



HEAT CONDUCTION FROM A FINE WIRE

by

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## ABSTRACT

Lees and Liu developed a method for calculating the heat conduction from a wire in rarefied gases assuming complete thermal accommodation at the surface. In this Thesis a modification of this method is presented to include all finite values of the accommodation coefficient. This modified method has been theoretically compared to the low-pressure and to the temperature-jump methods. For experimental verification of the method an apparatus was designed and built. A detailed description of the apparatus and a complete procedure are included in the Thesis. Preliminary calibration of the filament resistance has been performed.

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## TABLE OF CONTENTS

	<u>Page</u>
1. Introduction	1
2. Theoretical Discussion	4
2a. The Low-pressure Method	8
2b. The Temperature-jump Method	10
2c. The Moment Method	12
2d. Theoretical Comparison of Methods	13
3. Apparatus	18
4. Procedure	28
4a. Evacuation of the System	28
4b. Bake-out and Flashing the Filament	29
4c. Calibration	30
4d. Measurements	32
4e. Shutting of the Vacuum System	33
5. Sample Data on the Calibration of the Filament	34
Appendix	
Modification of the Moment Method for all Values of $\alpha$	40
Equipment	46
Bibliography	48

## LIST OF FIGURES

	<u>Page</u>
1. Cross section of experimental test tube	4
2. Pressure regimes as defined by the moment method	6
3. Values of pressure, $N_k$ , and mean free path for $r_2/r_1 = 340$	7
4. Theoretical curves showing heat conduction by the low-pressure, the temperature-jump, and the moment method	16
5. Percentage differences between the low-pressure, temperature-jump, and the moment methods	17
6. Photograph of the apparatus	18
7. Apparatus	19
8. Constant temperature bath	21
9. The test tube	22
10. Circuit for filament flashing	23
11. Electrical circuit for voltage measurements	24
12. Wiring arrangement for test tube	26
13. Cross-section of an oven	27
14. Voltage measurements across the filament	31
15. Typical calibration curve	37
16. Nonlinear calibration curve	38
A-1. Division of velocity space	40

## LIST OF TABLES

1. A sample set of voltage measurements	35
2. Sample calculation of calibration data	36

## 1. Introduction.

In order to calculate the heat conducted from a surface, it is necessary to know the thermal accommodation coefficient\* at the surface. In the past many theoretical and experimental investigations were performed to obtain a relationship between the heat conduction and the A. C.

Most of the theoretical models developed for the investigation of heat conduction in rarefied gases are restricted to certain pressure regimes. In the low pressure region, where the molecular mean free path is much larger than the characteristic dimensions of the apparatus, the "low-pressure" method has been used. This method was developed in 1910 by Knudsen<sup>1</sup>. In the pressure region near the continuum the "temperature-jump method" developed by Kennard<sup>2</sup> and later modified by Thomas and Golike<sup>3</sup> is used.

Many attempts have been made in the past to develop a theoretical model for calculating the conductivity of a gas at all pressure levels. G. K. Bienkowski<sup>4</sup> developed a theory for calculating the heat transfer between two parallel plates in supersonic flow. For still gases, a model was proposed by L. Lees<sup>5</sup> based on the division of velocity space to two separate regions. Lees' method was extended by L. Lees and C. Y. Liu<sup>6</sup> for cylindrical geometry, the geometry that has been most widely used in past experiments. It is appropriate to call this method the "moment method" because it utilizes Maxwell's moment equations. The form in which L. Lees and C. Y. Liu presented their method is only applicable when the A. C. is unity.

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\* From here on referred to as A. C. or  $\alpha$ .

One of the first scientists to perform experiments in heat conduction was Smoluchowski,<sup>7</sup> who measured the heat conducted by hydrogen from a thermometer bulb. The majority of the experiments that followed were performed in the low-pressure region, and calculations of the A. C. were based on the low-pressure method. Knudsen,<sup>1</sup> Weber,<sup>8</sup> and Keesom and Smidt<sup>9</sup> used the low-pressure method to get values of the A. C. on glass.

A comparative study of the A. C. by the low-pressure and the temperature-jump methods was performed by Thomas and Golike<sup>3</sup> in 1953. Their results showed very good agreement between the two methods. Recently Peterson<sup>10</sup> compared the two methods and found that both methods gave the same value for the A. C.

Relatively few experiments had been performed in the transition region. G. J. Bienkowski<sup>4</sup> performed experiments in the transition region, but he has few results and these are not conclusive. He says that his apparatus was "dirty" and consequently his results could be used only "as a partial reference for comparison of different theoretical methods." Bomelburg<sup>11</sup>, and Schafer, Ratings and Eucken<sup>12</sup> measured the heat conduction in the transition regime but they did not measure  $\alpha$ . C. F. Dewey<sup>13</sup> measured the heat loss and the recovery temperature of a hot wire in hypersonic flow. He did not report values of A. C. and did not mention whether he kept his apparatus clean.

In earlier investigations no serious attempt was made to maintain a clean surface. It was Roberts<sup>14</sup> who first pointed out the necessity of having a clean surface, and compared values of A. C. on clean and dirty surfaces. He showed, for example, that the A. C. of helium on clean tungsten is 0.05 while, if the surface is dirty, the A. C. is 0.19.

Later work by Thomas and Schofield<sup>15</sup> confirmed the arguments of Roberts.

As of now, a carefully controlled experiment whose result could be used to verify the moment method has not been performed in all three pressure regimes. The aim of the present investigation is two-fold. It is to extend the moment method to incorporate all possible values of  $\alpha$ ; and to design and build an apparatus for future controlled experiments.

2. Theoretical Discussion.

A cross-section of the cylindrical geometry used in the present investigation is shown in Fig. 1.

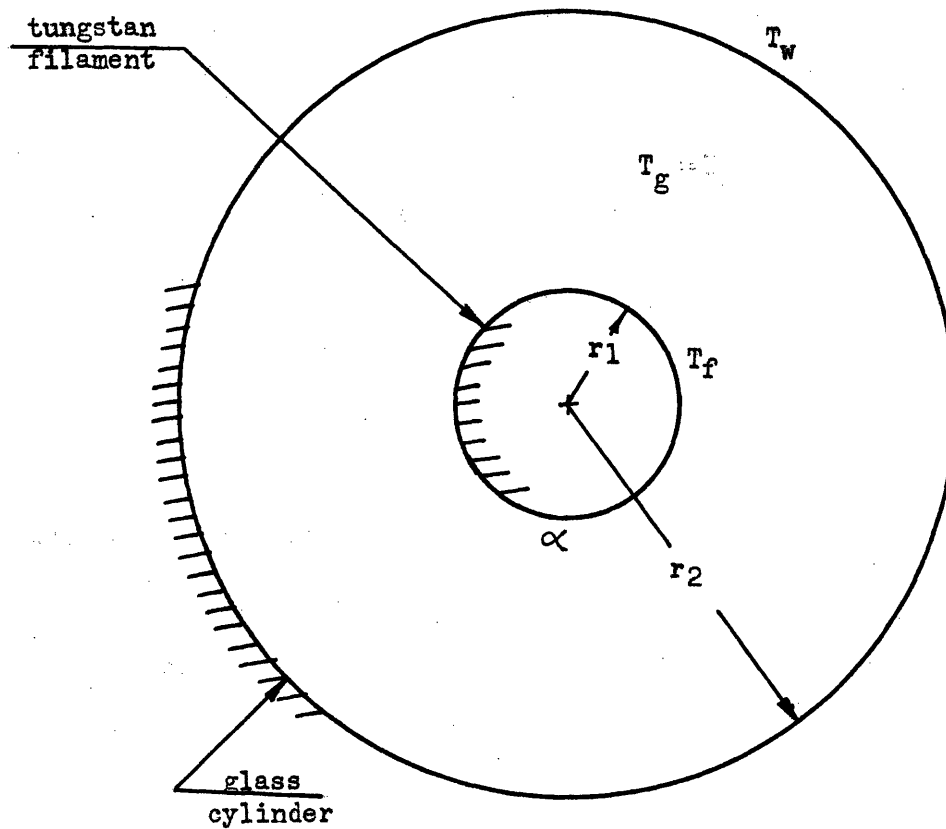


Figure 1  
Cross-section of experimental test tube

In Fig. 1,  $r_1$  is greatly exaggerated. Actually,  $r_2/r_1$  is approximately equal to 340.

Following Knudsen's definition, the average efficiency of the energy exchange per collision at the interface of a monatomic gas and a solid

is called the accomodation coefficient and is expressed as

$$\alpha = \frac{T_r - T_g}{T_f - T_g} \quad (1)$$

In this expression  $T_g$ ,  $T_f$ , and  $T_r$  are the temperatures of the incoming gas, the filament surface, and the rebounding gas respectively. The mean energy per molecule for monatomic gases corresponding to temperature  $T_i$  is  $\epsilon_i = 2kT_i$  where  $k$  is the Boltzmann constant. Utilizing the equation  $W = N\epsilon$  where  $N$  is the number of molecules per unit area per second, Eq. (1) can be rewritten for cylindrical symmetry as

$$\alpha = \frac{W_r - W_w}{W_f - W_w} \quad (2)$$

In Eq. (2) it is assumed that there is perfect accomodation at the surface of the outside cylinder, i.e.,  $T_g = T_w$  where  $T_w$  is the temperature of the outside cylinder.

Generally the Knudsen number is defined as the ratio of the mean free path to the diameter of the filament. In this paper, however, reference to pressure will be made by the reciprocal of the Knudsen number and will be denoted by

$$N_k = \frac{2r}{\lambda} \quad (3)$$

Although the moment method has not been verified experimentally yet, one of its results is shown in Fig. 2. It merely serves to illustrate some ideas.  $\lambda$  is assumed to be an average value of the mean free path,

and  $r_1$  and  $r_2$  are the radii of the inner and the outer cylinders respectively. As is seen from Fig. 2, the pressure regimes are functions of not only  $\lambda$  but also of the diameter ratios.

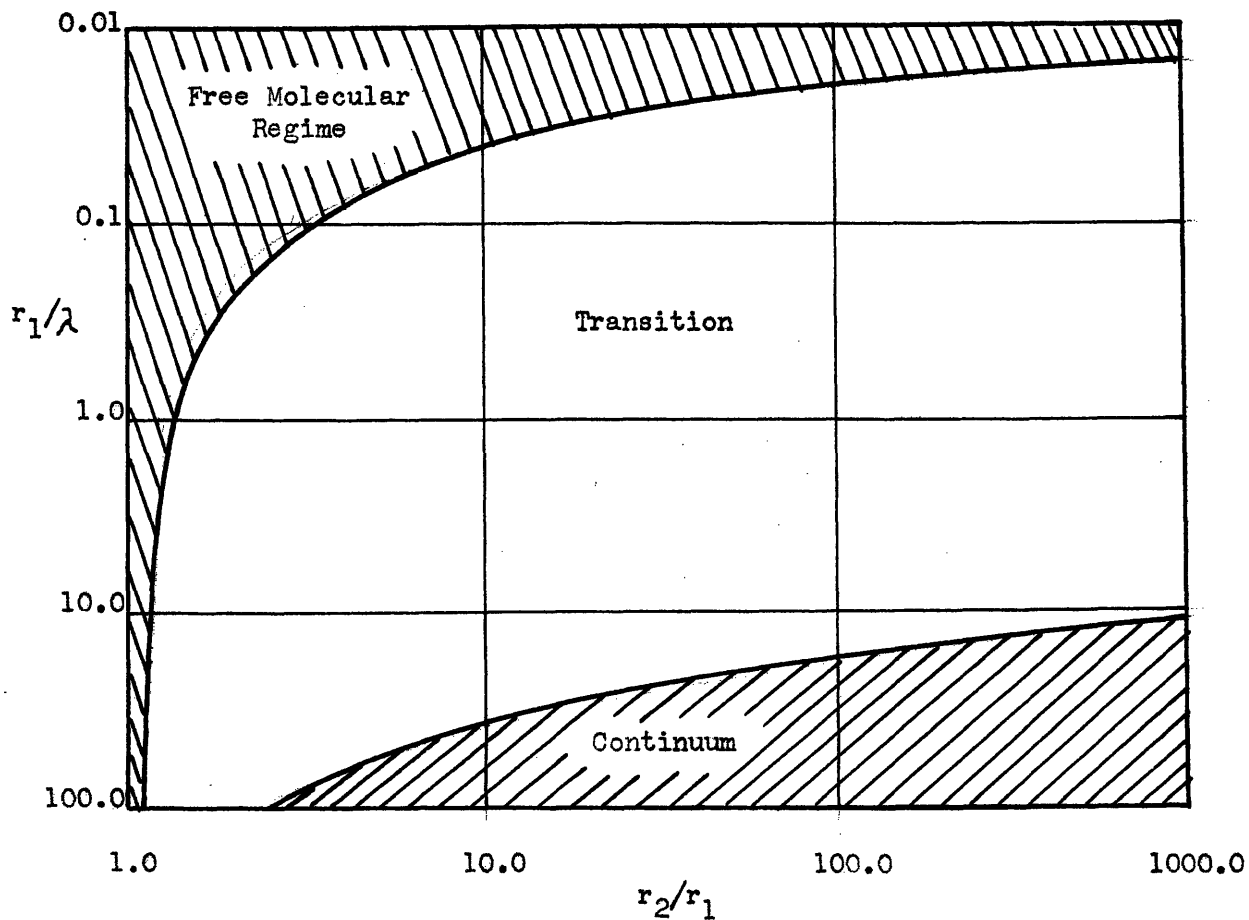


Figure 2  
Pressure regimes as defined by the moment method

An application of Fig. 2 to the diameter ratio used in this experiment is shown in Fig. 3.

