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TIME-DOMAIN COMPENSATION FOR CLOSED-LOOP SYSTEMS BY A DELAY LINE METHOD

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TECHNICAL REPORT 288

JANUARY 17, 1955

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RESEARCH LABORATORY OF ELECTRONICS
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE, MASSACHUSETTS

The Research Laboratory of Electronics is an interdepartmental laboratory of the Department of Electrical Engineering and the Department of Physics.

The research reported in this document was made possible in part by support extended the Massachusetts Institute of Technology, Research Laboratory of Electronics, jointly by the Army Signal Corps, the Navy Department (Office of Naval Research), and the Air Force (Office of Scientific Research, Air Research and Development Command), under Signal Corps Contract DA36-039 sc-42607, Project 132B; Department of the Army Project 3-99-12-022.

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TIME-DOMAIN COMPENSATION FOR CLOSED-LOOP SYSTEMS BY A DELAY LINE METHOD

Yu-Chi Ho

This report is based on a thesis submitted to the Department of Electrical Engineering, M.I.T., 1955, in partial fulfillment of the requirements for the degree of Master of Science.

Abstract

This report presents a new approach to the synthesis and analysis of closed-loop systems in the time domain. In general, it investigates the effect of the discrete filter

$$D(s) = \sum_{k=0}^{n} a_k e^{-skT} = a_o + a_1 e^{-sT} + \ldots + a_n e^{-snT}$$

in compensating for feedback systems, and its resultant transient response. First, the generality, simplicity, and flexibility of the discrete filter are demonstrated. The properties of D(s) when cascaded with other elements are then studied in both the time and the frequency domain. The procedure for designing D(s) to compensate a given system is carried out entirely in the time domain. It is based on the "reshaping of the open-loop impulse response of the system." Saturation and other constraints can be taken into account during the determination of D(s). The procedure is iterative and is carried out graphically in terms of the time functions involved; it is simple, effective, and down-to-earth. The basis of this method is analyzed in terms of servo and network theory. Several methods are presented for calculating the closed-loop response of the compensated system. A specific example is designed according to the procedure and its transient response calculated. The results agree well with similar ones obtained by computer simulation. An actual experimental servo system was designed on this basis. The realization of a practical D(s) for servo systems is discussed; several methods of doing this are presented. The performance of the experimental system further verifies the validity of this approach.

I. INTRODUCTION

In any closed-loop system, the evaluation of the transient response and the determination of the required compensation network are often two very important problems facing the engineer. This report introduces some theories and new techniques for the solution of these problems directly in the time domain. Analog computer and experimental results will be presented to verify the analytical method.

Time-domain analysis and synthesis is a relatively new field but is becoming increasingly popular. The reasons are obvious. Since we live in a time domain, it is only natural that a time-domain solution to a problem will appear to be more straightforward and down-to-earth than a frequency-domain solution. The synthesis problem has been approached in several different ways. First, efforts have been made to correlate more closely the relationship between the time and frequency domains so that what one does in one domain can be interpreted readily in the other. Graphical and analytical transformation techniques (1,2,3) have been developed. Second, several attempts have been made to develop a generalized synthesis procedure (4, 5, 6); each has attained a certain degree of success. Third, the time series method by Tustin (7) partially covers the problem. Fundamentally, it uses the difference equation techniques and is closely related to sampled-data theory (8,9,10). Most of the papers referred to above deal with network synthesis on an open-loop basis. However, it is possible to modify them for the solution of closed-loop problems. In fact, Linvill has successfully applied these techniques to transient analysis of sampled-data servomechanisms.

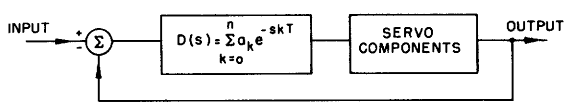


Fig. 1.1
The general closed-loop system.

It should be emphasized that up to now there has not been a real solution for the open-loop time-domain synthesis problem — to say nothing of the closed-loop case. This report does not claim to have completely solved the proposed problem. However, it does present a new, consistent, and workable procedure for the design and evaluation of the general closed-loop system in the time domain as shown in Fig. 1.1.

In the following sections, methods for synthesizing D(s) as a compensating network and the analysis of a given system will be presented. It will be shown that D(s), used as a compensating network, suffers no loss of generality but provides ease and flexibility for time-domain analysis.

II. DISCRETE FILTERS AND THE OPEN-LOOP SYSTEM RESPONSE

Before we discuss the properties of a discrete filter, we shall present certain ideas (4, 6) in time-domain synthesis that will be used later on.

1. THE OPEN-LOOP TIME-DOMAIN PROBLEM

In linear systems, the input and output time functions are related by the well-known convolution integral

$$g(t) = \int_0^{\infty} f(t-T)\, h(T)\, dT \tag{2.1}$$

where g(t) is the output time function, f(t) is the input time function, and h(t) is the impulse response of the system. When g(t) and f(t) are given, and h(t) is to be found, we have the problem of time-domain synthesis. While an exact solution of it is often very difficult if not impossible, attempts have been made to solve the problem on an approximation basis. There are two notable approaches to it:

a. The Cerrillo Method (11)

In Eq. 2.1 let us assume that h(t) is finite and normalized in time from t = 0 to t = 1. Now, expanding f(t-T) in a Taylor series about t = 0, we have

$$f(t-T) = \sum_{k=0}^{p} f^k(t) \frac{(-T)^k}{k!} + \text{remainder} \tag{2.2}$$

Substituting Eq. 2.2 into Eq. 2.1, we have

$$g(t) = \int_0^1 \sum_{k=0}^{p} f^k(t) \frac{(-T)^k}{k!}\, h(T)\, dT + \text{remainder} \tag{2.3}$$

The remainder term in Eq. 2.2 can be made as small as possible by merely increasing the order of p. For approximation purposes, we can neglect it.

$$g(t) \approx \sum_{k=0}^{p} f^k(t) \frac{(-1)^k}{k!} \int_0^1 T^k\, h(T)\, dT$$

$$= \sum_{k=0}^{p} f^k(t) \frac{(-1)^k}{k!} y_k \tag{2.4}$$

where

$$y_k = \text{weighting factor} = \int_0^1 T^k\, h(T)\, dT \tag{2.5}$$

Assume that h(t) is divided into q strips as in Fig. 2.1 and that the interval d is small, so that T^k does not vary much over the interval. To this extent, T^k in each interval can be replaced by a singular value $(nd)^k$

$$y_k = \int_0^1 T^k\, h(T)\, dT = \sum_{n=1}^{q} \int_{(n-1)d}^{nd} T^k\, h(T)\, dT$$

$$= \sum_{n=1}^{q} (nd)^k A_{nd} \tag{2.6}$$

where A_{nd} is the area of the n^{th} strip of h(T). Rewriting Eq. 2.6, we have

$$\left.\begin{aligned} y_o &= A_d + A_{2d} + \dots + A_{qd} \\ y_1 &= \frac{1}{q}(A_d + 2A_{2d} + \dots + qA_{qd}) \\ y_2 &= \frac{1}{q^2}\left(A_d + 2^2 A_{2d} + \dots + q^2 A_{qd}\right) \\ &\dots\dots\dots\dots\dots\dots \\ y_q &= \frac{1}{q^q}\left(A_d + 2^q A_{2d} + \dots + q^q A_{qd}\right) \end{aligned}\right\} \tag{2.7}$$

The determination of h(t) for a prescribed transfer characteristic means the solution of the set of simultaneous equations, given above, for A_{nd}s. For example, let $y_o = 1$, $y_k = 0$ for $k = 1, 2, \dots, p$; then $g(t) = Af(t)$; or, let $y_1 = 1$, $y_k = 0$ for $k = 0, 2, 3, \dots, p$; then $g(t) = Bf'(t)$. In general, it is possible to determine h(t) so that

$$g(t) = a_o f(t) + a_1 f'(t) + a_2 f''(t) + \dots + a_n f^n(t)$$

or

$$G(s) = \left(a_o + a_1 s + a_2 s^2 + \dots + a_n s^n\right) F(s) \tag{2.8}$$

by assigning appropriate values for y_ks. The synthesis procedure is completed.

b. The Ba Hli Method (4)

The first step in this method is the reduction of the specified input and output time functions to a single function which is the impulse response of the network to be synthesized. This is done, first, by sampling the output and input time functions at discrete intervals of time and reducing them to two sets of numbers. Then, by straightforward numerical division of the two sets of time sequences, one obtains the impulse response of the required network in the form of

$$H(s) = \sum_{k=0}^{n} a_k e^{-skT} \tag{2.9}$$

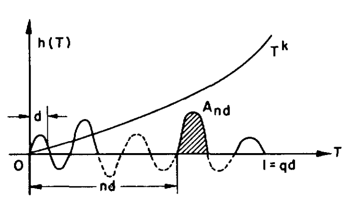


Fig. 2.1
Discrete impulse response.

where T is the sampling interval and a_k is the numerical time sequence obtained from the division. In the next step, Ba Hli expands Eq. 2.9 in terms of a power series expansion. $H(s)$ is obtained in powers of s. Then, a rational function $P(s)/Q(s)$ with undetermined coefficients is chosen and equated to $H(s)$. The coefficients are determined through the solution of a finite set of linear simultaneous equations. The synthesis procedure is completed.

Although the two methods described above are not completely general, they do point out a common approach to the time-domain problem that is logical and practical. In Eq. 2.6 if one assumes that the variation of T^k (thus, indirectly, the variation of $f(t)$) is very small over the interval d, then the integral $\int_{(n-1)d}^{nd} T^k h(T)\,dT$ can be replaced by $(nd)^k A_{nd}$ where A_{nd} is the area of the n^{th} strip of $h(T)$. Since only the area A_{nd} is of importance here, it is possible to replace A_{nd} by $A_n e^{-nds}$ where $A_n e^{-nds}$ is a delayed impulse at $T = nd$ with its magnitude equal to A_{nd}. The solution of Eq. 2.7 then gives $h(t)$ in terms of a set of delayed impulses, and we have

$$H(s) = \sum_{k=0}^{n} a_k e^{-skT} \approx \sum_{k=0}^{m} a_k s^k \tag{2.10}$$

This is in agreement with Ba Hli's result from the power series expansion. In fact, since Ba Hli showed the possibility of finding a rational function $P(s)/Q(s)$ which approximates $\sum_{k=0}^{m} a_k s^k$ for all ranges of t in the time domain, we can go one step further and arrive at the result

$$H(s) = \sum_{k=0}^{n} a_k e^{-skT} \approx \frac{\sum_{k=0}^{p} a_k s^k}{\sum_{k=0}^{q} c_k s^k} = \frac{P(s)}{Q(s)} \tag{2.11}$$

This agrees with physical reasoning. Here, one is dividing the impulse response of $P(s)/Q(s)$ into a number of equal time intervals and approximating the area of each interval by an impulse of equal magnitude delayed by the proper interval. As we decrease the interval (increase the division) and use more impulses to approximate the time function, Eq. 2.11 becomes more and more exact. In fact, this procedure is the basis for the numerical solution of integral equations.

