(36)

INTACT STABILITY CRITERIA FOR NAVAL SHIPS

by

Frédéric Deybach

Engineer Degree in Naval Architecture

Ecole Nationale Supérieure des Ingénieurs des Etudes et Techniques d'Armement, 1992

SUBMITTED TO THE DEPARTMENT OF OCEAN ENGINEERING IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE IN NAVAL ARCHITECTURE AND MARINE ENGINEERING

AT THE

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

FEBRUARY 1997

Signature of Author ....

Department of Ocean Engineering
January 17, 1997

Certified by ....

Alan J. Brown, Professor of Naval Architecture
Thesis Supervisor, Department of Ocean Engineering

Accepted by ....

J. Kim Vandiver, Professor of Ocean Engineering
Chairman, Ocean Engineering Departmental Graduate Committee

INTACT STABILITY CRITERIA FOR NAVAL SHIPS

by

Frédéric Deybach

Submitted to the Department of Ocean Engineering

on January 15, 1997 in Partial Fulfillment of the

Requirements for the Degree of Master of Science in

Naval Architecture and Marine Engineering

ABSTRACT

The United States Navy uses an intact stability criteria that was developed during World War II. This criteria is empirical and does not consider ship characteristics other than the righting arm curve. It does not directly consider the Sea State or the wind gusts. The International Maritime Organization is using a criteria developed in the 1970's and 1980's. This criteria is also empirical, and does not directly consider the Sea State. In this thesis the fundamentals of both the US Navy and IMO criteria are discussed. The equations of the roll motion from the linear theory of ship motion are reviewed. From these equations a new intact stability criteria is proposed which considers the ship characteristics influencing the roll motion, the Sea State and a wind heeling arm adapted to modern naval ships. This proposed criteria is validated with model test results from four ships and compared with the US Navy and IMO criteria. Both its validity and its superiority over the existing criteria are demonstrated.

Thesis Supervisor : Alan J. Brown

Title : Professor of Naval Architecture

TABLE OF CONTENTS

1. EXISTING CRITERIA HISTORICAL ANALYSIS

1.1 US Navy

The stability criteria of the United States Navy are adapted and used by many other navies. These criteria are found with few modifications in the Design Data Sheets or equivalent documents of most of the industrial navies; the criteria are summarized in Table 1 for four countries. However, much of the historical and fundamental basis of these criteria are lost and, as is often the case, our navies mutually assure themselves by seeing others using similar criteria. The focus of the naval community is more on damaged stability, which is generally more constraining and therefore the traditional concern of warship designers [Surko, Stephen W. (1994)].

1.1.1 Lessons of WW II [1]

The defining period for naval ship intact stability criteria was World War II. Before the war, the intact stability criteria was based on GM, range of stability and maximum righting arm. The damaged stability criteria required that a ship with two flooded compartments (symmetrical flooding) would not sink beyond the margin line.

---

1 Historical information about the evolution of the stability criteria has been provided by Jerry Posshel of the Weights Department in the Naval Sea System Command (NAVSEA).

Table 1 : Naval Intact Stability Criteria

CRITERIA		U.S. [1]	France [2]	Canada [3]	U.K. [4]
Conditions of loading		Minimum Operating and Full Load	Minimum Operating	Operational Light Loading	Light Seagoing
Criteria on righting arm curve (GZ)					
Area under GZ Curve with heel angle:					
Area under GZ curve with heel angle	• from 0° to 30°	NA	≥ 0.080 m.rad	NA	≥ 0.080 m.rad
	• from 0° to 40°	NA	≥ 0.133 m.rad	NA	≥ 0.133 m.rad
	• from 30° to 40°	NA	≥ 0.048 m.rad	NA	≥ 0.048 m.rad
Maximum righting arm (GZ max)		NA	≥ 0.3 m	≥ 0.3 m	≥ 0.3 m
Heel angle corresponding to maximum righting arm		NA	≥ 30°	NA	≥ 30°
Transverse metacentric height (GM) with free-surface correction		NA	≥ 0.3 m	≥ 0.05 m	≥ 0.3 m
Capsizing angle		NA	≥ 60°	NA	≥ 60°

(Continued on page 10)

1 [DDS 079-1 (1 August 1975)] from Naval Ship Engineering Center.

2 [IT 6014] from DCN.

3 [C-03-001-024 MS-002, (05 June 1986)]

4 [NES 109 (August 1989)] from Sea Systems Controllerate.

(Continued from page 9)

CRITERIA	U.S. (NAVSEA)	France (DCN)	Canada	U.K.
Beam wind combined with rolling				
Wind heeling arm	$\frac{0.0195 V^2 Az\cos^2\theta}{1000\Delta}$	$\frac{0.0195 V^2 Az\cos^2\theta}{1000\Delta}$	$\frac{0.0195 V^2 Az\cos^2\theta}{1000\Delta}$	$\frac{0.0195 V^2 Az\cos^2\theta}{1000\Delta}$
Ratio between righting arm at equilibrium and maximum righting arm ($\frac{GZ(\theta_0)}{GZmax}$)	≤ 0.6	≤ 0.6	≤ 0.6	≤ 0.6
Equilibrium heel angle θ_0 with wind = 100 knots	NA	≤ 30°	≤ 30°	NA
with wind = 90 knots	NA	NA	NA	≤ 30°
Windward roll-back angle (θ_1)	25°	25°	25°	25°
Ratio between capsizing and restoring energy ($\frac{A_2}{A_1}$)	≥ 1.4	≥ 1.4	≥ 1.4	≥ 1.4
Maximum angle for A_2 area	NA	NA	≤ 70°	≤ 70°

During and after the war many damaged ships were studied. Analysis of these ships showed that most of those with a damage length up to 15% of the length between perpendiculars survived (did not sink). For intact stability the primary source of information was the typhoon of December 1944. All the Pacific fleet was caught in a major tropical typhoon and many ships were lost due to the high winds [Colhoun, Raymond]. An extensive analysis of how the ships weathered the typhoon was made. The results were compared with the characteristics of the ships to determine the relevant variables. Three destroyers that capsized and ships that only marginally survived provided particularly useful data (some had heel angles up to 80°, one survived because the loss of its stack reduced the sail area). In 1946, the results of this analysis were summarized in an internal memo by Section 456 of the Bureau of Ships. This memo also proposed a tentative intact stability requirement.

The first variable studied for this new criteria was the wind speed. From the data gathered during the typhoon, a wind speed of 90 knots was identified as a reasonable criteria for survival in tropical storms. Therefore, the proposed minimum wind velocity for ships already in service was 90 knots. The proposed minimum wind velocity selected for new design purposes was 100 knots, but this objective had much less importance at the time, since the US Navy had many ships built during the war and no new designs were needed. The main concern was to modify existing ships to survive a wind velocity of 90 knots. The wind heeling arm was calculated as follows :

$$\text{WindUnsettingMoment} = \frac{l \cdot A \cdot (0.004 \cdot V^2) \cdot \cos^2\theta}{2240} \text{ft} \cdot \text{tons} \qquad (1)$$

with $\quad l$ = height of centroid of topside area above 1/2 the draft in ft

A = topside area in ft^2 plus 10%

V = wind velocity in knots

θ = angle of heel in degrees

The cosine square multiplication factor considers the reduction, with the heel of the ship, of the area above waterline and the height of the centroid of the area.

The second analysis concerned the righting arm curve. The ratio of the righting arm at the static heel angle to the maximum righting arm was 0.67 and greater for the destroyers that capsized; ships that survived had a ratio of 0.51 to 0.54. Therefore the proposed maximum ratio was 0.6.

The final analysis concerned a criterion for the reserve of dynamic stability. The reserve of stability is determined by comparing the heeling and the restoring energy, or comparing the area under the righting arm curve between the roll back and the equilibrium heel angle with the area under the righting arm curve between the angle of equilibrium and the intersection between righting arm and wind heeling arm curves as shown in Figure 1. The destroyers that capsized had only a 15% margin. The surviving ships had an 80% to 110% margin. The proposed margin was 40%. The roll back angle to windward was 25°; no justification of this angle is found in the memo. It seems it was coherent with the data observed during the typhoon, however in 1946 it was not technically possible to give a technical justification to the angle.

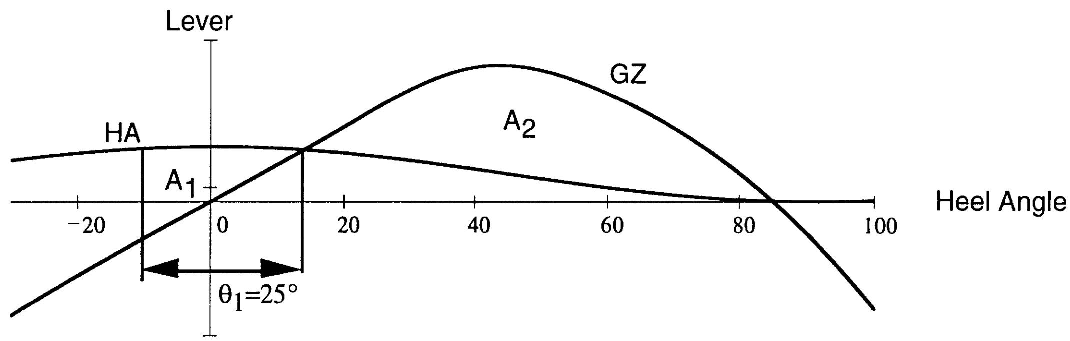


Figure 1 : Navy Stability Curves

1.1.2 Refining the criteria

In 1948 the tentative criteria were included in a Design Data Sheet (DDS). A curve specifying the required energy or energy below the damaged righting arm curve was included in the DDS. This curve was also determined from the data of W.W.II typhoon-damaged ships.

In 1962, the criteria were described by [Sarchin and Goldberg]. This became the standard used by the stability division in NAVSEA, but it was not sufficient to refer to a SNAME paper in official documents, in particular to contractors. Therefore, the content of the paper was put in a DDS in 1975. Included in the DDS was a required area ratio of 1.4 for the damaged righting arm curve with a wind heeling arm curve calculated using a reduced wind speed. The 1.4 ratio for damaged stability had no other justification than symmetry with intact stability. The included area is limited to an inclination of 45° or the angle of the first down-flooding point. No justification was given for this limitation.

1.1.3 Current criteria

Since 1975, the only attempt to scientifically justify the stability criteria was a series of model tests conducted by NAVSEA between 1991 and 1994 [Jones, Harry, D. (1991)&(1992)]. These tests included intact and damaged ships in heavy seas with 100 knots wind. The tests showed that the ships satisfying the criteria had an adequate stability. They also showed that the roll back angle of 25° was reasonable.

1.2 IMO

While the main interest of the military community was damaged stability, the civilian community, not concerned with weapon damage, first concentrated on the study of intact stability.

In order to create, improve and harmonize the technical requirements for the safety of ships from different countries, the International Maritime Organization (IMO, formerly Inter-governmental Maritime Consultative Organization, IMCO) was created prior to the 1960 International Conference on the Safety of Life at Sea (SOLAS 60). IMO is an agency of the United Nations and has currently over 130 member nations.

A Sub-committee on Subdivision and Stability was formed in 1962 and the first international stability criteria was adopted in 1968 with Resolution A167. These criteria are based on work by [Rahola] in 1935 on the capsizing of ships in the Baltic Sea. The requirements of the resolution were:

- Area under the righting arm curve ≥ 0.055 m.rad, up to a 30° angle of heel.
- Area under the righting arm curve ≥ 0.090 m.rad, up to a 40° angle of heel or to the downflooding angle θ_f, whichever is less.
- Area under the righting arm curve ≥ 0.030 m.rad, for angles of heel between 30° and 40° (or θ_f).
- Righting arm GZ ≥ 0.20 m at an angle of heel ≥ 30°
- Maximum righting arm GZ_{max} should occur at an angle of heel preferably ≥ 30° but no less then 25°.
- Initial metacentric height (GM_0) ≥ 0.15 m.
- Static heel angle with crowding of passengers on one side or in gyration ≤ 10°.

In the 1970's and 1980's, IMO has been involved in the development of a criteria considering wind and an energy balance between capsizing and restoring energy. Such criteria were adopted in 1985 with Resolution A.562 and modified in 1993 with Resolution A.749 which combines the requirements of Resolutions A.167, A.206 (ships carrying deck cargo), A.168 (fishing vessels) and A.562. The "weather" criteria in these recommendations use simplified equations and tables that do not show the method of calculation from fundamental principles. To compare these criteria with others, or to modify the level of requirement, the basis of the equations is required. Working papers presented at annual meetings of the Sub-committee on Subdivision, Stability and Load Lines during the late 1970's and early 1980's were used to reconstruct as much of this rationale as possible. This is described in the following sections.

1.2.1 Japanese weather criterion

In September 1979, in IMCO paper Stab XXIV/4, [Japan (1979)] proposed a weather criteria that would complement the GZ criterion of resolution A.167 and consider wind with gust. The complete criteria proposed by Japan was:

- Metacentric height : $GM > \dfrac{\text{heeling moment due to wind and shifting of passengers}}{\Delta \cdot \text{heeling angle when 70\% of free board immersed}}$
- Dynamic stability : $\dfrac{\text{restoring energy}}{\text{capsizing energy}} = \dfrac{\text{area } A_2}{\text{area } A_1} \geq 1$
- Maximum righting arm : $0.275 \text{ m} \geq GZ_{max} \geq 0.0215*B$ (with B = beam) [1]

The dynamic stability criterion considers the ship rolling with an amplitude of θ_1 in waves around an equilibrium heel angle due to a steady wind (θ_0). The ship is subjected to a gust when at maximum heel to windward (θ_0 - θ_1). The dynamic stability must be sufficient to prevent the ship from heeling beyond the flooding angle θ_2. Dynamic stability is represented by the area between the GZ curve and the wind heeling curve as shown in Figure 2. Sufficient stability is achieved when area A_2 is equal to or greater than area A_1. The wind speed proposed by Japan was 26 m/s for ocean-going ships and 19 m/s for coastal ships. The gust considered is $\sqrt{1.5}$ times

---

[1] No justification for the maximum limit for GZ_{max} was given in the document.

the steady wind speed (or 1.5 times the steady healing moment). No justification for the wind speed or the gust was given in any working paper.

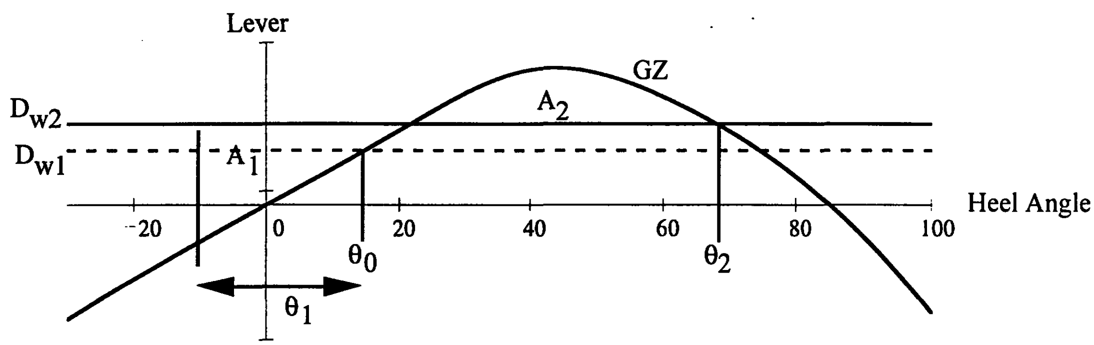


Figure 2 : Japanese Stability Curves

Japan proposed the following calculation of the heeling moment due to wind and the maximum rolling amplitude (θ_1).

1.2.1.1 Wind heeling moment

The heeling moment due to steady wind is:

$$Dw_1 = \frac{\frac{1}{2}\rho C_D V^2 AZ}{\Delta} \qquad (2)$$

with A (m^2) = total windage area of the ship

Z (m) = distance between the centroid of A and the centroid of the under-water projected area

V (m/s) = steady wind speed

ρ (kg/m^2) = air density

C_D (m^{-1}s^2) = drag coefficient

Δ (tons) = displacement

Japan also proposed a drag coefficient for ships of ordinary proportions (no definition of "ordinary" was provided) : $1/2\rho C_D = 0.76*10^{-4}$ (metric units). To be dimensionally correct this is equivalent to $C_D = 0.117$ m^{-1}s^2. This is a standard equation for the drag of bluff bodies. It is well understood and accepted.

The wind heeling moment is considered independent of the heel angle of the ship and a factor of 1.5 is applied for the gust wind heeling moment: $D_{w2} = 1.5\ D_{w1}$.

1.2.1.2 Roll amplitude

The roll amplitude is based on the resonant roll amplitude in regular waves which IMCO Stab XXIV/4 defines as :

$$\vartheta = \sqrt{\frac{\pi \cdot r \cdot \theta_w}{2N}} \qquad (3)$$

with r = effective wave slope factor = $0.73 + 0.6\frac{OG}{T}$ where OG is the vertical distance between G and the water line (positive axis up) and T the draft [1]

θ_w (°) = wave slope = $180*s$

---

[1] The draft was represented by d in the original formula

s = wave steepness = wave height / wave length = $\frac{h}{\lambda}$

N = Bertin's roll damping coefficient = 0.02

This equation does not appear to be dimensionally correct. The result in degrees is obtained by multiplying a non-dimensional number by the square root of an angle in degrees.

The standard rolling amplitude in irregular waves (θ_1) is derived from the resonance rolling in regular waves by taking the maximum amplitude in 1000 rolls :

$$\theta_1 = 0.7\vartheta = \sqrt{\frac{138rs}{N}} \qquad (4)$$

The proposed method of calculating the wave steepness is based on a relation between wave steepness and wave age, given by [Sverdrup and Munk]. The wave age is represented by the wave speed / wind speed ratio. For a given wind speed (26 m/s in this case), wave steepness is given as a function of the wave period (T) by using the deep water wave relationship :

$$\text{WaveSpeed} = C = \sqrt{\frac{g}{2\pi}\lambda} = \frac{g}{2\pi}T$$

Japan proposed a linear approximation of the resulting curve with:

$$s = 0.151 - 0.0072T \qquad (5)$$

Resonant roll amplitude is achieved when the wave excitation period is equal to the natural roll period of the ship. The wave steepness is calculated at this natural period. For ships in the design stage, for which the natural roll period cannot be measured, roll period is calculated using :

$$T = 2\pi \frac{k_x}{\sqrt{gGM}} \qquad (6)$$

with: k_x = radius of gyration about the longitudinal axis, or radius of inertia

k_x is usually expressed as $k_x = CB$ where B is the beam of the ship. For average cargo ships, the coefficient C is about 0.4, but Japan also proposed an empirical formula to calculate a precise value of C :

$$C^2 = f \cdot \left[C_B \cdot C_U + 1.1 C_U (1 - C_B) \left(\frac{H}{T} - 2.2 \right) + \frac{H^2}{B^2} \right] \qquad (7)$$

with: f = 0.125 for passenger and cargo ships

0.133 for tankers

0.200 for fishing boats

C_B = block coefficient

C_U = upper-deck area coefficient

$H \text{ (m)} = D + \frac{A}{L_{pp}}$

D (m) = depth

1.2.2 Refining the criteria

Between 1980 and 1982, the members of the Sub-committee studied the weather criteria submitted by Japan and proposed some modifications in particular to the angle to which the dynamic reserve of stability should be considered.

1.2.2.1 Japanese complement

In November 1981, with IMCO Stab/95, [Japan (1981)] proposed a new empirical formula to calculate the natural roll period of a ship (no basis for this formula was given in the document) :

$$C = 0.3725 + 0.0227\frac{B}{T} - 0.0043\frac{L_{pp}}{10} \qquad (8)$$

and when the ship side shell at midship is inclined or flared :

$$C = 0.3085 + 0.0227\frac{B}{T} - 0.0043\frac{L_{pp}}{10}$$

The factor C is used in the formula described previously :

$$T = 2\pi\frac{CB}{\sqrt{gGM}} \qquad (6)$$

The same document proposed to limit the calculation of the restoring energy to $\theta < 50°$. 50° is the angle beyond which the cargo is susceptible to move. It also mentioned the alternative method proposed by USSR (see below).

1.2.2.2 Criteria from USSR

In February 1982, IMCO Stab 27/5/3, [USSR] proposed an alternative method to calculate the amplitude of rolling. This method is included in the "Rules of the Register of Shipping of the USSR" and it was obtained by approximation from calculations of rolling amplitudes in irregular seas for different types of ships. The calculations are based on the fundamental equations of motion with the coefficients obtained from experimental data. In contrast to the criteria of Japan, this method takes into account the dependence of the roll damping coefficient on the hull form and the presence of appendages, and as it is based on real data, it also takes into account the dynamic components of the disturbing moment. The rolling amplitude in degrees is calculated as follows :

$$\theta = kX_1X_2Y \qquad (9)$$

with : k, X_1 and X_2 dimentionless factors given by Table 2, Table 3 and Table 4

Y (°) given by Table 5

Table 2 : Factor k

A_k/LB	0.0 %	1.0 %	1.5 %	2.0 %	2.5 %	3.0 %	3.5 %	≥ 4.0 %
k	1.00	0.98	0.95	0.88	0.79	0.74	0.72	0.70

with : A_k (m^2) = total area of bilge keels, or the area of the side projection of the bar keel, or a sum of those areas.

Table 3 : Factor X_1

B/T	≤ 2.4	2.5	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	≥ 3.5
X_1	1.0	0.98	0.95	0.96	0.93	0.91	0.90	0.88	0.86	0.84	0.82	0.80

Table 4 : Factor X_2

C_B	≤ 0.45	0.50	0.55	0.60	0.65	≥ 0.70
X_2	0.75	0.82	0.89	0.95	0.97	1.00

Table 5 : Factor Y for unrestricted navigation

$\frac{\sqrt{GM}}{B}$	≤ 0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	≥ 0.13
Y	24.0	25.0	27.0	29.0	30.7	32.0	33.4	34.4	35.3	36.0

1.2.2.3 Calculations and combinations

The members of the Sub-committee submitted the results of calculations for a number of existing ships using the weather criteria of Japan and of USSR, as well as the method of Recommendation A.167. These analyses showed the necessity of a weather criteria as it was more constraining than A.167 at low displacement [1]. To include the advantages of the Japanese and USSR methods, a combination of both methods for the calculation was proposed by the Sub-committee [STAB 27/WP.3] to be evaluated in another set of test calculations and reevaluation.

[1] Except for one tanker studied by the Peoples Republic of China, that showed A.167 to be more constraining at all displacements. No explanation of this phenomenon was proposed.

The resulting formula for the calculation of the roll-back angle was:

$$\theta_1 = \frac{kX_1X_2}{C}\theta_{1Japan} \qquad (10)$$

with : θ_{1Japan} = rolling amplitude calculated with the Japanese method described in § 1.2.1

k, X_1, and X_2 = correction factors described in the USSR method

$C = 0.76$ = coefficient accepted for test calculations

1.2.3 Recommendation A.749

The adopted recommendation proposes a combination of the area requirement under the GZ curve required by A.167 and an energy balance with wind requirement or "weather criteria". The requirements of the "weather criteria" are as follow :

Heeling arm due to steady wind (l_{w1}) and to gust wind (l_{w2}):

$$l_{w1} = \frac{PAZ}{1000g\Delta} \qquad (11)$$
$$l_{w2} = 1.5 \cdot l_{w1}$$

with : P (Nm^{-2}) = wind pressure [1] =504 Nm^{-2}

A (m^2) = projected lateral area above waterline

Z (m) = vertical distance from the center of A and the center of the underwater lateral projection or approximately half the draft

---

[1] Same as Japanese wind heeling moment with $\frac{1}{2}C_D\rho V^2 = 504\ Nm^{-2}$ for V = 26m/s

$\Delta(t)$ = displacement

g (ms^{-2}) = 9.81

Roll-back angle [1]:

$$\theta_1 = 109kX_1X_2\sqrt{rs} \qquad (12)$$

with : • k, X_1 and X_2, non-dimensional factors as used with the USSR criteria:

k = 1.0 for round-bilged ship with no bilge or bar keels

= 0.7 for ships with sharp bilges

= factor as shown in Table 2 for ships with bilge or bar keels

X_1 = factor as shown in Table 3

X_2 = factor as shown in Table 4

• r and s are the factors used with the Japanese criteria :

$r = 0.73 + 0.6\frac{OG}{T}$ = effective wave slope factor

OG (m) = vertical distance between G and the water line (positive axis up)

T (m) = mean moulded draft (d represented the draft in the original formula)

---

[1] The equation is the combination of the Japan and USSR criteria, its developed form is:

$\theta_1 = \frac{kX_1X_2}{C} \cdot 0.7\sqrt{\frac{\pi \cdot r \cdot 180s}{2N}}$ where k, X_1, X_2, r and s are as defined before, C = 0.7635 (test calculation factor) and N = 0.02 (Bertin's damping factor).

s = wave steepness = factor as shown in Table 6 [1]

Table 6 : Factor s

T_0	≤ 6	7	8	12	14	16	18	≥ 20
s	0.100	0.098	0.093	0.065	0.053	0.044	0.038	0.035

with : T_0 = natural rolling period of the ship [2] $= 2\frac{CB}{\sqrt{GM}}$ (13)

where : $C = 0.373 + 0.023\frac{B}{T} - 0.043\frac{L_{PP}}{100}$ (14)

[1] This table gives a better approximation to the curve of Sverdrup and Munk than the Japanese linear equation presented above.

[2] The coefficient $\frac{\pi}{\sqrt{g}}$ of the previous formula for T_0 is included in coefficient C. No formula is proposed for ships with inclined or flared side shell at midship.

2. WIND CAPSIZING FORCE

The capsizing forces for an intact ship are due to the action of the waves and the action of the wind. The wind creates a drag force on the area of the ship above water; this force creates a heeling moment around the center of flotation. The Navy and the IMO criteria described in the previous chapter both propose a formula to calculate the corresponding heeling arm. However, these formula do not detail the individual terms of the force and are not easy to modify for different requirements. We propose to use a fundamental expression where each term can be adapted to the specified requirement.

The following formula is used for the drag force :

$$F = \tfrac{1}{2} C_D \rho A V^2 \qquad (15)$$

where : C_D = drag coefficient

ρ (kg.m^{-3}) = air density = 1.293 kg.m^{-3}

A (m^2) = area exposed to wind

V (m.s^{-1}) = wind speed

The heeling moment created by this drag force around the center of flotation is given by :

$$M_W = F \cdot z$$

where : z (m) = vertical distance between the center of the area above the water and the center of flotation or approximately half the draft (see Figure 3).

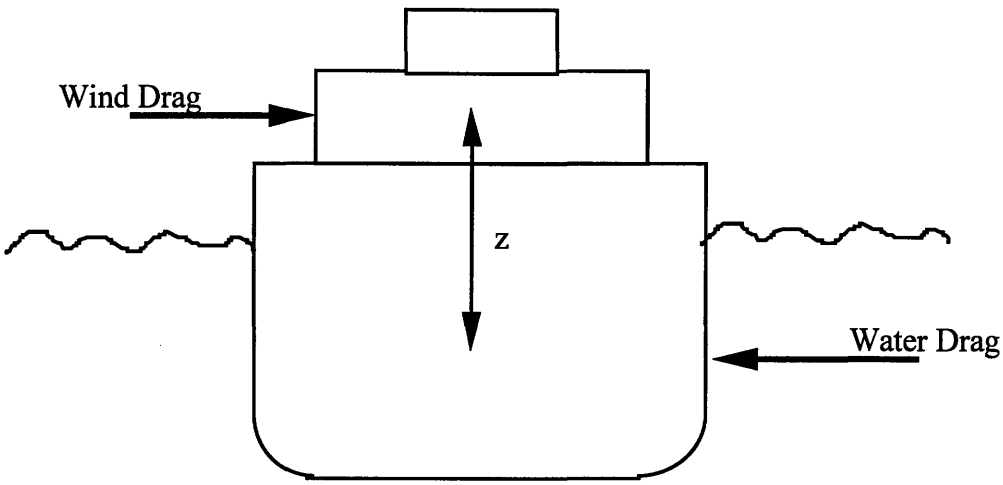


Figure 3 : Heeling moment

To compare the capsizing moment with the hydrostatic restoring moment, a normalized wind heeling arm is used :

$$H_W = \frac{F \cdot z}{1000 \cdot g \cdot \Delta} = \frac{\frac{1}{2} C_D \rho A V^2 \cdot z}{1000 \cdot g \cdot \Delta} \qquad (16)$$

with : Δ (t) = displacement of the ship (metric tons = t)

In order to evaluate the wind heeling arm in the proposed criteria, values for the individual terms in Equation (16) are determined as discussed in the following paragraphs.

2.1 Drag coefficient

2.1.1 US Navy

The formula used by the US Navy to calculate the wind heeling arm does not explicitly include a nondimensional drag coefficient. In order to compare the US Navy and IMO criteria, C_D for the US Navy is calculated as follows:

The US Navy formula, with HA in ft, V in knots, A in ft^2, l in ft and Δ in long tons, is :

$$HA = \frac{0.004V^2 Al\cos^2\theta}{2240\Delta}\cdot ft$$

Converting to metric units (HA in m, A in m^2, l in m, Δ in metric tons and V in m.s^{-1} :

$$HA = \frac{0.4536}{0.3048^3}\cdot\left(\frac{3600}{1852}\right)^2\cdot 0.3048\cdot\frac{0.004V^2 Al\cos^2\theta}{1000\Delta} \approx \frac{0.0737V^2 Al\cos^2\theta}{1000\Delta}\cdot m$$

Comparing with the standard form for body drag, Equation (16), and neglecting the $\cos^2$ term:

$$\frac{0.0737V^2 Al}{1000\Delta} = \frac{\frac{1}{2}C_D\rho\cdot V^2 Al}{1000g\Delta}$$

$$\Rightarrow \frac{\frac{1}{2}C_D\rho}{g} = 0.0737 \Rightarrow C_D = 0.0737\cdot\frac{2g}{\rho} = 0.0737\cdot\frac{2\cdot 9.81}{1.293} \approx 1.12$$

This indicates that the nondimensional drag coefficient used by the US Navy criteria is C_D = 1.12, with g = 9.81 m.s^{-2} and ρ = 1.293 kg.m^{-3}.

2.1.2 IMO

Resolution A.749 uses the following formula for the wind heeling arm :

$$l_W = \frac{PAz}{1000g\Delta} \qquad \text{with } P = 504 \text{ N.m}^{-2}.$$

Comparing with Equation (16) :

$$P = \tfrac{1}{2} C_D \rho \cdot V^2$$

With V = 26 m.s-1 (see § 1.2.3) and $\rho = 1.293$ kg.m^{-3}, we have :

$$C_D = \frac{2P}{\rho \cdot V^2} = \frac{2 \cdot 504}{1.293 \cdot 26^2} = 1.15$$

The non dimensional drag coefficient used by the IMO criteria is $C_D = 1.15$. It is similar to the naval coefficient and to the drag coefficient of a plate with sharp edges ($C_D = 1.2$).

2.1.3 Proposed criteria

For naval ships, the same drag coefficient currently used by the US Navy criteria is preferred: $C_D = 1.12$.

2.2 Heeling Arm Curve

2.2.1 US Navy

The US Navy uses a heeling arm curve that varies with a cosine square function of the angle of heel. The theory behind this function is that the wind pressure acts on the lateral area of the ship, and, as the ship is inclined, on the projected area that varies with a cosine function of the angle of heel. The same is true for the distance between the center of above water sail area (where the wind force acts) and the center of underwater area (where the water resistance force acts). The resulting wind heeling arm is :

$$HA = \frac{0.004V^2 Al\cos^2\theta}{2240\Delta} \cdot ft \qquad (17)$$

2.2.2 IMO

In Resolution A.749, IMO considers a constant heeling arm through the range of heel angles. The justification for this approach is that, for commercial vessels, the deckhouse is usually small and the projected area should be considered constant as the reduction in topside area is compensated with the area of the emerging hull. The IMO approach is very conservative for naval ships which have a large superstructure and projected area.

2.2.3 Proposed criteria

In the US Navy approach, the cosine squared function of heel angle reduces the moment to zero at 90°. This would be the case for a two dimensional plate rotating around its waterline, but when a three dimensional ship rotates the area of the emerging hull increases with the angle.

We propose to use a method more realistic than the current US Navy method, but not as conservative as IMO's. We consider that the projected area decreases with a cosine function from the upright position to a minimum corresponding to the ship laying on its side with half its hull emerged. The sail area is as follows :

$$A_{Proposed} = c_w \frac{L_{pp}B}{2} + \left(A - c_w \frac{L_{pp}B}{2} \right) \cos\theta \qquad (18)$$

where : A (m^2) = projected area above water

L_{pp} (m) = length between perpendiculars

B (m) = beam

c_w = waterplane coefficient

The lever is calculated by a similar method. It decreases with a cosine function from the upright position to a minimum corresponding to the ship laying on its side. In the latter configuration, the centers of the above and the below water areas are approximately at one quarter of the beam.

The lever is given by the following formula :

$$z_{Proposed} = \frac{B}{2} + \left(z - \frac{B}{2} \right) \cos\theta \qquad (19)$$

where : z (m) = vertical distance between the center of the area above the water and the center of flotation or approximately half the draft.

The wind heeling arm is then calculated with the following formula :

$$H_W = \frac{\frac{1}{2} C_D \rho V^2 \cdot \left[c_w \frac{L_{pp}B}{2} + \left(A - c_w \frac{L_{pp}B}{2} \right) \cos\theta \right] \cdot \left[\frac{B}{2} + \left(z - \frac{B}{2} \right) \cos\theta \right]}{1000 \cdot g \cdot \Delta} \qquad (20)$$

2.3 Wind speed

2.3.1 US Navy

The US Navy uses a wind speed of 100 knots (51.44 m/s) for new designs. Wind gust is not considered in the wind heeling calculation, but in a margin on the dynamic stability ($A_2 = 1.4\ A_1$).