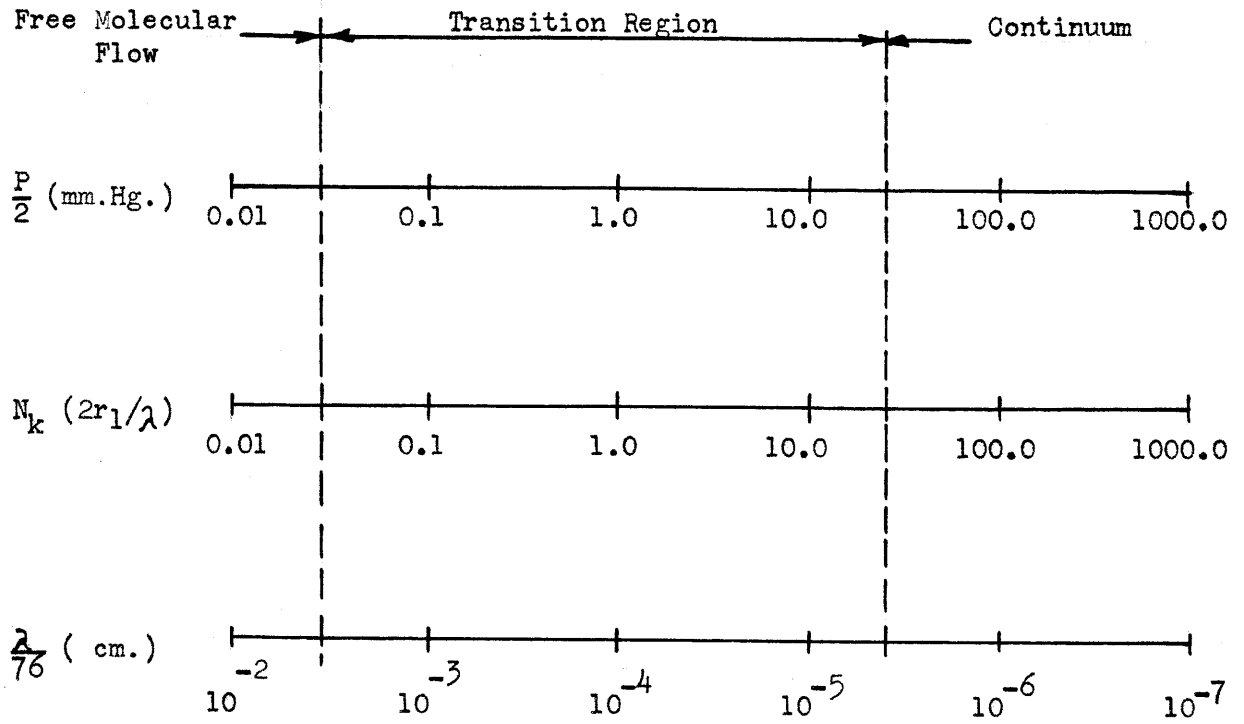


Figure 3

Values of pressure,  $N_k$ , and mean free path for  $r_2/r_1 = 340$

2a. The low-pressure method.

The expression for the accommodation coefficient as given by Wachmann<sup>16</sup> is

$$\alpha = \frac{W}{2PR(T_f - T_w)} \sqrt{2\pi MRT_w} \quad (4)$$

where  $W$  is the steady state power loss from unit filament area to gas.

At atmospheric pressure

$$W_\infty = \frac{\bar{K}(T_f - T_w)}{r_1 \ln r_2/r_1} \quad (5)$$

where  $\bar{K}$  is an average thermal conductivity expressed by

$$\bar{K} = \frac{\int_{T_f}^{T_w} k dT}{T_w - T_f} \cdot$$

The relation between conductivity and mean free path can be written as

$$\frac{\bar{K}}{\lambda_{\bar{K}}} = \frac{15}{4} P \sqrt{\frac{2R}{\pi M \bar{T}_{\bar{K}}}} \quad (6)$$

where

$$\bar{T}_{\bar{K}} = \frac{T_f + T_w}{2} \cdot$$

With the assumption  $\lambda_{\bar{K}} = \lambda_f = \lambda_w = \lambda$ , Eq. (6) can be written as

$$\frac{\bar{K}}{\lambda} = \frac{15}{4} P \sqrt{\frac{2R}{\pi M T_R}} \quad (7)$$

Combining Eqs. (4), (5) and (7), the final result for the theoretical heat conduction in the free molecular flow regime is given by

$$\frac{W}{W_\infty} = \frac{1}{\frac{1}{\alpha} + \frac{15}{2} \cdot \frac{1}{\ln r_2/r_1} \cdot \frac{1}{2r_1/\lambda} \sqrt{\frac{2}{T_R/T_w + 1}}} \quad (8)$$

This dimensionless form is useful for comparison with other methods.

2b. The temperature-jump method.

Following the derivations of Thomas and Golike<sup>3</sup> the heat conducted in a cylindrical cell per unit area per second is given by

$$W = \frac{\bar{K} (T_P - T_w)}{h_1 \ln r_2/r_1 + g_1/\rho + (r_1/r_2) \cdot g_2} \quad (9a)$$

If it is assumed that  $r_1/r_2$  is a very small number, then Eq. (9a) can be written as

$$W = \frac{\bar{K} (T_P - T_w)}{h_1 \ln r_2/r_1 + g_1/\rho} \quad (9b)$$

where

$$g_1 = \frac{1}{\beta} \cdot \frac{2-\alpha}{\alpha} \bar{K} \quad (10)$$

$\beta$  is defined as

$$\beta = \frac{\left(\frac{T_w + T_P}{2}\right)^{1/2}}{T_P} \cdot \left(\frac{8R}{\pi M}\right)^{1/2} \quad (11)$$

Combining Eqs. (9b), (10) and (11) one gets

$$W = \frac{\bar{K} (T_P - T_w)}{h_1 \ln r_2/r_1 + \frac{2-\alpha}{\alpha} \cdot \frac{1}{2\rho} \bar{K} \left(\frac{\pi M}{2R} \cdot \frac{T_P + T_w}{2}\right)^{1/2} \cdot \frac{T_P}{T_w}} \quad (12)$$

Again, combining Eqs. (5), (7) and (12) results in

$$\frac{W}{W_{\infty}} = \frac{1}{1 + \frac{2-\alpha}{\alpha} + \frac{15}{2} \frac{1}{\text{Mach}^2} + \frac{1}{2} \frac{1}{\lambda} + \frac{2}{1 + T_p/T_w} + \frac{T_p}{T_w}} \quad (13)$$

Equation (13) is valid only near the continuum region.

2c. The moment method.

Since this method is outlined in detail in the Appendix, only a brief description is presented here. This method is based on the division of the velocity space between the two cylinders to two distinct regions. The average properties of the gas are found by the use of velocity distribution functions related to each region. The form in which the final equation is presented by L. Lees and C. Y. Liu<sup>6</sup> is

$$\frac{W}{W_{\infty}} = \frac{1}{1 + \frac{15}{2} \frac{1}{\ln r_2/r_1} + \frac{1}{2r_1/\lambda}} \quad (14)$$

In the derivation of Eq. (14) it was assumed that the temperature ratio of the filament and the wall is nearly unity. Also it was assumed that there is perfect accommodation between the filament and the gas.

One of the objectives of the present investigation is to extend the moment method for all values of  $\alpha$ . The detailed derivation of the extension is presented in the Appendix. Only the final result is shown below:

$$\frac{W}{W_{\infty}} = \frac{1}{1 + \frac{1}{\alpha} \times \frac{15}{2} \cdot \frac{1}{\ln r_2/r_1} + \frac{1}{2r_1/\lambda}} \quad (15)$$

Unlike Eqs. (8) and (13), the above expression is valid at any pressure. If this method is experimentally verified and found to be good, it would serve as the most general method for calculating heat conduction.

2d. Theoretical Comparison of Methods.

The similarity between Eqs. (8), (13) and (15) is obvious. The equations could be further simplified if the assumption is made that the temperature ratio of filament and gas is nearly unity. This assumption has been made already in the case of the moment method. That is why the temperature ratio does not appear in Eq. (15). The simplified form of the equations can be written as

$$\begin{aligned}
 \left( \frac{W}{W_{ob}} \right)_{\text{low pressure}} &= \frac{1}{\frac{1}{\alpha} \times \frac{15}{2} \times \frac{1}{\ln r_2/r_1} \times \frac{1}{2r_1/\lambda}} \\
 \left( \frac{W}{W_{ob}} \right)_{\text{temp. jump}} &= \frac{1}{1 + \frac{2-\alpha}{\alpha} \times \frac{15}{2} \times \frac{1}{\ln r_2/r_1} \times \frac{1}{2r_1/\lambda}} \\
 \left( \frac{W}{W_{ob}} \right)_{\text{moment meth.}} &= \frac{1}{1 + \frac{1}{\alpha} \times \frac{15}{2} \times \frac{1}{\ln r_2/r_1} \times \frac{1}{2r_1/\lambda}}
 \end{aligned} \tag{16}$$

A further simplification is possible if it is assumed that the diameter ratio is constant. Let

$$Z = \frac{15}{2} \times \frac{1}{\ln r_2/r_1} \tag{17}$$

Then Eqs. (16) become

$$\left. \begin{aligned}
 \left(\frac{W}{W_\infty}\right)_{\text{low pressure}} &= \frac{\alpha}{Z} N_k \\
 \left(\frac{W}{W_\infty}\right)_{\text{temp. jump}} &= \frac{1}{1 + \frac{2-\alpha}{\alpha} \frac{Z}{N_k}} \\
 \left(\frac{W}{W_\infty}\right)_{\text{moment meth.}} &= \frac{1}{1 + \frac{1}{\alpha} \frac{Z}{N_k}}
 \end{aligned} \right\} \quad (18)$$

In Eqs. (18) one can consider  $\alpha$  and  $Z$  as constants and examine the variation with  $N_k$ .

