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A SIMPLE THEORY OF FINANCIAL RATIOS
AS PREDICTORS OF FAILURE

by
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A B S T R A C T

This paper presents a simple, intuitive theory of business risk. The results are used to explain empirical observations of Beaver on the power of various financial ratios to predict failure of firms, and to hypothesize improved predictive ratios for use in selecting attractive risk situations and in determining appropriate risk premiums.

A C K N O W L E D G E M E N T S

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Introduction

Though financial accounting's use for measuring income has been extensively, if not thoroughly, explored, comparatively little academic attention has been given to its use in measuring risk.

Several years ago William Beaver published a very interesting article reporting an empirical study of various financial ratios as predictors of failure.¹ Using matched samples of "failed" firms versus "non-failed" firms, he found that several easily available financial ratios were good predictors of failure, while others, probably more widely used, were mediocre predictors.² Specifically the criterion ratios cash flow/total assets, net income/total assets, total debt/total assets and particularly cash flow/total debt were good predictors of failure, the latter even up to five years before the event, while such widely used ratios as the "current" ratio were of only mediocre value until the final year before failure, and even then inferior to the aforementioned ratios.

Unfortunately, the reader is left with no particular rationale or theoretical explanation as to why certain ratios should be good predictors of failure. Of course, one explanation is that management of firms in trouble

¹See Beaver [1]. See also Lemke [3].

²To quote Beaver: "Of the 79 failed firms studied, 59 were bankrupt; 16 involved non-payment of preferred stock dividends; 3 were bond defaults; and 1 was an overdrawn bank account".

spuriously manipulates widely attended ratios such as the current ratio as long as possible so as to maintain availability of credit, thus rendering them not useful for predictive purposes until a crisis is nearly at hand. It is also possible that the determinants of real business risk are generally somewhat imprecisely understood, so that the well-known financial ratios based on this understanding, such as the current ratio, would be inferior predictors even under circumstances of no management attention given to their spurious manipulation. The purpose of this brief paper is to sketch the basis of a theoretical model which might explain Beaver's results in this latter light and give rise to hypotheses as to even better predictors.

In the model which follows, a number of very crude simplifying assumptions are made. Refinements would be in most cases straightforward.³

A Theoretical Model

Let us withdraw for the moment to the world of simple probability models. Suppose a system can exist in only one of a finite number N of states S_j , $j=0, 1, 2, \dots, N-1$, at any time t ; suppose further that the probability of being in state S_j is completely defined by the state S_i of the system at the previous time $t-k$, where k is a constant for all t . Suppose N is allowed to increase without bound, but remains countably infinite. This is a Markov process. Define $P(S_j|S_i)$ as the probability of S_j at time t given S_i at time $t-k$.

³Recently, a very interesting model has been published based on the same idea, but employing more involved mathematics and with a different emphasis; see Tinsley [4].

Let: $P(S_0 | S_0) = 1$

$$P(S_j | S_i, i = j-1, i \neq 0) = p$$

$$P(S_j | S_i, i = j+1) = 1-p$$

and all other $P(S_j | S_i) = 0$.

This is a specification of a one-dimensional random walk which has an absorbing barrier at one end and no barrier at the other, the classic gambler's ruin model.

Let $q = 1-p$. Then, according to random walk theory, the probability of the system starting at state S_z and ultimately visiting state S_0 , and thus being ruined, or failing, is:

$$P(\text{Ultimate Failure}) = \begin{cases} 1, & \text{if } p \leq q \\ (q/p)^z, & \text{otherwise.} \end{cases}^4$$

Now let us re-interpret this simple model in a more realistic, though radically simplified, picture of the firm. In particular; we will model losses and gains as arising from a binominal distribution with mass p at $+\sigma$ and q at $-\sigma$.

Suppose there exists a firm of wealth C which every year plays a game which nets it a gain or loss of constant size $\pm \sigma$, where the probability of a gain equals, p , and of a loss, q . Suppose $p > q$.

Then the probability of this firm's ultimate failure is:

$$P(\text{Ultimate Failure}) = (q/p)^{C/\sigma},$$

⁴See Feller [2, p. 347].

where the number of losses z the firm can take in a row before being ruined is C/σ . This is the basic simplified model we will use to predict failure of firms. The next section talks about the degree to which it can be made more realistic by appropriate restrictions and choice of real-world measures of q/p , C , and σ .

Mapping the Model into Reality

We have to recognize that many, if not all, business failures are not irretrievable if sufficient outside equity capital and managerial talent can be brought into the picture. Thus, our model will be most realistic where there are "barriers to entry" of new capital and management or where such entry is in itself viewed as a kind of failure. Second, we must recognize that the firm's wealth, C , must in reality be regarded as some function of both assets and liabilities of the firm. Depending on the availability of loans in the situation, the relevant C will be different -- perhaps total assets minus total liabilities for large firms in times of ready credit, perhaps current assets minus current liabilities for less established firms or in tight credit conditions, or even cash during financial panic. Thus, our measure of the probability of failure will differ depending on general credit conditions in the economy and on the credit resources of the firm. In general, $C = \bar{\lambda}_1 (\overline{\text{Assets}}) \text{ minus } (\bar{\lambda}_2) (\overline{\text{Liabilities}})$, where $\bar{\lambda}_1$ and $\bar{\lambda}_2$ are vectors of weights depending on the convertibility of different classes of assets to wealth and liabilities to negative wealth. Of course, there are some hidden assumptions as to the conversion transient period, but their consideration would lead us too far afield in a first simple model.

