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The Option to Try Again: Valuing a Sequence of Dependent
Trials

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Valuing a Sequence of Dependent Trials

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Abstract

In various fields of economic endeavor, agents enjoy the option to "try, try again."
Failure in a particular pursuit often brings renewed effort to finally succeed. Many areas
of R&D could be characterized in this fashion. Our purpose is to define and measure the
value of this option to try again. The value of repeated trials is closely related to the
extent of statistical dependence among them. We describe the solution to this valuation
problem, examine the behavior of the option premium, and characterize potential errors
that are inherent in two *ad hoc* procedures that are often used to obtain bounds on the true
value of the prospect. To be concrete, the problem is framed in terms of petroleum
exploration, but the methods we employ are general and could be applied to various
forms of R&D and other types of risky investments.

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1. Introduction:

In various fields of economic endeavor, agents enjoy the option to "try, try again."
Failure in a particular pursuit often brings renewed or repeated efforts to finally succeed.
Many areas of research and development could be characterized in this fashion. Our
purpose is to define and measure the value of this option to try again. The value of
repeated trials is closely related, of course, to the extent of statistical dependence among
them. An early failure that completely negates the chance for subsequent success would
render the option worthless. More interesting (and realistic) are cases which involve
imperfect dependence among trials.

We describe the solution to this valuation problem, examine the behavior of the
option premium, and characterize potential errors that are inherent in two *ad hoc*
procedures that are often used to obtain bounds on the true value of the prospect. To be
concrete, the problem is framed in terms of petroleum exploration, but the methods we
employ are general and could be applied to various forms of R&D and other types of
risky investments.

2. Case Study: A Petroleum Exploration Prospect

It is customary to evaluate petroleum exploration and development prospects in
terms of three parameters: the probability of success (p), the expected gross profit
conditional on success (V), and the cost of the drilling trial (C). Usually, the expected

reward associated with the trial is set against the cost of conducting that trial to obtain the

expected economic value of the prospect:[1]

$$EV_a \quad = \quad p \cdot V - C.$$

Alternatively, the evaluator may recognize the operator's ability to follow up an

unsuccessful trial with further attempts. If $n = 1/p$ independent trials were attempted,

then the expected number of successes would be given by $np = 1$, which suggests an

alternative calculation of expected economic value:[2]

$$EV_b \quad = \quad V - C/p,$$

in which the full reward for achieving a drilling success is set against the expected cost of

a sufficient number of trials to obtain that success.

The correspondence between these two estimates depends entirely upon the value

of p: they tend to converge if the probability of success is high and to diverge otherwise.

For example, if the probability of success is 1/3, we find that $EV_a = 3EV_b$. If the

probability of success were judged to be only 1/10, which is entirely plausible for many

exploration prospects, the two estimates of value diverge by an order of magnitude. In

cases where the two approaches diverge significantly, it is important to look more closely

at the underlying assumptions of the valuation process.

Which of the two valuations is correct? The answer, in most cases, is "neither."

Both approaches ignore the influence of dependence among trials. By "dependence

among trials" we mean that the conditional probability of success at later trials is reduced

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[1] This formulation appears in all the standard manuals on prospect valuation. See, for example, Megill (1988, p. 163), Newendorp (1975, pp. 64-83), and Lerche and MacKay (1999, p. 18). Kemp and Rose (1984), Hendricks and Kovenock (1989), Pickles and Smith (1993), and Laughton (1998) are among the many applications that have followed this approach.

[2] Among the major treatises on exploration economics, only Newendorp (1975, pp. 115-122) recognizes the option to drill again after failure, but even there the discussion and analysis is confined to a rather limited example from which no general conclusions are drawn.

by earlier failures. We show in this paper that it is impossible to obtain an unbiased estimate of prospect value without accounting for the degree of dependence among trials, and we demonstrate that the penalty for overlooking this aspect of the valuation problem can be quite large. We also show that the two formulations introduced above (EV_a and EV_b) provide lower and upper bounds, respectively, on the expected economic value of the prospect regardless of the degree of dependence, and that the expected value declines monotonically as the degree of dependence among trials increases. Finally, we offer some observations on the likely strength and pattern of dependence among trials that are suggested by the structure of exploration and development drilling uncertainties.

