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MICROWAVE FILTERS USING QUARTER WAVE COUPLINGS

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TECHNICAL REPORT NO. 8

JUNE 28, 1946

NDRC DIVISION 14
RESEARCH LABORATORY OF ELECTRONICS
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Contract OEMsr-262

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MICROWAVE FILTERS USING QUARTER WAVE COUPLINGS

by

R. M. Fano and A. W. Lawson*

Abstract

This paper presents a method of designing band-pass and band rejection microwave filters by appropriately transforming lumped-element filters. Such microwave filters are realized physically as chains of resonant elements (either cavities or irises) coupled by quarter-wave sections of line. A structure of this type has a number of practical advantages over other types of filter structures, such as a chain of directly coupled cavities, because it permits the construction of the resonant elements as separate units, and yields more liberal tolerances on the dimensions of the coupling irises.

*Now at the Institute for the Study of Metals, University of Chicago, Chicago, Ill.

Title Page
21 numbered pages
18 drawings

MICROWAVE FILTERS USING QUARTER WAVE COUPLINGS

Introduction

The design of a filter having specified frequency characteristics is a problem of network synthesis. A number of synthesis procedures have been developed in the last two decades,[1, 2, 3] which are applicable to lumped-element networks, that is to networks consisting of inductances, capacitances and resistances. In microwave systems, however, these lumped elements cannot be used for obvious reasons and must therefore be replaced by more suitable elements such as sections of transmission lines or waveguides, cavity resonators, resonant irises, etc. Unfortunately these microwave elements behave with frequency in such a complex manner that the development of direct synthesis procedures applicable to microwave networks seem quite remote. On the other hand, if one is interested in a limited frequency band, the same microwave elements can be made to approximate rather well the frequency behaviors of lumped elements, either alone or in resonant combinations. On this basis, one is lead to consider the possibility of designing microwave filters by appropriately transforming lumped-element filters.

In addition to theoretical limitations on filter design, the actual systems applications and the manufacturing techniques available often impose widely varying geometrical and physical requirements. These requirements are largely responsible for the ultimate appearance of microwave filters, and they also determine the practical usefulness of any design procedure. It follows that no single design procedure can be expected to be applicable in all cases and that the solution of any specific problem depends largely on the ingenuity of the designer. This paper will discuss a particular design procedure which is applicable in a variety of cases, and meets most of the usual practical requirements. This procedure is limited to the design of band-pass and band-elimination filters; fortunately, however, filters of other types are seldom needed in microwave systems.

General Theory of Quarter-Wave Coupling

The method of design discussed below leads to filters consisting of a chain of resonant elements, such as cavity resonators, linked by quarter-wave sections of line. This chain-like structure is obtained by appropriately transforming a lumped-element ladder structure which is characteristic of most practical low-frequency filters. To understand the mechanism of the transformation, consider first a quarter-wave lossless line terminated in a normalized* impedance Z. The

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1. Guillemin, E. A., "Communication Networks" Vol. II, John Wiley and Sons, New York, 1935.
2. Bode, H. W., "Network Analysis and Feedback Amplifier Design", D. Van Nostrand Co., New York, 1945.
3. Darlington, S., "Synthesis of Reactance 4 Poles", Journal of Mathematics and Physics, Vol. XVIII, p. 257-353, Sept. 1939.

* The normalized impedance is the ratio of the actual impedance to the characteristic impedance of the line. The normalized admittance is defined in a similar manner. Normalized impedances and admittances are used throughout this paper in connection with microwave structures, although the word "normalized" will be dropped in most instances for the sake of brevity.

normalized input impedance of such a line is:

$$Z' = \frac{Z + j \tan\frac{\pi}{2}}{1 + j Z \tan\frac{\pi}{2}} = \frac{1}{Z} \quad . \qquad (1)$$

It follows that the circuits of Figure 1 are equivalent if the normalized impedance Z' is numerically equal to the normalized admittance Y and if both lines are one-quarter of a wavelength long. In fact the two networks have the same open-circuit and short circuit impedances.

Consider now the structures of Figure 2, in which Y_2, Y_4, Y_6, etc. are numerically equal, respectively, to Z_2', Z_4', Z_6', etc. and the coupling lines are all identical. These two networks are exactly equivalent at the frequency ω_o for which the coupling lines are one-quarter wave long. At other frequencies, however, the equivalence does not hold, but the characteristics of the two networks will not differ appreciably over a frequency band small compared to ω_o. Therefore, in the following discussion, we shall neglect the frequency dependence of the electrical length of the line and assume that the two structures of Figure 2 are exactly equivalent. The error introduced by this approximation when the frequency band of interest is relatively large will be discussed later.

To proceed from this point one must be more specific about the elements of the general structures of Figure 2. We shall consider the particular case of the ladder structure shown in Figure 3a in which the series branches are series tuned circuits resonating at the frequency ω_o, and the shunt branches are parallel tuned circuits also resonating at the frequency ω_o. This structure is characteristic of band-pass filters whose insertion loss in the attenuation band is a monotonic function(3); for instance, constant-k band-pass filters,(1) are of this type. Filters of the m-derived type, and band elimination filters will be considered later.

Suppose then that a lumped element filter of the type shown in Figure 3a has been designed following any one of the available methods of network synthesis. The corresponding filter employing quarter wave coupling lines is shown in Figure 3b. The next step in the design procedure is to substitute for each tuned circuit a microwave element which behaves approximately in the same manner over the frequency band of interest. Such an element may be a cavity resonator, a resonant iris or a shorted section of line depending on the particular application and on the bandwidth of the filter.

It can be shown(4) that a cavity resonator with an input and an output line can be represented, approximately, in the vicinity of one of its resonant frequencies by the two equivalent circuits of Figure 4. The reactances X_1 and X_2 are very closely equal to the input and output reactances when the cavity is detuned. In most practical cases they are very small and can be neglected to a first approximation. The other parameters of the equivalent circuits are usually expressed in terms of the "loaded Q's" of the cavity defined as follows. Q_{L1} is the Q of the cavity when a resistance equal to the characteristic impedance Z_{01} of the input line is connected

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4. Schelkunoff, S. A., "Representation of Impedance Functions in Terms of Resonant Frequencies", I.R.E. Proc. Vol. 32, No. 2, February 1944.