2. DISCRETE FILTERS

Let us now define the discrete filter $D(s)$ as a filter of the form

$$D(s) = \sum_{k=0}^{n} a_k e^{-skT} \tag{2.12}$$

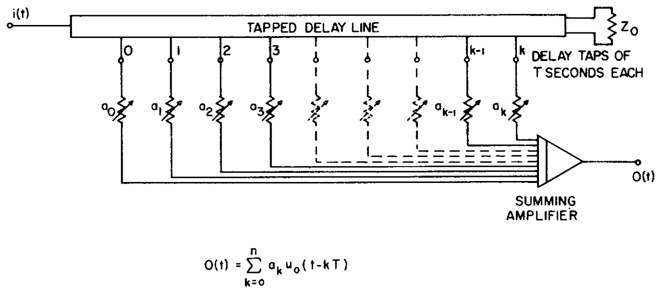


Fig. 2.2
Tapped delay line.

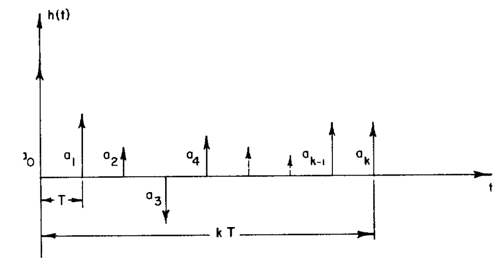


Fig. 2.3
Impulse response of a delay line.

In reality, this expression can be realized in the form of a tapped delay line as shown in Fig. 2.2. It is the theme of this report to propose the use of this type of discrete filter D(s) for compensating servo systems. The reasons are threefold:

a. Equation 2.11 shows that it is possible to approximate any lumped parameter filter with a discrete filter. Thus, no generality is lost by the introduction of the discrete filter; that is, D(s) can do anything that a conventional filter can do. In fact, we shall show that it can do many things in a much simpler fashion.

b. Furthermore, the properties of D(s) make it particularly adaptable for time-domain analysis. For example, the inverse transform d(t) of a general discrete filter D(s) is simply

$$d(t) = \sum_{k=0}^{n} a_k u_o(t - kT) \tag{2.13}$$

where $u_o(t)$ is the unit impulse. This is shown in Fig. 2.3. The impulse response d(t) can be determined from D(s) almost by inspection, while in general we cannot readily inverse-transform a rational transfer function P(s)/Q(s). This point will be further demonstrated.

c. The flexibility of a discrete filter is not matched by conventional filters. The impulse response d(t) is directly controlled by the coefficients a_ks in Eq. 2.12. In practice this means the setting of the potentiometers in Fig. 2.2. One does not have such a degree of direct control over the lumped parameter filter. Moreover, this advantage offers a flexibility for precise experimental adjustment without redesigning parameter values. One can transform from one set of transfer characteristics to another by merely changing the setting of a few potentiometers. The properties of D(s) will be discussed in the following section.

3. THE CASCADED DH SYSTEM

In this section, we shall study the properties of the system in Fig. 2.4 in both the frequency and the time domain. We shall call this a "DH system." H(s) is usually the

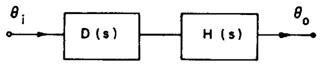


Fig. 2.4
The cascaded DH system.

fraction of two rational polynomials in s and represents the transfer function of servo components in the closed-loop system. D(s) is a discrete filter having a transfer function rational in e^{-sT} and is used for compensation purposes.

a. Time-Domain Behavior: Let h(t) be the impulse response of H(s); let d(t) be that of D(s). Then the composite impulse response $h_d(t)$ of the DH system is

$$h_d(t) = a_o h(t) + a_1 h(t-T) + a_2 h(t-2T) + \ldots + a_k h(t-kT) + \ldots \tag{2.14}$$

h(t) is usually given or can be evaluated. It is then a relatively simple matter to compute $h_d(t)$ by graphical or analytical methods. This is especially true, since in practical cases, as will be shown, the number of a_ks seldom exceeds four or five, and h(t) is usually either finite in duration or approaches a constant. A specific example is shown in Fig. 2.5.

b. Frequency-Domain Behavior: It is obvious that the frequency response of D(s) is periodic in nature. Thus, for purposes of investigation it is sufficient to evaluate the response for one period of the frequency band only. For example, e^{-2sT} has the same periodic response as e^{-sT} except that its period is half as long.

In Appendix A, a table of the real and imaginary parts of e^{-sT} as a function of normalized frequency is included. It can be used for computing the frequency response of any D(s). The procedure is as follows. Let

$$\begin{aligned} D(s) &= a_o + a_1 e^{-sT} + a_2 e^{-2sT} + \ldots + a_k e^{-ksT} \\ &= \left[a_o + \mathrm{Re}\left(a_1 e^{-sT}\right) + \mathrm{Re}\left(a_2 e^{-2sT}\right) + \ldots + \mathrm{Re}\left(a_k e^{-ksT}\right)\right] \\ &\quad + \left[\mathrm{Im}\left(a_1 e^{-sT}\right) + \mathrm{Im}\left(a_2 e^{-2sT}\right) + \ldots + \mathrm{Im}\left(a_k e^{-ksT}\right)\right] \\ &= \mathrm{Re}(D(s)) + \mathrm{Im}(D(s)) \end{aligned} \tag{2.15}$$

We know that

$$\begin{aligned} \mathrm{Re}\left(a_k e^{-ksT}\right)_{s=1/T} &= a_k \times \mathrm{Re}(e^{-sT})_{s=k/T} \\ \mathrm{Im}\left(a_k e^{-ksT}\right)_{s=1/T} &= a_k \times \mathrm{Im}(e^{-sT})_{s=k/T} \end{aligned} \tag{2.16}$$

Since values of the right-hand part of Eq. 2.16 are given, one can easily calculate Re(D(s)) and Im(D(s)).

$$\begin{aligned} \mathrm{Amp}(D(s)) &= \left[\mathrm{Re}(D(s))^2 + \mathrm{Im}(D(s))^2\right]^{1/2} \\ \underline{/(D(s))} &= \tan^{-1}\left[\mathrm{Im}(D(s))/\mathrm{Re}(D(s))\right] \end{aligned} \tag{2.17}$$

Graphically, the procedure is illustrated in Fig. 2.6.

The frequency response of the DH system used in the example given above is calculated and plotted in Fig. 2.7.

Thus we see how any DH system can be analyzed by frequency- and time-domain methods.

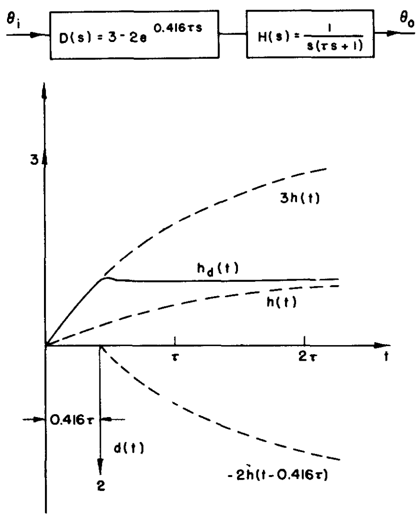


Fig. 2.5
Impulse response of a DH system.

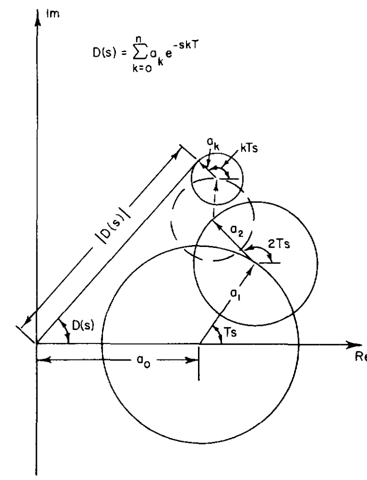


Fig. 2.6
Polar plot of D(s).

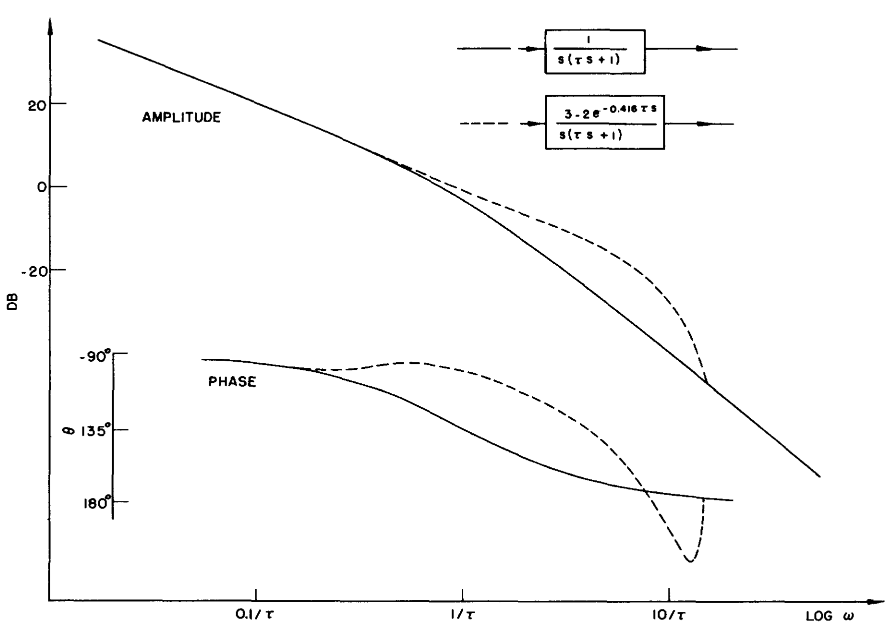


Fig. 2.7
Frequency response of a DH system.