When  $N_k$  is large, i.e., the pressure is near atmospheric

$$\lim_{N_k \rightarrow \text{large}} \left(\frac{W}{W_\infty}\right)_{\text{moment meth.}} = \left(\frac{W}{W_\infty}\right)_{\text{temp. jump}} \quad (19)$$

When  $N_k$  is small, i.e., the pressure is within the free molecular flow regime,

$$\lim_{N_k \rightarrow \text{small}} \left(\frac{W}{W_\infty}\right)_{\text{moment meth.}} = \left(\frac{W}{W_\infty}\right)_{\text{low pressure}} \quad (20)$$

The conclusion from the above argument is that the moment method is possibly valid when the pressure is either very low or very high, since then the result agrees with that of two other methods which already have been accepted as good ones. This implies that the smooth transition between the free molecular flow and the continuum regimes, given by the

moment method, may represent a very good approximation to the actual heat transfer. In Fig. 4 theoretical curves are drawn within the transition region using the above mentioned three methods. Figure 4 shows that the moment method is slowly approaching the low-pressure method and the temperature-jump method as the pressure is decreased or increased respectively. A more accurate evaluation is possible by investigating the percentage difference between methods. This is shown in Fig. 5. The percentage difference between the moment method and the other two methods is not more than one percent within their respective regions.

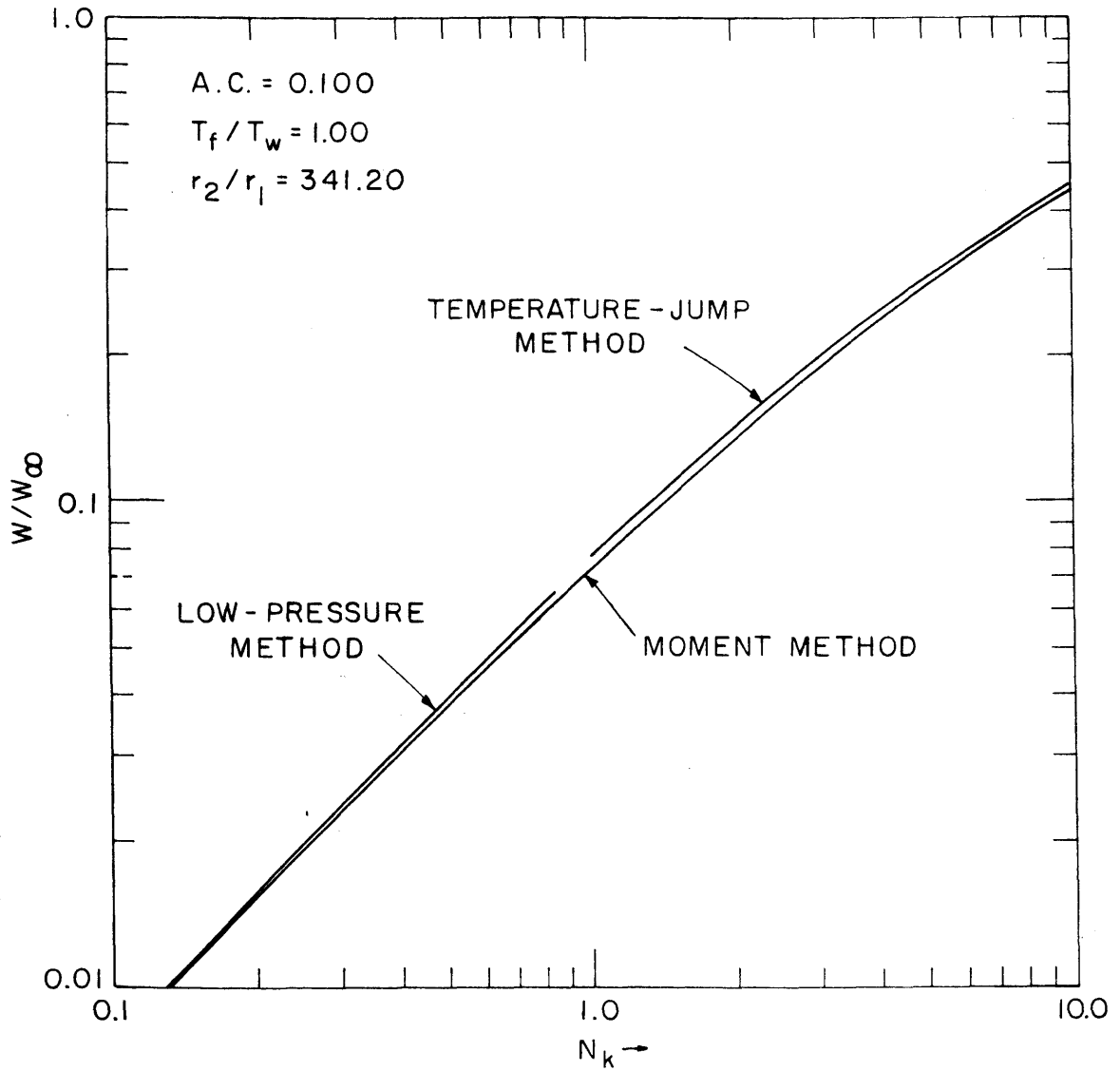


FIGURE 4. THEORETICAL CURVES SHOWING HEAT CONDUCTION BY THE LOW-PRESSURE, THE TEMPERATURE JUMP AND THE MOMENT METHOD IN THE TRANSITION REGION

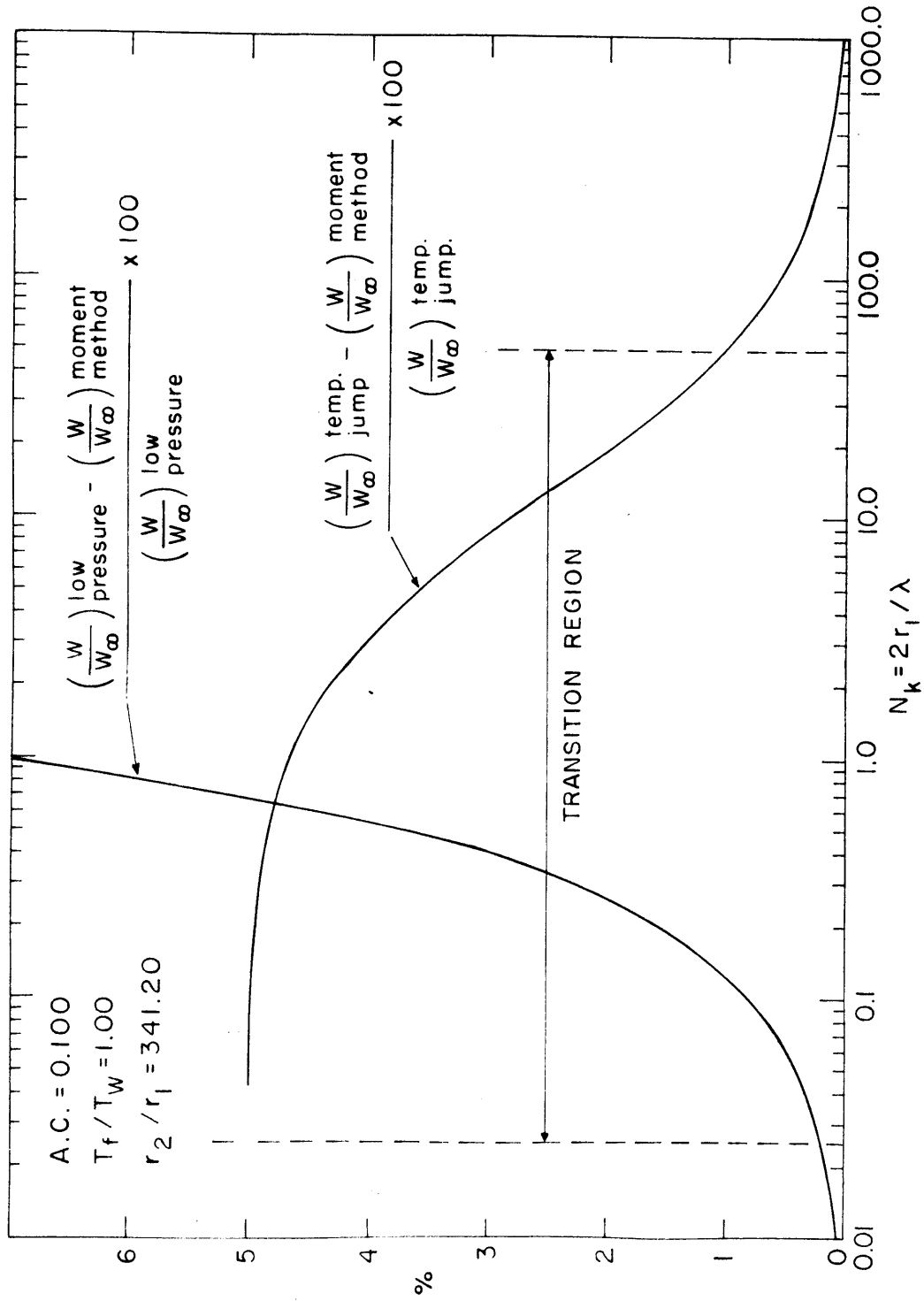


FIGURE 5. PERCENTAGE DIFFERENCES BETWEEN THE LOW-PRESSURE, TEMPERATURE JUMP AND THE MOMENT METHODS

### 3. Apparatus.

A photograph and a schematic diagram of the apparatus are shown in Fig. 6 and Fig. 7 respectively.

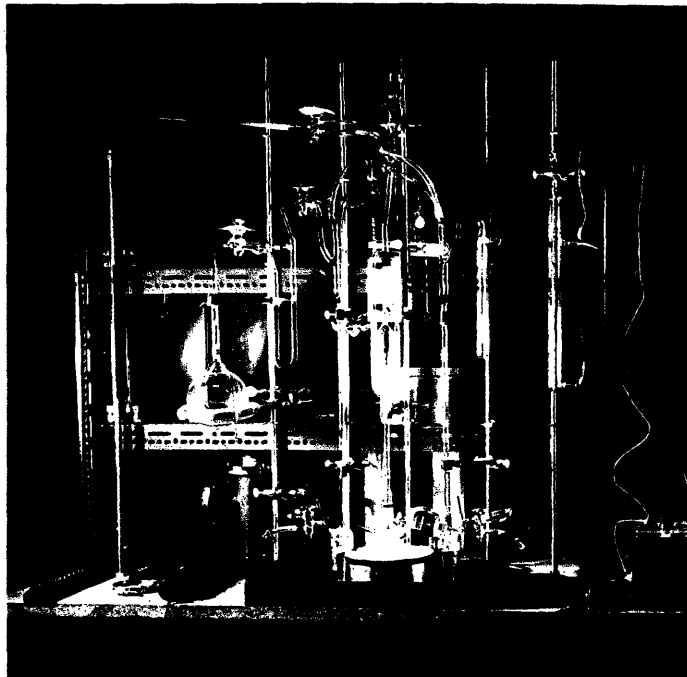


Figure 6  
Photograph of the apparatus

Essentially the apparatus consists of a 1/2 inch diameter pyrex glass vacuum system. The apparatus includes a gas supply system, the test tube with a tungsten filament, constant temperature bath for the test tube, and electrical measuring apparatus.

