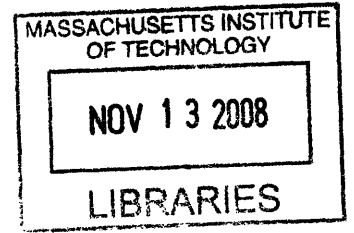


Physical Representation of Tension caused by Harmonic
Progression

by

ShiLing Seow

B.S., Massachusetts Institute of Technology (2006)



Submitted to the Department of Electrical Engineering and Computer Science
in partial fulfillment of the requirements for the degree of

Master of Engineering in Electrical Engineering and Computer Science

at the

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Author
Department of Electrical Engineering and Computer Science
August 22, 2008

Certified by
Dale Joachim
Visiting Professor
Thesis Supervisor

Accepted by
Arthur C. Smith
Chairman, Department Committee on Graduate Theses

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Abstract

Studies have shown that language is an essential tool for forming complicated concepts. The syntactical similarities between music and language have led some researchers to focus on perception of music in order to further understand the form, syntax, and development of language. A key component of music is tonality. Theories regarding the perception of musical tonality have been formulated based on acoustics [1], culture [2, 3], physiology [4, 5], and psychology¹. Lerdahl proposed a model which formally quantifies the terms *tension* and *relaxation* commonly used to describe the element of tonality in music. His harmonic tension model articulates the perception of harmonic progression in music.

In an attempt to tangibly explain the concepts of tension and relaxation, a physical representation of Lerdahl's harmonic tension model is presented. The physical representation is created by mapping *tension* caused by harmonic progression onto the surface tension of a visco-elastic sphere. The level of distortion on the sphere surface is made to correspond to the amount of tension in the music. Additionally, the elastic property of the sphere reflects the *relaxation* or resolution phases in tonal music which normally follow periods of high tension. Two demonstration systems were developed based on the physical representation of harmonic tension as an evaluation of the concept's effectiveness. The first system is a simulation of the visco-elastic sphere written using the Netlogo 3D Preview program. In this system, the surface tension of the simulated sphere is manipulated according to the harmonic tension of Pachelbel's *Canon in D* and Rimsky-Korsakov's *The Flight of the Bumblebee*. The simulated sphere provides users with visual feedback on the tension/relaxation phases of the music. The second system is an interactive tool for intuitively learning the harmonic tension model using the sphere representation. This interactive system consists of a visco-elastic sphere simulation controlled by an approximated spherical input device. The only difference between this simulation and the first system's is that the second system's input is controlled by the user. Using simulation and audio output, users explore different tension values and observe the associated chord progressions. The interactive system was found not to be as intuitive a learning tool as expected because highly dissonant chord progressions with different tension values are indistinguishable by hearing alone. The question of how sensitive humans are to tension/dissonance and the factors that affect that sensitivity certainly warrants more attention.

The physical model of Lerdahl's harmonic tension model presented explores tangible means for grasping the relation between harmonic progression and tonality. The simulated model provides visual feedback of the tension/relaxation patterns in music thus enhancing ones listening experience.

Thesis Supervisor: Dale Joachim
Title: Visiting Professor

¹ See University of California Berkeley's Center for New Music & Audio Technologies (CNMAT) for additional publications related to the psychophysics of music perception

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Special thanks to my caring family and wonderful friends. The completion of this thesis would not have been possible without their loving support and encouragement.

Terminology

| | |
|-----------------------------|---|
| Atonal | Lacks tonal center or key. |
| Chord | Three or more distinct notes played simultaneously (only triads and seventh chords are used in this thesis). |
| Chord quality | Determined by the intervals between the notes that make up the chord (major, minor, augmented, diminished). |
| Chromatic | Notes not belonging to the diatonic scale. |
| Consonance | Harmonious, stable, pleasant sounds. |
| Diatonic | The seven notes belonging in the scale. |
| Dissonance | High tension, unstable, jarring sounds. |
| Harmonic progression | Sequence of chords. |
| Interval | Distance between two notes. |
| Key | The major or minor scale in a piece of music. |
| Nonharmonic note | Note that does not belong in a chord. |
| Note | Pitch class. |
| Pitch class | Set of all notes that are whole octaves apart (referred to as <i>notes</i> in thesis). |
| Root note | Note upon which a chord is built. |
| Diatonic scale | Group of seven notes belonging to a particular key. |
| Scale degree | Name of chord in relation to position of root note in scale (used differently in <i>Surface tension rule</i>). |
| Semitone | Smallest interval used in traditional Western music. |
| Sequential harmonic tension | Local analysis of harmonic progression. Involves only current and adjacent chords. |
| Seventh chord | Created by adding a fourth note at the interval of a seventh above root note of the original triad. |
| Tonal | Music revolves around particular keys. |
| Tonal pitch space | Model to fully describe the tonal space [6]. |
| Triad | Chord made of three notes. |

Notation

| | |
|--------------------------------|-------------------------|
| Uppercase Roman Numeral e.g. I | Major chord |
| Lowercase Roman Numeral e.g. i | Minor chord |
| + | Augmented chord |
| ◦ | Diminished chord |
| 7 | Seventh chord |
| ⁷ M | Major seventh |
| ⁷ D | Dominant seventh |
| ⁷ MM | Minor major seventh |
| ⁷ m | Minor seventh |
| ⁷ AM | Augmented major seventh |
| ⁷ A | Augmented seventh |
| ◦7 | Half-diminished seventh |
| ◦7 | Diminished seventh |
| x/y | Chord x in key y |

Rules and equations

The following rules are used in the calculation of harmonic tensions. Further details and usage of these rules can be found in Chapter 2 and in Lerdahl's *Tonal Pitch Space* [6].

Regional circle-of-fifths rule

Move the notes at level d of the the basic space seven steps to the right or left (mod 12) on level e or seven steps to the left.

Chordal circle-of-fifths rule

Move the notes at levels a-c of basic space four steps to the right (mod 7) on level d or four steps to the left until the basic space for the desired chord is achieved.

Chord distance rule

$\delta(x \rightarrow y) = i + j + k$, where

$\delta(x \rightarrow y)$ = distance between chords x and y

i = number of applications of the regional circle-of-fifths rule needed to shift the diatonic collection that supports x into the diatonic collection that supports y .

j = smallest number of applications of the chordal circle-of-fifths rule needed to shift x into y

k = number of distinctive notes in the basic space of y compared to those in the basic space of x

Surface tension rule

$T_{diss}(y)$ = scale degree (add 1) + inversion (add 2) + nonharmonic tone (add 1 for sevenths, 3 for diatonic nonharmonic tones, and 4 for chromatic)

$T_{diss}(y)$ = the surface tension associated with chord y ,

scale degree = chords with non root note in the melodic voice

inversion = chords with non root note in the base

nonharmonic tone = any note that does not belong to y

Sequential tension rule

$T_{seq}(y) = T_{diss}(y) + \delta(x_{prec} \rightarrow y)$, where

y is the target chord

x_{prec} is the chord that immediately precedes y in the sequence

$T_{seq}(y)$ is the tension associated with y

T_{diss} is the surface tension rule

$\delta(x_{prec} \rightarrow y)$ is the distance from x_{prec} to y

(using $\delta[x \rightarrow y] = i + j + k$)

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Chapter 1

Introduction

Patterns of tension and relaxation experienced when listening to tonal music are caused in part by harmonic progressions or chord changes. Lerdahl proposed a harmonic tension model based on tonal pitch space to calculate the relative tension and relaxation between two chords in a piece of music [6]. In this model, the tension between two chords is dependent on two factors: the distance between them in pitch space, and the dissonance and surface tension of the melodic line. These two factors will be discussed in detail in Chapter 2. The goal of this thesis is to explore the representation of these tension and relaxation patterns in a more tangible manner, thus providing a more intuitive means to understanding the theory behind harmonic progression and tonality.

Chapter 2 presents relevant contextual background on Western music, pitch space, and sequential harmonic tension model to better understand the content of this thesis.

Chapter 3 describes the expression of Lerdahl's sequential harmonic tension model as surface tension of a visco-elastic sphere. The translation from theoretical tension to physical surface tension is intuitive - the more deformed the surface, the higher the harmonic tension and potential to return to the original form (relaxation). This translation is demonstrated using surface tension simulations of Pachelbel's *Canon in D* and Rimsky-Korsakov's *The Flight of the Bumblebee*.

Chapter 4 describes an interactive system based on the idea developed in Chapter 3. The system comprises of two parts: hardware (ATMega168 microcontroller - Arduino, SD card, Bluetooth, push buttons, wave shield, and accelerometer) and simulation (Netlogo). The hardware is encased in a sphere to enable users to control tension values and the simulated sphere intuitively. Information transferred from the hardware to the simulation modifies the simulated sphere surface to reflect the specified tension values. As the simulated sphere surface changes, the user hears a series of chords which match the changes in surface tension value.

Chapter 5 is a discussion of the results and possible future expansions to the project.

1.1 Motivations

The main goal of this thesis is to investigate whether Lerdahl’s theoretical harmonic tension model can be expressed realistically as a physical model. If such an expression were possible, the physical model would provide an intuitive way to learn the effects of harmonic progression in music tonality, and shed light on how we perceive consonance and dissonance. On a broader scale, the understanding of how music is perceived also provides insight on human language and culture [2].

The tension model proposed by Lerdahl could be used to intuitively explain the concept of consonance and dissonance. There are many theories on how humans perceive consonance/dissonance and how our perceptions are affected by factors such as culture [9], personality [10], and musical training [11]. Cazden [9] proposed that the consonance/dissonance of a chord is relative to the expectation of the listener, making it culture specific. Based on a study by Van de Geer [11], the term *consonant* is used differently by musicians and non-musicians. Generally, musicians associate consonance with unisons, octaves, fifths, and fourths (see Section 2.1.2) whereas non-musicians prefer thirds, sixths, and typically associate consonance with pleasantness.¹ Cazden’s expectation theory is reflected in Lerdahl’s harmonic tension model which incorporates the context of the music to explain tension. Therefore, a physical model of the harmonic tension model is useful in expressing how the overall context of the music affects the perception of consonance and dissonance.

As support for the relationship between music and language, Patel [2] quoted the Pirahã, a small tribe from Brazil Amazon, which lacks many aspects of civilization but still has music. He argues that this phenomenon is an indication that music is the universal language and that both language and music define us as humans. Although many aspects of music such as tonal material, the number of pitches per octave, interval patterns, and standardized tuning are culture-specific, there are many cross-cultural similarities as well. For example, the fifth interval appears in Western, Chinese, and Indian music [3] even though the scales in each culture differ in many respects. Empirical studies [13] show that this interval is perceived as very consonant due to the nature of the auditory system. Other commonalities include the number of tones per octave (usually 5 to 7), size distribution of intervals (between 1-3 semitones), and asymmetric scale intervals.²

The connection between language and music has also been shown in several neurological studies reported by [2, 4, 5]. According to research conducted by Maess et al. [5], the *Broca’s area* in the brain and its right hemisphere homologue might not be as language-specific as previously thought. The fact that these areas are involved in the processing of musical syntax suggests that they might be performing general functions such as processing complex rule-based information not exclusive to language. This report hints at a strong relationship that exists between language and music, and at least partly account for influences of musical training on verbal abilities [14].

¹ An interesting discussion of consonance/dissonance can be found in [12].

² See [2] for a more detailed comparison of music in different cultures.

Chapter 2

Background

2.1 Background on western music

The music theory provided in this section assumes the western interpretation of music. The background presented here is by no means comprehensive, but highlights core information needed for the understanding of Lerdahl's concept of pitch space and the harmonic tension model used in this thesis.¹

An octave is the interval between a note and another half its frequency. For example, the frequency of the middle A is 440Hz. The interval between 440Hz and 880Hz is one octave. In the twelve tone system, an octave is divided into twelve distinct notes: C, C#/Db, D, D#/Eb, E, F, F#/Gb, G, G#/Ab, A, A#/Bb, and B. The distance between two adjacent notes is called a semitone and the difference in frequency between these two notes depends on the octave in which the note resides (see Appendix A for examples of notes and the corresponding frequencies). A chord is defined as three or more notes that are played simultaneously. Chords are characterized by the number of distinct notes, type of intervals between these notes, the scale degree, chord quality, and inversion.

2.1.1 Pitch classes

A pitch class is a set of all notes that are whole octaves apart. For example, A₃ (220Hz), A₄ (440Hz), and A₅ (880Hz) are all in pitch class A. Therefore the chord C-E-G-B-C has only four distinct pitch classes even though there are five notes. I will focus only on chords with three or four distinct pitch classes. From here on, the term *notes* will be used in place of pitch class for conciseness.

¹ For a detailed analysis of pitch space, see Lerdahl's *Tonal Pitch Space* [6].

Table 2.1: List of interval names used in this thesis. See [8] for details on the naming convention for intervals.

| No. of semitones | Interval name |
|------------------|---|
| 0 | Unison |
| 1 | Minor 2 nd |
| 2 | Major 2 nd |
| 3 | Minor 3 rd |
| 4 | Major 3 rd |
| 5 | Perfect 4 th |
| 6 | Diminished 5 th |
| 7 | Perfect 5 th |
| 8 | Minor 6 th |
| 9 | Major 6 th or Diminished 7 th |
| 10 | Minor 7 th |
| 11 | Major 7 th |
| 12 | Octave |

2.1.2 Intervals

The distance between two notes in a chord is called an interval. In this thesis, I will only be focusing on *triads* which contain three notes: a root note, a second note that is either 3 or 4 semitones higher than the root note, and a third note that is either 3 or 4 semitones higher than the second note. An interval of 3 semitones between two notes is called a minor third while an interval of 4 semitones is called a major third. For example, the C-E interval is a major third while the C-E \flat interval is a minor third. Note that there are many subtleties involved in the interval naming system but for the sake of this discussion, the interval names in Table 2.1 will be used. See [8] for details on the naming convention for intervals.

2.1.3 Chord quality

The quality of a chord is determined by the intervals between the notes that make up the chord. Table 2.2 is a list of all the triads and intervals between the three notes that make up the triads. Major chords are notated with uppercase Roman numerals, minor chords with lowercase Roman numerals, augmented chords with uppercase Roman numerals and a + sign, and diminished chords with lowercase Roman numerals and a \circ sign.

Table 2.2: List of all triad types (in C major) and corresponding note intervals.

| Chord name | Intervals | Example | Chord symbol |
|------------------|--------------------------|------------------------|----------------|
| Major triad | Major third, Minor third | C-E-G | I |
| Minor triad | Minor third, Major third | C-E \flat -G | i |
| Augmented triad | Major third, Major third | C-E-G \sharp | I ⁺ |
| Diminished triad | Minor third, Minor third | C-E \flat -G \flat | i \circ |

2.1.4 Scale degree

The scale degree of a chord is relative to the key that it is in. Scales in traditional Western music consist of seven notes of varying intervals. For simplicity, the discussions in this thesis will always be in the scale of C major unless noted otherwise. The C major scale consists of pitch classes C, D, E, F, G, A, and B. Other scales are shown in Appendix B.

Chords are normally notated in Roman numerals using the scale degree of their root note. The root note of a chord is the note upon which the chord is built on. For example, the root note of C-E-G is C whereas the root note for B-G-D is G. The actual position of the notes does not affect the scale degree because the root note is constant for each chord. Table 2.3 shows the seven types of scale degrees. The roman numerals are based on the distance of the root note from the first note of the scale. Therefore, in C major, a C-E-G chord would be the tonic (I) and a G-B-D chord would be the dominant (V).

Table 2.3: Scale degree of chords in C major.

| Root note | Scale degree | Symbol |
|-----------|-----------------------|------------------|
| C | Tonic | I |
| D | Supertonic | ii |
| E | Mediant | iii |
| F | Subdominant | IV |
| G | Dominant | V |
| A | Submediant | vi |
| B | Leading tone/Subtonic | vii ^o |

2.1.5 Seventh chords

A seventh chord is created by adding a fourth note at the interval of a seventh above the root note of the original triad. The quality of the seventh chord is determined by the quality of the original triad and the type of seventh interval. Table 2.4 shows the different types of seventh chords. A “7” superscript indicates a seventh note. The interval of the seventh note (major, minor, or diminished) is implied from the current key. So a I^7 in C major is a C-E-G-B (major 7th) but a I^7 in C minor is a C-E-G-B \flat (minor 7th) because the B in C minor scale is flattened (see Appendix B). Notice the V^7/IV notation for the dominant seventh chord. The bold Roman numeral on the right indicates the scale that the chord on the left should be interpreted in. Therefore, in this case, the **IV** chord is a F major (relative to the current C major scale) and the V^7 chord is the dominant major seventh chord in F major (C-E-G-B \flat because B is flattened in F major). The vii chord is a special exception this rule. The \circ symbol indicates a diminished seventh chord and the \emptyset symbol indicates a half diminished seventh chord [8].

Table 2.4: List of seventh chords (in C major). For more information on seventh chords, see [8].

| Chord name | Original Chord | Interval (Root and 7 th) | Example | Chord symbol |
|-------------------------|----------------|--------------------------------------|-------------------------------------|-------------------------------------|
| Major seventh | Major | Major 7 th | CEGB | I ⁷ |
| Dominant seventh | Major | Minor 7 th | CEGB \flat | V ⁷ /IV |
| Minor major seventh | Minor | Major 7 th | CE \flat GB | i ⁷ |
| Minor seventh | Minor | Minor 7 th | CE \flat GB \flat | v ⁷ IV |
| Augmented major seventh | Augmented | Major 7 th | CEG \sharp B | I ⁺⁷ |
| Augmented seventh | Augmented | Minor 7 th | CEG \sharp B \flat | V ⁺⁷ /IV |
| Half-diminished seventh | Diminished | Minor 7 th | CE \flat G \flat B \flat | v ^{\flat7} IV |
| Diminished seventh | Diminished | Diminished 7 th | CE \flat G \flat B $\flat\flat$ | v ^{\circ7} |

2.1.6 Inversion

A chord is "inverted" when the lowest note played is not the root note of the chord. An inverted chord is less stable than a non-inverted chord. In a triad, a first inversion happens when the third note is at the bass (E-G-C or E-C-G) and a second inversion happens when the fifth note is at the bass (G-C-E or G-E-C).

2.1.7 Nonharmonic notes

A note is termed *nonharmonic* if it does not belong in the chord. Sometimes, a nonharmonic note may be introduced into a chord to smooth out the melody line or to create dissonance. Dissonance is created when two or more notes sounded at the same time produce jarring effects which increase tension. In tonal music, these dissonances are resolved usually by bringing the piece back to the original key, thus creating a feeling of relaxation. One of the main difference between tonal and atonal music is this resolution or relaxation. The dissonances in atonal music may not be resolved at all, leaving the music tense throughout [15]. A discussion of the history of tonality and the evolution in the use of dissonance can be found in [16].

For the purpose of understanding the harmonic tension model, Lerdahl [6] categorizes these nonharmonic tones into: 1) seventh notes, 2) diatonic (notes belonging to the current scale), and 3) chromatic (notes outside of the current scale). For example, the B note in C-E-G-B is a diatonic seventh note in C major but a chromatic seventh note in C minor. There can be more than one nonharmonic note in a chord and these notes do not necessarily have to be categorized as a seventh note. The definition and use of nonharmonic tones will be expanded upon later in this chapter when Lerdahl's ideas of pitch space and harmonic tension model are discussed.

Events: 1 2 3 4 5 6 7 8 9 10 11 12 13

E♭: I V⁶ vii⁶/_V V V²/_{ii} ii⁶ V² I⁶ ii⁷ V⁷ I

Figure 2-1: Quarter-note level of the time-span reduction, bars 1-4 from Mozart’s Sonata, K.282, I [6].

2.2 Harmonic tension model

The tension and relaxation in a piece of music are caused by many factors such as surface tension, metrical tension, psychological tension and so on [6]. The work in this thesis is focused solely on chordal or harmonic tension based on Lerdahl’s sequential harmonic tension model [6].

In reality, as discussed in Chapter 1, the perception of music is affected by a lot of factors. For simplicity, Lerdahl assumes that there are two distinct manners in which music is perceived. Naive listeners analyze music locally (using only nearby chords) while experienced listeners take into account more global features (overall key, phrasing, etc.) when analyzing music. The sequential model reflects the local analysis of the naive listeners while the hierarchical model reflects the global analysis of the experienced listeners. In the sequential model, pitch-space distances are calculated from one chord to the next whereas the hierarchical model uses the prolongational tree to take into account tension values that are inherited from previous relevant chords. A comparison of tension calculations from both models based on bars 1-4 of Mozart’s Sonata, K.282², I (Figure 2-1) is shown in Figure 2-2. Although the trend calculated using the sequential model is not as pronounced as the one calculated using the hierarchical model, the general pattern is quite similar. Therefore, in this thesis, the sequential model will be used instead of the hierarchical model in the interest of reducing computational cost and memory usage.

The rest of this chapter contains brief summaries of relevant sections in Lerdahl’s *Tonal Pitch Space* [6] pertaining basic pitch space and the sequential harmonic tension model. The sequential harmonic tension model defines the tension between two chords as the sum of the chord surface tension (scale degree, inversion, and nonharmonic tones) and the distance between the two chords in pitch space. These two components will be expounded in the following sections.

² The letter “K” refers to the Köchel catalogue for Mozart’s compositions. Created by Ludwig von Köchel, this numbering system is used to catalogue Mozart’s work chronologically.

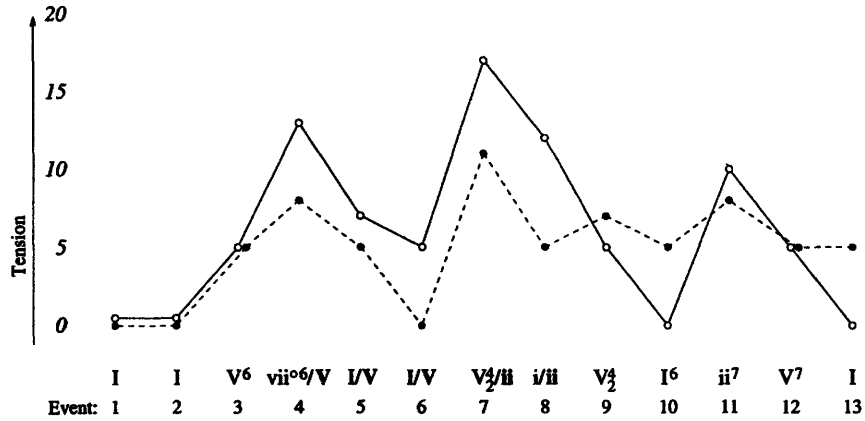


Figure 2-2: Graphical representation of the sequential and hierarchical tension values. The filled in nodes connected by dashed lines record the sequential values, and the open nodes connected by solid lines record the hierarchical values [6]. The trends for both models are similar although the hierarchical model trend has more pronounced slopes.

2.2.1 Chord distance

The calculation of chord distance is based on the concept of pitch space. The pitch space was introduced by Lerdahl [6] as a model to fully describe the tonal space because of the incompleteness of traditional geometric concepts of tonal hierarchy (Heinichen’s regional circle, Kellner’s regional circle, Weber’s regional chart, Shepard’s melodic map, etc.). The basic pitch space for the tonic (C-E-G) chord in C major (I/I) is shown in Figure 2-3. Level a is the octave space (note C), level b is the fifth space (notes C and G), level c is the triadic space (notes C, E, and G), level d is the diatonic space (all seven notes in C major), and level e is the chromatic space (all twelve notes). The stability of each space decreases from level a to level e. Figure 2-3(b) shows a numerical representation (C=0, C#=1, D=2, and so on) of the space shown in Figure 2-3(a).

The distance between two chords can be calculated using the following steps. An example is included at the end of the explanations.

1. Apply regional circle-of-fifths rule (changing keys)
2. Apply chordal circle-of-fifths rule
3. Calculate number of distinctive notes
4. Apply chord distance rule

Step 1: Apply regional circle-of-fifths rule

Figure 2-4 shows the chromatic circle of fifths that corresponds to repeated application of the regional circle-of-fifths rule. Examples of applying the regional circle-of-fifths rule are shown in Figure 2-5. When the regional circle-of-fifths rule is applied once to the right, the notes in C major (level d) are

a)

| | | | | | | | | | | | | | |
|----------|---|-----------|---|-----------|---|---|------------|---|-----------|---|-----------|-----|-----|
| level a: | C | | | | | | | | | | | (C) | |
| level b: | C | | | | | | G | | | | | (C) | |
| level c: | C | | | E | | | G | | | | | (C) | |
| level d: | C | D | | E | F | | G | A | | B | | (C) | |
| level e: | C | D \flat | D | E \flat | E | F | F \sharp | G | A \flat | A | B \flat | B | (C) |

I/I (= I/C)

b)

| | | | | | | | | | | | | |
|----------|---|---|---|---|---|---|---|---|---|----|----|-------------|
| level a: | 0 | | | | | | | | | | | (12 = 0) |
| level b: | 0 | | | | | | 7 | | | | | (12 = 0) |
| level c: | 0 | | | 4 | | | 7 | | | | | (12 = 0) |
| level d: | 0 | 2 | | 4 | 5 | | 7 | 9 | | 11 | | (12 = 0) |
| level e: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 (12 = 0) |

I/I

Figure 2-3: The basic space: (a) using letter names; (b) in numerical format. [6] Level a is the octave space, level b is the fifth space, level c is the triadic space, level d is the diatonic space, and level e is the chromatic space. The stability of each space decreases from level a to e.

Regional circle-of-fifths rule

Move the notes at level d of the the basic space seven steps to the right or left (mod 12) on level e or seven steps to the left.

shifted seven steps to the right on level e, creating the diatonic space of G major. Similarly, when the rule is applied three times to the left, the notes in C major are shifted 3 steps (21 mod 12) to the left as shown in Figure 2-4, creating the diatonic space of C minor. Note that only the natural minor scales are treated (harmonic and melodic scales are excluded). See B for reference on scales.

Step 2: Apply chordal circle-of-fifths rule

After applying the regional circle-of-fifths rule, apply the chordal circle-of-fifths rule on the basic space of the first chord until the basic space for the second chord is achieved. Figure 2-7 shows an example of this step for calculating the distance between the I and V chords, and between the I and ii chords. According to the chord circle (Figure 2-6), chord V is one step to the right of chord I. Therefore, apply the chordal circle-of-fifths rule once and shift levels a-c four (level d) steps to the right to achieve the basic space for the V chord. On the other hand, ii is two steps to the right of I in the chord circle. Therefore, levels a-c are shifted eight (level d) steps to the right to achieve the basic space for ii. Note that steps could also be taken to the left. For example, the basic space for ii could also be achieved by applying the chordal circle-of-fifths rule five times to the left (shift twenty level d steps to the left in basic space).

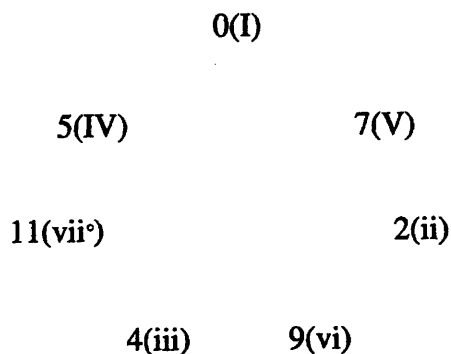


Figure 2-6: The diatonic cycle of fifths, or chord circle [6]. The numbers indicate the root notes in chromatic space relative to the current key and the Roman numerals indicate the scale-degree. For example, in C major, 7(V) indicates the 7th note in C major's chromatic space (G) which is the root note for the dominant chord, G-B-D.

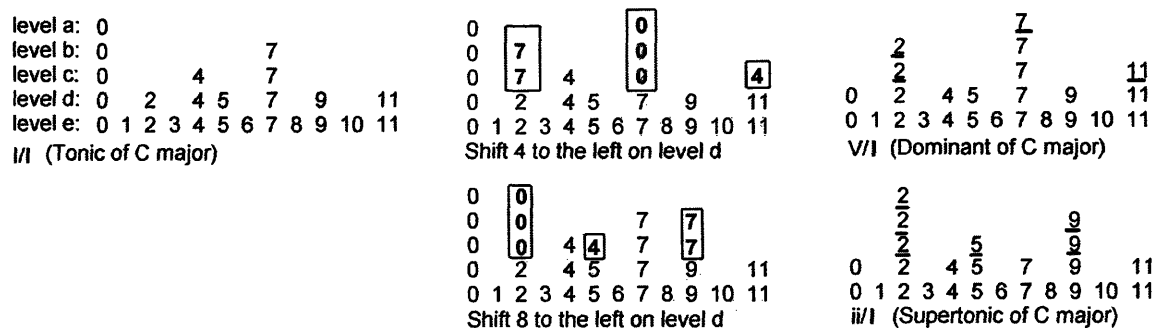


Figure 2-7: Examples of Step 2: Applying the chordal circle-of-fifths rule to calculate the distance between chords I and V, and chords I and ii. According to the chord circle (Figure 2-6), chord V is one step to the right of chord I. Therefore, in the basic space, levels a-c are shifted four (level d) steps to the right to achieve the basic space for the V chord. The boxed numbers in the second column indicate the new locations of the shifted space. The underlined numbers are the distinctive notes compared to the original chord, I.

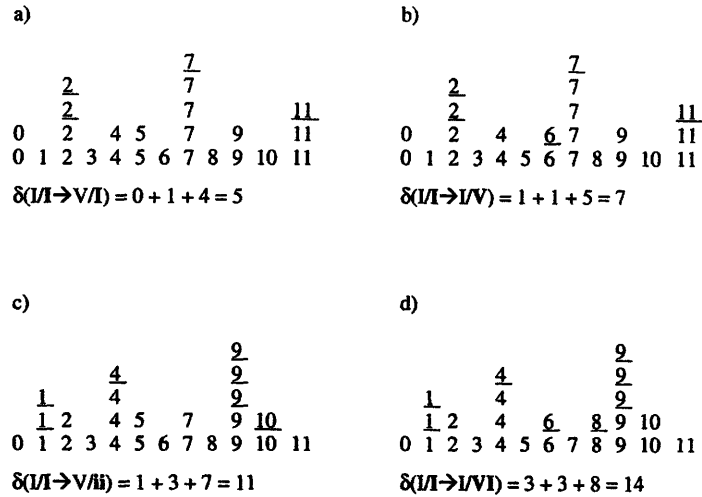


Figure 2-8: Examples of interregional chordal distances [6]. i is indicated by the number of underlines on level d , j is indicated by the number of steps needed to shift levels a - c to the desired chord, and k is indicated by the number of underlines on levels a - c .

Step 3: Calculate number of distinctive notes

This step involves comparing the basic space of the two chords for distinctive notes. The underlined numbers in Figure 2-7 are notes that do not exist in the basic space of the chord that is being compared (I/I). So in the case of I - V , there are two distinctive notes on level c (D and B), one on level b (D), and one on level a (G).