The current study is related to the approach introduced by Paddock, Siegel, and Smith (1988), who examined the option component of value inherent in petroleum exploration and development prospects. Whereas they focused on the value of the option to *delay drilling* pending the arrival of updated price information (through the value of *V*, which they treated as stochastic), we focus on the value of the option to *drill again* pending the outcome of previous drilling attempts (where we treat *V* as non-stochastic). Although any given prospect potentially holds both types of option value, our results suggest that the two are substitutes to some degree: prospects most likely to benefit substantially from the option to drill again are least likely to benefit substantially from the option to delay drilling, and vice versa.

3. Dependent Trials:

Let the sequence $\{p_1, p_2, p_3, \ldots\}$ represent the conditional probability of success on the first, second, third, ... trials, given no previous success. We will assume:

$$p_1 \geq p_2 \geq p_3 \geq \ldots$$

and note that the case of *independent* trials is characterized by strict equalities throughout. For completeness we define $p_0 = 0$.

The strength of dependence between trials is measured, for our purposes, by the extent to which a failure at one trial diminishes the probability of success on the succeeding trial. Thus, $d_t = (p_t - p_{t+1})/p_t = 1 - p_{t+1}/p_t$ represents the strength of dependence at the tth stage in the sequence. The probability of initial success, p_1, and the set $\{d_t\}$ are sufficient to describe any pattern of dependence among trials. For the case of independent trials, $d_t = 0$ for all t.[3] Another useful benchmark is the case of "complete dependence," by which we mean that the probability of success falls immediately to zero after the first failure: $d_1 = 1$ and $d_t = 0$ for all $t \geq 2$.[4]

Each trial is assumed to cost C, and success on any trial brings expected value V, which is assumed to be constant across all trials. Therefore, the marginal expected profit generated by the tth trial is $\pi_t = p_t V - C$. The sequence of trials is assumed to be truncated at the first success, or when marginal expected profit becomes negative, whichever comes first. Truncation due to negative marginal profit, if it does occur, will come after trial T, where:

$$p_{T+1} < C / V \quad \text{(i.e. } \pi_{T+1} < 0) \qquad \text{while } p_T \geq C / V.$$

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[3] If the strength of dependence remains constant throughout the sequence, probabilities evolve according to a geometric series:

$$p_{t+1} = \lambda\, p_t, \qquad \text{for all } t, \text{ with } 0 \leq \lambda < 1;$$

and the probability of success converges to zero. Constant dependence is perhaps not the most likely case, however. Consider, for example, a sequence whereby the probability of success *conditional on the presence of oil in the prospect* does not change from trial to trial. We show later (see Section 6) that, in such cases, the degree of dependence steadily increases throughout the sequence; i.e., $d_{t+1} > d_t$.

[4] Other names have been used to describe "complete dependence," including "full dependence," "shared risk," and "common risk." See Wang, Kokolis, Rapp, and Litvak (2000), for example.

The truncation point (T) may not be finite (trials may continue indefinitely if the probabilities converge to a number that exceeds C / V), but if it is finite, then T must be unique due to the monotonic behavior of the probabilities of success. We note that $\pi_t \geq 0$ for all $t \leq T$, and $\pi_t < 0$ for all $t > T$. To avoid degenerate cases, we will assume that $p_1 > C / V$; i.e., the marginal value of the first trial is positive. Otherwise the prospect should be discarded.

Finally, we represent the probability of reaching the tth trial by q_t, where:

$$q_t \quad = \quad \prod_{j=0}^{t-1}\left(1 - p_j\right), \qquad\qquad t \ = \ 1, 2, \dots$$

and where again for completeness we define: $q_0 = 1$.

4. Prospect Valuation:

We start by formulating the expected value of the prospect; i.e., the expression that properly captures the impact of whatever dependence exists among successive trials:

$$EV \quad \equiv \quad \sum_{t=0}^{T} \pi_t q_t \tag{1}$$

We wish to develop a simple, yet exact expression for EV, and to establish that: (a) EV_a and EV_b (defined previously) provide bounds on EV; (b) any increase in the degree of dependence among trials decreases EV; and (c) the bounds established in part (a) are exact: $EV = EV_a$ in the case of complete dependence and $EV = EV_b$ in the case of independence.

