#5

MICROWAVE DETERMINATIONS OF AVERAGE ELECTRON ENERGIES AND THE FIRST TOWNSEND COEFFICIENT IN HYDROGEN

LAWRENCE J. VARNERIN, JR.
SANBORN C. BROWN

LOAN COPY

only

TECHNICAL REPORT NO. 158

MAY 24, 1950

RESEARCH LABORATORY OF ELECTRONICS
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

The research reported in this document was made possible through support extended the Massachusetts Institute of Technology, Research Laboratory of Electronics, jointly by the Army Signal Corps, the Navy Department (Office of Naval Research) and the Air Force (Air Materiel Command), under Signal Corps Contract No. W36-039-sc-32037, Project No. 102B; Department of the Army Project No. 3-99-10-022.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

RESEARCH LABORATORY OF ELECTRONICS

Technical Report No. 158 May 24, 1950

MICROWAVE DETERMINATIONS OF AVERAGE ELECTRON ENERGIES AND THE FIRST TOWNSEND COEFFICIENT IN HYDROGEN

Lawrence J. Varnerin, Jr.
Sanborn C. Brown

Abstract

A microwave method of measuring the average energy of electrons in a gas in the presence of a d-c field has been developed through a measurement of the ratio between the electron diffusion and mobility. By solving for a-c and d-c distribution functions it is found possible to compare quantitatively the effective a-c ionization coefficient and the first Townsend coefficient in hydrogen. It is then possible to calculate the first Townsend coefficient from a-c breakdown data in hydrogen and these values are compared with previously determined values. Measurements of average energy have been made and values calculated from theory compared with previous measurements of average energy. All measurements are in substantial agreement with each other and with the theoretically determined values.

MICROWAVE DETERMINATIONS OF AVERAGE ELECTRON ENERGIES AND THE FIRST TOWNSEND COEFFICIENT IN HYDROGEN

Solutions for the energy distribution function of electrons in a gas show a very close similarity between the distribution functions under the action of a-c and d-c fields. This similarity makes it possible to compare quantitatively the first Townsend coefficient of a gas with the a-c ionization coefficient. In addition, it makes possible an experiment to determine the average energy of electrons by measuring the ratio of the diffusion coefficient of electrons, D, to the mobility coefficient, μ. The gas in a resonant cavity will break down when the losses of electrons to the walls are replenished by ionization in the body of the gas (1). When an a-c field is applied, the breakdown depends upon D. If a small d-c field is applied to the gas as well, a greater number of electrons will be lost and the breakdown of the gas will depend upon μ as well as D. Breakdown measurements with and without d-c fields applied permit a determination of the ratio D/μ.

I. THE BOLTZMANN EQUATION

The electron energy distribution function f can be determined with the aid of the Boltzmann transport equation which is the phase space continuity equation for electrons (2)

$$C = \frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f + \vec{a} \cdot \nabla_v f \tag{1}$$

where C is the net rate at which electrons appear in an element in phase space, $\vec{v}$ is the velocity, $\vec{a}$ the acceleration, t the time, and ∇_v the gradient operator in velocity space.

The distribution function may be expanded in spherical harmonics in velocity

$$f = f_0 + \frac{\vec{v} \cdot \vec{f}_1}{v} + \ldots \quad . \tag{2}$$

The spherically symmetric term f_0 is much larger than the higher order terms because of the disordering effect of collisions. This expansion is rapidly convergent for the cases to be considered here.

The production term C arises from collisions and may be expanded in spherical harmonics. The electric field is given by $\vec{E}$, and an energy variable $u = mv^2/2e$, in electron volts, is introduced; m is the mass and e the charge of an electron. Substitution of these terms in Eq. 1 gives a scalar and a vector equation

$$C_0 = \frac{\partial f_0}{\partial t} - \frac{v}{3u} \frac{\partial}{\partial u} (u\vec{E} \cdot \vec{f}_1) + \frac{v}{3} \nabla \cdot \vec{f}_1 \tag{3}$$

$$\vec{C}_1 = \frac{\partial \vec{f}_1}{\partial t} + v \nabla f_0 - v\vec{E} \frac{\partial f_0}{\partial u} \quad . \tag{4}$$

Elastic collisions contribute to C_0 and $\vec{C}_1$. By considering an elastic collision as an instantaneous process transferring an electron from one point in phase space to another, Morse, Allis, and Lamar (2) have shown that

$$C_{0,el} = \frac{2m}{M}\,\frac{v}{u}\,\frac{\partial}{\partial u}\left(\frac{u^2 f_0}{\ell}\right) \tag{5}$$

and

$$\vec{C}_{1,el} = -\frac{v}{\ell}\,\vec{f}_1 \tag{6}$$

where ℓ is the electronic mean free path and M is the mass of the gas molecule. The remaining contribution to the C_0 term results from inelastic collisions and is given by $-(h_x + h_i)\nu_c f_0$ where h_x and h_i are efficiencies of excitation and ionization and ν_c is used to represent v/ℓ, the collision frequency. We assume that electrons suffering inelastic collisions have no preferred direction after collision and there is no contribution to the $\vec{C}_1$ term.