Step 4: Apply chord distance rule

Chord distance rule, $\delta(x \rightarrow y) = i + j + k$, where
 $\delta(x \rightarrow y)$ = distance between chords x and y
 i = number of applications of the regional circle-of-fifths rule needed to shift the diatonic collection that supports x into the diatonic collection that supports y . The simplest way to calculate this is to count the number of changes in sharps and flats in the key signature.
 j = smallest number of applications of the chordal circle-of-fifths rule needed to shift x into y
 k = number of distinctive notes in the basic space of y compared to those in the basic space of x

Figure 2-8 shows examples of calculating chord distances using the four steps outlined in this section. The notation $\delta(I/I \rightarrow I/V)$ indicates the chordal distance between the tonic of C major (C-E-G) and the tonic of G major (G-B-D) (recall that C major is used by default in this thesis).

A note that doesn't belong to the original chord could either be "borrowed" or caused by a

regional shift (change in key). For example, a C-E \flat -G chord could either be the tonic of a C minor scale, I/i, or the tonic of a C major scale, I/I with a borrowed E \flat . Although this chord distance model expresses the concept of borrowing as well as regional shift, it does not indicate which of the two is preferred during an analysis of a piece. The choice between the two has to be determined using the Preference Rule [6] (Chapter 5 of Lerdahl's *Tonal Pitch Space*) which involves the context in which the event occurs.

It has been noted in [6] that the chord distance rule only works well for nearby regions. The analysis of distant regions will not be addressed in this thesis.

2.2.2 Chord surface tension

Another factor that affects the sequential harmonic tension is the chord surface tension. The surface tension rule is used to take into account the dissonance and surface tension of the melodic line. Surface dissonance is subdivided into three variables: scale degree (tracks melodic note over the triad that supports it), inversion, and nonharmonic tone (seventh, diatonic, or chromatic).

Surface Tension Rule, $T_{diss}(y)$ = scale degree (add 1) + inversion (add 2) + nonharmonic tone (add 1 for sevenths, 3 for diatonic nonharmonic tones, and 4 for chromatic nonharmonic tones)

$T_{diss}(y)$ = the surface tension associated with chord y ,
 scale degree = chords with non root note in the melodic voice
 inversion = chords with non root note in the base
 nonharmonic tone = any note that does not belong to y

2.2.3 Sequential tension rule

The sequential tension rule is obtained by combining the two factors described earlier: chord distance in pitch space and chord surface tension.

Sequential tension rule, $T_{seq}(y)$ = $T_{diss}(y) + \delta(x_{prec} \rightarrow y)$
 y is the target chord
 x_{prec} is the chord that immediately precedes y in the sequence
 $T_{seq}(y)$ is the tension associated with y
 T_{diss} is the surface tension rule
 $\delta(x_{prec} \rightarrow y)$ is the distance from x_{prec} to y
 (using $\delta[x \rightarrow y] = i + j + k$)

Figure 2-9 shows the analysis of the four-bar Mozart piece used previously (Figure 2-1) using the sequential tension model. The chord in event 1 is a non-inverted (root note E is at the bass of

| | surface dissonance | | | pitch-space distance | | | total |
|---|-----------------------|------|-------|-------------------------|---|---|-------|
| | sc.-deg. | inv. | nh.t. | i | j | k | |
| T _{seq} (0→1) [I/I] | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| T _{seq} (1→2) [I] | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| T _{seq} (2→3) [V ⁶] | 0 | 2 | 0 | 0 | 1 | 4 | 7 |
| T _{seq} (3→4) [vii ^{o6} /V] | 1 | 2 | 0 | 1 | 1 | 6 | 11 |
| T _{seq} (4→5) [I/V] | 1 | 0 | 0 | 0 | 1 | 4 | 6 |
| T _{seq} (5→6) [I/V] | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| T _{seq} (6→7) [V ₂ /II] | 0 | 2 | 1 | 1 | 2 | 8 | 14 |
| T _{seq} (7→8) [i/ii] | 1 | 2 | 0 | 0 | 1 | 4 | 8 |
| T _{seq} (8→9) [V ⁴ ₂] | 0 | 2 | 1 | 1 | 1 | 5 | 10 |
| T _{seq} (9→10) [I ⁶] | 1 | 2 | 0 | 0 | 1 | 4 | 8 |
| T _{seq} (10→11) [ii ⁷] | 0 | 0 | 1+3 | 0 | 2 | 6 | 12 |
| T _{seq} (11→12) [V ⁷] | 1 | 0 | 1 | 0 | 1 | 4 | 7 |
| T _{seq} (12→13) [I] | 1 | 0 | 0 | 0 | 1 | 4 | 6 |

Figure 2-9: Sequential tension: surface dissonance plus sequential pitch-space distance “Sc.-deg” = scale degree; “inv” = inversion; “nh.t.”= nonharmonic tone. [6]

the chord) tonic in Eb major (Eb-G-Bb). Because the top-most note is the fifth note (Bb) instead of the root (Eb), the scale degree is 1 in the the calculations. Event 11 shows examples of a seventh and a diatonic nonharmonic tone. Event 11 is a non-inverted supertonic dominant seventh of Eb major (F-Ab-C(missing)-Eb). A triad is assumed and the missing C was added automatically during the analysis. The presence of a diatonic seventh and the nonharmonic diatonic Bb adds 1 and 3 respectively to the calculations.

Chapter 3

Mapping Harmonic Tension Model to Surface Tension

The main purpose of this thesis is to examine whether the harmonic tension model proposed by Lerdahl can be expressed as tangible physical tension to facilitate the intuitive understanding of harmonic tension. To determine the feasibility of the mapping, I simulated the surface tension of an elastic sphere using the Netlogo multi-agent simulation program. The amount of tension calculated using the harmonic tension model is proportional to the distortion or surface tension of the simulated sphere. This idea of using surface tension as an equivalent to the harmonic tension model is tested on Pachelbel's *Canon in D* and Rimsky-Korsakov's *The Flight of the Bumblebee*.

3.1 Physical representation of harmonic tension

The surface of a visco-elastic sphere is chosen as the medium for expressing harmonic tension because of the sphere's perfect symmetry. Because of this unique property, there are no borderline cases and every point on the surface of the sphere can be treated similarly. Moreover, a sphere seems to be a suitable representation for the state of equilibrium (no tension) because the shape occurs naturally due to the minimization of surface area to volume ratio.

Tension values calculated using the harmonic tension model is translated directly into the surface tension of the sphere. Therefore, the more distorted the sphere, the higher the tension value. To simulate increase in surface tension, the simulated sphere is distorted by making protrusions on the surface (akin to having something poking out from within the sphere). Parts of the sphere surface that has been stretched would then return slowly to its original form. The visco-elasticity of the sphere surface simulates the relaxation phases which typically follows phases with high tension in tonal music.

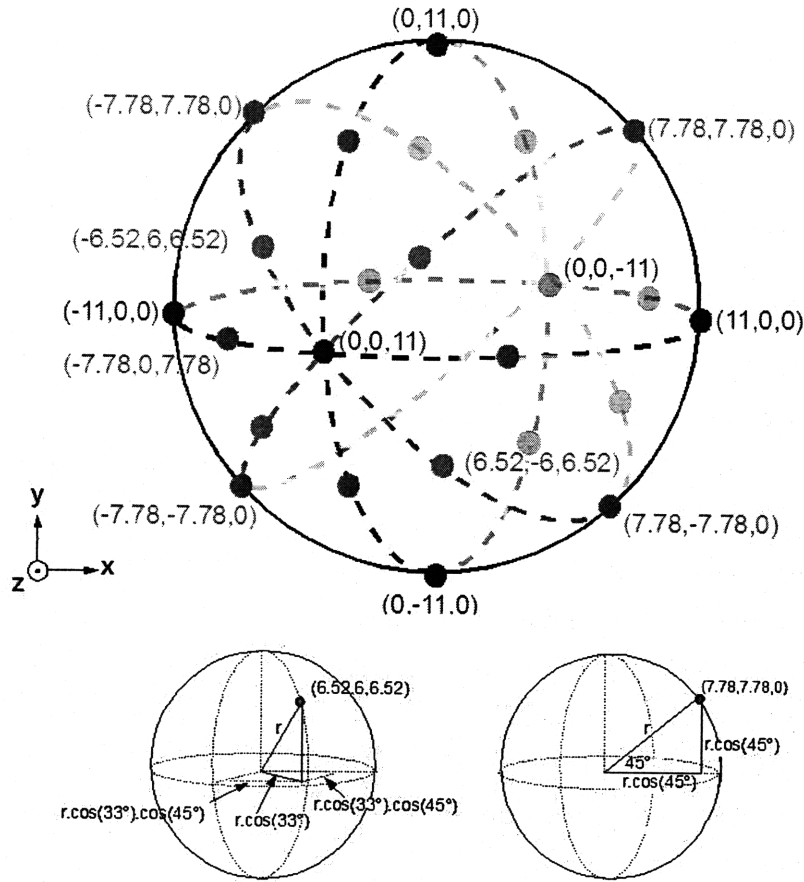


Figure 3-1: Points on the surface of the simulated sphere.

The sphere surface is subdivided into 26 non-overlapping areas as shown in Figure 3-1 and only one protrusion is generated from each area. Although the result is not as realistic because of these restrictions, the simulation is simplified because there is no need to account for interaction between patches. Because the translation of the harmonic tension value to surface tension value is relative, for simplicity, one protrusion is equivalent to a tension value of 1. Therefore, the maximum achievable tension value is 26, which suffices for the chosen examples: Pachelbel's *Canon in D* and Rimsky-Korsakov's *The Flight of the Bumblebee*. These limitations merely simplify the simulation but do not affect the result of the translation as shown at the end of this chapter. Possible future expansion of this physical translation will be discussed in Chapter 5.

3.2 Simulation of physical mapping

The idea of translating harmonic tension to physical tension described above is examined using the Netlogo 3D Preview simulation. This section describes the harmonic tension analysis of two pieces, Pachelbel's *Canon in D* and Rimsky-Korsakov's *The Flight of the Bumblebee*, and a simulation of

CEG GBD ACE EGB FAC CEG FAC GBD
 I V vi iii IV I IV V

Figure 3-2: First four measures of Variations on the *Canon in D* transcribed by George Winston for the piano. A more extensive analysis is shown in Appendix C.

the physical tension corresponding to the results of the analysis.

3.2.1 Harmonic analysis

As mentioned in Chapter 2, for the purpose of harmonic analysis, chords are always treated as triads. Therefore, notes which are not explicitly used in the piece will be automatically added into the triad while notes not belonging to the triad will be treated as nonharmonic notes. For example, as shown in the analysis of *Canon in D* (Figure 3-2), event 2 is assumed to be the dominant chord G-B-D even though the pitch class B is not present in the piece. An example of nonharmonic notes can be found in Figure 3-3 (*The Flight of the Bumblebee*) event 7. Notes D#, D \flat , and C# which do not belong in the A-C-E minor tonic chord are treated as nonharmonic notes in the harmonic tension model.

As seen in Figure 3-2, the first phrase of *Canon in D* is very straightforward. The piece remains in C major throughout and consists mainly of the I-V-vi-iii-IV-I-IV-V harmonic sequence which is repeated throughout the piece with only slight variation in the melody line and accompaniment. These four measures will be referred to as part of the analysis in this chapter. A more extensive harmonic analysis can be found in Appendix C.

Figure 3-3 shows the harmonic analysis for measures 9 to 12 of *The Flight of the Bumblebee*. Despite the fairly straightforward harmonic progression, the appearance of multiple nonharmonic notes in the melody line creates high tension throughout the piece and accounts for the dissonance. Note that in several cases, the chord can either be treated as a seventh or a triad with a nonharmonic note. The distinction is not crucial because in the harmonic tension model, the difference between the two choices is merely 1 which is negligible compared to the average tension values shown in the following calculations.

Figure 3-3: Measures 9-12 of Rimsky-Korsakov’s *The Flight of the Bumblebee*. Nonharmonic notes are circled. A more extensive analysis is shown in Appendix C.

Table 3.1: Harmonic tension analysis for four measures of Pachelbel’s *Canon in D* (C major) transcribed by George Winston for the piano. Sequential tension: Surface dissonance + sequential pitch-space distance. “Sc.-deg” = scale degree; “inv” = inversion; “nh.t.”= nonharmonic tone.
 $T_{seq}(x_{prec} \rightarrow y) = T_{diss}(y) + \delta(x_{prec} \rightarrow y)$

| | | Surface dissonance | | | Pitch space distance | | | |
|----------------------------|----------|--------------------|------|-------|----------------------|---|---|-------|
| | | sc.-deg | inv. | nh.t. | i | j | k | Total |
| $T_{seq}(0 \rightarrow 1)$ | I → I | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| $T_{seq}(1 \rightarrow 2)$ | I → V | 1 | 0 | 0 | 0 | 1 | 4 | 6 |
| $T_{seq}(2 \rightarrow 3)$ | V → vi | 1 | 0 | 0 | 0 | 2 | 6 | 9 |
| $T_{seq}(3 \rightarrow 4)$ | vi → iii | 1 | 0 | 0 | 0 | 1 | 4 | 6 |
| $T_{seq}(4 \rightarrow 5)$ | iii → IV | 1 | 0 | 0 | 0 | 2 | 6 | 9 |
| $T_{seq}(5 \rightarrow 6)$ | IV → I | 0 | 0 | 0 | 0 | 1 | 4 | 5 |
| $T_{seq}(6 \rightarrow 7)$ | I → IV | 1 | 0 | 0 | 0 | 2 | 4 | 7 |
| $T_{seq}(7 \rightarrow 8)$ | IV → V | 1 | 0 | 0 | 0 | 1 | 6 | 8 |

3.2.2 Lerdahl’s Harmonic Tension Model

Tables 3.1 and 3.2 show the harmonic tension calculation for the four measures shown in Figures 3-2 and 3-3 respectively. The tension values in both pieces correspond to the patterns of tension and relaxation. The *Canon in D* has fairly low tension values as reflected by the pleasant harmonies throughout the four measures, whereas the *Flight of the Bumblebee* has higher tension values which correspond to dissonance. Full calculations of tension values are shown in Appendix E. As shown in Table 3.2, the high tension values are caused by the nonharmonic notes in the melody line.

The tension values for approximately the first 60 events for both pieces are shown in Figure 3-4. As you can see, the tension values in *Canon in D* are fairly low with small deviations in beginning but has higher tension and variations later on due to the introduction of several nonharmonic notes to create movement in the melody line. *The Flight of the Bumblebee* has higher tension values and larger variations throughout because of the many nonharmonic passing notes in the melody line. From this analysis, it can be seen that higher tension values and transitions between tension and relaxation contribute greatly to the movement and pace of the pieces.

Table 3.2: Harmonic tension analysis for four measures of Rimsky-Korsakov's *The Flight of the Bumblebee* (A minor). Sequential tension: Surface dissonance + sequential pitch-space distance. "Sc.-deg" = scale degree; "inv" = inversion; "nh.t." = nonharmonic tone. $T_{seq}(x_{prec} \rightarrow y) = T_{diss}(y) + \delta(x_{prec} \rightarrow y)$

| | | Surface dissonance | | | Pitch space distance | | | Total |
|------------------------------|-------------------------|--------------------|------|-------|----------------------|---|---|-------|
| | | sc.-deg | inv. | nh.t. | i | j | k | |
| $T_{seq}(5 \rightarrow 6)$ | $IV^7 \rightarrow IV^7$ | 1 | 0 | 18 | 0 | 0 | 0 | 19 |
| $T_{seq}(6 \rightarrow 7)$ | $IV^7 \rightarrow IV^7$ | 1 | 0 | 15 | 0 | 0 | 0 | 16 |
| $T_{seq}(7 \rightarrow 8)$ | $IV^7 \rightarrow IV^7$ | 1 | 0 | 16 | 0 | 0 | 0 | 17 |
| $T_{seq}(8 \rightarrow 9)$ | $IV^7 \rightarrow IV^7$ | 1 | 0 | 15 | 0 | 0 | 0 | 16 |
| $T_{seq}(9 \rightarrow 10)$ | $IV^7 \rightarrow iv^7$ | 0 | 2 | 11 | 0 | 0 | 2 | 15 |
| $T_{seq}(10 \rightarrow 11)$ | $iv^7 \rightarrow i$ | 1 | 2 | 15 | 0 | 1 | 5 | 24 |
| $T_{seq}(11 \rightarrow 12)$ | $i \rightarrow V^7$ | 0 | 2 | 16 | 0 | 1 | 4 | 23 |

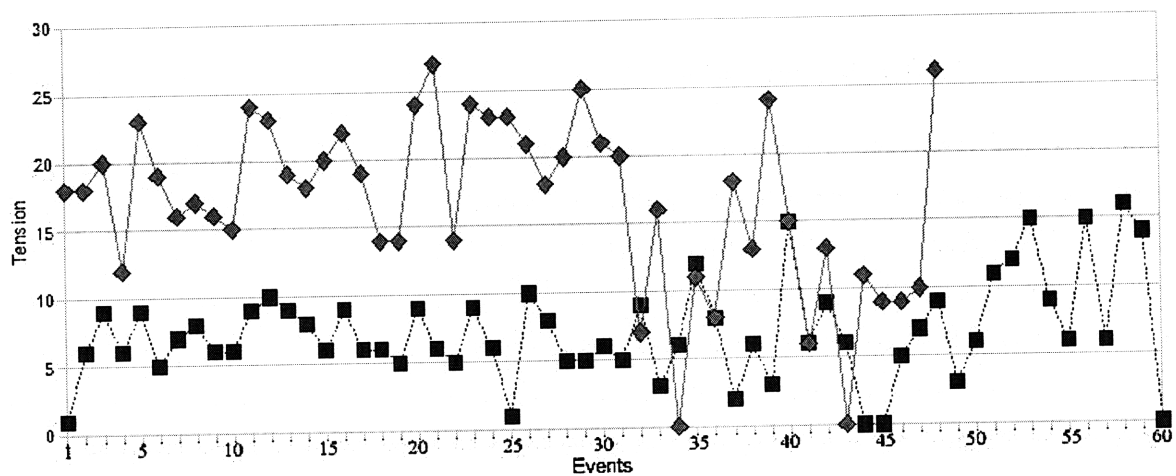


Figure 3-4: Graphical representation of the calculated tension values for *Canon in D* (square points, with dotted lines) and *The Flight of the Bumblebee* (diamond, solid lines). On average, the tension values of *The Flight of the Bumblebee* is higher than those of *Canon in D*.

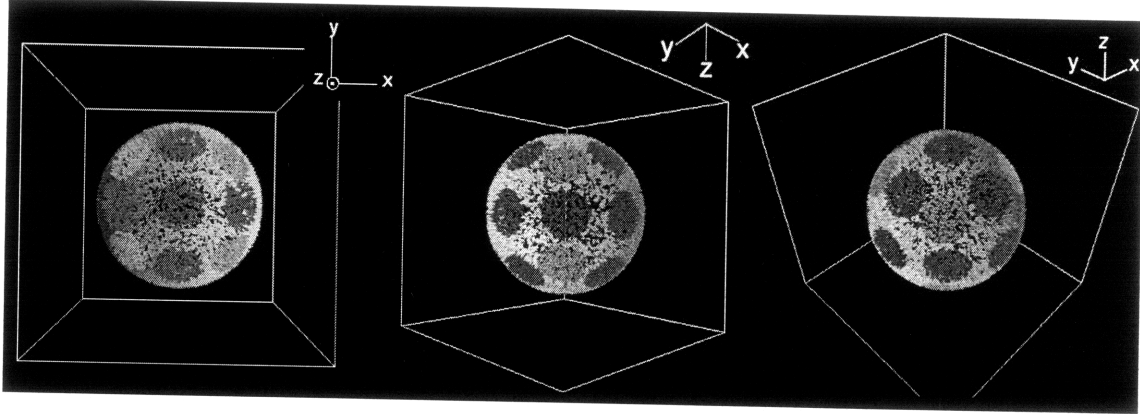


Figure 3-5: Non-overlapping and non-interacting circular patches of agents on the simulated sphere shown in different angles using Netlogo's 3D perspective manipulation.

3.2.3 Simulation

The Netlogo 3D Preview 5 multi-agent modeling platform was chosen for the simulation because the program is well-suited for the task of manipulating individual groups of agents (patches) based on user interaction [17]. The sphere was first created with 10000 individual agents. These agents were then subdivided into 26 circular, non-overlapping, and non-interacting patches where each patch would simulate one protrusion. Figure 3-1 shows the 26 points on the sphere. These points are chosen to maximize the area of the patches as shown in Figure 3-5 and each patch has a radius of $r\sqrt{2}/5$ from each of the 26 center points.

Each patch is responsible for creating only one protrusion. The magnitude of the protrusion is determined by the user's input. To mimic protrusion on a elastic surface, the center of a patch is the point where force is exerted and therefore has the largest displacement. The amount of displacement decays exponentially for agents further away from the center point. The protrusion of the patches as shown in Figures 3-11 and 3-12 are calculated based on the perpendicular distance of the individual agents within the patch to the plane tangent to the center point in the patch as shown in Figure 3-6. The equation used to calculate the amount of displacement needed is $val \times e^{6(dist-r/2)} / (e^{3r})$ where *val* is the maximum displacement determined by the user (maximum of 4 and minimum of 0 - no displacement). Figure 3-7 shows a plot of the equation with *val* = 0, 2, and 4.

The input to the simulation is in the form of a text file. The system goes through a prepared text file (see Figure 3-8) containing information on which patches to move and how much to move at given intervals. Each line contains 27 numbers: The first 26 numbers indicate the maximum displacement for each patch and the last number indicates the waiting time before resuming simulation to synchronize the simulation with the actual music. Arbitrarily, a protrusion with maximum displacement of 2 corresponds to tension value 1. Therefore the total tension of all 26 patches corresponds to the tension value of the music at that time. For example, the text input in Figure 3-8 shows a tension value of 2 with a protrusion on patches 1 and 10 for 0.85 seconds, followed by a

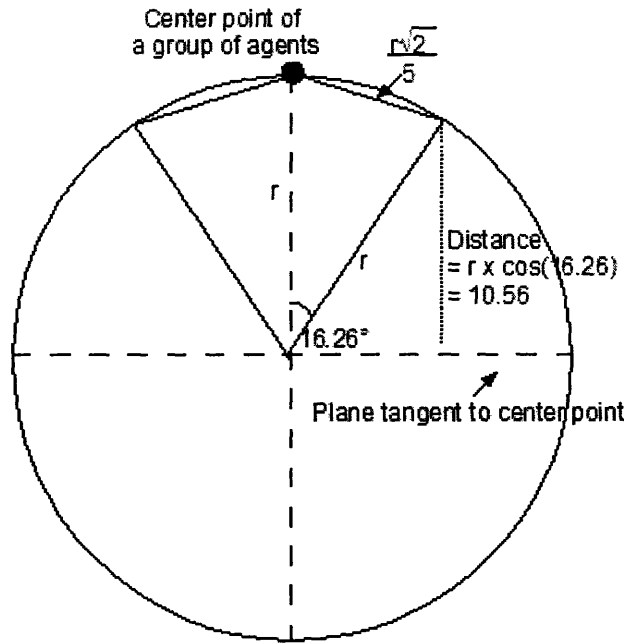


Figure 3-6: Relationship between distance of agent from plane tangent to center point to displacement to create protrusion on surface of sphere.

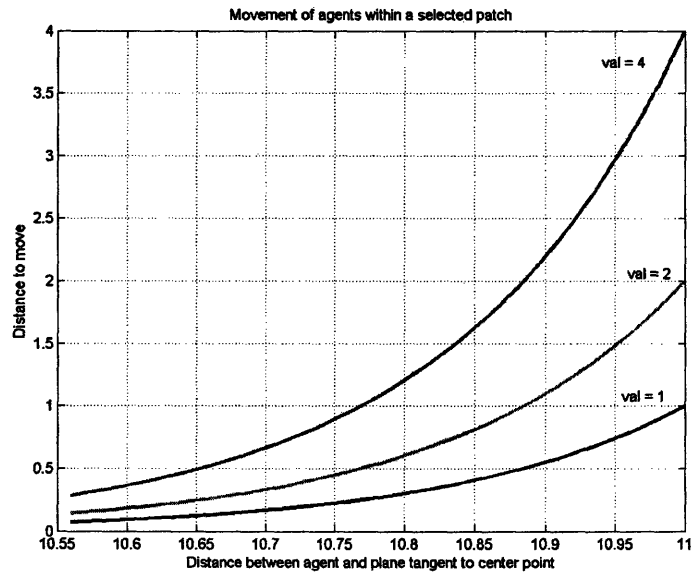


Figure 3-7: Plot of $val \times e^{\delta(dist-r/2)} / (e^{3r})$ with $val = 0, 2, \text{ and } 4$. Maximum displacement at center point where distance of agent from plane is equal to radius of sphere. The amount of displacement decays exponentially for agents further away from the center point.

```
2 0 0 0 0 0 0 0 0 0 2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.85
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1.00
```

Figure 3-8: Example of an input text file used to control the simulation. The first 26 numbers indicate the maximum displacement for each patch and the last number indicates the waiting time before resuming simulation (to synchronize simulation with actual music and chord progression).

tension value of 0 (no protrusions) for 1 second. The input text files for the simulation of *Canon in D* and *The Flight of the Bumblebee* is shown in Appendix F. Figure 3-10 is a pseudocode walkthrough of the simulation written in Netlogo. The actual code is provided in Appendix D. A simple interface is used to control the simulation (see Figure 3-9). The buttons on the interface are merely used to set up the sphere and start the simulation by going through the prepared input text file. A slider is provided to change the color of the sphere. To give an overall view of the simulation during runtime, the sphere is set to orbit round at every step.

The following Figures 3-11 and 3-12 show the simulated harmonic progression and tension values for the *Canon in D* (events 1-8) and *The Flight of Bumblebee* (events 6-12) respectively. As expected, the simulation of *Canon in D* looks more or less spherical throughout because of the straightforward harmonic progression. On the other hand, the simulation for *The Flight of the Bumblebee* looks very distorted because of the dissonance.

3.3 Conclusion

From observation, a physical representation of the harmonic tension model provides an intuitive view of the model and the relationship between harmonic progression and tonality within different contexts. Viewing the simulation while listening to the piece enables users to visualize the tonality of the piece, making them more aware of the tension/relaxation patterns and enhancing their listening experiences.

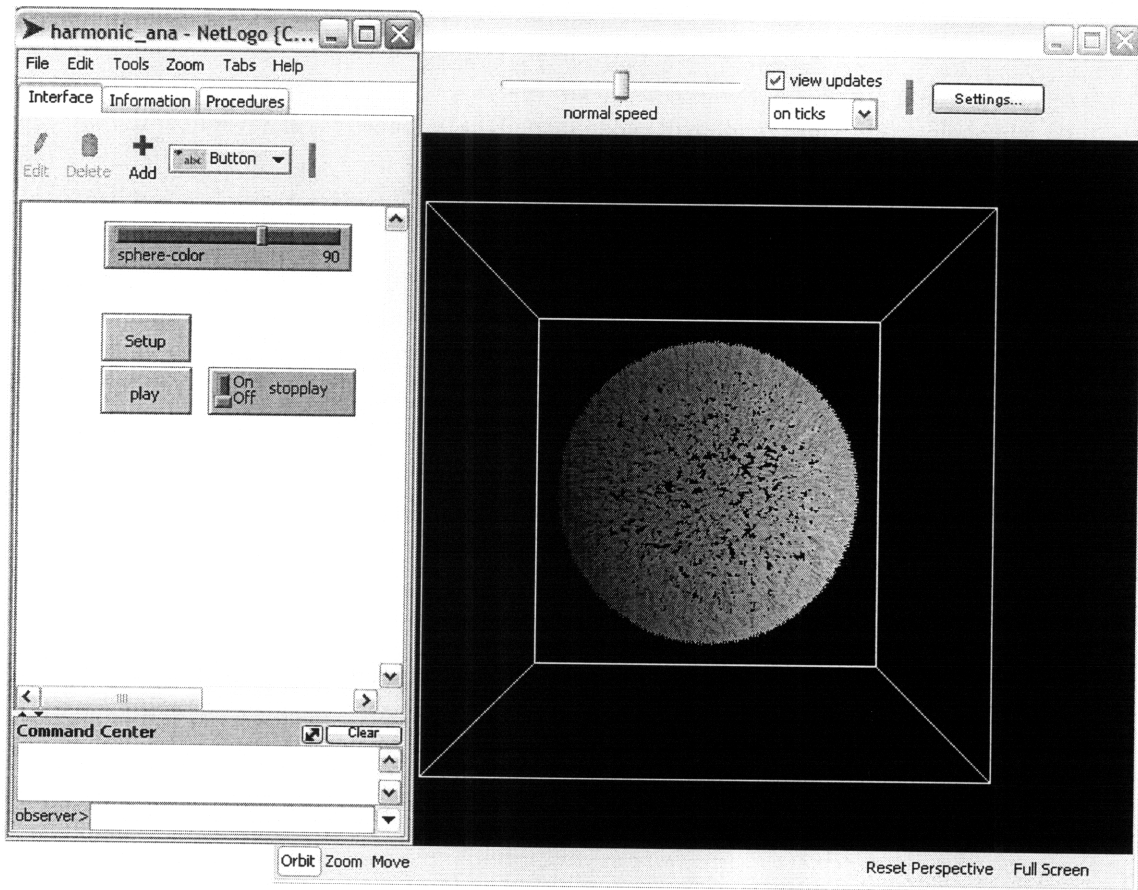


Figure 3-9: Interface for the simulation in NetLogo and preview of the sphere.

```

to setup
  Create initial sphere
  Set up agent patches on sphere
  Set up files for receiving input
end

to play ;; Loop forever until stopped or file ends
  while [ not file-at-end? ]
    Go through text input and protrude accordingly (see to make-dent)
    Change perspective
  end

to select [val] ;; Select one of the 26 patches
  return patch to original position (sphere)
  so that previous condition of patch will not affect current input value
  value determines how far agents should move (max 4, min 0)
  patch move according to selected value
end

to-report inrange? [x y z]
  report all agents within set radius of center point
end

to make-dent
  read file
  set data-list [patch1 patch2 ... patch 26 orbit-up orbit-right]
end

to make-dent
  select patch1 (value = first item in data-list)
  .
  .
  select patch26 (value = item 26 in data-list)
  wait (value = item 27 in data-list)
end

```

Figure 3-10: Simulation pseudocode. The physical mapping of Lerdahl's harmonic tension model is visualized using a simulation written in Netlogo. See Appendix D for the complete code.

