MIT

CEEPR   Center for Energy and Environmental Policy Research

Forward Trading in Exhaustible-Resource Oligopoly

by
Matti Liski and Juan-Pablo Montero

08-006   June 2008

A Joint Center of the Department of Economics,
MIT Energy Initiative, and Sloan School of Management

Forward trading in exhaustible-resource oligopoly

Matti Liski and Juan-Pablo Montero[*]

June 4, 2008

Abstract

We analyze oligopolistic exhaustible-resource depletion when firms can trade forward contracts on deliveries, a market structure prevalent in many resource commodity markets. We find that this organization of trade has substantial implications for resource depletion. As firms' interactions become infinitely frequent, resource stocks become fully contracted and the symmetric oligopolistic equilibrium converges to the perfectly competitive Hotelling (1931) outcome. Asymmetries in stock holdings allow firms to partially escape the procompetitive effect of contracting: a large stock provides commitment to leave a fraction of the stock uncontracted. In contrast, a small stock provides commitment to sell early, during the most profitable part of the equilibrium.

(JEL classification: G13, L13, Q30).

1   Introduction

Hotelling's (1931) theory of exhaustible-resource depletion is a building block for understanding intertemporal allocation of a finite resource stock. The theory is used in myriad of applications which, without exceptions known to us, assume implicitly or explicitly

that the commodity stock is sold in the spot market only, thereby ruling out forward trading despite the fact that it is commonly observed in many commodity markets and markets for exhaustible-stocks in particular. Forward trading is typically associated to the desire of some groups of agents to hedge risks but it can also arise in oligopoly settings without uncertainty. As shown by Allaz and Vila (1993) for the case of reproducible commodities, the mere possibility of forward trading forces firms to compete both in the spot and forward markets, creating a prisoner's dilemma for firms in that they voluntarily sell forward contracts (i.e., take short positions in the forward market) and end up producing more than in the absence of the forward market. In this paper we are interested in understanding the strategic role of forward trading in an oligopolistic exhaustible-resource market.[1]

In exhaustible-resource markets firms face an intertemporal capacity constraint coming from their finite stocks. Hotelling (1931) establishes a simple principle for monopolistic allocation of the capacity over time: marginal value of using the capacity in different periods should be equalized in present value. Under standard assumptions, resource depletion becomes more conservative. Compared to the perfectly competitive path, monopoly sales are shifted towards the future as a way to increase the value of early sales. An oligopoly follows the same (spot) allocation principle as the monopoly, with differences in outcome analogous to those that arise between static monopoly and oligopoly. Furthermore, this intertemporal capacity constraint rules out the output-expanding effect of forward contracting found by Allaz and Vila (1993) for the reproducible case. One may then conjecture that for exhaustible resources forward contracting leaves oligopoly rents intact (e.g., Lewis and Schmalensee, 1980).[2]

Our results depart from the above conjecture, however. We find, for example, that the symmetric subgame-perfect delivery path converges to the perfectly competitive path as firms interactions become infinitely frequent, i.e., in the continuous-time limit. To understand the logic of this result, consider first a stock so small, or period length so large, that the one-period demand absorbs the stock without any storage. Forward contracting then plays no strategic role because the overall supply is in any case to be consumed in one period. Reduce now the period length, or increase the stock size, so

---

[1]Phlips and Harstad (1990) already mentioned that forward contracting can have an important effect on oligopolistic exhaustible-resource markets but they did not explain whether and to what extent firms will sign forwards in equilibrium.

[2]Without explicitly studying forward markets, Lewis and Schmalensee (1980) suggest that the existence of futures markets could validate the use of "path strategies", i.e., it could allow firms to commit to production plans.

that consumption takes place over two periods. Contracting preceding spot sales now plays a role: it induces firms to race for a higher capacity share in the first period, the more profitable of the two periods. In effect, forward contracting moves supplies towards the present, leading to a more efficient allocation of the capacity. In the limit, when a given overall stock is sold arbitrarily frequently, firms have a large number of forward openings to race for the more profitable spot markets. The race ends when all spot markets are equally profitable, i.e., when the allocation is perfectly competitive, as in Hotelling (1931).

We also find that the competitive pressure from forward contracting is somewhat alleviated when firms have resource stocks of different sizes. The smaller firm can credibility use the forward market to increase its presence in the earlier (more profitable) markets because it knows that the large firm will react by reallocating part of its stock to later markets in an effort to soften competition. Forward contracting will then play a "stretching" role in equilibrium: the small firm will increase its deliveries to earlier periods and so will the large firm to later periods. In the simplest (two-period) case, for example, the smaller firm can commit to exhaust early by contracting its entire stock. The larger firm then has no contracting incentives, and hence, the prisoners' dilemma from contracting is greatly diminished (in fact the small firm strictly benefits from the forward market in that it allows it to implement its most profitable, i.e., Stackelberg, outcome). In general, the larger firm has contracting incentives that decline over time and vanish entirely after the small firm exit from the market. In this asymmetric equilibrium, firms can sustain some oligopoly rents along a depletion path that has qualitatively similar phases as those in Salant's (1976),[3] although our equilibrium is considerably more competitive.

Our strategy of exposition is to start (in Section 2) with a two-period model illustrating both of the above symmetric and asymmetric equilibria. While helpful in explaining the basic mechanism, the extensive form of the two-period model is somehow incomplete, because firms should be able to choose how long the market interaction lasts in equilibrium. For example, firm i may respond to firm j's heavy contracting in period t by avoiding own contracting at t and allocating more capacity to a less contracted period $t + 1$ instead. This difference in extensive form is an important difference to the basic Allaz and Vila (1993) model where firms are trapped to face the prisoners' dilemma in a

---

[3]Salant (1976) considers a game in which a large supplier and a fringe of competitive suppliers choose simultaneously their entire production path at time zero. He shows that there will be two distinctive phases in equilibrium: a "competitive" phase with both type of players serving the market followed by a monopoly phase in which only the large supplier serves the market.

particular spot market.

In Section 3, we set up the general version of the model where deliveries and future contract positions are chosen on a period-by-period basis depending on current physical stocks and positions inherited from the past. In section 4, we first present a discrete-time version of the model and characterize the properties of the subgame-perfect equilibrium. We also describe the contracting dynamics showing that contract positions are altered for all future dates in each forward market interaction. Then, we solve the continuous-time limit of the discrete model for the symmetric case and show how the equilibrium path converges to perfectly competitive path. In Section 5, we describe the asymmetric-stock case within the general framework showing how the small firm's commitment to sell early arises through aggressive contracting. In the concluding remarks, we discuss why collusion cannot be sustained in this setting.

We are aware that our results may not apply to many of the more conventional non-renewal resources (e.g., oil, copper, etc.) because (overall) stock depletion is not as nearly evident as envisioned by Hotelling (1931). There are other oligopolistic commodity markets, however, where we observe not only important forward trading activity but also that stock depletion enters into today's decisions (as indicated by the evolution of current prices, for example). A good example are markets for storable pollution permits; and in particular, the one created under the US Acid Rain Program in 1995. In order to gradually reach the long-term emissions goal of the acid rain legislation firms were allocated a stock permits that is expected to be depleted around 2012 (Ellerman and Montero, 2007). Another example is the depletion of water rights for hydropower development in rapidly growing electricity systems (e.g., Chile's central interconnected system).[4]

We conclude this introductory section with a brief discussion of how this research relates to three strands of literature. First, our work is closely related to the basic exhaustible-resource theory under oligopolistic market structure. This literature has focused on developing less restrictive production strategies for firms (from "path" to "decision rule" strategies)[5] and also on including more realistic extraction cost structure (towards stock-dependent costs).[6] None of the papers in this literature explicitly con-

---

[4]There are also electricity markets where hydro stocks are actively traded in forward markets and have features of an exhaustible resource. See, e.g., Kauppi and Liski (2008).

[5]Loury (1986), Polansky (1992), and Lewis and Schmalensee (1980) use path strategies; Salo and Tahvonen (2001), for example, use decision-rule strategies. For a recent survey on the Hotelling model and its extensions, see Gaudet (2007).

[6]Salo and Tahvonen (2001) solve their model with stock-dependent costs, so that the overall amount

sider the effect of forward trading on the equilibrium path. However, it is interesting that our resource-depletion path is qualitatively similar to that in Salant (1976) which also has two distinct phases. In Salant's model, there is a large supplier and fringe of competitive suppliers. All suppliers are active in the competitive phase, which is followed by a monopoly phase where only the large firm is active. Forward contracting among asymmetric firms leads to a qualitatively similar equilibrium pattern, although the mechanism is very different as well as the degree of competition arising from a given division of stocks.

Second, there is a recent literature on organization of trade in dynamic oligopolistic competition under capacity constraints (e.g., Dudey, 1992; Biglaiser and Vettas, 2005; Bhaskar, 2006). These papers focus on dynamic price competition and also on the efficiency losses and changes in division of surplus caused by strategic buyers. We depart from this literature by assuming non-strategic but forward looking buyers, and we consider quantity competition in two dimensions (spot and forward markets). Our result that the firm with smaller capacity sells first and at higher prices sounds similar to Dudey's (1992) but is, in fact, quite different. In our case the large firm is active throughout the equilibrium and makes larger profits overall; the small firm is only free-riding on the large firm's market power, much the same way the fringe is free-riding on the large firm's market power in Salant (1976).

Third, there is a literature on forward trading starting with Allaz and Vila (1993) who analyze a static Cournot market. Mahenc and Salanie (2005) show that price competition can reverse the effect of forward trading on competition. Liski and Montero (2006) explain that forward contracting by a firm can be seen as strategic investment in firm's own production, which explains the dependence of implications on the form of competition;[7] it is clear that our current model would produce different results under price competition. Liski and Montero (2006) also develop a repeated interaction model of forward contracting, and this modeling approach is also used in the current paper. There is also a recent empirical literature looking at the effect of forward contracting

---

of the resource used is endogenously determined in equilibrium. In this sense, the resource is only economically exhausted. In our model, the resource is physically exhausted as the cost of using it is independent of the stock level. We leave it open for future research how replacing physical capacity with economic capacity would alter the contracting incentives.

[7]Selling forward contracts is a tough investment in the sense that it lowers the rival's profit all else equal; thus, the strategic-investment models of Bulow et al. (1985) and Fudenberg and Tirole (1984) predict that firms over-invest in forwards (i.e., go short) when they compete in quantities and under-invest in forwards (i.e., sell fewer forwards or go long) when they compete in prices.

on the performance of some oligopoly markets, in particular, electricity markets (e.g., Wolak, 2000; Bushnell *et al.*, 2008).

2 Two-period illustration

The implications of forward contracting for the equilibrium of a depletable-stock oligopoly can be best explained by first considering a simple example with only two periods and then extending the analysis to the general case in which the number of periods is endogenously determined. This section will also introduce the notation and assumptions that will be used throughout the paper. We progress towards the general model assuming first two symmetric firms. Then, in Section 2.3, we allow firms to have resource stocks of different sizes.

