

**A Phase-Based Approach to Satellite Constellation
Analysis and Design**

by

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B.S.A.E., United States Air Force Academy
(1989)

Submitted in Partial Fulfillment

of the Requirements for the

Degree of

Master of Science

in Aeronautics and Astronautics

at the

Massachusetts Institute of Technology

June, 1991

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Paul B. DiDomenico

Submitted to the Department of Aeronautics and Astronautics on May 10, 1991
in partial fulfillment of the requirements for the degree of
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Abstract

An approach to the performance analysis of a satellite constellation providing intermittent coverage of the earth is developed. The approach uses only the phasing of satellites in terms of mean anomaly and site hour angle as the independent variables. A study of the revisit phenomenon is carried out using the phase-based approach, yielding an efficient method for evaluating the revisit-interval quality of a constellation. Computer software implementing the method makes constellation studies possible on a personal computer, giving advantages over the traditional technique of propagating orbit equations.

The method is applied to example problems, to demonstrate the effects of constellation design parameters on coverage performance. An algorithm is developed to design "optimal" intermittent-coverage constellations to meet mission constraints, using the methods developed for revisit-interval evaluation. A 10-satellite constellation is optimized on a personal computer to provide intermittent regional earth coverage within stated mission requirements. Finally, a brief study is made of the current constellation design for the "Iridium" worldwide cellular telephone system, to evaluate the design and propose possible improvements.

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Acknowledgements

I would like to take this opportunity to express my gratitude to all those who have had a part in the creation of this thesis, and who have supported me throughout the course of my graduate work at MIT and the Draper Laboratory.

To Dr. Matthew Bottkol, who was the inspiration for much of the original IR&D work, for providing invaluable assistance with the technical basis of this thesis, and for generously giving his time for help with any conceivable mathematical question.

To Dr. George Schmidt, my thesis supervisor and Division Leader at the Draper Laboratory, for making the Draper Fellowship possible for me, and for his patience and confidence during the writing of this thesis. I am also grateful for his approval of Independent Research and Development funding, which allowed me to present part of this work at a professional conference.

To Ron Harris, who proposed the original IR&D project, out of which this thesis evolved.

To friends--Draper Fellows Dennis Morton, for word-processing assistance, and Jesse Gossner, for reading initial drafts of the writing. Dennis' expertise with all aspects of Macintosh use was a great help. Also thanks to Jason Cuevas, for allowing me to use his prized Macintosh.

This report was prepared at the Charles Stark Draper Laboratory, Inc., under internal Draper Independent Research and Development funding.

Publication of this report does not constitute approval by the Draper Laboratory of the findings or conclusions contained herein. It is published for the exchange and stimulation of ideas.

Portions of this report were presented at the Spaceflight Mechanics Meeting of the American Institute of Aeronautics and Astronautics, held February 11-13, 1991, in Houston, Texas.

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Paul B. DiDomenico

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CHAPTER 1

Introduction

1.1 Problem Background

In recent years, more and more satellite missions are requiring multiple satellites acting in concert with one another to accomplish mission objectives. Single satellites were once considered satisfactory for tasks such as weather monitoring, scientific investigation, or telephone transponders. Current mission concepts, however, are becoming more dependent upon the coverage capabilities of multiple satellite systems. Furthermore, the advantages of systems such as the Space Shuttle and Pegasus launch system have only recently been realized, making access to low-earth orbits relatively easy. Missions requiring multiple-satellite constellations are, therefore, now gaining popularity. Notable examples include the Global Positioning System (GPS) and the Tracking and Data Relay Satellite System (TDRSS). Even private industry is eyeing the potential profit in deploying constellations of satellites--the Motorola Corporation plans to deploy a network of 77 satellites, called "Iridium," to facilitate a worldwide cellular phone system.

The advantages of constellations are readily apparent, and some missions simply cannot be performed without multiple satellites. Keeping a ground site under continuous or nearly-continuous observation requires many satellites, because they must be in low-earth orbits (LEO) for cameras, radars, or other sensors to distinguish objects. Satellites in LEO cannot view a large area of the earth at once, and hence many satellites are needed to cover large areas or to view a point on the earth frequently. A satellite in a high, perhaps geosynchronous, orbit can observe a larger portion of the earth and could even remain stationary over a target. Unfortunately, limitations on sensor capabilities, such as camera resolution, would render a satellite incapable of detailed earth observation in such a high orbit. Satellites which transmit

and/or receive radio signals must also be in LEO to eliminate the need for high-power transmitters. Clearly, then, many practical satellite missions require low altitude orbits, and therefore many satellites must work together to view a point on the earth continuously or nearly continuously. Moreover, some missions will require more than single-satellite viewing. The GPS navigation technique, for example, requires four simultaneous range measurements in order to provide the user with a reasonably accurate position estimate. Hence four satellites must be visible to a receiver at the same time to receive radio signals. The GPS constellation must provide continuous four-fold coverage of the earth to fulfill its mission.

Many satellite constellation applications fulfill missions where continuous observation of ground sites is necessary. However, it is desirable to utilize an absolute minimum number of satellites in a constellation, while still meeting mission requirements. Fewer satellites will clearly save on total system cost and maintenance. If the mission can be accomplished without requiring continuous earth coverage, the number of satellites needed in the constellation can potentially be reduced. A constellation that provides intermittent coverage might be acceptable.

For satellite constellations that provide intermittent coverage, any point on the earth's surface will be contacted at irregular intervals by satellites in the constellation. Contact is defined as when a satellite is above a minimum specified elevation angle, as measured by an observer at the ground site, and within appropriate functional range of either a camera, transmitter/receiver, or other sensor. Depending upon the application, there will usually be bounds on acceptable elevation angle and range. This contact requirement will tend to determine the constellation altitude and inclination, and the latitude limits in which the constellation can provide satisfactory coverage. The intervals between contacts, however, are influenced by the arrangement of the satellites in the constellation, as well as the contact requirement.

The interval of time during which no satellite in the constellation can observe a particular ground site is referred to as "revisit time." Such intervals are irregular, and the longest of these intervals, for a given constellation and ground site, is the "maximum revisit time." A required upper bound on maximum revisit time for points within a region of the earth's surface must be determined based on mission objectives. This thesis addresses the effects of such a requirement on satellite constellation coverage analysis and design, extending to consideration of numbers of satellites, orbital inclination, altitude, and elevation-angle requirements.

The search for an optimal constellation configuration is potentially tedious and can be computationally impractical. The "brute-force" computer simulation approach is undesirable, due to the computation required to integrate the orbital differential equations, which is then multiplied by the many possible constellation permutations. This study develops and exploits a new approach that not only reduces the computational demands of the problem, but also provides the analyst with a means of obtaining qualitative and quantitative insight into the satellite revisit problem.

1.2 Prior Research

Prior research dealing with this problem is fairly sparse, with few publications in the open literature. Although there is much literature on the continuous global and continuous regional earth coverage problem, there is little on the intermittent coverage problem. Recently (Fall 1990), authors Hanson, Evans, and Turner published the results of an independent study directed at the problem posed similar to this thesis. The methodology they develop exhibits some of the same techniques developed here at the C.S. Draper Laboratory. The strategy of employing these techniques, however, is an open matter. Prior to that study, no unified framework for dealing with the intermittent coverage problem as posed in this study is to be found in

the open literature. The journal articles most relevant to revisit coverage are cited in the References section, and a description of significant authors' work is included in Appendix A for further background.

The approach to the constellation selection problem that is usually seen in the literature is either "brute-force" computer simulation, or a geometrical approach. The former is based on propagating the orbital differential equations, while continuously evaluating viewing conditions such as altitude and elevation angle. This approach can become computationally burdensome if the constellation contains large numbers of satellites. Also, it provides little insight into the coverage problem, since the interaction of the satellites and the ground site is difficult to visualize. Constellation synthesis is not made intuitive by this process. Figure 1.1 gives an idea of this method, where the dashes represent contact periods between the satellites and ground site.

The geometrical approaches usually extend from Walker's method of analysis, or, as seen in more recent journal articles, "street of coverage" techniques. The Walker approach is to use the coverage geometry as illustrated in Figure 1.2 to find the optimum satellite system. The idea is to find the largest value of d during an orbit, and then adjust the constellation until the lowest value of d_{\max} is obtained. In this way, the "best" meshing of coverage circles is obtained, and therefore the best constellation. Walker assumes that all satellites are arranged in the constellation symmetrically. That is, the orbit planes are spaced equally in longitude of ascending node, and satellites within a particular plane are separated equally in mean anomaly. Phasing of satellites between different planes is arbitrary.

The "street of coverage" method involves projecting the viewing cone of each satellite onto the earth, and then analyzing the intersections of these projections as the satellites move in their orbits. The resulting patterns are strips or "streets"

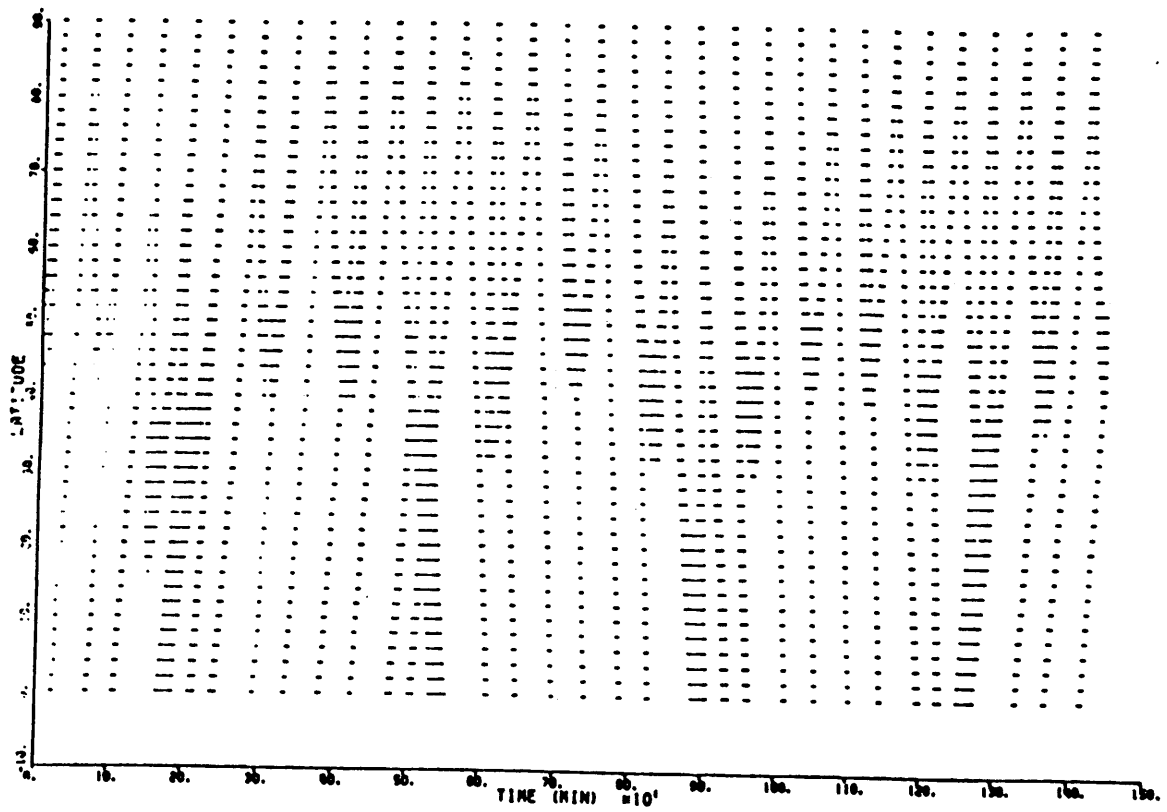


Figure 1.1: Output of Typical Computer Simulation Showing Revisit Patterns

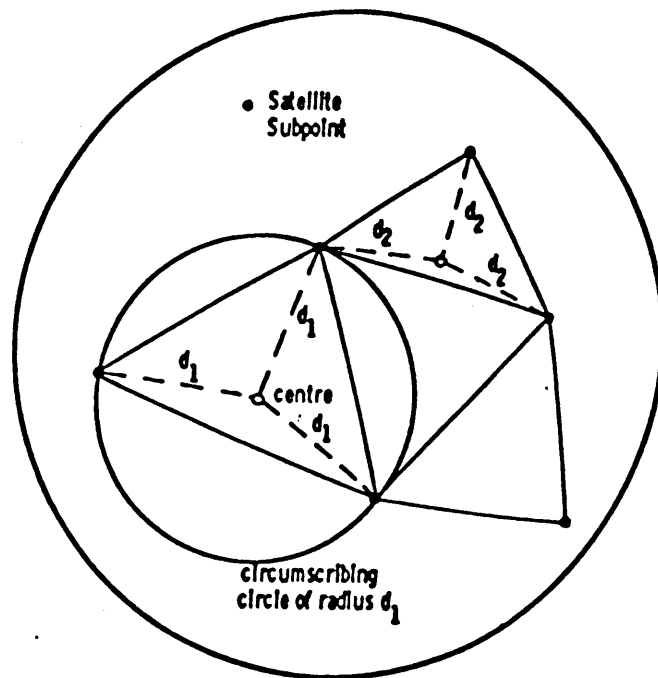


Figure 1.2: Walker Satellite Coverage Geometry

covering the earth's surface, and coverage is evaluated by looking for gaps between the streets. Constellation design is performed by deriving formulas for the "best" arrangement of these streets which will provide satisfactory coverage. Figures 1.3 and 1.4 (Ref [13]) give an idea of the street of coverage approach.

The goal of these geometrical approaches is usually to synthesize optimal constellations for continuous global or zonal earth coverage, where it is assumed that enough satellites are available to do the job. The meaning of the word "optimal" can be elusive in these analyses, as it is necessarily related to the presumptions on which the orbit selection analysis is based. Generally, the closed-form constellation solutions require some simplification or restriction of all possible constellations in order to make a closed-form solution possible. These restrictions may include the use of circular orbits, equal numbers of satellites in each plane, or minimum numbers of satellites per plane to allow a continuous "street of coverage" to be associated with each plane. The resulting constellations will be optimal based on these assumptions, but solutions to the same problem with fewer restrictions may be more "optimal" (Ref [13]). It then becomes an issue of whether to develop a computer search routine to find a "more optimal" constellation, or use an already available "less optimal" closed-form solution.

Compared to what is found in the literature, this study is no more restrictive in its presumptions. The initial simplification of using circular orbits is shown to be reasonable for the LEO missions considered in later chapters. All satellites are initially assumed to have a common period and inclination, and this too will be shown to be acceptable (and actually desirable) for analysis of coverage.

1.3 Thesis Outline

This thesis grew out of an Independent Research and Development project conducted at the Draper Laboratory starting in November of 1989. The project had

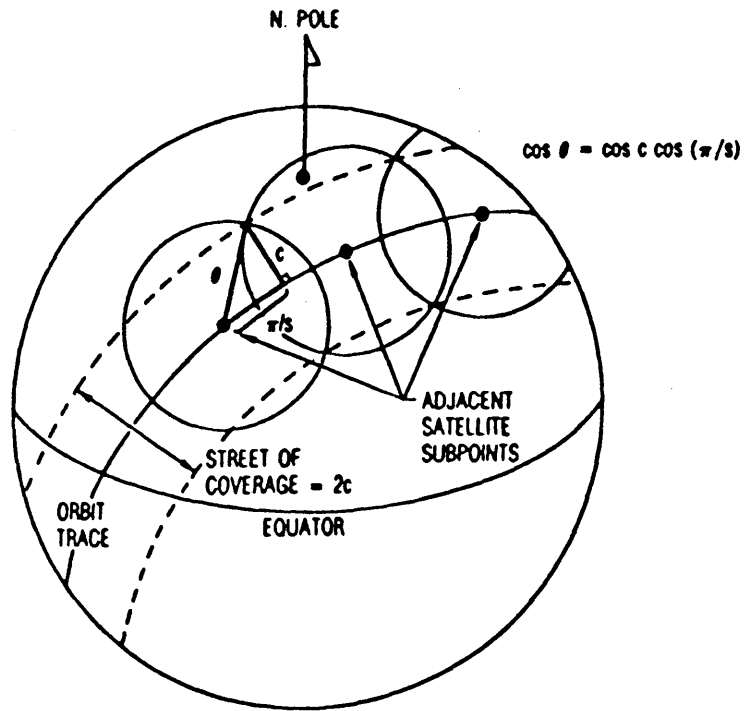


Figure 1.3: "Street of Coverage" Geometry

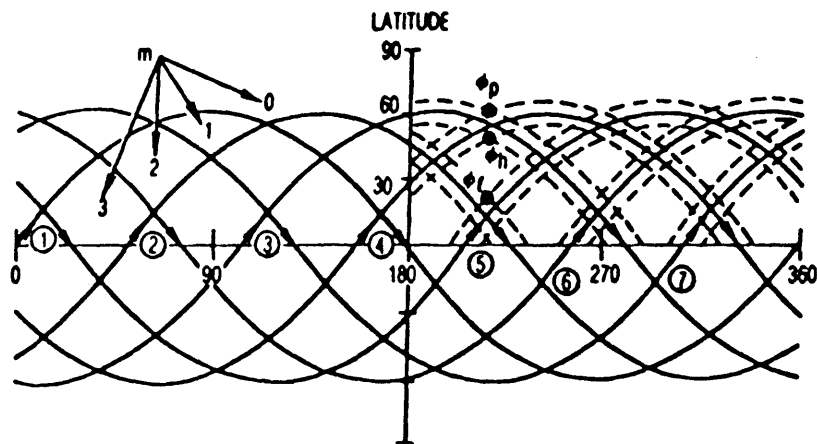


Figure 1.4: Street of Coverage Mesh Pattern Geometry

three objectives: (1) to develop a technique for searching for a constellation to meet a maximum revisit time requirement, (2) to generate and document the software needed to implement the search technique, and (3) to perform studies of "optimal" satellite constellations that satisfy maximum revisit time requirements, and to determine how relaxation of requirements on revisit time can reduce the number of satellites required. This thesis will cover how these objectives were approached and met.

Chapter 2 will develop the mathematical tools necessary to attack the problem, defining the assumptions made and conventions used. Chapter 3 will show how the concepts of Chapter 2 can be used to analyze the coverage properties of a single satellite, and provide examples. Chapter 4 will develop the framework for analyzing constellation coverage performance, and Chapter 5 will use the technique to study example problems and investigate how coverage is affected by different constellation designs. Chapter 6 will develop an algorithm for searching for an optimal constellation using the new methods. In this chapter, an example design problem is solved, where it is found that the latitude region 28-38 degrees North can be covered by a particular 10-satellite constellation with satellites in 2-hour orbits, and stipulating a maximum revisit time of 20 minutes. Chapter 7 will present the results of a case study: applying the new methods to the Iridium satellite constellation. Finally, Chapter 8 discusses the conclusions of this thesis and recommendations for further research.

The major conclusions reached in this thesis are (1) that the satellite constellation coverage problem can be formulated in a new way using a phase-based approach, (2) that this formulation can be exploited to provide a computationally efficient way of analyzing the coverage of a given constellation, to include intermittent (revisit) coverage and redundant coverage, and (3) that an automated constellation design algorithm can be developed using the new approach. Although the original

intent of the study was to analyze the intermittent coverage (revisit) problem, the methodology that emerged was found to be equally applicable to continuous and redundant coverage problems. The Motorola case study of Chapter 7 is an example of a continuous coverage problem that was investigated using the methods developed in this thesis.

CHAPTER 2

Mathematical Foundations

2.1 Introduction/Approach

The mathematics necessary for this study is driven by the need to describe the interaction of an orbiting satellite and a ground site. The approach of this study is to formulate this interaction in terms of only the angular positions of the satellite and ground site. This approach requires some original mathematical development, and this development will be described in detail. Other mathematical tools which will be needed deal with topics such as the effects of earth oblateness on orbital period, and the interval of time until ground trace repetition occurs for a given orbit. The mathematics dealing with these latter concepts is mostly found in journal articles, which will be referenced when used. The union of existing mathematical tools for dealing with satellite coverage, with a new formulation for the interaction of an orbiting satellite and a ground site, yields a methodology which can incorporate a great deal of information in a concise framework.

A survey of the open literature shows that previous work has often relied on "brute-force" computer analysis. Traditional approaches also typically emphasize ground track geometry, making it difficult to gain insight into the interaction of orbital phasing with the earth's rotation. A new formulation that emphasizes phase effects provides this insight, and exposes the influence of orbital inclination, target latitude/longitude, and orbital radius on the revisit phenomenon.

2.2 Assumptions

The approach of this study makes several assumptions and uses some possibly unfamiliar notation. For initial simplification, short orbital periods will be assumed since earth-observing satellites are usually placed in low earth orbits. Consequently,

circular orbits are assumed since only very small orbital eccentricity is allowable in a low earth orbit. Mean anomaly will be referenced from the ascending node crossing. Again, for initial simplification, only symmetric satellite constellations are considered. Here "symmetric" means that satellites within a single orbital plane are evenly distributed in mean anomaly, and orbital planes are at a common inclination and are evenly distributed in longitude of ascending node. Symmetric constellations have been used by Walker (Ref [14]) and Ballard (Ref [2]) to obtain continuous coverage with a minimum number of satellites. Since small revisit times are much like continuous coverage, it is assumed that symmetric constellations will also provide effective intermittent coverage. These assumptions of circular orbits and symmetric constellations will be maintained, as the framework for coverage analysis evolves. The fact that the constellation is symmetric permits the coverage to be analyzed initially in terms of only one satellite. It will be shown that the coverage provided by the constellation is easily found after that, since it is only a matter of repeating the same coverage according to how the satellites are arranged in the constellation.

2.3 Motion of a Ground Site

A vector defining the position of a site on the earth's surface, referenced to an earth-centered inertial coordinate frame, is as follows:

$$\vec{V}_T(t) = (x_T(t), y_T(t), z_T(t)) \quad (1)$$

where

$$\begin{aligned} x_T &= R_E \cos \lambda_T \cos(\omega_E t) \\ y_T &= R_E \cos \lambda_T \sin(\omega_E t) \\ z_T &= R_E \sin \lambda_T \end{aligned} \quad (2)$$

R_E and ω_E are the radius and rotation rate of the earth, respectively. λ_T is the latitude of the ground site, and time is measured from equinox passage. The mean radius of the earth is taken to be 6378.135 km.

2.4 Satellite Motion

The orbital plane of the satellite is defined by two angles: the angle from the first point of Aries to the line of nodes Ω , and the angle of orbital inclination i . Two orthonormal vectors which span the orbital plane can be written as follows:

$$\vec{X}_S = (\cos\Omega, \sin\Omega, 0) \quad (3)$$

$$\vec{Y}_S = (-\cos i \sin\Omega, \cos i \cos\Omega, \sin i) \quad (4)$$

Then the satellite in a circular orbit is located by the vector

$$\vec{V}_S(t) = (x_S(t), y_S(t), z_S(t)) \quad (5)$$

where

$$\vec{V}_S(t) = a(\cos(\omega_S t + \phi) \cdot \vec{X}_S + \sin(\omega_S t + \phi) \cdot \vec{Y}_S) \quad (6)$$

The orbital semi-major axis is a , ω_S is the orbital frequency, and ϕ is a phase angle representing true anomaly, as measured from the line of nodes. For a spherical earth, the orbital parameters are related through Kepler's equation

$$a_S^3 \omega_S^3 = \mu \quad (7)$$

where μ is the gravitational constant of the earth.

2.5 The J_2 Perturbation Effect

The fact that the earth is not a perfect sphere, but an oblate spheroid, causes variations in the symmetry of the earth's gravitational field. This affects the satellite motion by causing the angular momentum vector to precess (Ref [3]). The precession is such that the satellite's longitude of ascending node will move from east to west (for a prograde orbit) as time progresses. Depending upon the orbital inclination, this movement of the longitude of ascending node can be seven degrees per day or more. As a result, the ground trace will shift, and the frequency at which a specific ground site is observed will be affected. Fortunately, this phenomenon can be dealt with by changing the value for the earth's rotation rate. Instead of simply using inertial rate, the value will be modified as follows (Ref [5]):

$$\omega_E = \omega_{E_{\text{inertial}}} + 9.9639(R_E/r)^{3.5} \cos i \quad (8)$$

where R_E is the mean radius of the earth, r is the orbital radius, and i is the orbital inclination. The idea of modifying earth's rotation rate can best be understood by realizing that the satellite ground traces are actually shifting from east to west due to orbit precession. By increasing the earth's rotation rate as in (10), the effect is to speed up the ground under the satellite, thereby artificially accounting for the J_2 effect. The orbital period is also affected by the gravitational torque caused by earth's oblateness, and also must be modified, as given by (Ref [5]):

$$Period = 2\pi \sqrt{\frac{r^3}{\mu}} \left(1 - 0.001624 R_E^2 \frac{\left[1 + \frac{(4 - 5\sin^2 i)}{2} \right]}{r^2} \right) \quad (9)$$

Using these equations in all calculations will ensure that orbit precession is taken into account for satellite/ground site interactions.

2.6 Elevation Angle

Since one of the requirements for contact is line-of-sight satellite visibility, the satellite must be above a minimum elevation angle, as measured by an observer at the site. Define this angle to be β , where β is measured from the local vertical, so that $\beta=90$ degrees is the observer's horizon. Assume for the moment that a 20 degree elevation angle is necessary for line-of-sight visibility, due to local viewing obstructions or camera limitations. Then the elevation-angle condition for viewing is

$$\cos\beta(t) = \frac{(\vec{V}_S - \vec{V}_T) \cdot \vec{V}_T}{|\vec{V}_S - \vec{V}_T| |\vec{V}_T|} > \cos 70^\circ \quad (10)$$

Figure 2.1 shows an elevation angle plot for an example problem: a 6-hour orbit, with site latitude λ_T equal to the orbital inclination i of 40 degrees. Since elevation angle is measured from local vertical, the satellite starts from directly overhead at $t=0$, where $\cos(\text{elevation}) = 1$. Note the irregular intervals of contact as the satellite rises above and falls below the horizon (0 degrees elevation angle). The amount of time spent below the horizon varies--note the long and short periods when the satellite is below 0 degrees elevation angle. The behavior exhibited in this example is typical of a function of two frequencies such as $\cos(\omega_1 t + \omega_2 t)$, where the two frequencies may or may not be commensurate with one another. This phenomenon is at the heart of the revisit problem: the irregularity of revisit intervals is driven by the commensurability of the satellite's orbital frequency with the earth's rotation frequency. Moreover, the amount of time until ground trace repetition occurs is influenced in the same way. The ratio of the two periods, earth rotational period and satellite orbital period, will most likely be an irrational number, and they will not be commensurate with one another. In the example of Figure 2.1, the two periods divided exactly (24/6). So although the contact intervals were irregular, at least the

phenomenon was periodic, and therefore known to repeat within a finite (hopefully short) amount of time. This is not likely to happen in reality, without some measure of active correction on-orbit. Figure 2.2 shows the same example as Figure 2.1, except the orbital period has been changed to 4.314159 hours. Since 24 and 4.314159 do not divide to produce an integer, it may take a very long time until the contact pattern repeats itself (actually, it never will exactly). Ideally, if the two periods were exact divisors of one another, or if the earth did not rotate, the problem of determining when repetition of the revisit interval pattern occurs would be much simpler. Fortunately, the earth does rotate, and determining ground trace repetition becomes an issue. This will be addressed in more detail in Chapter 3, section 3.

2.7 New Satellite Coverage Problem Formulation

Given the orbital elements of a Keplerian orbit, the relative satellite/target geometry can be described by two angles:

$$\theta_S = \text{satellite's orbital mean anomaly} = \omega_S t$$

$$\theta_T = \text{ground site hour angle} = \omega_E t$$

Of interest is the set γ of (θ_T, θ_S) which satisfy the inequality

$$\gamma: \cos\beta(\theta_T, \theta_S) = \frac{(\vec{V}_S - \vec{V}_T) \cdot \vec{V}_T}{|\vec{V}_S - \vec{V}_T| |\vec{V}_T|} > \cos 70^\circ \quad (11)$$

where again it is assumed that a 20 degree viewing horizon criterion is sufficient. Note the cosine β function is now a function of the angles θ_T and θ_S , which is obtained by substituting $\omega_E t = \theta_T$ and $\omega_S t = \theta_S$ into the vector functions given by equations (1) and (6).

The two angles may then be viewed as coordinates on a torus, which is the realization of the product of two circles. A visualization is provided in Figure 2.3.

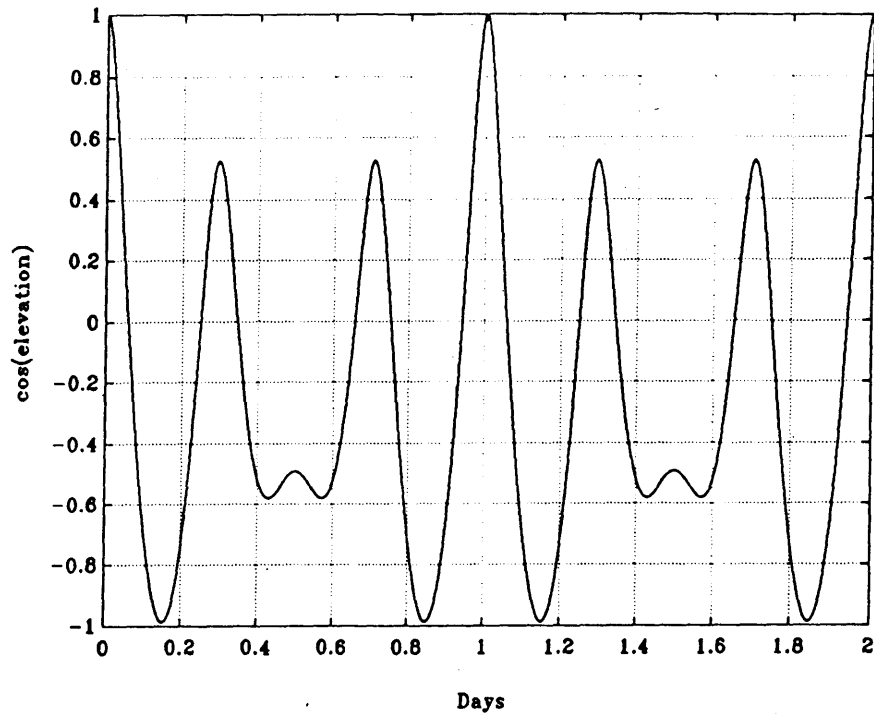


Figure 2.1: Cosine Elevation Angle Behavior. Orbital Period = 6 hrs.
 Latitude = Inclination = 40 deg; $\theta = -90$ deg; $\phi = 90$ deg.