The remaining problems have to do with restrictions on and measures of σ and q/p . The approach taken in the following is very crude, but a serviceable beginning for the problem at hand.

Our first task is to estimate q/p . Intuitively, this parameter has something to do with the population mean "drift rate" of the firm's wealth. If $q/p = 0$, the firm will gain σ every time period. If $q/p = \infty$, the firm will lose σ every time period. If $q/p = 1$, there will be no tendency for the firm to either lose or gain wealth on the average, but the random-walk, even though directionless, will eventually bring failure.

On the other hand, in a real firm the observed drift rate would reflect the average rate of return on total capital invested.

We would like to estimate the parameters (q/p) and σ underlying the drift rate with a statistic based on the observed drift rate of a real firm.

In the random-walk model, the average drift rate per time period is $(p - q)\sigma$. A real-world measure of drift is $A\theta\delta\gamma$, where A is the total assets employed, θ is the average return on total assets per time period, $(1 - \delta)$ is the dividend pay-out rate, and $(1 - \gamma)$ represents the average fraction of net cash after dividends re-invested in illiquid capital expenditures.⁵

Thus we have, setting the two drift rates equal,

$$(p - q)\sigma = A\theta\delta\gamma$$

and also, since $p + q = 1$, we have

$$q/p = \frac{1 - (A\theta\delta\gamma/\sigma)}{1 + (A\theta\delta\gamma/\sigma)}$$

If we regard σ as roughly, though somewhat imperfectly, measureable by $\hat{\sigma}$,

⁵ $A\theta$ refers to a return after taxes. Throughout this paper, net cash flow is taken to be after taxes, and, of course, after interest.

the estimated standard deviation of the firms net cash flow less capital expenditures on illiquid assets less dividends per time period, we can now estimate $\left(\frac{q}{p}\right)^{\frac{C}{\hat{\sigma}}}$ in a real situation.⁶

Let us relabel

$$\frac{A\theta\hat{\sigma}Y}{\hat{\sigma}} \text{ as } x,$$

and

$$\frac{C}{\hat{\sigma}} \text{ as } y.$$

Then our estimate \hat{P} of P (Ultimate Failure) is the statistic

$$\left(\frac{1-x}{1+x}\right)^y.$$

Of course, this is an extreme simplification; it does not take into account the important effect of government tax asymmetries in the treatment of losses and gains, nor of the increasing size of $\hat{\sigma}$ as C increases. Also, in reality the net cash flow less illiquid capital expenditures less dividends is probably distributed as a continuous distribution more like a gamma distribution than a binomial, with a central peak at a modest positive value of cash inflow.

A simpler statistic for use in discriminating among degrees of risk in the upper (more risky) regions of the function may be approximated by the following algebraic manipulations.

If $\theta \leq 0$, the estimated probability of failure is 1. Otherwise, $\theta \geq 0$, and since $p - q \leq 1$, it will be true that $0 < x < 1$. Also $y > 0$. Suppose that $xy < 1$. That is,

⁶The accuracy of $\hat{\sigma}$ as a measure of our σ depends on the adequacy of the assumption that the mean drift rate is small in comparison with $\hat{\sigma}$.

$$\left(\frac{C}{\sigma}\right) \left(\frac{A\theta\delta\gamma}{\sigma}\right) < 1.$$

This will be the case for average to high risk firms. Then:

$$\left(\frac{1-x}{1+x}\right)^y = \left(1 - \frac{2x}{1+x}\right)^y.$$

By the binomial theorem, if $|t| < 1$, $a \geq 0$,

$$(1+t)^a = 1 + \binom{a}{1}t + \binom{a}{2}t^2 + \dots + \binom{a}{a}t^a,$$

where $\binom{a}{2} = \frac{a!}{2!(a-2)!}$, etc.

Thus

$$\left(\frac{1-x}{1+x}\right)^y = 1 + \binom{y}{1}\left(\frac{-2x}{1+x}\right) + \binom{y}{2}\left(\frac{-2x}{1+x}\right)^2 + \dots + \binom{y}{y}\left(\frac{-2x}{1+x}\right)^y.$$

Under the assumptions given above, this series will be dominated by the first few terms, giving

$$\left(\frac{1-x}{1+x}\right)^y \approx 1 - \frac{2xy}{1+x} + \frac{2x^2y(y-1)}{(1+x)^2} - \dots.$$

In the region of very high risk, $x \ll 1$, $xy \ll 1$, the following approximation will be an adequate discriminator:

$$\left(\frac{1-x}{1+x}\right)^y \approx 1 - 2xy$$

This yields

$$\hat{P}(\text{Ultimate Failure}) \approx 1 - 2 \frac{A\theta\delta\gamma C}{\sigma^2}$$