2.1 Notation and assumptions

Consider two symmetric firms (i, j), each holding a stock of a perfectly storable homogenous good, denoted by $s^i = s^j$, to be sold in two periods $(t = 1, 2)$. That the firms will sell their stocks in exactly in two periods requires a restriction on stocks which we explain below. In the general case, where the number of periods is endogenously determined, the stock left for the last two periods is always consistent with exhaustion in the last period, so that no stock is left unused. There are no production (or extraction) costs other than the shadow cost of not being able to sell tomorrow what is sold today. Firms discount future profits at the common discount factor $\delta < 1$.

Firms attend the spot market in both periods $t = 1, 2$ simultaneously by choosing quantities q_t^i and q_t^j.[8] For simplicity, we assume that the spot price at t, which is denoted by p_t^s, is given by the linear inverse demand function $p_t^s = p^s(q_t^i + q_t^j) = a - (q_t^i + q_t^j)$. Firms are also free to simultaneously buy or sell forward contracts that call for delivery of the good at any of the spot markets that follow.

For each period we assume a two-stage structure: the forward market precedes the spot market. In a forward market, firms can take positions for any future spot market, including the present period spot market (in this two period illustration no spot markets will open after $t = 2$). Forward contracts by firm i at period $t = 1$ for the first and second spot markets are denoted by $f_{1,1}^i$ and $f_{1,2}^i$, respectively. Similarly, forward contracts at

---

[8]In this two-period example, we find it convenient to call periods by 1 and 2; in the general model, periods run from 0 to infinity.

$t = 2$ for period 2 are denoted by $f_{2,2}^i$. We adopt the convention that $f^i > 0$ when firm i is selling forward contracts (i.e., taking a short position) and $f^i < 0$ when is buying forwards (i.e., taking a long position).[9] We further assume that forward positions are observable and the delivery of contracts is enforceable.[10] For clarity, it may be useful to think of forward contracts as physical delivery commitments, although the results do not depend on this, i.e., contracts can be purely financial (as in Liski and Montero [2006]). Note that while position $f_{1,1}^i$ calls for delivery of the good at $t = 1$, position $f_{1,2}^i$ need not be equal to the actual delivery at $t = 2$ since the forward market at $t = 2$ allows the firm to change its overall position for the spot market at $t = 2$. For example, firm i can nullify its overall forward position at $t = 2$ (i.e., $f_{1,2}^i + f_{2,2}^i = 0$) by buying/selling $f_{2,2}^i = -f_{1,2}^i$. The forward price at t for delivery at $\tau \geq t$ is denoted by $p_{t,\tau}^f$.

To assure that in equilibrium stocks are sold in two periods for any forward contracting profile, the total stock must satisfy[11]

$$a(1 - \delta) \leq s^i + s^j \leq \frac{a}{2}(2 - \delta - \delta^2). \tag{1}$$

In fact, if $a(1 - \delta) < s^i + s^j$, perfectly competitive agents will sell their stocks in just one period. If, on the other hand, $s^i + s^j > a(2 - \delta - \delta^2)/2$, a monopoly holding both stocks would find it optimal to exhaust in three or more periods. The equilibrium rate of extraction will be bounded by these two market structures, so condition (1) assures depletion in just two periods.

2.2 Equilibrium

To facilitate the exposition, suppose for a moment that $f_{1,2}^i = f_{2,2}^i = 0$, so that firms sell forwards only for the first spot market. The equilibrium outcome derived under this assumption will be equivalent in terms of physical deliveries and payoffs to the outcome derived when $f_{1,2}^i$ and $f_{2,2}^i$ are unconstrained. The reason is that the deliveries in the first period determine what is left to be sold in the second period, i.e., $q_2^i = s^i - q_1^i$, so

---

[9]In equilibrium, the possibility of taking a long position is not used since forward positions can be interpreted as strategic investments in firm's own production, and these investments will be positive (i.e., positions will be short) as long as firms compete in quantities. However, it is important to allow for this possibility, because otherwise firms might be able to commit to aggressive behavior in some future spot market by the fact the positions cannot be adjusted downwards.

[10]The assumptions for the contract market are the same as in Allaz and Vila (1993), Mahenc and Salanie (2005), and Liski and Montero (2006).

[11]Note that this is particular to the two-period model. If the stock is to be depleted in three or more periods the monopoly and competitive solution will be of different duration.

that the size of the stocks constrains firms' actions and, thus, there will be no strategic decisions at $t = 2$. We can therefore focus on strategic interaction at period $t = 1$. Working backwards, consider first the spot subgame in $t = 1$. Given the forward contract commitments $f_{1,1}^i$ and $f_{1,1}^j$ made in the forward stage, firm i's present-value payoff from sales at $t = 1$ is given by

$$\pi_1^{s,i} = p^s(q_1^i + q_1^j)(q_1^i - f_{1,1}^i) + \delta p^s(q_2^i + q_2^j)q_2^i$$

Since the firm has already pocketed the revenue from forward contracts, it is selling only $q_1^i - f_{1,1}^i$ to the spot market at $t = 1$.

Because of the capacity constraint $s^i = q_1^i + q_2^i$, the subgame that starts at the spot market in $t = 1$ reduces to a static (Nash-Cournot) game of simultaneous choice of q_1^i and q_1^j. Firm i's best response to q_1^j (and q_2^j) satisfies the intertemporal optimization principle that discounted marginal revenues should be equalized across periods, that is,

$$a - 2q_1^i - q_1^j + f_{1,1}^i = \delta(a - 2q_2^i - q_2^j). \tag{2}$$

Solving, we obtain the (subgame-perfect) equilibrium allocation

$$q_1^i(f_{1,1}^i, f_{1,1}^j) = \left(a(1 - \delta) + 3\delta s^i + 2f_{1,1}^i - f_{1,1}^j\right) \frac{1}{3(1 + \delta)} \tag{3}$$

$$q_2^i(f_{1,1}^i, f_{1,1}^j) = \left(-a(1 - \delta) + 3s^i - 2f_{1,1}^i + f_{1,1}^j\right) \frac{1}{3(1 + \delta)}. \tag{4}$$

Before moving to the forward subgame, it is useful to see how the contract coverage affects the intensity of the spot competition. If firms sign no contracts, i.e., $f_{1,1}^i = f_{1,1}^j = 0$, we obtain the pure-spot oligopoly equilibrium. Unlike the perfectly competitive equilibrium where spot prices are the same in present value (i.e., $p_1^* = \delta p_2^*$),[12] in pure-spot oligopoly prices decline in present value over time:

$$p_1^s > \delta p_2^s,$$
$$p_1^s > p_1^* \text{ and } p_2 < p_2^*.$$

As can be seen from (2), this derives directly from the equilibrium condition that marginal revenues go up at the rate of interest. In other words, the oligopolists depart from competitive pricing by shifting production from the present to the future.[13]

When firms go short in the forward market, $f_{1,1}^i > 0$ and $f_{1,1}^j > 0$, the spot market becomes more competitive in that firms are credibly committing more production to the

---

[12]The perfectly competitive total deliveries are $q_1^* = [a(1 - \delta) + \delta(s^i + s^j)]/(1 + \delta)$ and $q_2^* = s - q_1^*$.

[13]This observation was already made by Hotelling (1931) for a monopoly.

present. This can be seen from condition (2): contracts increase firms' marginal revenues making them to behave more aggressively in the spot market. In fact, if $f_{1,1}^i = f_{1,1}^j = a(1-\delta)/2$, the perfectly competitive solution is implemented. Conversely, if firms go long in the forward market, i.e., $f_{1,1}^i, f_{1,1}^j < 0$, the spot market becomes less competitive; and when $f_{1,1}^i = f_{1,1}^j = -a(1-\delta)/4$, the monopoly solution is implemented.[14]

Obviously, in equilibrium firms do not trade any arbitrary amount of forwards. Firms, speculators and consumers are assumed to have rational expectations in that they correctly anticipate the effect of forward contracting on the spot market equilibrium. Thus, in deciding how many contracts to buy/sell in the forward market at $t = 1$, firm i evaluates the following payoff

$$\pi_1^i = p_{1,1}^f f_{1,1}^i + \pi_1^{s,i}(f_{1,1}^i, f_{1,1}^j)$$

where $\pi_1^{s,i}(f_{1,1}^i, f_{1,1}^j)$ are the spot (subgame-perfect) profits. Rearranging terms, firm i's overall profits as a function of $f_{1,1}^i$ and $f_{1,1}^j$ can be written as

$$\pi_1^i = (p_{1,1}^f - p_1^s)f_{1,1}^i + p_1^s q_1^i(f_{1,1}^i, f_{1,1}^j) + \delta p_2^s q_2^i(f_{1,1}^i, f_{1,1}^j)$$

where $p_t^s = p^s(q_t^i(f_{1,1}^i, f_{1,1}^j) + q_t^j(f_{1,1}^i, f_{1,1}^j))$ for $t = 1, 2$. As in Allaz and Vila (1993), the arbitrage payoff $(p_{1,1}^f - p_1^s)f_{1,1}^i$ is zero since speculators and/or consumers share the same information as producers and thus $p_{1,1}^f = p_1^s$. Therefore, firms are left with the contract-coverage dependent Cournot profit from the two periods, $p_1^s q_1^i + \delta p_2^s q_2^i$.

Solving, we obtain firm i's best response function in the forward subgame

$$f_r^i(f_{1,1}^j) = \frac{a}{4}(1-\delta) - \frac{1}{4}f_{1,1}^j \tag{5}$$

which, after imposing symmetry, leads to the equilibrium forward sales

$$f_{1,1}^i = f_{1,1}^j = \frac{a}{5}(1-\delta) \tag{6}$$

and equilibrium deliveries[15]

$$q_1^i = \frac{1}{3(1+\delta)}\left(\frac{6}{5}a(1-\delta) + 3\delta s^i\right) \tag{7}$$

$$q_2^i = \frac{1}{3(1+\delta)}(-\frac{6}{5}a(1-\delta) + 3s^i). \tag{8}$$

---

[14]The monopoly allocations are $q_1^m = [a(1-\delta) + 2\delta(s^i + s^j)]/2(1+\delta)$ and $q_2^m = s - q_1^m$.

[15]Note that since $\delta < 1$, $f_{1,1}^i < q_1^i$.

The mere opportunity of trading forward has created a prisoner's dilemma for the two firms bringing them closer to competitive pricing. Forward trading makes both firms worse off relative to the case in which they stay away from the forward market. If firm j does not trade any forwards, then firm i has all the incentives to make forward sales (i.e., $f_{1,1}^i > 0$) as a way to allocate a larger fraction of its total stock s^i to the first period, which is the most profitable of the two (recall that $p_1^s > \delta p_2^s$). In the reproducible commodity (Cournot) game, forward trading allows a firm to capture Stackelberg profits —given that the other firm has not sold any forwards— by credibly committing in advance to the Stackelberg production. In our depletable-stock game, forward trading allows a firm to capture Stackelberg profits by committing a larger fraction of its overall stock to the first period. This is the pro-competitive effect of forward contracts —first documented by Allaz and Vila (1993) for reproducible goods.

Let us now relax the assumption that contract positions can only be taken for the first spot market (i.e., $f_{1,1}^i$, $f_{1,2}^i$ and $f_{2,2}^i$ are unconstrained).