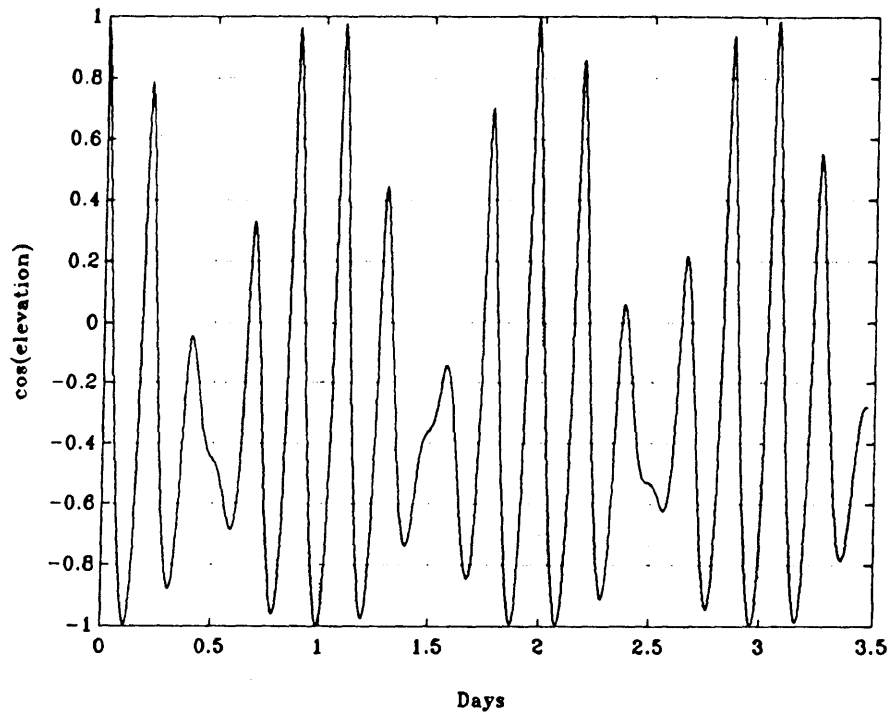


Figure 2.2: Cosine Elevation Angle Behavior. Orbital Period = 4.314159 hrs.
 (Other parameters same as Fig. 2.1)

When the satellite's orbital position θ_S is parameterized by mean anomaly, the motion of the target and the satellite is described by a straight line on the surface of the torus:

$$\begin{aligned}\theta_S &= \omega_S t + \theta_S(0) \\ \theta_T &= \omega_E t + \theta_T(0)\end{aligned}\tag{12}$$

where ω_S = satellite orbital frequency, ω_E = earth rotation frequency, and $\theta_S(0)$, $\theta_T(0)$ determine phasing. If the torus is then "unwrapped," the motion may be

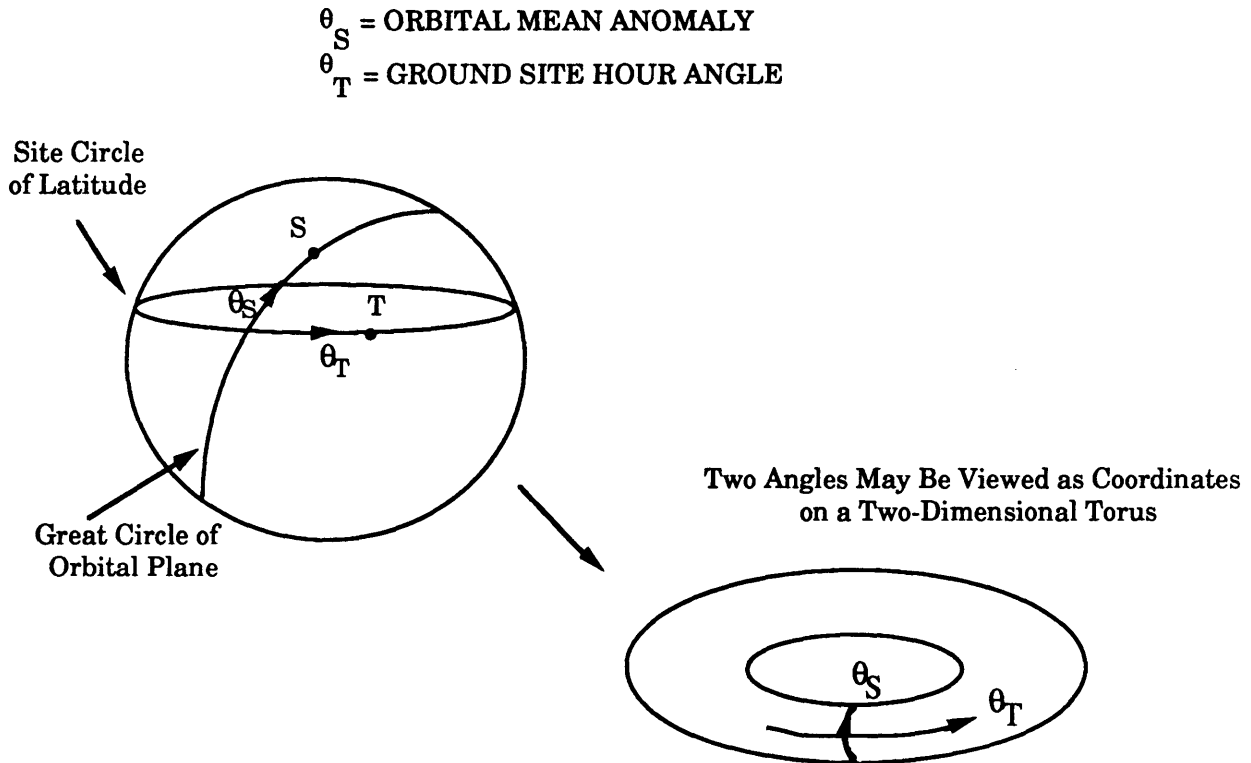


Figure 2.3: Two Phase Angles Produce a Torus

represented by a line in the plane of θ_T, θ_S (taken modulo 2π) whose slope is ω_S/ω_E . This plane will be referred to as the "period square," and is illustrated in Figure 2.4.

Recalling the function $\cos \beta(\theta_T, \theta_S)$ as the line of sight angle of the satellite above the target's local horizon, the "region of visibility" is now the set ρ of (θ_T, θ_S) satisfying

$$\rho: \beta(\theta_T, \theta_S) > \beta_0 \tag{13}$$

where β_0 is the minimum elevation angle necessary for the satellite to view the target, or the "viewing horizon," as defined by sensor capabilities. It is assumed that $\beta_0 = 10$ degrees is sufficient for analytical work. Figure 2.5 illustrates the visibility region on the torus.

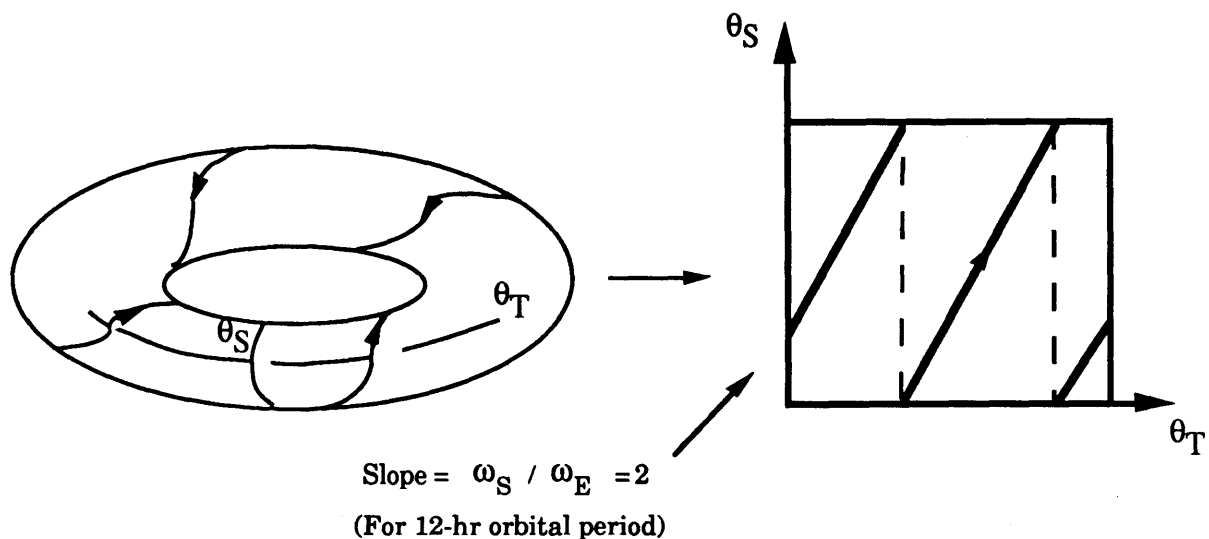


Figure 2.4: Unwrapping the Torus Into a Period Square

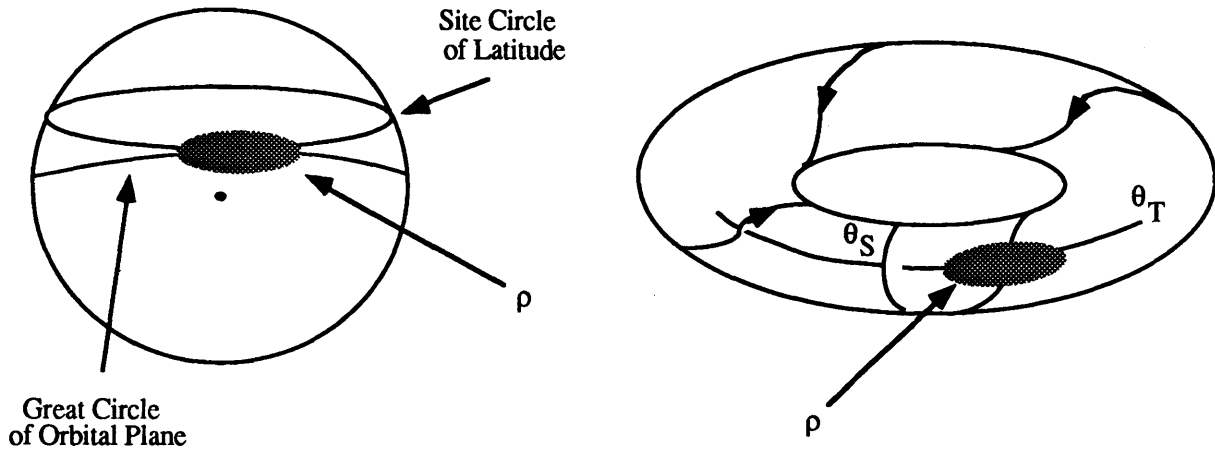


Figure 2.5: Locus of Points Defining "Visibility Region" Lives on the Torus

The boundary of the visibility region is computed as follows: first, find the level curves of the function

$$F(\theta_T, \theta_S) = \cos\beta(\theta_T, \theta_S) = \frac{(\vec{V}_S - \vec{V}_T) \cdot \vec{V}_S}{|\vec{V}_S - \vec{V}_T| |\vec{V}_S|} \quad (14)$$

on the torus. For simplicity, the function

$$H(\theta_T, \theta_S) = \cos\alpha(\theta_T, \theta_S) = \frac{\vec{V}_S \cdot \vec{V}_T}{|\vec{V}_S| |\vec{V}_T|} \quad (15)$$

can be used, which is dependent on F and so has the same level curves. Level curves $H(\theta_T, \theta_S) = \text{constant}$, one of which corresponds to the minimum required elevation angle and hence bounds the region of visibility, may be found by locating one point on the region boundary and integrating the orthogonal gradient around the boundary. Therefore, the task is to integrate the differential equations

$$\frac{d\theta_T}{dt} = \frac{\partial H}{\partial \theta_S} \quad ; \quad \frac{d\theta_S}{dt} = -\frac{\partial H}{\partial \theta_T} \quad (16)$$

These equations can be written out explicitly as follows: recall

$$\vec{V}_T = (\cos\lambda_T \cos\theta_T, \cos\lambda_T \sin\theta_T, \sin\lambda_T) \quad (17)$$

and

$$\vec{V}_S = a(\cos(\theta_S + \phi) \cdot \vec{X}_S + \sin(\theta_S + \phi) \cdot \vec{Y}_S) \quad (18)$$

from before. Define

$$g_1(\theta_T) = \vec{V}_T \cdot \vec{X}_S = \cos\lambda_T \cos(\theta_T - \Omega)$$

$$g_2(\theta_T) = \vec{V}_T \cdot \vec{Y}_S = \cos\lambda_T \sin(\theta_T - \Omega) + \sin\lambda_T \sin\Omega \quad (19)$$

then

$$H(\theta_T, \theta_S) = \cos(\theta_S + \phi) g_1(\theta_T) + \sin(\theta_S + \phi) g_2(\theta_T) \quad (20)$$

and the differential equations may be written

$$\frac{d\theta_T}{dt} = \frac{\partial H}{\partial \theta_S} = -\sin(\theta_S + \phi) g_1(\theta_T) + \cos(\theta_S + \phi) g_2(\theta_T) \quad (21)$$

$$\frac{d\theta_S}{dt} = -\frac{\partial H}{\partial \theta_T} = -\cos(\theta_S + \phi) g_1'(\theta_T) - \sin(\theta_S + \phi) g_2'(\theta_T) \quad (22)$$

These differential equations are nonlinear, and must be solved numerically. If one point (θ_1^*, θ_2^*) can be found on the boundary of the visibility region, i.e., satisfying

$F(\theta_1^*, \theta_2^*) = \cos \beta(\theta_1^*, \theta_2^*) = \cos 80$ degrees, then the equations may be integrated via a Runge-Kutta integration scheme using this point as an initial condition. Solution curves of these equations are level curves, or contours, of the function H . Since H and F are dependent, they are also contours of the function F .

An example visibility region problem, solved via computer software, is plotted on the period square in Figure 2.6. When a satellite is within the visibility region as it moves along the orbit lines, it is in view of the target. In Figure 2.6, when the satellite is at coordinates $(0,0)$, it is directly over the ground site. Therefore, 0 degrees mean anomaly is defined as when the satellite is vertically over the site. Figure 2.6 shows that if the satellite starts from directly over the target, it will almost miss seeing the target on its next pass, will miss on the next, almost miss again on the next, and finally pass again directly overhead as the orbit completes four periods and returns to its initial state. This behavior is consistent with what is seen in Figure 2.1--time $t=0$ in Figure 2.1 is the same as the center of Figure 2.6

The solution of Equations (21) and (22) is what determines all contact opportunities between the satellite and ground site. Therefore, as orbital elements are changed, it is useful to understand the following behavior of this solution:

- Changing longitude of ascending node or location of a satellite in its orbit merely shifts the visibility region within the period square without changing its shape.
- Orbital radius a and orbital frequency ω_S are dependent: an increase in a will decrease ω_S and therefore decrease the slope of the orbital lines.
- Increasing a enlarges the visibility region since more of the earth can be seen at once, as shown in Figure 2.7a.
- Changing the orbital inclination and target latitude influences the visibility region in more complicated ways; the general effects can be inferred from Figures 2.7a through 2.7d, where target latitude is fixed at 40 degrees and

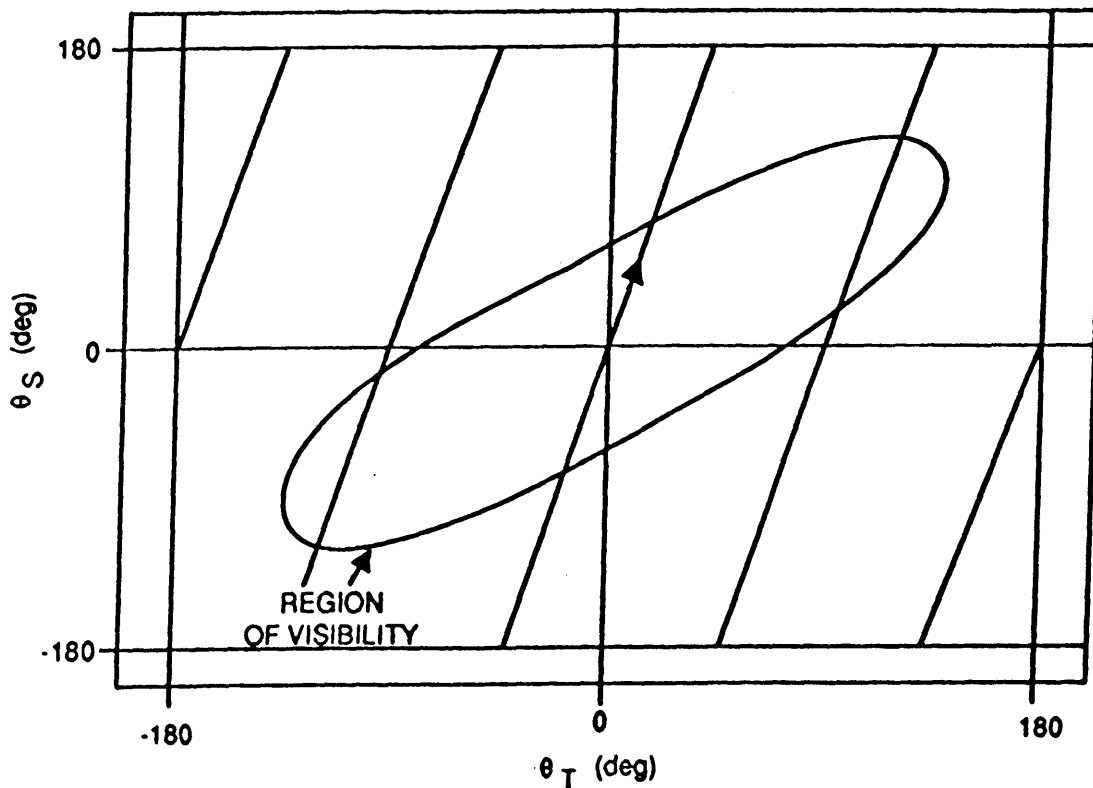


Figure 2.6: Example Visibility Region Solution. Period = 6 hrs; Latitude = Inclination = 40 deg; Viewing Horizon = 10 deg.

inclination is varied. Varying latitude has an equivalent effect if inclination is fixed. Two cases are to be distinguished:

$i < \text{Latitude}$

Assuming the orbit is inclined enough to see the target at all, there will be a single visibility region, as in Figure 2.6. For large enough orbital radius, the target is always visible at some time during an orbital period. This intuitive result is confirmed in Figure 2.7a.

$i >$ Latitude

For large i and small α the target may be viewed on two distinct occasions during one earth rotation--on the ascending and descending passes. This is manifested as a bifurcation of the visibility region, as in Figure 2.7d. Note that either increasing i or decreasing α enhances the bifurcation. This situation is also depicted in Figure 2.8.

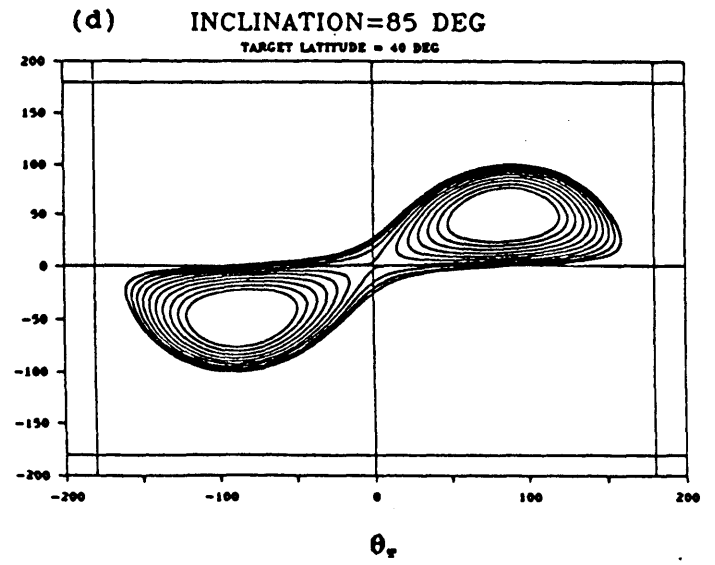
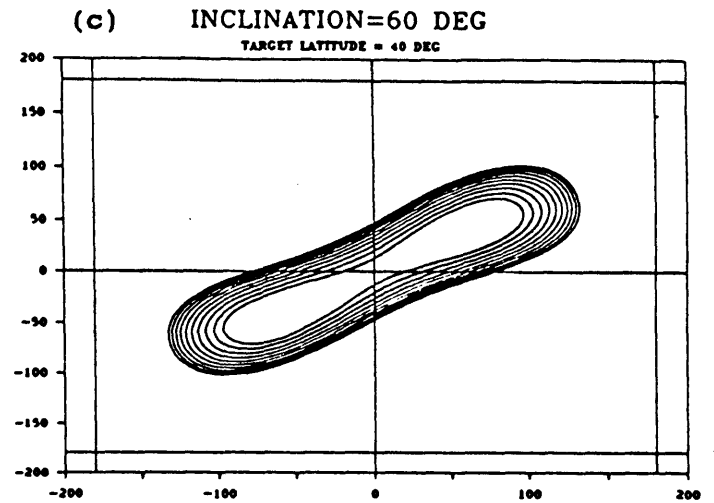
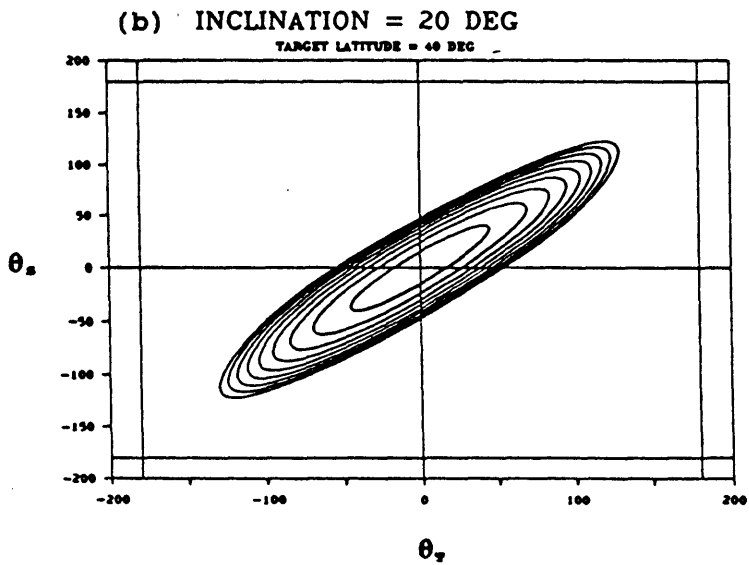
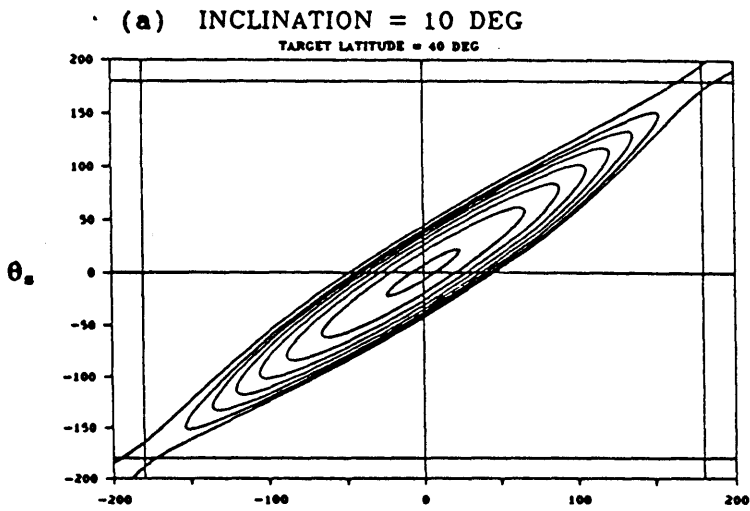
Authors Hanson, Evans, and Turner encountered the same bifurcation phenomenon in their 1990 study. What they describe as one or two "lobes" of coverage, is exactly the same as the visibility region bifurcation described here. It occurs when the orbital inclination is greater than the highest latitude visible to the satellite when it is directly vertical over the target. This is illustrated in Figure 2.9, where the satellite viewing geometry for an arbitrary ground site is related as follows:

$$\cos(\Theta + \alpha) = \frac{\cos(\alpha)}{1 + \frac{h}{R_E}} \quad (23)$$

Here α is the elevation angle as measured from the local horizon, and h is the satellite altitude. The earth-central angle θ defines how much of the earth's surface can be seen at once by the satellite. When the orbit is inclined greater than θ plus the site latitude, the visibility region is bifurcated. When it is less than θ minus the site latitude, the satellite will never see the target, and the visibility region disappears. When the inclination is between these two limits, there is a single visibility region.

This behavior of the visibility region solution is useful in understanding the coverage provided by a satellite or constellation of satellites. For example, the number of satellite passes possible per day is directly affected by the bifurcation phenomenon. This topic will be discussed in more detail in Chapter 3.

Figure 2.7: Visibility Region Topology for Varying Orbital Inclination. Site Latitude = 40 deg. $a = 1.1 \dots 2$ Earth Radii



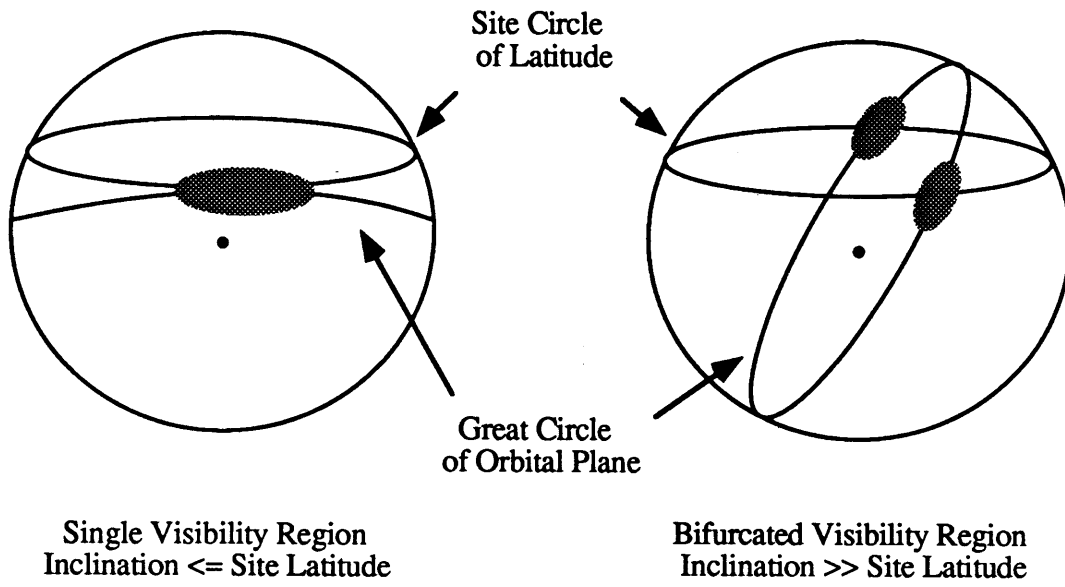


Figure 2.8: Visibility Region May Bifurcate

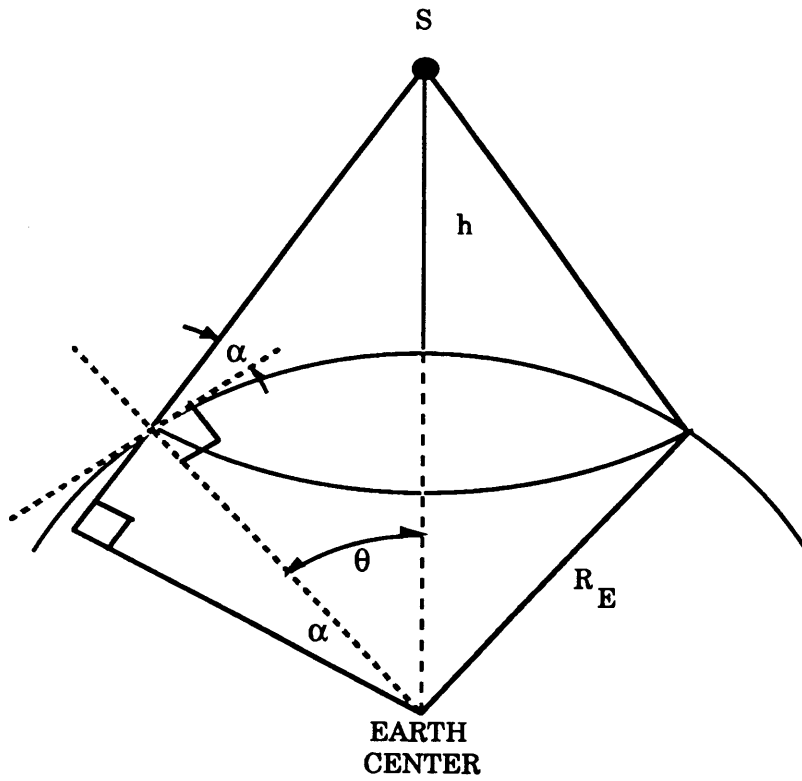


Figure 2.9: Satellite Viewing Geometry

2.8 Chapter Summary and Conclusions

This chapter has presented some of the basic mathematics for the method of satellite coverage analysis to be developed in later chapters. By formulating the contact opportunities between a satellite and ground site as a function of mean anomaly and earth rotation angle, the solution of a "visibility region" can be obtained. The visibility region defines the locus of all points in the space defined by mean anomaly and earth rotation angle. Its shape is influenced by the orbital elements and target latitude. The motion of the satellite and the ground site is defined by diagonal lines on the period square where the visibility region lies.

The J_2 perturbation effect has been considered and accounted for in the problem analysis. The next chapter will use the visibility region concepts of this chapter to develop a method for analyzing the coverage provided by a single satellite.

CHAPTER 3

Single Satellite Coverage

3.1 Introduction

Chapter 2 defined the interaction of a satellite and ground site in terms of the two angles, mean anomaly and earth rotation angle. It was shown how the locus of all points in these coordinates where the satellite can view the target is a region of visibility, whose boundary can be computed. The visibility region solution is influenced by all orbital parameters, as well as the contact requirements for observing the ground site. The motion of the satellite and earth is represented by diagonal lines intersecting the visibility region. When the lines pass into the region, the satellite is in view of the target.

3.2 Revisit Intervals

With the satellite/target interaction formulated as in Chapter 2, it is easy to visualize (and compute) revisit intervals. The longest orbital line segment that remains outside the visibility region (as in Figure 2.5) corresponds to the longest interval when the satellite cannot see the target. Here it is useful to realize that we can add 360 degrees to each period square and visibility region. Multiple redundant period squares can be drawn as in Figure 3.1, where each square is to be identified with every other. The figure indicates how the line ($\theta_T(t)=\omega_E t$, $\theta_S(t)=\omega_S t$) intersects the visibility region, as the earth rotates and the satellite moves in its orbit.

An orbital line segment which just misses the visibility region in Figure 3.1 by passing between tangency points P1 and P2 corresponds to a worst case: moving the line right or left slightly would cause an intersection with the visibility region. At some intermediate point, the line achieves a maximum revisit time for this orbit/ground site combination. It is important to note in Figure 3.1 that there is only

one visibility region--the figure is drawn simply by adding 360 degrees several times in mean anomaly and earth rotation angle.