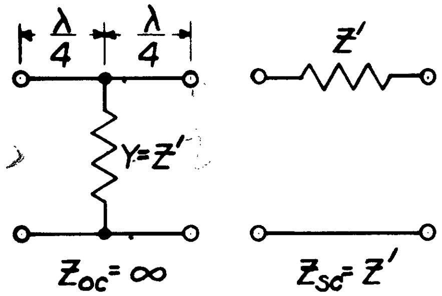

FIG. 1

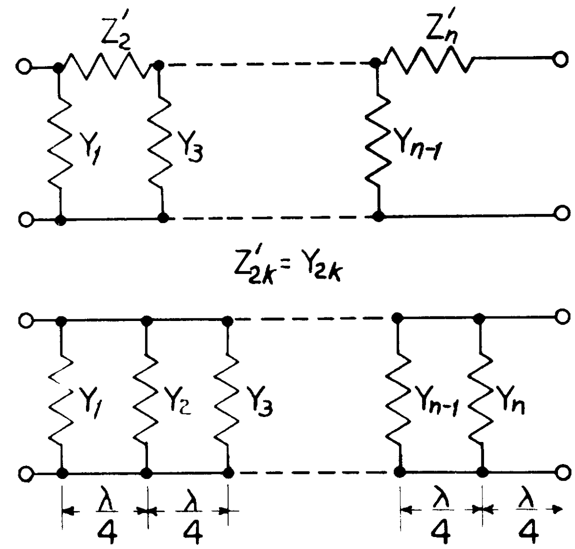

FIG. 2

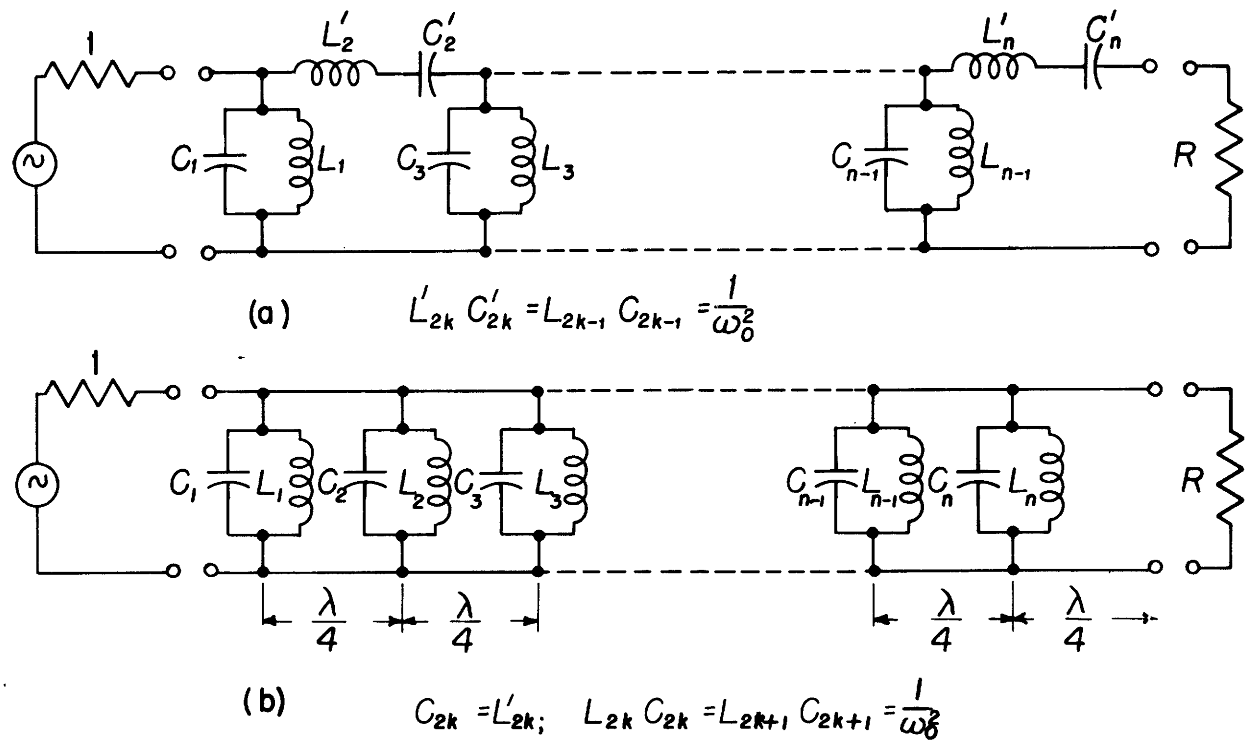

FIG. 3

to the input terminals and the cavity itself is assumed to be lossless; Q_{L2} is defined in a similar manner when a resistance equal to the characteristic impedance Z_{02} of the output line is connected to the output terminals. Finally Q_L is the Q of the cavity when both pairs of terminals are loaded by the characteristic impedances of the input and output lines. The unloaded Q[5,6] is the Q resulting from the losses in the cavity itself. According to these definitions and neglecting X_1 and X_2, one obtains for the loaded Q's of the cavity:

$$Q_{L1} = \omega_o C Z_{01} \tag{2}$$

$$Q_{L2} = \omega_o C \frac{Z_{02}}{a^2} = \omega_o C \left(\frac{M_1}{M_2}\right)^2 Z_{02} \tag{3}$$

$$Q_L = \frac{Q_{L1} Q_{L2}}{Q_{L1} + Q_{L2}} \tag{4}$$

In the particular case of a symmetrical cavity one has:

$$Q_{L1} = Q_{L2} = 2Q_L = \omega_o C Z_o \tag{5}$$

and the equivalent circuit reduces to a simple parallel tuned circuit in shunt to the line. Note that CZ_o is the normalized capacitance of the resonant circuit.

