, that are focused primarily on a few sites in the vicinity of the Santa Maria Basin, including Grassy GRAS.

We will utilize observations from the experiment (TREX 11) conducted during the March 1988 campaign which constituted a major reoccupation of the central California network. Four sessions (March 15–18, 1988; days 075–078) and twelve field receivers contributed measurements for this experiment. All but one site (Cotar) occupied in Experiment 2 were reoccupied as well as Palos Verdes and Twin on San Nicolas Island. The continental network was also included many sites, including Algonquin, Churchill, Platteville, Richmond (Florida), Tromso (Sweden), Westford, and Yellowknife. This is the last experiment that includes observations at the Fort Ord monument that had also been occupied by VLBI, due to the destruction of the monument by the U.S. Army in order to erect some barracks.

We include observations from two of the experiments conducted in the March and April 1989 campaign. The first experiment (TREX 14) reoccupied the central California network for four sessions during March 28–31, 1989 (days 087–090), and included eleven field receivers that again essentially reoccupied the January 1987 network. Another site collocated with VLBI was added occupied at the Jet Propulsion Laboratory

(JPL1). The TI4100 antenna at the Mojave CIGNET site was replaced by a FRPA-2 antenna. The continental network was degraded, however, due to problems with the Mini-Mac receivers that had replaced the CIGNET TI4100 receivers at Richmond and Westford, and the lack of the Canadian and Platteville receivers. Additional continental sites were added, one at Kokee KOKE on Kauai, Hawaii, and several in Europe (Onsala ONSA, Tromsø TROM, and Wettzell WETT), all of which are collocated with VLBI. However, we have only included observations from the six to eight hour sessions used by the California receivers. The large distance between California and Europe implies that the respective receivers observe substantially different satellite geometries. The result is that fewer doubly-differenced observations are possible with the European sites. This experiment was also conducted during one of the most intense periods of ionospheric disturbances at around the eleven-year maximum in sunspot activity.

The second experiment (TREX 15) from the March and April 1989 campaign concentrated on the sites in the Santa Maria Basin as part of the Vandenberg deformation study. A total of four receivers occupied various monuments over three sessions during April 1–3, 1989 (days 91–93), including reoccupations of Gaviota and Grassy. A few sessions were obtained from the Mini-Mac receivers at Mojave MOJM and Westford WSFM.

In February 1990, K. Feigl conducted another more extensive occupation of sites in the Santa Maria Basin. The 10-session experiment (VF 1) included only two field receivers, one of which always occupied Vandenberg, and continental observations at Kokee and at the improved Westford WSFM, Richmond RICM, and Mojave MOJM CIGNET Mini-Mac receivers.

The observations from one experiment (TREX 17) during the March 1990 campaign have been included. This four session experiment, conducted during March 25–28, 1990 (days 085–087) was the last experiment to rely primarily on TI4100 receivers in the field (experiment (TREX 18) used Trimble 4000 SST receivers, but these observations have

not been incorporated into this work). Field receiver observations from seven sites are included: Black Hill, Center, JPL1, Madre, Owens Valley, Palos Verdes, and Vandenberg. In addition, Mini-Mac observations were obtained from Mojave, Richmond, and Westford, and additional continental observations were obtained from Algonquin AROT. The Department of Defense activated SA dithering of the oscillator frequencies in January 1990 on the three Block II satellites that were observed in this experiment. Since the TI4100 and Mini-Mac receiver sample measurement times differ by about 0.92 seconds, the effect of SA adds a significant component of noise to the doubly-differenced phase observables, which we were required to model using the techniques of *Feigl et al.* [1990].

During September 6–19, 1990 (days 249–262), K. Feigl conducted a 14-session experiment (VF 2) in the Santa Maria Basin using three Trimble receivers. Once again, Vandenberg was occupied during all sessions. Eight other sites were occupied for one or two sessions, including Gaviota, Grassy, Lospe, and Madre. Observations from Mojave, Richmond, and Westford provide continental fiducial control. Due to the Persian Gulf crisis precipitated by the invasion of Kuwait by Iraq in August, the Department of Defense turned *off* oscillator dithering by SA, because the military did not have enough receivers capable of receiving the undithered signals to capitalize on its potential advantage.

The last experiment (TREX 20) that will be analyzed in this work was a four-session survey conducted during February 7–10, 1991 (days 038–041) and constitutes a major occupation of the central California network. Sixteen Trimble 4000 SST receivers were used to reoccupy all the primary sites established in January 1987, including Piedras Blancas, Black Hill, Center, Fibre, La Cumbre, Lospe, Madre, Owens Valley, and Vandenberg. The continental network is also fairly well observed, with measurements from Kokee, Mojave, Richmond, and Westford. SA effects were still turned off. We have also included an additional session just before the ones described (day 037), that

was used to perform a local survey about the Vandenberg monument and included the VLBI reference point 7223.

The remainder of this chapter will discuss analysis of the observations from these 71 sessions collected in 13 experiments over a 4.2 year interval. They provide a rich source of information for assessing the capabilities of high-precision GPS surveying using networks with baselines between 30 and 450 km while incorporating information from continental scale fiducial networks. Approximately 16 sites have observations spanning more than 1.5 years that allow their relative motions to be estimated using GPS.

ANALYSIS OF GPS OBSERVATIONS

Our analysis of the GPS observations can be broadly separated into two parts, each of which was accomplished with a different software package. The first package, called GAMIT, is used to remove or minimize the effects of outliers and cycle slips in the phase and pseudorange observations and to perform least-square adjustments of the observations, most typically on those collected during a single session. GAMIT provides estimates of satellite orbit parameters, site coordinates, and atmospheric parameters, and attempts to resolve phase ambiguities. The second package, call GLOBK, uses the single-session station positions and satellite orbit parameters derived by GAMIT to estimate mean site coordinates, site velocities, and multi-session orbital arc parameters. The scatter of the single-session estimates about the GLOBK-derived mean values are used to assess the precision of the GPS measurements during each campaign and over the entire 4.2-yr observation program in central and southern California. In this section, we briefly describe each of these packages.

GAMIT

The GAMIT software package, developed at M.I.T. and Scripps over the last eight years, is a general purpose analysis system to perform high-precision relative position geodesy with Global Positioning System observations. It is capable of editing and combining observations collected from different receiver types, numerically integrating satellite orbits, and modeling phase and pseudorange observations. Its primary function is to linearly estimate adjustments to station positions and satellite orbits based on the differences between measured and theoretical observations, while taking into account clock errors, atmospheric effects and integer-cycle ambiguities in the carrier phases. Several adjustments are performed using linearly independent, doubly differenced phase observations, usually from a single session, in order to optimally resolve phase ambiguities. Tightly weighted constraints are placed on the adjustment of *a priori* coordinates for several sites and, possibly, some satellite orbit parameters in order to define a suitable terrestrial reference frame. Once ambiguities have been resolved, two additional adjustments can be performed in which loosely weighted constraints are placed on all site and satellite parameters. The results from these two adjustments of ionosphere-free (*LC*) observations—one in which no ambiguities are resolved and one which uses the previously determined integer ambiguities—form the set of single-session observations that are further analyzed using GLOBK.

The first step in the GAMIT processing scheme is to transform the observations originally in receiver-dependent formats into the format suitable for GAMIT. This step has been greatly facilitated in recent years by the introduction of a standard exchange format, RINEX [Gurtner *et al.*, 1989]. In the reformatting stage, satellite clocks and ephemeris information obtained from the 50 Hz modulated code signal (broadcast ephemeris) are retained and nominal station clock corrections based on *a priori* site coordinates, pseudorange and ephemeris information are calculated. Other site-dependent information, such as the measured antenna heights, are also introduced.

The second data processing step is to detect and remove cycle slips. An *a priori* model of the site coordinates and satellite orbits (usually based on the broadcast ephemeris) is used to calculate theoretical observables that are compared to observed phase and pseudorange measurements. Cycle slips are then detected by computing epoch-to-epoch differences and by examining linear combinations of the phase observations that remove the effects of clock instabilities and the ionosphere; if both *L1* and *L2* pseudorange observations are available, additional linear combinations including these observations can enhance the capabilities of automatic editing algorithms [e.g., *Blewitt*, 1990]. GAMIT has the capability of adding additional parameters representing integer cycle slips that cannot be confidently removed, but estimating these parameters weakens the estimates of other geodetic parameters of interest.

Several physical models are adopted by GAMIT in order to calculate theoretical phase and pseudorange observations based on *a priori* estimates of the site coordinates and satellite ephemerides. These include the force models used to estimate the motions of the satellites over time in a quasi-inertial reference frame, the relationship between the quasi-inertial reference frame and the terrestrial reference frame used to locate the sites, and models that describe the propagation delay between the satellite and receivers.

A satellite orbit can be estimated given a state vector describing its initial position and velocity and the forces acting on the satellite, which include those due to the gravitational potential of the Earth, Sun, and Moon, and to solar radiation pressure. GAMIT has the capability to numerically integrate the motion of a satellite and to provide estimates of linear perturbations to the six-parameter satellite state vector and up to three components of the solar radiation pressure based on the phase and pseudorange estimates [*Ash*, 1972; *Abbot et al.*, 1985]. In general, the models used to integrate the orbits in this work are consistent with the MERIT standards [*Melbourne et al.*, 1983]. The GEM-L2 gravity field [*Lerch et al.*, 1985] complete to degree and order 8 is used, but with a gravitational constant for the Earth consistent with the scale of the terrestrial reference used to locate

the sites ($GM = 398600.4401 \text{ km}^3/\text{s}^2$). The motions of the Sun and Moon with respect to the Earth are provided by a tabulated ephemeris derived from lunar laser ranging and planetary observations by the Harvard Center for Astrophysics [King *et al.*, 1976; J. F. Chandler, personal communication, 1989].

The relationship between the quasi-inertial satellite reference frame and the terrestrial site reference frame can be described by a series of rotation matrices representing motions due to precession and nutation of the Earth, and Earth orientation parameters (polar motion and UT1). Precession is described by the IAU (1976) expressions with respect to 1950.0 [Lieske *et al.*, 1977]. Nutation is given by the IAU 1980 model [Wahr, 1981] and Earth orientation parameters are interpolated from values provided by the International Earth Rotation Service (e.g., IERS, [1990]).

Theoretical propagation delays are derived from the *a priori* locations of the receiver and satellite, and atmospheric refraction models. The height of the receiver antenna above the *a priori* monument location is added with respect to the WGS84 reference ellipsoid. A time-varying correction is added to the site coordinates due to solid Earth tides based on the location of the Sun and the Moon; site and frequency-independent Love numbers ($h_2 = 0.6090$ and $l_2 = 0.0852$) are assumed. In general, second-order polynomial corrections are applied to the receiver and satellite clocks. More complete models of the satellite clocks whose oscillator frequencies have been dithered by the implementation of SA follow the techniques described by Feigl *et al.* [1991]; estimates of the frequency dithering are obtained by comparison with hydrogen maser oscillators used by several CIGNET receivers. Atmospheric propagation delays due to both hydrostatic (dry) and water vapor (wet) components are estimated by two models. The Saastamoinen model is used to calibrate the zenith delay [Saastamoinen, 1972], while the CfA 2.2 model is used to map the delay to other elevations [Davis *et al.*, 1985]. Nominal values are used for meteorological conditions (sea-level barometric pressure = 1013.25 mbars, temperature = 20°C, relative humidity = 50%), with elevation corrections applied to the

barometric pressure. No surface meteorological or water vapor radiometer observations have been used.

Once the theoretical phase and pseudorange observations have been calculated from *a priori* models of the site and satellite locations, a sequence of least-square adjustments to these *a priori* values is performed using GAMIT, based on the observed phases and pseudoranges collected during each six to eight hour session. GAMIT uses the algorithm described by *Schaffrin and Bock* [1988] to form an independent set of doubly-differenced observations. In general, some—but not necessarily all—of the following parameters are estimated: (1) three-dimensional coordinates, (2) six initial conditions and three radiation pressure parameters for each satellite, (3) a zenith delay parameter for each site to account for unmodeled atmospheric refraction effects averaged over the session, and (4) an independent set of double-difference phase ambiguity parameters. Adjustments to the coordinates of several sites, and possibly some satellite parameters, are tightly constrained to establish a suitable reference frame for ambiguity resolution, as described above.

Several adjustments are performed to optimally resolve ambiguities, generally following the methodology described in [*Dong and Bock*, 1989]. In the first adjustment, which uses doubly differenced ionosphere-free (*LC*) observations, all parameters and ambiguities are estimated. The independent set of double-difference phase ambiguity parameters estimated in the first adjustment are generated from the original carrier-beat-phase ambiguities by means of a mapping based on baseline length and the distribution of phase data. The mapping constructs an independent set of double-difference ambiguities according to the shortest independent path (no closed loops) through the network to take advantage of the greater confidence of ambiguity resolution obtained on short baselines.

The next step employs a bootstrapping procedure to determine the best-fitting integer values for the doubly differenced wide-lane ambiguity parameters using the *L1* and *L2* phase observations independently. In this step, all the parameters, with the exception of

the ambiguities whose integer values are to be determined, are fixed to the values obtained by the first ionosphere-free adjustment. Two more techniques are used improve the resolution of the wide-lane ambiguities. *P*-code pseudorange observations, if available, are used to help indicate appropriate integers in marginal cases and constraints are placed on the residual ionospheric refraction effects weighted by baseline length following the methodology described by *Schaffrin and Bock* [1988]. For the experiments described in this chapter, the ionospheric constraint applied varied between 0.5 and 5.0 parts per million (ppm) of baseline length with an average of 1.0 ppm, depending on an empirical assessment of the level of ionospheric activity during the session. For the adjustments considered in this work, ambiguity resolution is attempted only on regional baselines; the longer continental-scale baseline ambiguities are always estimated. Once the wide-lane ambiguities that can be confidently resolved are fixed to their correct integer values, another adjustment is performed on the doubly differenced ionosphere-free observations to estimate all parameters and to sequentially resolve narrow-lane ambiguities.

For these single-session adjustments that are performed to resolve ambiguities, tightly weighted constraints are applied to the coordinate adjustments of a small number of sites (typically three) in order to provide a suitable definition of a terrestrial reference frame. As the derivation in Chapter 2 of this work demonstrated, we know that these coordinates, based on other space-geodetic techniques and local survey ties, have uncertainties associated with them, which are incorporated as *a priori* constraints on the coordinates for sites collocated with VLBI reference monuments. Once the ambiguities have been resolved, two more adjustments are performed in which all adjustments to site and satellite parameters are loosely constrained. In these adjustments all the site coordinates are assumed to have 10 m of uncertainty in all components, initial satellite positions are assumed to have 500 m uncertainty, and the initial satellite velocities and three radiation pressure parameters for all satellites are similarly unconstrained. These

levels were chosen to provide sufficiently high *a priori* uncertainties not to bias the GLOBK estimates while avoiding numerical problems in the GAMIT analysis. In the first adjustment, all ambiguities are freely estimated and no attempt is made to resolve their integer values; we call this the *ionosphere-free bias-free* adjustment. In the second adjustment, the integer ambiguities are fixed to their previously determined values from the tightly constrained adjustments and all other parameters are estimated; we call this the *ionosphere-free bias-fixed* adjustment. These adjustments form the set of single-session observations that are analyzed with GLOBK to determine weighted mean site coordinates, site velocities, and multi-session orbital parameters, and to assess the short-term and long-term precision of the GPS observations.

GLOBK

The adjustments performed with loosely weighted constraints provide solution vectors and covariances suitable for use in the Kalman filtering analysis routines, called GLOBK, that were developed by T. Herring at MIT. A basic description of the Kalman filtering is given in *Leibelt* [1967] and *Gelb* [1974] and the particular algorithms used in GLOBK are given in *Herring et al.* [1990]. A Kalman filter provides a method for sequentially estimating parameters. Given an initial state vector and covariance, a Kalman filter is able to incorporate additional information to improve the estimate of the state vector and covariance. In particular, the application of constraints, which the Kalman filter treats as additional information, to a state vector with a high *a priori* uncertainty yields an improved state vector and covariance that reflects the effects of the constraints. Two types of Kalman filter adjustments are typically performed: a “forward” solution that sequentially estimates the weighted mean of the observations and other model parameters, such as velocities, and a “back” solution that provides an assessment of the observation residuals with respect to the forward solution.

To take advantage of these capabilities, loosely weighted *a priori* constraints are applied in GAMIT to all the site coordinates and satellite state vectors. The resulting GAMIT estimate of the site and satellite vector has a covariance matrix with high uncertainties and correlations, but retains the intrinsic geometrical relationships of the tightly constrained solutions. For example, if this state vector is combined using Kalman filtering techniques with tightly weighted constraints on three site locations, the resulting solution vector will be essentially the same as would have been estimated using GAMIT using the same constraints if the *a priori* model is linearly close to the estimated model. This powerful technique is capable of combining the adjustments from several sessions to yield a simultaneous estimate of coordinates and provides a useful method for testing the effect of using different *a priori* constraints on both the site and satellite parameters. These capabilities are especially useful for uniformly combining adjustments that were derived using different fiducial networks. If the orbital arcs used to estimate the individual sessions within a GPS campaign are all derived with the same initial conditions, application of tightly weighted constraints on the satellite orbits is equivalent to using a multi-session ephemeris, which theoretically should provide more precise coordinate estimates. Parameters describing the velocities of the sites over time can be easily incorporated into this framework, and the scatter of site coordinate estimates about these rates can be estimated by introducing stochastic (random walk) processes to unconstrain the site positions between sessions.

Ideally, the constrained sites that make up the fiducial network for a single-session adjustment are geographically well distributed; that is, the sites are separated by thousands of kilometers so that errors in their locations will have less effect proportionally on the estimated baselines, and they have good spatial distribution both east-west and north-south to improve the estimates of satellite orbits. With a good distribution of baseline lengths and a robust fiducial network, the majority of ambiguities for baselines in California are typically resolved. As long as all the ambiguities are

properly resolved in the single-session adjustments, the day-to-day differences in the geometry of the fiducial network and the constraints placed on their coordinates are not be significant since they are uniformly combined in the GLOBK Kalman filtering analysis. However, if the distribution of the fiducial network is not well distributed, or if the bootstrapping ambiguity resolution technique is inhibited by a poor range of baselines, some ambiguities may remain unresolved. In this case, since GLOBK has no capability for improving the resolution of ambiguities, heterogeneities in the fiducial networks used in the single-session adjustments have a more significant effect and increase the day-to-day scatter of the parameter estimates.

SINGLE-SESSION ADJUSTMENTS

The measurements from all the sites observing during an individual session of an experiment were adjusted according to the methodology provided in the previous section. In this section, we describe the general characteristics of the tightly-constrained single-session adjustments to examine the quality of the experiments. In the following sections, we will combine the loosely-constrained adjustments to assess the precision and long-term stability of the measurements and estimate relative motions.

The *a priori* site coordinates and their uncertainties used in the single-session adjustments are derived from the SV5 terrestrial reference frame [Murray *et al.*, 1990]. The SV5 reference frame is the predecessor of the SV6 terrestrial reference frame described in Chapter 2. SV5 is derived from the same SLR and VLBI adjustments (CSR8902 and GLB659, respectively [Schutz *et al.*, 1989; Caprette *et al.*, 1990]) and local survey ties between collocated SLR and VLBI reference points given in Table 2.2. The SV5 reference points were estimated using a slightly different methodology than SV6 that did not fully incorporate geologic information or simultaneously estimate all site coordinates. For the purposes of this work, however, the differences between the two

reference frames are not significant because a majority of the fiducial sites are collocated with VLBI reference points with independent velocity estimates, thus preserving the high precision of the relative locations. Several exceptions should be noted, however. Several sites have poorly-resolved VLBI vertical velocities, which result in high uncertainties in site coordinates. These sites had significant adjustments in the vertical in the SV6 adjustment. The uncertainties on the SV5 site coordinates are derived solely from the VLBI formal uncertainties. Since the coordinates for Westford 7209 were held fixed to perform the GLB659 adjustment, the SV5 uncertainties for Westford are equal to zero. Therefore, Westford was held fixed whenever its observations were available. This implies that the uncertainties for the other sites, which are given in Table 3.3, are with respect to Westford.

The occupation history of the sites is given in Table 3.2. Excluding the experiments devoted to collecting observations in the vicinity of Vandenberg (TREX 9 and 15, and VF 1 and 2), each experiment lasted four to five days. Most sites obtained sufficient observations during every session during the experiment to enable their coordinates to be estimated with reasonable confidence. Exceptions to this usually occurred when the receiver failed to operate properly due to either power failure or local radio interference. Occupations that did not obtain measurements over the complete six to eight hour session generally provide less robust estimates of position, often due to the lack of one or more satellites.

We begin, by way of example, by examining the results from the single-session adjustment of the measurements collected from the session beginning on December 31, 1986 (day 365). Fifteen TI4100 receivers collected observations during this session, at eleven sites in California and at the continental sites Algonquin ALGO, Churchill CHUR, Platteville PLAT, and Westford WSFD (Table 3.2). The sky tracks for the six Block I satellites (PRN 3, 6, 9, 11, 12, 13), which indicate the azimuth and zenith angle of the satellite as viewed from a receiver in central California over the 7.5 hour nighttime

session, are shown in Figure 3.3. The measurements were collected at 30-second intervals, but were decimated to 120-second intervals, allowing a maximum of 225 sample times ('epochs') during which up to four satellites might be observed by the TI4100 receiver; only measurements that were made simultaneously with at least two receivers observing simultaneously to at least two satellites are used to form a total of 6211 independent doubly differenced phase observations. All the receivers successfully collected throughout the session. The satellite PRN 12 contributed the least number of observations since it was observed for only 1.4 hours. Undifferenced $L1$ and $L2$ measurements are assumed to be uncorrelated and were assigned an uncertainty of 10 mm.

A total of 184 parameters were estimated in the ionosphere-free bias-free adjustment, including the three-dimensional site coordinates and an atmospheric zenith delay parameter for the 15 sites, six state vectors and three radiation pressure parameters for each of the 6 satellites, and 70 independent, doubly differenced, ionosphere-free, linearly-combined integer cycle ambiguities. The SV5 coordinates of Algonquin, Owens Valley, Platteville, and Westford were given *a priori* constraints using the values in Table 3.4 to establish a stable reference frame. The weighted sum of the squared residuals ('chi-square') is 369.39, and thus the chi-square divided by the degrees of freedom ($6211 - 184 = 6027$), also known as the variance of unit weight or sampling variance, is 0.061. We define the normalized rms (root mean square) of the residuals to be the square root of the chi-square per degrees of freedom, which in this case, is 0.248.

If the assumptions that we made regarding the model and the errors associated with the observations were correct, and no grossly inconsistent measurements ('outliers') were included, both the sampling variance and the normalized rms would be equal to 1.0. A normalized rms of 0.248 is significantly smaller than unity and suggests that the assumed 10-mm formal uncertainty assigned to the undifferenced $L1$ and $L2$ observations is too large. We can roughly explain the observed normalized rms, which is typically 0.25–0.32

(see Table 3.3), by recalling that the ionosphere-free linear LC combination increases the variance of random measurements by a factor of 10.418 and by realizing that double differencing phase measurements results in a factor of 4 increase in variance from propagation of error [e.g., *Bock et al.*, 1986b]. These factors imply that a 10 mm uncertainty assigned to undifferenced $L1$ and $L2$ phase measurements is equivalent to 68.4 mm uncertainty for doubly differenced LC observations. The observed scatter the postfit measurement residuals is roughly 0.1 cycles or 20 mm, so that the normalized rms is $20/68.4 = 0.29$, as observed.

The small normalized rms suggests that 3 mm would be a more appropriate measurement precision for undifferenced $L1$ and $L2$ phase measurements. However, the postfit residuals are not random, but exhibit systematic trends with periods from about 10 to 20 minutes. We believe that these fluctuations are predominantly caused by short-term atmospheric effects and by signal multipath interference. To obtain appropriate statistics for our adjustments, our data covariance would need to include the correlations between measurements caused by these processes. Since we do not adequately understand the underlying causes of these fluctuations, modeling the correlations is difficult. We can account approximately for the correlations by recognizing that only every tenth observation or so is independent (using 30-s interval observations). Thus the formal uncertainties of the baseline estimates need to be multiplied by a factor of $\sqrt{10}$ to be consistent with Gaussianly distributed random errors. Thus, the underestimate of uncertainty due to ignoring correlations between measurements is offset by overestimating the random error on the individual measurements. Due to these considerations, the formal uncertainties estimated by the GAMIT adjustment are not scaled by the *a posteriori* scatter of the residual as given by the normalized rms. We will also assess the precision of GPS by comparing the scatter of the single-session adjustment estimates of baseline vectors.

The unscaled formal uncertainties of the relative site coordinates estimated by the biases-free adjustment for this session are all on the order of 5 mm in the local north component, 20 mm in the local east component and 35 mm in the local vertical component for sites in California (hereafter, we shall refer to these components as simply north, east, and up). They also partly reflect the geometry of the measurements. The predominantly north-south satellite sky tracks above central California, as indicated in Figure 3.3, provide the strongest constraints in the north-south components of baselines in California. The uncertainties in the vertical components are a function of both the geometry (since satellite signals cannot be received through the Earth and in general, are not accepted below 15° elevation above the horizon due to multipathing effects), and a function of the strong trade-off between atmospheric errors and vertical [e.g., *Herring*, 1986]. The estimated atmospheric zenith delays in propagation are equivalent to approximately $60\text{--}90 \pm 25$ mm in path length for all sites. The uncertainty of the orbital parameters is equivalent to approximately 1–2 parts in 10^7 for all satellites, except PRN 12, which has uncertainties on the order of 5 parts in 10^7 .

Using the ionosphere-free bias-free estimates of the station coordinates, satellite state vectors, and atmospheric parameters, the second adjustment sequentially attempts to resolve the wide-lane integer ambiguities for the $L1$ and $L2$ doubly-differenced observations corresponding to regional-scale baselines (lengths less than 500 km). Ionospheric effects were constrained to be 0.5 ppm of the baseline length, which has been found to be a reasonable value for nighttime observations and low sunspot activity. For the shortest baseline in the network observing during this session, Piedras Blancas BLAN to Black Hill BLHL (53 km), the estimates of the five independent wide-lane ambiguities formed by differencing between satellites are all within 0.04 ± 0.10 of an integer, with the exception of the double difference including PRN 12, which had an uncertainty of 0.14. Similar estimates were obtained for all the other regional baselines of the independent set

chosen by the algorithm described in *Dong and Bock* [1989]. All the wide-lane ambiguities corresponding to regional baselines were resolved to their integer values.

The ionosphere-free bias-fixed adjustment was performed by assigning the wide-lane ambiguities to their resolved integer values and attempting to sequentially resolve the narrow-lane ambiguities and estimate all the other parameters using the ionosphere-free observations. All the narrow-lane ambiguities were also resolved, enabling a final adjustment to be made on 134 parameters comprising the site coordinates, zenith delay parameters, satellite state vectors, and 20 ionosphere-free integer ambiguities corresponding to continental-scale baseline lengths. The final chi-square was 399.59 with 6077 degrees of freedom, implying a normalized rms of 0.256. The small increase of the normalized rms indicates that the integer values assigned to the doubly-differenced ambiguities are consistent with their real-valued estimates. The unscaled formal uncertainties on the estimated site coordinates for this session are on the order of 7 mm in the local north component, 13 mm in the local east component and 40 mm in the local vertical component for sites in California. The resolution of integer ambiguities, which effectively changes the phase measurement into a range measurement, improves the uncertainties in the east component of site position by a factor of 2. The north and vertical coordinate component uncertainties are only marginally improved. The uncertainties on the orbital position and velocity parameters improve by a factor of 1.5–2.

The December 31, 1986 (day 365) adjustment represents a typical result given a well-designed experiment and high-quality observations. Four sites are available to define a fiducial network that has a continental-scale distribution, although it should be noted that the four sites (Westford, Algonquin, Platteville, and Owens Valley) all lie approximately along a great circle. The regional network has a good variety of baseline lengths ranging from 53 to 460 km, which improves the ability of the sequential ambiguity resolution algorithm to bootstrap to longer lengths. The observations were obtained at nighttime and at low solar sunspot activity.

The statistics for the adjustments of the other sessions considered in this work are given in Table 3.3. In general, the normalized rms for each adjustment varies between 0.20 and 0.33. Exceptions include day 35, 1990 with a normalized rms of 0.066 for the ionosphere-free bias-free adjustment and 0.097 for the ionosphere-free bias-fixed adjustment, and day 38, 1991 with a normalized rms of 1.03. Table 3.3 also includes the value of the distance-dependent ionospheric constraint. These values were determined empirically after examining the measurements, and through tests of the ambiguity resolution algorithm. In general, the ionosphere was most tightly constrained at 0.5 ppm during January, 1987 when the observations were at nighttime. Observations in March typically were begun in the early evening when the ionosphere changes from daytime to nighttime levels of activity, so a constraint of 1.0 ppm was applied. A 5.0 ppm constraint was applied to accommodate increased solar sunspot activity in 1988 and 1989. Looser constraints were also applied on a few occasions when the ionosphere appeared to be unusually active (e.g., day 364, 1987 and day 265, 1987).

The ionospheric constraint is applied during the second stage of the adjustment when the *L1* and *L2* observations are not linearly-combined in order to determine the wide-lane ambiguities. Table 3.3 indicates the number of ionosphere-free ambiguities corresponding to regional baselines that were not resolved in the ionosphere-free bias-fixed adjustment. The experiments in December 1986 and January 1987, and the experiments in March 1988 and March 1989 all successfully resolved 95–100% of the regional ambiguities. In general, the experiments with fewer regional sites, and hence poorer distribution of baseline lengths, were unable to resolve some regional ambiguities. For example, the ambiguities corresponding to the long baselines to Mojave were frequently unresolved using observations from the experiments in the vicinity of Vandenberg (TREX 9, VF 1, VF 2). The May 1987 observations were collected in the afternoon and evening, and the September 1987 observations from sunrise to midafternoon, the time of peak ionospheric gradients. Also, the integer ambiguities

associated with sites or satellites with short observation spans are more difficult to resolve (e.g., Lospe on days 1 and 2, 1987).

Another source of heterogeneity between the session adjustments is the availability of fiducial sites to establish a reference frame with respect to which the remaining station coordinates and satellite orbital parameters are estimated. Table 3.4 shows the observations obtained from the continental sites plus the sites in California that are collocated with VLBI reference points (Fort Ord FTOR, JPL1, Mojave MOJA, Owens Valley OVRO, Palos Verdes PVER, Santa Paula SNPA, and Vandenberg VNDN). It also shows the typical constraints, some of which are based on the formal uncertainties obtained from the GLB659 adjustment of VLBI observations or a nominal 10 mm in all components, applied in the north, east, and up direction to *a priori* SV5 coordinates. During each session, observations were obtained from at least one regional fiducial station (e.g., Owens Valley, Mojave, or Vandenberg) and two North American sites (including Algonquin, Austin, Platteville, Richmond, Westford, and Yellowknife).

In general, the coordinates of at three sites were held constrained at their SV5 values during the adjustments. The choice of fiducial stations was motivated primarily by data availability. To provide the most stable reference system for an adjustment, we chose the fiducial stations having as few data omissions as possible for each session during an experiment. However, it is not possible to use the same set of fiducial sites for all experiments. Only Vandenberg was observed during every experiment. Other sites, such as Algonquin, Kokee Park, Westford, Mojave, and Richmond have changed receiver types and in the process moved the antennas. We have made use of local survey ties to collocate these different monuments at the same site for the Kalman filtering analysis performed by GLOBK to assess the long-term repeatability and estimate velocities. We will explore the sensitivity of inhomogeneous fiducial network effects on the long-term repeatability and accuracy of the baseline estimates in a later section. We shall now examine the day-to-day repeatability within each experiment to assess the short-term

repeatability of GPS measurements in California under the various conditions described above.

SHORT-TERM REPEATABILITY

We determined the short-term repeatability of the single-session baseline estimates from nine experiments with a wide range of baseline lengths. For each experiment, weighted mean estimates of the single-session baselines were obtained from a GLOBK forward solution. For each GLOBK adjustment, we assumed: (1) the position and velocity of Vandenberg were constrained at the 0.1 mm and 0.1 mm/yr level, respectively; (2) the positions and velocities of two continental sites (e.g., Algonquin, Platteville, and Westford) were constrained at the 10 mm and 1 mm/yr level, respectively; and (3) satellite orbits were constrained in position and velocity at the 500 m and 50 m level, respectively, and were allowed to vary between sessions at the 10 m/day level, and two radiation pressure parameters (direct radiation and y-bias) were similarly constrained. Note that the velocity constraints are irrelevant for the short times spanned by the individual experiments, but we used these same constraints to account for relative site motions when we adjusted all experiments simultaneously. Unless otherwise noted, the *a priori* positions of sites that are collocated with VLBI are derived from the SV5 terrestrial reference frame.

The statistics for the GLOBK adjustments of the nine experiments with sufficient observations are shown in Table 3.5. The number of observations refers to the total number of doubly differenced ionosphere-free observations used in the single-session adjustments. The degrees of freedom f is approximately represented by the difference between the number of input parameters (station coordinates and orbit parameters) and the number of estimated 'global' parameters (again, station coordinates and orbit

parameters). The χ^2/f estimates for the bias-free and bias-fixed adjustments are given. These represent the weighted norm of the residuals divided by the degrees of freedom and should be near unity. However, this is not the case for the Kalman filter estimates due to the incorporation of *a priori* constraints and stochastic processes. We shall use χ^2/f only to provide a relative sense of the consistency in baseline and orbital parameters between sessions. TREX 1 has the worst consistency followed by the experiments beginning with TREX 14 in 1989.

Individual baseline vector lengths, as well as components in north, east, and up are compared with the weighted mean estimate to assess the precision the estimates. We define repeatability of a baseline component x to be the weighted rms scatter S of n estimates with formal uncertainties σ with respect to the weighted mean estimate $\langle x \rangle$:

$$S_x = \left(\frac{\frac{n}{n-1} \sum_{i=1}^n \frac{(x_i - \langle x \rangle)^2}{\sigma_i^2}}{\sum_{i=1}^n \frac{1}{\sigma_i^2}} \right)^{1/2} \quad (3.3)$$

This statistic assumes that the true baseline components do not vary over the course of the experiment. The day-to-day rms scatter of baseline estimates is a useful measure of the precision of the measurements because it is sensitive to some of the systematic errors, such as orbit and atmospheric effects, that are not reflected in the formal uncertainties of the estimates, which are derived by propagation of data noise and, perhaps, scaling by postfit residuals. However, short-term repeatabilities are not sensitive to systematic errors that are constant over a four day experiment; for example, signal multipath interference can be highly correlated from day to day since the same satellite-receiver geometry typically is repeated at 23 hours and 56 minutes intervals.

The uncertainty σ of geodetic measurements of baseline components is frequently represented by the model

$$\sigma^2 = a^2 + b^2 L^2 \quad (3.4)$$

where a and b represent constant and length-dependent sources of error, respectively, and L is baseline length [Savage and Prescott, 1973]. This representation of measurement precision provides a convenient method for quantifying length-dependent effects, such as orbital errors which are roughly proportional to baseline length. Dixon [1991] elaborates on this model by including a third term to account independently for tropospheric errors

$$\sigma^2 = a^2 + b^2 L^2 + c^2 (1 - e^{-L/\lambda}) \quad (3.5)$$

where c is the asymptotic tropospheric error, and λ is the tropospheric correlation length. This model assumes that tropospheric errors approach a constant value as atmospheric variations become uncorrelated at large distances [e.g., Truehaft and Lanyi, 1987]. Dixon [1991] uses a correlation length of $\lambda = 10$ km for observations in California; for distances greater than the correlation length, tropospheric errors are essentially constant. Since the regional baseline lengths measured in the TREX experiments vary between 30 and 500 km, we assumed that the nearly constant tropospheric error is highly correlated with the constant random measurement error and cannot be independently estimated. We therefore used the two-parameter model (Equation 3.4) to characterize GPS measurement precision.

We performed a GLOBK back solution to determine the short-term repeatabilities of the vector baseline components (north, east, and up) with respect to their weighed means. We estimated the constant and length-dependent error source coefficients from a linearized least-square fit to the repeatabilities of all regional baselines less than 500 km

in length that were observed in three or more single sessions during each of the nine experiments. The estimates for both the bias-free and bias-fixed baselines are given in Table 3.6. The two-parameter measurement precision model assumes that errors increase with baseline length. In a few cases where the linearized least-square estimate became numerically unstable due to repeatabilities that remained roughly constant or slightly decreased with length, we estimated only a constant error source. The baseline component repeatabilities for the nine experiments with their estimated models of precision are shown versus baseline length in Figures 3.4a–j. For one experiment, TREX 2, both the bias-free and bias-fixed repeatabilities are provided for comparison (Figures 3.4a–b); the bias-fixed repeatabilities for the other eight experiments are also provided (Figures 3.4c–j).