2.3.2 IMO

In Resolution A.749, IMO specifies a steady wind speed of 26 m/s (50.54 knots) with gusts to $\sqrt{1.5} \cdot 26 = 31.84$ m/s.

2.3.3 Proposed criteria

The IMO approach considering steady wind and gusts separately is favored as it clearly defines the relevant factors instead of using a margin that includes all the unknowns. However, for naval ships, a higher wind speed is preferred. Bibliographic research did not provide elements determining the gust to consider for 100 knots wind speed, therefore the ratio proposed by IMO is adopted. The proposed criteria considers a steady wind of 100 knots (or 51.44 m/s) with gusts to 122.46 knots (or 63 m/s).

3. LINEAR SHIP MOTION

In addition to wind forces, the second set of capsizing forces are from the sea waves. The action of these waves on the hull creates ship motions, and in particular a roll motion which combined with lateral wind can capsize the ship. The classic equilibrium approach discussed in § 1, is at best a good ordinal criteria for intact stability. The actual motion of a ship is dynamic and non-linear, particularly in extreme conditions. In order to take a more fundamental approach to this problem while maintaining the intuitive simplicity of the equilibrium approach, linear theory is used to refine and quantify the existing criteria.

3.1 Axis System

To describe the motion of the ship with six degrees of freedom we consider three axis systems:

- $(x_0\, y_0\, z_0)$ is fixed on the earth with the x_0 axis in the direction of advance of the ship. This is used to define the incident waves.
- $(x\ y\ z)$ moves at constant velocity U_0 with the ship. This is called the inertial system.
- $(\bar{x}\,\bar{y}\,\bar{z})$ is fixed in the ship, therefore, the motions of the ship are measured as motions of the $(\bar{x}\,\bar{y}\,\bar{z})$ system relatively to the $(x\ y\ z)$ system with the three translations called surge, sway and heave, and the three rotations of the $\bar{x}, \bar{y}, \bar{z}$ axes called roll, pitch and yaw, respectively (see Figure 4).

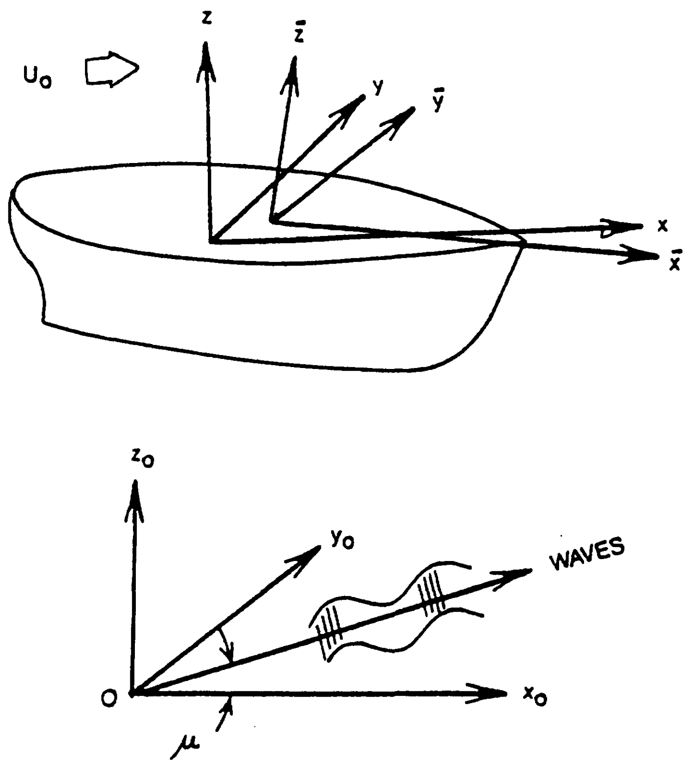


Figure 4 : Axis System

3.2 Equations of Motion

The linearized Euler equations of motion with six degrees of freedom in the body axes are:

$$\sum_{k=1}^{6} \Delta_{jk}\ddot{\eta}_k(t) = \mathbf{F}_j(t) \qquad j=1,\ 2\ ...6 \qquad (21)$$

where : Δ_{jk} = the components of the inertia matrix for the ship (mass, moment of inertia, and all possible couplings)

$\ddot{\eta}_k$ = acceleration in the k direction

$\mathbf{F}_j$ = forces and moment acting on the body in the j direction

A ship has lateral symmetry and [Abkowitz (1969)] has shown that these equations can be simplified as :

$$
\begin{aligned}
&\Delta(\ddot{\eta}_1 + \bar{z}_C\ddot{\eta}_5) = \mathbf{F}_1 && \text{(surge)}\\
&\Delta(\ddot{\eta}_2 - \bar{z}_C\ddot{\eta}_4 + \bar{x}_C\ddot{\eta}_6) = \mathbf{F}_2 && \text{(sway)}\\
&\Delta(\ddot{\eta}_3 - \bar{x}_C\ddot{\eta}_5) = \mathbf{F}_3 && \text{(heave)}\\
&I_{44}\ddot{\eta}_4 - I_{46}\ddot{\eta}_6 - \Delta\bar{z}_C\ddot{\eta}_2 = \mathbf{F}_4 && \text{(roll)}\\
&I_{55}\ddot{\eta}_5 + \Delta\left[\bar{z}_C\ddot{\eta}_1 - \bar{x}_C\ddot{\eta}_3\right] = \mathbf{F}_5 && \text{(pitch)}\\
&I_{66}\ddot{\eta}_6 - I_{64}\ddot{\eta}_4 + \Delta\bar{x}_C\ddot{\eta}_2 = \mathbf{F}_6 && \text{(yaw)}
\end{aligned}
\qquad (22)
$$

where : Δ = displacement of the ship

$(\bar{x}_C, 0, \bar{z}_C)$ = coordinates of the center of gravity in the $(\bar{x}\,\bar{y}\,\bar{z})$ system

I_{jj} (with j = 4,5,6) = moment of inertia around the axes $\bar{x}, \bar{y}, \bar{z}$ respectively

$I_{46} = I_{64}$ = roll - yaw product of inertia

$\mathbf{F}_j$ (with j = 1,2,3) = forces in the $\bar{x}, \bar{y}, \bar{z}$ directions respectively

$\mathbf{F}_j$ (with j = 4,5,6) = moments about the $\bar{x}, \bar{y}, \bar{z}$ axes respectively (positive counterclockwise)

The motions presented in Equation (21) are in the body axes system $(\bar{x}\,\bar{y}\,\bar{z})$, however the difference between this system and the inertial system (x y z) is lost in the linearization and for convenience we will use the inertial system in the following paragraphs.

3.3 Sinusoidal Response

Since we consider the linear response of the ship to sinusoidal waves, the response is also sinusoidal and can be represented in complex notation :

$$\eta_j(t) = \overline{\eta}_j e^{i\omega_e t} \tag{23}$$

where : $\overline{\eta}_j$ = complex amplitude of the response in the j direction

ω_e = frequency of encounter [1] defined as :

$$\omega_e = \omega_0 - \frac{\omega_0^2}{g} U_0 \cos\mu$$

ω_0 = frequency of the waves (in the ($x_0\, y_0\, z_0$) axis system)

U_0 = forward speed of the ship

μ = angle between the direction of the waves and the direction of the ship

Using this notation the acceleration in the j direction can be represented by:

$$\ddot{\eta}_j(t) = -\omega_e^2 \overline{\eta}_j e^{i\omega_e t}$$

3.4 Forces

Only gravitational and fluid forces are acting on the ship, we can write :

$$\mathbf{F}_j(t) = F_{Gj} + F_{Hj} \tag{24}$$

where : F_{Gj} = gravitational force acting in the j direction

F_{Hj} = fluid force acting in the j direction

---

[1] Traditionally, intact stability considers ships with zero speed in beam seas. In this case $\omega_e = \omega_0$. However, we will continue to use the general notation in this analysis.

The fluid forces can also be subdivided into hydrostatic and hydrodynamic forces. However, since the mean gravitational force and the mean buoyancy cancel one another, they are usually combined to give a net hydrostatic force (F_{HSj}). The forces acting on the ship are then expressed as :

$$\mathbf{F}_j(t) = F_{HSj} + F_{HDj} \tag{25}$$

where : F_{HSj} = net hydrostatic force

F_{HDj} = hydrodynamic force

3.5 Net Hydrostatic Forces

The detailed integration of pressure along the body can be found in [Newman (1977)]. The components of the forces in each direction are :

$$F_{HSj} = -\sum_{k=1}^{6} C_{jk}\overline{\eta}_j e^{i\omega_e t} \tag{26}$$

with :

$$\begin{aligned} &C_{jk} = 0 \quad \text{for all } j,k = 1,2..6 \text{ except for :} \\ &C_{33} = \rho g \int B(x)dx \\ &C_{35} = C_{53} = -\rho g \int xB(x)dx \\ &C_{44} = g\Delta\overline{GM}_T \\ &C_{55} = g\Delta\overline{GM}_L + \rho g S\overline{LCF}^2 \approx \rho g \int x^2 B(x)dx \end{aligned} \tag{27}$$

where : B(x) = full breadth of waterplane at x

Δ = displacement of the ship

S = waterplane area

$\overline{GM}_T$ = transverse metacentric height

$\overline{GM}_L$ = longitudinal metacentric height

$\overline{LCF}$ = longitudinal center of flotation

3.6 Hydrodynamic Forces

The hydrodynamic forces can be separated in two distinct components, the excitation forces and the radiation forces. These components can be calculated independently and superimposed.

$$F_{HDj} = F_{EXj} + F_{Rj} \qquad (28)$$

where : F_{EXj} = exiting forces in the j direction

$= F_j^I + F_j^D$

F_j^I = incident exiting forces

F_j^D = diffracted exiting forces

F_{Rj} = radiation forces in the j direction

The exiting forces are the forces and moments that excite the motion of the ship. One component of the exiting forces is due to incident waves (F_j^I), they are usually called the Froude-Krylov exiting forces and are calculated by integrating the pressure of the incident wave along the body surface with the ship not present. The second component of the exiting forces is due to the diffracted waves (F_j^D), they are called diffracted excitation forces. They are caused by the diffraction of the incident waves by the presence of the ship, considered fixed. It is not necessary to calculate these forces separately, they can be determined from the incident excitation forces and the radiation forces.

The radiation forces are the result of the waves radiating away from the ship when forced to oscillate in a calm water. These forces are represented by:

$$F_{Rj} = \sum_{k=1}^{6} \left(\omega_e^{\ 2} A_{jk} - i\omega_e B_{jk} \right) \overline{\eta}_k e^{i\omega_e t} \qquad (29)$$

where : A_{jk} = added mass in the j mode due to motion in the k direction

B_{jk} = damping coefficient in the j mode due to motion in the k direction

3.7 Linearized Equations of Motion

We have shown in the previous paragraphs that the forces applied to the ship can be decomposed as :

$$\mathbf{F}_j(t) = F_{Gj} + F_{Hj} = F_{HSj} + F_{HDj} = F_{HSj} + F_{EXj} + F_{Rj} = F_{HSj} + F_j^I + F_j^D + F_{Rj}$$

The expression of the different components of the forces are substituted in the equation of motion : Equation (21), using the complex notation for the motion :

$$\mathbf{F}_j = \sum_{k=1}^{6} -\omega_e^{\ 2} \Delta_{jk} \overline{\eta}_k \, e^{i\omega_e t} = F_{HSj} + F_j^I + F_j^D + F_{Rj} \qquad (30)$$

We replace F_{HSj} and F_{Rj} by their components using (26) and (29). The equation becomes:

$$\sum_{k=1}^{6} -\omega_e^{\ 2} \Delta_{jk} \overline{\eta}_k \, e^{i\omega_e t} = -\sum_{k=1}^{6} C_{jk} \overline{\eta}_j \, e^{i\omega_e t} + F_j^I e^{i\omega_e t} + F_j^D e^{i\omega_e t} + \sum_{k=1}^{6} \left(\omega_e^{\ 2} A_{jk} - i\omega_e B_{jk} \right) \overline{\eta}_k e^{i\omega_e t} \qquad (31)$$

If we group the coefficients of η_k on the left side of the equation and if we eliminate the $e^{i\omega_e t}$ the equation becomes :

$$\sum_{k=1}^{6}\left[-\omega_e^2\left(\Delta_{jk}+A_{jk}\right)+i\omega_e B_{jk}+C_{jk}\right]\overline{\eta}_k = F_j^I + F_j^D \qquad (32)$$

where : $j = 1,2...6$

Δ_{jk} = component of the inertia matrix as detailed in (22)

A_{jk} = added mass in the j mode due to motion in the k direction

B_{jk} = damping coefficient in the j mode due to motion in the k direction

C_{jk} = hydrostatic restoring force coefficients as detailed in (27)

F_j^I = incident exiting forces

F_j^D = diffracted exiting forces

3.8 Uncoupled Longitudinal and Transversal Motions

For an unrestrained ship with port/starboard symmetry the six equations of motion can be uncoupled into a set of three equations for longitudinal motions (surge, heave and pitch) and a set of three equations for transversal motions (sway, roll and yaw). The uncoupling of longitudinal and transversal motions results in the absence of the correspondent cross-coupling terms in the equations of motion (j = 1,3,5 and k = 2,4,6). We have shown in (22)and (27) that the cross coupling values of the inertia matrix (Δ_{jk}) and the hydrostatic restoring force coefficients (C_{jk}) are equal to zero if j = 1,3,5 and k = 2,4,6. The same is true for the added mass (A_{jk}) and the damping coefficients (B_{jk}).

The uncoupling of longitudinal and transversal motions is accepted as a good assumption for typical ships in moderate seas but if extreme seas are considered all six motions should be studied simultaneously. For simplicity, we will restrict our study to the set of transversal motions decoupled from the vertical motions.

3.9 Added Mass and Damping Coefficients

The added mass and damping coefficients can be calculated by using strip theory as a ship is considered a slender body (the length dimension is much larger the width and depth). The analysis will not be reproduced here, a simplified form can be found in Chapter 8 of the Principles of Naval Architecture (PNA) by [Beck, et al], and a detailed form in the work of [Salvesen, et al]. Strip theory is also used by the US Navy Ship Motion Program (SMP). The results are summarized in Table 7.

Table 7 : Transversal motions coefficients [1]

$$A_{22} = \int a_{22}dx \qquad B_{22} = \int b_{22}dx$$

$$A_{24} = A_{42} = \int a_{24}dx \qquad B_{24} = B_{42} = \int b_{24}dx$$

$$A_{26} = \int x\, a_{22}dx + \frac{U_0}{\omega_e^2} B_{22} \qquad B_{26} = \int x\, b_{22}dx - U_0 A_{22}$$

$$A_{44} = \int a_{44}dx \qquad B_{44} = \int b_{44}dx$$ [2]

$$A_{46} = \int x\, a_{24}dx + \frac{U_0}{\omega_e^2} B_{24} \qquad B_{46} = \int x\, b_{24}dx - U_0 A_{24}$$

$$A_{62} = \int x\, a_{22}dx - \frac{U_0}{\omega_e^2} B_{22} \qquad B_{62} = \int x\, b_{22}dx + U_0 A_{22}$$

$$A_{64} = \int x\, a_{24}dx - \frac{U_0}{\omega_e^2} B_{24} \qquad B_{64} = \int x\, b_{24}dx + U_0 A_{24}$$

$$A_{66} = \int x^2 a_{22}dx + \frac{U_0^2}{\omega_e^2} A_{22} \qquad B_{66} = \int x^2 b_{22}dx + \frac{U_0^2}{\omega_e^2} B_{22}$$

[1] All integrals are taken over the ship length.

[2] Because roll is lightly damped, viscous effects have to be added to the strip theory to give an more accurate prediction of the damping coefficient. We will use an equivalent linear coefficient $B^{*}_{44} = B_{44} + B_e$

3.10 Roll Motion

Roll motion of a ship is difficult to predict as the hydrodynamic damping effect predominant in other motions is very small in roll. Nonlinear viscous effects are of the same order and cannot be neglected. It is therefore common to include in the linear equations a correction for non-linearities. This correction is included as an equivalent damping coefficient B^*_{44} (see Table 7) and an equivalent restoring coefficient C^*_{44}. These coefficients are dependent on the amplitude of the response : $B^*_{44} = B^*_{44}(|\bar{\eta}_4|)$ and $C^*_{44} = C^*_{44}(|\bar{\eta}_4|)$. These nonlinearities become negligible for small amplitudes : when $|\bar{\eta}_4| \to 0$ then $B^*_{44} \to B_{44}$ and $C^*_{44} \to C_{44} = g\Delta\overline{GM}_T$.

It is shown above that cross-coupling between roll and longitudinal motions can be neglected. The remaining coupling is between roll, sway and yaw as shown in the following roll equation of motion:

$$\left[-\omega_e^2(\Delta_{42}+A_{42})+i\omega_e B_{42}\right]\bar{\eta}_2+\left[-\omega_e^2(\Delta_{44}+A_{44})+i\omega_e B^*_{44}+C^*_{44}\right]\bar{\eta}_4 +\left[-\omega_e^2(\Delta_{46}+A_{46})+i\omega_e B_{46}\right]\bar{\eta}_6=F_{EX4} \qquad (33)$$

If we select a different coordinate system $\hat{x}\hat{y}\hat{z}$, fixed to the ship, we can isolate the effect of roll. The origin of the new coordinate system is selected so that in the new system $\hat{\Delta}_{42}+\hat{A}_{42}=0$. The axes of the new system are also rotated by an angle ψ about the $\bar{y}$ axis so that $\hat{\Delta}_{46}+\hat{A}_{46}=0$. The coefficients represented with a circumflex accent are now calculated in the new coordinate system. Their value varies from those calculated previously. The new origin is called the center of roll. It is important to remember that as $\hat{A}_{ij}$ varies with the frequency of encounter, the position of the center of roll changes. However, the angle ψ is very small and the roll angle is considered similar in both coordinate systems, the same is true for the hydrostatic coefficient C^*_{44}.

In the new system the roll equation of motion becomes :

$$\left[-\omega_e^2\left(\hat{I}_{44}+\hat{A}_{44}\right)+i\omega_e\hat{B}^*_{44}+C^*_{44}\right]\overline{\eta}_4+i\omega_e\hat{B}_{42}\overline{\eta}_2+i\omega_e\hat{B}_{46}\overline{\eta}_6=\hat{F}_{EX4} \tag{34}$$

Roll is still coupled with sway and yaw but only through $\hat{B}_{42}$ and $\hat{B}_{46}$. This coupling is weak and we can ignore it. The roll equation of motion then becomes that of a simple harmonic oscillator with non-linear damping and restoring :

$$\left[-\omega_e^2\left(\hat{I}_{44}+\hat{A}_{44}\right)+i\omega_e\hat{B}^*_{44}+C^*_{44}\right]\overline{\eta}_4=\hat{F}_{EX4} \tag{35}$$

This representation of roll motion is the most commonly adopted since the work of [Conolly (1969)].

3.11 Excitation Moment

A good estimate of the excitation moment is :

$$\hat{F}_{EX4}=g\Delta\overline{GM}\cdot\overline{\dot{\xi}} \tag{36}$$

where : $\overline{\dot{\xi}}$ = amplitude of the wave slope

If we consider the complex notation for sinusoidal waves described in Equation (23), we have: $\xi(t)=\overline{\xi}e^{i\omega_e t}$, and as for deep water waves, the encounter wave speed is $\frac{dx}{dt}=\frac{g}{2\pi}T_e=\frac{g}{\omega_e}$, we can describe the wave slope as : $\dot{\xi} = \frac{d\xi}{dx} = \frac{d\xi}{dt} \cdot \frac{dt}{dx} = i\frac{\omega_e^2}{g}\bar{\xi} e^{i\omega_e t}$. Therefore the wave slope amplitude can be written as :

$$\bar{\dot{\xi}} = i\frac{\omega_e^2}{g}\bar{\xi} \qquad (37)$$

3.12 Nondimensional Roll Equation of Motion

In order to simplify the use of the roll equation of motion, particularly when introducing sea spectrum response, the equation is represented in a non dimensional form by dividing all terms of Equation (35) by $g\Delta\overline{GM}$:

$$\left[-\omega_e^2 \frac{\hat{I}_{44} + \hat{A}_{44}}{g\Delta\overline{GM}} + i\omega_e \frac{\hat{B}^*_{44}}{g\Delta\overline{GM}} + \frac{C^*_{44}}{g\Delta\overline{GM}}\right] \bar{\eta}_4 = \bar{\dot{\xi}} \frac{g\Delta\overline{GM}}{g\Delta\overline{GM}}$$

Resulting in :

$$\left[-\left(\frac{\omega_e}{\omega_n}\right)^2 + 2i\frac{\omega_e}{\omega_n} \cdot \beta^* + \gamma^*\right] \bar{\eta}_4 = \bar{\dot{\xi}} = i\frac{\omega_e^2}{g}\bar{\xi} \qquad (38)$$

where : $\frac{\omega_e}{\omega_n}$ = nondimentional encounter frequency

$\omega_n = \sqrt{\frac{g\Delta\overline{GM}}{\hat{I}_{44} + \hat{A}_{44}}}$ = roll resonance frequency

$\beta^* = \beta^*\left(\left|\bar{\eta}_4\right|\right) = \frac{\omega_n \hat{B}^*_{44}}{2g\Delta\overline{GM}}$ = nondimensional damping factor

$\gamma^* = \gamma^*\left(\left|\bar{\eta}_4\right|\right) = \frac{C^*_{44}}{g\Delta\overline{GM}} = \frac{C^*_{44}}{C_{44}} \approx 1$ = nondimensional roll restoring term

The roll response amplitude is then :

$$|\bar{\eta}_4| = \frac{1}{\left|1-\left(\frac{\omega_e}{\omega_n}\right)^2 + 2i\frac{\omega_e}{\omega_n}\cdot\beta^*\right|}\cdot\left|i\frac{\omega_e^2}{g}\right|\cdot|\bar{\xi}|$$

or

$$\frac{|\bar{\eta}_4|}{|2\bar{\xi}|} = \frac{1}{\sqrt{\left(1-\left(\frac{\omega_e}{\omega_n}\right)^2\right)^2 + \left(2\frac{\omega_e}{\omega_n}\cdot\beta^*\right)^2}}\cdot\frac{\omega_e^2}{2g} \qquad (39)^{[1]}$$

3.13 Roll Resonance Period

In the previous set of equations the ship roll resonance frequency or natural period is required. When a full or reduced model of the ship exists, this period can be measured after an initial impulse. However, for preliminary designs, the resonance period can be calculated as :

$$T_n = \frac{2\pi k}{\sqrt{g\overline{GM}}} \qquad (40)$$

where : $k = \sqrt{\frac{\hat{I}_{44}+\hat{A}_{44}}{\Delta}}$ = radius of gyration or radius of inertia

The factor k is usually represented as k = CB, where B = beam of the ship. However, different formula to calculate C exist in the literature:

[1] The introduction of $2\bar{\xi}$ in the roll amplitude preludes the use of wave height for the proposed method.

- PNA proposes C = 0.3613 for typical ships [1], but warns that the constant can vary by as much as 20%, an aircraft carrier for example has a constant C = 0.41, because of its flight deck.

- Resolution A.749 from IMO proposes $C = 0.3725 + 0.0227\frac{B}{T} - 0.043\frac{Lpp}{100}$ (see § 1.2.3)

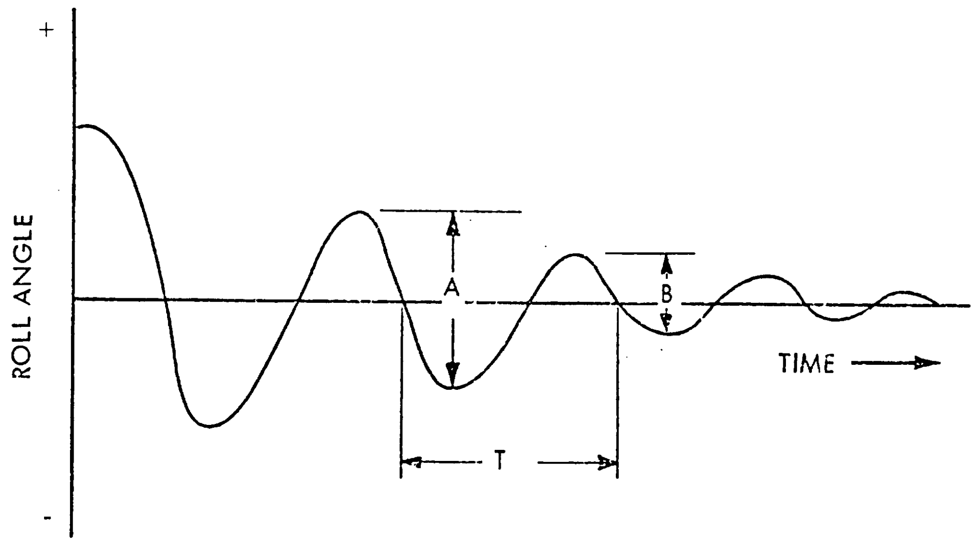
- IMCO Stab/95 by Japan (see § 1.2.2), proposes a specific factor for ships with inclined or flared side shell : $C = 0.3085 + 0.0227\frac{B}{T} - 0.043\frac{Lpp}{100}$

Appendix 1 compares the different formula for existing naval ships. These comparisons show Resolution A.749 to give the most accurate result for these ships and since this is the commercial standard, this formula is used for the proposed stability criteria.

3.14 Roll Damping

For a ship in service or for which a model exists, the damping factor can be determined by experiment. The study of the decay of the roll after an initial impulse is used to determine the damping factor at resonance. This method was described by [Miller, et al (1974)] and is described in Figure 5.

[1] The formula proposed for the natural period is $T_n \approx \frac{2.27 \cdot B}{\sqrt{g\overline{GM}}}$, it corresponds to a factor C = 0.3613 in the form we use.



$$\text{Effective Roll Damping Coefficient} = \frac{C}{C_c} = \frac{1}{2\pi} \cdot \ln\left(\frac{A}{B}\right)$$

Roll Period in Calm Water = T

Figure 5 : Roll Damping

However, for preliminary design stages, an analytical method is required. The same authors proposed an empirical formula to calculate the damping factor of naval vessels with long and slender hulls. The formula is based on statistical data of ships used by the US Navy.

The formula is divided into two parts, the first involves the calculation of the zero speed damping factor $\beta^{(0)}$, the second gives the damping factor at forward speed β^{*}.

Zero speed factor :

$$\beta^{(0)} = \beta^{(0)}\left(\left|\bar{\eta}\right|\right) = 19.25 \cdot \left[A_k \sqrt{b_k} + 0.0024 \cdot L B \sqrt{d}\,\right] \cdot \frac{d^2 \sqrt{d \left|\bar{\eta}\right|}}{C_B\, L\, B^3\, T} \qquad (41)$$

where : A_k = total area of the bilge keels (port + starboard)

b_k = width of the bilge keel

d = distance between the centerline at the waterline and the trace of the bilge keel (see Figure 6).

$\left|\bar{\eta}\right|$ = roll angle

C_B = block coefficient

Factor with forward speed :

$$\beta^{*} = \beta^{*}\left(\left|\bar{\eta}\right|\right) = \beta^{(0)} + 0.00085 \cdot \frac{L}{B} \cdot \sqrt{\left(\frac{L}{\overline{GM}}\right)} \cdot \frac{\mathbf{F}}{C_B}\left(1 + \frac{\mathbf{F}}{C_B} + 2\left(\frac{\mathbf{F}}{C_B}\right)^2\right) \qquad (42)$$

where : $\mathbf{F} = \dfrac{U_0}{\sqrt{gL}}$ = Froude number

For the purpose of intact stability the zero speed factor is used.

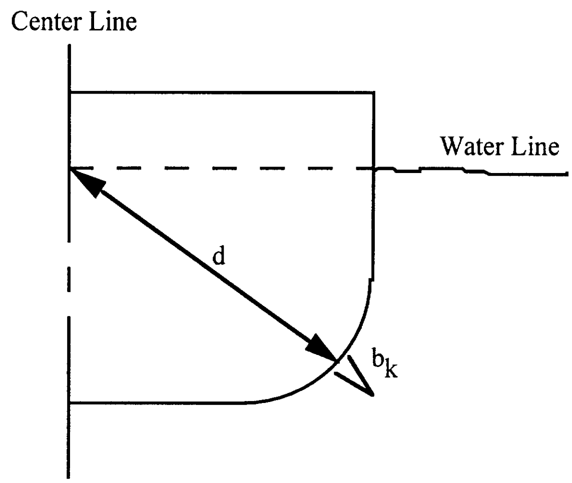


Figure 6 : Bilge Keel

4. PROPOSED NAVAL SHIP CRITERIA

The proposed naval ship criteria is based on the analysis of the wind capsizing force developed in § 2 and on the linear theory for ship motion developed in § 3. It is as follows :

The ability of a ship to withstand a combination of wind and rolling must be demonstrated for all loading conditions as follows:

1. The ship is subjected to a steady wind acting perpendicular to its longitudinal axis. This wind results in a heeling arm H_{W1} as calculated by Equation (43) below.
2. Around the resultant static heel angle (θ_0), the ship is assumed to roll from the action of the waves. The amplitude of the rolling to windward (θ_1), is assumed to be equal to the amplitude of rolling of the ship with no wind, as given by Equation (48) below.
3. When at its maximum angle to windward, the ship is subjected to a wind gust, which results into a heeling arm H_{W2} as given by Equation (44) below.
4. Under these circumstances, the restoring energy from the ship hull form must be greater then or equal to the capsizing energy. This is represented by the area A_2 being greater then or equal to area A_1 in Figure 7. Area A_2 is considered only up to the downflooding angle (θ_2).

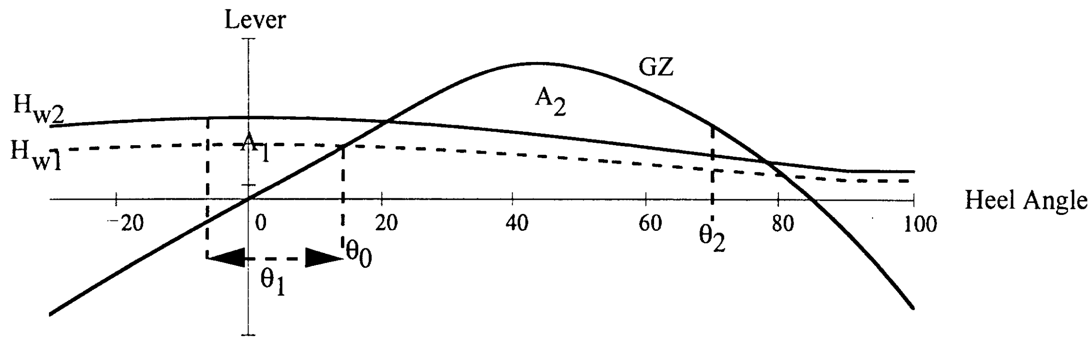


Figure 7 : Dynamic Stability

Wind heeling

The steady and the gust wind heeling arms are given by :

$$H_{W1} = \frac{\frac{1}{2} C_D \rho V^2 \cdot \left[c_w \frac{L_{pp} B}{2} + \left(A - c_w \frac{L_{pp} B}{2} \right) \right] \cdot \left[\frac{B}{2} + \left(z - \frac{B}{2} \right) \right] \cos^2 \theta}{1000 \cdot g \cdot \Delta} \qquad (43)$$

$$H_{W2} = 1.5 \cdot H_{W1} \qquad (44)$$

where : C_D = lateral drag coefficient = 1.12

ρ ($kg.m^{-3}$) = air density = 1.293 $kg.m^{-3}$

L_{pp} (m) = length between perpendiculars

B (m) = beam

c_w = waterplane coefficient

A (m^2) = area exposed to wind

z (m) = vertical distance between the center of the area above the water and the center of flotation or approximately half the draft

V ($m.s^{-1}$) = steady wind speed = 51.44 $m.s^{-1}$ = 100 knots

Δ (t) = displacement of the ship

Angle of roll windward

The response spectrum for angle of roll to windward is calculated by multiplying the wave spectrum of the specified sea state by the square of the roll transfer function, also called Response Amplitude Operator (RAO), developed from the linearized equations of motion.

$$SR(\omega) = SE(\omega) \cdot (RAO(\omega))^2 \qquad (45)$$

We propose to model the excitation using an Ochi North Atlantic wave spectrum (9 modal period family, two parameters model) [Ochi (1978)].