The vacuum is maintained by a mechanical forepump and an oil diffusion pump. A large liquid nitrogen trap is placed between the oil diffusion pump and the rest of the system. This trap prevents the oil from going into the vacuum system. An air-bleed stopcock is placed between the oil diffusion pump and the trap for letting the air into the system after the pumps are

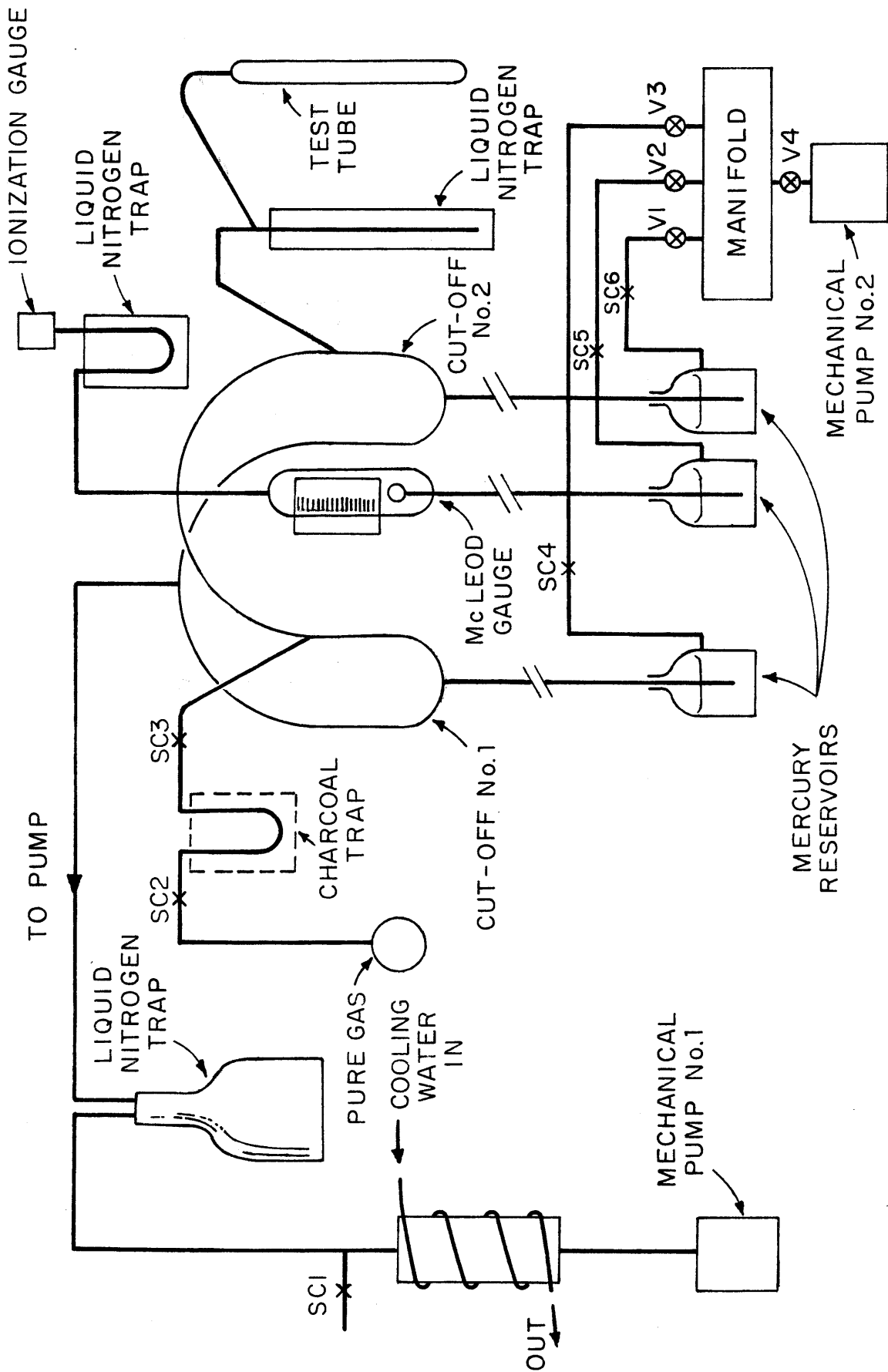


FIGURE 7 APPARATUS

shut down.

There are two pressure gauges installed into the system. The ionization gauge merely serves for indicating the approximate pressures beyond the limits of the McLeod gauge. For accurate pressure measurements a McLeod gauge is used. The range of the McLeod gauge is from 1.6 mm Hg to .1 micron. Above 1.6 mm Hg, a mercury manometer is used. One of the mercury cut-offs serves as a manometer as described in the procedures. Eastern triple distilled mercury is used for the cut-offs and the McLeod gauge. The maximum limit of impurities is 0.0000%.

The gas supply system consists of a charcoal trap, two stopcocks and a flask of helium welded onto the glass system. The helium is Airco reagent grade and it is passed through a charcoal trap which is cooled by liquid nitrogen. Two mercury-seal type stopcocks are used to regulate the amount of gas let into the test tube. All stopcocks are lubricated with Apiezan-N high vacuum grease. This grease is also used to fill up the cups of the stopcocks.

The constant temperature bath consists of a tin can, SAE 10 motor oil, heater, stirrer and a thermoregulator. A cross-section of the constant temperature bath is shown in Fig. 8. A 300 watts knife type immersion heater is used. The stirrer is driven by a small DC motor at about 60 rpm. A bimetallic thermoregulator is used. Temperature readings are taken by two thermometers with smallest divisions of 0.1 C.

The test tube used in the experiment is shown in Fig. 9. The inside diameter of the glass tube is 2.6 cm. The filament diameter is 0.003 inches and its total length is 28.655 cm. The filament is held tight by two identical tungsten springs. The spring wire diameter is 0.007 inches and its length is about 5 cm when stretched. A small 0.001 inch diameter tungsten lead wire is attached to the filament at 5.890 cm from the top

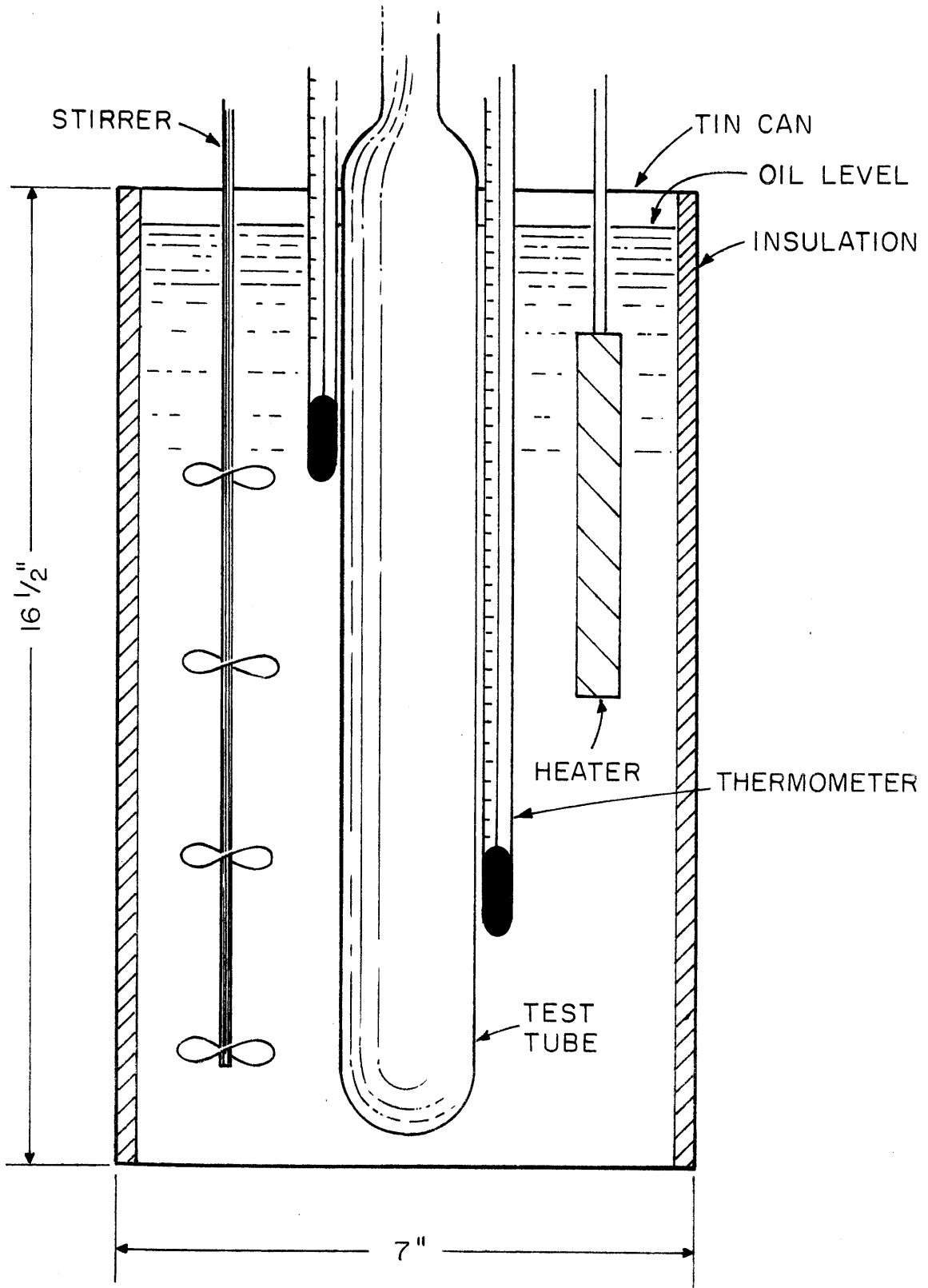


FIGURE 8 CONSTANT TEMPERATURE BATH

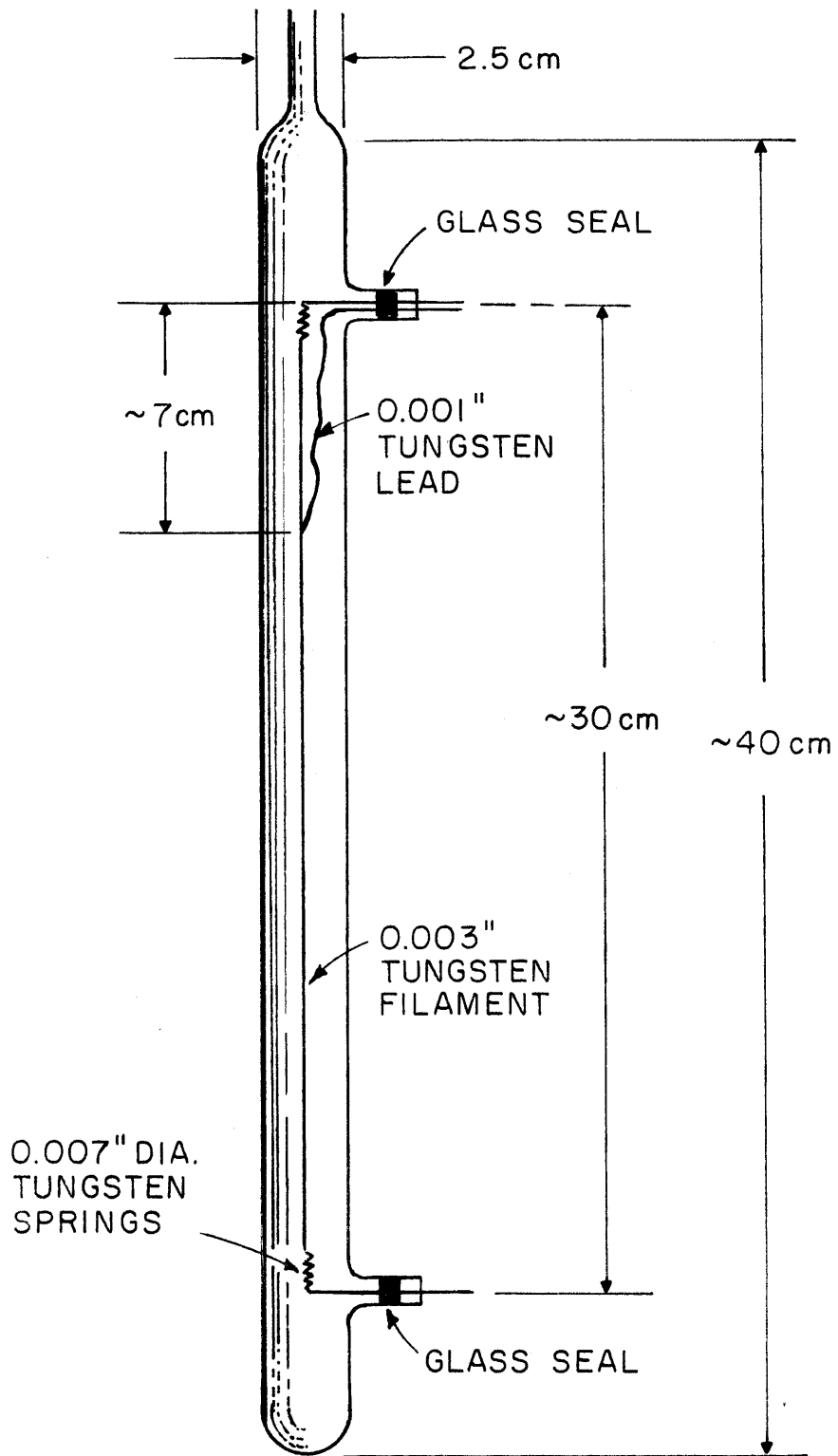


FIGURE 9 THE TEST TUBE

end of the filament. End losses are estimated by the use of this wire. Electrical connections are made by lead wires embedded in a vacuum tight glass seal.

