SOUND POWER MEASUREMENT IN

A SEMI-CONFINED SPACE

by

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S.B., General Motors Institute

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ABSTRACT

A sound field can be adequately described if sufficient measurements are made at separate points. A broad-band noise field requires only one measurement while a pure-tone or narrow-bandwidth noise field may require a burdensome number of measurements to get a measured mean value of squared sound pressure. If the mean value can be adequately determined, then the sound power level of a source, in reverberant conditions, can be predicted through a direct relationship between sound pressure level and sound power level. This is done for a point source in a channel.

Thesis Supervisor: Richard H. Lyon

Title: Professor of Mechanical Engineering

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I. Introduction

The measurement of the sound produced by a machine or other device is a topic which has been the subject of various papers and the basis of a standard of the American National Standards Institute (ANSI). The American Standard Method for the Physical Measurement of Sound [1] outlines in some detail the method to be used in the determination of sound power under four conditions: 1) in a free field; 2) in a free field above a reflecting surface; 3) in a diffuse (reverberant) field; and 4) in a semireverberant field.

The determination of the sound power of a source requires that an estimate, p^2, be made of the true, space-averaged value of squared sound pressure $\langle p^2 \rangle_{\bar{x}}$ where all squared pressures are assumed to be time averaged. The uncertainty in p^2 is estimated by its standard deviation σ_{p^2}.

A limitation of the ANSI standard is the requirement that the sound output of the source be primarily broad-band noise for the cases involving measurements in a reverberant or semireverberant field. For a source emitting line or narrow bandwidth spectra, the ANSI standard recommends against determining sound power under diffuse field or semireverberant field conditions. There are cases, however, when it is necessary to make measurements under these conditions when a source has pure tone components and still be reasonably confident of the results.

The purpose of this thesis is to present the work done in attempting to determine the sound power of a source located between two parallel walls, a case similar to a car in a city street which is flanked by tall, closely packed buildings. The problem involves both broad band noise and superimposed pure tone components where phase interference in reflected portions of the sound field cause large fluctuations in sound pressure level with small changes in position of source or observer.

The general format of this thesis is a) a general conclusion of the results of this work, b) an analysis of sound power determination for broad-band noise from a source in a channel, c) a description of the experimental program for measuring the mean and variance of the squared sound pressure, d) the experimental results giving an optimal measurement technique for minimizing the variance in p^2, and e) a brief outline for continuing the work begun in this thesis.

II. Conclusions

It has been shown that for the case of an omnidirectional point source of sound in a channel, there is a direct relationship between the space-averaged sound pressure level around a given location and the sound power level of the source. A single measurement position is sufficient to define the sound field when the sound is primarily broad-band noise. When a pure tone predominates in the sound field, several discrete sample points will give an average mean value of sound pressure that is as accurate as the average obtained by a continuous line average. The discrete points should be chosen at one-half wavelength spacings to get uncorrelated samples. A total of 6 sample points spaced at $\lambda/2$ apart will give a mean value of sound pressure that has no more variance in the mean value than a continuous line average over 3 wavelengths (or 1/4 street width) distance.

III. Determination of Sound Power

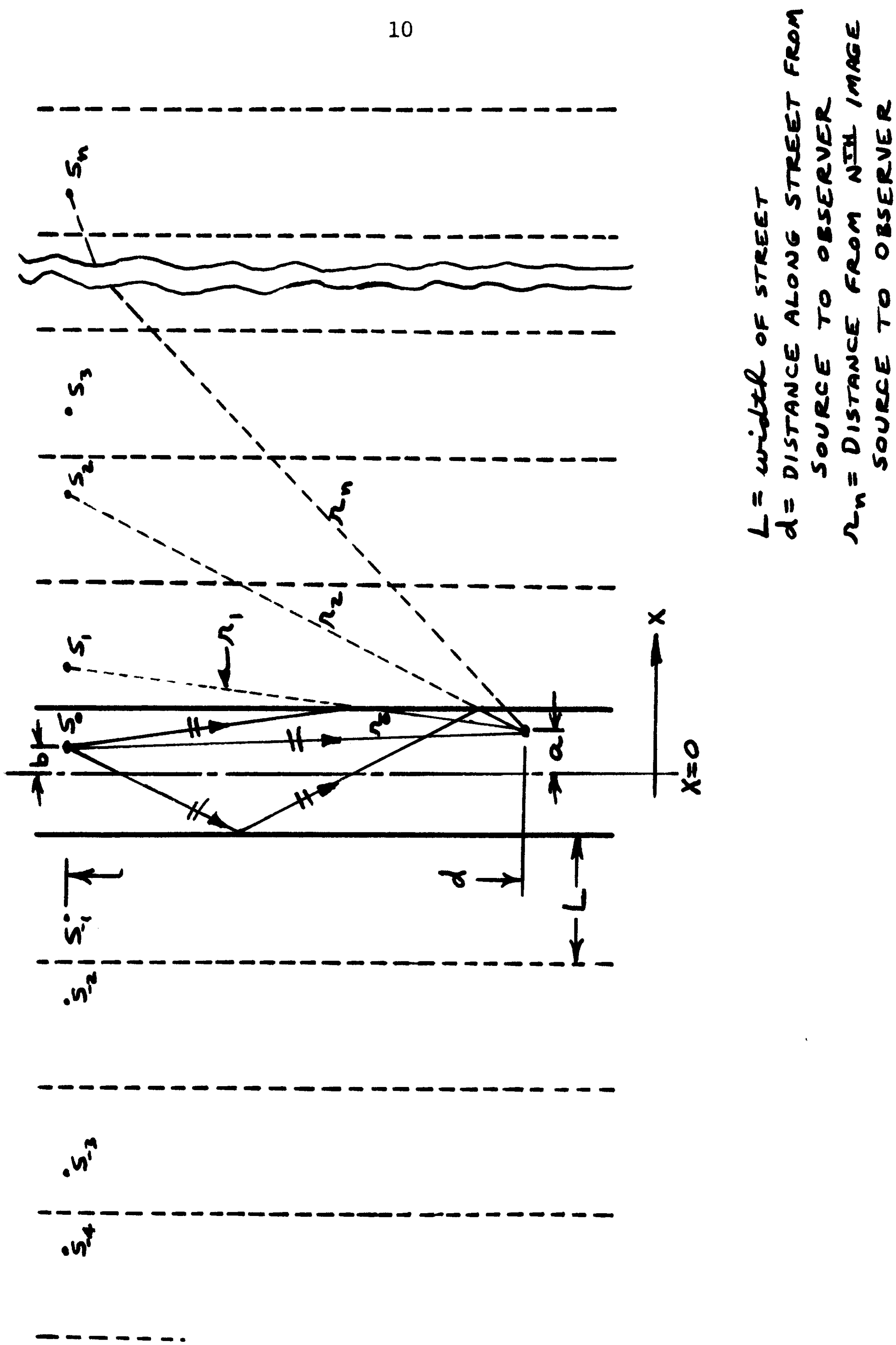
The total acoustical power radiated from a sound source operating in a given acoustic environment is frequently a more useful quantity than the sound pressure measured at a certain position. If the environment of the sound source should change, the sound pressure measured at a given position could change greatly whereas the radiated acoustical power is generally not changed appreciably. However, if the source is near one or more reflecting surfaces, the radiation impedance may differ appreciably from that of free space [2]. Under the condition that the reflecting surfaces remain fixed with respect to the source location, these surfaces may be considered a part of the source and the sound power determined as such. A car on a city street is an example of this since the street remains a fixed surface with respect to the car even though the car may be moving. This problem is further discussed in Section IV where directionality of the source is considered.

The radiated power of a sound source can be indirectly measured since the acoustic power is proportional to the mean-square sound pressure. The sound pressure level obtained by a single measurement in a reverberant sound field depends greatly upon the position of the microphone if the sound field contains significant pure tone or narrow bandwidth components.

This section deals only with those cases of wide band noise and a later section covers the cases of pure tone sources.

The computation of sound power can be made using the following relationship:

$$L_w = L_p - 10 \log X - C \qquad (1)$$



GEOMETRY OF POINT SOURCE & REFLECTED IMAGES FOR PARALLEL REFLECTING WALLS

FIGURE 1

where

$L_w \equiv 10 \log_{10} (W/10^{-12} \text{ watts}), \text{ (dB)}$

$W \equiv$ sound power of the source in watts

$L_p \equiv 20 \log_{10} (p/0.0002 \text{ microbar}), \text{ (dB)}$

p = sound pressure in microbars

$$C = 10 \log_{10} [(\frac{B}{30})(\frac{528}{460+\theta})^{\frac{1}{2}}] + .5\text{dB} \qquad [1]$$

θ = the air temperature in degrees Fahrenheit

B = the barometric pressure in inches of mercury

X = a factor determined by the geometry of the source, the source environment, and the directivity factor.

The main part of the following discussion centers around the determination and evaluation of the factor X for the semireverberant case considered in this paper.

At this point we describe the environment and type of source being considered. We consider the situation of a car being driven down a city street flanked by tall buildings spaced very close together. Figure 1 shows a graphic representation of the propagation model. The ray theory of acoustics is employed and multiple images replace the actual single source and reflecting surfaces. We shall also ignore source directionality at first and treat the car as an onmidirectional point source.

For a single point source, the sound pressure is related to the sound power by the formula:

$$p^2 = \rho c W/4\pi r^2 \qquad (2)$$

where

$$p^2/\rho_o cW = X = 1/4\pi r^2 \qquad (3)$$

W = The acoustical power of the omnidirectional source in watts

r = the distance from the source in meters.

The case of a single point source can readily be extended to that for many point sources by summing the contributions produced by each of the sources at the observer's position. When each of the point sources is producing incoherent or wide band noise so that phase interference between sources can be ignored, we can simply add the mean square pressures due to each source to obtain the total mean square pressure. For the multiple image case shown in Figure 1, the contribution of the n-th image to the mean square pressure is

$$p_n^2 = \rho c W R^{|n|}/4\pi r_n^2 \qquad (4)$$

where r_n = the distance from the n-th image

R = the energy reflection coefficient of the walls; W(reflected)/W(incident)

$WR^{|n|}$ = the effective sound power of the n-th image.

The total contribution, at a point, of all the point sound sources would be the summation of all p_n^2 from Equation 4:

$$p^2 = \sum_{n=-\infty}^{\infty} p_n^2 = \sum_{n=-\infty}^{\infty} \rho_o c W R^{|n|}/4\pi r_n^2 \qquad (5)$$

and

$$X = \sum_{n=-\infty}^{\infty} R^{|n|}/4\pi r_n^2 \quad . \qquad (6)$$

X is proportional to the sum of the inverse square of the distance to each source for the case of R = 1:

$$X \propto \sum_{n=-\infty}^{\infty} 1/r_n^2 = \sum_{n=-\infty}^{\infty} 1/[d^2 + (nL - a)^2] \qquad (7)$$

where (a) is as shown in Figure 1. The summation in Equation 7 has been evaluated by Johnson and Saunders [3] for the case of equal strength noise sources (R = 1) equally spaced:

$$\sum_{n=-\infty}^{\infty} 1/[d^2 + (nL - a)^2] = \frac{\pi}{dL} \cdot \frac{\sinh\,(2\pi d/L)}{\cosh\,(2\pi d/L) - \cos\,(2\pi a/L)} \qquad (8)$$

and the quantity X for the case R = 1 becomes

$$X = \frac{1}{4L^2\delta} \cdot \frac{\sinh\,(2\pi\delta)}{\cosh\,(2\pi\delta) - \cos\,(2\pi\alpha)} \qquad (9)$$

where δ and α are distances normalized to the street width (or source separation distance).

Equation 9 can be made even more general for the case shown in Figure 1 in which the sources are not evenly spaced. In this case, the series of points are broken up into two separate series of odd and even numbered locations. These two new series each have evenly spaced points 2L apart and a new value for X is given by

$$X = \frac{1}{8L^2\delta}\left[\frac{\sinh\pi\delta}{\cosh\pi\delta - \cos\pi\,\frac{(a - b)}{L}} + \frac{\sinh\pi\delta}{\cosh\pi\delta + \cos\,\frac{\pi(a + b)}{L}}\right] \qquad (10)$$

which reduces to Equation 9 for the case when b = 0. Equations 9 and 10 are listed in Table 1 along with a few other simple cases for comparison purposes.