The bias-free and bias-fixed repeatabilities for TREX 2 are shown in Figures 3.4a and 3.4b, respectively. Both the bias-free and bias-fixed north baseline components have repeatabilities that are less than 12 mm and show little dependence with baseline length; the estimated constant and length-dependent coefficients are: $a = 4\text{--}5\text{ mm}$, $b = 1 \times 10^{-8}$. The vertical components have higher rms scatters and higher dependence with baseline length ($a = 15\text{--}16\text{ mm}$, $b = 7 \times 10^{-8}$). Neither the north or vertical components show any improvement in repeatability despite the fact that nearly all the regional ambiguities were resolved (see Table 3.3). The major difference between the TREX 2 bias-free and bias-fixed results is the improvement in repeatability in the east component. The bias-free east repeatabilities are roughly twice the magnitude of the north repeatabilities and have slightly more dependence on baseline length ($a = 11\text{ mm}$, $b = 3 \times 10^{-8}$); however, after resolving the ambiguities, the east repeatabilities improve and become comparable with the north repeatabilities ($a = 5\text{ mm}$, $b = 1 \times 10^{-8}$). This improvement is consistent with the reduction in the formal uncertainties of the east component by a factor of two in the individual adjustments. Successful resolution of ambiguities for this experiment results

in short-term repeatabilities on the order of 5 mm in both the horizontal components and 15–25 mm in the vertical.

In general, the results from the other major experiments (TREX 3, 11, 17, and 20), except for TREX 14, are similar to those obtained for TREX 2 (see Figures 3.4d, 3.4g, 3.4i–j). The north-component repeatabilities for all baselines in these experiments are less than 10 mm and show little dependence with length; the best-fitting constant and length-dependent error source coefficients are: $a = 2\text{--}5$ mm, $b = 0\text{--}1.3 \times 10^{-8}$. The east-component repeatabilities improve when nearly all the regional ambiguities are successfully resolved (TREX 2, 3, 11), although they usually remain greater than the north-component repeatabilities, with constant and length-dependent error coefficients: $a = 3\text{--}6$ mm, $b = 0.6\text{--}1.8 \times 10^{-8}$. For those experiments less successful in resolving ambiguities (TREX 17 and 20), the length-dependent errors improve less ($b = 3.0\text{--}3.6 \times 10^{-8}$). The vertical-component repeatabilities generally are not affected by the resolution of ambiguities, and have a mean value of approximately 22 mm ($a = 12\text{--}24$ mm, $b = 0\text{--}7 \times 10^{-8}$).

The horizontal repeatabilities for TREX 14 show roughly 2–3 parts in 10^8 variation with baseline length (Figure 3.4h), which are higher than those observed in the other major experiments with successfully resolved ambiguities.. We did not include the observations from Richmond RICH and Westford WSFM due to a problem interpreting the observations supplied by early versions of the Mini-Mac receiver software. As a result, the fiducial network for this experiment consists of sites at Yellowknife, whose observations are affected by severe high-latitude ionospheric variations, Kokee Park in Hawaii, several sites in Europe with little mutual visibility with California sites, and several sites in California. We constrained the location of three sites in California (JPL1, Mojave MOJF, and Vandenberg VNDN) to perform the single-session adjustments. Such a regional fiducial network, with baseline lengths on the order of 350 km, theoretically should be sufficient to define a reference frames suitable for estimating satellite orbits.

However, position errors within such a network will have proportionally greater effect on relative precision compared to 3500-km scale continental networks. The lack of a continental-scale fiducial network is probably responsible for the higher length-dependence of baseline repeatabilities in TREX 14.

Experiments with fewer regional sites or similarly weak fiducial networks (TREX 1, 7, and 8) also have higher repeatabilities. While the bias-fixed repeatability in east for TREX 1 is comparable with the repeatability in north (all regional repeatabilities were resolved), the consistency of the baseline estimates for this experiment is worse than any other experiment (Figure 3.4c). In particular, the repeatabilities of all the components have the highest dependence on baseline length ($b = 6 \times 10^{-8}$ for the horizontal components). We tightly constrained the locations of three sites in California (Fort Ord FTOR, Owens Valley OVRO, and Vandenberg VNDN) to perform the single-session adjustments for TREX 1 (Table 3.4). Again, the lack of more widely separated (continental-scale) fiducial sites used in the other experiments is probably responsible for the high length-dependence of the repeatabilities. The baseline estimates for TREX 7, which included only a few sites in California, did not show any improvement in the east-component repeatability as a result of the poor resolution of ambiguities (Figure 3.4e). The horizontal-component repeatabilities for TREX 7 have the highest dependence on baseline length ($b = 3\text{--}6 \times 10^{-8}$ for the horizontal components) of all experiments with the exception of TREX 1. The north and east component repeatabilities for TREX 8 (Figure 3.4f) are similar and have a relatively high dependence on baseline length ($a = 3\text{--}5$ mm, $b = 3 \times 10^{-8}$). TREX 8 has a continental-scale fiducial network and a reasonable range of baseline lengths between eight sites in California. However, it is possible that long-baseline ambiguities many have been poorly resolved due to the effect of higher ionospheric variations experienced during daytime observations.

We conclude that the short-term precision of GPS relative position measurements in California, as determined by the day-to-day repeatability of baseline component

estimates, is approximately 3–5 mm with 1×10^{-8} dependence on baseline length (in km) for horizontal components. This assessment is valid for baselines with lengths between 30–500 km that are estimated in conjunction with a well-defined fiducial network both in relative precision and in geographic distribution, and under conditions that enable all regional ambiguities to be resolved. Under less ideal situations (e.g., poor fiducial network, poor range of regional baseline lengths, high ionospheric effects), the dependence of repeatabilities on baseline length can increase to $3\text{--}6 \times 10^{-8}$, especially in the east components, when ambiguities are less successfully resolved. The repeatabilities of vertical components generally vary between 15–25 mm. The highest dependence on length of the vertical repeatabilities ($5\text{--}9 \times 10^{-8}$) mostly occur during experiments that lack observations from Richmond that strengthen the north-south geometry of the fiducial network (e.g., TREX 2, 7, 8, 14), in agreement with the observations of *Larson* [1990a] .

LONG-TERM REPEATABILITY

The California GPS campaigns were conducted over a 4.2-yr interval under various ionospheric and meteorological conditions. During that time, the number of satellites observed during the experiments has increased from six to nine, including several Block II satellites that have operated under SA conditions during some of the experiments (e.g., March 1990). The continental fiducial network has continually evolved from the original Mojave, Platteville, and Westford sites, to include several more continuously operating sites in North America, Europe, and Hawaii. Several of these sites have replaced the original receivers and moved antennas to new monuments, and several different types of receivers have collected data. All these changes have the potential to affect the long-term precision of the GPS measurements and the estimates of crustal deformation that are conducted over several-year intervals.

To examine the long-term precision of GPS during the California campaigns, we combine 70 sessions from all the experiments and examine the weighted rms scatter of the baseline vector estimates for those sites with observations spanning greater than 1.5 years. In the Kalman forward solution we made the following assumptions: (1) the position and velocity of Vandenberg, Richmond, and Westford are constrained at the 10 mm and 1 mm/yr level in position and velocity, respectively; (2) the positions of all other sites are constrained at the 10 m level and their velocities are fixed at *a priori* values; (3) satellite orbits are constrained in position and velocity at the 500 m and 50 m level, respectively, and are allowed to vary between sessions at the 10 m/day level, and two radiation pressure parameters are similarly constrained, and (4) the positions of sixteen sites with observations spanning more than 1.5 years are allowed to vary between sessions at the 10 m/day level by incorporating a random-walk stochastic process. These constraints are essentially same as the single-experiment adjustment discussed in the previous section, although we now allow only 16 sites to adjust freely over time while the remaining sites, excluding the fiducials, are constrained to move with *a priori* velocities.

Several continental sites that are part of continuously operating orbit tracking networks collocated with VLBI reference points, such as CIGNET, have changed the receiver and antennas during the course of the California campaigns. These include Westford (WSFD and WSFM), Algonquin (ALGO and AROT), Kokee Park (KOKE and KOKT), Richmond (RICH and RICM), and Mojave (MOJA, MOJF, and MOJM). Local survey ties have been estimated between these monuments, either by terrestrial or GPS surveys (see Table 2.9). We have applied these ties to translate the more recent occupations back to the original reference points, thus reducing by six the number of sites whose positions and velocities are estimated.

Incorporation of local survey ties can, however, introduce additional discrepancies if the ties are not well known or in error. The CIGNET site at Mojave was occupied by a TI4100 receiver and antenna from November 1986 to July 1988, at which point the

TI4100 antenna was replaced by a FRPA-2 antenna over approximately the same monument. In February 1989, a Mini-Mac receiver and antenna were installed nearby, and the TI4100 receiver was discontinued the following July. Local survey ties to determine the location of the antennas with respect to the Mojave VLBI reference point 7222 were conducted by locating similar GPS receivers over a nearby reference monument (NCMN No. 1 1983). Observations from all three antennas are included in the California campaigns, and examination of the baseline vectors to Mojave indicate possible discrepancies at the 100-mm level in the vertical between the differential estimated positions and those predicted by the local survey ties. To minimize the impact of these discrepancies, we allowed all three position components of Mojave to vary according to a random-walk process at about the 50 mm/day level between sessions, while the velocity was constrained at the VLBI rate. Therefore, the relative motion of Mojave is given essentially by the VLBI estimate, rather than estimated from the observations and we will not discuss its relative motion further.

The statistics of the bias-free and bias-fixed GLOBK adjustments of all 70 sessions are shown in Table 3.5. The final χ^2/ν for the bias-free and bias-fixed adjustments (1.414 and 2.066, respectively) are greater than those obtained for the individual experiments with the exception of TREX 1. We did not attempt to evaluate whether these values are statistically greater than unity or whether they are statistically different from the single-experiment adjustments due to the difficulty of determining the correct degrees of freedom when *a priori* stochastic constraints are applied.

We determined the long-term repeatabilities of the baseline estimates by performing a Kalman filtering back solution in which the coordinates of the 16 sites with observations spanning more than 1.5 years were allowed to vary daily. The weighted rms scatters are estimated with respect to a linear trend representing uniform relative motion between the sites. The bias-free and bias-fixed weighted rms scatter repeatabilities of the baseline components are shown in Figures 3.5a and 3.5b, respectively, and the constant and

length-dependent error sources (according to Equation 3.4) that best fit the repeatabilities are given in Table 3.6. The long-term repeatability in the north component is the same for both bias-free and bias-fixed adjustments, with the best-fit constant and length-dependent error source coefficients given by: $a = 4$ mm, $b = 1.3 \times 10^{-8}$. The long-term repeatability in the east improves after ambiguities are resolved, although they remain higher than the north component repeatabilities in both the constant and the length-dependent error sources ($a = 5.6$ mm, $b = 2.6 \times 10^{-8}$). The repeatability in the vertical is unimproved by ambiguity resolution and is similar to the short-term repeatability ($a = 22$ mm, $b = 4.5 \times 10^{-8}$).

The long-term repeatabilities of the horizontal bias-fixed baseline components are similar to the short-term repeatability results derived from the major experiments. The repeatability of the north component has a slightly higher constant contribution ($a = 4.0$ mm) than the individual experiments, with the exception of TREX 2 ($a = 5.0$ mm). The long-term constant contribution to the east component repeatabilities ($a = 5.6$ mm) is also slightly higher than the individual experiments. These results suggest that GPS measurements have good stability over time, and that error sources, such as fiducial network inhomogeneities, satellite orbits, atmospheric effects, and signal multipath interferences which may be highly correlated over several-day intervals, do not contribute significantly to the long-term error budget.

Davis et al. [1989] suggest a method for quantifying the short-term and long-term contributions to the precision of GPS measurements. They assume that the total error e_k for a measurement in the k th observing session is the sum of the short-term error $e_{S,k}$ for the k th session and a long-term error e_L :

$$e_k = e_{S,k} + e_L \quad (3.6)$$

If these errors are uncorrelated, the variance of the total error is the sum of the short-term and long-term variances:

$$\sigma_k^2 = \sigma_{S,k}^2 + \sigma_L^2 \quad (3.7)$$

The short-term variance may be estimated using the same method that we used in the previous section; that is, based on the rms scatter of estimated components with respect to their weighted means over a 3–5 session experiment. The long-term contribution to the total variance may be estimated from the rms scatter of the weighted means of each experiment about the long-term linear trend.

We adopted an approach similar to *Davis et al.* [1989]. The estimates of long-term repeatability given in Table 3.6 are based on the rms scatter of every individual session about the long-term linear trend. We assumed that these repeatabilities represent the standard deviation of the total error sources, and estimate the long-term contributions to the total error by subtracting short-term contributions from the total. To obtain an average estimate for the short-term contribution of error, we combined all the short-term rms scatters of baseline components from the means of their respective experiments and determined the best-fit two-parameter precision model. The constant and length-dependent error source coefficients for the average short-term bias-fixed repeatabilities are: $a = 2.9$ mm, $b = 1.8 \times 10^{-8}$ in north; $a = 4.6$ mm, $b = 3.1 \times 10^{-8}$ in east and; $a = 19.7$ mm, $b = 5.3 \times 10^{-8}$ in up. For all components, the average short-term length-dependence of repeatability is slightly greater than the total error dependence with length, which suggests that long-term effects do not contribute significantly to length-dependent sources of error. On the other hand, the average short-term constant sources of error do not completely account for the total constant sources of errors. Using Equation 3.7, the long-term contribution to constant error sources are: 2.7 mm in north, 3.2 mm in east, and 9.3 mm in up.

In summary, the long-term precision of GPS relative position measurements in California is approximately equal to the average short-term repeatability plus a small constant contribution in all components due to long-term error sources. The long-term repeatability in the horizontal components of the biased-fixed baselines has a constant error source of 4–5.6 mm and $1.3\text{--}2.6\times 10^{-8}$ dependence on baseline length (in km), with the east components having slightly higher repeatabilities than the north. The long-term error sources that might be responsible for the several millimeter level increases in constant effects include tropospheric effects (approximately constant for baselines with lengths greater than 30 km), signal multipath interference effects that might vary with changes in the satellite constellation, and monument stability [e.g., *Wyatt*, 1982, 1989; *Langbein*, 1989; *Langbein et al.*, 1990]. Long-term length-dependent error sources do not significantly affect the long-term precision determined over 4.2 years of GPS measurements in California.

RELATIVE SITE MOTION

In the preceding section we assessed the long-term precision of GPS baseline components by determining the weighted rms scatters about linear trends that represent estimates of constant relative site motions. We will now discuss the estimated site motions and their uncertainties. An alternative adjustment of the ionosphere-free biased session results will be used to estimate and assess the relative motions. The main reason for performing another adjustment stems from the recent discovery of another potential discrepancy in the local survey ties at Westford.

The GPS receivers at Westford, which are part of the CIGNET orbit tracking network, provide some of the principal fiducial network observations used in this study due to the availability of measurements from there in all but two experiments (TREX 1

and 14). Like Mojave, Westford has been occupied by a TI4100 receiver with a TI4100 antenna (WSFD) beginning in October 1986, which was switched to a FRPA-2 antenna (WSDF) mounted on the same pole in August 1988. In February 1989, a Mini-Mac receiver and antenna (WSFM), mounted close to the TI4100 antenna pole, began to operate and measurements using the TI4100 were soon discontinued. The California campaigns use data from the TI4100 antenna and the Mini-Mac antenna. Some local survey tie information is provided in the May 1989 *International Coordination of Space Techniques for Geodesy and Geodynamics (CSTG) GPS Bulletin* distributed by the National Geodetic Survey. The vector survey ties are referenced to a ground monument (MICRO) whose location is known with respect to the Westford VLBI reference point 7209. The TI4100 antenna location with respect to MICRO was determined by a GPS survey in 1987. After the TI4100 antenna was struck by lightning, a FRPA-2 antenna was mounted on the same pole and the vertical height offset between the two GPS antenna phase centers was estimated. When the Mini-Mac antenna was installed, the local survey tie between the Mini-Mac and FRPA-2 antennas was estimated from simultaneous observations acquired by both receivers. Thus the location of the Mini-Mac antenna with respect to MICRO provided in the *CSTG GPS Bulletin* is the result of three separate baseline estimates. However, a direct estimate of the baseline via a Mini-Mac-to-Mini-Mac survey has recently become available, and a 10-cm level discrepancy with the result from the old surveys was discovered. If the direct survey is given preference, the true location of the Westford antenna should move by 92 mm to the north, 8 mm to the west, and 25 mm down with respect to the original SV5 location. The direct survey tie is given in Table 2.9 under the site name WSFN.

We used the direct tie for the adjustment to estimate relative site motions. In the process of converting the Westford ties, the data from one session in TREX 20 (day 40, 1991) were corrupted and were dropped from the solution. While both the ionosphere-free bias-free and bias-fixed parameter estimates were adjusted, we will discuss only the

bias-fixed results, unless otherwise noted. The constraints on Vandenberg were loosened to the 10 mm level in position and 1 mm/yr level in velocity. The velocity of Mojave was allowed to adjust at the 1 mm/yr level, in order to retain an estimated velocity—albeit fairly tightly constrained—for that site in the solution; we will not present any analysis of its velocity. The statistics of the forward Kalman filter adjustments are given in Table 3.5; the overall χ^2/f is 2.1 for the bias-fixed parameters. The additional three global parameters correspond to the tightly constrained Mojave velocity (the 336 global parameters include the coordinates for 53 sites, the velocities for 16 sites, six satellite state vectors and three radiation parameters for 14 satellites, and 3 Earth orientation parameters).

Since one of the principal goals of the California campaigns is to monitor crustal deformation west of the San Andreas fault, we have chosen to present all relative velocities, unless otherwise noted, with respect to Vandenberg VNDN whose observations are available in all but three of the sessions. The VLBI estimates for the relative velocity of Vandenberg with respect to the ‘stable’ North America plate indicate that it is consistent at the 1–2 mm level with NUVEL-1 predictions for Vandenberg assumed to be on the Pacific plate [e.g., *Sauber*, 1988; *Argus and Gordon*, 1990]. Thus, the reference frame we have chosen is essentially a Pacific-plate fixed frame.

The estimated site velocities with respect to Vandenberg are given in Table 3.7 for each of the other 15 sites. Westford, Richmond, and Mojave had tight *a priori* constraints placed on their relative velocities (1 mm/yr level). The velocities for all of these sites adjusted with respect to the *a priori* VLBI estimates at approximately the 3 sigma level, with some similarity in direction (3 mm/yr to the east, and 2–5 mm/yr south). The source of this systematic relative motion is not well understood. It is possible that discrepancies, such as other local survey offsets at one or more of the sites may be responsible for the observed systematic motions. Another possibility is that Vandenberg is not well described by its *a priori* velocity model, and that its adjustment is being mapped into all

the other sites when motions relative to Vandenberg are examined. The only unconstrained continental site was Algonquin. While its horizontal adjustments are relatively small, it has a large vertical adjustment of -27 mm/yr; one possible explanation for this large adjustment is again due to ties since the height of the AROT permanent tracker phase center is not well documented.

The remaining sites with sufficient observations to estimate their relative motion are in California. Figure 3.6a shows the horizontal components of the estimated velocities with their 95% confidence error ellipse scaled by the square root of the χ^2/f for the bias-free adjustments. As expected from considerations of geometry, the bias-free relative motions have uncertainties in the east component that are roughly twice the uncertainties in the north component. The bias-fixed estimates are shown in Figure 3.6b. The bias-fixed estimates are consistent with the 95% confidence region of the bias-free estimates in all cases, except for Owens Valley and marginally Fibre, both of which have bias-fixed relative velocity estimates that are more westerly than those predicted by the bias-free estimates. The most significant relative motions with respect to Vandenberg are those associated with motion across the San Andreas fault to Owens Valley and Fibre (recall that Mojave is constrained at the 1 mm/yr level to move at its *a priori* VLBI velocity). Several sites to the west of the San Andreas fault also appear to show relative motions with respect to Vandenberg that are significantly greater than zero at the 95% confidence level. In general, the most significant relative motions are associated with sites that are closest to the San Andreas fault (Madre MADC, La Cumbre LACU, and Palos Verdes PVER), and become less significant as the distance to the San Andreas fault increases (e.g., Center CENT, Lospe LOSP, Black Hill BLHL, and Piedras Blancas BLAN).

We now examine some of these baselines in detail to provide more insight into the robustness of the estimates. Figures 3.7a–k show the length and north, east, and up components for the bias-fixed baseline estimates between each site and Vandenberg. In each of these figures, length is plotted in the upper left-hand corner, the east component is

plotted in the upper right-hand corner, the north component is plotted in the lower left-hand corner, and up is plotted in the lower right-hand corner. The individual session bias-fixed estimates with respect to a nominal mean are plotted as a function of time with their unscaled formal uncertainties. Two linear trends are fit to the observations. The solid line is the formal Kalman filter estimate, whose slope and slope uncertainty are printed on the top line of each plot. The estimated rates are for the motion of Vandenberg with respect to each site, except in four cases where the rates are for the site with respect to Vandenberg: Black Hill BLHL, Piedras Blancas BLAN, Grassy GRAS, and Gaviota GAVI. These rates are also given consistently with respect to Vandenberg in Table 3.7. The dashed line is the weighted fit to the observations as they appear on the plot, without taking into account correlations between the components of the baseline. The weighted rms scatter and the χ^2/f of the residuals about the dashed line are printed on the second line of each plot.

The components of the 363-km Vandenberg with respect to Owens Valley baseline estimates are shown in Figure 3.7a. The weighted rms scatters about the uncorrelated linear trend are the highest noted on any of the baselines considered here, about a factor of two larger than the average values determined in the previous section. One estimate in the February 1991 (TREX 20) experiment appear to be inconsistent with the other two estimates in east and length and is a greater than two-sigma departure from the linear trend. If the two consistent TREX 20 estimates are used, the March 1989 (TREX 14) estimates would become more inconsistent with the trend. The May 1987 experiment shows a large scatter about the line. Systematic differences in east are also noted for the March 1988 and March 1989 experiments, which show internal precision but taken alone would indicate that Owens Valley was moving moving *west* with respect to Vandenberg. The Vandenberg to Owens Valley baseline is one of the longest in the network and would be more affected than shorter lines by fiducial site problems.

Figure 3.7b shows the motion of Vandenberg with respect to Fibre which is the only other site east of the San Andreas fault. The length of this baseline is 145 kilometers, less than half the length of the Vandenberg to Owens Valley baseline. The scatter about the lines is about 5 mm in the north and 7 mm in the east, and most of the observations appear to be consistent with the trend in the horizontal components, except for one day again in February 1991. The estimated rate of change of this baseline appears to be the most robust available from these experiments.

The components of the Vandenberg to Palos Verdes baseline estimates are shown in Figure 3.7c. The scatter about the lines is about 6 mm in the horizontal components, although the March 1989 (Trex 14) experiment again appears to be inconsistent in the north component. Palos Verdes is one of the few sites whose vertical component of relative motion with respect to Vandenberg is formally significant (-7.8 ± 2.0 mm/yr); however, Figure 3.7c indicates that this estimate is strongly influenced by inconsistent and widely scattered estimates obtained during the December 1986 experiment.

The components of the Vandenberg to La Cumbre baseline estimates are shown in Figure 3.7d. The estimate of the east component of relative motion may be slightly decreased due to the February 1991 outlier, but otherwise the estimate appears to be robust with weighted rms scatters of 3 and 5 mm in the north and east.

The components of the Vandenberg to Madre baseline estimates are shown in Figure 3.7e. Madre was one of the sites observed in the September 1990 experiment (VF 2) devoted to estimating local deformation in the vicinity of Vandenberg. The two estimates from that experiment are not consistent; nor is one estimate from the March 1990 experiment. These inconsistencies do not, however, appear to seriously degrade estimates of the north and east rates, about which the measurements have weighted rms scatters of about 5 mm.

Figure 3.7f shows the motion of Vandenberg with respect to Center, the only offshore site in our analysis. While the weighted rms scatter about the estimated north rate is 3

mm, the east component is more problematic. Both the March 1989 and May 1990 experiments appear to be inconsistent with the first four experiments. Given these difficulties, the estimate of the east component of the Vandenberg to Center baseline is probably not very well determined from these data.

The components of the 91-km Black Hill with respect to Vandenberg baseline estimates are shown in Figure 3.7g. The weighted rms scatters for the horizontal components are 5–6 mm, but the experiments that were conducted after 1989 again appear to be inconsistent at the 10 mm level, with the March 1989 and March 1990 experiments most widely diverging in the north component. A similar situation holds for the Piedras Blancas with respect to Vandenberg baseline estimates shown in Figure 3.7h, except that Piedras Blancas was less frequently observed. The March 1989 (TREX 14) experiment, however, appears to be significantly different from the other experiments in the north component, which has the second highest weighted rms of 9 mm except for the Vandenberg to Owens Valley baseline.

Finally, the components of the baselines from Vandenberg to Lospe, Gaviota and Grassy are shown in Figure 3.7i–k, respectively. Lospe has been observed most frequently; however, its east component with respect to Vandenberg has a 7 mm weighted rms scatter due to the several experiments, especially the September 1990 (VF 2) experiment. Gaviota and Grassy have high weighted rms scatters in their horizontal components (e.g., 8 mm in north at Grassy due mainly to the September 1990 scatter). The vertical component of Grassy with respect to Vandenberg has the lowest rms scatter in the vertical for all sites (13 mm), and its estimated rate is significantly large (15 ± 4 mm/yr).

The formal uncertainties associated with the horizontal velocities of the regional sites with respect to Vandenberg are 0.2–0.8 mm/yr. However, these uncertainties, derived by propagation of the individual baseline errors, do not take into account the postfit scatter of the residuals and probably underestimate the true uncertainties of the estimates. One

estimate for the standard deviation of unit weight $\hat{\sigma}_0$ required to scale the uncertainties to make them consistent with the postfit scatter is 1.45, the square root of the χ^2/f for the GLOBK adjustment. We used this value, multiplied by 2.456, to scale the 95% confidence error ellipses depicted in Figures 3.6a–b. As noted earlier, the estimated χ^2/f is only an approximation, since the *a priori* constraints used in the Kalman filtering estimation technique affect the true degrees of freedom. Other estimates for the *a posteriori* scaling of uncertainties can be obtained from the χ^2/f derived from the rms scatter of each baseline component about their best-fit linear trends; these values are provided in Figures 3.7a–k. For example, the north and east velocities of Vandenberg with respect to Owens Valley (Figure 3.7a), have χ^2/f of 5.4 and 6.0 (implying $\hat{\sigma}_0$ equal to 2.3 and 2.4), respectively. In general, $\hat{\sigma}_0$ is 1.5–3 for the horizontal components and around 1 for the vertical component. Based on these considerations, we consider the relative motions estimated over a 4.2-yr interval to have approximate uncertainties of 1–1.5 mm/yr in the horizontal, and 2–3 mm/yr in the vertical.

DISCUSSION

Precision

Davis et al. [1989] provide the first systematic study of long-term repeatability of GPS using observations spanning two years in California. They measure several baselines less than 50 km in length and the 223-km Palos Verdes to Vandenberg baseline. They simultaneously adjust all the phase data collected during several sessions (“multisession” adjustment), but are not able to resolve ambiguities on the longer baselines due to the lack of a short and intermediate-length baselines in their networks. The long-term repeatabilities of these baselines are 2–12 mm in north, 2–19 mm in east, and 4–49 mm in vertical. Based on the bias-free repeatability of the Palos Verdes to Vandenberg baseline, they conclude that both long-term and short-term error sources are

significant effects. Since this limited study was conducted, several other investigators have analyzed GPS observations from more extensive experiments in California and their results are largely in agreement with our estimates of the short-term and long-term precision of GPS relative positioning [e.g., *Dong and Bock*, 1989; *Blewitt*, 1989; *Dixon et al.*, 1990; *Larson*, 1990a,b].

Dong and Bock [1989] use a similar, but slightly different, method from ours to study ambiguity resolution and derive baseline repeatabilities for one of the January 1987 experiments (TREX 3). They perform multisession adjustments to estimate more precise satellite orbits that are constrained to be continuous over four or five days [e.g., *Lichten and Bertiger*, 1989; *Lichten*, 1990c]. The bootstrapping approach advocated by *Dong and Bock* [1989] uses the more precise orbital information to help resolve the ambiguities on longer baselines which in turn improves the precision of the orbits. Using this method, they successfully resolve all integer-cycle phase ambiguities over the California baselines and over a majority of the continental-scale baselines to obtain subcentimeter horizontal precision with $0.5\text{--}0.9 \times 10^{-8}$ baseline length dependence and several-centimeter vertical precision with 8×10^{-8} baseline length dependence. Due to the computational intensity of such adjustments, we did not attempt to resolve integer ambiguities in GAMIT using multisession adjustments, nor did we attempt to resolve ambiguities on baselines to sites outside of California.

The Kalman filtering approach used by GLOBK also has the potential to perform multisession adjustments if the satellite orbital parameters are sufficiently constrained to force the orbit to be continuous over the course of the experiment (although additional ambiguities cannot be resolved using these techniques). In the GLOBK adjustments described earlier, however, the satellites were given very loose constraints (e.g., 500 meters on initial position and 50 mm/s on initial velocity), and had a random-walk stochastic process applied with an equivalent uncertainty of 10 m/day. We adopted these loose constraints in order to better account for the effects of non-gravitational forces

which are more pronounced during periods when the satellites pass through the Earth's shadow. Future changes in GLOBK will allow different stochastic constraints to be applied to satellites for different time periods, allowing us to make better use of multisession arc when non-gravitational perturbations on the satellites are less severe.

Blewitt [1989] and *Dixon et al.* [1990] analyze GPS observations collected over four days during June 1986 at seventeen sites located in southern California. This experiment constitutes the original occupation of the southern half of the crustal monitoring network established during the TREX campaigns, but is not included in our analysis which has concentrated on the northern half of the network. They determine short-term repeatabilities that are comparable to our results with the north component given by $a = 4$ mm, $b < 1 \times 10^{-9}$, the east component given by $a = 5$ mm, $b = 2.2 \times 10^{-8}$, and vertical repeatabilities of 11–47 mm with a mean of 29 mm.

The June 1986 experiment is included also in the studies by *Larson* [1990a,b] who analyzes 2.7 years of observations from the TREX campaigns in both central and southern California. *Larson* [1990a] uses the observations from the March 1988 experiment (TREX 11) to assess short-term precision and obtains results that are almost identical to ours in the north component ($a = 2\text{--}3$ mm, $b = 0$) and east components ($a = 3\text{--}4$ mm, $b = 1 \times 10^{-8}$), while the vertical repeatabilities have a slightly smaller mean of 17 mm. Long-term precision, based on observations over 2.2 to 2.7 years, is 5 mm in north with no dependence on baseline length, 4 mm in east with 1.5×10^{-8} length dependence, and 24 mm in vertical with 4×10^{-8} length dependence. *Larson* [1990b] uses a subset of these baselines observed over 1.2 years to obtain long-term repeatabilities with no length dependence in all components and mean scatters of 5.0, 4.6, and 20.4 mm in north, east, and up, respectively. In general, the long-term (~ 2.5 year) repeatability results of *Larson* [1990a,b] are similar to the 4.2-yr interval results presented in this work, but have slightly less dependence on baseline length.

Blewitt [1989], *Dixon et al.* [1990], and *Larson* [1990a,b] use the GIPSY software developed at the Jet Propulsion Laboratory to analyze the GPS observations [*Lichten and Border*, 1987; *Blewitt*, 1989; *Lichten*, 1990a]. GIPSY uses Kalman filtering techniques to sequentially estimate relative site coordinates and satellite orbital parameters while applying stochastic noise processes to undifferenced transmitter and receiver clocks, atmospheric delay effects, and unmodeled satellite accelerations. Undifferenced clock behavior is modeled by a white noise process which is equivalent to double-differencing approach used by GAMIT. On the other hand, the ability of GIPSY to accommodate time-varying atmospheric and orbital effects should enable it to provide more precise estimates than GAMIT of vertical baseline components, for example. We are in the process of implementing Kalman filtering techniques in GAMIT.

Estimating a value with great precision does not necessarily guarantee that it is determined accurately; that is, it may not agree with “truth”. Systematic effects, such as those invoked to account for the difference between short-term and long-term precision, can bias estimates and reduce their accuracy. We have not attempted a systematic investigation of GPS measurement accuracy, but several other investigators have done so by comparing baseline estimates obtained using GPS with other geodetic techniques, such as trilateration and VLBI. *Davis et al.* [1989] find agreement in estimates of slip on the San Andreas fault at Parkfield, California, at the 1–2 mm/yr level between creep meters sensitive to motions on a 10-m scale, alignment arrays sensitive to motions on a 100-m scale, and GPS estimates over 1–10 km baseline lengths. They also compare Geodolite trilateration measurements up to 50 km in distance and with an assumed precision of $a = 3$ mm, $b = 20 \times 10^{-8}$, to GPS determinations of length and found no significant differences within the uncertainty of the measurements.

At distance scales greater than 50 kilometers, several investigators have made comparisons with VLBI baseline estimates, although the two space-geodetic techniques are not completely independent. For example, highly precise VLBI relative positions

often are used to establish the fiducial network used by GPS, which implies that the orientation and scale of baselines estimated by these two techniques will be strongly correlated. Also, both techniques have similar sensitivity to atmospheric effects. *Dong and Bock* [1989] examine a five-day multisession estimate of the 245-km Owens Valley to Mojave baseline that differs from VLBI by 4 mm in the north component, and does not differ in the east and vertical components. *Blewitt* [1989] and *Dixon et al.* [1990], in their analyses of the June 1986 observations that include six sites collocated with VLBI, find mean repeatabilities with respect to the VLBI values of 5 mm in the north and east components and 23 mm in the vertical for regional baselines.

Studies that span several years also provide estimates of relative motion that can be compared to VLBI rates. *Davis et al.* [1989] examine six bias-free estimates of the length of the 223-km Palos Verdes to Vandenberg baseline over a 2-yr interval; the mean difference in length between VLBI and GPS is 1 mm, and their estimates of the rate-of-change of length do not differ significantly. *Larson* [1990a,b] compute baseline components and rates for several baselines in California over a 2.7-yr interval. She finds no significant difference between the GPS and VLBI-derived horizontal rates, but finds differences in the mean component estimates of 10–30 mm in the horizontal, and about 80 mm in the vertical. Such large offsets are most likely due to errors in the local vector ties between the VLBI and GPS reference points. Despite these problems, GPS and VLBI baseline estimates generally show good agreement and suggest that the precision and accuracy of GPS measurements are comparable.

Reference Frame Sensitivity

The relative site motions derived in this chapter were estimated using specific assumptions regarding the definition of a terrestrial reference frame in which all the non-fiducial site coordinates and satellite orbit parameters are estimated. In this reference frame, Vandenberg, Richmond, and Westford were constrained with uncertainties of 10

mm and 1 mm/yr in position and velocity, respectively, and the satellite orbits were allowed to freely adjust between sessions. The *a priori* coordinates used in those adjustments were derived from the SV5 terrestrial reference frame, and the *a priori* velocities were defined by a set of North America plate-fixed velocities derived from VLBI observations through 1990 (T. A. Herring, personal communication, 1991). In particular, it was noted that the velocities of Westford and Richmond all adjusted by approximately 3 mm/yr in a similar direction (Table 3.7). To examine these effects with respect to the estimated relative site motions, we therefore performed some sensitivity tests on the definition of the velocity component of the reference frame.

The first suite of three tests that we performed varied the *a priori* constraints on the fiducial site velocities. The first test set the *a priori* constraint at a value of 0.5 mm/yr, which is consistent with the best estimates of horizontal velocities currently derived by VLBI. The second test set the *a priori* constraint at 1 mm/yr, which was used to derive the velocities in the previous section, and the third test set the *a priori* constraint at 10 mm/yr to allow the sites to adjust their velocities more freely. The second suite of tests applied the same set of *a priori* constraints, but used *a priori* values for position and velocity derived from the SV6 terrestrial reference frame in Chapter 2 in order to examine the effect of changing the relative site coordinates and velocities of the fiducial network. In these adjustments, the velocities of Westford and Richmond with respect to Vandenberg varied by as much as 30 mm/yr, suggesting inconsistencies up to the 100 mm level in the GPS determined positions of these sites. However, 100 mm relative position variations for baselines on the order of 3000 km correspond to changes in the reference frame of about 3 parts in 10^8 , or only a few millimeters for the baselines in the regional network. In none of the six test adjustments did the relative velocities estimated for California sites change by more than 1 mm/yr.

Relative Site Motions

The relative site motions derived in this work generally agree with other geodetic and geologic estimates of tectonic motion and deformation in California. In this section, we compare our results to previous geodetic results derived with GPS measurements alone [e.g., *Larson*, 1990a] and in combination with triangulation and trilateration measurements [*Feigl et al.*, 1989; *Larsen*, 1990; *Webb*, 1991], and with VLBI [e.g., *Caprette et al.*, 1990]. In the next section, we will discuss some of the tectonic implications of the estimated rates and how well they compare with geologic information.