It will be useful to consider a measure that recognizes the dependence created by failures among only the first τ trials (and ignores any further dependence created by subsequent failures). This measure is denoted EV_τ, where for all $\tau \geq 0$ for which $p_{\tau+1} > 0$:

$$EV_\tau \quad = \quad \sum_{t=0}^{\tau} \pi_t q_t + \pi_{\tau+1} q_{\tau+1} \left[1 + (1 - p_{\tau+1}) + (1 - p_{\tau+1})^2 + ... \right]$$

$$= \quad \sum_{t=0}^{\tau} \pi_t q_t + \pi_{\tau+1} q_{\tau+1} / p_{\tau+1} . \qquad (2)$$

This formula deviates from equation (1) in that all p_t and q_t beyond $t=\tau+1$ are held

constant at their previous values, and no truncation of trials is presumed to occur.

The benchmark valuation that ignores the impact of dependence altogether, EV_b,

is obtained from equation (2) as the special case where $\tau = 0$:

$$EV_b \quad = \quad EV_0 \quad = \quad \pi_1 q_1 / p_1 = \quad V - C / p_1 . \qquad (3)$$

Advancing the index in equation (2) shows the impact of dependence through $\tau+1$ trials

(again we assume that $p_{\tau+2} > 0$):

$$EV_{\tau+1} \quad = \quad \sum_{t=0}^{\tau+1} \pi_t q_t + \pi_{\tau+2} q_{\tau+2} \left[1 + (1 - p_{\tau+2}) + (1 - p_{\tau+2})^2 + ... \right]$$

$$= \quad \sum_{t=0}^{\tau+1} \pi_t q_t + \pi_{\tau+2} q_{\tau+2} / p_{\tau+2} . \qquad (4)$$

In comparing EV_τ and $EV_{\tau+1}$, it is convenient to rewrite (2) by grouping the first term of

the infinite series with the first summation, which yields the equivalent expression:

$$EV_\tau \quad = \quad \sum_{t=0}^{\tau+1} \pi_t q_t + \pi_{\tau+1} q_{\tau+1} \left[(1 - p_{\tau+1}) + (1 - p_{\tau+1})^2 + ... \right]$$

$$= \quad \sum_{t=0}^{\tau+1} \pi_t q_t + \pi_{\tau+1} q_{\tau+1} \left(\frac{1}{p_{\tau+1}} - 1 \right)$$

$$= \sum_{t=0}^{\tau+1} \pi_t q_t + \pi_{\tau+1} q_{\tau+2} / p_{\tau+1} . \tag{5}$$

Subtracting (5) from (4) then shows the incremental impact of dependence associated with the $\tau+1^{st}$ trial:

$$EV_{\tau+1} - EV_\tau \quad = \quad q_{\tau+2}\left(V - \frac{C}{p_{\tau+2}}\right) - q_{\tau+2} C\left(V - \frac{C}{p_{\tau+1}}\right)$$

$$= \quad q_{\tau+2} C\left(\frac{1}{p_{\tau+1}} - \frac{1}{p_{\tau+2}}\right)$$

$$= \quad -q_{\tau+2} C \frac{d_{\tau+1}}{p_{\tau+2}} \qquad (\leq 0 \quad \text{since} \ d_{\tau+1} \geq 0). \tag{6}$$

Upper Bound for Prospect Value:

The benchmark valuation EV_b, as defined by equation (3), gives the expected value of a prospect that affords independent trials. If trials are actually dependent, then this benchmark provides an estimate of value that is biased upwards by ignoring the impact of dependence.[5] To see this, we consider first the case where truncation of trials would never occur; i.e., where dependence is sufficiently weak that the probability of success converges to a number greater than C / V. In this case, we may write the expected value of the prospect as the infinite series:

$$EV \quad \equiv \quad EV_0 + (EV_1 - EV_0) + (EV_2 - EV_1) + (EV_3 - EV_2) + \dots \tag{7}$$

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[5] Although this result may be intuitively clear, there is some complexity to the proof owing mainly to the fact (cf. equation 1) that while a reduced probability of success at trial t tends to diminish the marginal value of any future trial (π_{t+1}), it also tends to increase the probability of continuing on to reap those marginal profits (q_{t+1}). The proof is a formal demonstration that the former effect outweighs the latter.

But, by (6) we have shown each component in this series after the first to be non-positive. Thus, EV_0 (the estimate that ignores dependence altogether) is an upper bound on the expected value of the prospect, no matter what pattern of dependence may follow.