The Ochi wave spectrum is described as follows :

$$SE(\omega) = \sum_{i=1}^{9} p_{\omega_{mi}} \cdot \frac{5}{16}\left(\frac{\omega_{mi}}{\omega}\right)^4 \frac{H^2}{\omega} e^{-1.25\left(\frac{\omega_{mi}}{\omega}\right)^4} \qquad (46)$$

where : ω_{mi} ($rad.s^{-1}$) = modal period i (see Table 8)

$p_{\omega_{mi}}$ = probability of modal period i (see Table 8)

H (m) = significant wave height

Table 8 : Ochi Modal Period Family

Modal period	Probability
$\omega_{m1} = 0.048\,(8.75 - \ln H)$	$p\omega_{m1} = 0.05$
$\omega_{m2} = 0.054\,(8.44 - \ln H)$	$p\omega_{m2} = 0.05$
$\omega_{m3} = 0.061\,(8.07 - \ln H)$	$p\omega_{m3} = 0.0875$
$\omega_{m4} = 0.069\,(7.77 - \ln H)$	$p\omega_{m4} = 0.1875$
$\omega_{m5} = 0.079\,(7.63 - \ln H)$	$p\omega_{m5} = 0.25$
$\omega_{m6} = 0.099\,(6.87 - \ln H)$	$p\omega_{m6} = 0.1875$
$\omega_{m7} = 0.111\,(6.67 - \ln H)$	$p\omega_{m7} = 0.0875$
$\omega_{m8} = 0.119\,(6.65 - \ln H)$	$p\omega_{m8} = 0.05$
$\omega_{m9} = 0.134\,(6.41 - \ln H)$	$p\omega_{m9} = 0.05$

The Response Amplitude Operator for roll per unit wave height is [1] :

$$\mathrm{RAO}(\omega) = \frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\frac{\omega}{\omega_n}\cdot\beta^*\right)^2}}\cdot\frac{\omega^2}{2g} \qquad (47)$$

where : $\omega_n = \frac{2\pi}{T_n}$ = roll resonance frequency. The resonance period T_n is either measured with a model or calculated with : $T_n = \frac{2\pi B}{\sqrt{g\overline{GM}}}\cdot\left(0.3725 + 0.0227\frac{B}{T} - 0.043\frac{L_{pp}}{100}\right)$

[1] Developed from Equation 39

β^* = nondimensional damping factor. β^* is either measured with a model or calculated with

$$\beta^* = 19.25 \cdot \left[A_k \sqrt{b_k} + 0.0024 \cdot L_{pp} B \sqrt{d} \right] \cdot \frac{d^2 \sqrt{d \cdot \theta_i}}{C_B L_{pp} B^3 T}$$

L_{pp} (m) = ship length between perpendiculars

B (m) = ship width

T (m) = ship draft

GM (m) = transverse metacentric height

C_B = ship block coefficient

A_k (m^2) = bilge keels area (port + starboard)

b_k (m) = bilge keel width

d (m) = distance between the centerline at the waterline and the trace of the bilge keel (see Figure 8)

θ_i (rad) = 0.262 rad (15°) or iterative process on the amplitude of roll [1]

---

[1] The damping increases with the amplitude of roll. Preliminary studies have shown that for Sea State 8 the amplitude of roll is always higher than 15°. The use of 15° gives a simplified conservative value for the damping factor. The more detailed studies should include an iterative process on the damping factor calculation.

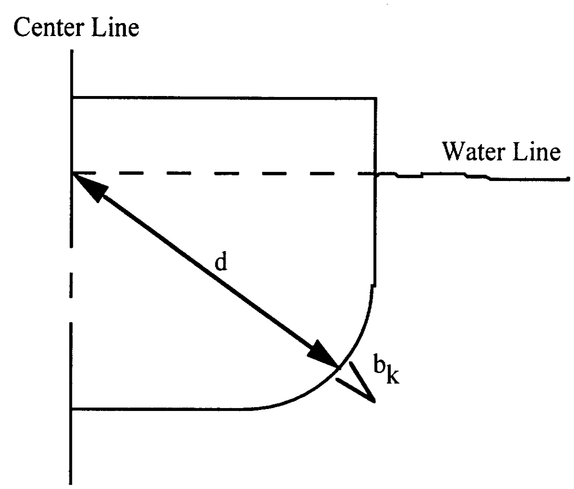


Figure 8 : Bilge Keel

Ocean waves are a stationary zero mean Gaussian random process, the applicable probability theory shows that the significant roll amplitude (mean value of the 1/3 highest amplitudes) is given by multiplying the square root of the response variance (the RMS response) by two [1]:

$$\theta_1 = 2\sqrt{m_0} \qquad (48)$$

where : $m_0 = E = \text{variance of the response} = \int_0^{\infty} SR(\omega)d\omega$

[1] See page 90 of [Beck, and al]

5. MODEL TEST RESULTS

The model tests referred to in § 1.1.3 are used for comparison with the ship motion predicted by the different methods. Four ships were considered : DD 963, DDG 51, MCM and FFG 7. The first three tests were conducted in 1991 and 1992 by the Naval Surface Warfare Center, Carderock Division (former David Taylor Research Center) in support of the Naval Sea Systems Command (NAVSEA). The last test was conducted by the Hydromechanics Laboratory at the United States Naval Academy.

5.1 DD 963

The objective of this test was to evaluate the intact and damaged stability of the DD 963 class ship [Jones, Harry D. (September 1991)]. A 6.91 m fiberglass model with all appendages was tested in beam waves and wind at zero speed. Long-crested irregular wave conditions were modeled for Sea States 5, 6, 7 and 8 with a wind speed of 60 knots; with a Bretschneider spectrum using the mean values of the significant wave amplitude and the most probable modal wave period for each sea state. An additional condition representing a storm was also tested with a wind speed of 100 knots using a sea spectrum measured during Hurricane Camille. The wind was generated using a fan. The fan speed was determined by matching, in calm water, the heel angle calculated with the wind heeling and righting arm curves. During this test, the wind speed was also measured above the model with anemometers. A mean speed of 13.7 knots was measured, this corresponds with a Froude number scale [1] to a full scale speed of

$13.56 \cdot \sqrt{24.824} = 67.56$ knots.

The ship characteristics for the model representing the real ship condition (GM2 in the test report) were (with a scale ratio of 24.824) :

- $\Delta =$ 8345 t
- $L_{pp} =$ 171.6 m
- B = 16.8 m
- T = 6.10 m
- GM = 1.37 m
- Roll Period : $T_0 =$ 12 s

The results of the tests, for the intact condition and the most probable modal period of each sea state, are presented in Table 9 :

---

[1] [Newman (1977)] shows in § 2.4 to 2.6 that model testing of drag forces should be scaled with a constant Reynolds number ($\mathbf{R} = \frac{V\sqrt{A}}{\upsilon}$, where V is the speed of the fluid, A the cross sectional area and υ the dynamic viscosity ($1.5\ 10^{-5}$ for air)). However for bluff bodies with sharp edges at high Reynolds number the drag becomes independent of the Reynolds number as long as the flow is fully turbulent. For our problem, the Reynolds number of the full scale ship with a wind speed of 60 knots is around 10^8, and around 10^6 for the model. This is sufficiently high to insure turbulent flow. We will consider the drag independent of the Reynolds number. In the equilibrium of drag and restoring forces, the latter are “gravitational forces’, therefore the experiment is scaled with a constant Froude number ($\mathbf{F} = \frac{U}{\sqrt{gl}}$).

Table 9 : DD 963 Model Test Results

Sea State	Significant Wave Height (m)	Most Probable Modal Period (s)	RMS Roll (deg)	Heel Angle (deg) [1]	Significant Roll Amplitude (deg) [2]
5	3.57	9.7	2.98	6.34	5.96
6	5.11	12.4	5.62	6.45	11.24
7	8.25	15	7.35	6.17	14.7
8	10.11	16.4	7.81	6.19	15.62
Climatic	12.34	13.4	10.88	13.61	21.76

5.2 DDG 51

The objective of this test was to evaluate the intact and damaged stability of the DDG 51 class ship [Jones, Harry D. (January 1992)]. As for DD 963, a fiberglass model with all appendages was tested in beam waves and wind at zero speed. Long-crested irregular wave conditions were modeled for Sea States 5, 6, 7 and 8 with a wind speed of 60 knots; with a Bretschneider spectrum using the mean values of the significant wave amplitude and the most probable modal wave period for each sea state. An additional condition representing a storm was also tested with a wind speed of 100 knots using a sea spectrum measured during Hurricane Camille. The wind was generated using a fan. The fan speed was determined by matching, on calm water, the heel angle calculated with the wind heeling and righting arm curves.

---

[1] The equilibrium heel angle is determined by taking the mean of the angles measured during the test.

[2] Significant roll amplitude or average of the 1/3 highest amplitudes. The waves follow a Raleigh distribution as do the roll amplitudes for a linear system. Therefore $\theta_{1/3} = 2*E^{1/2} = 2*RMS$

The ship characteristics of the model representing the real ship condition (Model Configuration 1 in the test report) were (with a scale ratio of 24.824) :

- $\Delta =$ 8693 t
- $L_{pp} =$ 142 m
- B = 18.9 m
- T = 6.34 m
- GM = 1.26 m
- Roll Period : $T_0 = 13$ s

The results of the test, for the intact condition and the most probable modal period of each sea state, are presented in Table 10 :

Table 10 : DDG 51 Model Test Results

Sea State	Significant Wave Height (m)	Most Probable Modal Period (s)	RMS Roll (deg)	Heel Angle (deg) [1]	Significant Roll Amplitude (deg) [2]
5	3.53	9.7	1.61	5.36	3.22
6	5.09	12.4	2.92	5.39	5.84
7	8.58	15	5.27	5.91	10.54
8	9.79	16.4	5.50	5.41	11.00
Climatic	10.66	13.4	8.02	13.63	16.04

[1] The equilibrium heel angle is determined by taking the mean of the angles measured during the test.

[2] Significant roll amplitude or average of the 1/3 highest amplitudes. The waves follow a Raleigh distribution as do the roll amplitudes for a linear system. Therefore $\theta_{1/3} = 2*E^{1/2} = 2*RMS$

5.3 MCM

The objective of this test was to evaluate the intact and damaged stability of the MCM class ship [Jones, Harry D. (September 1992)]. As for DD 963 and DDG 51, a fiberglass model with all appendages was tested in beam waves and wind at zero speed. Long-crested irregular wave conditions were modeled for Sea States 5, 6, 7 and 8 with wind speeds of 70 and 90 knots with a Bretschneider spectrum using the mean values of the significant wave amplitude and the most probable modal wave period for each sea state. An additional condition representing a storm was also tested with wind speeds of 90 and 112 knots with a spectrum measured during Hurricane Camille. The wind was generated using a fan. The fan speed was determined by matching, on calm water, the heel angle calculated with the wind heeling and righting arm curves.

The ship characteristics of the model representing the real ship condition (GM1 in the test report) were (with a scale ratio of 24.824) :

- $\Delta =$ 1261 t
- $L_{pp} =$ 62.6 m
- B = 11.9 m
- T = 3.14 m
- GM = 1.55 m
- Roll Period : $T_0 = 8$ s

The results of the test, for the intact condition and the most probable modal period of each sea state, are presented in Table 11 :

Table 11 : MCM Model Test Results

Wind Speed (knots)	Sea State	Significant Wave height (m)	Most Probable Modal Period (s)	RMS Roll (deg)	Heel Angle (deg)[1]	Significant Roll Amplitude (deg)[2]
70	5	3.20	9.7	4.38	6.84	8.76
70	6	4.75	12.4	4.79	7.25	9.58
70	7	8.06	15	5.79	7.84	11.58
70	8	9.71	16.4	5.73	7.04	11.46
90	8	10.04	16.4	5.68	11.57	11.36
90	Climatic	11.19	13.4	6.39	11.45	12.78
112	Climatic	10.94	13.4	7.01	16.57	14.02

5.4 FFG 7

The objective of this test was to evaluate the intact stability of the FFG 7 class ship and the influence of the modeling of the superstructure (2D or 3D) [Chong, et al. (June 1993)]. A fiberglass model with all appendages was tested in beam waves and wind at zero speed. A long-crested irregular wave condition was modeled for Sea State 8 with a wind speed of 80 knots with a modified ITTC spectrum using the mean value of the specific wave amplitude and the most probable modal wave period. The wind was generated using a fan. The fan speed was determined

[1] The equilibrium heel angle is determined by taking the mean of the angles measured during the test.

[2] Significant roll amplitude or average of the 1/3 highest amplitudes. The waves follow a Raleigh distribution as do the roll amplitudes for a linear system. Therefore $\theta_{1/3} = 2*E^{1/2} = 2*RMS$

by measuring the wind speed above the model with anemometers and scaling it with a constant Froude number.

The ship characteristics of the model were (3D with a scale ratio of 36.00) :

- $\Delta =$ 4182.8 t
- T = 4.91 m
- KG = 5.82 m
- GM = 0.93 m
- Roll Period : $T_0 = 11.34$ s

The results of the test for the intact condition are presented in Table 12 :

Table 12 : FFG 7 Model Test Results

Wind Speed (knots)	Sea State	Significant Wave Height (m)	Most Probable Modal Period (s)	RMS Roll (deg)	Heel Angle (deg) [1]	Significant Roll Amplitude (deg)[2]
100	0	11.52	16.21	2.72	6.08	5.44
0	8	11.52	16.21	9.92	0.23	19.84
100	8	11.52	16.21	7.18	6.78	14.36

[1] The equilibrium heel angle is determined by taking the mean of the angles measured during the test.

[2] Significant roll amplitude or average of the 1/3 highest amplitudes. The waves follow a Raleigh distribution as do the roll amplitudes for a linear system. Therefore $\theta_{1/3} = 2*E^{1/2} = 2*RMS$

6. COMPARATIVE ANALYSIS

To evaluate the proposed criteria its requirements for four naval ships are compared with the requirements of other existing criteria : the current US Navy criteria and the civilian IMO criteria [1]. The ship motion considered by the different criteria is also compared with the model test results described in § 5. The four ships considered are, by increasing displacement :

- MCM
- FFG 7
- DDG 51
- DD 963

Each method is programmed using the software [Mathcad]. The ship conditions used for the calculations are as close as possible to the conditions used for the model tests, to allow comparisons between model results and calculation results. An Ochi North Atlantic wave spectrum (nine modal period family, two parameter model) is used in the proposed criteria and applied to the Response Amplitude Operator (RAO) calculated using the linearized equations of motion. The damping factor is adjusted to the roll angle by an iterative process. When the criteria are compared with test results, the measured significant wave height is used as the significant wave height parameter of the Ochi spectrum. A Design Condition is also calculated for Full Load and Minimum Operating conditions, with the NATO significant wave height used as the significant wave height parameter of the Ochi spectrum and with the wind speed specified by

---

[1] For the purpose of comparison, the wind speed specified by Resolution A.749 (50.54 knots) is replaced by the wind speed used for the tests or specified for the Design Condition.

[DDS 079]. The down-flooding angle is not available for the ships considered. An angle of 70° is used for the calculations.

6.1 MCM

6.1.1 Ship Characteristics

To calculate the dynamic stability using the proposed method, various ship characteristics are required. These characteristics are found in four sources : the model test report by [Jones (1992)], the model test report by [Chong, et al., June 1993], ship drawings and results calculated using [POSSE].

The first model test report provides the following characteristics :

- Displacement : $\Delta = 1261$ t
- Length between perpendiculars : $L_{pp} = 62.6$ m
- Beam : B = 11.9 m
- Draft : T = 3.14 m
- Depth D = 7.33 m
- Transverse metacentric height : GM = 1.55 m

The second model test report provides the following characteristics (the same model was used in both tests) :

- Area above waterline : $A = 471\ m^2$

- Distance between the center of the projected underwater area and the center of the projected area above the water : $z = 5.96$ m

The following characteristics are measured from ship drawings [1]:

On NAVSEA J 53711 802 5364552 A, Molded lines and offsets :

- Skeg area : $A_s = 19.81\ m^2$
- Bilge keel length : $L_k = 21.34$ m
- Bilge keel height : $b_k = 0.76$ m
- Distance between the center-line and the bilge keel : $d = 5.35$ m

On NAVSEA H 53711 601 5976943 D, Outboard profile :

- Block coefficient : $C_B = 0.528$

Finally the characteristics calculated by POSSE are :

- Height of center of gravity : KG = 4.99 m
- Righting arm curve [2] : GZ (see Table 13)
- Water-plane coefficient : $C_W = 0.80$

Table 13 : MCM - GZ for Test Load

θ (°)	0	5	10	15	20	25	30	35	40	50	60	70	80
GZ (m)	0.00	0.14	0.28	0.40	0.51	0.59	0.65	0.70	0.73	0.67	0.46	0.17	-0.18

[1] The drawings were in US units and were converted into metric units using 1 ft = 0.3048 m.

[2] The GZ curve corresponding to a full load KG was used as the test load displacement is closer to that condition.

For the Design Condition calculations, supplementary data are provided by POSSE :

	For Full Load	Minimum Operating
• Displacement	Δ = 1289 t	Δ = 1200 t
• Draft :	T = 3.53 m	T = 3.39 m
• Height of center of gravity :	KG = 4.99 m	KG = 5.12 m
• Transverse metacentric height :	GM = 1.58 m	GM = 1.55 m
• Righting arm curve (see Table 14) :	GZ	GZ

Table 14 : MCM - GZ for Full Load and Minimum Operating Conditions

θ (°)	0	5	10	15	20	25	30	35	40	50	60	70	80
GZ (m) Full	0.00	0.14	0.27	0.39	0.50	0.59	0.65	0.69	0.73	0.66	0.45	0.15	-0.19
GZ (m) MinOp	0.00	0.14	0.27	0.38	0.48	0.55	0.60	0.63	0.65	0.59	0.38	0.08	-0.27

6.1.2 Calculations

With these ship characteristics, for each method, the ship motions and the energy balance are calculated for the sea states and wind used by the model tests. The calculations are similar to those presented in Appendix 2. The results are summarized in Table 15.

Table 15 : MCM - Comparison Test Load Condition

Wind	Sea State	Method	θ_0	θ_1	θ_w	A_1	A_2	A_2/A_1
70	5	Navy	7.41	25.00	-17.59	8.59	25.78	3.00
		IMO	7.76	14.87	-7.11	5.17	13.67	2.64
		Proposed	7.53	7.25	0.28	1.66	15.75	9.49
		Test	8.84	8.76	0.08	NA	NA	NA
70	6	Navy	7.41	25.00	-17.59	8.59	25.78	3.00
		IMO	7.76	14.87	-7.11	5.17	13.67	2.64
		Proposed	7.53	9.14	-1.61	2.32	15.75	6.79
		Test	7.25	9.58	-2.33	NA	NA	NA
70	7	Navy	7.41	25.00	-17.59	8.59	25.78	3.00
		IMO	7.76	14.87	-7.11	5.17	13.67	2.64
		Proposed	7.53	12.08	-4.55	3.53	15.75	4.46
		Test	7.84	11.58	-3.74	NA	NA	NA
70	8	Navy	7.41	25.00	-17.59	8.59	25.78	3.00
		IMO	7.76	14.87	-7.11	5.17	13.67	2.64
		Proposed	7.53	13.46	-5.93	3.99	15.75	3.95
		Test	7.04	11.46	-4.42	NA	NA	NA
90	8	Navy	12.28	25.00	-12.72	8.65	20.98	2.43
		IMO	13.30	14.87	-1.57	6.33	4.19	0.66
		Proposed	12.79	13.77	-0.98	5.36	6.55	1.22
		Test	11.57	11.36	0.21	NA	NA	NA

Results are also calculated for a wind speed of 80 knots (wind speed to be considered for ships that will be expected to avoid centers of tropical storms [DDS 079] which is appropriate for MCM) and for a sea state 8/9 (14 m is the maximum specific wave height considered by NATO standards) at both Full Load and Minimum Operating conditions. These calculations are in Appendix 2, the results are summarized in Table 16.

Table 16 : MCM Design Conditions

Load	Wind	Sea State	Method	θ_0	θ_1	θ_w	A_1	A_2	A_2/A_1
Full	80	8/9	Navy	9.75	25.00	-15.25	8.65	23.18	2.68
			IMO	10.38	16.04	-5.66	6.00	8.91	1.48
			Proposed	10.00	17.25	-7.25	6.91	11.18	1.62
MinOp	80	8/9	Navy	10.50	25.00	-14.50	8.48	19.19	2.26
			IMO	11.31	15.40	-4.10	6.17	5.15	0.83
			Proposed	10.88	17;04	-6.17	7.04	7.18	1.02

6.1.3 Analysis

All three methods calculate values for the static heel angle close to the measured values (within 1.3 ° for the proposed method), however the proposed method is the only one to provide a reasonable estimate of roll angle as compared to model tests. The difference between the results given by the proposed method and the model test is never greater than 2.4°. This shows a very good correlation between the proposed calculation and the tests and confirms the validity of the method.

The analysis of the energy balance also shows an important advantage of the proposed method, the area ratio decreases with the increasing sea state and gives a much better appreciation of the effective stability of the ship. The proposed method considers the specific ship roll characteristics. Wind gusts are also considered specifically, which eliminates the necessity for the 40% safety factor for the area ratio. The current US Navy method is very constraining for ships not operating in severe conditions and does not consider ship roll characteristics other than the righting arm curve. The very constraining wind heeling arm of the IMO method is shown in the test with a wind speed of 90 knots : the criteria shows a energy balance below one and predicts insufficient stability when the model test and the proposed method predict adequate stability.

The Design Condition case shows that MCM meets the intact stability requirement for both Full Load and Minimum Operating Conditions ($A_2/A_1 > 1$). This case also shows the limits of the IMO method for naval ships. From a Sea State 8/9 and above, the IMO method predicts smaller roll amplitudes than the proposed method.

6.2 FFG 7

6.2.1 Ship Characteristics

To calculate the dynamic stability using the proposed method, various ship characteristics are required. These characteristics are found in three sources : the model test report by [Chong. et al. (June 1993)], results from the ship design tool [ASSET] and results calculated using POSSE.

The following characteristics are found in the model test report :

- Displacement : $\Delta = 4182.8$ t
- Draft : $T = 4.91$ m
- Transverse metacentric height : $GM = 0.93$ m
- Height of center of gravity : $KG = 5.82$ m
- Area above waterline : $A = 1297.45\ m^2$
- Distance between the center of the projected underwater area and the center of the projected area above the water : $z = 9.33$ m

The following characteristics are calculated using ASSET (displacement similar to the model displacement) :

- Length between perpendiculars : $L_{pp} = 124.4$ m
- Beam : $B = 13.7$ m
- Depth $D = 9.10$ m
- Block coefficient : $C_B = 0.446$
- Water-plane coefficient : $C_W = 0.74$

- Skeg area : $A_s = 18.2\ m^2$
- Bilge keel length : $L_k = 29.38$ m
- Bilge keel height : $b_k = 0.91$ m
- Distance between the center-line and the turn of the bilge : d = 5.66 m
- Stabilizing fins projected area [1] : $A_f = 11.2\ m^2$

Finally the characteristics calculated by POSSE are :

- Righting arm curve : GZ (see Table 17)

Table 17 : FFG 7 - GZ for Test Load

θ (°)	0	5	10	15	20	25	30	35	40	50	60	70	80
GZ (m)	0.00	0.09	0.19	0.28	0.39	0.49	0.60	0.72	0.80	0.84	0.75	0.60	0.42

For the Design Condition calculations, supplementary data are provided by POSSE :

	For Full Load	Minimum Operating
• Displacement	Δ = 4031 t	Δ = 3807 t
• Draft :	T =4.83 m	T = 4.65 m
• Height of center of gravity :	KG = 5.73 m	KG = 5.93 m
• Transverse metacentric height :	GM = 0.99 m	GM = 0.86 m

[1] FFG 7 has one pair of stabilizing fins. The projected area of the fins is added to the bilge keel area to calculate the damping coefficient.

		For Full Load	Minimum Operating
•	Righting arm curve (see Table 18) :	GZ	GZ

Table 18 : FFG 7 - GZ for Full Load and Minimum Operating Conditions

θ (°)	0	5	10	15	20	25	30	35	40	50	60	70	80
GZ (m) Full	0.00	0.09	0.19	0.28	0.38	0.48	0.59	0.71	0.79	0.85	0.77	0.63	0.45
GZ (m) MinOp	0.00	0.08	0.16	0.24	0.31	0.39	0.48	0.57	0.66	0.71	0.64	0.51	0.33

6.2.2 Calculations

With these ship characteristics, for each method, the ship motions and the energy balance are calculated for the sea states and wind used by the model tests. The calculations are similar to those presented in Appendix 3. The results are summarized in Table 19.

Table 19 : FFG 7 - Comparison Test Load Condition

Wind	Sea State	Method	θ_0	θ_1	θ_w	A_1	A_2	A_2/A_1
100	8	Navy	24.04	25.00	-0.96	7.57	21.55	2.84
		IMO	29.18	14.13	15.05	NA	NA	NA
		Proposed	26.46	15.74	10.72	8.46	5.88	0.69
		Test	6.78	14.36	-7.58	NA	NA	NA

Results are also calculated for a wind speed of 100 knots (wind speed to be considered for ships that will weather full force of tropical cyclones [DDS 079]) and for a sea state 8/9 (14 m is the maximum specific wave height considered by NATO standards) at both Full Load and Minimum Operating Conditions. These calculations are in Appendix 3, the results are summarized in Table 20.

Table 20 : FFG 7 - Design Condition

Load	Wind	Sea State	Method	θ_0	θ_1	θ_w	A_1	A_2	A_2/A_1
Full	100	8/9	Navy	25.04	25.00	0.04	7.56	21.36	2.82
			IMO	30.51	14.11	16.40	0.00	0.00	NA
			Proposed	27.63	17.90	9.73	10.09	5.49	0.54
MinOp	100	8/9	Navy	29.46	25.00	4.46	7.28	15.11	2.07
			IMO	38.63	13.27	25.35	NA	NA	NA
			Proposed	33.49	16.72	16.76	9.52	0.65	0.07

6.2.3 Analysis

As concluded in the model test report [Chong. et al., (June 1993)], the static heel angle measured during the tests is not consistent with the values calculated by the other methods or the value measured for other similar ships. The test report proposes the following explanation : the size of the model is not adapted to the size of the wind profile and the extremities of the model are in zones of reduced wind. However, the large difference between the expected and measured value seem to indicate a test error, possibly in the Froude number scale of the wind speed. The measured heel angle is obtained with the proposed method for a wind speed around 50 knots.

The roll amplitude is very similar to the test value for both the IMO and the proposed method (greater by 2.10° for the proposed method). The proposed method is more constraining than IMO. This good correlation gives confidence in the method.

The analysis of the energy balance however is disappointing at first sight. The IMO wind heeling arm is larger than the maximum righting arm, therefore no energy balance calculation is possible. This again demonstrates the extremely constraining heeling arm in some cases considered by IMO. The proposed method shows an area ratio below one (0.60 for the test sea state). This means inadequate stability. The current US Navy criteria shows an energy balance similar to that found for MCM in § 6.1.2.

The Design Condition case shows that FFG 7 has inadequate stability for both Full Load and Minimum Operating conditions. It seems important to determine the level of stability met by FFG 7. Table 21 presents the results for the maximum wind for which the stability is adequate using the proposed method ($A_2/A_1 > 1$). It is reassuring that these wind speeds are close to the wind speed authorized by the US Navy (DDS 079) for ships in service (90 knots). It is also important to realize that FFG 7 is a ship operating with dynamic stabilizing fins , therefore, it has a reduced stability without the use of its stabilizers.

Table 21 : FFG 7 - Maximum Design Wind

Load	Wind	Sea State	Method	θ_0	θ_1	θ_w	A_1	A_2	A_2/A_1
Full	91	8/9	Navy	21.88	25.00	-3.12	7.42	23.58	3.18
			IMO	25.90	14.09	11.80	7.03	1.62	0.23
			Proposed	23.70	17.90	5.79	8.88	9.37	1.06
MinOp	82	8/9	Navy	22.83	25.00	-2.17	6.01	19.11	3.18
			IMO	27.18	13.27	13.90	5.71	0.95	0.17
			Proposed	24.88	16.72	8.15	6.47	7.17	1.11

The model test had an abnormally low static heel angle and was not able to properly demonstrate the level of stability of the ship. It is possible that FFG 7 has an insufficient stability that is not indicated by the current criteria. Additional model tests or full scale measurements are necessary to properly evaluate FFG 7 static stability.

6.3 DDG 51

6.3.1 Ship Characteristics

To calculate the dynamic stability using the proposed method, various ship characteristics are required. These characteristics are found in two sources : the model test report [Jones (1992)], results from the ship design tool ASSET.

The following characteristics are found in the model test report :

- Displacement : $\Delta = 8693$ t
- Length between perpendiculars : $L_{pp} = 142$ m
- Beam : B = 18 m
- Draft : T = 6.34 m
- Transverse metacentric height : GM = 1.26 m

The following characteristics are calculated using ASSET :

- Depth D = 12.70 m
- Height of center of gravity : KG = 6.90 m
- Block coefficient : $C_B = 0.505$
- Water-plane coefficient : $C_W = 0.79$
- Area above waterline : $A = 1442.5\ m^2$
- Distance between the center of the projected underwater area and the center of the projected area above the water : z = 9.52 m
- Skeg area : $A_s = 14\ m^2$
- Bilge keel length : $L_k = 49.22$ m
- Bilge keel height : $b_k = 0.91$ m
- Distance between the center-line and the bilge keel : d = 8.25 m
- Righting arm curve : GZ (see Table 22)

Table 22 : DDG 51 - GZ for Test Load

θ (°)	0	5	10	15	20	25	30	35	40	50	60	70	80
GZ (m)	0.00	0.14	0.28	NA	0.59	NA	0.76	NA	1.25	1.45	1.34	1.19	NA

DDG 51 is a compensated ship (fuel is continuously replaced by sea water in the tanks), therefore the most constraining load case is Full Load (sea water density is greater than fuel density). For the Design Condition calculation, supplementary data are provided by ASSET :

- Displacement — $\Delta = 8304$ t
- Draft : — $T = 6.22$ m
- Height of center of gravity : — $KG = 7.21$ m
- Transverse metacentric height : — $GM = 1.66$ m
- Righting arm curve : — GZ (see Table 23)

Table 23 : DDG 51 - GZ for Full Load Condition

θ (°)	0	5	10	15	20	25	30	35	40	50	60	70	80
GZ (m)	0.00	0.14	0.28	NA	0.59	NA	0.76	NA	1.25	1.45	1.34	1.19	NA

6.3.2 Calculations

With these ship characteristics, for each method, the ship motions and the energy balance are calculated for the sea states and wind used by the model tests. The calculation sheets are similar to the sheet presented in Appendix 4. The results are summarized in Table 24.

Table 24 : DDG 51 - Comparison

Wind	Sea State	Method	θ_0	θ_1	θ_w	A_1	A_2	A_2/A_1
60	5	Navy	3.87	25.00	-21.13	10.72	59.41	5.54
		IMO	4.00	13.34	-9.34	3.75	52.57	14.00
		Proposed	3.89	6.28	-2.40	1.00	53.78	53.55
		Test	5.36	3.22	2.14	NA	NA	NA
60	6	Navy	3.87	25.00	-21.13	10.72	59.41	5.54
		IMO	4.00	13.34	-9.34	3.75	52.57	14.00
		Proposed	3.89	8.04	-4.15	1.58	53.78	33.93
		Test	5.39	5.84	-0.45	NA	NA	NA
60	7	Navy	3.87	25.00	-21.13	10.72	59.41	5.54
		IMO	4.00	13.34	-9.34	3.75	52.57	14.00
		Proposed	3.89	12.09	-8.20	3.19	53.78	16.86
		Test	5.91	10.54	-4.63	NA	NA	NA
60	8	Navy	3.87	25.00	-21.13	10.72	59.41	5.54
		IMO	4.00	13.34	-9.34	3.75	52.57	14.00
		Proposed	3.89	13.26	-9.38	3.68	53.78	14.61
		Test	5.41	11.00	-5.59	NA	NA	NA

A calculation was also made for a wind speed of 100 knots (wind speed to be considered for ships that will weather full force of tropical cyclones [DDS 079]) and for a sea state 8/9 (14 m is the maximum specific wave height considered by NATO standards) for Full Load condition. This calculation is in Appendix 4, the results are summarized in Table 25.

Table 25 : DDG 51 - Full Load Design Condition

Wind	Sea State	Method	θ_0	θ_1	θ_w	A_1	A_2	A_2/A_1
100	8/9	Navy	11.13	25.00	-13.87	9.34	51.64	5.53
		IMO	11.85	14.40	-2.55	5.78	33.48	5.79
		Proposed	11.47	18.13	-6.66	8.06	36.83	4.57

6.3.3 Analysis

All three methods calculate values for the static heel angle close to the measured values (within 2.1° for the proposed method), however the proposed method is the only one for which roll amplitude increases with increasing sea state. The difference between the roll amplitude predicted by the proposed method and the model test is never greater than 3.1°. This shows a very good correlation between the proposed calculation and the tests, it confirms the validity of the method.

The analysis of the energy balance also shows an important advantage of the proposed method. The area ratio decreases with the increasing sea state and gives a much better prediction of the effective stability of the ship. The area ratio remains very large, but the wind speed is reduced compared to the design wind speed.

The Design Condition case shows that the design meets the stability requirement.

6.4 DD 963

6.4.1 Ship Characteristics

To calculate the dynamic stability using the proposed method, various ship characteristics are required. These characteristics are found in three sources : the model test report by [Jones (Sept. 1991)], ship drawings and results calculated using POSSE.

The following characteristics are found in the model test report :

- Displacement : $\Delta = 8345$ t
- Beam : B = 16.80 m
- Draft : T = 6.10 m
- Transverse metacentric height : GM = 1.37 m

The following characteristics are measured on ship drawings [1]:

On NAVSEA H 53711 802 5000179 B. Lines and Molded Offsets :

- Length between perpendiculars : $L_{pp} = 161.6$ m
- Block coefficient : $C_B = 0.461$

On NAVSHIP 845-4539497. Outboard Profile :

- Depth D = 12.95 m
- Area above waterline : A = 2039.7 m^2

[1] The drawings were in US units and were converted into metric units using 1 ft = 0.3048 m.