III. DESIGN OF COMPENSATION FOR CLOSED-LOOP SYSTEMS

1. GENERAL CONSIDERATIONS

The ideal compensation for a system having a transfer function H(s) will be 1/H(s). This result is intuitively obvious and can be proved by methods of the calculus of variation. However, in practice, no physical system has an infinite dynamic linear range. One has to design the compensation in the presence of saturation limits and other constraints. This subject was treated in detail by Newton (12). This thesis shows the theoretical optimum compensating network, but in practice the procedure is relatively difficult to carry out. Regular design procedure is still a matter of trial and error. It is also a matter of compromise, for the designer must face the problem of finding out statistically how often the system will saturate and how much saturation is tolerable for any compensating scheme. Since saturations, like velocity and acceleration limits, are essentially time-domain phenomena, a time-domain solution is naturally appealing.

2. DISCRETE FILTER COMPENSATION DESIGN

The method that will be presented is simple but effective. It is based on the reshaping of the open-loop impulse response of the system; at the same time, it deals directly, from the start, with the saturation problem in the time domain. It does not yield the theoretical optimum design; it gets reasonably close to it with a minimum of work and computation. From a practical viewpoint, this is very desirable. Since the representation of a system by a transfer function H(s) is in itself an idealization, optimum design based on H(s) does not always give optimum performance when applied to actual physical components. Some parameter adjustments are necessary. The flexibility provided by a discrete filter (mentioned in Sec. II) enables one to adjust experimentally the D(s) for best actual performance — provided, of course, that the original design of D(s) is reasonably close to the optimum. Furthermore, one can always use Newton's method as a guide.

So much for the preliminaries. Some reader may well wish to inquire at this point: On what types of performance criteria is this method based? This is not, actually, of great importance. One can choose almost any type of performance criteria because the method is mostly graphical and is carried out directly in terms of time functions. This point will become self-evident later on.

The central idea of this compensation scheme is, in a phrase, the "reshaping of the open-loop impulse response of the system." Working from this reshaped response, one proceeds to calculate the closed-loop step response of the system. (Methods for doing so are given in the next section.) The reader will notice that this procedure is the exact time-domain parallel of the familiar frequency-domain method for closed-loop system synthesis. This is only natural. We shall show how these two procedures complement each other. For a given system, let us call H(s) the system function to be compensated, h(t) the impulse response of H(s), $h_d(t)$ the reshaped impulse response (see Sec. II), and

$h_i(t)$ the desired impulse response of the system.

Our purpose here is to find a D(s) so that the resultant $h_d(t)$ of the cascaded DH system is as close an approximation to $h_i(t)$ as possible, judged by any one of the many performance criteria (absolute error, rms error, and so on) one may choose. This is best done by a simple iterative procedure. But, before we show an example of this, a word or two about $h_i(t)$ is necessary. It is here that we inject the problem of saturation and other physical constraints. For example, one may wish to have an ideal $h_i(t)$ with zero rise time. But this is impossible for physical systems. One must determine the desired rise time from the velocity limit of the system, the acceleration limit, and so forth. Furthermore, the occurrence of saturation will depend on the input level. Again, some statistical knowledge about the nature of input is necessary. These factors will determine and limit the waveform of $h_i(t)$. The following procedure is indicated.

a. Decide on an ideal impulse response of the system without saturation considerations.

b. Determine the rms magnitude of discontinuities in the input function. This gives a measure of the input level the system will be subject to. Normalize this value to unity.

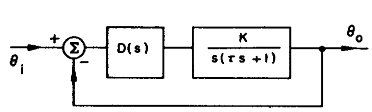
c. Now, the peak amplitude of $h_i(t)$ measures the maximum velocity that will be reached by the system in response to a unit step input. This is so because the peak amplitude represents the maximum slope of the $\int h_i(t)\,dt$ curve which is the system step response. Similarly, the maximum slope of $h_i(t)$ measures the top acceleration of the system to a step input.

d. From c, one has an approximate idea of the maximum velocity and acceleration of the system caused by the given statistical input. The designer must now make a compromise between these values and the given velocity and acceleration limits of the system. This is exactly the same idea as Newton's "trading curve." One must decide what percentage of saturation is acceptable. A good criterion is to equate this maximum velocity and acceleration to the given limits. (This insures, approximately, that the system will not saturate in the rms sense.) Other criteria could be used in different situations. These limits will, in general, determine the shape of $h_i(t)$ from the original ideal one in a.

e. It is not hard to see how other constraints can be incorporated into $h_i(t)$ in the same fashion.

Once $h_i(t)$ is fixed by physical considerations, the determination of $h_d(t)$ from h(t) with d(t) unknown is merely the inverse of the procedure presented in Section II, 2 and 3. This inverse procedure can be quickly carried out on an iterative basis. Let us now illustrate the entire process by a specific example.

The same system used in Section II, 3 is employed. Given



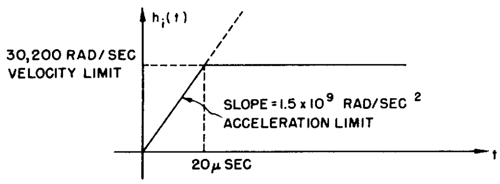


Fig. 3.1
Physical constraints on $h_i(t)$.

The hypothetical motor characteristics are: $\tau = 72\ \mu$sec, velocity limit = 30,200 rad/sec, acceleration limit = 1.5×10^9 rad/sec^2, rms input signal level = 1 unit, and $D(s) = a + be^{-sT}$. (For practical reasons, only one delay tap is available. This is a realistic assumption.) It is required that we determine a, b, T, and K.

The ideal impulse response for the motor in this case would be a simple step and infinite forward gain. However, this is clearly impossible. The velocity and acceleration constraints given will limit $h_i(t)$ as shown in Fig. 3.1. Thus K_{max} equals 30,200. Now let us try to determine a, b, and T:

1. $a = 2$ $\quad b = -1$ $\quad T = \tau$
2. $a = 3$ $\quad b = -2$ $\quad T = 0.416\tau$

The two responses $h_d(t)/_1$ and $h_d(t)/_2$ are shown in Fig. 3.2. How one arrives at these results is quite obvious, which is the beauty of this technique. The response of $h_d(t)/_2$ is superior to that of h(t) and approximates $h_i(t)$ very well (the maximum slope is 1.25×10^9 rad/sec^2). It is not hard to see how the response can be further improved iteratively by decreasing T and increasing a and b. In fact, as $T \to 0$, $a \to \infty$, $b \to \infty - 1$ in the limit, $h_d(t)$ becomes a perfect step function. For our purpose, let us say $h_d(t)/_2$ is good enough.

We should check our design at this point in the frequency domain. Figure 3.3 shows the frequency characteristics of $h_d(t)/_1$ and $h_d(t)/_2$. According to conventional servo techniques, the determination of T in $h_d(t)/_2$ is a trial-and-error process whereby one tries to establish the best phase characteristic (corresponding to maximum bandwidth) for the system. The optimum T was found to lie between 0.4τ to 0.45τ. This verifies our time-domain choice of $T = 0.416\tau$.

Once the optimum open-loop compensated characteristic is determined, the gain K can be calculated by phase margin rule. This turns out to be around 100,000. The closed-loop response can be calculated by the familiar Nichols chart.

It is not difficult now to see some of the advantages of the time-domain approach:

a. It gives the designer a physical picture of the actual effect of compensation. An optimum result is almost intuitively obvious.

b. It gives the designer a clear-cut way to take the saturation problem into account. As we have just shown, K based on the phase margin rule clearly leads to saturation in the system.

c. The flexibility and control one has over the system response in this type of compensation cannot be emphasized too strongly. The mere change of setting of one or two potentiometers directly dictates the transient response of the system. The system can be adjusted empirically for best performance.

Here is a summary of the compensation procedure:

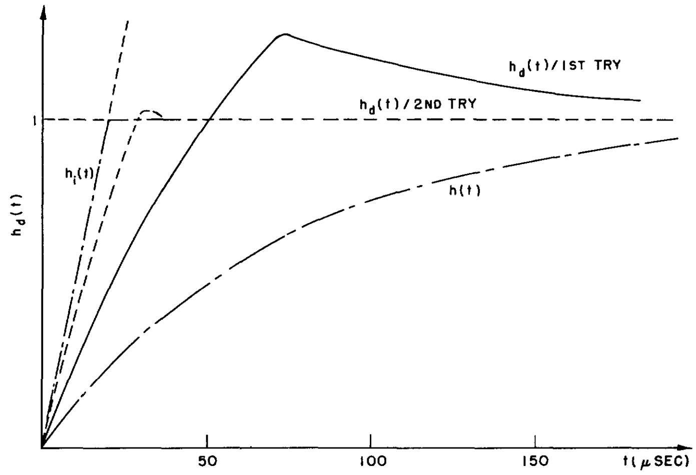


Fig. 3.2
Iterative time-domain compensation procedure.

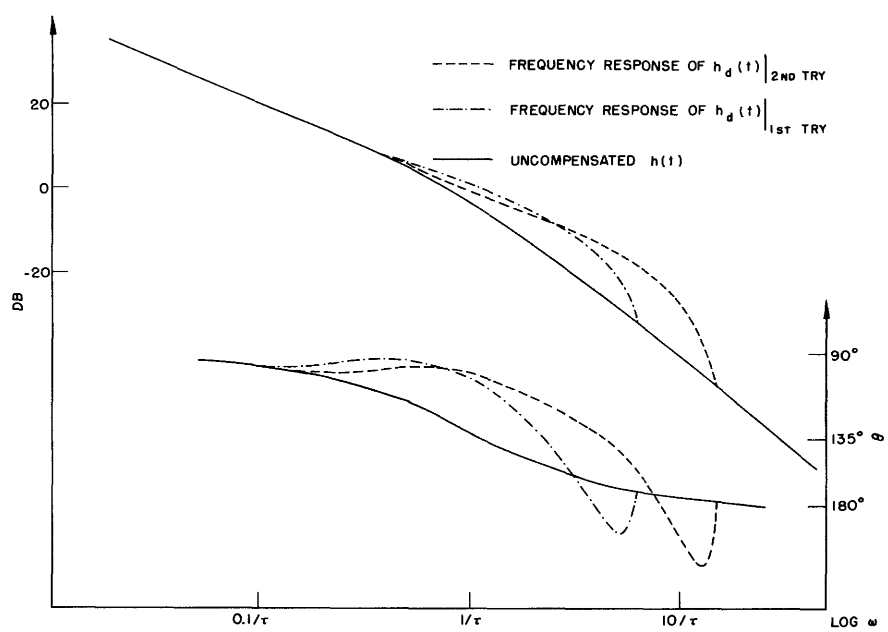


Fig. 3.3
Frequency response of the compensated system.