We have refined the results of a similar study using the GPS measurements from January 1987 to March 1989 (TREX 2, 7, 8, 11, and 14) by *Larson* [1990a]. Our refinements to this study include the use of two additional years of observations (TREX 17 and 20), and the application of network adjustment techniques to derive self-consistent relative motions. *Larson* [1990a] estimates the rate-of-change of each baseline component individually without taking into account the correlations between the components, which is equivalent to the method used to determine the dashed lines shown in Figure 3.7. Based on the comparisons offered in Figure 3.7, we do not believe the estimated rates are significantly affected by ignoring correlations (less than 2 mm/yr using 4.2 years of observations). However, network closure is not maintained by individually estimating baseline component rates. For example, *Larson* [1990a] estimates motions with respect to Vandenberg of La Cumbre (−6.6 mm/yr north and −1.9 mm/yr east) and Center (−2.5 mm/yr north and −0.8 mm/yr east). Closure predicts that the motion of La Cumbre with respect to Center is −4.1 mm/yr north and −1.1 mm/yr east; however, her individually estimated rate is −6.0 mm/yr north and −3.3 mm/yr east. The uncertainties on these motions are about 1.5–2 mm/yr.

The adjustment we performed with GLOBK strictly enforces network closure. For comparison, our estimate for the horizontal motion of La Cumbre with respect to Center is -5.2 ± 0.4 mm/yr north and 0.7 ± 0.4 mm/yr east (where the quoted errors are formal

unscaled uncertainties and probably should be multiplied by 1.5–2.0 to reflect more accurately the postfit scatter of the observations, as noted earlier). In this example, the north rates differ by less than 1 mm/yr and consistently indicate shortening across the Santa Barbara channel, whereas the east rates differ by 4 mm/yr. In general, the estimated rates between the two solutions often differ at the level of several millimeters per year, especially in the east component which is less well resolved and more sensitive to ambiguity resolution effects. For example, our estimate of the motion of Black Hill with respect to Piedras Blancas is -0.9 ± 0.4 mm/yr north, -1.1 ± 0.5 mm/yr east, whereas *Larson* [1990a] estimates -1.2 mm/yr north, -4.9 mm/yr east (uncertainties on the order of 1.5–2.5 mm/yr); however, neither of these estimates is significant at the 95% confidence level. An example in which the north rates also differ by several millimeters per year is the motion of Madre with respect to Vandenberg, given in Table 3.7 as -5.8 ± 0.4 mm/yr north and 5.1 ± 0.4 mm/yr east, and given in *Larson* [1990a] as -8.4 ± 1.1 mm/yr north and 1.8 ± 1.5 mm/yr east (see also *Feigl* [1991]).

Thus, the differences between the relative site motions estimated in this work and in *Larson* [1990a] typically are at the level of several millimeters per year and most prevalent in the east component, although the differences do not appear to be significant at the 95% confidence level of their respective uncertainties. The rates estimated in this work benefit from the availability of two more years of observations, and it is evident from long-term scatter of the baseline components in Figure 3.7 that the possible inconsistencies of these experiments and TREX 14 with the previous experiments may be largely responsible for the estimated rate differences. Clearly, a conservative approach is warranted when interpreting rates based on minimal observations collected over short time intervals. Keeping this caveat in mind, we can confidently assert that both sets of estimates share many of the same gross features and do provide a few useful and unique constraints on tectonic deformation in California.

The most significant relative motions are associated with baselines that cross the San Andreas fault to Owens Valley and Fibre (see Figure 3.6b). The horizontal motion of Owens Valley with respect to Vandenberg (-31.5 ± 0.5 mm/yr north, 25.3 ± 0.8 mm/yr east) is consistent with the estimate of *Larson* [1990a] based on a 1.2-yr interval (-28.3 ± 3.6 mm/yr north, 25.1 ± 6.9 mm/yr east). Also, the GPS receivers at these sites are collocated with the VLBI reference points Owens Valley 7207 and Vandenberg 7223; 46 measurements of this baseline during 1983–1989 are included in the GLB659 adjustment [*Caprette et al.*, 1990] and provide an independent estimate of the relative motion (-29.2 ± 0.3 mm/yr north, 23.2 ± 0.2 mm/yr east). The difference between the GPS and VLBI rates is ~ 2.2 mm/yr in both horizontal components, which is not significant at the 95% confidence level. The motion of Fibre with respect to Vandenberg (-24.8 ± 0.4 mm/yr north, 20.0 ± 0.5 mm/yr east) is also consistent with the estimate of *Larson* [1990a] (-23.8 ± 1.6 mm/yr north, 21.6 ± 2.4 mm/yr east). The relative motions of both Fibre and Owens Valley are essentially in the same direction ($S38.8^\circ E$), which is several degrees clockwise of the average central San Andreas fault azimuth of $S41.0^\circ E$ [*Minster and Jordan*, 1984], but they have different magnitudes with Fibre (31.8 mm/yr) moving more slowly than Owens Valley (40.4 mm/yr).

The only other site collocated with VLBI whose relative motion we estimated is Palos Verdes, located west of the San Andreas fault in the vicinity of Los Angeles. Our estimate for its horizontal motion with respect to Vandenberg (-6.7 ± 0.4 mm/yr north, 4.7 ± 0.5 mm/yr east) is not significantly different from the estimate of *Larson* [1990a] (-10.0 ± 2.2 mm/yr north, 1.0 ± 4.0 mm/yr east) at the 95% confidence level. Seven VLBI measurements of the Palos Verdes 7268 to Vandenberg 7223 baseline during 1983–1989 included in the GLB659 adjustment provide an independent estimate of the relative motion of -9.0 ± 1.5 mm/yr north, 2.1 ± 1.1 mm/yr east. Due to the small number observations, the VLBI-derived rate is relatively weakly determined and does not differ significantly from the GPS rate.

Two remaining sites, La Cumbre and Madre, have significant motions with respect to Vandenberg. We provided comparisons of their horizontal motions with *Larson* [1990a] in the preceding discussion, noting the several millimeter per year level differences that are not statistically significant. Combined with the small relative motion of Center with respect to Vandenberg, north-south convergence across the Santa Barbara channel between La Cumbre and Center at 5.2 ± 0.4 mm/yr is well resolved by the GPS observations. The estimated strain rates in both the eastern and central sections of the Santa Barbara channel are consistent with the results of *Larsen* [1990], who combined trilateration observations obtained in 1970–1971 with GPS measurements from 1987–1988, including TREX 3). To compare these results, we calculated uniform horizontal velocity field gradients within triangular regions defined by three sites and separated the gradients into their symmetric strain rate tensor and rotation rate components [e.g., *Malvern*, 1969]. We then determined the magnitudes of the strain rate $\dot{\epsilon}_1$ and $\dot{\epsilon}_2$ in the directions of the principal axes of the strain tensor, where, following the notation of *Feigl et al.* [1989], extension is positive, $\dot{\epsilon}_1 > \dot{\epsilon}_2$, and θ is the direction of principle axis corresponding to $\dot{\epsilon}_2$ measured clockwise from north.

Our estimate for the uniform strain rate in the eastern Santa Barbara channel, in the triangular region defined by Palos Verdes–La Cumbre–Center, is $\dot{\epsilon}_1 = 0.025 \pm 0.004$ $\mu\text{strain/yr}$, $\dot{\epsilon}_2 = -0.095 \pm 0.008$ $\mu\text{strain/yr}$, $\theta = 11.9^\circ$; for a region that does not extend as far south along the coast, *Larsen* [1990] finds $\dot{\epsilon}_1 = 0.02$ $\mu\text{strain/yr}$, $\dot{\epsilon}_2 = -0.14$ $\mu\text{strain/yr}$, $\theta = 25.1^\circ$ (the equivalent average rate of convergence is 6.4 mm/yr across the 45-km wide channel). Both of these results indicate the predominance of nearly north-south convergence. On the other hand, in the central section of the Santa Barbara channel, which we defined by Gaviota–La Cumbre–Center, both analyses show a strong component of left-lateral shear (we obtained $\dot{\epsilon}_1 = 0.118 \pm 0.019$ $\mu\text{strain/yr}$, $\dot{\epsilon}_2 = -0.098 \pm 0.009$ $\mu\text{strain/yr}$, $\theta = 12.3^\circ$, while *Larsen* [1990] finds $\dot{\epsilon}_1 = 0.94$ $\mu\text{strain/yr}$, $\dot{\epsilon}_2 = -0.94$ $\mu\text{strain/yr}$, $\theta = 37.7^\circ$). *Larsen* [1990] suggests that these results are consistent with

seismic and topographic observations which indicate north to north-east compression in the eastern Santa Barbara channel and left-lateral shear along east-west trending faults in the central Santa Barbara channel.

Another geodetic study of deformation across the Santa Barbara channel is given by *Webb* [1991], who combines GPS and trilateration observations with triangulation measurements collected since the 1880's, to estimate shear-strain rates on the order of 0.2–0.4 $\mu\text{rad/yr}$. If these estimates are modeled as uniaxial convergence, they indicate $18 \pm 5 \text{ mm/yr}$ at $\text{N}20 \pm 5^\circ\text{E}$ across the eastern part of the channel and $13 \pm 5 \text{ mm/yr}$ at $\text{N}23 \pm 5^\circ\text{W}$ across the western part of the channel. These results are somewhat inconsistent with our results, which predict much slower convergence in the eastern part ($5.2 \pm 0.4 \text{ mm/yr}$), and no convergence in the western part (at least, for Center with respect to Gaviota or Vandenberg). The source of these discrepancies is not clear, although the assumption of uniaxial compression in the western Santa Barbara channel may not be reasonable.

The motion of Madre with respect to Vandenberg provides some constraints on deformation in the Santa Maria fold and thrust belt. *Feigl et al.* [1989] combine triangulation and trilateration measurements, and GPS observations predominantly from TREX 1 and 3, to estimate a uniform compressive strain rate of $0.13 \pm 0.03 \mu\text{strain/yr}$ oriented at $\text{N}17 \pm 5^\circ\text{E}$. Assuming the velocity field has no rotational component, the equivalent motion of a site relatively near Madre (TEPW, Table 3.1) with respect to Vandenberg is estimated to $7 \pm 1 \text{ mm/yr}$ at $\text{S}03 \pm 13^\circ\text{W}$. *Feigl et al.* [1989] decompose this motion to show $6 \pm 2 \text{ mm/yr}$ of crustal shortening and $3 \pm 1 \text{ mm/yr}$ right-lateral shear on the general structural trend of the folds in the area ($\text{N}30^\circ\text{E}$), and show that this motion is consistent with earthquake focal mechanisms and geological information that indicate the Santa Maria fold and thrust belt is actively deforming. Our estimate of the relative motion of Madre with respect to Vandenberg is $-5.8 \pm 0.4 \text{ mm/yr}$ north, and 5.1 ± 0.4 east. While the motion in the north component is similar to the previous result, the GPS

measurements predict significant motion in the east as well. Projecting this motion on the structural trend predicts smaller crustal shortening of 2.5 ± 0.4 mm/yr of and larger right-lateral shear of 7.3 ± 0.4 mm/yr. The smaller crustal shortening estimate is more consistent with a geological estimate of 2.3–4.6 mm/yr based on a balanced cross section indicating 9.2 km shortening during the late Pliocene and Quaternary across the fold and thrust belt [Namson and Davis, 1990]. However, the larger right-lateral shear estimate of 7.3 mm/yr is more difficult to justify, since it is not consistent with the earthquake focal mechanisms which are predominantly thrust events and there is no surface evidence for a strike-slip fault of that magnitude. At the present, we are unable to determine whether the estimated motions are tectonic in origin or are in error, due possibly to poorly resolved ambiguities or reference frame effects on the east component; Feigl [1991] provides further discussion.

Six of the sites (Piedras Blancas, Black Hill, Lospe, Grassy, Gaviota, and Center) show no significant horizontal motions with respect to each other. The largest differences in motion (~ 4 mm/yr) are associated with the poorly resolved east components of Grassy and Gaviota. There is a suggestion that Vandenberg is moving to the northwest with respect to these six sites at ~ 2 – 3 mm/yr. The San Gregorio–San Simeon–Hosgri fault system is a possible candidate to accommodate this motion. This fault system, which runs principally offshore parallel to the central California coast, can be traced on land near San Simeon Point (near Piedras Blancas) before extending south offshore to near Point Sal (near Lospe) where its continuation becomes unclear [Hall, 1975]. Estimates of current motion on this system vary, although a minimum estimate of 1–3 mm/yr based on geologic information seems justifiable [e.g., Sedlock and Hamilton, 1991]. Due to the uncertainties of the GPS estimates, it would be premature to further interpret this relative motion, however, we see no evidence for an onshore extension of the Hosgri fault near Vandenberg that accommodates 10–15 mm/yr, as previously proposed [e.g., Weber and Lajoie, 1977; Graham and Dickinson, 1978; Hall, 1978]. The only significant vertical

motion within the network occurs at Grassy (Figure 3.7k) which is moving up 15.5 ± 3.5 mm/yr with respect to Vandenberg. *Feigl* [1991] speculates that this large rate may be due either to nearby extraction of oil and pumping of water in the Lompoc Oil Field, activities thought to be responsible in some cases for vertical motions over 100 mm/yr [e.g., *Cline et al.*, 1984], or to uplift associated with ongoing anticlinal folding [*Namson and Davis*, 1990].

In summary, the relative motions derived from 4.2 years of GPS measurements in central California generally agree with previous geodetic observations at the several millimeter per year level. In some cases, such as at Palos Verdes, they are beginning to improve upon the previous VLBI estimates. In other cases, such as at Madre, where they disagree with previous geodetic estimates and with geologic considerations, the estimated rates pose more questions. Our future efforts to remove some of the uncertainties associated with these rates will focus on including additional data, and refining the estimation procedures. For example, the lack of observations from Richmond and Westford in TREX 14 introduces possible fiducial network effects that require further investigation; we now believe we understand the receiver problems associated with these observations and will be able to incorporate them into future adjustments. We constrained the motion of Mojave to VLBI rates due to possible local survey ties errors between experiments. We are currently investigating these ties and are adding additional stochastic parameterizations to GLOBK to allow us to accommodate such problems when performing multi-experiment adjustments. We also intend to incorporate observations from the other TREX experiments, including the observations from the southern half of the network, and the March 1990 experiment (TREX 18) which was the major 1990 occupation of the central California network. These additions should help to resolve some of the apparent inconsistencies in the most recent experiments and improve the precision of the estimated rates.

The uncertainty of the GPS-derived relative motions we estimated is approximately equal to the value expected from propagating the precision of the baseline measurement by the number and frequency of the measurements. For example, assume a baseline component measurement x_i obtained at time t_i is described by the linear model

$$x_i = x_o + v t_i + e_i \quad (3.7)$$

where x_o is the initial value of the baseline component, v is the rate of change of the component, and e_i is random measurement error. If the random error is Gaussianly distributed with constant variance (i.e., $\langle e_i \rangle = 0$, $\langle e_i^2 \rangle = \sigma^2$), the least-square estimate of the baseline rate of change will have an uncertainty σ_v given by

$$\sigma_v^2 = \sigma^2 \frac{N}{N \sum_{i=1}^N t_i^2 - \left(\sum_{i=1}^N t_i \right)^2} \quad (3.8)$$

If we assume a constant time interval Δt between observations so that the measurement times are $t_i = (i - 1) \Delta t$, and the total time interval $T = (N - 1) \Delta t$, the rate uncertainty can be determined from

$$\begin{aligned} \sigma_v^2 &= \frac{12}{N(N-1)(N+1)} \times \frac{\sigma^2}{\Delta t^2} \\ &= \frac{12 \frac{T}{\Delta t}}{\left(1 + \frac{T}{\Delta t}\right) \left(2 + \frac{T}{\Delta t}\right)} \times \frac{\sigma^2}{T^2} \\ &= \frac{12 \left(1 - \frac{1}{N}\right)}{N \left(1 + \frac{1}{N}\right)} \times \frac{\sigma^2}{T^2} \end{aligned} \quad (3.9)$$

where the second expression is given in *Coates et al.* [1985]. For $N \gg 1$, we note from the third expression that the rate uncertainty is proportional to $\sim 1/T$ and $\sim 1/\sqrt{N}$. The long-term horizontal measurement precision we obtained from the California GPS observations is $\sigma \approx 5$ mm. Assuming annual observations ($\Delta t = 1$ yr), and observations over $T = 4.2$ yr, the expected uncertainty of the horizontal relative motions is $\sigma_v \approx 1.5$ mm/yr, which agrees with our assessment. If we were to continue to make GPS measurements annually for another five years, horizontal motions would be determined with an uncertainty of ~ 0.5 mm/yr.

The San Andreas Discrepancy

One of our primary motivations for establishing the geodetic network principally within the California margin west of the San Andreas fault was to find the “missing” motion represented by the San Andreas discrepancy, the difference between the North America–Pacific relative plate motion and the observed displacement accommodated by the San Andreas fault. *Minster and Jordan* [1984] rigorously formulated this problem by treating global plate tectonic models as vector-boundary constraints to integrations of velocity gradient tensor fields along paths crossing deformation zones. That is, the sum of the displacement vectors encountered along any path whose endpoints are on stable plates must equal the relative plate motion. They examined a path passing through a central California fiducial point (36°N , 120.6°) where the San Andreas fault, based on geodetic and neotectonic observations, accommodates 34 ± 3 mm/yr relative motion oriented at $\text{N}41 \pm 2^\circ\text{W}$. On the other hand, the RM2 global plate tectonic model [*Minster and Jordan*, 1978] predicts 56 ± 3 mm/yr, $\text{N}35 \pm 2^\circ\text{W}$. The difference between these vector rates, 22 ± 4 mm/yr, $\text{N}27 \pm 10^\circ\text{W}$, is the discrepancy vector that represents relative motion distributed east and west of the San Andreas fault. They estimated Basin and Range extension from a variety of geophysical and geological constraints and predicted that the remaining deformation (the so-called “modified” San Andreas discrepancy) of 4–

13 mm/yr of crustal shortening and 6–25 mm/yr right-lateral motion with respect to the San Andreas, is accommodated on faults west of the San Andreas fault. Similar plate kinematic studies by *Bird and Rosenstock* [1984] and *Weldon and Humphreys* [1986] that predict significant deformation (10–20 mm/yr) along the southern California margin and offshore provided further motivation for establishing a deformation monitoring network along the margin.

Since the initiation of the GPS measurement program in 1986, several studies have modified the predicted San Andreas discrepancy vector for central California. The development of the NUVEL-1 plate tectonic model [*DeMets et al.*, 1990] significantly revised the predicted North America–Pacific plate motion, primarily from an analysis of marine magnetic anomalies in the Gulf of California [*DeMets et al.*, 1987]. NUVEL-1 predicts 48 ± 1 mm/yr, $N36 \pm 2^\circ W$ relative plate motion at the central California fiducial point, implying a San Andreas discrepancy of 14 mm/yr, $N23^\circ W$. The studies of *Bird and Rosenstock* [1984] and *Weldon and Humphreys* [1986] are similarly affected, since they also assumed RM2 boundary constraints.

Observations at several VLBI antennas located on the eastern edge of the Sierra Nevada (e.g., Owens Valley, Quincy, and Hat Creek) provide estimates of Basin and Range extension. For example, *Minster and Jordan* [1987] estimated three rates of extension: one based solely on geological constraints (8.0 ± 3.3 , $N64 \pm 10^\circ W$), one based solely on VLBI measurements (9.0 ± 3.9 mm/yr, $N48 \pm 17^\circ W$), and one based on a combination of the two (9.7 ± 2.1 mm/yr, $N56 \pm 10^\circ W$) so that the direction of motion is roughly parallel to the direction of maximum extension observed geologically in the Great Basin. *Ward* [1990] used VLBI observations to construct several rigid plate models of western U.S. and Pacific deformation. Basin and Range extension in one of his models, in which North America–Pacific motion is constrained to be NUVEL-1, is 9.0 ± 1.0 mm/yr, $N33 \pm 6^\circ W$. The direction of this VLBI-only estimate significantly differs from the direction of maximum extension and is more parallel to the predicted

relative plate motion than the estimates of *Minster and Jordan* [1987]. Therefore, the resulting modified San Andreas discrepancy is not required to accommodate as much crustal shortening. *Argus and Gordon* [1989,1991] argue that the direction of motion in general does not parallel the direction of maximum extension in a deforming continuum. The modified San Andreas discrepancy using the *Argus and Gordon* [1991] estimate of Basin and Range extension, nearly identical to the estimate of *Ward* [1990] (9 ± 1 mm/yr, $N35 \pm 3^\circ W$), is 6 ± 3 mm/yr, $N18^\circ W$, implying 2 ± 2 mm/yr crustal shortening and 6 ± 3 mm/yr right-lateral motion with respect to the San Andreas fault.

Thus, improvements both in the global plate tectonic model and in the estimate of Basin and Range extension have greatly reduced the magnitude of the modified San Andreas discrepancy vector in central California. No significant fault-normal compression is required. Right-lateral motion with respect to the San Andreas is only marginally significant; however, relative motions of greater than about 4 mm/yr should be resolvable using GPS observations. In Figure 3.8, the bias-fixed relative motions with respect to Vandenberg have been plotted in an oblique Mercator projection about the NUVEL-1 North America–Pacific Euler pole. The VLBI-derived motion of Vandenberg with respect to North America is consistent at the few millimeters per year level with the NUVEL-1 prediction assuming that Vandenberg is on the stable Pacific plate [e.g., *Sauber*, 1988; *Ward*, 1990]. The horizontal line at Owens Valley lacking an error ellipse in Figure 3.8 is the predicted NUVEL-1 relative motion, which follow lines of latitude in this projection. Most of the sites with significant relative motions appear to closely aligned lines of latitude and have little motion perpendicular to the relative plate motion direction. In Figure 3.9, the parallel and perpendicular components of these motions with respect to the relative plate motion direction are plotted as a function of distance from Vandenberg projected onto the normal to the plate motion. The maximum motion perpendicular to plate motion is 3 mm/yr at Owens Valley. Recent uplift in the Coast Ranges and active thrust faults in the vicinity of the San Andreas offer abundant

geological evidence for compression normal to the San Andreas fault in central California [e.g., *Zoback et al.*, 1987]; however, we see no significant convergence in the GPS-derived relative motions. Our result is consistent with the observations of *Lisowski et al.* [1991] based on trilateration measurements in the vicinity of the San Andreas fault in central and southern California.

The parallel components of the GPS site motions with respect to Vandenberg all indicate right-lateral shear varying from ~1–3 mm/yr for sites in the vicinity of Vandenberg to 40 mm/yr at Owens Valley (Figure 3.9). These motions provide several constraints on the distribution of deformation across the plate boundary. The discrepancy between the observed motion of Owens Valley and the NUVEL-1 prediction is principally due to the relative motion of Owens Valley with respect to stable North America caused by extension in the Basin and Range. Fibre has 8 mm/yr less right-lateral shear than Owens Valley. This relative motion on the east side of the San Andreas fault may be due to two causes. First, Fibre is within 35 km of the San Andreas fault, while Owens Valley is 200 km more distant. Trilateration measurements in the vicinity of the San Andreas predict relative site motions consistent with models of continuous slip at depth on the fault [e.g., *Eberhart-Phillips et al.*, 1990]. For example, screw dislocation theory roughly approximates the surface deformation due to continuous slip at depth on a fault. *Savage and Burford* [1973] show that the surface velocity field $v(x)$ resulting from slip with velocity V at depth D in an infinite elastic half-space is

$$v = \frac{V}{\pi} \tan^{-1} \left(\frac{x}{D} \right) \quad (3.10)$$

Using typical values for the San Andreas in central California ($V = 34$ mm/yr, and $D = 10$ km) [e.g., *Lisowski et al.*, 1991], this simple model predicts 3.5 mm/yr relative motion between Fibre and Owens Valley. Thus, while these models show that relative motions

decrease rapidly at distances greater than one-fault depth from the fault, we can expect motions at the level of 3 mm/yr between Fibre and Owens Valley [e.g., *Lisowski et al.*, 1991]. Second, *Sauber et al.* [1986] and *Savage et al.* [1990] have shown that a north-northwest trending shear zone in the Mojave and Owens Valley may be accommodating as much as 7–8 mm/yr of the Pacific–North America relative plate motion. If this is so, the combination of this motion and 35 mm/yr across the San Andreas nearly account for the entire relative plate motion.

The 24 mm/yr of relative motion between Fibre and the two sites 35–45 km to the west of the San Andreas, Madre and La Cumbre, accommodates the majority of the predicted San Andreas motion. This level of relative motion is often observed across the trilateration networks spanning a similar width (~70 km) in the vicinity of the San Andreas; for example, a trilateration network located just north of Madre in the Carrizo Plain accommodates 21 ± 3 mm/yr in 70 km [*Lisowski et al.*, 1991]. Equation 3.10 predicts 28 mm/yr, which slightly overestimates the relative motion.

La Cumbre and Madre are 60–70 km from Vandenberg and are moving ~7 mm/yr with respect to it. The remaining sites are 20–30 km from Vandenberg (projected onto the normal of the plate direction), and are moving ~1–3 mm/yr with respect to it. One explanation for these motions is that the eastern shear zone and the San Andreas fault accommodate all relative motion in California, and that the motions of the sites in the vicinity of Vandenberg are simply a response to the slip at depth on the San Andreas. However, Equation 3.10 predicts that the relative motion of Madre with respect to Vandenberg due to slip on the San Andreas is only 3 mm/yr, while the relative motions of the other sites are less than 0.3 mm/yr. Thus, deformation due to buried slip on the San Andreas is insufficient to account for the observed motions. A second possibility is that some of the relative motion is accommodated on faults in the Coast Ranges and western Transverse Ranges, such as the San Gregorio–Hosgri fault whose possible extension onshore in the vicinity of Vandenberg is unclear. *Feigl et al.* [1989] favor a model in

which the Hosgri fault comes onshore north of Vandenberg and its right-lateral motion is transformed into the Santa Barbara channel by the Santa Maria fold and thrust belt. Considering the differences between the predicted and observed relative motions, 2–4 mm/yr might be accommodated by faults in the vicinity of Vandenberg. Considering the errors of the GPS estimates, and the inconsistency of the motion of Madre with geological and geophysical indications of convergence across the Santa Maria fold and thrust belt, it is difficult at this time to resolve these questions.

In summary, the relative site motions estimated by GPS show no significant convergence normal to the San Andreas fault, and are consistent with a simple shear model for deformation across the Pacific–North America plate boundary. Deformation due to slip at depth on the San Andreas fault is too small to accommodate fully the observed relative motion of Fibre with respect to Owens Valley, and of Madre with respect to Vandenberg. Therefore, additional relative motion on the order of 2–4 mm/yr may be required both east of the San Andreas fault, possibly within the shear zone geodetically measured by *Sauber et al.* [1986] and *Savage et al.* [1990], and west of the San Andreas, possibly along the San Gregorio–Hosgri fault.

CONCLUSIONS

We have analyzed GPS measurements collected in California spanning a 4.2-year interval. The data offer sufficient redundancy to assess the short and long-term precision of GPS baseline estimates for regional-scale networks with continental-scale fiducial control networks, and to estimate relative site motions. The short-term (four-to-five day) precision is approximately 3–5 mm with 1×10^{-8} dependence on baseline length (in km) for horizontal components under good conditions. When the fiducial network is weak or ambiguities are poorly resolved, the length dependence of the precision can increase to $3\text{--}6 \times 10^{-8}$. The long-term precision of the GPS measurements based on the day-to-day

scatter of the estimated baselines about their best-fit linear trend is 4–5.6 mm/yr in the horizontal components and $1.3\text{--}2.6\times 10^{-8}$ dependence on baseline length, with the east components having slightly worse precision than the north. The small increases in the long-term repeatabilities are probably due to constant error sources that are highly correlated over short-term intervals, such as tropospheric effects, multipath, and monument stability. The precision of the vertical is approximately 15–25 mm.

From the GPS baseline measurements, we have estimated relative site motions with uncertainties at the 1–1.5 mm/yr level. Confidence in some of the estimated rates is limited by apparent inconsistencies between experiments, especially TREX 14 in March 1989 which had poor fiducial control. The estimated relative motions of Owens Valley and Palos Verdes with respect to Vandenberg are consistent with VLBI-derived rates, and the rates-of-change of baselines crossing the San Andreas fault are well resolved. We estimate significant north-south convergence across the eastern Santa Barbara channel between La Cumbre and Center at 5.2 mm/yr, and our results are consistent with a change to left-lateral shear in the central Santa Barbara channel. The estimated deformation across the Santa Maria fold and thrust belt has less convergence and more right-lateral shear than had been previously estimated from triangulation and trilateration measurements and geological and geophysical constraints. The relative motion of six sites along the western margin in the vicinity of Vandenberg are consistent with 2–4 mm/yr motion on the Hosgri fault, if it comes ashore between Lospe and Vandenberg, but are not consistent with previously proposed significant motions (>10 mm/yr) on the Hosgri. Deformation due to continuous slip at depth on the locked San Andreas fault does not fully account for the motion of Fibre with respect to Owens Valley, and suggests that an additional 2–4 mm/yr motion is being accommodated east of San Andreas, possibly in a shear zone located in Owens Valley and the Mojave Desert [e.g., *Savage et al.*, 1990].

Table 3.1 Approximate NAD83 Coordinates of GPS Monuments

Monument	Site	Lat°N	Lon°E	Height, m
7223	VNDN7223	34.5561	-120.6164	-12.3187
ALAM	ALAMO 1925	34.7985	-120.2568	457.1624
ALGO	ALGORMA	45.9559	-78.0714	198.7755
ALVA	ALVADO 1933	34.5927	-120.6170	293.9618
AROT	ARO T141	34.5927	-120.6170	293.9618
AUST	AUSTIN GPS	30.3117	-97.7563	191.1261
BLAN	PIEDRAS BL	35.6646	-121.2845	-24.8568
BLHL	BLAK HILL	35.3587	-120.8317	166.4610
BLUF	S.Clemente	32.9268	-118.5185	297.3143
BRSH	S.CLEMENTE	33.4070	-118.4049	448.5315
CENT	S.CRUIZ IS.	33.9948	-119.7529	389.4936
CHAF	CHAFEE	34.3006	-119.3310	307.1545
CHUR	CHURCHILL	58.7592	-94.0884	-16.8612
COTR	PT. MAGU	34.1202	-119.1540	-34.0805
DEVL	DEVILS PK	34.0291	-119.7844	704.5505
FIBR	BUTTONWILL	35.3985	-119.3940	54.6695
FTOR	FTOR7266	36.6698	-121.7733	23.7641
GAVI	GAVIOTA	34.5018	-120.1988	713.3459
GRAS	Grassy	34.7306	-120.4141	331.1302
JPL1	JPL1ARI1	34.2047	-118.1710	441.5214
KOKE	KOKEFRPA	22.1267	-159.6649	1167.3330
KOKT	KOKETSST	22.1267	-159.6649	1166.5938
LACU	LA CUMBRE	34.4944	-119.7139	1164.8051
LIND	LINDA 1955	34.9599	-120.2997	181.5456
LOSP	LOSP 1927	34.8937	-120.6062	463.0934
MADC	MADRE ECC	35.0756	-120.0671	958.0447
MILL	MILLER CADT	34.5101	-120.2297	85.3316
MOJA	MOJATI41	35.3316	-116.8882	904.7205
MOJF	MOJAFRPA	35.3316	-116.8882	904.7205
MOJM	MOJAMMAC	35.3316	-116.8882	904.7205
NIGU	S. Niguel	33.5145	-117.7303	235.6844
ONSA	ONSATI41	57.3953	11.9255	47.0240
OVRO	OVRO7114	37.2326	-118.2938	1178.0107
PARG	Pt. Arguello	34.5549	-120.6160	-13.0652
PLAT	PLAT7258	40.1828	-104.7263	1501.3157
POZE	POZO ECC	35.3474	-120.2955	769.7786
POZO	POZO	35.3460	-120.2987	728.6094
PVER	PVER7268	33.7438	-118.4035	69.3917
RICH	RICHFRPA	25.6141	-80.3841	-19.3672
RICM	RICHMMAC	25.6141	-80.3841	-19.3672
RUS1	RUSTAD RM1	34.5708	-120.6270	176.6131
SCRE	SCruz East	34.0547	-119.5647	60.6796
SCRW	SCruz West	34.0732	-119.9180	182.0058
SNPA	SNPA7255	34.3879	-118.9988	184.7509
SOLE	S. ROSA IS	33.9509	-120.1057	443.4255
SOLI	SOLIMAR	34.2983	-119.3427	-11.6882
SYNZ	SYNEZ11917	34.5305	-119.9859	1217.8618
TEPT	Tepusquet T	34.9098	-120.1865	952.5544
TEPW	Tepusquet W	34.9100	-120.1867	953.1967
TROM	TROMGPSM	69.6628	18.9383	134.8139
TWIN	San Nichola	33.2318	-119.4790	200.1256
VND2	VNDNRM2	34.5558	-120.6163	-12.7389
VNDN	VNDNRM1	34.5563	-120.6162	-11.5327
VSLR	VNDN7880	34.5560	-120.6164	-12.2773
WETM	WETTMMAC	49.1446	12.8787	664.0512
WETT	WETTI41	49.1446	12.8787	664.0512
WSFD	WESTTI41	42.6133	-71.4933	86.3720
WSFF	WESTFRPA	42.6133	-71.4933	86.3720
WSFM	WESTMMAC	42.6133	-71.4933	86.3720
YKNF	Yellowknife	62.4769	-114.4689	183.8306

Table 3.2. Receiver-Session Occupations by Experiment

TREX	1	2	3	7	8	9	11	14	15	V1	17	V2	20
YEAR	88888 66666	88888 66677	88888 77777	88888 77777	88888 77777	88 77	8888 8888	8888 9999	888 999	9999999999 0000000000	9999 0000	9999999999999999 0000000000000000	99999
DAY	33333 55555 01234	33300 66600 34512	00000 00000 34567	11111 44444 45678	22222 66666 56789	22 77 45	0000 7777 5678	0000 8889 7890	000 999 123	0000000000 3333333344 1234578901	0000 8888 4567	2222222222222222 455555555555666 90123456789012	00000 33344 78901
7223						..			..				*.....
ALAM						..			..				
ALGO		*****	*****			..	*****		..				
ALVA						..			..	*			
AROT						..			..		*****		
AUST	***..	*.....	..**	*****		..			..				
BLAN		*****				..	*****	*****	..				*****
BLHL	*****	*****		*****	*****	..	*****	*****	..		*****		*****
BLUF						..			..				***.
BRSH			*****			..			..				***.
CENT		*****		*****	*****	..	*****	*****	..		*****		*****
CHAF			*			..			..				
CHUR		*****	*****		*****	..	*****		..				
COTR		*****				..			..				
DEVL			*****			..			..				
FIBR		*****	*****			..	*****	*****	..				*****
FTOR	*****	*****	*****		*****	..	*****		..				
GAVI			***.			..			***		*****		
GRAS						**		***	..*			***	
JPL1						..		*****			*****		
KOKE						..		*****	*.*	*.*			
KOKT						..			..				*****
LACU		*****	*****			..	*****	***.*	..				*****
LIND						..			..			***	
LOSP	*****	**.*				..	*****	*****	..	***.		***	*****
MADC		*****				..	*****	*****	..		***.		*****
MILL			*			..			**				
MOJA		*..	..**	**.*	*****	**	..**		..				
MOJF						..		*****	..				
MOJM						..			*.*	*****	*****	*****	*****
NIGU						..			..				*****
ONSA						..		*****	..				
OVRO	*****	*.*	*****	*****	*****	..	*****	*****	..		*****		*****
PARG						..			*				
PLAT	**.*	*****	*****		*****	..	*.*		..				
POZE	*.....					..			..				
POZO	*****					..		*****	..				*****
PVER	*****		*****	*****	*****	*	*****	*****	..		*****		*****
RICH						..	**.*		..				
RICM						..			*	*****	*****	*****	*****
RUS1						..			**			**	
SCRE					*	..			..				
SCRW					***.	..			..				
SNPA	*.*					..			..				
SOLE			*****			..			..				
SOLI			*****			..			..				*****
SYNZ						..			..	*			
TEPT						*			..				
TEPW						*			..				
TROM						..	***	*****	*				
TWIN						..	*****		..				*****
VND2						..			..				*
VNDN	*****	*****	***.*	*****	*****	**	..**	*****	***	***	*****	*****	*****
VSLR			*			..			..				
WETT						..		*****	..*				
WSFD		*****	*****	*****	*****	**	*****		..				
WSFM						..			***	*****	*****	*****	*****
YKNE				***.*	***.*	*	***	***	*				