TABLE I

X FOR DIFFERENT SOURCE CONFIGURATIONS

TYPE OF SOURCE	SOURCE DESCRIPTION	X	PARAMETERS **
POINT	ACOUSTIC STRENGTH (W)	$X = 1/4\pi r^2$	r = DISTANCE FROM SOURCE (IN METERS)
LINE	COHERENT LINE	$X = 1/2\pi d L$	L = STREET WIDTH WITH W DISTRIBUTED OVER L
LINE	INCOHERENT SERIES OF POINTS	$X = 1/4 d L$	d = DISTANCE (⊥) FROM LINE
MULTIPLE POINTS	EQUALLY SPACED NOISE SOURCES ALL OF POWER W SEPARATED BY L	$X = \frac{1}{4Ld}\left[\frac{\sinh 2\pi d/L}{\cosh\frac{2\pi d}{L} - \cos\frac{2\pi a}{L}}\right]$	a = DISTANCE OF OBSERVER TO ⊥ FROM NEAREST POINT
MULTIPLE POINT	EQUAL STRENGTH NOISE SOURCES OF AVERAGE SPACING L, CONFIGURATION OF FIG. 1	$X = \frac{1}{8Ld}\left[\frac{\sinh \pi d/L}{\cosh\frac{\pi d}{L} - \cos\frac{\pi(a-b)}{L}} + \frac{\sinh \pi d/L}{\cosh\frac{\pi d}{L} + \cos\frac{\pi(a+b)}{L}}\right]$	b = DEVIATION OF SOURCES FROM EQUAL SPACING
LINE + POINT *	LINE OF DECAYING STRENGTH	$X = \frac{\beta}{4\pi d^2} + \frac{\alpha}{2\pi d L}\left[\sin(\mu d) \times ci(\mu d) - \cos(\mu d)\, si(\mu d)\right]$	$\beta = \frac{1-\sqrt{R}}{1+\sqrt{R}}$ $\alpha = \frac{\sqrt{R}\ln R}{R-1}$ $\mu = -\frac{1}{L}\ln R$
POINT	IN REVERBERANT ROOM	$X = 4/\mathcal{R}$	R = REFLECTION COEFF. $\mathcal{R}$ = ROOM CONSTANT

* SEE APPENDIX A

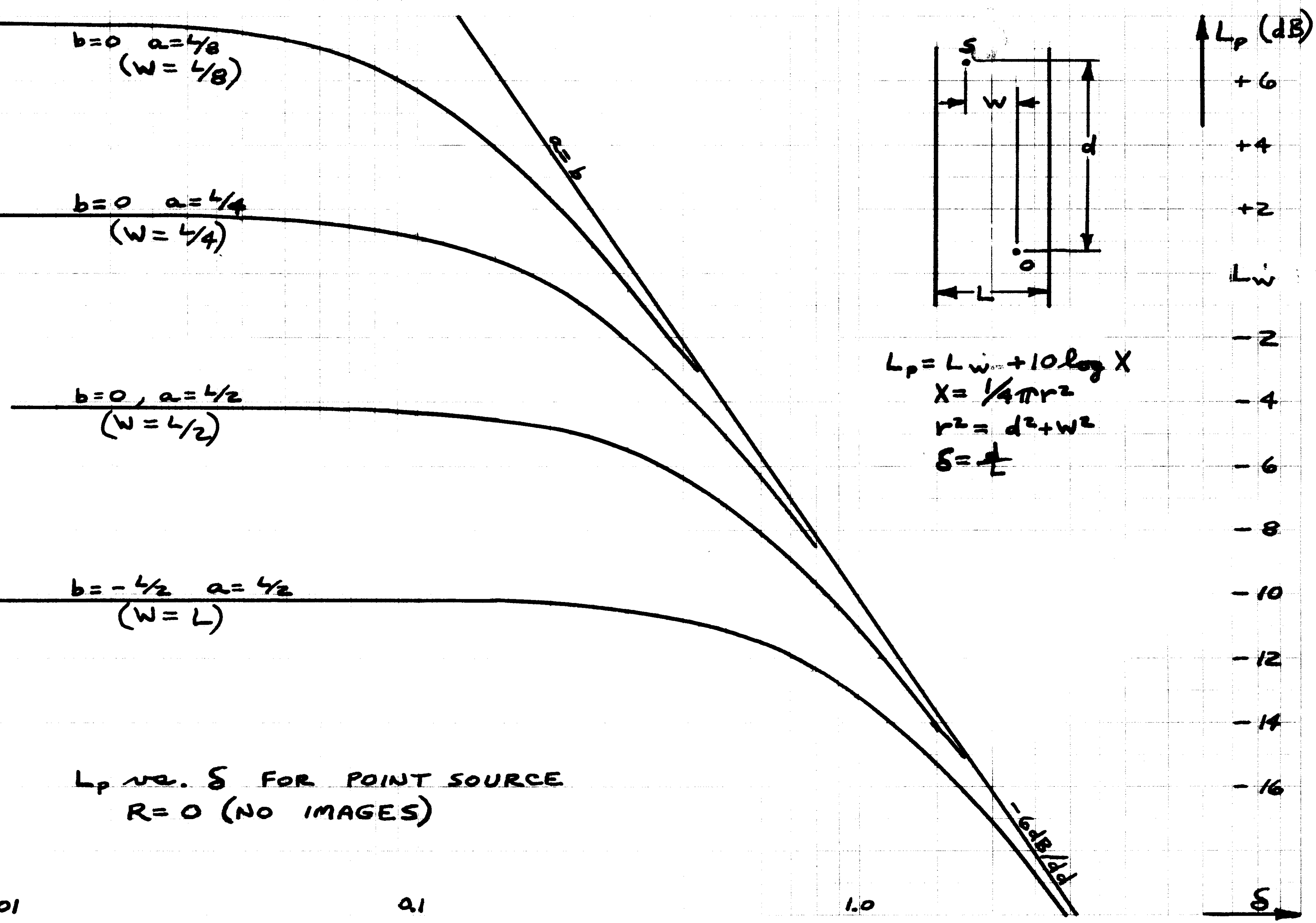
** ALL MKS UNITS

A plot of the simplest case, a single point source (R = 0), shown in Figure 2 shows some interesting results. Beyond two street widths from the source in the direction along the street, the mean-square pressure level, L_p, is within a 1 dB range regardless of the positions of the source and observer across the width of the street. The limiting value of L_p is determined by the separation w as the distance d, or $\delta = d/L$, decreases. The result in Figure 2 has only reference value. As the reflection coefficient, R, increases from 0 to 1, the mean-square pressure level, L_p, for the same source will always be equal to or higher than the level shown by Figure 2.

When R = 1, a plot of Equation 10 (the relation graphed in Figure 3) shows the maximum values one can obtain for L_p as a function of source and observer positions in the street. Some important points shown in Figure 3 are that when the sources are evenly spaced (i.e., b = 0) and the observer is also centered in the street (a = 0), the level L_p approaches that of a single point source for values of δ less than 0.2. When the source is against one wall, the first image corresponds with the original source and the levels along that wall approach a limit of 3 dB above that of the single source in the same position. Likewise, having the source and observer on opposite sides of the street will produce a minimum level of greater than 6 dB above that of a single source due to two double images spaced at one street width from the observer at the closest position.

Again, in the case of multiple points (R > 0), the levels all approach a limit where source and observer positions across the street

FIGURE 2



Lp vs. δ FOR POINT SOURCE
R = 0 (NO IMAGES)

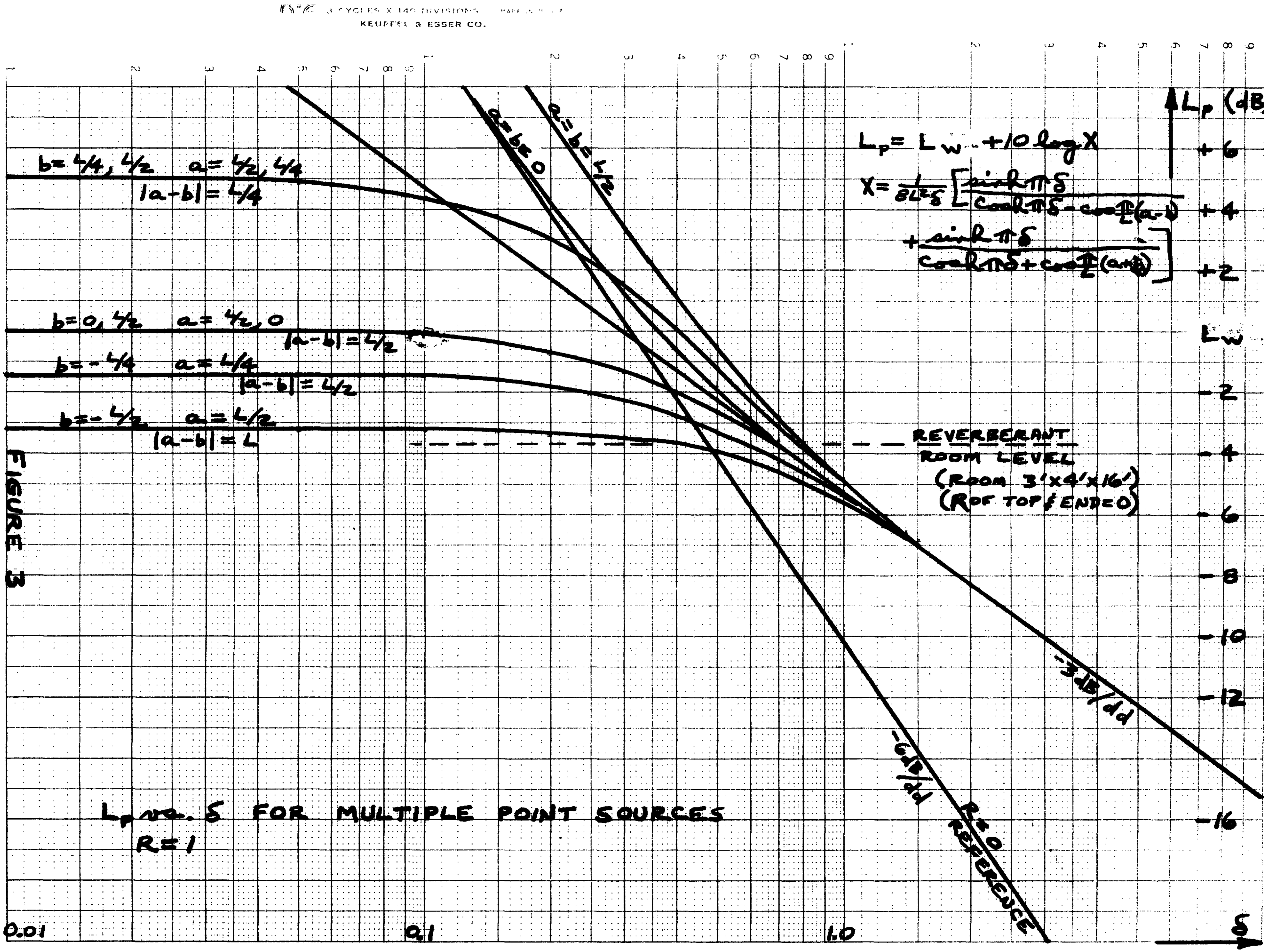
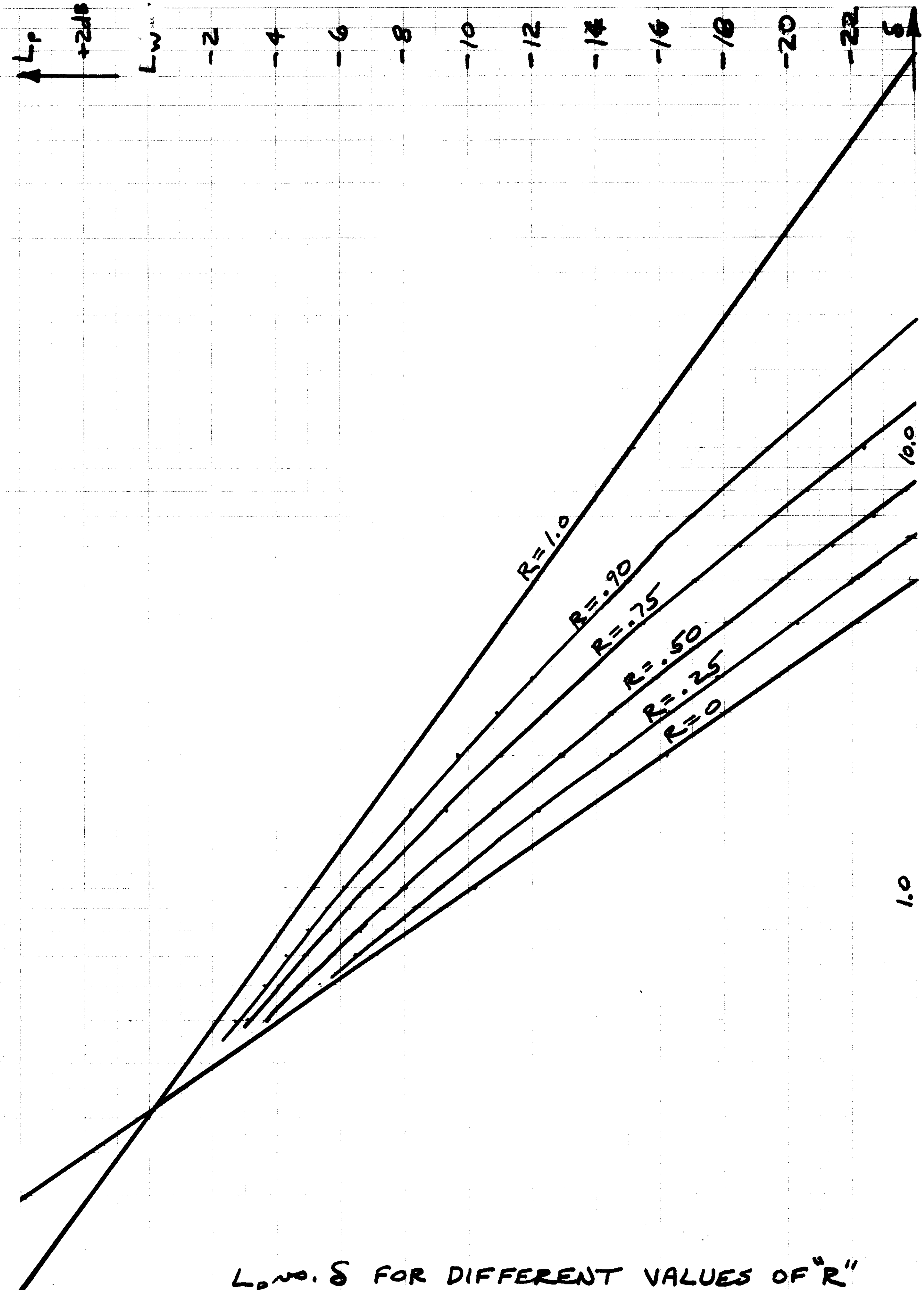