Equations 3 and 4 now give

$$-(h_x + h_i)\nu_c f_0 = \frac{\partial f_0}{\partial t} + \frac{v}{3}\left[\nabla \cdot \vec{f}_1 - \frac{1}{u}\frac{\partial}{\partial u}\left(u\vec{E} \cdot \vec{f}_1\right) - \frac{6m}{M}\,\frac{1}{u}\,\frac{\partial}{\partial u}\left(\frac{u^2}{\ell} f_0\right)\right] \tag{7}$$

$$0 = \nu_c \vec{f}_1 + \frac{\partial \vec{f}_1}{\partial t} + v\left[\nabla f_0 - \vec{E}\,\frac{\partial f_0}{\partial u}\right] \quad . \tag{8}$$

The electric field will be taken as

$$\vec{E} = \vec{E}_{DC} + \sqrt{2}\,\vec{E}_{AC}\,e^{j\omega t} \tag{9}$$

where $\vec{E}_{DC}$ represents the d-c field, $\vec{E}_{AC}$ the rms value of the a-c field and ω its radian frequency. Each term in the distribution function can be expanded in a Fourier series in time

$$f_0 = f_0^0 + \sqrt{2}\, f_0^1\, e^{j\omega t} + \dots \tag{10}$$

$$\vec{f}_1 = \vec{f}_1^0 + \sqrt{2}\, \vec{f}_1^1\, e^{j\omega t} + \dots \tag{11}$$

for the steady state condition. The f_0^1 term will not be retained since solutions will be obtained only for those cases where the mechanisms for density and energy decay are so slow that neither density nor energy fluctuations can occur in an r-f cycle. If terms of the same frequency are grouped and the second harmonic terms dropped, the product of E with f_0 and f_1 gives

$$\vec{E} f_0 = \vec{E}_{DC} f_0^0 + \sqrt{2}\,\vec{E}_{AC} f_0^0\, e^{j\omega t} + \dots \tag{12}$$

$$\vec{E}\cdot\vec{f}_1 = \vec{E}_{DC}\cdot\vec{f}_1^{\,0} + \vec{E}_{AC}\cdot\vec{f}_1^{\,1} + \sqrt{2}\left[\vec{E}_{DC}\cdot\vec{f}_1^{\,1} + \vec{E}_{AC}\cdot\vec{f}_1^{\,0}\right] e^{j\omega t} + \dots \quad . \tag{13}$$

The time independent component of Eq. 7 is now given by

$$-(h_x + h_i)\nu_c f_0^0 = \frac{v}{3}\left[\nabla\cdot\vec{f}_1^{\,0} - \frac{1}{u}\frac{\partial}{\partial u}(u\vec{E}_{DC}\cdot\vec{f}_1^{\,0}) - \frac{1}{u}\frac{\partial}{\partial u}(u\vec{E}_{AC}\cdot\vec{f}_1^{\,1}) - \frac{6m}{M}\frac{1}{u}\frac{\partial}{\partial u}\left(\frac{u^2}{\ell}f_0^0\right)\right] \quad . \tag{14}$$

The time independent and first frequency components of Eq. 8 give two equations

$$0 = \nu_c\vec{f}_1^{\,0} + v\left[\nabla f_0^0 - \vec{E}_{DC}\frac{\partial f_0^0}{\partial u}\right] \tag{15}$$

$$0 = (\nu_c + j\omega)\,\vec{f}_1^{\,1} - v\vec{E}_{AC}\frac{\partial f_0^0}{\partial u} \quad . \tag{16}$$

$\vec{f}_1^{\,0}$ and $\vec{f}_1^{\,1}$ can be obtained from Eqs. 15 and 16 and substituted in Eq. 14. If we choose coordinates so that $\vec{E}_{DC}$ is directed along the z axis, we obtain

$$\begin{aligned} -(h_x + h_i)\nu_c f = \frac{v}{3}\Bigg\{ & -\ell\nabla^2 f + \ell E_{DC}\frac{\partial}{\partial u}\frac{\partial f}{\partial z} + \frac{E_{DC}}{u}\frac{\partial}{\partial u}\left(\ell u\frac{\partial f}{\partial z}\right) \\ & - \frac{1}{u}\frac{\partial}{\partial u}\ell u\left[E_{DC}^2 + \frac{\nu_c^2}{\nu_c^2 + \omega^2}E_{AC}^2\right]\frac{\partial f}{\partial u} - \frac{6m}{M}\frac{1}{u}\frac{\partial}{\partial u}\left(\frac{u^2}{\ell}f\right)\Bigg\} \end{aligned} \tag{17}$$

where f is now understood to represent f_0^0.