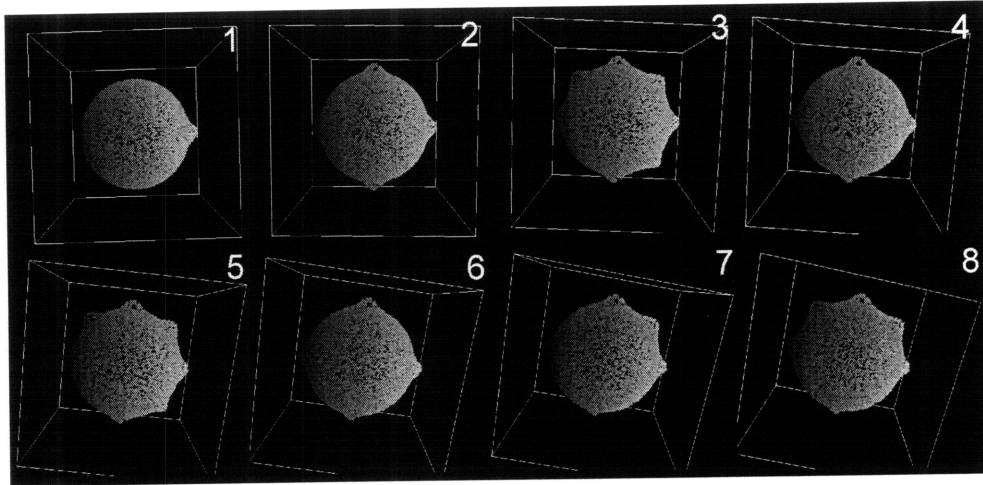


Figure 3-11: Screenshots of simulation for events 1-8 (*Canon in D*). Sphere distortion is low which reflects low tension in the piece. Perspective of simulation is rotated throughout to give overall view of the shape.

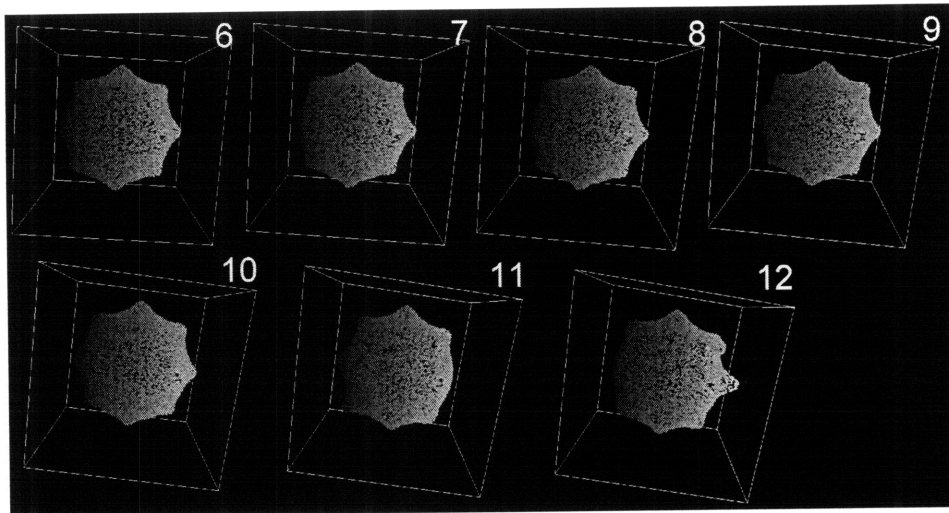


Figure 3-12: Screenshot of simulation for events 6-12 (*The Flight of the Bumblebee*). Sphere distortion is high which reflects high tension in the piece. Perspective of simulation is rotated throughout to give an overall view of the shape.

Chapter 4

Interactive Learning System

Based on observations of the harmonic tension simulations (*Canon in D* and *The Flight of the Bumblebee* in Chapter 3), the expression of Lerdahl's harmonic tension model into physical tension could potentially be used as an educational tool for understanding music tonality, dissonance, and harmonic progression. Therefore, to expand upon the idea of translating the harmonic tension model to physical surface tension, an interactive system was developed to allow users to explore the idea of tension and relaxation in relation to harmonic progression.

The system described in Chapter 3 demonstrates the physical mapping of harmonic tension by manipulating the simulated surface tension according to the harmonic analysis of a piece. However, for the interactive learning system described in this chapter, a different approach is used to demonstrate the physical mapping. In this system, users first choose a particular tension value using push buttons and subsequently observe the results in a simulation and listen to chords corresponding to that tension value. This interactive system consists of a simulation similar to the one described in Chapter 3 and a hardware component that is used to interact with the simulation. A block diagram of the system is shown in Figure 4-1. The user manipulates the tension values using six push buttons located on the surface of a solid sphere. When a button is pressed, the microcontroller assigns tension value to a coordinate specific to the button and calculates the total tension value. Based on this data, a protrusion is created on the simulated sphere at the specified coordinate. The total tension value is also used to select a recorded chord segment to be played, giving the user auditory as well as visual representation of the chosen tension value. An accelerometer is also used to give users control of the 3D view in the simulation.

4.1 Tension and chord relationship

The relationship between selected tension values and chords are based on Lerdahl's sequential harmonic tension model as described in Chapter 2. A session always begins with the tonic (I) C-E-G-C

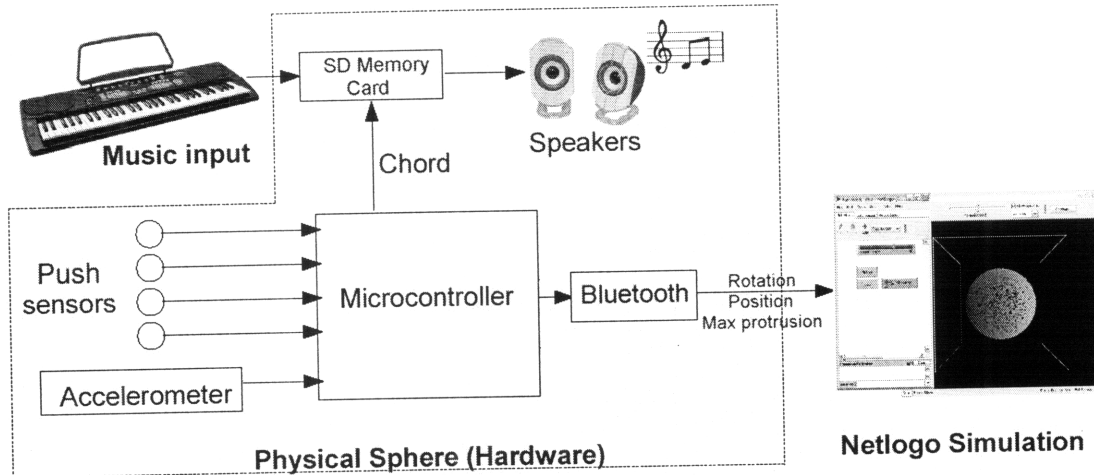


Figure 4-1: Block diagram of the interactive harmonic tension learning system. The system consists of a hardware component and a simulation. Information is transferred from a microcontroller to the sphere simulation via Bluetooth.

chord and a tension value of 0. Each subsequent button press increases the tension value by 2. Therefore, the more buttons that are pressed (active) simultaneously, the larger the tension, distortion, and dissonance. At each processing cycle, a chord is chosen from a table based on the previous chord and the current tension value and played on the speaker. After that, the tension value is reduced by one for each active button. For example, if two buttons had been pressed (active) in the previous cycle, the tension value is reduced by two; one for each button. This process is repeated until the total tension is reduced to zero. The reduction of tension values imitates the relaxation phases in tonal music.

Recall that the tension between two adjacent events (chords) can be calculated using the sequential harmonic tension model (Chapter 2):

$$\text{Sequential tension rule, } T_{seq}(y) = T_{diss}(y) + \delta(x_{prec} \rightarrow y)$$

y is the target chord

x_{prec} is the chord that immediately precedes y in the sequence

$T_{seq}(y)$ is the tension associated with y

T_{diss} is the surface tension rule

$\delta(x_{prec} \rightarrow y)$ is the distance from x_{prec} to y (using $\delta[x \rightarrow y] = i + j + k$)

Surface Tension Rule, $T_{diss}(y) = \text{scale degree (add 1) + inversion (add 2) + nonharmonic tone (add 1 for sevenths, 3 for diatonic nonharmonic tones, and 4 for chromatic nonharmonic tones)}$

$T_{diss}(y)$ = the surface tension associated with chord y ,

scale degree = chords with non root note in the melodic voice

inversion = chords with non root note in the base

nonharmonic tone = any pitch class that does not belong to y

In terms of this application, y is the chord to be chosen given x_{prec} (chord that was previously played) and $T_{seq}(y)$ (chosen tension value). To reduce the number of possible variations for a given chord, the scale degree and inversion variables are set to zero i.e. the melody and bass notes are set to root. For example, the I chord is fixed to C-E-G-C only instead of having variations in the scale degree such as C-E-G or C-G-E, and inversions such as E-G-C, G-E-C, E-C-G, or G-C-E. Note that because the key remains in C major throughout, i is always 0. Appendix G shows a full list of chord pairs and corresponding tension values based on the sequential harmonic tension model. Observe that one tension value could correspond to several chords. For example, given that the previous chord is I and the tension value is 18, the current chord could be, among others, i^7 half diminished, ii, or III^7 major. The choice is made based on the proximity of the the chord to the previous chord in the circle of fifth. In this example, the i^7 chord is chosen because the distance between this chord and the previous chord is zero in the circle-of-fifths. Notice that the seventh chords have the same tension values as their basic triads because the seventh note is treated as nonharmonic. Take the $I \rightarrow ii$ and $I^7 \text{ Major} \rightarrow ii$ progressions for comparison. The $T_{diss}(y)$ is 0 for both because events y , ii, do not have any nonharmonic notes and the variable j is 2 for both progressions in the circle-of-fifths. Because k is calculated based on the comparison of only the basic triads, C-E-G and D-F-A, the contribution of the seventh note, B, in the $I^7 \text{ Major}$ is immaterial. Table 4.1 shows the chord pair and tension values used in the system. Only a subset of the table shown in Appendix G and is used because of hardware memory limitations. In this table, only chords in the tonic (I, i, I+, i° , I^7M , I^7D , i^7MM , i^7m , I^7AM , I^7A , i^7HD , $i^{\circ 7}$), dominant (V, v, V+, v° , V^7M , V^7D , v^7MM , v^7m , V^7AM , V^7A , v^7HD , $v^{\circ 7}$), submediant (VI, vi, VI+, vi° , VI^7M , VI^7D , vi^7MM , vi^7m , VI^7AM , VI^7A , vi^7HD , $vi^{\circ 7}$), and supertonic (VII, vii, VII+, vii° , VII^7M , VII^7D , vii^7MM , vii^7m , VII^7AM , VII^7A , vii^7HD , $vii^{\circ 7}$) are used.

The following sections describe, in more detail, the modifications made to the simulation in Chapter 3 and the hardware component used to interact with the simulation. Using this system, users will be able to learn about dissonance and its relation to tension, and experiment with chord progression in relation to tension and relaxation.

4.2 Simulation

As previously mentioned, the simulation used in this system is very similar to the one described in Chapter 3. This section describes the major changes made to the simulation program to accommodate the hardware restrictions and new functions.

Because of memory restrictions on the hardware, only 6 points are used instead of 26. The radius of the patch is increased to $r\sqrt{2}/2$ as shown in Figure 4-2 but the displacement calculation is still the same as before - the displacement is proportional to the perpendicular distance of the individual

Table 4.1: Chord pairs and corresponding tension values. Notations: Uppercase roman numeral (Major), Lowercase roman numeral (Minor), + (Augmented), ° (Diminished), M (seventh, Major), D (seventh, Dominant), MM (seventh, Minor Major), m (seventh, Minor), AM (seventh, Augmented Major), A (seventh, Augmented), HD (seventh, Half Diminished), °7 (seventh, Diminished). A full list of chord pairs and tension values is presented in Appendix G.

| Current Chord | Tension Values | | | | | | | | | | | | |
|------------------|------------------|------------------|---------------------|-------------------|-------------------|--------------------|---------------------|---------------------|---------------------|--------------------|--------------------|--------------------|--------------------|
| | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| I | I | I | I [♯] M | i | vii [°] | i [♯] MM | I [♯] A | v [♯] m | VII [♯] AM | i [♯] HD | i [♯] HD | VII [♯] D | VII [♯] D |
| i | I | I | i | I [♯] M | i [♯] MM | v | vii [♯] HD | v [♯] m | i [♯] HD | VI [♯] A | VII [♯] D | VI [♯] M | VII [♯] M |
| I ⁺ | I | I | I ⁺ | I ⁺ | I [♯] D | vi [♯] m | V [♯] D | i [♯] m | i [♯] HD | v [°] | v [°] | v [♯] HD | VII [♯] M |
| i [°] | I | I | I | I | i [°] | i [♯] D | i [♯] m | I [♯] A | V [♯] AM | v [°] | VI [♯] M | v [♯] HD | v [♯] HD |
| V | V | V | V [♯] D | v | vi | v [♯] m | V [♯] AM | i [♯] MM | I [♯] AM | VI [♯] M | i ^{°7} | i [♯] HD | VII [♯] M |
| v | V | V | v | V [♯] D | v [♯] m | i | I ⁺ | V [♯] AM | I [♯] AM | VI [♯] M | VII | i [♯] HD | VII [♯] D |
| V ⁺ | V | V | V ⁺ | I | V [♯] M | I [♯] D | I ⁺ | v [♯] MM | I [♯] AM | VI [♯] M | i [♯] HD | VI [♯] M | VII [♯] M |
| v [°] | V | V | V | V | v [°] | V [♯] M | v [♯] MM | V [♯] AM | I [♯] AM | vi ^{°7} | i ^{°7} | i [♯] HD | VII [♯] D |
| VI | vi | vi | VI | VI ⁺ | VI [♯] D | VI [♯] A | V [♯] D | vi ^{°7} | i | v [♯] MM | v [♯] HD | VII [♯] D | i ^{°7} |
| vi | vi | vi | vi [♯] m | VI | V | VI ⁺ | VI [♯] M | VI [♯] AM | I [♯] AM | v [°] | v [♯] MM | v [♯] HD | i [♯] HD |
| VI ⁺ | vi | vi | vi | VI | vi [♯] m | VI [♯] D | V [♯] D | vi ^{°7} | v [°] | v [♯] MM | i [°] | VII [♯] D | i ^{°7} |
| vi [°] | vi | vi | vi [°] | vi [♯] m | VI ⁺ | VI | VI [♯] A | VI [♯] M | i [°] | I [♯] AM | VII [♯] D | v ^{°7} | v ^{°7} |
| VII | vii [♯] | vii [°] | vii [°] | vii | VII | vii [♯] m | VII [♯] D | VI ⁺ | vi ^{°7} | VI [♯] M | v [♯] MM | v ^{°7} | v ^{°7} |
| vii | vii [♯] | vii [°] | vii | vii | VII ⁺ | VII | VII [♯] A | VI [♯] D | VI [♯] M | i [♯] m | i [♯] HD | v ^{°7} | v ^{°7} |
| VII ⁺ | vii [°] | vii [♯] | VII ⁺ | I | VI [♯] A | I [♯] M | V [♯] M | vii [♯] MM | VI [♯] M | I [♯] A | i ^{°7} | v [♯] HD | v [♯] HD |
| vii [°] | vii [°] | vii [♯] | vii [♯] HD | vii ^{°7} | VII ⁺ | vii [♯] m | vii [♯] MM | VII [♯] AM | V [♯] AM | VII [♯] M | v [°] | i ^{°7} | i [♯] HD |

agents within the patch to the plane tangent to the center point in the patch. The equation used to calculate the amount of displacement needed is $val \times e^{3(dist-r/2)} / (e^{1.5r})$, where val is the maximum displacement determined by the user (maximum of 4 and minimum of 0 - no displacement). Figure 4-3 shows a plot of the displacement equation with $val = 0, 2, \text{ and } 4$.

Unlike the simulation in Chapter 3, the protrusions in this simulation are made to incrementally return to the original sphere shape to simulate the relaxation phases described earlier in this chapter. This visco-elastic property is simulated with an additional function in the code shown in Figure 4-4. This **return-shape** function is called in every simulation cycle. If there are any protrusions, the distances between agents in the protruded patch and the sphere origin are reduced gradually until the original spherical surface is recovered. The distance reduction is calculated using $3\sin(\text{distance of agent from original surface})$ to approximate a visco-elastic material: the further the agents are from the original position (higher tension), the faster the return.

The input text file is transmitted from the microcontroller to the simulation via Bluetooth. The data stream is logged from a terminal emulator (SecureCRT) into a text file which is then read by the simulator. An example of the input text file is shown in Figure 4-5. In order to synchronize the simulation with the microcontroller, the simulation is set to start once the constant 99 has been transmitted by the microcontroller. The simulation then goes through the text file which contains

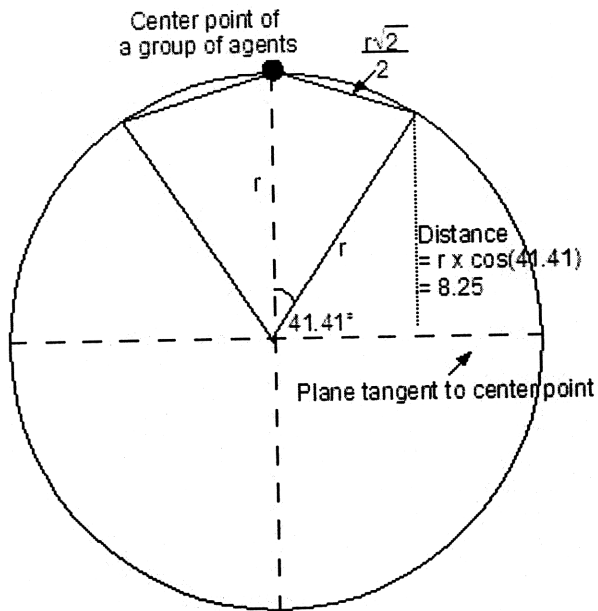


Figure 4-2: Relationship between distance of agent from plane tangent to center point to displacement to create protrusion on surface of sphere.

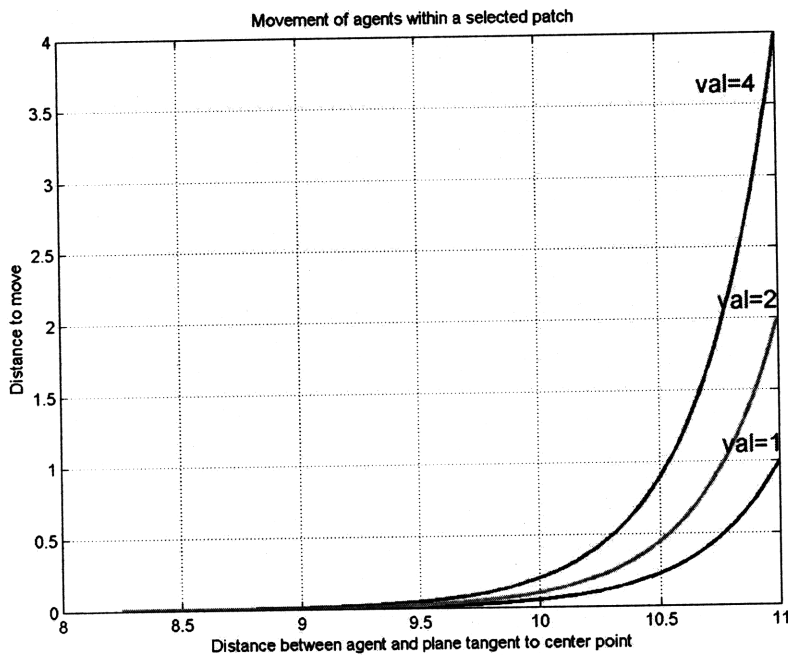


Figure 4-3: Plot of $val \times e^{3(dist-r/2)} / (e^{1.5r})$ with $val = 0, 2, \text{ and } 4$. Maximum displacement at center point where distance of agent from plane is equal to radius of sphere. The amount of displacement decays exponentially for agents further away from the center point.

```

to return-shape
  if (any? agents not in original position)
    return
end

to return
  ask agents in patch reduce distance
  [move closer to origin by 3 * sin ( r - distance from origin )]
end

```

Figure 4-4: Pseudocode of the return function written using Netlogo. The return function was added to the simulation program described in Chapter 3 to simulate the elastic property of the sphere. The full code for the simulation can be found in Appendix H.

information on which patches to move and how much to move at given intervals. Each line in the text file contains 8 numbers. The first 6 numbers correspond to specific points on the simulated sphere and a protrusion is created at a point only when the input to that point is 2. Otherwise, the protrusion is gradually reduced until the original sphere shape is recovered. The last two numbers in a line correspond to vertical and horizontal rotation obtained from the actual movement of the accelerometer in the hardware. These two numbers are used as input to the commands `orbit-up` and `orbit-right` in Netlogo to rotate the 3D view of the simulated sphere. The 1.8 input is used to stop the simulation while the microcontroller plays the 1.8 second long WAV file.

4.3 Hardware

All the components are contained within a solid sphere (see Figure 4-6) which can be held by the user to control the simulated sphere. Six push buttons are attached at locations matching points (11,0,0), (-11,0,0), (0,11,0), (0,-11,0), (0,0,11), and (0,0,-11) in Figure 3-1. The microcontroller performs two main functions: process data from sensors (push buttons and accelerometer) and read WAV files from SD Memory Card.

4.3.1 Microcontroller

The Arduino Diecimila prototyping board was used as the backbone of the hardware. This board uses the ATmega168 microcontroller which has 16K Byte of self-programming Flash Program Memory, 1K Byte SRAM, 512 Bytes EEPROM, and 8 Channel 10-bit A/D-converters. The 16kB of flash memory is used to store the table of chords (see Table 4.1), audio library, and code while the 600 bytes of the SRAM are used for audio buffers and to store WAV file data. Figure 4-7 shows the board. The microcontroller is powered by an external 9V battery power supply regulated at 5V and 3.3V using an on-board FTDI chip. Full schematics of the board using ATmega8 is shown in Appendix

```

99 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 -2 -2
0 0 0 0 0 0 -6
0 0 0 0 0 0 0
0 0 0 0 0 0 0
0 0 0 0 0 0 0
0 0 0 0 0 0 0
0 0 0 0 0 0 -1
0 0 0 0 0 0 1
0 0 0 0 2 0 0 0
1.8
1.8
0 0 0 0 1 0 0 0
1.8
0 0 0 0 0 0 0 0
1.8
0 0 0 0 0 0 0 0

```

Figure 4-5: Example of an input text file used to control the simulation. The first 6 numbers indicate the maximum displacement for each patch and the last 2 numbers are used to control the 3D preview perspective. The 1.8 indicates a wait time of 1.8 seconds to allow the WAV file to finish playing. During this waiting period, the simulation can only run the return-shape function.

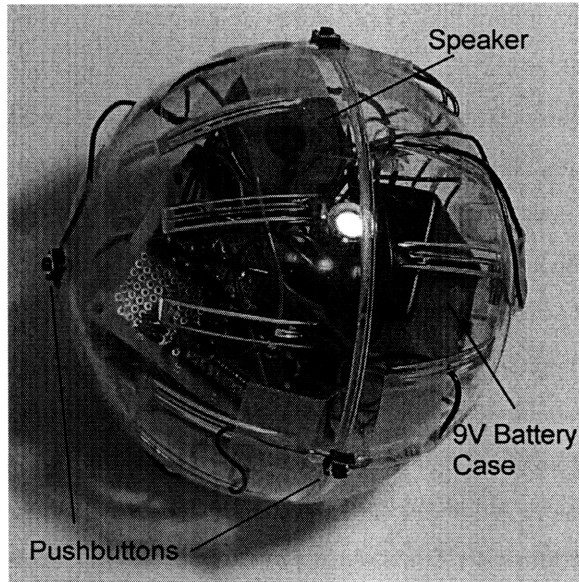


Figure 4-6: Simulation control. Microcontroller and other hardware components (Bluetooth, wave shield, accelerometer, speaker, and battery) encased in a plastic sphere. Six push buttons are attached on the sphere's outer surface.

I. The microcontroller is programmed using the open-source Arduino programming language (v10) which contains convenient macros for pin manipulation. This programming language is based on Processing/Wiring language [18].

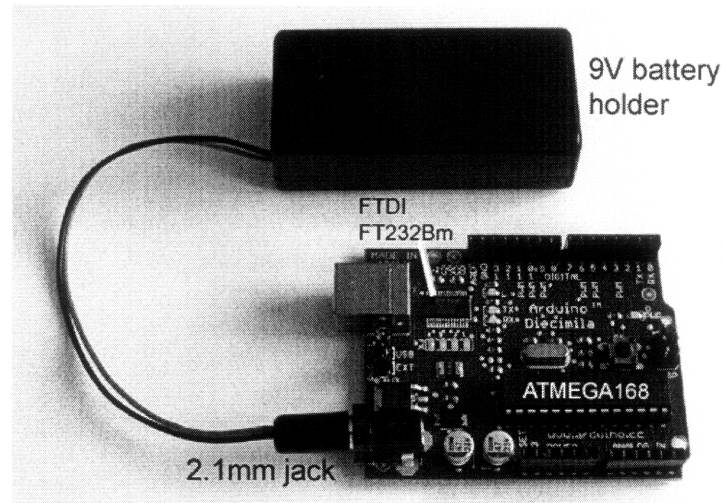


Figure 4-7: Arduino Diecimila board with ATmega168 microcontroller powered by a 9V battery.

The pin mapping of the ATmega168 to the Arduino board is shown in Figure 4-8. Push buttons are assigned to digital pins 7,8, and 9. Due to the limited amount of digital pins, push buttons are also assigned to analog pins 3,4, and 5. Digital pins 0 and 1 (RX and TX) are assigned to the Bluetooth chip for data transmission while digital pins 2, 3, 4, 5, 6, 11 (MOSI), 12 (MISO), and 13 (SCK) are assigned to the audio board. The accelerometer uses analog pins 0 and 1 for the Y-axis and X-axis respectively.

All sensors and the Bluetooth module are mounted on a perfboard as shown in Figure 4-9 which is connected to the microcontroller using pin headers. See Figure 4-10 for schematics of the Bluetooth, accelerometer, and push buttons. The sensors are sampled once every 300ms which is far below the capability of the microcontroller but is necessary to coordinate with the much slower simulation speed. Further discussion of the timing and possible improvements can be found in Chapter 5.

When a push button is pressed, the pin is set as active and subsequent presses will be ignored until the tension value associated with the button has decreased to zero. The SparkfunADXL330 3-Axis $\pm 3g$ accelerometer breakout board was used to control the rotation of the simulated sphere. To detect rotation, only the X and Y axes are needed (see Figure 4-11). According to the datasheet, the ADXL330 output is ratiometric. For the supply voltage 3.3V used in this application, the output sensitivity is approximately 330mV/g. To obtain only the relative movements of the accelerometer, only the difference of the current and previous acceleration values are transmitted to the simulation. These differences are transmitted as input to the function `orbit-up` and `orbit-right` of the simulation which will in turn rotate the view according to the values given. For wireless microcontroller

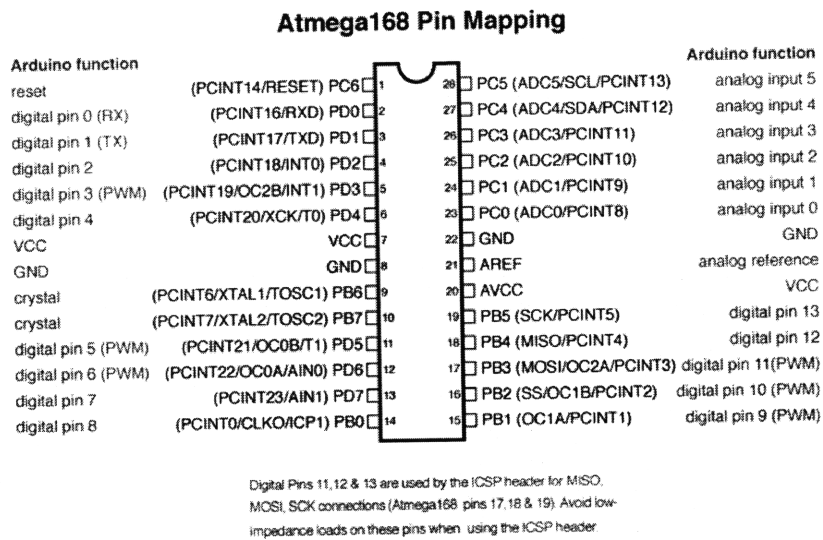


Figure 4-8: Pin mapping from Arduino board to ATmega168 microcontroller.

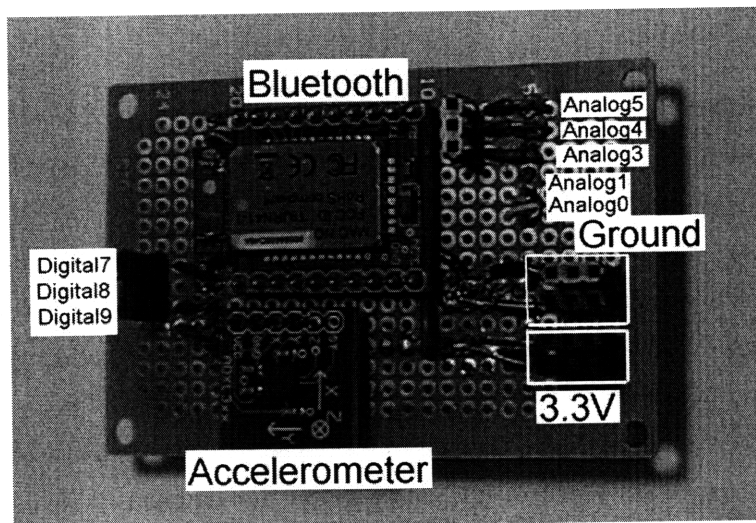


Figure 4-9: Perfboard with Bluetooth module, accelerometer, and pin header connections for push buttons.

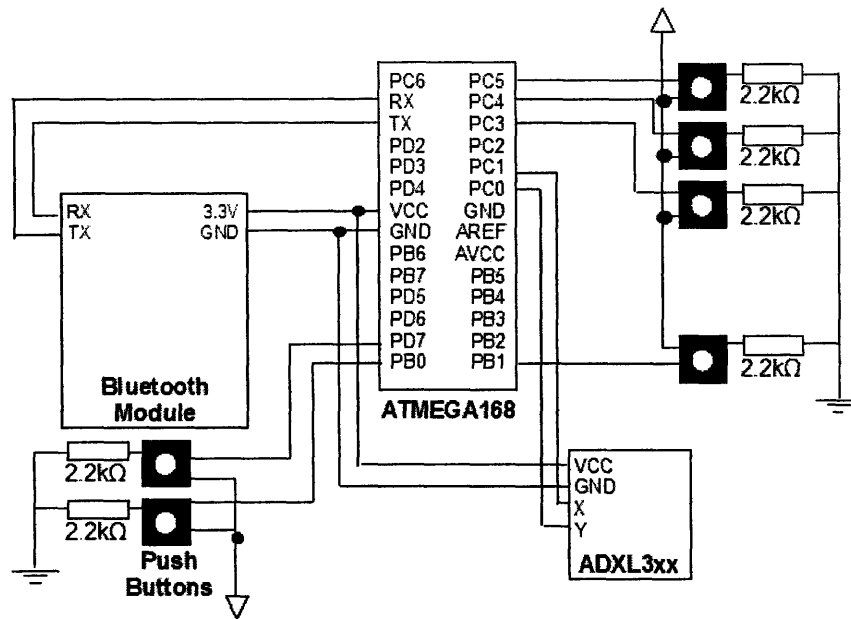


Figure 4-10: Schematics of circuit for perfboard (Bluetooth, accelerometer, push buttons) and microcontroller.

to simulation transmission, the Bluetooth DIP module by Roving Networks is used. The RN-41 chip has over air data rate of 721kbps to 2.0Mbps, uses 3.3V supply, and has Class 1 power output (100mW (20dBm), 100 meter range). The baud rate is set to 9600bps.