KEUFFEL & ESSER CO.
L_p (dB)
$L_p = L_W + 10 \log X$
$X = \frac{1}{8L^2\delta}\left[\frac{\sinh \pi\delta}{\cosh \pi\delta - \cos\frac{\pi}{L}(a-b)} + \frac{\sinh \pi\delta}{\cosh \pi\delta + \cos\frac{\pi}{L}(a+b)}\right]$
+6
+4
+2
L_W
-2
-4
-6
-8
-10
-12
-16
a=b=0
a=b=L/2
b= L/4, L/2   a= L/2, L/4
|a-b|= L/4
b=0, L/2   a= L/2, 0
|a-b|= L/2
b= -L/4   a= L/4
|a-b|= L/2
b= -L/2   a= L/2
|a-b|= L
REVERBERANT ROOM LEVEL
(ROOM 3'x4'x16')
(R OF TOP & END=0)
-3dB/dd
-6dB/dd
R=0 REFERENCE
L_p vs. δ FOR MULTIPLE POINT SOURCES
R=1
0.01
0.1
1.0
δ

FIGURE 3

are unimportant. In this case the distance where the levels are less than 1 dB from the limit is reached at $\delta = 1.0$ (or $d = L$) whereas this limit occurs at $\delta = 2.0$ (or $d = 2L$) for a single point source when $R = 0$. The limit for the case of $R = 1$ turns out to be the -3 dB per doubling of distance line one would get if an infinite line source had a strength of W watts per unit length where the unit length is equal to one street width. This is a useful result since it shows that beyond a distance proportional to the street width, the mean-square sound pressure level is independent of observer position across the street and of whether the source and its images are discrete points of arbitrary spacing or distributed in a continuous line. This result enables one to evaluate the intermediate cases where R is between 0 and 1 by distributing the point sources across the street in a continuous line. This is done in Appendix A to get the results shown in Figure 4.

The noise models used have resulted in three graphs showing the relationship between mean-square sound pressure and sound power of a source for measurements taken at various positions. Because of approximations for the intermediate values of R, that plot is only valid for $\delta \gtrsim 1.5$ or 2.0. The next step is to see if the results found can be verified experimentally. The following section presents experimental results.



L_p vs. δ FOR DIFFERENT VALUES OF "R"

FIGURE 4

IV. Experimental Measurement of Sound Power

In the preceding section, the relationship between sound pressure level and sound power level was given for the case of a sound source placed between two parallel reflecting surfaces of various reflection coefficients. The relationship

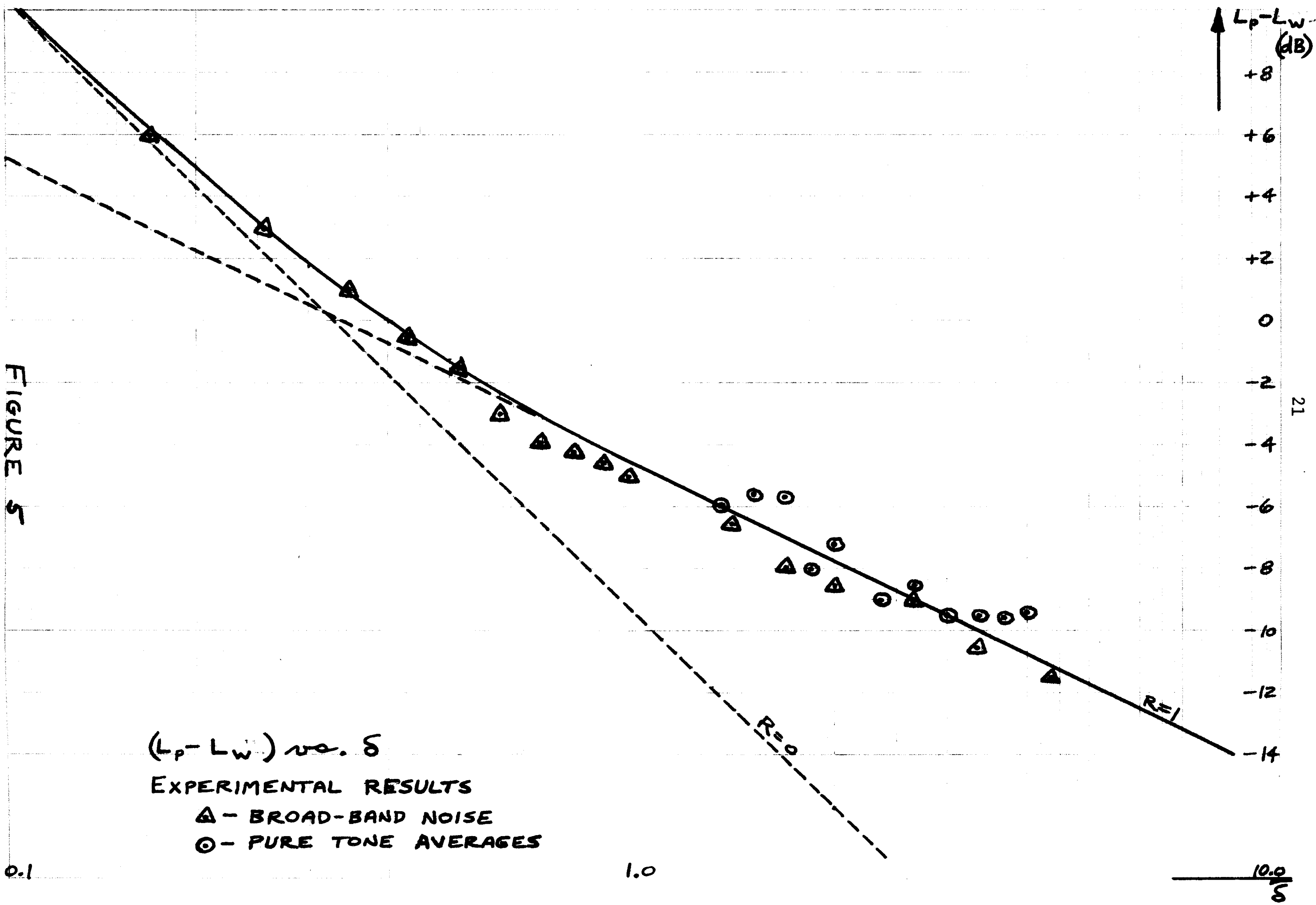
$$L_w = L_p - 10 \log X - c \qquad (1)$$

with R = 1.0 giving the result

$$X = \frac{1}{8L^2\delta}\left(\frac{\sinh\pi\delta}{\cosh\pi\delta - \cos\frac{\pi}{L}(a-b)} + \frac{\sinh\pi\delta}{\cosh\pi\delta + \cos\frac{\pi}{L}(a+b)}\right) \qquad (10)$$

has been experimentally verified for both broad-band noise and pure tone sources. Figure 5 shows the theoretical curve for Equations 1 and 10 with the experimental results plotted on the same graph (c = 0.5 dB).

The experimental setup used to get these results was a 32:1 scale model using a point sound source located midway between two parallel walls (Figure 6). As shown, the sound source used was a dynamic microphone driven by a voltage equivalent to the type of sound desired (e.g., pure tone-sine wave, white noise, filtered octave band or 1/3 octave band of white noise, etc.). The sound source was placed at the floor level pointed into the edge between the end and floor to get an omnidirectional source in the field of interest. The sound power and the directionality of the source were easily determined by making sound pressure level readings on the surface of an imaginary quarter sphere centered on the source and with a radius small enough that "free" field measurements were made in the region where the -6 dB/dd law applies, and was observed. The distance used



Lp-Lw (dB)
+8
+6
+4
+2
0
-2
-4
-6
-8
-10
-12
-14
0.1
1.0
10.0
δ
R=0
R=1
(Lp - Lw) vs. δ
EXPERIMENTAL RESULTS
△ - BROAD-BAND NOISE
⊙ - PURE TONE AVERAGES

FIGURE 5

EXPERIMENTAL SETUP

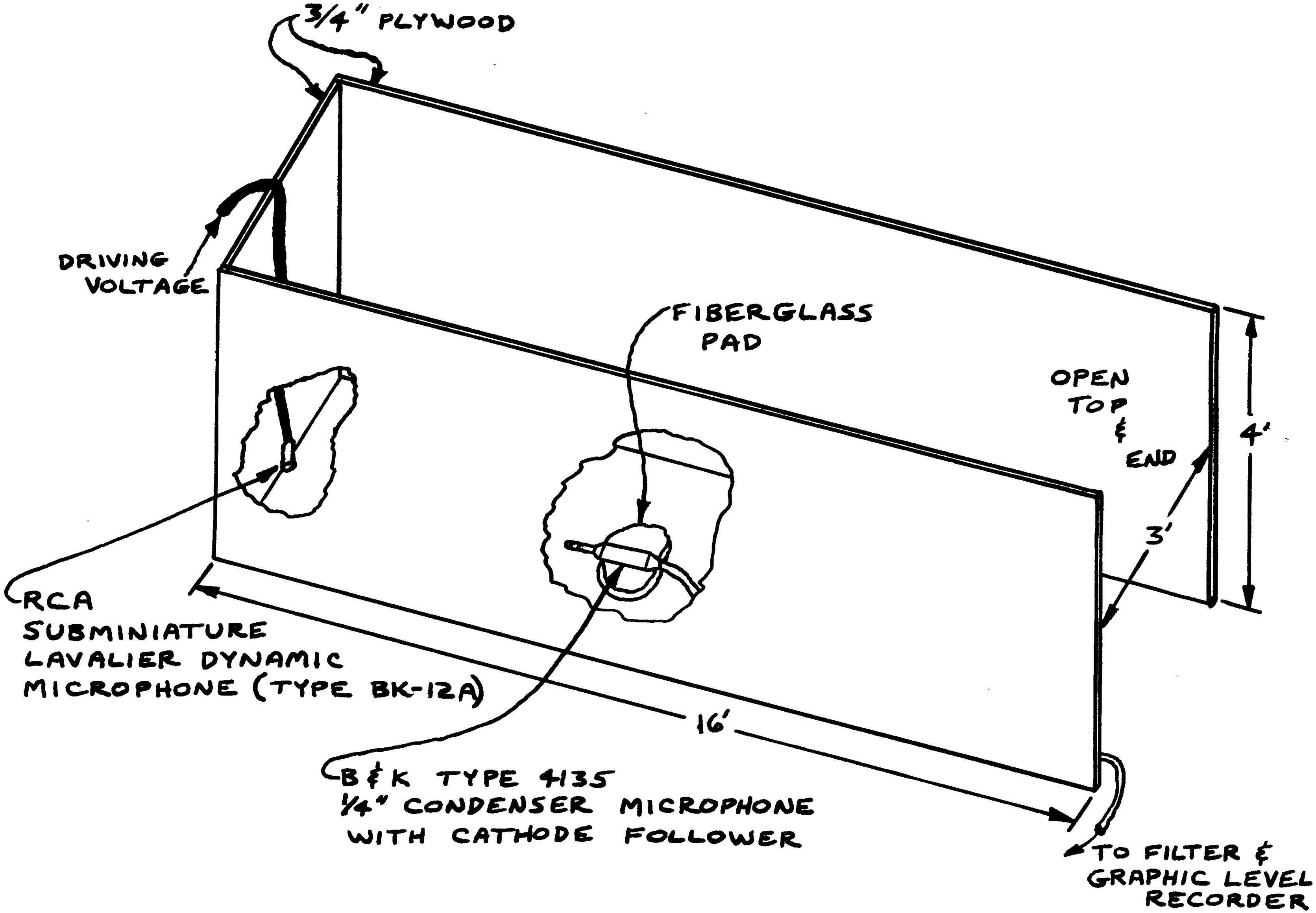


FIGURE 6

in this experiment was a radius of 3 inches.

The problem of source directivity enters when source power is determined. In this problem, the source has three mirrored images located at the same point as the source. Therefore, the sound power observed is that of a single point source in free space having four times the sound power or equivalently, a single source having a directionality of four in the region of space in the channel and zero elsewhere. Thus, in measuring the sound pressure level in the free-field part of the far field, Equation 9.1 from L.L. Beranek's Noise Reduction book

$$L_p \doteq L_w + DI_\theta - 20 \log r - 1 \text{ dB} \qquad (11)$$

where L_w = sound-power level of source, dB re 10^{-12} watt

r = distance of receiver from source in feet

DI_θ = directivity index of source in direction θ, dB

shows that if the true power level of the source is used with a directivity of four, the same result could be found using a directivity of one and a source strength of four times the true strength. In the experimental determination of sound power in this work, directionality was ignored, and the sound pressure level observed on one-quarter of a sphere centered on the source was implicitly assumed to be the same on the other three quarters. This assumption can be made for the work in this thesis but in stating the sound power of a source in field work, care must be taken to use this directionality so as not to overrate the sound power output of a particular source.