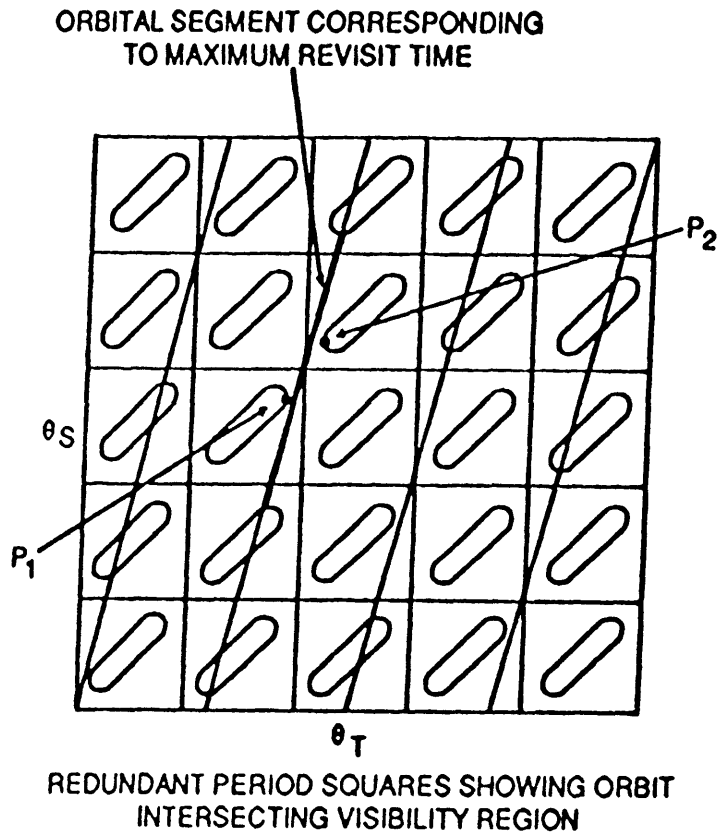


Figure 3.1: Visualization of Revisit Intervals for a Single Satellite

3.2.1 Effect of Ground Site Longitude on Revisit Intervals

Shifting the orbital lines right or left in Figure 3.1 is equivalent to changing the orbit's longitude of ascending node, or, equivalently, moving a ground site along a line of latitude (changing its longitude). This can be visualized as follows: by holding the vertical coordinate constant (satellite mean anomaly) and changing earth rotation

angle, the moment when then satellite passes over the equator can be arbitrarily chosen. This point also applies when dealing with a constellation of satellites: a horizontal shift on the period square corresponds to a change from one orbital plane in the constellation to another, since it is the same as choosing a different longitude of ascending node. Now consider that in Figure 3.1, shifting the lines left or right can conceivably change the worst-case line segment length. Hence, maximum revisit time is dependent upon the longitude of ascending node (or site longitude). Since an orbit's longitude of ascending node is essentially fixed in Figure 3.1--recall the J_2 effect has been accounted for, and it is assumed on-orbit plane changes will not be performed--the effect of ground site longitude on revisit time is revealed by the figure. It will be seen in a subsequent chapter how an appropriately phased constellation of satellites can effectively remove this revisit time dependency on longitude.

3.2.2 Effect of Ground Site Latitude on Revisit Intervals

Recall that ground site latitude was a parameter in computing the visibility region solution. Therefore, analyzing coverage performance at a new latitude requires computing a new visibility region solution, while a change to a new longitude a matter of shifting the orbit lines as discussed in the previous section. This point will be important in subsequent constellation coverage analysis. While there is no analytical solution for creating an appropriate visibility region, some qualitative observations are useful in obtaining good coverage performance.

If a target is at a high latitude, the orbital plane must be inclined enough to view this latitude. Unfortunately, this makes coverage at lower latitudes worse, because of the bifurcation of the visibility region. Clearly, then, the choice of orbital inclination with respect to the range of latitude of interest is a key factor in orbit selection. Other factors are more obvious, such as increasing satellite altitude to the maximum allowable by sensor limitations. All these effects are reflected in the

resulting visibility region solution. A larger visibility region, produced by increasing satellite altitude, obviously implies better coverage since there are more contact opportunities for greater lengths of time. Target latitude is not usually optional, and so a good orbital inclination choice with respect to the desired latitude is the key.

3.2.3 Effect of Orbital Inclination on Revisit Intervals

The choice of orbital inclination is more complicated than other orbit design parameters, and therefore is usually the final design variable in an optimization algorithm. As was shown in Chapter 2, a sufficiently high orbital inclination with respect to target latitude can cause a bifurcation of the visibility region. The bifurcation is a factor that can be of use when analyzing single or multiple satellite coverage. It was found through the course of this study that, in general, a bifurcation of the visibility region is a disadvantage. This is because a bifurcation creates an opportunity for the satellite to miss seeing the ground site during an orbit--the line representing the motion could conceivably pass between the two bifurcations. If this opportunity can be eliminated, more consistent coverage is obtained. Since bifurcation is caused by high orbital inclination with respect to ground site latitude (the actual limit being where inclination exceeds the highest latitude visible to a satellite vertically over the site, as discussed in Chapter 2), a good "rule of thumb" appears to be to choose inclination that is low enough so as not to cause visibility region bifurcation at the highest latitude of interest. As inclination is increased, the region covered on the period square becomes wider. Eventually the region will bifurcate. After this point, the maximum number of passes through the visibility region decreases below what is available from a single region. Since the orbit lines are equally spaced horizontally, they are fixed in relative position to one another, and a bifurcation will always produce a worse coverage situation than for a single region. This observation was also noted by Lang (Ref [10]), and Hanson, et al (Ref [5]).

3.2.4 Computing Revisit Times

Computing maximum revisit time is a matter of locating the two points of intersection of the orbital lines with the visibility region. Figure 3.2, which is similar to Figure 3.1, shows how maximum revisit time may be computed. Here the visibility region has bifurcated due to the high orbital inclination with respect to target latitude. As in Figure 3.1, there is one satellite viewing one ground site. By locating the longest orbital line segment outside the visibility region, projecting it onto the vertical axis, and then dividing by satellite orbital frequency, a value for maximum revisit time is obtained. Computer software was developed in the course of this study (to be discussed later) that implements this method--the algorithm has shown to be efficient and readily envisioned by the analyst.

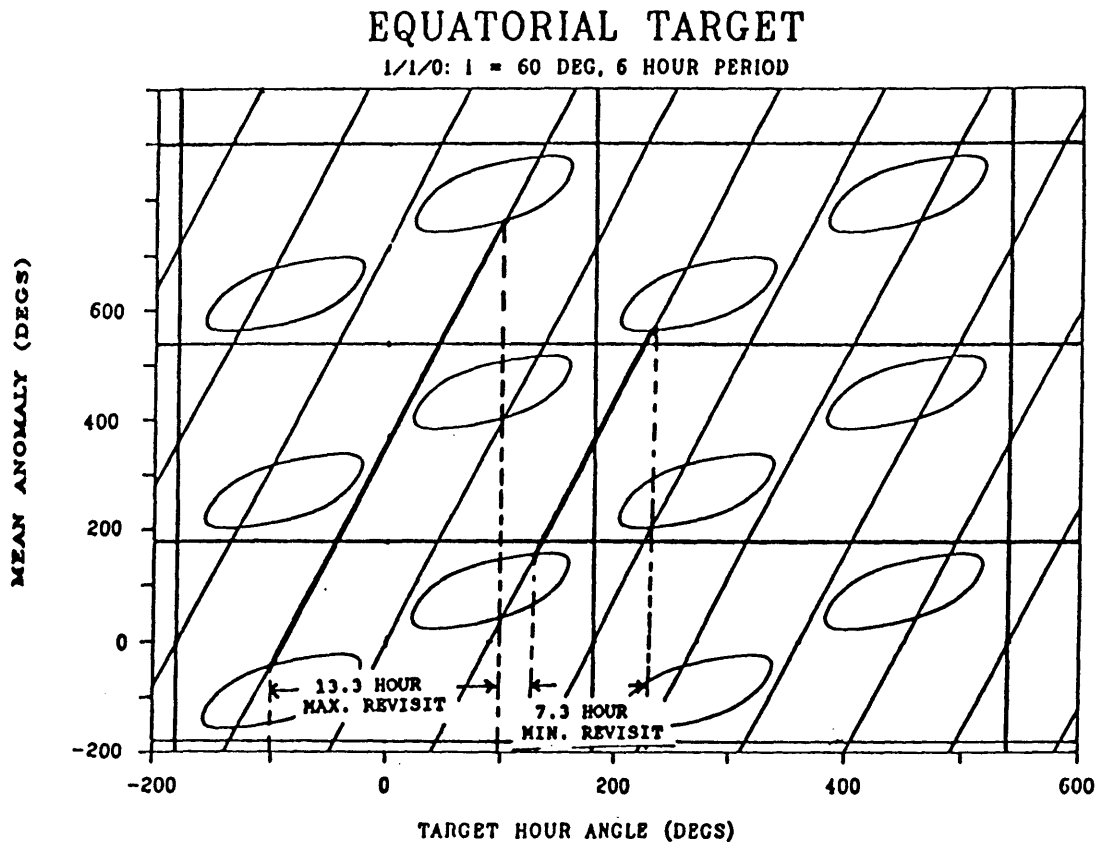


Figure 3.2: Method of Computing Maximum Revisit Time

3.3 Ground Trace Repetition

It has been shown how revisit intervals can be evaluated by adding 360 degrees several times in both mean anomaly and earth rotation angle on the period square. But it is important to know how far in the future to search for a maximum revisit interval. The obvious answer is: until the ground traces repeat, since the satellite(s) will be passing over the same ground points after returning to their initial state. Exactly when a ground trace repeats itself is not immediately clear, however, especially when the earth oblateness perturbation is considered.

3.3.1 The Ground Trace Repetition Parameter Q

If the orbital period is an exact divisor of earth's rotation period, the ground trace will repeat after a certain number of orbits. For example, 24 hrs / 2 hr period = 12 orbits until ground trace repetition. However, it is not appropriate to assume that the orbital period will be an exact divisor of earth's rotation period. In reality, the ratio of the two periods will most likely be an irrational number, and the ground trace will never repeat itself exactly. Fortunately, it is possible to calculate a repetition parameter, which is the ratio of the number of orbits to the number of days until the ground trace repeats. This number can then be resolved into a ratio of two integers, which will approximate the repetition parameter.

The derivation of the ground trace repetition parameter is as follows (Ref [8]): the ground trace will repeat after R orbits in D days. The longitudinal separation between equatorial crossings of the ground trace is (Ref [9])

$$\Delta\lambda = 2\pi D/R \tag{1}$$

This separation can also be defined in terms of the satellite orbital elements. If we take into account the J_2 perturbation effect, the separation can be expressed as

$$\Delta\lambda = P_N(\omega_E - \frac{d\Omega}{dt}) \quad (2)$$

where P_N is the nodal period, $d\Omega/dt$ is the regression rate of the longitude of ascending node, and ω_E is the inertial rotation rate of the earth. The nodal period, which is the period of nodal regression due to earth oblateness, is given as (Ref [5])

$$P_N = 2\pi \sqrt{a^3/\mu} [1 - \frac{3}{2} J_2 (R_E / a)^2 (3 - \frac{5}{2} \sin^2 i)] \quad (3)$$

where μ is the gravitational parameter of the earth--the product of earth's mass and the universal gravitational constant. The nodal regression rate is

$$\frac{d\Omega}{dt} = \frac{3}{2} \sqrt{\mu/a^3} J_2 \left[\frac{R_E}{a(1-e^2)} \right]^2 \cos i \quad (4)$$

where e is the orbital eccentricity. Luders (Ref [12]) first referred to the ground trace repetition parameter as Q , where

$$Q = R/D = \frac{2\pi}{P_N (\omega_E - \frac{d\Omega}{dt})} \quad (5)$$

The parameter Q can be represented as a ratio of two integers, just as the number π can be represented approximately by the ratio 22/7, or, more accurately, by 355/113. To resolve a number into a ratio of two integers is an ideal application of continued fractions, and a brief discussion of them is appropriate.

3.3.2 Continued Fractions

Any real number x can be represented as a continued fraction, which is of the form:

$$x = a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \xi_3 \dots}}} \quad (6)$$

where a_0, a_1, \dots are integers. The so-called continued fraction algorithm, as described in Ref [3], is the system of equations

$$x = a_0 + \xi_0 \quad (0 \leq \xi_0 < 1)$$

$$\frac{1}{\xi_0} = a_1 + \xi_1 \quad (0 \leq \xi_1 < 1)$$

$$\frac{1}{\xi_1} = a_2 + \xi_2 \quad (0 \leq \xi_2 < 1)$$

$$\frac{1}{\xi_2} = a_3 + \xi_3 \quad (0 \leq \xi_3 < 1) \quad \text{etc.} \quad (7)$$

By iterating this algorithm, a number can be approximated by the ratio of two integers, to within a specified accuracy. This algorithm is implemented in the supporting software, by separating the ground trace repetition parameter Q into its integer and fractional parts and proceeding through the algorithm (7) to find the integers a_0, a_1, \dots until the fractional part is less than an arbitrarily small tolerance. Then, the continued fraction is simplified into a ratio of two integers, which is the number of orbits over the number of days until the ground trace approximately repeats. The issue is to choose a ratio that is "reasonably" accurate. As in the example of the number π , the ratio $22/7$ is not as accurate as $355/113$, but it is probably not reasonable to search 113 days into the future for a maximum revisit interval.

During this length of time (over 3 months), the orbit would most likely be perturbed enough, due to gravity gradients, solar pressure, atmospheric drag, or other effects, that the original orbital parameters are no longer accurate. This is especially true if the satellite is in a low-earth orbit. The software therefore makes the arbitrary assumption that a ground trace repetition interval of more than 30 days is not reasonable, and the accuracy requirement is relaxed in order to obtain a rougher, but sooner, estimate of when ground trace repetition occurs. If, for example, Q were equal to π , the program would choose the ratio 22/7 versus the next most accurate ratio, 355/113.

3.4 Maximum Revisit Time Analysis

3.4.1 Software

As mentioned earlier, computer software was developed for this study that implements the method of revisit interval analysis using visibility regions. The program RevMap, written in TURBO Pascal for the IBM PC, iterates the algorithm illustrated by figures 3.1 and 3.2, cycling through a region of the earth's surface at arbitrary degree intervals in longitude and latitude, and evaluating maximum revisit time at each point. A listing is included in Appendix B. The user provides the program with the desired constellation altitude, inclination, viewing horizon, and latitude limits to search through. The program calculates the amount of time until the ground trace begins to repeat, and searches the equivalent of that length of time in satellite mean anomaly angle on the period square. The program can generate results over a region of 60 degrees in latitude by 90 degrees in longitude (5400 points) in about 15 minutes on the PC. As will be seen later, it is not necessary to search over a large range of longitude in all cases. The amount of longitude that must be analyzed is dependent on the number of separate ascending nodes of the constellation. A

routine in the program evaluates what the minimum necessary range of longitude is, in order so save computer search time.

3.4.2 Example Problems

The first example problem chosen was to analyze the coverage provided by a single satellite with a 6 hr period, inclination of 60 degrees, and viewing horizon of 10 degrees. The output data is represented by the contour map shown in Figure 3.3, where the contours are drawn at 1 hr intervals. The outline of the continents is included for reference. Note that there are two high plateaus of 14 hr maximum revisit time, separated by drastically lower plateaus of 7-9 hr maximum revisit time. The sudden jump from 14 hrs down to 8 hrs is not due to the plotting program--it is

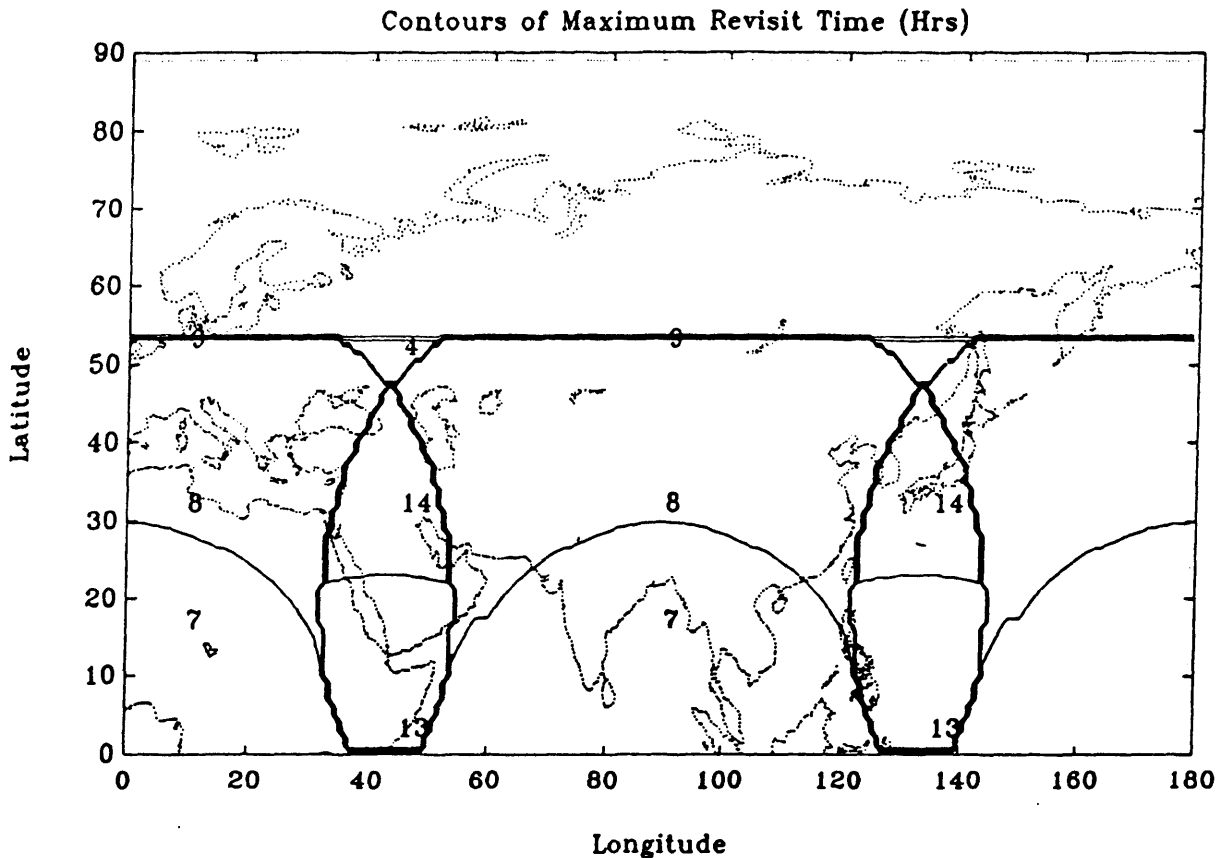


Figure 3.3: Maximum Revisit Time Data for A Single Satellite. 6-Hour Orbit ,
 $i = 60$ deg, Viewing Horizon = 10 deg.

Contours of Maximum Revisit Time

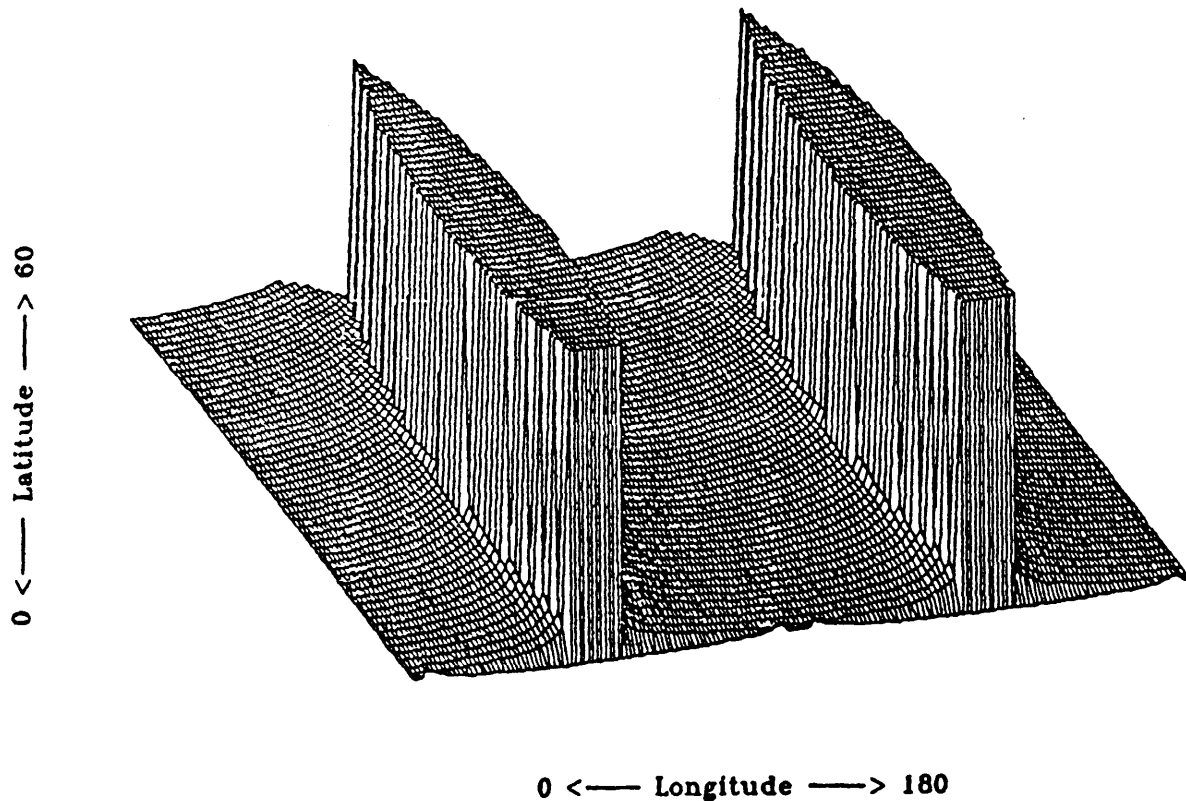


Figure 3.4: Relative Topology of Maximum Revisit Time Phenomenon
(for Case of Figure 3.3)

evidence of the discontinuity of the revisit phenomenon. This effect is illustrated more clearly in Figure 3.4, which shows a 3-dimensional plot of the same maximum revisit time data. The computer program MATLAB was used to generate the contour plots.

Figure 3.5 shows the results obtained for the same problem as in Figure 3.3, by authors Hanson and Lang (Ref [11]) in a previous study. Their map shows similar orders of magnitude for maximum revisit time, and serves to corroborate the results of the program RevMap. However, their map is slightly misleading, in that the contour plotting program has forced smooth changes from large to small values of maximum revisit time. The transition from 14 hours to 8 hours is actually a discontinuous jump, not a ramp as Figure 3.5 seems to imply.

The second example problem used a shorter orbital period of 2 hrs, with a viewing horizon of 20 degrees, in order to more realistically simulate the revisit behavior of an earth-observing satellite. The resulting contour map is shown in Figure 3.6, where the maximum revisit time is shown to range from about 11.3 to 13.3

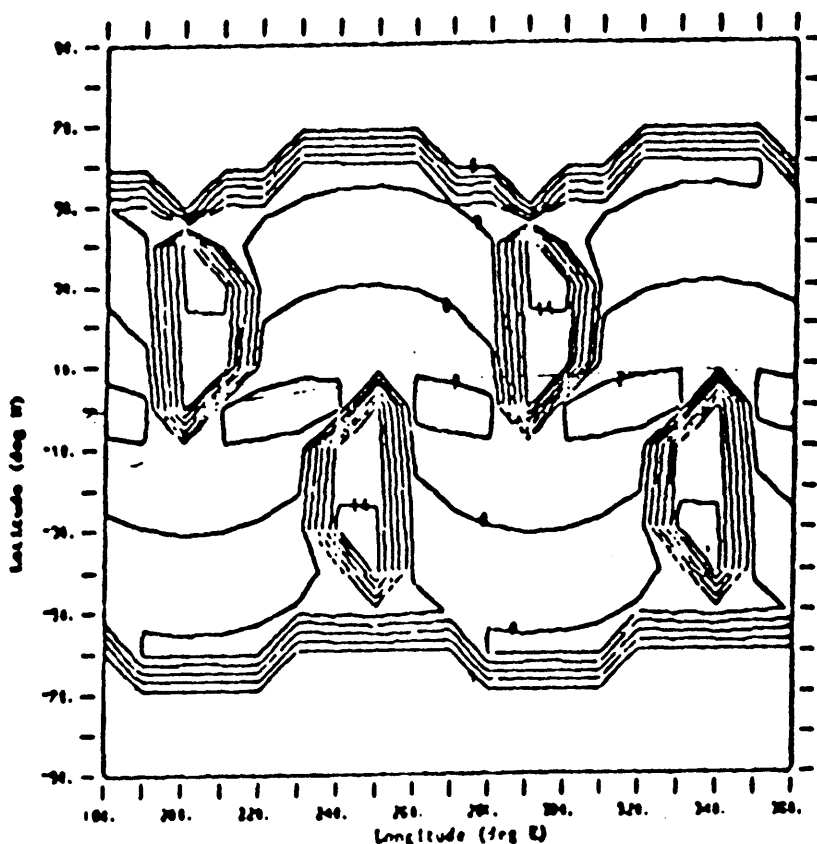


Figure 3.5: Results of Hanson & Lang for Case of Figure 3.4

hours. The fact that the difference is approximately one orbital period is easily explained using the visibility region concepts: recall that the visibility region can be phased horizontally to reflect a change in target longitude. This phasing can cause the loss (or gain) of a viewing opportunity by removing (or adding) an orbit line

intersection with the visibility region. Note, for example, that shifting the lines slightly to the left in Figure 3.1 would cause an intersection with point P1, while maintaining all other contact opportunities. This idea explains why moving along the equator in Figure 3.6 causes jumps of approximately one orbital period in maximum

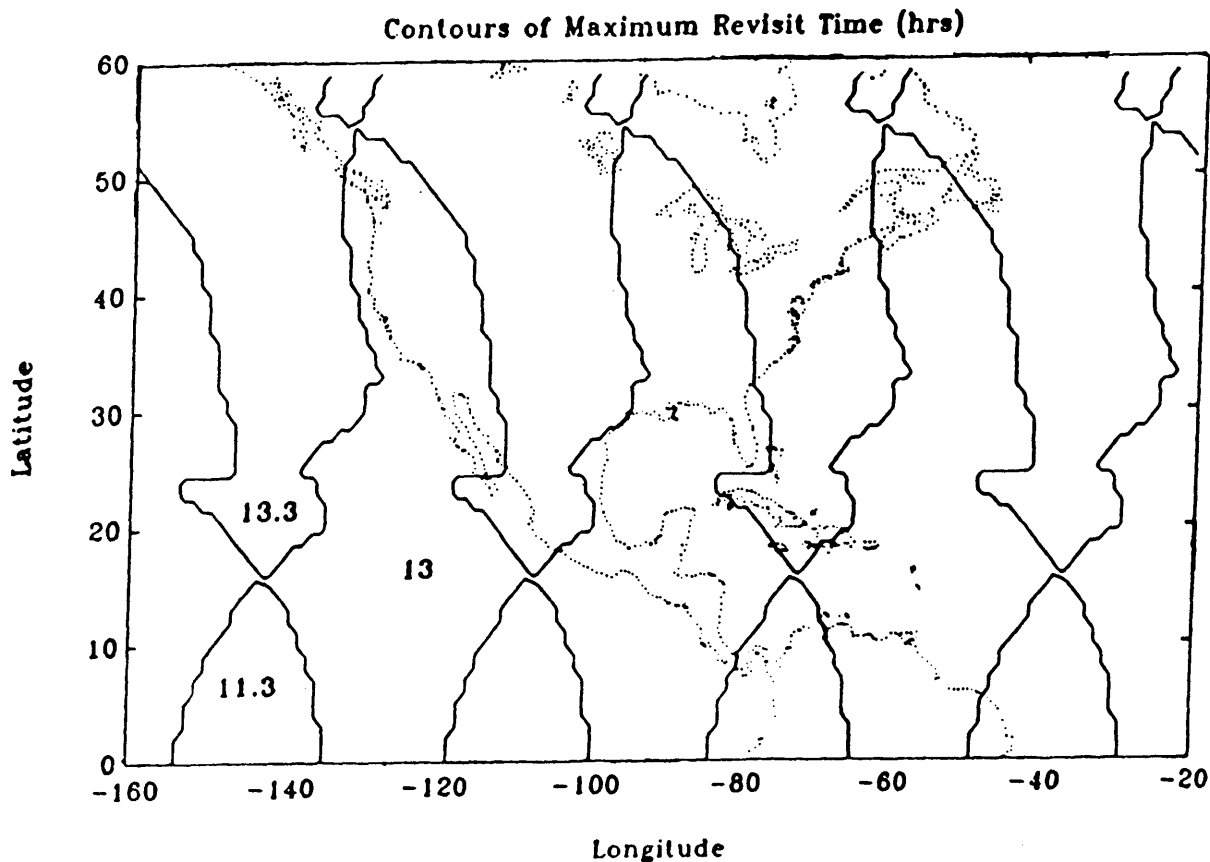


Figure 3.6: Maximum Revisit Time Data for A Single Satellite. 2-Hour Orbit, $i = 60$ deg, Viewing Horizon = 20 deg.

revisit time. This idea can also be exploited when it comes to constellation design, by choosing a longitude of ascending node that gives the maximum number of intersections with the visibility region. It is a simple calculation when using the visibility region problem formulation.

3.4.3 Properties of the Maximum Revisit Time Function

An important point to be noted in looking at contour maps such as Figures 3.3 and 3.6 is that the maximum revisit time is a strong function of longitude, and a weaker function of latitude. This is inevitable when only one satellite with a repeating ground trace is used. By using a constellation, however, the dependence on target longitude can be effectively removed, to where moving along a line of constant latitude causes very little (if any) change in the maximum revisit time. This property is not true of all constellations, however. The phasing of satellites in different planes becomes a critical issue in obtaining consistent coverage. This will be discussed in more detail in subsequent chapters.

Another observation worth noting is that the pattern of the maximum revisit time function exhibits a repetitive property as one moves along a line of constant latitude. This is clearly seen in Figures 3.3 and 3.6. The patterns repeat with a frequency that is the ratio of the earth's rotation period to the satellite's orbital period. For the example of Figure 3.3, the patterns repeat 4 times as one moves through 360 degrees of longitude, and 4:1 is the ratio of the earth's rotation period to the satellite rotation period. The same phenomenon is evident in Figure 3.6. If the maximum revisit time function has this resonance property, then this implies that it is not necessary to search for the maximum revisit time until the ground trace repeats. For the case of Figure 3.3, it takes 24 hours for the ground trace to repeat, but the figure implies that it is only necessary to search over 6 hours (90 degrees of longitude). Likewise, for Figure 3.6 it is only necessary to search 2 hours, instead of 24. It appears that this property can be used to limit search time in general.