- Distance between the center of the projected underwater area and the center of the projected area above the water : $z = 10.76$ m
- Skeg area : $A_s = 20.90$ m^2
- Bilge keel length : $L_k = 53.34$ m
- Bilge keel height : $b_k = 1.016$ m
- Distance between the center-line and the bilge keel : $d = 7.92$ m

Finally the characteristics calculated by POSSE are :

- Height of center of gravity : KG = 6.97 m
- Righting arm curve : GZ (see Table 26)
- Water-plane coefficient : $C_W = 0.74$

Table 26 : DD 963 - GZ for Test Load

θ (°)	0	5	10	15	20	25	30	35	40	50	60	70	80
GZ (m)	0.00	0.09	0.19	0.28	0.37	0.47	0.57	0.68	0.80	1.00	1.01	0.94	0.82

DD 963 is a compensated ship (fuel is continuously replaced by sea water in the tanks), therefore the worse load case is Full Load (sea water density is greater than fuel density). For the Design Condition calculation, supplementary data are provided by POSSE :

- Displacement $\Delta = 9038$ t

- Draft : T = 6.45 m
- Height of center of gravity : KG = 7.00 m
- Transverse metacentric height : GM = 1.01 m
- Righting arm curve : GZ (see Table 27)

Table 27 : DD 963 - GZ Full Load Condition

θ (°)	0	5	10	15	20	25	30	35	40	50	60	70	80
GZ (m)	0.00	0.09	0.18	0.27	0.36	0.47	0.58	0.70	0.83	0.99	0.97	0.85	0.74

6.4.2 Calculations

With these ship characteristics, for each method, the ship motions and the energy balance are calculated for the sea states and wind used by the model tests. The calculations are similar to those presented in Appendix 5. The results are summarized in Table 28.

Results are also calculated for a wind speed of 100 knots (wind speed to be considered for ships that will weather full force of tropical cyclones [DDS 079]) and for a sea state 8/9 (14 m is the maximum specific wave height considered by NATO standards) at Full Load condition. This calculation is in Appendix 5, the results are summarized in Table 29.

Table 28 : DD 963 - Comparison

Wind	Sea State	Method	θ_0	θ_1	θ_w	A_1	A_2	A_2/A_1
60	5	Navy	9.16	25.00	-15.84	5.72	37.90	6.63
		IMO	9.68	13.17	-3.50	3.03	27.71	9.13
		Proposed	9.35	6.57	2.78	1.24	30.95	25.00
		Test	6.34	5.96	0.38	NA	NA	NA
60	6	Navy	9.16	25.00	-15.84	5.72	37.90	6.63
		IMO	9.68	13.17	-3.50	3.03	27.71	9.15
		Proposed	9.35	8.40	0.95	1.71	30.95	18.10
		Test	6.45	11.24	-4.79	NA	NA	NA
60	7	Navy	9.16	25.00	-15.84	5.72	37.90	6.63
		IMO	9.68	13.17	-3.50	3.03	27.71	9.13
		Proposed	9.35	11.53	-2.18	2.56	30.95	12.10
		Test	6.17	14.70	-8.53	NA	NA	NA
60	8	Navy	9.16	25.00	-15.84	5.72	37.90	6.63
		IMO	9.68	13.17	-3.50	3.03	27.71	9.13
		Proposed	9.35	13.23	-3.88	2.88	30.95	10.75
		Test	6.19	15.62	-9.43	NA	NA	NA

Table 29 : DD 963 - Full Load Design Condition

Wind	Sea State	Method	θ_0	θ_1	θ_w	A_1	A_2	A_2/A_1
100	8/9	Navy	21.48	25.00	-3.52	6.79	29.27	4.31
		IMO	24.74	12.63	12.11	5.59	7.95	1.42
		Proposed	23.09	14.68	8.41	6.34	15.16	2.39

6.4.3 Analysis

All three methods calculate values for the static heel angle close to the measured values (within 3.5° for the proposed method), however the proposed method is the only one for which roll amplitude increases with increasing sea state. The difference between the roll amplitude predicted by the proposed method and the model test is never greater than 3.2°. This shows a very good correlation between the proposed calculation and the tests, it confirms the validity of the method. It shows also that the IMO method is less conservative than the proposed method above Sea State 8.

The analysis of the energy balance shows also an important advantage of the proposed method. The area ratio decreases with the increasing sea state and gives a much better prediction of the effective stability of the ship. The area ratio remains very large, but the wind speed is reduced compared to the design wind speed.

The Design Condition case shows that the design meets the stability requirement, but has a lower dynamic stability than DDG 51.

7. CONCLUSION

The intact stability criteria used by the US Navy and other countries has significant limitations :

1. It is empirical and dimensionally incorrect.
2. It considers an unrealistic wind heeling arm that reduces to zero at 90° of heel.
3. It does not vary the requirement with the sea state considered.
4. It considers only a steady wind, but includes a safety factor of 40%.
5. It does not include in its analysis the elements on a ship that improve its stability, like bilge keels.

The intact stability criteria used by the International Maritime Organization (IMO) also has limitations :

1. It is not suited for naval ships which are required to withstand more severe conditions.
2. The wind heeling arm constant with angle of heel is very constraining for ships with a large superstructure as is the case for naval ships.
3. The formula used to calculate the roll amplitude is empirical and dimensionally incorrect.
4. The criteria considers ship characteristics which reduce its roll motion, but the method is based on empirical data from commercial vessels.

5. The criteria does not consider specific sea states. A simplified conservative approach is used, but this approach is not conservative for the sea and wind conditions considered for naval ships.

The method proposed in this thesis presents many advantages :

1. It is based on the fundamental linear theory of motion while remaining simple.
2. It considers a wind heeling arm better adapted to naval ships, but is still simple and conservative.
3. Gusts are explicitly considered with the wind, therefore the dynamic stability is specified by $A_2/A_1 = 1$ without the 40% safety factor required by the US Navy criteria.
4. It considers ship characteristics which influence the ship roll motion.
5. It considers a required sea state using its wave spectrum.

These characteristics allow more flexibility with the requirements and rationale for future modifications of the criteria.

The comparison of the proposed criteria with the existing criteria and model test results shows in § 6 the validity of the proposed criteria and its superiority over the other criteria.

Additional studies are recommended based on the results presented in this thesis :

1. Study of the sensibility of the roll amplitude to the wave spectra considered.
2. Proposition of a damaged stability criteria based on the same method.
3. Complementary studies on models or full scale ship for FFG 7.
4. Update the formula for damping using modern naval ships.

REFERENCES

- A.167, IMO Resolution, "Recommendation on Intact Stability for Passenger and Cargo Ships under 100 meters in Length", International Maritime Organization, adopted 1968
- A.562, IMO Resolution, "Recommendations on a Severe Wind and Rolling Criterion for the Intact Stability of Passenger and Cargo Ships of 24 meters in Length and Over", International Maritime Organization, adopted 1985
- A.749, IMO Resolution, "Code on Intact for all Type of Ships Covered by IMO Instruments", International Maritime Organization, adopted 1993
- ASSET, "Advanced Surface Ship Evaluation Tool", Version 4.0, Naval Surface Warfare Center, Carderock Division
- Beck, Robert F., Cummins, William E., Dalzell, John F., Mandel, Philip and Webster, William C., "Motions In Waves", Chapter 8 of the Principal of Naval Architecture (PNA), 1989 by the Society of Naval Architects and Marine Engineers
- C-03-001-024 MS-002, (5 June 1986), "Stability and Buoyancy Requirements, Canadian Armed Forces Surface Ships"
- Chong, Keng Shin, Escamilla, Christopher R., and Compton, Roger H., (June 1993), "Experimental Studies of Ship Stability in Beam Wind and Waves", United States Naval Academy, Hydromechanics Laboratory, Division of Engineering and Weapons, Report EW-07-93
- Colhoun, Raymond (Capt.), "Typhoon: The other enemy."

- Conolly, J.E., and Goodrich, G.J., (1970), "Sea Trials of Antipitching fins", Transactions RINA, Vol. 112
- DDS 079-1, (1 August 1975), "Design Data Sheet : Stability and Buoyancy of U.S. Naval Surface Ships", Naval Ship Engineering Center, U.S. Department of the Navy
- Japan, (25 November 1981), "Intact Stability, Results of Calculation for the Sample Ship", IMCO - Sub-Committee on Subdivision, Stability and Load Lines -, STAB/95
- Japan, (27 September 1979), "Intact Stability, Stability of Ships in Ballast Condition, Weather Criteria", IMCO - Sub-Committee on Subdivision, Stability and Load Lines - 24th session, STAB XXIV/4
- Jones, Harry D., (September 1991) "DD963 Intact and Damage Stability in Beam Waves and Wind", David Taylor Research Center, Ship Hydromechanics Department, Report DTRC/SHD-1360-01
- Jones, Harry D., (January 1992) "DDG-51 Intact and Damage Stability in Beam Waves and Wind", David Taylor Research Center, Ship Hydromechanics Department, Report DTRC/SHD-1360-03
- Jones, Harry D., (September 1992) "MCM Intact and Damage Stability in Beam Waves and Wind", Carderock Division, Naval Surface Warfare Center, Ship Hydromechanics Department, Report CARDEROCKDIV/SHD-1360-04
- Mathcad Plus 6 for Macintosh (1996), Calculation software by MathSoft, Cambridge MA, USA

- Miler, E.R., Slager, J.J., and Webster, W.C., (1974), "Phase 1 Report on the Development of a Technical Practice for Roll Stabilization System Selection", NAVSEA Report 6136-74-280
- NES 109, (August 1989), "Stability Standards for Surface Ships", Issue 3, Sea Systems Controllerate, U.K. Ministry of Defence
- Newman, J. N., (1977), "Marine Hydrodynamics", The MIT Press
- Ochi, Michel K., (1978), "Wave Statistics for the Design of Ships and Ocean Structures", SNAME Transactions
- POSSE, "Program of Ship Salvage Engineering", Version 1.0, by the Naval Sea Systems Command, U.S. Department of the Navy
- Rahola, J., (1935), "The Judging of the Stability of Ships and the Determination of the Minimum Amount of Stability", Transactions, Institute of Naval Architects
- Sarchin and Goldberg, (1992), "Stability and buoyancy criteria for U.S. Naval surface ships" - SNAME
- Surko, Stephen W., (May 1994), "An Assessment of Current Warship Damaged Stability Criteria", Naval Engineers Journal
- Sverdrup, H. U., and Munk, W. H., (March 1947), "Wind, Sea and Swell: Theory of Relations for Forecasting", US Hydrographic Office Pub. No. 601
- URSS, (10 February 1982), IMCO - Sub-Committee on Subdivision, Stability and Load Lines - 27 th session, STAB 27/5/3

APPENDIX 1

NATURAL ROLL PERIOD

Different methods to calculate the roll natural period of a ship exist in the literature. This analysis compares three of these methods with measured natural period. As our study concerns different categories of naval ships, calculations are done for four ships to determine how accurate the formulas are for these ships. The ships considered are :

- MCM
- FFG-7
- DDG-51
- DD-963
- HOLYHEAD Ferry

The following ship characteristics are used:

- L_{pp} = ship length between perpendiculars
- B = ship beam
- T = ship draft
- GM = ship transversal metacentric height

1. METHODS

Principal of Naval Architecture (PNA) method

$$T_n = \frac{2\pi CB}{\sqrt{g\overline{GM}}} = \frac{2\pi B}{\sqrt{g\overline{GM}}} \cdot 0.3613$$

IMO Resolution A.749 method

$$T_n = \frac{2\pi CB}{\sqrt{g\overline{GM}}} = \frac{2\pi B}{\sqrt{g\overline{GM}}} \cdot \left(0.3725 + 0.0227\frac{B}{T} - 0.043\frac{L_{pp}}{100}\right)$$

Japan Stab/95 method for ships with inclined or flared side shell

$$T_n = \frac{2\pi CB}{\sqrt{g\overline{GM}}} = \frac{2\pi B}{\sqrt{g\overline{GM}}} \cdot \left(0.3085 + 0.0227\frac{B}{T} - 0.043\frac{L_{pp}}{100}\right)$$

2. COMPARISON

SHIP	MCM	FFG-7	DDG-51	DD-963	HOLYHEAD
Δ (t)	1 261.	4182.8	8 693.	8 345.	4 082.
L_{PP} (m)	62.6	124.4	142.	171.6	105.7
B (m)	11.9	13.7	18.9	16.8	16.78
T (m)	3.14	4.8	6.64	6.1	3.87
GM (m)	1.55	0.93	1.26	0.88	1.4
T_n (Measured)	8.	11.34	13.	15.	12.2
T_n (PNA)	6.93	10.3	12.2	12.98	10.28
T_n (IMO)	8.28	10.94	12.7	12.98	12.1
T_n (Flare)	7.05	9.11	10.54	10.68	10.28

2. CONCLUSION

The IMO method provides the best estimate of roll natural period for naval ships.

APPENDIX 2

MCM COMPARISON

This Appendix gives an example of the calculations of the dynamic stability for MCM with the US Navy, IMO and the proposed method. The Appendix is divided into three parts :

- the section giving the ship characteristics for the Test Condition
- the sections giving the ship characteristics and the results for the Full Load Condition with a wind speed of 80 knots
- the complete example of the calculations for the Minimum Operating Condition with a wind speed of 80 knots

MCM TEST

UNITS AND CONSTANTS

Second :	$s := sec$
Metric ton :	$t := 1000 \cdot kg$
Air density :	$\rho_a := 1.293 \cdot kg \cdot m^{-3}$
Sea Water density :	$\rho_w := 1025 \cdot kg \cdot m^{-3}$
Gravity :	$g := 9.81 \cdot m \cdot s^{-2}$
Range of angles	$\theta := -30, -29 .. 90$
Constant for graphs	$cte := -1 .. 1$
Matrix origin	$ORIGIN := 1$

SHIP CHARACTERISTICS

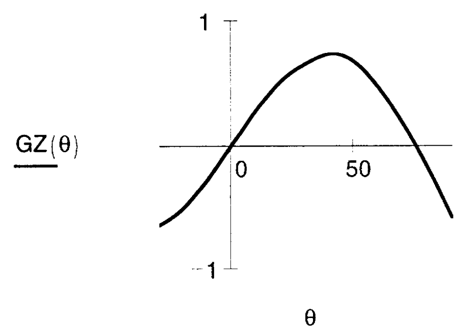
Displacement :	$\Delta := 1261 \cdot t$	
Length between perpendiculars :	$L := 62.6 \cdot m$	
Beam :	$B := 11.9 \cdot m$	
Draft :	$T := 3.14 \cdot m$	
Depth :	$D := 7.33 \cdot m$	
Block coefficient	$C_B := 0.528$	
Waterplane coefficient	$C_{wp} := 0.8$	
Height of center of gravity :	$KG := 4.99 \cdot m$	
Transverse methacentric height	$GM := 1.55 \cdot m$	
Area above waterline :	$A := 471 \cdot m^2$	
Distance between the center of the projected underwater area and the center of the projected area above the water	$z := 5.96 \cdot m$	
Skeg area	$A_s := 19.8 \cdot m^2$	
Length of Bilge Keels	$L_k := 21.34 \cdot m$	
Height of Bilge Keels	$b_k := 0.76 \cdot m$	
Total Area of Bilge Keels	$A_k := 2 \cdot L_k \cdot b_k$	$A_k = 32.437 \cdot m^2$
Distance between center line at DWL and bilge keel	$d := 5.35 \cdot m$	
Total area of bilge keels + skeg	$A_t := A_s + A_k$	$A_t = 52.237 \cdot m^2$

Righting arm curve (GZ)

$$\theta := \begin{bmatrix} -30 \\ -25 \\ -20 \\ -15 \\ -10 \\ -5 \\ 0 \\ 5 \\ 10 \\ 15 \\ 20 \\ 25 \\ 30 \\ 35 \\ 40 \\ 50 \\ 60 \\ 70 \\ 80 \end{bmatrix} \qquad GZ := \begin{bmatrix} -0.65 \\ -0.59 \\ -0.51 \\ -0.40 \\ -0.28 \\ -0.14 \\ 0.00 \\ 0.14 \\ 0.28 \\ 0.40 \\ 0.51 \\ 0.59 \\ 0.65 \\ 0.70 \\ 0.73 \\ 0.67 \\ 0.46 \\ 0.17 \\ -0.18 \end{bmatrix}$$

Cubic interpolation of GZ curve

$$GZ(\theta) := \mathrm{interp}(\mathrm{cspline}(\theta, GZ), \theta, GZ, \theta)\cdot m$$



First intersept default value: $\theta_{d1} := 0$

Second intersept default value: $\theta_{d2} := 90$

Downflooding angle : $\theta_{f} := 70$

MCM FULL LOAD 80 KNOTS

UNITS AND CONSTANTS

Second :	$s := sec$
Metric ton :	$t := 1000 \cdot kg$
Air density :	$\rho_a := 1.293 \cdot kg \cdot m^{-3}$
Sea Water density :	$\rho_w := 1025 \cdot kg \cdot m^{-3}$
Gravity :	$g := 9.81 \cdot m \cdot s^{-2}$
Range of angles	$\theta := -30, -29 .. 90$
Constant for graphs	$cte := -1 .. 1$
Matrix origin	$ORIGIN := 1$

SHIP CHARACTERISTICS

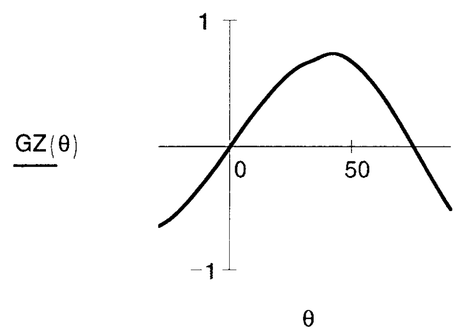
Displacement :	$\Delta := 1289 \cdot t$	
Length between perpendiculars :	$L := 62.6 \cdot m$	
Beam :	$B := 11.9 \cdot m$	
Draft :	$T := 3.53 \cdot m$	
Depth :	$D := 7.33 \cdot m$	
Block coefficient	$C_B := 0.528$	
Waterplane coefficient	$C_w := 0.8$	
Height of center of gravity :	$KG := 4.99 \cdot m$	
Transverse methacentric height	$GM := 1.58 \cdot m$	
Area above waterline :	$A := 471 \cdot m^2$	
Distance between the center of the projected underwater area and the center of the projected area above the water	$z := 5.96 \cdot m$	
Skeg area	$A_s := 19.8 \cdot m^2$	
Length of Bilge Keels	$L_k := 21.34 \cdot m$	
Height of Bilge Keels	$b_k := 0.76 \cdot m$	
Total Area of Bilge Keels	$A_k := 2 \cdot L_k \cdot b_k$	$A_k = 32.437 \cdot m^2$
Distance between center line at DWL and bilge keel	$d := 5.35 \cdot m$	
Total area of bilge keels + skeg	$A_t := A_s + A_k$	$A_t = 52.237 \cdot m^2$

Righting arm curve (GZ)

$$\theta := \begin{bmatrix} -30 \\ -25 \\ -20 \\ -15 \\ -10 \\ -5 \\ 0 \\ 5 \\ 10 \\ 15 \\ 20 \\ 25 \\ 30 \\ 35 \\ 40 \\ 50 \\ 60 \\ 70 \\ 80 \end{bmatrix} \qquad GZ := \begin{bmatrix} -0.65 \\ -0.59 \\ -0.50 \\ -0.39 \\ -0.27 \\ -0.14 \\ 0.00 \\ 0.14 \\ 0.27 \\ 0.39 \\ 0.50 \\ 0.59 \\ 0.65 \\ 0.69 \\ 0.73 \\ 0.66 \\ 0.45 \\ 0.15 \\ -0.19 \end{bmatrix}$$

Cubic interpolation of GZ curve

$$GZ(\theta) := \text{interp}(\text{cspline}(\theta, GZ), \theta, GZ, \theta) \cdot m$$



First intersept default value: $\theta_{d1} := 0$

Second intersept default value: $\theta_{d2} := 90$

Downflooding angle : $\theta_f := 70$

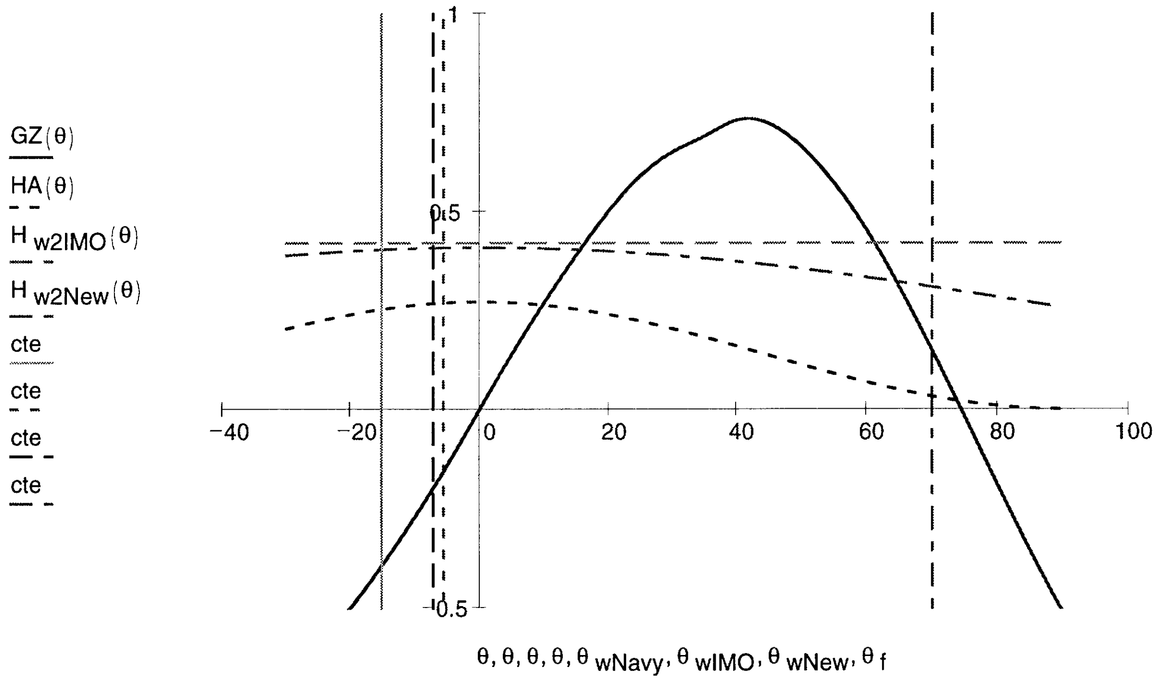
COMPARISON OF THE ANGLES BETWEEN THE THREE METHODS

$\theta_{0Navy} = 9.753$ $\theta_{1Navy} = 25$ $\theta_{2Navy} = 70$ $\theta_{wNavy} = -15.247$

$\theta_{0IMO} = 10.384$ $\theta_{1IMO} = 16.041$ $\theta_{2IMO} = 61.171$ $\theta_{wIMO} = -5.656$

$\theta_{0New} = 9.997$ $\theta_{1New} = 17.252$ $\theta_{2New} = 64.532$ $\theta_{wNew} = -7.255$

COMPARISON OF ENERGY BALANCES BETWEEN THE THREE METHODS



$A_{1Navy} = 8.655 \cdot m$ $A_{2Navy} = 23.183 \cdot m$ $\frac{A_{2Navy}}{A_{1Navy}} = 2.679$

$A_{1IMO} = 6.004 \cdot m$ $A_{2IMO} = 8.909 \cdot m$ $\frac{A_{2IMO}}{A_{1IMO}} = 1.484$

$A_{1New} = 6.914 \cdot m$ $A_{2New} = 11.179 \cdot m$ $\frac{A_{2New}}{A_{1New}} = 1.617$

MCM MINIMUM OPERATING 80 KNOTS

UNITS AND CONSTANTS

Second :	$s := sec$
Metric ton :	$t := 1000 \cdot kg$
Air density :	$\rho_a := 1.293 \cdot kg \cdot m^{-3}$
Sea Water density :	$\rho_w := 1025 \cdot kg \cdot m^{-3}$
Gravity :	$g := 9.81 \cdot m \cdot s^{-2}$
Range of angles	$\theta := -30, -29 .. 90$
Constant for graphs	$cte := -1 .. 1$
Matrix origin	$ORIGIN := 1$

SHIP CHARACTERISTICS

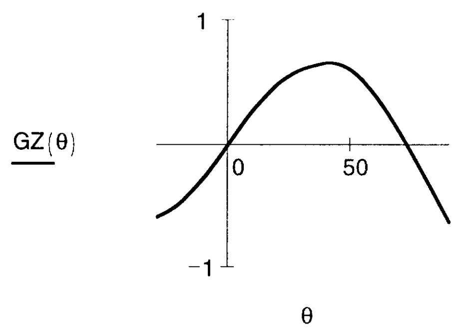
Displacement :	$\Delta := 1200 \cdot t$	
Length between perpendiculars :	$L := 62.6 \cdot m$	
Beam :	$B := 11.9 \cdot m$	
Draft :	$T := 3.39 \cdot m$	
Depth :	$D := 7.33 \cdot m$	
Block coefficient	$C_B := 0.528$	
Waterplane coefficient	$C_w := 0.8$	
Height of center of gravity :	$KG := 5.12 \cdot m$	
Transverse methacentric height	$GM := 1.55 \cdot m$	
Area above waterline :	$A := 471 \cdot m^2$	
Distance between the center of the projected underwater area and the center of the projected area above the water	$z := 5.96 \cdot m$	
Skeg area	$A_s := 19.8 \cdot m^2$	
Length of Bilge Keels	$L_k := 21.34 \cdot m$	
Height of Bilge Keels	$b_k := 0.76 \cdot m$	
Total Area of Bilge Keels	$A_k := 2 \cdot L_k \cdot b_k$	$A_k = 32.437 \cdot m^2$
Distance between center line at DWL and bilge keel	$d := 5.35 \cdot m$	
Total area of bilge keels + skeg	$A_t := A_s + A_k$	$A_t = 52.237 \cdot m^2$

Righting arm curve (GZ)

$$\theta := \begin{bmatrix} -30 \\ -25 \\ -20 \\ -15 \\ -10 \\ -5 \\ 0 \\ 5 \\ 10 \\ 15 \\ 20 \\ 25 \\ 30 \\ 35 \\ 40 \\ 50 \\ 60 \\ 70 \\ 80 \end{bmatrix} \qquad GZ := \begin{bmatrix} -0.60 \\ -0.55 \\ -0.48 \\ -0.38 \\ -0.27 \\ -0.14 \\ 0.00 \\ 0.14 \\ 0.27 \\ 0.38 \\ 0.48 \\ 0.55 \\ 0.60 \\ 0.63 \\ 0.65 \\ 0.59 \\ 0.38 \\ 0.08 \\ -0.27 \end{bmatrix}$$

Cubic interpolation of GZ curve

$$GZ(\theta) := \mathrm{interp}(\mathrm{cspline}(\theta, GZ), \theta, GZ, \theta) \cdot m$$



First intersept default value: $\theta_{d1} := 0$

Second intersept default value: $\theta_{d2} := 90$

Downflooding angle : $\theta_f := 70$

WEATHER CHARACTERISTICS

Wind speed : $V := 80 \cdot \frac{1852}{3600} \cdot m \cdot s^{-1}$ $V = 41.156 \cdot m \cdot s^{-1}$

Sea state 8/9, short term analysis, Average North Atlantic storm
H_s = 14 m; duration 5 hours; 9 modal period family; 2 parameter model

Description of family

$H := 14 \cdot m$ $i := 1..9$ $\omega m_1 := 0.048 \cdot \left(8.75 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$

$\omega m_2 := 0.054 \cdot \left(8.44 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$ $\omega m_3 := 0.061 \cdot \left(8.07 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$

$\omega m_4 := 0.069 \cdot \left(7.77 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$ $\omega m_5 := 0.079 \cdot \left(7.63 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$

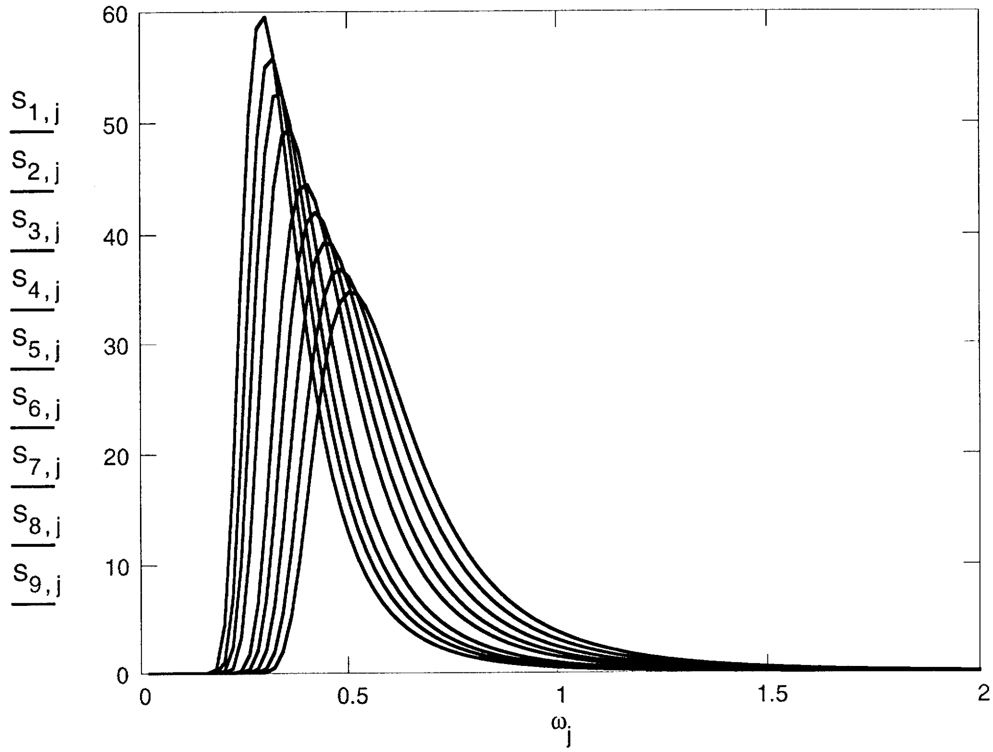
$\omega m_6 := 0.099 \cdot \left(6.87 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$ $\omega m_7 := 0.111 \cdot \left(6.67 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$

$\omega m_8 := 0.119 \cdot \left(6.65 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$ $\omega m_9 := 0.134 \cdot \left(6.41 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$

$$p\omega m := \begin{bmatrix} 0.05 \\ 0.05 \\ 0.0875 \\ 0.1875 \\ 0.25 \\ 0.1875 \\ 0.0875 \\ 0.05 \\ 0.05 \end{bmatrix}$$

$j := 1..100$ $\omega_j := j \cdot 0.02 \cdot rad \cdot s^{-1}$

$$S_{i,j} := \frac{5}{16} \cdot \left[\frac{\omega m_i}{\omega_j}\right]^4 \cdot \left[\frac{H^2}{\omega_j} \cdot e^{-1.25 \cdot \left[\frac{\omega m_i}{\omega_j}\right]^4}\right]$$



US NAVY METHOD

Wind heeling arm :
$$HA(\theta) := \frac{0.0737 \cdot V^2 \cdot A \cdot z \cdot (\cos(\theta \cdot deg))^2}{1000 \cdot \Delta} \cdot \frac{t}{m^4 \cdot s^{-2}}$$

Angle of equilibrium :
$$\theta_{0Navy} := root\left(HA\left(\theta_{d1}\right) - GZ\left(\theta_{d1}\right), \theta_{d1}\right)$$

$$\theta_{0Navy} = 10.504$$

Maximum angle :
$$\theta_{intercept} := root\left(HA\left(\theta_{d2}\right) - GZ\left(\theta_{d2}\right), \theta_{d2}\right)$$

$$\theta_{2Navy} := \begin{cases} \theta_{intercept} & \text{if } \theta_{intercept} \le \theta_f \\ \theta_f & \text{otherwise} \end{cases}$$

$$\theta_{2Navy} = 70$$

Roll back angle : $\theta_{1Navy} := 25$

Maximum windward angle $\theta_{wNavy} := \theta_{0Navy} - \theta_{1Navy}$

$\theta_{wNavy} = -14.496$

Energy balance calculations

$$\mathrm{Diff1}_{Navy}(\theta) := \mathrm{if}\left(\theta_{wNavy} \le \theta \le \theta_{0Navy}, |HA(\theta) - GZ(\theta)|, 0\right)$$

$$\mathrm{Diff2}_{Navy}(\theta) := \mathrm{if}\left(\theta_{0Navy} \le \theta \le \theta_{2Navy}, |HA(\theta) - GZ(\theta)|, 0\right)$$

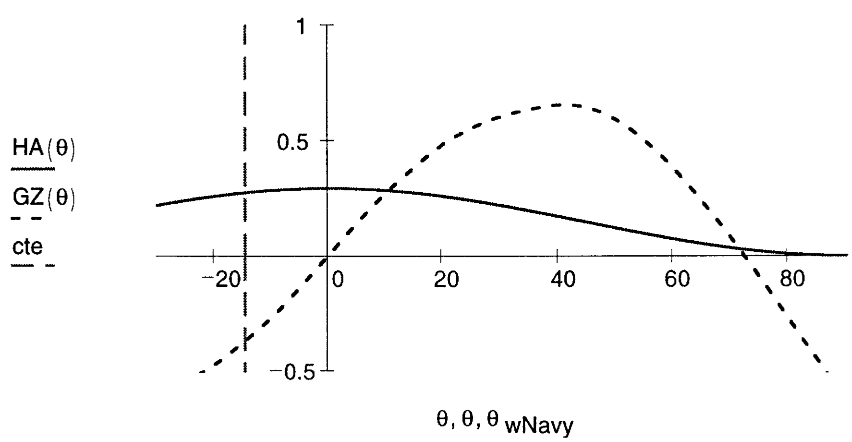
$$A_{1Navy} := \sum_{\theta} \mathrm{Diff1}_{Navy}(\theta) \cdot 1$$

$$A_{2Navy} := \sum_{\theta} \mathrm{Diff2}_{Navy}(\theta) \cdot 1$$

$A_{1Navy} = 8.483 \cdot m$

$A_{2Navy} = 19.191 \cdot m$

$$\frac{A_{2Navy}}{A_{1Navy}} = 2.262$$



IMO METHOD

Wind heeling arm :