In the experimental work done, measurements of L_p were made with the top and far end of the channel open. The measuring microphone was protected from floor vibrations by a pad of fiberglass. In all parts of the experimental work, the source was kept in the center of the channel and the receiver was either in the center of the channel or at a distance down the channel where position across the channel was unimportant.

Broad Band Noise - In the first part of the experimental work, one octave of white noise centered at 4000 Hz was used. This broad band of noise results in a frequency average which is equivalent to a spatial average for discrete frequencies [4]. Measurements of L_p as a function of distance along the center of the channel were made and compared to the sound power level of the source. The result plotted in Figure 5 shows fairly close correlation between theory and experiment. Measured values are listed in Table 3 in the appendix.

Pure Tone Sound - To compare theory to experiment for pure tone sound, a tone of 4000 Hz was used and a spatial average was required to arrive at a single number level L_p as a function of distance down the channel. To do this, a sweep across the channel was made at distances where average level was predicted to be independent of receiver position across the channel. A continuous average was made of this sweep and a level L_p was determined for each of several locations δ and the results are listed in Table 4 and plotted in Figure 5.

The following section is an analysis in more detail of how the spatial average can be most easily made when a pure tone source is used.

V. Optimal Measurement Technique

In the preceding section, some values of sound pressure level were given from measurements made in a pure tone sound field. The accuracy of these values is dependent upon how the measurements were made. This section contains an analysis of the previous work done by various authors to determine an optimum measurement technique for arriving at a reasonably precise value for the average squared sound pressure without making the measurement work too tedious. An experimental analysis is also made of the particular sound field being described in this thesis (a pure tone point source located in a channel) to determine an acceptable means of measuring the average sound pressure level.

Theory - The time-averaged value of squared sound pressure (called sound intensity here, for convenience) is taken as a random variable whose statistical properties can be determined. The variance of sound intensity is the best single index of the size of the sound level fluctuations. The variance is generally emphasized for this reason. Furthermore, a normalized variance is used so that results are independent of the magnitude of sound intensity. Lubman [5] makes an analysis of the normalized variance (V^2) in sound intensity in a reverberant room and arrives at some results showing that it has unit value for a pure tone:

$$V_1^2 = 1 \tag{12}$$

and when a pair of tones are introduced in the reverberant room, V_2^2 has a range of values bounded by:

$$\frac{1}{2} < V_2^2 \leq 1 \ . \tag{13}$$

The limits on V_2^2, where the subscript stands for the number of tones, are reached under several conditions. In order to achieve minimum variance ($V_2^2 = 1/2$), the amplitude of the two tones must be equal and the frequency separation of the tones must be larger than the bandwidth of a typical room resonance. The limit of 1 is approached by equating the frequencies or making the amplitudes unequal by an order of magnitude or more. Extending the variance of intensity for a multitone field of M tones, Lubman gives the bounds as:

$$1/M < V_M^2 \leq 1 \ . \tag{14}$$

Finally, the result is extended for a narrow-band noise field to the approximate equation,

$$V_n^2 \simeq [1 + BT_{60}/6.9]^{-1} \tag{15}$$

which is valid for the product $BT_{60} \geq 20$

where T_{60} = the 60 dB reverberation time of the room

B = the modal bandwidth of the room.

These results will be useful for comparison to the experimental values of normalized variance.

The normalized variances described so far have been for single measurement positions. In a later paper by Lubman, he derived the normalized variance for spatial averaging in a diffuse sound field [6]. When a spatial average is made over a continuous straight-line traverse, a normalized variance in a pure-tone or extremely narrow-band noise field is found to be:

$$V^2 = 1/(1 + 2L/\lambda) \tag{16}$$

where L = the length of traverse of the averaging path

λ = the wavelength of the tone.

Researchers at Carrier Corporation [7] have arrived at a slightly modified form of Equation 16 through experimental means for an experimental normalized variance in a reverberant room of:

$$V^2 = 1/1.56(1 + .55L/\lambda)^2 \qquad (17)$$

Continuous-line averaging in a reverberant sound field will give reasonable results for the mean sound intensity with a low variance in intensity. Averaging at discrete, well-separated points will give results very nearly the same as the continuous line averaging. Waterhouse and Lubman [8] published a paper in which they argue three points: 1) continuous averaging is not always better than discrete averaging at a fixed interval; 2) a discrete average taken in a certain way is always better than a continuous average; and 3) the sample variance decreases with the size of the discretely sampled region in one, two, or three dimensions. They conclude that discrete averaging is generally to be preferred to continuous averaging, as the former is simpler to perform experimentally and, in addition, yields better results.

The key to the accuracy in discrete point averaging is in choosing points in the sound field where the cross-correlation function (also called the covariance function) for the mean-square sound pressure is zero or as small as possible. If all points are chosen so that zero correlation exists between points, then the result will be the most accurate value of the mean.

Cook et. al. [9] in 1953 showed theoretically and confirmed experimentally that the cross-correlation function between the sound pressures in a diffuse sound field of narrow bandwidth was equal to:

$$R(r) = (\sin kr)/kr \tag{18}$$

where k = the wavenumber

r = the distance between the points at which the pressure measurements are taken.

Thus, for readings taken at points spaced $\lambda/2$ apart, the two values of sound pressure (and mean-square pressure) have zero correlation.

In considering the case of a point source in a channel, a cross-correlation function between sound pressures was derived (see Appendix B):

$$R(r) = \sum_{n=-\infty}^{\infty} \frac{R^{|n|}}{r_{n_1} r_{n_2}} \cos k(r_{n_1} - r_{n_2}) \Big/ \sum_{n=-\infty}^{\infty} \frac{R^{|n|}}{r_{n_1} r_{n_2}} \tag{19}$$

where the parameters have been defined at Equation 4. This expression has not yet been evaluated and the optimal point spacing has not been determined.

Experiment - Experimental measurements in both continuous-line averaging and discrete point averaging have been made. For the experimental case of this thesis (described in Section III), a slow sweep of the microphone across the channel produced a graphic level recorder output of the following form in Figure 7.

Each chart was broken up into approximately 50 intervals of equal spacing. Each interval represented a microphone sweep distance of approximately 0.70 inches. A quasi-continuous line average was made by reading the average sound pressure level (not a dB average) of each interval and

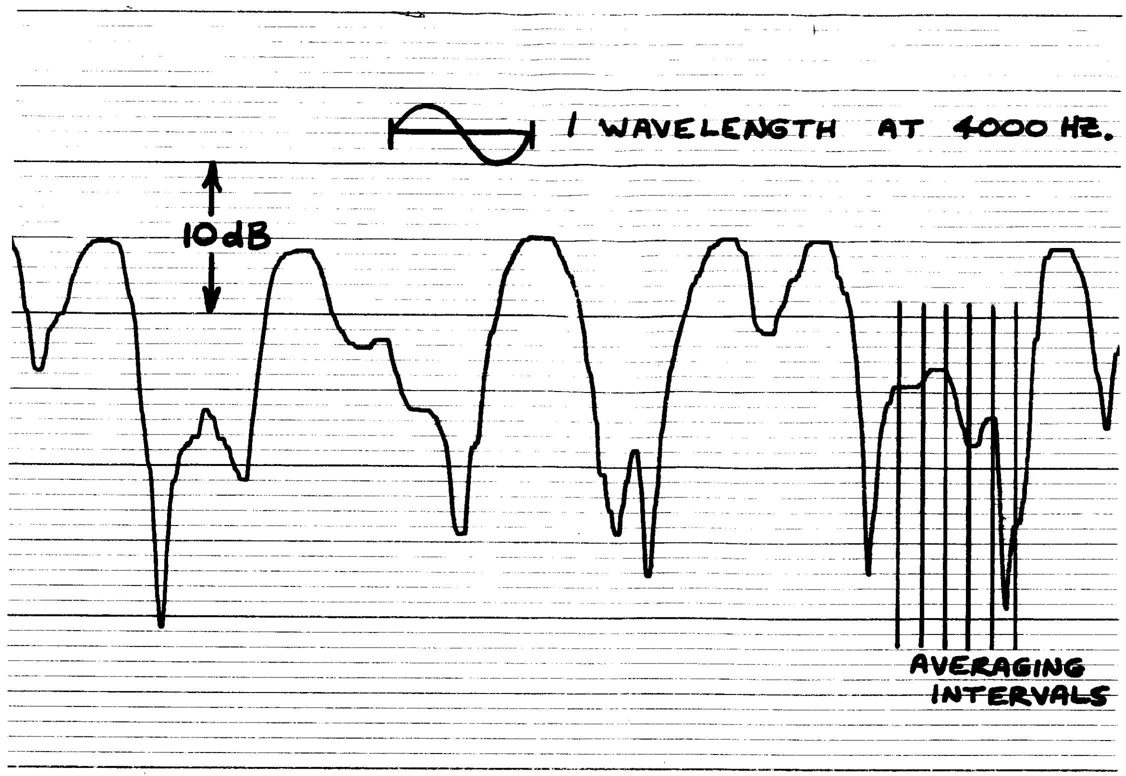


FIGURE 7

Sound Pressure Level as a Function of Position Across the Channel

recording it in Table 4 in the appendix. Each level was then converted to a squared sound pressure, in microbars squared, in Table 6. The figures in Table 6 were manipulated to get the average levels plotted in Figure 5 and the variances discussed below.

For the discrete point readings used, the same charts were read at points representing the ends of the adjoining continuous intervals and the levels recorded in Table 5 and converted to squared sound pressures in Table 7.

Variances normalized to the mean squared pressures predicted by theory are calculated and listed in Table 2 below for both continuous-line averaging and discrete point averaging. Values of theoretical variance come from Equation 16.

TABLE 2: AVERAGE VARIANCES

CONTINUOUS LINE:	% CHANNEL WIDTH	EXP. VAR.	THEORY
	100%	.0384	.0457
	50%	.0495	.0873
	25%	.1594	.1605

DISCRETE POINT:	POINT SPACING	NO. OF POINTS	VARIANCE
	.22λ	48	.0309
		24	.0488
		12	.1388
	.44λ	24	.0365
		12	.0495
		6	.1663
	.87λ	12	.1444
		6	.1963
		3	.4764

A conclusion from the above results is that discrete point readings taken at spacings of $\lambda/2$ will provide uncorrelated samples. The number of readings needed will have to be determined by the accuracy required (i.e., more readings mean better accuracy).

VI. Continuation of Thesis Work

The conclusion stated in the preceding section is based on a limited amount of data. Further work must be done to get enough values of variance so that curves can be plotted showing the amount of variance as a function of the number of sample points and the spacing of the points. Enough data is available in Tables 4 - 7 in the appendix for this to be done.

A computer program for evaluating the correlation function between points in the sound field (Appendix B) is needed for an analytic result for optimal spacing of the sample points. The number of sample points needed is based on the accuracy required. Waterhouse and Lubman [8,10] have attacked the problem of determining the probability of a measured value of mean-square pressure lying within ±1 dB, or some other range, of the mean value as a function of the number of sample points used for each measurement. This work will have to be extended for the purpose of this thesis to determine the minimum acceptable number of sample points required.

A further step in the continuation of this work is to investigate the effects of surface irregularities on the channel walls to find what effect sound diffusion will have on measurement accuracy. A moving source should also be incorporated in future studies to simulate real life conditions.

Appendix A

Distribution of Point Sources in Infinite Line

For all cases of R between 0 and 1, the expression for X is given by Equation 6:

$$X = \sum_{n=-\infty}^{\infty} R^{|n|}/4\pi r_n^2 \qquad (6)$$

which has not been evaluated for any case other than R = 0 and R = 1. An approximation can be made by replacing the series of points by a continuous line source having a decaying acoustical strength so that the net acoustic power in each interval of the line source is the same as the net power of the discrete point source which would have occupied that interval of space. An expression of the form

$$w(x) = \alpha \frac{W}{L} e^{-\mu|x|} \qquad (A\text{-}1)$$

where $\mu = -\frac{1}{L} \ln R$ will be used.