Next we consider the other possibility: that dependence is strong enough to force truncation after the Tth trial (if success has not already occurred by then). In this event, the expected value of the prospect may be written:

$$EV \quad \equiv \quad \sum_{t=0}^{T} \pi_t q_t \quad = \quad EV_{T-1} + \pi_T q_T - \pi_T q_T / p_T, \tag{8}$$

where we have used equation (2) to obtain the expression on the right. Thus:

$$EV - EV_{T-1} \quad = \quad -\frac{\pi_T q_{T+1}}{p_T} \quad < \quad 0, \tag{9}$$

since $\pi_T \geq 0$ by definition if truncation is to occur after trial T.

Where dependence forces truncation after trial T, we can write (cf. equation 7):

$$EV \quad = \quad EV_0 + (EV_1 - EV_0) + (EV_2 - EV_1) + \dots + (EV - EV_{T-1}), \tag{10}$$

where all terms in this summation after the first have been shown via equations (6) and (9) to be non-positive. The first term, EV_0, which is the valuation that ignores dependence altogether, therefore serves as an upper bound for the expected value of the prospect in the truncated case, as well as in the non-truncated case.

<u>Lower Bound for Prospect Value:</u>

Recall from equation (1) that the expected value of the prospect, *EV*, may be written:

$$EV \equiv \sum_{t=0}^{T} \pi_t q_t ;$$

from which we observe:

$$EV = \pi_1, \qquad\qquad \text{if } T = 1, \qquad\qquad (11)$$

$$= \pi_1 + \sum_{t=2}^{T} \pi_t q_t , \qquad\qquad \text{otherwise.}$$

Whether the truncation point (*T*) is finite or infinite, each term in the summation on the right is non-negative by construction, which implies:

$$EV \geq \pi_1 = p_1 V - C = EV_a. \qquad\qquad (12)$$

This result establishes a lower bound for the expected value of the prospect, regardless of the strength and pattern of dependence. That bound, EV_a, corresponds to the expected value of the prospect if trials are completely dependent, in which case only one trial is attempted regardless of the outcome. Thus, the results so far have demonstrated that the benchmark cases of *independence* and *complete dependence* provide upper and lower bounds, respectively, for the expected value of any sequence of dependent trials.

An Exact Expression for Prospect Value:

We consider first the case without truncation. Successive substitution from equation (6) into (7) yields:

$$EV \quad = \quad EV_0 + Cq_2\left(\frac{1}{p_1} - \frac{1}{p_2}\right) + Cq_3\left(\frac{1}{p_2} - \frac{1}{p_3}\right) + \dots \tag{13}$$

which becomes, after rearrangement:

$$EV \quad = \quad EV_0 + \frac{Cq_2}{p_1} + \frac{C(q_3 - q_2)}{p_2} + \frac{C(q_4 - q_3)}{p_3} + \dots \tag{14}$$

This expression can be simplified by using the identity: $q_t - q_{t-1} \equiv -q_{t-1}p_{t-1}$, plus the facts that $EV_0 = V - C/p_1$ and $q_2 = (1-p_1)$, which after substitution in (14) yields:

$$EV \quad = \quad V \ - \ C\sum_{t=1}^{\infty}q_t \quad = \quad V \ - \ C\cdot\bar{n}, \tag{15}$$

where $\bar{n}$ represents the expected number of trials actually attempted. (Recall that q_t corresponds to the probability of reaching trial t, at which point one additional well will be drilled).

Thus, in the special case of non-truncated trials, the expected value of the prospect is given, and not surprisingly, by the expected gross value of the item less the cost per trial times the expected number of trials that will conducted. This expression holds generally for any pattern of dependence among trials, as long as the residual probability of success never falls below the economic threshold for truncation.

In the other case, where the sequence would be truncated after trial T, an analogous expression describes the expected value of the prospect. It is obtained by substituting from equations (6) and (9) into (10), which yields:

$$EV \;\; = \;\; EV_0 + Cq_2\left(\frac{1}{p_1} - \frac{1}{p_2}\right) + Cq_3\left(\frac{1}{p_2} - \frac{1}{p_3}\right) + ... + Cq_T\left(\frac{1}{p_{T-1}} - \frac{1}{p_T}\right) - \frac{\pi_T q_{T+1}}{p_T},$$

which, after simplification via the same procedure used above, finally reduces to:

$$EV \;\; = \;\; (1 - q_{T+1})V \;\; - \;\; C\sum_{t=1}^{T} q_t \;\; = \;\; (1 - q_{T+1})V \;\; - \;\; \bar{n}C. \tag{16}$$

In other words, regardless of the particular pattern and strength of dependence among trials, the expected value of the prospect is given by the expected gross value of the prospect times the probability it is discovered before giving up, less the cost per trial times the expected number of wells drilled before giving up.