Proposition 1 *In the two-period equilibrium, symmetric equilibrium deliveries are given by (7) and (8), and equilibrium forward positions satisfy*

$$f_{1,1}^i - \delta f_{1,2}^i = \frac{a}{5}(1 - \delta).$$

For the proof, let us work backwards and consider the last spot subgame ($t = 2$): firms can only sell what is left of the stocks so there are no decisions to make, other than meeting delivery commitments and putting the rest to the spot market; under the constraint on initial stocks (1), firms do not find it profitable to extend the sales path by an additional period. The same capacity constraint dictates behavior at the forward subgame at $t = 2$. Selling contracts at this point cannot change delivery allocations and thus $f_{2,2}^i = 0$.[16] Consider then the first spot subgame ($t = 1$), where the delivery allocation is still open. Given what has been contracted for the two periods ($f_{1,1}^i$ and $f_{1,2}^i$), the condition equalizing present-value marginal revenues must hold,

$$a - 2q_1^i - q_1^j + f_{1,1}^i = \delta(a - 2q_2^i - q_2^j + f_{1,2}^i), \text{ or}$$
$$a - 2q_1^i - q_1^j + (f_{1,1}^i - \delta f_{1,2}^i) = \delta(a - 2q_2^i - q_2^j)$$

Therefore, the payoff-relevant variables in the forward subgame are not the individual positions $f_{1,1}^i$ and $f_{1,2}^i$ but the composite position $f_{1,1}^i - \delta f_{1,2}^i$. By the same backward induction arguments laid out before, in equilibrium firms will choose $f_{1,1}^i$ and $\delta f_{1,2}^i$ as to

---

[16]More precisely, contacting at this stage is payoff-irrelevant, so we can set $f_{2,2}^i = f_{2,2}^j = 0$.

satisfy $f_{1,1}^i - \delta f_{1,2}^i = a(1-\delta)/5$, which leads to the same equilibrium delivery allocation found earlier.

It is irrelevant how firms trade in the contract market as long as their overall position satisfies $f_{1,1}^i - \delta f_{1,2}^i = a(1-\delta)/5$ (and, of course, $f_{1,1}^i \leq q_1^i$ and $f_{1,2}^i \leq q_2^i$, where q_1^i and q_2^i are the equilibrium quantities given by (7) and (8), respectively). For example, firm j can fully contract its period-two deliveries (i.e., $f_{1,2}^j = q_2^j$) and simultaneously take a short position in period-one spot market equal to $f_{1,1}^j = a(1-\delta)/5 + \delta q_2^j$. Firm i, on the other hand, might just take a short position in period one equal to $f_{1,1}^i = a(1-\delta)/5$, or alternatively, go long in period two in an amount equal to $f_{1,2}^i = -a(1-\delta)/5\delta$.

This analysis of the symmetric case tells us that in the general model it is sufficient to start working backwards from the next to the last period. We can thus ignore the forward sales to the very last spot market and set $f_{1,2}^i = f_{2,2}^i = 0$ as a perfectly valid backward induction hypothesis.

2.3 Asymmetric stocks

Maintaining assumption (1) that ensures the exhaustion of the overall stock in just two periods, we now look at the case in which stocks are of different sizes. Letting firm j be the smaller of the two firms, we will study how the equilibrium in two periods changes as we move from $s^j = 0$ to the symmetric case $s^j = s^i > a(1-\delta)/2$. Understanding this is important for the general model because even though firms' stocks may be very similar at the start, asymmetries are necessary large near depletion.

The case $s^j = 0$ is immediate. A monopolist (i.e., firm i) will never sign forward contracts because this would only introduce more competition to the spot market (recall that selling forwards has the same competition effect as selling part of the stock to a fringe of competitive suppliers). Now, to understand how stock asymmetries affect the equilibrium path when both firms hold some initial stock, it is useful to recall what firms seek to implement through forward markets: if one firm does not sell forwards, the other can achieve Stackelberg profits by entering the forward market. Consider first the Stackelberg outcome for the larger firm. Firm i's first-best is to induce $q_1^j = s^j$ and $q_2^j = 0$, i.e., it is optimal for i to let j exhaust in period 1, if

$$s^j \leq \frac{1}{4}a(1-\delta).\tag{9}$$

Thus, when j is small enough, i will let j to sell only to the more profitable first period, even if i could commit part of its sales before j takes any action.[17]

---

[17]The proof is immediate and ignored here. Set $f_{1,1}^j = 0$ and solve i's best response in the forward

Consider then firm j's Stackelberg outcome. If allowed to move first, j would like to sell its entire stock in the first period as long as

$$s^j \leq \frac{1}{2}a(1-\delta). \tag{10}$$

It is intuitively clear that when we consider j's own stock, j's first-best threshold for leaving capacity for the less profitable second period is larger than in (9).

These inequalities imply that both firms prefer j's early exhaustion in period $t = 1$ when j is small enough such that (9) holds. Thus, both firms' best-responses to no contracting by the other firm is not to contract. In equilibrium, when (9) holds, j's small stock gives it commitment to sell only the more profitable market, which in effect solves the prisoners' dilemma problem presented by the forward market. However, j can use the forward market for extending its commitment to sell early even when its stock exceeds the level identified by (9) as stated next:

Proposition 2 *If*

$$\frac{1}{4}a(1-\delta) \leq s^j \leq \frac{5-2\sqrt{2}}{5}a(1-\delta) \tag{11}$$

and $s^i + s^j$ satisfies (1), then, there is a two-period equilibrium where the larger firm does not contract at all (i.e., $f_{1,1}^i = 0$) and the smaller firm commits to sell only in the more profitable first period by contracting $f_{1,1}^j$ according to

$$s^j \geq f_{1,1}^j \geq f_{\min}^j(s^j) \equiv \frac{4}{3}s^j - \frac{1}{3}a(1-\delta)$$

Proof. See Appendix.[18] ∎

Proposition 2 says that j needs to contract at least $f_{\min}^j(s^j)$ to achieve its first-best. Note that if j contracts nothing when its stock is above the threshold in (9), i could achieve its first-best by contracting which would shift part of j's sales to $t = 2$. But j can prevent this by making the spot market in $t = 1$ less profitable to i through its own contracting —minimum contracting $f_{\min}^j(s^j)$ is calculated as a position that keeps i unwilling to sign contracts. Contracting more than $f_{\min}^j(s^j)$, e.g., $f_{1,1}^j = s^j$, is more than enough to keep i away from the forward market until (10) holds as an equality. This "excessive" contracting, $s^j - f_{\min}^j(s^j)$, does not affect profits since j's entire stock is sold in all cases in the first market. When j's stock is above the upper limit in (11), i's first-best is to make j to deliver also at $t = 2$ by selling contracts to $t = 1$. Then,

market and then use the chosen position to solve for equilibrium deliveries. Alternatively, one can change the timing in the pure spot market model to find the Stackelberg allocations.

[18]Note that $(5 - 2\sqrt{2})/5 = 0.434$.

firm j contracts according to (5), i.e., $f_r^j(\ f_{1,1}^i) > 0$, which leads to j delivering in both periods. But if j is expected to deliver in both periods, firm i's best contracting response must also be given by (5). Therefore, when both firms are active in both periods the only possible equilibrium is the symmetric one with both firms signing $a(1-\delta)/5$ in the contracting stage.[19]

This two-period model illustrates how asymmetries can help firms to escape the competitive pressure introduced by the forward market. In fact, the smaller firm can greatly benefit from the forward market in that it may be able to implement its first-best (Stackelberg) solution (unlike the larger firm which has nothing to gain from the opening of the forward market). A similar result, although not so advantageous for the smaller firm, will emerge in the general model that we study next. The two-period model also illustrates how forward contracting reinforces the fact that asymmetric firms will generally exit the market at different times. In our two-period model forward trading expands the stock threshold for which firm j would exit the market in the first period from $s_j < a(1-\delta)/3$, the threshold under pure-spot trading, to $s_j < (5 - 2\sqrt{2})a(1-\delta)/5$. This is because forward contracting plays an "stretching" role in equilibrium when firms are of different sizes: the small firm increases its deliveries to earlier periods ($t = 1$ in the example above) while large firm does the same to later periods (where the smaller firm is absent).[20]

3   The model

In the original reproducible-good model of Allaz and Vila (1993), the model structure is such that all forward markets open before any spot delivery takes place. This timing implies that firms are trapped to face the prisoners' dilemma in a single spot market

---

[19] Note that the symmetric contracting equilibrium extends below the threshold $(5 - 2\sqrt{2})a(1-\delta)/5$ in (11). In fact, for $s^j \in [2a(1-\delta)/5, (5 - 2\sqrt{2})a(1-\delta)/5]$ both equilibria coexist (and perhaps with one in mixed strategies) but the asymmetric equilibrium Pareto dominates (i.e., better for both firms) the symmetric one. Likewise, the asymmetric equilibrium extends above $(5 - 2\sqrt{2})a(1-\delta)/5$ up to the threshold $a(1-\delta)/2$ in (10); within this range there is no Pareto ranking of equilibria, however. In any case, this multiplicity is specific to the two-period setting and is inconsequential more generally because even small asymmetries in initial stocks will generate large asymmetries in the future as the smaller firm exhausts its stock.

[20] To see the latter consider any s^j such that under pure-spot trading firm j would attend both periods ($t = 1, 2$) but that with forward trading would only attend $t = 1$. Firm i's deliveries in $t = 1$ under pure-spot-trading and with forward-contracting are, respectively, $q_1^{i(s)} = [a(1-\delta) + 3\delta s^i]/3(1+\delta)$ and $q_1^{i(f)} = [a(1-\delta) + 2\delta s^i - s^j]/2(1+\delta)$. Then $q_1^{i(f)} < q_1^{i(s)}$ (and $q_2^{i(f)} > q_2^{i(s)}$) iff $s^j > a(1-\delta)/3$, which precisely indicates the range where j attends both periods in pure-spot equilibrium.

as many times as there are forward market openings. This extensive form is critical to the result that forward markets enhance competition. It is not reasonable to assume that all contracting takes place before stock consumption begins; contracts should be traded as stock depletion progresses. This opens up possibilities that are not present in the two period model. For example, firms are not by definition trapped to deliver their stocks in some given periods but, rather, free to open new spot markets as a response to heavy contracting by other firms. Therefore, in the true stock-depletion equilibrium with contracting, the time horizon of consumption is endogenously determined. Our plan is to introduce such a general model structure. We introduce the model in discrete time so that the extensive form of the game becomes clear, and then by letting the period length vanish we characterize the continuous time version. The continuous time limit identifies the most competitive sales path of a given pair of resource stocks in the sense that there are no *a priory* restrictions on firms' possibilities to trade forward contracts.

