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LIGHT

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
RESEARCH LABORATORY OF ELECTRONICS
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Abstract

This work is based on an article for publication in the McGraw-Hill Encyclopedia of Science and Technology. The following topics are covered: General Nature and Theories of Light, Classification of Phenomena and Theories Involving Light, Electromagnetic Wave Character of Light, Relativistic Effects in the Behavior of Light, and the Corpuscular Nature of Light.

The treatment places particular emphasis on light as a group of electromagnetic radiations. It singles out many of the fundamental properties of light that require further investigation – for the study of light in its own right, and for the information that it gives about the subatomic world, as well as about the universe. The well-known electromagnetic and corpuscular characters of light, which form the subject matter of the classical electromagnetic theory and of quantum mechanics, are recalled, and a detailed description of the relativistic effects in the behavior of light is given. The effects that are describable by the General Theory of Relativity are not incorporated in the present quantum theories dealing with light. A need appears, therefore, for the formulation of a theory of light that would extend beyond the present quantum theories and incorporate all of the known properties of electromagnetic radiations.

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I. INTRODUCTION

1.1 GENERAL NATURE AND THEORIES OF LIGHT

"Light" is the descriptive term that is applied to the group of phenomena in physics called electromagnetic radiations when certain of their common characteristics are discussed.

The radiations known as gamma-rays, X-rays, ultraviolet rays, visible light, infrared rays, microwaves, and radio waves are involved. In a more restricted sense, the term is used only for radiations affecting the eye (1, 2).

The principal common characteristics of electromagnetic radiations (1, 3) are: the propagation of "signals" with a finite velocity, wave motion, and the property of a corpuscular, discrete exchange of energy and momentum between radiations and matter in interactions involving light and matter. The creation of electromagnetic radiation from matter and the creation of matter from radiation, both of which are being experimentally achieved, provide a fascinating insight into the unity of the physical world (11). The same is true for the deflection of light beams by strong gravitational fields such as the directional deviation of starlight passing near the sun (4, 16).

The study of light deals with some of the most fundamental properties of the physical world and is intimately linked with studies of the properties of matter (quantum theory and nuclear physics) and also with studies of the properties of the universe (unified field theory, cosmology).

An entirely satisfactory and comprehensive theory of light still remains to be formulated, but the theories sometimes referred to as "the quantum theory" incorporate a theory of light that provides a fairly satisfactory account of a large number of experimental facts involving light within our definition (3, 14, 12).

The Special Theory of Relativity (4, 16) and the General Theory of Relativity (4, 16) have to be called upon to explain certain phenomena involving light, such as the Doppler effect (21), the value of the velocity of light in a moving medium or in media with differing gravitational potentials, and the gravitational red shift (4, 16, 22).

1.2 CLASSIFICATION OF PHENOMENA AND THEORIES

A classification of phenomena involving light according to their theoretical interpretations provides the clearest insight into the nature of light. When a detailed accounting of experimental phenomena is required, such a classification appears to draw mainly upon two groups of theories which, in most cases, account separately, but not simultaneously, for the "wave character" and the "corpuscular character" of light. The quantum theories (17, 10, 12, 14) seem to obviate questions concerning this dual or complementary character of light and make the classical wave theories appear as one of the two very useful limiting theories of the quantum theories. It happens

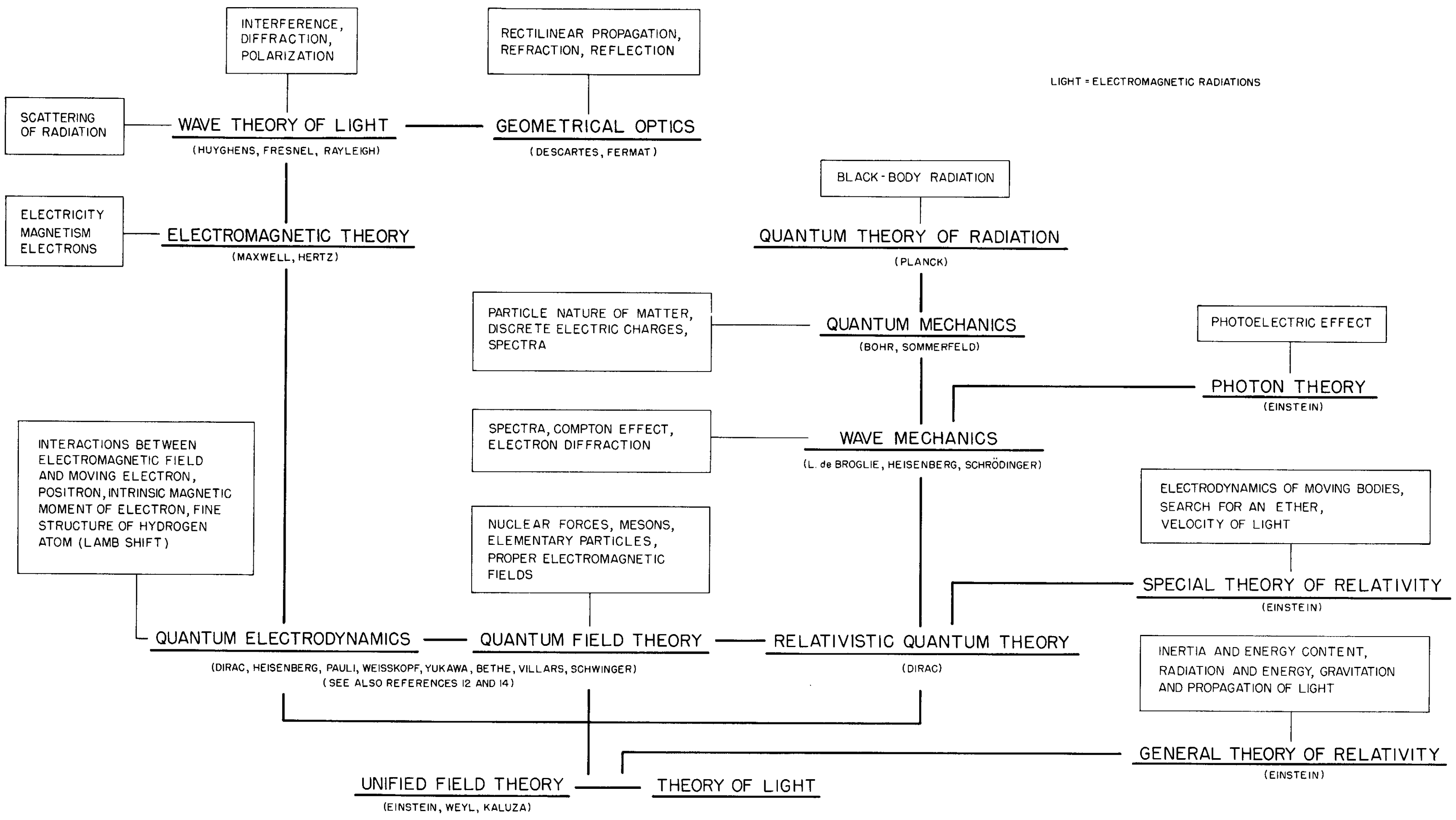


Fig. 1. Theories of light and of its interactions with matter and fields.

that the wave theories can cope with a considerable part of the phenomena involving electromagnetic radiations, somewhat as geometrical optics (33), which is a limiting theory of the wave theory of light, can cope with the solution of many of the more common problems involving the propagation of electromagnetic radiations, such as refraction, provided that the limits of the geometrical optics theory, based in this case on the wave character of light, are not disregarded. Ordinary quantum mechanics (17), dealing with the corpuscular character of light and of matter, forms the other limiting theory.

This apparent hierarchy in the theories of light (Fig. 1) is not a measure of the extent to which solutions to all of the important problems in the various fields dealing with light have been found or facilitated by the development of new theories. Some of the most difficult questions in all fields, for example, the exact nature of the measured velocities of propagation of electromagnetic radiations, the diffraction phenomena at the passage of electromagnetic radiations through apertures that are small compared with the wavelengths (13), and the detailed calculation of the light distribution in images formed by the diffraction of light originating at wave fronts (34) whose shapes deviate in a complicated way from perfect spheres, have not been completely answered.

There are three groups of phenomena involving light: electromagnetic wave phenomena, corpuscular or quantum phenomena, and relativistic effects. Of these, the third group appears to affect the observation of corpuscular and wave phenomena in a similar way.

A distinction has been drawn here between three groups of phenomena and only two theories dealing with light. This is due to the fact that the results of Einstein's special theory of relativity are incorporated in advanced developments of both electromagnetic and quantum theories (11, 12, 14). However, the results of Einstein's general theory of relativity are not incorporated in the quantum theories and will have to be considered in any formulation of a comprehensive theory of light (14).

The corpuscular, or quantum, phenomena are best studied in connection with quantum theory and quantum mechanics of which they form the experimental basis (17), and a detailed treatment of the manifestations of the electromagnetic character of light would include interference, diffraction, and polarization (33, 34, 1, 35).