4.3.2 Wave Shield

The audio component of the system could be implemented by either: 1) generating audio using Pulse-Width Modulation (PWM) from microcontroller, 2) use embedded MP3 player, or 3) play uncompressed WAV audio from memory card (or similar storage devices).

Table 4.2 shows a comparison of the advantages and disadvantages of the three methods. For the purpose of my application, the third method (audio from memory card) provides the necessary audio resolution with relatively low cost and easy implementation. The WAV audio player used in this application is a wave shield designed by Ladyada [7] for the Arduino Diecimila (see Figure 4-12). This audio player is capable of playing 22kHz, 16 bit, mono, uncompressed WAV files using either small speakers or headphones. Further details of the design and Arduino audio library can be found in the wave shield website [7]. Appendix J shows the schematics of the wave shield.

The audio files are stored in a 1GB FAT16 formatted SD Memory Card. Figure 4-13 shows all the chords that were recorded as 1.8 second WAV files using a USB M-Audio MIDI keyboard and GarageBand (software application developed by Apple Inc. for Mac OS X). These chords correspond to the ones in Table 4.1 (tonic, dominant, submediant, and supertonic). Audacity was used to edit

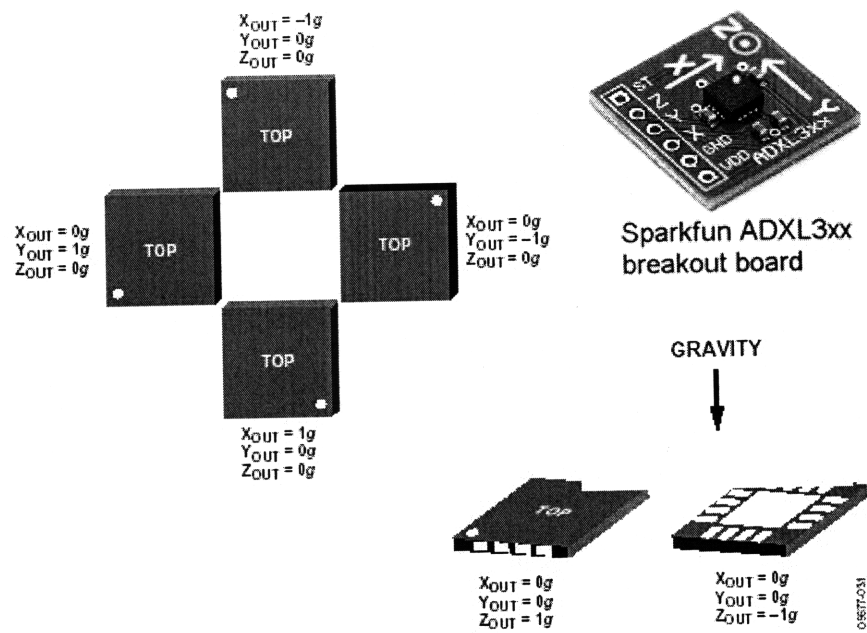


Figure 4-11: Accelerometer output responses versus orientation to gravity (modified from ADXL330 datasheet). The breakout board by Sparkfun is shown in the top right corner for comparison.

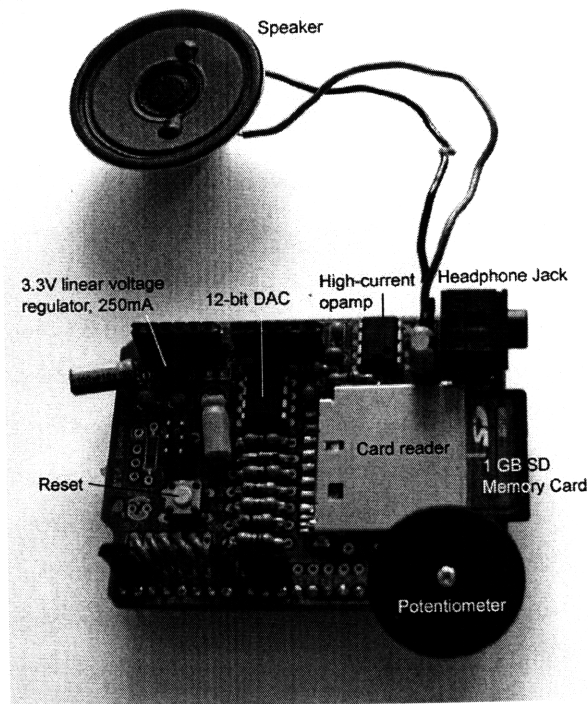


Figure 4-12: Wave shield designed by Ladyada [7] for audio playback.

Table 4.2: Comparison of audio output methods.

| Method | Implementation | Cost | Audio Quality |
|-------------------|---|-----------------|---------------|
| Direct PWM output | Easiest (many reference sources) Insufficient EEPROM storage | Free | Very low |
| Embedded MP3 | Difficult Need MP3 encoding/licensing | \$40 (Sparkfun) | High |
| Uncompressed WAV | Easy (well designed and documented) | \$22 (Adafruit) | Medium |

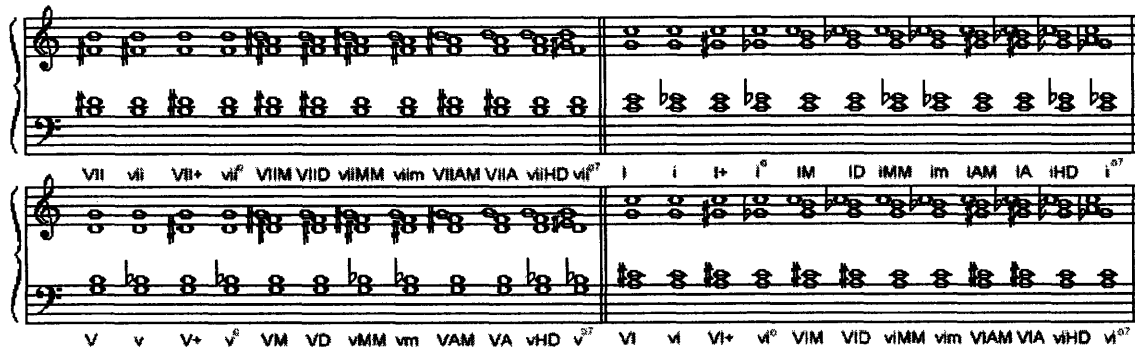


Figure 4-13: Chord segments stored in SD memory card as WAV files. These chords were recorded using a USB keyboard on GarageBand.

the chord segments and modify the audio format to suit the player specifications. The 512 bytes of audio data provided by the SD card is double-buffered (256 bytes) in the ATmega's SRAM for smooth playback.

The chord segment files are named using three character strings as shown in Table 4.3.2. The first character indicates the root of the chord which is either C (tonic), G (dominant), A (submediant), or B (supertonic). The second character indicates the quality of the triad: M (major), I (minor), A (augmented), D (diminished). The last character is a number which indicates the interval of the seventh note: 1 (no seventh), 2 (major seventh - 11 semitones), and 3 (minor seventh - 10 semitones). The diminished chords are an exception to this naming system: 2 (minor seventh) and 3 (diminished seventh - 9 semitones). Using this naming system, Table 4.1 is stored in the microcontroller's flash memory as 16 arrays each containing a string of 36 characters (12 filenames) using the code shown in Figure 4-14. For every tension value, three characters are read from the table and a .wav is attached to the end to form the filename of the audio segment to be played through the speaker. For example, given that the previous chord is V (Gmaj) and the tension value is 10, the characters in array Gmaj [12:14], the file GI3.wav is obtained. This result matches the G minor chord (v⁷m) from Table 4.1. Note that the column for tension value 0 is not included to save memory space because of the redundancy (chords associated with tension value 0 are the same as the chords associated with tension value 2).

```

const char Cmaj[] PROGMEM = "CM1CM2CI1BI1CI2CA3GI3BA2CD2CD2BM3BM3";
const char Cmin[] PROGMEM = "CM1CI1CM2CI2GI1BD2GI3CD2AA3BM3BM2BM2";
const char Caug[] PROGMEM = "CM1CA1CA1CM3AI3GM3CI3CD2GD1GD1GD2BM2";
const char Cdim[] PROGMEM = "CM1CM1CM1CD1CM3CI3CA3GA2GD1AM2GD2GD2";

const char Gmaj[] PROGMEM = "GM1GM3GI1AI1GI3GA2CI2CA2AM2CD3CD2BM2";
const char Gmin[] PROGMEM = "GM1GI1GM3GI3CI1CA1GA2CA2AM2BM1CD2BM3";
const char Gaug[] PROGMEM = "GM1GA1CM1GM2CM3CA1GI2CA2AM2CD2BM2BM2";
const char Gdim[] PROGMEM = "GM1GM1GM1GD1GM2GI2GA2CA2AD3CD3CD2BM3";

```

Figure 4-14: Table of chord-pairs and tension values stored in ATmega168 flash memory. See full code for ATmega168 in Appendix K.

Table 4.3: Filename of wav files on SD card and corresponding chords. Each filename contains three characters: first character indicates scale degree, second character indicates chord quality, and the third character indicates interval of the seventh note.

| Filename | Chord | Filename | Chord |
|----------|--------------------------------|----------|---------------------------------|
| CM1.wav | I C-E-G-C | GM1.wav | V G-B-D-G |
| CM2.wav | I ⁷ M C-E-G-B-C | GM2.wav | V ⁷ M G-B-D-F#-G |
| CM3.wav | I ⁷ D C-E-G-Bb-C | GM3.wav | V ⁷ D G-B-D-F-G |
| CI1.wav | i C-Eb-G-C | GI1.wav | v G-Bb-D-G |
| CI2.wav | i ⁷ MM C-Eb-G-B-C | GI2.wav | v ⁷ MM G-Bb-D-F#-G |
| CI3.wav | i ⁷ m C-Eb-G-Bb-C | GI3.wav | v ⁷ m G-Bb-D-F-G |
| CA1.wav | I+ C-E-G#-C | GA1.wav | V+ G-B-D#-G |
| CA2.wav | I ⁷ AM C-E-G#-B-C | GA2.wav | V ⁷ AM G-B-D#-F#-G |
| CA3.wav | I ⁷ A C-E-G#-Bb-C | GA3.wav | V ⁷ A G-B-D#-F-G |
| CD1.wav | i ^o C-Eb-Gb-C | GD1.wav | v ^o G-Bb-Db-G |
| CD2.wav | i ⁷ HD C-Eb-Gb-Bb-C | GD2.wav | v ⁷ HD G-Bb-Db-F-G |
| CD3.wav | i ^{o7} C-Eb-Gb-A-C | GD3.wav | v ^{o7} G-Bb-Db-E-G |
| AM1.wav | VI A-C#-E-A | BM1.wav | VII B-D#-F#-B |
| AM2.wav | VI ⁷ M A-C#-E-G#-A | BM2.wav | VII ⁷ M B-D#-F#-A#-B |
| AM3.wav | VI ⁷ D A-C#-E-G-A | BM3.wav | VII ⁷ D B-D#-F#-A-B |
| AI1.wav | vi A-C-E-A | BI1.wav | vii B-D-F#-B |
| AI2.wav | vi ⁷ MM A-C-E-G#-A | BI2.wav | vii ⁷ MM B-D-F#-A#-B |
| AI3.wav | vi ⁷ m A-C-E-G-A | BI3.wav | vii ⁷ m B-D-F#-A-B |
| AA1.wav | VI+ A-C#-F-A | BA1.wav | VII+ B-D#-G-B |
| AA2.wav | VI ⁷ AM A-C#-F-G#-A | BA2.wav | VII ⁷ AM B-D#-G-A#-B |
| AA3.wav | VI ⁷ A A-C#-F-G-A | BA3.wav | VII ⁷ A B-D#-G-A-B |
| AD1.wav | vi ^o A-C-Eb-A | BD1.wav | vii ^o B-D-F-B |
| AD2.wav | vi ⁷ HD A-C-Eb-G-A | BD2.wav | vii ⁷ HD B-D-F-A-B |
| AD3.wav | vi ^{o7} A-C-Eb-F#-A | BD3.wav | vii ^{o7} B-D-F-G#-B |

4.4 Conclusion

While the system performs as expected, the objective of using the system as an educational tool for learning the relationship between harmonic progression, tension, and dissonance was not achieved. From observation, it is difficult to distinguish between two chord progressions that have high but different tension values. For example, it is somewhat more difficult to determine whether the $I \rightarrow v^7 m$ progression (tension value 14) or the $I \rightarrow VII^7 D$ progression (tension value 22) has higher tension value just by listening to the chord segments. Possible explanations for this condition are presented in Chapter 5.

Chapter 5

Discussion and Future Work

The goal of this thesis is to explore a more tangible and intuitive physical representation of tension/relaxation patterns in music. In the past few chapters, I described two learning systems based on the mapping of Lerdahl's harmonic tension model onto a visco-elastic sphere to help articulate the correlation between harmonic progression and tonality. These two systems are: 1) a simulation that displays the surface tension of a sphere based on calculated harmonic tension of Pachelbel's *Canon in D* and Rimsky-Korsakov's *The Flight of the Bumblebee*, and 2) an interactive learning system which enables users to experiment with different tension values and observe the results using simulation and audio output.

Based on observations of the sphere simulations for *Canon in D* and *The Flight of the Bumblebee*, the translation of harmonic tension to physical tension is indeed useful for visualizing tension and relaxation patterns in music. This intuitive representation can be helpful in understanding the theory of harmonic progression and exploring how these progressions affect the mood and consonance/dissonance of a piece.

As mentioned in Chapter 4, the effectiveness of the interactive system for exploring harmonic progression is dampened by the fact that the tension difference between two highly dissonant chords is not obvious. Based on current theories, there are two possible explanations for this condition. The first explanation is derived from Cazden's work [9]. According to him, consonance and dissonance are highly dependent on the context and expectations of the listener:

[Dissonance] identifies rather the functional moment of any sonorous event that is expected to resolve, while the moment to which it ultimately resolves is then deemed consonant. Should the framework for the normative expectations of this kind not be present, or should the apparent resolution tendencies and outcomes be thwarted consistently, as may happen in some compositional styles of twentieth century art music, neither consonance nor dissonance can be said to exist.

In the interactive system, there was no context because the chords were merely played according to given tension values. Given this situation, the listener could not form expectations on the resolution of the chords hence the inability to distinguish between chord progressions that have different tension values.

Another explanation is based on the idea of relative dissonance presented in [12]. The perceived consonance and dissonance of a chord depends largely on the chords that surround it. For example, a consonant chord may sound dissonant if placed in the context of chords which are more consonant. On the other hand, a slightly dissonant chord may even sound consonant if placed within a stream of highly dissonant chords. Because the relaxation stage of the interactive system is gradual, the relative difference between the successive chords during this stage is small and perhaps even imperceptible. Therefore, the chords played during the relaxation stage merely sound like a series of similar dissonant chords.

5.1 Future work

The simulation system could be expanded to real-time processing and analysis of audio signals. With a real-time system, users will be able to experiment with different types of music to better understand the differences in the tension/relaxation patterns and the context of the music. The processing could be done using either hardware (DSP or FPGA) or software programs such as Mathematica which is supported for use with Netlogo. The tension analysis could also include more factors such as rhythm, phrasing, and global context.

To improve the effectiveness of the interactive learning system, I propose a different approach. Instead of creating chords by changing tension values, users can modify existing audio by adding or reducing tension. Tension can be added by simply inserting nonharmonic notes into chords or shifting the key of the piece. The rhythm of the audio could also be modified to increase or decrease tension. This way, users will be able to experience the effects of tension and relaxation in context.

The simulation could be expanded to allow users to control the shape without limiting distortions to certain patches only. In fact, different shapes such as ellipsoids could also be explored. The system could be modified such that users will only hear two chords when two points on the ellipsoid are pressed. The choice of chords depends on the tension values associated with the two points that are being pressed on the ellipsoid. The effect of the output will be similar to original system but without the relaxation phase. This modification could resolve the problem of differentiating two highly dissonant chords during the relaxation phase.

There are also several smaller improvements that could be made to the system:

Table 5.1: Bluetooth device class specifications.

| Class | Maximum Permitted Power mW(dBm) | Range (approximate) |
|---------|------------------------------------|------------------------|
| Class 1 | 100mW (20 dBm) | 100 meters |
| Class 2 | 2.5mW (4 dBm) | 10 meters |
| Class 3 | 1mW (0 dBm) | 1 meter |

Simulation

The current Netlogo sound extension can only support one task at a given moment, making it impossible to play audio while running the simulation. There are two options for solving this problem: 1) write an extension for playing music using Netlogo while running the simulation simultaneously, or 2) coordinate Netlogo with an audio player software such as Realplayer or Winamp.

Another extension could be written to accept serial input directly from the microcontroller. The speed at which Netlogo processes instructions varies depending on the capacity of the machine that it is running on. Therefore, the current system of logging input on the terminal emulator and passing that input to the simulator is suboptimal because there is no way to coordinate the flow of data. Moreover, a lot of bandwidth and power is wasted because the microcontroller has to consistently send redundant data to the simulation.

Hardware

In hindsight, a Class 1 output bluetooth module is an overkill for the system and drains too much power for such a small system. Table 5.1 shows the class specifications for Bluetooth modules. Based on the specifications, a Class 3 output Bluetooth would have been sufficient for this application.

The controller developed with low cost in mind and has limited processing power. Upgrades can be made to the following parts: 1) use microcontroller with higher memory capacity and pins so that more points (push buttons) can be added to the simulation, 2) the push buttons could be changed to pressure sensors so that the users will have better control of the sphere's distortion, and 3) an MP3 player could be used for higher quality audio output.

Chapter 6

Contributions

In recent years, studies have hypothesized possible relations between language and human intelligence. Wittgenstein, a philosopher on logic, mathematics, mind, and language, associated language to the formulation of complicated concepts. The connection between language and the ability to grasp complicated spatial concepts was also expressed in a study by Spelke [19]¹. Syntactical similarities [20, 21, 2, 22] as well as parallels in human brain processing [2, 4, 5] suggest that understanding music perception might shed light on the form, syntax, and development of language.

In this thesis, I proposed a physical representation of Lerdahl's harmonic tension model for intuitive understanding of music tonality perception. In order to evaluate the effectiveness of the physical mapping, I presented two interactive systems: 1) a surface tension simulation of a visco-elastic sphere which changes depending on the harmonic tension, and 2) an interactive simulation and microcontroller-based system for intuitive understanding of harmonic progression and its effects on tonality. The translation of Lerdahl's harmonic tension model to an intuitive physical model is a step forward in the exploration of the human perception of music and tonality. The physical representation of harmonic tension allows non-musicians to explore the idea of tension and relaxation in a tangible manner in hope of enhancing their listening experiences.

¹The Society of Music Perception and Cognition (SMPC) is an organization for researchers interested in the scientific and scholarly understanding of music from a broad range of disciplines, including music theory, psychology, psychophysics, linguistics, neurology, neurophysiology, ethology, ethnomusicology, artificial intelligence, computer technology, physics and engineering.

Appendix A

Notes and frequencies

Notes in three octaves and their corresponding frequencies. C4 is middle C.

| Note | Frequency (Hz) | Note | Frequency (Hz) | Note | Frequency (Hz) |
|------|----------------|------|----------------|------|----------------|
| C3 | 130.81 | C4 | 261.63 | C5 | 523.25 |
| C#3 | 138.59 | C#4 | 277.18 | C#5 | 554.37 |
| D3 | 146.83 | D4 | 293.66 | D5 | 587.33 |
| D#3 | 155.56 | D#4 | 311.13 | D#5 | 622.25 |
| E3 | 164.81 | E4 | 329.63 | E5 | 659.26 |
| F3 | 174.61 | F4 | 349.23 | F5 | 698.46 |
| F#3 | 185.00 | F#4 | 369.99 | F#5 | 739.99 |
| G3 | 196.00 | G4 | 392.00 | G5 | 783.99 |
| G#3 | 207.65 | G#4 | 415.30 | G#5 | 830.61 |
| A3 | 220.00 | A4 | 440.00 | A5 | 880.00 |
| Bb3 | 233.08 | Bb4 | 466.16 | Bb5 | 923.33 |
| B3 | 246.94 | B4 | 493.88 | B5 | 987.77 |

Appendix B

Major and Minor Scales

All major and minor scales.

| Major Scale | Minor Scale | Key Signature |
|-------------|-------------|----------------------|
| C | A | |
| G | E | F# |
| D | B | F# C# |
| A | F# | F# C# G# |
| E | C# | F# C# G# D# |
| B | G# | F# C# G# D# A# |
| F# | D# | F# C# G# D# A# E# |
| C# | A# | F# C# G# D# A# E# B# |
| F | D | Bb |
| Bb | G | Bb Eb |
| Eb | C | Bb Eb Ab |
| Ab | F | Bb Eb Ab Db |
| Db | Bb | Bb Eb Ab Db Gb |
| Gb | Eb | Bb Eb Ab Db Gb Cb |
| Cb | Ab | Bb Eb Ab Db Gb Cb Fb |

List of all major and minor scales. [23]

C major G major D major A major E major B major F# major C# major
A minor E minor B minor F# minor C# minor G# minor D# minor A# minor

C major F major B^b major E^b major A^b major D^b major G^b major C^b major
A minor D minor G minor C minor F minor B^b minor E^b minor A^b minor

Appendix C

Harmonic Analysis

Variations On The Kanon By Johann Pachelbel — Arranged by George Winston
 캐논 변주곡

Andante Moderato

| Measure | Treble Note | Bass Chord | Roman Numeral |
|---------|-------------|------------|------------------|
| 1 | G4 | CEG | I |
| 2 | B4 | GBD | V |
| 3 | A4 | ACE | vi |
| 4 | G4 | EGB | iii |
| 5 | F4 | FAC | IV |
| 6 | E4 | CEG | I |
| 7 | D4 | FAC | IV |
| 8 | C4 | GBD | V |
| 9 | G4 | CEG | I |
| 10 | B4 | GBD | V |
| 11 | A4 | ACE | vi |
| 12 | G4 | EGBD | iii ⁷ |
| 13 | F4 | FAC | IV |
| 14 | E4 | CEG | I |
| 15 | D4 | FAC | IV |
| 16 | C4 | GBD | V |

17 18 19 20 21 22 23 24 25 26 27 28

CEG GBD GBD F ACE EGB EGB D FAC CEG CEG DFA GBD GBD F

I V V⁷ VI III III⁷ IV I I II V V⁷

29 30 31 32 33 34 35 36 37 38 39 40 41 42 43

CEG GBD GBD F ACE ACE EGB GBD FAC FAC CEG CEG DFA ACE

I V V⁷ VI VI III V IV II I I II⁷ VI I I⁷

44 45 46 47 48 49 50 51 52 53 54 55 56 57 58

CEG CEG GBD GBD F ACE ACE EGB GBD FACE DFA CEG CEG DFA ACE

I I V V⁷ VI VI III V IV⁷ II⁷ I I⁷ II⁷ VI V⁷ I⁷

59 60 61 62 63

CEG

I

The Flight of the Bumble-Bee

RIMSKY-KORSAKOV

A minor Vivace $\text{♩} = 160 - 144$

Piano

f

dim.

EG#E
V

EG#BD
V⁷

5

pp

3

4 5

ACE
i

ACE
i

DF#AC
IV⁷

9

6 7 8 9 10 11 12

ACE
i

ACE
i

DF#AC
IV⁷

ACE
i

DFAC
iv⁷

ACE
i

EG#BD
V⁷

13

13 14 15 16 17 18 19 20

ACE
i

DFA
iv

ACE
i

EG#BD
V⁷

AC#EG
I⁷

DFA
iv

DFA
iv

GBDF
VII⁷

17

21 22 23 24 25 26 27

DFA
iv

DFA
iv

GBDF
VII⁷

DFA
iv

GB^bDF
VII⁷

DFA
iv

ACEG
I⁷

©1999 Anne Christopherson GRSM ARCM www.music-scores.com

2

21

28 29 30 31 32 33

DFA iv GB^bDF vii⁷ DFA iv AC[#]EG I⁷ DFA iv ACEG[#] i⁷

25

34 35 36 37 38

ACE i ACEG[#] i⁷ ACE i ACEG[#] i⁷ ACEG[#] i⁷ ACEG[#] i⁷

29

39 40 41 42

FAC[#] vi⁷ FAC[#] vi⁷ DF[#]A iv DFAC[#] iv⁷

33

43 44 45 46 47

DFA iv DFAC[#] iv⁷ DFAC[#] iv⁷ DFAC[#] iv⁷ DFAC[#] iv⁷

37

48 49 50 51

mf

GB^bDF[#] vii⁷

www.music-scores.com
 GB^bDF[#]
 vii⁷

Appendix D

Simulation code in Netlogo

The physical mapping of Lerdahl's harmonic tension model is visualized using a simulation written in Netlogo.

```
globals [
  stopvar
  r          ;; Radius
  range
  inrangex inrangey inrangez inrangex inrangeny inrangenz  ;; 1 Axis (6)
  inrangexy inrangexny inrangexny inrangexny inrangexz inrangexz inrangexnz
  inrangexnz inrangeyz inrangeny inrangeyz inrangeny inrangeny  ;; 2 Axes (12)
  inrangexyz inrangexyz inrangexny inrangexny inrangexny inrangexny
  inrangexny inrangexny  ;; 3 Axes (8)
  data-number  ;; Data stream elements inrange [x y z nx ny nz xy nxy ... ] (26)
  data-list
]

;; To setup initial sphere (reset)
to setup
  ;; Setup shape
  ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
  ca
  set r 11
  set range ( r * sqrt(2)) / 5
  set data-number 27 set data-list []
  set-default-shape turtles "default"
  crt 10000  ;; Number of turtles
  [
    tilt-up asin (1.0 - random-float 2.0)  ;; turtles have random heading
    roll-right random-float 360
    jump r  ;; change in model setting (currently 16)
    set color scale-color sphere-color xcor (-1 * max-pxcor) max-pxcor
  ]
  set inrangex turtles with [inrangex? xcor ycor zcor]
  set inrangey turtles with [inrangey? xcor ycor zcor]
  set inrangez turtles with [inrangez? xcor ycor zcor]
  set inrangex turtles with [inrangex? xcor ycor zcor]
  set inrangeny turtles with [inrangeny? xcor ycor zcor]
  set inrangenz turtles with [inrangenz? xcor ycor zcor]
  set inrangexy turtles with [inrangexy? xcor ycor zcor]
```



```

to make-list
  set data-list []
  let i 1
  while [ i <= data-number ]      ;; Process list [reset sensor1 ... sensor6]
  [
    ifelse (file-at-end?)          ;; In case file ends before set is complete
    [ user-message ("Opps, something wrong with data.") ]
    [
      set data-list lput file-read data-list
      set i ( i + 1 )
    ]
  ]
]
end

```

```

to dent
  let i 0
  selectx (item i data-list)
  selecty (item ( i + 1) data-list)
  selectz (item ( i + 2) data-list)
  selectnx (item ( i + 3) data-list)
  selectny (item ( i + 4) data-list)
  selectnz (item ( i + 5) data-list)
  selectxy (item ( i + 6) data-list)
  selectnxy (item ( i + 7) data-list)
  selectxny (item ( i + 8) data-list)
  selectnxny (item ( i + 9) data-list)
  selectxz (item ( i + 10) data-list)
  selectnxz (item ( i + 11) data-list)
  selectxnz (item ( i + 12) data-list)
  selectnxnz (item ( i + 13) data-list)
  selectyz (item ( i + 14) data-list)
  selectnyz (item ( i + 15) data-list)
  selectynz (item ( i + 16) data-list)
  selectnynz (item ( i + 17) data-list)
  selectxyz (item ( i + 18) data-list)
  selectnxyz (item ( i + 19) data-list)
  selectxnyz (item ( i + 20) data-list)
  selectnxnyz (item ( i + 21) data-list)
  selectxynz (item ( i + 22) data-list)
  selectnxynz (item ( i + 23) data-list)
  selectxnynz (item ( i + 24) data-list)
  selectnxnynz (item ( i + 25) data-list)
end

```

```

;-----
; Single Axis Only
;-----
;; opt can be 4 2 0

```

```

to selectx [opt]
  return-ori (inrangex)
  ask inrangex [jump opt * ( exp ( 6 * ( xcor - r / 2) ) / exp ( r * 3 ))]
  set-color
end

```

```

to-report inrangex? [x y z]
  report sqrt ( ( x - 11 ) ^ 2 + ( y - 0 ) ^ 2 + ( z - 0 ) ^ 2 ) < range
end

```

```

;-----
to selectnx [opt]
  return-ori (inrangex)
  ask inrangex [jump opt * ( exp ( 6 * ( ( -1 * xcor ) - r / 2)) / exp ( r * 3))]
  set-color
end

to-report inrangex? [x y z]
  report sqrt (( y - 0 ) ^ 2 + ( x + 11 ) ^ 2 + ( z - 0 ) ^ 2 ) < range
end

;-----

to selecty [opt]
  return-ori (inrangey)
  ask inrangey [jump opt * ( exp ( 6 * ( ycor - r / 2)) / exp ( r * 3 ))]
  set-color
end

to-report inrangey? [x y z]
  report sqrt ( ( x - 0 ) ^ 2 + ( y - 11 ) ^ 2 + ( z - 0 ) ^ 2 ) < range
end

;-----

to selectny [opt]
  return-ori (inrangeny)
  ask inrangeny [jump opt * ( exp ( 6 * ( ( -1 * ycor ) - r / 2)) / exp ( r * 3))]
  set-color
end

to-report inrangeny? [x y z]
  report sqrt (( x - 0 ) ^ 2 + ( y + 11 ) ^ 2 + ( z - 0 ) ^ 2 ) < range
end

;-----

to selectz [opt]
  return-ori (inrangez)
  ask inrangez [jump opt * ( exp ( 6 * ( zcor - r / 2)) / exp ( r * 3 ))]
  set-color
end

to-report inrangez? [x y z]
  report sqrt ( ( z - 11 ) ^ 2 + ( y - 0 ) ^ 2 + ( x - 0 ) ^ 2 ) < range
end

;-----

to selectnz [opt]
  return-ori (inrangenz)
  ask inrangenz [jump opt * ( exp ( 6 * ( ( -1 * zcor ) - r / 2)) / exp ( r * 3))]
  set-color
end

to-report inrangenz? [x y z]
  report sqrt (( y - 0 ) ^ 2 + ( z + 11 ) ^ 2 + ( x - 0 ) ^ 2 ) < range
end