3.1 Strategies and payoffs

The discrete-time framework can be described as follows. Periods run from zero to infinity, and each period has the same two-stage structure as in the two-period illustration above. In the following we describe the states and the payoff-relevant variables for each state separately. In any given period t, the spot market opens with contract commitments made at earlier dates $0, 1, 2, .., t-1$ plus the commitments made at the forward market t. For firm i, we denote the commitments made prior to t for market t by F_t^i (the existing aggregate position for market t) and contract sales made at t by $\{f_{t,\tau}^i\}_{\tau \geq t}$. Thus, the contract coverage of firm i at spot market t is $F_t^i + f_{t,t}^i$. We define the state at the beginning of period t forward subgame as

$$I_t = (s_t^i, s_t^j, \mathbf{F}_t^i, \mathbf{F}_t^j)$$

where $\mathbf{F}_t^i = (F_t^i, F_{t+1}^i, F_{t+2}^i, ...)$ denotes aggregate positions that firm i is holding for all future dates at t. The state at period t spot subgame is then $(I_t, \mathbf{f}_t^i, \mathbf{f}_t^j)$ where we adopt the notation $\mathbf{f}_t^i = (f_{t,t}^i, f_{t,t+1}^i, f_{t,t+2}^i, ...)$ to denote what firm i contracted at period t forward market opening. We are interested in equilibria where strategies depend on the current state only and therefore look for forward-contracting strategies that are functions of the form

$$\mathbf{f}_t^i = \mathbf{f}^i(I_t).$$

Given the state at period t forward subgame, this vector-valued function determines the forward transactions made for all periods $\tau \geq t$ at period t. Similarly, we look for spot

14

market strategies of the form

$$q_t^i = q^i(I_t, \mathbf{f}_t^i, \mathbf{f}_t^j).$$

Deliveries to market t depend on the remaining stocks, positions inherited from previous periods, and contracting made at period t.

Let $V^i(I_t)$ denote firm i's equilibrium payoff at the forward stage, in the beginning of period t when state is I_t. Let $\pi^i(I_t, \mathbf{f}_t^i, \mathbf{f}_t^j)$ denote the firm's payoff at the spot stage in the same period t, given the contract commitments $(\mathbf{f}_t^i, \mathbf{f}_t^j)$ made in the forward stage of t. Firm i's best response $\mathbf{f}_t^i$ to $\mathbf{f}_t^j$ defines $V^i(I_t)$ as

$$V^i(I_t) = \max_{\mathbf{f}_t^i}\{\sum_{\tau=t} \delta^{\tau-t} p_{t,\tau}^f f_{t,\tau}^i + \pi^i(I_t, \mathbf{f}_t^i, \mathbf{f}_t^j)\}, \tag{12}$$

where

$$\pi^i(I_t, \mathbf{f}_t^i, \mathbf{f}_t^j) = \max_{q_t^i}\{p_t^s \cdot (q_t^i - F_t^i - f_{t,t}^i) + \delta V^i(s_t^i - q_t^i, s_t^j - q_t^j, \mathbf{F}_{t+1}^i, \mathbf{F}_{t+1}^j)\}. \tag{13}$$

We can express the equilibrium payoff at time zero as

$$V^i(I_0) = \sum_{\tau=0} \delta^\tau p_\tau^s \cdot (q_\tau^i - \sum_{t=0}^\tau f_{t,\tau}^i) + \sum_{\tau=0}^\tau \sum_{t=0} \delta^t p_{t,\tau}^f f_{t,\tau}^i.$$

Since all parties share the same information, there is no arbitrage profit: $\delta^t p_{t,\tau}^f = \delta^\tau p_\tau^s$ for all (t, τ). Therefore,

$$V^i(I_0) = \sum_{\tau=0} \delta^\tau p_\tau^s q_\tau^i.$$

where quantities and prices are evaluated along the equilibrium path. At some $t > 0$, we can express the equilibrium payoff as

$$V^i(I_t) = p_t^s \cdot (q_t^i - F_t^i) + \delta V^i(I_{t+1}). \tag{14}$$

Effectively, we are finding contracting profiles $(\mathbf{f}_t^i, \mathbf{f}_t^j)$ starting with $\mathbf{F}_0^i = \mathbf{F}_0^j = 0$ and generating the above values such that no shot-deviations are profitable.

3.2   Spot subgames

In each spot subgame t, (interior) equilibrium quantities delivered satisfy

$$\frac{\partial p_t^s}{\partial q_t^i}(q_t^i - f_{t,t}^i - F_t^i) + p_t^s = \delta\frac{\partial}{\partial s_{t+1}^i}V^i(I_{t+1}') \tag{15}$$

for i, j. We write I_{t+1}' for the state at the spot stage to distinguish it from the state at the forward stage: due to contracting for future markets at t, the state changes from

$$I_{t+1} = (s_{t+1}^i, s_{t+1}^j, \mathbf{F}_{t+1}^i, \mathbf{F}_{t+1}^j)$$

15

to

$$I'_{t+1} = (s^i_{t+1}, s^j_{t+1}, (F^i_{t+1} + f^i_{t,t+1}, F^i_{t+1} + f^i_{t,t+2}, ...), (F^j_{t+1} + f^j_{t,t+1}, F^i_{t+1} + f^i_{t,t+2}, ...), ...)$$

between the forward and spot markets at t. The difference in payoffs between the two stages is just

$$V^i(I_{t+1}) - V^i(I'_{t+1}) = \sum_{\tau=t+1} \delta^{\tau-t} p^s_\tau f^i_{t,\tau}, \tag{16}$$

which is the equilibrium value of forward sales made at t.

Note that if there were no contracting, condition (15) for firm i would be satisfied when marginal revenues from different periods are equalized in present value. When there is contracting, $\partial V^i(I'_{t+1})/\partial s^i_{t+1}$ does not equal the equilibrium marginal revenue from the next spot sale but, rather, the value of the stock at the beginning of the next forward subgame.

3.3   Forward subgames

Consider the choices in the forward stage at period t. Recall that the firms are simultaneously choosing $\mathbf{f}^i_t = (f^i_{t,t}, f^i_{t,t+1}, f^i_{t,t+2}, ...)$ so that in principle there is a very large set of first-order conditions. The (interior) sale $f^i_{t,t} > 0$ by firm i for the current spot market satisfies

$$\frac{\partial q^i_t}{\partial f^i_{t,t}} \{ \frac{\partial p^s_t}{\partial q^i_t}(q^i_t - F^i_t) + p^s_t + \delta \frac{\partial}{\partial s^i_{t+1}} V^i(I_{t+1}) \frac{\partial s^i_{t+1}}{\partial q^i_t} \} \tag{17}$$

$$+ \frac{\partial q^j_t}{\partial f^i_{t,t}} \{ \frac{\partial p^s_t}{\partial q^j_t}(q^i_t - F^i_t) \} \tag{18}$$

$$+ \frac{\partial q^j_t}{\partial f^i_{t,t}} \{ \delta \frac{\partial}{\partial s^j_{t+1}} V^i(I_{t+1}) \frac{\partial s^j_{t+1}}{\partial q^j_t} \} = 0 \tag{19}$$

First line (17) gives the loss in revenues due to the fact that i's own behavior becomes more competitive. To illustrate, assume no future contracting at t, i.e., assume $f^i_{t,t+1} = f^i_{t,t+2} = ... = 0$. Then, $V^i(I_{t+1}) = V^i(I'_{t+1})$, and, by (15),

$$\frac{\partial p^s_t}{\partial q^i_t}(q^i_t - F^i_t) + p^s_t - \delta \frac{\partial}{\partial s^i_{t+1}} V^i(I_{t+1}) = \frac{\partial p^s_t}{\partial q^i_t} f^i_{t,t}, \tag{20}$$

so that the first line reduces to
$$\frac{\partial p^s_t}{\partial q^i_t} \frac{\partial q^i_t}{\partial f^i_{t,t}} f^i_{t,t} < 0.$$

16

In equilibrium, there will be contracting for future periods, $f_{t,t+\tau}^i > 0$ with $\tau \geq 1$, and this will affect the above loss in firm i's revenues. However, since a monopoly would always choose not to contract, $f_{t,t+\tau}^i = 0$ for all τ, the expression on line (17) must be negative in equilibrium.

Second line (18) is the 'strategic investment' effect in spot market t and thus positive. It measures the gain from shifting competitor j away from the current market. However, if firm j reduces supply today due to i's contracting, firm j must sell more in the future, otherwise it would not exhaust its capacity. This capacity substitution implies that the effect on last line (19) is negative.

Recall that at t firms are choosing not only $f_{t,t}^i$ but also $(f_{t,t+1}^i, f_{t,t+2}^i, ...)$. A positive sale at period t for period $\tau > t$, $f_{t,\tau}^i > 0$, needs to satisfy the first-order condition,

$$\frac{\partial q_t^i}{\partial f_{t,\tau}^i}\{\frac{\partial p_t^s}{\partial q_t^i}(q_t^i - F_t^i) + p_t^s + \delta\frac{\partial}{\partial s_{t+1}^i}V^i(I_{t+1})\frac{\partial s_{t+1}^i}{\partial q_t^i}\}$$

$$+\frac{\partial q_t^i}{\partial f_{t,\tau}^i}\{\frac{\partial p_t^s}{\partial q_t^j}(q_t^i - F_t^i) + \delta\frac{\partial}{\partial s_{t+1}^j}V^i(I_{t+1})\frac{\partial s_{t+1}^j}{\partial q_t^j}\}$$

$$+...$$

$$+\frac{\partial q_\tau^i}{\partial f_{t,\tau}^i}\{\frac{\partial p_\tau^s}{\partial q_\tau^i}(q_\tau^i - F_\tau^i) + p_\tau^s + \delta\frac{\partial}{\partial s_{\tau+1}^i}V^i(I_{\tau+1})\frac{\partial s_{\tau+1}^i}{\partial q_\tau^i}\}$$

$$+\frac{\partial q_\tau^i}{\partial f_{\tau,\tau}^i}\{\frac{\partial p_\tau^s}{\partial q_\tau^j}(q_\tau^i - F_\tau^i) + \delta\frac{\partial}{\partial s_{\tau+1}^j}V^i(I_{\tau+1})\frac{\partial s_{\tau+1}^j}{\partial q_\tau^j}\} = 0.$$

A marginal change in the equilibrium contracting for some future date $f_{t,\tau}^i$ has above-discussed effects (see (17)-(19)) for each period between t and τ.

4 Competitive outcome: symmetric stocks

In this section we use the above-discussed equilibrium conditions, symmetry, and the linear demand to solve for the equilibrium deliveries explicitly, first in discrete and then in continuous time. A main result of the paper follows: equilibrium allocations become perfectly competitive in the continuous time limit.

4.1 Deliveries in discrete time

Solving by backward induction, as shown in the Appendix, we find the symmetric equilibrium deliveries and contracting levels. The overall number of periods needed for symmetric stock exhaustion, denoted by T, depends on the size of the stocks. If the forward

markets were absent, the equilibrium delivery per firm in the next to the last market $T-1$, for example, would be a unique number independently of the overall number of periods, T. This can no longer hold when forward markets exist, because the delivery at $T-1$ depends on how many times firms have an opportunity to trade contracts for period $T-1$ deliveries before period $T-1$ opens. In this sense, the size of the stocks, which determines T and thereby the number for market openings for forwards, influences the actual deliveries in the last two periods.

Let $k=1,...,T-1$ denote the backward-induction step, and let $t_k=0,...,T-1$ denote the associated period in real time.