It follows from the above discussion that each resonant branch of the structure shown in Figure 3b can be physically realized by means of a symmetrical cavity resonator whose loaded Q is given by:

$$(Q_L)_k = \frac{1}{2}\omega_o C_k \tag{6}$$

where C_k is the normalized capacitance of the resonant branch. The only difficulty which may arise in this transformation concerns the terminating resistance which, in general, is different from the line impedance. Since it is desirable to match the filter to the output line, the last element of the structure is further transformed as shown in Figure 5 to obtain the proper change of impedance level. The corresponding cavity resonator is then no longer symmetrically coupled, and in this case the loaded Q's are given by:

$$Q_{L1} = C_n \omega_o \tag{7}$$

$$Q_{L2} = \frac{C_n}{R}\omega_o \quad . \tag{8}$$

If the last element of the structure of Figure 3a were a shunt branch, one would obtain:

$$Q_{L1} = C_n \omega_o \tag{9}$$

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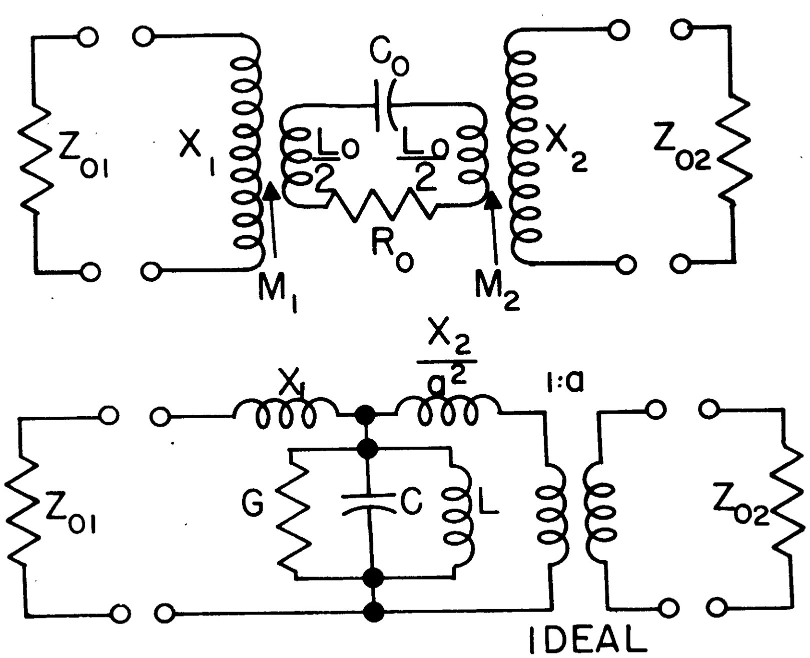
5. Staff M. I. T. Radar School, "Principles of Radar", The Technology Press, Cambridge, Mass., 1944
6. Sarbacher, R. I. and Edson, W. A., "Hyper and Ultrahigh Frequency Engineering", John Wiley and Sons, New York, 1943.



$C_0L_0 = CL = \dfrac{1}{\omega_0^2}$

$C = \dfrac{L_0}{\omega_0^2 M_0^2}$ $\quad a = \dfrac{M_2}{M_1}$

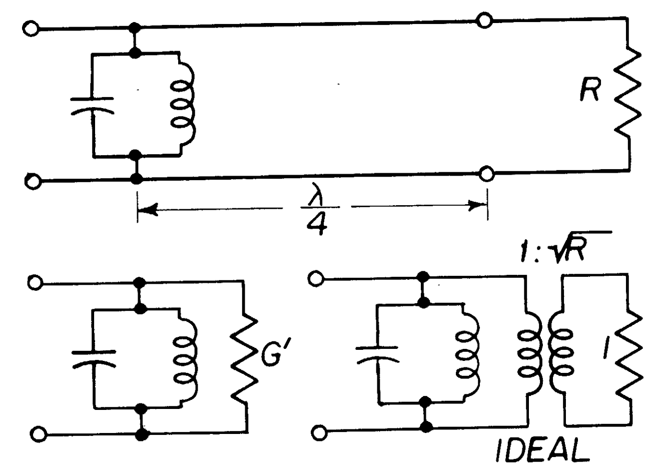
$Q_0 = \dfrac{\omega_0 L_0}{R_0} = \dfrac{\omega_0 C}{G}$

$Q_{L1} = \omega_0 C Z_{01}$

$Q_{L2} = \omega_0 C \dfrac{Z_{02}}{a^2}$

$Q_L = \dfrac{Q_{L1}\, Q_{L2}}{Q_{L1}+Q_{L2}}$

FIG. 4



$G' = R$

FIG. 5

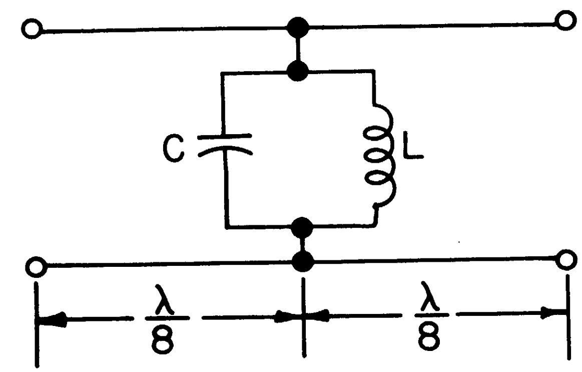


FIG. 6

$$Q_{L2} = RC_n\omega_o \qquad (10)$$

Changes of impedance level similar to the one just described can be performed at other points of the structure, if so desired.

In the case of waveguide filters, resonant irises[57] can be used instead of cavity resonators to approximate the behavior of the resonant branches of Figure 3b. However, since resonant irises have relatively low unloaded Q's they are not used in connection with narrow band filters. On the other hand, in the case of bandwidths of a few percent, the use of resonant irises instead of cavities results in a considerable saving of space and weight. The design procedure for such filters is the same as for cavity filters, although no change of impedance level can be performed in this case because resonant irises are inherently symmetrical.

Filters Employing Identical Elements

The construction of a microwave filter can be considerably simplified by making all the resonant elements identical. Such a design is not optimum in the sense that better filter characteristics could be obtained with the same number of cavities, but very often the resulting simplicity of construction is worth the loss of performance.

The power-loss ratio of a filter consisting of a number of identical resonant elements can be easily computed by noting that such a filter is a cascade connection of n identical sections of the type shown in Figure 6. The susceptance of the shunt branch can be expressed in terms of the loaded Q as follows:

$$B = 2Q_L\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right) = 2x \qquad (11)$$

where x is a normalized frequency variable which is zero at the resonance frequency and becomes ± 1 at the two half power frequencies. By making use of image-parameter theory[1,8] one obtains for the power-loss ratio:

$$\left(\frac{P_o}{P_L}\right)_n = 1 + x^2U_n^2(x) \qquad (12)$$

where the function $U_n(x)$ is a Tschebyscheff polynomial of the second kind and order n. The polynomials corresponding to n = 1, 2, 3, 4 are given below with a recurrence formula from which they may be successively derived.

$$\begin{aligned} U_1(x) &= 1 \\ U_2(x) &= 2x \\ U_3(x) &= 4x^2 - 1 \\ U_4(x) &= 8x^3 - 4x \end{aligned} \qquad (13)$$

$$U_{n+1}(x) = 2xU_n - U_{n-1} \qquad (14)$$

Plots of the insertion loss are shown in Figure 7 for n = 1, 2, 3, 4. These curves

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7. Leiter, H. A., "A Microwave Band-Pass Filter in Waveguide," M. I. T. Radiation Laboratory Report No. 814.
8. Richards, P. I., "Applications of Matrix Algebra to Filter Theory", I. R. E. Proc. Vol. 34, No. 3, March, 1946.