II. A-C AND D-C DISTRIBUTION FUNCTIONS

Solutions will be sought for the case where the collision frequency ν_c can be assumed a constant. This holds particularly well for helium and hydrogen (3,4). The calculations to be made will consider the particular case of hydrogen for which Brode's data (5) give $\nu_c = 5.93\times 10^9 p$ (sec^{-1}) (p in mm Hg). It is then possible to introduce an effective field defined by

$$E_e^2 = E_{DC}^2 + \frac{\nu_c^2}{\nu_c^2 + \omega^2}E_{AC}^2 \tag{18}$$

and we shall also use the letter q to represent the quantity

$$\begin{aligned} q &= \frac{m}{e}\frac{3m}{M}\frac{\nu_c^2}{E_e^2} \\ &= \frac{16.17}{(E_e/p)^2} \quad . \quad \text{(for hydrogen)} \end{aligned} \tag{19}$$

We shall now consider separately the equations for the purely a-c and purely d-c cases in order to compare the effective distribution functions for the same fields in the two cases. We may separate the distribution functions into a space times an energy function. The difference in the spatial function comes from the factor $\exp(\alpha z)$, where α is the first Townsend coefficient, in the d-c case because of the multiplication of the electrons in their direction of motion. This differs in the a-c case for which a solution of the diffusion equation gives $\nabla^2 f_{AC} = - f_{AC}/\Lambda^2$, where Λ is the characteristic diffusion length of the container. Substituting

$$\eta = \frac{\alpha}{E_{DC}}$$

Eq. 17 yields for the separate cases

$$0 = u \frac{d^2 f_{AC}}{du^2} + \left[qu + \frac{3}{2} \right] \frac{df_{AC}}{du} + \left[\frac{3}{2} q - \frac{u}{E_e^2 \Lambda^2} - \frac{(h_x + h_i) Mq}{2m} \right] f_{AC} \tag{20}$$

$$0 = u \frac{d^2 f_{DC}}{du^2} + \left[(q - 2\eta)u + \frac{3}{2} \right] \frac{df_{DC}}{du} + \left[\frac{3}{2} q - \frac{3}{2} \eta + \eta^2 u - (h_x + h_i) \frac{Mq}{2m} \right] f_{DC} \quad . \tag{21}$$

The substitution of

$$f_{DC} = \phi_{DC} \exp(\eta u) \tag{22}$$

yields

$$0 = u \frac{d^2 \phi_{DC}}{du^2} + \left[qu + \frac{3}{2} \right] \frac{d\phi_{DC}}{du} + \left[\frac{3}{2} q + q\eta u - (h_x + h_i) \frac{Mq}{2m} \right] \phi_{DC} \quad . \tag{23}$$

If a differential operator L is defined as

$$L = u \frac{d^2}{du^2} + \left[qu + \frac{3}{2} \right] \frac{d}{du} + \frac{3}{2} q - (h_x + h_i) \frac{Mq}{2m} \tag{24}$$

Eqs. 20 and 23 become

$$0 = \left[L - \frac{u}{E_e \Lambda^2} \right] f_{AC} \tag{25}$$

$$0 = \left[L + \eta q u \right] \phi_{DC} \quad . \tag{26}$$

The term $\eta q u$ is of the order of magnitude of $10^{-5} u$. Since this is much smaller than other terms in L, it may be dropped.

$$0 = L \phi_{DC} \quad . \tag{27}$$

Equations 25 and 27 demonstrate that $f_{AC} = \phi_{DC}$ when $E_e \Lambda$ is very large. This corresponds to a range of small E_e/p. In this range, however, η is very small as well. From Eq. 22, we deduce that

$$f_{DC} = f_{AC} \tag{28}$$

and the quantities D, μ, and ν become identical for the a-c and d-c cases.

When E_e/p is large, a comparison of f_{DC} with f_{AC} requires solutions for the distribution functions. Solutions have already been obtained for the a-c case (4) and what is now required are solutions for the d-c case. We shall treat separately from the higher energy part of the function that part of the distribution function below 8.9 volts where $h_x + h_i = 0$.

When u < 8.9 volts, MacDonald and Brown (4) found the solution for the a-c case in terms of confluent hypergeometric functions. The d-c equation may be solved by substituting

$$\phi_{DC} = \psi_{DC} \exp(-\tfrac{q}{2} u) \tag{29}$$

in Eq. 27 to give

$$\frac{d^2\psi_{DC}}{du^2} + \frac{3/2}{u}\frac{d\psi_{DC}}{du} + \left[\frac{3/4}{u} q - \frac{q^2}{4}\right] \psi_{DC} = 0 \quad . \tag{30}$$

Since q^2 is extremely small for E_e/p large, the term in q^2 may be dropped to give

$$\frac{d^2\psi_{DC}}{du^2} + \frac{3/2}{u}\frac{d\psi_{DC}}{du} + \frac{3/4}{u} q \, \psi_{DC} = 0 \quad . \tag{31}$$