$C_D := 1.15$

Steady wind : $$H_{w1IMO}(\theta) := \frac{\frac{1}{2} \cdot C_D \cdot \rho_a \cdot V^2 \cdot A \cdot z}{1000 \cdot g \cdot \Delta} \cdot \left(\frac{t}{kg}\right)$$

Gust wind $$H_{w2IMO}(\theta) := 1.5 \cdot H_{w1IMO}(\theta)$$

Angle of equilibrium : $$\theta_{0IMO} := \mathrm{root}\left(H_{w1IMO}(\theta_{d1}) - GZ(\theta_{d1}), \theta_{d1}\right)$$

$$\theta_{0IMO} = 11.308$$

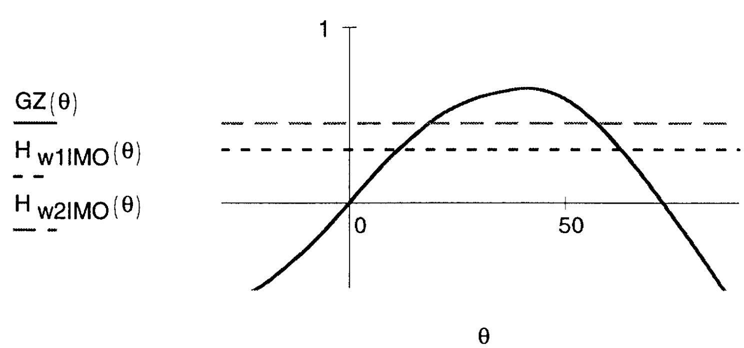
Equilibrium with gust : $$\theta_{gIMO} := \mathrm{root}\left(H_{w2IMO}(\theta_{d1}) - GZ(\theta_{d1}), \theta_{d1}\right)$$

$$\theta_{gIMO} = 18.378$$

Maximum angle : $$\theta_{intercept} := \mathrm{root}\left(H_{w2IMO}(\theta_{d2}) - GZ(\theta_{d2}), \theta_{d2}\right)$$

$$\theta_{2IMO} := \begin{cases} \theta_{intercept} & \text{if } \theta_{intercept} \le \theta_f \\ \theta_f & \text{otherwise} \end{cases}$$

$$\theta_{2IMO} = 57.212$$



Coefficients necessary for Roll back angle :

Calculation of coefficient k

$$At/LB := \begin{bmatrix} 0.00 \\ 0.01 \\ 0.015 \\ 0.02 \\ .025 \\ 0.03 \\ 0.035 \\ 0.04 \\ 1 \end{bmatrix} \quad k := \begin{bmatrix} 1.00 \\ 0.98 \\ 0.95 \\ 0.88 \\ 0.79 \\ 0.74 \\ 0.72 \\ 0.70 \\ 0.70 \end{bmatrix}$$

$f(x) := \mathrm{linterp}(At/LB, k, x)$ Linear interpolation based on given values

$$k := f\left(\frac{A_t}{L \cdot B}\right)$$

$k = 0.7$

Calculation of coefficient X_1

$$B/T := \begin{bmatrix} 0 \\ 2.4 \\ 2.5 \\ 2.7 \\ 2.8 \\ 2.9 \\ 3.0 \\ 3.1 \\ 3.2 \\ 3.3 \\ 3.4 \\ 3.5 \\ 10 \end{bmatrix} \quad X_1 := \begin{bmatrix} 1.00 \\ 1.00 \\ 0.98 \\ 0.96 \\ 0.95 \\ 0.93 \\ 0.91 \\ 0.90 \\ 0.88 \\ 0.86 \\ 0.82 \\ 0.80 \\ 0.80 \end{bmatrix}$$

$f(x) := \mathrm{linterp}(B/T, X_1, x)$ Linear interpolation based on given values

$$X_1 := f\left(\frac{B}{T}\right)$$

$X_1 = 0.8$

Calculation of coefficient X_2

$$C_B := \begin{bmatrix} 0.00 \\ 0.45 \\ 0.5 \\ 0.55 \\ 0.60 \\ .65 \\ 0.70 \\ 1.00 \end{bmatrix} \quad X_2 := \begin{bmatrix} 0.75 \\ 0.75 \\ 0.82 \\ 0.89 \\ 0.95 \\ 0.97 \\ 1 \\ 1 \end{bmatrix}$$

$f(x) := \mathrm{linterp}(C_B, X_2, x)$ Linear interpolation based on given values

$X_2 := f(C_B)$

$X_2 = 0.859$

Calculation of the ship natural period T_n

$$C := \left(0.373 + 0.023\cdot\frac{B}{T} - 0.043\cdot\frac{L}{100\cdot m}\right)\cdot\left(m^{-0.5}\cdot s\right)$$

$$T_n := 2\cdot\frac{C\cdot B}{\sqrt{GM}} \qquad T_n = 8.159\cdot s$$

Calculation of wave slope *s*

$$T_n := \begin{bmatrix} 0 \\ 6 \\ 7 \\ 8 \\ 12 \\ 14 \\ 16 \\ 18 \\ 20 \\ 40 \end{bmatrix}\cdot s \qquad S := \begin{bmatrix} 0.100 \\ 0.100 \\ 0.098 \\ 0.093 \\ 0.065 \\ 0.053 \\ 0.044 \\ 0.038 \\ 0.035 \\ 0.035 \end{bmatrix}$$

$$f(x) := \text{linterp}\left(T_n, S, x\right)$$ Linear interpolation base on given values

$$s := f\left(T_n\right)$$

$$s = 0.092$$

Test results coefficient : $C := 0.7635$

Bertin damping coefficient : $N := 0.02$

Height of center of gravity above water line $OG := KG - (D - T)$

Roll back angle

$$\theta_{1IMO} := \left[\frac{k\cdot X_1\cdot X_2}{C}\cdot 0.7\cdot\sqrt{\frac{\pi\cdot\left(0.73 + 0.6\cdot\frac{OG}{T}\right)\cdot 180\cdot s}{2\cdot N}}\right]$$

$$\theta_{1IMO} = 15.405$$

Maximum windward angle

$$\theta_{wIMO} := \theta_{0IMO} - \theta_{1IMO}$$

$$\theta_{wIMO} = -4.098$$

Energy balance calculations

$$\mathrm{Diff1}_{IMO}(\theta) := \mathrm{if}\left(\theta_{wIMO} \le \theta \le \theta_{gIMO}, \left|H_{w2IMO}(\theta) - GZ(\theta)\right|, 0\right)$$

$$\mathrm{Diff2}_{IMO}(\theta) := \mathrm{if}\left(\theta_{gIMO} \le \theta \le \theta_{2IMO}, \left|H_{w2IMO}(\theta) - GZ(\theta)\right|, 0\right)$$

$$A_{1IMO} := \sum_{\theta} \mathrm{Diff1}_{IMO}(\theta) \cdot 1$$

$$A_{2IMO} := \sum_{\theta} \mathrm{Diff2}_{IMO}(\theta) \cdot 1$$

$$A_{1IMO} = 6.172 \cdot m \qquad \frac{A_{2IMO}}{A_{1IMO}} = 0.834$$

$$A_{2IMO} = 5.146 \cdot m$$

PROPOSED NEW METHOD

Drag coefficient : $C_D := 1.12$

Projected Area :
$$A_p(\theta) := \begin{cases} \left[C_{wp} \cdot \frac{B \cdot L}{2} + \left(A - C_{wp} \cdot \frac{B \cdot L}{2}\right) \cdot \cos(\theta \cdot deg) \right] & \text{if } \theta \le 90 \\ C_{wp} \cdot \frac{B \cdot L}{2} & \text{otherwise} \end{cases}$$

Projected Lever :
$$z_p(\theta) := \begin{cases} \left[\frac{B}{2} + \left(z - \frac{B}{2}\right) \cdot \cos(\theta \cdot deg) \right] & \text{if } \theta \le 90 \\ \frac{B}{2} & \text{otherwise} \end{cases}$$

Steady wind heeling arm
$$H_{w1New}(\theta) := \frac{\frac{1}{2} \cdot C_D \cdot \rho_a \cdot V^2 \cdot A_p(\theta) \cdot z_p(\theta)}{1000 \cdot g \cdot \Delta} \cdot \left(\frac{t}{kg}\right)$$

Gust wind heeling arm
$$H_{w2New}(\theta) := 1.5 \cdot H_{w1New}(\theta)$$

Angle of equilibrium :
$$\theta_{0New} := \mathrm{root}\left(H_{w1New}(\theta_{d1}) - GZ(\theta_{d1}), \theta_{d1}\right)$$

$$\theta_{0New} = 10.878$$

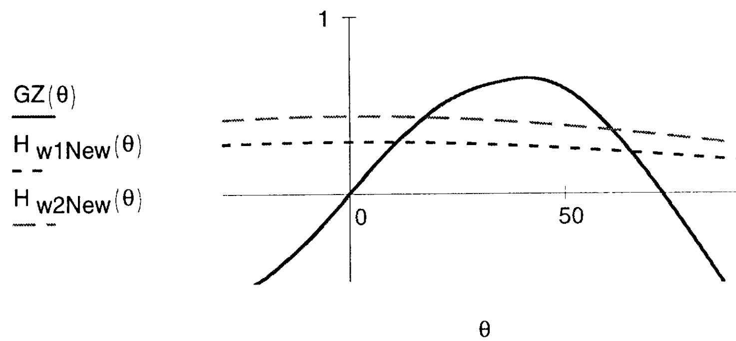
Equilibrium with gust : $\theta_{gNew} := \mathrm{root}\left(H_{w2New}(\theta_{d1}) - GZ(\theta_{d1}), \theta_{d1}\right)$

$\theta_{gNew} = 17.433$

Maximum angle : $\theta_{intercept} := \mathrm{root}\left(H_{w2New}(\theta_{d2}) - GZ(\theta_{d2}), \theta_{d2}\right)$

$$\theta_{2New} := \begin{cases} \theta_{intercept} & \text{if } \theta_{intercept} \le \theta_f \\ \theta_f & \text{otherwise} \end{cases}$$

$\theta_{2New} = 60.91$



Calculation of the roll back angle

Natural roll period

$$C := \left(0.3725 + 0.0227 \cdot \frac{B}{T} - 0.043 \cdot \frac{L}{100 \cdot m}\right)$$

$$T_n := \frac{2\pi \cdot C \cdot B}{\sqrt{g \cdot GM}} \qquad T_n = 8.154 \cdot s \qquad \omega_n := \frac{2\pi}{T_n}$$

Non dimensional damping factor

$$\beta^* := 19.25 \cdot \left(A_k \cdot \sqrt{b_k} + 0.0024 \cdot L \cdot B \cdot \sqrt{d}\right) \cdot d^2 \cdot \frac{\sqrt{d \cdot \theta_i}}{C_B \cdot L \cdot B^3 \cdot T}$$

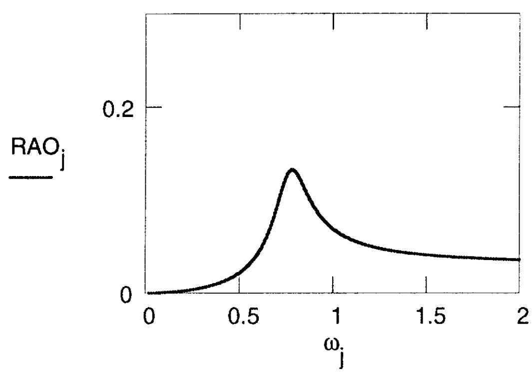
$\beta^* = 0.116$

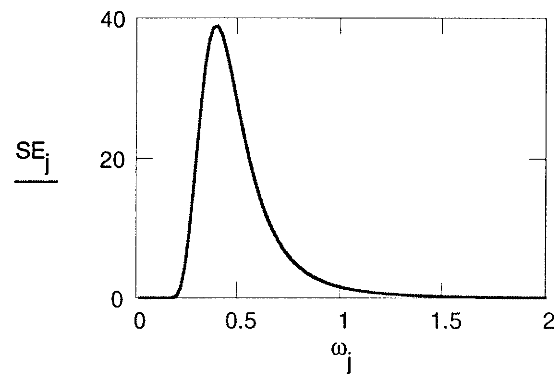
RAO

$$RAO_j := \frac{1}{\sqrt{\left[1 - \left(\frac{\omega_j}{\omega_n}\right)^2\right]^2 + \left(2 \cdot \frac{\omega_j}{\omega_n} \cdot \beta^*\right)^2}} \cdot \frac{(\omega_j)^2}{2 \cdot g}$$

$$SE_j := \sum_{i=1}^{9} p\omega_{m_i} \cdot S_{i,j}$$

$$SR_j := (RAO_j)^2 \cdot SE_j$$





Roll back angle: $$m_0 := \sum_{j=2}^{100} SR_j \cdot (\omega_j - \omega_{j-1})$$

$$\theta_{1New} := 2 \cdot \sqrt{m_0} \cdot \frac{180}{\pi} \qquad \theta_{1New} = 17.045$$

With the initial roll angle used to calculate the damping factor $\theta_i \equiv 16 \cdot deg$

Maximum windward angle $\theta_{wNew} := \theta_{0New} - \theta_{1New}$

$$\theta_{wNew} = -6.167$$

Energy balance calculations

$$Diff1_{New}(\theta) := if\left(\theta_{wNew} \le \theta \le \theta_{gNew}, \left|H_{w2New}(\theta) - GZ(\theta)\right|, 0\right)$$

$$Diff2_{New}(\theta) := if\left(\theta_{gNew} \le \theta \le \theta_{2New}, \left|H_{w2New}(\theta) - GZ(\theta)\right|, 0\right)$$

$$A_{1New} := \sum_{\theta} Diff1_{New}(\theta) \cdot 1$$

$$A_{2New} := \sum_{\theta} Diff2_{New}(\theta) \cdot 1$$

$$A_{1New} = 7.044 \cdot m \qquad \frac{A_{2New}}{A_{1New}} = 1.02$$

$$A_{2New} = 7.186 \cdot m$$

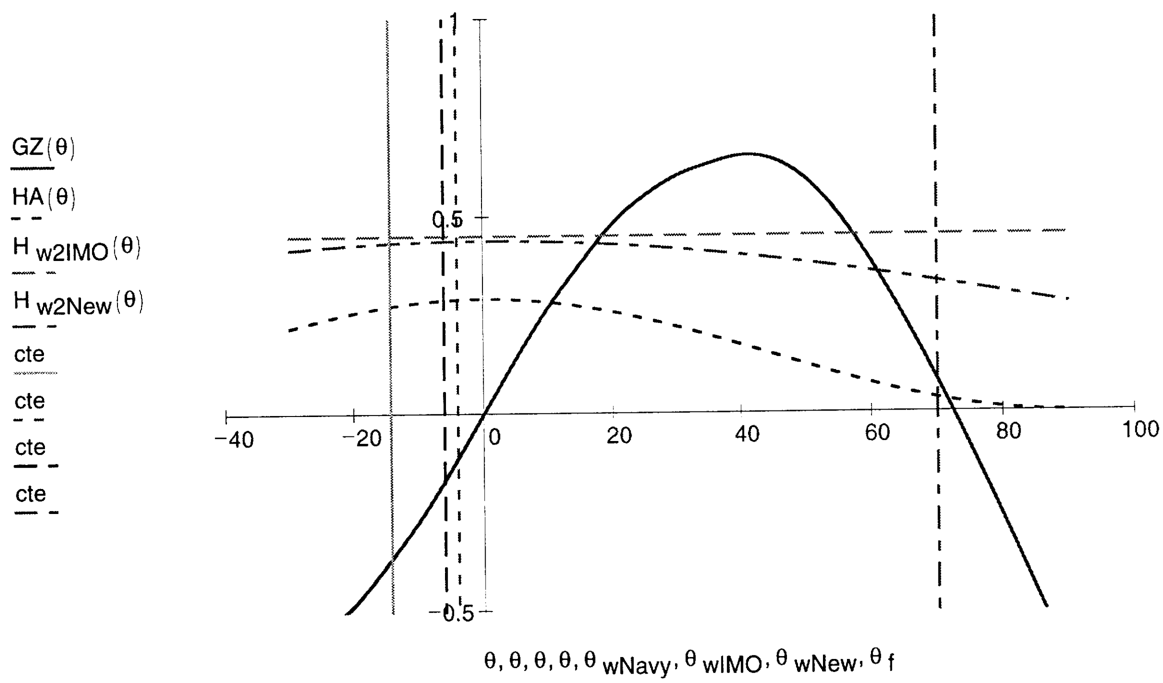
COMPARISON OF THE ANGLES BETWEEN THE THREE METHODS

$\theta_{0Navy} = 10.504$ $\theta_{1Navy} = 25$ $\theta_{2Navy} = 70$ $\theta_{wNavy} = -14.496$

$\theta_{0IMO} = 11.308$ $\theta_{1IMO} = 15.405$ $\theta_{2IMO} = 57.212$ $\theta_{wIMO} = -4.098$

$\theta_{0New} = 10.878$ $\theta_{1New} = 17.045$ $\theta_{2New} = 60.91$ $\theta_{wNew} = -6.167$

COMPARISON OF ENERGY BALANCES BETWEEN THE THREE METHODS



$A_{1Navy} = 8.483 \cdot m$ $A_{2Navy} = 19.191 \cdot m$ $\frac{A_{2Navy}}{A_{1Navy}} = 2.262$

$A_{1IMO} = 6.172 \cdot m$ $A_{2IMO} = 5.146 \cdot m$ $\frac{A_{2IMO}}{A_{1IMO}} = 0.834$

$A_{1New} = 7.044 \cdot m$ $A_{2New} = 7.186 \cdot m$ $\frac{A_{2New}}{A_{1New}} = 1.02$

APPENDIX 3

FFG 7 COMPARISON

This Appendix gives an example of the calculations of the dynamic stability for FFG 7 with the US Navy, IMO and the proposed method. The Appendix is divided into five parts :

- the section giving the ship characteristics for the Test Condition
- the sections giving the ship characteristics and the results for the Full Load Condition with a wind speed of 100 knots
- the sections giving the ship characteristics and the results for the Minimum Operating Condition with a wind speed of 100 knots
- the sections giving the ship characteristics and the results for the Full Load Condition with a wind speed of 91 knots
- the complete example of the calculations for the Minimum Operating Condition with a wind speed of 82 knots

FFG 7 TEST

UNITS AND CONSTANTS

Second :	$s := sec$
Metric ton :	$t := 1000 \cdot kg$
Air density :	$\rho_a := 1.293 \cdot kg \cdot m^{-3}$
Sea Water density :	$\rho_w := 1025 \cdot kg \cdot m^{-3}$
Gravity :	$g := 9.81 \cdot m \cdot s^{-2}$
Range of angles	$\theta := -30, -29 .. 90$
Constant for graphs	$cte := -1 .. 1$
Matrix origin	$ORIGIN := 1$

SHIP CHARACTERISTICS

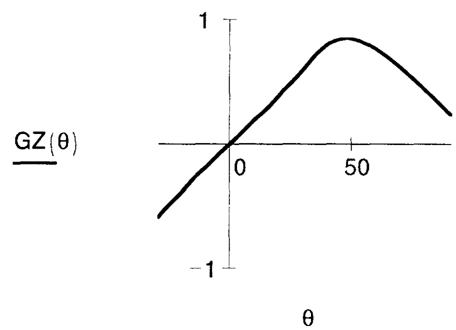
Displacement :	$\Delta := 4182.8 \cdot t$	
Length between perpendiculars :	$L := 124.4 \cdot m$	
Beam :	$B := 13.7 \cdot m$	
Draft :	$T := 4.9 \cdot m$	
Depth :	$D := 9.1 \cdot m$	
Block coefficient	$C_B := 0.446$	
Waterplane coefficient	$C_w := 0.74$	
Height of center of gravity :	$KG := 5.82 \cdot m$	
Transverse methacentric height	$GM := 0.93 \cdot m$	
Area above waterline :	$A := 1297.45 \cdot m^2$	
Distance between the center of the projected underwater area and the center of the projected area above the water	$z := 9.33 \cdot m$	
Skeg area	$A_s := 18.2 \cdot m^2$	
Length of Bilge Keels	$L_k := 29.38 \cdot m$	
Height of Bilge Keels	$b_k := 0.91 \cdot m$	
Total Area of Bilge Keels	$A_k := 2 \cdot \left(L_k \cdot b_k + 5.6 \cdot m^2\right)$	$A_k = 64.672 \cdot m^2$
Distance between center line at DWL and bilge keel	$d := 5.66 \cdot m$	
Total area of bilge keels + skeg	$A_t := A_s + A_k$	$A_t = 82.872 \cdot m^2$

Righting arm curve (GZ)

$$\theta := \begin{bmatrix} -30 \\ -25 \\ -20 \\ -15 \\ -10 \\ -5 \\ 0 \\ 5 \\ 10 \\ 15 \\ 20 \\ 25 \\ 30 \\ 35 \\ 40 \\ 50 \\ 60 \\ 70 \\ 80 \end{bmatrix} \qquad GZ := \begin{bmatrix} -0.60 \\ -0.49 \\ -0.39 \\ -0.28 \\ -0.19 \\ -0.09 \\ 0.00 \\ 0.09 \\ 0.19 \\ 0.28 \\ 0.39 \\ 0.49 \\ 0.60 \\ 0.72 \\ 0.80 \\ 0.84 \\ 0.75 \\ 0.60 \\ 0.42 \end{bmatrix}$$

Cubic interpolation of GZ curve

$$GZ(\theta) := \mathrm{interp}(\mathrm{cspline}(\theta, GZ), \theta, GZ, \theta)\cdot m$$



First intersept default value: $\theta_{d1} := 0$

Second intersept default value: $\theta_{d2} := 90$

Downflooding angle : $\theta_f := 70$

FFG 7 FULL LOAD 100 KNOTS

UNITS AND CONSTANTS

Second :	$s := sec$
Metric ton :	$t := 1000 \cdot kg$
Air density :	$\rho_a := 1.293 \cdot kg \cdot m^{-3}$
Sea Water density :	$\rho_w := 1025 \cdot kg \cdot m^{-3}$
Gravity :	$g := 9.81 \cdot m \cdot s^{-2}$
Range of angles	$\theta := -30, -29 .. 90$
Constant for graphs	$cte := -1 .. 1$
Matrix origin	$ORIGIN := 1$

SHIP CHARACTERISTICS

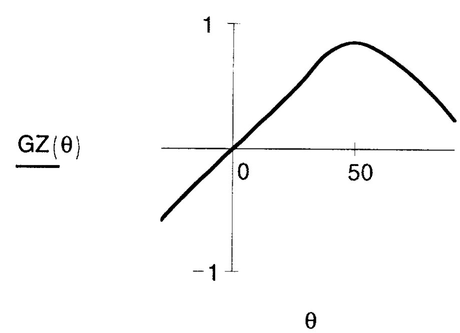
Displacement :	$\Delta := 4031 \cdot t$	
Length between perpendiculars :	$L := 124.4 \cdot m$	
Beam :	$B := 13.7 \cdot m$	
Draft :	$T := 4.83 \cdot m$	
Depth :	$D := 9.1 \cdot m$	
Block coefficient	$C_B := 0.446$	
Waterplane coefficient	$C_w := 0.74$	
Height of center of gravity :	$KG := 5.73 \cdot m$	
Transverse methacentric height	$GM := 0.99 \cdot m$	
Area above waterline :	$A := 1297.45 \cdot m^2$	
Distance between the center of the projected underwater area and the center of the projected area above the water	$z := 9.33 \cdot m$	
Skeg area	$A_s := 18.2 \cdot m^2$	
Length of Bilge Keels	$L_k := 29.38 \cdot m$	
Height of Bilge Keels	$b_k := 0.91 \cdot m$	
Total Area of Bilge Keels	$A_k := 2 \cdot \left(L_k \cdot b_k + 5.6 \cdot m^2\right)$	$A_k = 64.672 \cdot m^2$
Distance between center line at DWL and bilge keel	$d := 5.66 \cdot m$	
Total area of bilge keels + skeg	$A_t := A_s + A_k$	$A_t = 82.872 \cdot m^2$

Righting arm curve (GZ)

$$\theta := \begin{bmatrix} -30 \\ -25 \\ -20 \\ -15 \\ -10 \\ -5 \\ 0 \\ 5 \\ 10 \\ 15 \\ 20 \\ 25 \\ 30 \\ 35 \\ 40 \\ 50 \\ 60 \\ 70 \\ 80 \end{bmatrix} \qquad GZ := \begin{bmatrix} -0.59 \\ -0.48 \\ -0.38 \\ -0.28 \\ -0.19 \\ -0.09 \\ 0.00 \\ 0.09 \\ 0.19 \\ 0.28 \\ 0.38 \\ 0.48 \\ 0.59 \\ 0.71 \\ 0.79 \\ 0.85 \\ 0.77 \\ 0.63 \\ 0.45 \end{bmatrix}$$

Cubic interpolation of GZ curve

$$GZ(\theta) := \mathrm{interp}(\mathrm{cspline}(\theta, GZ), \theta, GZ, \theta) \cdot m$$



First intersept default value: $\theta_{d1} := 0$

Second intersept default value: $\theta_{d2} := 90$

Downflooding angle : $\theta_f := 70$

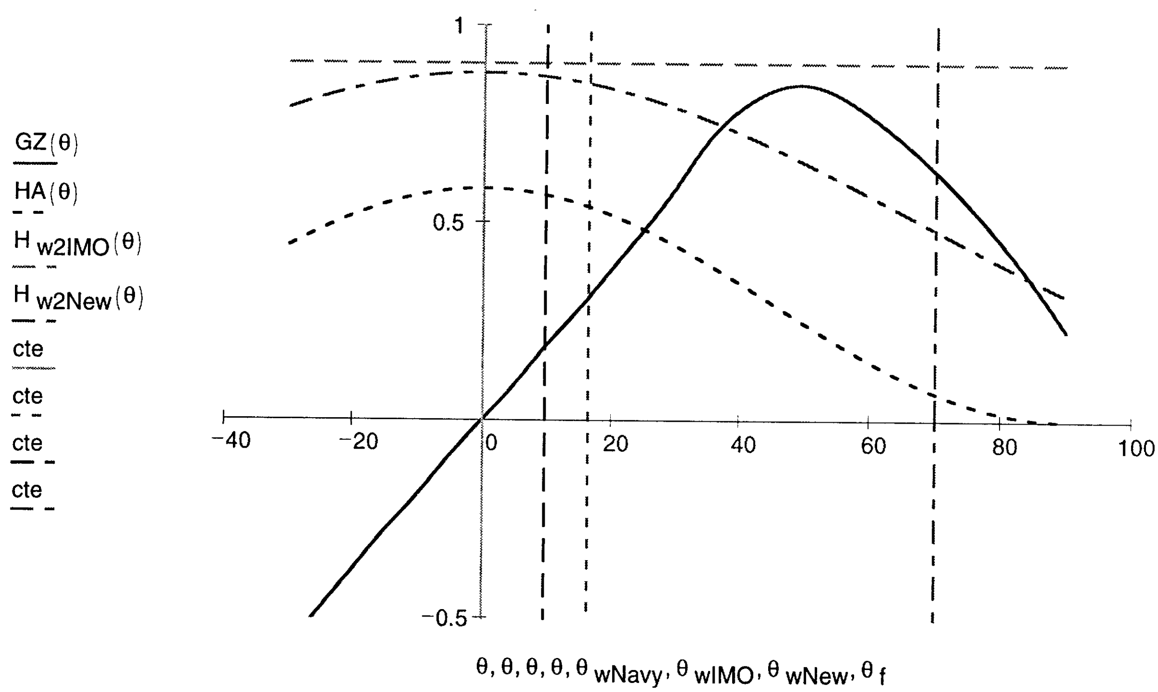
COMPARISON OF THE ANGLES BETWEEN THE THREE METHODS

$\theta_{0Navy} = 25.045$ $\theta_{1Navy} = 25$ $\theta_{2Navy} = 70$ $\theta_{wNavy} = 0.045$

$\theta_{0IMO} = 30.511$ $\theta_{1IMO} = 14.09$ $\theta_{2IMO} = 70$ $\theta_{wIMO} = 16.421$

$\theta_{0New} = 27.634$ $\theta_{1New} = 17.902$ $\theta_{2New} = 70$ $\theta_{wNew} = 9.732$

COMPARISON OF ENERGY BALANCES BETWEEN THE THREE METHODS



$A_{1Navy} = 7.562 \cdot m$ $A_{2Navy} = 21.36 \cdot m$ $\frac{A_{2Navy}}{A_{1Navy}} = 2.825$

$A_{1IMO} =$ $A_{2IMO} =$ $\frac{A_{2IMO}}{A_{1IMO}} =$

$A_{1New} = 10.087 \cdot m$ $A_{2New} = 5.492 \cdot m$ $\frac{A_{2New}}{A_{1New}} = 0.544$

FFG 7 MINIMUM OPERATING 100 KNOTS

UNITS AND CONSTANTS

Second :	$s := sec$
Metric ton :	$t := 1000 \cdot kg$
Air density :	$\rho_a := 1.293 \cdot kg \cdot m^{-3}$
Sea Water density :	$\rho_w := 1025 \cdot kg \cdot m^{-3}$
Gravity :	$g := 9.81 \cdot m \cdot s^{-2}$
Range of angles	$\theta := -30, -29 .. 90$
Constant for graphs	$cte := -1 .. 1$
Matrix origin	$ORIGIN := 1$

SHIP CHARACTERISTICS

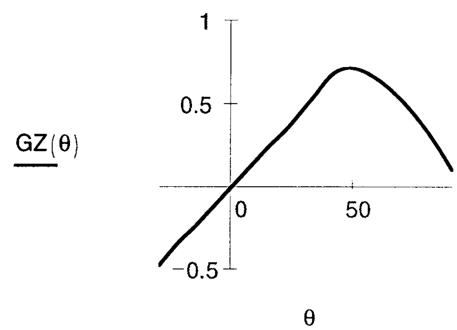
Displacement :	$\Delta := 3807 \cdot t$	
Length between perpendiculars :	$L := 124.4 \cdot m$	
Beam :	$B := 13.7 \cdot m$	
Draft :	$T := 4.65 \cdot m$	
Depth :	$D := 9.1 \cdot m$	
Block coefficient	$C_B := 0.446$	
Waterplane coefficient	$C_W := 0.74$	
Height of center of gravity :	$KG := 5.93 \cdot m$	
Transverse methacentric height	$GM := 0.86 \cdot m$	
Area above waterline :	$A := 1297.45 \cdot m^2$	
Distance between the center of the projected underwater area and the center of the projected area above the water	$z := 9.33 \cdot m$	
Skeg area	$A_s := 18.2 \cdot m^2$	
Length of Bilge Keels	$L_k := 29.38 \cdot m$	
Height of Bilge Keels	$b_k := 0.91 \cdot m$	
Total Area of Bilge Keels	$A_k := 2 \cdot \left(L_k \cdot b_k + 5.6 \cdot m^2\right)$	$A_k = 64.672 \cdot m^2$
Distance between center line at DWL and bilge keel	$d := 5.66 \cdot m$	
Total area of bilge keels + skeg	$A_t := A_s + A_k$	$A_t = 82.872 \cdot m^2$

Righting arm curve (GZ)

$$\theta := \begin{bmatrix} -30 \\ -25 \\ -20 \\ -15 \\ -10 \\ -5 \\ 0 \\ 5 \\ 10 \\ 15 \\ 20 \\ 25 \\ 30 \\ 35 \\ 40 \\ 50 \\ 60 \\ 70 \\ 80 \end{bmatrix} \qquad GZ := \begin{bmatrix} -0.48 \\ -0.39 \\ -0.31 \\ -0.24 \\ -0.16 \\ -0.08 \\ 0.00 \\ 0.08 \\ 0.16 \\ 0.24 \\ 0.31 \\ 0.39 \\ 0.48 \\ 0.57 \\ 0.66 \\ 0.71 \\ 0.64 \\ 0.51 \\ 0.33 \end{bmatrix}$$

Cubic interpolation of GZ curve

$$GZ(\theta) := \text{interp}(\text{cspline}(\theta, GZ), \theta, GZ, \theta)\cdot m$$