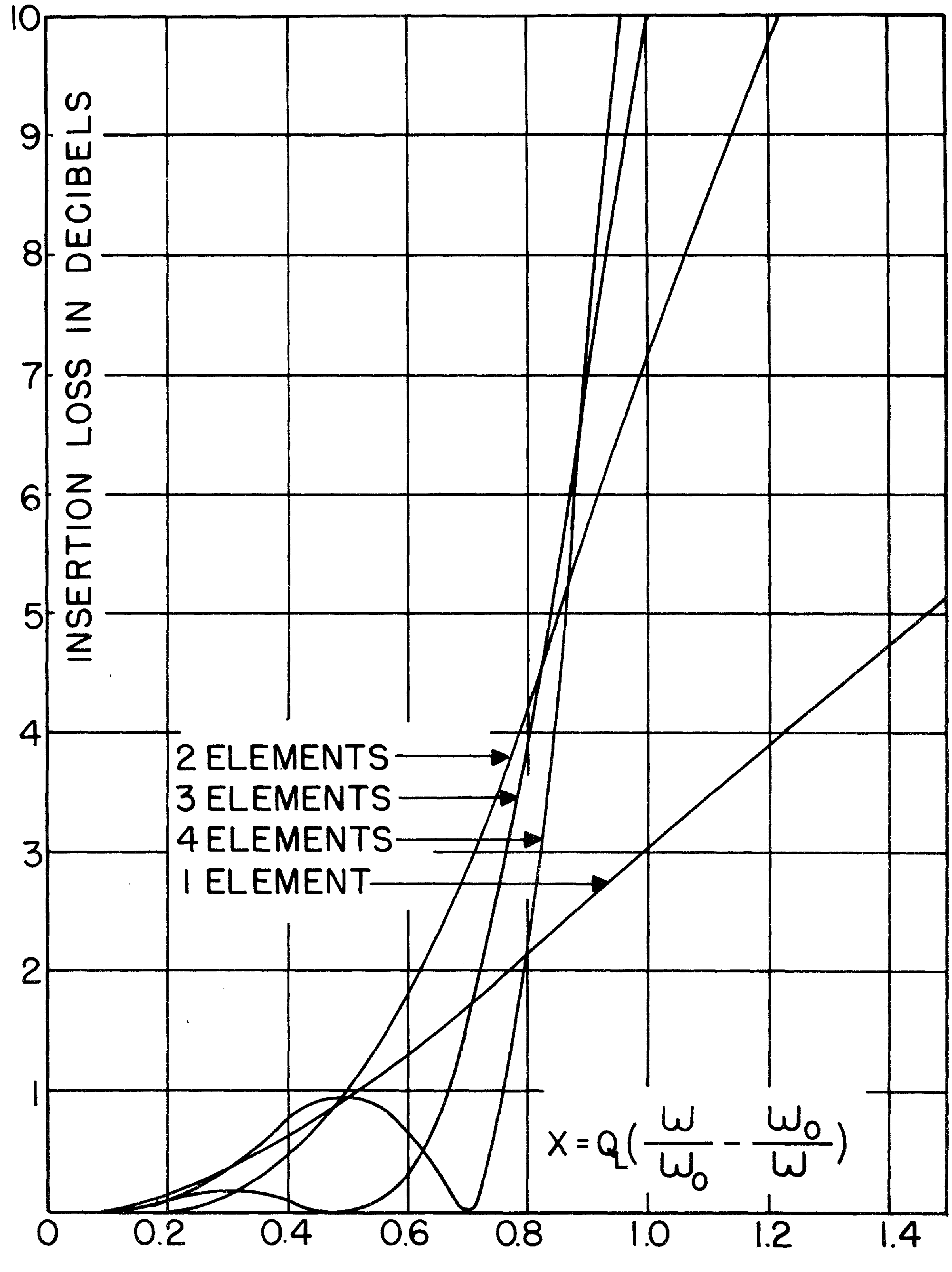
INSERTION LOSS IN DECIBELS
10
9
8
7
6
5
4
3
2
1
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
2 ELEMENTS
3 ELEMENTS
4 ELEMENTS
1 ELEMENT
$X = Q_L\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$

FIG. 7

show that the pass-band tolerance increases with n, consequently values of n larger than 4 are seldom used. For large values of x, the off band insertion loss becomes approximately

$$L_D \approx 6(n - 1) + 20n \log x \text{ db} . \qquad (15)$$

Practical Design Considerations

The main advantage of quarter wave coupled filters is that the resonant elements can be built, tested and tuned separately. Only minor tuning adjustments are necessary after assembly since the cavities can be joined to each other by means of standard connectors[5] without need of any further soldering. This great advantage results from the fact that the coupling lines are non-resonant and, therefore, their continuity can be broken by standard connectors without causing appreciable additional loss. Such a construction would not be possible, on the other hand, if the lines were an integral number of half-waves long.

The structure employing quarter-wave coupling lines is not the only microwave structure equivalent to the lumped-element ladder network. The ladder network of Figure 3, for instance, can be transformed[1] into a chain of resonant loops tuned to the same frequency and coupled to one another by mutual inductances. The microwave realization of this structure consists of a chain of directly coupled cavity resonators. Filters of this type, however, must be built in a single unit, and moreover, require closer machining tolerances than the corresponding filters employing quarter-wave coupling lines. It can be shown, in fact, that, in the case of directly coupled cavities, the coupling susceptances are equal to the square of the coupling susceptances required when the cavities are quarter-wave spaced. It must be pointed out, however, that directly coupled filters are superior from the point of view of space and weight requirements. This advantage may outweigh, in some instances, the construction difficulties mentioned above.