Proposition 3 *Let T be the last period of consumption in a symmetric equilibrium, starting with stocks $s_0^i = s_0^j$. Then, the equilibrium delivery is given by*

$$q_{t_k}^i = q_{t_k}^j = \{\frac{a}{3}[\sum_{h=1}^{k} \delta^{h-1} - k\delta^k][1 + \frac{T-k}{3+2(T-k)}] + \delta^k s_{t_k}^i\}\frac{1}{\sum_{h=0}^{k} \delta^h}. \tag{21}$$

Proof. See Appendix. ∎

For the economics of deliveries, it proves useful to rewrite (21) as

$$q_{t_k}^i = q_{t_k}^j = \{\frac{a}{3}[\sum_{h=1}^{k} \delta^{h-1} - k\delta^k] + \delta^k s_{t_k}^i\}\frac{1}{\sum_{h=0}^{k} \delta^h} + \frac{H(T,k)}{\sum_{h=0}^{k} \delta^h} \tag{22}$$

where

$$H(T,k) = \frac{a}{3}[\sum_{h=1}^{k} \delta^{h-1} - k\delta^k]\frac{T-k}{3+2(T-k)}.$$

Without forward markets, $H(T,k) = 0$, and the delivery per firm equals the path obtained in pure spot-sale equilibrium. The term $H(T,k)$ thus expresses directly how contracting increases supplies, compared to pure spot equilibrium, in a given period t_k that is preceded by $T-k$ forward market openings (at periods $0, 1, ..., t_k$), and followed by $k-1$ periods of deliveries (at $t_k, ..., T$). The term

$$\frac{T-k}{3+2(T-k)}$$

in $H(T,k)$ indicates how many times firms face the prisoners' dilemma from contracting, and the term

$$\frac{a}{3}[\sum_{h=1}^{k} \delta^{h-1} - k\delta^k]$$

in $H(T,k)$ weights the importance of the competitive pressure by taking into account what fraction of the remaining supply is at stake in the current market. For example, if T is very large and $k=1$, then $H(T,k)$ approaches

$$\frac{1}{6}a(1-\delta),$$

18

and deliveries become

$$\frac{1}{1+\delta}\left(\frac{a}{2}(1-\delta) + \delta s^i_{T-1}\right),$$

which equals the (symmetric) efficient delivery per firm in a two-period model (see foot-note 12).

4.2 Continuous-time limit

We have seen that the number of periods, or the size of the stocks, has an effect on the degree of competition along the equilibrium path. Alternatively, we can take the stocks as given, and vary the period length. Recall that when the period length is sufficiently large, any given initial holdings are consumed in just two periods in equilibrium, and the firms face the prisoners' dilemma from contracting only once. The depletion of the same holdings require increasingly many periods if the period length becomes shorter; in the limit, the two-period model is transformed into a continuous time version. In the latter, after any positive interval of time, firms face the prisoners' dilemma arbitrarily many times, but it is not *a priory* clear if the overall capacity constraint puts a limit to the competitive pressure. We will explore this next.

It proves useful to explain first how the period length can be incorporated into the standard spot sale equilibrium. Let Δ denote the period length and assume it takes three periods to exhaust the initial holdings in equilibrium. To be concrete, conditions

$$
\begin{aligned}
a - 2q^i_0 - q^j_0 &= \delta(a - 2q^i_1 - q^j_1), \\
a - 2q^i_1 - q^j_1 &= \delta(a - 2q^i_2 - q^j_2), \\
\Delta(q^i_0 + q^i_1 + q^i_2) &= s^i_0,
\end{aligned}
$$

for i, j must hold in equilibrium (marginal revenues equalized in present value, and stocks depleted). The conditions lead to the following first-period delivery:

$$q^i_0 = q^j_0 = \frac{a}{3}\left\{(1 + \delta - 2\delta^2) + \delta^2 \frac{s^i_0}{\Delta}\right\}\frac{1}{(1 + \delta + \delta^2)}.$$

More generally, if the symmetric pure-spot equilibrium lasts for T periods, then period t_k equilibrium delivery is

$$q^i_{t_k} = q^j_{t_k} = \left\{\frac{a}{3}\left[\sum_{h=1}^{k}\delta^{h-1} - k\delta^k\right] + \delta^k \frac{s^i_{t_k}}{\Delta}\right\}\frac{1}{\sum_{h=0}^{k}\delta^h},$$

where $k = 1, ..., T - 1$ as defined in the previous section. It thus clear that period length only scales the stock size in the expression for deliveries. But this same conclusion

holds for deliveries in the contracting equilibrium: the effect of contracts on deliveries, measured through $H(T,k)$ in (22), depends only on the number of times the market opens before and after t_k, but not on how short or long these openings are. Therefore, we can immediately rewrite the delivery rule (22) as follows, for a given period length:

$$q_{t_k}^i = q_{t_k}^j = \{\frac{a}{3}[\sum_{h=1}^k \delta^{h-1} - k\delta^k] + \delta^k \frac{s_{t_k}^i}{\Delta}\}\frac{1}{\sum_{h=0}^k \delta^h} + \frac{H(T,k)}{\sum_{h=0}^k \delta^h} \qquad (23)$$

Let τ denote the time used for consumption of stocks, and let r be the continuous time discount rate.

Proposition 4 *As $\Delta \to 0$, the symmetric subgame-perfect equilibrium deliveries approach the socially efficient deliveries at any given $t > 0$.*

Proof. Note that $T = \tau/\Delta$ is the number of discrete steps of size Δ associated with total consumption time τ. At time $t_k > 0$ when the stock is $s_{t_k}^i$ $(= s_{t_k}^j)$, the remaining time is $\tau - t_k$, and the implied induction step is

$$k = \frac{\tau - t_k}{\Delta} - 1.$$

Recall that $H(T,k)$ measures the impact of contracts on deliveries in (23). The spot market at $t_k > 0$ is preceded by t_k/Δ forward markets, when $\Delta = \tau/T$, implying that we can replace

$$T - k = t_k/\Delta$$

when evaluating $H(T,k)$ at time t_k. The continuous-time discount factor is $\delta = e^{-r\Delta}$. We can now write equilibrium deliveries at time t_k as follows

$$
\begin{aligned}
q_{t_k}^i = q_{t_k}^j &= \{\frac{a}{3}[\sum_{h=1}^{\frac{\tau-t_k}{\Delta}-1} e^{-r\Delta(h-1)} - (\frac{\tau - t_k}{\Delta} - 1)e^{-r(\tau-t_k-\Delta)}][1 + \frac{t_k/\Delta}{3 + 2t_k/\Delta}] \qquad (24)\\
&\quad + e^{-r\Delta(\tau-t_k-\Delta)}\frac{s_{t_k}^i}{\Delta}\}\frac{1}{\sum_{h=0}^k e^{-r\Delta h}}.
\end{aligned}
$$

Note how to read this expression: when the total time τ and time point t_k from the equilibrium path is fixed, we know what is the associated k. Obviously, given (s_0^i, Δ) is consistent with a particular τ. Whatever is the time point $t = t_k > 0$ before exhaustion, the deliveries must satisfy the above equation. In particular, it must hold in the limit $\Delta \to 0$, obtained from (24) for a fixed τ and $t = t_k$:

$$q_t^i = \frac{a}{2}\frac{(e^{r(\tau-t)} - 1 - r(\tau - t))}{e^{r(\tau-t)} - 1} + \frac{rs_t^i}{e^{r(\tau-t)} - 1}. \qquad (25)$$

20

(The limiting expression converges to a point on the equilibrium path since all time points $t < \tau$ are on the equilibrium path).

Consider then the socially optimal delivery starting with overall stock $s_t^i + s_t^j = s_t$ at time t. Denote the socially optimal total delivery by $q_{t'}^*$ at any time $t \leq t' \leq \tau$. It must satisfy

$$a - q_{t'}^* = (a - q_t^*)e^{r(t'-t)},$$

because socially optimal prices grow at the rate of interest over the depletion period $t \leq t' < \tau$. Solving for $q_{t'}^* = Q(q_t^*, t, t')$ and using the exhaustion condition

$$\int_t^\tau Q(q_t^*, t, t')dt' = s_t,$$

yields

$$q_t^* = a\frac{(e^{r(\tau-t)} - 1 - r(\tau - t))}{e^{r(\tau-t)} - 1} + \frac{rs_t}{e^{r(\tau-t)} - 1}. \tag{26}$$

Thus, equilibrium delivery per firm at each (s_t^i, s_t^j) given by (25) is equal to one half of the total socially efficient delivery q_t^* at $s_t^i + s_t^j = s_t$. ∎

Let us now go back to general first-order conditions to find the contracting path associated to this result. When $\Delta \to 0$, it must be the case that $f_{t,t}^i \to 0$ for any given $t > 0$: the cumulative contract positions F_t^i and F_t^j almost instantly converge to their equilibrium levels due to the infinitely large number of forward openings between 0 and $t > 0$. Then, in the limit, the first-order condition for $f_{t,t}^i$ must be consistent with the choice $f_{t,t}^i = 0$. With no further contracting taking place, the continuation value $V^i(I_{t+1})$ is only affected by actions at the spot stage, and hence, (equilibrium) contracting positions F_t^i and F_t^j must be consistent with spot market equilibrium condition (15) and $V^i(I'_{t+1}) = V^i(I_{t+1})$. The optimality of spot actions, given profiles F_t^i and F_t^j, requires

$$a - q_t^i - q_t^j - (q_t^i - F_t^i) = e^{-r\Delta}[a - q_{t+\Delta}^i - q_{t+\Delta}^j - (q_{t+\Delta}^i - F_{t+\Delta}^i)], \tag{27}$$

i.e., marginal revenues, after controlling for contract coverage, grow at the rate of interest.

Denoting the uncovered deliveries by $u_t^i = q_t^i - F_t^i$, condition (27) can be rewritten as

$$p_t^s - e^{-r\Delta}p_{t+\Delta}^s = u_t^i - e^{-r\Delta}u_{t+\Delta}^i \tag{28}$$

But from Proposition 4 we know that when $\Delta \to 0$ prices grow at the rate of interest (i.e., $p_t^s = e^{-r\Delta}p_{t+\Delta}^s$), which implies

$$u_t^i = e^{-r\Delta}u_{t+\Delta}^i \tag{29}$$

21

for i, j. In equilibrium, uncovered deliveries u_t^i also grow up at the rate of interest as $\Delta \to 0$. Furthermore, since $q_t^i \to 0$ as $t \to \tau$ (the exhaustion time), it must also hold that $u_t^i = q_t^i - F_t^i \to 0$ as $t \to \tau$. It then follows that $u_t^i = 0$ for all $t > 0$, that is, firms are fully contracted as soon as F_t^i and F_t^j have converged to their equilibrium level, which happens almost instantaneously when $\Delta \to 0$.

5   Source of oligopoly rents: asymmetric stocks

The two-period example of Section 2.3 illustrated how a firm with a small stock can credibly commit to deliver only in the most profitable period through aggressive contracting displacing part of the large firm's stock to a later period. In this way asymmetries helped firms to alleviate somewhat the prisoners' dilemma presented by the forward market. We now explore this result in the general model.