First intersept default value: $\theta_{d1} := 0$

Second intersept default value: $\theta_{d2} := 90$

Downflooding angle : $\theta_f := 70$

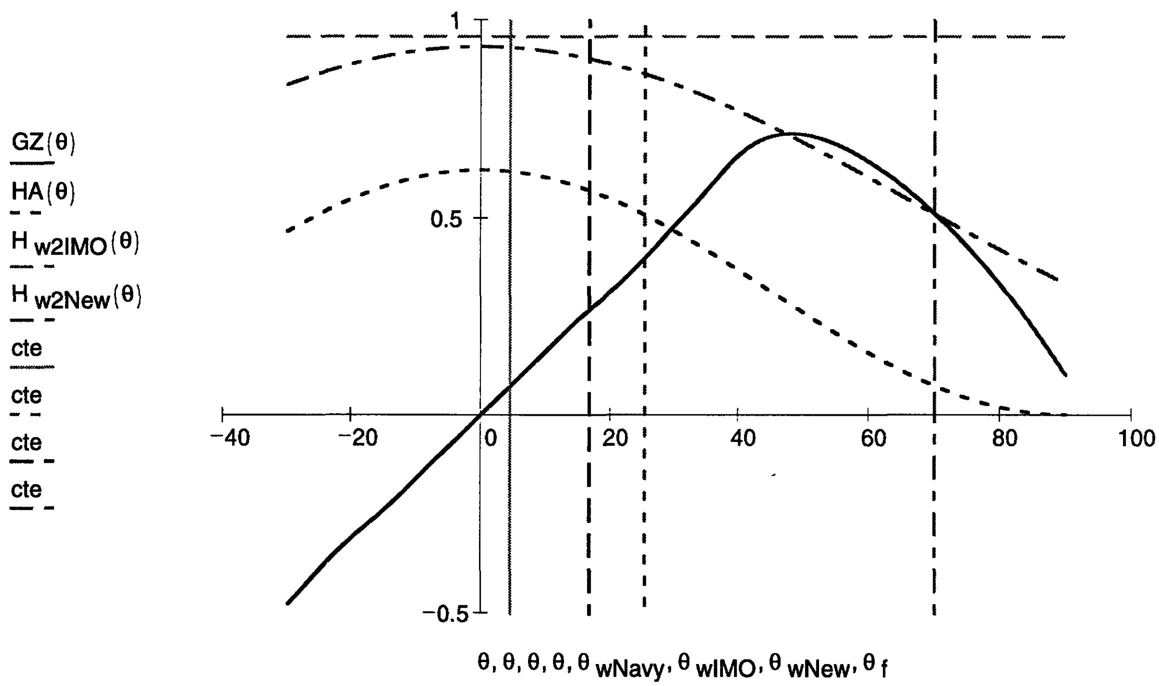
COMPARISON OF THE ANGLES BETWEEN THE THREE METHODS

$\theta_{0Navy} = 29.457$ $\theta_{1Navy} = 25$ $\theta_{2Navy} = 70$ $\theta_{wNavy} = 4.457$

$\theta_{0IMO} = 38.629$ $\theta_{1IMO} = 13.274$ $\theta_{2IMO} = 70$ $\theta_{wIMO} = 25.355$

$\theta_{0New} = 33.49$ $\theta_{1New} = 16.725$ $\theta_{2New} = 70$ $\theta_{wNew} = 16.765$

COMPARISON OF ENERGY BALANCES BETWEEN THE THREE METHODS



$A_{1Navy} = 7.282 \cdot m$ $A_{2Navy} = 15.11 \cdot m$ $\frac{A_{2Navy}}{A_{1Navy}} = 2.075$

$A_{1IMO} =$ $A_{2IMO} =$ $\frac{A_{2IMO}}{A_{1IMO}} =$

$A_{1New} = 9.521 \cdot m$ $A_{2New} = 0.647 \cdot m$ $\frac{A_{2New}}{A_{1New}} = 0.068$

FFG 7 FULL LOAD 91 KNOTS

UNITS AND CONSTANTS

Second :	$s := sec$
Metric ton :	$t := 1000 \cdot kg$
Air density :	$\rho_a := 1.293 \cdot kg \cdot m^{-3}$
Sea Water density :	$\rho_w := 1025 \cdot kg \cdot m^{-3}$
Gravity :	$g := 9.81 \cdot m \cdot s^{-2}$
Range of angles	$\theta := -30, -29 .. 90$
Constant for graphs	$cte := -1 .. 1$
Matrix origin	$ORIGIN := 1$

SHIP CHARACTERISTICS

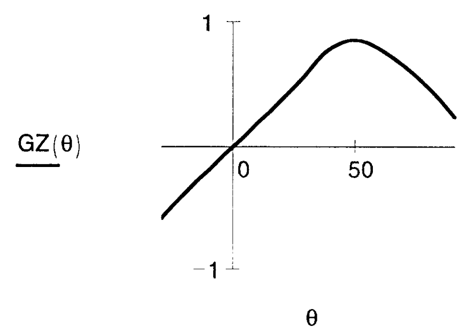
Displacement :	$\Delta := 4031 \cdot t$	
Length between perpendiculars :	$L := 124.4 \cdot m$	
Beam :	$B := 13.7 \cdot m$	
Draft :	$T := 4.83 \cdot m$	
Depth :	$D := 9.1 \cdot m$	
Block coefficient	$C_B := 0.446$	
Waterplane coefficient	$C_w := 0.74$	
Height of center of gravity :	$KG := 5.73 \cdot m$	
Transverse methacentric height	$GM := 0.99 \cdot m$	
Area above waterline :	$A := 1297.45 \cdot m^2$	
Distance between the center of the projected underwater area and the center of the projected area above the water	$z := 9.33 \cdot m$	
Skeg area	$A_s := 18.2 \cdot m^2$	
Length of Bilge Keels	$L_k := 29.38 \cdot m$	
Height of Bilge Keels	$b_k := 0.91 \cdot m$	
Total Area of Bilge Keels	$A_k := 2 \cdot \left(L_k \cdot b_k + 5.6 \cdot m^2\right)$	$A_k = 64.672 \cdot m^2$
Distance between center line at DWL and bilge keel	$d := 5.66 \cdot m$	
Total area of bilge keels + skeg	$A_t := A_s + A_k$	$A_t = 82.872 \cdot m^2$

Righting arm curve (GZ)

$$\theta := \begin{bmatrix} -30 \\ -25 \\ -20 \\ -15 \\ -10 \\ -5 \\ 0 \\ 5 \\ 10 \\ 15 \\ 20 \\ 25 \\ 30 \\ 35 \\ 40 \\ 50 \\ 60 \\ 70 \\ 80 \end{bmatrix} \qquad GZ := \begin{bmatrix} -0.59 \\ -0.48 \\ -0.38 \\ -0.28 \\ -0.19 \\ -0.09 \\ 0.00 \\ 0.09 \\ 0.19 \\ 0.28 \\ 0.38 \\ 0.48 \\ 0.59 \\ 0.71 \\ 0.79 \\ 0.85 \\ 0.77 \\ 0.63 \\ 0.45 \end{bmatrix}$$

Cubic interpolation of GZ curve

$$GZ(\theta) := \text{interp}(\text{cspline}(\theta, GZ), \theta, GZ, \theta) \cdot m$$



First intersept default value: $\theta_{d1} := 0$

Second intersept default value: $\theta_{d2} := 90$

Downflooding angle : $\theta_f := 70$

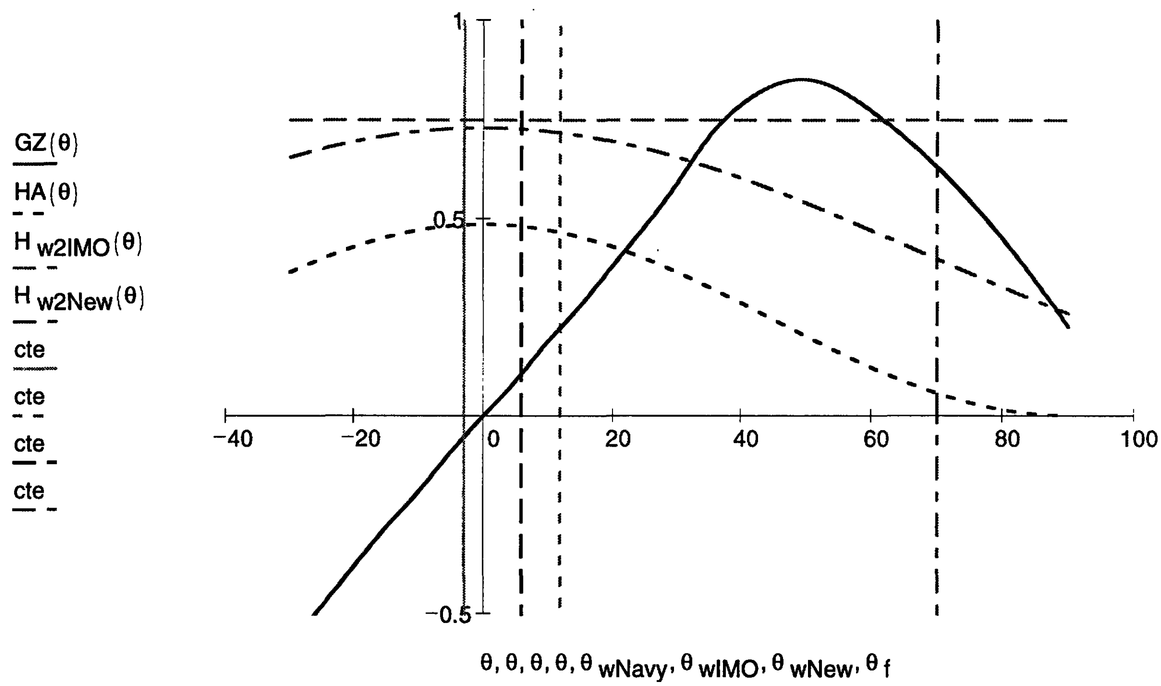
COMPARISON OF THE ANGLES BETWEEN THE THREE METHODS

$\theta_{0Navy} = 21.884$ $\theta_{1Navy} = 25$ $\theta_{2Navy} = 70$ $\theta_{wNavy} = -3.116$

$\theta_{0IMO} = 25.896$ $\theta_{1IMO} = 14.09$ $\theta_{2IMO} = 61.758$ $\theta_{wIMO} = 11.806$

$\theta_{0New} = 23.697$ $\theta_{1New} = 17.902$ $\theta_{2New} = 70$ $\theta_{wNew} = 5.795$

COMPARISON OF ENERGY BALANCES BETWEEN THE THREE METHODS



$A_{1Navy} = 7.419 \cdot m$ $A_{2Navy} = 23.581 \cdot m$ $\frac{A_{2Navy}}{A_{1Navy}} = 3.178$

$A_{1IMO} = 7.028 \cdot m$ $A_{2IMO} = 1.623 \cdot m$ $\frac{A_{2IMO}}{A_{1IMO}} = 0.231$

$A_{1New} = 8.876 \cdot m$ $A_{2New} = 9.373 \cdot m$ $\frac{A_{2New}}{A_{1New}} = 1.056$

FFG 7 MINIMUM OPERATING 82 KNOTS

UNITS AND CONSTANTS

Second :	$s := sec$
Metric ton :	$t := 1000 \cdot kg$
Air density :	$\rho_a := 1.293 \cdot kg \cdot m^{-3}$
Sea Water density :	$\rho_w := 1025 \cdot kg \cdot m^{-3}$
Gravity :	$g := 9.81 \cdot m \cdot s^{-2}$
Range of angles	$\theta := -30, -29 .. 90$
Constant for graphs	$cte := -1 .. 1$
Matrix origin	$ORIGIN := 1$

SHIP CHARACTERISTICS

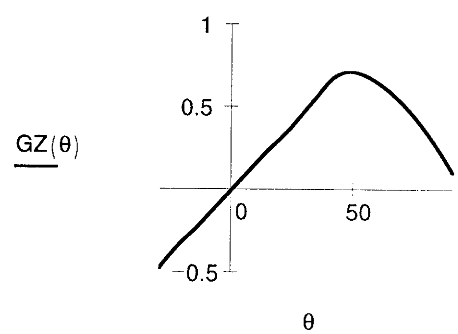
Displacement :	$\Delta := 3807 \cdot t$	
Length between perpendiculars :	$L := 124.4 \cdot m$	
Beam :	$B := 13.7 \cdot m$	
Draft :	$T := 4.65 \cdot m$	
Depth :	$D := 9.1 \cdot m$	
Block coefficient	$C_B := 0.446$	
Waterplane coefficient	$C_W := 0.74$	
Height of center of gravity :	$KG := 5.93 \cdot m$	
Transverse methacentric height	$GM := 0.86 \cdot m$	
Area above waterline :	$A := 1297.45 \cdot m^2$	
Distance between the center of the projected underwater area and the center of the projected area above the water	$z := 9.33 \cdot m$	
Skeg area	$A_s := 18.2 \cdot m^2$	
Length of Bilge Keels	$L_k := 29.38 \cdot m$	
Height of Bilge Keels	$b_k := 0.91 \cdot m$	
Total Area of Bilge Keels	$A_k := 2 \cdot \left(L_k \cdot b_k + 5.6 \cdot m^2\right)$	$A_k = 64.672 \cdot m^2$
Distance between center line at DWL and bilge keel	$d := 5.66 \cdot m$	
Total area of bilge keels + skeg	$A_t := A_s + A_k$	$A_t = 82.872 \cdot m^2$

Righting arm curve (GZ)

$$\theta := \begin{bmatrix} -30 \\ -25 \\ -20 \\ -15 \\ -10 \\ -5 \\ 0 \\ 5 \\ 10 \\ 15 \\ 20 \\ 25 \\ 30 \\ 35 \\ 40 \\ 50 \\ 60 \\ 70 \\ 80 \end{bmatrix} \qquad GZ := \begin{bmatrix} -0.48 \\ -0.39 \\ -0.31 \\ -0.24 \\ -0.16 \\ -0.08 \\ 0.00 \\ 0.08 \\ 0.16 \\ 0.24 \\ 0.31 \\ 0.39 \\ 0.48 \\ 0.57 \\ 0.66 \\ 0.71 \\ 0.64 \\ 0.51 \\ 0.33 \end{bmatrix}$$

Cubic interpolation of GZ curve

$$GZ(\theta) := \mathrm{interp}(\mathrm{cspline}(\theta, GZ), \theta, GZ, \theta) \cdot m$$



First intersept default value: $\theta_{d1} := 0$

Second intersept default value: $\theta_{d2} := 90$

Downflooding angle : $\theta_f := 70$

WEATHER CHARACTERISTICS

Wind speed : $V := 82 \cdot \frac{1852}{3600} \cdot m \cdot s^{-1}$ $V = 42.184 \cdot m \cdot s^{-1}$

Sea state 8/9, short term analysis, Average North Atlantic storm
H_s = 14 m; duration 5 hours; 9 modal period family; 2 parameter model

Description of family

$H := 14 \cdot m$ $i := 1 .. 9$

$\omega m_1 := 0.048 \cdot \left(8.75 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$

$\omega m_2 := 0.054 \cdot \left(8.44 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$ $\omega m_3 := 0.061 \cdot \left(8.07 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$

$\omega m_4 := 0.069 \cdot \left(7.77 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$ $\omega m_5 := 0.079 \cdot \left(7.63 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$

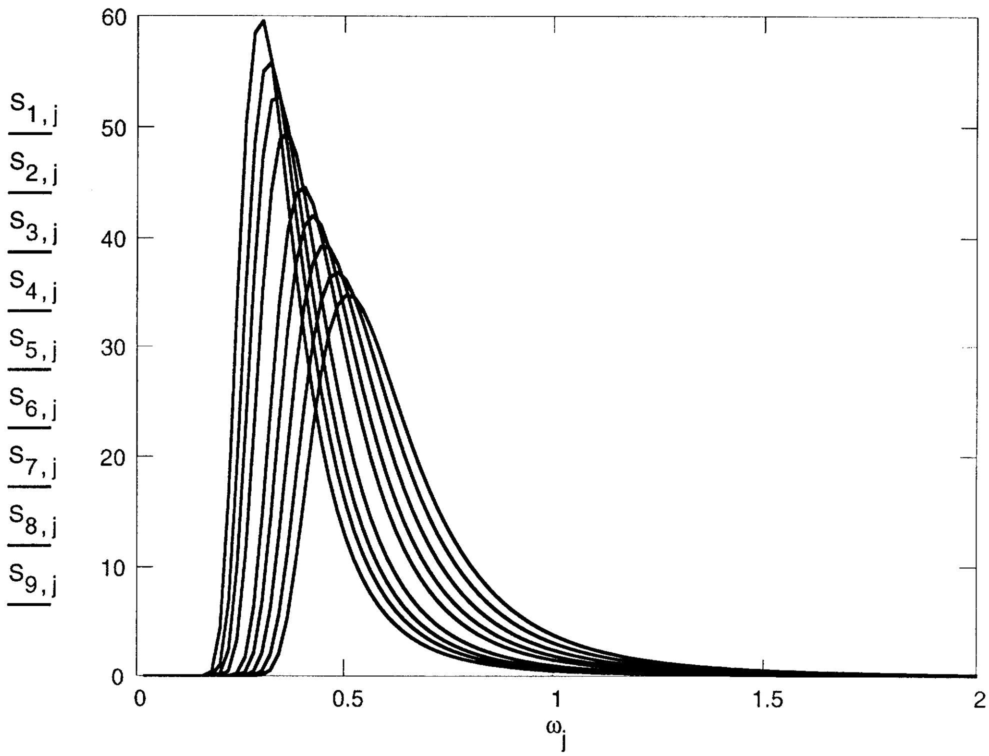
$\omega m_6 := 0.099 \cdot \left(6.87 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$ $\omega m_7 := 0.111 \cdot \left(6.67 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$

$\omega m_8 := 0.119 \cdot \left(6.65 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$ $\omega m_9 := 0.134 \cdot \left(6.41 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$

$$p\omega_m := \begin{bmatrix} 0.05 \\ 0.05 \\ 0.0875 \\ 0.1875 \\ 0.25 \\ 0.1875 \\ 0.0875 \\ 0.05 \\ 0.05 \end{bmatrix}$$

$j := 1 .. 100$ $\omega_j := j \cdot 0.02 \cdot rad \cdot s^{-1}$

$$S_{i,j} := \frac{5}{16} \cdot \left[\frac{\omega m_i}{\omega_j}\right]^4 \cdot \left[\frac{H^2}{\omega_j} \cdot e^{-1.25 \cdot \left[\frac{\omega m_i}{\omega_j}\right]^4}\right]$$



US NAVY METHOD

Wind heeling arm : $HA(\theta) := \frac{0.0737 \cdot V^2 \cdot A \cdot z \cdot (\cos(\theta \cdot deg))^2}{1000 \cdot \Delta} \cdot \frac{t}{m^4 \cdot s^{-2}}$

Angle of equilibrium : $\theta_{0Navy} := root(HA(\theta_{d1}) - GZ(\theta_{d1}), \theta_{d1})$

$\theta_{0Navy} = 22.834$

Maximum angle : $\theta_{intercept} := root(HA(\theta_{d2}) - GZ(\theta_{d2}), \theta_{d2})$

$$\theta_{2Navy} := \begin{cases} \theta_{intercept} & \text{if } \theta_{intercept} \le \theta_f \\ \theta_f & \text{otherwise} \end{cases}$$

$\theta_{2Navy} = 70$

Roll back angle : $\theta_{1Navy} := 25$

Maximum windward angle $\theta_{wNavy} := \theta_{0Navy} - \theta_{1Navy}$

$\theta_{wNavy} = -2.166$

Energy balance calculations

$$Diff1_{Navy}(\theta) := if\left(\theta_{wNavy} \le \theta \le \theta_{0Navy}, |HA(\theta) - GZ(\theta)|, 0\right)$$

$$Diff2_{Navy}(\theta) := if\left(\theta_{0Navy} \le \theta \le \theta_{2Navy}, |HA(\theta) - GZ(\theta)|, 0\right)$$

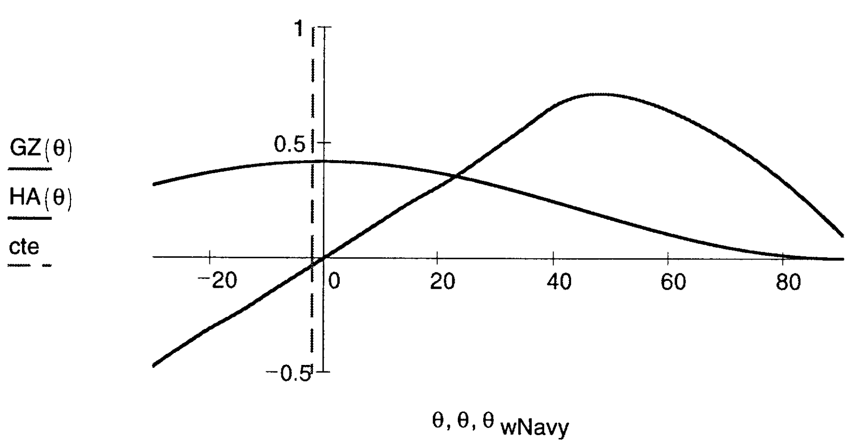
$$A_{1Navy} := \sum_{\theta} Diff1_{Navy}(\theta) \cdot 1$$

$$A_{2Navy} := \sum_{\theta} Diff2_{Navy}(\theta) \cdot 1$$

$A_{1Navy} = 6.007 \cdot m$

$A_{2Navy} = 19.114 \cdot m$

$$\frac{A_{2Navy}}{A_{1Navy}} = 3.182$$



IMO METHOD

Wind heeling arm :

$$C_D := 1.15$$

Steady wind :

$$H_{w1IMO}(\theta) := \frac{\frac{1}{2} \cdot C_D \cdot \rho_a \cdot V^2 \cdot A \cdot z}{1000 \cdot g \cdot \Delta} \cdot \left(\frac{t}{kg}\right)$$

Gust wind

$$H_{w2IMO}(\theta) := 1.5 \cdot H_{w1IMO}(\theta)$$

Angle of equilibrium :

$$\theta_{0IMO} := root\left(H_{w1IMO}(\theta_{d1}) - GZ(\theta_{d1}), \theta_{d1}\right)$$

$$\theta_{0IMO} = 27.178$$

Equilibrium with gust :

$$\theta_{gIMO} := root\left(H_{w2IMO}(\theta_{d1}) - GZ(\theta_{d1}), \theta_{d1}\right)$$

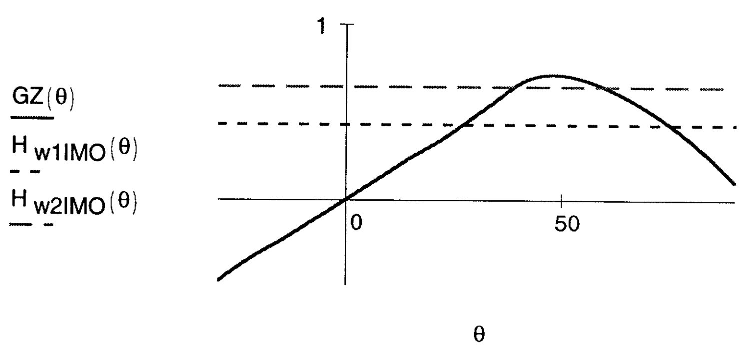
$$\theta_{gIMO} = 38.995$$

Maximum angle :

$$\theta_{intercept} := root\left(H_{w2IMO}(\theta_{d2}) - GZ(\theta_{d2}), \theta_{d2}\right)$$

$$\theta_{2IMO} := \begin{cases} \theta_{intercept} & \text{if } \theta_{intercept} \le \theta_f \\ \theta_f & \text{otherwise} \end{cases}$$

$$\theta_{2IMO} = 59.73$$



Coefficients necessary for Roll back angle :

Calculation of coefficient k

$$At/LB := \begin{bmatrix} 0.00 \\ 0.01 \\ 0.015 \\ 0.02 \\ .025 \\ 0.03 \\ 0.035 \\ 0.04 \\ 1 \end{bmatrix} \quad k := \begin{bmatrix} 1.00 \\ 0.98 \\ 0.95 \\ 0.88 \\ 0.79 \\ 0.74 \\ 0.72 \\ 0.70 \\ 0.70 \end{bmatrix}$$

$f(x) := \text{linterp}(At/LB, k, x)$ Linear interpolation based on given values

$$k := f\left(\frac{A_t}{L \cdot B}\right)$$

$k = 0.7$

Calculation of coefficient X_1

$$B/T := \begin{bmatrix} 0 \\ 2.4 \\ 2.5 \\ 2.7 \\ 2.8 \\ 2.9 \\ 3.0 \\ 3.1 \\ 3.2 \\ 3.3 \\ 3.4 \\ 3.5 \\ 10 \end{bmatrix} \quad X_1 := \begin{bmatrix} 1.00 \\ 1.00 \\ 0.98 \\ 0.96 \\ 0.95 \\ 0.93 \\ 0.91 \\ 0.90 \\ 0.88 \\ 0.86 \\ 0.82 \\ 0.80 \\ 0.80 \end{bmatrix}$$

$f(x) := \text{linterp}(B/T, X_1, x)$ Linear interpolation based on given values

$$X_1 := f\left(\frac{B}{T}\right)$$

$X_1 = 0.921$

Calculation of coefficient X_2

$$C_B := \begin{bmatrix} 0.00 \\ 0.45 \\ 0.5 \\ 0.55 \\ 0.60 \\ .65 \\ 0.70 \\ 1.00 \end{bmatrix} \quad X_2 := \begin{bmatrix} 0.75 \\ 0.75 \\ 0.82 \\ 0.89 \\ 0.95 \\ 0.97 \\ 1 \\ 1 \end{bmatrix}$$

$f(x) := \text{linterp}(C_B, X_2, x)$ Linear interpolation based on given values

$$X_2 := f(C_B)$$

$X_2 = 0.75$

Calculation of the ship natural period T_n

$$C := \left(0.373 + 0.023\cdot\frac{B}{T} - 0.043\cdot\frac{L}{100\cdot m}\right)\cdot\left(m^{-0.5}\cdot s\right)$$

$$T_n := 2\cdot\frac{C\cdot B}{\sqrt{GM}} \qquad T_n = 11.442\cdot s$$

Calculation of wave slope *s*

$$T_n := \begin{bmatrix} 0 \\ 6 \\ 7 \\ 8 \\ 12 \\ 14 \\ 16 \\ 18 \\ 20 \\ 40 \end{bmatrix}\cdot s \qquad S := \begin{bmatrix} 0.100 \\ 0.100 \\ 0.098 \\ 0.093 \\ 0.065 \\ 0.053 \\ 0.044 \\ 0.038 \\ 0.035 \\ 0.035 \end{bmatrix}$$

$f(x) := \text{linterp}(T_n, S, x)$ Linear interpolation based on given values

$s := f(T_n)$

$s = 0.069$

Test results coefficient : $C := 0.7635$

Bertin damping coefficient : $N := 0.02$

Height of center of gravity above water line $OG := KG - (D - T)$

Roll back angle

$$\theta_{1IMO} := \left[\frac{k\cdot X_1\cdot X_2}{C}\cdot 0.7\cdot\sqrt{\frac{\pi\cdot\left(0.73 + 0.6\cdot\frac{OG}{T}\right)\cdot 180\cdot s}{2\cdot N}}\right]$$

$\theta_{1IMO} = 13.274$

Maximum windward angle

$\theta_{wIMO} := \theta_{0IMO} - \theta_{1IMO}$

$\theta_{wIMO} = 13.904$

Energy balance calculations

$$\text{Diff1}_{IMO}(\theta) := \text{if}\left(\theta_{wIMO} \le \theta \le \theta_{gIMO}, \left|H_{w2IMO}(\theta) - GZ(\theta)\right|, 0\right)$$

$$\text{Diff2}_{IMO}(\theta) := \text{if}\left(\theta_{gIMO} \le \theta \le \theta_{2IMO}, \left|H_{w2IMO}(\theta) - GZ(\theta)\right|, 0\right)$$

$$A_{1IMO} := \sum_{\theta} \text{Diff1}_{IMO}(\theta) \cdot 1$$

$$A_{2IMO} := \sum_{\theta} \text{Diff2}_{IMO}(\theta) \cdot 1$$

$$A_{1IMO} = 5.71 \cdot m \qquad \frac{A_{2IMO}}{A_{1IMO}} = 0.167$$

$$A_{2IMO} = 0.953 \cdot m$$

PROPOSED NEW METHOD

Drag coefficient : $C_D := 1.12$

Projected Area : $$A_p(\theta) := \begin{cases} \left[C_w \cdot \frac{B \cdot L}{2} + \left(A - C_w \cdot \frac{B \cdot L}{2}\right) \cdot \cos(\theta \cdot deg)\right] & \text{if } \theta \le 90 \\ C_w \cdot \frac{B \cdot L}{2} & \text{otherwise} \end{cases}$$

Projected Lever : $$z_p(\theta) := \begin{cases} \left[\frac{B}{2} + \left(z - \frac{B}{2}\right) \cdot \cos(\theta \cdot deg)\right] & \text{if } \theta \le 90 \\ \frac{B}{2} & \text{otherwise} \end{cases}$$

Steady wind heeling arm $$H_{w1New}(\theta) := \frac{\frac{1}{2} \cdot C_D \cdot \rho_a \cdot V^2 \cdot A_p(\theta) \cdot z_p(\theta)}{1000 \cdot g \cdot \Delta} \cdot \left(\frac{t}{kg}\right)$$

Gust wind heeling arm $$H_{w2New}(\theta) := 1.5 \cdot H_{w1New}(\theta)$$

Angle of equilibrium : $$\theta_{0New} := \text{root}\left(H_{w1New}(\theta_{d1}) - GZ(\theta_{d1}), \theta_{d1}\right)$$

$$\theta_{0New} = 24.877$$

Equilibrium with gust :

$$\theta_{gNew} := \mathrm{root}\left(H_{w2New}(\theta_{d1}) - GZ(\theta_{d1}), \theta_{d1}\right)$$

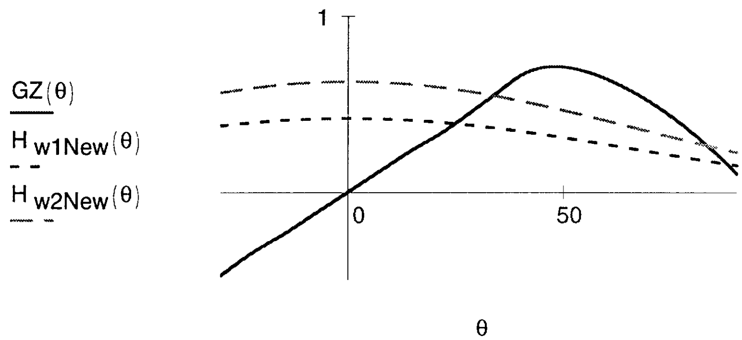
$$\theta_{gNew} = 33.702$$

Maximum angle :

$$\theta_{intercept} := \mathrm{root}\left(H_{w2New}(\theta_{d2}) - GZ(\theta_{d2}), \theta_{d2}\right)$$

$$\theta_{2New} := \begin{vmatrix} \theta_{intercept} & \text{if } \theta_{intercept} \le \theta_f \\ \theta_f & \text{otherwise} \end{vmatrix}$$

$$\theta_{2New} = 70$$



Calculation of the roll back angle

Natural roll period

$$C := \left(0.3725 + 0.0227\cdot\frac{B}{T} - 0.043\cdot\frac{L}{100\cdot m}\right)$$

$$T_n := \frac{2\,\pi\cdot C\cdot B}{\sqrt{g\cdot GM}} \qquad T_n = 11.436\cdot s \qquad \omega_n = \frac{2\,\pi}{T_n}$$

Non dimensional damping factor

$$\beta^* := 19.25\cdot\left(A_k\cdot\sqrt{b_k} + 0.0024\cdot L\cdot B\cdot\sqrt{d}\right)\cdot d^2\cdot\frac{\sqrt{d\cdot\theta_i}}{C_B\cdot L\cdot B^3\cdot T}$$

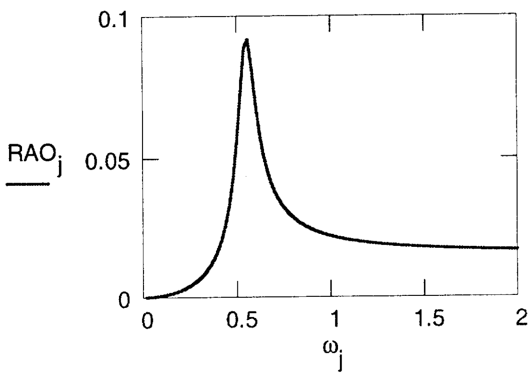
$$\beta^* = 0.083$$

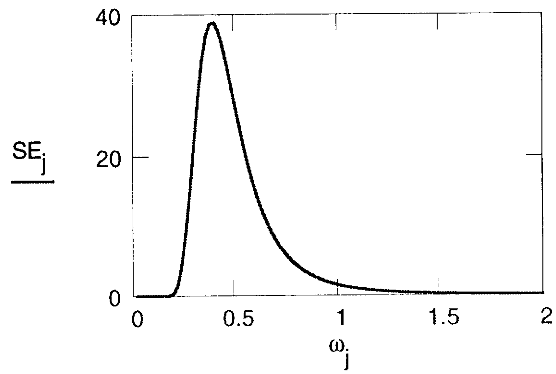
RAO

$$RAO_j := \frac{1}{\sqrt{\left[1 - \left(\frac{\omega_j}{\omega_n}\right)^2\right]^2 + \left(2\cdot\frac{\omega_j}{\omega_n}\cdot\beta^*\right)^2}}\cdot\frac{(\omega_j)^2}{2\cdot g}$$

$$SE_j := \sum_{i=1}^{9} p\omega_{m_i} \cdot S_{i,j}$$

$$SR_j := (RAO_j)^2 \cdot SE_j$$





Roll back angle: $$m_0 := \sum_{j=2}^{100} SR_j \cdot (\omega_j - \omega_{j-1})$$

$$\theta_{1New} := 2 \cdot \sqrt{m_0} \cdot \frac{180}{\pi} \qquad \theta_{1New} = 16.725$$

With the initial roll angle used to calculate the damping factor $\theta_i \equiv 16 \cdot deg$