The solutions of Eq. 31 are given by

$$\psi_{DC} = u^{-1/2} \left[\sin \sqrt{3qu} + H \cos \sqrt{3qu}\right] \tag{32}$$

giving

$$f_{DC} = u^{-1/2} \left[\sin \sqrt{3qu} + H \cos \sqrt{3qu}\right] \exp\,(\eta - \tfrac{q}{2})u \quad . \tag{33}$$

When u > 8.9 volts, the excitation terms become very important. MacDonald and Brown (4) have shown for the a-c case that an asymptotic expansion

$$f_{AC} = Du^T \exp(-su) \left[1 + \frac{d}{u} + \dots\right] \tag{34}$$

can be used for this part of the function, where

$$T = \frac{B}{2A} - \frac{3}{4}$$

$$s = (A + \frac{1}{2b})\, qb$$

$$d = \frac{1}{qb}\,\frac{B}{4A^2}\,(1 - \frac{B}{2A})$$

$$b = \left[1 + \left(\frac{2}{qE_e\Lambda}\right)^2 \right]^{1/2}$$

$$A = \left[\frac{M}{2m} \frac{h_1}{qb^2} + \frac{1}{4} \right]^{1/2}$$

$$B = \frac{1}{b} \left[\frac{M}{2m} h_1 u_x + \frac{3}{4} \right]$$

and the excitation-plus-ionization function has been approximated by

$$h_x + h_i = h_1 (u - u_x)$$

$$= 9.0 \times 10^{-3} (u - 8.9) \quad \text{(for hydrogen)}$$

where u_x is the excitation potential.

Since the excitation term completely dominates the equation for f_{AC}, Eq. 34 serves equally well for f_{DC} for the same E_e/p. That this is correct was checked in the following manner. From Eqs. 25 and 27, we see that ϕ_{DC} and f_{AC} are identical if $\Lambda \to \infty$. ϕ_{DC} determined in this manner determines f_{DC} with the aid of Eq. 22. The distribution function f_{DC} so obtained agrees with Eq. 34 within 0.5 percent at $E_e/p = 70$ and within 1 percent at $E_e/p = 100$ for energies up to 20 volts. This is sufficiently accurate for our purposes. There are so few electrons at higher energies that they have little bearing on integrals over the distribution function.

The constants D and H are adjusted to join solution (34) smoothly with solution (33) at the patching point (4) $u_p = 9.5$ volts.

$$H = \frac{1 - 2\sqrt{\frac{u_p}{3q}} \left[\frac{qb}{\Phi} + \frac{1}{2u_p} + \frac{q}{2} - \eta \right] \tan \sqrt{3qu_p}}{2\sqrt{\frac{u_p}{3q}} \left[\frac{qb}{\Phi} + \frac{1}{2u_p} + \frac{q}{2} - \eta \right] + \tan \sqrt{3qu_p}}$$

$$D = \frac{u_p^{-1/2} \left[\sin \sqrt{3qu_p} + H \cos \sqrt{3qu_p} \right] \exp\left(\eta - \frac{q}{2}\right) u_p}{u_p^T \exp(-su_p)\left(1 + \frac{d}{u_p} + \ldots\right)}$$

where

$$\frac{qb}{\Phi} = \left[\frac{1}{f} \frac{df}{du} \right]_{u = u_p} .$$

III. IONIZATION COEFFICIENTS

We are now able to determine ζ_e and η from the distribution functions. The effective a-c ionization coefficient ζ_e is defined by

$$\zeta_e = \frac{\nu_{AC}}{D_{AC}E_e^2}$$

where the subscripts AC refer to the fact that such quantities are to be calculated from f_{AC}. This quantity is equal to $1/\Lambda^2E_e^2$ for breakdown in the a-c case. Experimental values of $1/\Lambda^2E_e^2$ determine ζ_e.

The first Townsend coefficient η may be calculated as well. The number of ionizations per centimeter of electron drift α is $\nu_{DC}/\mu_{DC}E_{DC}$ since $\mu_{DC}E_{DC}$ is the drift velocity. Thus,

$$\eta = \frac{\nu_{DC}}{\mu_{DC}E_{DC}^2}$$

where the subscripts DC refer to the fact that such quantities are to be calculated from f_{DC}. If we divide η by ζ_e, for the same value of E_e (E_{DC} is the effective field for the d-c case) in both cases, we obtain

$$\frac{\eta}{\zeta_e} = \frac{\nu_{DC}}{\nu_{AC}}\frac{D_{AC}}{\mu_{DC}} \quad . \tag{35}$$

The ionization rate $n\nu$ may be calculated (6)

$$n\nu = -8\pi(e/m)^2\int_{u_i}^{\infty}\left(\frac{u}{v}\right)\nu_c h_i f\,du$$

where u_i is the lowest ionization potential. Since $f_{AC} = f_{DC}$ for the complete range of integration, it follows that

$$n_{AC}\,\nu_{AC} = n_{DC}\nu_{DC} \quad . \tag{36}$$

The coefficient of mobility of electrons μ is given by

$$\mu = -\frac{1}{n}\int_0^{\infty}\frac{\ell v}{3}\,4\pi v^2\,\frac{\partial f}{\partial u}\,dv \quad . \tag{37}$$