To facilitate the exposition, consider first a three-period model, $t = 0, 1, 2$ (some properties of the general model cannot be illustrated in two periods). Suppose, as before, that firm j is the smaller of the two (i.e., $s_0^j < s_0^i$) and that the division of the stocks is such that under pure-spot trading j sells only in two periods ($t = 0, 1$) while i sells in all three periods. If firms have no access to the forward market, equilibrium deliveries are obtained from the first-order conditions

$$a - 2q_0^i - q_0^j = \delta(a - 2q_1^i - q_1^j) = \delta^2(a - 2q_2^i)$$

$$a - 2q_0^j - q_0^i = \delta(a - 2q_1^j - q_1^i)$$

subject to $q_0^i + q_1^i + q_2^i = s_0^i$ and $q_0^j + q_1^j = s_0^j$. When stocks are sufficiently asymmetric, it is not possible to have marginal revenues growing at the rate of interest and both firms exhausting at the same time. Rather, the smaller firm must exhaust first, leaving the larger firm alone for some final *monopoly phase*. Thus, qualitatively, the equilibrium consists of a Cournot phase, where prices grow at some rate smaller than the interest rate, and of a monopoly phase, where prices grow at even lower rate (see, e.g., Lewis and Schmalensee [1980]).

Let us now introduce forward contracting. From the two-period model we know that forward contracting reinforces the fact that firms will exit the market at different times. Thus, if under pure-spot trading j was only serving the market at $t = 0, 1$, the introduction of forwards will at best make firm j to continue serving the market at $t = 0, 1$, and eventually only at $t = 0$. To keep the model instructive, however, we will

assume that the division of stocks is such that j will continue serving at $t = 0, 1$. From the two-period model we also know that if in equilibrium firm j is only present in $t = 0, 1$, then, the only contracts that are relevant for the analysis are the ones sold at $t = 0$; more specifically, $f_{0,0}^i$ and $f_{0,0}^j$.[21]

On the other hand, we learned from the symmetric case that as we reduce the period length (i.e., $\Delta \to 0$) and spot markets are preceded by a large number of forward openings, firms will stop selling contracts only when all spot markets become equally profitable, i.e., when all prices are equal in present value (see Proposition 4). We can use the three-period model to show that the same result must hold for asymmetric firms during the time in which both firms are serving the market (i.e., $t = 0, 1$). In so doing, let the spot markets be preceded by $N \to \infty$ forward openings and look for equilibrium positions (F_0^i, F_0^j) that would induce firms to sell no contracts at the opening of the forward market at $t = 0$ (i.e., $f_{0,0}^i = f_{0,0}^j = 0$).

Letting F_0^i and F_0^j be any given firms' contract coverage right before the opening of the forward at $t = 0$, the subgame perfect equilibrium conditions for $f_{0,0}^i$ and $f_{0,0}^j$, satisfy, respectively

$$F_0^i + f_{0,0}^i = \frac{a(1 - \delta^3) + (2 + \delta)(2 + 4\delta)F_0^i - (1 + \delta + \delta^2)F_0^j}{5 + 11\delta + 5\delta^2}$$

$$F_0^j + f_{0,0}^j = \frac{a(1 - \delta)(2 + 5\delta + 2\delta^2) + (8 + 17\delta + 8\delta^2)F_0^j - (2 + \delta)(1 + 2\delta)F_0^i}{2(5 + 11\delta + 5\delta^2)}$$

Imposing $f_{0,0}^i = f_{0,0}^j = 0$, we obtain that the converging positions F_0^i and F_0^j must satisfy the unique equilibrium condition

$$F_0^i + F_0^j = a(1 - \delta) \tag{30}$$

(the exact equilibrium values of F_0^j and F_0^i are to be found with additional (sequential) equilibrium conditions). Adding the spot first-order conditions (for any given F_0^i and F_0^j) for i and j, respectively,

$$a - 2q_0^i - q_0^j + F_0^i = \delta(a - 2q_1^i - q_1^j)$$

$$a - 2q_0^j - q_0^i + F_0^j = \delta(a - 2q_1^j - q_1^i)$$

_____

[21] Note that in equilibrium we have $f_{0,0}^j > f_{0,0}^i$ with $f_{0,0}^i = a(1 - \delta^3)/(5 + 11\delta + 5\delta^2)$ and $f_{0,0}^j = f_{0,0}^i + 3\delta(1 - \delta)/2(5 + 11\delta + 5\delta^2)$.

and using (30) we obtain[22]

$$p_0^s \equiv a - q_0^i - q_0^j = \delta(a - q_1^i - q_1^j) \equiv \delta p_1^s$$

Consistent with Proposition 4, during the periods in which both firms are active (i.e., $t = 0, 1$) prices grow up at the rate of interest. Once the level of contracting in (30) is reached no firm wants to sign additional contracts because that would only introduce more competition to the spot market.[23]

The three-period model conveys two important results that obviously extend to the general model, namely, that (asymmetric) firms will exit the market at different times and that prices will grow up at the rate of interest while both firms are active (provided that there is an infinitely large number of forward openings). Making use of these two results, we can now complete the description of the equilibrium path for the general model with $\Delta \rightarrow 0$. Since equilibrium contract positions (F_t^i, F_t^j) will converge rather quickly as the period length vanishes, we can restrict attention to positions (F_t^i, F_t^j) from past contracting such that both firms are willing to choose $f_{t,t}^i = 0$ in the current period $t > 0$. As in the symmetric case, when no further contracting takes place, the continuation values $V^i(I_{t+1})$ and $V^j(I_{t+1})$ are only affected by actions in the spot subgame and, hence, we can concentrate on the spot market equilibrium conditions (15) for both i and j.

Following the arguments given in the symmetric case, we know that for a given contracting profile (F_t^i, F_t^j), firms' spot market choices must satisfy (29) while both firms are producing. Thus, if the smaller firm j exhausts at some t' and the larger firm i at time $t'' > t'$, it must hold that

$$u_{t'-\Delta}^j = e^{-r\Delta} u_{t'}^j \longrightarrow 0 \tag{31}$$

$$u_{t'-\Delta}^i = e^{-r\Delta} u_{t'}^i \longrightarrow e^{-r\Delta} q_{t'}^i \tag{32}$$

as $\Delta \rightarrow 0$. Condition (31) follows since $q_{t'}^j \rightarrow 0$, which implies, as in the symmetric case, that j is fully contracted in equilibrium, i.e., $u_t^j = 0$ for all $t > 0$. On the other hand, the larger firm i has no reason to sell contracts to and during the monopoly phase starting at t', so its uncontracted quantity u_t^i must be equal to the delivered quantity at any $t \geq t'$. Furthermore, since q_t^i decreases with t, it is not difficult to infer from (29) and (32)

---

[22]Note that there is a range of stock divisions for which firm j always attends both markets, $t = 0, 1$, regardless of the equilibrium solution under consideration (i.e., pure-spot trading, forward-contracting with a single opening, and forward-contracting with numerous openings).

[23]If for any reason $F_0^i + F_0^j > a(1 - \delta)$, competitive agents will store part of firms' deliveries making sure that $p_0^s = \delta p_1^s$ holds in equilibrium.

that the large firm's contracting incentives (i.e., contract coverage) decline over time to ultimately disappear at t'.

It remains to determine the exact equilibrium values of F_t^i for all $t < t'$, without which we would be unable to obtain equilibrium deliveries and prices. Unlike in the symmetric case, it is not immediately obvious how to proceed here other than explicitly solving for the subgame perfect path for a given Δ.[24]

In concluding this section, it may be helpful to contrast this asymmetric equilibrium with the one described in Salant (1976) who considered an extreme oligopoly with one (large) seller and a continuum of price takers. In Salant's equilibrium, the large agent is also a monopoly at the end, and the small sellers free-ride on the large agent's market power by selling at the present-value monopoly price during a "competitive phase" where all firms are active. Qualitatively, similar free-riding by the smaller firm occurs here, and our price path has the Salant shape in the sense we have described above. However, our outcome is more competitive. For example, if s_0^j is close to s_0^i, our competitive phase would extend to almost the end (the monopoly phase would be quite brief), implying a reasonably competitive outcome; in Salant, such holdings by the fringe and the large firm would involve considerable market power.

6   Concluding remarks

We have found that forward contracting can have substantial implications for resource depletion in a non-cooperative oligopolistic environment. It is yet to be discussed whether and to what extend forward contracting could also affect the possibility of collusion in this market. Recall that for a reproducible-commodity market, Liski and Montero (2006)

---

[24]An approximate solution that is relatively simple to solve in continuous time is the following. Consider a game in which there are no forward markets but right before the opening of the spot markets firms have a one-time opportunity to simultaneously sell a fraction of their stocks to perfectly competitive agents. Due to the same strategic forces working under forward contracting, firms will sell positive quantities in equilibrium, say, $\alpha^i s_0^i$ and $\alpha^j s_0^j$, with $\alpha^i < \alpha^j$ because $s_0^i > s_0^j$. We can now use these α coefficients to obtain a reasonable estimate of the fraction of the stock that firm i would have contracted in equilibrium in our original model, i.e., $\Sigma_0^{t'} F_t^i$. The expansion factor that moves firm j from a "partial contracting" of $\alpha^j s_0^j$ to "full contracting" of s_0^j is $1/\alpha^j$. Assuming identical "forward" expansion rates for the two firms, firm i's overall contracting level would then be $\Sigma_0^{t'} F_t^i = \alpha^i s_0^i / \alpha^j$. With this contracting level and $F_t^j = q_t^j$ for all $t < t'$, the next step is to find a contracting profile F_t^i, along with equilibrium deliveries q_t^i and q_t^j, that adds to $\alpha^i s_0^i / \alpha^j$ and satisfies (15). The profile, which is unique, is to be found iteratively.

have already shown that forward trading increases the scope of collusion —independently of the form of competition— by allowing firms to either construct harsher punishments or limit the deviation profits.

Unfortunately, the lessons from Liski and Montero (2006) are not easily exported to this market because the intertemporal capacity constraints associated to the stocks introduce new elements into the analysis. It is possible for nonstationary and collusive strategies to arise in equilibrium when the overall consumption horizon is infinite, for example, due to (high) stock-dependent extraction costs or an infinite choke price —the price at which demand falls to zero. But when the choke price is finite and there is a gap between this price and the cost of extracting the last unit, as in our model and the examples in the introduction,[25] the consumption horizon is finite; either under perfect competition or monopoly. In fact, following the monopoly path to the very end is not sustainable because in the period before the last one, firms will surely deviate from the monopoly path by increasing their sales; and this deviation incentive will "propagate" to the very first period.

One may still borrow an insight from the durable-good monopoly literature and ask whether firms could sustain a collusive path that only asymptotically approaches the choke price, very much in the spirit of Ausubel and Deneckere (1989) and Gul (1987). Suppose for example that firms follow the monopoly path and only connect to an asymptotic path when the remaining stock is very small. Such collusive path is not sustainable either because at some point along the asymptotic portion, jumping to the punishment path (perfectly competitive pricing if firms are symmetric) is strictly more profitable than continuing along the collusive path.[26] This does not happen in the models of Ausubel and Deneckere (1989) and Gul (1987) because in those models the punishment path entails zero profits for firms (so it is always possible to fashion an asymptotic collusive path where the present value from colluding is greater than the one-shot deviation profit). Consequently, we are only left with a finite-horizon backward-induction equilibrium where the competitive pressure from contracting can be severe, if not perfect.

---

[25]Note that it costs nothing to "extract" water rights or pollution quotas (of course there is an opportunity cost associated to their use: the market price).

[26]Consider, for example, the inverse demand $p(q) = 1 - q$ and the following collusive path for a remaining stock of size ε: $p^m(t) = 1 - \varepsilon\eta e^{-\eta t}$ for $t \geq 0$. It can be shown that for any $\eta > 0$ and $\varepsilon > 0$ there will always be a time $t > 0$ at which becomes more profitable for firms to follow the perfectly competitive path than continuing along the collusive path.