```

end

```
-----  
; Between Positive axes ( y = -x )  
-----
```

```
to selectxy [opt]  
  return-ori (inrangexy)  
  ask inrangexy [jump opt * ( exp ( 6 * ( ( ( ycor + xcor )  
    / sqrt ( 2 ) ) - r / 2)) / exp ( r * 3 ))]  
  set-color  
end  
  
to-report inrangexy? [x y z]  
  report sqrt ( ( x - r * cos(45)) ^ 2 + ( y - r * cos(45)) ^ 2 + ( z - 0 ) ^ 2 )  
    < range  
end
```

```
-----
```

```
to selectxz [opt]  
  return-ori (inrangexz)  
  ask inrangexz [jump opt * ( exp ( 6 * ( ( ( zcor + xcor )  
    / sqrt ( 2 ) ) - r / 2)) / exp ( r * 3 ))]  
  set-color  
end  
  
to-report inrangexz? [x y z]  
  report sqrt ( ( x - r * cos(45)) ^ 2 + ( z - r * cos(45)) ^ 2 + ( y - 0 ) ^ 2 )  
    < range  
end
```

```
-----
```

```
to selectyz [opt]  
  return-ori (inrangeyz)  
  ask inrangeyz [jump opt * ( exp ( 6 * ( ( ( ycor + zcor )  
    / sqrt ( 2 ) ) - r / 2)) / exp ( r * 3 ))]  
  set-color  
end  
  
to-report inrangeyz? [x y z]  
  report sqrt ( ( z - r * cos(45)) ^ 2 + ( y - r * cos(45)) ^ 2 + ( x - 0 ) ^ 2 )  
    < range  
end
```

```
-----  
; Between Negative axes ( y = -x )  
-----
```

```
to selectnxny [opt]  
  return-ori (inrangexny)  
  ask inrangexny [jump opt * ( exp ( 6 * ( ( ( -1 * ( ycor + xcor ) )  
    / sqrt ( 2 ) ) - r / 2)) / exp ( r * 3 ))]  
  set-color  
end
```

```

to-report inrangexny? [x y z]
  report sqrt ( (x + r * cos(45)) ^ 2 + (y + r * cos(45)) ^ 2 + (z - 0) ^ 2 )
    < range
end

;-----

to selectxnz [opt]
  return-ori (inrangexnz)
  ask inrangexnz [jump opt * ( exp ( 6 * ( ( ( -1 * ( zcor + xcor ) )
    / sqrt ( 2 ) ) - r / 2)) / exp ( r * 3 ))]
  set-color
end

to-report inrangexnz? [x y z]
  report sqrt ( (x + r * cos(45)) ^ 2 + (z + r * cos(45)) ^ 2 + (y - 0) ^ 2 )
    < range
end

;-----

to selectnynz [opt]
  return-ori (inrangenyxz)
  ask inrangenyxz [jump opt * ( exp ( 6 * ( ( ( -1 * ( ycor + zcor ) )
    / sqrt ( 2 ) ) - r / 2)) / exp ( r * 3 ))]
  set-color
end

to-report inrangenyxz? [x y z]
  report sqrt ( (z + r * cos(45)) ^ 2 + (y + r * cos(45)) ^ 2 + (x - 0) ^ 2 )
    < range
end

;-----
; Between Pos-Neg axes ( y = x )
;-----

to selectnxz [opt]
  return-ori (inrangexxz)
  ask inrangexxz [jump opt * ( exp ( 6 * ( ( ( zcor - xcor )
    / sqrt ( 2 ) ) - r / 2)) / exp ( r * 3 ))]
  set-color
end

to-report inrangexxz? [x y z]
  report sqrt ( (x + r * cos(45)) ^ 2 + (z - r * cos(45)) ^ 2 + (y - 0) ^ 2 )
    < range
end

;-----

to selectxnz [opt]
  return-ori (inrangexnz)
  ask inrangexnz [jump opt * ( exp ( 6 * ( ( ( xcor - zcor )
    / sqrt ( 2 ) ) - r / 2)) / exp ( r * 3 ))]
  set-color
end

to-report inrangexnz? [x y z]

```

```

    report sqrt ( ( x - r * cos(45) ) ^ 2 + ( z + r * cos(45) ) ^ 2 + ( y - 0 ) ^ 2 )
      < range
end

;-----

to selectnxy [opt]
  return-ori (inrangexy)
  ask inrangexy [jump opt * ( exp ( 6 * ( ( ( ycor - xcor )
    / sqrt ( 2 ) ) - r / 2)) / exp ( r * 3 ))]
  set-color
end

to-report inrangexy? [x y z]
  report sqrt ( ( x + r * cos(45) ) ^ 2 + ( y - r * cos(45) ) ^ 2 + ( z - 0 ) ^ 2 )
    < range
end

;-----

to selectxny [opt]
  return-ori (inrangeyny)
  ask inrangeyny [jump opt * ( exp ( 6 * ( ( ( xcor - ycor )
    / sqrt ( 2 ) ) - r / 2)) / exp ( r * 3 ))]
  set-color
end

to-report inrangeyny? [x y z]
  report sqrt ( ( x - r * cos(45) ) ^ 2 + ( y + r * cos(45) ) ^ 2 + ( z - 0 ) ^ 2 )
    < range
end

;-----

to selectynz [opt]
  return-ori (inrangeynz)
  ask inrangeynz [jump opt * ( exp ( 6 * ( ( ( ycor - zcor )
    / sqrt ( 2 ) ) - r / 2)) / exp ( r * 3 ))]
  set-color
end

to-report inrangeynz? [x y z]
  report sqrt ( ( z + r * cos(45) ) ^ 2 + ( y - r * cos(45) ) ^ 2 + ( x - 0 ) ^ 2 )
    < range
end

;-----

to selectnyz [opt]
  return-ori (inrangenyzy)
  ask inrangenyzy [jump opt * ( exp ( 6 * ( ( ( zcor - ycor )
    / sqrt ( 2 ) ) - r / 2)) / exp ( r * 3 ))]
  set-color
end

to-report inrangenyzy? [x y z]
  report sqrt ( ( z - r * cos(45) ) ^ 2 + ( y + r * cos(45) ) ^ 2 + ( x - 0 ) ^ 2 )
    < range
end

```

```

;-----
; 3 Axes
;-----

to selectxyz [opt]
  return-ori (inrangexyz)
  ask inrangexyz [jump opt * ( exp ( 6 * ( ( ( xcor + ycor + zcor )
    / sqrt ( 3 ) ) - r / 2)) / exp ( r * 3 ))]
  set-color
end

to-report inrangexyz? [x y z]
  report sqrt ( (x - r * cos(45) * cos(33)) ^ 2 + (y - r * sin(33)) ^ 2
    + (z - r * cos(45) * cos(33)) ^ 2 ) < range
end

;-----

to selectnxyz [opt]
  return-ori (inrangexyz)
  ask inrangexyz [jump opt * ( exp ( 6 * ( ( ( ycor + zcor - xcor )
    / sqrt ( 3 ) ) - r / 2)) / exp ( r * 3 ))]
  set-color
end

to-report inrangexyz? [x y z]
  report sqrt ( (x + r * cos(45) * cos(33)) ^ 2 + (y - r * sin(33)) ^ 2
    + (z - r * cos(45) * cos(33)) ^ 2 ) < range
end

;-----

to selectxnyz [opt]
  return-ori (inrangexnyz)
  ask inrangexnyz [jump opt * ( exp ( 6 * ( ( ( xcor - ycor + zcor )
    / sqrt ( 3 ) ) - r / 2)) / exp ( r * 3 ))]
  set-color
end

to-report inrangexnyz? [x y z]
  report sqrt ( (x - r * cos(45) * cos(33)) ^ 2 + (y + r * sin(33)) ^ 2
    + (z - r * cos(45) * cos(33)) ^ 2 ) < range
end

;-----

to selectnxnyz [opt]
  return-ori (inrangexnyz)
  ask inrangexnyz [jump opt * ( exp ( 6 * ( ( ( zcor - xcor - ycor )
    / sqrt ( 3 ) ) - r / 2)) / exp ( r * 3 ))]
  set-color
end

to-report inrangexnyz? [x y z]
  report sqrt ( (x + r * cos(45) * cos(33)) ^ 2 + (y + r * sin(33)) ^ 2
    + (z - r * cos(45) * cos(33)) ^ 2 ) < range
end

```

```

;-----
to selectxynz [opt]
  return-ori (inrangexynz)
  ask inrangexynz [jump opt * ( exp ( 6 * ( ( ( xcor + ycor - zcor )
                    / sqrt ( 3 ) ) - r / 2)) / exp ( r * 3 ))]
  set-color
end

to-report inrangexynz? [x y z]
  report sqrt ( ( x - r * cos(45) * cos(33)) ^ 2 + ( y - r * sin(33)) ^ 2
                + ( z + r * cos(45) * cos(33)) ^ 2 ) < range
end

;-----

to selectnxynz [opt]
  return-ori (inrangexynz)
  ask inrangexynz [jump opt * ( exp ( 6 * ( ( ( ycor - zcor - xcor )
                    / sqrt ( 3 ) ) - r / 2)) / exp ( r * 3 ))]
  set-color
end

to-report inrangexynz? [x y z]
  report sqrt ( ( x + r * cos(45) * cos(33)) ^ 2 + ( y - r * sin(33)) ^ 2
                + ( z + r * cos(45) * cos(33)) ^ 2 ) < range
end

;-----

to selectnxnynz [opt]
  return-ori (inrangexnynz)
  ask inrangexnynz [jump opt * ( exp ( 6 * ( ( (-1 * ( ycor + zcor + xcor )
                    / sqrt ( 3 ) ) - r / 2)) / exp ( r * 3 ))]
  set-color
end

to-report inrangexnynz? [x y z]
  report sqrt ( ( x + r * cos(45) * cos(33)) ^ 2 + ( y + r * sin(33)) ^ 2 +
                ( z + r * cos(45) * cos(33)) ^ 2 ) < range
end

;-----

to selectxnynz [opt]
  return-ori (inrangexnynz)
  ask inrangexnynz [jump opt * ( exp ( 6 * ( ( ( xcor - zcor - ycor )
                    / sqrt ( 3 ) ) - r / 2)) / exp ( r * 3 ))]
  set-color
end

to-report inrangexnynz? [x y z]
  report sqrt ( ( x - r * cos(45) * cos(33)) ^ 2 + ( y + r * sin(33)) ^ 2 +
                ( z + r * cos(45) * cos(33)) ^ 2 ) < range
end

```


Appendix E

Tension Calculations

Canon in D

| Event | x → y | Surface dissonance | | | Pitch space distance | | | Total |
|---------|-----------------------|--------------------|-----|--------|----------------------|---|---|-------|
| | | Sc.Deg | Inv | Nh. t. | i | j | k | |
| 0 → 1 | I → I | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 → 2 | I → V | 1 | 0 | 0 | 0 | 1 | 4 | 6 |
| 2 → 3 | V → vi | 1 | 0 | 0 | 0 | 2 | 6 | 9 |
| 3 → 4 | vi → iii | 1 | 0 | 0 | 0 | 1 | 4 | 6 |
| 4 → 5 | iii → IV | 1 | 0 | 0 | 0 | 2 | 6 | 9 |
| 5 → 6 | IV → I | 0 | 0 | 0 | 0 | 1 | 4 | 5 |
| 6 → 7 | I → IV | 1 | 0 | 0 | 0 | 2 | 4 | 7 |
| 7 → 8 | IV → V | 1 | 0 | 0 | 0 | 1 | 6 | 8 |
| 8 → 9 | V → I | 1 | 0 | 0 | 0 | 1 | 4 | 6 |
| 9 → 10 | I → V | 1 | 0 | 0 | 0 | 1 | 4 | 6 |
| 10 → 11 | V → vi | 1 | 0 | 0 | 0 | 2 | 6 | 9 |
| 11 → 12 | vi → iii ⁷ | 1 | 0 | 4 | 0 | 1 | 4 | 10 |
| 12 → 13 | iii ⁷ → IV | 1 | 0 | 0 | 0 | 2 | 6 | 9 |
| 13 → 14 | IV → I | 1 | 2 | 0 | 0 | 1 | 4 | 8 |
| 14 → 15 | I → IV | 1 | 0 | 0 | 0 | 1 | 4 | 6 |
| 15 → 16 | IV → V | 1 | 0 | 0 | 0 | 2 | 6 | 9 |
| 16 → 17 | V → I | 1 | 0 | 0 | 0 | 1 | 4 | 6 |
| 17 → 18 | I → V | 1 | 0 | 0 | 0 | 1 | 4 | 6 |
| 18 → 19 | V → V ⁷ | 1 | 0 | 4 | 0 | 0 | 0 | 5 |
| 19 → 20 | V ⁷ → vi | 1 | 0 | 0 | 0 | 2 | 6 | 9 |

| | | | | | | | | |
|---------|-----------------------------------|---|---|---|---|---|---|----|
| 20 → 21 | vi → iii | 1 | 0 | 0 | 0 | 1 | 4 | 6 |
| 21 → 22 | iii → iii ⁷ | 1 | 0 | 4 | 0 | 0 | 0 | 5 |
| 22 → 23 | iii ⁷ → IV | 1 | 0 | 0 | 0 | 2 | 6 | 9 |
| 23 → 24 | IV → I | 1 | 0 | 0 | 0 | 1 | 4 | 6 |
| 24 → 25 | I → I | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 25 → 26 | I → iii | 1 | 2 | 0 | 0 | 3 | 4 | 10 |
| 26 → 27 | iii → V | 1 | 0 | 0 | 0 | 3 | 4 | 8 |
| 27 → 28 | V → V ⁷ | 1 | 0 | 4 | 0 | 0 | 0 | 5 |
| 28 → 29 | V ⁷ → I | 0 | 0 | 0 | 0 | 1 | 4 | 5 |
| 29 → 30 | I → V | 1 | 0 | 0 | 0 | 1 | 4 | 6 |
| 30 → 31 | V → V ⁷ | 1 | 0 | 4 | 0 | 0 | 0 | 5 |
| 31 → 32 | V ⁷ → vi | 1 | 0 | 0 | 0 | 2 | 6 | 9 |
| 32 → 33 | vi → vi | 1 | 2 | 0 | 0 | 0 | 0 | 3 |
| 33 → 34 | vi → iii | 1 | 0 | 0 | 0 | 1 | 4 | 6 |
| 34 → 35 | iii → V | 1 | 0 | 4 | 0 | 3 | 4 | 12 |
| 35 → 36 | V → IV | 0 | 0 | 0 | 0 | 2 | 6 | 8 |
| 36 → 37 | IV → IV | 0 | 2 | 0 | 0 | 0 | 0 | 2 |
| 37 → 38 | IV → I | 1 | 0 | 0 | 0 | 1 | 4 | 6 |
| 38 → 39 | I → I | 1 | 2 | 0 | 0 | 0 | 0 | 3 |
| 39 → 40 | I → ii ⁷ | 1 | 2 | 4 | 0 | 2 | 6 | 15 |
| 40 → 41 | ii ⁷ → vi | 1 | 0 | 0 | 0 | 1 | 4 | 6 |
| 41 → 42 | vi → I | 0 | 2 | 0 | 0 | 3 | 4 | 9 |
| 42 → 43 | I → I ⁷ | 0 | 2 | 4 | 0 | 0 | 0 | 6 |
| 43 → 44 | I ⁷ → I | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 44 → 45 | I → I | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 45 → 46 | I → V | 0 | 0 | 0 | 0 | 1 | 4 | 5 |
| 46 → 47 | V → V ⁷ | 1 | 2 | 4 | 0 | 0 | 0 | 7 |
| 47 → 48 | V ⁷ → vi | 1 | 0 | 0 | 0 | 2 | 6 | 9 |
| 48 → 49 | vi → vi | 1 | 2 | 0 | 0 | 0 | 0 | 3 |
| 49 → 50 | vi → iii | 1 | 0 | 0 | 0 | 1 | 4 | 6 |
| 50 → 51 | iii → V | 0 | 0 | 4 | 0 | 3 | 4 | 11 |
| 51 → 52 | V → IV ⁷ | 0 | 0 | 4 | 0 | 2 | 6 | 12 |
| 52 → 53 | IV ⁷ → ii ⁷ | 0 | 2 | 4 | 0 | 3 | 6 | 15 |
| 53 → 54 | ii ⁷ → I | 1 | 0 | 0 | 0 | 2 | 6 | 9 |
| 54 → 55 | I → I ⁷ | 0 | 2 | 4 | 0 | 0 | 0 | 6 |
| 55 → 56 | I ⁷ → ii ⁷ | 1 | 2 | 4 | 0 | 2 | 6 | 15 |
| 56 → 57 | ii ⁷ → vi | 1 | 0 | 0 | 0 | 1 | 4 | 6 |
| 57 → 58 | vi → V ⁷ | 1 | 0 | 7 | 0 | 2 | 6 | 16 |
| 58 → 59 | V ⁷ → I ⁷ | 0 | 2 | 7 | 0 | 1 | 4 | 14 |
| 59 → 60 | I ⁷ → I | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

The Flight of the Bumblebee

| Event | x → y | Surface dissonance | | | Pitch space distance | | | Total |
|---------|------------------------------------|--------------------|-----|--------|----------------------|---|---|-------|
| | | Sc.Deg | Inv | Nh. t. | i | j | k | |
| 0 → 1 | V → V | 0 | 0 | 18 | 0 | 0 | 0 | 18 |
| 1 → 2 | V → V ⁷ | 0 | 0 | 18 | 0 | 0 | 0 | 18 |
| 2 → 3 | V ⁷ → i | 1 | 0 | 14 | 0 | 1 | 4 | 20 |
| 3 → 4 | i → i | 1 | 0 | 11 | 0 | 0 | 0 | 12 |
| 4 → 5 | i → IV ⁷ | 1 | 0 | 16 | 0 | 1 | 5 | 23 |
| 5 → 6 | IV ⁷ → IV ⁷ | 1 | 0 | 18 | 0 | 0 | 0 | 19 |
| 6 → 7 | IV ⁷ → IV ⁷ | 1 | 0 | 15 | 0 | 0 | 0 | 16 |
| 7 → 8 | IV ⁷ → IV ⁷ | 1 | 0 | 16 | 0 | 0 | 0 | 17 |
| 8 → 9 | IV ⁷ → IV ⁷ | 1 | 0 | 15 | 0 | 0 | 0 | 16 |
| 9 → 10 | IV ⁷ → iv ⁷ | 0 | 2 | 11 | 0 | 0 | 2 | 15 |
| 10 → 11 | iv ⁷ → i | 1 | 2 | 15 | 0 | 1 | 5 | 24 |
| 11 → 12 | i → V ⁷ | 0 | 2 | 16 | 0 | 1 | 4 | 23 |
| 12 → 13 | V ⁷ → i | 1 | 2 | 11 | 0 | 1 | 4 | 19 |
| 13 → 14 | i → iv | 0 | 2 | 10 | 0 | 1 | 5 | 18 |
| 14 → 15 | iv → i | 1 | 2 | 11 | 0 | 1 | 5 | 20 |
| 15 → 16 | i → V ⁷ | 0 | 2 | 15 | 0 | 1 | 4 | 22 |
| 16 → 17 | V ⁷ → I ⁷ | 0 | 2 | 12 | 0 | 1 | 4 | 19 |
| 17 → 18 | I ⁷ → iv | 1 | 0 | 8 | 0 | 1 | 4 | 14 |
| 18 → 19 | iv → iv | 1 | 2 | 11 | 0 | 0 | 0 | 14 |
| 19 → 20 | iv → VII ⁷ | 1 | 2 | 12 | 0 | 1 | 8 | 24 |
| 20 → 21 | VII ⁷ → iv | 1 | 2 | 15 | 0 | 1 | 8 | 27 |
| 21 → 22 | iv → iv | 1 | 2 | 11 | 0 | 0 | 0 | 14 |
| 22 → 23 | iv → VII ⁷ | 1 | 2 | 12 | 0 | 1 | 8 | 24 |
| 23 → 24 | VII ⁷ → iv | 1 | 2 | 11 | 0 | 1 | 8 | 23 |
| 24 → 25 | iv → vii ⁷ | 0 | 0 | 14 | 0 | 1 | 8 | 23 |
| 25 → 26 | vii ⁷ → iv | 1 | 0 | 11 | 0 | 1 | 8 | 21 |
| 26 → 27 | iv → I ⁷ | 0 | 0 | 12 | 0 | 1 | 5 | 18 |
| 27 → 28 | I ⁷ → iv | 1 | 2 | 11 | 0 | 1 | 5 | 20 |
| 28 → 29 | iv → vii ⁷ | 0 | 2 | 14 | 0 | 1 | 8 | 25 |
| 29 → 30 | vii ⁷ → iv | 1 | 0 | 11 | 0 | 1 | 8 | 21 |
| 30 → 31 | iv → I ⁷ | 0 | 2 | 12 | 0 | 1 | 5 | 20 |
| 31 → 32 | I ⁷ → iv | 1 | 0 | 0 | 0 | 1 | 5 | 7 |
| 32 → 33 | iv → I ⁷ | 0 | 2 | 9 | 0 | 0 | 5 | 16 |
| 33 → 34 | I ⁷ → i | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 34 → 35 | i → I ⁷ | 0 | 2 | 9 | 0 | 0 | 0 | 11 |
| 35 → 36 | I ⁷ → i | 0 | 0 | 8 | 0 | 0 | 0 | 8 |
| 36 → 37 | i → VI+ | 0 | 0 | 9 | 0 | 2 | 7 | 18 |
| 37 → 38 | VI+ → VI+ | 0 | 2 | 11 | 0 | 0 | 0 | 13 |
| 38 → 39 | VI+ → iv | 1 | 0 | 14 | 0 | 3 | 6 | 24 |
| 39 → 40 | iv → iv ⁷ | 1 | 0 | 14 | 0 | 0 | 0 | 15 |
| 40 → 41 | iv ⁷ → IV | 0 | 2 | 4 | 0 | 0 | 0 | 6 |
| 41 → 42 | IV → iv ⁷ | 0 | 2 | 9 | 0 | 0 | 2 | 13 |
| 42 → 43 | iv ⁷ → iv | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 43 → 44 | iv → iv ⁷ | 0 | 2 | 9 | 0 | 0 | 0 | 11 |
| 44 → 45 | iv ⁷ → iv ⁷ | 0 | 0 | 9 | 0 | 0 | 0 | 9 |
| 45 → 46 | iv ⁷ → iv ⁷ | 0 | 0 | 9 | 0 | 0 | 0 | 9 |
| 46 → 47 | iv ⁷ → iv ⁷ | 0 | 2 | 8 | 0 | 0 | 0 | 10 |
| 47 → 48 | iv ⁷ → vii ⁷ | 1 | 2 | 14 | 0 | 1 | 8 | 26 |

Appendix G

Tension and chord mapping

T = Tension
N = Nonharmonic

| Current chord | | | Previous Chord | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---------------|----------------------------------|---------------------|----------------|----|---|----|----|---|----|----|---|----|----|---|----|----|---|----|----|---|----|-----|---|----|-----|---|----|----|---|----|----|
| | | | N | I | | | i | | | I+ | | | i° | | | II | | | ii | | | II+ | | | ii° | | | | | | |
| | | | | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | |
| I | I | CEG | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 3 | 3 | 0 | 5 | 5 | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 6 | 8 | |
| | i | C(E)G | 4 | 0 | 2 | 6 | 0 | 0 | 4 | 0 | 5 | 9 | 0 | 3 | 7 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | |
| | I+ | CEG# | 4 | 0 | 3 | 7 | 0 | 5 | 9 | 0 | 0 | 4 | 0 | 5 | 9 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 4 | 10 | |
| | i° | C(E)(G) | 8 | 0 | 5 | 13 | 0 | 3 | 11 | 0 | 5 | 13 | 0 | 0 | 8 | 2 | 6 | 16 | 2 | 8 | 18 | 2 | 6 | 16 | 2 | 8 | 18 | 2 | 6 | 12 | |
| | I ⁷ Major | I ⁷ M | CEGB | 4 | 0 | 0 | 4 | 0 | 2 | 6 | 0 | 3 | 7 | 0 | 5 | 9 | 2 | 6 | 12 | 2 | 6 | 12 | 2 | 6 | 12 | 2 | 6 | 12 | 2 | 6 | 12 |
| | I ⁷ Dominant | I ⁷ D | CEG(B) | 5 | 0 | 0 | 5 | 0 | 2 | 7 | 0 | 3 | 8 | 0 | 5 | 10 | 2 | 6 | 13 | 2 | 6 | 13 | 2 | 6 | 13 | 2 | 6 | 13 | 2 | 6 | 13 |
| | I ⁷ Minor Major | I ⁷ MM | C(E)GB | 8 | 0 | 2 | 10 | 0 | 0 | 8 | 0 | 5 | 13 | 0 | 3 | 11 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 |
| | I ⁷ Minor | I ⁷ m | C(E)G(B) | 9 | 0 | 2 | 11 | 0 | 0 | 9 | 0 | 5 | 14 | 0 | 3 | 12 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 |
| | I ⁷ Augmented Major | I ⁷ AM | CEG#B | 8 | 0 | 3 | 11 | 0 | 5 | 13 | 0 | 0 | 8 | 0 | 5 | 13 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 4 | 14 |
| | I ⁷ Augmented | I ⁷ A | CEG#(B) | 9 | 0 | 3 | 12 | 0 | 5 | 14 | 0 | 0 | 9 | 0 | 5 | 14 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 4 | 15 |
| | I ⁷ Half Diminished | I ⁷ HD | C(E)(G)(B) | 13 | 0 | 5 | 18 | 0 | 3 | 16 | 0 | 3 | 16 | 0 | 0 | 13 | 2 | 6 | 21 | 2 | 8 | 23 | 2 | 6 | 21 | 2 | 8 | 23 | 2 | 8 | 23 |
| | I ⁷ Diminished | I ⁷ ° | C(E)(G)A | 12 | 0 | 5 | 17 | 0 | 3 | 15 | 0 | 3 | 15 | 0 | 0 | 12 | 2 | 6 | 20 | 2 | 8 | 22 | 2 | 6 | 20 | 2 | 8 | 22 | 2 | 8 | 22 |
| II | II | DF#A | 4 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 5 | 11 | 0 | 0 | 4 | 0 | 2 | 6 | 0 | 3 | 7 | 0 | 5 | 9 | 0 | 5 | 9 | |
| | ii | DFA | 0 | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 6 | 8 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 5 | 5 | 0 | 3 | 3 | 0 | 3 | 3 | |
| | II+ | DF## | 8 | 2 | 8 | 18 | 2 | 8 | 18 | 2 | 8 | 18 | 2 | 6 | 16 | 0 | 3 | 11 | 0 | 5 | 13 | 0 | 0 | 8 | 0 | 5 | 13 | 0 | 5 | 13 | |
| | ii° | DF(A) | 4 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 4 | 10 | 2 | 7 | 13 | 0 | 5 | 9 | 0 | 3 | 7 | 0 | 5 | 9 | 0 | 0 | 4 | 0 | 0 | 4 | |
| | II ⁷ Major | II ⁷ M | DF#AC# | 9 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 5 | 16 | 0 | 0 | 9 | 0 | 2 | 11 | 0 | 3 | 12 | 0 | 5 | 14 | 0 | 5 | 14 |
| | II ⁷ Dominant | II ⁷ D | DF#AC | 8 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 5 | 15 | 0 | 0 | 8 | 0 | 2 | 10 | 0 | 3 | 11 | 0 | 5 | 13 | 0 | 5 | 13 |
| | II ⁷ Minor Major | II ⁷ MM | DFAC# | 5 | 2 | 6 | 13 | 2 | 6 | 13 | 2 | 6 | 13 | 2 | 6 | 13 | 0 | 2 | 7 | 0 | 0 | 5 | 0 | 5 | 10 | 0 | 3 | 8 | 0 | 3 | 8 |
| | II ⁷ Minor | II ⁷ m | DFAC | 4 | 2 | 6 | 12 | 2 | 6 | 12 | 2 | 6 | 12 | 2 | 6 | 12 | 0 | 2 | 6 | 0 | 0 | 4 | 0 | 5 | 9 | 0 | 3 | 7 | 0 | 3 | 7 |
| | II ⁷ Augmented Major | II ⁷ AM | DF##C# | 13 | 2 | 8 | 23 | 2 | 8 | 23 | 2 | 8 | 23 | 2 | 6 | 21 | 0 | 3 | 16 | 0 | 5 | 18 | 0 | 0 | 13 | 0 | 5 | 18 | 0 | 5 | 18 |
| | II ⁷ Augmented | II ⁷ A | DF##C | 12 | 2 | 8 | 22 | 2 | 8 | 22 | 2 | 8 | 22 | 2 | 6 | 20 | 0 | 3 | 15 | 0 | 5 | 17 | 0 | 0 | 12 | 0 | 5 | 17 | 0 | 5 | 17 |
| | II ⁷ Half Diminished | II ⁷ HD | DF(A)C | 8 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 4 | 14 | 2 | 7 | 17 | 0 | 5 | 13 | 0 | 3 | 11 | 0 | 5 | 13 | 0 | 0 | 8 | 0 | 0 | 8 |
| | II ⁷ Diminished | II ⁷ ° | DF(A)B | 8 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 4 | 14 | 2 | 7 | 17 | 0 | 5 | 13 | 0 | 3 | 11 | 0 | 5 | 13 | 0 | 0 | 8 | 0 | 0 | 8 |
| III | III | EG#B | 4 | 3 | 6 | 13 | 3 | 8 | 15 | 3 | 4 | 11 | 3 | 8 | 15 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 5 | 11 | 2 | 5 | 11 | |
| | iii | EGB | 0 | 3 | 4 | 7 | 3 | 6 | 9 | 3 | 6 | 9 | 3 | 8 | 11 | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 6 | 8 | |
| | III+ | EG#C | 4 | 3 | 4 | 11 | 3 | 6 | 13 | 3 | 2 | 9 | 3 | 6 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 5 | 11 | 2 | 5 | 11 | |
| | iii° | EG(B) | 4 | 3 | 5 | 12 | 3 | 7 | 14 | 3 | 7 | 14 | 3 | 9 | 16 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 4 | 10 | 2 | 7 | 13 | 2 | 7 | 13 | |
| | III ⁷ Major | III ⁷ M | EG#BD# | 9 | 3 | 6 | 18 | 3 | 8 | 20 | 3 | 4 | 16 | 3 | 8 | 20 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 5 | 16 | 2 | 5 | 16 |
| | III ⁷ Dominant | III ⁷ D | EG#BD | 8 | 3 | 6 | 17 | 3 | 8 | 19 | 3 | 4 | 15 | 3 | 8 | 19 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 5 | 15 | 2 | 5 | 15 |
| | III ⁷ Minor Major | III ⁷ MM | EGBD# | 5 | 3 | 4 | 12 | 3 | 6 | 14 | 3 | 6 | 14 | 3 | 8 | 16 | 2 | 6 | 13 | 2 | 6 | 13 | 2 | 6 | 13 | 2 | 6 | 13 | 2 | 6 | 13 |
| | III ⁷ Minor | III ⁷ m | EGBD | 4 | 3 | 4 | 11 | 3 | 6 | 13 | 3 | 6 | 13 | 3 | 8 | 15 | 2 | 6 | 12 | 2 | 6 | 12 | 2 | 6 | 12 | 2 | 6 | 12 | 2 | 6 | 12 |
| | III ⁷ Augmented Major | III ⁷ AM | EG#CD# | 9 | 3 | 4 | 16 | 3 | 6 | 18 | 3 | 2 | 14 | 3 | 6 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 5 | 16 | 2 | 5 | 16 |
| | III ⁷ Augmented | III ⁷ A | EG#CD | 8 | 3 | 4 | 15 | 3 | 6 | 17 | 3 | 2 | 13 | 3 | 6 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 5 | 15 | 2 | 5 | 15 |
| | III ⁷ Half Diminished | III ⁷ HD | EG(B)D | 8 | 3 | 5 | 16 | 3 | 7 | 18 | 3 | 7 | 18 | 3 | 9 | 20 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 4 | 14 | 2 | 7 | 17 | 2 | 7 | 17 |
| | III ⁷ Diminished | III ⁷ ° | EG(B)C# | 9 | 3 | 5 | 17 | 3 | 7 | 19 | 3 | 7 | 19 | 3 | 9 | 21 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 4 | 15 | 2 | 7 | 18 | 2 | 7 | 18 |