II. ELECTROMAGNETIC WAVE CHARACTER OF LIGHT

2.1 EXPERIMENTAL EVIDENCE

The phenomena of interference and diffraction, such as the colors of thin oil films and the appearance of diffraction bands that are seen at a monochromatic light source, observed through a narrow slit, are the most striking manifestations of the wave character of light (1, 13, 33, 34). The phenomena of polarization (34, 1), such as the variation of intensity of a light beam transmitted through two parallel polarizers when their relative orientation about the beam is varied, demonstrate the transverse character of electromagnetic vibrations. The fundamental identity of diffraction and interference has been demonstrated in a number of experiments (34). The wave aspect of the entire group of electromagnetic radiations is most convincingly apparent in the similarity of diffraction pictures produced on a photographic plate placed at some distance behind a diffraction grating by radiations of different frequencies, such as X-rays and visible light, when beams of these radiations pass through or are reflected by the grating (10). A convincing proof of the electromagnetic character of visible light is found in the exact similarity between the interference phenomena observed in experiments with waves produced by electromagnetic means, for example, with two radio-wave or microwave antennas fed coherently, and the phenomena involved in experiments with visible light waves originating at an excited atom or molecule and reaching a screen or photoelectric detector after traversing two parallel slits (Young's experiment).

Further proof of the electromagnetic character of light is found in the possibility of inducing in a transparent body that is being traversed by a beam of plane-polarized light the property of rotating the plane of polarization of the beam when the body, otherwise free from this property, is placed in a magnetic field (Faraday effect). Still another proof of the electromagnetic character of light is found in the change of frequency of light (visible, ultraviolet or infrared) emitted from atoms under the action of a strong magnetic field (Zeeman effect) or electric field (Stark effect). Parenthetically, the similar action of gravitational fields on the observed frequency of light, which is of relatively smaller experimental magnitude and therefore more difficult to demonstrate terrestrially, again emphasizes the importance of including all possible fields in a comprehensive treatment of light phenomena. The wavelengths of electromagnetic radiations range from 2×10^{-16} cm for gamma rays to 1×10^{4} cm for radio waves, with corresponding frequencies of from 1.5×10^{26} cps to 3×10^{7} cps.

The fact that the experimentally measured velocity of visible light had been calculated to a good approximation with the aid of exclusively electrical and magnetic parameters (permittivity and permeability) was at the root of Maxwell's conclusion in 1865 that "light (including heat and other radiations if any) is a disturbance in the form of waves propagated ... according to electromagnetic laws" (24). Finally, the fact that electrons, generally considered as elementary particles in physics, can give rise to diffraction pictures quite similar to those produced by X-rays and visible light, and the fact that they lend themselves to interference experiments, have made it necessary to regard particles as having also a wave character (10).

2.2 VELOCITY OF PROPAGATION OF ELECTROMAGNETIC WAVES AND OF SIGNALS TRANSMITTED BY ELECTROMAGNETIC WAVES

a. The Velocity of Light – Measured, Calculated, and Theoretical

The finite velocity of the transmission of signals by means of electromagnetic waves in free space, devoid of matter, is a well-established fact for visible light and microwaves, as well as for radio waves and gamma rays, in a variety of terrestrial and astronomical experiments (19, 20). Another quantity known as the free-space velocity of propagation of electromagnetic waves, usually called c, is determined by calculation from more or less direct terrestrial measurements of the frequencies and free-space wavelengths of the radiations involved in a particular experiment, and from measurements of the ratio of electromagnetic to electrostatic units. The measured magnitudes of the velocity of transmission and of the phase velocity seem to be in reasonably good agreement, possibly to a few parts in 10^5, and the most recent value given for the velocity of light is $c = 299,792.5 \pm 0.1$ km/sec. However, there is an admitted, and disturbing, variance in the values given by the experimenters, and diversely weighted reviews of past measurements have been given by the same experimenters and by others. The velocity of light c is <u>not</u> as invariable or as steady a constant as it is sometimes thought to be. While it is true, according to the theories of relativity, that the velocity of light will be measured by the same magnitude c in any frame of reference, independently of its state of motion, it should be equally true, according to the same theories, that the measured magnitudes will be equal to c in all frames only if these frames have the same gravitational potential Φ, where $\Phi = -GM/R$, with G, the universal constant of gravitation; M, the mass of the body; and R, the radius of the body (4, 16). According to Einstein

$$c = c_o(1 + \Phi/c^2) \qquad (1)$$

Conceivably, local secular variations of the gravitational potential Φ could lead to variations of the measured velocity of light, although experiments for measuring these variations, if there are any, remain to be devised. As the theory now stands, the Φ/c^2 part is approximately 7×10^{-10} on earth, but 3×10^3 times as great on the sun (2×10^{-6}), and considerably greater elsewhere in the universe. The differences are far from small, as compared with the precision of many experiments for the measurement of c.

b. Types of Measurement and of Determination of c

Measurements of c fall basically into two groups: the group-velocity (or signal-velocity) measurements of the form c = length/time, and the phase-velocity determinations of the form c = frequency/wavelength.

Group-velocity measurements are: (a) attempts to measure the average time required for a light signal, that is, a modulated electromagnetic wave train, to traverse a given distance in one direction (example: Rømer's astronomical measurement (37), in 1676, of the time required for a signal produced by the eclipse of Jupiter's first satellite to traverse the distance between two positions of the earth in its orbit); (b) terrestrial attempts, with the light source and the observer in the same frame of reference, to measure the time required for a light signal to traverse a given distance between a source and one or more reflecting mirrors and back again (examples: Fizeau's toothed wheel, Foucault's and Michelson's revolving mirrors, Bergstrand's electronically modulated light beam and photoelectric receiver). In reality, these measurements only give c if the experiments are carried out in a nondispersive medium (vacuum); otherwise, the group velocity is measured. The group velocity is smaller than the phase velocity by 0.7×10^{-5} in air, by 1.5×10^{-2} in water, and by 2.4×10^{-2} in ordinary crown glass (values for the middle of the visible light range).

Phase-velocity determinations are indirect and incorporate the assumption that $c = f\lambda$, where f is the frequency, and λ is the free-space wavelength of an electromagnetic radiation. Most, but not all, of the measurements of this kind involve microwave interference in various forms (20): Floorman with two sources; Frome with an apparatus similar to a Michelson interferometer; Essen and Gordon-Smith with a microwave resonant cavity. Rank and others, and Plyer and others, determine c by calculation from microwave and infrared spectroscopic measurements of the ratio of the same molecular constant

$$\left(\frac{B_o \text{ cycles/sec}}{B_o \text{ waves/cm}} = \frac{\text{cm}}{\text{sec}} = c \right)$$

A detailed description of experiments will be found in the bibliography, particularly in Bergstrand (19) and Dupeyrat (20). The velocity of light in moving media will be discussed in Section III.

c. Various Velocities of Propagation Involving Electromagnetic Radiations

Electromagnetic waves can be propagated not only through free space, devoid of matter and fields, and with a constant gravitational potential, but also through space devoid of matter with varying gravitational potentials, as well as through more or less absorbing, homogeneous, nonconducting, dielectric material media (solids, liquids, gases) that can be either isotropic or birefringent. Electromagnetic waves can also be propagated through waveguides with cylindrical, elliptical, rectangular or other conducting boundaries, the insides of which can be either "free-space" (vacuum) or a dielectric medium.

A great variety of parameters for describing the medium and its boundaries is required for an adequate description of the various velocities of propagation involving electromagnetic waves that are relevant to the particular experiment. A great variety of velocities is involved and each one has a definite meaning, for example, group velocity and phase velocity in the medium; group velocity and phase velocity in the waveguide; "velocity" of electromagnetic waves (defined as $1/(\mu_o\epsilon_o)^{1/2}$, where ϵ_o is the permittivity, and μ_o the permeability of free space); free-space group velocity; free-space signal velocity; and free-space phase velocity. The velocity of propagation of electromagnetic energy is also included.