Maximum windward angle $$\theta_{wNew} := \theta_{0New} - \theta_{1New}$$

$$\theta_{wNew} = 8.152$$

Energy balance calculations

$$Diff1_{New}(\theta) := if\left(\theta_{wNew} \le \theta \le \theta_{gNew}, \left|H_{w2New}(\theta) - GZ(\theta)\right|, 0\right)$$

$$Diff2_{New}(\theta) := if\left(\theta_{gNew} \le \theta \le \theta_{2New}, \left|H_{w2New}(\theta) - GZ(\theta)\right|, 0\right)$$

$$A_{1New} := \sum_{\theta} Diff1_{New}(\theta) \cdot 1$$

$$A_{2New} := \sum_{\theta} Diff2_{New}(\theta) \cdot 1$$

$$A_{1New} = 6.469 \cdot m \qquad \frac{A_{2New}}{A_{1New}} = 1.109$$

$$A_{2New} = 7.172 \cdot m$$

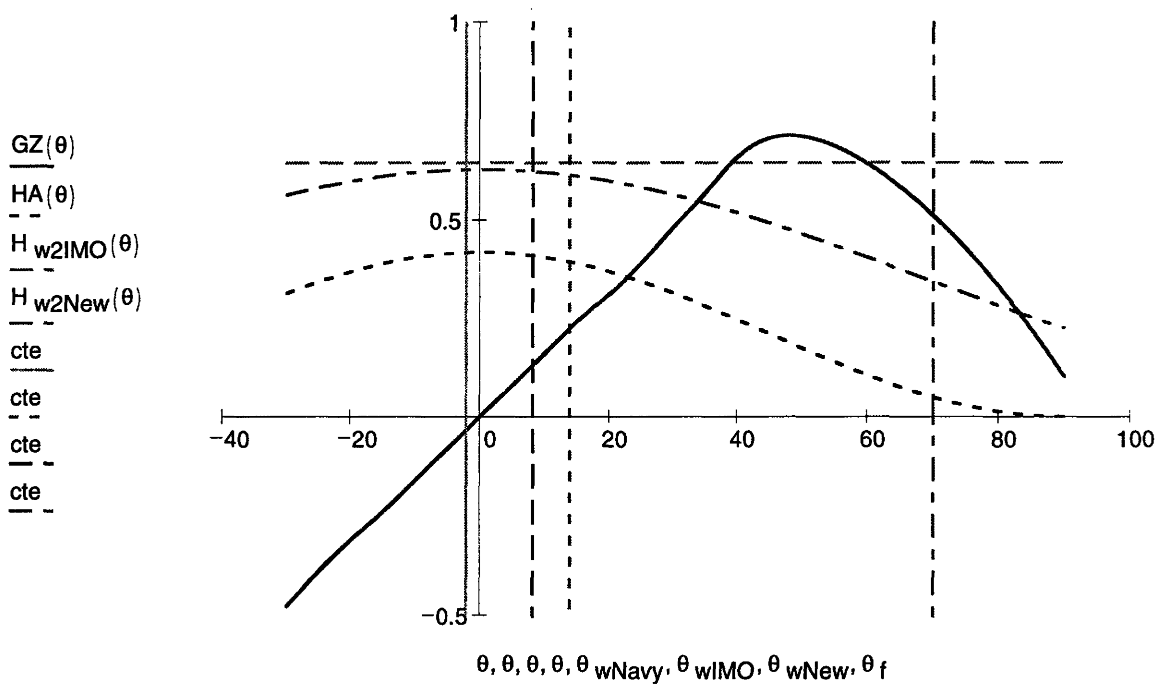
COMPARISON OF THE ANGLES BETWEEN THE THREE METHODS

$\theta_{0Navy} = 22.834$ $\theta_{1Navy} = 25$ $\theta_{2Navy} = 70$ $\theta_{wNavy} = -2.166$

$\theta_{0IMO} = 27.178$ $\theta_{1IMO} = 13.274$ $\theta_{2IMO} = 59.73$ $\theta_{wIMO} = 13.904$

$\theta_{0New} = 24.877$ $\theta_{1New} = 16.725$ $\theta_{2New} = 70$ $\theta_{wNew} = 8.152$

COMPARISON OF ENERGY BALANCES BETWEEN THE THREE METHODS



$A_{1Navy} = 6.007 \cdot m$ $A_{2Navy} = 19.114 \cdot m$ $\frac{A_{2Navy}}{A_{1Navy}} = 3.182$

$A_{1IMO} = 5.71 \cdot m$ $A_{2IMO} = 0.953 \cdot m$ $\frac{A_{2IMO}}{A_{1IMO}} = 0.167$

$A_{1New} = 6.469 \cdot m$ $A_{2New} = 7.172 \cdot m$ $\frac{A_{2New}}{A_{1New}} = 1.109$

APPENDIX 4

DDG 51 COMPARISON

This Appendix gives an example of the calculations of the dynamic stability for DDG 51 with the US Navy, IMO and the proposed method. The Appendix is divided into two parts :

- the section giving the ship characteristics for the Test Condition
- the complete example of the calculations for the Full Load Condition with a wind speed of 100 knots

DDG 51 TEST

UNITS AND CONSTANTS

Second :	$s := sec$
Metric ton :	$t := 1000 \cdot kg$
Air density :	$\rho_a := 1.293 \cdot kg \cdot m^{-3}$
Sea Water density :	$\rho_w := 1025 \cdot kg \cdot m^{-3}$
Gravity :	$g := 9.81 \cdot m \cdot s^{-2}$
Range of angles	$\theta := -30, -29 .. 90$
Constant for graphs	$cte := -1 .. 1$
Matrix origin	$ORIGIN := 1$

SHIP CHARACTERISTICS

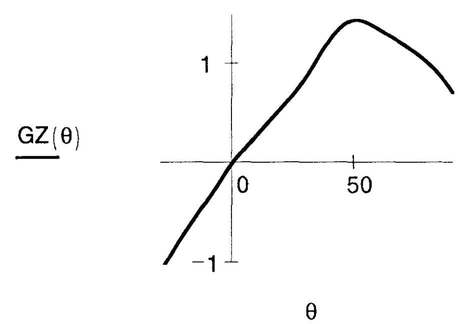
Displacement :	$\Delta := 8693 \cdot t$	
Length between perpendiculars :	$L := 142 \cdot m$	
Beam :	$B := 18 \cdot m$	
Draft :	$T := 6.34 \cdot m$	
Depth :	$D := 12.7 \cdot m$	
Block coefficient	$C_B := 0.505$	
Waterplane coefficient	$C_w := 0.79$	
Height of center of gravity :	$KG := 6.9 \cdot m$	
Transverse methacentric height	$GM := 1.26 \cdot m$	
Area above waterline :	$A := 1442.5 \cdot m^2$	
Distance between the center of the projected underwater area and the center of the projected area above the water	$z := 9.52 \cdot m$	
Skeg area	$A_s := 14 \cdot m^2$	
Length of Bilge Keels	$L_k := 49.22 \cdot m$	
Height of Bilge Keels	$b_k := 0.91 \cdot m$	
Total Area of Bilge Keels	$A_k := 2 \cdot L_k \cdot b_k$	$A_k = 89.58 \cdot m^2$
Distance between center line at DWL and bilge keel	$d := 8.25 \cdot m$	
Total area of bilge keels + skeg	$A_t := A_s + A_k$	$A_t = 103.58 \cdot m^2$

Righting arm curve (GZ)

$$\theta := \begin{bmatrix} -30 \\ -20 \\ -10 \\ -5 \\ 0 \\ 5 \\ 10 \\ 20 \\ 30 \\ 40 \\ 50 \\ 60 \\ 70 \\ 80 \end{bmatrix} \qquad GZ := \begin{bmatrix} -1.11 \\ -0.71 \\ -0.36 \\ -0.18 \\ 0.00 \\ 0.14 \\ 0.28 \\ 0.55 \\ 0.86 \\ 1.25 \\ 1.44 \\ 1.34 \\ 1.19 \\ 1 \end{bmatrix}$$

Cubic interpolation of GZ curve

$$GZ(\theta) := \mathrm{interp}(\mathrm{cspline}(\theta, GZ), \theta, GZ, \theta)\cdot m$$



First intersept default value: $\theta_{d1} := 0$

Second intersept default value: $\theta_{d2} := 90$

Downflooding angle : $\theta_f := 70$

DDG 51 FULL LOAD 100 KNOTS

UNITS AND CONSTANTS

Second :	$s := sec$
Metric ton :	$t := 1000 \cdot kg$
Air density :	$\rho_a := 1.293 \cdot kg \cdot m^{-3}$
Sea Water density :	$\rho_w := 1025 \cdot kg \cdot m^{-3}$
Gravity :	$g := 9.81 \cdot m \cdot s^{-2}$
Range of angles	$\theta := -30, -29 .. 90$
Constant for graphs	$cte := -1 .. 1$
Matrix origin	$ORIGIN := 1$

SHIP CHARACTERISTICS

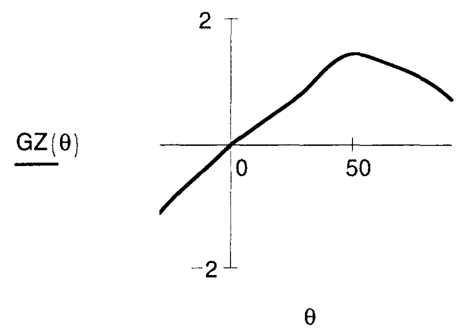
Displacement :	$\Delta := 8304 \cdot t$	
Length between perpendiculars :	$L := 142 \cdot m$	
Beam :	$B := 18 \cdot m$	
Draft :	$T := 6.22 \cdot m$	
Depth :	$D := 12.7 \cdot m$	
Block coefficient	$C_B := 0.505$	
Waterplane coefficient	$C_W := 0.79$	
Height of center of gravity :	$KG := 7.21 \cdot m$	
Transverse methacentric height	$GM := 1.66 \cdot m$	
Area above waterline :	$A := 1442.5 \cdot m^2$	
Distance between the center of the projected underwater area and the center of the projected area above the water	$z := 9.52 \cdot m$	
Skeg area	$A_s := 14 \cdot m^2$	
Length of Bilge Keels	$L_k := 49.22 \cdot m$	
Height of Bilge Keels	$b_k := 0.91 \cdot m$	
Total Area of Bilge Keels	$A_k := 2 \cdot L_k \cdot b_k$	$A_k = 89.58 \cdot m^2$
Distance between center line at DWL and bilge keel	$d := 8.25 \cdot m$	
Total area of bilge keels + skeg	$A_t := A_s + A_k$	$A_t = 103.58 \cdot m^2$

Righting arm curve (GZ)

$$\theta := \begin{bmatrix} -30 \\ -20 \\ -10 \\ -5 \\ 0 \\ 5 \\ 10 \\ 20 \\ 30 \\ 40 \\ 50 \\ 60 \\ 70 \\ 80 \end{bmatrix} \qquad GZ := \begin{bmatrix} -1.11 \\ -0.71 \\ -0.36 \\ -0.18 \\ 0.00 \\ 0.14 \\ 0.28 \\ 0.55 \\ 0.86 \\ 1.25 \\ 1.44 \\ 1.34 \\ 1.19 \\ 1 \end{bmatrix}$$

Cubic interpolation of GZ curve

$$GZ(\theta) := \mathrm{interp}(\mathrm{cspline}(\theta, GZ), \theta, GZ, \theta)\cdot m$$



First intersept default value: $\theta_{d1} := 0$

Second intersept default value: $\theta_{d2} := 90$

Downflooding angle : $\theta_f := 70$

WEATHER CHARACTERISTICS

Wind speed : $V := 100 \cdot \frac{1852}{3600} \cdot m \cdot s^{-1}$ $V = 51.444 \cdot m \cdot s^{-1}$

Sea state 8/9, short term analysis, Average North Atlantic storm
H_s = 14 m; duration 5 hours; 9 modal period family; 2 parameter model

Description of family

$H := 14 \cdot m$ $i := 1..9$

$\omega m_1 := 0.048 \cdot \left(8.75 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$

$\omega m_2 := 0.054 \cdot \left(8.44 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$ $\omega m_3 := 0.061 \cdot \left(8.07 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$

$\omega m_4 := 0.069 \cdot \left(7.77 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$ $\omega m_5 := 0.079 \cdot \left(7.63 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$

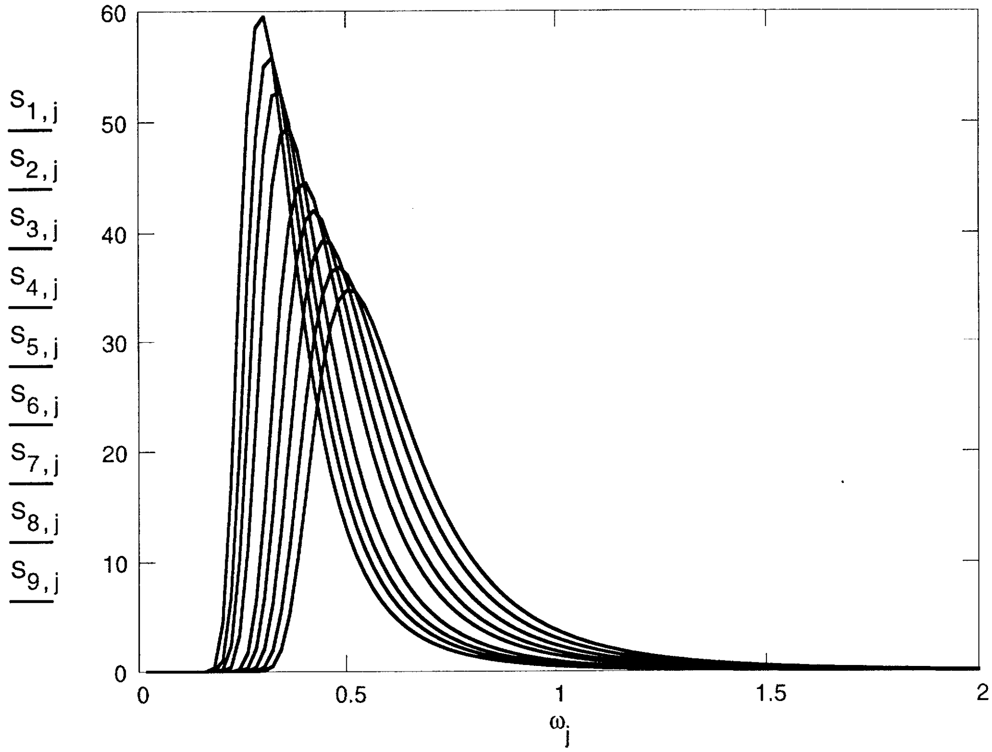
$\omega m_6 := 0.099 \cdot \left(6.87 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$ $\omega m_7 := 0.111 \cdot \left(6.67 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$

$\omega m_8 := 0.119 \cdot \left(6.65 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$ $\omega m_9 := 0.134 \cdot \left(6.41 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$

$$p\omega m := \begin{bmatrix} 0.05 \\ 0.05 \\ 0.0875 \\ 0.1875 \\ 0.25 \\ 0.1875 \\ 0.0875 \\ 0.05 \\ 0.05 \end{bmatrix}$$

$j := 1..100$ $\omega_j := j \cdot 0.02 \cdot rad \cdot s^{-1}$

$$S_{i,j} := \frac{5}{16} \cdot \left[\frac{\omega m_i}{\omega_j}\right]^4 \cdot \left[\frac{H^2}{\omega_j} \cdot e^{-1.25 \cdot \left[\frac{\omega m_i}{\omega_j}\right]^4}\right]$$



US NAVY METHOD

Wind heeling arm : $HA(\theta) := \frac{0.0737 \cdot V^2 \cdot A \cdot z \cdot (\cos(\theta \cdot deg))^2}{1000 \cdot \Delta} \cdot \frac{t}{m^4 \cdot s^{-2}}$

Angle of equilibrium : $\theta_{0Navy} := \text{root}\left(HA(\theta_{d1}) - GZ(\theta_{d1}), \theta_{d1}\right)$

$\theta_{0Navy} = 11.13$

Maximum angle : $\theta_{intercept} := \text{root}\left(HA(\theta_{d2}) - GZ(\theta_{d2}), \theta_{d2}\right)$

$$\theta_{2Navy} := \begin{cases} \theta_{intercept} & \text{if } \theta_{intercept} \le \theta_f \\ \theta_f & \text{otherwise} \end{cases}$$

$\theta_{2Navy} = 70$

Roll back angle : $\theta_{1Navy} := 25$

Maximum windward angle $\theta_{wNavy} := \theta_{0Navy} - \theta_{1Navy}$

$\theta_{wNavy} = -13.87$

Energy balance calculations

$$\text{Diff1}_{Navy}(\theta) := \text{if}\left(\theta_{wNavy} \le \theta \le \theta_{0Navy}, |HA(\theta) - GZ(\theta)|, 0\right)$$

$$\text{Diff2}_{Navy}(\theta) := \text{if}\left(\theta_{0Navy} \le \theta \le \theta_{2Navy}, |HA(\theta) - GZ(\theta)|, 0\right)$$

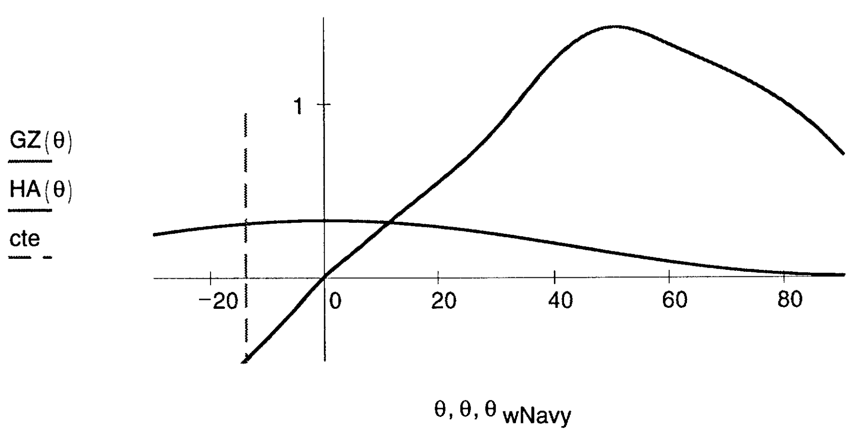
$$A_{1Navy} := \sum_{\theta} \text{Diff1}_{Navy}(\theta) \cdot 1$$

$$A_{2Navy} := \sum_{\theta} \text{Diff2}_{Navy}(\theta) \cdot 1$$

$A_{1Navy} = 9.342 \cdot m$

$A_{2Navy} = 51.641 \cdot m$

$$\frac{A_{2Navy}}{A_{1Navy}} = 5.528$$



IMO METHOD

Wind heeling arm :

$$C_D := 1.15$$

Steady wind :

$$H_{w1IMO}(\theta) := \frac{\frac{1}{2}\cdot C_D\cdot \rho_a\cdot V^2\cdot A\cdot z}{1000\cdot g\cdot \Delta}\cdot\left(\frac{t}{kg}\right)$$

Gust wind

$$H_{w2IMO}(\theta) := 1.5\cdot H_{w1IMO}(\theta)$$

Angle of equilibrium :

$$\theta_{0IMO} := root\left(H_{w1IMO}\left(\theta_{d1}\right) - GZ\left(\theta_{d1}\right), \theta_{d1}\right)$$

$$\theta_{0IMO} = 11.848$$

Equilibrium with gust :

$$\theta_{gIMO} := root\left(H_{w2IMO}\left(\theta_{d1}\right) - GZ\left(\theta_{d1}\right), \theta_{d1}\right)$$

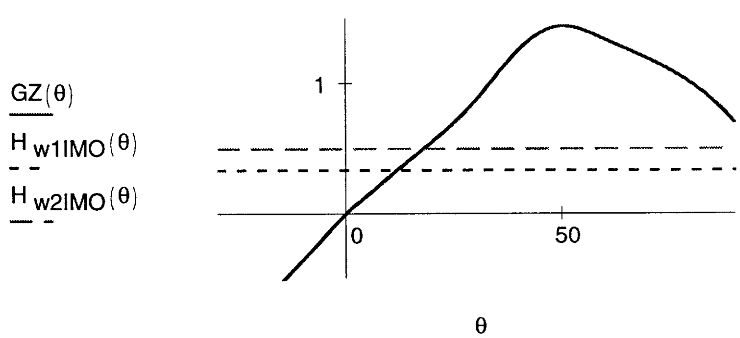
$$\theta_{gIMO} = 18.055$$

Maximum angle :

$$\theta_{intercept} := root\left(H_{w2IMO}\left(\theta_{d2}\right) - GZ\left(\theta_{d2}\right), \theta_{d2}\right)$$

$$\theta_{2IMO} := \begin{cases} \theta_{intercept} & \text{if } \theta_{intercept} \le \theta_f \\ \theta_f & \text{otherwise} \end{cases}$$

$$\theta_{2IMO} = 70$$



Coefficients necessary for Roll back angle :

Calculation of coefficient k

$$At/LB := \begin{bmatrix} 0.00 \\ 0.01 \\ 0.015 \\ 0.02 \\ .025 \\ 0.03 \\ 0.035 \\ 0.04 \\ 1 \end{bmatrix} \quad k := \begin{bmatrix} 1.00 \\ 0.98 \\ 0.95 \\ 0.88 \\ 0.79 \\ 0.74 \\ 0.72 \\ 0.70 \\ 0.70 \end{bmatrix}$$

$f(x) := \text{linterp}(At/LB, k, x)$ Linear interpolation based on given values

$$k := f\left(\frac{A_t}{L \cdot B}\right)$$

$k = 0.7$

Calculation of coefficient X_1

$$B/T := \begin{bmatrix} 0 \\ 2.4 \\ 2.5 \\ 2.7 \\ 2.8 \\ 2.9 \\ 3.0 \\ 3.1 \\ 3.2 \\ 3.3 \\ 3.4 \\ 3.5 \\ 10 \end{bmatrix} \quad X_1 := \begin{bmatrix} 1.00 \\ 1.00 \\ 0.98 \\ 0.96 \\ 0.95 \\ 0.93 \\ 0.91 \\ 0.90 \\ 0.88 \\ 0.86 \\ 0.82 \\ 0.80 \\ 0.80 \end{bmatrix}$$

$f(x) := \text{linterp}(B/T, X_1, x)$ Linear interpolation based on given values

$$X_1 := f\left(\frac{B}{T}\right)$$

$X_1 = 0.931$

Calculation of coefficient X_2

$$C_B := \begin{bmatrix} 0.00 \\ 0.45 \\ 0.5 \\ 0.55 \\ 0.60 \\ .65 \\ 0.70 \\ 1.00 \end{bmatrix} \quad X_2 := \begin{bmatrix} 0.75 \\ 0.75 \\ 0.82 \\ 0.89 \\ 0.95 \\ 0.97 \\ 1 \\ 1 \end{bmatrix}$$

$f(x) := \text{linterp}(C_B, X_2, x)$ Linear interpolation based on given values

$$X_2 := f(C_B)$$

$X_2 = 0.827$

Calculation of the ship natural period T_n

$$C := \left(0.373 + 0.023\cdot\frac{B}{T} - 0.043\cdot\frac{L}{100\cdot m}\right)\cdot\left(m^{-0.5}\cdot s\right)$$

$$T_n := 2\cdot\frac{C\cdot B}{\sqrt{GM}} \qquad T_n = 10.576\cdot s$$

Calculation of wave slope *s*

$$T_n := \begin{bmatrix} 0 \\ 6 \\ 7 \\ 8 \\ 12 \\ 14 \\ 16 \\ 18 \\ 20 \\ 40 \end{bmatrix}\cdot s \qquad S := \begin{bmatrix} 0.100 \\ 0.100 \\ 0.098 \\ 0.093 \\ 0.065 \\ 0.053 \\ 0.044 \\ 0.038 \\ 0.035 \\ 0.035 \end{bmatrix}$$

$f(x) := \mathrm{linterp}(T_n, S, x)$ Linear interpolation base on given values

$s := f(T_n)$

$s = 0.075$

Test results coefficient : $C := 0.7635$

Bertin damping coefficient : $N := 0.02$

Height of center of gravity above water line $OG := KG - (D - T)$

Roll back angle

$$\theta_{1IMO} := \left[\frac{k\cdot X_1\cdot X_2}{C}\cdot 0.7\cdot\sqrt{\frac{\pi\cdot\left(0.73 + 0.6\cdot\frac{OG}{T}\right)\cdot 180\cdot s}{2\cdot N}}\right]$$

$\theta_{1IMO} = 14.396$

Maximum windward angle

$\theta_{wIMO} := \theta_{0IMO} - \theta_{1IMO}$

$\theta_{wIMO} = -2.547$

Energy balance calculations

$$\mathrm{Diff1}_{IMO}(\theta) := \mathrm{if}\left(\theta_{wIMO} \le \theta \le \theta_{gIMO}, \left|H_{w2IMO}(\theta) - GZ(\theta)\right|, 0\right)$$

$$\mathrm{Diff2}_{IMO}(\theta) := \mathrm{if}\left(\theta_{gIMO} \le \theta \le \theta_{2IMO}, \left|H_{w2IMO}(\theta) - GZ(\theta)\right|, 0\right)$$

$$A_{1IMO} := \sum_{\theta} \mathrm{Diff1}_{IMO}(\theta) \cdot 1$$

$$A_{2IMO} := \sum_{\theta} \mathrm{Diff2}_{IMO}(\theta) \cdot 1$$

$$A_{1IMO} = 5.784 \cdot m \qquad \frac{A_{2IMO}}{A_{1IMO}} = 5.788$$

$$A_{2IMO} = 33.476 \cdot m$$

PROPOSED NEW METHOD

Drag coefficient : $C_D := 1.12$

Projected Area :

$$A_p(\theta) := \begin{cases} \left[C_w \cdot \frac{B \cdot L}{2} + \left(A - C_w \cdot \frac{B \cdot L}{2}\right) \cdot \cos(\theta \cdot deg)\right] & \text{if } \theta \le 90 \\ C_w \cdot \frac{B \cdot L}{2} & \text{otherwise} \end{cases}$$

Projected Lever :

$$z_p(\theta) := \begin{cases} \left[\frac{B}{2} + \left(z - \frac{B}{2}\right) \cdot \cos(\theta \cdot deg)\right] & \text{if } \theta \le 90 \\ \frac{B}{2} & \text{otherwise} \end{cases}$$

Steady wind heeling arm

$$H_{w1New}(\theta) := \frac{\frac{1}{2} \cdot C_D \cdot \rho_a \cdot V^2 \cdot A_p(\theta) \cdot z_p(\theta)}{1000 \cdot g \cdot \Delta} \cdot \left(\frac{t}{kg}\right)$$

Gust wind heeling arm

$$H_{w2New}(\theta) := 1.5 \cdot H_{w1New}(\theta)$$

Angle of equilibrium :

$$\theta_{0New} := \mathrm{root}\left(H_{w1New}(\theta_{d1}) - GZ(\theta_{d1}), \theta_{d1}\right)$$

$$\theta_{0New} = 11.465$$

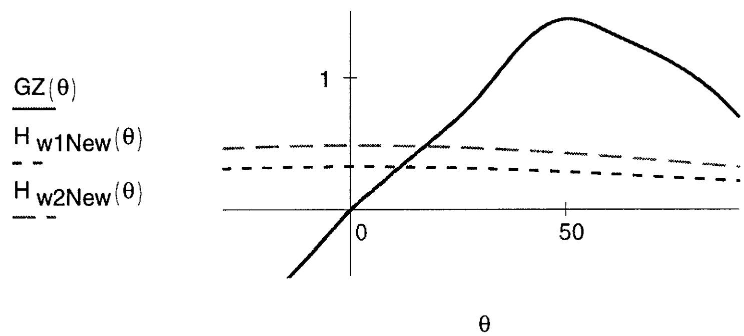
Equilibrium with gust : $\theta_{gNew} := \text{root}\left(H_{w2New}(\theta_{d1}) - GZ(\theta_{d1}), \theta_{d1}\right)$

$\theta_{gNew} = 17.281$

Maximum angle : $\theta_{intercept} := \text{root}\left(H_{w2New}(\theta_{d2}) - GZ(\theta_{d2}), \theta_{d2}\right)$

$$\theta_{2New} := \begin{cases} \theta_{intercept} & \text{if } \theta_{intercept} \le \theta_f \\ \theta_f & \text{otherwise} \end{cases}$$

$\theta_{2New} = 70$



Calculation of the roll back angle

Natural roll period $C := \left(0.3725 + 0.0227 \cdot \frac{B}{T} - 0.043 \cdot \frac{L}{100 \cdot m}\right)$

$T_n := \frac{2\pi \cdot C \cdot B}{\sqrt{g \cdot GM}}$ $T_n = 10.57 \cdot s$ $\omega_n := \frac{2\pi}{T_n}$

Non dimensional damping factor

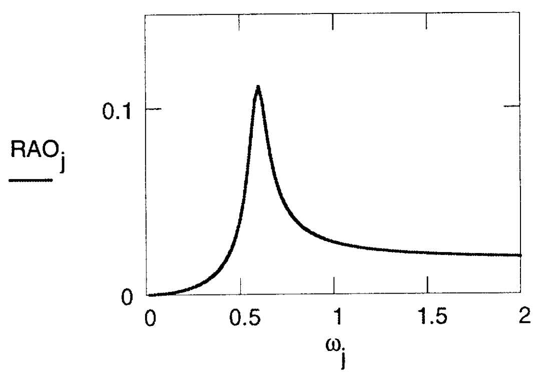
$$\beta^* := 19.25 \cdot \left(A_k \cdot \sqrt{b_k} + 0.0024 \cdot L \cdot B \cdot \sqrt{d}\right) \cdot d^2 \cdot \frac{\sqrt{d \cdot \theta_i}}{C_B \cdot L \cdot B^3 \cdot T}$$

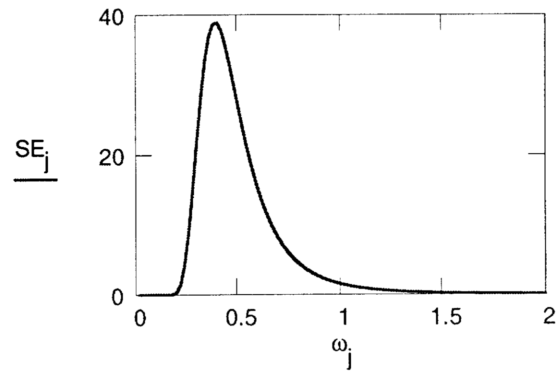
$\beta^* = 0.081$

RAO $$RAO_j := \frac{1}{\sqrt{\left[1 - \left(\frac{\omega_j}{\omega_n}\right)^2\right]^2 + \left(2 \cdot \frac{\omega_j}{\omega_n} \cdot \beta^*\right)^2}} \cdot \frac{(\omega_j)^2}{2 \cdot g}$$

$$SE_j := \sum_{i=1}^{9} p\omega\, m_i \cdot S_{i,j}$$

$$SR_j := (RAO_j)^2 \cdot SE_j$$





Roll back angle: $$m_0 := \sum_{j=2}^{100} SR_j \cdot (\omega_j - \omega_{j-1})$$

$$\theta_{1New} := 2 \cdot \sqrt{m_0} \cdot \frac{180}{\pi} \qquad \theta_{1New} = 18.129$$

With the initial roll angle used to calculate the damping factor $\theta_i \equiv 17 \cdot deg$

Maximum windward angle $\theta_{wNew} := \theta_{0New} - \theta_{1New}$

$$\theta_{wNew} = -6.664$$

Energy balance calculations

$$Diff1_{New}(\theta) := if(\theta_{wNew} \le \theta \le \theta_{gNew}, |H_{w2New}(\theta) - GZ(\theta)|, 0)$$

$$Diff2_{New}(\theta) := if(\theta_{gNew} \le \theta \le \theta_{2New}, |H_{w2New}(\theta) - GZ(\theta)|, 0)$$

$$A_{1New} := \sum_{\theta} Diff1_{New}(\theta) \cdot 1$$

$$A_{2New} := \sum_{\theta} Diff2_{New}(\theta) \cdot 1$$

$$A_{1New} = 8.057 \cdot m \qquad \frac{A_{2New}}{A_{1New}} = 4.58$$

$$A_{2New} = 36.896 \cdot m$$

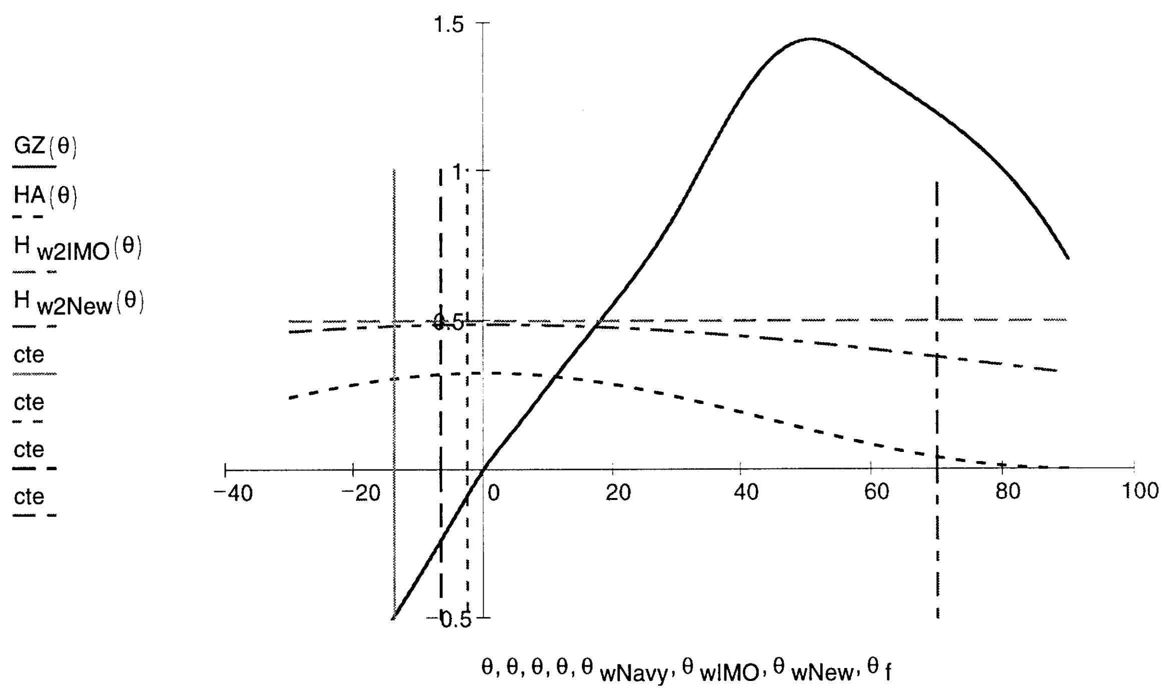
COMPARISON OF THE ANGLES BETWEEN THE THREE METHODS