In connection with the design of quarter-wave coupled filters, the determination of the effective location of the terminals of a cavity on the input and output lines requires further explanations, particularly in the case of loop couplings to coaxial lines. Such a determination can be performed experimentally with the help of a standing-wave detector by finding the position of the voltage zero in the line when the cavity is detuned. Under these conditions the input terminals of the cavity are effectively short circuited so that their location must be an integral number of half wavelengths from the position of any voltage zero. This experimental determination may be done in such a way as to include the equivalent line length of whatever fittings are used to connect the cavity to the coaxial line. Moreover, the locations of the terminals are automatically shifted to compensate approximately for the presence of the reactances X_1 and X_2 of Figure 4, which have been neglected in the theoretical analysis.

The effective length of the coupling lines may be increased to $\frac{3}{4}\lambda$ or $\frac{5}{4}\lambda$ when larger mechanical spacing between cavities is required. These longer lines are, of course, more frequency sensitive but their frequency behavior can still be neglected in most practical cases. The lengths of the coupling lines are not critical, as one would expect from the fact that the lines are not resonant. It was found in practice that the characteristics of a narrow-band two-cavity filter were not appreciably changed by variations of the coupling line length as large as $\lambda/40$.

The actual characteristics of a number of quarter-wave coupled filters were found to be very close to those predicted apart from the effect of incidental dissipation which was neglected in the theoretical analysis. The theoretical and measured characteristics of a four-cavity filter are compared in Figure 12. The main effect of dissipation is a finite loss in the pass band, approximately equal, in the case of n identical cavities, to n times the loss of one cavity; the insertion loss for a symmetrical cavity at resonance is given by:

$$L_o = 20 \log \left(1 + \frac{Q_L}{Q_o}\right) \text{ db} \tag{16}$$

In view of the fact that the bandwidth of a filter is inversely proportional to the loaded Q's of the cavities employed, the effect of incidental dissipation becomes increasingly important in narrow-band filters. For detailed computations of the effects of incidental dissipation the reader is referred to the literature on this subject in the case of lumped element filters.[1,2,3].

The value of Q_o for a cavity is limited, in practice, by space considerations as well as by the requirement that the cavity must approximate the behavior of a simple resonant circuit over a specified frequency band. The unloaded Q_o is roughly proportional to the ratio of the volume of the cavity to its surface area,[5,6] so that Q_o increases with the volume. On the other hand, an increase of volume always results in a reduction of the separation between resonance frequencies, and therefore in a reduction of the useful frequency band. In general this fact sets up an upper limit on the size of the cavity and consequently on the unloaded Q for a given surface material.

A discussion of the design of cavity resonators is beyond the scope of this paper. It will suffice here to describe two types of cavities which are particularly suitable for use in microwave filters. The first type of cavity, shown in Figure 8, has loop coupled coaxial terminals. The loaded Q of such a cavity is roughly proportional to the square of the area of the coupling loop projected over a plane normal to the magnetic lines of force[5]. It follows that the loaded Q can be varied by simply rotating the coupling loops. The cavity may be tuned over a broad band by varying the length of the center conductor. Figure 8 is scaled from the dimensions of a cavity designed to operate over a 10% band in the neighborhood of 3000 mc. The Q_o of such a cavity is about 5000 and the loaded Q for the loop size shown is about 500.

A cavity particularly suitable for waveguide filters is shown in Figure 9. The proper distance between the two inductive irises, for a given resonance frequency, depends on the dimensions of the irises, and it is always smaller than one half of the guide wavelength. In practice this distance is made a little shorter than required and the cavity is then tuned to the desired frequency by means of the screw shown in Figure 9. The loaded Q and the length ℓ of a symmetrical cavity of this type can be computed by means of the equations:

$$Q_L = \frac{1 + b_o^2}{4} \left(\frac{\lambda_g}{\lambda_o}\right)^2 \tan^{-1} \frac{2b_o}{b_o^2 - 1} \tag{17}$$