Not all are measurable quantities, but all are involved in one way or another in determinations of the velocity of light in the two kinds of measurement described in section 2.2b. When velocity-of-light values have been obtained for a considerable frequency range, they should permit further verification of the frequency independence of the velocity-of-light value deduced from electromagnetic theory and equal to $c = 1/(\mu_o\epsilon_o)^{1/2}$. It is not inconceivable that unexpected facts might be brought to light in measurements that are designed to involve other frequencies, fields, and frames of reference than the ones of past experiments.

d. Mathematical Description of the Propagation of Electromagnetic Wave Trains

From electromagnetic theory, a plane-wave disturbance of a single frequency f, ($\omega = 2\pi f$), propagated in the direction x with a phase velocity $V = \lambda f = \lambda\omega/2\pi$, where λ is the free-space wavelength, can be described by

$$Y = A \cos \omega\left(t - \frac{x}{V}\right) \tag{2}$$

Two disturbances of the same amplitude A, of angular frequencies ω_1 and ω_2, and velocities V_1 and V_2, propagated in the same direction, yield the resulting disturbance Y':

$$Y' = Y_1 + Y_2 = 2A \cos \frac{1}{2}\left[(\Delta\omega)\, t - \Delta\left(\frac{\omega}{V}\right) x\right] \cos\left(\omega t - \frac{\omega x}{V}\right) \tag{3}$$

where $\Delta\omega = \omega_1 - \omega_2$, and $\omega = 1/2(\omega_1 + \omega_2)$. The ratio $U = \Delta\omega/\Delta(\omega/V)$ is defined as the group velocity, just as the ratio $\omega/(\omega/V)$ is defined as the phase velocity. In the limit for small $\Delta\omega$, we have

$$U = \frac{d\omega}{d}\left(\frac{\omega}{V}\right) \tag{4}$$

and noting that $\omega = 2\pi V/\lambda$, $d\omega = 2\pi(\lambda dV - V d\lambda)/\lambda^2$, and $d(\omega/V) = -2\pi d\lambda/\lambda^2$, we obtain

$$U = V - \lambda \frac{dV}{d\lambda} \tag{5}$$

which shows that the group velocity U is different from the phase velocity V in a medium with dispersion $dV/d\lambda = -c/n^2\, dn/d\lambda$, where n is the refractive index of the medium. In vacuo $V = c = U$. With the help of Fourier theorems, expression 5 can be shown to apply to the propagation of a wave group of infinite length, but with frequencies extending over a finite small domain. Furthermore, even if a wave train starts by being emitted with an infinite length in a perfectly monochromatic form (which is never the case), modulation or chopping results in an extension of the frequency range that is required in its representation, and hence in the appearance of a group velocity (18). Considerations of this nature are not trivial nor are they of the nature of refinements in any velocity-of-light measurement; they are quite fundamental in the transition from the instrument readings in the experiments to the expression of a value for c. Similar considerations apply to the adequate incorporation of the parameters describing the effects of the medium and the boundaries involved in the experiments. For the rather involved situation that arises in the regions of anomalous dispersion (absorption regions) where the phase velocity can exceed c and $dV/d\lambda > 0$, consult Ditchburn (1) and Sommerfeld (13).

e. Phase Velocity and Refractive Index

A plane-wave front (surface of constant electromagnetic phase) incident from a medium in which its phase velocity is V onto the surface of a dielectric medium forming an angle i with the surface, is refracted to form an angle i' with the surface in the dielectric medium in which its velocity becomes V'. Geometry then shows that

$$\frac{\sin i}{V} = \frac{\sin i'}{V'} \qquad (6)$$

Since Snell's law gives $n \sin i = n' \sin i'$, we obtain

$$\frac{V}{V'} = \frac{n'}{n} \qquad (7)$$

and, in particular, if $n = 1$ and $V = c$, then $n' = c/V'$. Thus the refractive index of a medium (n) is simply the ratio of the phase velocity of light in vacuo (c) to the phase velocity of light in the medium (V').

f. Dispersion of Free Space

The weight of experimental evidence, based on astronomical observations, is against a dispersion in vacuo. Observation of the modulation of light that reaches the earth from the eclipsing binary Algol, 120 light years distant, shows that the light intensity is modulated simultaneously for all colors. The eclipse occurs every 68 hours and 49 minutes. Were there a difference in velocity for red light and for blue light in interstellar space as great as one part in a million, this star would show a measurable time difference in the occurrence of the eclipses if they were observed in the two colors (7).

g. Independence of the Velocity of Light from the Velocity of Source

Astronomical observations of eclipsing binaries, both visual and spectroscopic, indicate that the velocity of light does not depend upon the velocity of the source (7).

2.3 MAXWELL'S DYNAMICAL THEORY OF THE ELECTROMAGNETIC FIELD

Light is an electromagnetic radiation propagated with a finite velocity in free space, devoid of matter, and through certain media. The energy carried by the electromagnetic field can exert pressure on material media, can interact with matter and with other electromagnetic radiations, and, under certain circumstances, can cause the emission of electromagnetic radiation from matter or, indeed, create matter by interaction with other radiation. The theoretical description of the structure of the electromagnetic field and of its propagation forms the subject of Maxwell's theory (24), while the need for expressing the quantum or photon nature of electromagnetic radiation that appears in interaction processes with matter and other radiations has resulted in the Unified Theories, or Quantum Theories, of the electromagnetic radiation field (11, 12, 14).

Maxwell's equations incorporate in compact and symmetrical form the elementary laws of electromagnetic induction (electric field caused by the moving magnet $\nabla \times E = \partial \bar{B}/\partial t$) and of the magnetic field caused by the flow of current ($\nabla \times \bar{H} = \bar{i}$), which had been established for fields of constant magnitude or of slow variation. Maxwell postulated the existence, in nonconducting space, of a displacement current (equal to $-\partial \bar{D}/\partial t$ and caused by the variation of the electric field), in order to arrive at a description of the structure of the electromagnetic field for free space, devoid of matter, which is expressed in equations that are equally applicable to electromagnetic radiations of all frequencies:

$$\nabla \times E = \frac{\partial B}{\partial t} \qquad \nabla \times H = -\frac{\partial D}{\partial t}$$

$$\nabla \cdot \bar{E} = 0 \qquad \nabla \cdot \bar{H} = 0$$

where $\bar{B} = \mu_o \bar{H}$.

A mathematical consequence of Maxwell's equations is seen in the equations

$$\nabla^2 \bar{E} = \mu_o \epsilon_o \frac{\partial^2 \bar{E}}{\partial t^2} \tag{8a}$$

and

$$\nabla^2 \bar{H} = \mu_o \epsilon_o \frac{\partial^2 \bar{H}}{\partial t^2} \tag{8b}$$

which express the fact (6) that a varying electromagnetic field will propagate in vacuo in the form of transverse electromagnetic waves with a finite velocity $c = 1/(\mu_o \epsilon_o)^{1/2}$.

The value of c calculated from electrical and magnetic parameters happened to be so nearly equal to the velocity of visible light, as it was conceived in 1865, that Maxwell found in this fact a proof for his hypothesis (previously expressed by Faraday, according to Maxwell) that visible light is an electromagnetic radiation, the properties of which are described by his equations. The experimentally determined laws of refraction and reflection, as well as the behavior of anisotropic crystalline media, fully bear out Maxwell's theory. Another fact for which the electromagnetic theory accounts is that a beam of electromagnetic radiation not only carries energy but also momentum, and that an electromagnetic wave impinging on the surface of a material medium exerts a force in the direction of the propagation of the wave (6, 24). The amount of energy flowing through a unit area per unit time is given by the Poynting vector $\overline{P} = \overline{E} \times \overline{H}$ in rationalized MKS units, and the pressure exerted by a plane wave impinging normally on a surface is given by $\overline{P}/c$. This pressure of light, which is of the order of 10^{-5} to 10^{-6} dynes/cm^2, can be experimentally demonstrated and can also be accounted for with the aid of the quantum theories. The extension of Maxwell's theory to include interactions with the electron was first carried out by H. A. Lorentz and finally led to quantum electrodynamics (11).

2.4 QUANTUM THEORIES OF ELECTROMAGNETIC RADIATION AND UNIFIED THEORIES INCORPORATING INTERACTION PROCESSES WITH MATTER

The need for reconciling Maxwell's theory of the electromagnetic field, which describes the electromagnetic wave character of light, with the quantum nature of photons, which demonstrates the equally important corpuscular character of light, has resulted in the formulation of several theories that go a long way toward giving a satisfactory unified treatment of the wave and the corpuscular characters of light (11, 12, 14, 1). These theories incorporate the theory of Quantum Electrodynamics, which was first set forth by Dirac, Jordan, Heisenberg and Pauli (12), and the ordinary theory of Quantum Mechanics of L. de Broglie, Heisenberg, and Schrödinger (17). They furnish a consistent account of the wave character of light, satisfying the requirements of the special theory of relativity, as well as of the greater part of the interaction processes between radiation and matter, such as refraction, dispersion, absorption, diffusion, scattering, the photoelectric effects, the Compton effect, spectroscopic emission and absorption processes, and so on (10, 17). Unresolved quantum theoretical difficulties persist, for example, in the higher than first approximations of the interactions between light and elementary particles (14). The incorporation of a theory of the nucleus into a theory of light is bound to call for additional formulation.