7 Appendix

7.1 Proposition 2

We derive $f_{\min}^j(s^j)$ as follows. First, we find the Stackelberg first-best payoff and deliveries for i (the larger firm), given that j is holding some contracts $f_{1,1}^j$. This defines the maximum for what i can achieve in the original game of contracting. Second, we find contracting level $f_{1,1}^j$ that induces the follower j to produce only in the first period. This will define $f_{\min}^j(s^j)$. Given this level of contracting by j, i can implement its first-best in the original game by not contracting and letting j to sell only at $t = 1$. Third, we will derive the threshold (11), under which this characterization holds.

Consider the first-best choice of q_1^i. Given q_1^i and $f_{1,1}^j$, j's best-response in the first-period quantities satisfies

$$a - 2q_1^j - q_1^i + f_{1,1}^j = \delta(a - (s^i - q_1^i) - (s^j - q_1^j)) - \delta(s^j - q_1^j),$$

giving

$$q_1^j(q_1^i, f_{1,1}^j) = \left(a(1 - \delta) + 2\delta s^j + f_{1,1}^j - (1 + \delta)q_1^i + \delta s^i\right) \frac{1}{2(1 + \delta)}.$$

Firm i's first-best payoff is, given $f_{1,1}^j$,

$$\max_{q_1^i}\{p^s(q_1^i, q_1^j(q_1^i, f_{1,1}^j))q_1^i + \delta p^s(s^i - q_1^i, s^j - q_1^j(q_1^i, f_{1,1}^j))(s^i - q_1^i)\}.$$

Solving

$$q_1^i(f_{1,1}^j) = \left(a(1 - \delta) + 2\delta s^i - f_{1,1}^j\right) \frac{1}{2(1 + \delta)},$$

and evaluating the follower's best-response gives

$$q_1^j(q_1^i(f_{1,1}^j), f_{1,1}^j) = \left(a(1 - \delta) + 4\delta s^j + 3f_{1,1}^j\right) \frac{1}{4(1 + \delta)}.$$

Contracting $f_{\min}^j(s^j)$ is defined by

$$q_1^j(q_1^i(f_{1,1}^j), f_{1,1}^j) = s^j.$$

Finally, note that the domain of the symmetric contracting is defined as follows: as long as condition (8) gives $q_2^j \geq 0$, both firms can be active at $t = 2$ and the symmetric equilibrium is valid. This defines a threshold $2a(1 - \delta)/5$ for the smaller stock such that the symmetric equilibrium is valid. Choosing $f_{1,1}^i = 0$ is indeed the best-response to $f_{\min}^j(s^j)$ provided that the symmetric contracting $f_{1,1}^i = f_{1,1}^j = a(1 - \delta)/5$ does not lead to a larger payoff for i. Comparing the payoffs shows that this is the case if

$$s^j \leq \frac{1}{5}(1 - 2\sqrt{2})a(1 - \delta) = 0.434a(1 - \delta),$$

which is the threshold in the proposition (to save space we do not report the payoff expressions). When s^j is larger than this quantity, then the larger firm will implement the symmetric equilibrium by contracting $a(1-\delta)/5$ (firm j's best-response to this is $a(1-\delta)/5$). When s^j is below the above threshold, i's best-response to $f_{min}^j(s^j)$ is $f_{1,1}^i = 0$. For firm j, $f_{min}^j(s^j)$ is best-response to $f_{1,1}^i = 0$ since (10) holds ($s^j \leq 0.434a(1-\delta) < 0.5a(1-\delta)$).

7.2 Proposition 3

We progress inductively backwards from T. At $T-1$, the firms face the symmetric two-period model we have already solved in section 2.

Spot stage of period $T-1$: deliveries for $i = 1, 2$ are chosen to satisfy

$$q_{T-1}^i(I_{T-1}, f_{T-1,T-1}^i, f_{T-1,T-1}^j) =$$
$$[a(1-\delta) + 3\delta s_{T-1}^i + 2(F_{T-1}^i + f_{T-1,T-1}^i) - (F_{T-1}^j + f_{T-1,T-1}^j)]\frac{1}{3(1+\delta)}. \quad (33)$$

Compare this expression with (3) where the only difference is that there are no contracts from the past, F_{T-1}^i and F_{T-1}^j.

Forward stage of period $T-1$: firm i's payoff is (see (14) in the text):

$$V^i(I_{T-1}) = \max_{f_{1,1}^i}\{p_{T-1}^s \cdot (q_{T-1}^i - F_{T-1}^i) + \delta p_T^s q_T^i\},$$

where q_{T-1}^i is defined in (33) and $q_T^i = s_{T-1}^i - q_{T-1}^i$.

Note that for reasons already explained for the two period model, we can ignore contracting for the very last spot market T. Using the linear demand and imposing symmetry, gives the equilibrium contracting at $T-1$,

$$f_{T-1,T-1}^i = \frac{1}{5}[a(1-\delta) - F_{T-1}^i - F_{T-1}^j]. \quad (34)$$

The overall coverage at the outset of spot market $T-1$ is therefore

$$F_{T-1}^i + f_{T-1,T-1}^i = \frac{1}{5}[a(1-\delta) + 4F_{T-1}^i - F_{T-1}^j]. \quad (35)$$

Using (35) in $q_{T-1}^i(I_{T-1}, f_{T-1,T-1}^i, f_{T-1,T-1}^j)$, gives the subgame-perfect equilibrium delivery

$$q_{T-1}^i = [2a(1-\delta) + 5\delta s_{T-1}^i + 3F_{T-1}^i - 2F_{T-1}^j]\frac{1}{5(1+\delta)}.$$

Spot stage of period $T-2$: Recall that $\mathbf{F}_{T-2}^i = (F_{T-2}^i, F_{T-2+1}^i)$ is what firm i is holding at $T-2$ for periods $T-2$ and $T-1$ from the past contracting (before contracting

at $T-2$). Given $\mathbf{F}^i_{T-2}$ and $f^i_{T-2,T-2}$, and $f^i_{T-2,T-1}$ for $i=1,2$, firm i's best-response in the spot market solves

$$\max_{q^i_{T-2}}\{p^s_{T-2}\cdot(q^i_{T-2}-F^i_{T-2}-f^i_{T-2,T-2})+\delta V^i(I_{T-1})\},$$

where $I_{T-1}=(s^i_{T-2}-q^i_{T-2},s^j_{T-2}-q^j_{T-2},F^i_{T-2+1}+f^i_{T-2,T-1},F^j_{T-2+1}+f^j_{T-2,T-1})$.

We find that firm i's best response in delivered quantities satisfies

$$\frac{\partial p^s_{T-2}}{\partial q^i_{T-2}}(q^i_{T-2}-F^i_{T-2}-f^i_{T-2,T-2})+p^s_{T-2}+\delta\frac{\partial V^i(I_{T-1})}{\partial s^i_{T-1}}\frac{\partial s^i_{T-1}}{q^i_{T-2}}=0. \qquad (36)$$

Using the first-order condition from the spot stage at $T-1$ and $\partial s^i_T/\partial s^i_{T-1}=1$, we have

$$
\begin{aligned}
\frac{\partial V^i(I_{T-1})}{\partial s^i_{T-1}} &= \frac{\partial}{\partial q^i_{T-1}}\{p^s_{T-1}\cdot(q^i_{T-1}-F^i_{T-1})+\delta p^s_T q^i_T\}\frac{\partial q^i_{T-1}}{\partial s^i_{T-1}}\\
&\quad+\delta\frac{\partial(p^s_T q^i_T)}{\partial q^i_T}\frac{\partial q^i_T}{\partial s^i_{T-1}}\frac{\partial s^i_T}{\partial s^i_{T-1}}\\
&= \frac{\partial p^s_{T-1}}{\partial q^i_{T-1}}f^i_{T-1,T-1}\frac{\partial q^i_{T-1}}{\partial s^i_{T-1}}+\delta\frac{\partial(p^s_T q^i_T)}{\partial q^i_T}\frac{\partial q^i_T}{\partial s^i_{T-1}}.
\end{aligned}
$$

Subgame-perfect $f^i_{T-1,T-1}$ is given in (34), q^i_{T-1} in (33), and $q^i_T=s^i_{T-1}-q^i_{T-1}$. Thus,

$$\frac{\partial V^i(I_{T-1})}{\partial s^i_{T-1}}=-2\frac{\delta}{1+\delta}s^i_{T-1}+\frac{\delta}{1+\delta}(2a+F^i_{T-1}-s^j_{T-1}). \qquad (37)$$

Combining (37) and (36) gives the best response in the spot market $T-2$:

$$q^i_{T-2}(I_{T-2},f^i_{T-2,T-2},f^i_{T-2,T-1},f^j_{T-2,T-2},f^j_{T-2,T-1})=$$
$$\{a(1+\delta-2\delta^2)+3\delta^2 s^i_{T-2}+2H^i_{T-2}-H^j_{T-2}\}\frac{1}{3(1+\delta+\delta^2)}.$$

where

$$
\begin{aligned}
H^i_{T-2} &= (1+\delta)(F^i_{T-2}+f^i_{T-2,T-2})-\delta^2 F^i_{T-1} \qquad (38)\\
&= (1+\delta)(F^i_{T-2}+f^i_{T-2,T-2})-\delta^2(F^i_{T-2+1}+f^i_{T-2,T-1}).
\end{aligned}
$$

<u>Forward stage of period $T-2$:</u> Firm i's payoff is

$$V^i(I_{T-2})=\max_{f^i_{T-2,T-2},f^i_{T-2,T-1}}\{p^s_{T-2}\cdot(q^i_{T-2}-F^i_{T-2})+\delta V^i(I_{T-1})\}.$$

Note that here q^i_{T-2} is a function of the contract choice, $(f^i_{T-2,T-2},f^i_{T-2,T-1})$, through (38). The choice $f^i_{T-2,T-1}$ determines q^i_{T-1} through

$$F^i_{T-1}+f^i_{T-1,T-1}=\frac{1}{5}[a(1-\delta)+4(F^i_{T-2+1}+f^i_{T-2,T-1})-(F^j_{T-2+1}+f^j_{T-2,T-1})],$$

29

which is (35) written as a function of the current choice $f^i_{T-2,T-1}$.