This equation can be integrated by parts to give

$$\mu = \frac{1}{n}\frac{e}{m}\int_0^{\infty}\frac{1}{\nu_c}\,4\pi v^2 f\,dv \quad . \tag{38}$$

Since

$$n = \int_0^{\infty}4\pi v^2 f\,dv = 2\pi\left(\frac{2e}{m}\right)^{3/2}\int_0^{\infty}u^{1/2}f\,du \tag{39}$$

and ν_c is a constant, Eq. 38 becomes

$$\mu = \frac{e}{m} \frac{1}{\nu_c}$$

whether we use f_{AC} or f_{DC}. Thus μ without subscript serves both a-c and d-c cases.

When ζ_e and η are small, as was discussed previously for low E_e/p, f_{AC} and f_{DC} become identical so that $\nu_{DC} = \nu_{AC}$, in which case

$$\frac{\eta}{\zeta_e} = \frac{D}{\mu} \quad . \tag{40}$$

For larger E_e/p, we now can determine the ratio ν_{DC}/ν_{AC}.

From Eq. 39, we see that the contribution to n in any energy interval is $u^{1/2} f$. This quantity (unnormalized) is plotted in Fig. 1 for E_e/p = 100 volts/cm-mm Hg for both the a-c and d-c functions. There are about 5 percent fewer electrons in the low energy range for the d-c than for the a-c case. From Eq. 36, we see that this implies that $\nu_{DC} \cong 1.05\ \nu_{AC}$. This fact has a physical interpretation. In the d-c case the density varies as exp (αz). This dependence makes for large density gradients which allow fast electrons to diffuse back into the gas against the d-c field, while those which are swept out by the field are drawn equally from all energy levels. The larger number of higher energy electrons contributes to the increased ν.

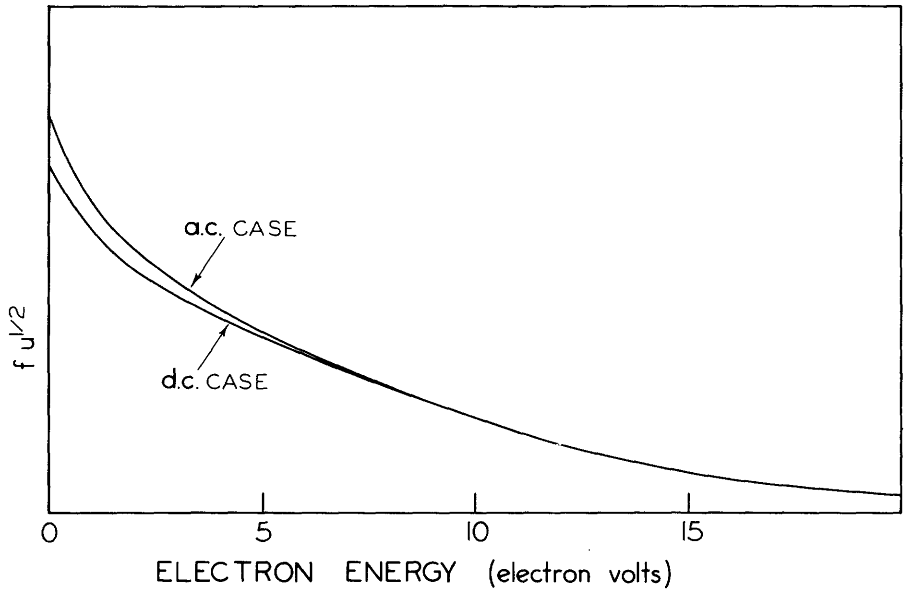

Fig. 1 Comparison of a-c and d-c energy distribution functions at E_e/p = 100 for hydrogen.

In a similar way it can be shown that the diffusion coefficient calculated from f_{DC} varies less than 5 percent from that calculated from f_{AC}. Since experimental errors in determinations of η and ζ_e are frequently larger than 10 percent, Eq. 40 may be taken as sufficiently accurate for $E_e/p < 100$ volts/cm-mm Hg.

From the integrals for D and the average energy $\overline{u}$

$$D = \frac{1}{n} \int_0^\infty \frac{\ell v}{3} 4\pi v^2 f \, dv$$

$$\overline{u} = \frac{1}{n} \int_0^\infty \frac{mv^2}{2e} 4\pi v^2 f \, dv \quad .$$

A comparison with Eq. 38 shows

$$\frac{D}{\mu} = \frac{2}{3} \overline{u} \tag{41}$$

when ν_c is a constant. Hence D/μ is a direct measure of the average energy of electrons. Even when ν_c is not a constant, Eq. 41 holds reasonably well. For a Druyvesteyn (7) distribution, $D/\mu = 0.763\, \overline{u}$, while for a Maxwellian distribution we have again, $D/\mu = 2/3\, \overline{u}$, whatever the dependence of ν_c on energy.