Table 3.3. Single-session Adjustment Statistics

yr	day	TREX	stn	sat	obs	Free		Fixed		ion	bias
						dof	nrms	dof	nrms		
86	350	1	10	6	15680	135	0.275	101	0.282	1.0	0
86	351	1	9	6	14255	127	0.275	97	0.278	1.0	0
86	352	1	9	6	15284	129	0.276	94	0.282	1.0	0
86	353	1	9	6	13926	125	0.258	98	0.262	1.0	0
86	363	2	12	6	4960	155	0.229	111	0.239	0.5	0
86	364	2	15	6	6405	184	0.266	139	0.272	5.0	0
86	365	2	15	6	6211	184	0.247	134	0.256	0.5	0
87	001	2	14	6	5396	171	0.259	134	0.264	0.5	9
87	002	2	15	6	5977	181	0.249	136	0.265	0.5	2
87	003	3	14	6	6030	175	0.261	130	0.273	0.5	0
87	004	3	15	6	5913	181	0.291	133	0.302	0.5	0
87	005	3	15	6	6157	182	0.290	132	0.294	0.5	0
87	006	3	16	6	6468	191	0.298	141	0.307	0.5	0
87	007	3	16	6	6780	192	0.318	138	0.328	0.5	0
87	144	7	9	7	3467	142	0.298	118	0.309	1.0	5
87	145	7	9	7	3665	143	0.310	120	0.317	1.0	7
87	146	7	7	7	2803	124	0.278	108	0.287	1.0	3
87	147	7	9	7	3885	144	0.318	117	0.328	1.0	3
87	148	7	8	7	3430	134	0.306	119	0.309	1.0	12
87	265	8	12	7	4302	169	0.300	144	0.311	5.0	10
87	266	8	12	7	4684	170	0.301	132	0.314	1.0	2
87	267	8	11	7	4592	164	0.271	127	0.281	1.0	2
87	268	8	11	7	4337	164	0.281	130	0.286	1.0	2
87	269	8	11	7	4607	166	0.278	132	0.293	1.0	2
87	274	9	6	7	8015	113	0.270	99	0.272	1.0	3
87	275	9	6	7	8279	114	0.309	107	0.311	1.0	8
88	075	11	18	7	7461	236	0.250	176	0.255	5.0	0
88	076	11	19	7	8148	245	0.274	175	0.281	5.0	0
88	077	11	19	7	8087	246	0.261	174	0.269	5.0	0
88	078	11	18	7	8185	236	0.265	165	0.272	5.0	0
89	087	14	16	7	26811	208	0.266	143	0.277	5.0	0
89	088	14	17	7	27244	215	0.283	151	0.298	5.0	0
89	089	14	16	7	26480	208	0.285	148	0.303	5.0	0
89	090	14	17	7	29395	219	0.289	153	0.300	5.0	0
89	091	15	9	7	11915	144	0.378	127	0.391	5.0	0
89	092	15	6	7	9365	116	0.328	94	0.335	1.0	0
89	093	15	8	7	11999	132	0.296	109	0.305	4.0	0
90	031	V1	5	8	1989	85	0.223	81	0.227	1.0	4
90	032	V1	7	8	12541	129	0.272	117	0.275	1.0	7
90	033	V1	6	8	13373	139	0.316	130	0.317	1.0	12
90	034	V1	6	8	9111	118	0.318	114	0.321	1.0	2
90	035	V1	5	8	9205	126	0.066	122	0.097	1.0	8
90	037	V1	5	9	9511	117	0.320	117	0.320	1.0	7
90	038	V1	6	9	14190	136	0.337	121	0.341	1.0	2
90	039	V1	6	9	11302	127	0.282	126	0.282	1.0	13
90	040	V1	6	9	12120	126	0.266	115	0.272	1.0	1
90	041	V1	5	9	10331	120	0.242	113	0.250	1.0	7
90	084	17	11	9	21434	198	0.308	175	0.312	1.0	30
90	085	17	11	9	21626	200	0.323	163	0.333	1.0	16
90	086	17	11	9	23046	201	0.321	161	0.333	1.0	17
90	087	17	10	9	20704	188	0.303	162	0.308	1.0	21
90	249	V2	6	9	10107	132	0.217	123	0.237	1.0	0
90	250	V2	6	9	13454	144	0.195	128	0.241	1.0	8

90	251	V2	6	9	13636	144	0.203	123	0.226	1.0	3
90	252	V2	6	9	13728	144	0.202	122	0.249	1.0	2
90	253	V2	6	9	11752	132	0.195	114	0.212	1.0	5
90	254	V2	6	9	13654	144	0.202	123	0.223	1.0	3
90	255	V2	6	9	13595	144	0.234	134	0.261	1.0	14
90	256	V2	6	9	13394	144	0.202	124	0.239	1.0	4
90	257	V2	5	9	13040	144	0.241	123	0.260	1.0	3
90	258	V2	7	9	13473	144	0.225	124	0.241	1.0	4
90	259	V2	19	9	12967	143	0.210	124	0.234	1.0	5
90	260	V2	20	9	12273	142	0.243	124	0.268	1.0	6
90	261	V2	19	9	12809	143	0.188	122	0.217	1.0	4
90	262	V2	18	9	9423	131	0.224	124	0.244	1.0	3
91	037	20	6	5	13815	153	0.256	132	0.273	1.0	4
91	038	20	6	8	46720	294	1.026	195	1.033	1.0	13
91	039	20	7	8	47966	303	0.392	183	0.433	1.0	0
91	040	20	5	8	49277	*	*	295	*	1.0	*
91	041	20	6	8	45517	282	0.399	173	0.425	1.0	2

* Statistics unknown, due to corrupted computer file

Table 3.4. Fiducial Sites by Experiment

Site	1	2	3	7	8	9	11	14	15	V1	17	V2	20	Constraints [†] , mm		
														n	e	u
ALGO		*	*				*							9	9	42
AROT											•					
AUST	•	◦	◦	•										13	29	41
CHUR		•	•		•		•									
FTOR	*	•	•		•		•							10	10	10
JPL1								*			*			10	10	10
KOKE								•	•	*				10	10	10
KOKT													•			
MOJA		◦	◦	•	•	*	•							10	10	10
MOJF								*						10	10	10
MOJM									•	*	*	•	•	10	10	10
ONSA								•								
OVRO	*	*	*	*	*		*	•			•		•	6	4	14
PLAT	•	*	*		*		*							2	2	10
PVER	•		•	•	•	•	•	•			•		•			
RICH							*							1	1	3
RICM									*	*	*	*	*	10	10	10
SNPA	•															
TROM							•	•	•							
VNDN	*	•	•	•	•	*	•	*	*	*	*	*	*	10	10	10
WETT								•	•							
WSFD		*	*	*	*	*	*							1	1	1
WSFM									*	*	*	*	*	10	10	10
YKNF				*	*	•	*	•	•					25	16	186

* Fiducial used in single-session adjustments

• Fiducial observations available

◦ Weak fiducial observations (1 day only)

[†] Typical constraint applied in the single-session adjustments

Table 3.5. Statistics for experiment adjustments

TREX	Sessions	Observ.	Parameters		χ^2/f^*	
			input	global	free	fixed
1	4	59145	327	93	4.849	7.838
2	5	28949	483	114	0.046	0.085
3	5	31348	498	123	0.127	0.150
7	5	17250	441	99	0.461	0.665
8	5	22522	486	111	0.107	0.163
11	4	31881	474	132	0.169	0.223
14	4	109930	450	120	0.739	0.975
17	4	86810	453	126	0.579	0.674
20	5	203285	654	159	0.792	1.383
All ^a	70	921671	6933	333	1.414	2.066
All ^b	69	872404	6795	336	1.425	2.077

* approximate estimate of chi-square per degrees of freedom f , where f equals input–global parameters.

^a Vandenberg tightly constrained: 0.1 mm in position, 0.1 mm/yr in velocity

^b Vandenberg loosely constrained: 10 mm in position, 1 mm/yr in velocity

Table 3.6. Baseline Component Repeatabilities

TREX	North		East		Up	
	<i>a</i> (mm)	<i>b</i> ($\times 10^{-8}$)	<i>a</i> (mm)	<i>b</i> ($\times 10^{-8}$)	<i>a</i> (mm)	<i>b</i> ($\times 10^{-8}$)
<i>bias-free</i>						
1	4.2	9.5	23.4	35.5	32.7	28.2
2	4.3	1.4	11.3	3.1	15.3	7.0
3	3.1	2.1	9.6	4.9	22.1	3.5
7	1.6	2.6	4.6	6.3	27.0	3.0
8	5.8	4.5	16.0	5.5	26.4	9.2
11	3.4	0.9	9.2	1.6	21.4	0*
14	2.5	3.6	9.4	8.3	20.6	3.9
17	3.0	1.3	8.0	3.6	15.1	3.1
20	2.5	0.8	5.5	2.7	13.4	3.0
All†	4.0	1.3	8.7	2.8	20.3	4.5
<i>bias-fixed</i>						
1	2.8	6.1	5.9	6.1	15.2	17.3
2	5.0	1.2	4.8	1.5	16.5	6.9
3	3.3	1.3	6.2	0.6	21.3	0*
7	0.3	3.0	7.5	5.8	27.8	5.5
8	3.4	2.9	5.5	3.1	23.5	8.5
11	2.8	0*	3.4	1.8	21.6	0*
14	2.3	2.1	3.4	3.0	23.6	6.9
17	3.0	0.9	3.9	3.0	12.7	2.0
20	1.8	1.3	4.3	3.6	17.1	2.0
All†	4.0	1.3	5.6	2.6	21.8	4.5

* Constrained to be zero due to apparent decrease in repeatability with baseline length.

† Long-term repeatabilities for 16 sites with observations spanning more than 1.5 years during all the experiments

Table 3.7. Estimated Site Velocities with respect to Vandenberg VNDN

Site	East, mm/yr	North, mm/yr	Cor(n,e)	Up, mm/yr
WSFD	30.4 ± 1.0	-37.5 ± 1.0	-0.006	-0.4 ± 1.3
ALGO	26.9 ± 3.2	-35.0 ± 1.9	-0.390	-26.6 ± 6.3
RICH	31.5 ± 1.0	-39.4 ± 1.0	0.000	4.3 ± 1.3
MOJA	28.7 ± 1.1	-31.4 ± 1.0	0.006	-0.7 ± 1.3
OVRO	25.3 ± 0.8	-31.5 ± 0.5	0.012	-2.4 ± 2.1
PVER	4.7 ± 0.5	-6.7 ± 0.4	0.105	-7.8 ± 2.0
FIBR	20.0 ± 0.5	-24.8 ± 0.4	0.110	1.2 ± 2.4
LACU	3.3 ± 0.4	-7.0 ± 0.3	0.164	0.2 ± 2.5
CENT	2.6 ± 0.4	-1.7 ± 0.3	0.086	2.9 ± 2.3
MADC	5.1 ± 0.4	-5.8 ± 0.4	0.065	1.8 ± 2.7
GAVI	-1.5 ± 0.7	-2.7 ± 0.5	0.152	-4.8 ± 4.2
GRAS	-0.1 ± 0.8	-1.5 ± 0.5	-0.114	15.5 ± 3.5
LOSP	2.0 ± 0.3	-1.8 ± 0.2	0.175	0.3 ± 1.8
BLHL	2.2 ± 0.3	-0.9 ± 0.3	0.143	4.7 ± 1.9
BLAN	3.4 ± 0.5	0.1 ± 0.4	0.138	5.5 ± 3.0

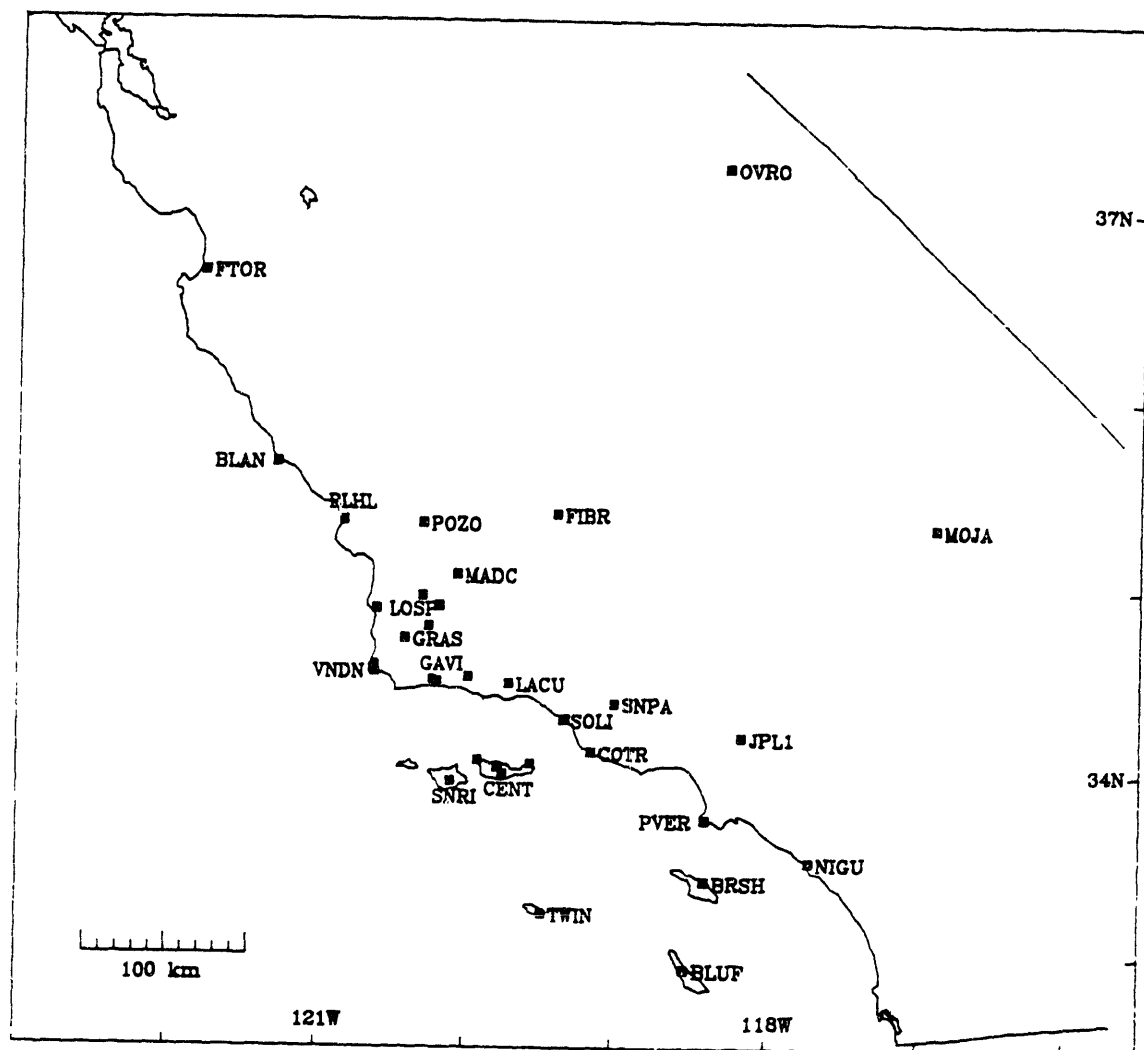


Figure 3.1. Principal sites in California occupied by GPS receivers in the California campaigns considered in this work. A subset of twelve sites in central California have sufficient observations to estimate relative site velocities.

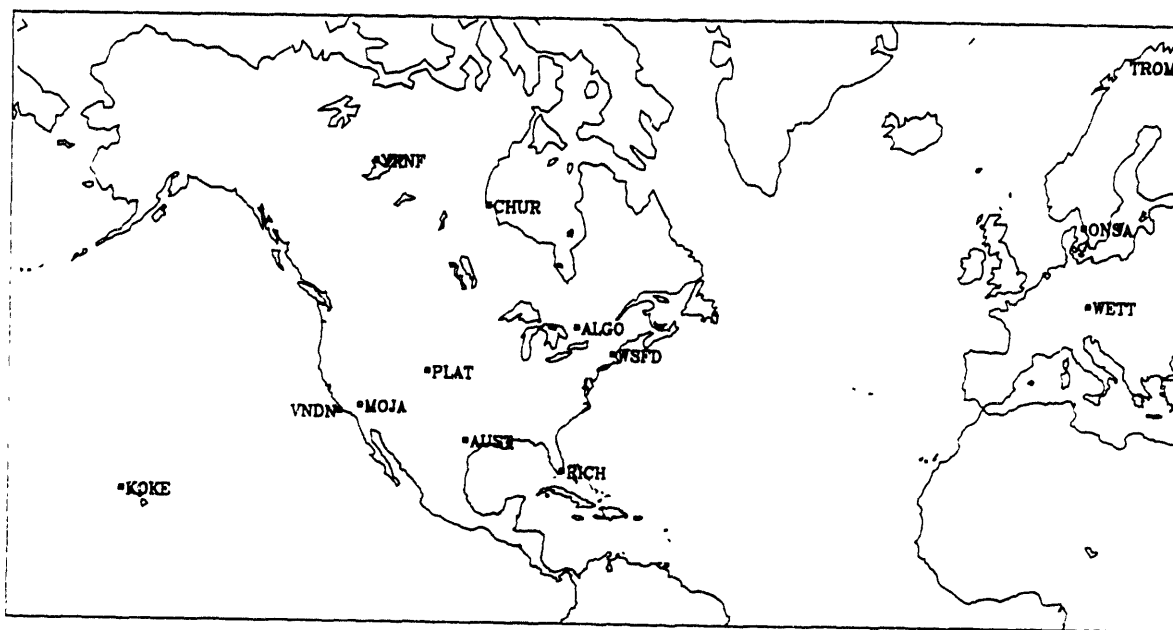


Figure 3.2. Principal sites occupied by GPS outside of California during the campaigns considered in this work. The CIGNET site at Mojave MOJA is also included. All sites, with the exception of Churchill CHUR and Austin AUST, are collocated with VLBI reference points that allow their relative locations to be determined in the SV5 or SV6 terrestrial reference frame.

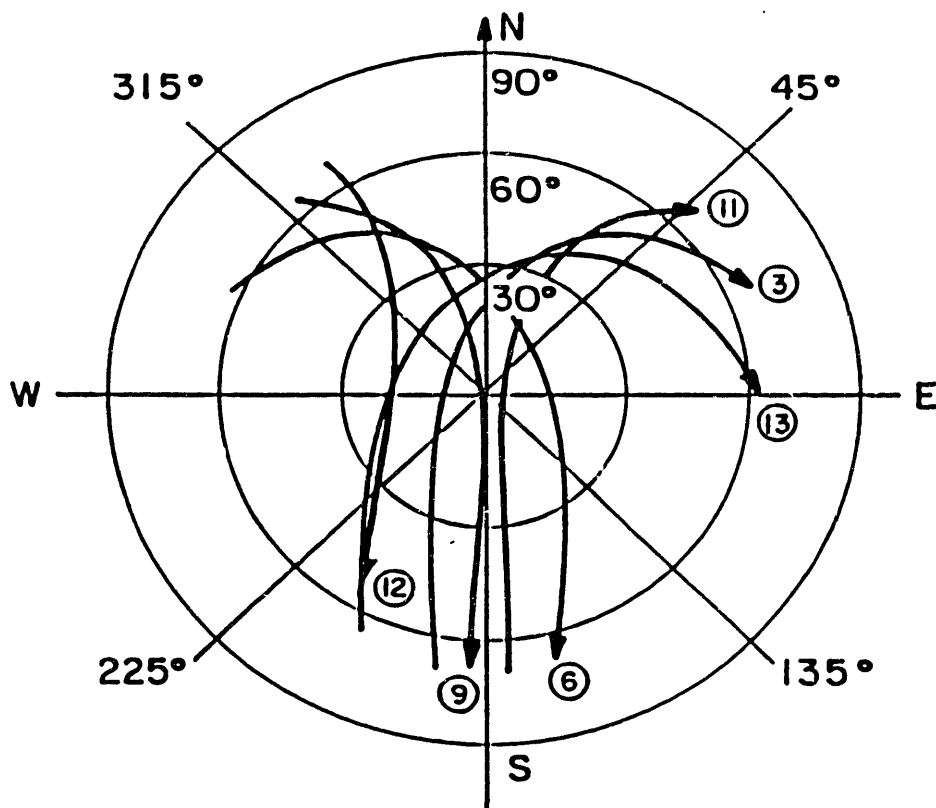


Figure 3.3. GPS satellite sky tracks by azimuth and zenith angle as viewed from central California in January 1987. The PRN numbers of the satellites are given in the circles. The sky tracks demonstrate the predominant north-south orientation of the satellite orbits over central California.

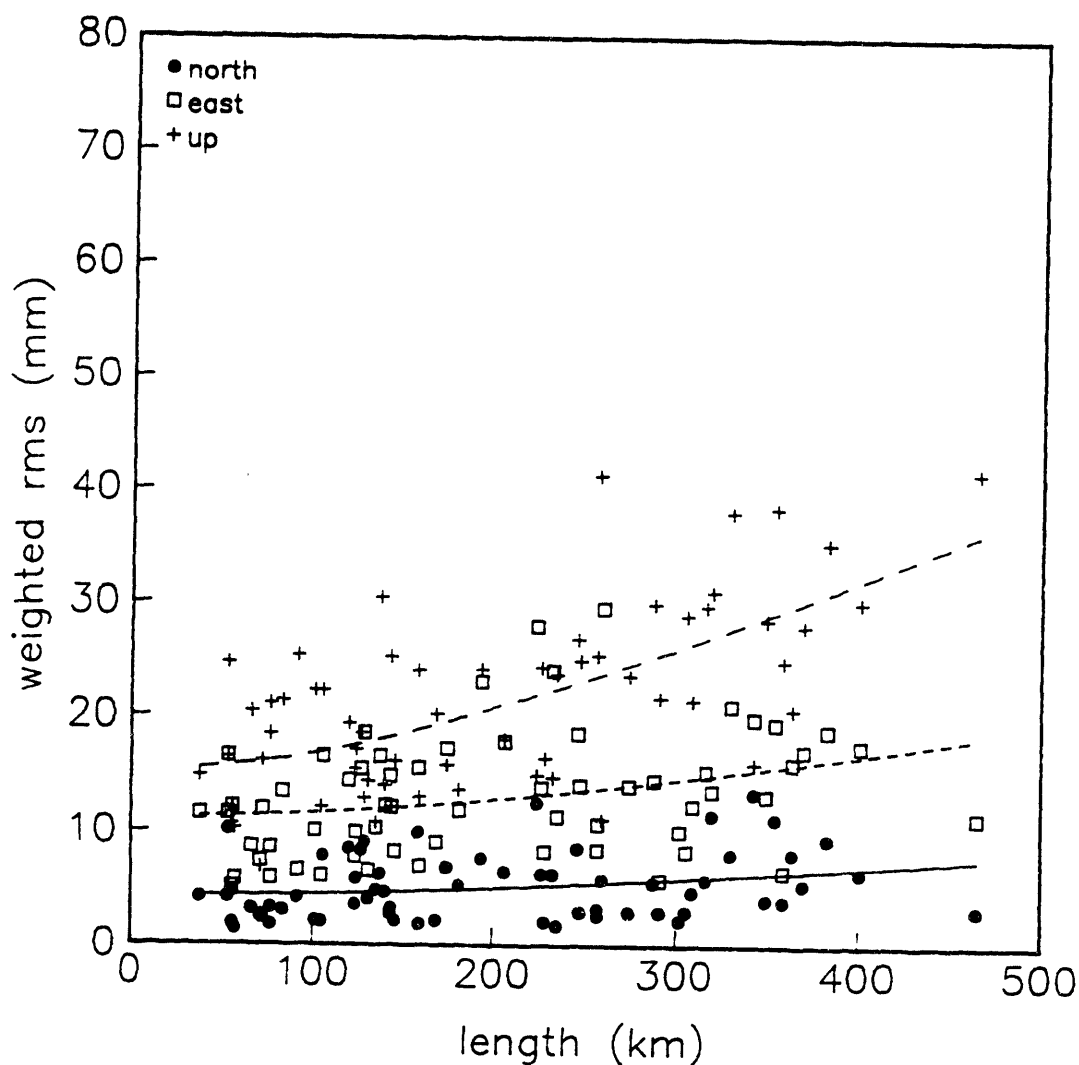


Figure 3.4a. TREX 2 (January 1987) weighted rms scatter of residuals about their weighted mean (repeatabilities) of single-session bias-free estimated baseline components as a function of baseline length. Estimated constant and length-dependent errors: north (solid line) $a = 4.3$ mm, $b = 1.4 \times 10^{-8}$; east (short-dashed line) $a = 11.3$ mm, $b = 3.1 \times 10^{-8}$; up (long-dashed line) $a = 15.3$ mm, $b = 7.0 \times 10^{-8}$.

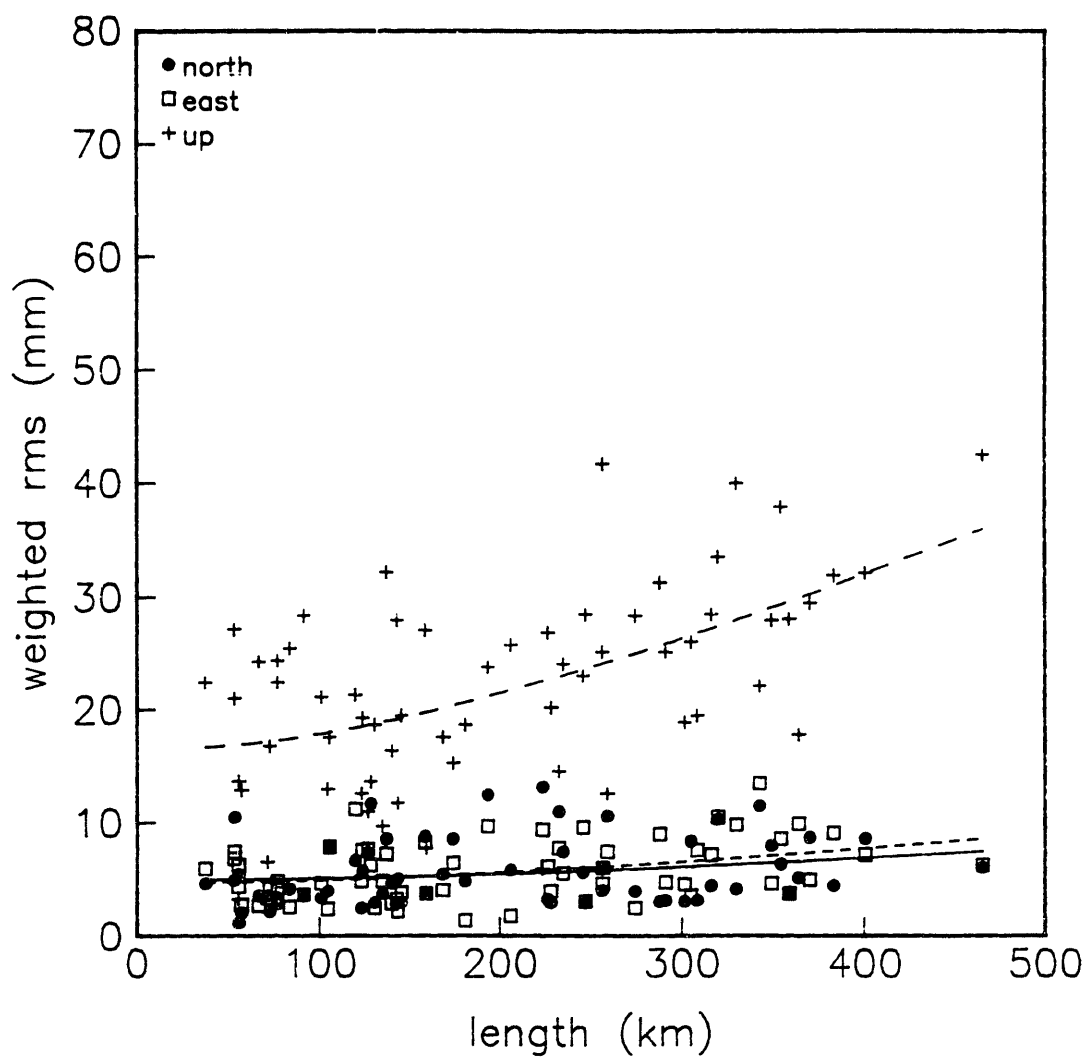


Figure 3.4b. TREX 2 (January 1987) biased-fixed baseline repeatabilities. Estimated constant and length-dependent errors: north (solid line) $a = 5.0$ mm, $b = 1.2 \times 10^{-8}$; east (short-dashed line) $a = 4.8$ mm, $b = 1.5 \times 10^{-8}$; up (long-dashed line) $a = 16.5$ mm, $b = 6.9 \times 10^{-8}$.

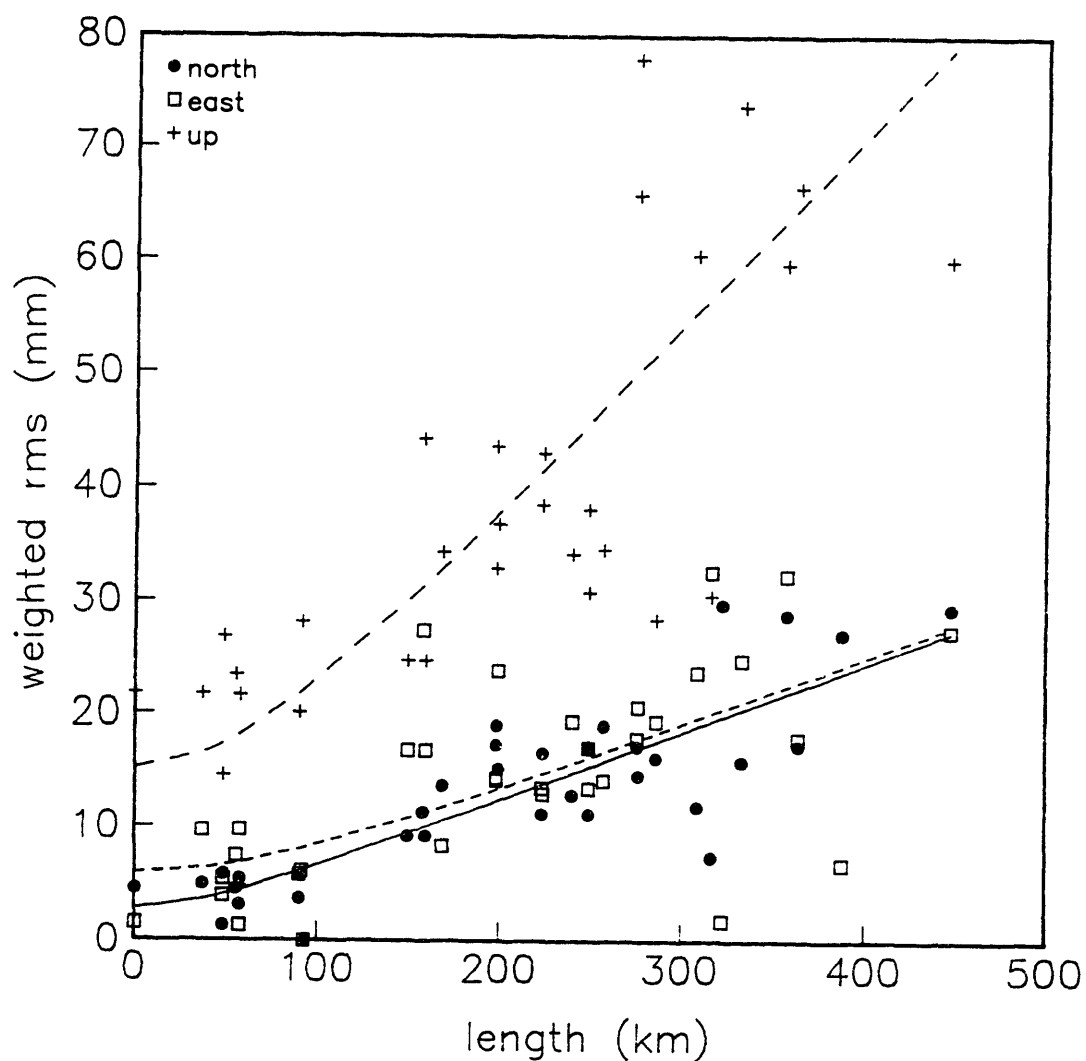


Figure 3.4c. TRES 1 (December 1986) biased-fixed baseline repeatabilities. Estimated constant and length-dependent errors: north (solid line) $a = 2.8$ mm, $b = 6.1 \times 10^{-8}$; east (short-dashed line) $a = 5.9$ mm, $b = 6.1 \times 10^{-8}$; up (long-dashed line) $a = 15.2$ mm, $b = 17.3 \times 10^{-8}$.

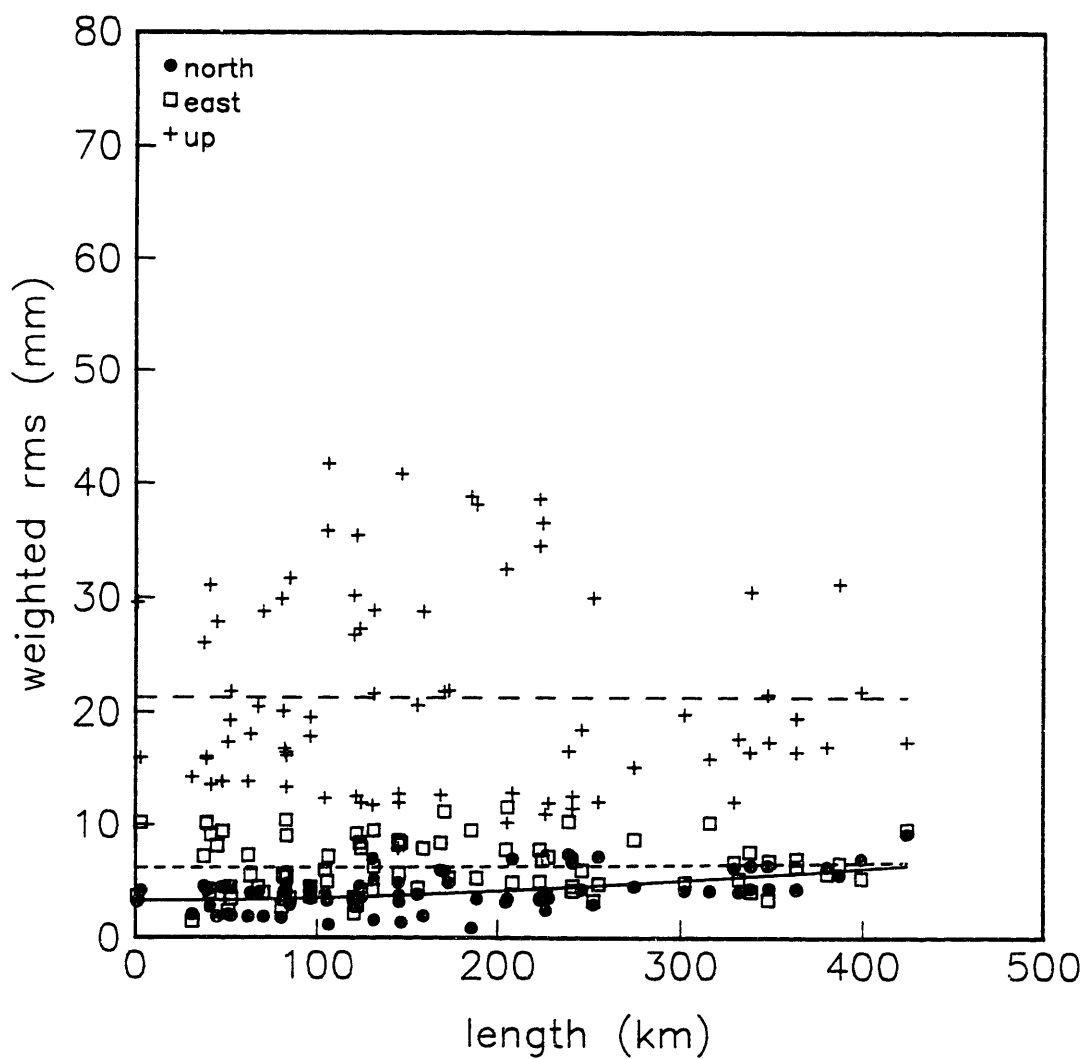


Figure 3.4d. TREX 3 (January 1987) biased-fixed baseline repeatabilities. Estimated constant and length-dependent errors: north (solid line) $a = 3.2$ mm, $b = 1.4 \times 10^{-8}$; east (short-dashed line) $a = 5.5$ mm, $b = 2.7 \times 10^{-8}$; up (long-dashed line) $a = 21.3$ mm, $b = 0$.