Fortunately, the visibility region problem formulation shows why this phenomenon occurs. It is only necessary to search over the equivalent of one orbital period in earth rotation angle for all possible maximum revisit times. Although the maximum revisit time changes as a new longitude is chosen, it is only necessary to

check over (360 degrees * orbital period / 24 hrs) in longitude. The pattern of maximum revisit time will then repeat. (It will be seen in a subsequent chapter that this number is divided by the number of separate ground traces for a constellation, which limits search time and explains how coverage becomes more consistent across longitude for constellations.) Figure 3.8 provides the explanation for this. Since

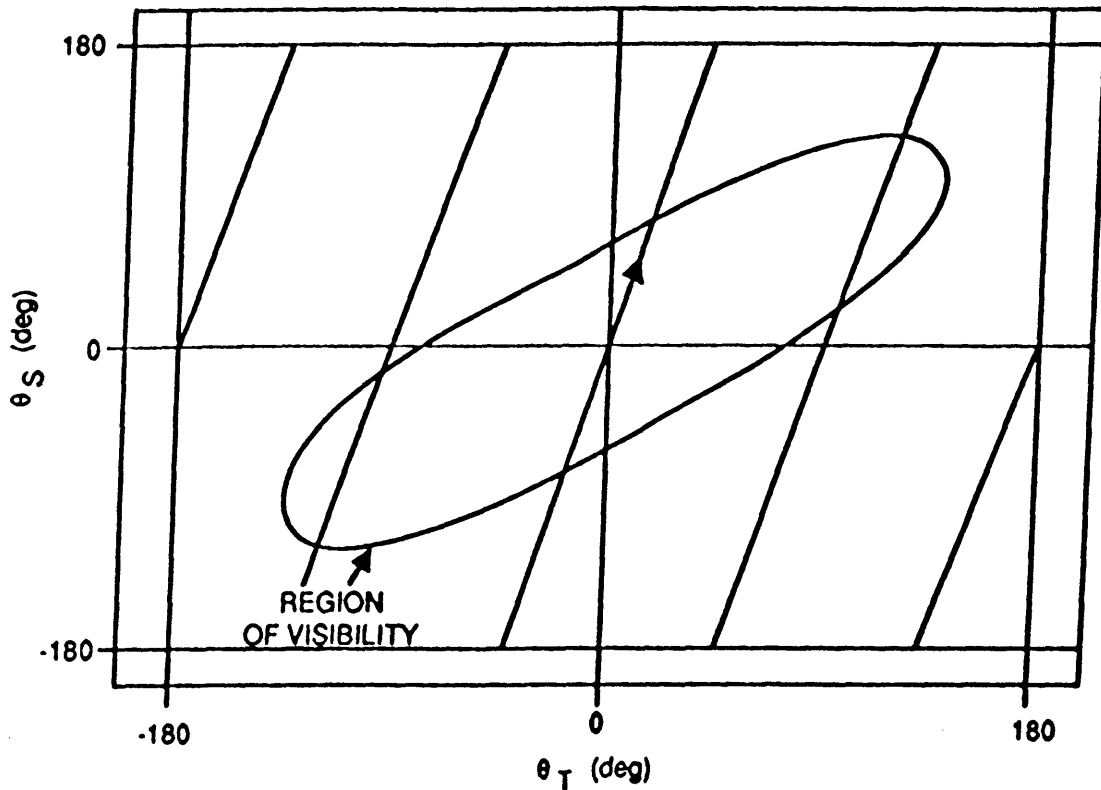


Figure 3.7: Visibility Region for 6-Hour Orbit

shifting to a new target longitude is the same as shifting the orbit lines horizontally on the period square, it is clear that this only needs to be done until the lines overlap. Since they are evenly spaced, the complete pattern of maximum revisit time can be evaluated through just one horizontal interval between the lines. With a shorter

orbital period, there will be more lines, and the search interval is shorter. Note, however, that these example problems use periods that are exact divisors of each other (24/6 and 24/2). In reality, the periods will not divide exactly, and an approximation has to be made, as was discussed in section 3.3. It is still necessary to make the approximation of how many orbits are needed until ground trace repetition occurs, in order to draw the orbit lines on the period square correctly. The ratio of the orbital period to earth rotation period is most likely irrational, and has no exact representation as ratio of two integers. Therefore, the orbit line actually winds around the torus without ever closing, and covers the torus completely. There is, then, only one true maximum revisit time, but it may take an exceedingly long time for it to occur for a given target. When the approximation for the number of orbits until ground trace repetition is made, it means that a local maximum is being found over a reasonable length of time, during which orbit perturbations or active orbit corrections will probably take place.

3.5 Chapter Summary and Conclusions

At this point, maximum revisit interval analysis can be conducted efficiently for a single satellite, by iterating the algorithm described in this chapter. The orbital elements and target location determine the shape of the visibility region--changing the target latitude requires computing a new visibility region, while changing target longitude is a matter of shifting the orbital lines on the period square. Computing maximum revisit time is done by locating the longest orbital segment that remains outside the visibility region. With the process automated in software, maps of maximum revisit time may be drawn, providing insight into the revisit phenomenon. The next chapter will attack the constellation coverage problem by extending the analysis framework already developed.

CHAPTER 4

Satellite Constellation Analysis

4.1 Introduction

Chapter 3 described the problem formulation for single satellite coverage. It was shown how the visibility region approach to the problem provides insight into the revisit phenomenon, and an efficient means for computing maximum revisit time and analyzing coverage when the process is automated in software. This chapter will develop the method further to deal with the coverage provided by a constellation.

4.2 Notation

To describe a satellite constellation, the following so-called "Walker" notation is used: T/P/F, where

T = total number of satellites in the constellation

P = number of orbital planes in the constellation

F = phasing of satellites between planes, in units of $360/T$

also,

i = common orbital inclination angle (degrees)

a = common orbital semi-major axis

e = orbital eccentricity (zero for circular orbits)

A constellation may therefore be described as follows: consider T/P/F = 12/3/1. This means there are 12 satellites in 3 orbital planes, with the 4 satellites in each plane separated symmetrically by 90 degrees in mean anomaly, and the 3 orbit planes separated symmetrically by 120 degrees in longitude of ascending node. The "1" notation indicates that when a satellite in the "first" orbital plane is at zero degrees mean anomaly (say, over the equator), a satellite in the "next" orbital

plane is at $1 \times 360/12 = 30$ degrees mean anomaly. Similarly, a satellite in the "third" orbital plane is at 60 degrees mean anomaly at the same instant. In this way, the phasing between satellites in different orbits is indicated by the notation. The satellite phasing within each plane is determined by the number of satellites in the plane, while the relative phasing between orbit planes is determined by F. It will be shown that this phasing between satellites in different orbits has important effects on the coverage properties of the constellation. This notation, commonly called "Walker" notation after its originator, is illustrated in Figure 4.1.

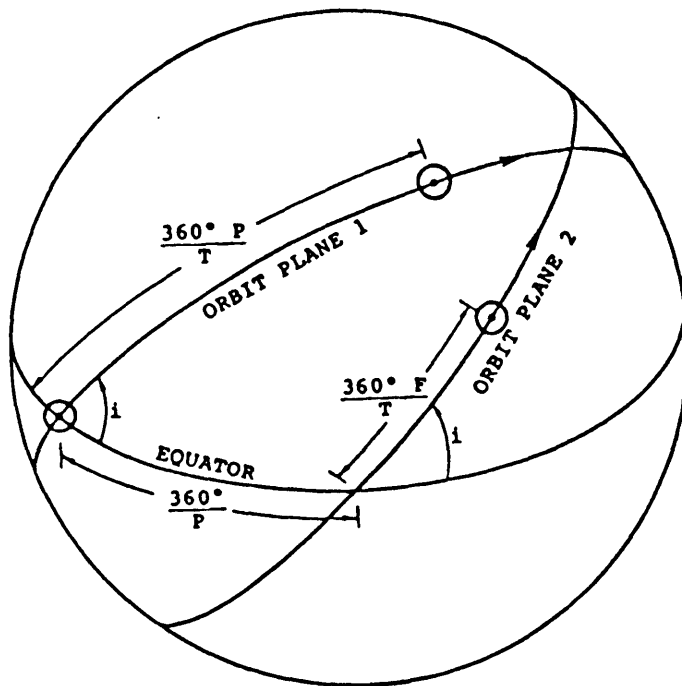


Figure 4.1: Illustration of Walker Constellation Notation

4.3 Standard Methods of Coverage Analysis

It is useful to understand the typical approaches to determining the coverage provided by a constellation, and methods of designing constellations, since they provide insight into how constellations of different orientation will cover the earth. These approaches were discussed briefly in Chapter 1, but will be reiterated in this section.

There are several underlying assumptions for these methods. They assume that a satellite will require a line-of-sight to the ground site, as determined by a minimum local elevation angle at the site. The satellite must be within functional range of a camera, radio beacon, radar, or other sensor, and this will determine the maximum altitude allowable for the constellation. As long as these requirements are met, it is assumed that the satellite can "cover" the ground site.

4.3.1 Circumcircle Approach

The goal of the circumcircle approach is to synthesize symmetric constellations using circular orbits, where all satellites are at the same altitude and inclination. These constellations are described by the Walker notation discussed earlier--each plane has the same number of satellites, and they are separated evenly in mean anomaly within the plane. All orbital planes are spaced evenly about the equator, and the phasing between satellites in different planes is the same for all planes. They are frequently referred to in the literature as Walker or "Walker-like" constellations. Optimization is usually performed by first cycling through all possible numbers of planes which are divisors of the total number of satellites, then varying the interplane satellite phasing, and then adjusting the common orbital inclination to give the best possible coverage by the constellation. This process can require a very lengthy computer search if more than a few satellites are needed. The number of constellation combinations becomes large, and the time required to find the optimal

coverage for each trial constellation multiplies the search time. Usually, the term "optimal" is taken to mean a minimum number of satellites required to provide satisfactory coverage, within constraints on altitude and/or sensor capabilities.

Walker-like constellations, having all satellites at a common altitude and inclination, offer the advantage of removing the problem of the earth's oblateness affecting the coverage. If all satellites are at the same altitude and inclination, then the lines of nodes for all orbital planes will move at the same rate due to precession. Also, if elliptical orbits are used, the arguments of perigee for all planes would precess at the same rate.

It will be shown in a subsequent chapter how the visibility region formulation greatly reduces the demands of designing Walker-like constellations by providing a fast means for evaluating coverage, and providing a very good initial guess (if not an optimal choice) for inclination. Furthermore, experience with example problems has provided some rules-of-thumb to follow when cycling through the constellation combinations.

4.3.2 Streets of Coverage Approach

This approach, developed by L. Rider, derives its name from the coverage pattern provided by multiple satellites in a single plane. The formulation requires that the coverage circles for the symmetrically distributed satellites in each plane overlap, so as to provide a continuous band or "street" of coverage along the projection of the orbital plane. The meshing of the streets from all orbital planes in the constellation determines the coverage, and the design of the constellation is driven by how many streets are required to cover the region of interest. The constellation design is usually Walker-like, with symmetric distribution of satellites and common altitude and inclination. The goal of this approach is normally to synthesize constellations for continuous coverage. It is a geometrical method, good for designing continuous-

coverage constellations, but not well-suited for revisit coverage, because it does not provide a numerical method for evaluating revisit intervals. If the coverage circles of satellites within a particular plane do not overlap, then there is no "street" of coverage.

4.3.3 Polyhedral Approach

The polyhedral approach of J.E. Draim involves the use of polyhedra, which are usually tetrahedral or prismatic, to obtain global coverage using inclined, non-circular orbits. This is another geometrical approach, which employs several "theorems" Draim proposes. The basic method is to arrange satellites in such a way that the projections of the triangular planes formed between any three satellites onto the earth will completely cover the earth. The goal of the method is to obtain continuous global coverage. Draim published several studies using his methods to propose improved Walker constellations that provide global coverage, such as a four satellite continuous global coverage constellation, etc. His methods are not well suited for evaluating revisit coverage or for designing intermittent coverage constellations.

4.4 Co-Rotating Versus Counter-Rotating Constellations

The most difficult region for an inclined, Walker-like constellation to cover is the equator. This is true because the satellites are spaced evenly in mean anomaly, and the planes are inclined, so there will be heavier coverage at the higher latitudes and sparse coverage at lower latitudes, with the worst case at the equator. Since planes in Walker constellations are phased evenly in ascending node around 360 degrees, satellites going north across the equator are near other satellites going south across the equator. This is characteristic of what is called a "counter-rotating" constellation, and was noted by Hanson and Linden (Ref [6]). The fact that the satellites are passing each other going north and south causes irregular coverage--

the coverage is good or bad depending on how the coverage circles of the individual satellites mesh.

If the satellites which are near each other as they cross the equator are travelling in the same direction, then it seems intuitive that the coverage would be much better. The coverage circles of the satellites would mesh together as time goes on, instead of passing through each other. This type of constellation is called "co-rotating," and instead of phasing the ascending nodes evenly around 360 degrees, the nodes are phased around 180 degrees. At 0 and 180 degrees, the satellites crossing the equator are travelling in opposite directions, in similar fashion to a counter-rotating constellation, but this only happens at these two locations. Figure 4.2 illustrates the idea of counter- and co-rotating constellations, and Figure 4.3 shows the coverage gaps which may occur above a certain latitude limit, when using a counter-rotating constellation that does not provide continuous whole-earth coverage. Although Figure 4.2 implies polar orbits, inclined orbits also apply. In fact, a Walker (counter-rotating) constellation with satellites in polar orbits, and using an even number of planes, would have satellites colliding over the poles. This situation is depicted in Figure 4.4. The constellation designer should be mindful of potential satellite collisions when using polar orbits.

The discussion in this section leads to the conclusion that traditional Walker constellations, which are counter-rotating, do not necessarily provide the best coverage if the requirement is for continuous coverage. It is better to use co-rotating constellations in order to avoid the irregular meshing of coverage circles. However, if gaps in coverage are tolerable, and one desires only a maximum bound on revisit times, then the choice of whether to use a co-rotating or counter-rotating constellation becomes an issue, and which provides the better coverage is an open matter. These considerations should be kept in mind when analyzing and designing constellations.

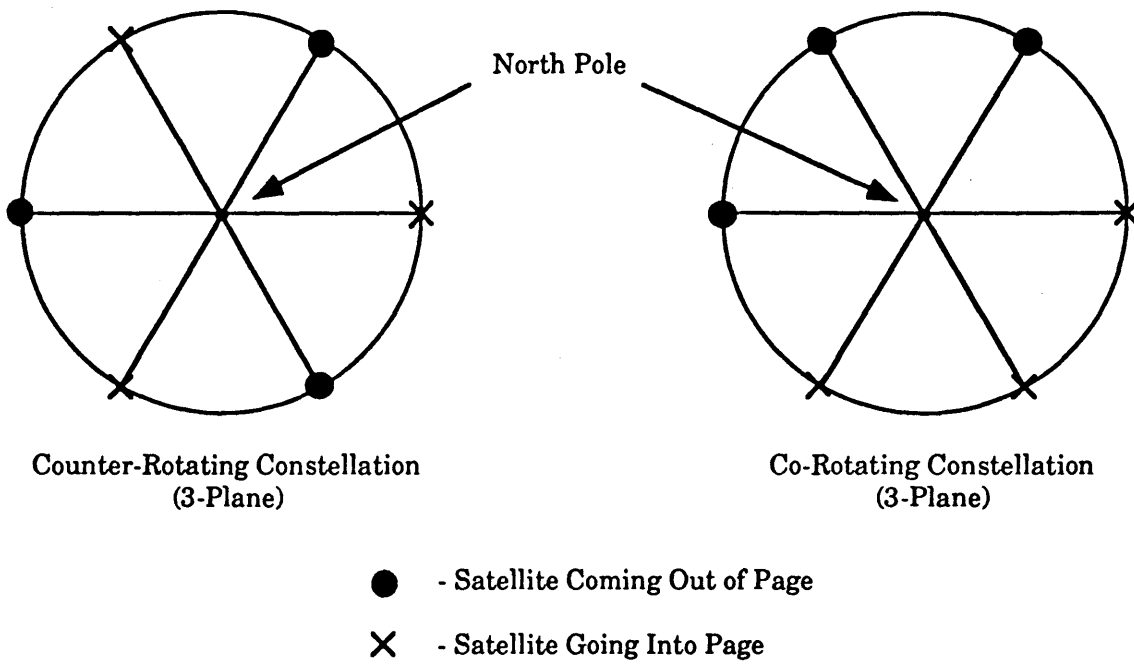


Figure 4.2: Different Constellation Orientations Using Polar Orbits

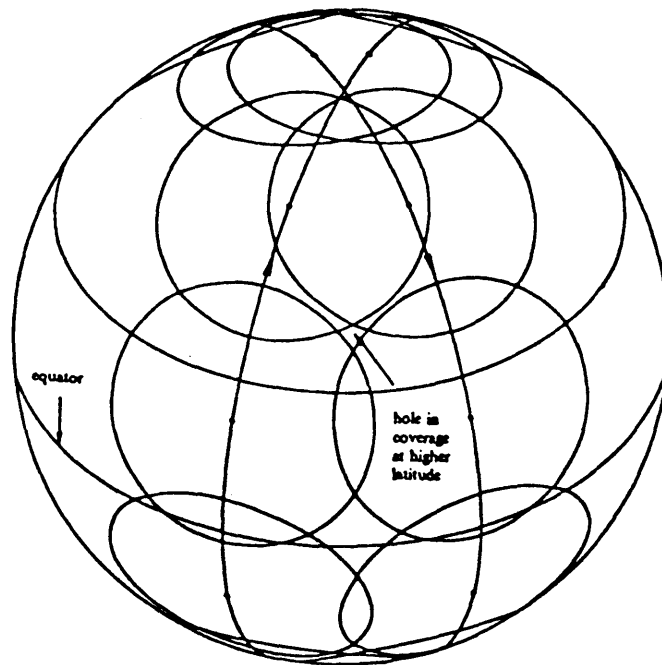


Figure 4.3: Gaps in Coverage due to Counter-Rotating Constellation Orientation

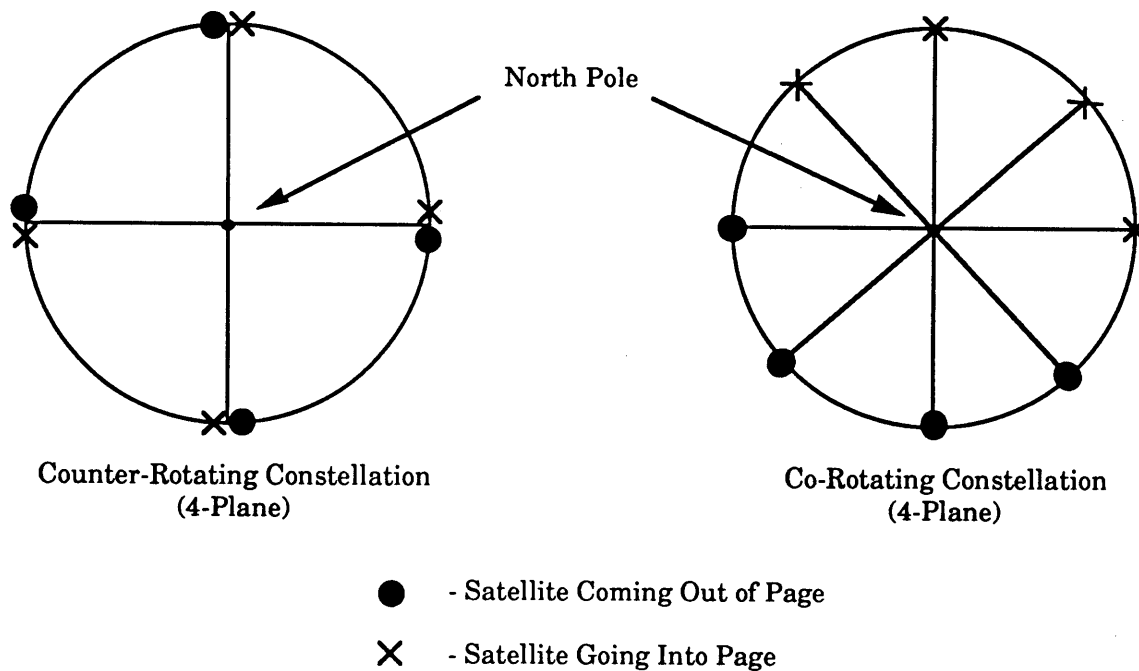


Figure 4.4: Using Polar Orbits and an Even Number of Planes

4.5 New Constellation Coverage Problem Formulation

The fact that the coverage provided by a satellite has been formulated in prior chapters in terms of mean anomaly and earth rotation angle is ideal for performing constellation analysis, since these two angles are used to define the positions of satellites within the constellation. Chapters 2 and 3 developed this formulation and applied it up to single satellite coverage.

The difference between a single satellite and a constellation of satellites is that, for a constellation, there will be several visibility regions within the same period square, and they will be phased in special ways. Each represents an opportunity for a different satellite in the constellation to view the same target. Horizontal phasing represents an orbital plane change by indicating a different ascending node, and vertical phasing represents changing a satellite's position within a particular orbit since it indicates a mean anomaly change. This idea can be illustrated as follows:

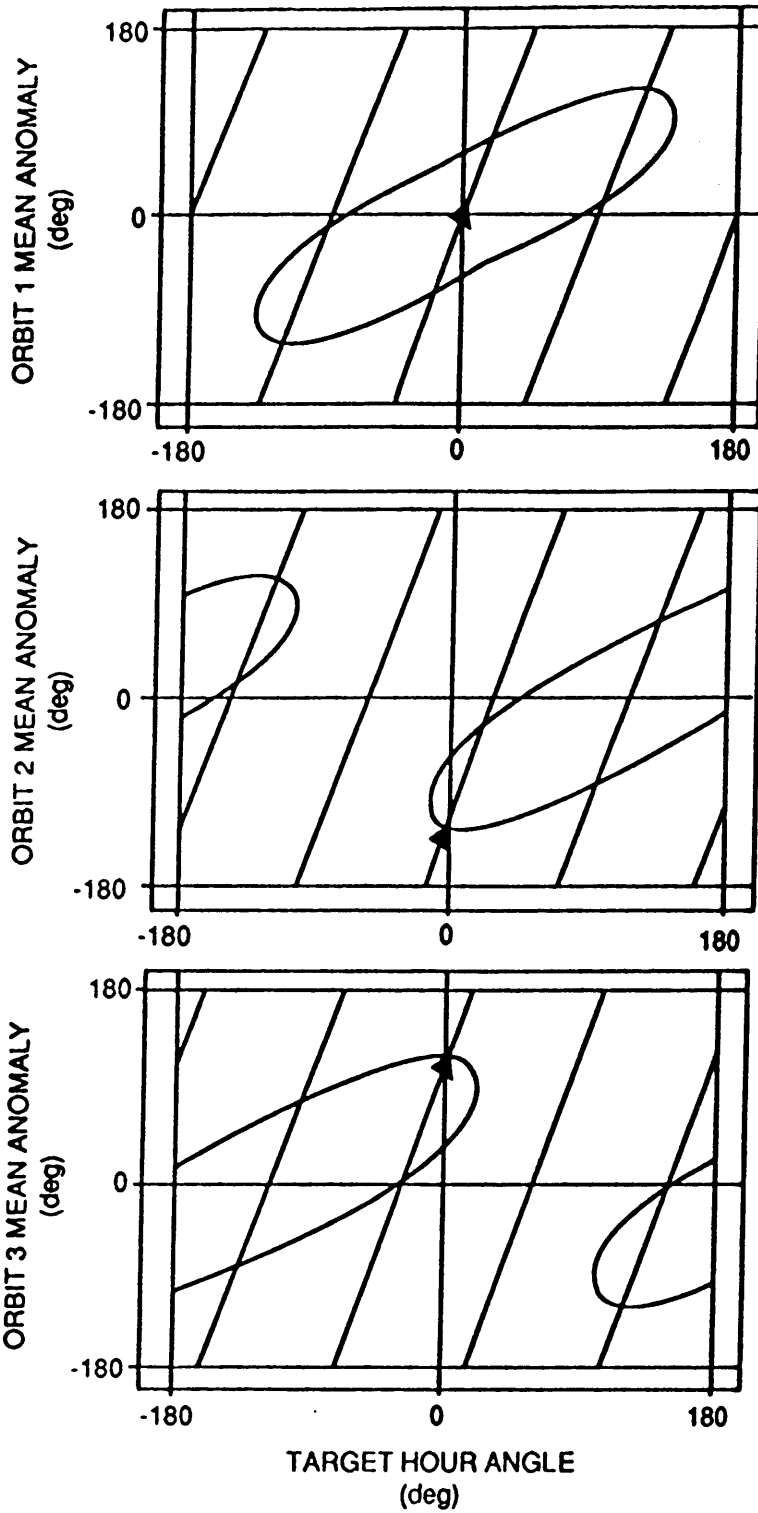
consider a 3/3/2 Walker satellite constellation. Here we have three satellites, one in each orbital plane, with the planes separated 120 degrees in longitude of ascending node. Because of the "2" phasing, when the "first" satellite is at zero degrees mean anomaly, the satellite in the "next" orbit is at $2 \times 360/3 = 240$ (equivalently, -120) degrees mean anomaly, and likewise the satellite in the "third" orbit is at $3 \times 360/3 = 120$ degrees mean anomaly. Figure 4.5 depicts this example situation for a 6 hour orbit, with common inclination 40 degrees, and a target located at 40 degrees latitude. Each orbit is shown individually--note each is shifted forward by 120 degrees in longitude of ascending node. At the instant of time when target hour angle is zero degrees, one satellite is directly overhead (orbit 1), one is entering into view of the target (at 240 degrees mean anomaly, orbit 2), and one is leaving from view of the target (at 120 degrees mean anomaly, orbit 3). Additionally, all three satellites in this constellation follow the same ground track. This phenomenon is evidenced by the fact that the visibility region is always "cut" in the same way. Figure 4.5 demonstrates that a great deal of information can be visualized by this methodology.

4.5.1 Constellation Coverage (Revisit to Single)

This section describes an analysis method for coverage by a constellation, where the requirement is for some level of intermittent viewing up to continuous viewing of a ground site.

The general method proposed in this study for analyzing the coverage of a constellation can be developed by continuing the same example used in Figure 4.5. When the three figures in Figure 4.5 are collapsed into one, Figure 4.6 results. Figure 4.6 contains the same information as Figure 4.5, but also reveals more general properties of the constellation. In this picture, the orbit line represents all three satellites at once--it is ambiguous which satellite is which until a visibility region is encountered. This formulation removes the issue of which satellite views the target--

ORBITAL ANOMALY MEASURED FROM
ASCENDING CROSSING THROUGH 40-deg LATITUDE



3/3/2: ALL THREE ORBITS
HAVE SAME GROUND TRACE

Figure 4.5: Phased Visibility Regions Representing a 3/3/2 Walker Constellation.
Example: 6-Hour Orbits, Latitude=Inclination=40 deg,
Viewing Horizon=10 deg.

it is assumed that it does not matter, only that at least one satellite in the constellation does view the target. Again, a maximum revisit interval is represented by the longest line segment outside the visibility region pattern, which can be computed as described in Chapter 3. Note that at the target latitude in Figure 4.6, the revisit interval is nearly independent of longitude--recall that shifting target longitude is equivalent to shifting the orbit lines on the figure. Figure 4.7 represents the constellation 3/3/1, with all other features the same as in Figure 4.6. Here it is apparent that the maximum revisit interval is reduced, but it is a strong function of longitude. Similar figures can be drawn for any constellation desired, and analyzed in terms of maximum revisit interval length and/or longitude dependency. The trade-offs between various constellations can be evaluated as parameters are changed. One problem that has not been circumvented is that analysis must be performed for each latitude individually. If mission requirements must be met for a region of latitude, analyzing constellation performance at one or two degree intervals within the region should be sufficient.

4.5.2 Constellation Coverage (Multiple or Redundant)

The analysis to this point has assumed that the mission requirement for coverage was to have one satellite viewing a site, either intermittently (revisit coverage) or continuously. Many applications, however, require several satellites to view a site at the same time. This idea is usually referred to in the literature as n-fold coverage. An example is the Global Positioning System, which requires four satellites to be in line-of-sight view of a receiver in order to obtain an accurate position fix. For the system to operate over the whole earth, the constellation must provide continuous four-fold global coverage. The n-fold coverage problem creates enormous computational demands if traditional orbit propagation techniques are used. Fortunately, the methods developed in this study can be extended easily to handle n-

fold coverage. Surprisingly, no additional computational load is required to analyze a constellation's performance beyond single coverage. The analysis is the same regardless of whether single satellite intermittent or multiple satellite continuous coverage is the case. Redundant coverage is indicated in the visibility region formulation by areas of overlapping regions, such as in Figures 4.6 and 4.7. Here, there are areas of intersection between two regions. The area of intersection indicates the locus of all coordinates in mean anomaly and earth rotation angle where two satellites view the ground site simultaneously. These "double visibility" regions can be treated the same as single regions. The level of double (two-fold) coverage provided by the constellation can be evaluated as for single coverage, by evaluating the orbital segments which remain outside the double regions. As these intervals approach zero, the constellation provides continuous two-fold coverage. Given a constellation that provides continuous single coverage, its n-fold coverage properties can be evaluated by looking for areas of n-overlapping visibility regions.