The value of α can be found by equating the sound power in the n-th interval

$$W_n = \int_{(n-\frac{1}{2})L}^{(n+\frac{1}{2})L} w(x)\, dx = WR^n \qquad (A\text{-}2)$$

or

$$\int_{(n-\frac{1}{2})L}^{(n+\frac{1}{2})L} \left(\frac{\alpha W}{L} e^{-\mu x}\right) dx = WR^n \qquad (A\text{-}3)$$

yields

$$\alpha = \frac{\sqrt{R}\, \ln R}{R - 1} \qquad (A\text{-}4)$$

In this simplified model, the infinite line of decaying strength is symmetric about the center of the street. In equating the sound power in the 0-th, or street interval, an additional factor must be added in the form of a point source of strength βW.

$$\beta W + 2 \int_0^{\frac{1}{2}L} \alpha \frac{W}{L} e^{-\mu x} \, dx = W \tag{A-5}$$

implies

$$\beta = \frac{1 - \sqrt{R}}{1 + \sqrt{R}} \tag{A-6}$$

and gives a factor of X for this model of:

$$X = \frac{\beta}{4\pi d^2} + 2 \int_0^{\infty} \left(\frac{\alpha e - \mu x}{4\pi L (d^2 + x^2)} \right) dx \tag{A-7}$$

which can be evaluated using Equation 3.354 on page 312 of I.S. Gradshteyn and I.M. Ryzhik's book of mathematical functions to get a result of

$$X = \frac{\beta}{4\pi d^2} + \frac{\alpha}{2\pi dL} [\sin(\mu d) \, \mathrm{ci}(\mu d) - \cos(\mu d) \, \mathrm{si}(\mu d)] \tag{A-8}$$

where ci (cosine integral) $= - \int_x^{\infty} \frac{\cos t}{t} \, dt$

si (sin integral) $= - \int_x^{\infty} \frac{\sin t}{t} \, dt = - \frac{\pi}{2} + \int_0^{x} \frac{\sin t}{t} \, dt$

Figure 8 is a plot of the magnitude vs. x for the function [sin x ci x - cos x si x] and Figure 9 is a plot of the $\log_{10}$ of the same function which approaches a limiting asymptote of -3 dB per doubling of distance. This is an interesting result for it points out that for any value of the reflection coefficient R, with the exception of R = 1, there exists some distance beyond which the level L_p drops off at 6 dB per doubling of distance since p^2 is also inversely proportional to distance. The distance beyond which L_p drops off at 6 dB per doubling of distance is found from Figure 8 to be:

$$\delta = \frac{-7}{\ln R} \qquad \text{(A-9)}$$

and this asymptote will be parallel to the R = 0 line and above it by an amount given by

$$10 \log \left(\frac{1 + R}{1 - R}\right) \text{, dB} \qquad \text{(A-10)}$$

which can be found by integrating Equation A-1 over all of x to find the total acoustic power for any particular value of R (the amount of βW must be added to the total, too). Figure 10 is a plot showing the limiting curve below which all curves for any value of R run parallel to the R = 0 line. The points marked on this curve are points where the various plots for R enter this region and were determined by Equations A-9 and A-10.

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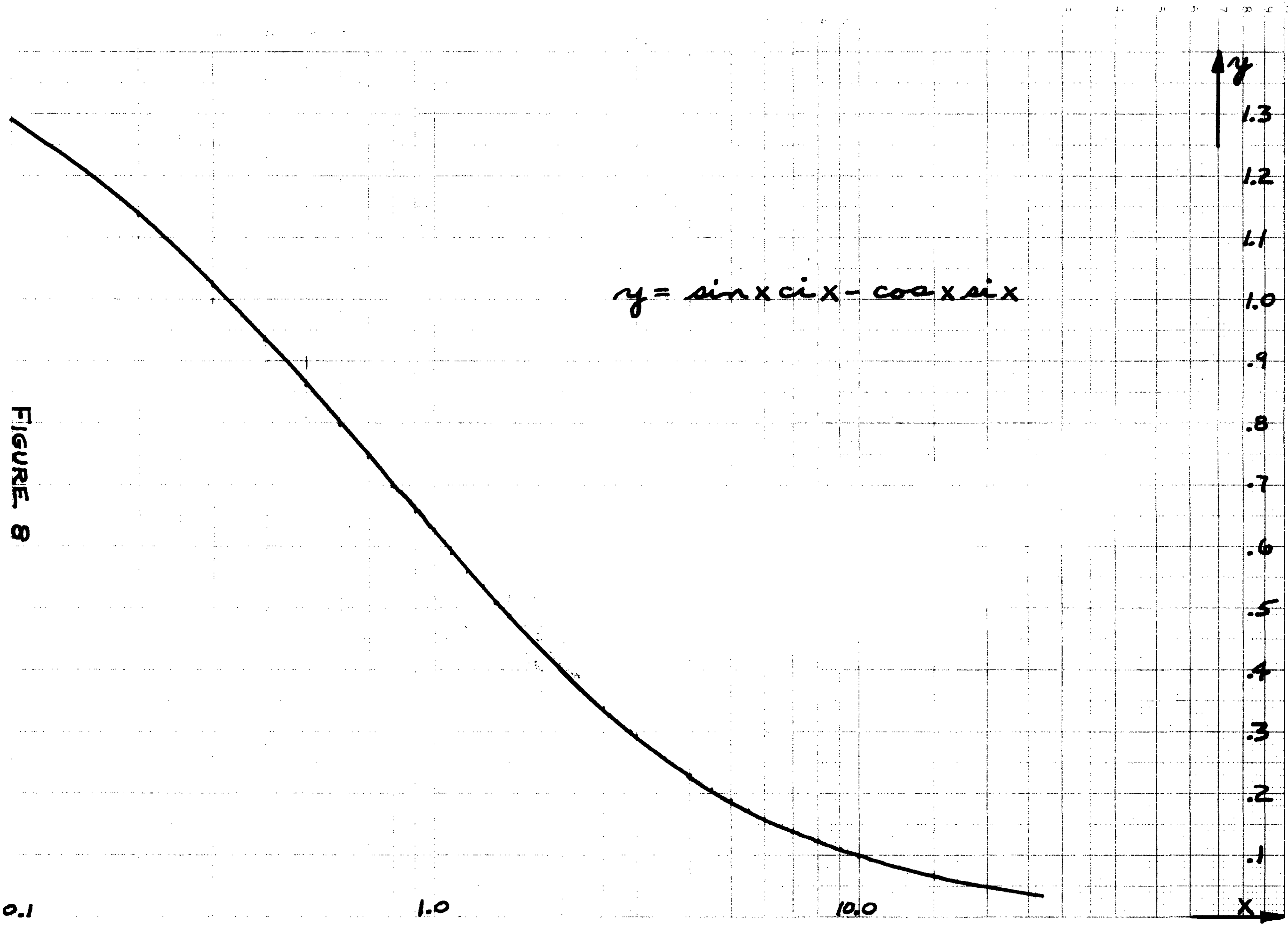


FIGURE 8

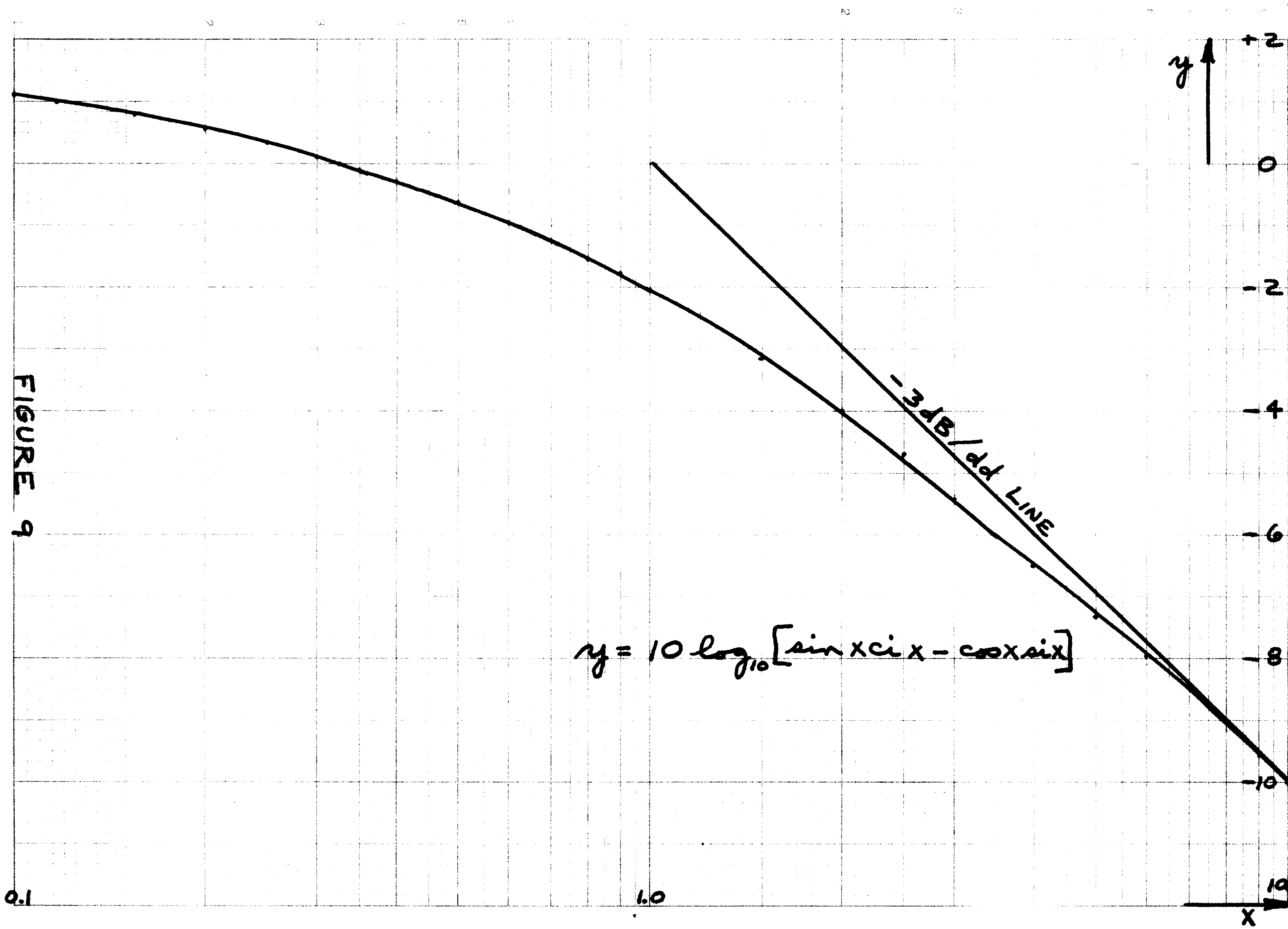
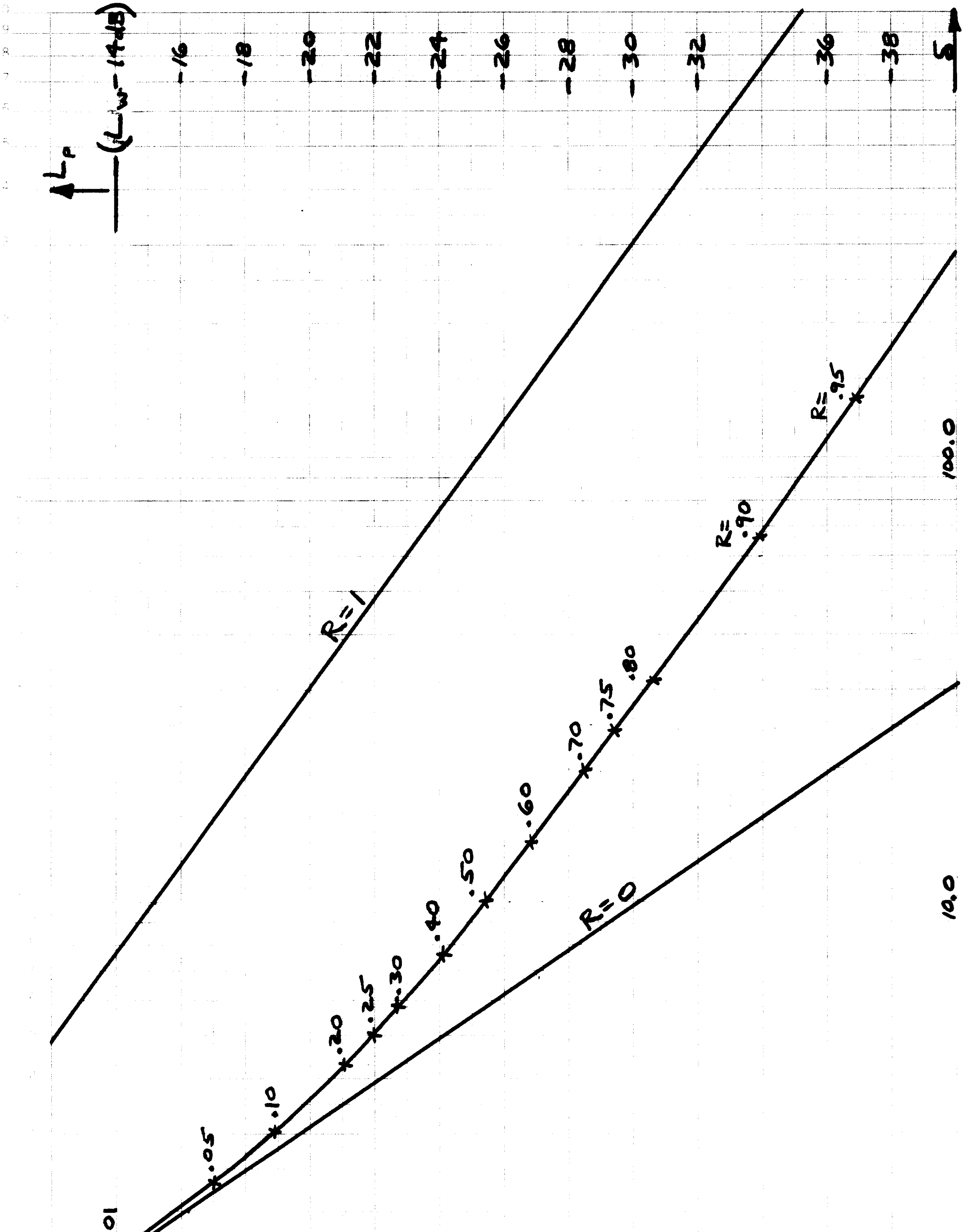