Dirac's synthesis of the wave and corpuscular theories of light is based on rewriting Maxwell's (wave) equations in Hamiltonian form that resembles the Hamiltonian equations of the classical mechanics for material systems (3, 11, 12, 14). Using the same formalism involved in the transformation of classical into wave-mechanical equations by the introduction of the quantum of action h, Dirac obtains a new equation of the electromagnetic field. The solutions of this equation are quantized waves, with their energy and momentum as integral multiples of $h\nu$, with a quantized angular momentum, and correspond to the photons. The superposition of these solutions represents the electromagnetic field. The quantized waves are subject to Heisenberg's uncertainty relation. The quantized description of radiation cannot be taken literally in terms of either photons or waves, but rather as a description of the probabilities or of the possibility of occurrence in a given region of the interactions to which its description applies, as well as of the magnitudes of the interactions, if they happen to occur. Otherwise it becomes very difficult to answer the question, How does a "photon," emitted as a "spherical wave" from a stellar source, hundreds of light years away and "extending" its wave front over an appreciable portion of the universe, "collapse" its entire, "indivisible" energy with an infinite speed, if an observer at some point on the earth's surface decides to offer it the possibility of being detected, and therefore requires for this detection process precisely the entire, indivisible energy of the photon? The description of the Michelson interferometer also requires the admission that before being detected a photon can be present with its "indivisible" energy spread out within the entire pertinent volume of the interferometer, on both sides of the divider plate, possibly as a system of standing waves.

In the emission of light by an atom, it is the total angular momentum of atom plus radiation that is conserved (10). It is interesting to point out, in conclusion, that the limiting classical theory for the quantum theory of radiation when $h \rightarrow 0$ is an electromagnetic wave theory, while the classical limiting theory for a beam of electrons described by wave mechanics is essentially a particle theory.

III. RELATIVISTIC EFFECTS IN THE BEHAVIOR OF LIGHT

The measured magnitudes of such characteristics as the wavelength and frequency, and the velocity and the direction of radiation in a light beam, are affected by a relative motion between the frames of the source and of the observer which occurs during the emission of the signal-carrying electromagnetic wave trains, as well as by a difference in gravitational potential between the two frames (4). Several important effects of this nature are listed in section 3.1, and the simple explanation provided by the special theory of relativity, as well as explanations according to the general theory of relativity, in later sections.

3.1 BASIC EXPERIMENTAL EVIDENCE

a. Velocity of Light in a Moving Medium

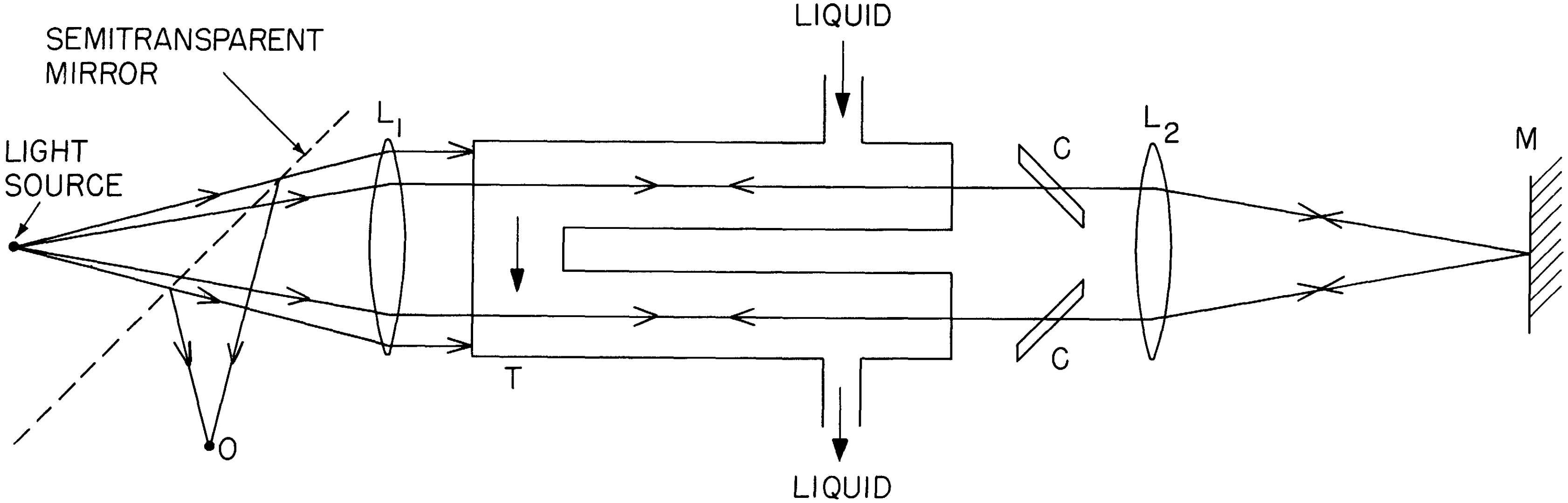
In 1818, Fresnel suggested that it ought to be possible to determine the velocity of light in a moving medium, for instance, the velocity of a beam of light traversing a column of liquid of length d and of refractive index n, flowing with a velocity v relative to the observer, by measuring the optical thickness nd. This experiment was carried out by Fizeau, in 1851, with the use of a modified Rayleigh interferometer, shown in Fig. 2, by measuring the fringe displacement in O that corresponds to the reversing of the direction of flow (1). If V' is the phase velocity of light in the medium deduced from the refractive index by the relation $V' = c/n$, we find that the measured velocity V in the moving medium can be expressed as $V = V' + v(1 - 1/n^2)$, rather than as $V = V' + v$ which might have been expected on the basis of a Newtonian velocity addition.

b. Aberration in the Apparent Location of Stars

Bradley (38) discovered, in 1725 and in 1729 communicated to the Royal Society, a yearly variation in the angular position of stars, the amplitude of the variation being 20".5 (total excursion 41"). This effect is distinct from the well-known parallax effect and was correctly ascribed to the combination of the proper motion of the earth in its orbit and the speed of light (7). Bradley used the amplitude of the variation and arrived at a value for the velocity of light. Airy and Hoek, in 1871, compared the angle of aberration in a telescope before and after filling the tube with water and found, contrary to their expectation, that there was no difference in angle.

c. Michelson-Morley Experiment (1887)

The famous Michelson-Morley experiment (27) was designed to measure the relative velocity of the earth through inertial space, the space in which Newton's laws of motion hold. Dynamically, an inertial frame is a frame in which the observed accelerations are zero if no forces act. A point on an orbit is the center of such a frame.



C = COMPENSATOR PLATES
M = MIRROR
L_1, L_2 = LENSES
T = TUBE
O = LOCATION OF INTERFERENCE FRINGES

Fig. 2. Measurement of the velocity of light in a moving medium.

The rotation of the earth on its axis, with tangential velocities never exceeding 0.5 km/sec, is easily demonstrated mechanically (Foucault's pendulum, precession of gyroscopes), and optically (Michelson's rectangular interferometer with two circuits): these experiments prove that the surface of the earth is not an inertial frame. In its orbit around the sun, on the other hand, the earth has translational velocities of the order of 30 km/sec, but this motion cannot be detected by mechanical experiments because of its orbital nature. The expectation existed, however, that optical experiments would permit the detection — and measurement — of the relative motion of the earth through inertial space, by comparing the times of travel of two light beams, one traveling in the direction of the translation through inertial space, and the other at right angles to it (27, 28, 29, 30).

This expectation was based on the hypothesis, which has now been disproved, that the velocity of a light beam is equal to a constant c only when it is measured with respect to the privileged inertial space. If a light beam were projected in the direction of and in the opposite direction to the translation of the earth, which moves with a velocity v in inertial space, then, according to this hypothesis, the velocity of light would be measured as (c−v) and (c+v), respectively. According to this classical velocity-addition theorem — which does not apply to light — a velocity difference of 2v would be detected under such circumstances. Not only does the earth move in an orbit around the sun, but it is carried with the sun in the galactic rotation toward the constellation Cygnus with a velocity of a few hundred kilometers per second, and the galaxy itself is moving with a great speed in its local spiral group. Speeds of hundreds, and possibly thousands, of kilometers per second should be detectable by measurements on the earth in two orthogonal directions (5), if we assume that the earth's motion is with respect to inertial space, or, indeed, that such a space has the physical meaning ascribed to it. The only unexpected result was that of the experiment: no relative motion could be detected by optical means.

This result is in agreement with Einstein's theories of relativity, according to which the velocity of light will be measured by the same magnitude c independently of the state of motion of the framework in which the measurement is being carried out (4, 16).