The fact that the period square becomes quite crowded with many overlapping visibility regions is not so daunting if the figure is drawn on a computer graphics screen. This way, the visibility regions can be placed according to their positions in the constellation, and the overlaps can be colored. For example, the single regions can be colored white, and the areas of double overlap blue, triple overlap red, etc. Many graphics packages allow images to be placed with a logical operator indicating how each individual point is to be compared to the point already on the screen. The TURBO Pascal package has a "PutImage" function, which can place an image with an AND, OR, NOT, or XOR operator. By increasing the color value by one for each overlap, the level of redundant coverage is indicated by the color. The coverage is then evaluated by looking for the intervals between regions of a particular color. TURBO Pascal has a function called "GetPixel," which can return the position and color value of a point on the screen. By moving along the orbit lines at a slope of ω_S/ω_E , revisit

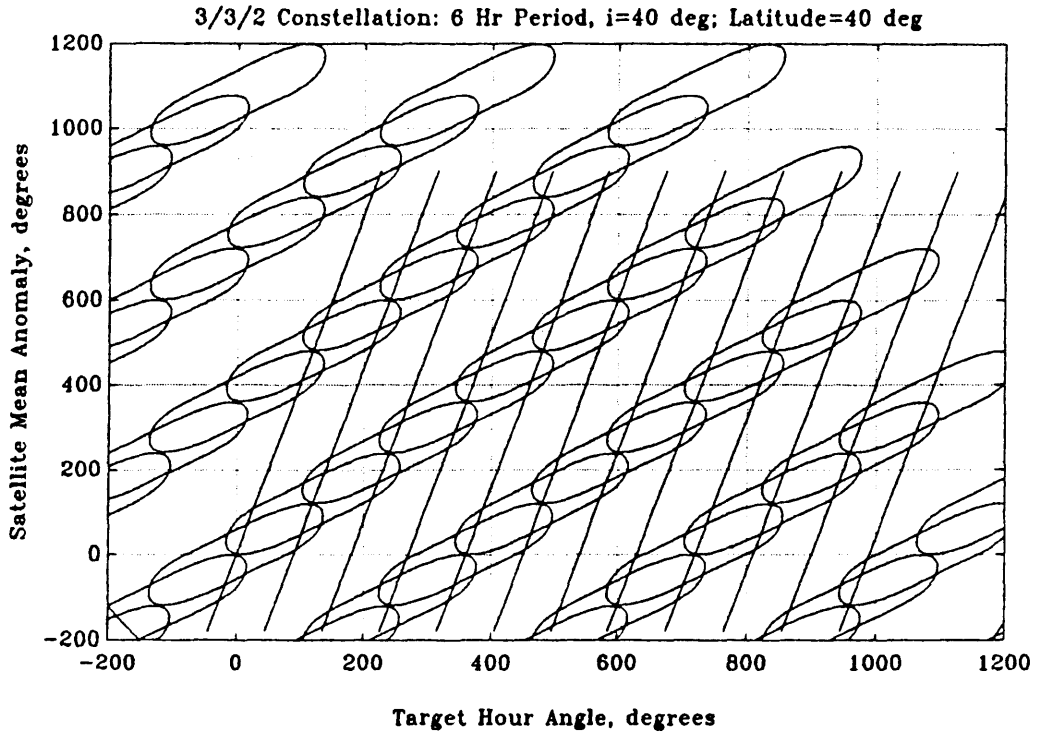


Figure 4.6: Visibility Region Pattern for Example of Figure 4.5

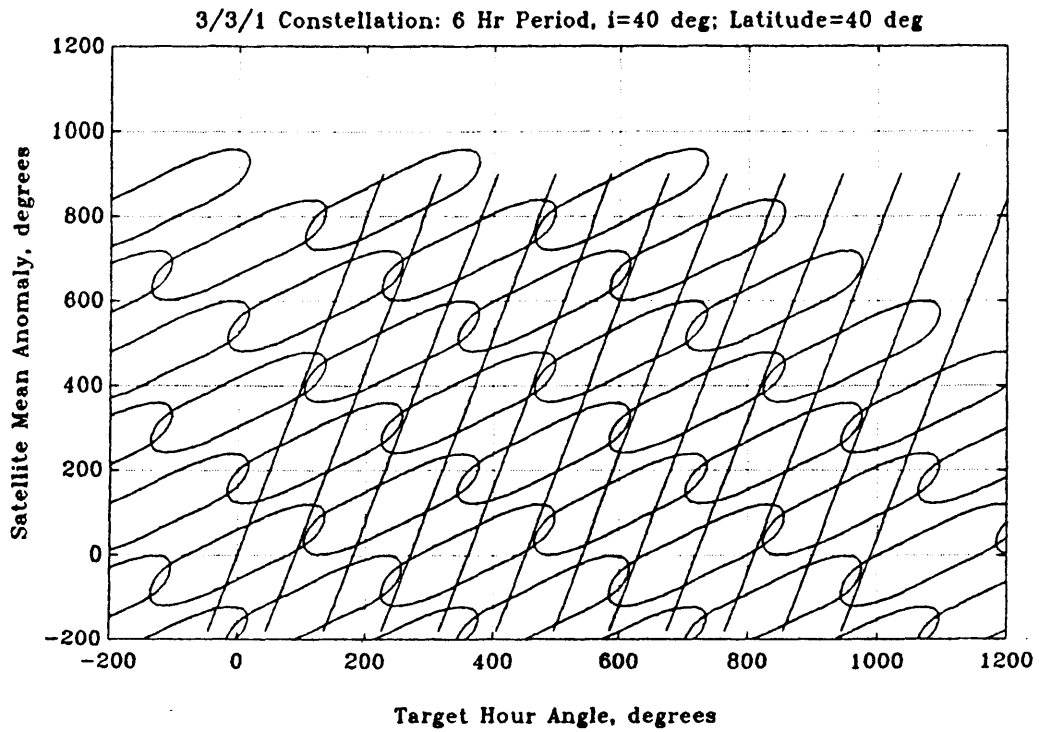


Figure 4.7: Effect of Inter-Plane Satellite Phasing on Coverage Pattern

intervals can be evaluated as they were for single satellite coverage in Chapter 3. The key point is that the computational demand is the same for evaluating revisit to n-fold coverage using this formulation, because all forms of contact between the constellation and the ground site, from none to n-fold, are recorded on the period square. Using a graphics screen as suggested here is a way of taking advantage of the computer hardware to store information for subsequent analysis.

The analog of the idea of placing visibility regions in various locations on the period square is to place one at the origin, and test whether the satellites view the target by checking if their phased locations on the period square fall within the visibility region. This method may present a more intuitive formulation, since the resulting figure will be simpler, and the computer graphics techniques suggested above may not be possible with available hardware. There is a slight sacrifice in speed using this approach, however, since the algorithm must perform one operation for each satellite in the constellation as the program searches the required interval in mean anomaly. It should not be a significant difference for constellations of fewer than about a dozen satellites. This method was used in the supporting software for this thesis, because of limitations on the graphics screen.

4.5.3 Using Non-Circular Orbits

Elliptical orbits can provide better coverage of specific regions of the earth, since a satellite will spend more time viewing a particular region as it approaches apocenter. However, elliptical orbits will experience a precession of periapsis due to earth oblateness, which would make the coverage of particular latitudes inconsistent over time. Also, the main emphasis of this study is on low-earth orbits for terrestrial surveillance or communications, making only small eccentricity permissible. Therefore, the assumption of circular orbits will be maintained for constellation coverage studies and constellation design.

4.6 Chapter Summary and Conclusions

This chapter has discussed some common types of constellation arrangements, and the visibility region problem formulation for satellite constellation coverage. Symmetrically phased constellations, with satellites at a common altitude and inclination, are referred to as "Walker" or "Walker-like." If continuous coverage is desired, a Walker-like constellation that is co-rotating can provide optimal coverage. A counter-rotating constellation does not provide optimal continuous coverage, but may be optimal for intermittent coverage.

By phasing multiple visibility regions on the same period square according to their positions in the constellation, all forms of contact between the constellation and the ground site, from none to n -fold, are recorded. With this formulation, the coverage provided by the constellation at any level can be evaluated with the same computational demand. The revisit intervals are evaluated in the same way as for single coverage in Chapter 3. Use of a computer graphics screen is an ideal way to store the coverage information--by using different colors for areas of visibility region overlap, the level of redundant coverage can be recorded. This reduces a large amount of information to a single picture which can be used for efficient coverage analysis.

CHAPTER 5

Constellation Coverage Studies

5.1 Introduction

Chapter 4 developed the theory necessary to analyze a constellation's coverage properties using the phase-based approach of the previous chapters. This chapter will use these concepts to study example constellations and their coverage behavior, comparing the effects of design issues such as constellation phasing and orientation (co-rotating vs. counter-rotating) on coverage.

5.2 Revisit Coverage

The case of revisit coverage usually results from a limitation on the total number of satellites in the constellation, such that the region of interest cannot be viewed continuously. In this situation, the goal is to find the best revisit coverage that the constellation can provide with the limited number of satellites. Although Chapter 6 will discuss methods of constellation design, this chapter will study how the coverage can change with different types of constellations. To study revisit coverage using the methodology of this study, example constellations of 3 and 6 satellites will be used to demonstrate different coverage phenomena.

5.2.1 Software

The software that was written to support this chapter is a program called "RevMap," written in TURBO Pascal for the IBM PC. TURBO Pascal was used because of its good graphics capabilities and ease of use. The editor and compiler are much faster and easier to use than FORTRAN. This program is the same as was mentioned in Chapter 3--the difference here is that the program is being used to study constellations.

The program listing is included in Appendix B. Its basic approach is to use the visibility region concepts as described in previous chapters to perform coverage analysis for a constellation. As inputs, the program accepts the latitude limits of interest, the common altitude and inclination of the constellation, the target viewing horizon, and the number of satellites, number of planes, and interorbit phasing parameter for the constellation. It then cycles through the necessary longitude limits, as determined by the number of ascending nodes of the constellation, and evaluates maximum revisit time using visibility region concepts. A new visibility region is solved for at each latitude, and longitude change is a matter of horizontal shifting of the orbital lines. The graphics capabilities of TURBO Pascal are useful in drawing the visibility region and performing the analysis. A fundamental subroutine, called "InView," takes advantage of the graphics functions to determine if a point on the period square is within the visibility region and hence in view of the target. By phasing all satellites according to their positions in the constellation as points on the graphics screen, the program can tell if any satellite in the constellation is in view of the target at any instant of time. The process is very rapid, because once the visibility region is solved for, no further calculations are required, and the program can cycle through large regions of longitude and latitude quickly. The process is slowed somewhat for large constellations. It is still, however, an efficient way of attacking the problem, and the fact that a large constellation of, say, 36 satellites can be run on a PC is a clear advantage.

5.2.2 Example Problems

Several example problems were run using the software, to validate the method and study constellation behavior. Unless changed to point out altitude or inclination effects on coverage, the common orbital period used was 6 hours (an altitude of 5600 nm), and the common inclination was 60 degrees. The constellations were assumed

to be symmetric, with phasing and orientation (co- versus counter-rotation) as free parameters.

The examples will include a computation of the number of distinct ground traces that each constellation produces. The pattern repeat interval of the maximum revisit time function should be inversely related to the number of distinct ground traces. A constellation with T satellites and a 6-hour common orbital period could have as many as $4T$ distinct ground traces per day. If all satellites follow the same ground trace, there could be as few as 4. Reference [15] contains an algorithm which allows the number of distinct ground traces to be determined for a constellation, if the parameters T , P , F , and the ground trace repetition interval (discussed in Chapter 3) are known. The algorithm, developed by J. Walker, is summarized here: the number of repetitive ground traces for a constellation is

$$E_{L,M} = T/K \tag{1}$$

where T is the total number of satellites in the constellation, and K is determined as follows: If the orbital elements are such that the ground trace repeats after L orbits in M days, let

$$G = SL + FM \tag{2}$$

$$J = \text{GCF}[S,M] \tag{3}$$

where S is the number of satellites in each plane, and GCF means "greatest common factor." Then

$$K = \text{GCF}[G,PJ] \tag{4}$$

For the 3/3/2 example constellation from Chapter 4 (6 hr period), $L=4$, $M=1$, so $G = 1(4) + 2(1) = 6$, $J = \text{GCF}[1,1] = 1$, and $K = \text{GCF}[6,3] = 3$. So the number of distinct ground traces is $E_{L,M} = T/K = 3/3 = 1$.

As explained in Chapter 4, the number of ascending nodes for a given constellation determines the repeat interval of the maximum revisit time function, and hence limits the longitude search space. The longitude repeat interval is given by $(360/E) \times (\text{Orbital Period (hrs)}/24)$, where E is the number of distinct ground traces. Hanson and Lang (Ref [15]) noted the same pattern repeat phenomenon, although their maximum revisit time plots exhibit some aliasing due to the low resolution of their data. They computed maximum revisit time every 10 degrees in longitude and latitude. The data used for the plots presented here were computed every 1 degree in latitude and longitude, producing plots that have 100 times greater resolution. The difference is presumably due to the much greater computational efficiency of the method developed in this thesis.

5.2.2.1 Phasing Effects

In the first example, shown in Figure 5.1, the constellation consists of 3 satellites in 3 planes, with an interorbit phasing parameter of 1. With an orbital period of 6 hours, there will be 4 orbits until the 3 separate ground traces repeat, and hence 12 ascending nodes per day. So the repeat interval of the maximum revisit time function should be 30 degrees in longitude. Note that the maximum revisit time is a strong function of latitude, but changes little with longitude, except in the vicinity of the equator. The pattern repeats every 30 degrees in longitude, as expected.

The second example shows the effects of interorbit phasing on revisit time performance. Here the constellation is 3/3/2, with all other parameters the same as for the 3/3/1 constellation in the first example. This constellation has all the satellites following a common ground trace, with a total of 4 ascending nodes per day. Note that

the pattern repeats every 90 degrees in longitude as expected, and that the function is now a strong function of longitude as well as latitude. The change would seem to indicate that phasing a constellation so as to maximize the number of ascending nodes will minimize the dependence of the maximum revisit time function on longitude. It will be shown through further examples that this conclusion is valid. It should be noted, however, that the maximum revisit time may be larger in magnitude, on the average or in certain areas, and just because it is weakly related to longitude, this does not necessarily mean the coverage is "better."

Figures 5.3 and 5.4 show the maximum revisit time patterns for constellations of 6 satellites: 6/6/1 and 6/6/2 respectively. Again, the constellation with more ascending nodes has a much smoother dependency on longitude as it affects maximum revisit time. The larger number of satellites at the same altitude makes the revisit times smaller overall as expected. Note, however, that the 6/6/1 constellation has large (170 minute) revisit times near the equator, and which exceeds the worst-case for the 6/6/2 constellation. So although the 6/6/1 coverage is smoother, it has a greater worst-case magnitude than the 6/6/2 coverage.

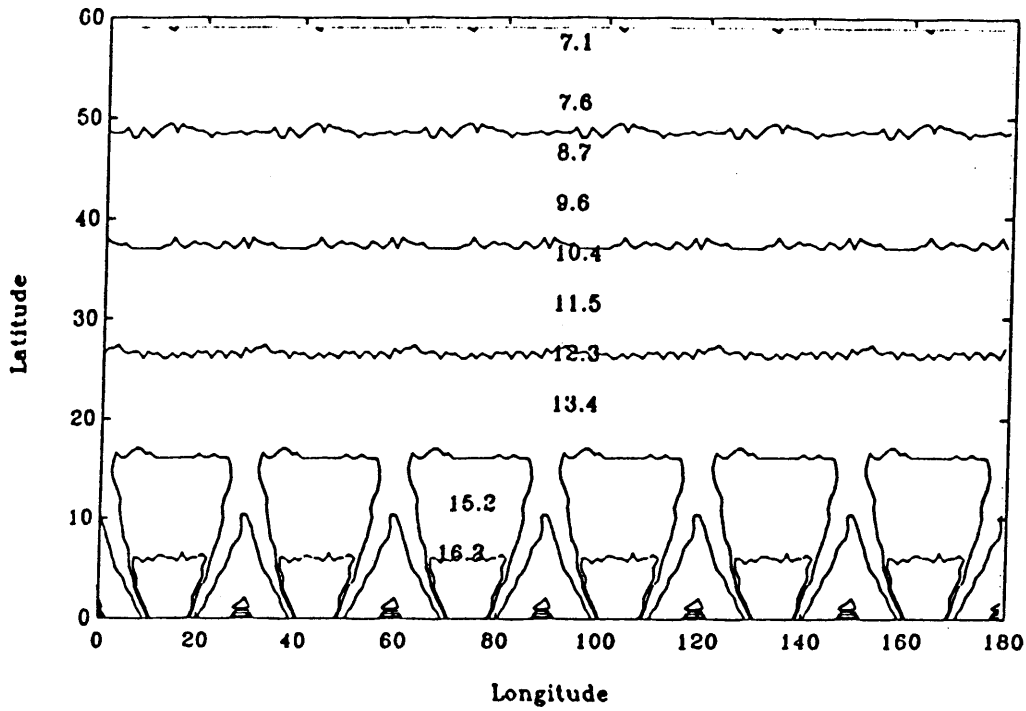


Figure 5.1: Contours of Maximum Revisit Time (units of 10 min). $T/P/F = 3/3/1$, 6 hour period, $i=60$ deg, Viewing Horizon=10 deg. 12 Ascending nodes/day.

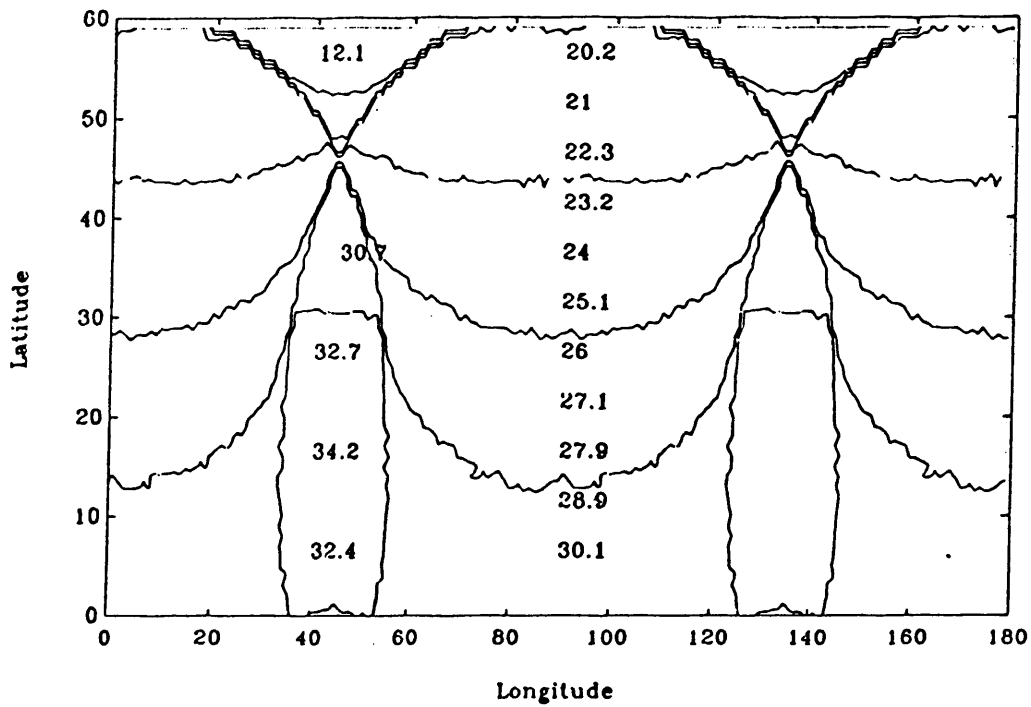


Figure 5.2: Contours of Maximum Revisit Time (units of 10 min). $T/P/F = 3/3/2$, 6 hour period, $i=60$ deg, Viewing Horizon=10 deg. 4 Ascending nodes/day.

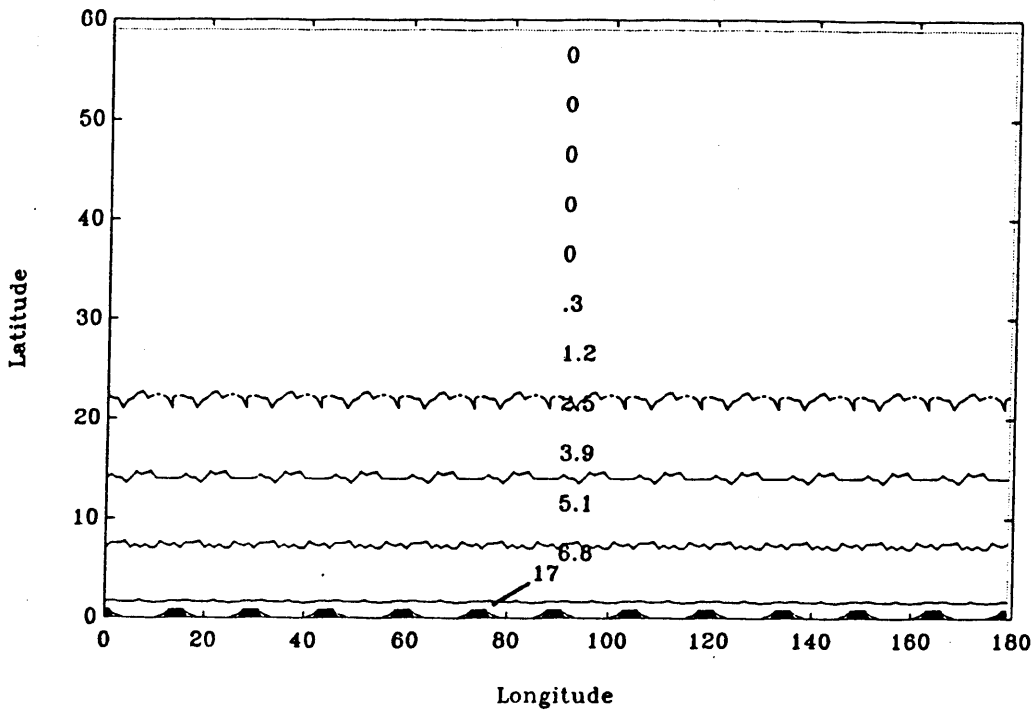


Figure 5.3: Contours of Maximum Revisit Time (units of 10 min). T/P/F = 6/6/1, 6 hour period, $i=60$ deg, Viewing Horizon=10 deg. 24 Ascending nodes/day.

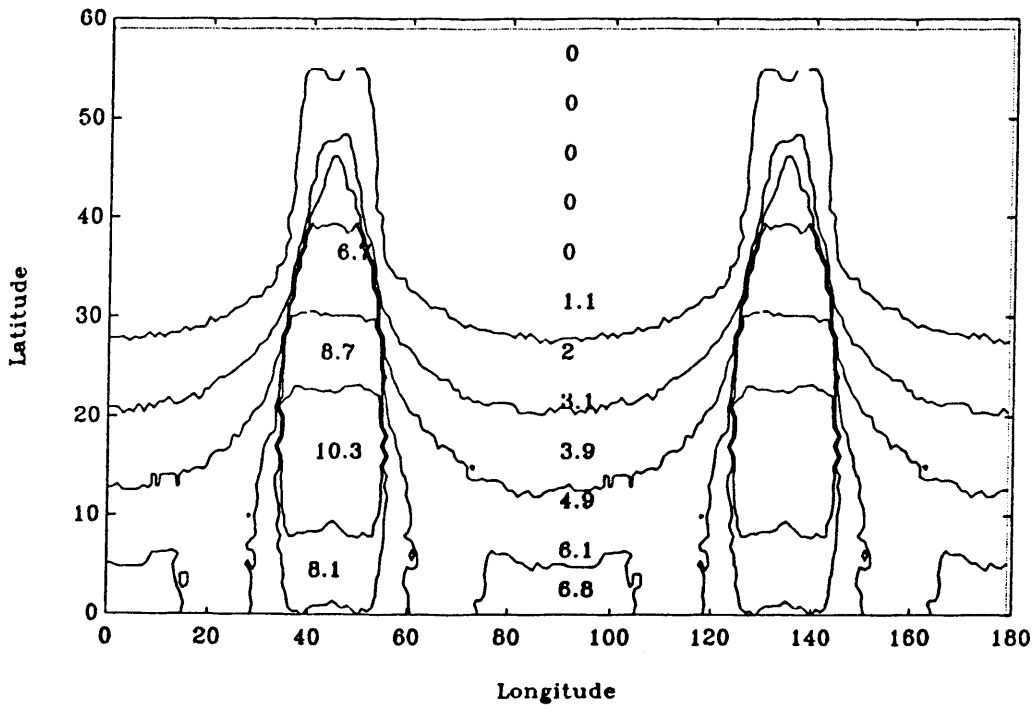


Figure 5.4: Contours of Maximum Revisit Time (units of 10 min). T/P/F = 6/6/2, 6 hour period, $i=60$ deg, Viewing Horizon=10 deg. 4 Ascending nodes/day.

5.2.2.2 Orbital Element Effects

From the examples of the last section, the altitude and inclination will be changed in order to verify intuition and assess the impact of changing these parameters on coverage. First, the orbital period will be reduced to lower the altitude of the constellation. Periods of 4 hours (3470 nm altitude) and 2 hours (910 nm altitude) are used for the 3/3/1 constellation of the last section, to produce Figures 5.5 and 5.6. Using the original 6-hour period, the inclination will be changed to 90 degrees and 30 degrees for Figures 5.7 and 5.8.

The results are as expected--note in Figures 5.5 and 5.6 how the revisit times increase overall with a shorter orbital period. With all other parameters kept the same, the revisit times should increase as the visibility region shrinks with a decrease in altitude. Figure 5.7 shows how the higher latitudes have shorter revisit times with high orbital inclination (90 degrees), and the equator has much higher revisit times. This is typical of a polar constellation, since the most difficult region for a polar constellation to cover is the equator. Conversely, the lower inclination used for Figure 5.8 shows the lower latitudes covered more efficiently. This constellation, in fact, views the equatorial region continuously.

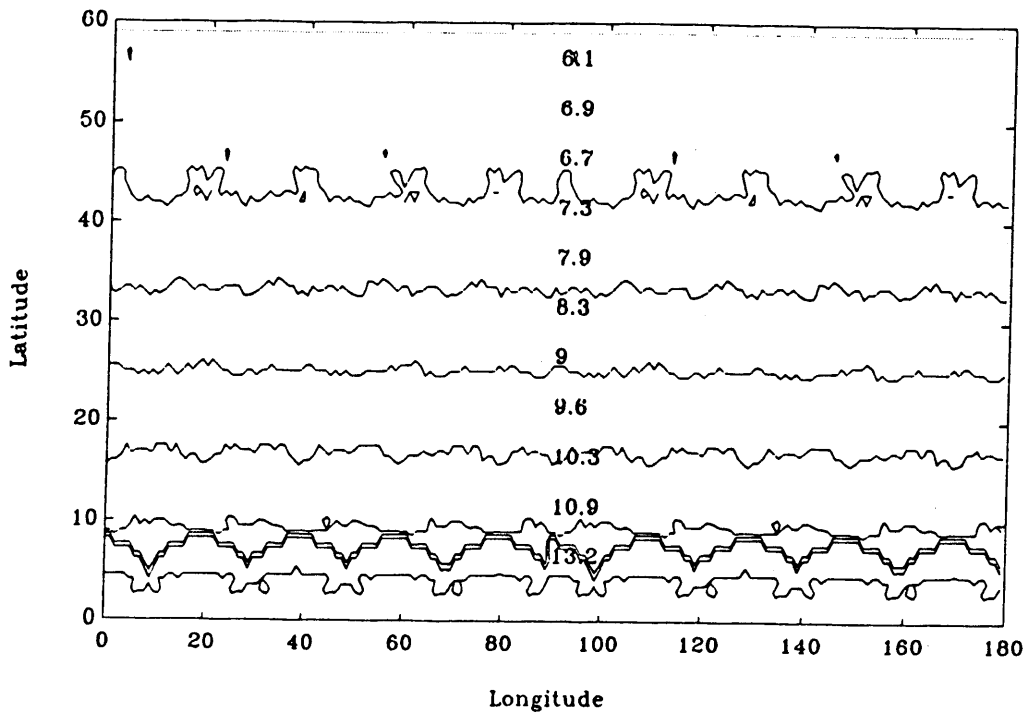


Figure 5.5: Contours of Maximum Revisit Time (units of 10 min). T/P/F = 3/3/1, 4 hour period, i=60 deg, Viewing Horizon=10 deg.

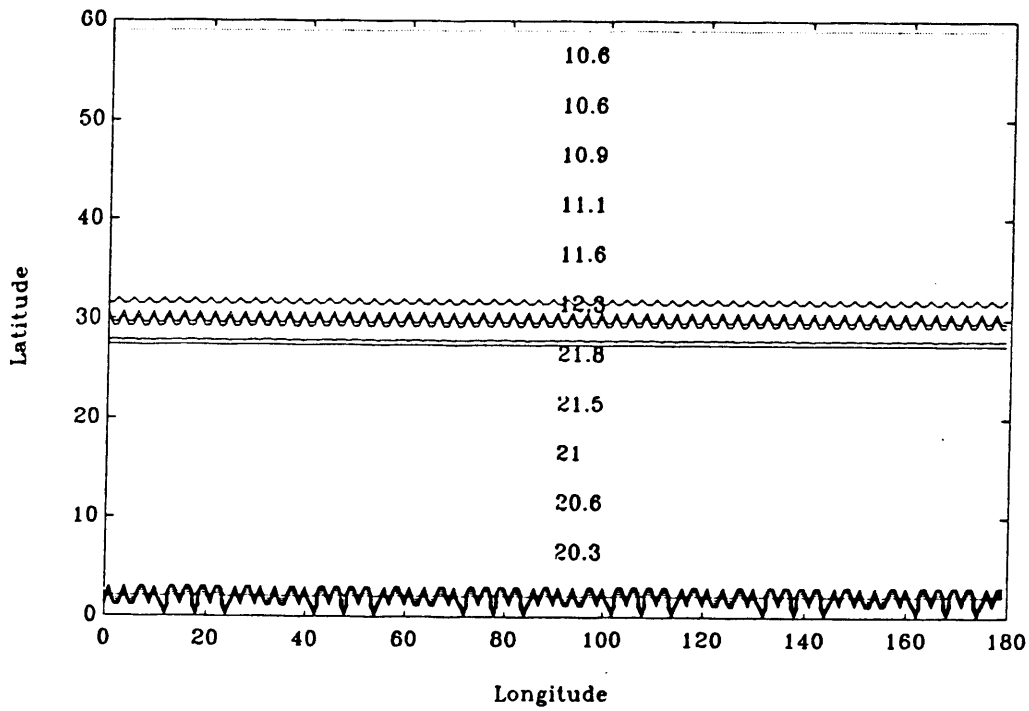


Figure 5.6: Contours of Maximum Revisit Time (units of 10 min). T/P/F = 3/3/1, 2 hour period, i=60 deg, Viewing Horizon=10 deg.

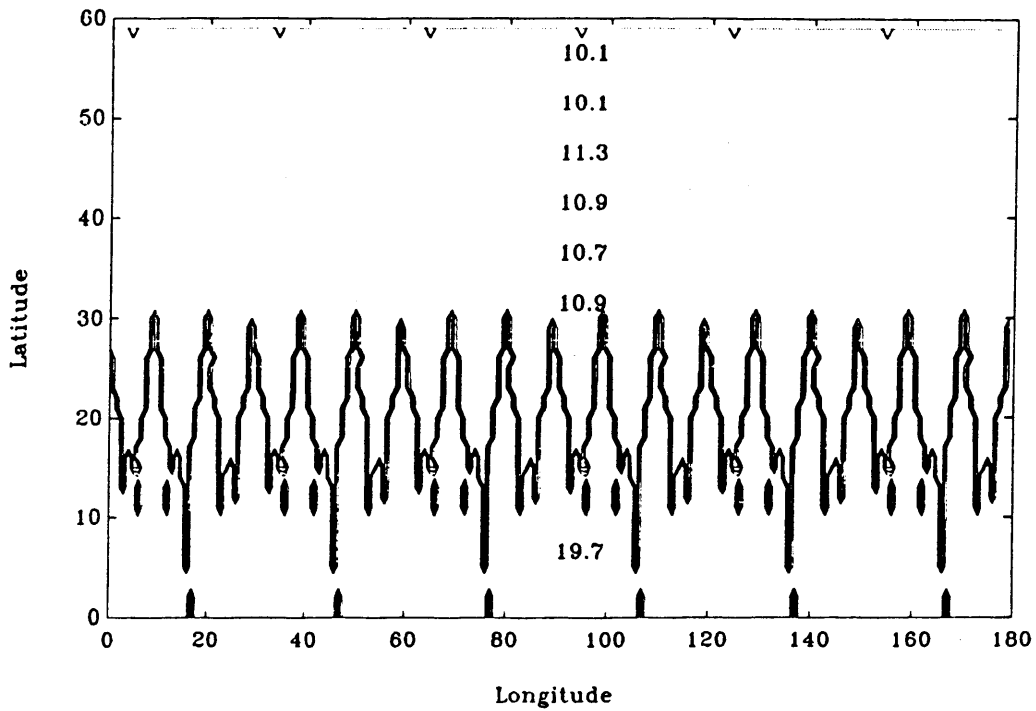


Figure 5.7: Contours of Maximum Revisit Time (units of 10 min). T/P/F = 3/3/1, 6 hour period, $i=90$ deg, Viewing Horizon=10 deg.