$$\ell = \frac{\lambda_g}{2\pi} \tan^{-1} \frac{2b_o}{b_o^2 - 1} \tag{18}$$

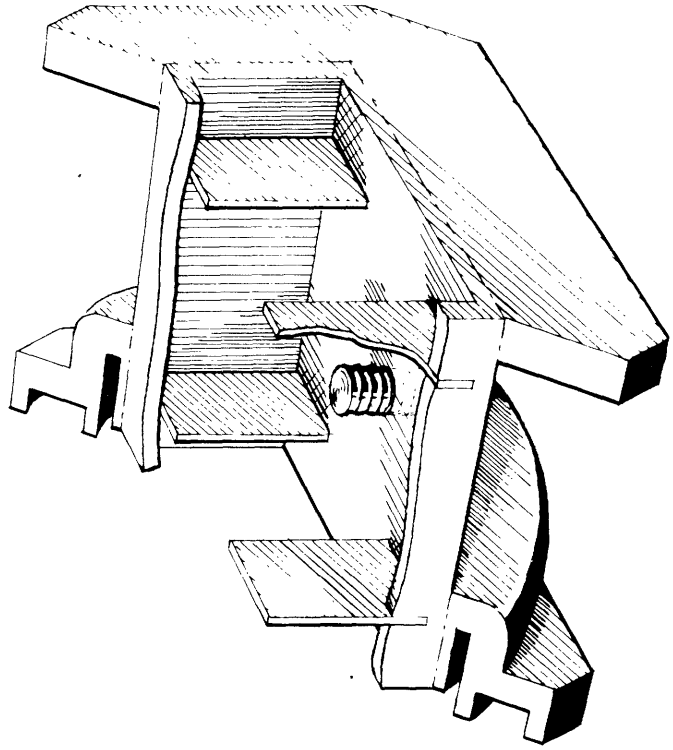

FIG. 9

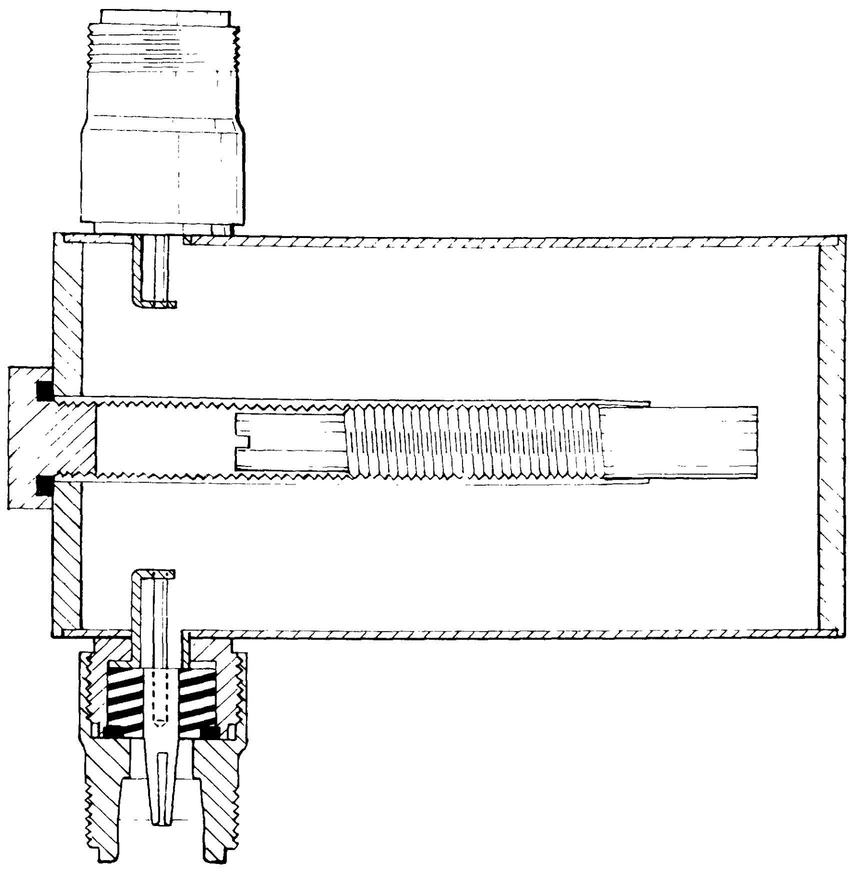

FIG. 8

where b_o, λ_o and λ_g are, respectively, the normalized susceptances of the irises, the resonance wavelength and the corresponding guide wavelength[5,6]. The cavity is provided with standard choke and flange connectors[5] spaced a distance equal to $\lambda_g/8$ from the two irises so as to yield the desired quarter-wave coupling when two cavities are connected together. In the case of waveguide filters, of course, the coupling lines must be $\lambda_g/4$ long rather than $\lambda/4$.

In Figure 10 is shown an experimental filter consisting of 4 identical brass cavities tuned to a mean wavelength λ_o = 3.308 cms; the dimensions of the cavities are given in Figure 11. The effective coupling susceptance of the inductive irises is 9.3, as obtained by means of a formula developed by Marcuvitz of the M.I.T. Radiation Laboratory[9]. The corresponding Q_L for each cavity is equal to 133, according to Equation 17. The proper length of the cavity, as given by Equation 18, should be approximately 0.880"; the actual length was made 0.050" shorter to permit screw tuning.

The theoretical curve for the insertion loss of the four-cavity filter neglecting dissipation is plotted in Figure 12 as curve B. The transmission loss of the individual cavitieswas measured at resonance and found to be approximately 0.44db. This value, according to Equation 16, corresponds to a ratio Q_L/Q_o = 0.05 and, therefore, to an unloaded Q_o equal to 2660. It must be pointed out in this connection that the actual Q_o of a cavity may differ considerably from the theoretical value because of the additional losses resulting from the presence of the coupling irises and of the tuning screw. In this particular case, for instance, the theoretical Q_o is almost twice the measured value.

Curve A in Figure 12 is a plot of the theoretical insertion loss in the presence of dissipation for a ratio Q_L/Q_o = 0.05. The crosses in the same figure represent measured values. In considering the rather large effect of dissipation one must remember that no attempt has been made in the design of this filter to reduce the losses. Silver plating alone would increase Q_o by a factor of approximately 2, thus reducing the mid-band loss by approximately the same factor. Further improvement could be obtained by reducing the losses in the tuning device.

The machining tolerance on the apertures of the irises was $\pm$0.003". Since the bandwidth of the filter is roughly proportional to the square of the iris susceptance and, therefore, to the fourth power of the iris aperture, the corresponding tolerance on the bandwidth is approximately $\pm$4%. It is interesting to note in this respect that the same machining tolerance in the case of a directly coupled filter with the same characteristics would yield a bandwidth tolerance of approximately $\pm$8%.

In many cases where loaded Q's of less than 30-40 are required, it is possible to reduce the size and weight of a filter by replacing the cavity resonators with resonant irises[5,7]. These irises, behave like paralle-tuned circuits in shunt to the line, and, therefore, may be considered as combinations of inductive and capacitative irises. Several types of such irises may be used, the most common of

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9. M.I.T. Radiation Laboratory Wave Guide Handbook, Report 43-2/7/44, Sections 2-1, 21-a, 26-a.