Pertinent Financial Ratios

We might use average reported net income after taxes as an estimator for $A\theta$. Then our proposed hypothetical financial ratio for discriminating between very high risk and lower risk firms would be simply xy , where

$$x = \frac{(\text{avg. net income}) (1 - \text{dividend payout ratio}) (1 - \text{avg. proportion of net cash flow less dividends re-invested in illiquid assets})}{\text{standard deviation of [net cash flow less capital expenditures for illiquid assets and less dividends]}}$$

and

$$y = \frac{\bar{\lambda}_1(\text{assets}) - \bar{\lambda}_2(\text{liabilities})}{\text{standard deviation of [net cash flow less capital expenditures for illiquid assets and less dividends]}}$$

Comparison with Beaver

Ignoring the information content of various components of the rather complex ratio suggested above yields the ratios found by Beaver to have predictive value. There are two major components: the first is x , a measure of the ratio of the observed drift rate to the standard deviation of that drift rate; the other is y , a measure of the ratio of the liquid wealth of the firm to what in some sense is the modal magnitude of setbacks in the drift, which latter we have somewhat crudely measured as the standard deviation of the drift rate. This last point relies on the standard deviation of the drift rate being large in comparison with the mean drift rate.

If we ignore the information in x , and also ignore the differences between firms in relative variability of the net cash flow less dividends and less capital

expenditures on illiquid assets, we get the discriminant statistic

$$\frac{\overline{\lambda_2(\text{liabilities})}}{\text{net cash flow}},$$

which is markedly similar to Beaver's best single ratio predictor. Other ratios such as $\frac{\text{cash flow}}{\text{assets}}$ (a component of y), net income/total assets (a component of x), and even total debt/total assets (a measure of [total debt/cash flow] given reasonably constant ratios of [cash flow/assets]) can all be linked to the underlying model.

Comparison with Tinsley

Using a normal distribution of the drift rate per time period, Tinsley gets the following results for an all-equity firm.

Defining C as the "cash reserve" (similar to our use of C), x_i as the "cash flow" (this is not strictly equivalent to accounting cash flow, but is similar to our use of net cash flow less dividends and capital expenditures on illiquid assets) of period i, \bar{x} as the mean x_i , and σ_x as the standard deviation of x_i , Tinsley obtains as the likelihood of ultimate failure the expression

$$e^{-\frac{2Cx}{(\sigma_x^2)}}$$

Now if this likelihood is large (high risk), one gets the approximation

$$1 - \frac{2C}{\sigma_x} \frac{x}{\sigma_x}$$

which corresponds to the results here, where his $\frac{x}{\sigma_x}$ corresponds to our



$\frac{A\theta\delta\gamma}{\hat{\sigma}}$ and his $\frac{C}{\sigma}$ corresponds to our $\frac{C}{\hat{\sigma}}$.

Better Predictive Results

One would expect that the major improvements in discriminant ability between high and lower risk firms would come through refinements in measures of C , $\hat{\sigma}$, and $A\theta\delta\gamma$ to take into account more precisely some of the differences between firms assumed away in the previous section. For example, different convertibility weights for the various classes of assets and liabilities, λ_1, λ_2 , could be estimated depending on the time horizon of the estimate. Measures of $\hat{\sigma}$ and $A\theta\delta\gamma$ might be better as exponential weighted averages of past performance rather than as their maximum likelihood estimates from a fixed sample -- that is, an adaptive model rather than one assuming a stationary process could be usefully attempted.

More sophisticated models developed along the lines indicated by Tinsley's work may take into account the precise shape of the probability distribution of the cash flow. Intuitively, for any given $\hat{\sigma}$, negative skew increases risk; positive skew decreases it. However, the extent of this and also the effect of higher moments is not so intuitive.

Directions for Research

A key component of the risk is, as we have seen, $\hat{\sigma}^2$, closely related to the variance of the net cash flow after taxes less dividends and less expenditures on illiquid assets. We have assumed that this parameter is at least relatively stable over time. If we can predict changes in this parameter, we could improve the total

risk prediction; we might do this by paying attention to three components of this variance:

- a. the variance of the cash inflows
- b. the variance of the cash outflows
- c. and especially the covariance of the inflows and outflows.

Another very different approach is, of course, to make use of auto-correlation properties of the cash flows to predict actual cash flow over the period at risk, but this would probably not be very feasible at present because of the great effort required if one desires cross-firm comparison.

A bank trying to construct an optimal portfolio of loans would need also to obtain estimates of the cash flow covariances among firms.

Most important, empirical studies measuring the variables and testing the hypotheses herein suggested are essential for practical use. The present paper essentially is merely a reflective comment on Beaver's data.

Conclusion

A simple theoretical model offering an explanation of Beaver's empirical results was presented. The model suggests hypotheses as to improved predictors of financial failure.

Better predictors will aid lenders and investors in selecting attractive risk situations and in determining appropriate risk premiums.

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