1. Determine the physical constraints on the system.

2. Determine the statistical nature of the input.

3. From a and b, determine a desired impulse response $h_i(t)$ by the procedure outlined above.

4. Determine the number of delay operators to be used. This will, in general, be fixed by practical considerations.

5. Use an iterative procedure to reshape h(t) so that the resultant $h_d(t)$ will be as close an approximation to $h_i(t)$ as possible. Usually, a good $h_d(t)$ will become obvious.

6. Once the loop gain and D(s) are determined, translate the design into the frequency-domain by the methods of Section II, 2 and 3, to check stability. This is optional, since the stability can be checked by other methods.

7. Proceed to calculate the transient response of the closed-loop system by the methods given in Section IV.

No attempt is made here to discard the frequency-domain method altogether. In fact, we have shown that these methods complement each other, and that they give the designer a better physical picture of the compensation process.

3. FREQUENCY-DOMAIN SPECIFICATION OF DESIGN CRITERIA

On occasion, design criteria are specified in the frequency domain – such as a prescribed frequency response characteristic, or a prescribed P(s)/Q(s) transfer function. In this case, the first step is to translate these specifications into an equivalent D(s) in the time domain. This can be done in a number of ways (3, 4, 5, 6). All of them involve the inversion of a specified matrix. The basic ideas behind these methods are the power series expansion and the method of undetermined coefficients (also known as Padé's approximation method).

$$
\begin{aligned}
D(s) &= \sum_{k=0}^{n} a_k e^{-skT} = \frac{P(s)}{Q(s)} = \sum_{k=0}^{\infty} p_k s^k \\
&= a_o + a_1\left[\sum 1 - (sT) + \frac{(sT)^2}{2!} + \dots\right] + a_2\left[\sum 1 - (2sT) + \frac{(2sT)^2}{2!} + \dots\right] \\
&\quad + [\quad] + \dots + a_n\left[\sum 1 - (nsT) + \frac{(nsT)^2}{2!} + \dots\right]
\end{aligned} \tag{3.1}
$$

Therefore

$$
\left.\begin{aligned}
a_o + a_1 + a_2 + \dots + a_n &= p_o \\
a_1 T + 2Ta_2 + \dots + nTa_n &= p_1 \\
\frac{a_1T^2}{2!} + \frac{4T^2a_2}{2!} + \dots + \frac{n^2T^2a_n}{2!} &= p_2 \\
\cdots\cdots\cdots\cdots\cdots\cdots \\
\frac{a_1T^n}{n!} + \frac{2^nT^na_2}{n!} + \dots + \frac{n^nT^na_n}{n!} &= p_n
\end{aligned}\right\} \tag{3.2}
$$

Solution of Eq. 3.2 is assured by the proof in Section II. Once the equivalent D(s) is determined, the procedure described above applies. However, it should be stated that if the design criteria are specified in the frequency domain, the ordinary frequency method will do just as well. The discussion given above is included to show the generality of the proposed method.

IV. CALCULATION OF THE TRANSIENT RESPONSE OF A CLOSED-LOOP SYSTEM

One problem in the procedure remains; that is, to evaluate the transient step response of a closed-loop system. Several approximating methods will be discussed in this section. All of them are simple and direct, involving only graphical and numerical calculation. For an average problem, the approximate transient response can be calculated in a matter of hours. Thus, an engineer is able to study a system completely, from its design to its evaluation, in a fairly short time. This is, in fact, our principal aim.

1. METHOD I

This method was originally described in reference 13. Basically, it is a step-by-step calculation of the convolution integral (Eq. 2.1). The following is a slightly modified procedure adapted for our case and presented without proof.

a. From $h_d(t)$, the reshaped impulse response, determine $\int h_d(t)\,dt$ graphically. This is the step response S(t).

b. Approximate S(t) by a series of delayed steps as shown in Fig. 4.1; that is,

$$S(t) = c_o u_{-1}(t) + c_1 u_{-1}(t-T) + c_2 u_{-1}(t-2T) + \dots \tag{4.1}$$

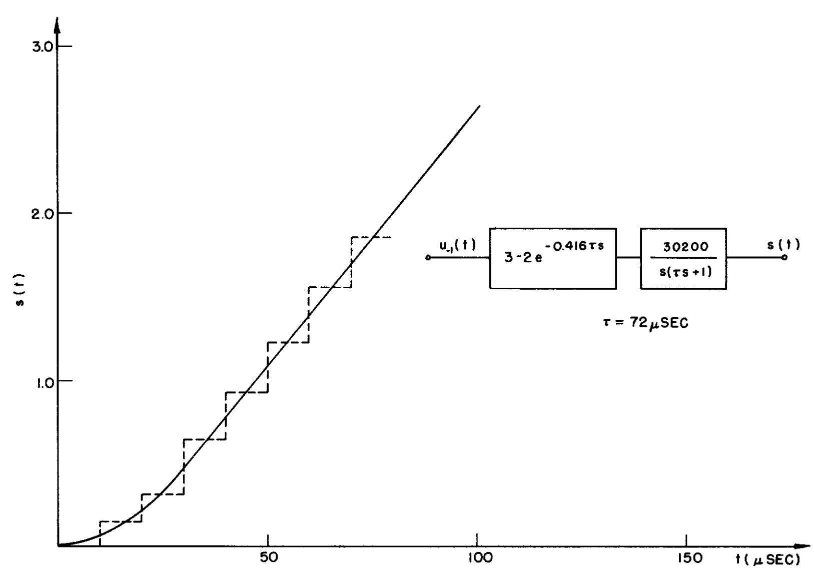


Fig. 4.1
Open-loop step response of a DH system.

where $u_{-1}(t)$ is the unit step, T is the calculation period, and c_k is the magnitude of the k^{th} incremental step.

c. Define

$\theta_o(t)$ = step response of the system with loop closed.

θ_{oko} = magnitude of $\theta_o(t)$ at the k^{th} calculation interval not corrected for effect of feedback.

θ_{okc} = magnitude of $\theta_o(t)$ at the k^{th} calculation interval corrected for effect of feedback.

E_k = average incremental error signal during the k^{th} calculation period.

d. Now

$$\theta_{ooo} = \theta_{ooc} = S(0) \qquad \text{for } t = 0$$

$$\theta_{olo} = S(T) \qquad E_o = 1 - \theta_{olo}\, c_o \qquad t = T$$

$$\theta_{o2o} = S(2T)\, E_o \qquad t = 2T$$

$$E_1 = (\theta_{o2o} + \theta_{olo})/2 - (\theta_{olo} + \theta_{ooo})/2$$

$$\theta_{olc} = \theta_{olo} - E_1 c_1$$

and so on.

e. The value of $\theta_o(t)$ at k^{th} interval will be

$$\theta_{okc} = S(kT)\, E_o - \sum_{k=0}^{k} \left[\frac{\theta_{o(k+1)o} + \theta_{oko}}{2} - \frac{\theta_{oko} + \theta_{o(k-1)o}}{2}\right] \times c_k \qquad (4.2)$$

The principle of this method is clear. Two rules of thumb should be remembered: the delay step approximation of S(t) must have the same net area as the original S(t); about twenty calculations per system response period are usually good for 10 per cent accuracy in the resultant calculation. Thus, it is necessary to have some idea about the highest significant frequency component in the system response before an intelligent choice of T can be made.

2. METHOD II

This method utilizes the frequency-domain behavior of a DH system. We have demonstrated in Section II how the open-loop frequency response curve can be calculated. The closed-loop frequency response of the DH system can then be calculated by use of the familiar Nichols chart with the loop gain determined by the methods in Section III. In most cases, the problem may be regarded as solved at this point. However, one can go one step further to compute the transient step response of the system. This is done by transforming the closed-loop response into the time domain.

3. FREQUENCY-TO-TIME-DOMAIN TRANSFORMATION

The classical method of inverse-transforming involves the solution for the roots of an algebraic equation and partial fraction expansion. This is often impractical, especially for closed-loop systems. Several methods of approximation exist. These can be divided roughly into two groups:

a. Analytical methods: The methods given in references 3, 4, and 7 belong in this group. Basically, all of them transform the frequency function into discrete equivalents by some sort of approximation procedure. From these discrete equivalent functions, the time-domain response is obtained. The process is illustrated schematically in Fig. 4.2. These methods are particularly suited for computer calculations, and require only simple programming.

b. Graphical methods: The methods given in references 1 and 2 belong in this group. These methods are, in general, somewhat simpler than the analytical ones. However, their accuracy cannot be improved as easily as the accuracy of those in group a, owing to the inherent limitation of graphical procedures.

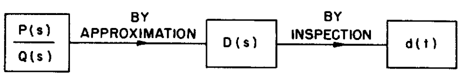


Fig. 4.2
Frequency-to-time domain transformation.

It would take too long to discuss the merits of each of these methods. They all have advantages and disadvantages. The experienced reader can use his own judgment in picking the particular method that suits his purpose best. A rule of thumb that should be remembered is that if the frequency response has some pronounced peaks (3 db or over), the transient response usually will be underdamped. In this case, a better-than-average approximation should be used in calculation. None of these methods work as well for an underdamped transient response as for a well-damped one. However, a good system in practice is usually well damped; underdamping rarely occurs.

4. VARIATION OF METHOD II

A variation of method II suggests itself. If the reshaped impulse response can be approximated by some simple well-known time function whose frequency-domain behavior is familiar, then one can by-pass the calculations required for the open-loop frequency response of the DH system. This does not occur so rarely as one might think. A well-reshaped impulse response almost invariably tends to become a simpler time function (analytically or graphically). This is, somewhat, a natural effect of compensation. The inverse-transforming process may also be speeded up because of the simpler frequency-domain expression. An example of this will be given in section 5.

There are other methods for calculating the transient response of a servo system. Some are directly applicable; some are not. One notable method is the time-series method (14).

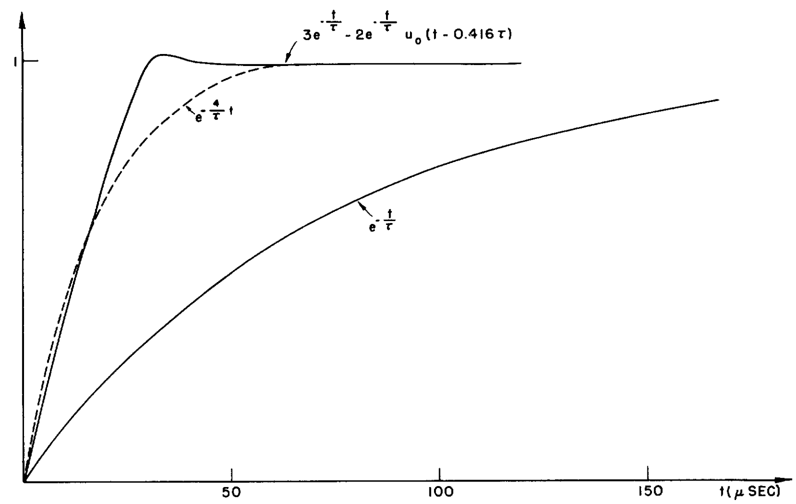


Fig. 4.3
Exponential approximation of $h_d(t)$.