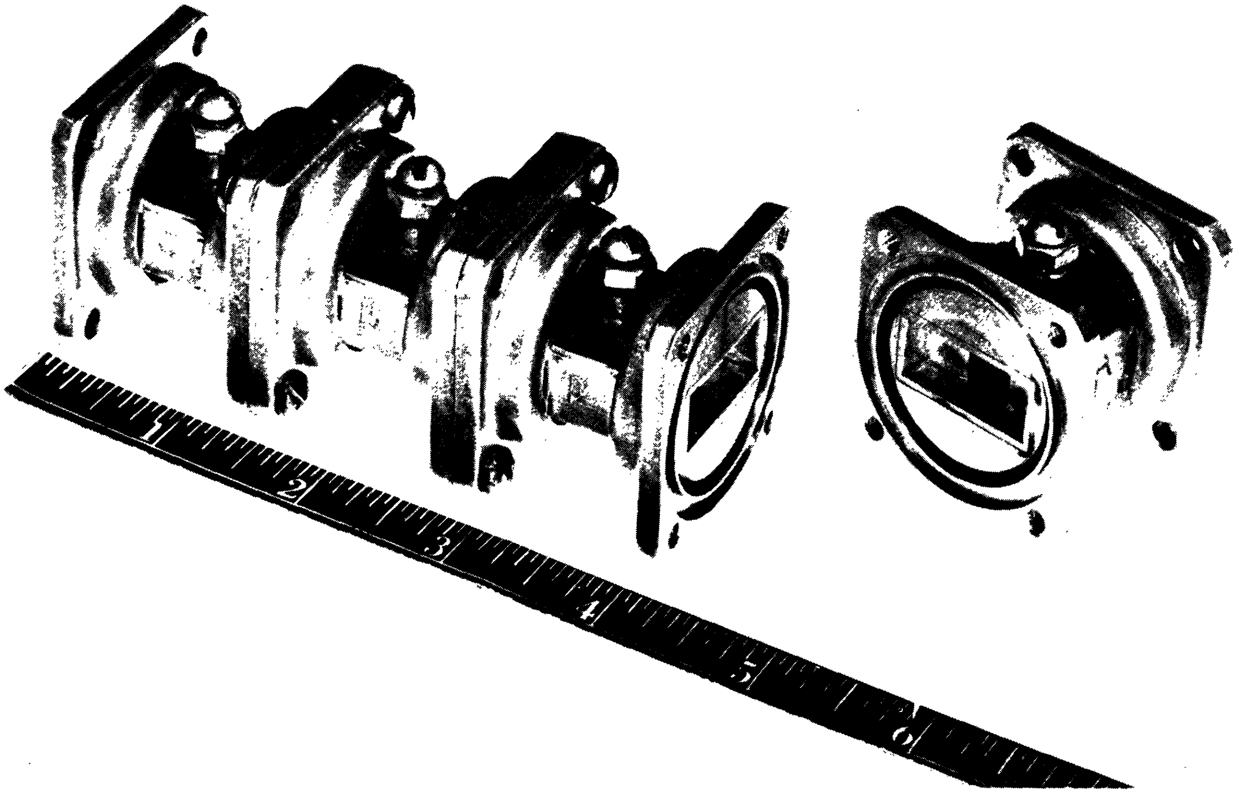

FIG. 10

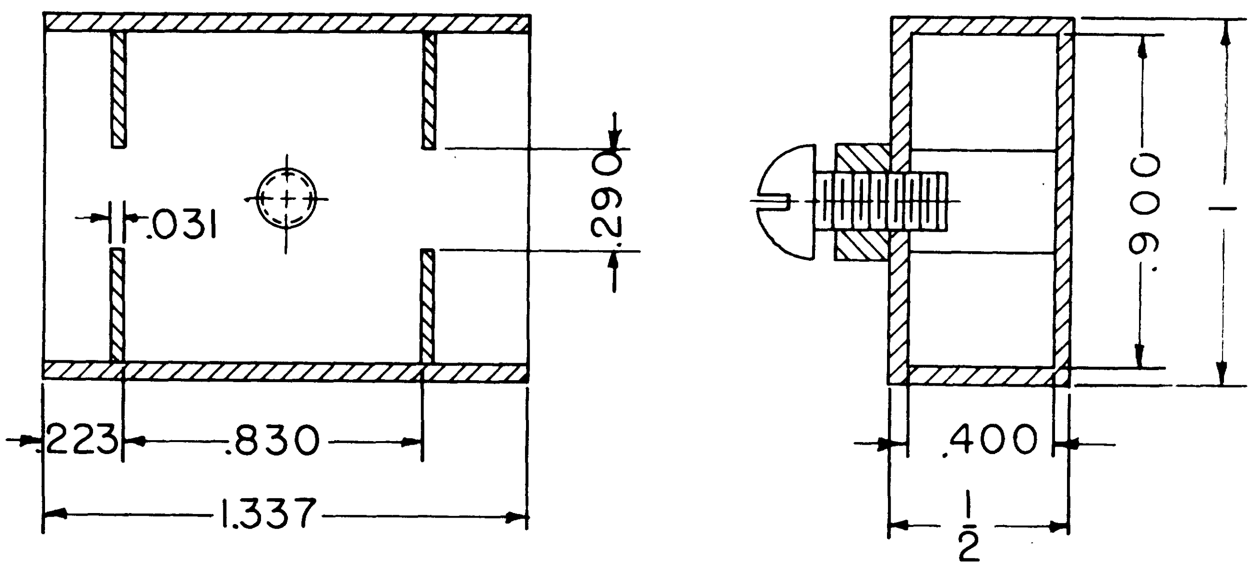

FIG. 11

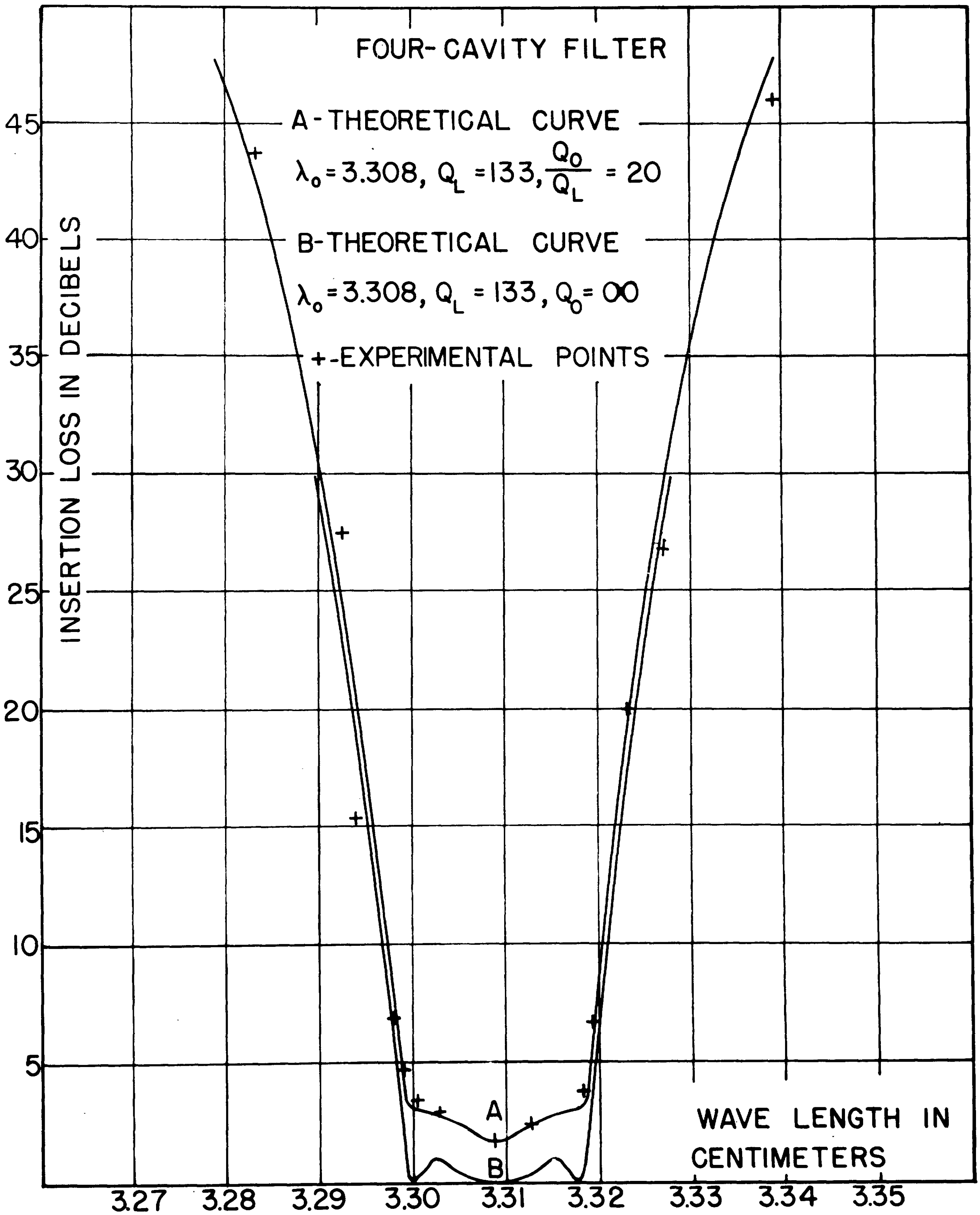
FOUR-CAVITY FILTER
A-THEORETICAL CURVE
$\lambda_0 = 3.308,\ Q_L = 133,\ \frac{Q_0}{Q_L} = 20$
B-THEORETICAL CURVE
$\lambda_0 = 3.308,\ Q_L = 133,\ Q_0 = \infty$
+-EXPERIMENTAL POINTS
INSERTION LOSS IN DECIBELS
WAVE LENGTH IN CENTIMETERS
A
B
45
40
35
30
25
20
15
10
5
3.27
3.28
3.29
3.30
3.31
3.32
3.33
3.34
3.35