FIGURE 9



BEGINNING OF -6 dB/dd DROPOFF OF L_p
PARAMETER IS "R" (L_p vs. S)
FIGURE 10

APPENDIX B

CORRELATION COEFFICIENT BETWEEN TWO POINTS IN A SOUND FIELD

$$R = \langle p_1 p_2 \rangle \Big/ \left[\langle p_1^2 \rangle \langle p_2^2 \rangle \right]^{1/2}$$

$$\langle p_1 p_2 \rangle = \frac{1}{T} \int_0^T p_1(t)\, p_2(t)\, dt$$

$$R = \int_0^T p_1(t)\, p_2(t)\, dt \Big/ \left[\int_0^T p_1^2(t)\, dt \int_0^T p_2^2(t)\, dt \right]^{1/2}$$

$$p(t) = \sum_{n=-\infty}^{\infty} \frac{P_s R^{\frac{1}{2}|n|}}{r_n} \cos(\omega t - k r_n)$$

$$R = \frac{\int_0^T \sum_n \frac{P_s R^{\frac{1}{2}|n|}}{r_{n_1}} \cos(\omega t - k r_{n_1}) \cdot \sum_n \frac{P_s R^{\frac{1}{2}|n|}}{r_{n_2}} \cos(\omega t - k r_{n_2})\, dt}{\left[\int_0^T \left(\sum_n \frac{P_s R^{\frac{1}{2}|n|}}{r_{n_1}} \cos(\omega t - k r_{n_1}) \right)^2 dt \cdot \int_0^T \left(\sum_n \frac{P_s R^{\frac{1}{2}|n|}}{r_{n_2}} \cos(\omega t - k r_{n_2}) \right)^2 dt \right]^{\frac{1}{2}}}$$

$$R = \sum_{n=-\infty}^{\infty} \frac{R^{|n|}}{r_{n_1} r_{n_2}} \cos k(r_{n_2} - r_{n_1}) \Big/ \sum_{n=-\infty}^{\infty} \frac{R^{|n|}}{r_{n_1} r_{n_2}}$$

APPENDIX C

TABLES 3- 11

TABLE 3

EXPERIMENTAL DATA (L_p vs. δ)

	BROAD-BAND	PURE-TONE
AVE. LEVEL L_p ON SURFACE OF QUARTER SPHERE OF RADIUS 3"	106 dB	96.5 dB
L_w re 10^{-12} WATT D.I. = 0 dB D.I. = 6 dB	 95 dB 89 dB	 85.5 dB 79.5 dB

		BROAD-BAND NOISE		PURE-TONE	
d (ft)	δ (d/3)	L_p	$L_p - L_w$	L_p	$L_p - L_w$
.51	.170	101 dB	6 dB		
.77	.257	98	3		
1.03	.344	96	1		
1.30	.430	94.5	-.5		
1.60	.517	93.5	-1.5		
1.80	.604	92	-3.0		
2.10	.690	91	-4.0		
2.33	.780	90.8	-4.2		
2.60	.865	90.5	-4.5		
2.86	.952	90	-5.0		
4	1.33	88.5	-6.5	79.5 dB	-6.0 dB
4½	1.50			80	-5.5
5	1.67	87	-8.0	79.8	-5.6
5½	1.83			77.6	-7.9
6	2.00	86.5	-8.5	77.9	-7.6
7	2.33			76.5	-9.0
8	2.67	86	-9.0	77.0	-8.5
9	3.00			75.9	-9.6
10	3.33	84.5	-10.5	76.0	-9.5
11	3.67			75.9	-9.6
12	4.00			76.1	-9.4
13	4.33	83.5	-11.5		

TABLE 4

SOUND PRESSURE LEVELS (L_p, dB)

CONTINUOUS SWEEP RECORDING

INTERVAL	d = 4′	d = 4½′	d = 5′	d = 5½′	d = 6′	d = 7′
1	79.0	76.5	82.0	75.5	73.0	77.0
2	75.0	71.5	85.5	74.5	69.0	74.0
3	73.5	68.0	84.0	72.0	75.0	75.0
4	76.0	76.0	72.0	68.0	78.0	76.0
5	78.0	80.0	78.5	78.5	64.0	65.0
6	78.5	82.5	79.0	76.5	73.0	70.0
7	79.5	81.5	81.5	75.5	80.5	65.0
8	77.5	78.5	81.5	75.5	81.5	62.0
9	76.0	75.0	81.0	65.0	78.0	68.0
10	75.5	79.0	79.5	68.0	65.5	65.0
11	79.0	76.0	80.0	73.0	66.0	68.0
12	77.0	71.0	76.0	80.0	72.0	78.0
13	80.5	79.0	79.0	82.0	73.0	78.5
14	86.0	82.5	83.0	73.0	75.0	78.5
15	86.5	83.5	82.5	73.0	75.5	78.0
16	81.5	77.0	67.0	79.5	73.0	73.0
17	74.0	76.0	70.0	77.0	74.0	78.0
18	72.0	75.0	74.0	76.0	72.0	81.5
19	75.0	80.0	73.0	81.0	81.0	81.5
20	81.0	83.0	79.5	81.5	84.0	77.0
21	79.5	79.0	77.0	76.0	75.0	67.0
22	80.0	80.0	76.0	72.0	75.0	77.0
23	81.5	83.0	82.5	66.0	72.0	80.5
24	80.0	80.5	83.5	72.0	78.0	74.0
25	75.0	83.5	75.0	74.0	79.5	63.0
26	76.5	85.0	74.0	73.5	77.0	74.0
27	75.5	82.5	79.0	71.5	80.0	78.0
28	67.0	72.0	81.0	75.0	80.0	74.5
29	80.0	73.5	76.0	72.5	83.5	75.0
30	83.0	78.0	70.0	78.5	84.5	70.0

TABLE 4 (CONT.)

SOUND PRESSURE LEVELS (L_p, dB)

CONTINUOUS SWEEP RECORDING

INTERVAL	d = 4'	d = 4½'	d = 5'	d = 5½'	d = 6'	d = 7'
31	82.5	79.0	72.0	80.0	83.0	69.0
32	79.0	80.0	73.5	81.5	70.0	73.0
33	80.5	82.0	73.0	83.0	76.0	82.0
34	76.0	80.0	82.5	78.0	78.5	81.0
35	72.5	70.0	83.5	78.0	71.0	75.0
36	74.0	76.5	77.0	81.5	75.0	79.0
37	75.5	76.0	73.0	81.0	78.0	81.5
38	80.5	81.0	72.0	75.0	75.5	79.0
39	84.0	82.0	73.0	81.5	69.0	72.0
40	84.0	82.0	76.5	83.0	67.0	72.0
41	78.0	83.0	77.0	78.0	81.0	72.0
42	77.0	82.5	81.5	73.5	83.5	75.5
43	81.0	77.5	81.5	74.5	78.5	74.0
44	80.0	78.0	77.5	76.0	78.0	63.0
45	71.0	73.0	82.0	66.0	78.0	75.0
46	71.5	78.0	81.5	70.0	78.0	78.5
47	74.0		78.0	74.0		74.0
48	78.0		84.0			
49	74.0					
50	74.0					

TABLE 4 (CONT.)

SOUND PRESSURE LEVELS (L_p, dB)

CONTINUOUS SWEEP RECORDING

INTERVAL	d=8'	d=9'	d=10'	d=11'	d=12'	
1	74.0	77.0	68.0	75.0	77.0	
2	79.5	75.0	72.0	75.0	77.0	
3	80.0	71.5	75.0	78.5	76.0	
4	77.5	71.5	78.5	75.0	76.0	
5	74.0	65.0	79.0	65.0	78.0	
6	68.0	61.0	75.0	73.0	78.5	
7	74.0	67.0	70.0	74.5	75.0	
8	72.0	72.5	75.5	64.0	65.0	
9	68.0	70.0	76.0	70.0	61.0	
10	78.0	68.0	76.5	74.5	58.0	
11	80.5	77.0	76.5	74.5	70.0	
12	79.0	79.0	75.0	72.0	76.0	
13	72.0	79.5	69.0	60.0	77.5	
14	76.0	75.0	63.0	70.0	75.5	
15	75.0	75.0	69.0	77.0	74.5	
16	76.0	79.5	78.0	80.0	71.0	
17	79.0	80.0	78.5	78.0	75.0	
18	83.0	77.0	76.5	70.0	76.5	
19	80.0	75.0	75.0	75.0	77.0	
20	68.0	75.5	75.5	76.0	77.5	
21	73.0	75.5	77.5	77.0	76.0	
22	67.0	70.0	80.0	71.0	75.5	
23	72.0	71.0	81.0	81.0	73.0	
24	76.0	73.5	74.5	82.5	64.5	
25	77.0	73.0	76.0	78.0	71.0	
26	80.0	74.0	70.0	77.0	74.0	
27	80.0	73.0	60.0	82.0	77.0	
28	75.5	69.0	72.5	79.0	80.5	
29	71.0	73.0	74.5	76.0	82.0	
30	68.5	64.0	77.0	72.0	82.0	

TABLE 4 (CONT.)

SOUND PRESSURE LEVELS (L_p, dB)

CONTINUOUS SWEEP RECORDING

INTERVAL	d = 8′	d = 9′	d = 10′	d = 11′	d = 12′	
31	71.5	78.0	74.0	75.0	79.0	
32	74.0	82.5	69.0	81.0	74.5	
33	75.5	80.0	70.0	80.5	71.0	
34	77.0	80.0	81.0	63.0	66.0	
35	77.0	81.0	81.0	70.0	71.5	
36	75.0	76.0	70.0	64.0	75.0	
37	79.0	77.0	72.0	63.0	76.5	
38	80.0	79.0	75.0	68.5	78.5	
39	70.0	74.0	79.0	70.5	70.0	
40	78.0	67.0	80.0	71.0	71.0	
41	79.5	71.0	70.0	65.0		
42	79.0	69.0	71.0	67.5		
43	73.0	67.0	76.0	70.0		
44	73.0	70.0	75.0	77.0		
45	68.0	70.0	72.5	76.0		
46	74.5	74.5	73.5	67.0		
47	76.0	77.0	79.0	70.0		
48	81.5		71.0			
49			76.0			
50						

TABLE 5

SOUND PRESSURE LEVELS (L_p, dB)

DISCRETE POINT READINGS

LOCATION #	d=4'	d=4½'	d=5'	d=5½'	d=6'	d=7'
1	80.0	78.0	78.0	75.5	76.0	77.5
2	77.5	73.0	84.5	76.5	62.0	76.0
3	68.0	62.0	85.5	73.0	71.0	69.5
4	73.5	73.5	75.0	67.5	78.5	78.0
5	77.5	78.5	75.0	75.0	70.0	70.5
6	78.0	82.0	74.0	79.5	69.0	69.0
7	79.5	82.5	75.5	75.0	78.0	68.5
8	79.0	80.0	81.5	76.0	81.5	55.0
9	76.0	75.5	81.5	72.5	81.0	67.5
10	75.0	77.5	80.0	66.0	70.0	64.0
11	77.5	79.0	80.0	69.0	64.0	67.5
12	79.0	66.0	78.5	76.5	68.5	74.0
13	74.0	73.0	75.0	82.0	73.5	79.5
14	84.0	81.5	82.0	79.0	70.5	78.5
15	86.5	83.5	83.5	63.5	76.0	78.5
16	84.5	82.0	76.5	77.5	66.5	77.0
17	73.0	73.0	67.5	80.5	75.5	73.0
18	74.5	76.0	73.0	72.5	68.5	80.5
19	68.0	73.5	74.0	79.0	78.0	81.5
20	79.5	82.5	76.5	81.5	83.5	80.0
21	81.0	83.0	80.0	79.5	82.0	74.0
22	79.0	77.5	69.0	75.0	71.0	72.0
23	81.0	82.0	80.0	70.0	77.0	79.5
24	81.5	82.5	83.5	69.0	74.0	80.0
25	78.5	81.5	80.0	74.0	79.0	69.0
26	74.0	84.5	73.5	74.0	80.0	71.5
27	77.5	84.5	75.0	73.5	77.0	74.5
28	65.5	68.0	80.0	74.5	80.0	76.0
29	74.0	74.5	80.0	75.5	82.0	77.5
30	82.0	74.0	66.0	75.5	84.0	74.5

TABLE 5 (CONT.)