Using equations (15) and (16), we can easily confirm that dependence among trials has a monotonic impact: any increase in the degree of dependence must decrease the expected value of the prospect. For the case of non-truncated trials, this result is immediately apparent. Holding all else constant, an increase in any one of the $\{d_t\}$ will cause one or more of the $\{q_t\}$ in equation (15) to increase, which increases the expected number of trials, $\bar{n}$, and thereby reduces the expected value.

Where trials would be truncated after trial T, there is something more to the argument. If the increase in dependence does not alter the truncation point, then the situation is again quite simple; the first term in equation (16) must fall (due to the rise in q_{T+1}), while the second term must rise, and the expected value of the prospect is

diminished. In addition, we must allow for the possibility that the truncation point will itself decline, say from T to $T\text{-}1$. The proof for this case can be sketched very briefly. Since T was the optimal point of truncation before dependence was increased, we know already that $EV(T\text{-}1) < EV(T)$, where $EV(t)$ represents the expected value of the original prospect (before the increase in dependence) if the sequence is to be truncated after trial t. But we also know (by the argument in the preceding paragraph) that $EV'(T\text{-}1) < EV(T\text{-}1)$, where the EV' notation represents the expected value of the revised prospect (based on the $\{p_t\}$ that result from the increase in dependence). By transitivity we must then have $EV'(T\text{-}1) < EV(T)$ for any increase in dependence among trials. The same argument works by extension if the optimal truncation point is reduced by more than one step.

5. An Illustration of the Potential Option Premium:

Although expected value of the prospect varies directly with the initial probability of success (through its impact on the $\{q_t\}$), the distance between our bounds on prospect value varies inversely with this parameter. We have shown the upper bound to be $EV_b = V - C / p_1$; and the lower bound $EV_a = p_1 V - C = p_1 EV_b$. Thus, the relative size of the interval within which the actual value must lie is: $EV_b / EV_a = 1 / p_1$.

If the probability of success were small, say 10%, then the upper bound would be an order of magnitude larger than the lower bound. Such cases merit the most detailed examination of the extent of dependencies among trials simply because the potential penalty from ignoring that dependence is the greatest. The economic lower limit on p_1 is given by C / V. If we define the inverse, V / C, as the "unrisked return" or "gross margin" of the project, then it follows that prospects with the highest unrisked returns (or gross margins) are the ones most prone to potential mis-estimation of value as a result of

ignoring or misstating the extent of dependencies (since those are the prospects that admit the lowest probabilities of success). Prospects with relatively low unrisked returns do not admit low success probabilities and therefore the bounds on project value will be much tighter. Consequently, detailed examination of the nature of dependencies for projects with low unrisked returns would have less impact on the assessment of prospect value.

These relations between probability of success, unrisked return, and width of the valuation interval are illustrated in Figures 1 and 2, below. For convenience, the prospect in each figure is characterized by $V = 100$, but that parameter can be scaled up or down without changing the geometrical shape of the diagrams—only the scale on the vertical axis would be affected.

It is evident from the figures that the value of the option to drill again, which accounts for the spread between the two curves, is of greater potential significance for high-margin prospects than for low-margin prospects. In contrast, the opposite relationship holds with regard to the option to delay drilling, as demonstrated previously by Paddock, Siegel, and Smith (1988, pp. 504-505). That is, prospects for which the reward exceeds the cost by a sufficiently wide margin should be drilled immediately because the potential gain from favorable future price developments is outweighed by the fact that undeveloped oil reserves appreciate more slowly than money in the bank. The implication is that petroleum prospects are likely to benefit significantly from one or the other option component, but rarely from both. In light of the fact that exploration wells, especially in frontier areas or immature plays, are particularly high-risk gambles with large rewards (relative to costs) if successful, exploration prospects would as a general rule be more likely to benefit from the option to drill again. Development wells, which

are drilled into known formations that carry less risk, are more likely to benefit from the option to delay. Each prospect is unique, however, so there could be many individual exceptions to this general pattern.