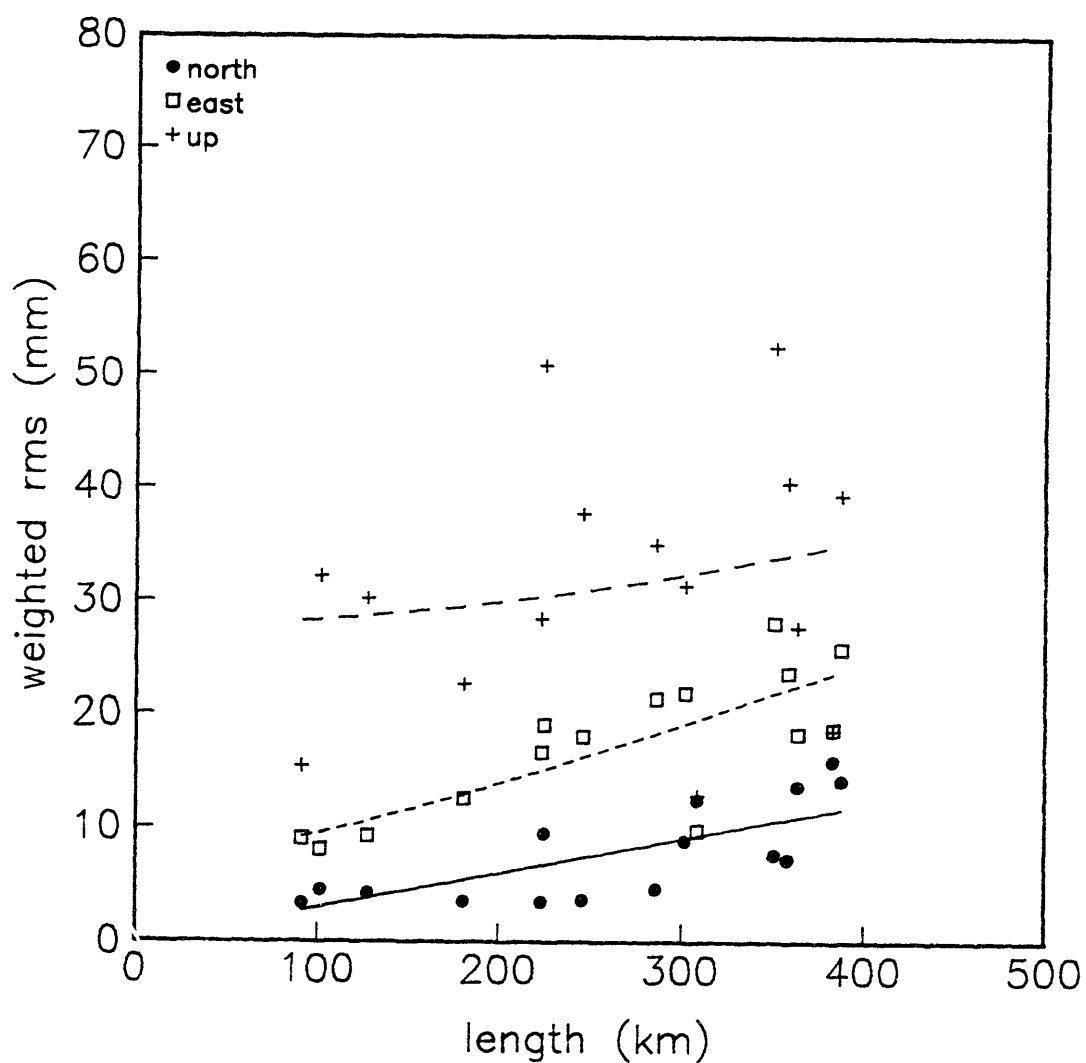


Figure 3.4e. TREX 7 (May 1987) biased-fixed baseline repeatabilities. Estimated constant and length-dependent errors: north (solid line) $a = 0.3$ mm, $b = 3.0 \times 10^{-8}$; east (short-dashed line) $a = 7.5$ mm, $b = 5.8 \times 10^{-8}$; up (long-dashed line) $a = 27.8$ mm, $b = 5.5 \times 10^{-8}$.

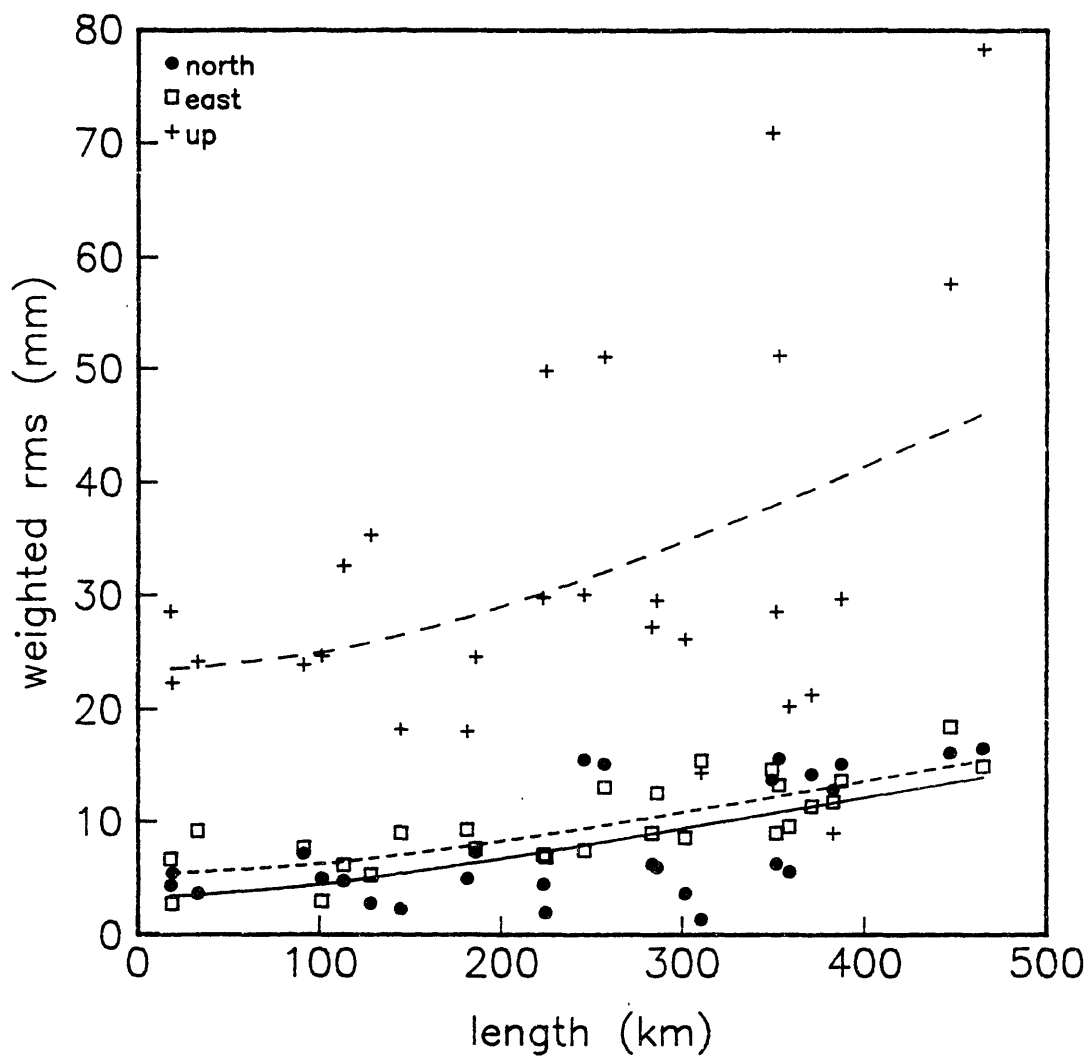


Figure 3.4f. TREX 8 (September 1987) biased-fixed baseline repeatabilities. Estimated constant and length-dependent errors: north (solid line) $a = 3.4$ mm, $b = 2.9 \times 10^{-8}$; east (short-dashed line) $a = 5.5$ mm, $b = 3.1 \times 10^{-8}$; up (long-dashed line) $a = 23.5$ mm, $b = 8.5 \times 10^{-8}$.

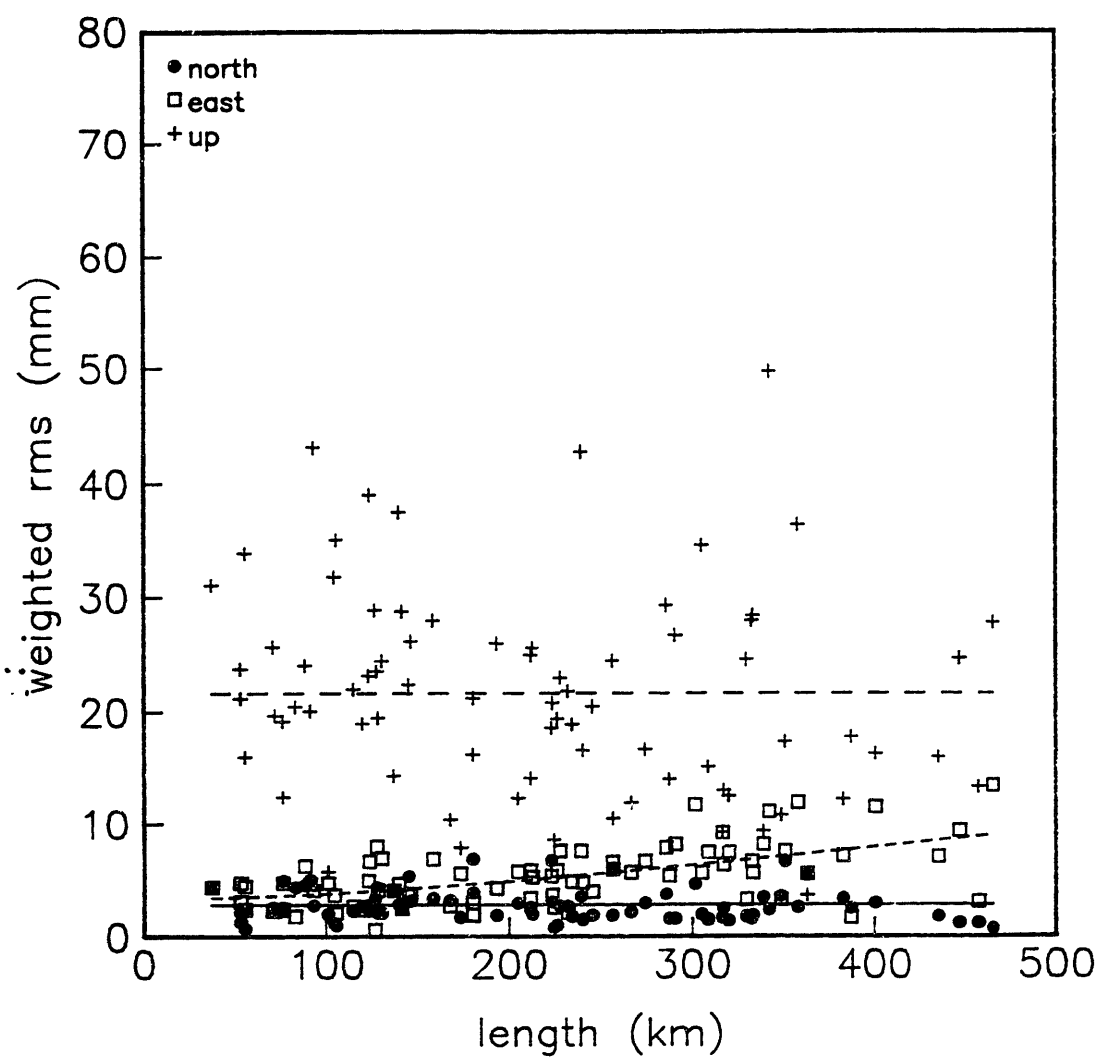


Figure 3.4g. TREX 11 (March 1988) biased-fixed baseline repeatabilities. Estimated constant and length-dependent errors: north (solid line) $a = 2.8$ mm, $b = 0$; east (short-dashed line) $a = 3.4$ mm, $b = 1.8 \times 10^{-8}$; up (long-dashed line) $a = 21.6$ mm, $b = 0$.

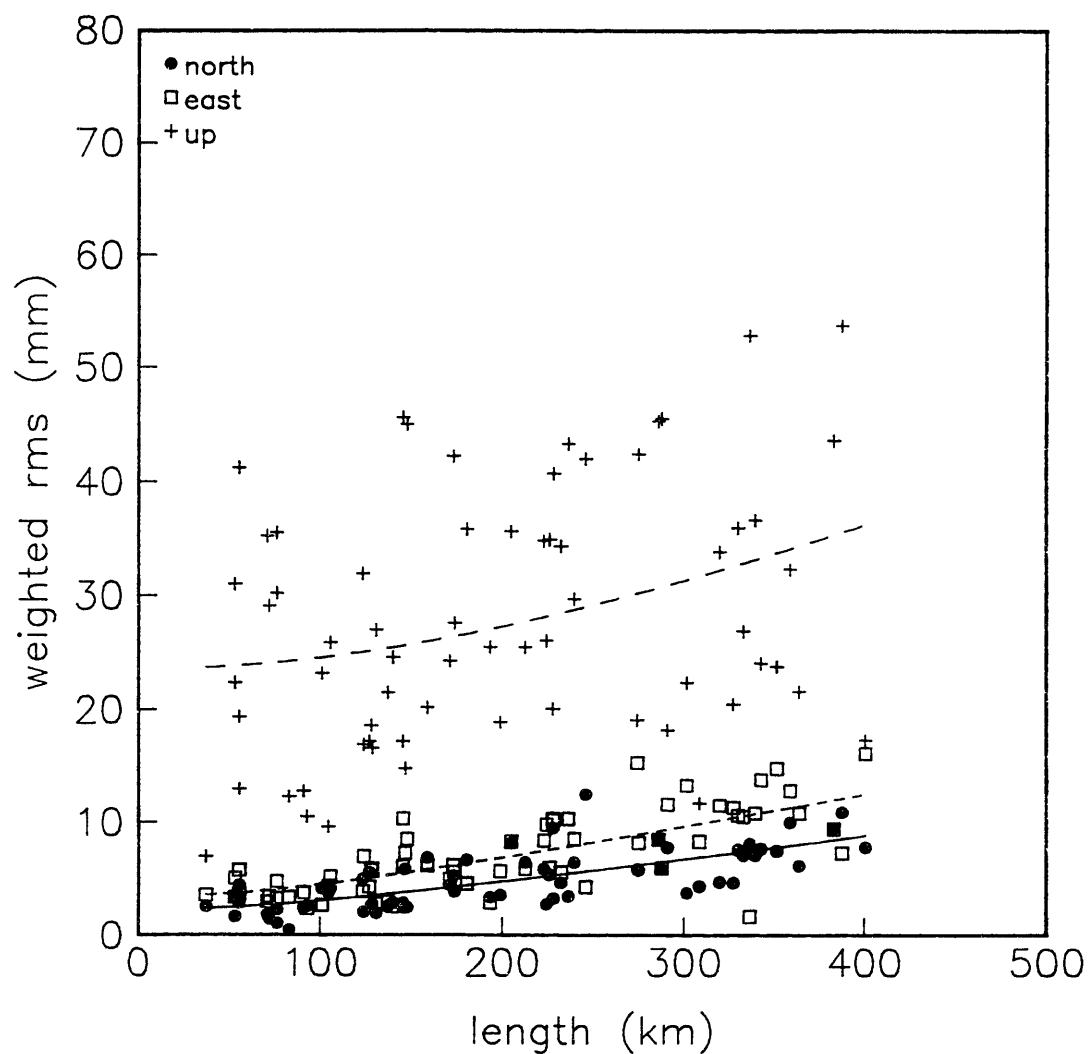


Figure 3.4h. TREX 14 (March 1989) biased-fixed baseline repeatabilities. Estimated constant and length-dependent errors: north (solid line) $a = 2.3$ mm, $b = 2.1 \times 10^{-8}$; east (short-dashed line) $a = 3.4$ mm, $b = 3.0 \times 10^{-8}$; up (long-dashed line) $a = 23.6$ mm, $b = 6.9 \times 10^{-8}$.

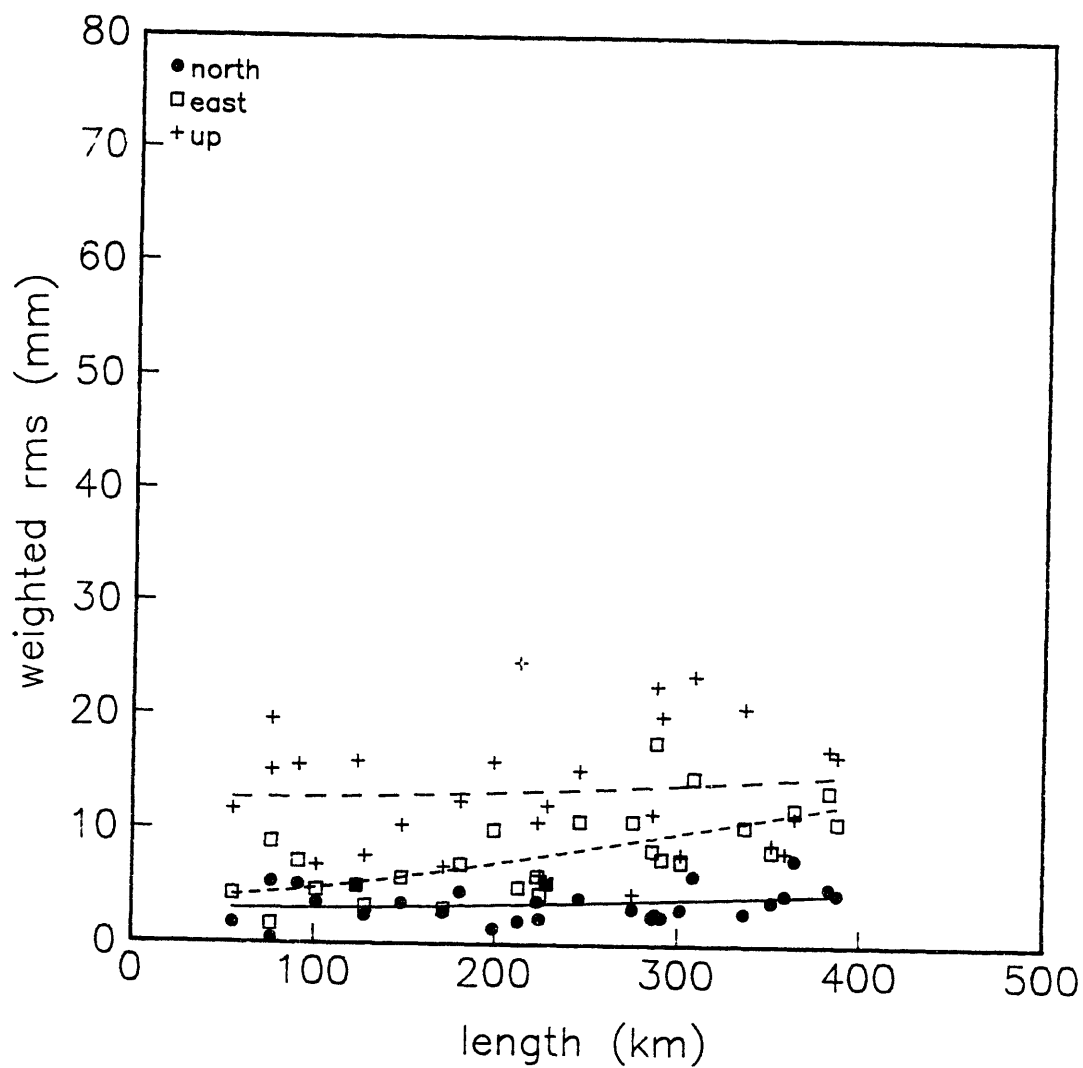


Figure 3.4i. TREX 17 (March 1990) biased-fixed baseline repeatabilities. Estimated constant and length-dependent errors: north (solid line) $a = 3.0$ mm, $b = 0.9 \times 10^{-8}$; east (short-dashed line) $a = 3.9$ mm, $b = 3.0 \times 10^{-8}$; up (long-dashed line) $a = 12.7$ mm, $b = 2.0 \times 10^{-8}$.

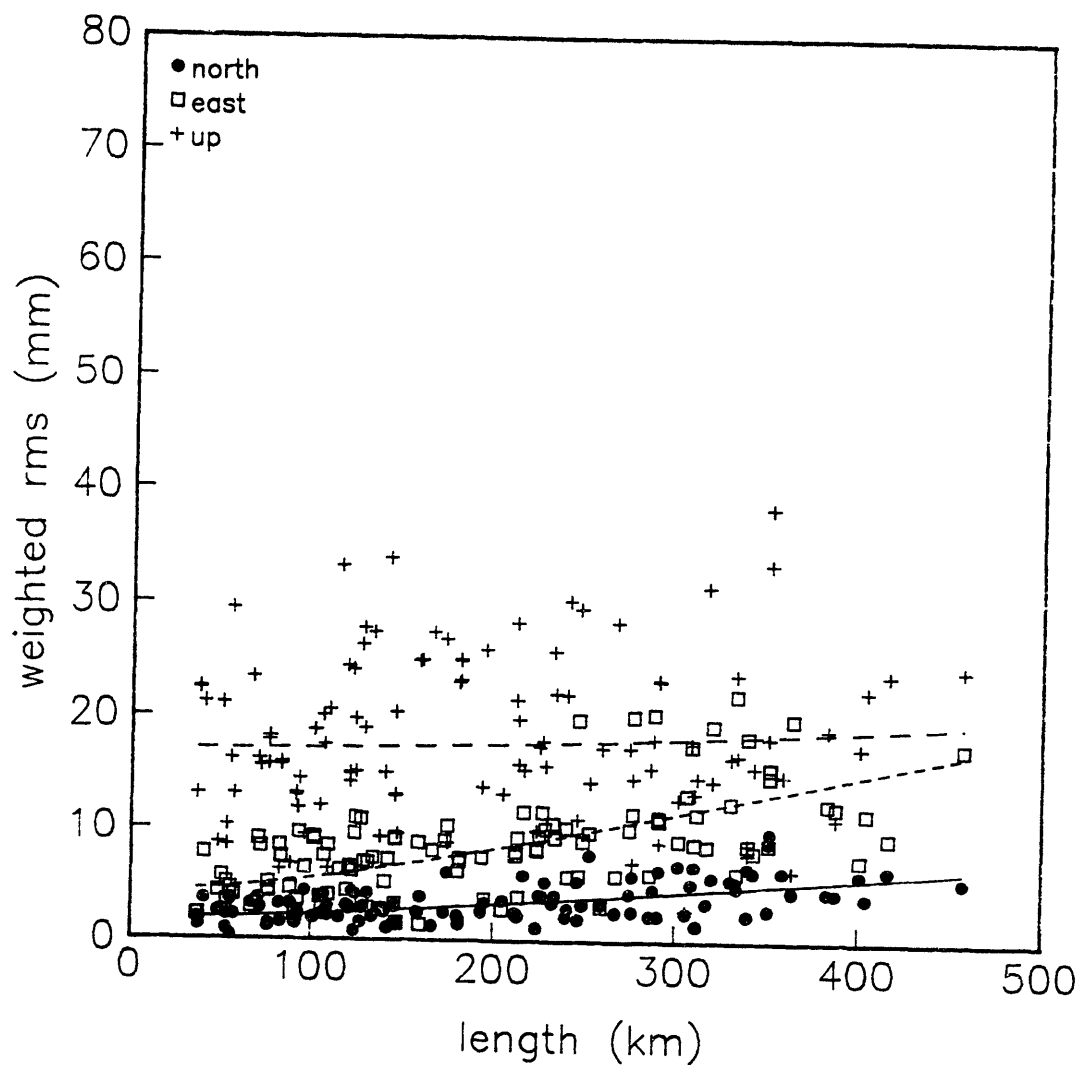


Figure 3.4j. TREX 20 (February 1991) biased-fixed baseline repeatabilities. Estimated constant and length-dependent errors: north (solid line) $a = 1.8$ mm, $b = 1.3 \times 10^{-8}$; east (short-dashed line) $a = 4.3$ mm, $b = 3.6 \times 10^{-8}$; up (long-dashed line) $a = 17.1$ mm, $b = 2.0 \times 10^{-8}$.

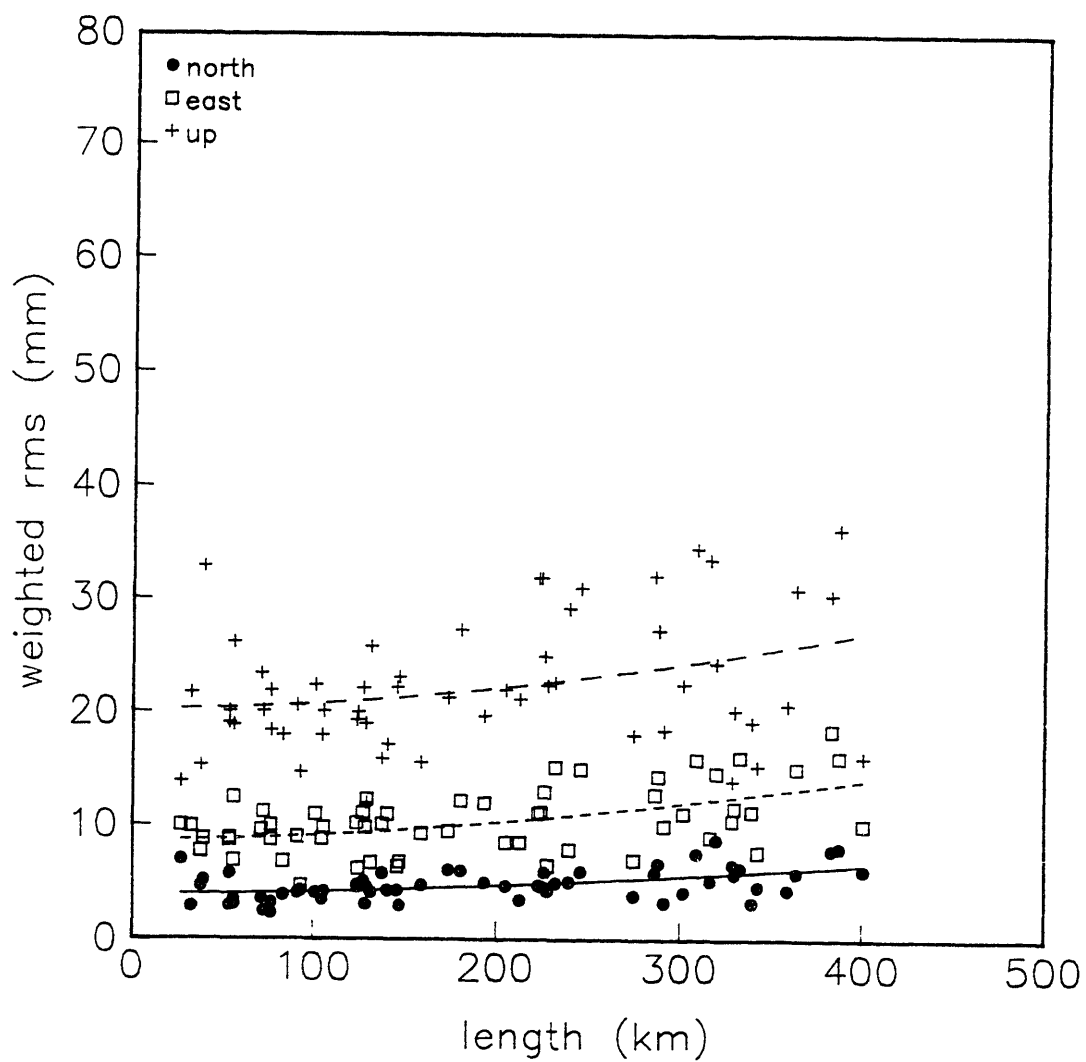


Figure 3.5a. Long-term weighted rms scatter repeatabilities about a constant velocity estimate for baseline rate of change based on bias-free baseline component estimates from all experiments as a function of baseline length. The length-independent mean value for each component is: (A) length = 7.5 mm, (B) north = 4.6 mm, (C) east = 10.1 mm, and (D) vertical = 22.6 mm.

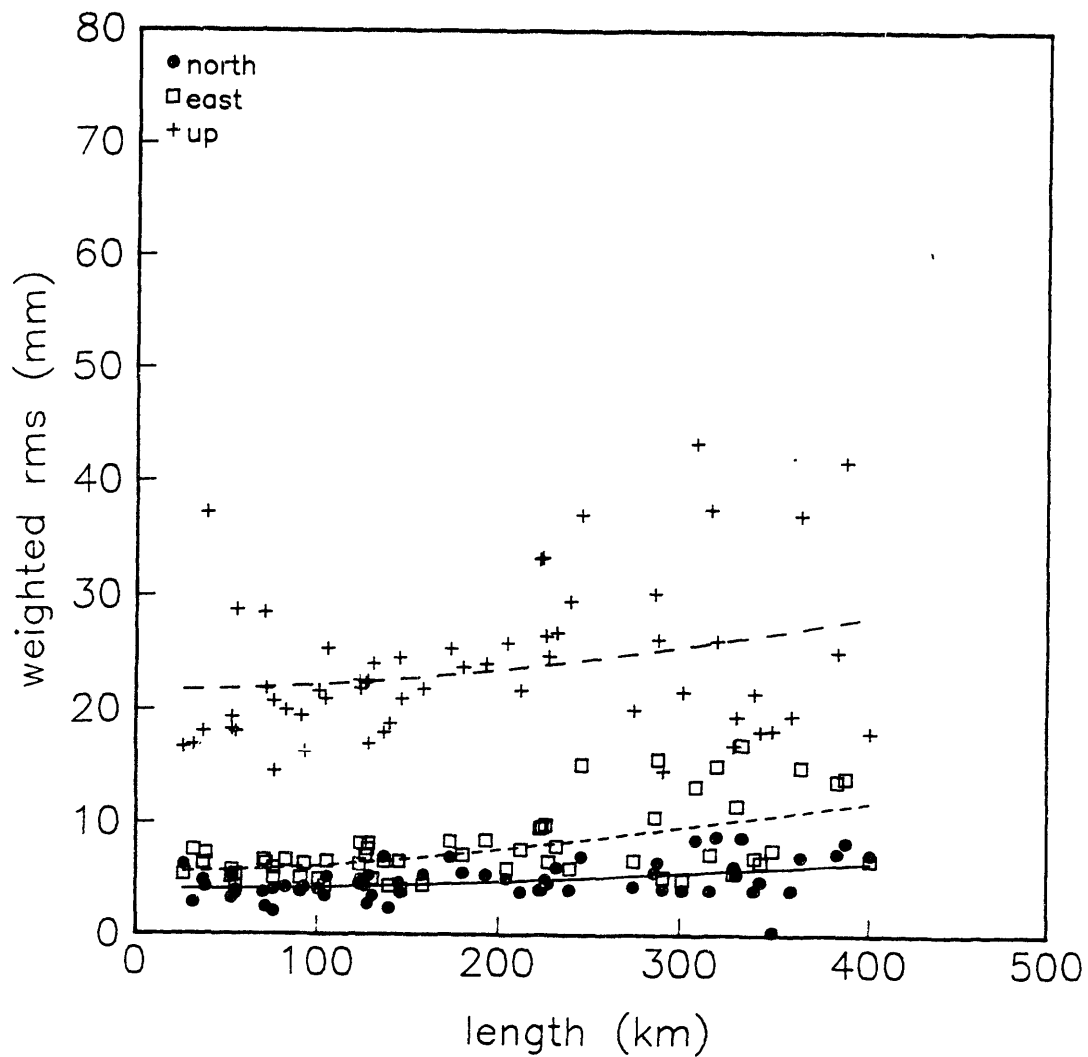


Figure 3.5b. Long-term weighted rms scatter repeatabilities about a constant velocity estimate for baseline rate of change based on bias-fixed baseline component estimates from all experiments as a function of baseline length. The length-independent mean value for each component is: (A) length = 6.2 mm, (B) north = 4.7 mm, (C) east = 7.8 mm, and (D) vertical = 23.9 mm.

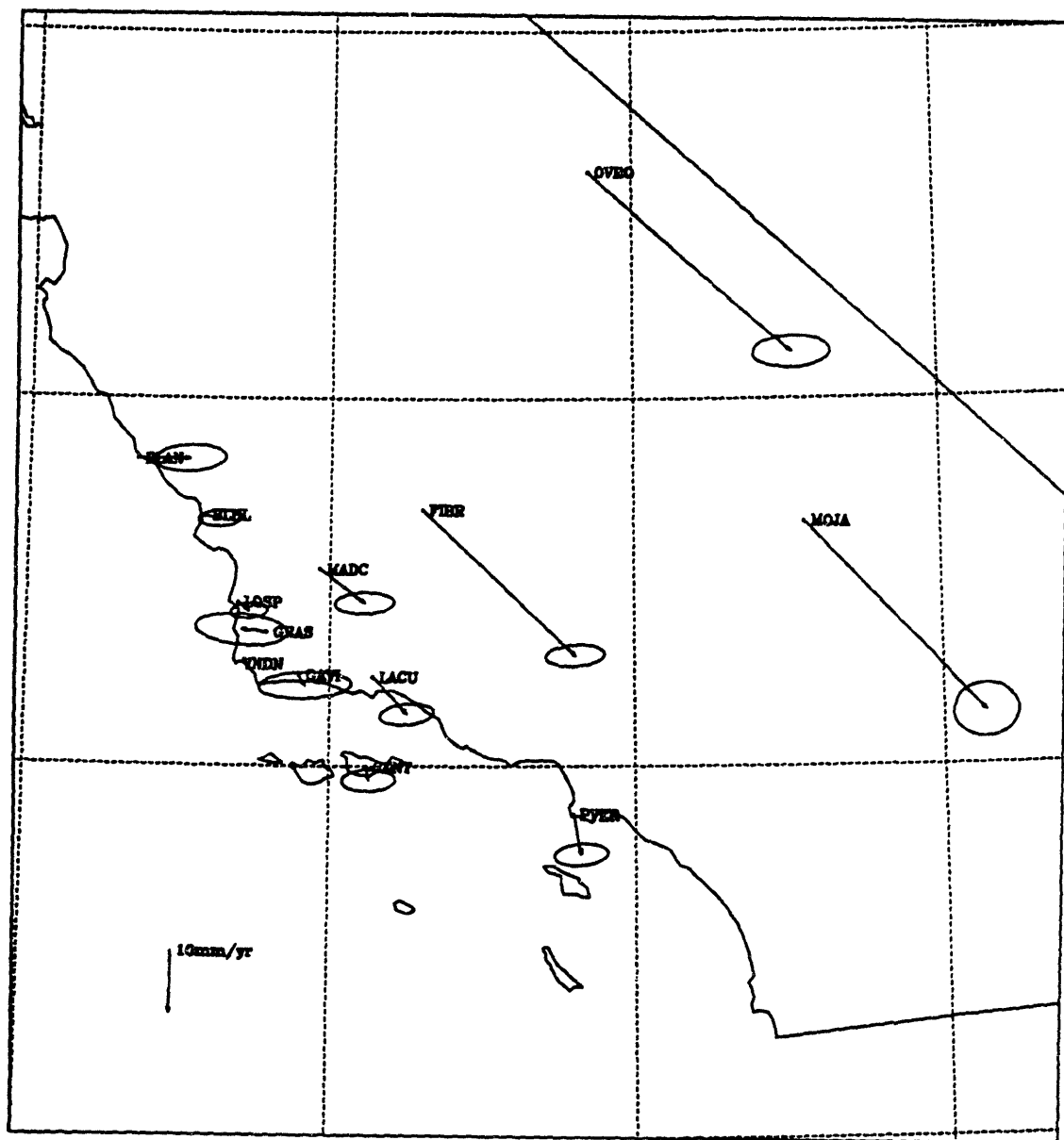


Figure 3.6a. Bias-free horizontal velocities for California sites with respect to Vandenberg. The error ellipse represent 95% confidence levels that have been scaled by the chi-square per degrees of freedom of the Kalman filter adjustment. A 10 mm/yr scale is shown.