Two separate electrical systems are used during the experiment. The filament flashing circuit is shown in Fig. 10. It includes 22 ohm variable resistor, three 62  $\Omega$  resistors connected parallel, an ammeter and a DC power supply. The circuit used for voltage measurements is more elaborate and it is shown in Fig. 11. A Leeds and Northrop potentiometer is used with a standard cell and a reflecting mirror type galvanometer. Depending on the sensitivity of the galvanometer an accuracy of 0.004% can be achieved with the potentiometer.

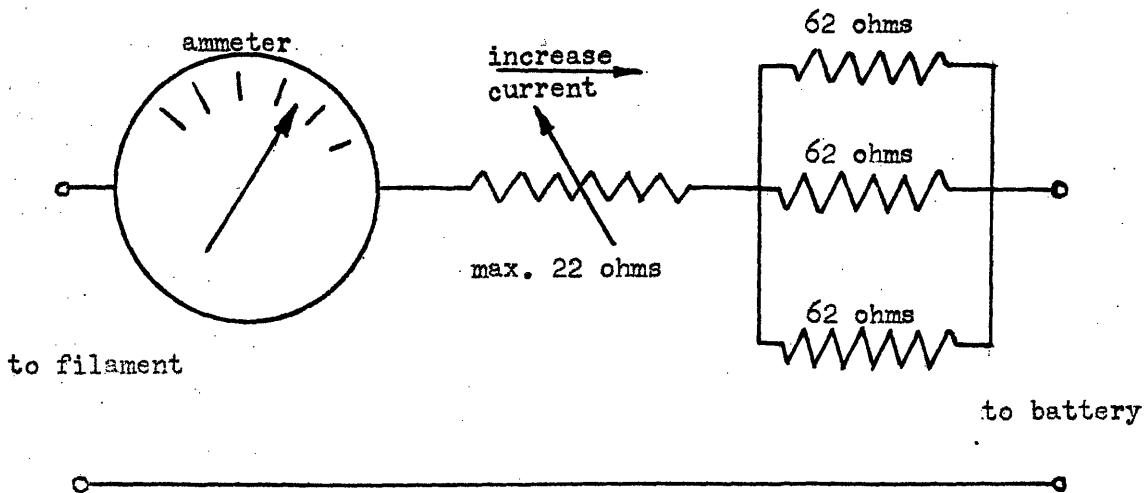


Figure 10  
Circuit for filament flashing

A terminal plate is mounted above the test tube. From the test tube to this terminal plate electrical connections are made by using stranded, gauge no. 12 nickel coated copper wire. The wires are silver soldered

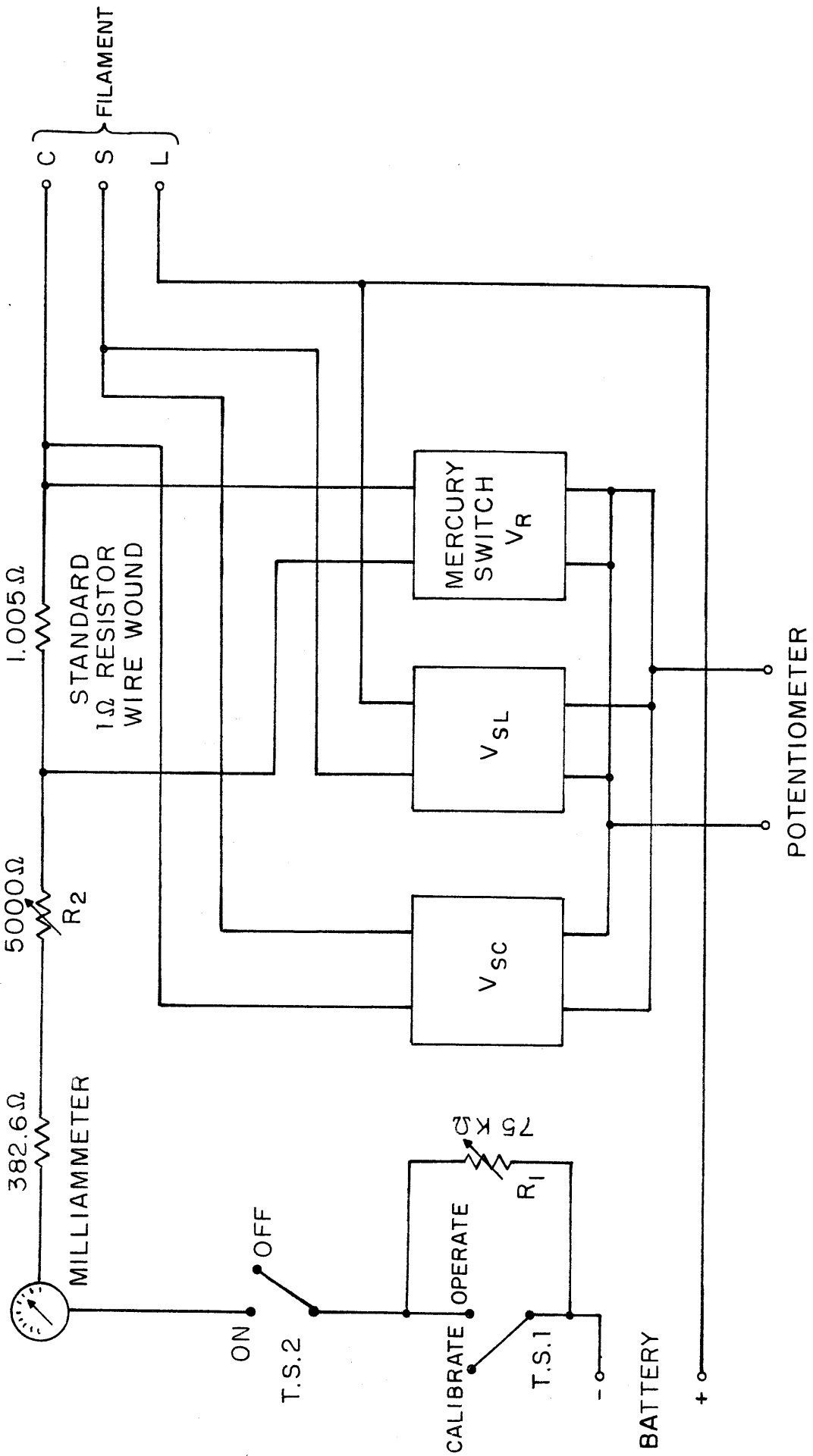


FIGURE 11. ELECTRICAL CIRCUIT FOR VOLTAGE MEASUREMENTS

to the leads coming out from the test tube. The terminal plate is connected to the electrical apparatus by gauge no. 12 plastic coated solid copper wires. This arrangement is shown in Fig. 12. Where possible, all the joints were soldered for better conduction.

There are two types of ovens for baking out the system. These were designed to heat up the glass system to at least 400°C. The traps and the test tube are baked out using cylindrical shaped ovens. The cross-section of one of these ovens is shown in Fig. 13. The ovens are regulated with powerstats.

An oven is built around the ionization gauge trap, the McLeod gauge, and the two mercury cut-offs. Essentially, the oven is a box made out of 1/4 inch thick asbestos boards. No. 20 nichrome wire is stretched out between standoff posts on the inside of the boards. The sides of the oven are removable, so that they can be taken off when the oven is not in operation.

To measure the temperature of the glass during the bake-out period copper-constant thermocouples are used with a ten-post selector switch. All clamps for supporting the glass are special cast-iron clamps which do not creep at high temperatures.