We can now consider the choice of $f^i_{T-2,T-2}$ which enters $V^i(I_{T-2})$ only through H^i_{T-2}. Differentiating we see that the interior choice satisfies:

$$
\begin{aligned}
&\{\frac{\partial p^s_{T-2}}{\partial q^i_{T-2}}(q^i_{T-2}-F^i_{T-2})+p^s_{T-2}+\delta\frac{\partial V^i(I_{T-1})}{\partial s^i_{T-1}}\frac{\partial s^i_{T-1}}{\partial q^i_{T-2}}\}\frac{\partial q^i_{T-2}}{\partial H^i_{T-2}}\frac{\partial H^i_{T-2}}{\partial f^i_{T-2,T-2}}\\
&+\{\frac{\partial p^s_{T-2}}{\partial q^j_{T-2}}(q^i_{T-2}-F^i_{T-2})+\delta\frac{\partial V^i(I_{T-1})}{\partial s^j_{T-1}}\frac{\partial s^j_{T-1}}{\partial q^j_{T-2}}\}\frac{\partial q^j_{T-2}}{\partial H^i_{T-2}}\frac{\partial H^i_{T-2}}{\partial f^i_{T-2,T-2}}\\
&=\ 0
\end{aligned}
$$

We have expressions for q^i_{T-2} and H^i_{T-2}, so we can solve for optimal $f^i_{T-2,T-2}$ if we know $\partial V^i(I_{T-1})/\partial s^i_{T-1}$. Note that when evaluated at $T-2$,

$$
\frac{\partial V^i(I_{T-1})}{\partial s^i_{T-1}}=-2\frac{\delta}{1+\delta}s^i_{T-1}+\frac{\delta}{1+\delta}(2a+F^i_{T-2+1}-s^j_{T-1}),
$$

because F^i_{T-2+1} is the contract position at $T-2$ for period $T-1$ before $f^i_{T-2,T-1}$ is sold (i.e., after this sale, $F^i_{T-1}=F^i_{T-2+1}+f^i_{T-2,T-1}$, and continuation value changes accordingly). Solving, after imposing symmetry on the contract choices, gives

$$
\begin{aligned}
f^i_{T-2,T-2}\ =\ &f^j_{T-2,T-2}=\\
&\frac{a}{5}\frac{(1+\delta-2\delta^2)}{(1+\delta)}+\frac{a}{7}\frac{\delta^2(1-\delta)}{(1+\delta)}\\
&-\frac{1}{5}F^i_{T-2}-\frac{1}{5}F^j_{T-2}\\
&+\frac{2}{35}\frac{\delta^2}{(1+\delta)}(F^i_{T-2+1}+F^j_{T-2+1}).
\end{aligned}
$$

Consider then the first-order condition for $f^i_{T-2,T-1}$, the optimal contract choice for the next period, given all other contracting:

$$
\begin{aligned}
&0\times\frac{\partial q^i_{T-2}}{\partial H^i_{T-2}}\frac{\partial H^i_{T-2}}{\partial f^i_{T-2,T-1}}\\
&+\{\frac{\partial p^s_{T-2}}{\partial q^j_{T-2}}(q^i_{T-2}-F^i_{T-2})+\delta\frac{\partial V^i(I_{T-1})}{\partial s^j_{T-1}}\frac{\partial s^j_{T-1}}{\partial q^j_{T-2}}\}\frac{\partial q^j_{T-2}}{\partial H^j_{T-2}}\frac{\partial H^j_{T-2}}{\partial f^i_{T-2,T-1}}\\
&+\frac{\partial V^i(I_{T-1})}{\partial F^i_{T-1}}\frac{\partial F^i_{T-1}}{\partial f^i_{T-2,T-1}}\\
&=\ 0
\end{aligned}
$$

where the first line disappears because of the first-order condition for $f^i_{T-2,T-2}$ (both $f^i_{T-2,T-2}$ and $f^i_{T-2,T-1}$ affect q^i_{T-2} through H^i_{T-2}). Evaluating and imposing symmetry

on the forward subgame gives:

$$f^i_{T-2,T-1} = f^j_{T-2,T-1} = \frac{a}{7}(1-\delta) - \frac{1}{5}F^i_{T-2+1} - \frac{1}{5}F^j_{T-2+1}.$$

We have now solved a three-period model: if the initial stocks $s^i_0 = s^j_0$ are such that they are exhausted in three periods, then $F^i_{T-2} = F^j_{T-2} = 0$ and $F^i_{T-2+1} = F^j_{T-2+1}$ ($T-2$ is the first period so there is no contracting from the past). Then, using the above solution,

$$
\begin{aligned}
H^i_{T-2} &= H^j_{T-2} = \frac{a}{5}(1+\delta-2\delta^2) \\
q^i_{T-2} &= q^j_{T-2} = \{a(1+\delta-2\delta^2) + 3\delta^2 s^i_{T-2} + \frac{a}{5}(1+\delta-2\delta^2)\}\frac{1}{3(1+\delta+\delta^2)} \\
q^i_{T-1} &= q^j_{T-1} = \{a(1-\delta) + 3\delta s^i_{T-1} + \frac{2a}{7}(1-\delta)\}\frac{1}{3(1+\delta)} \\
q^i_T &= q^j_T = s^i_0 - q^i_{T-1}.
\end{aligned}
$$

Here, period $T-1$ spot market is served twice in the forward market (at $T-2$ and $T-1$) and period $T-2$ only once. In general, if the next to the last period, period $T-1$, is served N times, then

$$q^i_{T-1} = q^j_{T-1} = \{a(1-\delta) + 3\delta s^i_{T-1} + \frac{Na}{3+2N}(1-\delta)\}\frac{1}{3(1+\delta)}.$$

This expression can be solved from a two-period model where deliveries are preceded by N forward markets. Similarly, we can solve a three-period model where forward market opens such that $T-1$ spot market is served N times. It then follows that period $T-2$ is served $N-1$ times, and we can solve

$$
\begin{aligned}
q^i_{T-2} &= q^j_{T-2} = \\
&\{a(1+\delta-2\delta^2) + 3\delta^2 s^i_{T-2} + \frac{(N-1)a}{3+2(N-1)}(1+\delta-2\delta^2)\}\frac{1}{3(1+\delta+\delta^2)}.
\end{aligned}
$$

Setting $N = 2$ gives the above expression for q^i_{T-2} in a three-period model. We can advance to a four-period model where period $T-1$ is served N times, period $T-2$ is served $N-1$ times, and period $T-3$ is served $N-2$ times in the forward market:

$$
\begin{aligned}
q^i_{T-3} &= q^j_{T-3} = \\
&\{a(1+\delta+\delta^2-3\delta^3) + 3\delta^3 s^i_{T-3} \\
&+ \frac{(N-2)a}{3+2(N-2)}(1+\delta+\delta^2-3\delta^3)\}\frac{1}{3(1+\delta+\delta^2+\delta^3)}.
\end{aligned}
$$

31

If it indeed takes four-periods to exhaust the stocks, then $N = 3$. In general, if there are T periods and $k = 1, ...T - 1$ is the backward induction step, then

$$q^i_{T-k} = q^j_{T-k} =$$
$$\{a(1 + \delta + ... + \delta^{k-1} - k\delta^k) + 3\delta^k s^i_{T-1}$$
$$+ \frac{(T-k)a}{3 + 2(T-k)}(1 + \delta + ... + \delta^{k-1} - k\delta^k)\} \frac{1}{3(1 + \delta + ... + \delta^k)}.$$

Then, in the next to the last spot market

$$q^i_{T-1} = q^j_{T-1} = \{a(1 - \delta) + 3\delta s^i_{T-1} + \frac{Ta}{3 + 2T}(1 - \delta)\} \frac{1}{3(1 + \delta)}.$$

The forward market opens in $T - 1$ preceding periods plus at T, so this market is served T times. In the first spot market of the T-period model we have

$$q^i_{t=0} = q^j_{t=0} =$$
$$\{a(1 + \delta + ... + \delta^{T-2} - (T-1)\delta^{T-1}) + 3\delta^{T-1} s^i_{T-1}$$
$$+ \frac{a}{5}(1 + \delta + ... + \delta^{T-2} - (T-1)\delta^{T-1})\} \frac{1}{3(1 + \delta + ... + \delta^{T-1})}.$$

We can express the delivery rule concisely as stated in the proposition:

$$q^i_{t_k} = q^j_{t_k} = \{\frac{a}{3}[\sum_{h=1}^{k} \delta^{h-1} - k\delta^k][1 + \frac{T-k}{3 + 2(T-k)}] + \delta^k s^i_{t_k}\} \frac{1}{\sum_{h=0}^{k} \delta^h},$$

where t_k is the time associated with the induction step k.

References

[1] Allaz, B., and J.-L. Vila (1993), Cournot competition, forward markets and efficiency, *Journal of Economic Theory* 59, 1-16.

[2] Ausubel, L. M., and R. J. Deneckere (1989), Reputation and bargaining in durable goods monopoly, *Econometrica* 57, 511-531.

[3] Bhaskar, V. (2006), Dynamic countervailing power: Public vs private monitoring, working paper.

[4] Biglaiser, G., and Vettas, N. (2005), Dynamic price competition with capacity constraints and strategic buyers, working paper.

[5] Bulow, J.I., J.D. Geanakoplos, and P.D. Kemplerer (1985), Multimarket oligopoly: strategic substitute and complements, *Journal of Political Economy* 93, 488-511.

[6] Bushnell, J., E. Mansur and C. Saravia (2008), Vertical arrangements, market structure and competition: An analysis of restructured U.S. electricity markets, *American Economic Review* 98, 237-266.

[7] Dudey, M. (1992), Dynamic Edgeworth-Bertrand competition, *Quarterly Journal of Economics* 107, 1461-1477.

[8] Ellerman, A.D., and J.-P. Montero (2007), The efficiency and robustness of allowance banking in the U.S. Acid Rain Program, *The Energy Journal* 28, 47-71.

[9] Fudenberg, D., and J. Tirole (1984), The fat cat effect, the puppy dog ploy and the lean and hungry look, *American Economic Review Papers and Proceedings* 74, 361-368.

[10] Gaudet, G. (2007), Natural resource economics under the rule of Hotelling, *Canadian Journal of Economics* 40, 1034-1059.

[11] Gul, F. (1987), Noncooperative collusion in durable goods oligopoly, *The RAND Journal of Economics* 18, 248-254.

[12] Hotelling, H. (1931), The economics of exhaustible resources, *Journal of Political Economy* 39, 137-175.

[13] Kauppi, O. and M. Liski (2008), Market power and storage: Evidence from hydro use in the Nordic Power Market, Helsinki School of Economics, working paper.

[14] Lewis, T. and R.Schmalensee (1980), On oligopolistic markets for nonrenewable resources, *Quarterly Journal of Economics* 95, 475-491.

[15] Liski, M., and J.-P. Montero (2006), Forward trading and collusion in oligopoly, *Journal of Economic Theory* 131, 212-230.

[16] Loury, G. (1986), A theory of 'oil'igopoly: Courtnot–Nash equilibrium in exhaustible resources markets with fixed supplies. *International Economic Review* 27, 285–301

[17] Mahenc, P., and F. Salanié (2004), Softening competition through forward trading, *Journal of Economic Theory* 116, 282-293.

[18] Phlips, L., and R.M. Harstad (1990), Interaction between resource extraction and futures markets: A game-theoretic analysis, in R. Selten (ed.), *Game Theory in the Behavioral Sciences*, Springer Verlag, Heidelberg.

[19] Polansky, S. (1992), Do oil producers act as 'oil'igopolists?, *Journal of Environmental Economics and Management* 23, 216–247.

[20] Salant, S.W. (1976), Exhaustible resources and industrial structure: A Nash-Cournot approach to the world oil market, *Journal of Political Economy* 84, 1079-1093.

[21] Salo, S., and O. Tahvonen (2001), Oligopoly equilibria in nonrenewable resource markets, *Journal of Economic Dynamics and Control,* Volume 25, Issue 5, 671-702

[22] Wolak, F. (2000), An empirical analysis of the impact of hedge contracts on bidding behavior in competitive electricity markets, *International Economic Journal* 14, 1-39.