6. Trials With Increasing Dependence

In this section we investigate the implications for dependence and prospect value of a plausible structure of geological uncertainty and its resolution. We begin with the postulate that the conditional probability of success (S_t) at trial t, given the presence of an oil-bearing structure (O) in the prospect, is independent of t:

$$P(S_t \mid O) \quad = \quad \alpha > 0 \qquad \text{for } t = 1, 2, \ldots$$

We also assume that the probability of success at any trial, conditional on the absence of an oil-bearing structure, is zero:

$$P(S_t \mid \overline{O}) \quad = \quad 0 \qquad \text{for } t = 1, 2, \ldots$$

Finally, we take the *a priori* probability of an oil-bearing structure to be $P(O) = \beta$, where $0 < \beta < 1$.

We may then write the probability of success at the first trial, p_1, in the following manner:

$$p_1 \quad = \quad P(S_1) \quad = \quad P(S_1 \mid O)P(O) + P(S_1 \mid \overline{O})P(\overline{O})$$

$$= \quad P(S_1 \mid O) \cdot P(O) \quad = \quad \alpha\beta .$$

Moreover, we can write the conditional probability of success on the second trial, given failure on the first, as:

$$p_2 \quad = \quad P(S_2 \mid \overline{S}_1) \;=\; P(S_2 \mid O) \cdot P(O \mid \overline{S}_1).$$

In general, the conditional probabilities at each stage in the sequence take the form:

$$p_t \quad = \quad P(S_t \mid \overline{S}_1 \cap \ldots \cap \overline{S}_{t-1}) \;=\; P(S_t \mid O) \cdot P(O \mid \overline{S}_1 \cap \ldots \overline{S}_{t-1}). \tag{17}$$

The last term in (17) can be written equivalently, per Bayes Rule, as:

$$P(O \mid \overline{S}_1 \cap \ldots \cap \overline{S}_{t-1}) \;=\; \frac{P(\overline{S}_1 \cap \ldots \cap \overline{S}_{t-1} \mid O) \cdot P(O)}{P(\overline{S}_1 \cap \ldots \cap \overline{S}_{t-1})}.$$

The numerator in this last expression equals $(1-\alpha)^t \beta$. The denominator is evaluated by rewriting in the form:

$$P(\overline{S}_1 \cap \ldots \cap \overline{S}_{t-1}) \;=\; P(\overline{S}_1 \cap \ldots \cap \overline{S}_{t-1} \mid O) \cdot P(O) + P(\overline{S}_1 \cap \ldots \cap \overline{S}_{t-1} \mid \overline{O}) \cdot P(\overline{O})$$

$$= \; (1-\alpha)^{t-1} \beta + (1-\beta).$$

Substituting these results back into (17) yields:

$$p_t \quad = \quad \frac{\alpha(1-\alpha)^{t-1}\beta}{(1-\alpha)^{t-1}\beta + (1-\beta)}. \tag{18}$$

The degree of dependence at trial t is then determined from (18) by the ratio $p_{t+1}/p_t, = 1 - d_t = \lambda_t$, which after simplification can be expressed as (cf. footnote 3):

$$\lambda_t \quad = \quad \frac{(1-\alpha)^t \beta + (1-\alpha)(1-\beta)}{(1-\alpha)^t \beta + (1-\beta)} \quad < \quad 1 \qquad \text{for all } t. \tag{19}$$

The fact that $\lambda_t < 1$ signifies that trials are dependent. That fact that dependence is *increasing* as trials continue follows from the fact that $\lambda_t/\lambda_{t-1} < 1$, a result which can be confirmed by using (19) to evaluate the ratio: λ_t/λ_{t-1}:

$$\frac{\lambda_t}{\lambda_{t-1}} \quad = \quad \frac{\beta^2 (1-\alpha)^{2t-2} + 2\beta(1-\beta)(1-\alpha)^{t-1} + (1-\beta)^2}{\beta^2 (1-\alpha)^{2t-2} + \beta(1-\beta)(1-\alpha)^{t-2} + \beta(1-\beta)(1-\alpha)^t + (1-\beta)^2}$$

$$= \quad \frac{\beta^2 (1-\alpha)^{2t-2} + 2\beta(1-\beta)(1-\alpha)^{t-1} + (1-\beta)^2}{\beta^2 (1-\alpha)^{2t-2} + \beta(1-\beta)(1-\alpha)^{t-1}[(1-\alpha)^{-1} + (1-\alpha)] + (1-\beta)^2}.$$

Only the middle term differs between numerator and denominator of the last expression. Thus:

$$\frac{\lambda_t}{\lambda_{t-1}} \quad < \quad 1 \quad \text{if and only if}: \quad (1-\alpha)^{-1} + (1-\alpha) > 2, \quad \text{which is true for all } \alpha \in (0,1).$$

In summary, under the maintained hypothesis that the conditional probability of success is constant *given the presence of an oil-bearing structure*, it follows that successive failures have increasing negative impacts on the relative probability of success.