$\theta_{0Navy} = 11.13$ $\theta_{1Navy} = 25$ $\theta_{2Navy} = 70$ $\theta_{wNavy} = -13.87$

$\theta_{0IMO} = 11.848$ $\theta_{1IMO} = 14.396$ $\theta_{2IMO} = 70$ $\theta_{wIMO} = -2.547$

$\theta_{0New} = 11.465$ $\theta_{1New} = 18.129$ $\theta_{2New} = 70$ $\theta_{wNew} = -6.664$

COMPARISON OF ENERGY BALANCES BETWEEN THE THREE METHODS



$A_{1Navy} = 9.342 \cdot m$ $A_{2Navy} = 51.641 \cdot m$ $\frac{A_{2Navy}}{A_{1Navy}} = 5.528$

$A_{1IMO} = 5.784 \cdot m$ $A_{2IMO} = 33.476 \cdot m$ $\frac{A_{2IMO}}{A_{1IMO}} = 5.788$

$A_{1New} = 8.057 \cdot m$ $A_{2New} = 36.896 \cdot m$ $\frac{A_{2New}}{A_{1New}} = 4.58$

APPENDIX 5

DD 963 COMPARISON

This Appendix gives an example of the calculations of the dynamic stability for DD 963 with the US Navy, IMO and the proposed method. The Appendix is divided into two parts :

- the section giving the ship characteristics for the Test Condition
- the complete example of the calculations for the Full Load Condition with a wind speed of 100 knots

DD 963 TEST

UNITS AND CONSTANTS

Second :	$s := sec$
Metric ton :	$t := 1000 \cdot kg$
Air density :	$\rho_a := 1.293 \cdot kg \cdot m^{-3}$
Sea Water density :	$\rho_w := 1025 \cdot kg \cdot m^{-3}$
Gravity :	$g := 9.81 \cdot m \cdot s^{-2}$
Range of angles	$\theta := -30, -29 .. 90$
Constant for graphs	$cte := -1 .. 1$
Matrix origin	$ORIGIN := 1$

SHIP CHARACTERISTICS

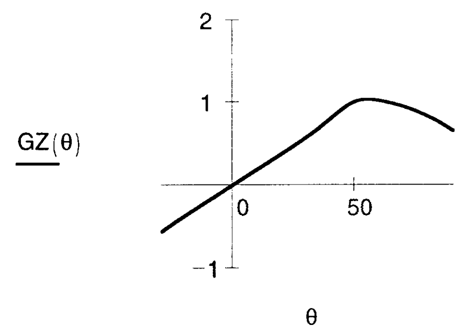
Displacement :	$\Delta := 8345 \cdot t$	
Length between perpendiculars :	$L := 161.6 \cdot m$	
Beam :	$B := 16.8 \cdot m$	
Draft :	$T := 6.1 \cdot m$	
Depth :	$D := 12.95 \cdot m$	
Block coefficient	$C_B := 0.461$	
Waterplane coefficient	$C_W := 0.74$	
Height of center of gravity :	$KG := 6.97 \cdot m$	
Transverse methacentric height	$GM := 1.37 \cdot m$	
Area above waterline :	$A := 1944 \cdot m^2$	
Distance between the center of the projected underwater area and the center of the projected area above the water	$z := 10.76 \cdot m$	
Skeg area	$A_s := 20.9 \cdot m^2$	
Length of Bilge Keels	$L_k := 53.34 \cdot m$	
Height of Bilge Keels	$b_k := 1.016 \cdot m$	
Total Area of Bilge Keels	$A_k := 2 \cdot L_k \cdot b_k$	$A_k = 108.387 \cdot m^2$
Distance between center line at DWL and bilge keel	$d := 7.92 \cdot m$	
Total area of bilge keels + skeg	$A_t := A_s + A_k$	$A_t = 129.287 \cdot m^2$

Righting arm curve (GZ)

$$\theta := \begin{bmatrix} -30 \\ -25 \\ -20 \\ -15 \\ -10 \\ -5 \\ 0 \\ 5 \\ 10 \\ 15 \\ 20 \\ 25 \\ 30 \\ 35 \\ 40 \\ 50 \\ 60 \\ 70 \\ 80 \end{bmatrix} \qquad GZ := \begin{bmatrix} -0.572 \\ -0.471 \\ -0.374 \\ -0.28 \\ -0.187 \\ -0.094 \\ 0.00 \\ 0.094 \\ 0.187 \\ 0.28 \\ 0.374 \\ 0.471 \\ 0.572 \\ 0.679 \\ 0.799 \\ 1.003 \\ 1.013 \\ 0.94 \\ 0.82 \end{bmatrix}$$

Cubic interpolation of GZ curve

$$GZ(\theta) := \text{interp}(\text{cspline}(\theta, GZ), \theta, GZ, \theta) \cdot m$$



First intersept default value: $\theta_{d1} := 0$

Second intersept default value: $\theta_{d2} := 90$

Downflooding angle : $\theta_f := 70$

DD 963 FULL LOAD 100 KNOTS

UNITS AND CONSTANTS

Second :	$s := sec$
Metric ton :	$t := 1000 \cdot kg$
Air density :	$\rho_a := 1.293 \cdot kg \cdot m^{-3}$
Sea Water density :	$\rho_w := 1025 \cdot kg \cdot m^{-3}$
Gravity :	$g := 9.81 \cdot m \cdot s^{-2}$
Range of angles	$\theta := -30, -29 .. 90$
Constant for graphs	$cte := -1 .. 1$
Matrix origin	$ORIGIN := 1$

SHIP CHARACTERISTICS

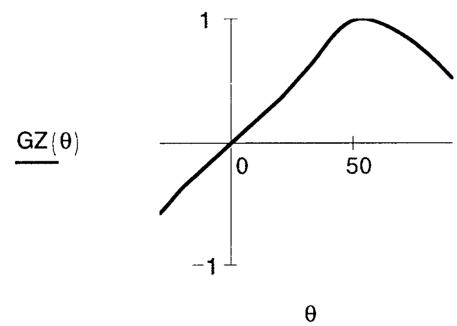
Displacement :	$\Delta := 9038 \cdot t$	
Length between perpendiculars :	$L := 161.6 \cdot m$	
Beam :	$B := 16.8 \cdot m$	
Draft :	$T := 6.45 \cdot m$	
Depth :	$D := 12.95 \cdot m$	
Block coefficient	$C_B := 0.461$	
Waterplane coefficient	$C_W := 0.74$	
Height of center of gravity :	$KG := 7.00 \cdot m$	
Transverse methacentric height	$GM := 1.01 \cdot m$	
Area above waterline :	$A := 1944 \cdot m^2$	
Distance between the center of the projected underwater area and the center of the projected area above the water	$z := 10.76 \cdot m$	
Skeg area	$A_s := 20.9 \cdot m^2$	
Length of Bilge Keels	$L_k := 53.34 \cdot m$	
Height of Bilge Keels	$b_k := 1.016 \cdot m$	
Total Area of Bilge Keels	$A_k := 2 \cdot L_k \cdot b_k$	$A_k = 108.387 \cdot m^2$
Distance between center line at DWL and bilge keel	$d := 7.92 \cdot m$	
Total area of bilge keels + skeg	$A_t := A_s + A_k$	$A_t = 129.287 \cdot m^2$

Righting arm curve (GZ)

$$\theta := \begin{bmatrix} -30 \\ -25 \\ -20 \\ -15 \\ -10 \\ -5 \\ 0 \\ 5 \\ 10 \\ 15 \\ 20 \\ 25 \\ 30 \\ 35 \\ 40 \\ 50 \\ 60 \\ 70 \\ 80 \end{bmatrix} \qquad GZ := \begin{bmatrix} -0.58 \\ -0.47 \\ -0.36 \\ -0.27 \\ -0.18 \\ -0.09 \\ 0.00 \\ 0.09 \\ 0.18 \\ 0.27 \\ 0.36 \\ 0.47 \\ 0.58 \\ 0.70 \\ 0.83 \\ 0.99 \\ 0.97 \\ 0.87 \\ 0.72 \end{bmatrix}$$

Cubic interpolation of GZ curve

$$GZ(\theta) := \text{interp}(\text{cspline}(\theta, GZ), \theta, GZ, \theta) \cdot m$$



First intersept default value: $\theta_{d1} := 0$

Second intersept default value: $\theta_{d2} := 90$

Downflooding angle : $\theta_f := 70$

WEATHER CHARACTERISTICS

Wind speed : $V := 100 \cdot \frac{1852}{3600} \cdot m \cdot s^{-1}$ $V = 51.444 \cdot m \cdot s^{-1}$

Sea state 8/9, short term analysis, Average North Atlantic storm
H_s = 14 m; duration 5 hours; 9 modal period family; 2 parameter model

Description of family

$H := 14 \cdot m$ $i := 1 .. 9$

$$\omega m_1 := 0.048 \cdot \left(8.75 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$$

$$\omega m_2 := 0.054 \cdot \left(8.44 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1} \qquad \omega m_3 := 0.061 \cdot \left(8.07 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$$

$$\omega m_4 := 0.069 \cdot \left(7.77 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1} \qquad \omega m_5 := 0.079 \cdot \left(7.63 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$$

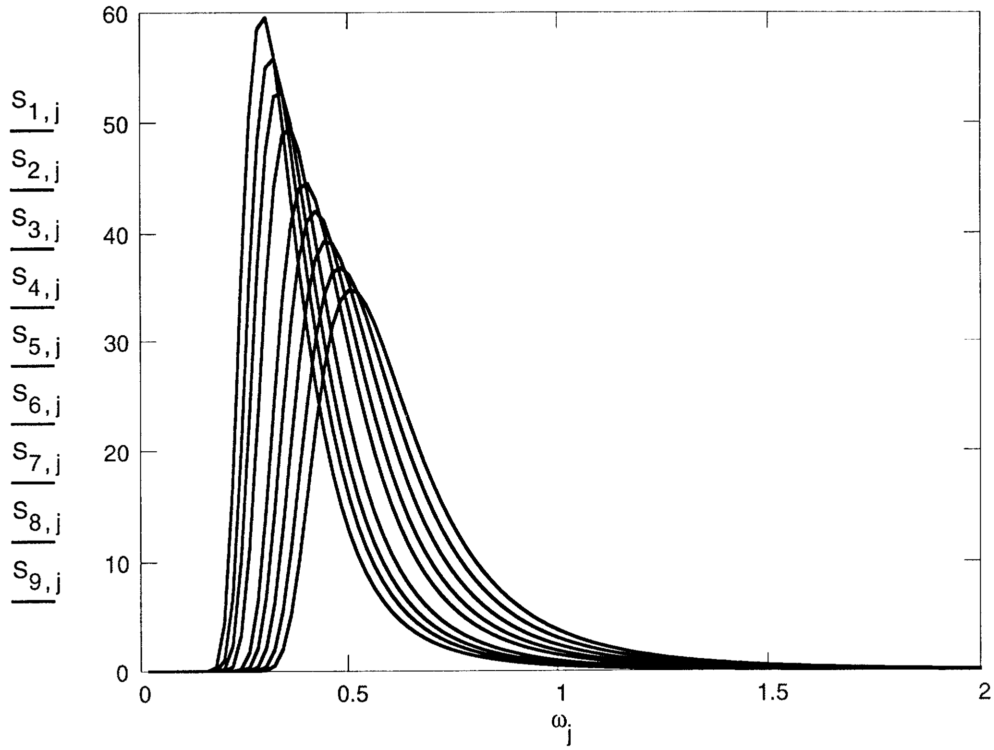
$$\omega m_6 := 0.099 \cdot \left(6.87 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1} \qquad \omega m_7 := 0.111 \cdot \left(6.67 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$$

$$\omega m_8 := 0.119 \cdot \left(6.65 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1} \qquad \omega m_9 := 0.134 \cdot \left(6.41 - \ln\left(\frac{H}{m}\right)\right) \cdot rad \cdot s^{-1}$$

$$p\omega m := \begin{bmatrix} 0.05 \\ 0.05 \\ 0.0875 \\ 0.1875 \\ 0.25 \\ 0.1875 \\ 0.0875 \\ 0.05 \\ 0.05 \end{bmatrix}$$

$j := 1 .. 100$ $\omega_j := j \cdot 0.02 \cdot rad \cdot s^{-1}$

$$S_{i,j} := \frac{5}{16} \cdot \left[\frac{\omega m_i}{\omega_j}\right]^4 \cdot \left[\frac{H^2}{\omega_j} \cdot e^{-1.25 \cdot \left[\frac{\omega m_i}{\omega_j}\right]^4}\right]$$



US NAVY METHOD

Wind heeling arm : $HA(\theta) := \frac{0.0737 \cdot V^2 \cdot A \cdot z \cdot (\cos(\theta \cdot deg))^2}{1000 \cdot \Delta} \cdot \frac{t}{m^4 \cdot s^{-2}}$

Angle of equilibrium : $\theta_{0Navy} := root\left(HA\left(\theta_{d1}\right) - GZ\left(\theta_{d1}\right), \theta_{d1}\right)$

$\theta_{0Navy} = 21.477$

Maximum angle : $\theta_{intercept} := root\left(HA\left(\theta_{d2}\right) - GZ\left(\theta_{d2}\right), \theta_{d2}\right)$

$$\theta_{2Navy} := \begin{vmatrix} \theta_{intercept} \text{ if } \theta_{intercept} \le \theta_f \\ \theta_f \text{ otherwise} \end{vmatrix}$$

$\theta_{2Navy} = 70$

Roll back angle : $\theta_{1Navy} := 25$

Maximum windward angle $\theta_{wNavy} := \theta_{0Navy} - \theta_{1Navy}$

$\theta_{wNavy} = -3.523$

Energy balance calculations

$$\text{Diff1}_{Navy}(\theta) := \text{if}\left(\theta_{wNavy} \le \theta \le \theta_{0Navy}, \left|HA(\theta) - GZ(\theta)\right|, 0\right)$$

$$\text{Diff2}_{Navy}(\theta) := \text{if}\left(\theta_{0Navy} \le \theta \le \theta_{2Navy}, \left|HA(\theta) - GZ(\theta)\right|, 0\right)$$

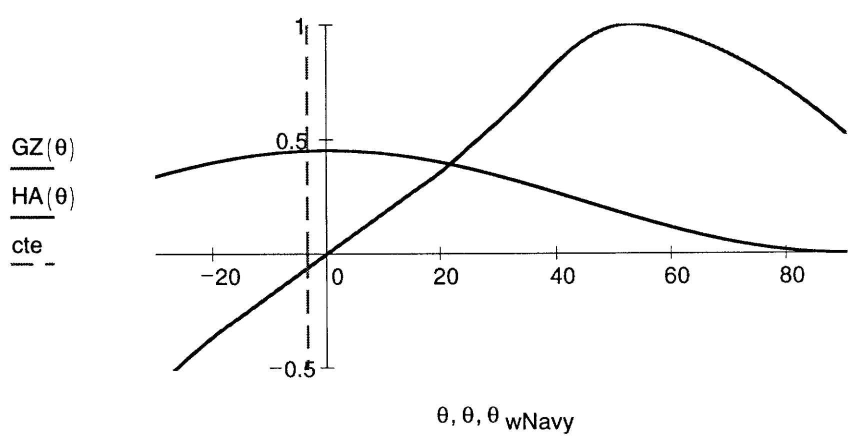
$$A_{1Navy} := \sum_{\theta} \text{Diff1}_{Navy}(\theta) \cdot 1$$

$$A_{2Navy} := \sum_{\theta} \text{Diff2}_{Navy}(\theta) \cdot 1$$

$A_{1Navy} = 6.792 \cdot m$

$A_{2Navy} = 29.276 \cdot m$

$\dfrac{A_{2Navy}}{A_{1Navy}} = 4.31$



IMO METHOD

Wind heeling arm :

$$C_D := 1.15$$

Steady wind :

$$H_{w1IMO}(\theta) := \frac{\frac{1}{2}\cdot C_D\cdot \rho_a\cdot V^2\cdot A\cdot z}{1000\cdot g\cdot \Delta}\cdot\left(\frac{t}{kg}\right)$$

Gust wind

$$H_{w2IMO}(\theta) := 1.5\cdot H_{w1IMO}(\theta)$$

Angle of equilibrium :

$$\theta_{0IMO} := \mathrm{root}\left(H_{w1IMO}(\theta_{d1}) - GZ(\theta_{d1}), \theta_{d1}\right)$$

$$\theta_{0IMO} = 24.743$$

Equilibrium with gust :

$$\theta_{gIMO} := \mathrm{root}\left(H_{w2IMO}(\theta_{d1}) - GZ(\theta_{d1}), \theta_{d1}\right)$$

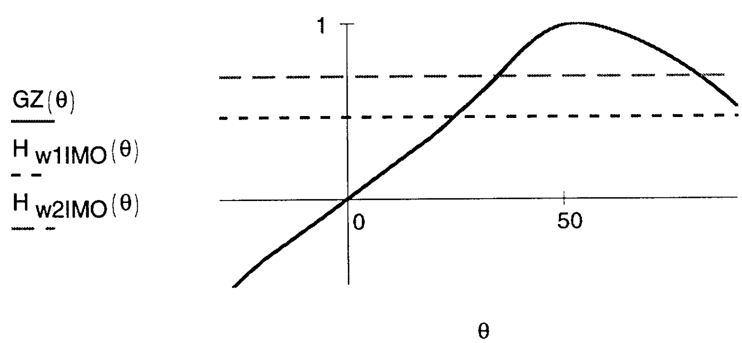
$$\theta_{gIMO} = 34.824$$

Maximum angle :

$$\theta_{intercept} := \mathrm{root}\left(H_{w2IMO}(\theta_{d2}) - GZ(\theta_{d2}), \theta_{d2}\right)$$

$$\theta_{2IMO} := \begin{cases} \theta_{intercept} & \text{if } \theta_{intercept} \le \theta_f \\ \theta_f & \text{otherwise} \end{cases}$$

$$\theta_{2IMO} = 70$$



Coefficients necessary for Roll back angle :

Calculation of coefficient k

$$At/LB := \begin{bmatrix} 0.00 \\ 0.01 \\ 0.015 \\ 0.02 \\ .025 \\ 0.03 \\ 0.035 \\ 0.04 \\ 1 \end{bmatrix} \quad k := \begin{bmatrix} 1.00 \\ 0.98 \\ 0.95 \\ 0.88 \\ 0.79 \\ 0.74 \\ 0.72 \\ 0.70 \\ 0.70 \end{bmatrix}$$

$f(x) := \text{linterp}(At/LB, k, x)$ Linear interpolation based on given values

$$k := f\left(\frac{A_t}{L \cdot B}\right)$$

$k = 0.7$

Calculation of coefficient X_1

$$B/T := \begin{bmatrix} 0 \\ 2.4 \\ 2.5 \\ 2.7 \\ 2.8 \\ 2.9 \\ 3.0 \\ 3.1 \\ 3.2 \\ 3.3 \\ 3.4 \\ 3.5 \\ 10 \end{bmatrix} \quad X_1 := \begin{bmatrix} 1.00 \\ 1.00 \\ 0.98 \\ 0.96 \\ 0.95 \\ 0.93 \\ 0.91 \\ 0.90 \\ 0.88 \\ 0.86 \\ 0.82 \\ 0.80 \\ 0.80 \end{bmatrix}$$

$f(x) := \text{linterp}(B/T, X_1, x)$ Linear interpolation based on given values

$$X_1 := f\left(\frac{B}{T}\right)$$

$X_1 = 0.97$

Calculation of coefficient X_2

$$C_B := \begin{bmatrix} 0.00 \\ 0.45 \\ 0.5 \\ 0.55 \\ 0.60 \\ .65 \\ 0.70 \\ 1.00 \end{bmatrix} \quad X_2 := \begin{bmatrix} 0.75 \\ 0.75 \\ 0.82 \\ 0.89 \\ 0.95 \\ 0.97 \\ 1 \\ 1 \end{bmatrix}$$

$f(x) := \text{linterp}(C_B, X_2, x)$ Linear interpolation based on given values

$$X_2 := f(C_B)$$

$X_2 = 0.765$

Calculation of the ship natural period T_n

$$C := \left(0.373 + 0.023 \cdot \frac{B}{T} - 0.043 \cdot \frac{L}{100 \cdot m}\right) \cdot \left(m^{-0.5} \cdot s\right)$$

$$T_n := 2 \cdot \frac{C \cdot B}{\sqrt{GM}} \qquad T_n = 12.15 \cdot s$$

Calculation of wave slope *s*

$$T_n := \begin{bmatrix} 0 \\ 6 \\ 7 \\ 8 \\ 12 \\ 14 \\ 16 \\ 18 \\ 20 \\ 40 \end{bmatrix} \cdot s \qquad S := \begin{bmatrix} 0.100 \\ 0.100 \\ 0.098 \\ 0.093 \\ 0.065 \\ 0.053 \\ 0.044 \\ 0.038 \\ 0.035 \\ 0.035 \end{bmatrix}$$

$$f(x) := \text{linterp}\left(T_n, S, x\right)$$ Linear interpolation based on given values

$$s := f\left(T_n\right)$$

$$s = 0.064$$

Test results coefficient : $C := 0.7635$

Bertin damping coefficient : $N := 0.02$

Height of center of gravity above water line $OG := KG - (D - T)$

Roll back angle

$$\theta_{1IMO} := \left[\frac{k \cdot X_1 \cdot X_2}{C} \cdot 0.7 \cdot \sqrt{\frac{\pi \cdot \left(0.73 + 0.6 \cdot \frac{OG}{T}\right) \cdot 180 \cdot s}{2 \cdot N}}\right]$$

$$\theta_{1IMO} = 12.633$$

Maximum windward angle

$$\theta_{wIMO} := \theta_{0IMO} - \theta_{1IMO}$$

$$\theta_{wIMO} = 12.11$$

Energy balance calculations

$$\mathrm{Diff1}_{IMO}(\theta) := \mathrm{if}\left(\theta_{wIMO} \le \theta \le \theta_{gIMO}, \left|H_{w2IMO}(\theta) - GZ(\theta)\right|, 0\right)$$

$$\mathrm{Diff2}_{IMO}(\theta) := \mathrm{if}\left(\theta_{gIMO} \le \theta \le \theta_{2IMO}, \left|H_{w2IMO}(\theta) - GZ(\theta)\right|, 0\right)$$

$$A_{1IMO} := \sum_{\theta} \mathrm{Diff1}_{IMO}(\theta) \cdot 1$$

$$A_{2IMO} := \sum_{\theta} \mathrm{Diff2}_{IMO}(\theta) \cdot 1$$

$$A_{1IMO} = 5.589 \cdot m \qquad \frac{A_{2IMO}}{A_{1IMO}} = 1.423$$

$$A_{2IMO} = 7.95 \cdot m$$

PROPOSED NEW METHOD

Drag coefficient : $C_D := 1.12$

Projected Area :
$$A_p(\theta) := \begin{cases} \left[C_w \cdot \frac{B \cdot L}{2} + \left(A - C_w \cdot \frac{B \cdot L}{2}\right) \cdot \cos(\theta \cdot deg)\right] & \text{if } \theta \le 90 \\ C_w \cdot \frac{B \cdot L}{2} & \text{otherwise} \end{cases}$$

Projected Lever :
$$z_p(\theta) := \begin{cases} \left[\frac{B}{2} + \left(z - \frac{B}{2}\right) \cdot \cos(\theta \cdot deg)\right] & \text{if } \theta \le 90 \\ \frac{B}{2} & \text{otherwise} \end{cases}$$

Steady wind heeling arm
$$H_{w1New}(\theta) := \frac{\frac{1}{2} \cdot C_D \cdot \rho_a \cdot V^2 \cdot A_p(\theta) \cdot z_p(\theta)}{1000 \cdot g \cdot \Delta} \cdot \left(\frac{t}{kg}\right)$$

Gust wind heeling arm
$$H_{w2New}(\theta) := 1.5 \cdot H_{w1New}(\theta)$$

Angle of equilibrium :
$$\theta_{0New} := \mathrm{root}\left(H_{w1New}(\theta_{d1}) - GZ(\theta_{d1}), \theta_{d1}\right)$$

$$\theta_{0New} = 23.093$$

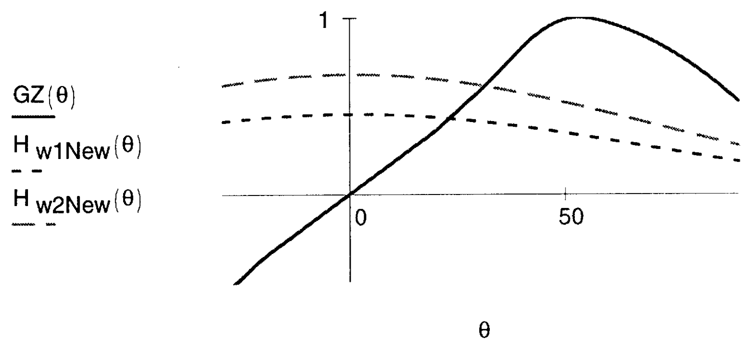
Equilibrium with gust : $\theta_{gNew} := root\left(H_{w2New}(\theta_{d1}) - GZ(\theta_{d1}), \theta_{d1}\right)$

$\theta_{gNew} = 31.328$

Maximum angle : $\theta_{intercept} := root\left(H_{w2New}(\theta_{d2}) - GZ(\theta_{d2}), \theta_{d2}\right)$

$$\theta_{2New} := \begin{cases} \theta_{intercept} & \text{if } \theta_{intercept} \le \theta_f \\ \theta_f & \text{otherwise} \end{cases}$$

$\theta_{2New} = 70$



Calculation of the roll back angle

Natural roll period

$$C := \left(0.3725 + 0.0227 \cdot \frac{B}{T} - 0.043 \cdot \frac{L}{100 \cdot m}\right)$$

$$T_n := \frac{2\,\pi \cdot C \cdot B}{\sqrt{g \cdot GM}} \qquad T_n = 12.144 \cdot s \qquad \omega_n := \frac{2\,\pi}{T_n}$$

Non dimensional damping factor

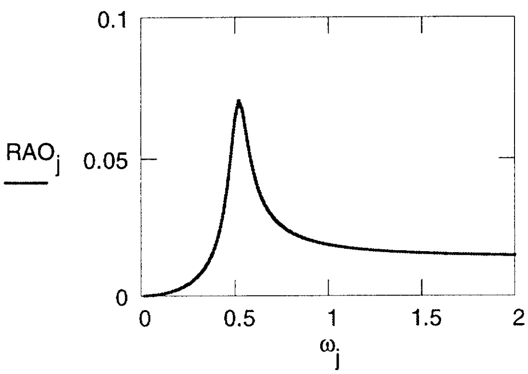
$$\beta^* := 19.25 \cdot \left(A_k \cdot \sqrt{b_k} + 0.0024 \cdot L \cdot B \cdot \sqrt{d}\right) \cdot d^2 \cdot \frac{\sqrt{d \cdot \theta_i}}{C_B \cdot L \cdot B^3 \cdot T}$$

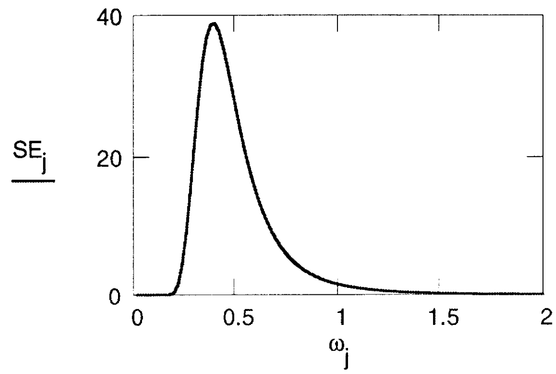
$\beta^* = 0.097$

RAO

$$RAO_j := \frac{1}{\sqrt{\left[1 - \left(\frac{\omega_j}{\omega_n}\right)^2\right]^2 + \left(2 \cdot \frac{\omega_j}{\omega_n} \cdot \beta^*\right)^2}} \cdot \frac{(\omega_j)^2}{2 \cdot g}$$

$$SE_j := \sum_{i=1}^{9} p\omega_{m_i} \cdot S_{i,j} \qquad SR_j := \left(RAO_j\right)^2 \cdot SE_j$$





Roll back angle: $$m_0 := \sum_{j=2}^{100} SR_j \cdot \left(\omega_j - \omega_{j-1}\right)$$

$$\theta_{1New} := 2 \cdot \sqrt{m_0} \cdot \frac{180}{\pi} \qquad \theta_{1New} = 14.382$$

With the initial roll angle used to calculate the damping factor $\theta_i \equiv 15 \cdot deg$

Maximum windward angle $\theta_{wNew} := \theta_{0New} - \theta_{1New}$

$$\theta_{wNew} = 8.711$$

Energy balance calculations

$$Diff1_{New}(\theta) := if\left(\theta_{wNew} \le \theta \le \theta_{gNew}, \left|H_{w2New}(\theta) - GZ(\theta)\right|, 0\right)$$

$$Diff2_{New}(\theta) := if\left(\theta_{gNew} \le \theta \le \theta_{2New}, \left|H_{w2New}(\theta) - GZ(\theta)\right|, 0\right)$$

$$A_{1New} := \sum_{\theta} Diff1_{New}(\theta) \cdot 1$$

$$A_{2New} := \sum_{\theta} Diff2_{New}(\theta) \cdot 1$$

$$A_{1New} = 6.341 \cdot m \qquad \frac{A_{2New}}{A_{1New}} = 2.39$$

$$A_{2New} = 15.158 \cdot m$$

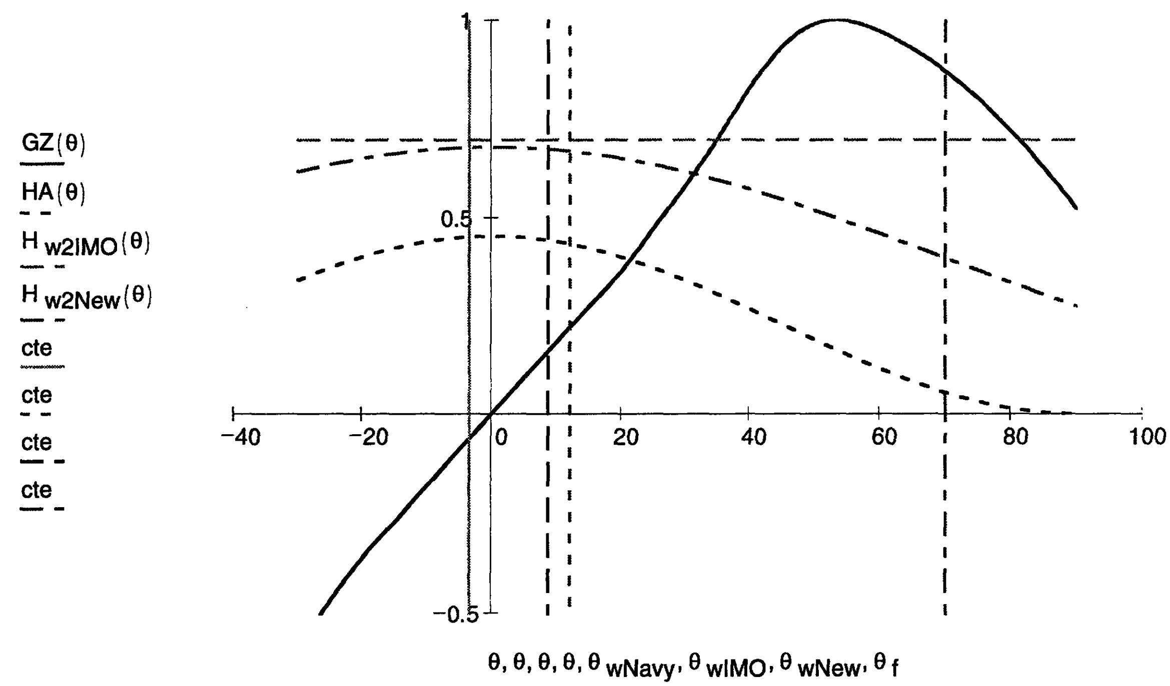
COMPARISON OF THE ANGLES BETWEEN THE THREE METHODS

$\theta_{0Navy} = 21.477$ $\theta_{1Navy} = 25$ $\theta_{2Navy} = 70$ $\theta_{wNavy} = -3.523$

$\theta_{0IMO} = 24.743$ $\theta_{1IMO} = 12.633$ $\theta_{2IMO} = 70$ $\theta_{wIMO} = 12.11$

$\theta_{0New} = 23.093$ $\theta_{1New} = 14.382$ $\theta_{2New} = 70$ $\theta_{wNew} = 8.711$

COMPARISON OF ENERGY BALANCES BETWEEN THE THREE METHODS



$A_{1Navy} = 6.792 \cdot m$ $A_{2Navy} = 29.276 \cdot m$ $\dfrac{A_{2Navy}}{A_{1Navy}} = 4.31$

$A_{1IMO} = 5.589 \cdot m$ $A_{2IMO} = 7.95 \cdot m$ $\dfrac{A_{2IMO}}{A_{1IMO}} = 1.423$

$A_{1New} = 6.341 \cdot m$ $A_{2New} = 15.158 \cdot m$ $\dfrac{A_{2New}}{A_{1New}} = 2.39$