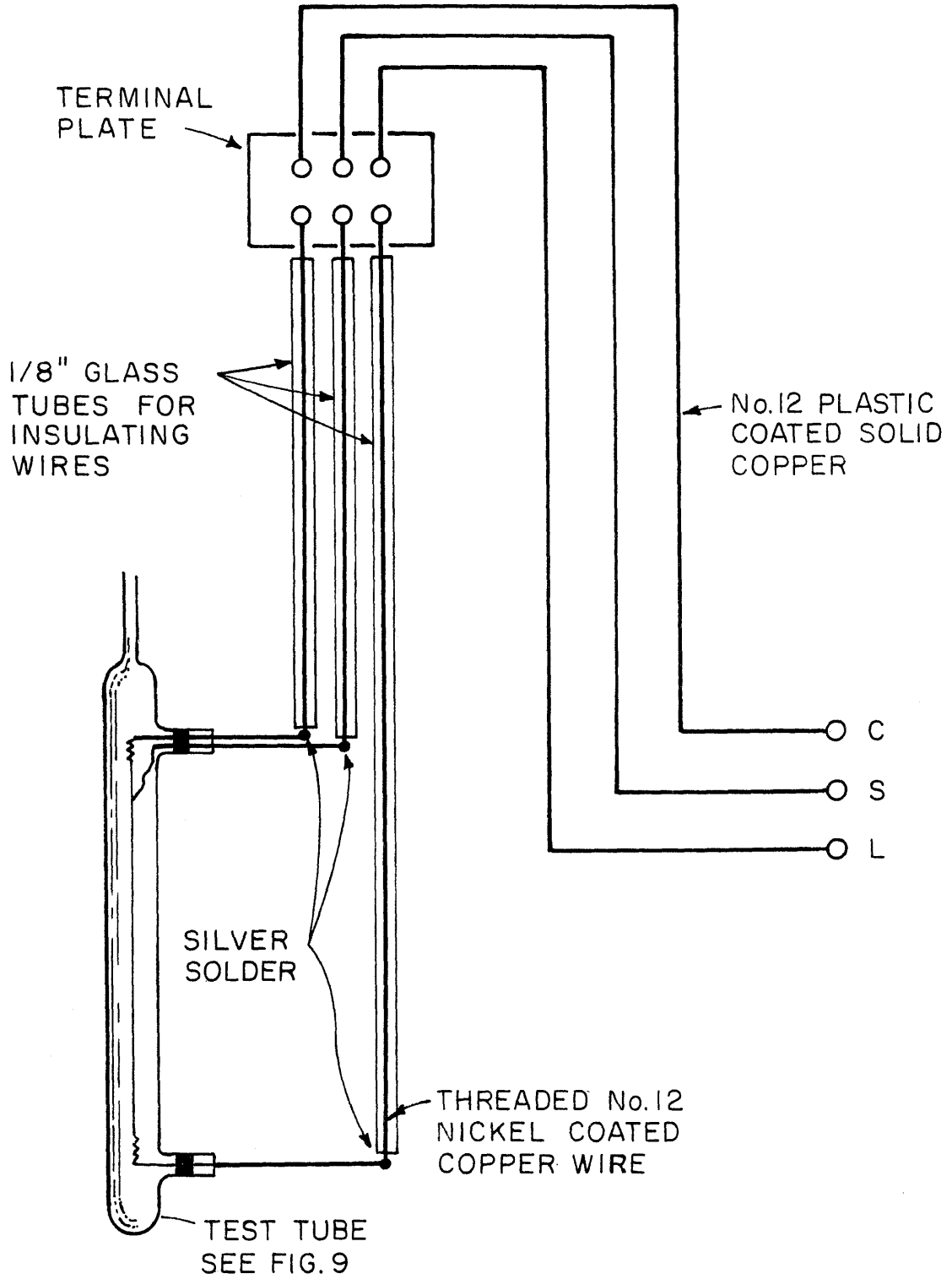


FIGURE 12. WIRING ARRANGEMENT FOR TEST TUBE

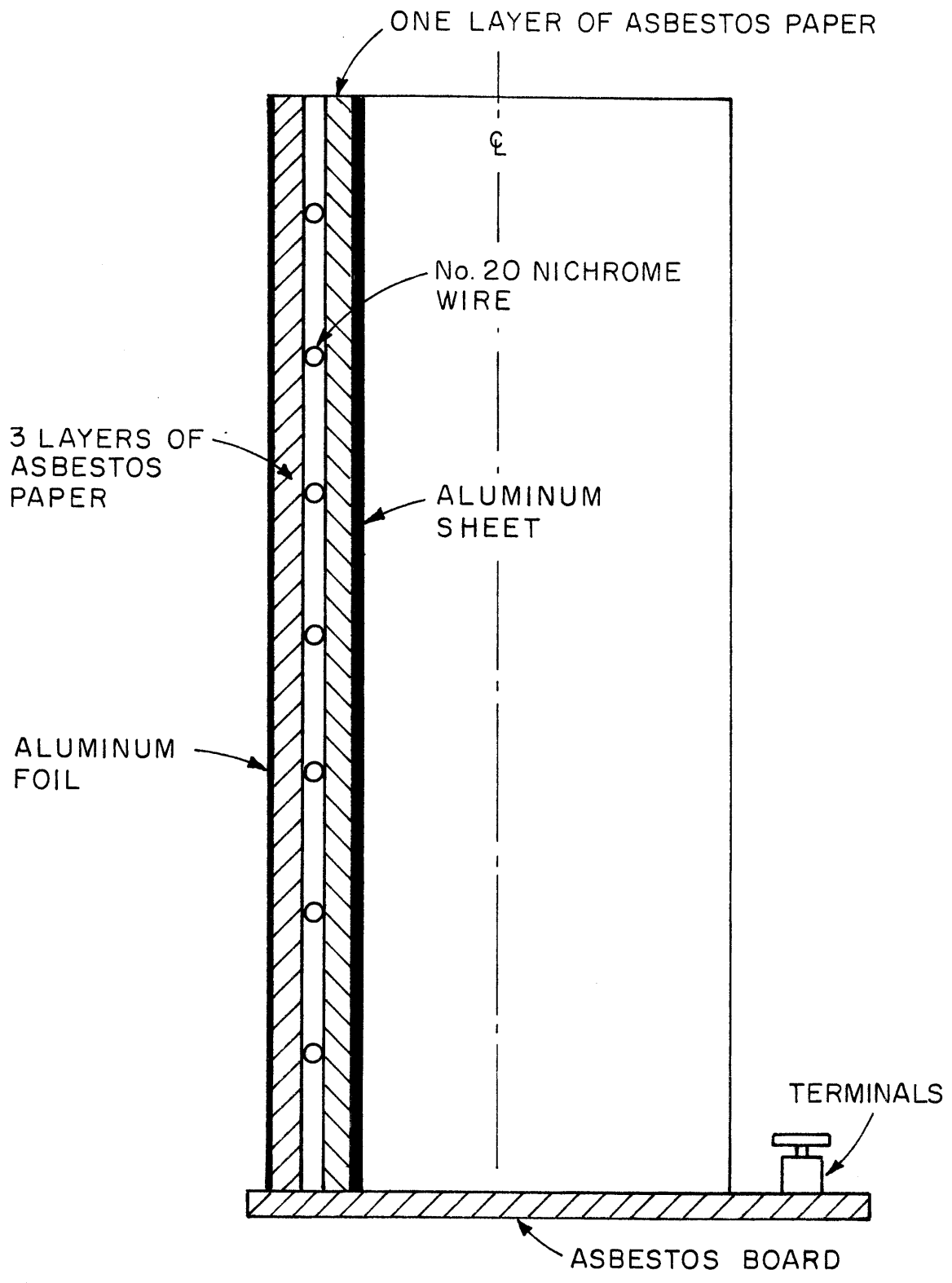


FIGURE 13 CROSS SECTION OF AN OVEN

4. Procedure.

4a. Evacuation of the system.

Before the mechanical pumps are turned on valves V1, V2, V3, V4, and stopcocks SC2, SC3, SC4, SC5, and SC6 are fully opened. SC1 is closed. Then both mechanical pumps are turned on at the same time. This way, the mercury in the reservoirs will stabilize itself and will not start bubbling. After a few minutes of pumping the pressure should be low enough (below 180 microns) so that the diffusion pump can be turned on. Before this is done the water cooling system has to be turned on. After the diffusion pump is on the pressure should rapidly go down to a few microns. This can be checked by the use of the McLeod gauge. At this point it is advisable to make a final leak detection on the system. Because the ionization gauge cannot be turned on above 1 micron, helium or acetone cannot be used for detecting leaks. However, the glass part of the system can be checked by using a high-frequency tesla coil. The small vacuum chamber of the stopcocks has to be evacuated too. This is done by closing them, i.e., turning them around 180 degrees. After a few minutes the stopcocks are opened again. This procedure has to be repeated a few times until the pressure does not increase suddenly when the stopcocks are opened after they have been closed for a few minutes.

The ionization gauge trap is filled with liquid nitrogen at all times when the gauge is in operation.

In order to get accurate pressure readings on the McLeod gauge the air bubbles inside the mercury have to be freed. This is done by raising the mercury column and then lowering it again. The same thing should be done with the mercury cut-offs.

It is advisable to run the system for a couple of days before baking it out. During this time the gases trapped inside the tubes, especially in the charcoal trap, will be evacuated. If this is not done, the pressure, during bake-out, could rise above 180 microns and thus burn out the diffusion pump. When the pressure is down to about  $5 \times 10^{-6}$  mm Hg, the bake-out procedure should start.

4b. Bake-out and flashing the filament.

With the exception of the following parts, the whole glass system is baked out: the tube between the diffusion pump and the liquid nitrogen trap, the trap itself, the gas containing flask, the mercury reservoirs, and the stopcocks.

The ovens are put into place and all other glass, outside of the ovens, is covered with heating tape. Asbestos tape is wrapped around the heating tapes, and then aluminum foil is wrapped around the asbestos. The stopcocks should be protected from overheating. For this reason, wet asbestos paper is wrapped around them covered with aluminum foil to trap the moisture. Cold water should be added to the asbestos paper to maintain cooling effects.

First, the oven around the charcoal trap is turned on to drive the bulk of the trapped gases out of the system. 2 hours later, the temperature of the ovens is slowly and uniformly raised up to about 350°C. Then the system is continuously baked for two or three days at this temperature.

The filament flashing is done just before the ovens are removed. The flashing circuit is connected to the system. By slowly adjusting the variable resistance a current of about .5 amps is put through the wire. At this current value the wire temperature is approximately 2000°C. After

half an hour the flashing current is cut back to zero and the flashing circuit is disconnected. Now the filament is annealed and its resistivity is set.

The first oven to be removed is the one around the liquid nitrogen trap next to the test tube. After this oven is removed a dewar is placed around the tube and is filled with liquid nitrogen. This dewar should be kept full at all times during measurements. All the heaters are turned off now except the one around the test tube. This is removed last.

#### 4c. Calibration.

The resistance of the filament is calculated by the formula

$$R = R_r [1 + \beta(t - t_r)]$$

In order to calculate  $R$ , the resistance of the wire ( $R_r$ ) is needed at a reference temperature ( $t_r$ ) together with the constant  $\beta$ . To obtain these unknowns the filament has to be calibrated.

After the ovens are removed the constant temperature oil bath is placed around the test tube. Then the pumps are cut off by raising the mercury in the two cut-offs. A few millimeters of helium are let into the system and the test tube is completely isolated from the rest of the system by further raising the mercury in cut-off no. 1. With the temperature of the oil bath set at the desired point, small amounts of current (less than 1ma) are passed through the filament. The switching procedure for this is the following:

1. Connect potentiometer and battery to circuit.
2. Turn both variable resistors to full scale. (clockwise)
3. Turn toggle switch no. 2 on.
4. Turn toggle switch no. 1 to "calibrate."
5. By turning one of the variable resistors counter-clockwise the current in the filament is increased.
6. Turn switch  $V_r$  on and measure the voltage across the standard resistor  $R_{st}$ .
7. Turn switch  $V_{sl}$  on and measure voltage. (see Fig. 14)
8. Turn switch  $V_{sc}$  on and measure voltage. (see Fig. 14)

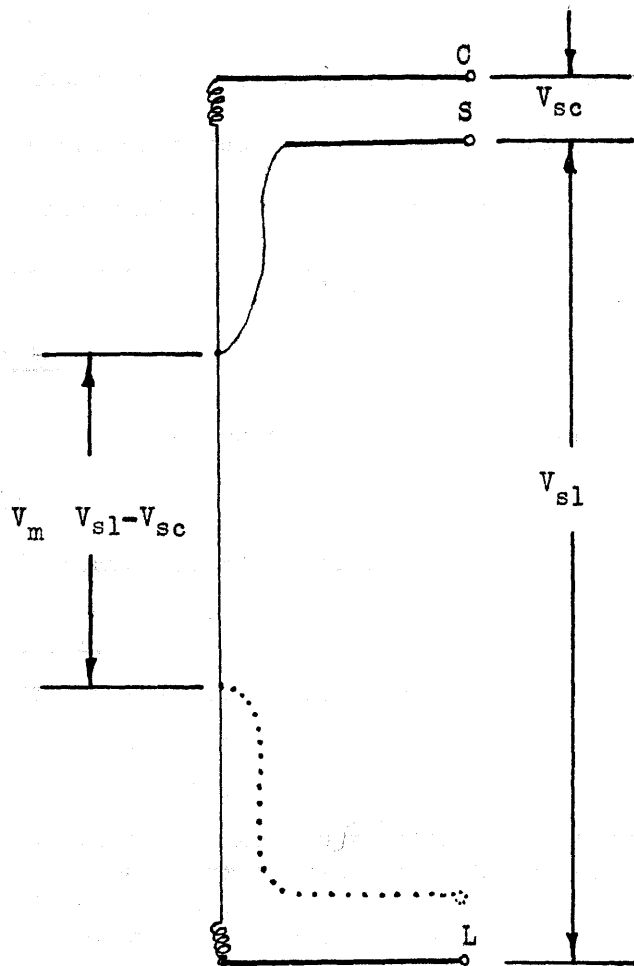


Figure 14

Voltage measurements across the filament

The current passing through the filament is  $i = V_r/R_{st}$ . If the voltage  $V_{sc}$  is subtracted from  $V_{sl}$  the voltage across the middle part of the filament,  $V_m$ , is obtained. The power input to the middle section of the filament is  $W = V_m \times i$ , and its resistance is  $R_m = V_m/i$ .  $W$  and  $R_m$  are plotted. For small values of current (below 1 ma) this gives a linear plot which is interpolated back to zero power input. This point gives the resistance of the filament at the temperature of the bath. It is important that the wire reach equilibrium after increasing the current. This check is made by measuring  $V_r$  at five or ten minute intervals until its value does not change.

The above procedure is repeated at different bath temperatures. The filament resistances obtained this way are plotted against the bath temperatures. This calibration curve is used in further measurements.

When the calibration is done, toggle switch no. I is reversed.

#### 4d. Measurements.

The following is the procedure to measure heat conduction at a given pressure level:

1. Evacuate and flash filament.
2. Let .02 mm to 1 mm helium into test tube and flash filament again.
3. Adjust pressure until required value is obtained.
4. Set bath temperature.
5. Pass current through filament.
6. Evaluate filament temperature by measuring  $V_r$ ,  $V_{sl}$ ,  $V_{sc}$ .  
If temperature is same as required calculate heat loss<sup>sc</sup> ( $W_m$ );  
if ~~temperature~~ temperature is not the same as required adjust current to get required value.
7. Evaluate power loss in vacuum ( $W_v$ ).

To change the pressure in the test tube:

1. Raise mercury in cut-offs nos. 1 and 2 to close system from pumps.
2. Close SC2.

3. Break open the gas containing flask.
4. Close SC3.
5. Open SC2 for a few seconds and then close it again. Now there is a small volume of helium trapped between the two stopcocks.
6. Slowly open SC3 and close it right away when the mercury column in the cut-offs has moved.
7. Measure the pressure using the McLeod gauge. If pressure is too high, use a steel scale with .5 mm divisions or better.
8. Close test tube off from the rest of the system by raising the mercury in cut-off no. 1.