5. CALCULATION OF THE TRANSIENT RESPONSE OF THE DESIGN EXAMPLE IN SECTION III BY METHODS I AND II

The unit step response of the system studied in Section III was calculated according to methods I and II. The results are shown in Fig. 6.5 compared with results obtained by computer simulation. Some very good agreements are demonstrated. The calculation data are included below.

a. By method I: The open-loop step response S(t) of the DH system is shown in Fig. 4.1. It is approximated by delayed steps of 5-μsec intervals. The same notations as defined above are used. A sample calculation of the 35-μsec interval is

$$\theta_{o8o} = 0.993 \times 0.783 - 0.030 \times 0.629 - 0.065 \times 0.475 - 0.076 \times 0.345$$

$$- 0.091 \times 0.222 - 0.109 \times 0.147 - 0.114 \times 0.059 = 0.658$$

$$E_7 = (0.658 + 0.553)/2 - (0.553 + 0.431)/2 = 0.114$$

$$\theta_{o7o} = 0.553$$

$$\theta_{o7c} = \theta_{o7o} - 0.015 \times 0.114 = 0.551$$

The entire calculation is tabulated in Table I.

b. By method II: A short cut is used in this calculation. The reshaped impulse response of the DH system $h_d(t)$ is approximated by the time function $1 - e^{-4t/\tau}$; that is

$$1 - e^{-4t/\tau} \approx 3\left(1 - e^{-t/\tau}\right) - 2\left(1 - e^{-t/\tau}\right) u_o(t - 0.416\tau) \tag{4.3}$$

Table I

Calculation data for method I.

$t(\mu sec)$	c_k	θ_{oko}	θ_{okc}	E_k
0	2.002	0	0	0.993
5	1.851	0.015	0.015	0.030
10	1.700	0.059	0.058	0.065
15	1.539	0.144	0.143	0.076
20	1.388	0.213	0.212	0.091
25	1.237	0.324	0.323	0.109
30	1.086	0.431	0.428	0.114
35	0.934	0.553	0.551	0.114
40	0.783	0.658	0.656	0.096
45	0.629	0.746	0.745	0.082
50	0.475	0.822	0.821	0.069
55	0.345	0.884	0.883	0.055
60	0.222	0.931	0.930	0.045
65	0.147	0.974	0.973	0.027
70	0.059	1.012	1.012	0.013
75	0.015	1.007	1.007	0.003
80	0.000	1.000	1.000	0.000
85		1.000	1.000	0.000

as shown in Fig. 4.3. This approximation is actually fairly reasonable, especially in view of the fact that there is very good agreement for the first few intervals of time. Thus

$$\frac{\theta_o(s)}{E(s)} = \frac{30,200}{s\left(\frac{\tau}{4} s + 1\right)} \tag{4.4}$$

$$\frac{\theta_o(s)}{\theta_i(s)} = \frac{\dfrac{30,200}{s\left(\frac{\tau}{4} s + 1\right)}}{1 + \dfrac{30,200}{s\left(\frac{\tau}{4} s + 1\right)}}$$

$$= \frac{30,200}{18 \times 10^{-6} s^2 + s + 30,000} \tag{4.5}$$

$$\theta_o(s) = \frac{1.67 \times 10^9}{s\left[s^2 + 55,500 s + 1.67 \times 10^9\right]} \tag{4.6}$$

$$\theta_o(t) = 1 - 1.36\, e^{-0.0278t} \sin(0.03t + 47.2°) \tag{4.7}$$

t is in μsec. The response calculated by the second method is somewhat sluggish as compared to the one calculated by method I. This is to be expected from the nature of the approximation made in the former case.

V. THEORY OF TIME-DOMAIN COMPENSATION

The preceding sections were written without elaborate mathematical background. They were primarily designed to show the simplicity and straightforwardness of the method. It is proper at this point that one return to examine the method in the light of the more familiar servo and network theories; a better appreciation and understanding of the entire process will result. For purposes of discussion, let us return to the example given in Section III.

1. THE SERVO THEORY VIEWPOINT

Readers who are familiar with servo theory must have recognized the similarity between $D(s) = 3 - 2e^{-sT}$ and the conventional lead network. Examination of Fig. 2.7 confirms this similarity. D(s) improves the amplitude and phase characteristics of H(s) in the way a lead network would, especially in the low-frequency region. From sampled-data theory

$$1 - e^{-sT} \approx sT \tag{5.1}$$

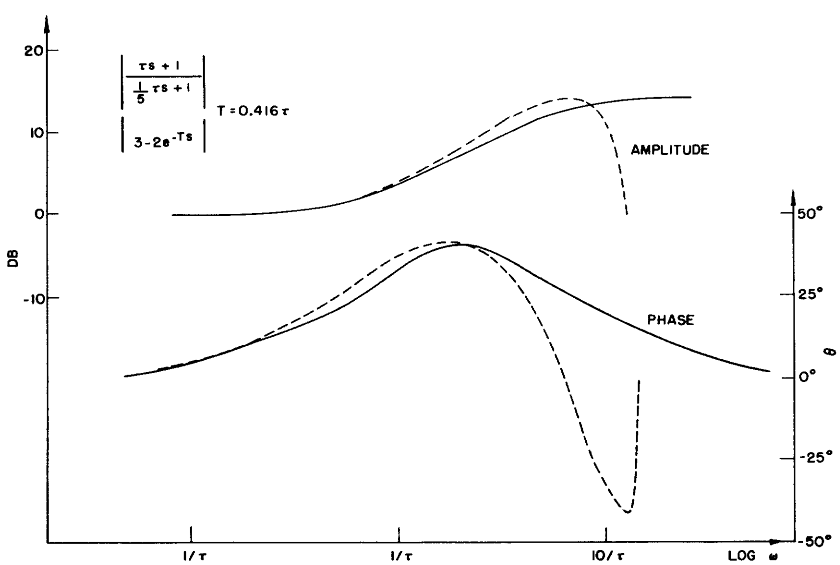


Fig. 5.1
D(s) and a lead network.

for $s << 2\pi/T$

$$D(s) = 3 - 2e^{-sT} = 1 + 2(1 - e^{-sT})$$
$$= 1 + 2(T)s = 1 + 2(0.416\tau)s$$
$$= 1 + 0.85\tau s \qquad (5.2)$$

This is true for frequencies which are small compared to $2\pi/0.416\tau$. A plot of $3 - 2e^{-0.416\tau s}$ and $(\tau s + 1)/\left(\frac{1}{5}\tau s + 1\right)$ vs. frequency is shown in Fig. 5.1. Derivative control is apparently characteristic.

2. THE NETWORK THEORY VIEWPOINT

The pole-zero pattern of the DH system is illustrated in Fig. 5.2. One finds that D(s) which is rational in e^{-sT} has an infinite sequence of zeros with a pattern of period $j2\pi/T$, where $T = 0.416\tau$. The H(s) has two poles. The pole at the origin signifies the part of the output which is a step function. The other pole on the negative real axis represents an exponential transient component.

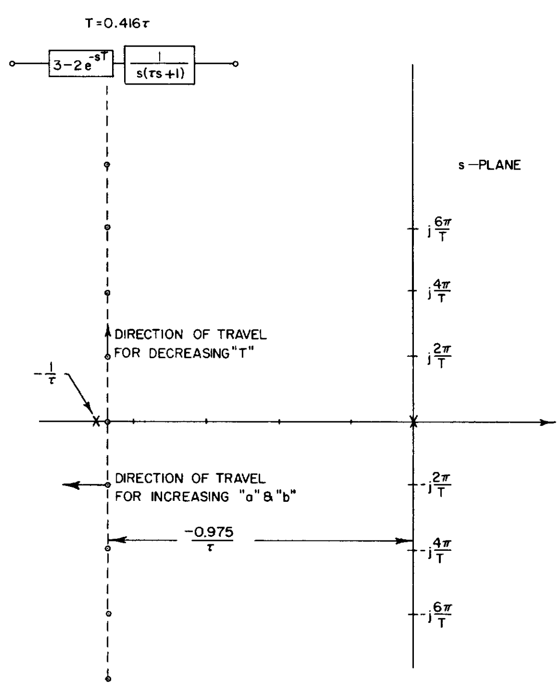


Fig. 5.2
Pole-zero pattern of the DH system.

Now for frequencies small compared to $2\pi/0.416\tau$, that is, for s in the vicinity of the origin of the s-plane, the effect of the transient pole $s = -1/\tau$ is approximately nullified by its adjacent zero. All of the zeros elsewhere in the top half-plane will be balanced by the ones in the bottom half-plane. Thus the pole at the origin completely dominates the system behavior (the ideal step response). As the frequencies go higher, although the pole and zero on the real axis will still approximately cancel each other, the zeros in the top and bottom half-plane will no longer be balanced. The system thus deviates from being an ideal integrator.

Now, for $D(s) = a + be^{-sT}$ one can exactly cancel the transient pole by further adjustments of the values of a and b.

Let $a - b = 1$, $b = a - 1$

$$D(s) = 1 + (a - 1)(1 - e^{-0.416\tau s})$$
$$\approx 1 + (a - 1)(0.416\tau)s$$
$$= 1 + \tau s \qquad (5.3)$$

for a = 3.4, b = 2.4.

Moreover, if T is made smaller, that is, if the period of the zeros of e^{-sT} becomes larger, then the frequency band in which the system behaves like an ideal integrator will also be extended, since the zeros are moving further away from the origin of the s-plane. Of course, now the a and b have to be further increased. In the limit as

$$\left.\begin{array}{l} a \rightarrow \infty \\ b \rightarrow \infty - 1 = \infty \\ T \rightarrow 0 \end{array}\right\} \qquad (5.4)$$

the only thing left in the finite s-plane will be the pole at the origin. The system impulse response is an ideal step and it behaves as an ideal integrator for all values of finite s. This is exactly the same conclusion we arrived at in Section III, 2.

At this stage, the reader may wonder, Why bother about a new method if it yields the same result that could be obtained approximately by a lead network? It should be emphasized that our example was merely used with the intention of demonstrating the method so that its results could be compared and analyzed in terms of known theories, and so that experimental and computer performances could be established for verification without an elaborate setup. However, this new approach gives us insight into the compensation process that is valuable in preliminary design. In analyzing more complicated systems than our example, this insight can be a powerful tool in guiding the design until it is checked by accurate computer solution and actual testing of the system; and even in as simple a case as our example the advantages of the method are obvious.