T = Tension

N = Nonharmonic

Current chord

Previous Chord

| Current chord | N | I | | | | i | | | | I+ | | | | i° | | | | II | | | | ii | | | | II+ | | | | ii° | | | | | | |
|----------------------------------|---------------------|----------|----|---|---|----|---|---|----|----|---|----|---|----|----|---|---|----|---|---|----|----|---|----|---|-----|----|---|---|-----|---|---|----|---|---|----|
| | | j | k | T | | j | k | T | | j | k | T | | j | k | T | | j | k | T | | j | k | T | | j | k | T | | j | k | T | | | | |
| IV | IV | FAC | 0 | 1 | 4 | 5 | 1 | 4 | 5 | 1 | 4 | 5 | 1 | 4 | 5 | 3 | 6 | 9 | 3 | 4 | 7 | 3 | 8 | 11 | 3 | 6 | 9 | 3 | 4 | 7 | 3 | 8 | 11 | 3 | 6 | 9 |
| iv | iv | F(A)C | 4 | 1 | 5 | 10 | 1 | 5 | 10 | 1 | 3 | 8 | 1 | 5 | 10 | 3 | 8 | 15 | 3 | 6 | 13 | 3 | 8 | 15 | 3 | 4 | 11 | 3 | 8 | 15 | 3 | 4 | 11 | 3 | 4 | 11 |
| IV+ | IV+ | FAC# | 4 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 7 | 12 | 3 | 7 | 14 | 3 | 5 | 12 | 3 | 9 | 16 | 3 | 7 | 14 | 3 | 7 | 14 | 3 | 7 | 14 | 3 | 7 | 14 |
| iv° | iv° | F(A)B | 4 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 5 | 10 | 1 | 7 | 12 | 3 | 8 | 15 | 3 | 6 | 13 | 3 | 8 | 15 | 3 | 4 | 11 | 3 | 8 | 15 | 3 | 4 | 11 | 3 | 4 | 11 |
| IV ⁷ Major | IV ⁷ M | FACE | 4 | 1 | 4 | 9 | 1 | 4 | 9 | 1 | 4 | 9 | 1 | 4 | 9 | 3 | 6 | 13 | 3 | 4 | 11 | 3 | 8 | 15 | 3 | 6 | 13 | 3 | 4 | 11 | 3 | 6 | 13 | 3 | 4 | 11 |
| IV ⁷ Dominant | IV ⁷ D | FAC(E) | 5 | 1 | 4 | 10 | 1 | 4 | 10 | 1 | 4 | 10 | 1 | 4 | 10 | 3 | 6 | 14 | 3 | 4 | 12 | 3 | 8 | 16 | 3 | 6 | 14 | 3 | 6 | 14 | 3 | 6 | 14 | 3 | 6 | 14 |
| iv ⁷ Minor Major | iv ⁷ MM | F(A)CE | 8 | 1 | 5 | 14 | 1 | 5 | 14 | 1 | 3 | 12 | 1 | 5 | 14 | 3 | 8 | 19 | 3 | 6 | 17 | 3 | 8 | 19 | 3 | 4 | 15 | 3 | 8 | 19 | 3 | 4 | 15 | 3 | 4 | 15 |
| iv ⁷ Minor | iv ⁷ m | F(A)C(E) | 9 | 1 | 5 | 15 | 1 | 5 | 15 | 1 | 3 | 13 | 1 | 5 | 15 | 3 | 8 | 20 | 3 | 6 | 18 | 3 | 8 | 20 | 3 | 4 | 16 | 3 | 8 | 20 | 3 | 4 | 16 | 3 | 4 | 16 |
| IV ⁷ Augmented Major | IV ⁷ AM | FAC#E | 8 | 1 | 7 | 16 | 1 | 7 | 16 | 1 | 7 | 16 | 1 | 7 | 16 | 3 | 7 | 18 | 3 | 5 | 16 | 3 | 9 | 20 | 3 | 7 | 18 | 3 | 7 | 18 | 3 | 7 | 18 | 3 | 7 | 18 |
| IV ⁷ Augmented | IV ⁷ A | FAC#(E) | 9 | 1 | 7 | 17 | 1 | 7 | 17 | 1 | 7 | 17 | 1 | 7 | 17 | 3 | 7 | 19 | 3 | 5 | 17 | 3 | 9 | 21 | 3 | 7 | 19 | 3 | 7 | 19 | 3 | 7 | 19 | 3 | 7 | 19 |
| iv ⁷ Half Diminished | iv ⁷ HD | F(A)B(E) | 9 | 1 | 7 | 17 | 1 | 7 | 17 | 1 | 5 | 15 | 1 | 7 | 17 | 3 | 8 | 20 | 3 | 6 | 18 | 3 | 8 | 20 | 3 | 4 | 16 | 3 | 8 | 20 | 3 | 4 | 16 | 3 | 4 | 16 |
| iv ⁷ Diminished | iv ⁷ | F(A)BD | 8 | 1 | 7 | 16 | 1 | 7 | 16 | 1 | 5 | 14 | 1 | 7 | 16 | 3 | 8 | 19 | 3 | 6 | 17 | 3 | 8 | 19 | 3 | 4 | 15 | 3 | 8 | 19 | 3 | 4 | 15 | 3 | 4 | 15 |
| V | V | GBD | 0 | 1 | 4 | 5 | 1 | 4 | 5 | 1 | 7 | 8 | 1 | 7 | 8 | 1 | 4 | 5 | 1 | 4 | 5 | 1 | 4 | 5 | 1 | 4 | 5 | 1 | 4 | 5 | 1 | 4 | 5 | 1 | 4 | 5 |
| v | v | G(B)D | 4 | 1 | 5 | 10 | 1 | 5 | 10 | 1 | 8 | 13 | 1 | 8 | 13 | 1 | 5 | 10 | 1 | 5 | 10 | 1 | 3 | 8 | 1 | 5 | 10 | 1 | 3 | 8 | 1 | 5 | 10 | 1 | 5 | 10 |
| V+ | V+ | GBD# | 4 | 1 | 5 | 10 | 1 | 5 | 10 | 1 | 8 | 13 | 1 | 6 | 11 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 6 | 11 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 7 | 12 |
| v° | v° | G(B)D | 8 | 1 | 6 | 15 | 1 | 6 | 15 | 1 | 9 | 18 | 1 | 9 | 18 | 1 | 8 | 17 | 1 | 8 | 17 | 1 | 6 | 15 | 1 | 8 | 17 | 1 | 6 | 15 | 1 | 8 | 17 | 1 | 8 | 17 |
| V ⁷ Major | V ⁷ M | GBDF# | 5 | 1 | 4 | 10 | 1 | 4 | 10 | 1 | 7 | 13 | 1 | 7 | 13 | 1 | 4 | 10 | 1 | 4 | 10 | 1 | 4 | 10 | 1 | 4 | 10 | 1 | 4 | 10 | 1 | 4 | 10 | 1 | 4 | 10 |
| V ⁷ Dominant | V ⁷ D | GBDF | 4 | 1 | 4 | 9 | 1 | 4 | 9 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 4 | 9 | 1 | 4 | 9 | 1 | 4 | 9 | 1 | 4 | 9 | 1 | 4 | 9 | 1 | 4 | 9 | 1 | 4 | 9 |
| v ⁷ Minor Major | v ⁷ MM | G(B)DF# | 9 | 1 | 5 | 15 | 1 | 5 | 15 | 1 | 8 | 18 | 1 | 8 | 18 | 1 | 5 | 15 | 1 | 5 | 15 | 1 | 3 | 13 | 1 | 5 | 15 | 1 | 3 | 13 | 1 | 5 | 15 | 1 | 5 | 15 |
| v ⁷ Minor | v ⁷ m | G(B)DF | 8 | 1 | 5 | 14 | 1 | 5 | 14 | 1 | 8 | 17 | 1 | 8 | 17 | 1 | 5 | 14 | 1 | 5 | 14 | 1 | 3 | 12 | 1 | 5 | 14 | 1 | 3 | 12 | 1 | 5 | 14 | 1 | 5 | 14 |
| V ⁷ Augmented Major | V ⁷ AM | GBD#F# | 9 | 1 | 5 | 15 | 1 | 5 | 15 | 1 | 8 | 18 | 1 | 6 | 16 | 1 | 7 | 17 | 1 | 7 | 17 | 1 | 6 | 16 | 1 | 7 | 17 | 1 | 6 | 16 | 1 | 7 | 17 | 1 | 7 | 17 |
| V ⁷ Augmented | V ⁷ A | GBD#F | 8 | 1 | 5 | 14 | 1 | 5 | 14 | 1 | 8 | 17 | 1 | 6 | 15 | 1 | 7 | 16 | 1 | 7 | 16 | 1 | 6 | 15 | 1 | 7 | 16 | 1 | 6 | 15 | 1 | 7 | 16 | 1 | 7 | 16 |
| v ⁷ Half Diminished | v ⁷ HD | G(B)D)F | 12 | 1 | 6 | 19 | 1 | 6 | 19 | 1 | 9 | 22 | 1 | 9 | 22 | 1 | 8 | 21 | 1 | 8 | 21 | 1 | 6 | 19 | 1 | 8 | 21 | 1 | 6 | 19 | 1 | 8 | 21 | 1 | 8 | 21 |
| v ⁷ Diminished | v ⁷ | G(B)D)E | 12 | 1 | 6 | 19 | 1 | 6 | 19 | 1 | 9 | 22 | 1 | 9 | 22 | 1 | 8 | 21 | 1 | 8 | 21 | 1 | 6 | 19 | 1 | 8 | 21 | 1 | 6 | 19 | 1 | 8 | 21 | 1 | 8 | 21 |
| VI | VI | AC#E | 4 | 3 | 6 | 13 | 3 | 6 | 13 | 3 | 6 | 13 | 3 | 8 | 15 | 1 | 5 | 10 | 1 | 5 | 10 | 1 | 8 | 13 | 1 | 8 | 13 | 1 | 8 | 13 | 1 | 8 | 13 | 1 | 8 | 13 |
| vi | vi | ACE | 0 | 3 | 4 | 7 | 3 | 6 | 9 | 3 | 4 | 7 | 3 | 6 | 9 | 1 | 4 | 5 | 1 | 4 | 5 | 1 | 7 | 8 | 1 | 7 | 8 | 1 | 7 | 8 | 1 | 7 | 8 | 1 | 7 | 8 |
| VI+ | VI+ | AC#F | 4 | 3 | 7 | 14 | 3 | 7 | 14 | 3 | 7 | 14 | 3 | 7 | 14 | 1 | 6 | 11 | 1 | 4 | 9 | 1 | 9 | 14 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 7 | 12 |
| vi° | vi° | AC(E) | 4 | 3 | 6 | 13 | 3 | 6 | 13 | 3 | 6 | 13 | 3 | 4 | 11 | 1 | 5 | 10 | 1 | 5 | 10 | 1 | 8 | 13 | 1 | 8 | 13 | 1 | 8 | 13 | 1 | 8 | 13 | 1 | 8 | 13 |
| VI ⁷ Major | VI ⁷ M | AC#EG# | 9 | 3 | 6 | 18 | 3 | 8 | 20 | 3 | 6 | 18 | 3 | 8 | 20 | 1 | 5 | 15 | 1 | 5 | 15 | 1 | 8 | 18 | 1 | 8 | 18 | 1 | 8 | 18 | 1 | 8 | 18 | 1 | 8 | 18 |
| VI ⁷ Dominant | VI ⁷ D | AC#EG | 8 | 3 | 6 | 17 | 3 | 8 | 19 | 3 | 6 | 17 | 3 | 8 | 19 | 1 | 5 | 14 | 1 | 5 | 14 | 1 | 8 | 17 | 1 | 8 | 17 | 1 | 8 | 17 | 1 | 8 | 17 | 1 | 8 | 17 |
| VI ⁷ Minor Major | vi ⁷ MM | ACEG# | 5 | 3 | 4 | 12 | 3 | 6 | 14 | 3 | 4 | 12 | 3 | 6 | 14 | 1 | 4 | 10 | 1 | 4 | 10 | 1 | 7 | 13 | 1 | 7 | 13 | 1 | 7 | 13 | 1 | 7 | 13 | 1 | 7 | 13 |
| VI ⁷ Minor | vi ⁷ m | ACEG | 4 | 3 | 4 | 11 | 3 | 6 | 13 | 3 | 4 | 11 | 3 | 6 | 13 | 1 | 4 | 9 | 1 | 4 | 9 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 7 | 12 |
| VI ⁷ Augmented Major | VI ⁷ AM | AC#FG# | 9 | 3 | 7 | 19 | 3 | 7 | 19 | 3 | 7 | 19 | 3 | 7 | 19 | 1 | 6 | 16 | 1 | 4 | 14 | 1 | 9 | 19 | 1 | 7 | 17 | 1 | 7 | 17 | 1 | 7 | 17 | 1 | 7 | 17 |
| VI ⁷ Augmented | VI ⁷ A | AC#FG | 8 | 3 | 7 | 18 | 3 | 7 | 18 | 3 | 7 | 18 | 3 | 7 | 18 | 1 | 6 | 15 | 1 | 4 | 13 | 1 | 9 | 18 | 1 | 7 | 16 | 1 | 7 | 16 | 1 | 7 | 16 | | | |
| vi ⁷ Half Diminished | vi ⁷ HD | AC(E)G | 8 | 3 | 6 | 17 | 3 | 6 | 17 | 3 | 6 | 17 | 3 | 4 | 15 | 1 | 5 | 14 | 1 | 5 | 14 | 1 | 8 | 17 | 1 | 8 | 17 | 1 | 8 | 17 | 1 | 8 | 17 | | | |
| vi ⁷ Diminished | vi ⁷ | AC(E)F# | 9 | 3 | 6 | 18 | 3 | 6 | 18 | 3 | 6 | 18 | 3 | 4 | 16 | 1 | 5 | 15 | 1 | 5 | 15 | 1 | 8 | 18 | 1 | 8 | 18 | 1 | 8 | 18 | 1 | 8 | 18 | | | |
| VII | VII | BD#F# | 8 | 2 | 8 | 18 | 2 | 6 | 16 | 2 | 8 | 18 | 2 | 3 | 13 | 3 | 6 | 17 | 3 | 8 | 19 | 3 | 6 | 17 | 3 | 8 | 19 | 3 | 6 | 17 | 3 | 8 | 19 | | | |
| vii | vii | BDF# | 4 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 4 | 10 | 3 | 4 | 11 | 3 | 6 | 13 | 3 | 4 | 11 | 3 | 6 | 13 | 3 | 4 | 11 | 3 | 6 | 13 | | | |
| VII+ | VII+ | BD#G | 4 | 2 | 5 | 11 | 2 | 3 | 9 | 2 | 8 | 14 | 2 | 6 | 12 | 3 | 7 | 14 | 3 | 7 | 14 | 3 | 7 | 14 | 3 | 7 | 14 | 3 | 7 | 14 | 3 | 7 | 14 | | | |
| vii° | vii° | BDF | 0 | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 6 | 8 | 3 | 6 | 9 | 3 | 4 | 7 | 3 | 6 | 9 | 3 | 4 | 7 | 3 | 6 | 9 | 3 | 4 | 7 | | | |
| VII ⁷ Major | VII ⁷ M | BD#F#A# | 13 | 2 | 8 | 23 | 2 | 6 | 21 | 2 | 8 | 23 | 2 | 3 | 18 | 3 | 6 | 22 | 3 | 8 | 24 | 3 | 6 | 22 | 3 | 8 | 24 | 3 | 6 | 22 | 3 | 8 | 24 | | | |
| VII ⁷ Dominant | VII ⁷ D | BD#F#A | 12 | 2 | 8 | 22 | 2 | 6 | 20 | 2 | 8 | 22 | 2 | 3 | 17 | 3 | 6 | 21 | 3 | 8 | 23 | 3 | 6 | 21 | 3 | 8 | 23 | 3 | 6 | 21 | 3 | 8 | 23 | | | |
| vii ⁷ Minor Major | vii ⁷ MM | BDF#A# | 9 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 4 | 15 | 3 | 4 | 16 | 3 | 6 | 18 | 3 | 4 | 16 | 3 | 6 | 18 | 3 | 4 | 16 | 3 | 6 | 18 | | | |
| vii ⁷ Minor | vii ⁷ m | BDF#A | 8 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 4 | 14 | 3 | 4 | 15 | 3 | 6 | 17 | 3 | 4 | 15 | 3 | 6 | 17 | 3 | 4 | 15 | 3 | 6 | 17 | | | |
| VII ⁷ Augmented Major | VII ⁷ AM | BD#GA# | 9 | 2 | 5 | 16 | 2 | 3 | 14 | 2 | 8 | 19 | 2 | 6 | 17 | 3 | 7 | 19 | 3 | 7 | 19 | 3 | 7 | 19 | 3 | 7 | 19 | 3 | 7 | 19 | 3 | 7 | 19 | | | |
| VII ⁷ Augmented | VII ⁷ A | BD#GA | 8 | 2 | 5 | 15 | 2 | 3 | 13 | 2 | 8 | 18 | 2 | 6 | 16 | 3 | 7 | 18 | 3 | 7 | 18 | 3 | 7 | 18 | 3 | 7 | 18 | 3 | | | | | | | | |

| Current | Previous Chord | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---------|----------------|---|----|-----|---|----|------|---|----|------|---|----|----|----|----|----|----|----|-----|----|----|-----|----|----|---|----|----|---|----|----|----|----|----|----|----|----|---|----|---|---|---|---|---|---|---|
| | III | | | iii | | | III+ | | | iii° | | | IV | | | iv | | | IV+ | | | iv° | | | V | | | v | | | V+ | | | v° | | | | | | | | | | | |
| | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T |
| I | 3 | 6 | 9 | 3 | 4 | 7 | 3 | 3 | 6 | 3 | 4 | 7 | 1 | 4 | 5 | 1 | 4 | 5 | 1 | 7 | 8 | 1 | 6 | 7 | 1 | 4 | 5 | 1 | 4 | 5 | 1 | 4 | 5 | 1 | 4 | 5 | | | | | | | | | |
| i | 3 | 8 | 15 | 3 | 6 | 13 | 3 | 5 | 12 | 3 | 6 | 13 | 1 | 5 | 10 | 1 | 5 | 10 | 1 | 8 | 13 | 1 | 7 | 12 | 1 | 5 | 10 | 1 | 5 | 10 | 1 | 3 | 8 | 1 | 5 | 10 | | | | | | | | | |
| I+ | 3 | 4 | 11 | 3 | 6 | 13 | 3 | 2 | 9 | 3 | 6 | 13 | 1 | 5 | 10 | 1 | 3 | 8 | 1 | 8 | 13 | 1 | 5 | 10 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 7 | 12 | | | | | | | | | |
| i° | 3 | 8 | 19 | 3 | 8 | 19 | 3 | 6 | 17 | 3 | 8 | 19 | 1 | 6 | 15 | 1 | 6 | 15 | 1 | 9 | 18 | 1 | 8 | 17 | 1 | 8 | 17 | 1 | 8 | 17 | 1 | 6 | 15 | 1 | 8 | 17 | | | | | | | | | |
| I°M | 3 | 6 | 13 | 3 | 4 | 11 | 3 | 3 | 10 | 3 | 4 | 11 | 1 | 4 | 9 | 1 | 4 | 9 | 1 | 7 | 12 | 1 | 6 | 11 | 1 | 4 | 9 | 1 | 4 | 9 | 1 | 4 | 9 | 1 | 4 | 9 | | | | | | | | | |
| I°D | 3 | 6 | 14 | 3 | 4 | 12 | 3 | 3 | 11 | 3 | 4 | 12 | 1 | 4 | 10 | 1 | 4 | 10 | 1 | 7 | 13 | 1 | 6 | 12 | 1 | 4 | 10 | 1 | 4 | 10 | 1 | 4 | 10 | 1 | 4 | 10 | | | | | | | | | |
| i°MM | 3 | 8 | 19 | 3 | 6 | 17 | 3 | 5 | 16 | 3 | 6 | 17 | 1 | 5 | 14 | 1 | 5 | 14 | 1 | 8 | 17 | 1 | 7 | 16 | 1 | 5 | 14 | 1 | 5 | 14 | 1 | 3 | 12 | 1 | 5 | 14 | | | | | | | | | |
| i°m | 3 | 8 | 20 | 3 | 6 | 18 | 3 | 5 | 17 | 3 | 6 | 18 | 1 | 5 | 15 | 1 | 5 | 15 | 1 | 8 | 18 | 1 | 7 | 17 | 1 | 5 | 15 | 1 | 5 | 15 | 1 | 3 | 13 | 1 | 5 | 15 | | | | | | | | | |
| I°AM | 3 | 4 | 15 | 3 | 6 | 17 | 3 | 2 | 13 | 3 | 6 | 17 | 1 | 5 | 14 | 1 | 3 | 12 | 1 | 8 | 17 | 1 | 5 | 14 | 1 | 7 | 16 | 1 | 7 | 16 | 1 | 7 | 16 | 1 | 7 | 16 | | | | | | | | | |
| I°A | 3 | 4 | 16 | 3 | 6 | 18 | 3 | 2 | 14 | 3 | 6 | 18 | 1 | 5 | 15 | 1 | 3 | 13 | 1 | 8 | 18 | 1 | 5 | 15 | 1 | 7 | 17 | 1 | 7 | 17 | 1 | 7 | 17 | 1 | 7 | 17 | | | | | | | | | |
| i°HD | 3 | 8 | 24 | 3 | 8 | 24 | 3 | 6 | 22 | 3 | 8 | 24 | 1 | 6 | 20 | 1 | 6 | 20 | 1 | 9 | 23 | 1 | 8 | 22 | 1 | 8 | 22 | 1 | 8 | 22 | 1 | 6 | 20 | 1 | 8 | 22 | | | | | | | | | |
| i°7 | 3 | 8 | 23 | 3 | 8 | 23 | 3 | 6 | 21 | 3 | 8 | 23 | 1 | 6 | 19 | 1 | 6 | 19 | 1 | 9 | 22 | 1 | 8 | 21 | 1 | 8 | 21 | 1 | 8 | 21 | 1 | 6 | 19 | 1 | 8 | 21 | | | | | | | | | |
| II | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 3 | 6 | 13 | 3 | 6 | 13 | 3 | 8 | 15 | 3 | 6 | 13 | 3 | 8 | 15 | 1 | 5 | 10 | 1 | 8 | 13 | 1 | 8 | 13 | | | | | | | | | |
| ii | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 6 | 8 | 3 | 4 | 7 | 3 | 6 | 9 | 3 | 4 | 7 | 3 | 6 | 9 | 1 | 4 | 5 | 1 | 4 | 5 | 1 | 7 | 8 | 1 | 7 | 8 | | | | | | | | | |
| II+ | 2 | 8 | 18 | 2 | 8 | 18 | 2 | 8 | 18 | 2 | 5 | 15 | 3 | 8 | 19 | 3 | 8 | 19 | 3 | 8 | 19 | 3 | 8 | 19 | 1 | 6 | 15 | 1 | 4 | 13 | 1 | 9 | 18 | 1 | 7 | 16 | | | | | | | | | |
| ii° | 2 | 5 | 11 | 2 | 7 | 13 | 2 | 5 | 11 | 2 | 7 | 13 | 3 | 6 | 13 | 3 | 4 | 11 | 3 | 6 | 13 | 3 | 4 | 11 | 1 | 5 | 10 | 1 | 5 | 10 | 1 | 8 | 13 | 1 | 8 | 13 | | | | | | | | | |
| II°M | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 3 | 6 | 18 | 3 | 8 | 20 | 3 | 6 | 18 | 3 | 8 | 20 | 1 | 5 | 15 | 1 | 5 | 15 | 1 | 8 | 18 | 1 | 8 | 18 | | | | | | | | | |
| II°D | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 3 | 6 | 17 | 3 | 6 | 17 | 3 | 8 | 19 | 3 | 6 | 17 | 3 | 8 | 19 | 1 | 5 | 14 | 1 | 8 | 17 | 1 | 8 | 17 | | | | | | | | | |
| ii°MM | 2 | 6 | 13 | 2 | 6 | 13 | 2 | 6 | 13 | 2 | 6 | 13 | 3 | 4 | 12 | 3 | 6 | 14 | 3 | 4 | 12 | 3 | 6 | 14 | 1 | 4 | 10 | 1 | 4 | 10 | 1 | 7 | 13 | 1 | 7 | 13 | | | | | | | | | |
| ii°m | 2 | 6 | 12 | 2 | 6 | 12 | 2 | 6 | 12 | 2 | 6 | 12 | 3 | 4 | 11 | 3 | 6 | 13 | 3 | 4 | 11 | 3 | 6 | 13 | 1 | 4 | 9 | 1 | 4 | 9 | 1 | 7 | 12 | 1 | 7 | 12 | | | | | | | | | |
| II°AM | 2 | 8 | 23 | 2 | 8 | 23 | 2 | 8 | 23 | 2 | 5 | 20 | 3 | 8 | 24 | 3 | 8 | 24 | 3 | 8 | 24 | 3 | 8 | 24 | 1 | 6 | 20 | 1 | 4 | 18 | 1 | 9 | 23 | 1 | 7 | 21 | | | | | | | | | |
| II°A | 2 | 8 | 22 | 2 | 8 | 22 | 2 | 8 | 22 | 2 | 5 | 19 | 3 | 8 | 23 | 3 | 8 | 23 | 3 | 8 | 23 | 3 | 8 | 23 | 1 | 6 | 19 | 1 | 4 | 17 | 1 | 9 | 22 | 1 | 7 | 20 | | | | | | | | | |
| ii°HD | 2 | 5 | 15 | 2 | 7 | 17 | 2 | 5 | 15 | 2 | 7 | 17 | 3 | 6 | 17 | 3 | 4 | 15 | 3 | 6 | 17 | 3 | 4 | 15 | 1 | 5 | 14 | 1 | 5 | 14 | 1 | 8 | 17 | 1 | 8 | 17 | | | | | | | | | |
| ii°7 | 2 | 5 | 15 | 2 | 7 | 17 | 2 | 5 | 15 | 2 | 7 | 17 | 3 | 6 | 17 | 3 | 4 | 15 | 3 | 6 | 17 | 3 | 4 | 15 | 1 | 5 | 14 | 1 | 5 | 14 | 1 | 8 | 17 | 1 | 8 | 17 | | | | | | | | | |
| III | 0 | 0 | 4 | 0 | 2 | 6 | 0 | 2 | 6 | 0 | 5 | 9 | 2 | 7 | 13 | 2 | 5 | 11 | 2 | 7 | 13 | 2 | 3 | 9 | 2 | 6 | 12 | 2 | 8 | 14 | 2 | 6 | 12 | 2 | 8 | 14 | | | | | | | | | |
| iii | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 4 | 4 | 0 | 3 | 3 | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 4 | 6 | 2 | 4 | 6 | 2 | 4 | 6 | 2 | 4 | 6 | 2 | 6 | 8 | | | | | | | | | |
| III+ | 0 | 2 | 6 | 0 | 4 | 8 | 0 | 0 | 4 | 0 | 4 | 8 | 2 | 5 | 11 | 2 | 3 | 9 | 2 | 8 | 14 | 2 | 5 | 11 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | | | | | | | | | |
| iii° | 0 | 5 | 9 | 0 | 3 | 7 | 0 | 5 | 9 | 0 | 4 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 6 | 12 | 2 | 4 | 10 | 2 | 6 | 12 | 2 | 4 | 10 | | | | | | | |
| III°M | 0 | 0 | 9 | 0 | 2 | 11 | 0 | 2 | 11 | 0 | 5 | 14 | 2 | 7 | 18 | 2 | 5 | 16 | 2 | 7 | 18 | 2 | 3 | 14 | 2 | 6 | 17 | 2 | 8 | 19 | 2 | 6 | 17 | 2 | 8 | 19 | | | | | | | | | |
| III°D | 0 | 0 | 8 | 0 | 2 | 10 | 0 | 2 | 10 | 0 | 5 | 13 | 2 | 7 | 17 | 2 | 5 | 15 | 2 | 7 | 17 | 2 | 3 | 13 | 2 | 6 | 16 | 2 | 8 | 18 | 2 | 6 | 16 | 2 | 8 | 18 | | | | | | | | | |
| iii°MM | 0 | 2 | 7 | 0 | 0 | 5 | 0 | 4 | 9 | 0 | 3 | 8 | 2 | 6 | 13 | 2 | 6 | 13 | 2 | 6 | 13 | 2 | 4 | 11 | 2 | 4 | 11 | 2 | 4 | 11 | 2 | 4 | 11 | 2 | 6 | 13 | | | | | | | | | |
| iii°m | 0 | 2 | 6 | 0 | 0 | 4 | 0 | 4 | 8 | 0 | 3 | 7 | 2 | 6 | 12 | 2 | 6 | 12 | 2 | 6 | 12 | 2 | 4 | 10 | 2 | 4 | 10 | 2 | 4 | 10 | 2 | 4 | 10 | 2 | 6 | 12 | | | | | | | | | |
| III°AM | 0 | 2 | 11 | 0 | 4 | 13 | 0 | 0 | 9 | 0 | 4 | 13 | 2 | 5 | 16 | 2 | 3 | 14 | 2 | 8 | 19 | 2 | 5 | 16 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | | | | | | | | | |
| III°A | 0 | 2 | 10 | 0 | 4 | 12 | 0 | 0 | 8 | 0 | 4 | 12 | 2 | 5 | 15 | 2 | 3 | 13 | 2 | 8 | 18 | 2 | 5 | 15 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | | | | | | | | | |
| iii°HD | 0 | 5 | 13 | 0 | 3 | 11 | 0 | 5 | 13 | 0 | 0 | 8 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 6 | 16 | 2 | 4 | 14 | 2 | 6 | 16 | 2 | 4 | 14 | | | | | | | | | |
| iii°7 | 0 | 5 | 14 | 0 | 3 | 12 | 0 | 5 | 14 | 0 | 0 | 9 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 6 | 17 | 2 | 4 | 15 | 2 | 6 | 17 | 2 | 4 | 15 | | | | | | | | | |
| IV | 2 | 7 | 9 | 2 | 6 | 8 | 2 | 4 | 6 | 2 | 6 | 8 | 0 | 0 | 2 | 2 | 0 | 3 | 3 | 0 | 4 | 4 | 3 | 6 | 9 | 3 | 6 | 9 | 3 | 6 | 9 | 3 | 6 | 9 | 3 | 6 | 9 | | | | | | | | |
| iv | 2 | 5 | 11 | 2 | 7 | 13 | 2 | 3 | 9 | 2 | 7 | 13 | 0 | 2 | 6 | 0 | 0 | 4 | 0 | 5 | 9 | 0 | 2 | 6 | 3 | 7 | 14 | 3 | 7 | 14 | 3 | 7 | 14 | 3 | 7 | 14 | | | | | | | | | |
| IV+ | 2 | 8 | 14 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 0 | 3 | 7 | 0 | 5 | 9 | 0 | 4 | 0 | 5 | 9 | 3 | 7 | 14 | 3 | 7 | 14 | 3 | 7 | 14 | 3 | 4 | 11 | | | | | | | | | | |
| iv° | 2 | 3 | 9 | 2 | 5 | 11 | 2 | 5 | 11 | 2 | 8 | 14 | 0 | 4 | 8 | 0 | 2 | 6 | 0 | 5 | 9 | 0 | 4 | 3 | 6 | 13 | 3 | 8 | 15 | 3 | 6 | 13 | 3 | 8 | 15 | | | | | | | | | | |
| IV°M | 2 | 7 | 13 | 2 | 6 | 12 | 2 | 4 | 10 | 2 | 6 | 12 | 0 | 4 | 0 | 2 | 6 | 0 | 3 | 7 | 0 | 4 | 8 | 3 | 6 | 13 | 3 | 6 | 13 | 3 | 6 | 13 | 3 | 6 | 13 | | | | | | | | | | |
| IV°D | 2 | 7 | 14 | 2 | 6 | 13 | 2 | 4 | 11 | 2 | 6 | 13 | 0 | 5 | 0 | 2 | 7 | 0 | 3 | 8 | 0 | 4 | 9 | 3 | 6 | 14 | 3 | 6 | 14 | 3 | 6 | 14 | 3 | 6 | 14 | | | | | | | | | | |
| iv°MM | 2 | 5 | 15 | 2 | 7 | 17 | 2 | 3 | 13 | 2 | 7 | 17 | 0 | 2 | 10 | 0 | 0 | 8 | 0 | 5 | 13 | 0 | 2 | 10 | 3 | 7 | 18 | 3 | 7 | 18 | 3 | 7 | 18 | 3 | 7 | 18 | | | | | | | | | |
| iv°m | 2 | 5 | 16 | 2 | 7 | 18 | 2 | 3 | 14 | 2 | 7 | 18 | 0 | 2 | 11 | 0 | 0 | 9 | 0 | 5 | 14 | 0 | 2 | 11 | 3 | 7 | 19 | 3 | 7 | 19 | 3 | 7 | 19 | 3 | 7 | 19 | | | | | | | | | |
| IV°AM | 2 | 8 | 18 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 0 | 3 | 11 | 0 | 5 | 13 | 0 | 0 | 8 | 0 | 5 | 13 | 3 | 7 | 18 | 3 | 7 | 18 | 3 | 7 | 18 | 3 | 4 | 15 | | | | | | | | | |
| IV°A | 2 | 8 | 19 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 0 | 3 | 12 | 0 | 5 | 14 | 0 | 0 | 9 | 0 | 5 | 14 | 3 | 7 | 19 | 3 | 7 | 19 | 3 | 7 | 19 | 3 | 4 | 16 | | | | | | | | | |
| iv°HD | 2 | 3 | 14 | 2 | 5 | 16 | 2 | 5 | 16 | 2 | 8 | 19 | 0 | 4 | 13 | 0 | 2 | 11 | 0 | 5 | 14 | 0 | 0 | 9 | 3 | 6 | 18 | 3 | 8 | 20 | 3 | 6 | 18 | 3 | 8 | 20 | | | | | | | | | |
| iv°7 | 2 | 3 | 13 | 2 | 5 | 15 | 2 | 5 | 15 | 2 | 8 | 18 | 0 | 4 | 12 | 0 | 2 | 10 | 0 | 5 | 13 | 0 | 0 | 8 | 3 | 6 | 17 | 3 | 8 | 19 | 3 | 6 | 17 | 3 | 8 | 19 | | | | | | | | | |