To lower the pressure, open cut-off no. 2 to the pump momentarily and close it again. Then lower the mercury in cut-off no. 1 just enough to open the test tube. This way the pressure is dropped to about half of its previous value. To raise the pressure the procedure described earlier has to be repeated starting from step no. 6.

4e. Shutting off the vacuum system.

The following is the procedure to use in shutting off the vacuum system:

1. Disconnect diffusion pump and wait until it is thoroughly cooled (approximately 40 minutes).
2. Lower mercury in both cut-offs and McLeod gauge.
3. Turn off all electrical apparatus.
4. Shut off both mechanical pumps simultaneously.
5. Slowly raise mercury in the two cut-offs and in the McLeod gauge to a height of about 30 cm.
6. Open air-bleed valve (SC1) slowly until mercury is lowered back to its original level (same height as in the reservoir).
7. Repeat steps No. 5 and 6 until the pressure in the system builds up to one atmosphere.

5. Sample Data on the Calibration of the Filament.

After the vacuum system was in operation, calibration of the filament was attempted. This was done following the procedure described earlier. The resistance of the filament was found to be approximately 1.7 ohms. The accuracy in determining this resistance is not better than 1%. The temperature of the oil bath was kept constant within  $\pm .5^{\circ}\text{C}$ . Better control of the oil bath could not be achieved with the bimetallic thermoregulator used during the calibration. It was found that the galvanometer did not provide its specified sensitivity ( $0.001 \mu\text{a}/\text{mm}$ ) but rather gave about  $0.02 \mu\text{a}/\text{mm}$ .

A sample set of measurements is shown in Table 1. The slight increase in temperature was uncontrollable. The fourth decimal figure of the voltage readings is an approximation made within the smallest division of the potentiometer. Measurements were repeated at five or ten minute intervals until there was no sign of drift and the voltage measurements were reproducible. Sample data calculations for the calibration curve are shown in Table 2. Using the measurements shown in Table 1 a calibration curve is drawn and it is shown in Fig. 15. Neglecting the points at very low currents a curve can be drawn through the other points. This is shown by the dotted line. The intersection of this curve with the vertical axis gives the resistance of the filament at the temperature of the oil bath. Figure 16 shows the nonlinearity of the calibration curve when currents above 1.0 ma are used.

Although the calibration measurements are not very accurate, they serve as a future reference. For example, it was found that calibration has to be done within the current range from 0.3 ma to 0.8 ma for the

Pressure: 7 mm.Hg.	VOLTAGES (MV)		
Bath Temp.	$V_r$	$V_{s1}$	$V_{sc}$
25.9°C	.2014	.5136	.1705
	.2015	.5135	.1704
26.0°C	.2502	.6374	.2100
	.2500	.6375	.2114
	.2502	.6375	.2116
26.1°C	.3005	.7650	.2536
	.3005	.7650	.2536
26.2°C	.3519	.8969	.2975
	.3519	.8970	.2975
	.4035	1.0275	.3410
	.4030	1.0285	.3409
	.4029	1.0290	.3410
	.4030	1.0280	.3410

Table 1

A sample set of voltage measurements

Standard ohm resistance	$R_{st}$	1.005 ohm
Voltage across standard ohm resistor	$V_r$	.3519 mv
Current across filament	$i = V_r/R_{st}$	.3501 ma
Voltage across long section of filament	$V_{sl}$	.8969 mv
Voltage across short section of filament	$V_{sc}$	.2975 mv
Voltage across middle section of filament	$V_m = V_{sl} - V_{sc}$	.6865 mv
Resistance of middle section of filament	$R_m = V_m/i$	1.7098 ohm
Power input to middle section of filament	$W = V_m \times i$	$.275 \times 10^{-6}$ watt

Table 2

Sample calculation of calibration data

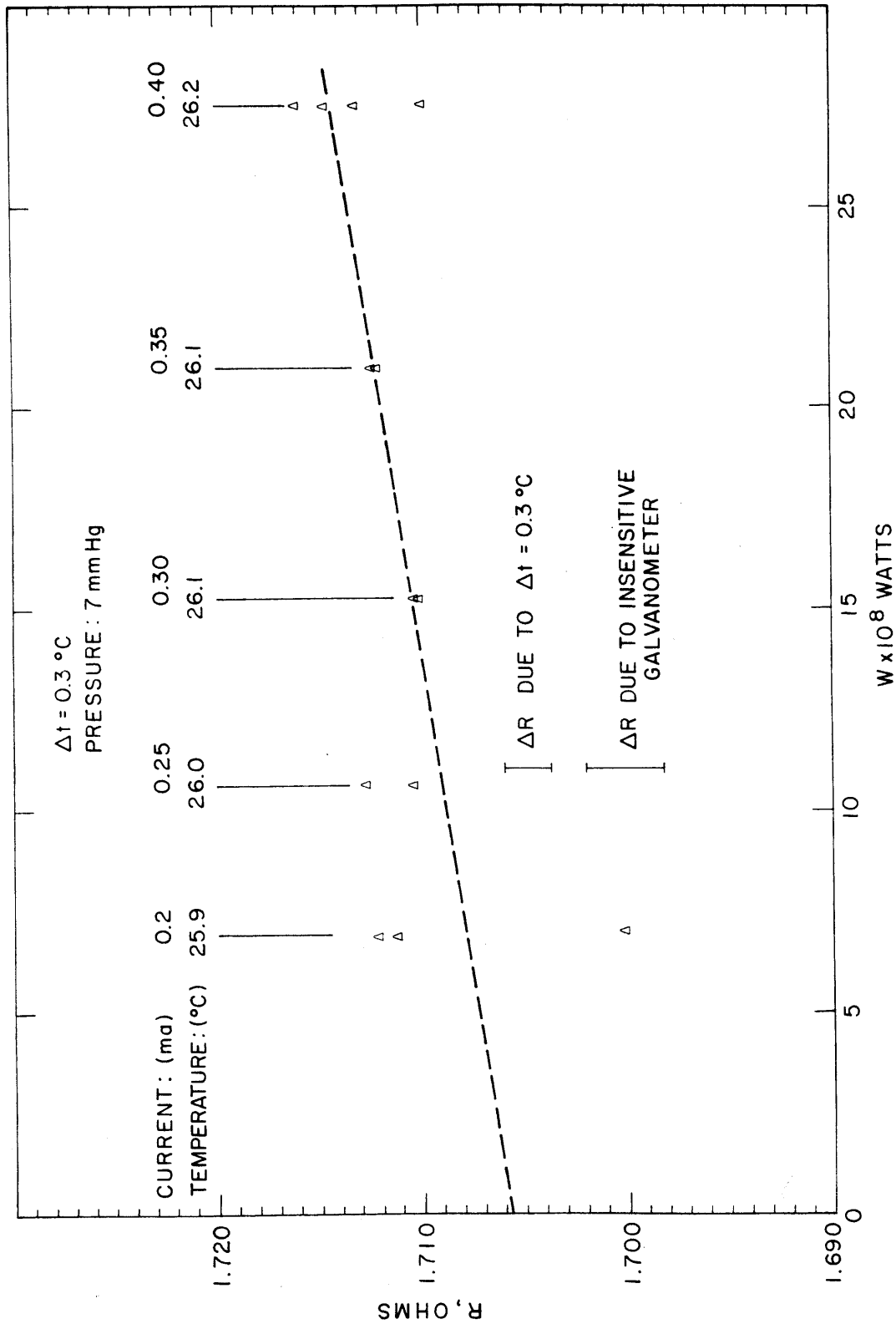


FIGURE 15. TYPICAL CALIBRATION CURVE

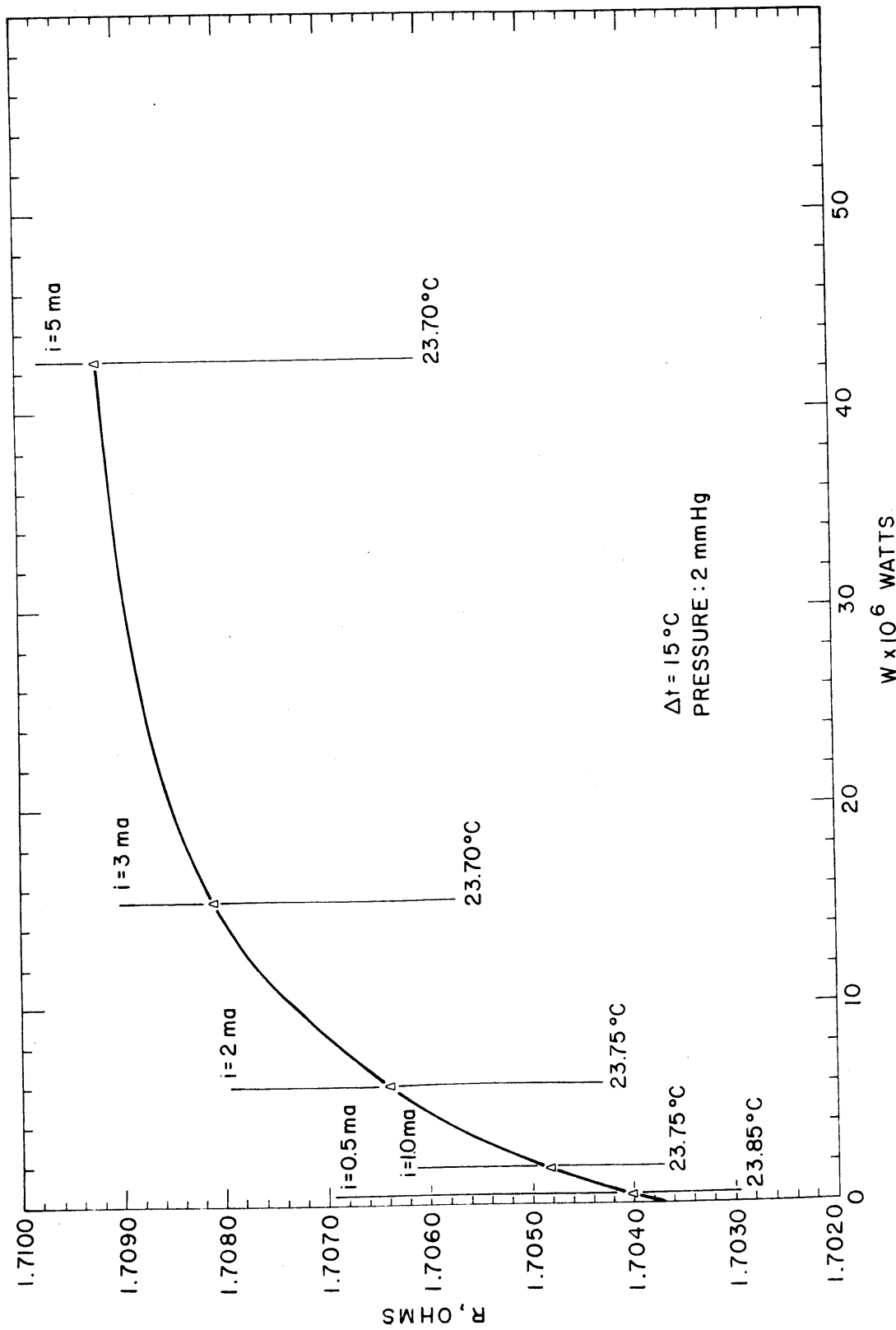


FIGURE 16. NONLINEAR CALIBRATION CURVE

particular wire used. Below 0.3 ma the uncertainties in the measurements are too high, and above 0.8 ma the plot of  $R_m$  vs.  $W$  is not linear any more.

In order to get more accurate measurements the temperature of the oil bath has to be controlled more accurately. The relationship between the temperature and the resistance is the following:

$$R = R_r [1 + \beta (t - t_r)] .$$

From the above equation

$$\Delta R = R_r \beta \Delta t .$$

Now, suppose that it is required to have a maximum  $\Delta R$  of  $\pm 0.001$  ohms.