IV. THE AC-DC BREAKDOWN CONDITION

The gas in a cavity will break down when the losses of electrons to the walls of the cavity are replaced by ionization in the body of the gas. When an a-c field alone is applied, electrons are lost by diffusion. When a small d-c sweeping field is applied, electrons are lost both by diffusion and mobility. The breakdown condition can be formulated mathematically by a consideration of these processes.

The flow of electrons $\vec{\Gamma}$ is given by

$$\vec{\Gamma} = - n\mu \vec{E}_{DC} - D\nabla n \quad . \tag{42}$$

When the electrons that are lost are replaced by new ones resulting from ionization,

$$\nabla \cdot \vec{\Gamma} = \nu n \quad . \tag{43}$$

If the divergence of Eq. 42 is equated to νn, and $\vec{E}_{DC}$ is directed along the z axis, we obtain

$$\nabla^2 n + \frac{E_{DC}}{D/\mu} \frac{\partial n}{\partial z} + \nu n = 0 \quad . \tag{44}$$

Equation 44 may be readily solved for the case of a cylinder of axial height L and axial coordinate z, radius R and radial coordinate r. Rigorous boundary conditions require the concentration to be small at a boundary and to extrapolate to zero outside the boundary at a distance of the order of a mean free path. In the range of pressures to be considered, the mean free path is very small compared to cavity dimensions and the condition of zero concentration on the cylinder walls is imposed.

By separating variables

$$n = M(r)N(z)$$

we obtain two equations

$$\nabla_r^2 M + k_1^2 M = 0$$

$$\frac{d^2N}{dz^2} + \frac{E_{DC}}{D/\mu}\frac{dN}{dz} + \left(\frac{\nu}{D} - k_1^2\right)N = 0 \tag{45}$$

where k_1^2 is the separation constant and ∇_r^2 is the two-dimensional Laplacian in the plane perpendicular to z. The solutions are

$$M = \text{const } J_0(k_1 r) \tag{46}$$

$$N = \text{const } \sin\frac{\pi}{L} z \exp\left(-\frac{\mu E_{DC}}{2D} z\right) \tag{47}$$

where $k_1 = 2.404/R$ and J_0 is the zero-order Bessel function. The exponential represents the deformation of the sine caused by the sweeping of electrons. This solution is subject to the condition

$$\frac{\nu}{D} = \frac{1}{\Lambda_{DC}^2} \tag{48}$$

where Λ_{DC} defines a modified diffusion length

$$\frac{1}{\Lambda_{DC}^2} = \frac{1}{\Lambda^2} + \left(\frac{E_{DC}}{2D/\mu}\right)^2 . \tag{49}$$

For this case, the characteristic diffusion length is given by

$$\frac{1}{\Lambda^2} = \left(\frac{\pi}{L}\right)^2 + \left(\frac{2.404}{R}\right)^2 . \tag{50}$$

The only difference between the breakdown condition (48) in the ac-dc case and the pure a-c case is the substitution of a modified diffusion length Λ_{DC} for the characteristic diffusion length Λ. It will be noted that the modified diffusion length of a cavity is smaller than the characteristic diffusion length. A cavity whose electron losses are increased by a d-c sweeping field is equivalent to a smaller cavity without a sweeping field.

V. USE OF BREAKDOWN CONDITION IN DETERMINING D/μ

If we divide Eq. 48 by E_e^2, we obtain

$$\frac{\nu}{DE_e^2} = \frac{1}{\Lambda_{DC}^2 E_e^2} . \tag{51}$$

When an a-c field alone is applied, this quantity defines the effective a-c ionization coefficient ζ_e previously considered. A curve of ζ_e is obtained from a-c breakdown measurements alone by plotting $1/\Lambda^2 E_e^2$ versus E_e/p. Equation 51 shows that this plot is the same as for $1/\Lambda_{DC}^2 E_e^2$. If an ac-dc breakdown is observed at a given E_e/p, it is then possible to obtain the appropriate ζ_e from data taken with pure a-c breakdown. The ζ_e so obtained is equated to $1/\Lambda_{DC}^2 E_e^2$ and Λ_{DC} is obtained. From the definition of Λ_{DC}, Eq. 49, D/μ is obtained. The implicit assumption that the distribution function of electrons is the same for the ac-dc case as for the a-c case alone is best understood from the similarity of the extreme cases of d-c field and a-c field alone.