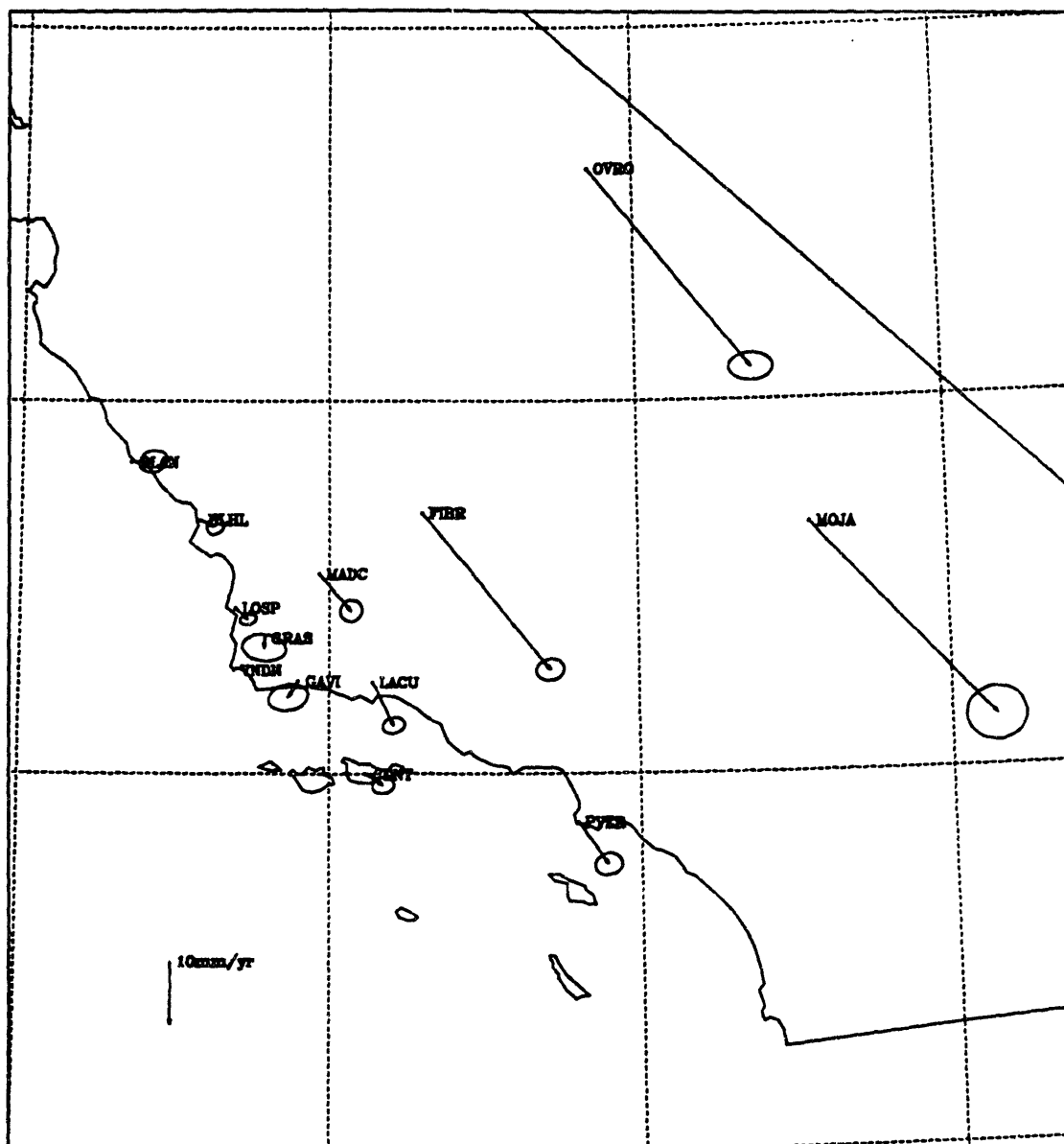


Figure 3.6b. Bias-fixed horizontal velocities for California sites with respect to Vandenberg. The error ellipse represent 95% confidence levels that have been scaled by the chi-square per degrees of freedom of the Kalman filter adjustment. A 10 mm/yr scale is shown.

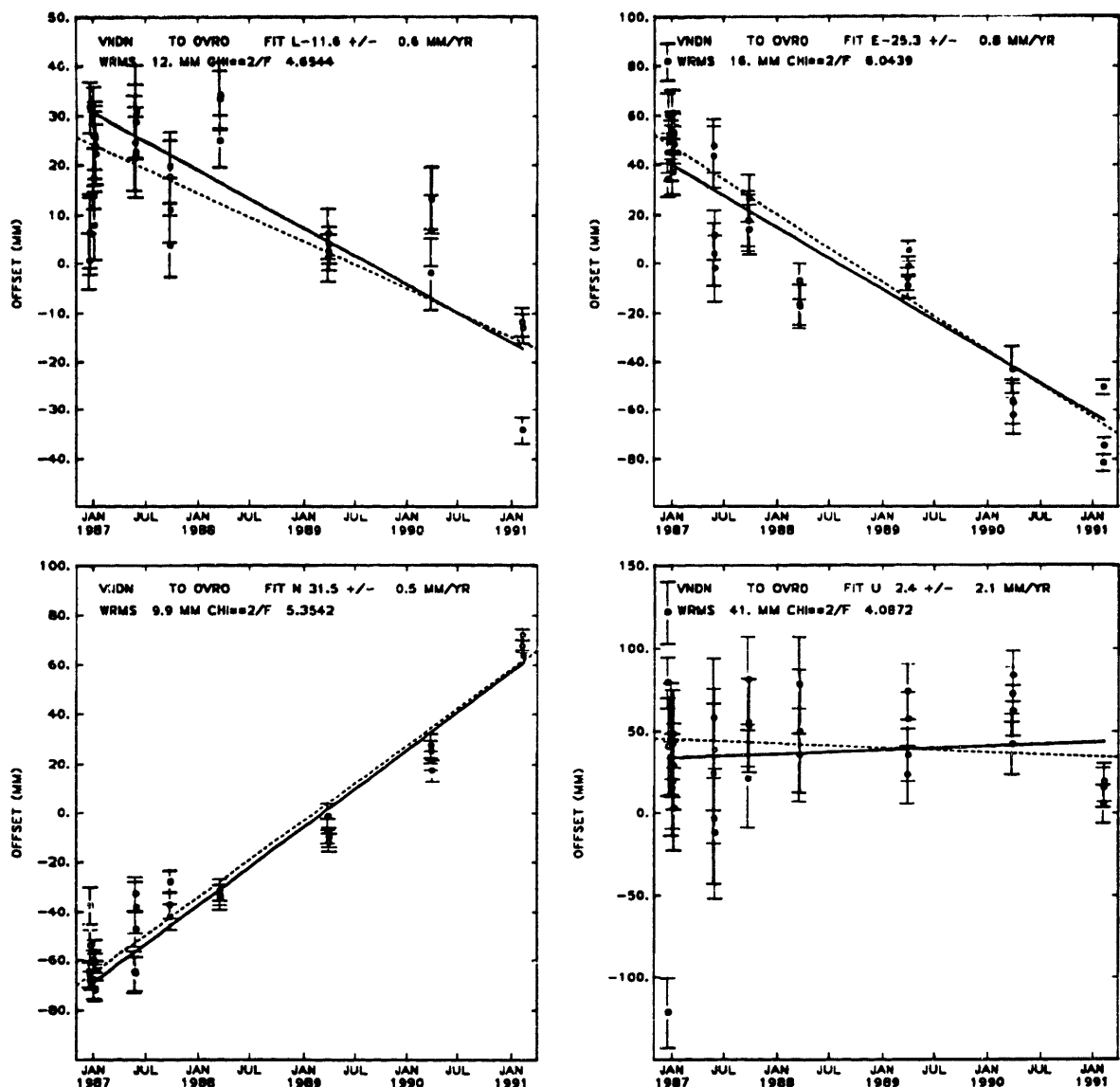


Figure 3.7a. The length and north, east, and up components of the Vandenberg to Owens Valley baseline. Length is plotted in the upper left-hand corner, the east component is plotted in the upper right-hand corner, the north component is plotted in the lower left-hand corner, and up is plotted in the lower right-hand corner. The bias-fixed estimates with respect to a nominal mean are plotted as a function of time with their unscaled formal uncertainties. The solid line is the formal Kalman filter estimate, whose slope and slope uncertainty are printed on the top line of each plot. The dashed line is the weighted fit to the observations without taking into account correlations with other components of the baseline. The weighted rms scatter and the χ^2/f of the residuals about the dashed line are printed on the second line of each plot.

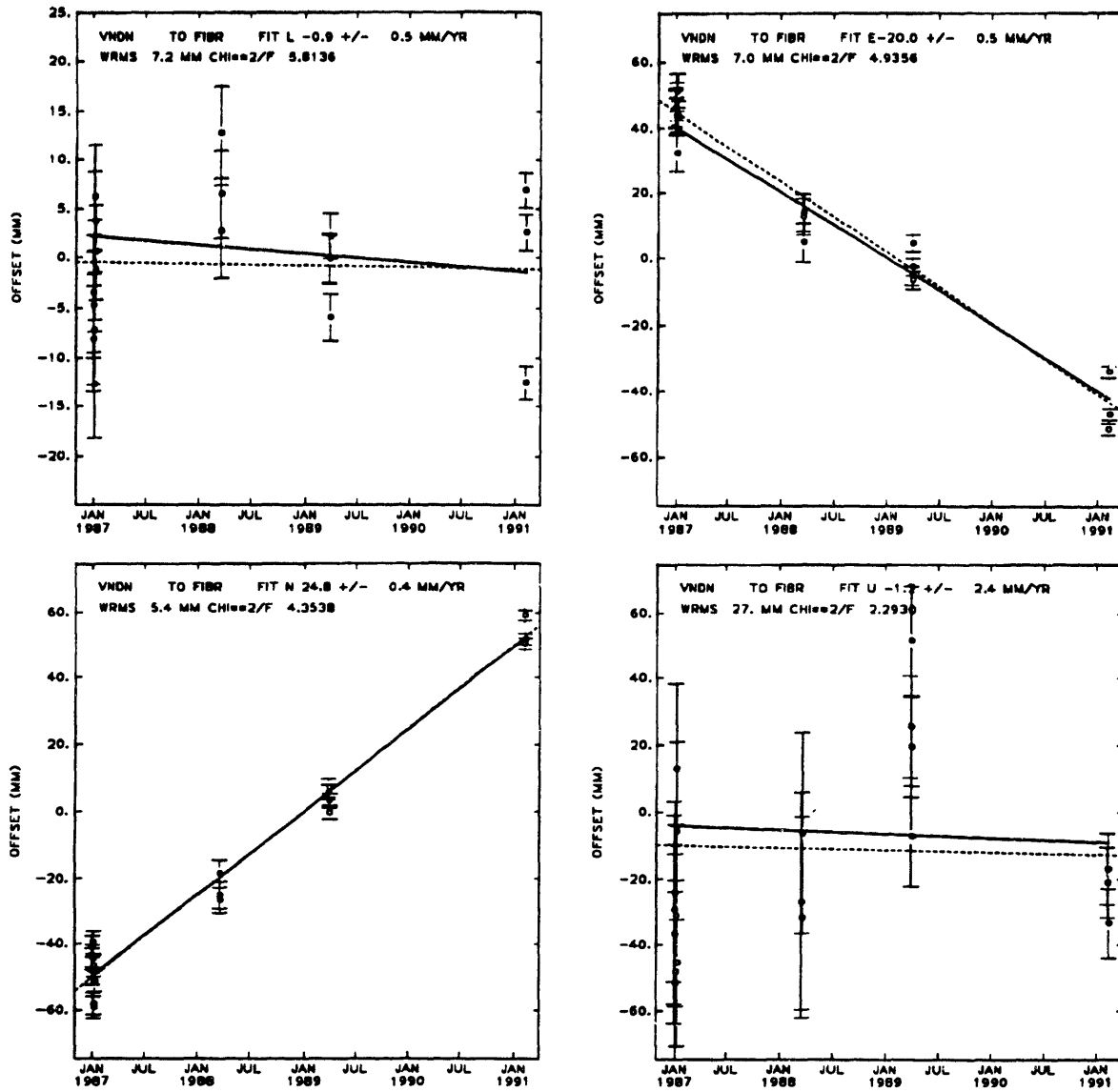


Figure 3.7b. Vandenberg with respect to Fibre (see Figure 3.7a for description)

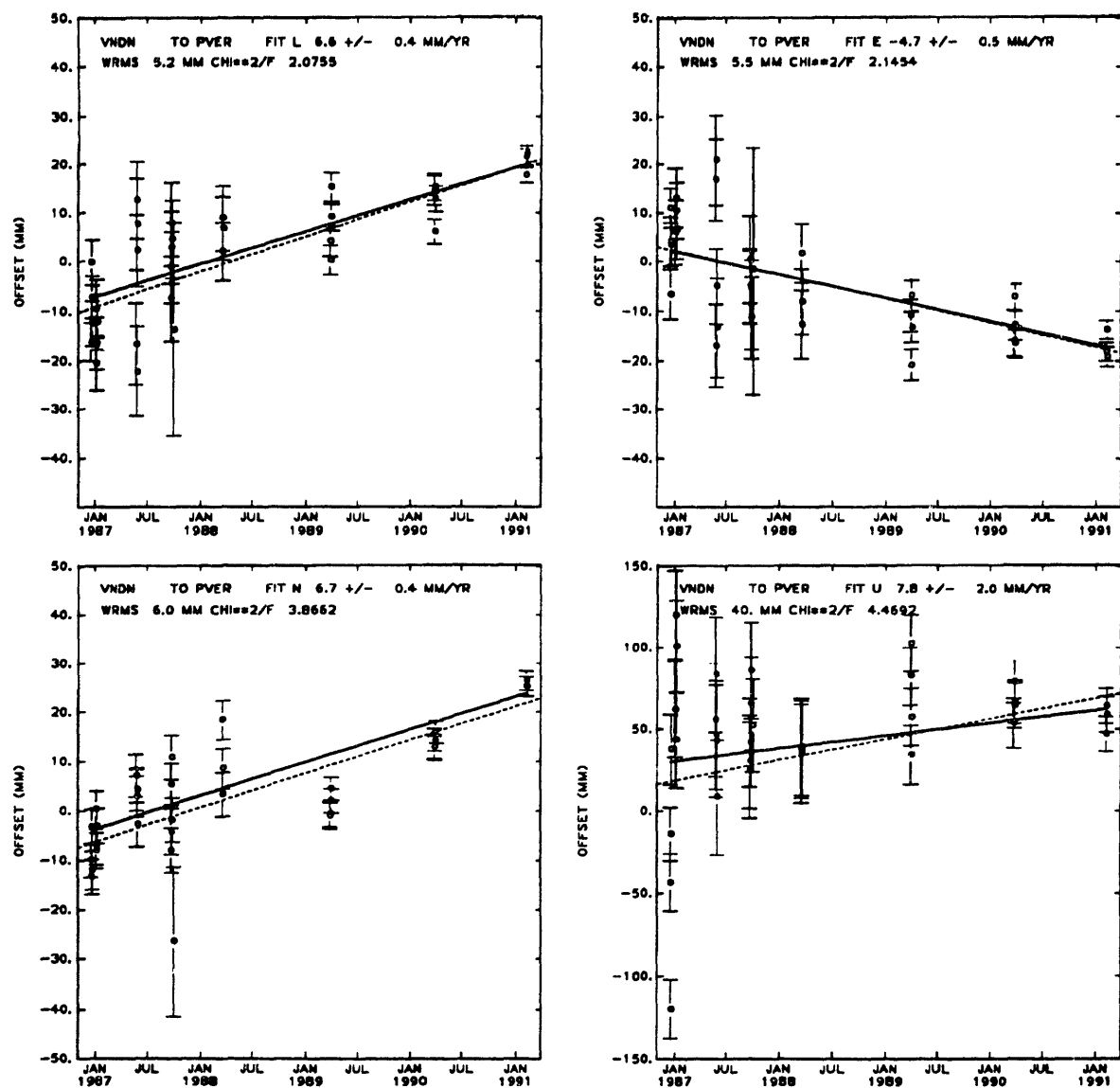


Figure 3.7c. Vandenberg with respect to Palos Verdes (see Figure 3.7a for description)

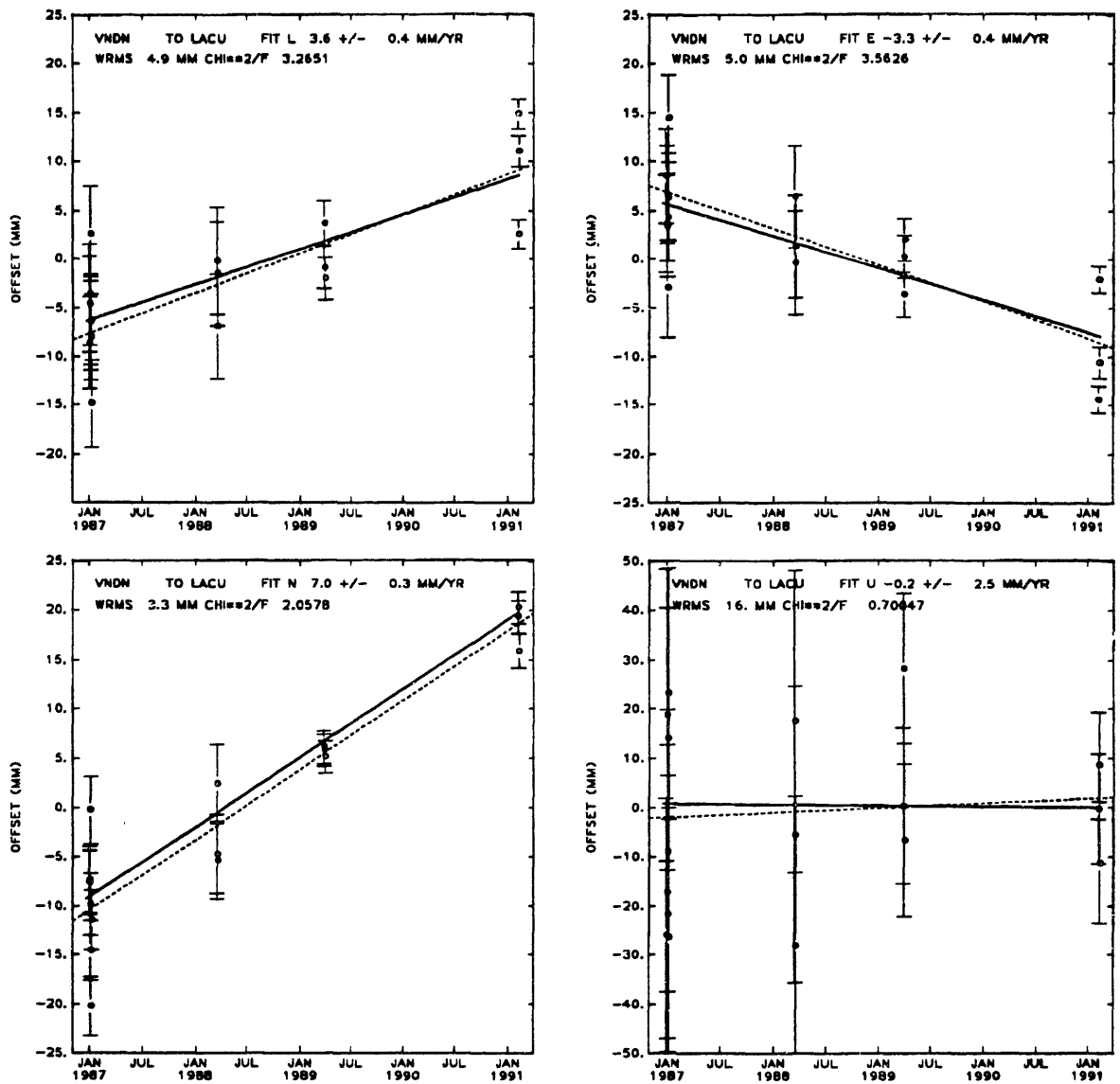


Figure 3.7d. Vandenberg with respect to La Cumbre (see Figure 3.7a for description)

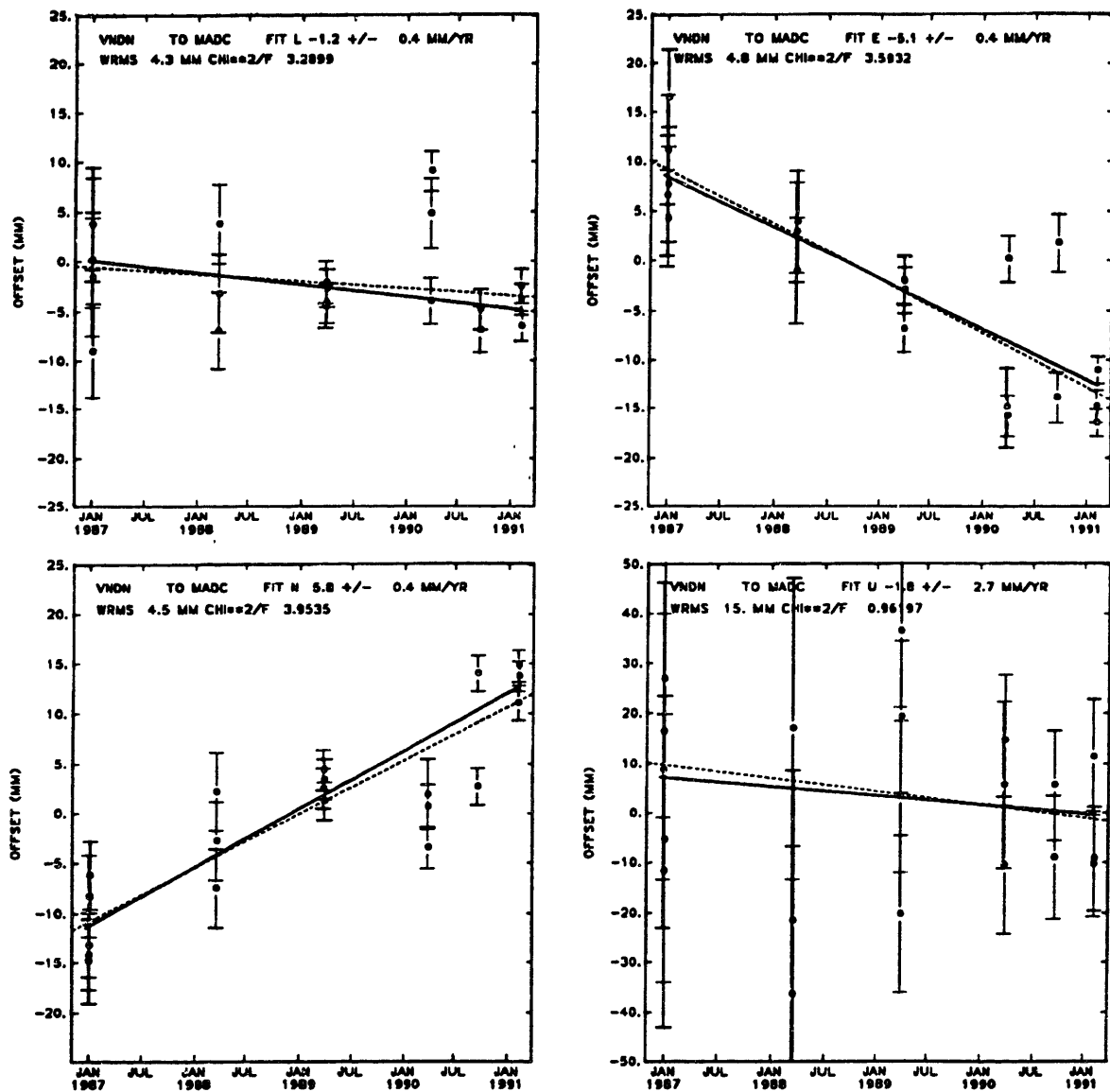


Figure 3.7e. Vandenberg with respect to Madre (see Figure 3.7a for description)

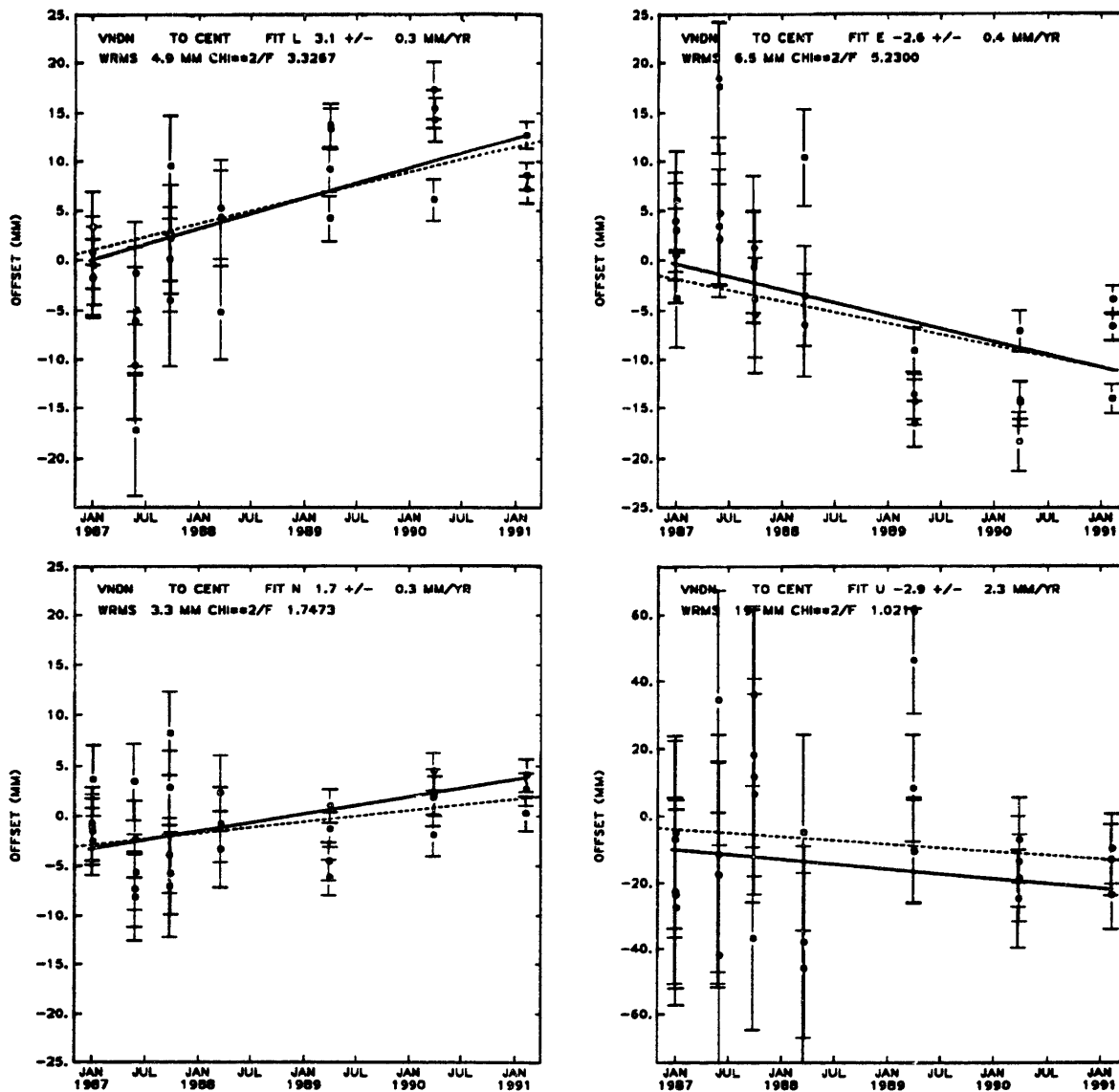


Figure 3.7f. Vandenberg with respect to Center (see Figure 3.7a for description)

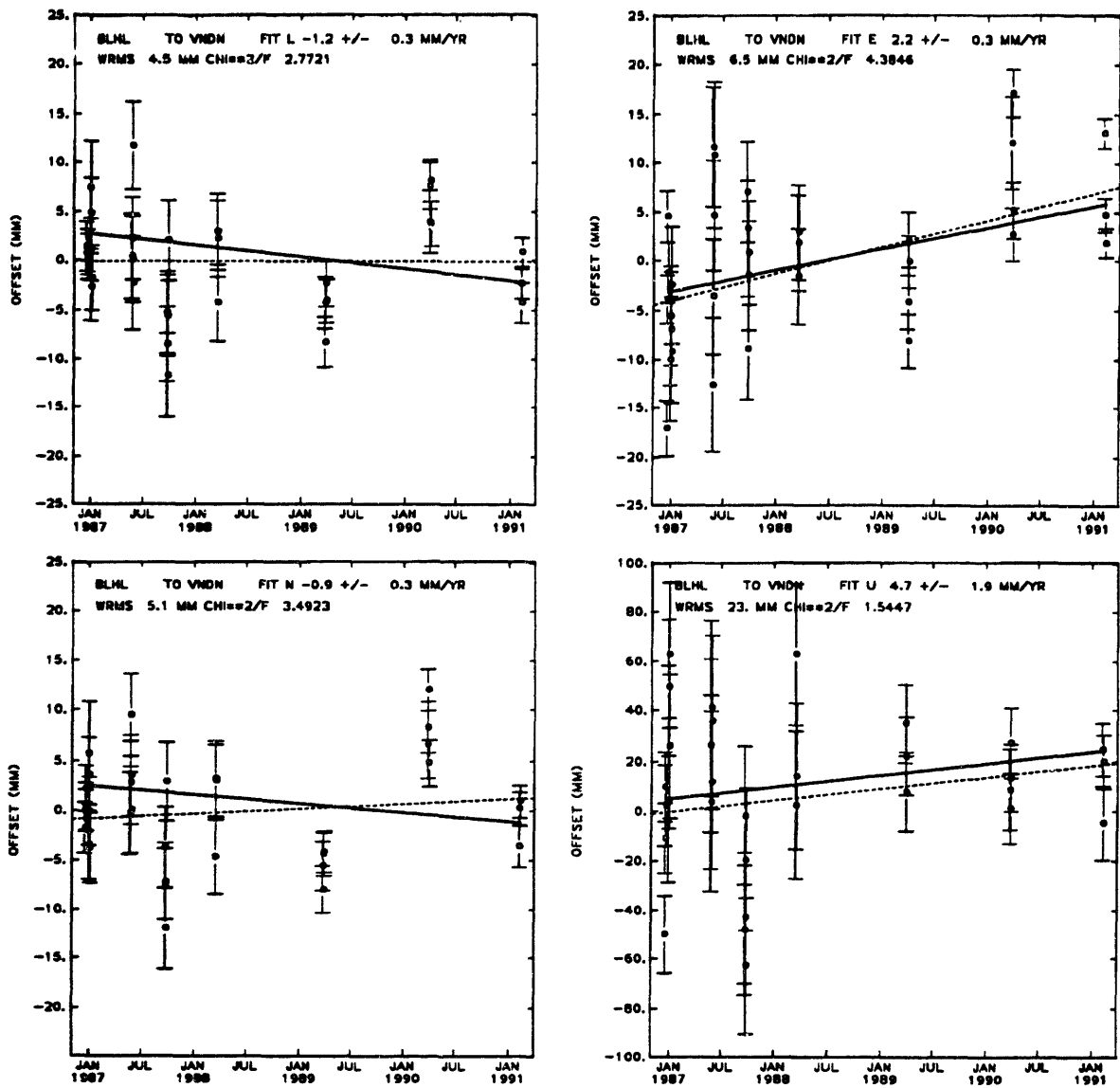


Figure 3.7g. Black Hill with respect to Vandenberg (see Figure 3.7a for description)

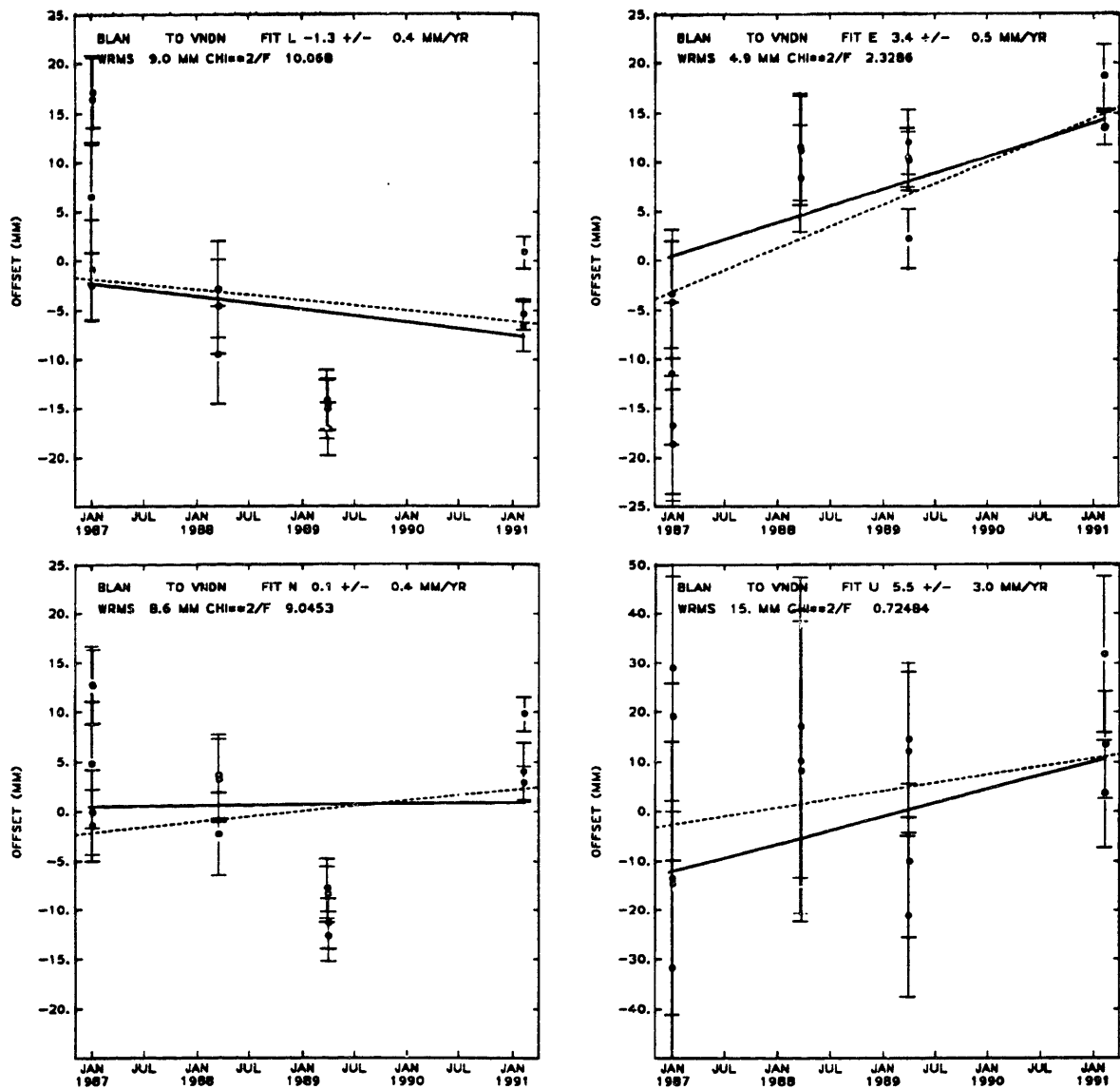


Figure 3.7h. Piedras Blancas with respect to Vandenberg (see Figure 3.7a for description)

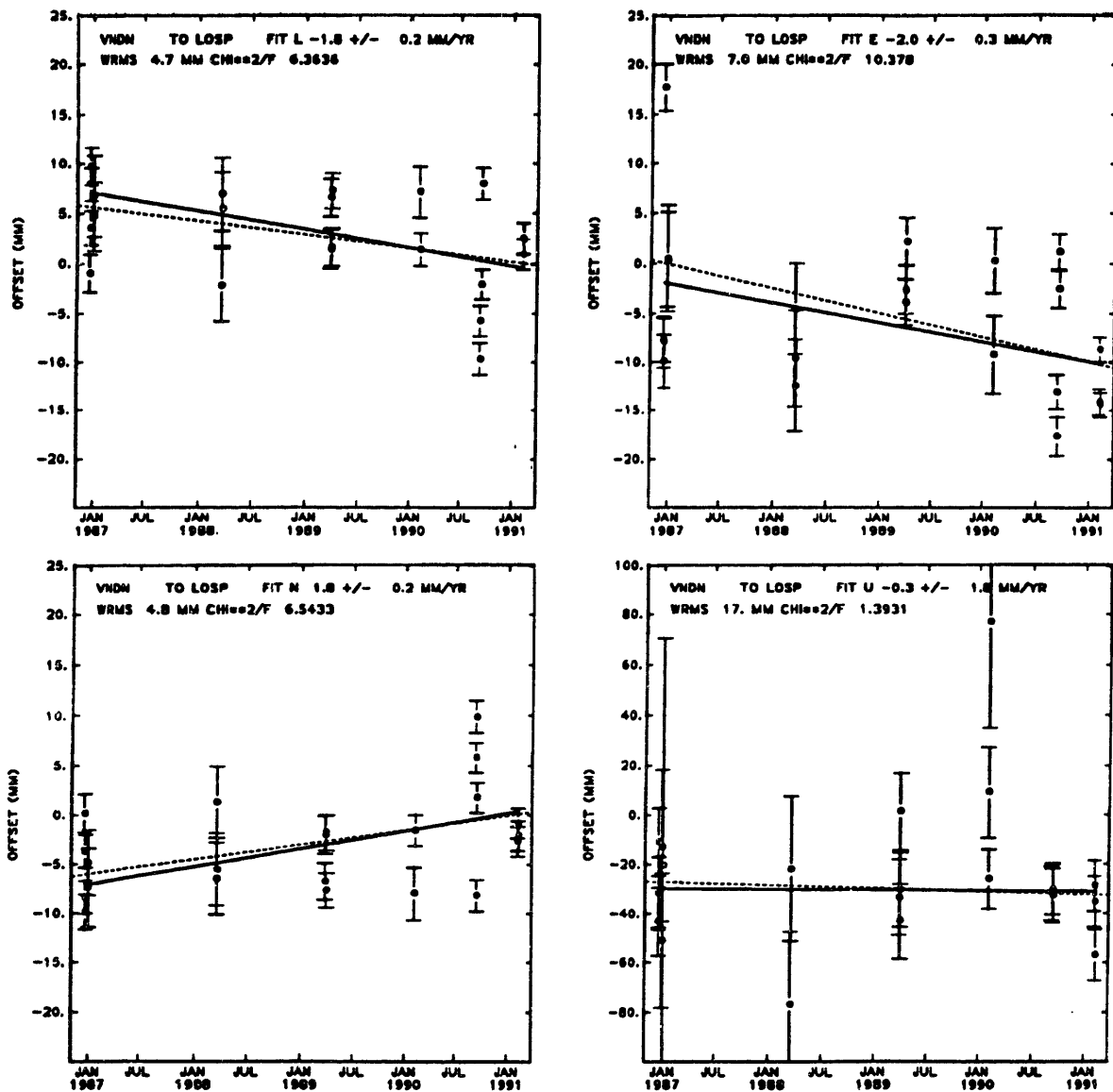


Figure 3.7i. Vandenberg with respect to Lospe (see Figure 3.7a for description)