Briefly, the Michelson-Morley apparatus consists of a horizontal Michelson interferometer with its two arms at right angles. The mirrors are adjusted so that the central white-light fringe falls on the cross hair of the observing telescope. This indicates equality of optical phase, and therefore equality of the times taken by the light beams in traveling from the beam-splitting surface to first one and then to the other of the two mirrors and back again. Rotation of the entire system 90°, or indeed through any angle, as well as repetition of the experiment at various times of year, have all been found to leave the central white-light fringe and associated fringe system undisplaced, which indicates that there is no change in the time required for light to traverse the two arms of the interferometer when their directions relative to the

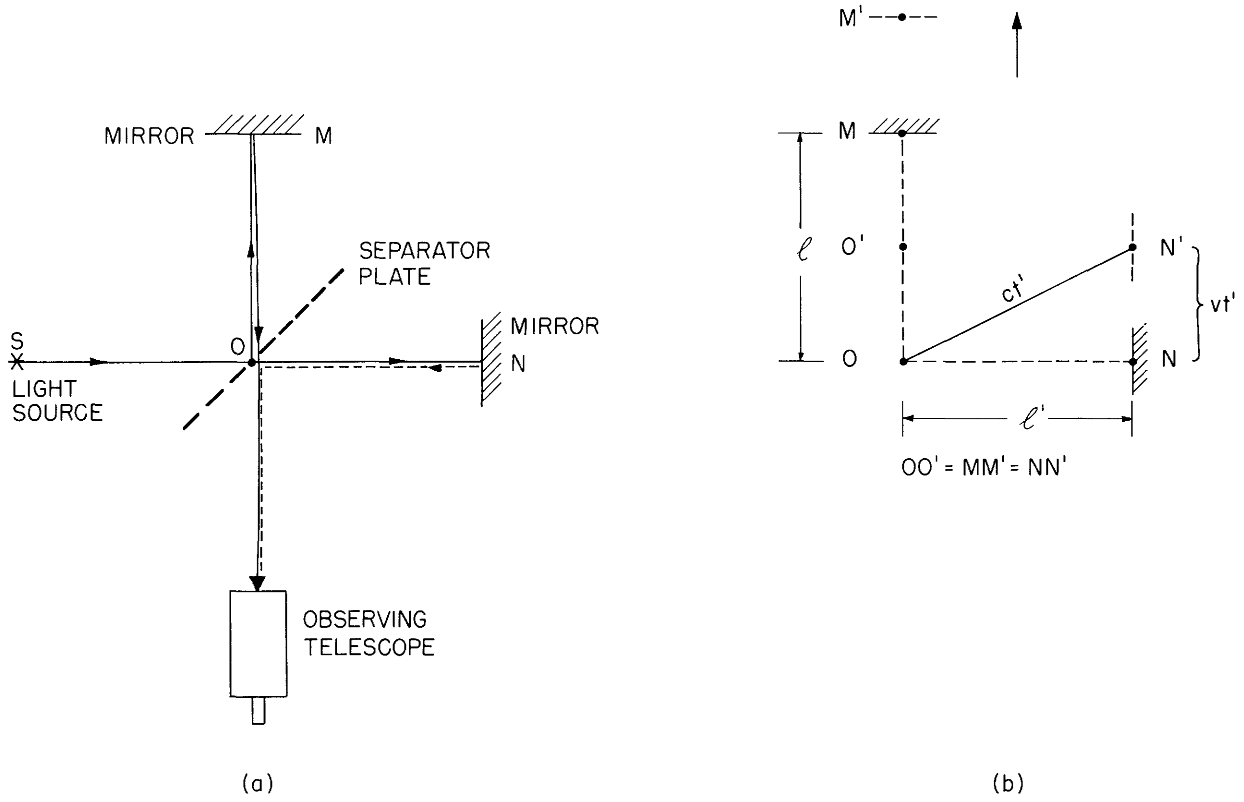


Fig. 3. Michelson-Morley experiment.

direction of the earth's motion are varied.

Had there been a difference in the velocity of light in the two directions OM and ON (Fig. 3), the two arms would be of unequal length in the initial adjustment. For example, if the light travels faster in the direction OM (on the average, going back and forth), then the corresponding arm would have to be longer to make the time of travel in the longer arm equal to the time of travel in the shorter arm, in which light travels slower. By turning the apparatus through 90°, the shorter arm takes the place of the longer arm, and the "faster" light now travels in the shorter arm, and the "slower" light in the longer arm; as a result, a noticeable fringe displacement should, but does not, take place.

The evaluation of the time intervals involved, according to classical mechanics, is simple, but the hypotheses involved are, of course, incorrect (5).

(i) If the earth were still in inertial space, the traversal back and forth along OM of length ℓ would take time

$$t = \frac{2\ell}{c} \tag{9}$$

(ii) If the earth moves with a velocity v in the direction OM, the light would reach the receding mirror in time $t_1 = \ell/(c-v)$, and would return to O in time $t_2 = \ell/(c+v)$.

The total traverse from O to M and back again would require time

$$T = t_1 + t_2 = \frac{\ell}{c - v} + \frac{\ell}{c + v} = \frac{2\ell}{c}\left[\frac{1}{1 - \frac{v^2}{c^2}}\right] \tag{10}$$

which depends on v and is greater than t.

(iii) On the basis of the same hypothesis, the mirror N moves a distance vt' to N' during the time that the beam from O tries to reach it. If ON = ℓ', geometry gives

$$c^2 t'^2 = \ell'^2 + v^2 t'^2 \tag{11}$$

or

$$t' = \left[\frac{\ell'}{c}\right]\left[\frac{1}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}\right] \tag{12}$$

The time for return is (for present purposes) the same, and the total time of traverse from O to N and back again is

$$T' = \frac{2\ell'}{c}\left[\frac{1}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}\right] \tag{13}$$

which is different from both t and T. The initial white-light fringe adjustment makes T = T' by an appropriate adjustment of ℓ and ℓ', and, in the example chosen, ℓ' should be greater than ℓ. The rotation through 90° should then destroy the equality of T and T'. This classical hypothesis of the addition of velocities c and v has been proved wrong by the experiment. Einstein's theory accounts for the null result by the simple remark that no relative motion existed in the experiment between the apparatus and the observer. No change in measured length has occurred in either direction, and since the propagation of light is isotropic, no velocity or detection of relative motion would be expected.

d. Gravitational and Nebular Red Shifts

Two different kinds of shift, or displacement, of spectral lines toward the red end of the spectrum are observed in spectrograms of starlight or of light from nebulae (7, 22, 1). One is the rare, but extremely significant, gravitational red shift observed in some spectra from white dwarfs, and the other, much more widely encountered, is the red shift in the spectra of nebulae, which is usually described as a radial Doppler effect characteristic of an expanding universe.

The most famous example of the gravitational red shift is that observed in the spectrum of the dark companion of Sirius (22). The existence of this companion star was predicted by Bessel, in 1844, on the basis of a perturbation in the motion of Sirius;

the prediction was not verified observationally until 1861 because of its weak luminosity (1/360 of our sun, 1/11,000 of Sirius). This companion is a white dwarf (8000° K), with mass comparable to that of the sun (approximately 0.95), with a very small astronomically inferred radius of 18,000 km, and a fantastic mass density of 61,000 times that of water. The notable fact is that the companion shows a shift in its spectral lines relative to the lines emitted by Sirius itself. This shift of 0.3 angstrom unit was reliably determined by W. S. Adams, of the Mount Wilson Observatory, for the H_β line.

The nebular red shift, a systematic shift observed in the spectra of all nebulae, is best measured with the calcium H and K absorption lines. Distances of nebulae are determined photometrically by measurements of their intensities, and it is found that the wavelength shift toward the red increases with the distance of a nebula from the earth. (The earth does not have a privileged position if "expansion of the universe" is indeed involved, and its position is somewhat like that of a man in a crowd that is dispersing: every individual in the crowd will find himself at an ever increasing distance from his neighbors.) The change of wavelength with distance has been given by Hubble as

$$\frac{\Delta\lambda}{\lambda} = 1.7 \times 10^{-9}\ d \tag{14}$$

where d is in parsecs (1 parsec = 3×10^{18} cm). Red shifts in nebulae that are as far distant as 1.1 billion light years have been measured in part of a vast program carried out at the Mount Wilson Observatory by Hubble and Humason (1). (A light year is the distance traversed by light, traveling at the rate of 3×10^{10} cm/sec, in 1 year.)

e. Other Spectroscopic Doppler Shifts and Broadenings

Spectral lines, in emission or absorption, show a finite width which indicates a finite frequency or wavelength distribution. This width is caused by various influences affecting the emission of absorption of light, such as gas pressure and velocity of atoms, ions or molecules. The part that is caused by the velocity is called "Doppler broadening" and is quite large, as a rule, except in atomic-beam sources when the light beams are made to emerge transversely to the direction of motion of the atoms. Another example is afforded by the rotation of the sun, with equatorial line-of-sight velocities of the order of 2 km/sec, which leads to detectable shifts toward the blue at the receding limb of the sun, and toward the red at the approaching limb.