VI. EXPERIMENTAL PROCEDURE

A block diagram of the experimental apparatus for observing ac-dc breakdown is shown in Fig. 2. A continuous-wave tunable magnetron in the range of 10-cm wave length supplies power to a coaxial line. For wave length determinations, a probe in the coaxial line is connected to a cavity wavemeter. A power divider provides a continuously variable control over the fraction of the power incident on the cavity; unused power is dissipated in a matched load. A known fraction of the power incident on the cavity is coupled from the line by a directional coupler to a power measuring thermistor and associate measuring bridge. A probe and slotted section in the coaxial line permit measurements of the standing wave pattern in the line immediately ahead of the cavity. Measurement of the standing wave ratio and voltage minimum as a function of frequency permits calculation of the cavity Q as well as the fraction of the incident power that is actually absorbed by the cavity. For convenience the power transmitted through the cavity is calibrated in terms of the incident power.

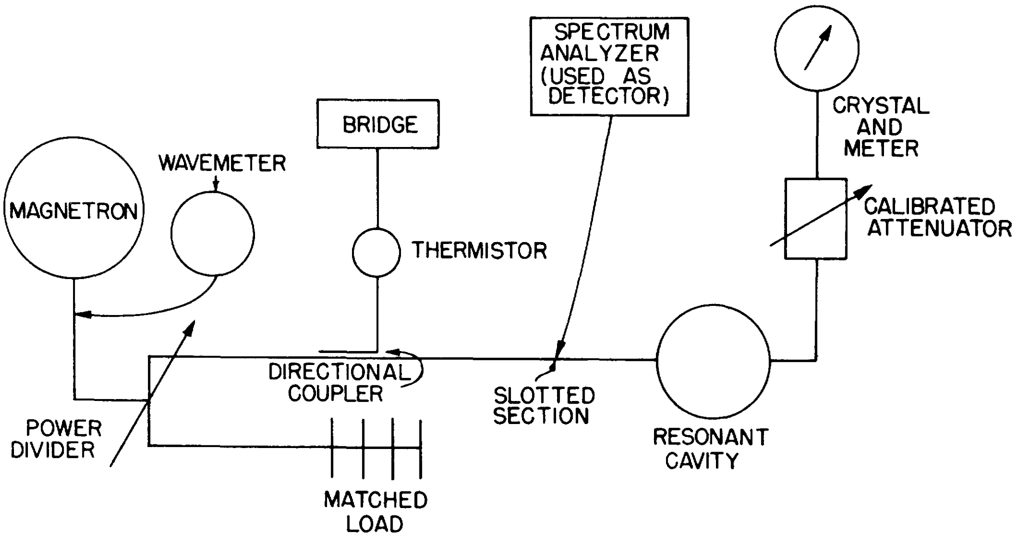

Fig. 2 Block diagram of the experimental apparatus.

Figure 3 shows an oxygen-free copper resonant cavity which can be outgassed and which permits the application of a d-c field to the discharge region. The r-f field configuration is identical with a cylindrical TM_{010}-mode cavity of 1/16-inch separation with the exception that there is a d-c break around the diameter backed up by an r-f choke.

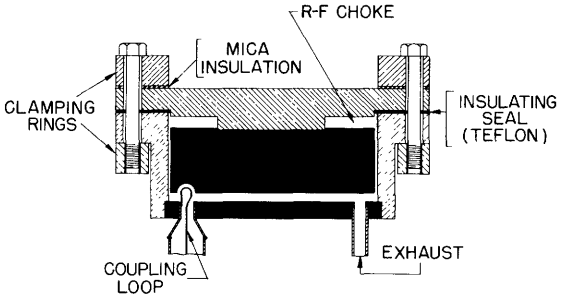

Fig. 3 Schematic diagram of the resonant cavity.

The output loop is not shown. An insulating gasket made of sheet Teflon served to position the inner cavity assembly and to make the vacuum seal. The flanges were clamped together on the gasket with clamping rings and bolts made of No. 303 stainless steel to match the expansion of the copper. This arrangement permitted baking the cavity at 290°C.

The field within the cavity was determined by constructing an identical cavity structure provided with a small axial plug which could be inserted into the cavity. By observing the change in resonant frequency with plug insertion, after the method of Slater (8), the relationship between the stored energy in the cavity and the square of the electric field at the point of insertion of the plug was determined. From the cavity Q and the power absorbed by the cavity, the field could be calculated. The field determinations were accurate to about 8 percent.

Measurements of D/μ converted to average energy of electrons have been carried out for hydrogen. The cavity was outgassed for a week at 290° C. Two liquid air traps were employed. With the vacuum system isolated from the oil diffusion pump by a stopcock, the system held at a pressure of the order of 3×10^{-7} mm Hg for over an hour as measured by an ionization gauge. Spectroscopically pure hydrogen was used. Figure 4 gives values of average energy as a function of E_e/p. The experimental error is approximately 8 percent. The broken curve was obtained by Townsend and Bailey (9) by observing the lateral diffusion of an electron beam moving in a uniform d-c field. Theoretical values of average energy were calculated here from the distribution functions. The measurements were taken for only those pressures where the mean free path was much smaller than the cavity size.