Previous Chord

| Current | III | | | iii | | | III+ | | | iii° | | | IV | | | iv | | | IV+ | | | iv° | | | V | | | v | | | V+ | | | v° | | | | |
|---------------------|-----|---|----|-----|---|----|------|---|----|------|---|----|----|---|----|----|---|----|-----|---|----|-----|---|----|---|---|----|---|---|----|----|---|----|----|----|----|---|---|
| | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k |
| V | 3 | 6 | 9 | 3 | 4 | 7 | 3 | 7 | 10 | 3 | 6 | 9 | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 5 | 7 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 3 | 3 | 0 | 5 | 5 | | |
| v | 3 | 8 | 15 | 3 | 6 | 13 | 3 | 8 | 15 | 3 | 4 | 11 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 0 | 2 | 6 | 0 | 0 | 4 | 0 | 5 | 9 | 0 | 3 | 7 | | |
| V+ | 3 | 7 | 14 | 3 | 5 | 12 | 3 | 8 | 15 | 3 | 7 | 14 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 6 | 12 | 0 | 3 | 7 | 0 | 5 | 9 | 0 | 0 | 4 | 0 | 5 | 9 | | |
| v° | 3 | 9 | 20 | 3 | 7 | 18 | 3 | 9 | 20 | 3 | 5 | 16 | 2 | 8 | 18 | 2 | 8 | 18 | 2 | 5 | 15 | 2 | 8 | 18 | 0 | 5 | 13 | 0 | 3 | 11 | 0 | 5 | 13 | 0 | 0 | 8 | | |
| V ⁷ M | 3 | 6 | 14 | 3 | 4 | 12 | 3 | 7 | 15 | 3 | 6 | 14 | 2 | 6 | 13 | 2 | 6 | 13 | 2 | 6 | 13 | 2 | 5 | 12 | 0 | 0 | 5 | 0 | 2 | 7 | 0 | 3 | 8 | 0 | 5 | 10 | | |
| V ⁷ D | 3 | 6 | 13 | 3 | 4 | 11 | 3 | 7 | 14 | 3 | 6 | 13 | 2 | 6 | 12 | 2 | 6 | 12 | 2 | 6 | 12 | 2 | 5 | 11 | 0 | 0 | 4 | 0 | 2 | 6 | 0 | 3 | 7 | 0 | 5 | 9 | | |
| v ⁷ MM | 3 | 8 | 20 | 3 | 6 | 18 | 3 | 8 | 20 | 3 | 4 | 16 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 0 | 2 | 11 | 0 | 0 | 9 | 0 | 5 | 14 | 0 | 3 | 12 | | |
| v ⁷ m | 3 | 8 | 19 | 3 | 6 | 17 | 3 | 8 | 19 | 3 | 4 | 15 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 0 | 2 | 10 | 0 | 0 | 8 | 0 | 5 | 13 | 0 | 3 | 11 | | |
| V ⁷ AM | 3 | 7 | 19 | 3 | 5 | 17 | 3 | 8 | 20 | 3 | 7 | 19 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 6 | 17 | 0 | 3 | 12 | 0 | 5 | 14 | 0 | 0 | 9 | 0 | 5 | 14 | | |
| V ⁷ A | 3 | 7 | 18 | 3 | 5 | 16 | 3 | 8 | 19 | 3 | 7 | 18 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 6 | 16 | 0 | 3 | 11 | 0 | 5 | 13 | 0 | 0 | 8 | 0 | 5 | 13 | | |
| v ⁷ HD | 3 | 9 | 24 | 3 | 7 | 22 | 3 | 9 | 24 | 3 | 5 | 20 | 2 | 8 | 22 | 2 | 8 | 22 | 2 | 5 | 19 | 2 | 8 | 22 | 0 | 5 | 17 | 0 | 3 | 15 | 0 | 5 | 17 | 0 | 0 | 12 | | |
| v ^{o7} | 3 | 9 | 24 | 3 | 7 | 22 | 3 | 9 | 24 | 3 | 5 | 20 | 2 | 8 | 22 | 2 | 8 | 22 | 2 | 5 | 19 | 2 | 8 | 22 | 0 | 5 | 17 | 0 | 3 | 15 | 0 | 5 | 17 | 0 | 0 | 12 | | |
| VI | 1 | 5 | 10 | 1 | 5 | 10 | 1 | 5 | 10 | 1 | 5 | 10 | 3 | 6 | 13 | 3 | 8 | 15 | 3 | 4 | 11 | 3 | 8 | 15 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 5 | 11 | | |
| vi | 1 | 4 | 5 | 1 | 4 | 5 | 1 | 3 | 4 | 1 | 4 | 5 | 3 | 4 | 7 | 3 | 6 | 9 | 3 | 6 | 9 | 3 | 7 | 10 | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 6 | 8 | | |
| VI+ | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 7 | 12 | 3 | 4 | 11 | 3 | 6 | 13 | 3 | 4 | 11 | 3 | 6 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 5 | 11 | | |
| vi° | 1 | 6 | 11 | 1 | 7 | 12 | 1 | 6 | 11 | 1 | 7 | 12 | 3 | 5 | 12 | 3 | 7 | 14 | 3 | 7 | 14 | 3 | 8 | 15 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 4 | 10 | 2 | 7 | 13 | | |
| VI ⁷ M | 1 | 5 | 15 | 1 | 5 | 15 | 1 | 5 | 15 | 1 | 5 | 15 | 3 | 6 | 18 | 3 | 8 | 20 | 3 | 4 | 16 | 3 | 8 | 20 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 5 | 16 | | |
| VI ⁷ D | 1 | 5 | 14 | 1 | 5 | 14 | 1 | 5 | 14 | 1 | 5 | 14 | 3 | 6 | 17 | 3 | 8 | 19 | 3 | 4 | 15 | 3 | 8 | 19 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 5 | 15 | | |
| vi ⁷ MM | 1 | 4 | 10 | 1 | 4 | 10 | 1 | 3 | 9 | 1 | 4 | 10 | 3 | 4 | 12 | 3 | 6 | 14 | 3 | 6 | 14 | 3 | 7 | 15 | 2 | 6 | 13 | 2 | 6 | 13 | 2 | 6 | 13 | 2 | 6 | 13 | | |
| vi ⁷ m | 1 | 4 | 9 | 1 | 4 | 9 | 1 | 3 | 8 | 1 | 4 | 9 | 3 | 4 | 11 | 3 | 6 | 13 | 3 | 6 | 13 | 3 | 7 | 14 | 2 | 6 | 12 | 2 | 6 | 12 | 2 | 6 | 12 | 2 | 6 | 12 | | |
| VI ⁷ AM | 1 | 7 | 17 | 1 | 7 | 17 | 1 | 7 | 17 | 1 | 7 | 17 | 3 | 4 | 16 | 3 | 6 | 18 | 3 | 4 | 16 | 3 | 6 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 5 | 16 | | |
| VI ⁷ A | 1 | 7 | 16 | 1 | 7 | 16 | 1 | 7 | 16 | 1 | 7 | 16 | 3 | 4 | 15 | 3 | 6 | 17 | 3 | 4 | 15 | 3 | 6 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 5 | 15 | | |
| vi ⁷ HD | 1 | 6 | 15 | 1 | 7 | 16 | 1 | 6 | 15 | 1 | 7 | 16 | 3 | 5 | 16 | 3 | 7 | 18 | 3 | 7 | 18 | 3 | 8 | 19 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 4 | 14 | 2 | 7 | 17 | | |
| vi ^{o7} | 1 | 6 | 16 | 1 | 7 | 17 | 1 | 6 | 16 | 1 | 7 | 17 | 3 | 5 | 17 | 3 | 7 | 19 | 3 | 7 | 19 | 3 | 8 | 20 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 4 | 15 | 2 | 7 | 18 | | |
| VII | 1 | 6 | 15 | 1 | 6 | 15 | 1 | 8 | 17 | 1 | 9 | 18 | 1 | 8 | 17 | 1 | 8 | 17 | 1 | 8 | 17 | 1 | 6 | 15 | 3 | 7 | 18 | 3 | 9 | 20 | 3 | 5 | 16 | 3 | 9 | 20 | | |
| vii | 1 | 5 | 10 | 1 | 5 | 10 | 1 | 7 | 12 | 1 | 8 | 13 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 5 | 10 | 3 | 5 | 12 | 3 | 7 | 14 | 3 | 7 | 14 | 3 | 9 | 16 | | |
| VII+ | 1 | 6 | 11 | 1 | 4 | 9 | 1 | 8 | 13 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 5 | 10 | 3 | 4 | 11 | 3 | 6 | 13 | 3 | 2 | 9 | 3 | 6 | 13 | | |
| vii° | 1 | 4 | 5 | 1 | 4 | 5 | 1 | 6 | 7 | 1 | 7 | 8 | 1 | 4 | 5 | 1 | 4 | 5 | 1 | 4 | 5 | 1 | 2 | 3 | 4 | 7 | 3 | 6 | 9 | 3 | 6 | 9 | 3 | 8 | 11 | | | |
| VII ⁷ M | 1 | 6 | 20 | 1 | 6 | 20 | 1 | 8 | 22 | 1 | 9 | 23 | 1 | 8 | 22 | 1 | 8 | 22 | 1 | 8 | 22 | 1 | 6 | 20 | 3 | 7 | 23 | 3 | 9 | 25 | 3 | 5 | 21 | 3 | 9 | 25 | | |
| VII ⁷ D | 1 | 6 | 19 | 1 | 6 | 19 | 1 | 8 | 21 | 1 | 9 | 22 | 1 | 8 | 21 | 1 | 8 | 21 | 1 | 8 | 21 | 1 | 6 | 19 | 3 | 7 | 22 | 3 | 9 | 24 | 3 | 5 | 20 | 3 | 9 | 24 | | |
| vii ⁷ MM | 1 | 5 | 15 | 1 | 5 | 15 | 1 | 7 | 17 | 1 | 8 | 18 | 1 | 7 | 17 | 1 | 7 | 17 | 1 | 7 | 17 | 1 | 5 | 15 | 3 | 5 | 17 | 3 | 7 | 19 | 3 | 7 | 19 | 3 | 9 | 21 | | |
| vii ⁷ m | 1 | 5 | 14 | 1 | 5 | 14 | 1 | 7 | 16 | 1 | 8 | 17 | 1 | 7 | 16 | 1 | 7 | 16 | 1 | 7 | 16 | 1 | 5 | 14 | 3 | 5 | 16 | 3 | 7 | 18 | 3 | 7 | 18 | 3 | 9 | 20 | | |
| VII ⁷ AM | 1 | 6 | 16 | 1 | 4 | 14 | 1 | 8 | 18 | 1 | 7 | 17 | 1 | 7 | 17 | 1 | 7 | 17 | 1 | 7 | 17 | 1 | 5 | 15 | 3 | 4 | 16 | 3 | 6 | 18 | 3 | 2 | 14 | 3 | 6 | 18 | | |
| VII ⁷ A | 1 | 6 | 15 | 1 | 4 | 13 | 1 | 8 | 17 | 1 | 7 | 16 | 1 | 7 | 16 | 1 | 7 | 16 | 1 | 7 | 16 | 1 | 5 | 14 | 3 | 4 | 15 | 3 | 6 | 17 | 3 | 2 | 13 | 3 | 6 | 17 | | |
| vii ⁷ HD | 1 | 4 | 9 | 1 | 4 | 9 | 1 | 6 | 11 | 1 | 7 | 12 | 1 | 4 | 9 | 1 | 4 | 9 | 1 | 4 | 9 | 1 | 2 | 7 | 3 | 4 | 11 | 3 | 6 | 13 | 3 | 6 | 13 | 3 | 8 | 15 | | |
| vii ^{o7} | 1 | 4 | 10 | 1 | 4 | 10 | 1 | 6 | 12 | 1 | 7 | 13 | 1 | 4 | 10 | 1 | 4 | 10 | 1 | 4 | 10 | 1 | 2 | 8 | 3 | 4 | 12 | 3 | 6 | 14 | 3 | 6 | 14 | 3 | 8 | 16 | | |

| Current | Previous Chord | | | | | | | | | | | | | | | | | | | | | | | | | |
|---------|----------------|---|----|----|---|----|-----|---|----|-----|---|----|-----|---|----|-----|---|----|------|---|----|------|---|----|---|---|
| | VI | | | vi | | | VI+ | | | vi° | | | VII | | | vii | | | VII+ | | | vii° | | | | |
| | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k |
| I | 3 | 6 | 9 | 3 | 4 | 7 | 3 | 7 | 10 | 3 | 6 | 9 | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 4 | 6 | 2 | 6 | 8 | | |
| i | 3 | 8 | 15 | 3 | 6 | 13 | 3 | 8 | 15 | 3 | 4 | 11 | 2 | 5 | 11 | 2 | 7 | 13 | 2 | 3 | 9 | 2 | 7 | 13 | | |
| I+ | 3 | 7 | 14 | 3 | 5 | 12 | 3 | 8 | 15 | 3 | 7 | 14 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | | |
| i° | 3 | 9 | 20 | 3 | 7 | 18 | 3 | 9 | 20 | 3 | 5 | 16 | 2 | 3 | 13 | 2 | 5 | 15 | 2 | 6 | 16 | 2 | 8 | 18 | | |
| I'M | 3 | 6 | 13 | 3 | 4 | 11 | 3 | 7 | 14 | 3 | 6 | 13 | 2 | 6 | 12 | 2 | 6 | 12 | 2 | 4 | 10 | 2 | 6 | 12 | | |
| I'D | 3 | 6 | 14 | 3 | 4 | 12 | 3 | 7 | 15 | 3 | 6 | 14 | 2 | 6 | 13 | 2 | 6 | 13 | 2 | 4 | 11 | 2 | 6 | 13 | | |
| i'MM | 3 | 8 | 19 | 3 | 6 | 17 | 3 | 8 | 19 | 3 | 4 | 15 | 2 | 5 | 15 | 2 | 7 | 17 | 2 | 3 | 13 | 2 | 7 | 17 | | |
| i'm | 3 | 8 | 20 | 3 | 6 | 18 | 3 | 8 | 20 | 3 | 4 | 16 | 2 | 5 | 16 | 2 | 7 | 18 | 2 | 3 | 14 | 2 | 7 | 18 | | |
| I'AM | 3 | 7 | 18 | 3 | 5 | 16 | 3 | 8 | 19 | 3 | 7 | 18 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | | |
| I'A | 3 | 7 | 19 | 3 | 5 | 17 | 3 | 8 | 20 | 3 | 7 | 19 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | | |
| i'HD | 3 | 9 | 25 | 3 | 7 | 23 | 3 | 9 | 25 | 3 | 5 | 21 | 2 | 3 | 18 | 2 | 5 | 20 | 2 | 6 | 21 | 2 | 8 | 23 | | |
| i°7 | 3 | 9 | 24 | 3 | 7 | 22 | 3 | 9 | 24 | 3 | 5 | 20 | 2 | 3 | 17 | 2 | 5 | 19 | 2 | 6 | 20 | 2 | 8 | 22 | | |
| II | 1 | 5 | 10 | 1 | 5 | 10 | 1 | 5 | 10 | 1 | 5 | 10 | 3 | 6 | 13 | 3 | 4 | 11 | 3 | 8 | 15 | 3 | 6 | 13 | | |
| ii | 1 | 4 | 5 | 1 | 4 | 5 | 1 | 3 | 4 | 1 | 4 | 5 | 3 | 8 | 11 | 3 | 6 | 9 | 3 | 7 | 10 | 3 | 4 | 7 | | |
| II+ | 1 | 8 | 17 | 1 | 8 | 17 | 1 | 8 | 17 | 1 | 8 | 17 | 3 | 7 | 18 | 3 | 5 | 16 | 3 | 9 | 20 | 3 | 7 | 18 | | |
| ii° | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 6 | 11 | 1 | 7 | 12 | 3 | 9 | 16 | 3 | 7 | 14 | 3 | 8 | 15 | 3 | 5 | 12 | | |
| II'M | 1 | 5 | 15 | 1 | 5 | 15 | 1 | 5 | 15 | 1 | 5 | 15 | 3 | 6 | 18 | 3 | 4 | 16 | 3 | 8 | 20 | 3 | 6 | 18 | | |
| II'D | 1 | 5 | 14 | 1 | 5 | 14 | 1 | 5 | 14 | 1 | 5 | 14 | 3 | 6 | 17 | 3 | 4 | 15 | 3 | 8 | 19 | 3 | 6 | 17 | | |
| ii'MM | 1 | 4 | 10 | 1 | 4 | 10 | 1 | 3 | 9 | 1 | 4 | 10 | 3 | 8 | 16 | 3 | 6 | 14 | 3 | 7 | 15 | 3 | 4 | 12 | | |
| ii'm | 1 | 4 | 9 | 1 | 4 | 9 | 1 | 3 | 8 | 1 | 4 | 9 | 3 | 8 | 15 | 3 | 6 | 13 | 3 | 7 | 14 | 3 | 4 | 11 | | |
| II'AM | 1 | 8 | 22 | 1 | 8 | 22 | 1 | 8 | 22 | 1 | 8 | 22 | 3 | 7 | 23 | 3 | 5 | 21 | 3 | 9 | 25 | 3 | 7 | 23 | | |
| II'A | 1 | 8 | 21 | 1 | 8 | 21 | 1 | 8 | 21 | 1 | 8 | 21 | 3 | 7 | 22 | 3 | 5 | 20 | 3 | 9 | 24 | 3 | 7 | 22 | | |
| ii'HD | 1 | 7 | 16 | 1 | 7 | 16 | 1 | 6 | 15 | 1 | 7 | 16 | 3 | 9 | 20 | 3 | 7 | 18 | 3 | 8 | 19 | 3 | 5 | 16 | | |
| ii°7 | 1 | 7 | 16 | 1 | 7 | 16 | 1 | 6 | 15 | 1 | 7 | 16 | 3 | 9 | 20 | 3 | 7 | 18 | 3 | 8 | 19 | 3 | 5 | 16 | | |
| III | 1 | 5 | 10 | 1 | 5 | 10 | 1 | 7 | 12 | 1 | 8 | 13 | 1 | 5 | 10 | 1 | 5 | 10 | 1 | 5 | 10 | 1 | 5 | 10 | | |
| iii | 1 | 4 | 5 | 1 | 4 | 5 | 1 | 6 | 7 | 1 | 7 | 8 | 1 | 4 | 5 | 1 | 4 | 5 | 1 | 3 | 4 | 1 | 4 | 5 | | |
| III+ | 1 | 6 | 11 | 1 | 4 | 9 | 1 | 8 | 13 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 5 | 10 | 1 | 7 | 12 | | |
| iii° | 1 | 5 | 10 | 1 | 5 | 10 | 1 | 7 | 12 | 1 | 8 | 13 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 6 | 11 | 1 | 7 | 12 | | |
| III'M | 1 | 5 | 15 | 1 | 5 | 15 | 1 | 7 | 17 | 1 | 8 | 18 | 1 | 5 | 15 | 1 | 5 | 15 | 1 | 5 | 15 | 1 | 5 | 15 | | |
| III'D | 1 | 5 | 14 | 1 | 5 | 14 | 1 | 7 | 16 | 1 | 8 | 17 | 1 | 5 | 14 | 1 | 5 | 14 | 1 | 5 | 14 | 1 | 5 | 14 | | |
| iii'MM | 1 | 4 | 10 | 1 | 4 | 10 | 1 | 6 | 12 | 1 | 7 | 13 | 1 | 4 | 10 | 1 | 4 | 10 | 1 | 3 | 9 | 1 | 4 | 10 | | |
| iii'm | 1 | 4 | 9 | 1 | 4 | 9 | 1 | 6 | 11 | 1 | 7 | 12 | 1 | 4 | 9 | 1 | 4 | 9 | 1 | 3 | 8 | 1 | 4 | 9 | | |
| III'AM | 1 | 6 | 16 | 1 | 4 | 14 | 1 | 8 | 18 | 1 | 7 | 17 | 1 | 7 | 17 | 1 | 7 | 17 | 1 | 5 | 15 | 1 | 7 | 17 | | |
| III'A | 1 | 6 | 15 | 1 | 4 | 13 | 1 | 8 | 17 | 1 | 7 | 16 | 1 | 7 | 16 | 1 | 7 | 16 | 1 | 5 | 14 | 1 | 7 | 16 | | |
| iii'HD | 1 | 5 | 14 | 1 | 5 | 14 | 1 | 7 | 16 | 1 | 8 | 17 | 1 | 7 | 16 | 1 | 7 | 16 | 1 | 6 | 15 | 1 | 7 | 16 | | |
| iii°7 | 1 | 5 | 15 | 1 | 5 | 15 | 1 | 7 | 17 | 1 | 8 | 18 | 1 | 7 | 17 | 1 | 7 | 17 | 1 | 6 | 16 | 1 | 7 | 17 | | |
| IV | 3 | 6 | 9 | 3 | 4 | 7 | 3 | 4 | 7 | 3 | 4 | 7 | 1 | 7 | 8 | 1 | 7 | 8 | 1 | 6 | 7 | 1 | 4 | 5 | | |
| iv | 3 | 8 | 15 | 3 | 6 | 13 | 3 | 6 | 13 | 3 | 6 | 13 | 1 | 8 | 13 | 1 | 8 | 13 | 1 | 7 | 12 | 1 | 5 | 10 | | |
| IV+ | 3 | 4 | 11 | 3 | 6 | 13 | 3 | 2 | 9 | 3 | 6 | 13 | 1 | 8 | 13 | 1 | 8 | 13 | 1 | 7 | 12 | 1 | 5 | 10 | | |
| iv° | 3 | 7 | 14 | 3 | 7 | 14 | 3 | 5 | 12 | 3 | 7 | 14 | 1 | 6 | 11 | 1 | 6 | 11 | 1 | 5 | 10 | 1 | 3 | 8 | | |
| IV'M | 3 | 6 | 13 | 3 | 4 | 11 | 3 | 4 | 11 | 3 | 4 | 11 | 1 | 7 | 12 | 1 | 7 | 12 | 1 | 6 | 11 | 1 | 4 | 9 | | |
| IV'D | 3 | 6 | 14 | 3 | 4 | 12 | 3 | 4 | 12 | 3 | 4 | 12 | 1 | 7 | 13 | 1 | 7 | 13 | 1 | 6 | 12 | 1 | 4 | 10 | | |
| iv'MM | 3 | 8 | 19 | 3 | 6 | 17 | 3 | 6 | 17 | 3 | 6 | 17 | 1 | 8 | 17 | 1 | 8 | 17 | 1 | 7 | 16 | 1 | 5 | 14 | | |
| iv'm | 3 | 8 | 20 | 3 | 6 | 18 | 3 | 6 | 18 | 3 | 6 | 18 | 1 | 8 | 18 | 1 | 8 | 18 | 1 | 7 | 17 | 1 | 5 | 15 | | |
| IV'AM | 3 | 4 | 15 | 3 | 6 | 17 | 3 | 2 | 13 | 3 | 6 | 17 | 1 | 8 | 17 | 1 | 8 | 17 | 1 | 7 | 16 | 1 | 5 | 14 | | |
| IV'A | 3 | 4 | 16 | 3 | 6 | 18 | 3 | 2 | 14 | 3 | 6 | 18 | 1 | 8 | 18 | 1 | 8 | 18 | 1 | 7 | 17 | 1 | 5 | 15 | | |
| iv'HD | 3 | 7 | 19 | 3 | 7 | 19 | 3 | 5 | 17 | 3 | 7 | 19 | 1 | 6 | 16 | 1 | 6 | 16 | 1 | 5 | 15 | 1 | 3 | 13 | | |
| iv°7 | 3 | 7 | 18 | 3 | 7 | 18 | 3 | 5 | 16 | 3 | 7 | 18 | 1 | 6 | 15 | 1 | 6 | 15 | 1 | 5 | 14 | 1 | 3 | 12 | | |