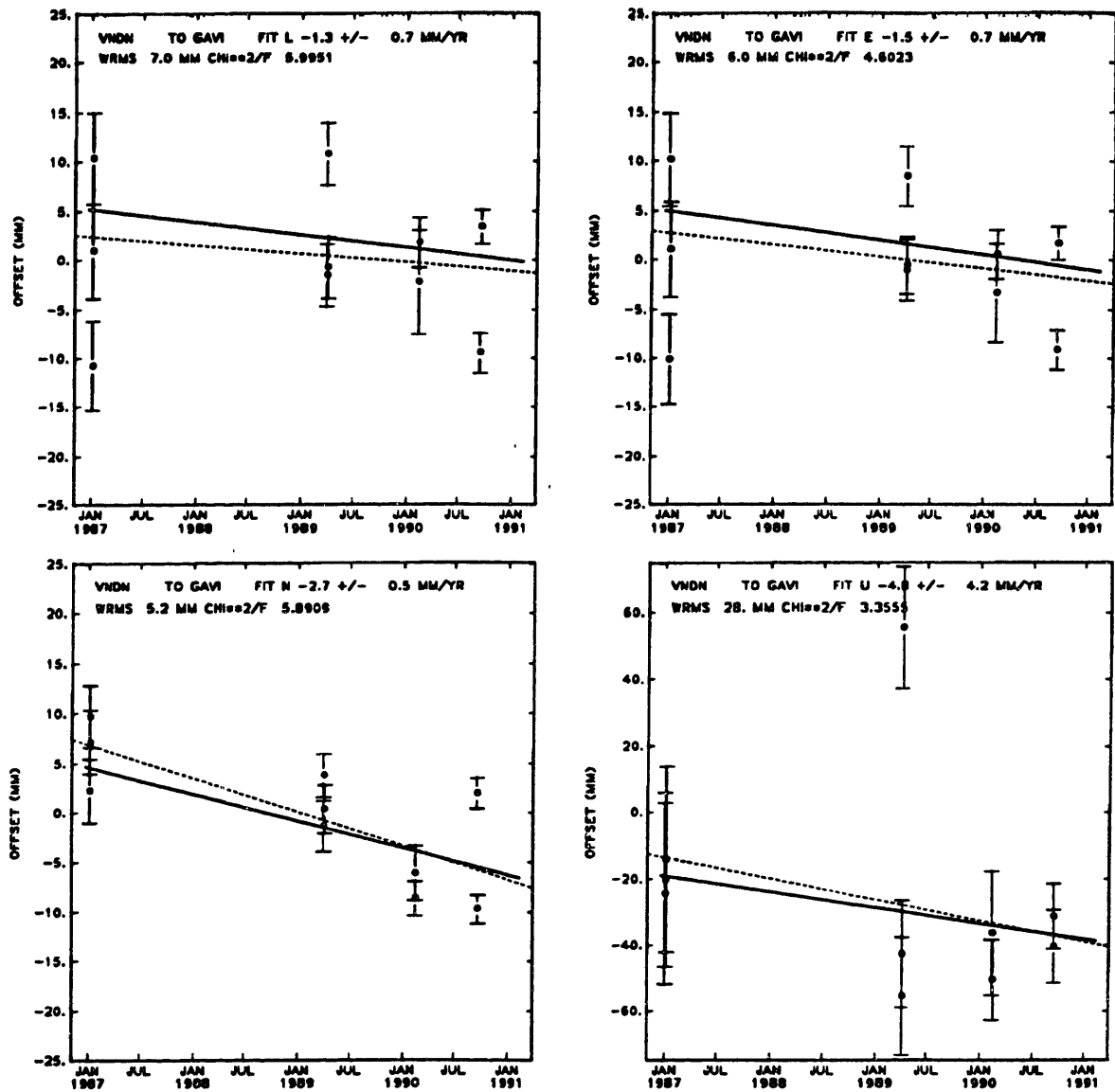


Figure 3.7j. Gaviota with respect to Vandenberg (see Figure 3.7a for description)

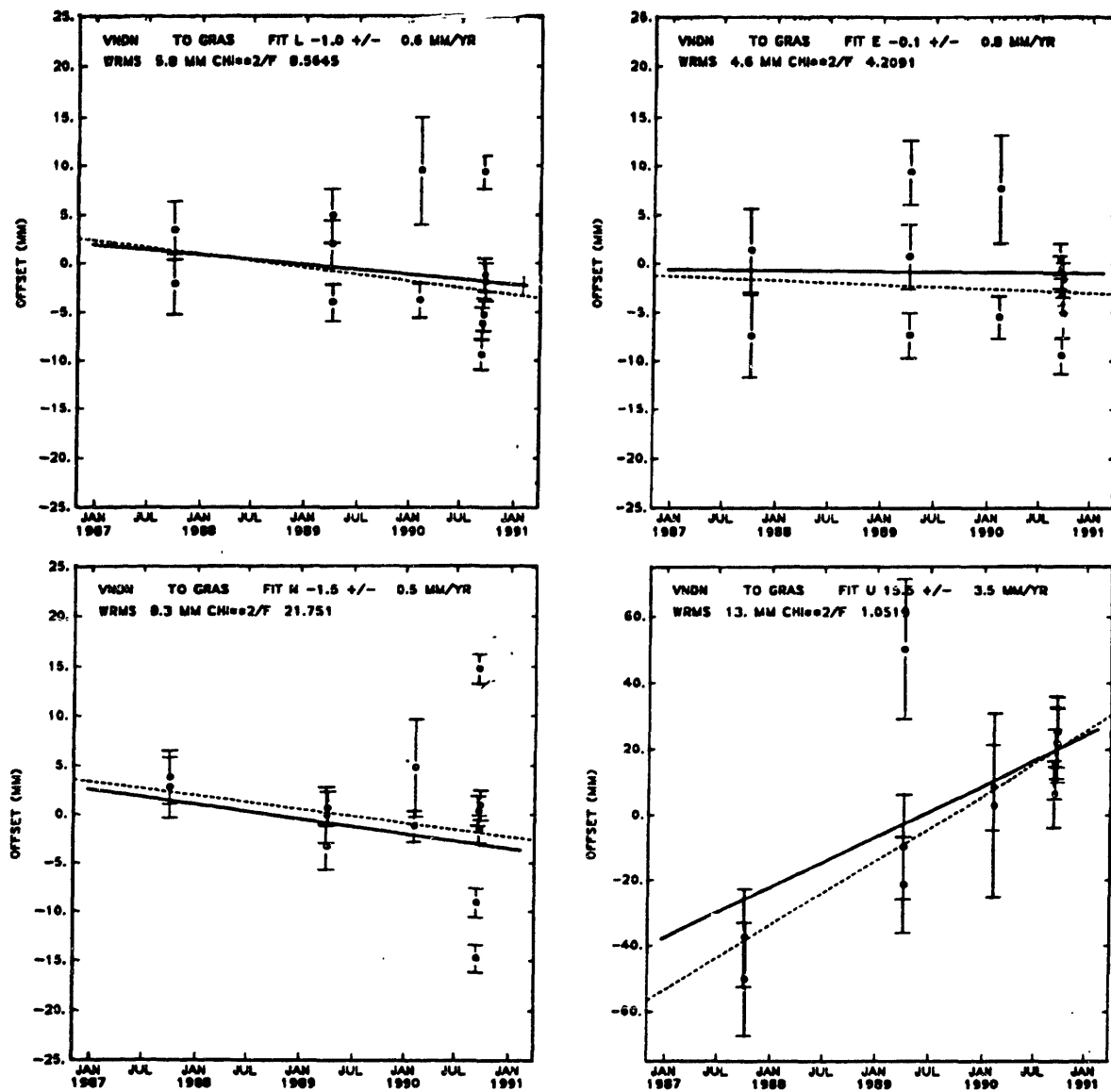


Figure 3.7k. Grassy with respect to Vandenberg (see Figure 3.7a for description)

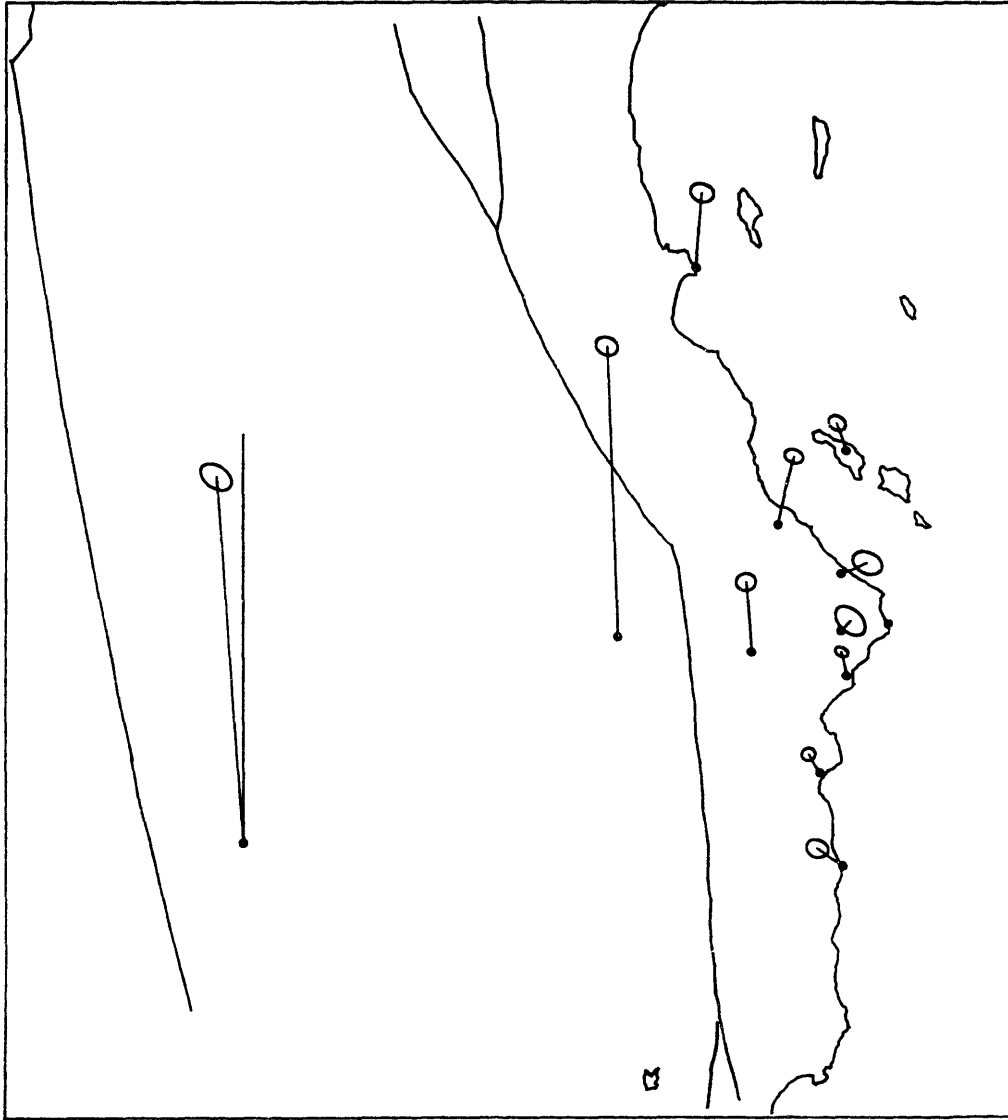


Figure 3.8. Oblique Mercator map of California and the San Andreas fault system about the NUVEL-1 North America-Pacific relative motion Euler pole (48.7°N , 78.2°). Relative site motions with respect to Vandenberg are the same as in Figure 3.6b, except that Mojave has been omitted. The Owens Valley vector lacking a 95% confidence error ellipse is the NUVEL-1 predicted relative motion (44.9 mm/yr) which in this projection follows a line of latitude.

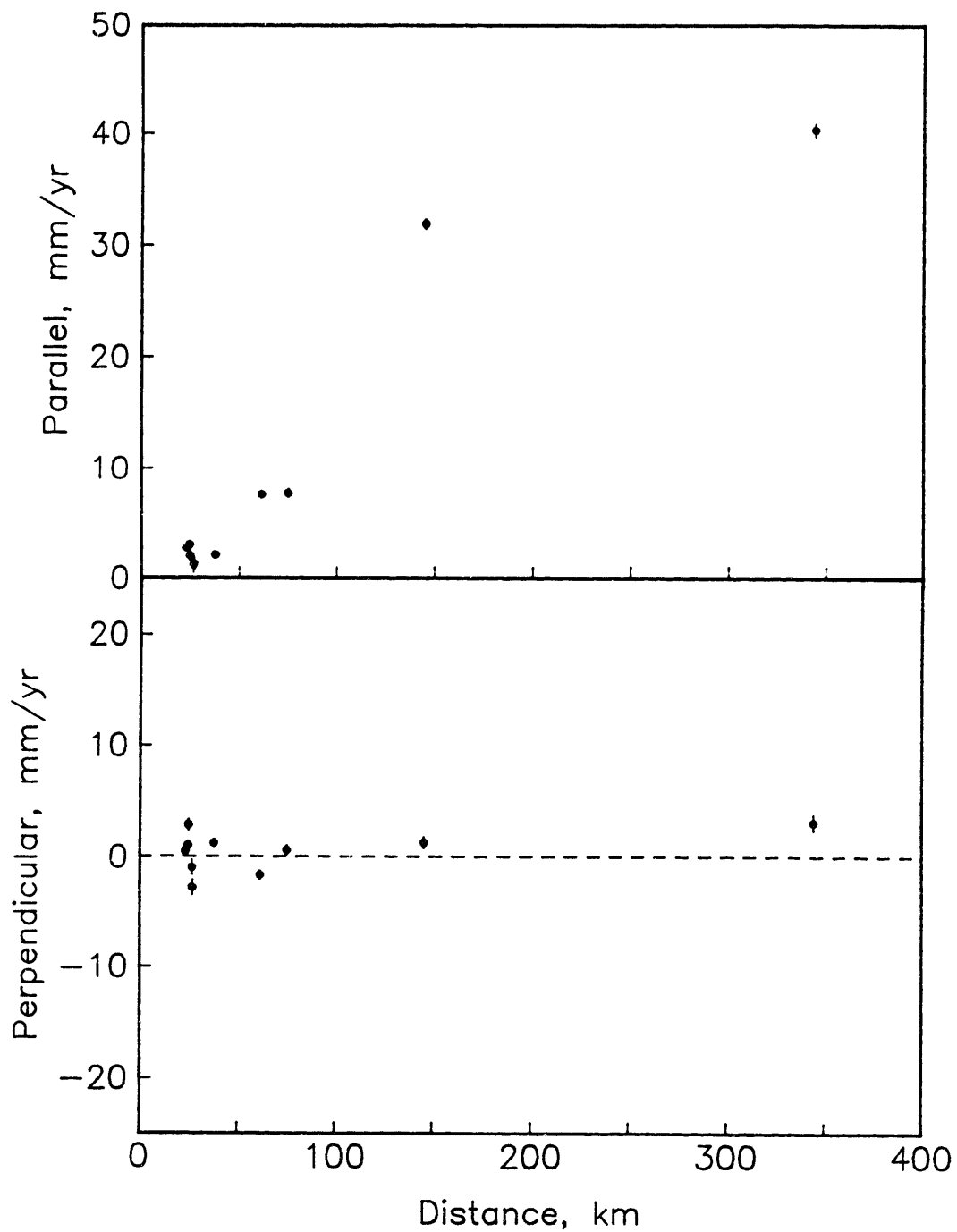


Figure 3.9. The parallel and perpendicular components of the relative motions in Figure 3.8 with respect to the relative plate motion direction are plotted as a function of distance from Vandenberg. One-sigma unscaled error bars are shown. Palos Verdes has been omitted for clarity.

CHAPTER 4. CONCLUSIONS

Geodetic techniques, such as the Global Positioning System (GPS), provide useful tools for studying the kinematics of crustal deformation, such as in the ‘wide, soft boundary’ between the North America and Pacific plates. These techniques make their measurements with respect to absolute reference frames that enable them to estimate all the components of the vector velocity gradient field within zones of deformation, including block rotations that cannot be resolved by terrestrial geodetic techniques. GPS, due to its simple and economical operating requirements and (eventual) continuous access to source signals, is ideally suited for continuously monitoring tectonic motions. The ongoing development of globally distributed fiducial networks promises to extend these capabilities to all points on the Earth’s surface. Such networks have the potential for continuously monitoring crustal deformation, and can thus be useful for studies of coseismic and postseismic slip on faults and with the possibility of detecting precursory seismic or volcanic activity.

In Chapter 2, we presented a methodology for deriving a terrestrial reference frame suitable for the requirements of monitoring crustal deformation with GPS. Precise knowledge of the locations of orbit-tracking (or fiducial) receivers improves the precision of the relative locations of other receivers of interest. To determine the precise relative locations of certain receivers, and hence to define a terrestrial reference frame, we utilize coordinates derived from laser ranging to LAGEOS and from very-long-baseline interferometry (VLBI) measurements. SLR coordinates, whose estimation also requires precise modeling of the orbit of LAGEOS, are sensitive to the location of the geocenter. VLBI coordinates lack all sensitivity to the geocenter (since the radio sources are essentially infinitely distant), but provide highly-precise relative locations. By combining the coordinates derived from these two space-geodetic techniques, we can derive a

terrestrial reference frame with high-precision relative locations and whose origin is located at a dynamically determined geocenter. We presented a methodology for systematically combining temporally heterogeneous space-geodetic observations with geophysical and geological information, such as global plate tectonic motion models. We apply this methodology to two recent adjustments of SLR and VLBI observations, and derive a terrestrial reference frame, designated SV6 (SV for SLR/VLBI), suitable for monitoring crustal deformation with GPS. SV6 provides estimates of the temporal evolution and uncertainties of the coordinates from VLBI and geophysical observations and we develop a methodology for systematically combining temporally heterogeneous space-geodetic observations with geophysical and geological information, such as global plate tectonic motion models.

In Chapter 3, we described a five-year program of GPS observations recently conducted along central and southern California margin. Two of the objectives of this program were to assess the long-term precision and accuracy of GPS measurements and to establish a network for monitoring crustal deformation. We analyzed the observations from 13 campaigns during 1986–1991, the majority of which were conducted over 4–5 days using between 12–18 receivers in California. The data offer sufficient redundancy to assess the short and long-term precision of GPS baseline estimates for regional-scale networks with continental-scale fiducial control networks, and to estimate relative site motions. The measurements from each single-day ‘session’ were first adjusted to estimate station positions, satellite state vectors, atmospheric parameters, and integer ambiguities. The day-to-day rms scatter of baseline components were then used to assess both the short-term baseline precision for each campaign and the long term precision over the course of the five year program. The short-term (~five day) and long-term precision of the GPS measurements based on the day-to-day scatter of the estimated baselines are 3–5 mm/yr in the horizontal components with 1×10^{-8} dependence on baseline length for well designed experiments and about 15–25 mm in the vertical. The slight increase in

long-term scatter over short-term scatter is suggestive of systematic constant error sources.

We estimated relative site motions from GPS observations spanning 4.2 years. Changes in the continental-scale fiducial network over time appear to limit the precision of the velocity estimates from our series of measurements to roughly 1.5 mm/yr. The motions predicted by the GPS experiments are, in general, consistent with models for deformation within the North America–Pacific plate boundary zone. The sites that are collocated with VLBI show good agreement with the VLBI-predicted rates at the 2 mm/yr level. North-south convergence is well resolved across the eastern Santa Barbara channel. On the other hand, the estimated motion across the Santa Maria fold and thrust fault has less convergence and more right-lateral shear than with previous results obtained using triangulation and trilateration observations and geological and geophysical evidence. The relative motion of six sites along the western margin in the vicinity of Vandenberg are consistent with 2–4 mm/yr motion on the Hosgri fault. Deformation due to continuous slip at depth on the locked San Andreas fault does not fully account for the motion of Fibre with respect to Owens Valley, and suggests that an additional 2–4 mm/yr motion is being accommodated east of San Andreas, possibly in a shear zone located in Owens Valley and the Mojave Desert [e.g., *Savage et al.*, 1990].

APPENDIX A. TERRESTRIAL DETERMINATION OF VLBI REFERENCE POINTS

INTRODUCTION

Very-long-baseline interferometric (VLBI) antennas, whose relative locations are currently known to high precision, play critical fiduciary roles for other geodetic techniques. For example, Global Positioning System (GPS) antennas in the Cooperative International GPS Network (CIGNET) are presently located in the vicinity of VLBI antennas in order to take advantage of the superior VLBI-derived reference frame [e.g., *Chin et al.*, 1987]; observations from these sites are used to improve the estimates of the GPS satellite orbits which, in turn, are used to improve the relative accuracy of other GPS-derived baselines [e.g., *Schutz et al.*, 1990]. In fact, the current network of collocated GPS and VLBI antennas provides sufficient redundancy to assess the consistency of the baselines estimated by each technique. Such comparisons have also been conducted between collocated VLBI and satellite laser ranging (SLR) systems and provide an important measure of the overall accuracy of the two techniques [e.g., *Kolenkiewicz et al.*, 1985; *Ray et al.*, 1991].

An important issue for comparisons between these space-geodetic techniques is the relative location of the GPS antennas or the SLR systems with respect to the VLBI antennas; the uncertainties on these local vector ties is often the severest limitation of the study. In general, local vector ties are themselves the combination of several other ties: from the reference point of the SLR system or the GPS antenna phase center to a nearby ground monument, from the ground monument near the GPS antenna or SLR system to a ground monument near the VLBI antenna, and from the ground monument to the reference point of the VLBI antenna. In general, the ground-to-ground ties, typically from tens to hundreds of meters in length, can be estimated to millimeter precision from

adjustments of terrestrial observations. The location of the space-geodetic system reference points (e.g., the GPS antenna phase center, or the intersection of axes on the VLBI antenna) with respect to ground monuments are more problematic, since the reference points are often not accessible to direct measurement. Because GPS antennas are comparatively small and mobile, the location of their phase centers, in principle, can be well established by short baseline tests, and the relationship between the GPS antenna and the ground is usually a trivial (but critical) tape measurement, or, if the antenna is inaccessibly mounted, a tie to another GPS antenna whose location can be reliably determined. The SLR and VLBI reference points are more difficult to establish. This report addresses the problem of determining the location of the VLBI antenna reference point with respect to nearby ground monuments.

A VLBI antenna is essentially a paraboloid-shaped dish which focuses electromagnetic radiation arriving in plane-waves from a distant source to a single point where it is collected, time-tagged, and stored for comparison with the data from other VLBI antennas. The shape of the antenna dishes are designed such that the delay of the radiation from the moment it passes the plane of the rim of the dish to the time of collection is constant, regardless of the orientation of the antenna. Because the delay is constant, and because the VLBI observations are always differenced with the data from other antennas, the reference point for the antenna is at the location nearest the focus which does not move as the antenna changes its orientation. Since the orientation of a VLBI antenna is usually determined by two (not necessarily intersecting) rotation axes, the reference point is always located on one of these axes, specifically at the intersection between the fixed axis and the plane perpendicular to the fixed axis containing the moving axis [e.g., *Ma et al.*, 1989]. The orientations of these axes vary from antenna to antenna. One arrangement, called an AZ-EL mount, consists of a fixed vertical axis which varies the azimuth of the antenna, and a horizontal axis which varies its elevation angle. In the simplest case, these two axes intersect at the reference point. More

generally, the horizontal axis is displaced from the vertical axis and the reference point is located at the intersection of the vertical axis with the horizontal plane containing the horizontal axis. Another arrangement, called an X-Y mount, consists of two orthogonal horizontal axes that are displaced vertically. The fixed X-axis is always closest to the ground and is usually oriented north-south (although it is sometimes oriented east-west). The reference point for an X-Y antenna is defined to be the intersection of the X-axis with the vertical plane containing the Y-axis.

Determining the location of the VLBI antenna reference point with respect to ground monuments is often complicated by the fact that the reference point is usually not accessible to direct measurement; this has lead some to half-jestingly observe that VLBI can measure a baseline several thousand kilometers long but not its last centimeter. In most cases the reference point location is derived from ground measurements to nearby points on the antenna whose relative location to the reference point are inferred from engineering drawings [e.g., *Carter et al*, 1980]. A second approach is to conduct special observation campaigns in which mobile VLBI antennas are located near ground monuments in the vicinity of fixed antennas; it is generally assumed that the eccentricities from the mobile VLBI reference point to the ground monument can be well determined and that the mobile to fixed VLBI tie can be estimated to high precision. Several such campaigns were conducted by the NASA Crustal Dynamics Project in 1988 to improve ties to the fixed antennas at Owens Valley, Mojave, and Fort Davis, and to provide a new collocation between the fixed VLBI antenna on Kauai and the satellite and lunar ranging systems on Maui. A third approach, which shall be examined in more detail in this report, is to make observations of the antenna dish as the antenna orientation is varied; the reference point can then be estimated by taking into account the kinematic constraints imposed by the physical dimensions of the antenna and the orientations of its rotation axes.

This last technique is neither complicated nor original, but it apparently is not practiced regularly. It has probably been applied to several antennas, such as Hatcreek and Tidbinbilla, although there appears to be no documentation on these previous studies. It is not without its limitations. The technique is only useful for antennas with reasonable visibility from ground marks and therefore not easily applicable to antennas covered by radomes, such as Westford and Haystack. Its accuracy is limited by the extent to which the antenna's motions can be described from geometrical considerations; non-geometric motions, such as deformations of the antenna, can bias estimates of the reference point, although some of the effects may be modelled as well [e.g., *Rogers et al.*, 1978; *Clark and Thomsen*, 1988]. On the other hand, for sites where it is applicable and deformations are minimal, it offers a reasonably accurate, albeit indirect, measurement of the reference point which does not rely on engineering specifications. It is also easily repeatable, and hence useful for estimating temporal variations of the location of the VLBI reference point with respect to the ground marks (i.e., small-scale footprints).

The impetus of this study was the discovery of a discrepancy between two terrestrial survey estimates of the relative location of the Kokee Park VLBI antenna reference point (CDP monument 1311) on Kauai, Hawaii. This report will focus on the Kokee test case, beginning with a brief description of the Kokee antenna and the terrestrial survey observations. A methodology for estimating the reference point from terrestrial data will then be presented and the results of the study will be discussed. Finally, the method will be generalized for use with other VLBI antenna types.

KOKEE VLBI REFERENCE POINT 1311

The antenna at Kokee is a 9-meter dish attached to an X-Y mount; the horizontal X-axis is oriented north-south and the Y-axis is oriented east-west. The vertical distance from the lower X-axis to the Y-axis is 8 feet or 2.4384 m, according to the engineering

specifications; this is nearly the same value used by the VLBI community to model the theoretical delay of the VLBI observations [T. A. Herring, personal communication, 1990]. The reference point is defined to be the intersection of the X-axis with the vertical plane containing the Y-axis. The design specifications required that the orientations of the axes, both horizontally and in azimuth, be accurate to 5 arc seconds.

Several terrestrial surveys have been conducted in the vicinity of Kokee. The first survey was conducted for NASA when the antenna was originally installed in 1965 for spacecraft tracking. It included only three ground marks (“MANU 1961” and two sites several kilometers distant), and three points associated with the X-axis of the VLBI antenna (labeled South, Central, and North); presumably, “Central X-axis” refers to the VLBI reference point 1311 and that the North and South X-axis marks are the intersections of the X-axis with the exterior of the antenna support structure. Currently, these intersections are covered by endplates; however, it is assumed that this survey was conducted during installation before the endplates were emplaced. This survey essentially utilized the first approach discussed above to locate the antenna reference point: it made observations to nearby points on the antenna whose relationship to the reference point was inferred (i.e., the mean of the North and South X-axis locations). The survey data were readjusted using the HAVAGO adjustment program [Vincenty, 1979] on June 29, 1987; the tie between MANU 1961 and the reference point 1311 determined by this survey is currently used by the GPS community to collocate an CIGNET receiver [Chin, 1989]. Although this tie has a formal uncertainty of about 10 mm, the observed elevation difference between the North and South X-axis monuments was 48 mm—greatly exceeding the 0.1 mm value predicted by the design specifications—suggesting that the true uncertainties are at least several centimeters.

Several other surveys have been conducted in the vicinity of the antenna during the last three years. An NGS leveling crew estimated the elevation difference between another nearby ground monument (“KOKEE 1961”) and the X- and Y-axes in 1987. By

this time, the endplates covered the actual axes, so measurements were with respect to punchmarks in the endplates that were located as close as possible to the inferred axes. In August 1989, Bendix Field Engineering, Corp. (BFEC) conducted an extensive horizontal and vertical survey of several of the historical monuments (including MANU 1961 and KOKEE 1961) and four NCMN monuments that were established at that time. No direct measurements were made to the X-axis, but a corner-cube reflector was attached to the VLBI antenna feedbox and was observed from the NCMN monuments as the antenna was moved to different azimuths and elevation angles. The observations from the 1987 and 1989 surveys were combined and adjusted using HAVAGO on January 16, 1990, and yielded several millimeter-level relative monument locations. A misinterpretation of the NGS leveling data caused a 300 mm bias to be introduced in the elevation of the 1311 reference point which was subsequently discovered in comparisons to the 1965 survey results and corrected. It should be noted that the only “direct” measurement to the VLBI reference point in this survey was the single level line from KOKEE 1961 to the X-axis punchmark. However, as we shall demonstrate, the observations to the antenna feedbox target can be used to indirectly estimate the horizontal and vertical location of the reference point.

The feedbox target was usually visible from at least three of the ground monuments and at each position direction, vertical angle, and absolute distance measurements were typically performed. Thus the position of the target at each antenna orientation can, in principle, be uniquely determined with respect to the ground. The adjustment includes observations of the antenna at zenith and at 5° zenith angle increments at north, south, east and west azimuths, i.e., when either the X or the Y-axis, but not both, were rotated; each target position is then incorporated into the adjustment as a separate monument. A total of 67 antenna positions are included in the adjustment. The target at the various antenna orientations was located at approximately the 3-mm uncertainty level. In order to relate these locations to the desired reference point, “position differences” were also input

to HAVAGO; the next section will discuss the properties of position differences, and how they may be used in conjunction with the other terrestrial observations to determine the location of 1311.

POSITION DIFFERENCES

A position difference is simply the vector difference between two locations expressed in plane coordinates (local north, east, up). HAVAGO allows position differences to be entered as input data with uncertainties; these are then adjusted in accordance with their *a priori* uncertainties along with the other observations. A position difference would, for example, be a suitable method for utilizing a three-dimensional vector derived from GPS or VLBI observations, although HAVAGO currently does not have the facility to incorporate correlations between the vector components. For the purposes of this report, the position differences provide a means to map all the target locations to a single point which ideally will be the reference point of the VLBI antenna. The problem, therefore, is to determine the relationship between the reference point and the target locations.

Note that, lacking any additional information, the definition of the origin of the position differences (the reference point) is completely undetermined. Thus, once the relative locations of the target positions have been estimated (from all the terrestrial observations), a set of position differences can be defined to any arbitrary point that will completely satisfy the observations. It is this fact that allowed the large vertical bias to go undetected in the 1990 adjustment. For that adjustment, the set of position differences were estimated in a several step process. First, the location of the target, when the antenna was pointed toward the zenith point, was visible from all four NCMN marks and its elevation difference to the ground mark KOKEE 1961 was estimated. The level line discussed above provided an estimate of the elevation difference between X-axis and KOKEE 1961. These two elevation differences were combined to provide an *a priori*

estimate of the height of the target above the X-axis at zenith of 10.660 m. The *a priori* origin of the position differences was therefore defined to be 10.660 m directly below the target at zenith. The remaining *a priori* position differences were estimated with respect to this point using subsets of the data and all the position differences were given large uncertainties to allow them to adjust to become self-consistent with the target locations. The final result was a set of position differences which satisfied the constraints imposed by the observations as represented by the target locations, but whose arbitrary origin was biased by the incorrect elevation difference.

The problem, of course, was that the only additional information added to form the position differences was the level line data and the assumption that the reference point was directly beneath the target at zenith. Fortunately, we *do* have additional information which will enable the origin of the position differences to be determined, and they are the kinematic constraints imposed on the VLBI antenna by its physical dimensions and the orientation of its X and Y rotation axes. In fact, the motions are theoretically a very simple set of rotations and translations that can be completely specified once the dimensions of the VLBI antenna are known. The relevant dimensions are the origin of the reference point in the X-axis, the distance between the X and Y-axes, and the relative location of the target, which may be located anywhere on the antenna dish; only the distance between the axes is known *a priori*. However, the 67 target locations provide sufficient redundancy to estimate these parameters, and hence define a set of position differences relative to the true reference point. The following sections describe the mathematical model used to estimate these parameters, and the results of the estimation.

VLBI ANTENNA KINEMATIC CONSTRAINTS

We begin by assuming that the origin is arbitrary and that the location of the true reference point with respect to the origin is given by the vector $\mathbf{x} = (x_1, x_2, x_3)$, where $1 =$

north (X-axis), 2 = west (Y-axis), and 3 = zenith. Let $\mathbf{a} = (a_1, a_2, a_3)$ be the vector describing the location of the center of the Y-axis with respect to the X-axis when the antenna is pointed towards the zenith; a_3 has an *a priori* estimate from the engineering specifications (and VLBI measurements) of 2.438 m. Similarly, let $\mathbf{b} = (b_1, b_2, b_3)$ be the vector from the center of the Y-axis to the center of the target on the feedbox when the antenna is at zenith; in general $\mathbf{b}$ will have non-zero horizontal components due to the arbitrary location of the target with respect to the zenith axis. Thus, the location of the target with respect to the origin when the antenna is at zenith is $\mathbf{t} = \mathbf{x} + \mathbf{a} + \mathbf{b}$. Assuming that the X-axis is pointing north, a positive rotation about the X-axis results in a positive east zenith angle α . Similarly, a positive rotation about the Y-axis, which is oriented west to form a right-handed coordinate system, results in a positive north zenith angle β (Figure A.1). We therefore define two fundamental rotation matrices:

$$\mathbf{R}_x(\alpha) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{pmatrix} \quad (\text{A.1})$$

$$\mathbf{R}_y(\beta) = \begin{pmatrix} \cos \beta & 0 & -\sin \beta \\ 0 & 1 & 0 \\ \sin \beta & 0 & \cos \beta \end{pmatrix} \quad (\text{A.2})$$

A rotation about the X-axis will affect both $\mathbf{a}$ and $\mathbf{b}$, whereas a rotation about the Y-axis will only affect $\mathbf{b}$. Therefore, the location of the target $\mathbf{t}$ as a function of α and β is:

$$\begin{aligned} \mathbf{t} &= \mathbf{x} + \mathbf{R}_x(\alpha) \mathbf{a} + \mathbf{R}_y(\beta) \mathbf{R}_x(\alpha) \mathbf{b} \\ &= \begin{pmatrix} (x_1 + a_1) + b_1 \cos \beta + b_2 \sin \alpha \sin \beta - b_3 \cos \alpha \sin \beta \\ x_2 + (a_2 + b_2) \cos \alpha + (a_3 + b_3) \sin \alpha \\ x_3 + b_1 \sin \beta - (a_2 + b_2 \cos \beta) \sin \alpha + (a_3 + b_3 \cos \beta) \cos \alpha \end{pmatrix} \end{aligned} \quad (\text{A.3})$$

The 1990 HAVAGO adjustment provides estimates of $\mathbf{t}$ (the adjusted position differences of the target with respect to the assumed origin) as of function of antenna orientation (α, β) that are self-consistent with all the observations from the NCMN

monuments to the target. The unknowns in above model (Equation A.3) are therefore the physical dimensions of the VLBI antenna (x , a , and b); we will assume that we do not know the physical separation a_3 between the X and Y-axes in order to verify our method.