$R_r$  is approximately 1.7 ohms and  $\beta$  for tungsten is approximately 0.0045 at room temperature. Using these values

$$\Delta t = \frac{\pm 00.001}{.0045 \times 1.7} = \pm 00.15^\circ C .$$

In other words, the oil bath has to be equipped with such instruments as to provide a maximum temperature fluctuation of  $\pm 0.015^\circ C$ .

APPENDIX

Modification of the Moment Method for all values of  $\alpha$ .<sup>6</sup>

In a cylindrical symmetry the space between two coaxial cylinders is divided into two regions as shown in Fig. 15.

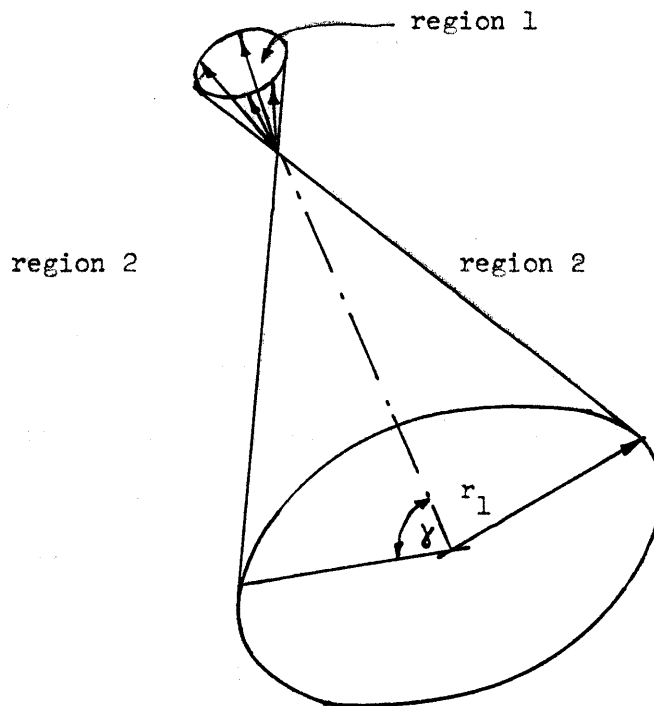


Figure A-1

Division of velocity space

Velocity vectors of all particles are described by two distribution functions  $f_1$  and  $f_2$ .

In region 1

$$f_1 = \frac{n_1(R,t)}{\left[2\pi k T_1(R,t)/m\right]^{3/2}} \cdot e^{-\left\{\frac{m\left[\frac{z}{r} - u_1(R,t)\right]^2}{2kT_1(R,t)}\right\}} \quad (A-1a)$$

In region 2

$$f_2 = \frac{n_2(R,t)}{\left[2\pi k T_2(R,t)/m\right]^{3/2}} \cdot e^{-\left\{\frac{m\left[\frac{z}{r} - u_2(R,t)\right]^2}{2kT_2(R,t)}\right\}} \quad (A-1b)$$

If these distribution functions are known all mean quantities of the gas can be calculated by averaging over all velocity spaces. The average radial heat transfer is found to be

$$(W)_{av} = \left(\frac{z}{\pi m}\right)^{1/2} \cos \gamma \left[ n_1 (kT_1)^{3/2} - n_2 (kT_2)^{3/2} \right] \quad (A-2)$$

By utilizing the Maxwell integral equation for cylindrical geometry, the following differential equations are obtained:

Continuity:

$$\bar{n}_1 (\bar{T}_1)^{1/2} = \bar{n}_2 (\bar{T}_2)^{1/2} \quad (A-3a)$$

Radial momentum:

$$(\sin 2\gamma - 2\gamma) \frac{d}{dr} (\bar{n}_1 \bar{T}_1 - \bar{n}_2 \bar{T}_2) + \pi \frac{d}{dr} (\bar{n}_1 \bar{T}_1 + \bar{n}_2 \bar{T}_2) = 0 \quad (A-3b)$$

Energy :

$$\bar{n}_1 \bar{T}_1^{3/2} - \bar{n}_2 \bar{T}_2^{3/2} = \beta \quad (A-3c)$$

Heat flux :

$$\begin{aligned} & (\sin 2\gamma - 2\gamma) \frac{d}{d\bar{r}} (\bar{n}_1 \bar{T}_1^2 - \bar{n}_2 \bar{T}_2^2) + \pi \frac{d}{d\bar{r}} (\bar{n}_1 \bar{T}_1 + \bar{n}_2 \bar{T}_2) + \\ & + \frac{4}{15} \frac{\pi}{\lambda_f} \frac{\beta}{\bar{r}} [\bar{n}_1 (\pi - 2\gamma) + \bar{n}_2 (\pi + 2\gamma)] = 0 \end{aligned} \quad (A-3d)$$

In Eqs. (A-3),  $\beta$  is the constant of integration and all quantities are normalized with respect to  $n_f$ ,  $T_f$ , and  $r_1$ .

L. Lees and C. Y. Liu applied the boundary conditions  $\alpha = 1$  at  $\bar{r} = 1$  to the above equations. Now, it is assumed that  $\alpha \neq 1$  at  $\bar{r} = 1$ . Then,

$$\alpha = \frac{T_2 - T_1}{T_2 - T_f} \quad (A-4)$$

After normalizing with respect to  $T_f$ ,

$$\alpha = \frac{\bar{T}_2 - \bar{T}_1}{\bar{T}_2 - 1} \quad (A-5)$$

If the assumption is made that  $T_w/T_f = 1 - \epsilon$  where  $\epsilon \ll 1$ , then it is true that

$$\begin{aligned} \bar{n}_1 &= 1 + N_1, \\ \bar{n}_2 &= 1 + N_2, \\ \bar{T}_1 &= 1 + t_1, \\ \bar{T}_2 &= 1 + t_2. \end{aligned} \quad (A-6)$$

where  $N_1$ ,  $N_2$ ,  $t_1$ , and  $t_2$  are much less than one. By substituting Eqs. (A-6) into Eq. (A-5), one finds

$$t_1 = (1 - \alpha)t_2 . \quad (A-7)$$

By applying Eqs. (A-6) and (A-7), one finds a set of equations governing the four quantities,  $N_1$ ,  $N_2$ ,  $t_1$ ,  $t_2$ :

$$t_1 - t_2 = \beta , \quad (A-8a)$$

$$N_1 - N_2 = -\frac{1}{2}\beta , \quad (A-8b)$$

$$t_1 = -\frac{\beta}{\alpha}(1 - \alpha) , \quad (A-8c)$$

$$t_1 + t_2 + \frac{8}{15} \frac{r_i}{\lambda_f} \beta \ln \bar{r} = K = \text{constant} . \quad (A-8d)$$

At  $\bar{r} = 1$ , Eq. (A-8d) becomes

$$t_1 + t_2 = K . \quad (A-9)$$

Substitution of Eq. (A-8a) and (A-8c) into Eq. (A-9) will yield

$$K = -\beta \left( \frac{2 - \alpha}{\alpha} \right) . \quad (A-10)$$

Combining Eq. (A-10) with Eqs. (A-8) leads to

$$t_2 = -\beta \left( \frac{1}{\alpha} + \frac{4}{15} \frac{r_1}{\lambda_f} \ln \bar{r} \right). \quad (\text{A-11})$$

At the boundary where  $\bar{r} = r_2/r_1$ ,  $t_2 = -\epsilon$ . Applying this boundary condition to Eq. (A-11), one gets

$$\beta = \frac{\epsilon}{\frac{1}{\alpha} + \frac{4}{15} \frac{r_1}{\lambda_f} \ln \bar{r}_2}. \quad (\text{A-12})$$

To find  $W/W_\infty$  one proceeds as follows:

Normalize Eq. (A-2) so that

$$W = \left( \frac{2}{\pi m} \right)^{1/2} k^{3/2} n_f T_f^{3/2} \frac{r_1}{r} \left[ \bar{n}_1 \bar{T}_1^{3/2} - \bar{n}_2 \bar{T}_2^{3/2} \right]. \quad (\text{A-13})$$

Combine the above equation with Eq. (A-3c) to get

$$W = \left( \frac{2}{\pi m} \right)^{1/2} k^{3/2} n_f T_f^{3/2} \frac{r_1}{r} \beta. \quad (\text{A-14})$$

Substitute Eq. (A-12) into Eq. (A-14):

$$W = \left( \frac{2}{\pi m} \right)^{1/2} k^{3/2} n_f T_f^{3/2} \frac{r_1}{r} \times \frac{\epsilon}{\frac{1}{\alpha} + \frac{4}{15} \frac{r_1}{\lambda_f} \ln r_2/r_1}. \quad (\text{A-15})$$

To find  $W$  at atmospheric pressure, Fourier's Law is used:

$$W_\infty = \frac{k_c (T_f - T_w)}{r \ln r_2/r_1} = \frac{k_c \epsilon T_f}{r \ln r_2/r_1}. \quad (\text{A-16})$$

where

$$k_c = \left(\frac{2}{\pi m}\right)^{1/2} k n_f T_f^{1/2} \lambda_f \cdot \frac{15}{4} . \quad (\text{A-17})$$

Combining Eqs. (A-15), (A-16), and (A-17) will give the final result:

$$\frac{W}{W_\infty} = \frac{1}{1 + \frac{1}{\alpha} \times \frac{15}{2} \times \frac{1}{\ln r_2/r_1} + \frac{1}{2r_1/\lambda}} . \quad (\text{A-18})$$

EQUIPMENT

1. Main vacuum pump, General Electric, 1/2 HP, 1725 rpm, Model 5KC42IG14A.
2. Secondary vacuum pump, General Electric, 1/3 HP, 1725 rpm, Model 5KH35KG113.
3. "Norescoil" mechanical vacuum pump fluid.
4. Diffusion pump, National Research Corporation, Type 0125, No. H-2-P.
5. Silicon oil for diffusion pump, No. DC 704.
6. Manifold with three outlets for secondary vacuum system.
7. Trap to condense oil from pumps, pyrex 19 mm.
8. Trap for test tube, pyrex 10 mm.
9. Charcoal trap, 1/2 inch pyrex tubing.
10. Trap for ionization gauge, 1/2 inch pyrex tubing.
11. Dewar flask for item #7, 5 liter.
12. 3 dewar flasks for items #8, 9 and 10, 500 ml.
13. 3 two-way mercury-seal stopcocks for main system, 2 mm.
14. 3 three-way stopcocks for secondary system, 2 mm.
15. Apiezon-N high vacuum grease for all stopcocks.
16. One liter of Helium, Airco no. A785-6-60.
17. Activated charcoal for trap, coarse, Norit.
18. 2 mercury cut-offs, 1/2 inch pyrex tubing.
19. Test tube, 26 mm pyrex tubing.
20. Ionization gauge, CENCO, No. 507.
21. Control for item #20, CENCO, 710B.
22. McLeod gauge, CENCO, 4A.
23. Mercury for cut-offs and McLeod gauge, EASTERN, triple distilled.

24. 2 immersion thermometers for oil bath, Range:  $-10^{\circ}\text{C}$  to  $30^{\circ}\text{C}$ , steps of  $.1^{\circ}\text{C}$ .
25. Thermocouple wire, copper-constant.
26. Potentiometer for temperature measurements, Leeds and Northrop, No. 8690.
27. Electrical switching circuit for measurements (built in M.E. Fluid Lab., M.I.T.).
28. Potentiometer for voltage measurements, Leeds and Northrop, No. 7553,K-3.
29. Rubicon galvanometer for item #28, no. 3414.
30. Standard cell for item #28, Epply Lab., no. 100.
31. Storage cell for item #28, Atlantic, 2 volts.
32. Storage cell for item #27, Eastern, 6 volts.
33. Tin can for oil bath, 1 gal.
34. Thermoregulator for oil bath, CENCO, no. 99005-3.
35. Motor oil for oil bath, SAE, No. 10.
36. Cast iron clamps.
37. Ovens (made in the M.E. Fluid Lab., M.I.T.).
38. 6 powerstats for ovens.
39. Electrical heating tapes.
40. DC motor for stirrer, 27 volts.

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