Previous Chord

| Current | VI | | | vi | | | VI+ | | | vi° | | | VII | | | vii | | | VII+ | | | vii° | | | | |
|---------------------|----|---|----|----|---|----|-----|---|----|-----|---|----|-----|---|----|-----|---|----|------|---|----|------|---|----|----|---|
| | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k | T | j | k |
| V | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 6 | 8 | 3 | 6 | 9 | 3 | 4 | 7 | 3 | 4 | 7 | 3 | 4 | 7 | | |
| v | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 6 | 12 | 3 | 8 | 15 | 3 | 6 | 13 | 3 | 7 | 14 | 3 | 6 | 13 | | |
| V+ | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 4 | 10 | 3 | 4 | 11 | 3 | 6 | 13 | 3 | 2 | 9 | 3 | 6 | 13 | | |
| v° | 2 | 6 | 16 | 2 | 8 | 18 | 2 | 6 | 16 | 2 | 8 | 18 | 3 | 8 | 19 | 3 | 8 | 19 | 3 | 6 | 17 | 3 | 8 | 19 | | |
| V ⁷ M | 2 | 6 | 13 | 2 | 6 | 13 | 2 | 6 | 13 | 2 | 6 | 13 | 3 | 6 | 14 | 3 | 4 | 12 | 3 | 4 | 12 | 3 | 4 | 12 | | |
| V ⁷ D | 2 | 6 | 12 | 2 | 6 | 12 | 2 | 6 | 12 | 2 | 6 | 12 | 3 | 6 | 13 | 3 | 4 | 11 | 3 | 4 | 11 | 3 | 4 | 11 | | |
| v ⁷ MM | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 6 | 17 | 3 | 8 | 20 | 3 | 6 | 18 | 3 | 7 | 19 | 3 | 6 | 18 | | |
| v ⁷ m | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 6 | 16 | 3 | 8 | 19 | 3 | 6 | 17 | 3 | 7 | 18 | 3 | 6 | 17 | | |
| V ⁷ AM | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 4 | 15 | 3 | 4 | 16 | 3 | 6 | 18 | 3 | 2 | 14 | 3 | 6 | 18 | | |
| V ⁷ A | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 4 | 14 | 3 | 4 | 15 | 3 | 6 | 17 | 3 | 2 | 13 | 3 | 6 | 17 | | |
| v ⁷ HD | 2 | 6 | 20 | 2 | 8 | 22 | 2 | 6 | 20 | 2 | 8 | 22 | 3 | 8 | 23 | 3 | 8 | 23 | 3 | 6 | 21 | 3 | 8 | 23 | | |
| v ⁷ | 2 | 6 | 20 | 2 | 8 | 22 | 2 | 6 | 20 | 2 | 8 | 22 | 3 | 8 | 23 | 3 | 8 | 23 | 3 | 6 | 21 | 3 | 8 | 23 | | |
| VI | 0 | 0 | 4 | 0 | 0 | 3 | 7 | 0 | 2 | 6 | 0 | 5 | 9 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | |
| vi | 0 | 2 | 2 | 0 | 0 | 0 | 4 | 0 | 4 | 0 | 3 | 3 | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 6 | 8 | | |
| VI+ | 0 | 2 | 6 | 0 | 5 | 9 | 0 | 0 | 4 | 0 | 4 | 8 | 2 | 8 | 14 | 2 | 8 | 14 | 2 | 7 | 13 | 2 | 5 | 11 | | |
| vi° | 0 | 5 | 9 | 0 | 3 | 7 | 0 | 5 | 9 | 0 | 0 | 4 | 2 | 5 | 11 | 2 | 7 | 13 | 2 | 5 | 11 | 2 | 7 | 13 | | |
| VI ⁷ M | 0 | 0 | 9 | 0 | 3 | 12 | 0 | 2 | 11 | 0 | 5 | 14 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | | |
| VI ⁷ D | 0 | 0 | 8 | 0 | 3 | 11 | 0 | 2 | 10 | 0 | 5 | 13 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | | |
| vi ⁷ MM | 0 | 2 | 7 | 0 | 0 | 5 | 0 | 4 | 9 | 0 | 3 | 8 | 2 | 6 | 13 | 2 | 6 | 13 | 2 | 6 | 13 | 2 | 6 | 13 | | |
| vi ⁷ m | 0 | 2 | 6 | 0 | 0 | 4 | 0 | 4 | 8 | 0 | 3 | 7 | 2 | 6 | 12 | 2 | 6 | 12 | 2 | 6 | 12 | 2 | 6 | 12 | | |
| VI ⁷ AM | 0 | 2 | 11 | 0 | 5 | 14 | 0 | 0 | 9 | 0 | 4 | 13 | 2 | 8 | 19 | 2 | 8 | 19 | 2 | 7 | 18 | 2 | 5 | 16 | | |
| VI ⁷ A | 0 | 2 | 10 | 0 | 5 | 13 | 0 | 0 | 8 | 0 | 4 | 12 | 2 | 8 | 18 | 2 | 8 | 18 | 2 | 7 | 17 | 2 | 5 | 15 | | |
| vi ⁷ HD | 0 | 5 | 13 | 0 | 3 | 11 | 0 | 5 | 13 | 0 | 0 | 8 | 2 | 5 | 15 | 2 | 7 | 17 | 2 | 5 | 15 | 2 | 7 | 17 | | |
| vi ⁷ | 0 | 5 | 14 | 0 | 3 | 12 | 0 | 5 | 14 | 0 | 0 | 9 | 2 | 5 | 16 | 2 | 7 | 18 | 2 | 5 | 16 | 2 | 7 | 18 | | |
| VII | 2 | 8 | 18 | 2 | 8 | 18 | 2 | 8 | 18 | 2 | 6 | 16 | 0 | 0 | 8 | 0 | 2 | 10 | 0 | 3 | 11 | 0 | 5 | 13 | | |
| vii | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 8 | 14 | 0 | 2 | 6 | 0 | 0 | 4 | 0 | 5 | 9 | 0 | 3 | 7 | | |
| VII+ | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 7 | 13 | 2 | 5 | 11 | 0 | 2 | 6 | 0 | 4 | 8 | 0 | 0 | 4 | 0 | 4 | 8 | | |
| vii° | 2 | 6 | 8 | 2 | 6 | 8 | 2 | 4 | 6 | 2 | 7 | 9 | 0 | 5 | 5 | 0 | 3 | 3 | 0 | 4 | 4 | 0 | 0 | 0 | | |
| VII ⁷ M | 2 | 8 | 23 | 2 | 8 | 23 | 2 | 8 | 23 | 2 | 6 | 21 | 0 | 0 | 13 | 0 | 2 | 15 | 0 | 3 | 16 | 0 | 5 | 18 | | |
| VII ⁷ D | 2 | 8 | 22 | 2 | 8 | 22 | 2 | 8 | 22 | 2 | 6 | 20 | 0 | 0 | 12 | 0 | 2 | 14 | 0 | 3 | 15 | 0 | 5 | 17 | | |
| vii ⁷ MM | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 8 | 19 | 0 | 2 | 11 | 0 | 0 | 9 | 0 | 5 | 14 | 0 | 3 | 12 | | |
| vii ⁷ m | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 8 | 18 | 0 | 2 | 10 | 0 | 0 | 8 | 0 | 5 | 13 | 0 | 3 | 11 | | |
| VII ⁷ AM | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 7 | 18 | 2 | 5 | 16 | 0 | 2 | 11 | 0 | 4 | 13 | 0 | 0 | 9 | 0 | 4 | 13 | | |
| VII ⁷ A | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 7 | 17 | 2 | 5 | 15 | 0 | 2 | 10 | 0 | 4 | 12 | 0 | 0 | 8 | 0 | 4 | 12 | | |
| vii ⁷ HD | 2 | 6 | 12 | 2 | 6 | 12 | 2 | 4 | 10 | 2 | 7 | 13 | 0 | 5 | 9 | 0 | 3 | 7 | 0 | 4 | 8 | 0 | 0 | 4 | | |
| vii ⁷ | 2 | 6 | 13 | 2 | 6 | 13 | 2 | 4 | 11 | 2 | 7 | 14 | 0 | 5 | 10 | 0 | 3 | 8 | 0 | 4 | 9 | 0 | 0 | 5 | | |

Appendix H

Simulation for Interactive Learning System in Netlogo

```
globals [ stopvar
          r           ;; Radius
          range
          inrangex inrangey inrangez inrangex inrangey inrangez
          data-number
          data-list ]

;; To setup initial sphere (reset)
to setup
  ;; Setup shape
  ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
  ca
  set r 11
  set range ((r * sqrt(2)) / 2)
  set data-number 8 set data-list []

  set-default-shape turtles "default"

  crt 10000    ;; Number of turtles
  [
    tilt-up asin (1.0 - random-float 2.0)
    roll-right random-float 360
    jump r           ;; change in model setting (currently 16)
    set color scale-color sphere-color xcor (-1 * max-pxcor) max-pxcor
  ]

  set inrangex turtles with [inrangex? xcor ycor zcor]
  set inrangey turtles with [inrangey? xcor ycor zcor]
  set inrangez turtles with [inrangez? xcor ycor zcor]
  set inrangex turtles with [inrangex? xcor ycor zcor]
  set inrangey turtles with [inrangey? xcor ycor zcor]
  set inrangez turtles with [inrangez? xcor ycor zcor]

  ;; Setup Files
  ;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
```

```

file-close
if ( file-exists? "my-file-in.txt" ) [ file-delete "my-file-in.txt" ]
end

to play
if ( file-exists? "my-file-in.txt" )      ;; Start reading when receive input
[
file-open "my-file-in.txt"
wait 2                                     ;; Wait for terminal to start output
let start? 0
while [ start? != 99 ]                    ;; To coordinate with hardware
[ set start? file-read ]

while [ not file-at-end? and (not stopplay?) ]
[
make-dents
return-shape
tick
]
]
set stopvar 0
end

to-report stopplay?
report stopplay
end

to set-color
ask turtles [set color scale-color sphere-color xcor (-1 * max-pxcor) max-pxcor]
end

to return-ori [sets]
ask sets [set xcor 0]
ask sets [set ycor 0]
ask sets [set zcor 0]
ask sets [jump r]
end

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
;;
;; Button presses - From file
;;
;;
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

to make-dents
let i file-read
ifelse ( i = 1.8)
[ wait 0.2
return-shape
wait 0.2
return-shape
wait 0.2
return-shape
wait 0.2
return-shape
wait 0.2
return-shape
wait 0.2
return-shape
]
end

```

```

    wait 0.2
    return-shape
    wait 0.2
    return-shape
    wait 0.2
    return-shape
    wait 0.2
    return-shape
    wait 0.2
    return-shape
    ]
  [ ifelse (file-at-end?)          ;; In case file ends before set is complete
    [ user-message ("Opps, something wrong with data.") ]
    [ dent ( i ) ]
  ]
]
end

to dent [ firstitem ]
  ifelse (firstitem = 2) [selectx firstitem] [return-shape]
  ifelse (file-read = 2) [selectnx 4] [return-shape]
  ifelse (file-read = 2) [selecty 4] [return-shape]
  ifelse (file-read = 2) [selectny 4] [return-shape]
  ifelse (file-read = 2) [selectz 4] [return-shape]
  ifelse (file-read = 2) [selectnz 4] [return-shape]
  orbit-up (file-read * 10)
  orbit-right (file-read * 10)
end

to return-shape
  if any? inrangex with [ distancexyz 0 0 0 != r ]
  [ returnx ]
  if any? inrangey with [ distancexyz 0 0 0 != r ]
  [ returny ]
  if any? inrangez with [ distancexyz 0 0 0 != r ]
  [ returnz ]
  if any? inrangex with [ distancexyz 0 0 0 != r ]
  [ returnnx ]
  if any? inrangeny with [ distancexyz 0 0 0 != r ]
  [ returnny ]
  if any? inrangenz with [ distancexyz 0 0 0 != r ]
  [ returnnz ]
end

to returnx
  ask inrangex [set xcor ( xcor + 3 * sin ( precision ( r - distancexyz 0 0 0 ) 1))]
end

to returnnx
  ask inrangex [set xcor ( xcor - 3 * sin ( precision ( r - distancexyz 0 0 0 ) 1))]
end

to returny
  ask inrangey [set ycor ( ycor + 3 * sin ( precision ( r - distancexyz 0 0 0 ) 1))]
end

to returnny
  ask inrangeny [set ycor ( ycor - 3 * sin ( precision ( r - distancexyz 0 0 0 ) 1))]
end

to returnz

```

```

ask inrangez [set zcor ( zcor + 3 * sin ( precision ( r - distancexyz 0 0 0 ) 1))]
end

to returnnz
ask inrangenz [set zcor ( zcor - 3 * sin ( precision ( r - distancexyz 0 0 0 ) 1))]
end

;-----
; Single Axis Only
;-----
;; opt can be 4 2 0

to selectx [opt]
return-ori (inrangex)
ask inrangex [jump opt * ( exp ( 3 * ( xcor - r / 2)) / exp ( r * 1.5 ))]
set-color
end

to-report inrangex? [x y z]
report sqrt ( ( x - 11) ^ 2 + ( y - 0) ^ 2 + ( z - 0) ^ 2 ) < range
end

to selectnx [opt]
return-ori (inrangex)
ask inrangex [jump opt * ( exp ( 3 * ( (-1 * xcor) - r / 2)) / exp ( r * 1.5))]
set-color
end

to-report inrangex? [x y z]
report sqrt (( y - 0) ^ 2 + ( x + 11) ^ 2 + ( z - 0) ^ 2 ) < range
end

to selecty [opt]
return-ori (inrangey)
ask inrangey [jump opt * ( exp ( 3 * ( ycor - r / 2)) / exp ( r * 1.5 ))]
set-color
end

to-report inrangey? [x y z]
report sqrt ( ( x - 0) ^ 2 + ( y - 11) ^ 2 + ( z - 0) ^ 2 ) < range
end

to selectny [opt]
return-ori (inrangeny)
ask inrangeny [jump opt * ( exp ( 3 * ( (-1 * ycor) - r / 2)) / exp ( r * 1.5))]
set-color
end

to-report inrangeny? [x y z]
report sqrt (( x - 0) ^ 2 + ( y + 11) ^ 2 + ( z - 0) ^ 2 ) < range
end

to selectz [opt]
return-ori (inrangez)
ask inrangez [jump opt * ( exp ( 3 * ( zcor - r / 2)) / exp ( r * 1.5 ))]
set-color
end

to-report inrangez? [x y z]

```

```

    report sqrt ( ( z - 11 ) ^ 2 + ( y - 0 ) ^ 2 + ( x - 0 ) ^ 2 ) < range
end

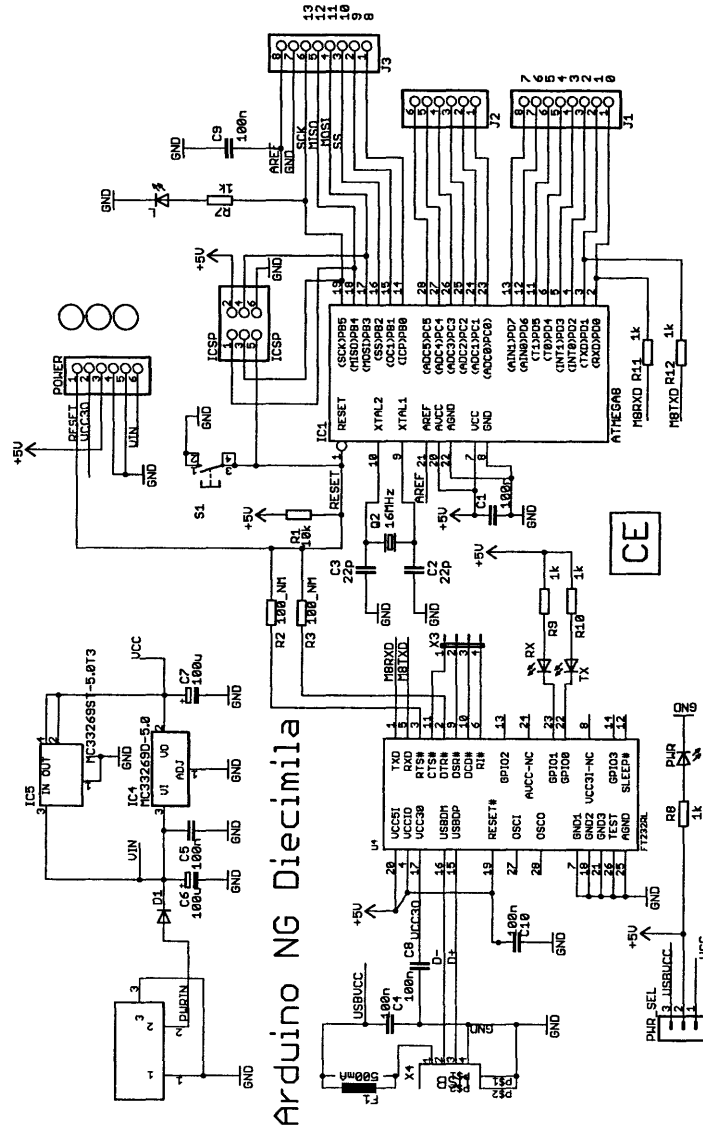
to selectnz [opt]
  return-ori (inrangenz)
  ask inrangenz [jump opt * ( exp ( 3 * ( ( -1 * zcor ) - r / 2 ) ) / exp ( r * 1.5 ) )]
  set-color
end

to-report inrangenz? [x y z]
  report sqrt ( ( y - 0 ) ^ 2 + ( z + 11 ) ^ 2 + ( x - 0 ) ^ 2 ) < range
end

```

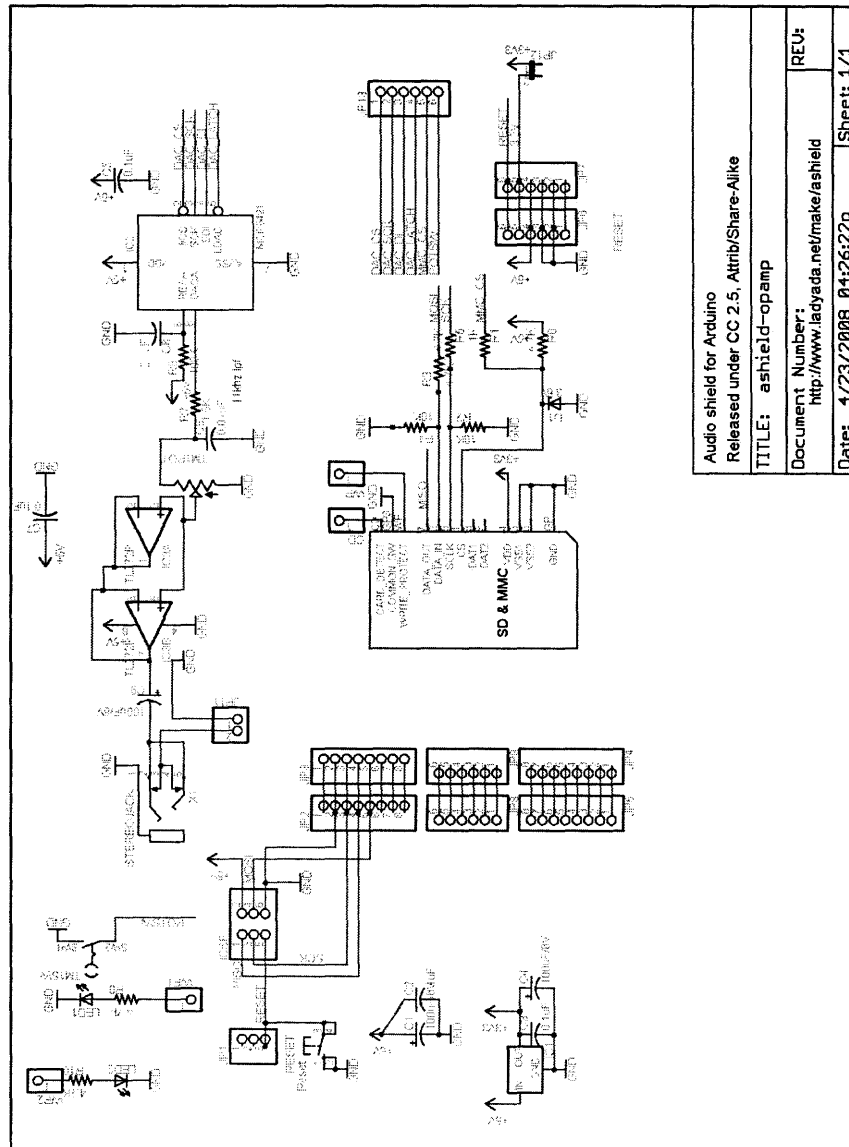

Appendix I

Arduino Diecimilla Schematics



Appendix J

Wave Shield Schematics (Ladyada)



Appendix K

Interactive Learning System Microcontroller Code (Arduino)

The code written for basic audio functions was obtained from Ladyada [7].

```
#include <AF_Wave.h>
#include <avr/pgmspace.h>
#include "util.h"
#include "wave.h"

// Initialize Audio Shield
AF_Wave card;
File f;
Wavefile wave;

char filename[13] = "CM1.wav"; // 3 (filename) + 4 (.wav)

// Wav file reference table in Flash Memory 16kB
const char Cmaj[] PROGMEM = "CM1CM2CI1BI1CI2CA3GI3BA2CD2CD2BM3BM3";
const char Cmin[] PROGMEM = "CM1CI1CM2CI2GI1BD2GI3CD2AA3BM3BM2BM2";
const char Caug[] PROGMEM = "CM1CA1CA1CM3AI3GM3CI3CD2GD1GD1GD2BM2";
const char Cdim[] PROGMEM = "CM1CM1CM1CD1CM3CI3CA3GA2GD1AM2GD2GD2";

const char Gmaj[] PROGMEM = "GM1GM3GI1AI1GI3GA2CI2CA2AM2CD3CD2BM2";
const char Gmin[] PROGMEM = "GM1GI1GM3GI3CI1CA1GA2CA2AM2BM1CD2BM3";
const char Gaug[] PROGMEM = "GM1GA1CM1GM2CM3CA1GI2CA2AM2CD2BM2BM2";
const char Gdim[] PROGMEM = "GM1GM1GM1GD1GM2GI2GA2CA2AD3CD3CD2BM3";

const char Amaj[] PROGMEM = "AI1AM1AA1AM3AA3GM3AD3CI1GI2GD2BM3CD3";
const char Amin[] PROGMEM = "AI1AI3AM1GM1AA1AM2AA2CA2GD1GI2GD2CD2";
const char Aug[] PROGMEM = "AI1AI1AM1AI3AM3GM3AD3GD1GI2CD1BM3CD3";
const char Adim[] PROGMEM = "AI1AD1AI3AA1AM1AA3AM2CD1CA2BM3GD3GD3";

const char Bmaj[] PROGMEM = "BD1BD1BI1BM1BI3BM3AA1AD3AM2GI2GD3GD3";
const char Bmin[] PROGMEM = "BD1BI1BI1BA1BM1BA3BM3BM2CI3CD2GD3GD3";
const char Baug[] PROGMEM = "BD1BA1CM1BA3CM2GM2BI2BM2CA3CD3GD2GD2";
const char Bdim[] PROGMEM = "BD1BD2BD3BA1BI3BI2BA2AA2BM2GD1CD3CD2";
```

```

// Initialize vars for pushbuttons
int start = 0;    // To start playchord

int tension1 = 0;
int tension2 = 0;
int tension3 = 0;
int tension4 = 0;
int tension5 = 0;
int tension6 = 0;
int tensionall = 0;
int tensiontotal = 0;

int sensor1 = 0;
int sensor2 = 0;
int sensor3 = 0;
int sensor4 = 0;
int sensor5 = 0;
int sensor6 = 0;

// Initialize vars for accelerometer
int y = analogRead(0);
int x = analogRead(1);
int xOld = x;
int yOld = y;
int ver = 0;    // Difference in axis readings
int hor = 0;

void setup() {

  Serial.begin(9600);    // set up Serial library at 9600 bps
  Serial.print(99);    // Coordinate starting point with simulation
  Serial.print(" ");
  // Wave Shield (Digital pins)
  pinMode(2, OUTPUT);
  pinMode(3, OUTPUT);
  pinMode(4, OUTPUT);
  pinMode(5, OUTPUT);
  pinMode(6, OUTPUT);

  // Push Buttons (Analog pins)
  pinMode(17, INPUT);
  pinMode(18, INPUT);    //Analog pin 4
  pinMode(19, INPUT);    //Analog pin 5

  // Digital
  pinMode(7, INPUT);
  pinMode(8, INPUT);
  pinMode(9, INPUT);

  // Analog Accelerometer
  pinMode(14, INPUT);    //Analog pin 2 y-axis
  pinMode(15, INPUT);    //Analog pin 3 x-axis

  // Troubleshoot wave shield
  card.init_card();
  card.open_partition();
  card.open_filesys();
  card.open_rootdir();

```

```

}

void loop() {
  // Read value from sensors
  sensor1 = adc(analogRead(3));
  sensor2 = adc(analogRead(4));
  sensor3 = adc(analogRead(5));
  sensor4 = digitalRead(7);
  sensor5 = digitalRead(8);
  sensor6 = digitalRead(9);

  tension1 = tension_calc(sensor1, tension1);
  tension2 = tension_calc(sensor2, tension2);
  tension3 = tension_calc(sensor3, tension3);
  tension4 = tension_calc(sensor4, tension4);
  tension5 = tension_calc(sensor5, tension5);
  tension6 = tension_calc(sensor6, tension6);

  // Print to terminal emulator for simulation
  Serial.print(tension1);
  Serial.print(" ");
  Serial.print(tension2);
  Serial.print(" ");
  Serial.print(tension3);
  Serial.print(" ");
  Serial.print(tension4);
  Serial.print(" ");
  Serial.print(tension5);
  Serial.print(" ");
  Serial.print(tension6);
  Serial.print(" ");
  Serial.print(ver);    // Modify to fit simulation values -360 to 360 ver
  Serial.print(" ");
  Serial.print(hor);
  Serial.println(" ");

  tensiontotal = tension1 + tension2 + tension3 + tension4 + tension5 + tension6;

  if (tensiontotal != 0 && start != 1 ) {    //Start playchord sequence
    Serial.print("1.8 ");
    playchord(0);
    start = 1;
  }

  if (tensiontotal == 0 && start == 1) {
    Serial.print("1.8 ");
    playchord(0);
    start = 0;
  }

  if (start == 1) {
  //   if (tensiontotal % 5 == 0) {          // Play at most once every second
    Serial.print("1.8 ");
    playchord(tensiontotal);    // Index starts at zero
  //   }
  }

  // Accelerometer

```

```

    if (abs(x-xOld) < 200 && abs(x-xOld) > 5) {
        hor = (x-xOld)/5;
    }
    else {
        hor = 0;
    }

    if (abs(y-yOld) < 200 && abs(y-yOld) > 5) {
        ver = (y-yOld)/5;
    }
    else {
        ver = 0;
    }

    xOld = x;
    yOld = y;
    y = analogRead(0);
    x = analogRead(1);

    delay(300);          // Keep short so can detect quick sensor presses
}

/////////////////////////////////////////////////////////////////
//
//          Pushbutton Functions
//
/////////////////////////////////////////////////////////////////

int adc (int val) {      // To convert analog signal into digital signal
    if (val > 100)
        return 1;
    else
        return 0;
}

int tension_calc (int sensor, int tension) {
    if (tension != 0)
        tension--;
    if (sensor == 1 && tension == 0)
        tension = 2;
    if (sensor == 0 && tension == 0)
        tension = 0;

    return tension;
}
/*
int change (int val) {
    val = (val - (val%5))/5;
    return val;
}
*/
/////////////////////////////////////////////////////////////////
//          Wave Shield Functions
/////////////////////////////////////////////////////////////////

void playchord(int tension) {

// C chords
    if (filename[0] == 'C') {

```

```

if (filename[1] == 'M') {
    char c1 = pgm_read_byte(&Cmaj[3*tension]);
    char c2 = pgm_read_byte(&Cmaj[3*tension+1]);
    char c3 = pgm_read_byte(&Cmaj[3*tension+2]);
    filename[0] = c1;
    filename[1] = c2;
    filename[2] = c3;
}
if (filename[1] == 'I') {
    char c1 = pgm_read_byte(&Cmin[3*tension]);
    char c2 = pgm_read_byte(&Cmin[3*tension+1]);
    char c3 = pgm_read_byte(&Cmin[3*tension+2]);
    filename[0] = c1;
    filename[1] = c2;
    filename[2] = c3;
}
if (filename[1] == 'A') {
    char c1 = pgm_read_byte(&Caug[3*tension]);
    char c2 = pgm_read_byte(&Caug[3*tension+1]);
    char c3 = pgm_read_byte(&Caug[3*tension+2]);
    filename[0] = c1;
    filename[1] = c2;
    filename[2] = c3;
}
if (filename[1] == 'D') {
    char c1 = pgm_read_byte(&Cdim[3*tension]);
    char c2 = pgm_read_byte(&Cdim[3*tension+1]);
    char c3 = pgm_read_byte(&Cdim[3*tension+2]);
    filename[0] = c1;
    filename[1] = c2;
    filename[2] = c3;
}
}
}

// G chords
if (filename[0] == 'G') {
    if (filename[1] == 'M') {
        char c1 = pgm_read_byte(&Gmaj[3*tension]);
        char c2 = pgm_read_byte(&Gmaj[3*tension+1]);
        char c3 = pgm_read_byte(&Gmaj[3*tension+2]);
        filename[0] = c1;
        filename[1] = c2;
        filename[2] = c3;
    }
    if (filename[1] == 'I') {
        char c1 = pgm_read_byte(&Gmin[3*tension]);
        char c2 = pgm_read_byte(&Gmin[3*tension+1]);
        char c3 = pgm_read_byte(&Gmin[3*tension+2]);
        filename[0] = c1;
        filename[1] = c2;
        filename[2] = c3;
    }
    if (filename[1] == 'A') {
        char c1 = pgm_read_byte(&Gaug[3*tension]);
        char c2 = pgm_read_byte(&Gaug[3*tension+1]);
        char c3 = pgm_read_byte(&Gaug[3*tension+2]);
        filename[0] = c1;
        filename[1] = c2;
        filename[2] = c3;
    }
}

```

```

    }
    if (filename[1] == 'D') {
        char c1 = pgm_read_byte(&Gdim[3*tension]);
        char c2 = pgm_read_byte(&Gdim[3*tension+1]);
        char c3 = pgm_read_byte(&Gdim[3*tension+2]);
        filename[0] = c1;
        filename[1] = c2;
        filename[2] = c3;
    }
}

// A chords
if (filename[0] == 'A') {
    if (filename[1] == 'M') {
        char c1 = pgm_read_byte(&Amaj[3*tension]);
        char c2 = pgm_read_byte(&Amaj[3*tension+1]);
        char c3 = pgm_read_byte(&Amaj[3*tension+2]);
        filename[0] = c1;
        filename[1] = c2;
        filename[2] = c3;
    }
    if (filename[1] == 'I') {
        char c1 = pgm_read_byte(&Amin[3*tension]);
        char c2 = pgm_read_byte(&Amin[3*tension+1]);
        char c3 = pgm_read_byte(&Amin[3*tension+2]);
        filename[0] = c1;
        filename[1] = c2;
        filename[2] = c3;
    }
    if (filename[1] == 'A') {
        char c1 = pgm_read_byte(&Aaug[3*tension]);
        char c2 = pgm_read_byte(&Aaug[3*tension+1]);
        char c3 = pgm_read_byte(&Aaug[3*tension+2]);
        filename[0] = c1;
        filename[1] = c2;
        filename[2] = c3;
    }
    if (filename[1] == 'D') {
        char c1 = pgm_read_byte(&Adim[3*tension]);
        char c2 = pgm_read_byte(&Adim[3*tension+1]);
        char c3 = pgm_read_byte(&Adim[3*tension+2]);
        filename[0] = c1;
        filename[1] = c2;
        filename[2] = c3;
    }
}

// B chords
if (filename[0] == 'B') {
    if (filename[1] == 'M') {
        char c1 = pgm_read_byte(&Bmaj[3*tension]);
        char c2 = pgm_read_byte(&Bmaj[3*tension+1]);
        char c3 = pgm_read_byte(&Bmaj[3*tension+2]);
        filename[0] = c1;
        filename[1] = c2;
        filename[2] = c3;
    }
    if (filename[1] == 'I') {
        char c1 = pgm_read_byte(&Bmin[3*tension]);

```

```

    char c2 = pgm_read_byte(&Bmin[3*tension+1]);
    char c3 = pgm_read_byte(&Bmin[3*tension+2]);
    filename[0] = c1;
    filename[1] = c2;
    filename[2] = c3;
}
if (filename[1] == 'A') {
    char c1 = pgm_read_byte(&Baug[3*tension]);
    char c2 = pgm_read_byte(&Baug[3*tension+1]);
    char c3 = pgm_read_byte(&Baug[3*tension+2]);
    filename[0] = c1;
    filename[1] = c2;
    filename[2] = c3;
}
if (filename[1] == 'D') {
    char c1 = pgm_read_byte(&Bdim[3*tension]);
    char c2 = pgm_read_byte(&Bdim[3*tension+1]);
    char c3 = pgm_read_byte(&Bdim[3*tension+2]);
    filename[0] = c1;
    filename[1] = c2;
    filename[2] = c3;
}
}

filename[3] = '.';
filename[4] = 'W';
filename[5] = 'A';
filename[6] = 'V';

playcomplete(filename);
}

void playcomplete(char *name) {
    playfile(name);
    while (wave.isplaying);
    card.close_file(f);
}

void playfile(char *name) {
    if (wave.isplaying) { // already playing something, so stop it!
        wave.stop(); // stop it
        card.close_file(f);
    }
    f = card.open_file(name);
    if (!f) {
        return; }
    if (!wave.create(f)) {
        return; }
    // ok time to play!
    wave.play();
}

```


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