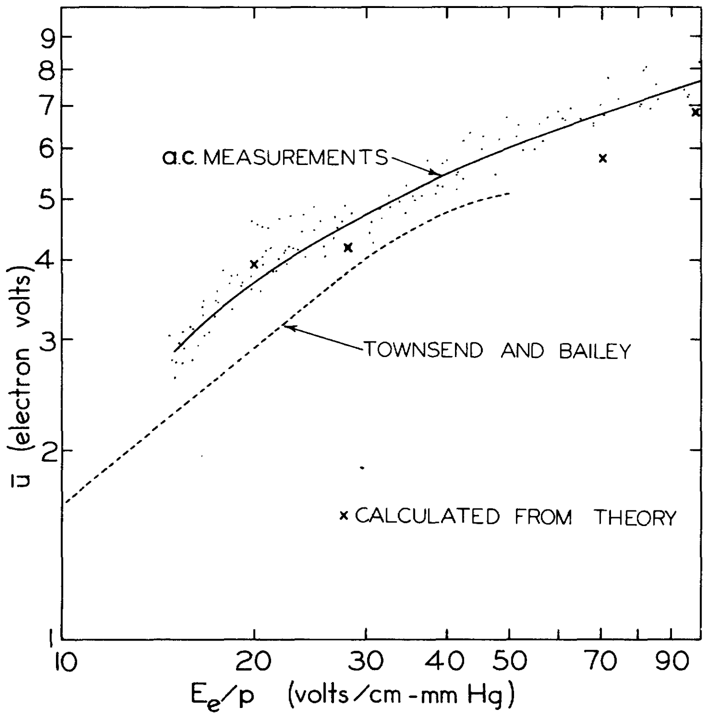

Fig. 4 Comparison of theory and experiments for the average electron energy.

From the observed value of D/μ and ζ_e, the first Townsend coefficient

η is calculated and plotted in Fig. 5 (it is shown as a smooth curve, since it was determined from two smooth curves); the maximum experimental error in this curve is about 20 percent. Theoretical values of η determined from those points for which the average energy was calculated are shown. It is seen that the present data are in substantial agreement with previous data of Ayers (10) and Hale (11).

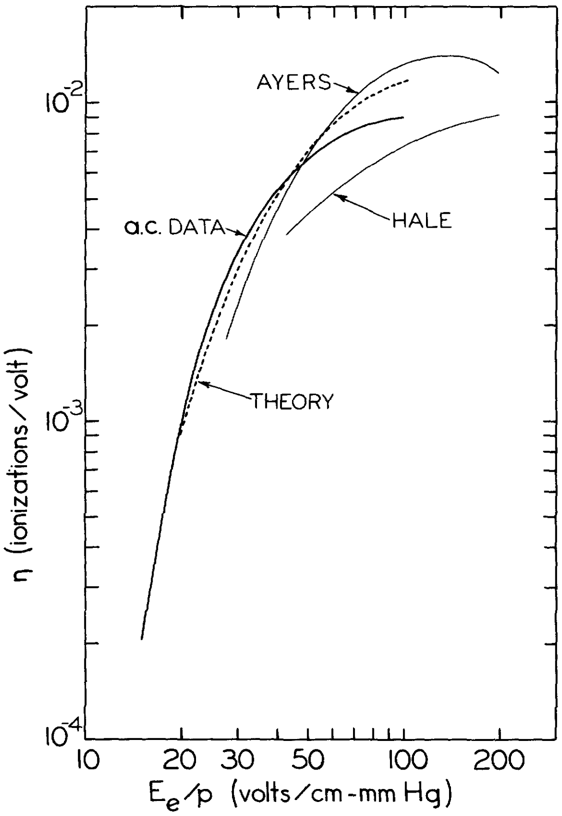

Fig. 5 Comparison of theory and experiments for the first Townsend ionization coefficients in hydrogen.

The authors wish to acknowledge the many valuable ideas contributed by Professor W. P. Allis in numerous discussions of the problems involved in this report.

References

(1) M. A. Herlin, S. C. Brown: Phys. Rev. 74, 291 (1948).

(2) P. M. Morse, W. P. Allis, E. S. Lamar: Phys. Rev. 48, 412 (1935).

(3) A. D. MacDonald, S. C. Brown: Phys. Rev. 75, 411 (1949).

(4) A. D. MacDonald, S. C. Brown: Phys. Rev. 76, 1634 (1949).

(5) R. B. Brode: Rev. Mod. Phys. 5, 257 (1933).

(6) Formula (16), reference 4.

(7) M. J. Druyvesteyn: Physica 10, 69 (1930).

(8) J. C. Slater: Rev. Mod. Phys. 18, 441 (1946), Eq. III.89.

(9) J. S. Townsend, V. A. Bailey: Phil. Mag. 42, 873 (1921).

(10) T. L. R. Ayers: Phil. Mag. 6, 23, 353 (1923).

(11) D. H. Hale: Phys. Rev. 55, 815 (1939); values of α/p versus E/p are given by L. B. Loeb, Fundamental Processes of Electrical Discharges in Gases, p. 356 (John Wiley and Sons, New York, 1939).