SOUND PRESSURE LEVELS (L_p, dB)

DISCRETE POINT READINGS

LOCATION #	d= 4′	d= 4½′	d= 5′	d= 5½′	d= 6′	d= 7′
31	83.5	79.0	69.0	79.5	84.5	70.0
32	79.5	79.0	74.5	80.0	76.0	68.0
33	80.0	80.5	63.5	83.0	70.0	79.5
34	79.5	82.5	79.0	83.0	78.5	83.0
35	73.5	75.5	83.5	69.0	77.5	78.0
36	73.0	72.0	81.0	80.5	67.0	74.5
37	74.0	77.0	70.5	82.0	77.5	81.5
38	78.0	79.0	73.5	76.5	77.0	81.0
39	82.0	82.0	72.0	77.5	74.5	76.0
40	85.5	82.0	74.5	83.0	60.5	72.0
41	80.0	82.5	77.0	81.5	76.5	72.0
42	73.0	83.5	78.0	74.5	83.0	73.0
43	79.5	79.0	82.0	73.5	82.0	76.0
44	82.0	78.0	80.0	75.5	78.0	65.5
45	73.5	75.0	78.0	75.5	78.0	71.0
46	71.0	75.5	83.0	66.5	78.0	78.5
47	71.5	78.0	75.0	67.5	76.5	78.5
48	77.0	79.0	85.0	79.5	70.0	56.0

TABLE 5 (CONT.)

SOUND PRESSURE LEVELS (L_p, dB)

DISCRETE POINT READINGS

LOCATION #	d=8′	d=9′	d=10′	d=11′	d=12′	
1	72.0	78.0	67.0	77.0	77.5	
2	78.0	76.0	69.5	74.5	76.5	
3	80.0	73.0	72.0	77.0	76.5	
4	79.0	71.5	77.5	79.0	76.0	
5	77.0	68.5	79.0	66.0	77.0	
6	67.5	60.0	78.5	70.0	78.5	
7	71.5	61.0	71.0	74.0	78.0	
8	74.5	71.0	74.5	73.0	70.0	
9	63.0	71.5	75.5	60.0	64.0	
10	75.0	66.0	76.5	72.0	56.5	
11	79.5	74.5	76.5	75.5	58.0	
12	80.5	78.5	76.0	72.0	73.5	
13	77.0	79.5	73.0	63.0	77.0	
14	73.5	79.0	66.0	62.0	77.0	
15	76.0	71.0	65.0	73.5	74.5	
16	76.5	78.5	75.5	79.5	71.5	
17	74.5	80.0	78.5	80.0	72.0	
18	82.0	78.5	77.5	71.0	76.0	
19	83.0	75.0	76.0	73.0	77.0	
20	75.5	75.5	75.0	75.0	77.0	
21	72.5	75.5	77.0	77.0	77.0	
22	67.5	73.5	78.5	74.0	75.0	
23	69.0	66.0	80.5	78.0	76.0	
24	75.0	73.5	78.0	82.5	64.0	
25	74.0	73.0	74.0	81.0	65.0	
26	79.5	73.5	76.0	74.5	73.0	
27	80.5	74.0	59.5	80.5	74.5	
28	78.0	67.0	69.0	82.0	79.0	
29	75.0	72.0	74.0	78.0	81.5	
30	67.0	67.0	74.0	73.5	82.0	

TABLE 5 (CONT.)

SOUND PRESSURE LEVELS (L_P, dB)

DISCRETE POINT READINGS

LOCATION #	d=8'	d=9'	d=10'	d=11'	d=12'	
31	69.5	73.0	77.0	60.0	80.5	
32	73.5	81.0	75.0	80.0	75.0	
33	75.0	82.5	61.0	81.0	73.0	
34	75.5	76.5	79.0	75.0	65.0	
35	77.0	81.0	81.0	67.0	69.0	
36	76.5	80.0	76.0	70.0	73.0	
37	77.5	66.0	69.5	62.0	76.0	
38	80.0	79.5	73.0	66.0	77.0	
39	75.0	76.5	78.0	69.5	77.0	
40	74.0	52.0	80.0	72.0	70.0	
41	78.5	70.0	78.5	68.0		
42	79.5	69.0	63.5	66.5		
43	77.5	68.5	76.0	68.5		
44	70.5	68.5	75.0	75.0		
45	74.0	69.5	72.5	77.0		
46	67.0	71.5	72.5	71.0		
47	75.5	76.5	77.0	68.5		
48	81.0	70.0	80.0	77.5		

TABLE 6

Sound Pressure Squared ($\overline{p^2}$, μ-bars)

$$\overline{p^2} = \text{Antilog}_{10}\left(\frac{L_p - 74}{10}\right), \quad L_p \text{ from Table 4}$$

Interval	d = 4′	d = 4½′	d = 5′	d = 5½′	d = 6′	d = 7′
1	3.16	1.78	6.30	1.41	.79	2.00
2	1.26	.56	14.10	1.12	.32	1.00
3	.89	.25	10.00	.63	1.26	1.26
4	1.58	1.58	.63	.25	2.51	1.58
5	2.51	3.98	2.82	2.82	.10	.13
6	2.82	7.08	3.16	1.78	.79	.40
7	3.55	5.62	5.62	1.41	4.46	.13
8	2.24	2.82	5.62	1.41	5.62	.06
9	1.58	1.26	5.00	.13	2.51	.25
10	1.41	3.16	3.55	.25	.14	.13
11	3.16	1.58	3.98	.79	.16	.25
12	2.00	.50	1.58	3.98	.63	2.51
13	4.46	3.16	3.16	6.30	.79	2.82
14	15.80	7.08	7.95	.79	1.26	2.82
15	17.80	8.91	7.08	.79	1.41	2.51
16	5.62	2.00	.20	3.55	.79	.79
17	1.00	1.58	.40	2.00	1.00	2.51
18	.63	1.26	1.00	1.58	.63	5.62
19	1.26	3.98	.80	5.00	5.00	5.62
20	5.00	7.95	3.55	5.62	10.00	2.00
21	3.55	3.16	2.00	1.58	1.26	.20
22	3.98	3.98	1.58	.63	1.26	2.00
23	5.62	7.95	7.08	.16	.63	4.46
24	3.98	4.46	8.91	.63	2.51	1.00
25	1.26	8.91	1.26	1.00	3.55	.08
26	1.78	12.60	1.00	.89	2.00	1.00
27	1.41	7.08	3.16	.56	3.98	2.51
28	.20	.63	5.00	1.26	3.98	1.12
29	3.98	.89	1.58	.71	8.91	1.26
30	7.95	2.51	.40	2.82	11.20	.40

TABLE 6 (CONT.)

SOUND PRESSURE SQUARED ($\overline{p^2}$, μ-bars)

$\overline{p^2} = \text{ANTILOG}_{10}\left(\frac{L_p - 74}{10}\right)$, L_p FROM TABLE 4

INTERVAL	d= 4'	d= 4½'	d= 5'	d= 5½'	d= 6'	d= 7'
31	7.08	3.16	.63	3.98	7.95	.32
32	3.55	3.98	.89	5.62	.40	.79
33	4.46	6.30	.79	7.95	1.58	6.30
34	1.58	3.98	7.08	2.51	2.51	5.00
35	.71	.40	8.91	2.51	.50	1.26
36	1.00	1.78	2.00	5.62	1.26	3.16
37	1.41	1.58	.79	5.00	2.51	5.62
38	4.46	5.00	.63	1.26	1.41	3.16
39	10.00	6.30	.79	5.62	.32	.63
40	10.00	6.30	1.78	7.95	.20	.63
41	2.51	7.95	2.00	2.51	5.00	.63
42	2.00	7.08	5.62	.89	8.91	1.41
43	5.00	2.24	5.62	1.12	2.82	1.00
44	3.98	2.51	2.24	1.58	2.51	.08
45	.50	.79	6.30	.16	2.51	1.26
46	.56	2.51	5.62	.40	2.51	2.82
47	1.00		2.51	1.00		1.00
48	2.51		10.00			
49	1.00					
50	1.00					

TABLE 6 (CONT.)

SOUND PRESSURE SQUARED ($\overline{p^2}$, μ-bars)

$\overline{p^2} = \text{ANTILOG}_{10}\left(\frac{L_p - 74}{10}\right)$, L_p FROM TABLE 4

INTER-VAL	d=8′	d=9′	d=10′	d=11′	d=12′	
1	1.00	2.00	.25	1.26	2.00	
2	3.55	1.26	.63	1.26	2.00	
3	3.98	.56	1.26	2.82	1.58	
4	2.24	.56	2.82	1.26	1.58	
5	1.00	.13	3.16	.13	2.51	
6	.25	.05	1.26	.79	2.82	
7	1.00	.20	.40	1.12	1.26	
8	.63	.71	1.41	.10	.13	
9	.25	.40	1.58	.40	.05	
10	2.51	.25	1.78	1.12	.03	
11	4.46	2.00	1.78	1.12	.40	
12	3.16	3.16	1.26	.63	1.58	
13	.63	3.55	.32	.04	2.24	
14	1.58	1.26	.08	.40	1.12	
15	1.26	1.26	.32	2.00	1.41	
16	1.58	3.55	2.51	3.98	.50	
17	3.16	3.98	2.82	2.51	1.26	
18	7.95	2.00	1.78	.40	1.78	
19	3.98	1.26	1.26	1.26	2.00	
20	.25	1.41	1.41	1.58	2.24	
21	.79	1.41	2.24	2.00	1.58	
22	.20	.40	3.98	.50	1.41	
23	.63	.50	5.00	5.00	.79	
24	1.58	.89	1.12	7.08	.11	
25	2.00	.79	1.58	2.51	.50	
26	3.98	1.00	.40	2.00	1.00	
27	3.98	.79	.04	6.30	2.00	
28	1.41	.32	.71	3.16	4.46	
29	.50	.79	1.12	1.58	6.30	
30	.28	.10	2.00	.63	6.30	

TABLE 6 (CONT.)

SOUND PRESSURE SQUARED ($\overline{p^2}$, μ-bars)

$$\overline{p^2} = \text{ANTILOG}_{10}\left(\frac{L_p - 74}{10}\right)$$, L_p FROM TABLE 4

INTERVAL	d=8′	d=9′	d=10′	d=11′	d=12′	
31	.56	2.51	1.00	1.26	3.16	
32	1.00	7.08	.32	5.00	1.12	
33	1.41	3.98	.40	4.46	.50	
34	2.00	3.98	5.00	.08	.16	
35	2.00	5.00	5.00	.40	.56	
36	1.26	1.58	.40	.10	1.26	
37	3.16	2.00	.63	.08	1.78	
38	3.98	3.16	1.26	.28	2.82	
39	.40	1.00	3.16	.45	.40	
40	2.51	.20	3.98	.50	.50	
41	3.55	.50	.40	.13		
42	3.16	.32	0.50	.22		
43	.79	.20	1.58	.40		
44	.79	.40	1.26	2.00		
45	.25	.40	.71	1.58		
46	1.12	1.12	.89	.20		
47	1.58	2.00	3.16	.40		
48	5.62		.50			
49			1.58			
50						

TABLE 7

SOUND PRESSURE SQUARED (p^2, μ-bars)

$p^2 = \text{ANTILOG}_{10}\left(\frac{L_p - 74}{10}\right)$ L_p FROM TABLE 5

LOCATION #	d=4′	d=4½′	d=5′	d=5½′	d=6′	d=7′
1	3.98	2.51	2.51	1.41	1.58	2.24
2	2.24	.79	11.20	1.78	.06	1.58
3	.25	.06	14.10	.79	.50	.36
4	.89	.89	1.26	.22	2.82	2.51
5	2.24	2.82	1.26	1.26	.40	.45
6	2.51	6.30	1.00	3.55	.32	.32
7	3.55	7.08	1.41	1.26	2.51	.28
8	3.16	3.98	5.62	1.58	5.62	.01
9	1.58	1.41	5.62	.71	5.00	.22
10	1.26	2.24	3.98	.16	.40	.10
11	2.24	3.16	3.98	.32	.10	.22
12	3.16	.16	2.82	1.78	.28	1.00
13	1.00	.79	1.26	6.30	.89	3.55
14	10.00	5.62	6.30	3.16	.45	2.82
15	17.80	8.91	8.91	.09	1.58	2.82
16	11.20	6.30	1.78	2.24	.18	2.00
17	.79	.79	.22	4.46	1.41	.79
18	1.12	1.58	.79	.71	.28	4.46
19	.25	.89	1.00	3.16	2.51	5.62
20	3.55	7.08	1.78	5.62	8.91	3.98
21	5.00	7.95	3.98	3.55	6.30	1.00
22	3.16	2.24	.32	1.26	.50	.63
23	5.00	6.30	3.98	.40	2.00	3.55
24	5.62	7.08	8.91	.32	1.00	3.98
25	2.82	5.62	3.98	1.00	3.16	.32
26	1.00	11.20	.89	1.00	3.98	.56
27	2.24	11.20	1.26	.89	2.00	1.12
28	.14	.25	3.98	1.12	3.98	1.58
29	1.00	1.12	3.98	1.41	6.30	2.24
30	6.30	1.00	.16	1.41	10.00	1.12

TABLE 7 (CONT.)