3.2 RESULTS OF THE SPECIAL THEORY OF RELATIVITY (1905)

The theory of special relativity (4, 5, 16) deals with the transformation properties of the laws of mechanics, electrodynamics, and optics when their mathematical formulation is referred to space and time coordinates in different frames of reference that are in uniform motion with respect to each other. In particular, it deals with the manner in which the magnitudes of a given length, time, and velocity, measured in a given frame of reference, are related to the magnitudes of these same quantities measured in another frame of reference that is in uniform motion with respect to the first frame. The space, time, and velocity transformation equations (Lorentz transformations) relating the measurements of these quantities are arrived at on the basis of two postulates:

(i) The Principle of Equivalence postulate, according to which the laws of physics are independent of the motion of the coordinate system to which they are referred.

(ii) The Velocity of Light postulate, according to which the velocity of light will be measured by the same magnitude c independent of the motion of the coordinate system in which the measurements are being performed.

According to the general theory of relativity, the motion can be accelerated, but, according to the same theory, the measured magnitude of the velocity of light is not independent of the gravitational potential at the place of measurement.

The transformation equations lead to the following conclusions describing the results of measurements performed in any one of two coordinate systems S and S' in relative motion with respect to each other:

(i) Relativity of simultaneity. Two events that occur simultaneously at different places in a system S do not occur simultaneously according to measurements performed in a system S', and vice versa.

(ii) Contraction of space. An observer in either of the two systems will measure the other observer's yardstick as shorter than an identical yardstick in his own system.

(iii) Dilation of time intervals. An observer in any one of the two systems will measure the other observer's clock as running more slowly than an identical clock in his own system. That is to say, for a given time interval in his own system, the observer finds the corresponding time interval in the other system to be greater.

It is essential to note that Einstein's fundamental argument (5) is that it is proper to speak only of "measured magnitudes" in physics. What are the "true" values of length, or of a time interval? They are the values measured by an observer at rest with respect to a phenomenon that is being measured (length of an object, time interval or period of a clock).

The mathematical formulation of these results is given in the following equations. It is important to note that v is the velocity of the system S' with respect to the system S. (The velocity of the system S with respect to the system S' is -v.) The relative velocity is assumed to be along the common x-direction of the two systems.

For the lengths

$$x' = \frac{x - vt}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}} \qquad \text{(measured in S)} \qquad (15)$$

from which

$$x'_2 - x'_1 = \left(1 - \frac{v^2}{c^2}\right)^{1/2} (x_2 - x_1) \qquad \text{(measured in S)} \qquad (16)$$

For time

$$t' = \frac{t - \left(\frac{v}{c^2}\right) x}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}} \qquad \text{(measured in S)} \qquad (17)$$

from which

$$\Delta t' = t'_2 - t'_1 = \frac{t_2 - t_1}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}} = \frac{\Delta t}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}} \qquad \text{(measured in S)} \qquad (18)$$

The velocity-addition equation is

$$U = \frac{U' + v}{1 + \frac{vU'}{c^2}} \qquad \text{(measured in S)} \qquad (19)$$

where U is the velocity measured by the observer in S, and U' is the velocity measured by the observer in S'.

3.3 RESULTS OF THE GENERAL THEORY OF RELATIVITY (1916)

The propagation of light is influenced by gravitation. This is one of the fundamental results of Einstein's general theory of relativity which has been put to experimental test and found to be valid (16).

Three important results involving light need to be singled out (4).

(i) The velocity of light, measured by the same magnitude c independently of the state of motion of the frame in which the measurement is being carried out, should depend on the gravitational potential Φ of the field in which it is being measured, according to the equation

$$c = c_o\left(1 + \frac{\Phi}{c^2}\right)$$

where $\Phi = -GM/R$ [G is the universal constant of gravitation (6.670×10^{-8} cgs units), M, the mass of the heavenly body (grams), and R, the radius of the body (cm)].

For example, the term Φ/c^2 is approximately 3000 times greater on the sun than on earth, so that the measurements of c are smaller by 2 parts in a million on the sun, as compared with measurements on earth.

(ii) The frequency of light emitted from a source in a gravitational field with the gravitational potential Φ is different from the frequency emitted by an identical source (atomic, nuclear, molecular, and so forth) in a field-free region, according to the equation

$$\nu = \nu_o\left(1 + \frac{\Phi}{c^2}\right) \qquad (20)$$

Spectral lines in sunlight should be displaced toward the red by a factor of 2×10^{-6}, as compared with light from terrestrial sources.

(iii) Light beams are deflected when they are passing near a heavenly body, according to the equation

$$\alpha = \frac{4GM}{c^2R} \qquad (21)$$

where α is the angular deflection (in radians), and R the distance of the beam from the center of the heavenly body of mass M. The deflection is so directed that the apparent angular distance of a star from the center of the sun is increased when starlight is passing near the sun. The deflection according to this equation should be 1.75 seconds of arc; eclipse measurements of the star field around the sun indicate values up to 2.2 seconds, as compared with photographs of the same field taken at night six months earlier [see Freundlich, von Klüber, and von Brunn (36)]. This sensational prediction of Einstein's theory might seem less surprising today when the corpuscular photon character of light is widely known, and when a Newtonian M/R^2 attraction might be considered as being involved in the motion of a corpuscle moving past the sun with a velocity c. However, application of Newton's law predicts a deviation only half as great as the reasonably well verified relativistic prediction.

According to certain views, the most reliable test of the general theory of relativity is found in the correct prediction (42.9 seconds of arc per century) of the observed advance of the perihelion of Mercury (43 seconds of arc per century).

3.4 EXPLANATION OF RELATIVISTIC EFFECTS IN THE BEHAVIOR OF LIGHT

a. Velocity of Light in a Moving Medium

No special assumptions are required to explain this effect. Simple application of the velocity-addition equation, with V and V' written in place of U and U', and with $V'/c = 1/n$, gives

$$V = \frac{V' + v}{1 + \left(\frac{vV'}{c^2}\right)} = \frac{V' + n}{1 + \frac{v}{nc}}$$

When v is small compared with nc

$$V \approx (V' + v)\left(1 - \frac{v}{nc}\right) = V' + v\left(1 - \frac{1}{n^2} + \frac{v}{nc}\right)$$

and, again neglecting v/nc, we obtain

$$V = V' + v\left(1 - \frac{1}{n^2}\right) \qquad (22)$$

which is identical with the experimentally determined value.

b. Acoustic Doppler Effect

In acoustical observations of a moving source emitting sound at a constant frequency its pitch appears higher when the source is approaching the listener, and lower when the source is receding from the listener.

This effect results from the basic fact that the listener perceives as frequency the value obtained by dividing the speed with which the sound waves arrive by the wavelength of these waves.

The acoustic Doppler effect deals with cases of relative motion between the listener and the source, and includes the effects of the motion of the medium itself relative to both the source and the listener (23). The wave velocity u of the sound in the medium is a property of the medium, and its value is referred to that medium. The wavelength λ, frequency f, and velocity v are generally related in wave propagation by the well-known equation $v = f\lambda$, in which values appropriate to the experiment that is being made need to be used.

A distinction must be drawn between the case of a source moving relative to the listener fixed in the medium, and the case of a listener moving with respect to the source fixed in the medium.

In the first case, if the source moves towards the fixed observer with a velocity v_s, waves emitted with a frequency f_s appear to have their wavelength shortened in the ratio $(u - v_s)/u$, because of a "crowding" of the waves which, however, still reach the listener with a velocity u.

In the second case, if the listener moves toward a fixed source, the waves appear to arrive with a velocity $(u + v_L)$. The wavelength of the sound in the medium is unchanged and is equal to that measured in the previous case.

If we now include the effect of the velocity of the medium relative to the listener and the source, and if v_M is the component of this velocity taken positive in the direction from the listener to the source, and if v_L and v_s are the velocity components along the line joining the listener to the source also taken positive in the direction from the listener to the source, then the general equation relating the observed frequency f_L and the source frequency f_s is

$$\frac{f_L}{u + v_L - v_M} = \frac{f_s}{u + v_s - v_M} \qquad (23)$$

c. Optical Doppler Effect

An effect, which at first sight seems analogous to the acoustic Doppler effect, appears when electromagnetic waves are involved. But the causes, detailed effects, and explanations are fundamentally different and result from the relativistic behavior of light.

Three fundamental differences exist between the acoustic and optical Doppler effects:

(i) The optical frequency change does not depend on whether the source or the observer is moving with respect to the other, but the acoustic frequency change is different in the two cases.

(ii) No effect is observable in the acoustical case when the source, or the observer, moves at right angles to the line connecting the source and the observer. There is an observable optical frequency change under such conditions.