The method is quick, simple, and down-to-earth. Even if the first try is not up to standard, one has only used up a few hours. And, at the same time, one has learned a lot about the system and knows how to improve the design on the next try.

It is easy to see how the method can be applied in a similar manner to more sophisticated and complex cases. The difficulties will not increase by any appreciable extent, since our approach is, essentially, a graphical one. A good example is the compensation of a hydraulic servo by this method.

3. CONCLUSIONS

The theoretical basis of the method has been completely presented at this stage save for a few suggestions and conclusions.

a. Use of the sampled-data theory: Sections 1 and 2 show an example of the relationship between this method and the conventional techniques. Those who are familiar with sampled-data control theory (8) will immediately see that many of its design concepts can be applied to this approach with little or no modification. However, they must realize that our DH system differs from a sampled-data system in the sense that the former is still a continuous system. Just for example, Figs. 5.3 and 5.4 show another

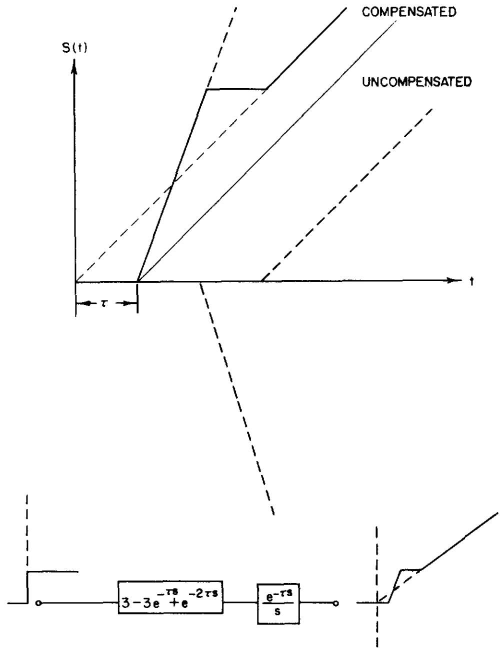


Fig. 5.3
Another DH system.

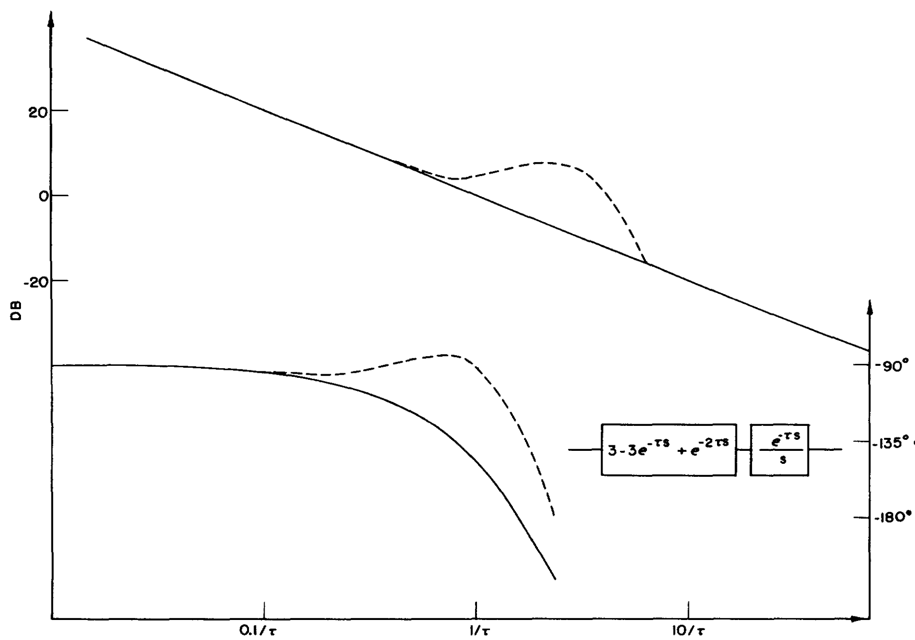


Fig. 5.4
Another DH system.

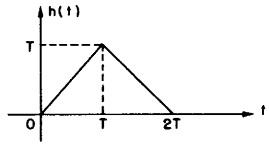
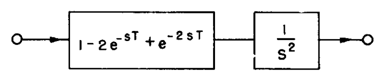


Fig. 5.5
Triangular impulse response.

system compensated by this method, and its responses. It is not necessary in this case to explain the design procedures, since they are self-evident. However, it is interesting to note that D(s) is exactly the prediction filter used in sampled-data systems (15).

b. Variations of D(s): Discrete filters can often be cascaded with conventional filters to produce desired impulse responses with far more ease and flexibility than could one type alone. For example, to produce an impulse response h(t) as shown in Fig. 5.5 one merely cascades



The designer should be constantly aware of such possibilities.

c. It is the belief of the author that, in general, the effect of compensation is to produce a simpler (graphically or analytically) and better-behaved system response. In frequency-domain terminology, compensation is aimed at canceling undesirable transient poles and zeros, as demonstrated in Section 2. Our open-loop impulse response approach described above is approximate in nature and involves a certain amount of trial and error. The aim here is to provide a simple yet approximate method which a designer can use in order to get quickly a feeling for the system, since nowadays system design invariably will be helped by the use of high-speed computers. Detail checks and design can be simulated on a computer after an initial study has been made. In such cases, a method like the one described above is useful. However, it should be pointed out that a more exact approach can be used by starting the design from a reshaped closed-loop transient response instead of an open-loop one, although such a procedure tends to complicate the nature of the method and becomes more time-consuming.

VI. COMPUTER SIMULATION OF A TIME-DOMAIN COMPENSATED SYSTEM

In this section, we shall describe the problem of simulating the example of Section III on a special computer setup and compare the results with the ones obtained by analytical methods in Sections III and IV.

1. DESCRIPTION AND BLOCK DIAGRAM OF THE COMPUTER

Figure 6.1 shows the schematic diagram of the computer setup.

a. General considerations: It is proposed to obtain the delay required in this problem by use of an ordinary RLC ladder delay line; it is not only simple, but also transmits dc signals, which is an important feature. The actual delay line used is a section of Stutt's delay line (16) which has a bandwidth of 1 Mc/sec. The available length is 30 μsec of delay. The distortion is relatively small for this delay time (see Fig. 3.12, p. 48 in ref. 16). Since 0.416τ equals 30 μsec, τ equals 72 μsec.

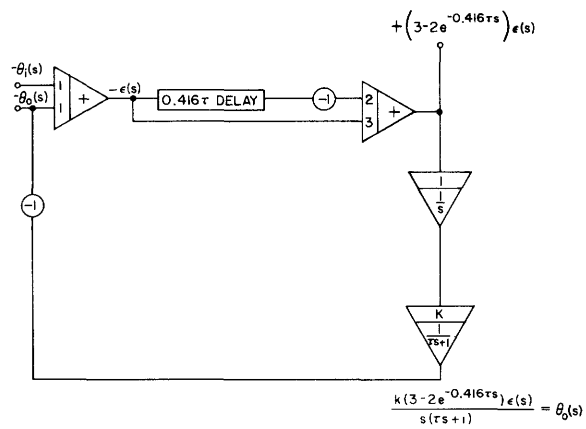


Fig. 6.1
Block diagram of computer.

A study of Fig. 3.4, the frequency response of the DH system, shows that the range of frequencies under investigation should reach 60 kc/sec. It is necessary that all the equipment used in this problem should have a flat response somewhat larger than this range.

b. Operational amplifier: The Reeves computer was suggested for simulation purposes. However, investigation showed that the amplifier had only a bandwidth of 1 kc/sec. Consequently, a special computer was built for simulation. The G. A. Philbrick Company markets a type K2-W plug-in operational amplifier which roughly satisfies the requirements. Its drift rate is reasonably small, and it has a bandwidth with a range up to 100 kc/sec when used as an inverter. For other values of gain, a narrower bandwidth will result. Thus, we just about meet the bandwidth requirement mentioned above, although the high-frequency responses are less ideal. This effect will show up in the results given in the next section.

c. Integrator: After due consideration, it was decided that a passive RC integrator should be used. This enabled us to operate the system continuously; an active integrator demanded a repetitive solution setup involving more equipment.

d. Gain control amplifier: A dc amplifier by Electromechanical Research, Inc., is used to provide loop gain adjustment and low-impedance output for driving the delay line, which has a characteristic impedance of 1000-900 ohms. The bandwidth of this amplifier is 1 Mc/sec.

The final computer diagram is shown in Fig. 6.2.

2. COMPUTER RESULTS

The computer was set up and tested for open- and closed-loop frequency responses. The transient response was tested by feeding into the system a 1200-cps square wave. These results are shown in Figs. 6.3, 6.4, 6.5, 6.6, and 6.7.

In Fig. 6.5 the computer transient response is a little bit slower than it should be. This is primarily because the bandwidth of the K2-W operational amplifier does not quite

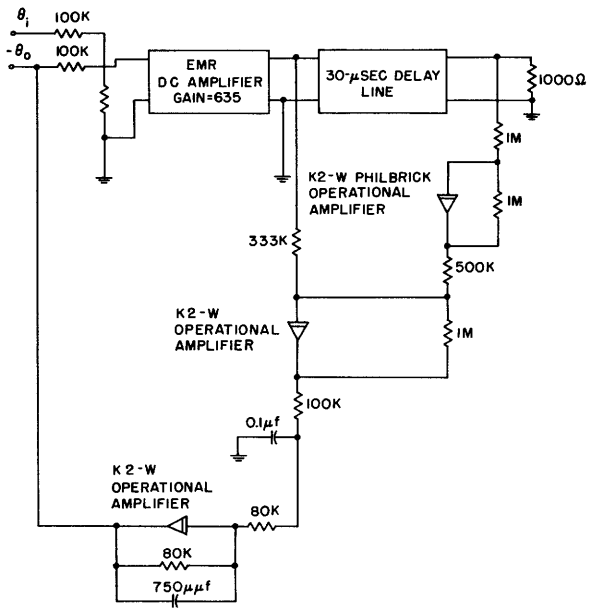


Fig. 6.2
Computer for simulation of the DH system.