A cursory examination of the model reveals that a_1 is perfectly correlated with x_1 ; less obviously, it turns out that a_2 is perfectly correlated with b_2 . These relationships are essentially a restatement of the definition that the VLBI reference point is located at the intersection of the X-axis with the vertical plane containing the Y-axis. Therefore, we set $a_1 = a_2 = 0$, and separating the unknowns from their partial derivatives, we obtain the model equations for the ideal geometric motions:

$$t = \begin{pmatrix} 1 & 0 & 0 & 0 & \cos \beta & \sin \alpha \sin \beta & -\cos \alpha \sin \beta \\ 0 & 1 & 0 & \sin \alpha & 0 & \cos \alpha & \sin \alpha \\ 0 & 0 & 1 & \cos \alpha & \sin \beta & -\sin \alpha \cos \beta & -\cos \alpha \cos \beta \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ a_3 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} \quad (A.4)$$

REFERENCE POINT ESTIMATION

Equation A.4 represents the idealized model prediction of the location of the target with respect to the assumed origin as a function of zenith angles; the estimate of x will determine the location of the true reference point. The adjusted position differences in the January 16, 1990 HAVAGO adjustment (which included the 300 mm vertical bias due to the levelling data) provide 67 samples of the model which satisfy the constraints imposed by the survey observations. We therefore are able to formulate a least squares estimate of the physical dimensions of the antenna and its true reference point by concatenating the t and its corresponding partial derivative matrix with the appropriate position differences and zenith angles. In general, the latitude and longitudes of the predicted antenna positions from the HAVAGO adjustment have an uncertainty of 1 mm,

whereas the heights have an uncertainty of 3 mm except for a few cases when the antenna has a high zenith angle and the uncertainty approaches 7 mm. For simplicity, we will assume that each of the components of the position differences has an uncertainty of 1 mm; since we are assuming that the component uncertainties are equal and uncorrelated, the sampling standard deviation (or standard deviation of unit weight) of each estimate, σ_o , will then essentially represent the weighted root mean square scatter of the residuals in millimeters (ignoring a near-unity scaling factor due to the difference in the number of observations and the degrees of freedom of the estimate).

To demonstrate the sensitivity of the estimated parameters, we consider three subsets of the seven-parameter model. The assumption used to derive the *a priori* position differences was that the reference point was vertically beneath the feedbox target, therefore the most simple model parametrization (Test 1) would assume that origin is at the reference point and that all the horizontal components are zero: $x_1 = x_2 = x_3 = b_1 = b_2 = 0$. Since a 300-mm level vertical offset of the reference point is anticipated, a more general parametrization allows x_3 be non-zero (Test 2). Finally, the most general parametrization, assumes that all the horizontal offsets are to be estimated (Test 3). The results of these three cases are given in Table A.1, along with the sampling standard deviation. The residuals for each latitude, longitude, and vertical component corresponding to either the north or east zenith angles for each of these cases are given in Figures A.2–A.4.

The results of Test 1, with a sampling standard deviation significantly greater than unity ($\sigma_o = 110.30$), clearly indicate an inadequacy of the model parametrization. The residuals (Figure A.2) show large systematics on most of the components; the two nearly flat residual sets correspond to components which are relatively insensitive to the motion of the antenna (e.g., varying the east zenith angle should result in very little change in latitude). Providing for a vertical offset from the assumed origin to the reference point (Test 2), significantly improves the overall fit ($\sigma_o = 11.092$) and shows good agreement

Table A.1. Estimated Dimensions of the Kokee VLBI Antenna, mm

Test	x_1	x_2	x_3	a_3	b_1	b_2	b_3	$\sigma_o^\dagger$
1	0.0	0.0	0.0	$2738.4 \pm 25.$	0.0	0.0	$7849.5 \pm 25.$	110.30
				3			3	
2	0.0	0.0	331.4 ± 2.4	2439.4 ± 3.3	0.0	0.0	7881.9 ± 2.6	11.092
3	22.3 ± 0.8	-20.4 ± 0.7	331.4 ± 0.7	2439.4 ± 1.0	-23.3 ± 0.8	22.7 ± 0.7	7881.9 ± 0.8	3.288
4	22.3 ± 0.4	-22.5 ± 1.3	334.6 ± 1.3	2437.7 ± 1.4	-23.2 ± 0.4	25.0 ± 1.3	7883.4 ± 0.4	1.672

[†]The *a priori* uncertainties of the position differences were 1 mm.

with the 300 mm discrepancy hypothesis. The residuals in Figure A.3, however, still show 30-mm level systematics, indicating that the model remains inadequate. Allowing either the reference point or the target to move horizontally, but not both, does not significantly improve the estimate; the large systematic residuals remain but are offset from the mean by varying amounts. The general geometric model, which allows both the reference point and the target to offset horizontally (Test 3), does significantly improve the agreement with the data ($\sigma_o = 3.288$) and all the parameters are now estimated at a sub-millimeter level. Note that the addition of horizontal parameters does not change the estimate of the vertical parameters, which agree well with our *a priori* knowledge: the distance from the X to the Y-axis $a_3 = 2439.4$ mm differs from the engineering specifications by 1 mm, the vertical discrepancy differs from our initial estimate (based on the 1965 survey) by 31 mm, and the sum of the three vertical components (10.652 m) agrees with the initial guess (10.660 m) which was used to derive the *a priori* position differences. The reference point translated 20 mm in both horizontal components, while the target translate in almost exactly the opposite direction; this is simply a reflection of the assumption used to derive the *a priori* position differences that the target was vertically above the assumed origin.

The residuals of the general model (Test 3), shown in Figure A.4, also show considerable improvement over the previous test cases, but several anomalies are evident. The latitude residuals show a several millimeter deviation from the mean for the 0–30°

north zenith angles (note that the zenith point, which is common to both north and east angle plots, appears to be an outlier in the latitude of the east zenith angles). The most likely explanation is a survey blunder, but all the positive north zenith angle locations were visible from the same three NCMN stations, which argues against inhomogeneous sampling. Removal of these data do not significantly alter the results. More troubling is the 5–10 mm level systematic behavior displayed in the longitude and vertical components of the east zenith angles. While surveying errors are a possibility, more likely explanations are either unmodeled geometric or non-geometric effects. The geometric model (Equation A.4) assumed that the rotation axes were horizontal and oriented exactly north-south and east-west. The engineering specifications require that these conditions be met to 5 arc seconds (25 ppm); a 5 mm deviation over 10 m implies a 100 arc second error. Alternatively, the geodetic network might have a different orientation than that implied by the plane coordinates. In any case, forward modelling did not indicate that the possible additional rotations were a likely explanation. Non-geometric effects include antenna deformations due to gravitational and wind loading forces. Modelling of these effects is considerably more complicated, especially since the deviations do not appear to be a simple function of zenith angle. It is notable that the north zenith angle residuals are more consistent than the east residuals: north zenith angles require only the antenna structure above the Y-axis to move, whereas east zenith angles require the entire dish structure to move. This suggests that the problem might be due to coupling motions associated with the Y-axis. Observations of the antenna when both the X and the Y-axes are moved might provide stronger evidence of this phenomenon. To test the sensitivity of the general geometric model to these problems, the low elevation east and west zenith angles were removed (Figure A.5). The results (Test 4 of Table A.1) show an improved fit, but higher uncertainties on the estimated parameters, due to the significant reduction in data. The estimate of the reference point also changed by 2–3 mm in the longitude and vertical directions, indicating that potential

biases due to the non-geometric effects may be several times larger than the formal errors. These problems present the most serious limitations for reliable estimation of VLBI reference points using terrestrial observations.

INCORPORATION OF GEOMETRIC CONSTRAINTS INTO HAVAGO

The above estimation procedure yields an estimate of the location of the reference point (i.e., $\mathbf{x}$) with respect to the position differences in plane coordinates. In order to determine the location of the reference point with respect to the ground monuments, these results need to be incorporated into the HAVAGO adjustment. The most self-consistent approach would be to incorporate the kinematic constraints directly into HAVAGO and estimate the antenna dimensions and monument locations simultaneously. This approach, while attractive theoretically, is beyond the scope of this report, and must be consigned to the category of “future work”. The approach we have taken, therefore, is to calculate position differences and uncertainties according to the theoretical model (Equation A.4) using the estimated general geometric model parameters, with the exception that the origin is now defined to be the estimated reference point (i.e., $\mathbf{x} = 0$). These theoretical position differences are then reintroduced to the adjustment in place of the original *a priori* estimates of the position differences. Since the theoretical position differences are given relatively significant weight, the estimated target locations will also adjust to accommodate both the terrestrial observations and the kinematic constraints. Thus, the process of estimating the antenna dimensions and the target locations should be iterated until the solution converges.

The results of the iterations are presented in Table A.2. For the first iteration, the results of Test 3 from Table A.1 were used to estimate the theoretical position differences and uncertainties; these are denoted Iteration 0 in Table A.2 where the reference point offset has been set to zero, but its uncertainties have been included in the formal error

Table A.2. Iterated Dimensions of the Kokee VLBI Antenna, mm

Iter.	x_1	x_2	x_3	a_3	b_1	b_2	b_3	$\sigma_o^\dagger$
0	0.0 ± 0.8	0.0 ± 0.7	0.0 ± 0.7	2439.4 ± 1.0	-23.3 ± 0.8	22.7 ± 0.7	7881.9 ± 0.8	3.288
1	-0.2 ± 0.3	-1.4 ± 0.3	1.4 ± 0.3	2438.3 ± 0.4	-23.1 ± 0.3	24.1 ± 0.3	7881.8 ± 0.3	0.472
2	-0.1 ± 0.2	-0.6 ± 0.2	0.3 ± 0.2	2438.0 ± 0.3	-23.0 ± 0.2	24.8 ± 0.2	7881.7 ± 0.2	0.275

[†]The *a priori* uncertainties of the position differences were 1 mm.

propagation to determine the position difference uncertainties. The uncertainties of the *a priori* theoretical position difference components were approximately 2 mm; for the second iteration they were approximately 1 mm. The improvement in the estimates results from the increasingly stronger weight given to the kinematic constraints. The adjustments to the reference point offset are submillimeter after the second iteration, which was deemed a reasonable criterion for convergence.

PREVIOUS RESULTS

The final adjustment, corresponding to Iteration 2 in Table A.2, yields an estimate of the Kokee VLBI reference point, with respect to the fixed station KOKEE 1961, that has formal uncertainties of 1.5 mm in the horizontal components and 2 mm in the vertical. Based on the evidence for antenna deformation, the true uncertainties in longitude and vertical are more likely on the order of 3 mm. Nominal coordinates for the reference point with respect to the NAD 83 reference ellipsoid are: N22°7'34".62651, W159°39'54".55345, 1167.796 m.

These results can be compared with several independent estimates of some of the baselines. The local vector tie to MANU 1961 was measured in the 1965 NASA survey. Table A.3 gives the estimate based on this survey from the HAVAGO adjustment dated June 29, 1987, as well as the difference between the old and new estimates in both Cartesian and plane coordinates. The differences in the horizontal coordinates are well within their formal uncertainties. The 44 mm vertical difference is too large to be

Table A.3. The local vector tie from 1311 to MANU 1961

HAVAGO Adjustment	x (mm)	y (mm)	z (mm)
900723	-115934.7 ± 3.9	-13316.3 ± 2.7	-276916.2 ± 1.8
870629	-115976.0 ± 9.0	-13326.0 ± 10.0	-276903.0 ± 15.0
870629–900723	–41.3	–9.7	13.2
	N (mm)	E (mm)	U (mm)
870629–900723	–3.6	–5.3	44.0

attributed to the uncertainties of the estimates; more likely the discrepancy is due to the 1965 survey data which also estimated a 48 mm elevation difference between the North and South X-axis points.

The July 23, 1990 HAVAGO adjustment did not include the elevation difference between 1311 and KOKEE 1961 determined by the 1987 NGS survey, which was the source of the original 300 mm discrepancy. Thus, the only constraints on the location of 1311 are the target locations combined with the estimated kinematic constraints. However, the misinterpretation of the 1987 levelling data has been resolved, providing independent estimates of the height of the X and Y-axes. The comparisons of these values with the adjustment estimate to the X and Y-axis (assuming the vertical distance between the axes is 2438 mm) are given in Table A.4. Both the levelling line data and the feedbox target observations show good agreement with the elevation difference to the Y-axis, but show a significant discrepancy to the X-axis, and therefore to the reference point. This discrepancy is most likely due to a problem with the levelling data since the estimated vertical offset between the X and Y-axis disagrees with the engineering specifications, VLBI observable modelling, and the kinematic constraint estimates by 31 mm. The level line was conducted in two steps. First, a line was run from KOKEE 1961 to one of the reference marks near the VLBI antenna and then to a couple of temporary monuments; reobservations of these segments by BFEC in 1989 showed agreement at the millimeter level. Second, vertical angles were observed from the temporary monuments

Table A.4. Elevation Difference from KOKEE 1961 to 1311, mm

Survey	X-axis	Y-axis
1987 Levelling Line	6491. $\pm$ 2	8960. $\pm$ 2
1989 Feedbox Target	6528. $\pm$ 3	8966. $\pm$ 3 [†]
1987–1989	–37.	–6.

[†]Assuming Y–X = 2438 mm

to punchmarks in the endplates of the north X-axis, and the east Y-axis (the other sides could not be easily observed). This second set of observations might be suspect for several reasons, including the difficulty of taking vertical angles from close ranges and the uncertain location of the endplate punchmarks with respect to the true horizontal rotation axes. Therefore, all the current direct measurements of the X-axis are suspect, and the indirect kinematic constraint measurements probably provide the most reliable estimate of its location until further direct measurements of the X-axis can be observed.

OTHER VLBI ANTENNA MOUNTS

The above methodology can easily be generalized to other VLBI antennas, provided their basic geometrical configuration is well understood. In this section, we derive appropriate mathematical models for two other types of antennas: an X-Y mount where the X-axis is oriented in the east-west direction, and a general AZ-EL mount.

The geometrical model for an X-Y mount where the X-axis is oriented in the east-west direction is a simple extension of the model given in Equations A.3 and A.4. To maintain a right-handed system, we define the positive X-axis to be towards east and the positive Y-axis to be towards north. These orientations imply that a positive east zenith angle α results from a positive rotation about the Y-axis, but that a positive north zenith angle β results from a *negative* rotation about the X-axis. Letting 1 = north (Y-axis), 2 =

east (X-axis), and 3 = zenith (note that the sign change in the longitude sign convention from before), the fundamental rotation matrices become:

$$\mathbf{R}_x(\beta) = \begin{pmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{pmatrix} \quad (\text{A.5})$$

$$\mathbf{R}_y(\alpha) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{pmatrix} \quad (\text{A.6})$$

which implies that the location of the target with respect to the assumed origin is given by:

$$\begin{aligned} \mathbf{t} &= \mathbf{x} + \mathbf{R}_x(\beta) \mathbf{a} + \mathbf{R}_y(\alpha) \mathbf{R}_x(\beta) \mathbf{b} \\ &= \begin{pmatrix} 1 & 0 & 0 & \sin \beta & \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 & 0 & -\sin \alpha \sin \beta & \cos \alpha & \sin \alpha \cos \beta \\ 0 & 0 & 1 & \cos \beta & -\cos \alpha \sin \beta & -\sin \alpha & \cos \alpha \cos \beta \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ a_3 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} \quad (\text{A.7}) \end{aligned}$$

The AZ-EL mount is characterized by a vertical rotation axis which varies the azimuth of the antenna, and a horizontal rotation axis which varies the elevation. In general, the horizontal axis may be offset from the vertical axis; the reference point is defined to be the intersection of the vertical axis with the horizontal plane containing the horizontal axis. This case is analogous to the X-Y mount, except that the offset between the axes is horizontal rather than vertical. The following conventions will be used to describe the antenna motions, letting 1 = north, 2 = east, and 3 = zenith: counterclockwise rotations about the vertical axis with respect to north define a positive azimuth angle θ ; the horizontal axis is oriented east-west when $\theta=0$ and the axis offset, oriented north-south, shall be denoted a_1 ; when $\theta=0$, a positive zenith angle ϕ is a rotation

about the horizontal axis towards the north with respect to the zenith direction. Given these definitions the fundamental rotation matrices are:

$$\mathbf{R}_{AZ}(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (\text{A.8})$$

$$\mathbf{R}_{EL}(\phi) = \begin{pmatrix} \cos \phi & 0 & \sin \phi \\ 0 & 1 & 0 \\ -\sin \phi & 0 & \cos \phi \end{pmatrix} \quad (\text{A.9})$$

which implies that the location of the target with respect to the assumed origin is given by:

$$\begin{aligned} \mathbf{t} &= \mathbf{x} + \mathbf{R}_{AZ}(\theta) \mathbf{a} + \mathbf{R}_{EL}(\phi) \mathbf{R}_{AZ}(\theta) \mathbf{b} \\ &= \begin{pmatrix} 1 & 0 & 0 & \cos \theta & \cos \theta \cos \phi & -\sin \theta \cos \phi & \sin \phi \\ 0 & 1 & 0 & \sin \theta & \sin \theta \cos \phi & \cos \theta & 0 \\ 0 & 0 & 1 & 0 & -\cos \theta \sin \phi & \sin \theta \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ a_1 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} \quad (\text{A.10}) \end{aligned}$$

Equivalent models for other antenna types, such as those with non-horizontal axes, are equally straightforward to derive.

CONCLUSIONS

The location of the reference point of the Kokee VLBI antenna with respect to nearby ground monuments has been estimated to approximately 1 millimeter in latitude and 3 mm in longitude and vertical. No direct observations of the reference point were made; instead, the location of the reference point was estimated in a two step process. First, terrestrial observations were conducted from well-located ground marks to a target attached to the antenna feedbox which enabled the location of the target to be determined as the antenna was positioned at different north-south and east-west zenith angles. The

reference point was then estimated with respect to the target locations by utilizing the kinematic constraints imposed on the motions of the antenna by its physical dimensions and the orientation of its rotation axes.

The kinematic constraint approach is robust and straightforward to implement, provided the antenna dish is readily visible from nearby ground monuments. Deformations of the antenna structure can potentially bias estimates based on kinematic constraints; in the case of the Kokee antenna, systematic residual behavior in the longitude and vertical components, which could not readily be characterized by standard deformation models, suggest that these components are more poorly resolved. On the other hand, the kinematic constraint approach offers a relatively precise method to estimate reference point locations which are not accessible to direct measurements, and provide a useful check for more traditional surveys which rely on assumptions derived from engineering specifications, or for surveys utilizing collocated mobile VLBI antennas. In addition to facilitating locations of the reference point, we believe that VLBI antenna kinematic constraint observations should be incorporated into any systematic survey designed to monitor the temporal motions of fixed VLBI antennas with respect to local and regional scale sites ("geodetic footprints"). The technique is not limited solely to the use of terrestrial observations. For example, GPS antennas attached to the VLBI antenna and acquiring measurements in a "kinematic" mode potentially offer a relatively inexpensive and powerful method for implementing the proposed surveys, assuming, of course, that undesired wave reflection ("multipath") effects can be minimized.

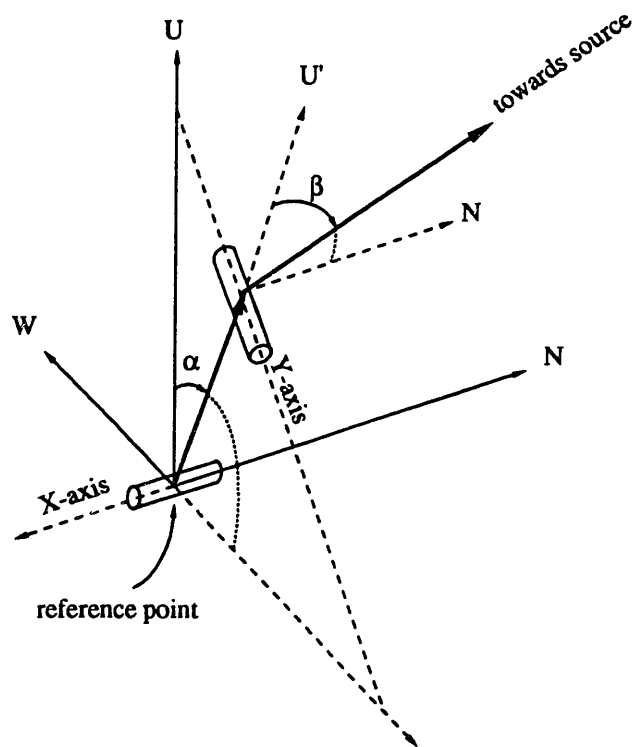


Figure A.1. Conventions used for an X-Y antenna geometry, where the fixed horizontal X-axis is oriented north-south, the moving Y-axis is oriented east-west, and vertical is with respect to the reference ellipsoid. For simplicity, the offset from the assumed origin to the reference point on the X-axis has been omitted.

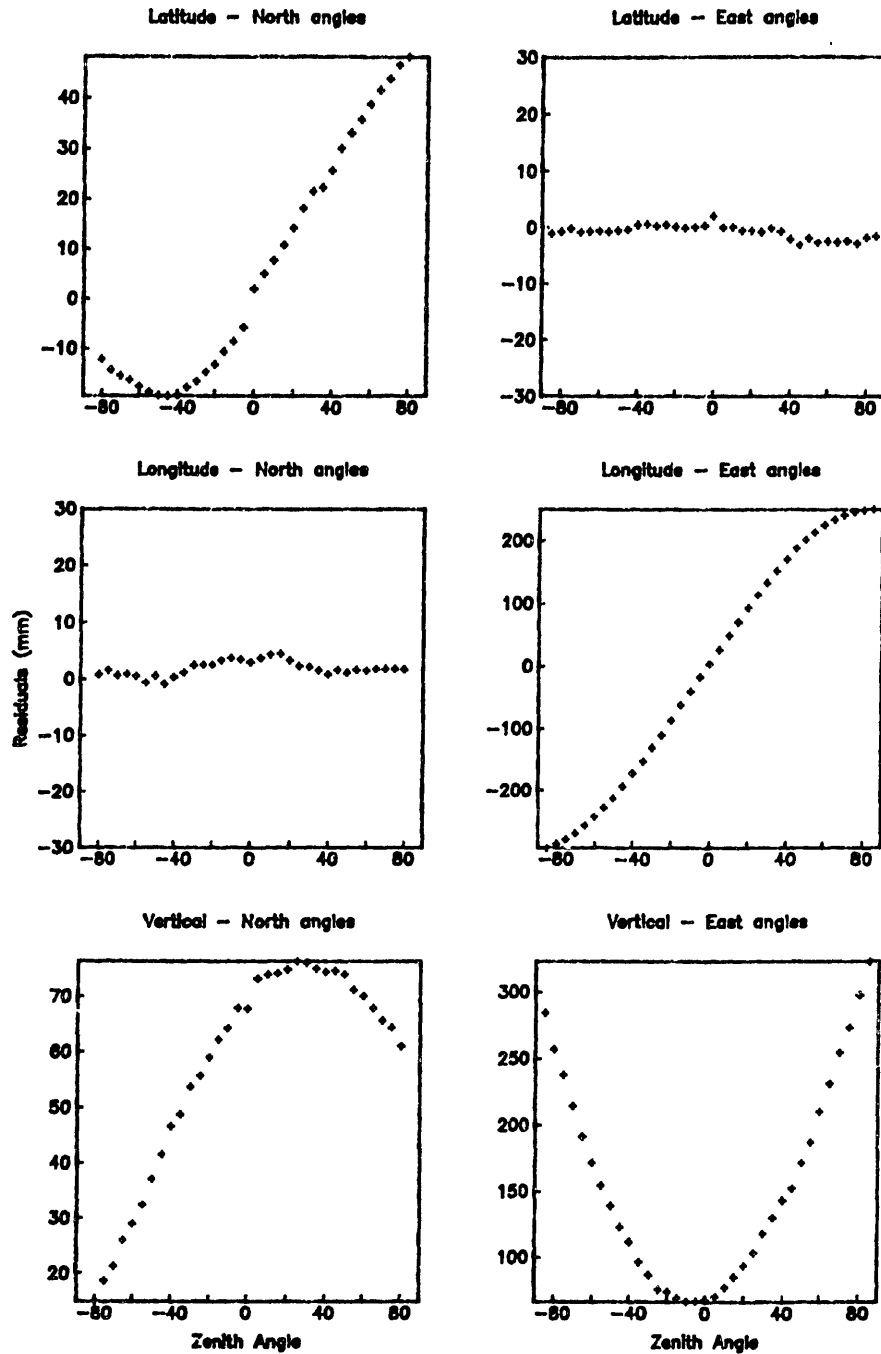


Figure A.2. Latitude, longitude, and vertical residuals when the antenna is oriented at either north or east zenith angles for kinematically constrained target locations minus target locations estimated from terrestrial observations. The kinematic model assumes that the origin as at the reference point and the target is aligned with the zenith axis (Test 1).

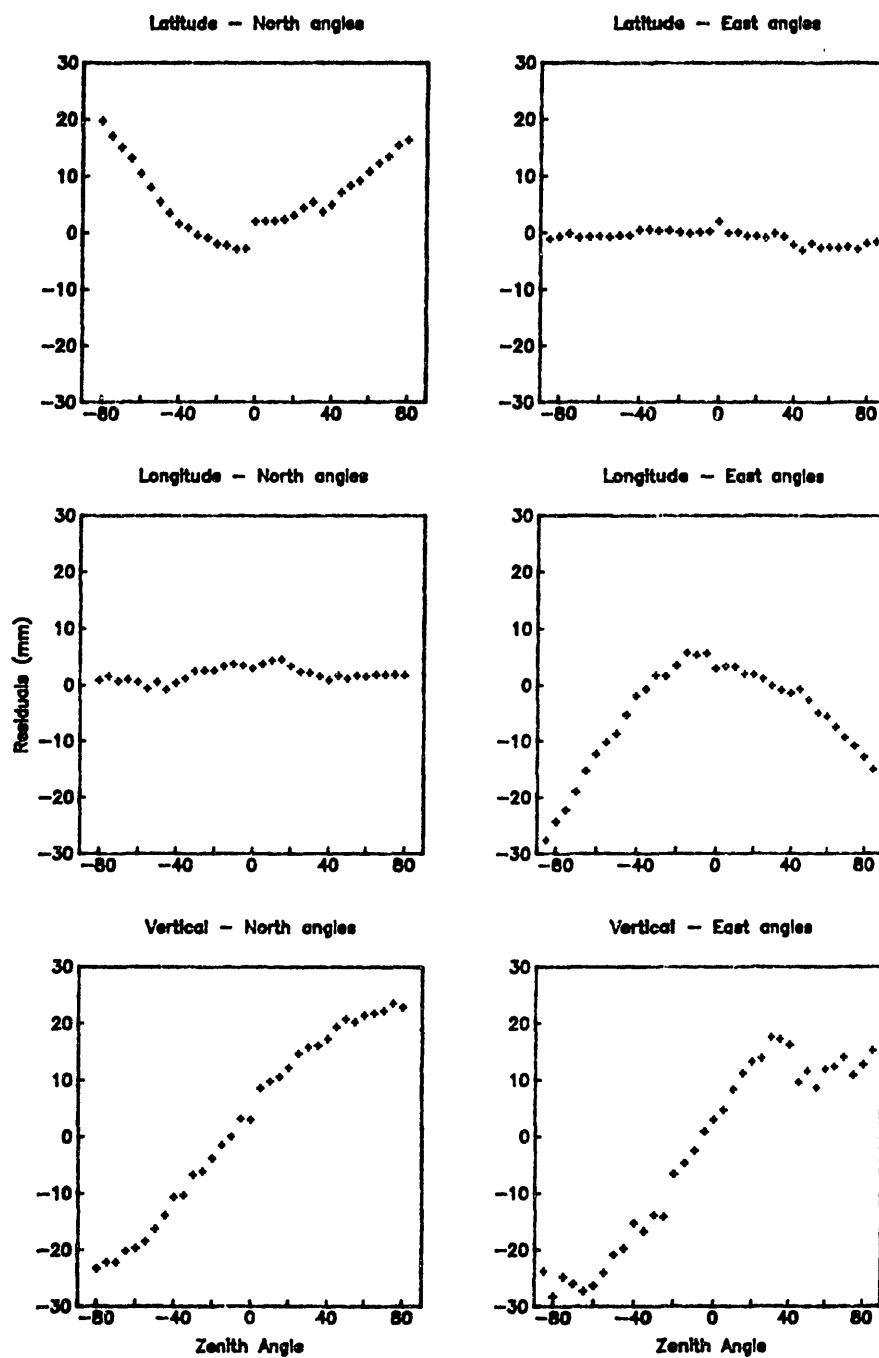


Figure A.3. Same as Figure A.2, except that the origin is allowed to displace vertically from the reference point (Test 2).

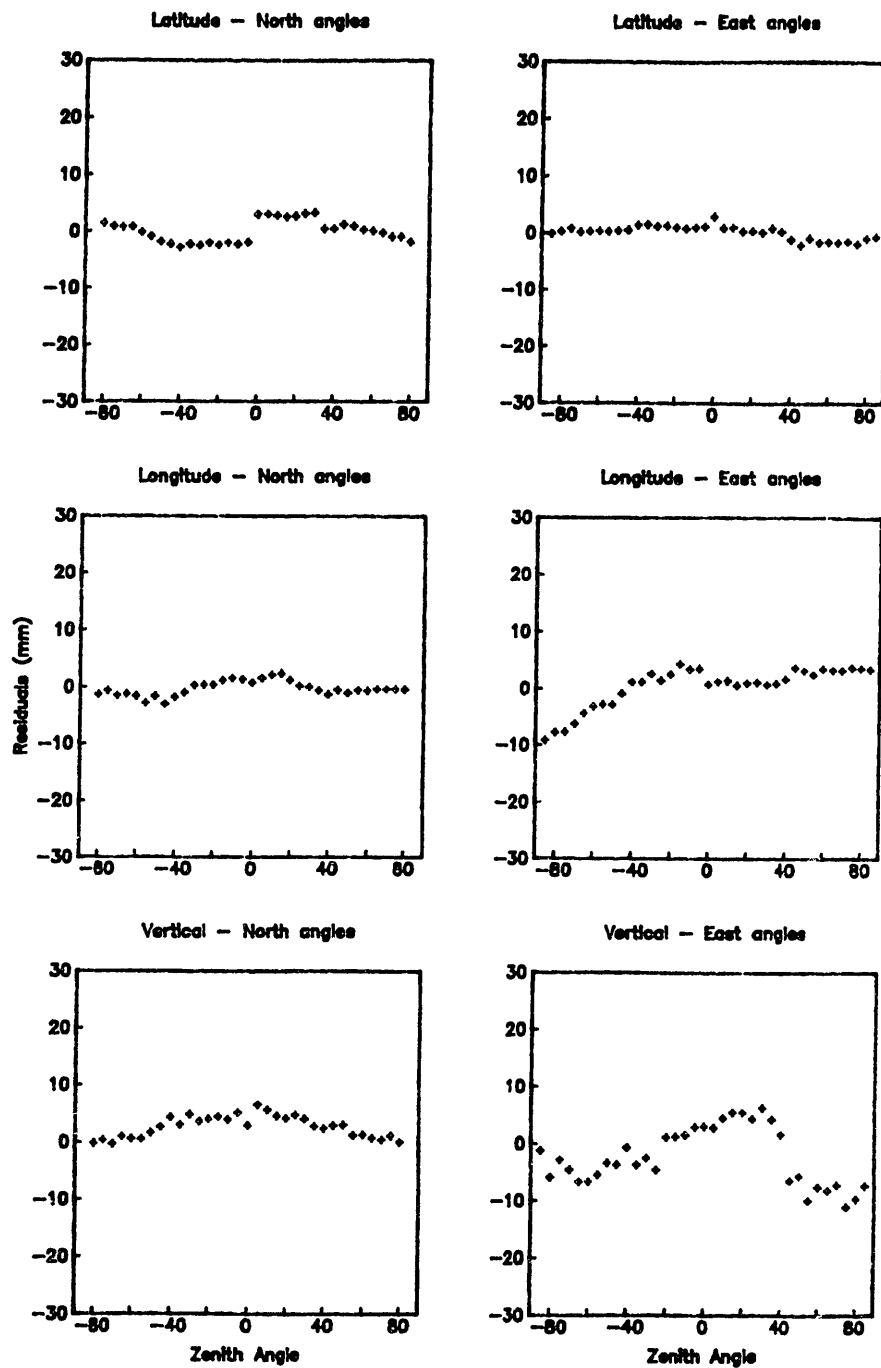


Figure A.4. Same as Figure A.2, except that the kinematic model assumes that both the origin and the target have vertical and horizontal displacements with respect to the reference point (Test 3).

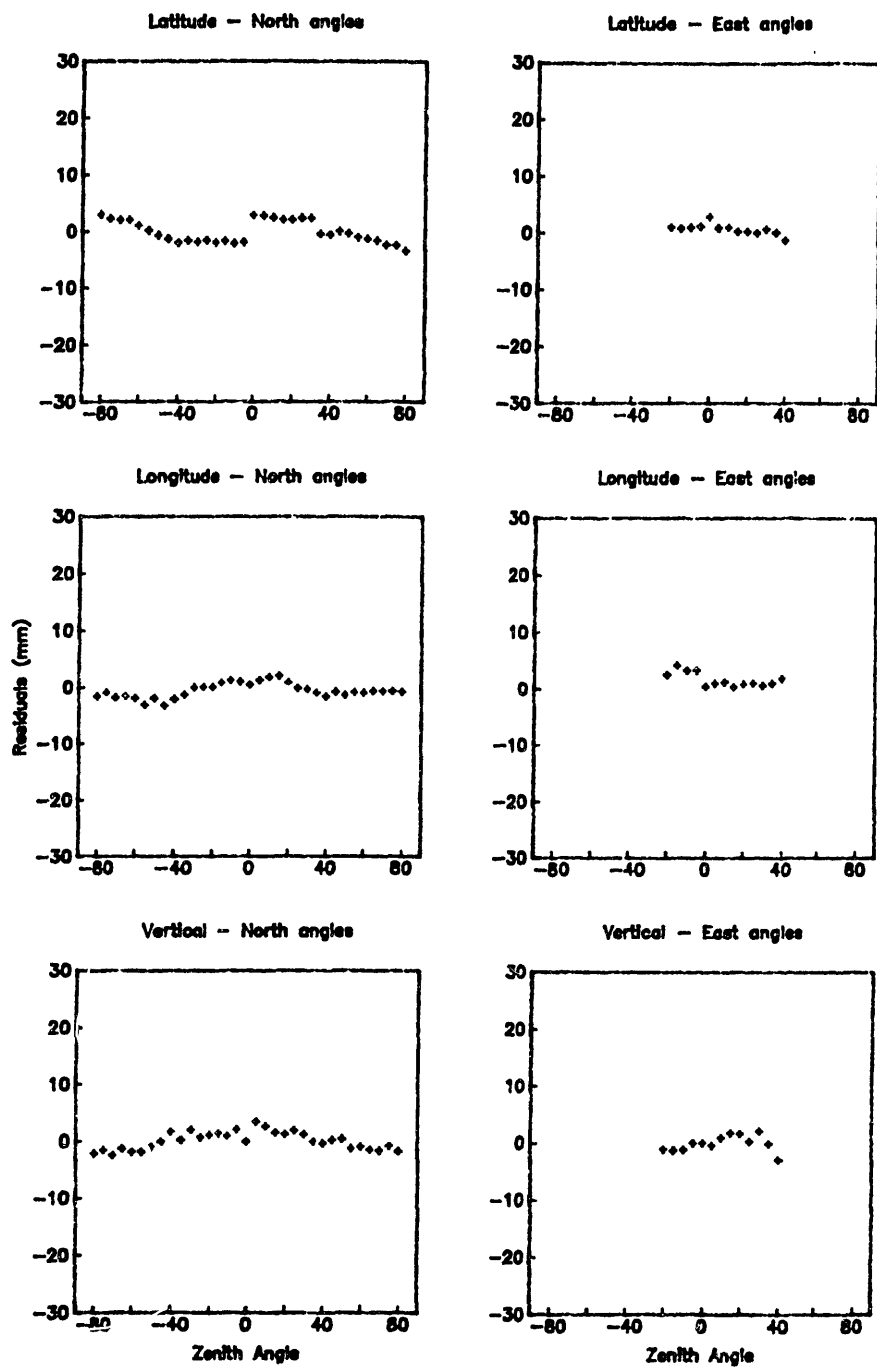


Figure A.5. Same as Figure A.4, except the low elevation east angles have been deleted to assess possible biases from antenna deformation (Test 4).

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