(iii) The motion of the medium through which the waves are propagated does not affect the observed optical frequencies, but it does affect the acoustic frequencies.

d. Apparent Frequency and Apparent Direction of Light from the Source in Relative Motion with Respect to the Observer

The mathematical expressions of the observable effects involving electromagnetic waves are arrived at by noting that the propagation of a given plane wave must be described by the same law in the source frame and in the observer frame according to the relativistic principle of equivalence (4). Accordingly, the equation of propagation of the plane wave written for the source frame of coordinates is transformed to the observer frame of coordinates with the aid of the Lorentz transformations, and the relevant factors on the two sides of the resulting equation, which identify the descriptions in the two frames, are identified. The result is expressed by two equations

$$f_o = \frac{f_s\left(1 - \frac{v^2}{c^2}\right)^{1/2}}{1 - \frac{v}{c}\cos\theta_o} \tag{24}$$

and

$$\cos\theta_o = \frac{\cos\theta_s + \frac{v}{c}}{1 + \frac{v}{c}\cos\theta_s} \tag{25}$$

which relate the frequency f_o and angle θ_o measured in the observer frame to the frequency f_s and angle θ_s that would be measured in the source frame, under the conditions in which the source frame is measured (in the observer frame) as moving with a velocity v relative to the observer frame.

Examination of the frequency relation shows that it incorporates two important factors:

(i) A direction-independent factor

$$f_o \sim f_s\left(1 - \frac{v^2}{c^2}\right)^{1/2}$$

according to which the observed frequency will be smaller than the source frequency regardless of the apparent direction of motion of the source (transverse Doppler effect).

(ii) A direction-dependent factor

$$\left[f_o \sim f_s/\left(1 - \frac{v}{c}\cos\theta_o\right)\right]$$

showing a further dependence on the direction of relative motion, an effect that is understandable on the basis of classical arguments, unlike the other effect which is purely relativistic (radial Doppler effect).

The part involving the direction of relative velocity (radial Doppler effect) can be derived classically in several ways, for example, by an argument based on counting as the observed frequency f_o the number of waves arriving in a time interval dt_o which corresponds to the difference in the times of arrival of a "first wave" and of a "last wave" traveling with a velocity c towards the observer, the waves having been emitted at a frequency f_s by a source traveling with a velocity v at an angle θ_o with respect to the observer during a given time interval dt_s in the course of which the source emitted a total of $f_s dt_s$ waves. The relativistic velocity-dependent part is then included by noting that the source frequency will appear to be $f_o = f_s(1 - v^2/c^2)^{1/2}$ according to the theory of relativity.

H. E. Ives and G. R. Stilwell (31, 32), sceptical about the conclusions of the special theory of relativity, set out to verify the velocity-dependent part of the frequency shift (transverse Doppler effect) which can be observed at zero angle ($\theta_o = 0$). (See Fig. 4.) By measuring the wavelengths of the H_β line in the direction of motion ($\theta_o = 90°$) of hydrogen canal rays at 18,000 volts, and in the opposite direction ($\theta_o = -90°$) for which the frequencies are $f_{o1} = f_s(1-\beta^2)^{1/2}/(1-\beta)$ and $f_{o2} = f_s(1-\beta^2)^{1/2}/(1+\beta)$, respectively, where $\beta = v/c$, they determined the average, $f_o = f_{o1} + f_{o2}/2 = f_s/(1-\beta^2)^{1/2}$, and found it to be in accord with the theoretical value, $f_o = f_s/(1-\beta^2)^{1/2}$. Thus a beautiful direct proof of the "dilation of time" was provided, according to which the observer thinks that the source's period, $T_s = [T_o/(1-\beta^2)^{1/2}] > T_o$, is greater than the observer's period, T_o. (In this experiment, the source frequency appears to be $f_s = [f_o(1-\beta^2)^{1/2}] < f_o$, where f_o is the frequency determined in the spectrographic measurement.)

Finally, if the observing spectrograph is <u>not</u> at rest with respect to the system S to which f_o applies, as, for example, in the measurement of the apparent frequency of the light from a fixed star at different places on the earth's surface, the following

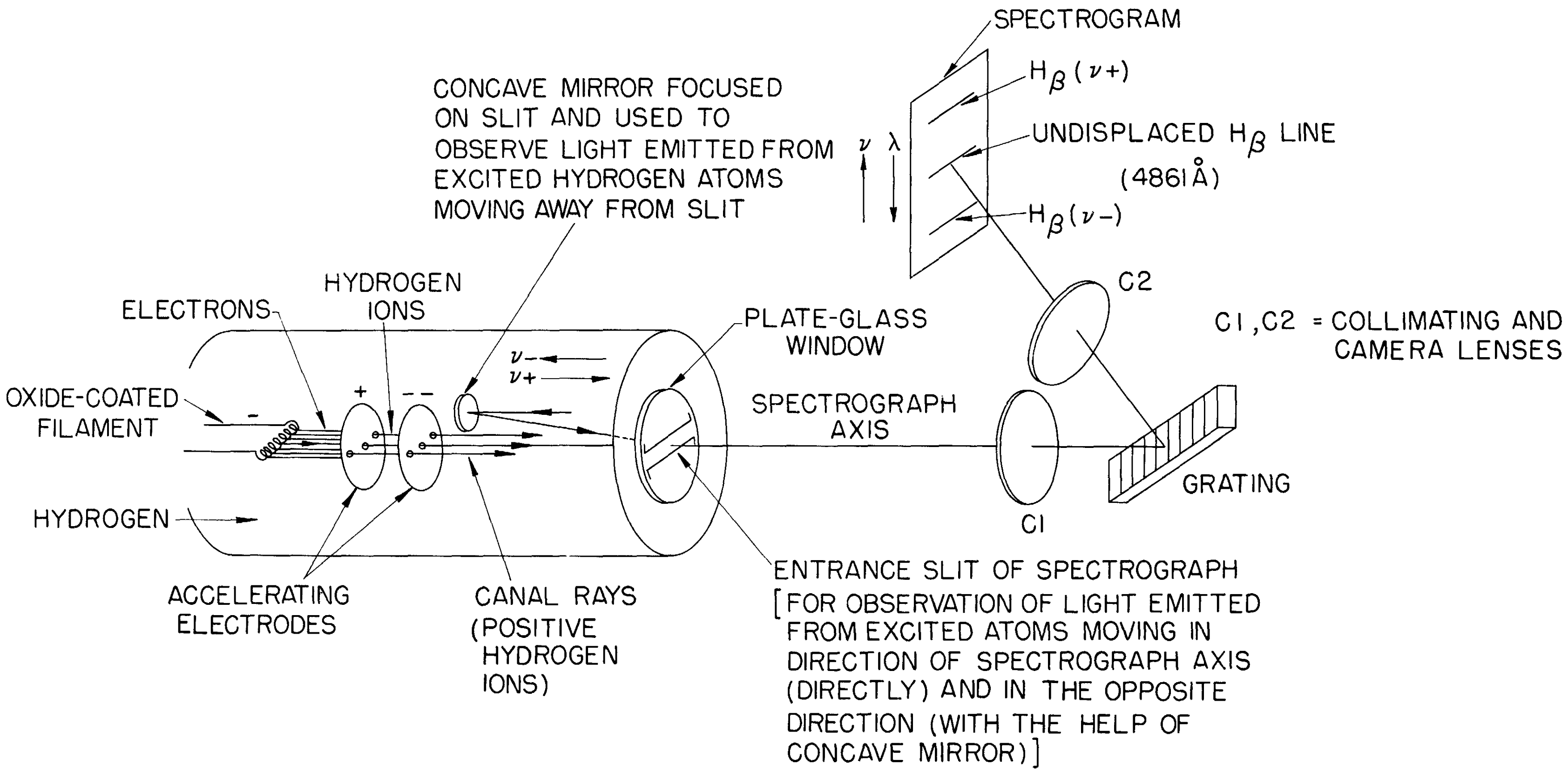


Fig. 4. Ives-Stilwell experiment.

equation can be shown (21) to apply to the frequency f_q measured by the spectrograph:

$$f_q = f_s \left(\frac{1 - \frac{v^2}{c^2}}{1 - \frac{V^2}{c^2}} \right)^{1/2} \frac{1 - \frac{V \cos \phi}{c}}{1 - \frac{v \cos \phi}{c}} \tag{26}$$

where V and ϕ apply to the velocity and angle of the spectrograph with respect to the system S. (If $V = 0$, and $\phi = 0$, then $f_q = f_o$.)

The calculations of the mathematical expressions of the Doppler effect follow.

A plane wave emitted in the system S' can be described in that system by

$$Y' = A' \cos \omega' \left(t' - \frac{x' \cos \theta'}{c} - \frac{y' \sin \theta'}{c} \right) \tag{27}$$

and in the system S by

$$Y = A \cos \omega \left(t - \frac{x \cos \theta}{c} - \frac{y \sin \theta}{c} \right) \tag{28}$$

If S' is moving with a velocity v with respect to S in the direction of the common x-axis of the two systems, we can write the Lorentz transformations

$$t' = \frac{t - \frac{vx}{c^2}}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}, \qquad x' = \frac{x - vt}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}, \qquad y' = y, \qquad z' = z \tag{29}$$

from which

$$Y' = A' \cos \omega' \left[\frac{t - \frac{vx}{c^2}}{(1 - \beta^2)^{1/2}} - \frac{(x - vt) \cos \theta'}{c(1 - \beta^2)^{1/2}} - \frac{y \sin \theta'}{c} \right] \tag{30}$$

or

$$Y' = A' \cos \omega' \left[\frac{t}{(1 - \beta^2)^{1/2}} \left(1 + \frac{v \cos \theta'}{c}\right) - \frac{x}{c(1 - \beta^2)^{1/2}} \left(\cos \theta + \frac{v}{c}\right) - \frac{y \sin \theta'}{c} \right] \tag{31}$$

According to the postulate of equivalence, we can identify Y and Y' of Eqs. 28 and 31 and, in particular, their frequency factors.