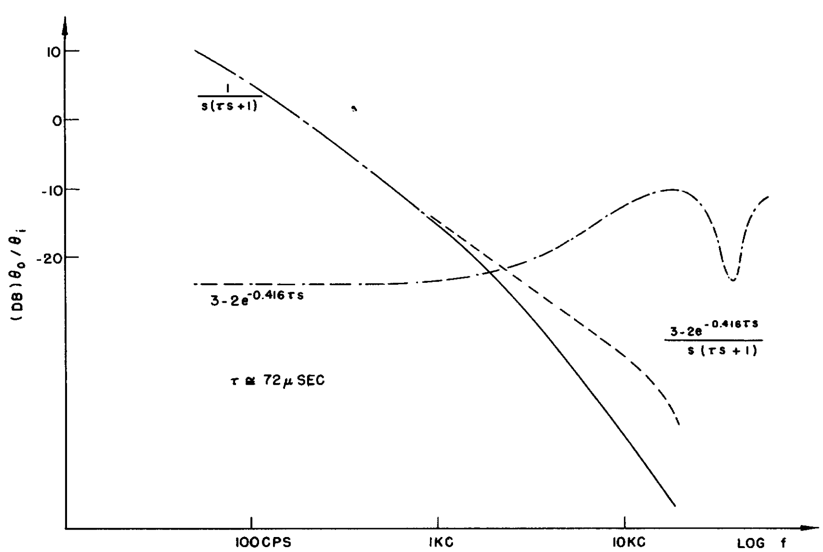


Fig. 6.3
Open-loop frequency response of the simulated DH system; open-loop frequency response of D(s).

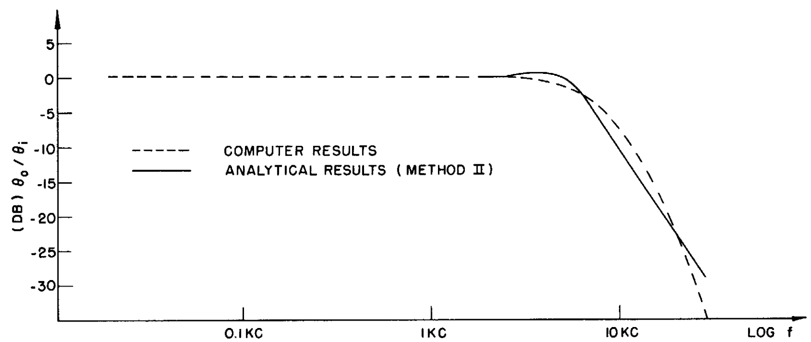


Fig. 6.4

Closed-loop frequency response of the DH system (computer-simulated and analytical).

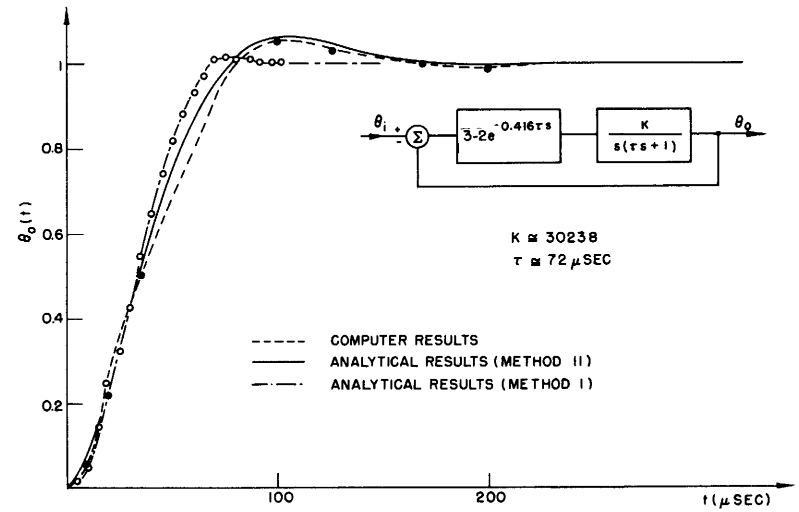


Fig. 6.5

Transient response of the simulated system as compared with the analytical results obtained in Section IV.

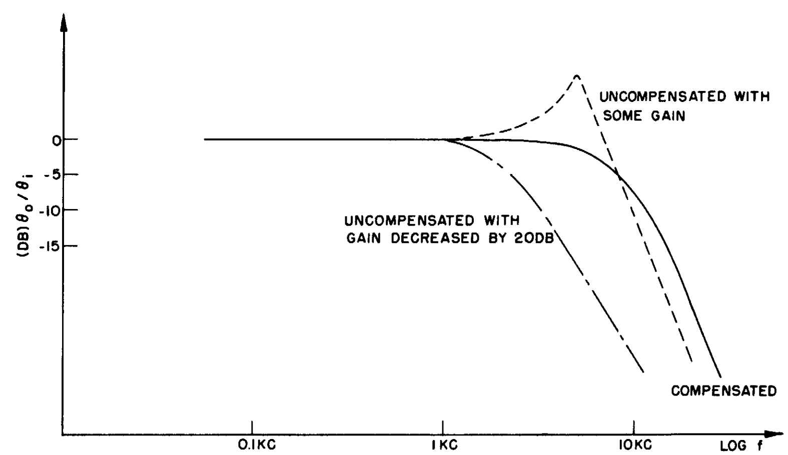
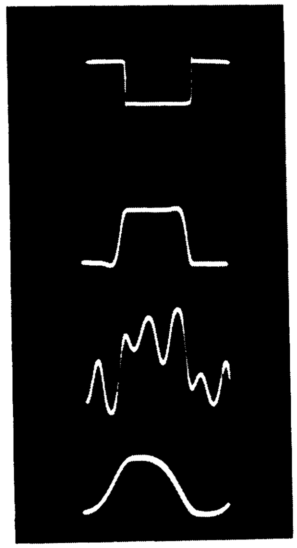


Fig. 6.6

Closed-loop response of the simulated system with and without compensation.



Input
(1200-cps square wave)

Output with
delay compensation

Output without
delay compensation
(with same gain)

Output without
delay compensation
(with decreased gain)

Fig. 6.7

Photographs of the transient response of the simulated system, with and without compensation.

meet our requirements (see Section VI, 1, b). Otherwise, we expect the transient response to lie somewhere in between the results from methods I and II. However, the agreements are good, considering all the approximations involved.

VII. EXPERIMENTAL DESIGN AND PERFORMANCE OF A TIME-DOMAIN COMPENSATED SYSTEM

1. DESIGN OF DELAY-LINE COMPENSATION

The design of D(s) means, in practice, the realization of a tapped delay line as shown in Fig. 2.1. In general, most feedback systems operate in the low-frequency region. The delay-line units involved will be of the order of milliseconds. This renders the ordinary RLC delay line impractical because of the prohibitively long physical length required. Several alternatives exist:

a. It is possible to program a digital computer to simulate a general discrete transfer function like Eq. 2.12. This is feasible when the system is elaborate, and many delay units are needed.

b. With analog computer elements (operational amplifiers, for example) it is possible to realize delay times up to the order of seconds within a limited frequency range (17).

c. Delay can be achieved mechanically through magnetic recording (18). A recording head and numerous pickup heads are placed along a continuous magnetic recording medium (tape loop or drum). The signal is recorded onto the medium through the recording head and is picked up later by the pickup heads. The delay is provided by

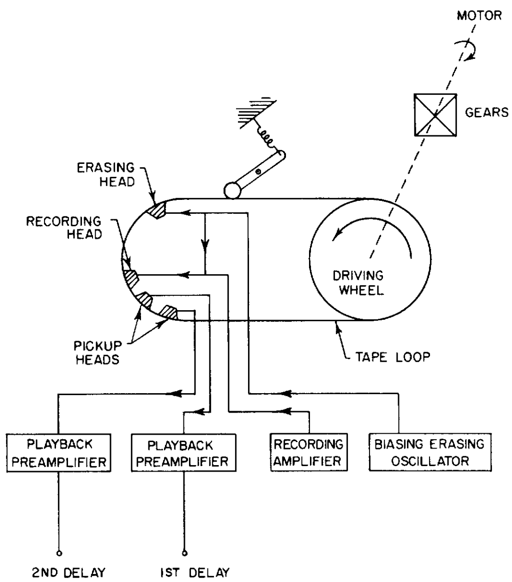


Fig. 7.1
Magnetic tape delay unit.

the speed of travel of the recording medium.

d. A special RC network can be constructed (19) which approximates the delay operator e^{-sT} in a limited frequency band.

Alternatives a and b satisfy our requirements for D(s), but are not economically feasible for our purpose. Alternatives c and d appear attractive. They were studied further.

a. Magnetic tape delay unit: The basic model is shown in Fig. 7.1. Since in almost all feedback systems the error signal is dc, some modulation scheme will be required to transmit the signal through the magnetic delay unit. At first sight, amplitude modulation seems to be a nice arrangement. For example, in a 400-cps servo system, one merely has to place the delay unit between the ac chopper and the power amplifier of the system for it to function properly. However, this scheme has certain subtle but inherent limitations:

Magnetic tape has an inherent amplitude uncertainty of about $\pm$ 2 db at its output for various reasons. This is not acceptable where accurate reproduction of the waveform is necessary.

Small variations in tape speed will result in a large variation of the phase of the delayed carrier. Thus, it becomes impossible to demodulate the signal with a phase-sensitive detector. Several ways were devised to avoid this difficulty. However, for reasons of economy or accuracy, they proved to be unsuitable.

Since the AM scheme was unsuccessful, possibilities for frequency modulation were investigated next. Here, the difficulties mentioned above are automatically eliminated. In fact, FM units for use with tape recorders are commercially available (though not intended for the purpose of feedback compensation). However, they proved again to be too expensive for a preliminary investigation.

b. RC delay network: It can be shown that

$$\lim_{n\to\infty} \frac{1}{(\tau s + 1)^n} = e^{-sT} \tag{7.1}$$

where $T = n\tau$. For finite n

$$\frac{1}{(\tau s + 1)^n} \approx e^{-n\tau s} \tag{7.2}$$

for a limited frequency band. Reference 19 describes the construction of such a delay

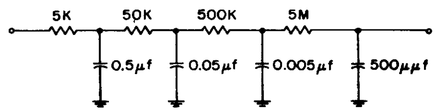


Fig. 7.2
RC delay network.

line and its performance. In this case, a four-stage RC ladder network was constructed to approximate the delay. This network is shown in Fig. 7.2. The τ for each section is 2.5 msec. The values of R and C for each section are so chosen that they do not load their previous stages. The characteristics of the RC delay network are shown in Fig. 7.3. The approximation is admittedly rough but is reasonable enough for the purpose of semi-quantitative investigation.