The evolution of success probabilities over successive trials, and the associated decline in λ_t, are illustrated below in Figures 3 and 4, respectively. We present two cases: Case 1 has $\alpha = 30\%$ and $\beta = 90\%$ (a strong *a priori* probability of an oil-bearing structure but a relatively weak test), whereas Case 2 has $\alpha = 70\%$ and $\beta = 50\%$ (a weaker

prior accompanied by a more powerful test). Clearly, the evolution of success probabilities varies significantly as these parameters are changed. What we have established in this section is that, subject to the structure of uncertainty described above, all such curves are monotonically decreasing.

The figures illustrate a further, rather intuitive lesson about the option to drill again. If there is a strong *a priori* belief in the presence of oil but the power of the drilling trial to confirm that belief is low, then the degree of dependence among trials is reduced. In the extreme, this situation would approximate the case of independent trials, wherein drilling continues until the original presumption of a deposit is proven correct. On the other hand, if the *a priori* probability of a deposit is low and the power of the drilling trial to detect that deposit is high, the situation would approximate the case of completely dependent trials in which the first well tells the whole story, and the value of the option to drill again would diminish.

7. Conclusion:

We have demonstrated that the value of the option to try again can be significant—in some cases greatly exceeding the expected value of the initial trial considered in isolation. The value of this option is diminished, however, by the influence of dependence among trials. Dependence essentially reduces the volatility of outcomes, and therefore also reduces option value, by reducing the upside potential of successive trials. We have provided exact formulas by which the expected value of any such prospect can be computed. We have also demonstrated that the impact of dependence is monotonic: any increase in the degree of dependence among trials must further reduce the expected value of the prospect.

Two relatively simple and familiar estimation procedures provide bounds on the actual value of the prospect. The most common estimation approach, here designated EV_a, errs by ignoring the option component completely. The other approach, EV_b, recognizes the option component but ignores the impact of dependence among trials. The gap between these two approaches is potentially very wide if the initial probability of success is not high.

We have also demonstrated that the option to try again tends to act as a substitute for the option to delay trials. Generally speaking, prospects that are "deep in the money" are most likely to benefit significantly from the option to try again, but least likely to benefit from the option to delay trials; the converse applies to prospects that are "even money" or below.

Our conclusions regarding the negative impact of dependence are specific to the particular problem at hand: valuation of a single prospect that may or may not yield its reward upon repeated testing. It is not correct to assume that dependence plays a similar role in all valuation problems. The expected value of a portfolio of multiple prospects, for example, is actually enhanced by dependence among prospects, owing to the different mechanism by which dependence augments the volatility of potential portfolio returns—but that is a more complicated problem that goes beyond the scope of the present paper.