Equating the coefficients of t and x in the frequency factors, we obtain

$$\omega = \frac{\omega' \left(1 + \frac{v}{c} \cos \theta'\right)}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}} \tag{32}$$

and

$$\cos \theta = \frac{\cos \theta' + \frac{v}{c}}{\left(1 + \frac{v}{c} \cos \theta'\right)^{1/2}} \tag{33}$$

By using the following notation:

$\omega = f_o$ = frequency measured by an observer at rest in the observer space S,
$\omega' = f_s$ = frequency measured by an observer in the source space S',
$\theta = \theta_o$ = angle of the normal to a plane wave front measured in the observer space S,
$\theta' = \theta_s$ = angle of the normal to a plane wave front measured in source frame S',

by letting $\beta = v/c$, and by noting the following important relation which can be easily derived,

$$\frac{1 - \beta \cos \theta}{(1 - \beta^2)^{1/2}} = \frac{(1 - \beta^2)^{1/2}}{1 + \beta \cos \theta'} \tag{34}$$

we obtain the two important optical-Doppler-effect equations (see Eqs. 24 and 25)

$$f_o = \frac{f_s (1 - \beta^2)^{1/2}}{1 - \beta \cos \theta_o}$$

and

$$\cos \theta_o = \frac{\cos \theta_s + \beta}{1 + \beta \cos \theta_s}$$

e. Gravitational Effects in the Behavior of Light

The observed frequency shift in the light from the companion of Sirius relative to the frequency of light from Sirius, and the deflection of light beams passing near the sun are accounted for by the general theory of relativity (4, 16, 22).

According to this same theory, however, the propagation of electromagnetic waves with an invariant frequency is not possible, nor is the definition of a relative velocity. Nevertheless, under well-defined circumstances of static, slowly time-variant gravitational fields, the equation relating f_o and f_s in section 3.4 can be shown (21) to hold, provided that the appropriate equations for the potential difference between the observer and the source frames are included. (These equations are included in section 3.3.)

Finally, according to relativistic cosmological theories, still other reasons exist for a change of both the frequency and the velocity of propagation of an electromagnetic radiation during the very course of its propagation from the source to the observer (21). For a detailed summary of these theories and effects, see von Laue (21).

IV. CORPUSCULAR NATURE OF LIGHT

In its interactions with matter or other electromagnetic radiations, light manifests the property of exchanging energy and momentum only in discrete, quantized amounts (10, 17). This corpuscular property, the third fundamental character of light, forms the experimental basis for the quantum theories. One result of these theories is the prediction of the possibility of the creation of matter from electromagnetic radiation and of the creation of electromagnetic radiation from matter, both of which are being experimentally observed (10). Only a brief summary of the corpuscular nature of light is given here.

4.1 BASIC EXPERIMENTAL EVIDENCE

a. Photoelectric Effect

When a monochromatic, single-frequency beam of light, X-rays or gamma rays illuminates a metallic surface, negative electrons are ejected and their kinetic energy can be measured electronically by setting up an opposing electrostatic potential between the metallic surface and a collector at a negative potential with respect to the surface. It is found (17, 9, 15) that

(i) The emission of photoelectrons is immediate, independent of the intensity of the light beam, even at very low light intensities, exclusive of the possibility of the accumulation of energy from the light beam until an amount corresponding to the kinetic energy of the ejected electron has been reached.

(ii) The number of electrons, however, is proportional to the light intensity of the incident beam.

(iii) The velocities of the electrons ejected by light at varying frequencies agree with the equation

$$\frac{1}{2} m v_m^2 = h(\nu - \nu_o)$$

where m is the mass of the electron, v_m is the maximum observed velocity, ν is the frequency of the illuminating light beam, ν_o is a threshold frequency characteristic of the metal, and h is Planck's constant.

b. Inverse Photoelectric Effect

Photoelectrons can also be emitted from a plate P_2 by an X-ray radiation that comes from a plate P_1 where it was itself produced by the impact of electrons (1). The energy of the photoelectrons emitted from P_2 is independent of the distance between P_2 and P_1, which "demonstrates" that the X-ray radiation, which is undoubtedly spreading out from P_1 in spherical electromagnetic waves, is somehow able to give up at P_2 its entire energy created by an electron impinging on P_1. The conclusions of the Unified Quantum Theories of Electromagnetic Radiation, according to which we exclude such "wave" and "photon" descriptions of experiments, are well illustrated by this inverse photoelectric effect. If the spherical electromagnetic wave is taken as describing the possibility that the X-ray radiation will reach P_2 from P_1, and hence produce the observed interchange of energy at P_2 which results in the ejection of a photoelectron, no inconsistency appears between the wave and the corpuscular characters of the electromagnetic radiation.

c. Compton Effects

The scattering of X-rays of frequency ν_0 by light (not heavy) elements (10) can be attributed to the collision of a light quantum (photon) with an electron, both of which are assumed to be elastic spheres with the kinetic energy of the photon taken as $h\nu_0$. Under such circumstances, both a scattered X-ray photon and a scattered electron are observed, and the scattered X-ray has a lower frequency than the impinging X-ray. The kinetic energies of the impinging X-ray, the scattered X-ray, and the scattered electron, as well as their relative directions, are in agreement with calculations involving the conservation of energy and conservation of momentum laws. Single collisions are being observed between various radiations and particles in Wilson cloud chambers and bubble chambers, and Geiger and Bothe have demonstrated the simultaneity of the scattering of the electron and the X-ray.

d. Black-Body Temperature Radiation (1, 9, 17)

This involves the exchange of energy between radiation and matter in an enclosed cavity, in which the matter is at some temperature T. A mathematical expression relating the observed frequency distribution of the radiation emitted by the cavity at a given temperature T can be brought into accord with experimental observations only if we assume the possibility of a radiation of frequency ν which possesses a set of energies that are all integral multiples of a smallest quantity of energy equal to $h\nu$, where h is Planck's constant.

4.2 UNIFIED THEORIES OF ELECTROMAGNETIC RADIATION: INTERACTIONS OF ELECTROMAGNETIC RADIATION WITH MATTER

For a basic treatment, see Section II.

For a general and detailed treatment, see the particular references listed (1, 3, 11, 12, 14).

4.3 CREATION OF MATTER FROM RADIATION

The possibility of creating a pair of electrons, one positive and the other negative, by a rapidly varying electromagnetic field (gamma rays of high energies) was predicted as a consequence of Dirac's wave equation for a free electron and has been verified experimentally (10). Irène Curie and F. Joliot, J. Chadwick, P. M. S. Blackett and G. P. S. Occhialini, and others have compared the number of positive electrons (positrons) and negative electrons ejected by gamma rays passing through a thin sheet of lead (and other materials), and have found these numbers to be the same, after taking care of two other groups of electrons which also appear in the experiment (photoelectrons, recoil electrons). Other examples of pair production include the results of

(i) the collision of two heavy particles (proton, α particle passing through matter),
(ii) a fast electron passing through the field of a nucleus,
(iii) the direct collision of two electrons,
(iv) the collision of two light quanta in vacuo, and
(v) the action of a nuclear field on a gamma ray emitted by the nucleus involved in the action.

4.4 MASS, ENERGY, AND MOMENTUM OF PHOTONS

Evidence of the creation of matter from radiation, as well as of radiation from matter, substantiates Einstein's equation

$$E = mc^2$$

which was first expressed in the following words: "If a body gives off the energy E in the form of radiation, its mass diminishes by E/c^2" (4).

In regard to exchanges of energy and momentum, electromagnetic waves behave like a set of particles of energy

$$E = mc^2 = h\nu$$

and of momentum

$$p = h\nu/c$$

Finally, many experiments with photons show that they also possess an intrinsic angular momentum, as do particles (10, 17, 12). Circularly polarized light, for example, carries an experimentally observable angular momentum, and it can be shown that, under certain circumstances, an angular momentum can be imparted to unpolarized or plane-polarized light (plane wave passing through a finite circular aperture) (10, 1). In any case, the angular momentum will be in units of $\hbar = h/2\pi$.

4.5 CREATION OF RADIATION FROM MATTER

The inverse process to the creation of electron pairs, the annihilation of a positron and an electron, results in a predicted production of two gamma-ray quanta (two-quantum annihilation) (10). Atomic and nuclear chain reactions are known to involve similar processes.

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