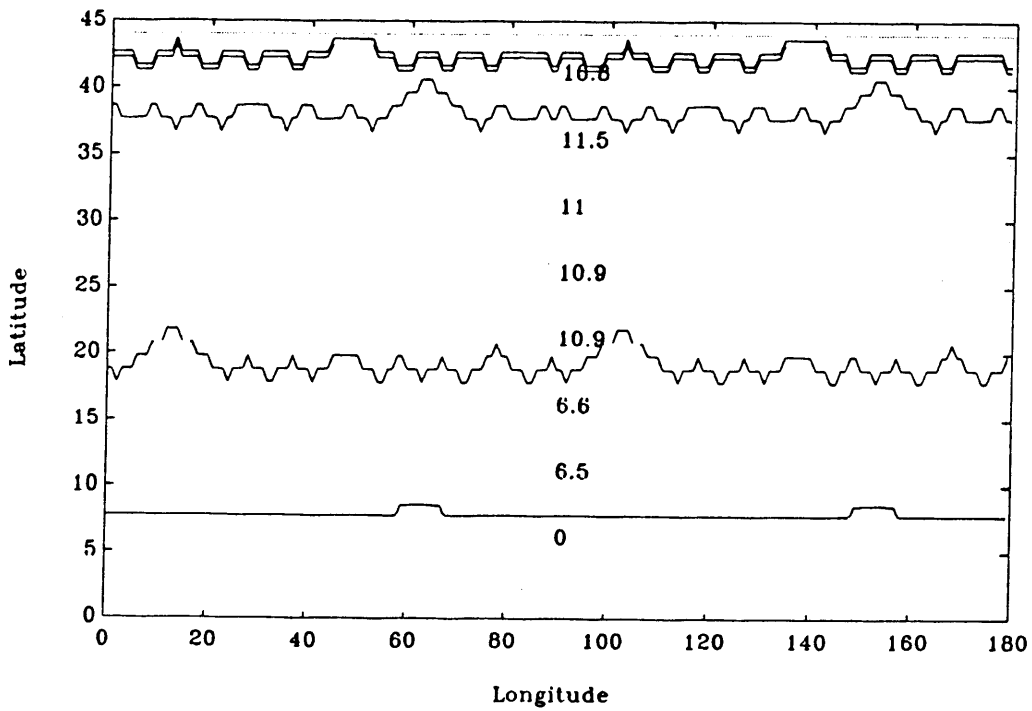


Figure 5.8: Contours of Maximum Revisit Time (units of 10 min). T/P/F = 3/3/1, 6 hour period, $i=30$ deg, Viewing Horizon=10 deg.

5.2.2.3 Orientation Effects

The examples of Figures 5.1 to 5.4 dealt with counter-rotating constellations, where the orbital planes were phased symmetrically in ascending node through 360 degrees. The program RevMap is easily modified to handle co-rotating constellations in order to analyze the effect on revisit coverage. A co-rotating constellation will have all its ascending nodes phased through 180 degrees in longitude.

Figure 5.9 shows the results for a 3/3/1 co-rotating constellation. The maximum revisit times are correlated to both longitude and latitude, and the overall magnitude of the worst-case revisits are greater than for the 3/3/1 counter-rotating constellation. Note that the dependency on latitude in Figure 5.9 is not as strong as for the counter-rotating constellation. For the 3/3/2 constellation, the co-rotating orientation in Figure 5.10 does not show much improvement over the counter-rotating version--there is strong revisit interval dependency on longitude and latitude for both designs, with roughly the same overall magnitude of maximum revisit time.

The coverage of the six-satellite constellation did not improve by using a co-rotating orientation. Figures 5.11 and 5.12 show the coverage for the 6/6/1 and 6/6/2 constellations, with both showing latitude and longitude dependency. Whereas the counter-rotating constellations showed large regions of consistent continuous coverage, the co-rotating versions do not exhibit any continuous coverage. Note, however, that the very large revisit times near the equator for the 6/6/1 counter-rotating constellation are not evident in the co-rotating version.

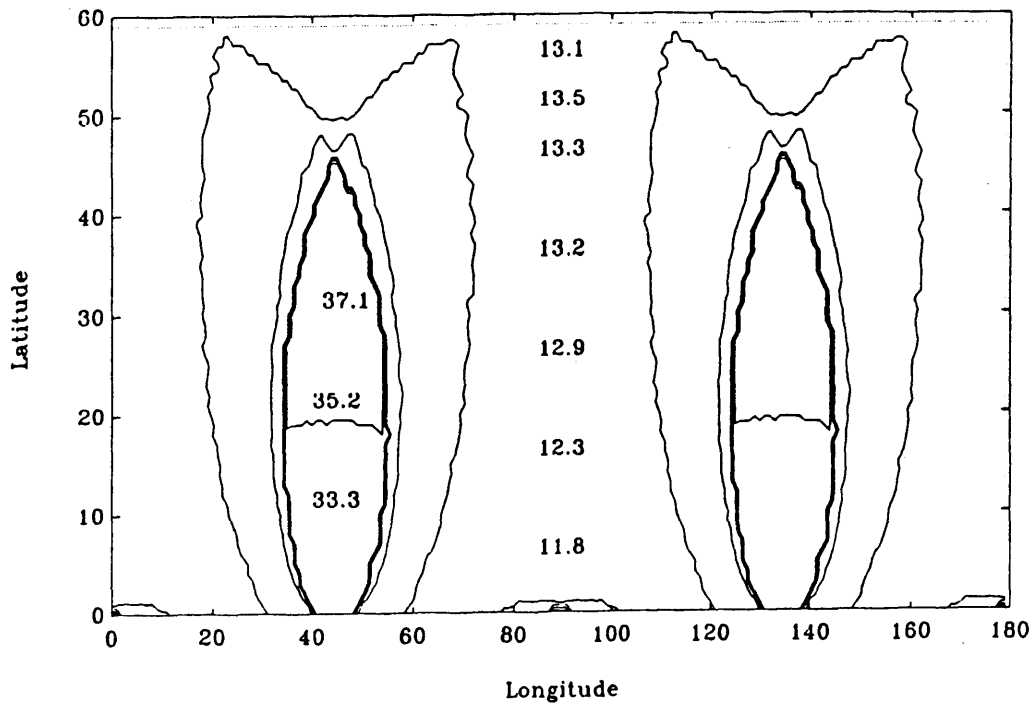


Figure 5.9: Contours of Maximum Revisit Time (units of 10 min). $T/P/F = 3/3/1$, (Co-Rotating), 6 hour period, $i=60$ deg, Viewing Horizon=10 deg.

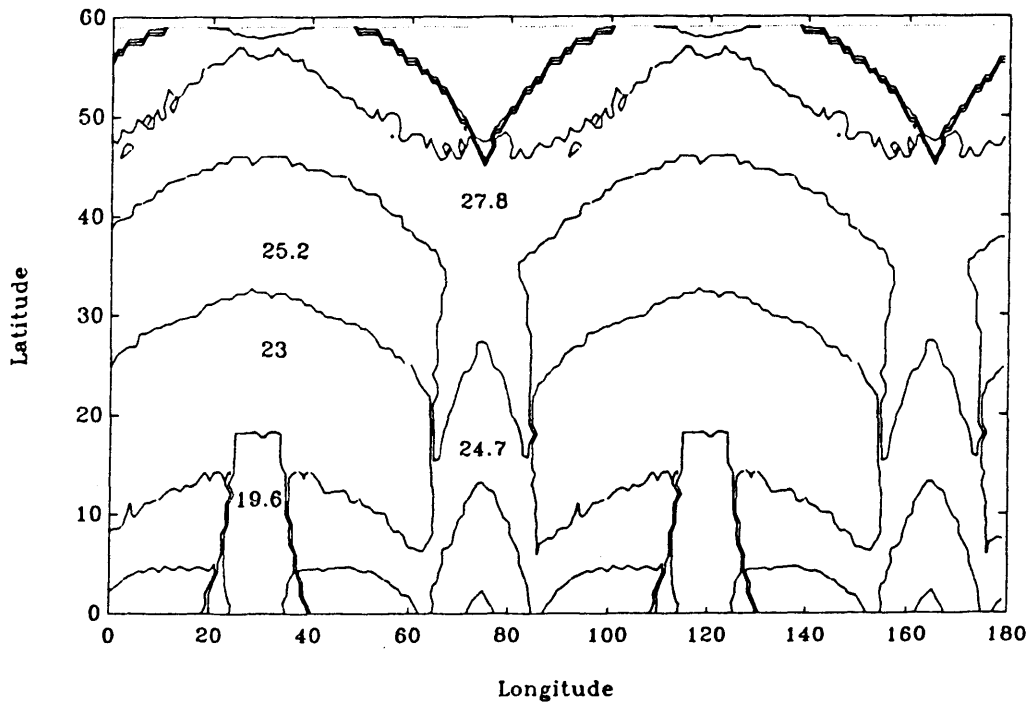


Figure 5.10: Contours of Maximum Revisit Time (units of 10 min). $T/P/F = 3/3/2$, (Co-Rotating), 6 hour period, $i=60$ deg, Viewing Horizon=10 deg.

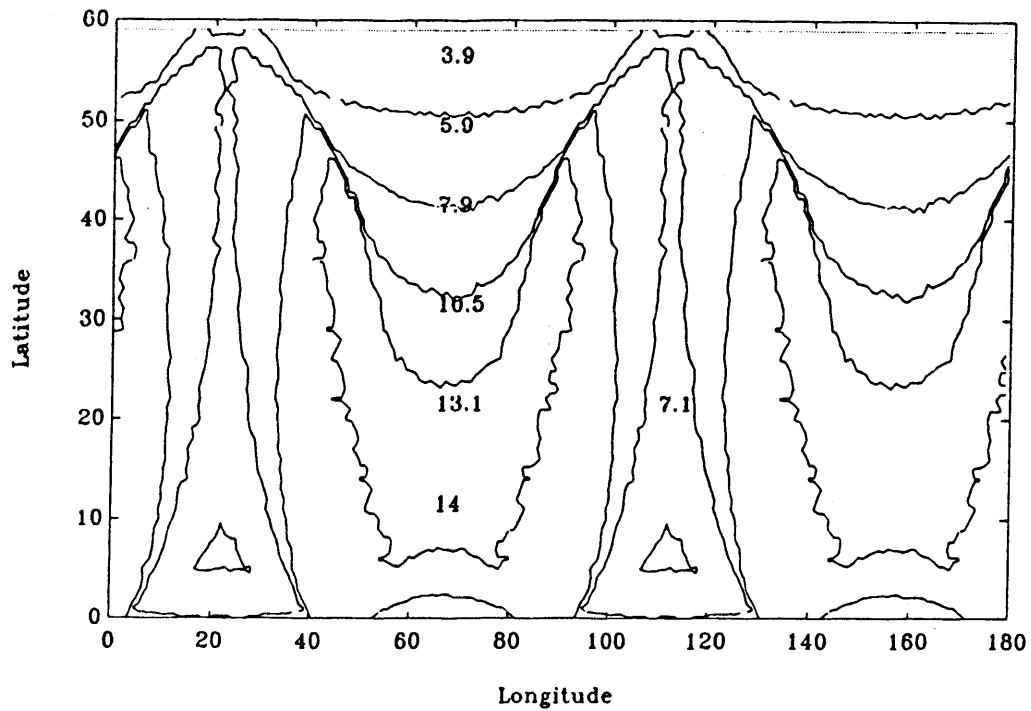


Figure 5.11: Contours of Maximum Revisit Time (units of 10 min). T/P/F = 6/6/1, (Co-Rotating), 6 hour period, $i=60$ deg, Viewing Horizon=10 deg.

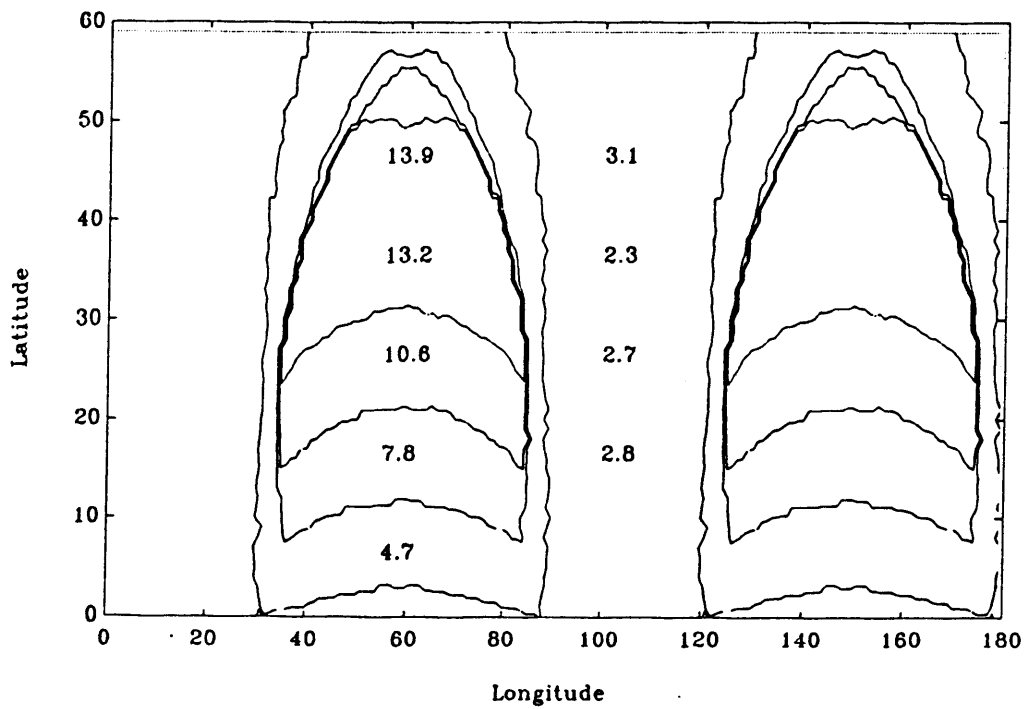


Figure 5.12: Contours of Maximum Revisit Time (units of 10 min). T/P/F = 6/6/2, (Co-Rotating), 6 hour period, $i=60$ deg, Viewing Horizon=10 deg.

5.3 Properties of the Maximum Revisit Time Function for Constellations

The preceding examples have shown maximum revisit time behavior for various test cases, to show how the coverage changes with different types of constellations.

The figures show that the pattern of revisit time contours repeats itself as a function of the number of separate longitudes of ascending node that occur. In general, the larger the number of longitudes of ascending node, the smaller the repeat interval will be, and therefore the maximum revisit time function will have less dependence on longitude. This conclusion is useful in constellation design, since it may be desirable to have as little revisit interval dependence on longitude as possible for a particular mission. The constellation should be phased such that the number of separate longitudes of ascending node per day is maximized. Hanson and Lang (Ref [11]) made an observation similar to this, stating that it would be reasonable to assume that maximum revisit time is a function only of latitude when the number of longitudes of ascending node per day for the constellation in question is greater than about 12 or 24. What Hanson and Lang concluded from observing data plots is confirmed through the methodology presented here.

Generally speaking, a counter-rotating constellation that provides smooth coverage at high latitudes will have large revisit intervals near the vicinity of the equator. A change to a co-rotating orientation does not appear to provide advantages--the revisit coverage tends to be irregular and of roughly the same magnitude. Although a co-rotating design is better for continuous coverage applications because it eliminates revisit intervals, it does not improve revisit intervals if they already exist.

Usually, the phasing parameter F that produces the best coverage (lowest revisit times) is small--perhaps 1 or 2. Although higher phasing parameters were experimented with during the research for this thesis, the lower phasing numbers

were always better than higher numbers. For example, a 10/5/2 would be better than a 10/5/5.

5.4 Chapter Summary and Conclusions

This chapter has shown how the visibility region methodology of this thesis can be automated in software and used to perform constellation coverage analysis. The example problems of 3 and 6-satellite constellations were used to observe the effects of different constellation designs on the resulting coverage.

The coverage that a constellation provides can be greatly influenced by the interorbit phasing of satellites. While altitude, inclination, and viewing horizon are intuitive effects on coverage, the interorbit phasing can have strong effects that are not as obvious. Co-rotating constellations, while superior for continuous coverage applications, are not necessarily better than counter-rotating constellations for intermittent coverage.

CHAPTER 6

Constellation Design

6.1 Introduction

Chapter 5 studied the revisit behavior of constellations as it is affected by constellation design. Software that implements the phase-based approach of this thesis was written, validated, and used to perform case studies. It showed that constellation coverage is affected in intuitive ways through adjusting orbital elements, and somewhat non-intuitive ways by changing phasing parameters and using co-rotating constellations. This chapter will develop a technique for designing constellations based on the methods and tools for evaluating coverage from the previous chapters.

6.2 Design Issues

A constellation whose mission is to provide intermittent coverage as efficiently as possible will be placed at an altitude low enough to allow communication and/or sensor operation, and high enough to allow each individual satellite to cover as large an area of the earth as possible. The altitude, then, will be selected as the highest altitude that still allows communications and sensor operations. This parameter will be considered in this analysis to be fixed.

The inclination of the constellation is generally the orbital element that is used as an optimization parameter. The inclination is adjusted, keeping all other parameters and constellation arrangement fixed, so as to achieve the "best" coverage over the region of interest that the constellation can provide.

Preceding chapters have shown that it is desirable to have a maximum number of separate longitudes of ascending node per day for a given constellation, in order to minimize the dependency of the maximum revisit time function on target

longitude. If this approach is taken, the algorithm should seek the constellation arrangement that maximizes the number of ascending nodes. However, it has been shown through examples in Chapter 5 that the average magnitude of maximum revisit time may be unacceptably large in the region of interest (say, the equator), and therefore the constellation may not be satisfactory for a particular mission.

As for the number of satellites, the goal is clearly to reduce to total number, in order to minimize cost. This analysis will assume, however, that for each design the total number of satellites is fixed, and the goal is to obtain the best possible coverage with the available satellites. The designer can then attempt to reduce the total number of satellites, to see if the mission requirements can still be met with an optimal arrangement of fewer satellites. The constellations are assumed to be symmetric (can be described by the parameters T/P/F), and the satellites are at common altitude and inclination in circular orbits. This assumption will limit the search space for each constellation.

6.3 Design Methods

6.3.1 Approach

Symmetric Walker constellations will be the candidates for the constellations in this chapter. As discussed in Chapter 4, considerations such as constant nodal precession for all satellites and consistent coverage over changing longitude make symmetric constellations attractive, and they are common in the literature. Furthermore, the search space is made discrete by using the Walker phasing parameters--instead of a continuum of interorbit phasing, there are limited choices. This simplification greatly reduces the search space.

The approach taken here is to define a cost function that will be evaluated for all ground sites of interest. The optimization algorithm will try to minimize this cost function by adjusting orbital inclination for each constellation arrangement, i.e., each

possible combination of T/P/F. The best constellation out of all the possibilities will have the lowest cost. The cost function is as follows: for N targets, the cost J is

$$J = \sum_{k=1}^N \{ \xi(k) + \alpha |\xi(k)| \} \quad (1)$$

where

$$\xi(k) = \text{AMR}(k) - \text{DMR}(k) \quad (2)$$

and AMR(k) and DMR(k) are the actual and desired maximum revisit times at target k. The weighting factor α determines the relative cost of a target which has lower than the desired maximum revisit time. This can be visualized from Figure 6.1. This cost function is similar to one posed in Reference 11, but is more general here through the use of the weighting parameter α .

As the figure indicates, the parameter α makes the targets where the maximum revisit time is greater than desired more heavily weighted in the cost function. This has the effect in the optimization of first driving the constellation to reduce the maximum revisit times that are above the desired limit, and then driving the revisit times below the desired limit. With an α of 1, the targets where the maximum revisit time is already below the desired limit are not weighted at all in the cost function. In this case, the policy is to drive all maximum revisit times to the desired limit, and there is no benefit to driving below the limit. An α of slightly less than 1 would probably be best, in order to place some cost on all targets.

As an initial "guess" for inclination, the algorithm will look for the lowest inclination where the highest target latitude is still in view, and use an interval-halving technique between this inclination and 90 degrees inclination, while searching for optimal inclination according to the lowest cost. This will ensure that the constellation is not inclined too high initially relative to the lowest latitude.

Depending upon the range of latitude of interest, the constellation may have to be inclined higher in order to cover the high latitudes.

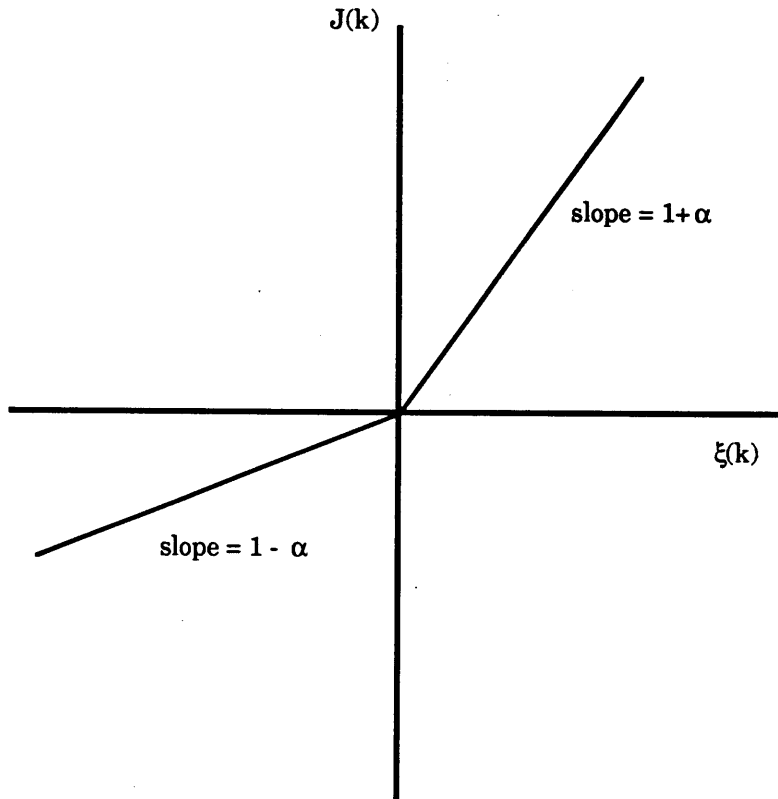


Figure 6.1: Effect of Weighting Parameter α on Cost Function

6.3.2 Software

The computer program written to perform constellation design implements the optimization algorithm described in the last section. It was written in TURBO Pascal for the IBM PC, in order to use some of the same subroutines as for the constellation coverage program written for Chapter 5. A listing is included in Appendix B. As inputs, the program accepts common orbital altitude, total number of satellites to be used, a target viewing horizon, and a set of targets to be observed by the constellation.

Also, the desired maximum revisit time for all targets is asked for. The program will then cycle through all possible constellation arrangements--all possible numbers of planes and interorbit phasing parameters--and find the "optimal" inclination for each constellation. The "best" constellation is the one with the lowest total cost for the given set of targets.

To study the behavior of the cost function described here, example problems were run using software written for this chapter. The software implements the design algorithm by evaluating the cost function described earlier, so it is important to verify that the cost function is "well behaved."

In the first example, a single target was placed at 40 degrees latitude, with a desired maximum revisit time of 15 minutes and a viewing horizon of 10 degrees. The constellation was a 3/3/1, with a 1.6 hr period (approx. 800 nm altitude). The software was set to search through all inclinations from 20 to 80 degrees. Figure 6.2 shows the result: the cost function has a minimum at slightly above 40 degrees, which is what would be expected for viewing a target at 40 degrees latitude. Note that the cost function is always positive, meaning that the constellation never achieves the goal of less than 15 minutes maximum revisit time for the single target.

For the next example, a set of six targets spaced at 5-degree intervals from 25 to 50 deg north latitude was input. This band of latitude roughly represents the continental United States. The constellation used was a 6/2/1, with the satellites in two-hour orbits. A 10-degree viewing horizon was used, and runs were made for three different values of α , the weighting parameter explained in Figure 6.1. Figure 6.3 shows the resulting cost function behavior for this example, with the constellation inclination shown on the horizontal axis and the cost on the vertical axis. Note the interior minimum of the function, which is interesting in that the optimal inclination is not at an extreme boundary of the optimization variable (inclination). The minimum is the same for all values of α , which is as it should be, because α only

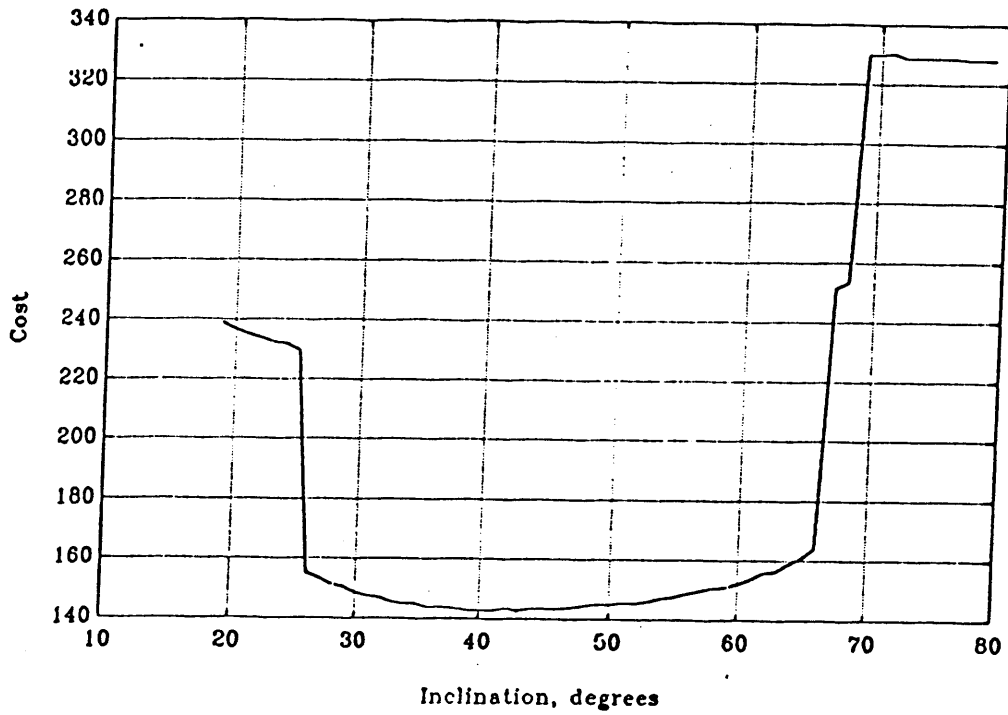


Figure 6.2: Cost Function Behavior for 3/3/1, 1.6 hr Period, 10 degree viewing horizon, 1 target at 40 deg Latitude, $\alpha=0.6$, DMR=15 minutes.

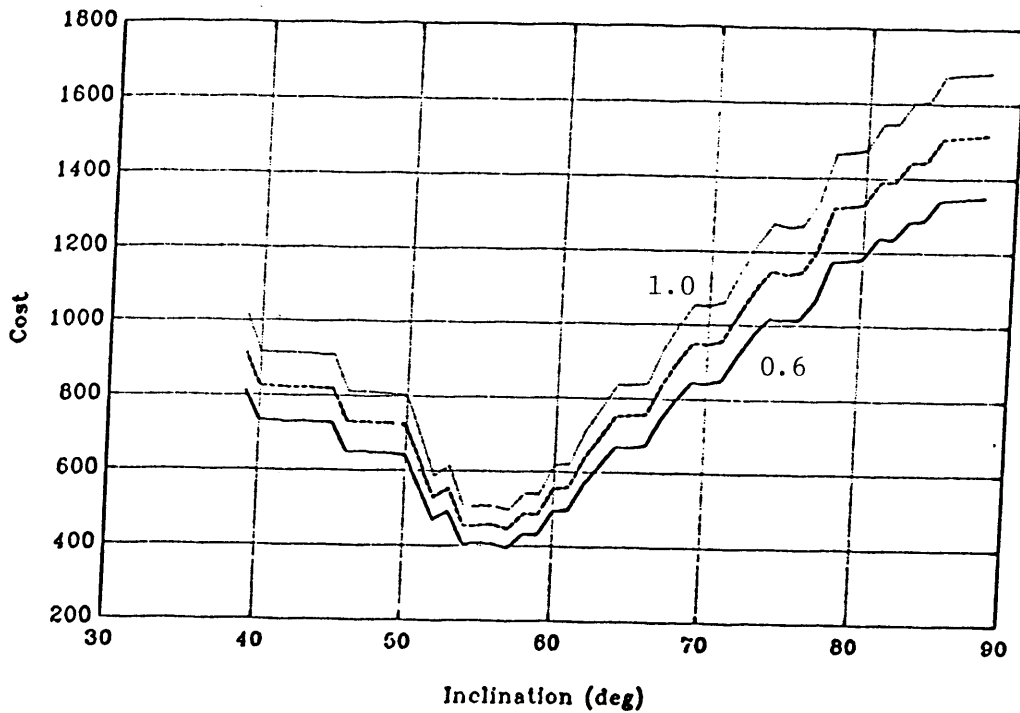


Figure 6.3: Cost Function Behavior for 6/2/1, 2 hr Period, 10 degree viewing horizon, 6 targets from 25-50 deg Latitude, $\alpha=0.6, 0.8, 1.0$, DMR=15 minutes.

changes how the optimal inclination is approached. As α approaches 1, the cost changes more rapidly with changing inclination. In Figure 6.3, as the inclination is increased beyond the optimum, the slope of the increasing cost is greater with larger α . This is because the larger α weights all targets with unsatisfactory revisit times more heavily, and the sum total increases at a faster rate. As in the first example, the cost function is again always positive, indicating that the constellation could not achieve the desired goal of less than 15 minutes maximum revisit time for the set of six targets.

For the final example, the 6/6/1 constellation from Figure 5.3 was used, since it provided a large area of continuous coverage when analyzed in Chapter 5. The satellites were in 6-hour orbits (5600 nm altitude). The set of five targets were spaced evenly in latitude from 30 to 50 degrees north, with 10 degree viewing horizon. As before, the desired maximum revisit time was 15 minutes. Figure 6.4 shows the resulting cost function behavior for inclinations from 30 to 70 degrees. Note that the cost function is negative (as would be expected) in the entire interval, meaning that the constellation is exceeding the goal with less than 15 minutes maximum revisit time for all five targets. From about 37 to 53 degrees inclination, the cost function bottoms out and becomes constant. This indicates that the constellation is providing continuous coverage of all five targets with any inclination in this interval. Since the constellation cannot do any better than continuous coverage, the cost function reaches an absolute minimum.