Figure 1: Bounds on Prospect Value (High Margin: C/V = 10%)
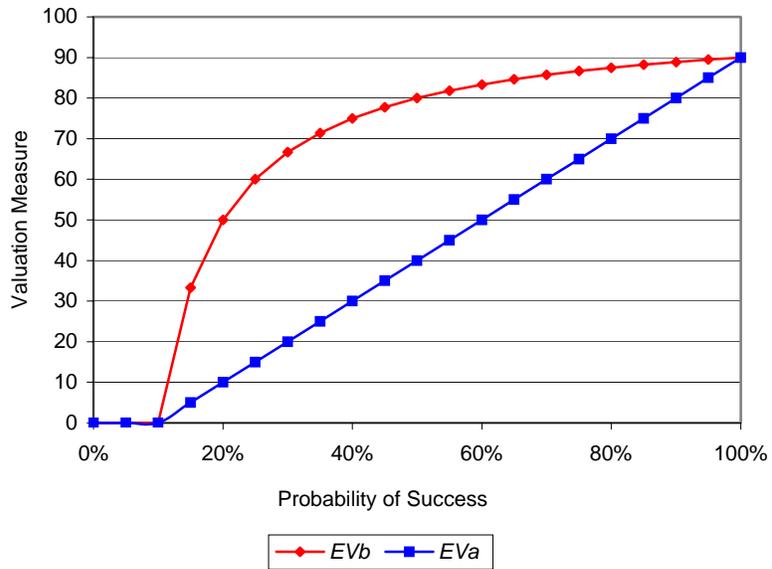
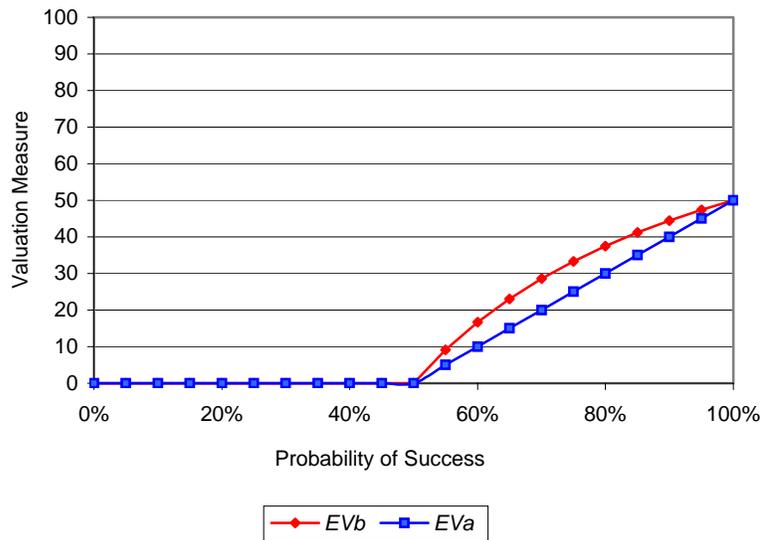**Figure 2: Bounds on Prospect Value (Low Margin: C/V = 50%)**
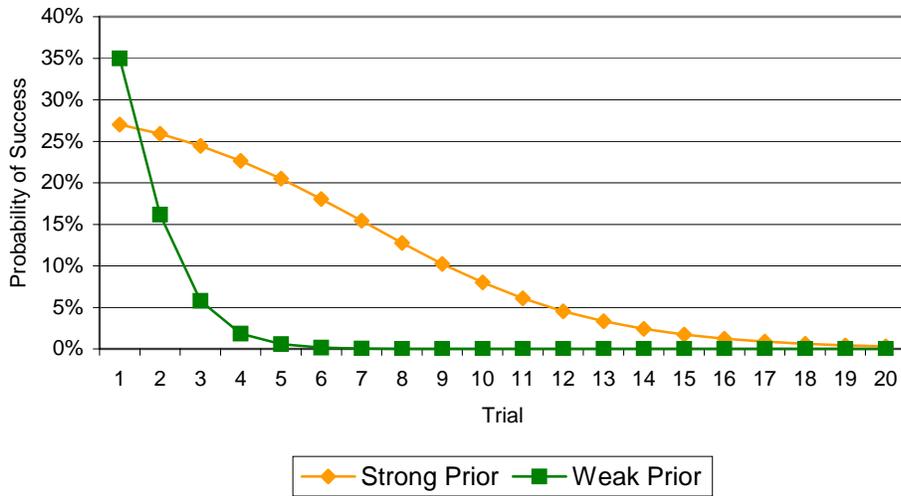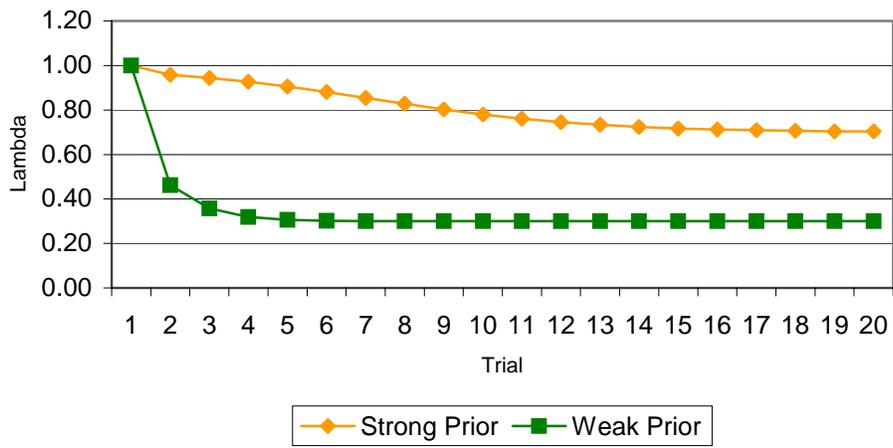
Figure 3: Success Probability Declines at Each Trial

Figure 4: Lambda Declines (Dependence Increases) at Each Trial

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