SOUND PRESSURE SQUARED (p^2, μ-bars)

$$p^2 = \text{ANTILOG}_{10}\left(\frac{L_p - 74}{10}\right)$$, L_p FROM TABLE 5

LOCATION #	d=4′	d=4½′	d=5′	d=5½′	d=6′	d=7′
31	8.91	3.16	.32	3.55	11.20	.40
32	3.55	3.16	1.12	3.98	1.58	.25
33	3.98	4.46	.09	7.95	.40	3.55
34	3.55	7.08	3.16	7.95	2.82	7.95
35	.89	1.41	8.91	.32	2.24	2.51
36	.79	.63	5.00	4.46	.20	1.12
37	1.00	2.00	.45	6.30	2.24	5.62
38	2.51	3.16	.89	1.78	2.00	5.00
39	6.30	6.30	.63	2.24	1.12	1.58
40	14.10	6.30	1.12	7.95	.05	.63
41	3.98	7.08	2.00	5.62	1.78	.63
42	.79	8.91	2.51	1.12	7.95	.79
43	3.55	3.16	6.30	.89	6.30	1.58
44	6.30	2.51	3.98	1.41	2.51	.14
45	.89	1.26	2.51	1.41	2.51	.50
46	.50	1.41	7.95	.18	2.51	2.82
47	.56	2.51	1.26	.22	1.78	2.82
48	2.00	3.16	12.60	3.55	.40	.02

TABLE 7 (CONT.)

SOUND PRESSURE SQUARED (p^2, μ-bars)

$p^2 = \text{ANTILOG}_{10}\left(\frac{L_p - 74}{10}\right)$, L_p FROM TABLE 5

LOCATION #	d=8′	d=9′	d=10′	d=11′	d=12′	
1	.63	2.51	.20	2.00	2.24	
2	2.51	1.58	.36	1.12	1.78	
3	3.98	.79	.63	2.00	1.78	
4	3.16	.56	2.24	3.16	1.58	
5	2.00	.28	3.16	.16	2.00	
6	.22	.04	2.82	.40	2.82	
7	.56	.05	.50	1.00	2.51	
8	1.12	.50	1.12	.79	.40	
9	.08	.56	1.41	.04	.10	
10	1.26	.16	1.78	.63	.02	
11	3.55	1.12	1.78	1.41	.03	
12	4.46	2.82	1.58	.63	.89	
13	2.00	3.55	.79	.08	2.00	
14	.89	3.16	.16	.06	2.00	
15	1.58	.50	.13	.89	1.12	
16	1.78	2.82	1.41	3.55	.56	
17	1.12	3.98	2.82	3.98	.63	
18	6.30	2.82	2.24	.50	1.58	
19	7.95	1.26	1.58	.79	2.00	
20	1.41	1.41	1.26	1.26	2.00	
21	.71	1.41	2.00	2.00	2.00	
22	.22	.89	2.82	1.00	1.26	
23	.32	.16	4.46	2.51	1.58	
24	1.26	.89	2.51	7.08	.10	
25	1.00	.79	1.00	5.00	.13	
26	3.55	.89	1.58	1.12	.79	
27	4.46	1.00	.04	4.46	1.12	
28	2.51	.20	.32	6.30	3.16	
29	1.26	.63	1.00	2.51	5.62	
30	.20	.20	1.00	.89	6.30	

TABLE 7 (CONT.)

SOUND PRESSURE SQUARED (p^2, μ-bars)

$p^2 = \text{ANTILOG}_{10}\left(\frac{L_p - 74}{10}\right)$, L_p FROM TABLE 5

LOCATION #	d=8'	d=9'	d=10'	d=11'	d=12'	
31	.36	.79	2.00	.04	4.46	
32	.89	5.00	1.26	3.98	1.26	
33	1.26	7.08	.05	5.00	.79	
34	1.41	1.78	3.16	1.26	.13	
35	2.00	5.00	5.00	.20	.32	
36	1.78	3.98	1.58	.40	.79	
37	2.24	.16	.36	.06	1.58	
38	3.98	3.55	.79	.16	2.00	
39	1.26	1.78	2.51	.36	2.00	
40	1.00	.01	3.98	.63	.40	
41	2.82	.40	2.82	.25		
42	3.55	.32	.09	.18		
43	2.24	.28	1.58	.28		
44	.45	.28	1.26	1.26		
45	1.00	.36	.71	2.00		
46	.20	.56	.71	.50		
47	1.41	1.78	2.00	.28		
48	5.00	.40	3.98	2.24		

TABLE 8

p^2 AVERAGED OVER DIFFERENT LENGTHS

CONTINUOUS AVERAGE (FROM TABLE 6)

d	AVERAGED OVER INTERVALS NUMBERED							THEORY AVERAGE $\langle p^2 \rangle_x$
	1-12	13-24	25-36	37-48	1-24	25-48	1-48	
4	2.18	5.72	2.91	3.28	3.95	3.11	3.52	3.55
4½	2.51	4.62	4.35	4.23	3.57	4.29	3.92	3.16
5	5.20	3.64	2.72	3.66	4.42	3.19	3.81	2.82
5½	1.33	2.39	2.95	2.50	1.86	2.74	2.29	2.57
6	1.61	2.21	3.15	2.87	1.91	3.02	2.44	2.34
7	0.81	2.69	1.93	1.66	1.75	1.80	1.78	2.04
8	2.00	1.96	1.70	2.24	1.98	1.97	1.98	1.78
9	0.94	1.79	2.32	1.03	1.36	1.70	1.53	1.58
10	1.47	1.90	1.50	1.51	1.68	1.50	1.59	1.41
11	1.00	2.23	2.29	0.57	1.61	1.46	1.54	1.29
12	1.40	1.45	2.45	1.23	1.42	1.84	1.63	1.18

TABLE 9

NORMALIZED VARIANCE IN p^2

$$\left[\left(p^2 - \langle p^2 \rangle_{\bar{x}}\right) / \langle p^2 \rangle_{\bar{x}}\right]^2 \qquad p^2 \text{ FROM TABLE 8}$$

d	VARIANCE OF AVE. p^2 OVER INTERVALS NUMBERED						
	1-12	13-24	25-36	37-48	1-24	25-48	1-48
4	.1490	.3720	.0324	.0058	.0121	.0151	.0001
4½	.0420	.2115	.1430	.1156	.0169	.1295	.0576
5	.7140	.0841	.0012	.0900	.3250	.0169	.1225
5½	.2320	.0048	.0225	.0007	.0762	.0042	.0119
6	.0980	.0030	.1210	.0511	.0338	.0841	.0018
7	.3630	.1024	.0029	.0346	.0202	.0137	.0169
8	.0149	.0106	.0020	.0665	.0123	.0114	.0119
9	.1640	.0174	.2210	.1225	.0182	.0062	.0010
10	.0016	.1210	.0040	.0050	.0365	.0040	.0164
11	.5060	.5330	.6000	.3140	.0620	.0182	.0372
12	.0350	.0530	1.1550	.0018	.0433	.3080	.1450
TOTAL	2.3195	1.5128	2.3050	0.8076	0.6565	0.6113	0.4223
AVE. VAR.	.2110	.1375	.2095	.0796	.0597	.0555	.0384
VAR.	.1594 — 12 READINGS (¼ SWEEP)				.0576 — ½ SWEEP		FULL SWEEP .0384

TABLE 10

AVE. VALUES OF p^2 FROM DISCRETE POINT READINGS

READINGS OF p^2 FROM TABLE 7

READINGS TAKEN APPROX. 0.7 IN. APART

d	NO. OF READINGS						
	1ST-12	2ND-12	3RD-12	4TH-12	1ST-24	2ND-24	48
4	2.255	5.370	2.930	3.540	3.810	3.235	3.530
4½	2.620	4.625	4.190	3.980	3.620	4.080	3.850
5	4.560	3.270	2.740	3.510	3.915	3.125	3.520
5½	1.235	2.605	2.920	2.720	1.920	2.820	2.370
6	1.630	2.165	3.990	2.595	1.900	3.290	2.595
7	.774	2.930	1.890	1.840	1.852	1.870	1.861
8	1.960	2.125	1.725	2.095	2.040	1.910	1.975
9	.915	1.905	2.275	.823	1.410	1.550	1.480
10	1.465	1.845	1.500	1.730	1.655	1.615	1.635
11	1.110	1.975	2.595	.683	1.540	1.640	1.590
12	1.523	1.281	2.206	1.373	1.402	1.790	1.596

READINGS TAKEN APPROX. 1.4 IN. APART

d	NO. OF. READINGS						
	1ST-6	2ND-6	3RD-6	4TH-6	1ST-12	2ND-12	24
4	2.320	4.980	3.310	2.715	3.640	3.010	3.325
4½	2.840	4.275	4.495	3.720	3.555	4.100	3.830
5	4.820	3.230	3.090	2.190	4.020	2.640	3.330
5½	.959	3.000	2.520	2.780	1.975	2.650	2.315
6	1.700	2.450	4.220	2.620	2.075	3.420	2.745
7	.629	2.890	1.690	2.120	1.755	1.905	1.830
8	1.800	2.280	1.725	1.830	2.040	1.775	1.910
9	.885	1.810	2.550	.793	1.345	1.670	1.510
10	1.280	1.960	1.515	1.665	1.620	1.590	1.605
11	1.100	1.710	2.870	.538	1.405	1.705	1.555
12	1.726	1.156	2.090	1.830	1.441	1.960	1.700

TABLE 11

NORMALIZED VARIANCE IN p^2

$$\left[\left(p^2 - \langle p^2 \rangle_{\bar{x}}\right) / \langle p^2 \rangle_{\bar{x}}\right]^2$$

READINGS OF p^2 FROM TABLE 10

d	VARIANCE OF AVE. p^2, READINGS 0.7 IN. APART						
	12	12	12	12	24	24	48
4	.1332	.2620	.0306	.0009	.0053	.0078	.0001
4½	.0292	.2160	.1075	.0676	.0213	.0848	.0346
5	.3810	.0256	.0008	.0600	.1505	.0119	.0615
5½	.2695	.0002	.0185	.0035	.0640	.0096	.0059
6	.0924	.0056	.4970	.0119	.0354	.1640	.0119
7	.3860	.1910	.0055	.0096	.0086	.0071	.0077
8	.0104	.0376	.0010	.0317	.0213	.0052	.0121
9	.1772	.0420	.1935	.2295	.0117	.0004	.0041
10	.0016	.0961	.0041	.0520	.0303	.0211	.0256
11	.0216	.2810	1.0260	.2210	.0380	.0739	.0538
12	.0847	.0072	.7550	.0266	.0354	.2662	.1231
AVE. VAR.	.1388 VARIANCE FOR 12 READINGS				.0488 24 READINGS		.0309 48 RDGS

d	VARIANCE OF AVE. p^2, READINGS 1.4 IN. APART						
	6	6	6	6	12	12	24
4	.1205	.1640	.0046	.0552	.0006	.0223	.0041
4½	.0102	.1245	.1790	.0317	.0156	.0888	.0449
5	.5040	.0210	.0092	.0502	.1805	.0041	.0328
5½	.3930	.0282	.0004	.0066	.0534	.0010	.0100
6	.0751	.0023	.6450	.0144	.0130	.0012	.0299
7	.4790	.1740	.0296	.0015	.0196	.0044	.0182
8	.0001	.0790	.0010	.0008	.0216	.0001	.0052
9	.1935	.0211	.3780	.2480	.0222	.0032	.0019
10	.0085	.1521	.0056	.0324	.0222	.0164	.0193
11	.0218	.1055	1.5000	.3395	.0081	.1037	.0421
12	.2125	.0004	.5930	.3025	.0484	.4360	.1935
AVE. VAR.	.1663 VARIANCE FOR 6 READING				.0495 12 READINGS		.0365 24 RDG

TABLE 10 (CONT.)

d	AVE. VALUES OF P^2, READINGS 2.8 IN. APART						
	3	3	3	3	6	6	12
4	2.403	6.790	1.493	7.467	4.597	4.480	4.538
4½	1.677	6.820	1.347	3.990	4.248	2.668	3.458
5	3.233	4.157	3.367	5.900	3.695	4.633	4.164
5½	1.193	2.727	3.187	4.303	1.960	3.745	2.852
6	2.907	3.363	1.920	.987	3.135	1.453	2.294
7	1.173	3.320	.983	.263	2.247	.623	1.435
8	2.913	1.483	1.727	2.150	2.198	1.938	2.068
9	1.293	1.707	3.060	.230	1.500	1.645	1.572
10	1.647	1.727	1.053	3.073	1.687	2.063	1.875
11	1.527	3.963	3.560	1.377	2.745	2.468	2.607
12	.957	.887	1.737	—	.922	1.312	1.193

TABLE 11 (CONT.)

d	VARIANCE OF AVE. P^2, READINGS 2.8 IN. APART						
	3	3	3	3	6	6	12
4	.1050	.8320	.3350	1.2210	.0876	.0697	.0778
4½	.2210	1.3450	.3295	.0692	.1186	.0244	.0088
5	.0213	.2255	.0376	1.1950	.0961	.4130	.2265
5½	.2873	.0037	.0581	.4530	.0562	.2090	.0121
6	.0591	.1905	.0320	.3340	.1155	.1436	.0004
7	.1806	.3932	.2682	.7580	.0104	.4825	.0879
8	.4030	.0276	.0009	.0428	.0552	.0079	.0262
9	.0331	.0064	.8775	.7300	.0026	.0017	.0001
10	.0282	.0502	.0640	1.3910	.0384	.2150	.1089
11	.0335	4.2800	3.0950	.0045	1.2780	.8340	1.0400
12	.0357	.0615	.2217	—	.0475	.0125	.0001
AVE. VAR.	.4764 VARIANCE FOR 3 READINGS				.1963 6 READINGS		.1444 12 RDGS.

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