6.4 Example Design Problem

The following example design problem illustrates the constellation design process using the methods and software developed to this point. The following mission requirements are given: It is desired to observe the region of the earth from 28 to 38 degrees north latitude. The observation payload on the satellites will consist of

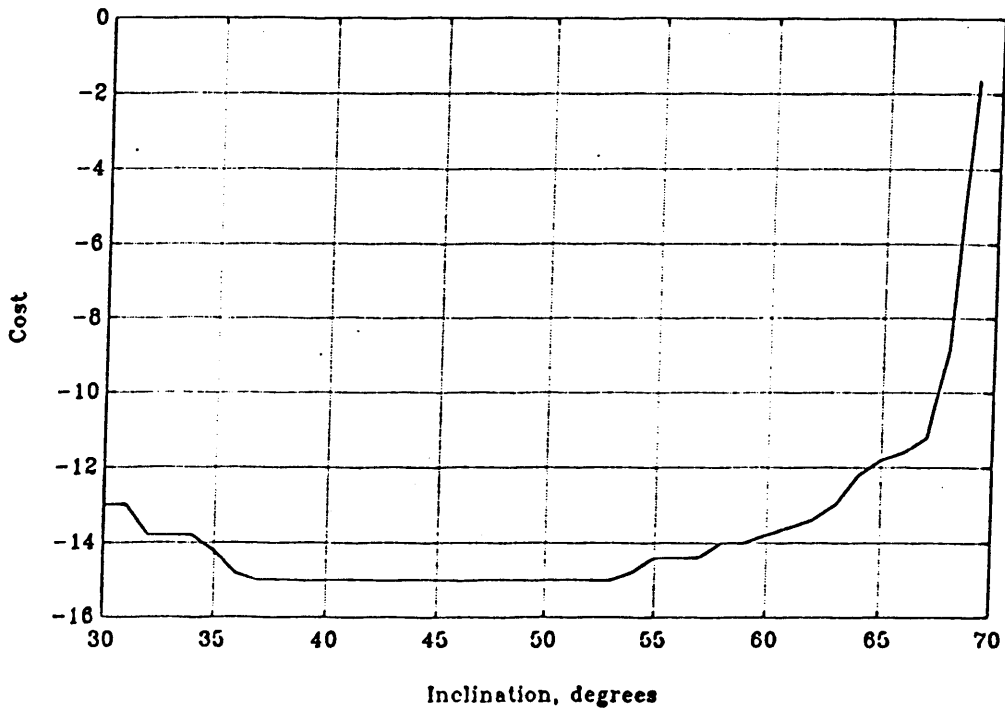


Figure 6.4: Cost Function Behavior for 6/6/1: 6 hr Period, 10 degree viewing horizon, 5 targets from 30-50 deg Latitude, $\alpha=0.8$, DMR=15 minutes.

cameras whose resolution restricts them to operation below 900 nm altitude, each with a telescope whose aperture requires that a satellite be 20 degrees above the local horizon before the ground site is viewed. The mission of this constellation will be to observe all points within the region of interest such that no point is out of view of the constellation for more than 20 minutes. Due to limited funding in an era of budget cuts, a maximum of 12 satellites are available to fulfill the mission.

Given these requirements, the constellation should be one that has little coverage dependency on longitude. The satellites will be placed at the highest allowable altitude, 900 nm.

As inputs to the computer program, targets will be placed at 28, 33, and 38 degrees latitude. This spread of targets should ensure satisfactory coverage of all latitudes of interest. The desired maximum revisit time is entered as 20 minutes, and the weighting parameter α discussed earlier is set to 0.8.

Although 12 satellites were available, the program was first run with 10 satellites. Surprisingly, the algorithm determined that a 10/10/7 constellation is capable of fulfilling the mission requirements at an inclination of 48 degrees. This constellation was unexpected after all prior attempts at a 10-satellite constellation (10/10/1 through 10/10/6, as well as 10/5/x constellations) had failed. The cost function for the 10/10/7 constellation just barely passes through zero at 48 degrees inclination. Figure 6.5 shows the cost function performance for the 10/10/7 and two other constellations that were attempted by the computer program. To verify the coverage of the chosen constellation, its coverage map is shown in Figure 6.6. Note that the constellation emphasizes only the latitudes of interest--where the maximum revisit times are below 20 minutes as required in the optimization routine.

The optimal (lowest cost) inclinations for the three constellations shown in Figure 6.5 vary greatly with changing phasing parameters and number of planes. This is typical of symmetric constellation design--the optimal inclination for a constellation is very dependent on the phasing parameter F , even when all other parameters are kept constant. A 10/10/1 constellation that is to observe a particular region may have a very different optimal inclination than a 10/10/2.

6.5 Chapter Summary and Conclusions

This chapter has shown how the methods developed in this thesis can be applied to solve constellation design problems, where the constellation is adjusted to provide optimal intermittent coverage of specified targets. The algorithm determines the best constellation arrangement for a given number of satellites to view a target or

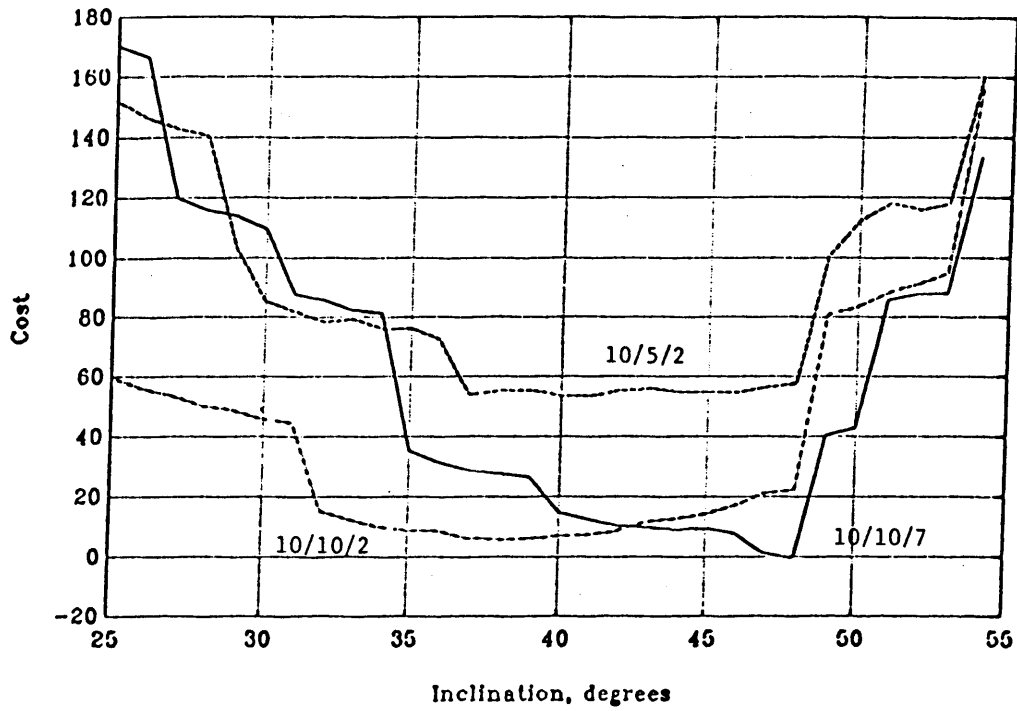


Figure 6.5: Cost Function Behavior for 10/10/7, 10/10/2, and 10/5/2, 900 nm altitude, 20 deg viewing horizon, 3 targets from 28-38 degrees Latitude, $\alpha = 1.0$, DMR = 20 minutes.

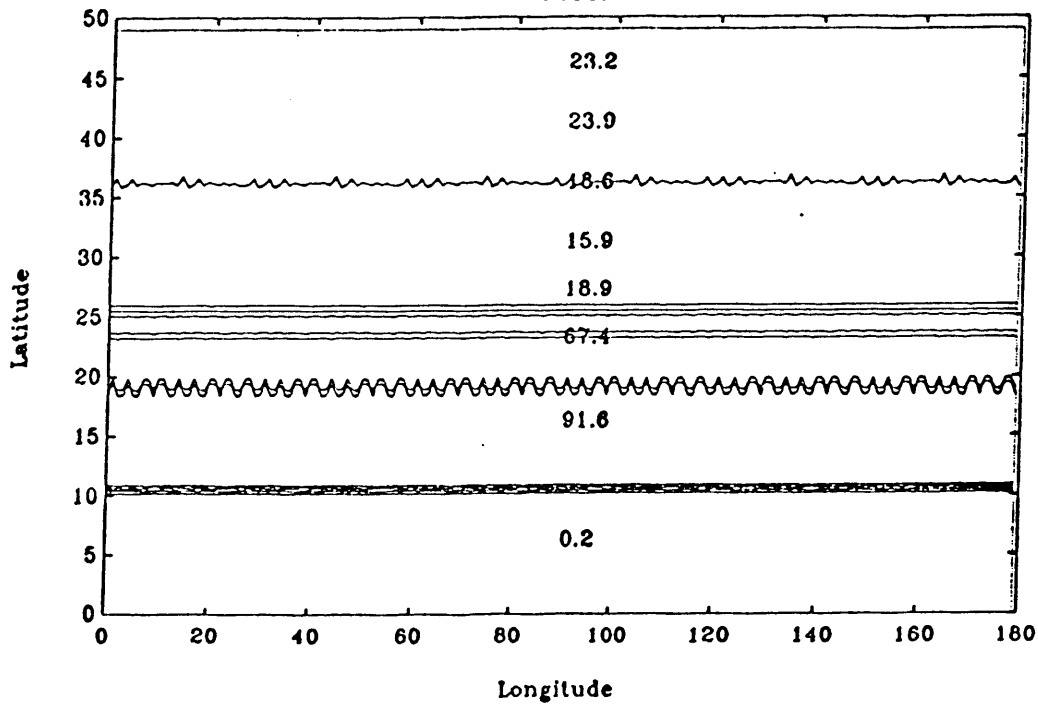


Figure 6.6: Contours of Maximum Revisit Time for 10/10/7 constellation from Figure 6.5. Inclination=48 degrees.

set of targets, while trying to meet a goal for the maximum revisit time at all targets. Example problems were presented to illustrate how the optimization behaves. A design problem was solved, using the software written for this thesis, on an IBM PC. Such problems have been solved in the past only through mainframe computer simulation runs or extensive manual analysis.

CHAPTER 7

The Iridium Constellation

7.1 Introduction

The preceding chapters have discussed constellation analysis and design. This chapter will apply those concepts to the "Iridium" constellation proposed recently (1990) by the Motorola Corporation. The intent is to investigate the suitability of the current constellation for the stated mission requirements, and to use the concepts of this thesis to study possible improvements to the constellation, such that the same mission might be accomplished at lower cost with fewer satellites.

7.2 The Iridium Constellation

7.2.1 Mission Concept

"Iridium" is the name given by the Motorola Corporation to its proposed worldwide satellite-based cellular phone system. The system will provide cellular telephone usage from any point on the earth, such that a user will be able to make and receive calls anywhere via portable battery-powered phone units. It is the inverse of a typical cellular phone system, where the transponders remain at fixed locations and the user moves physically through the system. In a typical system, the radio signal from the phone unit is handed-off from one transponder to another, depending on the strength of the signal being received from the user. The signal is then routed into the existing phone system, or is transmitted to another cellular phone. Iridium, on the other hand, is a constellation of orbiting transponders placed on small satellites, which provides continuous single-satellite accessibility from any point on the earth. The user remains in a relatively fixed position, while the satellites (and transponders) move. The source signal is first verified by a ground station as coming from an authorized user, and is then re-transmitted to the constellation. The signal is passed

from one satellite to another until it can be transmitted to stations on the ground to be routed into the existing terrestrial phone system. The advantages of such a system are obvious--the system can be accessed from any point on the earth, not just within the service area of a particular vendor. It is not intended to replace the existing telephone system (Ref [12]), but to complement the existing cellular phone systems, by extending the radio telephone coverage area to the entire world.

7.2.2 Constellation Requirements

The constellation must provide single-satellite continuous coverage of the entire earth. A viewing horizon of 10 degrees or more is required for contact between the satellite antenna and portable unit, because of the fixed-geometry beams employed by the satellites. The units will have sufficient power to penetrate some obstacles, but building penetration is not guaranteed by the system. Because of the fixed-geometry beams (which simplifies the antenna design), it is important to keep the coverage provided by each satellite as equal in area as possible. Also, it is desirable to have the satellites in low-earth orbits, to keep the power required for the portable units as low as possible. The last two considerations imply that low, circular orbits are desirable. On the other hand, the satellites should not be too low, because the station-keeping fuel requirements would be higher due to increased atmospheric drag.

With these considerations in mind, a constellation was chosen by a Motorola design team from a study done by Adams and Rider (Ref [1]). The Walker-like constellations in that study provide whole-earth coverage using circular, polar orbits, with arbitrary interorbit phasing. The constellation selected by Motorola employs 77 satellites, with 11 satellites in each of 7 orbital planes. The constellation is co-rotating, as described in Chapter 4. The first and last planes are separated slightly less than the others, in order to improve the coverage at the two counter-rotating

seams in the constellation. The common orbital altitude is 413 nm, which provides the desired 10 degree viewing horizon.

7.3 Proposed Improvements

It has been seen in earlier chapters that a constellation using inclined orbits will tend to provide the worst coverage at the equator. A highly inclined constellation will tend to provide the best coverage at the higher latitudes, and the polar constellations are an extreme case. They provide highly redundant coverage of the polar regions, and many satellites must be used in order to provide satisfactory coverage at the equator. The use of polar orbits for an application such as telephone communications is wasteful of satellites in the sense that about one-fourth of the satellites at any time are inside the polar circles (i.e., above 66 degrees latitude).

In the case of Iridium, a heavy price is paid in the number of satellites required, in order to cover the whole earth continuously while using polar orbits. Even if it were necessary to view the North and South poles with enormous redundancy, it is still not necessary to use polar orbits. An orbital inclination of roughly 79 degrees is sufficient for satellites at 413 nm altitude to see poles with a 10-degree viewing horizon. An inclination slightly above this would be enough to view the poles continuously, with 77 satellites sharing the viewing time. Clearly, then, polar orbits are not necessary, and with lower orbital inclination, there will be some overlap of coverage circles at the equator. This overlap could be viewed in one of two ways: either the number of satellites could potentially be reduced, or the number could be maintained, with the overlap in coverage providing a degree of "robustness" for the constellation. This last point is one that needs to be considered in any real-world constellation application. The 77 satellite constellation taken from the Adams and Rider study has been optimized to exactly provide continuous global coverage with for the specified altitude and viewing horizon. A slight change in altitude, or a higher

viewing horizon caused by local obstructions, could cause a gap in coverage and interrupt communication with the ground. This situation is similar to the 10/10/7 example constellation from Chapter 6--recall that the cost function barely passed through zero at a specific inclination. A slight change in inclination or other orbital or target parameters would disrupt the finely tuned "optimum" coverage performance. For Iridium, the use of inclined orbits would increase the amount of useful time per satellite, and improve the coverage somewhat, so as to require fewer satellites or at least offer some redundancy.

Another advantage would be to increase altitude in order to increase the coverage of each individual satellite. If an altitude of 490 nm were used, then the viewing horizon would be raised to 13 degrees (Ref [12]). This means that continuous coverage would be maintained with one satellite above 13 degrees elevation for all points on the earth. On the other hand, this would mean fewer satellites would be required for the 10 degrees elevation angle requirement. Reference [12] states that altitudes of up to 600 nm are permissible.

Given these considerations, it seems that the current Iridium constellation is not as efficient as it could be. At a projected cost of \$25 million per satellite, a constellation which provides equivalent coverage performance with fewer satellites would be quite a financial advantage, and would make maintenance of the constellation easier, with fewer satellites to maintain.

The computer program written to perform constellation optimization for Chapter 6 used inclination as the optimization parameter. The program is easily modified, however, to adjust other parameters for optimization. Since altitude for the Iridium constellation is a variable, it would be the best choice to attempt to reduce the number of required satellites by increasing altitude. Since there are almost limitless combinations of altitudes and choices for T/P/F other than the stated 77 satellite

constellation, this study will attempt only to propose a feasible alternative, with the goal of providing equivalent coverage with a reduced number of satellites.

The software was modified to adjust orbital altitude, starting from a low of 410 nm, and not exceeding 600 nm. Many constellations were attempted, by first observing the visibility region patterns as seen in Chapter 4. Eventually a 67/67/4 constellation was chosen as a candidate for the program, since it is a modest reduction in the number of satellites, and the 4 phasing provided a good pattern of visibility region coverage. The viewing horizon was kept at 10 degrees, and a set of six targets was spread from the equator to 75 degrees latitude. This approach places less emphasis on viewing the poles, which would be viewed anyway by a constellation inclined above roughly 79 degrees for the stated range of altitude.

The program arrived at a constellation whose cost function passes through zero at an altitude of 517 nm. Figure 7.1 shows how the cost function decreased rapidly as the constellation altitude was increased.

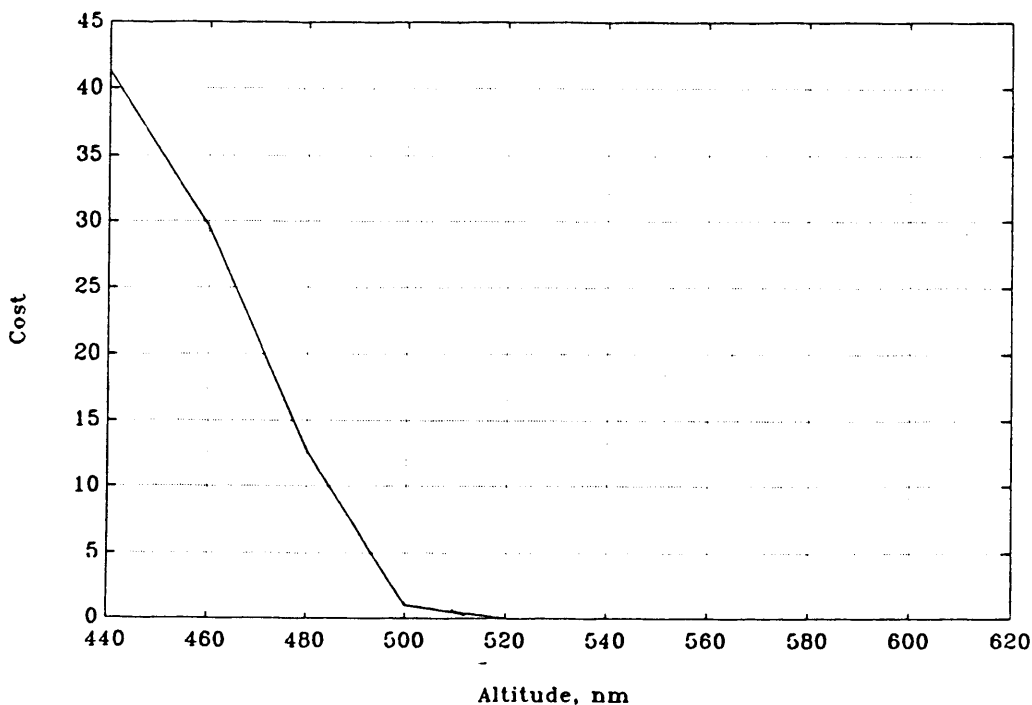


Figure 7.1: 67/67/4 Constellation Cost Function. Inclination = 79 degrees, Viewing Horizon = 10 degrees, 6 Targets from 0-75 degrees Latitude.

7.4 Chapter Summary and Conclusions

This chapter has presented the Iridium constellation for analysis and possible improvement. It was pointed out that the use of a polar constellation is wasteful of satellite coverage for a cellular phone system, because the satellites spend roughly one-fourth of the time within the arctic circles. Since the most difficult region for a polar constellation to cover is the equator, this drives up the number of required satellites for a low-altitude, polar constellation. By using a lower inclination, and increasing the constellation altitude, a possible alternative constellation was proposed using the optimization software developed for this thesis. The alternative, a 67/67/4, represented a 10 satellite savings over the original 77 satellite constellation. It should be pointed out that there are many viable alternative constellations, when inclination is a free parameter and an increase in altitude is allowed. Although previous chapters have mentioned the speed at which the phase-based approach allows constellation design to be performed, the run times for the very large constellations in this chapter were quite lengthy. Most runs for this chapter had to be accomplished overnight.

CHAPTER 8

Conclusion

8.1 Conclusions

To reiterate what was stated in the Introduction, the major conclusions reached in this thesis are (1) that the satellite constellation coverage problem can be formulated in a new way using a phase-based approach, (2) that this formulation can be exploited to provide a computationally efficient way of analyzing the coverage of a given constellation, to include intermittent (revisit) coverage and redundant coverage, and (3) that an automated constellation design algorithm can be developed using the new approach. Although the original intent of the study was to analyze the intermittent coverage (revisit) problem, the methodology that emerged was found to be equally applicable to continuous and redundant coverage problems. The Motorola case study of Chapter 7 is an example of a continuous coverage problem that was investigated using the methods developed in this thesis.

The intermittent coverage problem appears to be best approached using the phase-angle ideas of this thesis. The continuous coverage problem is probably best suited to the geometrical approaches discussed in Chapter 4, since a numerical method for evaluating coverage is not required. Where a numerical method is necessary, the phase-based approach for analyzing coverage provides qualitative and quantitative insight into the problem. The method is efficient enough that constellation design optimization can be performed on a PC.

The behavior of constellation revisit coverage has been investigated, in terms of worst-case revisit times, consistent revisit coverage across a span of longitude, and preferential revisit coverage in a particular latitude band. It has been shown how the longitude of a target is much less important than latitude, for constellations which have a large number of ascending nodes. The interorbit phasing of a constellation

has a significant impact on revisit coverage, insofar as it affects the number of ascending nodes. The co-rotating constellation, which is advantageous for continuous coverage applications, does not necessarily improve intermittent coverage for a constellation.

A constellation's intermittent coverage can be optimized to try to view a particular region of the earth with a specified upper bound on maximum revisit time. Choosing inclination as the optimization parameter reveals that an interior optimal choice for inclination exists, as opposed to an optimum at the limit of the parameter's range, as would be the case for altitude. The optimal choice for inclination can be affected by the interorbit phasing of the constellation.

All analytical work for this thesis was conducted on a Compaq Deskpro 386/25, which is a moderately fast IBM PC-compatible computer. These ideas of this thesis made it possible to conduct analyses that, prior to this work, were performed on mainframe computers. The run-times were, of course, dependent upon the size of the constellation of interest, but most runs for constellations of fewer than a dozen satellites lasted under an hour. A large revisit-time map or a large constellation optimization would have to be run overnight.

8.2 Recommendations for Further Research

The use of a symmetric constellation, although simplifying the search process in constellation design, may not always be the best choice for the most efficient constellation. The asymmetry of a constellation could be exploited to give preference to covering a particular region. For example, using a uneven ascending node spacing could have desirable effects on coverage. Another possibility would be to make the orbital planes mutually orthogonal to one another.

All the constellations addressed in this thesis were inclined relative to the equator, up to a maximum of 90 degrees. It would be interesting to study

constellations that are inclined relative to some plane other than the equator. For example, a constellation inclined at 90 degrees relative to a plane that passes through the north and south poles would have its point of intersection over the equator.

The cost function used for optimization in Chapter 6 is certainly not the only measure that could be used for improving coverage. A method which is driven by the average revisit time could be used. Also, the program is not restricted to optimizing in terms of maximum revisit time only. It could be modified to maximize the time spent viewing a target, for example.

Recently, some authors have proposed using Walker constellations with an equatorial orbit added to cover the large revisit gaps typically seen in the vicinity of the equator. These are sometimes referred to as "modified Walker constellations," and would seem to be promising for synthesizing efficient constellations. A typical Walker constellation could be optimized as in this thesis, and an equatorial orbit added to complete the coverage. This approach has been given little attention in the open literature, due to the difficulty of selecting an appropriate equatorial orbit. A computer optimization would seem ideally suited to this problem, using the phase-based methods developed in this thesis.

APPENDIX A

Description of Prior Research

Some of the significant authors who have done research relevant to the topic of this thesis are identified in this section, for anyone conducting a literature survey. This is not meant as a substitute for the References section.

Authors J. Hanson and T. Lang published in 1983 the only study that specifically addresses the intermittent coverage problem and proposes a cost-function method for dealing with it (see Ref [11]). This thesis starts with a very different approach, and arrives at some of the same conclusions. Any similar ideas in this thesis are cited with regard to Hanson & Lang. Hanson has published several studies (e.g., Ref [5], [6], and [11]) and is a well-known researcher in the field of constellation design.

E. Hayes of Lockheed published several AIAA papers (e.g., Ref [7]) which deal with revisit time in an average sense. Her approach is to use a computer simulation to identify all contact opportunities between a certain latitude and a satellite, and record these on a plot. These plots are visibility regions as described in this thesis, except with a different coordinate system and drawn point-by-point with a simulation. Her method of constellation design involves overlaying these plots by hand to try and achieve satisfactory constellation coverage.

Hanson, Evans, and Turner published a significant study (Ref [5]) during the course of the research for this thesis, in the Fall of 1990. They show some visibility region plots, which are found geometrically through identifying all ascending nodes which will result in viewing the site. They use the visibility region idea to select a reference longitude of ascending node for the constellation, and proceed into constellation design using other methods.

Anyone interested in the topic of satellite constellations should read the work of British author J. Walker of the Royal Aircraft Establishment (Ref [15]), who did much of the early conceptual groundwork in the 1960's and has published more than any other author to date. Every recent study into satellite constellation design will cite Walker as a reference. Many studies will seek to develop so-called "modified Walker constellations," using Walker's work as a starting point. Authors such as T. Lang (Ref [10]) will develop their own methods in order to make Walker constellations more efficient.

Finally, authors W. Adams, A. Ballard, and L. Rider deserve mention as well. Ballard's "rosette constellations" are frequently noted by other authors, and the Adams and Rider study cited as Reference 1 is a very sophisticated paper, used by the Motorola Corporation for choosing its "Iridium" constellation. Rider was the first to develop the "street of coverage" techniques described in Chapter 4.

APPENDIX B

Computer Program Listings

Since the methods developed in this thesis enable constellation analysis and design to be performed on a PC, listings of the software that was written to implement the methods are included. The first program was used to evaluate the coverage for a given constellation over a given region of the earth. The second was used for constellation design. Since the second program uses many routines that are in the first, they are edited out for brevity. All software was written in TURBO Pascal for the IBM PC. The computer used was a Compaq Deskpro 386/25, which at this writing is a moderately fast PC, but certainly not the fastest available.

The first program presented was used to generate the revisit time coverage maps seen in this thesis, mostly in Chapter 5.

Program RevMap;

```
{ This program was written to perform revisit interval
  coverage analysis for a constellation of satellites.
  It is written in TURBO Pascal for the IBM PC with EGA
  graphics capability. Given a satellite constellation's
  orbital parameters and arrangement, a viewing horizon
  specification, and a region of latitude, the program will
  calculate the maximum revisit time at every degree in
  latitude and longitude, and output the data to a disk file.
```

```
Author: Paul B. DiDomenico
Date: May 1991
```

```
C.S. Draper Laboratory, Cambridge, MA, 02139 }
```

Uses Crt, Graph;

Const

```
Grav : Real = 398600.8; { mu in km^3/sec^2 }
Re   : Real = 6378.135; { earth radius in km }
```

Type

```
Vec2 = Array[1..2] of Real;
TargetType = Record { Record of target parameters }
```

```

    Angle: Real; Lat: Real; Lon: Real;
  End;
CommonType = Record { Record of common constants }
  OmegaE: Real; TwoPi: Real; D2R: Real; R2D: Real;
  CenterX: Integer; CenterY: Integer;
  End;
ConType = Record { Record of constellation parameters }
  As      : Real;   Period  : Real;   Incl   : Real;
  OmegaS  : Real;   Phi     : Real;   OmegaN : Real;
  T       : Integer; P      : Integer; F      : Integer;
  Slope   : Real;   Satphase: Integer; Nodephase: Integer;
  Meanphase: Integer; SPP   : Integer;
  End;
LoopType = Record { Record of latitude/longitude limits }
  LatLow: Real; LatInt: Real; LatHigh: Real;
  LonLow: Real; LonInt: Real; LonHigh: Real;
  Alt   : Real; Incl  : Real; VH    : Real;
  End;

Var
  RevInt: Array[1..180,1..90] of Real;
  { a 180 by 90 matrix of revisit times }
  F : Text;
  { output data file }
  Y, DY : Vec2;
  { Y is a point on the boundary of the visibility region.
    DY is the orthogonal gradient at that point. }
  S, Line1 : String;
  X, Xc, Yc, ThetaE, ThetaS, P, Q : Real;
  { Xc and Yc locate the center of the visibility region.
    ThetaE and ThetaS are coordinates in earth rotation angle
    and satellite mean anomaly. }
  Gd, Gm, M, I, J, a, b, Orbits, Days, LatCount, LonCount : Integer;
  { Orbits is the number of orbits until ground trace repetition.
    Days is the number of days until ground trace repetition.
    LatCount and LonCount keep track of latitude and longitude. }
  Target : TargetType;
  { A target is defined by its latitude/longitude position and a
    local viewing horizon. }
  Common : CommonType;
  { Constants, Conversion Factors. }
  Constellation : ConType;
  { The orbital elements of the constellation, and the
    constellation design and phasing parameters. }
  LoopParm : LoopType;
  { Latitude and Longitude search limits. }

{ Several basic math functions must be defined in TURBO Pascal, as
follows... }

Function ArcSin(var x: Real):Real;
Var z : Real;
Begin
  z := x/(Sqrt(1.0-Sqr(x)));
  ArcSin := ArcTan(z);
  { Quadrant check is not necessary. }

```

```

End;

Function ArcCos(var x: Real):Real;
Begin
  ArcCos := Pi/2.0 - ArcSin(x);
End;

Function Tan(var x: Real):Real;
Begin
  Tan := Sin(x)/Cos(x);
  { Quadrant check is not necessary. }
End;

Function Cube(var x: Real):Real;
Begin
  Cube := x*x*x;
End;

Function Seventh(var x: Real):Real;
Begin
  Seventh := x*x*x*x*x*x*x;
End;

Function RealMod(x, y: Real):Real;
Begin
  RealMod := Frac(x/y)*y;
  { Returns the modulo remainder after dividing all multiples of y
  out of x. }
End;

Function Region(var Constellation: ConType; var Target:
TargetType):Integer;
Var
  Alt, Ang, Theta, High, Low, x: Real;

{ This routine determines if there exists a single, bifurcated, or
no visibility region, returning a result of 1, 2, or 0
respectively. }

Begin
  { Figure 2.9 in Chapter 2 illustrates the geometry that is solved here. }
  Alt := Constellation.As - Re;
  { Alt is the satellite altitude. }
  Ang := Pi/2.0 - Target.Angle;
  x := Cos(Ang)/(1.0 + Alt/Re);
  Theta := ArcCos(x) - Ang;
  High := Target.Lat + Theta;
  { The highest latitude the satellite can view. }
  Low := Target.Lat - Theta;
  { The lowest latitude the satellite can view. }
  If Constellation.Inclin < Low then
    Region := 0 { No visibility region }
  Else If Constellation.Inclin > High then
    Region := 2 { Bifurcated visibility region }
  Else
    Region := 1 { Single visibility region }
End; {End of Region Function }