2. SYSTEM DESIGN

The servo is intended to be a servomultiplier for use in analog computation. The design of the basic components follows regular procedures (20, 21, 22). The final circuit diagram of the system incorporating the RC delay network is shown in Fig. 7.4. The diagram is self-explanatory and shows the function of each component used. Two K2-W Philbrick operational amplifiers were used for the error-sensing and the addition of the error voltage and its time-delayed signal. They were chosen not only because they serve as a stable adding circuit without loss of gain, but also because they provide the very low output impedance necessary at places where they were employed. The ac chopper was operated at low impedance for minimum noise pickup.

3. PERFORMANCE

The performance of the system is shown in Figs. 7.5, 7.6, and 7.7. These results agree well with the ones obtained by computer simulation except that the latter have

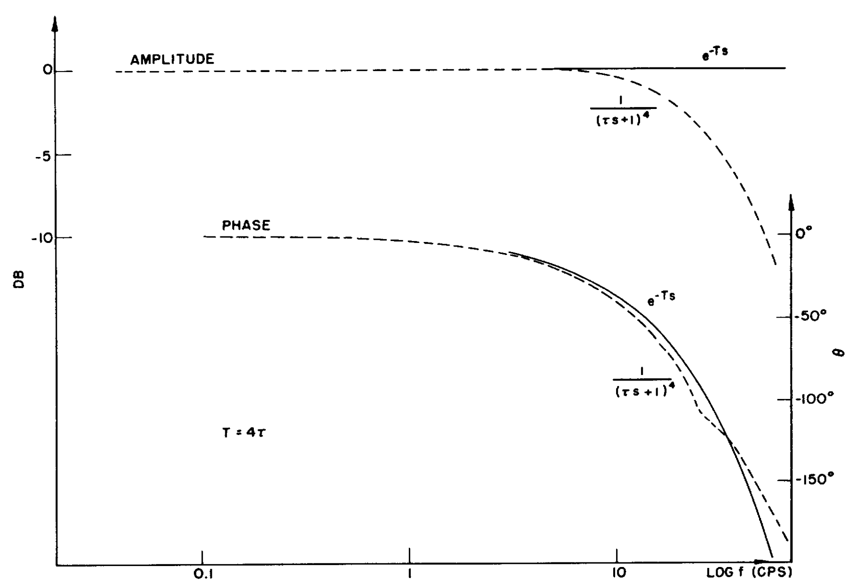


Fig. 7.3
Frequency response of RC delay network.

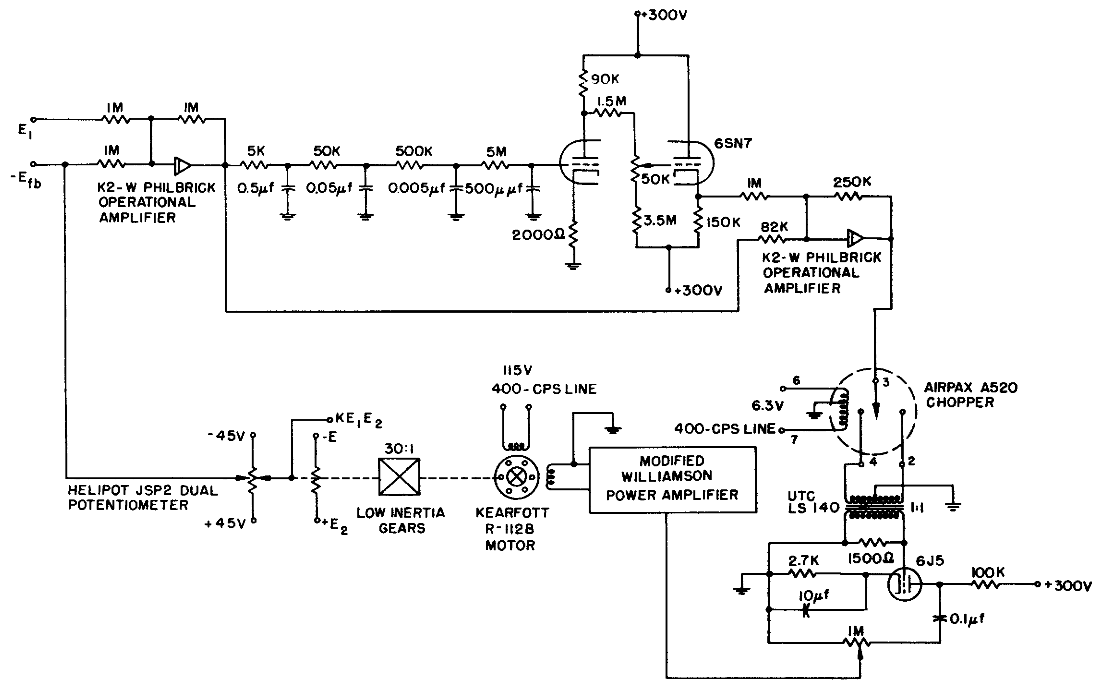


Fig. 7.4
Circuit diagram of the compensated servomultiplier.

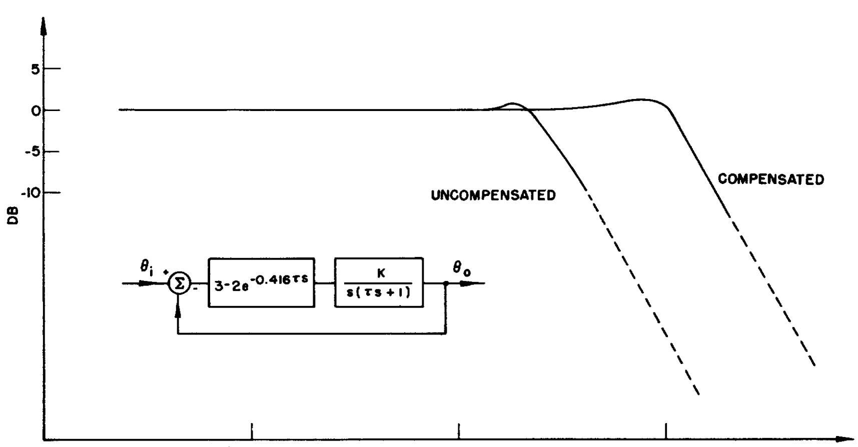
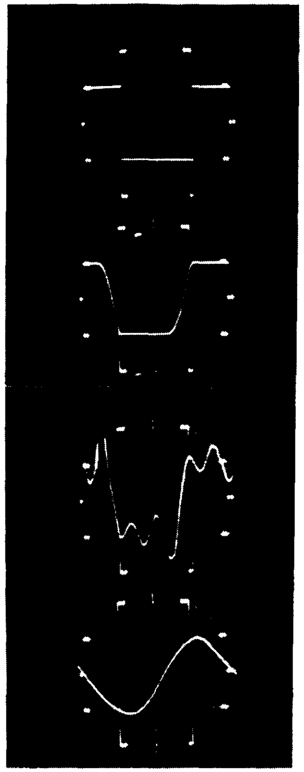


Fig. 7.5
Frequency response of the servomultiplier.



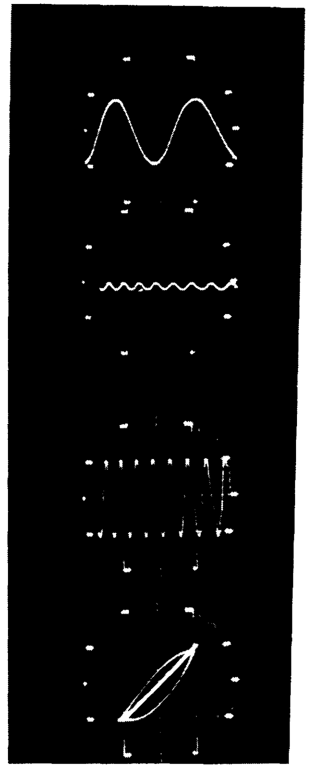
Input
(3-cps square wave)

Output with
delay compensation

Output without
delay compensation
(with same gain)

Output without
delay compensation
(with decreased gain)

Fig. 7.6
Transient response of the DH system.



2-cps response

10-cps response
uncompensated

10-cps response
compensated

Phase shift at 2 cps
compensated and
uncompensated

Fig. 7.7
Frequency response of the DH system.

been scaled up in frequency by a factor of approximately 350. Experimentally, the scale factor measures around 400-450. This discrepancy is quite reasonable considering the approximations involved.

VIII. CONCLUSION

This report has presented a simple and approximate method for feedback compensation design in the time-domain. The method is complementary to the existing frequency-domain techniques. It is useful in preliminary system investigation because of its simplicity and straightforwardness. After this initial study, detail design can then be carried out by computer simulation and actual testing of bandwave. This is the approach that has been taken in the design example of this report.

APPENDIX A

TABLE OF THE REAL AND IMAGINARY PARTS OF e^{-sT}

s	sT	-θ	Re	Im
0	0	0°	1	0
0.2/T	0.2	11.45	0.979	-0.199
0.4/T	0.4	22.9	0.921	-0.389
0.6/T	0.6	34.37	0.825	-0.565
0.8/T	0.8	45.83	0.697	-0.717
1.0/T	1.0	57.28	0.540	-0.842
1.2/T	1.2	68.73	0.363	-0.932
1.4/T	1.4	80.18	0.170	-0.985
1.6/T	1.6	90.7	-0.012	-0.999
1.8/T	1.8	103.1	-0.226	-0.974
2.0/T	2.0	114.6	-0.416	-0.909
2.2/T	2.2	126.1	-0.589	-0.808
2.4/T	2.4	137,6	-0.738	-0.674
2.6/T	2.6	149.0	-0.857	-0.515
2.8/T	2.8	160.5	-0.943	-0.334
3.0/T	3.0	172.0	-0.990	-0.139
3.2/T	3.2	183.5	-0.998	0.061
3.4/T	3.4	195.0	-0.965	0.258
3.6/T	3.6	206.5	-0.895	0.446
3.8/T	3.8	218.0	-0.788	0.616
4.0/T	4.0	229.5	-0.649	0.760
4.2/T	4.2	240.5	-0.492	0.870
4.4/T	4.4	252.0	-0.309	0.951
4.6/T	4.6	263.5	-0.113	0.994
4.8/T	4.8	275.0	0.087	0.996
5.0/T	5.0	286.5	0.284	0.959
5.2/T	5.2	298.0	0.469	0.883
5.4/T	5.4	309.0	0.632	0.774
5.6/T	5.6	320.5	0.771	0.636
5.8/T	5.8	332.0	0.891	0.454
6.0/T	6.0	343.7	0.960	0.281
2π/T	2π	360.0	1.000	0.000

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