```

```

Function CosBeta(var Constellation: ConType; var Target: TargetType;
                var ThetaE, ThetaS: Real):Real;
Var
  Qs, Qt : Array[1..3] of Real;
  C1, C2, C3, C4, C5, C6, Result, Norm : Real;
  I : Integer;

{ This routine computes the cosine of the complementary elevation angle to
a satellite from a given target. The vectors Qs and Qt are explained in
Chapter 2. }

Begin
With Constellation do
Begin
  C1 := Cos(Target.Lat);
  Qs[1] := C1*Cos(ThetaE);
  Qs[2] := C1*Sin(ThetaE);
  Qs[3] := Sin(Target.Lat);

  C1 := As/Re;
  C2 := Sin(ThetaS + Phi);
  C3 := Cos(ThetaS + Phi);
  C4 := Cos(OmegaN);
  C5 := Cos(Inclin);
  C6 := Sin(OmegaN);
  Qt[1] := C1*(C3*C4 - C2*C6*C5);
  Qt[2] := C1*(C3*C6 + C2*C4*C5);
  Qt[3] := C1*C2*Sin(Inclin);

  Result := 0.0;
  Norm := 0.0;
  For I := 1 to 3 Do
  { Take the dot product of Qs and (Qt-Qs) - see Chapter 2 }
  Begin
    Result := Result+Qs[I]*(Qt[I]-Qs[I]);
    Norm := Norm+Sqr( Qt[I]-Qs[I] );
  End;
  { Cosine is the dot product over the norm. }
  CosBeta := Result/Sqr(Norm);
End;
End; {End of CosBeta Function }

Procedure CalcPeriod(var Constellation: ConType; var Common: CommonType);

Begin
With Constellation do
Begin
  Period := Common.TwoPi*Sqrt(Cube(As)/Grav)*(1.0-0.001624*Sqr(Re)
    *(1.0+(4.0-5.0*Sqr(Sin(Inclin)))/2.0)/Sqr(As));

    { ...accounts for earth oblateness effect on }
    { orbital period by modifying it. }
End;
End; {End of CalcPeriod Procedure }

```

```

Procedure SwitchGraphPage(var a,b: Integer);

{ This procedure switches the graphics "page" that the user is viewing. }
Begin
  If (a=0) then a:=1
    Else If (a=1) then a:=0;
  If (b=0) then b:=1
    Else If (b=1) then b:=0;
  Setactivepage(a);
  Setvisualpage(b);
End; {End of SwitchGraphPage Procedure }

```

```

Procedure Init(var Constellation:ConType; var Target:TargetType;
              var Common:CommonType; var LoopParm:LoopType);
Var
  Temp : Real;
  LineNum, code : Integer;
  s,s1,s2,s3 : string;

{ This routine reads and/or initializes all target and constellation
parameters. }

```

```

Begin
With Constellation do
Begin
  LineNum := 8;
  TextBackground(0); ClrScr;
  TextColor(10);
  GotoXY(16,LineNum);
  Writeln('*** Define Target/Constellation Parameters ***');
  Writeln;

  {Constants }
  Common.TwoPi := 2.0*Pi;
  Common.D2R := Pi/180.0;
  Common.R2D := 180.0/Pi;
  Common.OmegaE := 0.0000729211514; {inertial earth rate }

  {Target Region }
  TextColor(10);
  GotoXY(16,LineNum + 2);
  Write('Target Latitude Limits = ? (deg) ');
  GotoXY(50,LineNum + 2);
  Readln(s);

With LoopParm do
Begin
  If Pos(',', s)=0 then
  Begin
    Val(s,LatLow,code);
    LatLow := LatLow*Common.D2R;
    LatHigh := LatLow;
    LatInt := Common.D2R;
  End
  Else
  Begin

```

```

s1 := copy(s,1,Pos(',', ' ', s) - 1);
Val(s1,LatLow,code);
LatLow := LatLow*Common.D2R;
Delete(s,1,Pos(',', ' ', s));
s2 := copy(s,1,Pos(',', ' ', s) - 1);
Val(s2,LatInt,code);
LatInt := LatInt*Common.D2R;
Delete(s,1,Pos(',', ' ', s));
Val(s,LatHigh,code);
LatHigh := LatHigh*Common.D2R;
End;

{Constellation }
GotoXY(16,LineNum + 4);
Write('Orbital Altitude = ? (nm) ');
GotoXY(50,LineNum + 4);
Readln(Alt);
Alt := Alt*1.852 + Re;
As := Alt;
Phi := pi/2.0;
OmegaN := -pi/2.0;
GotoXY(16,LineNum + 5);
Write('Orbital Inclination = ? (deg) ');
GotoXY(50,LineNum + 5);
Readln(Incl);
Incl := Incl*Common.D2R;
Inclin := Incl;
CalcPeriod(Constellation, Common);
OmegaS := Common.TwoPi/Period;
Temp := Re/As;
Common.OmegaE := Common.OmegaE + 9.9639/86400.0*Common.D2R*
                Sqrt(Seventh(Temp))*Cos(Inclin);

                { ...takes orbit plane precession effect into account }
                {   by modifying earth rate. }

GotoXY(16,LineNum + 6);
Write('Viewing Horizon = ? (deg) ');
GotoXY(50,LineNum + 6);
Readln(VH);
VH := (90.0 - VH)*Common.D2R;
Target.Angle := VH;

End;
GotoXY(16,LineNum + 7);
Write('Total Number of Satellites = ?');
GotoXY(50,LineNum + 7);
Readln(T);
GotoXY(16,LineNum + 8);
Write('Number of Orbital Planes = ?');
GotoXY(50,LineNum + 8);
Readln(P);
GotoXY(16,LineNum + 9);
Write('Interorbit Phasing = ?');
GotoXY(50,LineNum + 9);
Readln(F);
Slope := OmegaS/Common.OmegaE;
{ The slope of the orbital motion lines as explained in Chapter 2. }

```

```

Satphase := F*360 div T;
{ Phasing in mean anomaly between satellites in different
  orbital planes. }
Nodephase := 360 div P;
{ Phasing in earth rotation angle between ascending nodes of different
  orbital planes. }
SPP := T div P;
{ Number of satellites per plane. }
Meanphase := 360 div SPP;
{ Mean anomaly separation between satellites in the same plane. }

End;
End; {End of Init Procedure }

Procedure IniCon(var Constellation: ConType; var Target: TargetType;
                var M: Integer; var Xc, Yc: Real; var Y: Vec2);

Var
  Temp, ThetaE, ThetaS, OldVal, NewVal, Lim, Epsilon : Real;

{ This routine searches for an initial condition--one point on the boundary
of the visibility region as a starting point for integrating the
differential equations whose solution defines the boundary. These
equations are derived in Chapter 2. }

Begin
  ThetaE := 0.0;
  ThetaS := 0.0;
  { Start at coordinates (0,0) on the period square. }
  M := Region(Constellation, Target);
  Epsilon := 0.01;
  If M=2 then
  {Compute the center of the bifurcated visibility region }
  Begin
    Temp := Sin(Target.Lat)/Sin(Constellation.Inclin);
    ThetaS := ArcCos(Temp);
    Temp := Tan(ThetaS)/Cos(Constellation.Inclin);
    ThetaE := ArcTan(Temp);
  End;
  Xc := ThetaE;
  Yc := ThetaS;
  { (Xc,Yc) define the center in mean anomaly and earth rotation angle. }
  Lim := Cos(Target.Angle);
  { By starting at the center of the region, the cosine of elevation angle
    is at its maximum, 1. Search until it passes through the desired
    value, which is called Lim. }

  If ((M=1) or (M=2)) then { be sure the region exists }
  Begin
    NewVal := CosBeta(Constellation, Target, ThetaE, ThetaS);
    Repeat
      {Search for an initial condition }
      OldVal := NewVal;
      ThetaE := ThetaE + Epsilon;
      NewVal := CosBeta(Constellation, Target, ThetaE, ThetaS);
    Until NewVal < Lim;
    {Interpolate between last two points }
  End;
End;

```

```

    Y[1] := ThetaE - Epsilon +
           (Cos(Target.Angle)-OldVal)/(NewVal-OldVal)*Epsilon;
    Y[2] := ThetaS;
End;
End; {End of IniCon Procedure }

```

```

Procedure DifEq(var Constellation: ConType; var Target: TargetType;
               var T: Real; var Y, DY: Vec2);

```

```

Var
  C1, C2, C3, C4, C5, C6, C7 : Real;

```

```

{ This routine evaluates the differential equations (derived in Chapter 2)
  which define the boundary of the visibility region. }

```

```

Begin
With Constellation do
Begin
  C1 := Sin(Y[2] + Phi);
  C2 := Cos(Y[1] - OmegaN);
  C3 := Sin(Y[1] - OmegaN);
  C4 := Cos(Y[2] + Phi);
  C5 := Cos(Target.Lat);
  C6 := Cos(Inclin);
  C7 := C5*C6;
  DY[1] := -C1*C5*C2 + C4*(C7*C3 +
                        Sin(Inclin)*Sin(Target.Lat));
  DY[2] := C4*Cos(Target.Lat)*C3 - C1*(C7*C2);
End;
End; {End of DifEq Procedure }

```

```

Procedure Integ(var H, T:Real; var Y: Vec2);

```

```

Var
  J : Integer;
  TT : Real;
  DY, K1, K2, K3, TempSts : Vec2;

```

```

{ This is a basic 4th Order Runge-Kutta Integrator }

```

```

Begin
  DifEq(Constellation, Target, T, Y, DY);
  For J := 1 to 2 do
    Begin
      K1[J] := DY[J];
      TempSts[J] := Y[J] + H*K1[J]/2.0;
    End;
    TT := T + H/2.0;
    DifEq(Constellation, Target, TT, TempSts, DY);
    For J := 1 to 2 do
      Begin
        K2[J] := DY[J];
        TempSts[J] := Y[J] + H*K2[J]/2.0;
      End;
      DifEq(Constellation, Target, TT, TempSts, DY);
      For J := 1 to 2 do
        Begin

```

```

        K3[J] := DY[J];
        TempSts[J] := Y[J] + H*K3[J];
    End;
    T := T + H;
    DifEq(Constellation, Target, T, TempSts, DY);
    For J := 1 to 2 do
        Begin
            Y[J] := Y[J] + H*(K1[J]+2.0*K2[J]+2.0*K3[J]+DY[J])/6.0;
        End;
    End; {End of Integ Procedure }

Procedure Draw(var Constellation:ConType; var Target:TargetType;
               var Common:CommonType);
Var
    Temp, T, H, ThetaE, ThetaS, Xscale, Yscale : Real;
    P1, P2, P3, P4, M, I, ICount, j, k,
    Plold, P2old, P3old, P4old, Xc, Yc, Xo, Yo,
    CenterX, CenterY : Integer;
    Y, DY : Vec2;
    S, Line1, Line2 : string;

{ This routine draws the visibility region on the graphics screen,
utilizing many of the TURBO Pascal graphics capabilities. }

Begin
With Common do
Begin
    SetViewport(50,40,590,310,ClipOff);
    ClearViewport;
    CenterX := GetMaxX div 2 - 50;
    CenterY := GetMaxY div 2 - 40;
    { Center of the screen }
    Xscale := 1.5;
    Yscale := 0.75;
    { Scaling is necessary because the graphics screen is not square. Xscale
and Yscale change period square coordinates to screen coordinates. }
    T := 0.0;
    H := Pi/180.0*10.0;
    I := 0;
    ICount := 0;
    IniCon(Constellation, Target, M, ThetaE, ThetaS, Y);

    Xc := round(ThetaE*R2D*Xscale) + CenterX; {Center of Region }
    Yc := -round(ThetaS*R2D*Yscale) + CenterY;
    Xo := round(Y[1]*R2D) + CenterX; {Initial Condition }
    Yo := -round(Y[2]*R2D) + CenterY;
    If M = 0 then {Region doesn't exist }
        Begin
            SetTextJustify(CenterText,CenterText);
            SetUserCharSize(2,1,2,1);
            SetColor(15);
            Line(-5,0,539,0);
            Line(539,0,539,274);
            Line(539,269,-5,269);
            Line(0,274,0,0);
            OutTextXY(135,CenterY,'The visibility region does not exist.');
```

```

Else
  Begin {Draw the visibility region }

  { (P1,P2) are coordinates of the single visibility region, or one of
  the bifurcations. (P3,P4) define the second bifurcation if
  necessary. }
  Setcolor(14);
  P1 := round(Xscale*Y[1]*R2D) + CenterX;
  P2 := -round(Yscale*Y[2]*R2D) + CenterY;
  P3 := -round(Xscale*Y[1]*R2D) + CenterX;
  P4 := round(Yscale*Y[2]*R2D) + CenterY;
  Repeat
    I := I + 1;
    P1old := P1; { Save the old point }
    P2old := P2;
    If M=2 then
      Begin
        P3old := P3;
        P4old := P4;
      End;
    Integ(H, T, Y); { Integrate to the next point }
    Y[1] := RealMod(Y[1],TwoPi);
    Y[2] := RealMod(Y[2],TwoPi);

    If M=1 then
      Begin
        P1 := round(Y[1]*R2D*Xscale) + CenterX;
        P2 := -round(Y[2]*R2D*Yscale) + CenterY;
      End
    Else
      Begin
        P1 := round(Y[1]*R2D*Xscale) + CenterX;
        P2 := -round(Y[2]*R2D*Yscale) + CenterY;
        P3 := -round(Y[1]*R2D*Xscale) + CenterX;
        P4 := round(Y[2]*R2D*Yscale) + CenterY;
      End;
    Line(P1old,P2old,P1,P2);
    If M=2 then Line(P3old,P4old,P3,P4);
    { Draws the regions }
    If ((P2old > Yc) and (P2 <= Yc)) then I := 380;
  until I = 400; { I limits the loop }

  SetFillStyle(SolidFill,14);
  SetColor(15);
  Line(-5,0,539,0);
  Line(539,0,539,274);
  Line(539,269,-5,269);
  Line(0,274,0,0);
  Floodfill(Xc,Yc,14);
  If M=2 then Floodfill(2*CenterX-Xc,2*CenterY-Yc,14);
  { Floodfill will color in the visibility region on the screen. }
End;

Line(135,269,135,274);
Line(405,269,405,274);
Line(0,67,-5,67);
Line(0,202,-5,202);
SetTextStyle(1,HorizDir,1);

```

```

SetUserCharSize(1,2,1,2);
SetTextJustify(CenterText,CenterText);
OutTextXY(269,296,'Site Hour Angle');
SetTextStyle(1,VertDir,1); SetUserCharSize(1,2,1,2);
OutTextXY(-37,CenterY,'Mean Anomaly');
SetTextStyle(2,HorizDir,4);
OutTextXY(269,280,'0'); OutTextXY(405,280,'90');
OutTextXY(539,280,'180'); OutTextXY(135,280,'-90');
OutTextXY(0,280,'-180');
SetTextStyle(2,VertDir,4);
OutTextXY(-15,0,'180'); OutTextXY(-15,67,'90');
OutTextXY(-15,CenterY,'0'); OutTextXY(-15,202,'-90');
OutTextXY(-15,269,'-180');
SetTextStyle(1,HorizDir,3); SetUserCharSize(1,1,1,2);
OutTextXY(CenterX,-35,'Revisit Time Analysis');
SetColor(14);
SetTextStyle(2,HorizDir,4);
SetTextJustify(LeftText,BottomText);
Temp := Constellation.Period / 3600.0;
Str(Temp:5:2,s);
Line1 := Concat('Period = ',s,' hrs, Altitude = ');
Temp := (Constellation.As - Re)/1.852;
Str(Temp:5:2,s);
Line1 := Concat(Line1,s,' nm, Inclination = ');
Temp := Constellation.Inclin * R2D;
Str(Temp:5:2,s);
Line1 := Concat(Line1,s,' deg, Viewing Horizon = ');
Temp := abs((Pi/2.0 - Target.Angle)*R2D);
Str(Temp:5:2,s);
Line1 := Concat(Line1,s,' deg');
SetFillStyle(1,0);
Bar(0,-22,550,-1);
OutTextXY(-2,-15,Line1);
Temp := Target.Lat * R2D;
Str(Temp:5:2,s);
Line2 := Concat('Latitude = ',s,' deg, Constellation: ');
Str(Constellation.T,s);
Line2 := Concat(Line2,s,' satellites in ');
Str(Constellation.P,s);
Line2 := Concat(Line2,s,' planes, interorbit phasing = ');
Str(Constellation.F:2,s);
Line2 := Concat(Line2,s,'*360/T');
OutTextXY(-2,-5,Line2);

End;
End; {End of Draw procedure }

```

```

Procedure QRD(var Constellation:ConType; var Common:CommonType;
var Orbits,Days : Integer);

```

```

Var
Q, Tol, Eps, Qo, Ecc, J2, Pn, OmgDot : Real;
I, R, K, x : Integer;
Coeff : Array[1..200] of Integer;

```

```

{ This routine will determine how many orbits are needed until ground trace
repetition occurs for the given orbital parameters. This algorithm is
explained in Chapter 3, section 3. }

```

```

Begin
With Common do
With Constellation do
Begin
  Ecc:= 0.0; { zero eccentricity }
  J2 := 0.001082627;
  Pn := TwoPi*Sqrt(Cube(As)/Grav)*(1.0 - 1.5*J2*Sqr(Re/As)
    *(4.0*Sqr(cos(Inclin)) - 1.0));
  OmgDot := -1.5*J2*Sqrt(Grav/Cube(As))*Sqr(Re/(As*(1.0-Sqr(Ecc))))
    *cos(Inclin);
  Qo := TwoPi/(Pn*(Omegae - OmgDot));

  {Continued Fraction Expansion of Repetition Parameter Q }

  Eps := 100.0;
  Repeat
    R := Trunc(Qo);
    Q := Qo - R;
    I := 1;
    Coeff[1] := R;
    Tol := 1.0/Eps;
    While Q>Tol Do
      Begin
        Q := 1.0/Q;
        I := I + 1;
        Coeff[I] := Trunc(Q);
        Q := Q - Coeff[I];
        Tol := Tol*Coeff[I];
      End;
    Orbits := Coeff[I];
    Days := 1;
    I := I - 1;
    For K := I Downto 1 do
      Begin
        x := Orbits;
        Orbits := Coeff[K]*Orbits + Days;
        Days := x;
      End;
    Eps:=Eps/10.0;
    Until Days <= 21;

End;
End; {of QRD Procedure }

Function InView(var Y:Vec2; var Constellation:ConType;
               var Common:CommonType):Boolean;
Var
  I, J, K, P1, P2, TP1, TP2 : Integer;
  x : Real;
  PixelColor : Word;

{ This routine checks if any satellite in the constellation is in view of
the target, by testing whether any screen coordinate corresponding to a
satellite is within the visibility region. The function is assigned "true"
if the target is in view. }

```

```

Begin
With Common do
With Constellation do
Begin
  x := 360.0;
  P1 := round(realmod(Y[1],x));
  P2 := round(realmod(Y[2],x));
  If P1 >= 180 then P1 := P1 - 360;
  If P2 >= 180 then P2 := P2 - 360;
  { Coordinates in range from -180 to 180 degrees. }

  InView := False;
  For I := 0 to P-1 do { all orbital plane phasings }
  Begin
    TP1 := P1 - I*Nodephase; { phase according to plane }
    For J := 0 to SPP-1 do { all satellites within each plane }
    Begin
      TP2 := P2 + round(J*Meanphase + I*Satphase);
      { add phasing according to satellite position within the plane,
        plus interorbit mean anomaly phasing. }

      TP1 := round(realmod(TP1,x));
      TP2 := round(realmod(TP2,x));
      If TP1 >= 180 then TP1 := TP1 - 360;
      If TP2 >= 180 then TP2 := TP2 - 360;

      If TP1 <= -180 then TP1 := TP1 + 360;
      If TP2 <= -180 then TP2 := TP2 + 360;

      TP1 := round(1.5*TP1) + CenterX;
      TP2 := -round(0.75*TP2) + CenterY;
      { ensure coordinates are from -180 to 180 and scaled to screen. }
      PixelColor := GetPixel(TP1,TP2);
      { test screen pixel color }
      If PixelColor = 14 then { a satellite sees the target }
        InView := True;
    End;
  End;
End;
End; {of InView Function }

```

```

Function MaxRev( var Constellation:ConType; var Target:TargetType;
                var Common:CommonType; var Orbits:Integer): Real;

```

```

Var

```

```

  YNew, YOld : Vec2;
  start, finish, RevIntOld, RevIntNew: Real;
  I, Count, Limit : Integer;

```

```

{ This routine evaluates the maximum revisit interval in minutes for a
given constellation and target. }

```

```

Begin
With Constellation do
With Target do
With Common do
  Begin

```

```

RevIntOld := 0.0;
RevIntNew := 0.0;
Limit := round(Orbits*360);
{ search until ground trace repeats }
Yold[1] := round(Lon);
Yold[2] := 0.0;
{ start at 0 mean anomaly, phase in earth rotation angle according to
  target longitude. }
Repeat
  Ynew[1] := Yold[1] + 1.0/Slope;
  Ynew[2] := Yold[2] + 1.0;
  Limit := Limit - 1;
  { search along diagonal and decrement 1 degree in mean anomaly }

  If InView(Yold,Constellation,Common) and
     not InView(Ynew,Constellation,Common) then
    { This is the start of a revisit interval }
    Begin
      start := Ynew[2];
      { Evaluate the interval }
      Repeat
        Ynew[1] := Ynew[1] + 1.0/Slope;
        Ynew[2] := Ynew[2] + 1.0;
        Limit := Limit - 1;
        { search further until in view again }
      Until InView(Ynew,Constellation,Common);
      Ynew[1] := Ynew[1] - 1.0/Slope;
      Ynew[2] := Ynew[2] - 1.0;
      { back off 1 unit to offset }
      finish := Ynew[2];
      RevIntNew := (abs(start - finish)*D2R/OmegaS)/60.0;
      If RevIntNew > RevIntOld then RevIntOld := RevIntNew;
      { save this interval if it's the largest so far }
    End;

    Yold[1] := Ynew[1];
    Yold[2] := Ynew[2];

  Until Limit <= 0;
End;
MaxRev := RevIntOld;

End; {End of MaxRev Function }

```

```

Begin {Main Program }

  LatCount := 0;
  CheckBreak := True;
  Init(Constellation, Target, Common, LoopParm);
  { Initializes constants, gets user inputs }
  QRD(Constellation, Common, Orbits, Days);
  { Compute ground trace repetition interval }
  DetectGraph(Gd,Gm);
  Gd := EGA;
  Gm := EGAHi;
  InitGraph(Gd,Gm,'graph');

```

```

{ Initializes graphics screen }
Assign(f,'c:\matlab\MapDat');
Rewrite(f);
{ Opens output file }

With Common do
With Target do
With LoopParm do
With Constellation do
  Begin
    GetLon(Constellation, Target, LoopParm);
    Lat := LatHigh;
    { Get longitude limits and start at highest latitude }
    Repeat { Latitude loop }
      LatCount := LatCount + 1;
      LonCount := 0;
      Lon := LonLow;
      Draw(Constellation, Target, Common);
      settextstyle(2, HorizDir, 4);
      settextjustify(lefttext, bottomtext);
      Repeat { Longitude loop }
        LonCount := LonCount + 1;
        x := MaxRev(Constellation, Target, Common, Orbits);
        { Find the maximum revisit interval at this target }
        Str(x:6:4, s);
        RevInt[LatCount, LonCount] := x;
        Line1:= Concat('Max Revisit Time = ', s);
        Line1:= Concat(Line1, ' min');
        SetColor(0); Bar(CenterX+100, 290, CenterX+270, 310);
        SetColor(14);
        OutTextXY(CenterX+100, 300, Line1);
        Lon := Lon + LonInt; { increment longitude }
      Until Lon > LonHigh;
      Lat := Lat - LatInt; { decrement latitude }
    Until Lat < LatLow;

  End;
RestoreCrtMode;
For I := 1 to LatCount do
  Begin
    For J := 1 to LonCount-1 do Write(f, RevInt[I, J]:5:1);
    Writeln(f, '');
    { Output all data to disk file }
  End;

Close(f);
Readln;
End.

```

The next program, "ConDes," was written to perform the constellation design studies of Chapter 6. Since it uses many of the routines from the previous program, these routines have been excluded from this listing.

```

Program ConDes;

{ Author: Paul DiDomenico, C.S. Draper Lab }

Uses Crt, Graph;

Const
  Grav   : Real = 398600.8; {mu in km^3/sec^2}
  Re     : Real = 6378.135; {earth radius in km}

Type
  Vec2 = Array[1..2] of Real;
  Vec20 = Array[1..20] of Integer;
  TargetType = Record
    Angle: Real; NTargets : Integer; Lat: Real; Lon:Real;
    Point: Array[1..100,1..2] of Real; DMR:Real;
  end;
  CommonType = Record
    OmegaE: Real; TwoPi: Real; D2R: Real; R2D: Real;
    CenterX: Integer; CenterY: Integer;
  end;
  ConType = Record
    As      : Real;   Period  : Real;   Incl   : Real;
    OmegaS  : Real;   Phi     : Real;   OmegaN : Real;
    T       : Integer; P      : Integer; F      : Integer;
    Slope   : Real;   Satphase: Integer; Nodephase: Integer;
    Meanphase: Integer; SPP   : Integer; Orbits  : Integer;
  end;
  LoopType = Record
    LatLow: Real; LatInt: Real; LatHigh: Real;
    LonLow: Real; LonInt: Real; LonHigh: Real;
    Alt   : Real; Incl  : Real; VH    : Real;
  end;

Var
  Outdat : Array[1..2,1..90] of Real;
  { Output data for cost versus optimization parameter }
  F : text;
  { Output file }
  Y, DY : Vec2;
  { Y is a point on the boundary of the visibility region.
    DY is the orthogonal gradient at that point. }
  Planes: Vec20;
  { The possible numbers of orbital planes for the given number of
    satellites }
  S, Line1 : String;
  IncLow, Xc, Yc, ThetaE, ThetaS, p, q, tc :real;

```

```

{ Xc and Yc locate the center of the visibility region.
  ThetaE and ThetaS are coordinates in earth rotation angle
  and satellite mean anomaly. }
Gd, Gm, M, I, J, a,b, Orbits, Days,
Count, ConCount, NPlanes : Integer;
{ Orbits is the number of orbits until ground trace repetition.
  Days is the number of days until ground trace repetition.
  ConCount is the number of possible constellations. NPlanes
  is the number of possible numbers of planes. }
Target : TargetType;
{ A target is defined by its latitude/longitude position and a
  local viewing horizon. }
Common : CommonType;
{ Constants, Conversion Factors. }
Constellation : ConType;
{ The orbital elements of the constellation, and the
  constellation design and phasing parameters. }
LoopParm : LoopType;
{ Latitude and Longitude search limits. }

Procedure Init(var Constellation:ConType; var Target:TargetType;
              var Common:CommonType; var LoopParm:LoopType);
Var
  Temp : Real;
  LineNum, Code : Integer;
  S, S1, S2, S3 : string;

{ This routine reads and/or initializes all target and constellation
  parameters. }

Begin
With Constellation do
With LoopParm do
With Target do
Begin
  LineNum := 5;
  TextBackground(0); ClrScr;
  TextColor(10);
  GotoXY(16,LineNum);
  Writeln('*** Define Target/Constellation Parameters ***');
  Writeln;

  {Constants}
  Common.TwoPi := 2.0*Pi;
  Common.D2R   := Pi/180.0;
  Common.R2D   := 180.0/Pi;
  Common.OmegaE := 0.0000729211514; {inertial earth rate}

  TextColor(10);

  {Constellation}
  GotoXY(16,LineNum + 4);
  Write('Orbital Altitude = ? (nm) ');
  GotoXY(50,LineNum + 4);
  Readln(Alt);

```

```

Alt := Alt*1.852 + Re;
As := Alt;
Phi := Pi/2.0;
OmegaN := -Pi/2.0;
Inclin := 89.0*Common.D2R;
CalcPeriod(Constellation, Common);
OmegaS := Common.TwoPi/Period;
Temp := Re/As;
Common.OmegaE := Common.OmegaE + 9.9639/86400.0*Common.D2R*
                Sqrt(Seventh(Temp))*Cos(Inclin);

                { ...takes orbit precession effect into account }
                {   by modifying earth rate.                       }

GotoXY(16,LineNum + 6);
Write('Viewing Horizon = ? (deg) ');
GotoXY(50,LineNum + 6);
Readln(VH);
VH := (90.0 - VH)*Common.D2R;
Angle := VH;

GotoXY(16,LineNum + 7);
Write('Total Number of Satellites = ?');
GotoXY(50,LineNum + 7);
Readln(T);
Slope := omegas/Common.omegae;
GotoXY(16,LineNum+10);
Write('Number of Targets = ?');
{ Get all targets the constellation is trying to observe }
GotoXY(50,LineNum+10);
Readln(NTargets);
For I := 1 to NTargets do
Begin
  GotoXY(16,LineNum+10+I);
  Write('Target # ',I,' Lat: ');
  Read(Point[I,1]);
  GotoXY(40,LineNum+10+I);
  Write(' Lon: ');
  Read(Point[I,2]);
End;
GotoXY(16,LineNum+12+NTargets);
Write('Desired Maximum Revisit Time = ? (min)');
GotoXY(55,LineNum+12+NTargets);
Readln(DMR);

End;
End; { of Init Procedure }

Function Cost(var Constellation:ConType; var Target:TargetType;
              var Common:CommonType):Real;
Var
  alpha, CMR, sum : real;

{ This routine evaluates the cost function used in constellation
optimization, as explained in Chapter 6 }

```

```

Begin
With Constellation do
With Target do
Begin
  sum := 0;
  alpha := 1.0;
  For I := 1 to NTargets do
  Begin
    Lat := Point[I,1]*Common.D2R;
    Lon := Point[I,2];
    Draw(Constellation, Target, Common);
    CMR := MaxRev(Constellation, Target, Common);
    { Get the computed maximum revisit time }
    sum := sum + CMR - DMR + alpha*abs(CMR-DMR);
  End;
End;

End;
Cost := sum;
End; { of Cost Function }

Procedure GetPlanes(var Constellation:ConType; var NPlanes: Integer;
                   var Planes: Vec20);
Var
  Count, I : Integer;

{ This routine finds all the possible numbers of planes for the given
number of satellites, i.e., all factors of T. }

Begin
With Constellation do
Begin
  Count := 1;
  Planes[1] := 1;
  For I := 2 to T do
  If T mod I = 0 then
  Begin
    Count := Count+1;
    Planes[Count] := I;
  End;
  NPlanes:=Count;
End;
End; { of GetPlanes Procedure }

Begin {Main Program}

  CheckBreak := True;
  Init(Constellation, Target, Common, LoopParm);
  DetectGraph(Gd,Gm);
  Gd := EGA;
  Gm := EGAHi;
  InitGraph(Gd,Gm,'graph');
  Assign(f,'c:\matlab\OutDat');
  Rewrite(f);

```

```

With Common do
With Target do
With LoopParm do
With Constellation do
Begin
QRD(Constellation, Common);
GetPlanes(Constellation, NPlanes, Planes);
Count:=0;
For ConCount := 1 to NPlanes do
Begin
P := Planes[ConCount];          {all P which are factors of T}
Nodephase := 360 div P;
SPP := T div P;

Meanphase := 360 div SPP;

For F := 0 to T-1 do {all possible phasings}
Begin
Satphase := F*(360 div T);
Repeat
Count:=Count+1;
tc := Cost(Constellation, Target, Common);
{ evaluate cost }
Inclin:=Inclin-1.0*D2R;
{ adjust Inclin }
OutDat[1,count]:=Inclin*R2D;
OutDat[2,count]:=tc;
str(tc:8:2,s);
Line1:=concat('Cost = ',s);
setcolor(0);bar(Centerx+100,290,centerx+270,310);
setcolor(14);
outtextxy(centerx+100,300,Line1);
Until Inclin <= IncLow;
End;
End;
End;
restorecrtmode;
For I := 1 to Count do
Begin
For J := 1 to 2 do Write(f,OutDat[J,I]:8:2);
{ output data }
Writeln(f,'');
end;
close(f);
readln;
End.

```

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