FIG. 12

which are illustrated schematically in Figure 13. The saving in space and weight

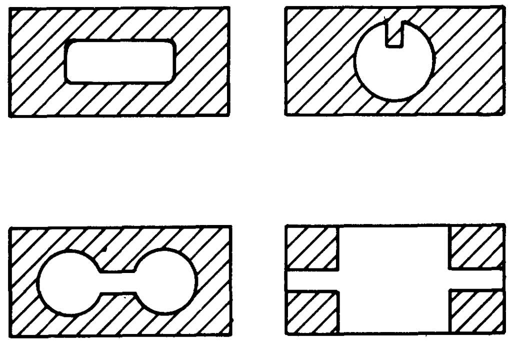

FIG. 13

associated with the use of a resonant iris results from the fact that the waveguide stores the energy associated with the iris, while still performing its function as a link to other components of the system. However, since the ratio of the effective volume to the surface is always smaller for an iris than for a cavity with the same loaded Q, the unloaded Q of the iris will always be smaller. Thus, the use of resonant irises is restricted to values of loaded Q sufficiently small to keep the transmission loss within reasonable limits. On the other hand, the design of cavities becomes rather difficult when small loaded Q's of the order of magnitude of 50 are desired because the coupling elements become a major portion of the cavity. It follows that resonant irises are complements to cavities rather than substitutes.

One of the outstanding uses of resonant irises is in the construction of broad band TR tubes[5]. The technique of quarter-wave coupling a number of identical resonant irises was first used by Fiske[10,11] of General Electric in the design of these tubes before the authors considered its application to cavity resonators and to the design of microwave filters. TR boxes and filters[7] employing identical irises were also designed at the Radiation Laboratory in the section headed by L. D. Smullin.

Further Theoretical Considerations

The design of a microwave filter involves first of all, as pointed out above, the synthesis of a lumped element structure with specified frequency characteristics. A discussion of the different synthesis procedures available for this purpose is certainly beyond the scope of this paper. However, a few remarks should be made on this subject because of their bearing in the particular case of microwave filters.

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10. Fiske, M. D., "A Broad-Band TR Switch", General Electric Report 10/18/43.
11. Fiske, M. D., "Characteristics of Single and Multiple Tuned Circuits", General Electric Report 5/25/45.

The characteristics of microwave filters must often satisfy special requirements resulting from the peculiarities and limitations of microwave systems. Two of these requirements have particular bearing on the theoretical part of the design of a filter. The first of them results from the fact that, at present, low-noise microwave amplifiers are not available and, therefore, the loss in the radio-frequency part of any microwave system must be kept to a minimum. This requirement excludes at once the use of dissipative filters and puts a premium on any reduction of incidental dissipation and reflection losses. The second requirement results from the fact that in general microwave generators[5] do not operate satisfactorily when the voltage standing wave ratio in the output line is larger than a certain value, in many cases as small as 1.5. Even assuming negligible reflection in the other components of the system, a maximum standing wave ratio of 1.5 which corresponds to 4% power reflection places a very stringent limitation on the pass-band characteristics of a filter.

When such stringent specifications on the electrical characteristics of a filter are added to the usual space and weight limitations, it becomes necessary to make the best possible use of any given number of resonant elements. In the simplest case, the optimum design corresponds to an insertion loss function which oscillates in the pass band between zero and a specified value, all the maxima being equal. It becomes evident, at this point that the familiar method of filter design based on the specification of image parameter functions is not satisfactory, since this method does not take into account the loss resulting from mismatched terminations. To overcome this limitation in filter design Darlington, of the Bell Telephone Laboratories, developed a direct synthesis procedure which permits the direct synthesis of non-dissipative networks having specified insertion loss characteristics. For a general discussion of this method of filter design, the reader is referred to the original paper by Darlington.[3] It seems worthwhile, however, to mention briefly the optimum design of filters of the type shown in Figure 3.

The power loss ratio of a band-pass ladder structure of the type shown in Figure 3 has always the functional form

$$\frac{P_o}{P_L} = 1 + P(x) \qquad (19)$$

where $P(x)$ is an even polynomial and x is a frequency variable normalized with respect to the mean frequency ω_o and the bandwidth w measured between the frequencies for which $x = \pm 1$.

$$x = \frac{\omega_o}{w}\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right) \qquad (20)$$

The degree of $P(x)$ is equal to twice the number of resonant elements in the ladder structure. It can be shown that the optimum design of a filter with n resonant elements corresponds to a power loss ratio of the form:

$$\frac{P_o}{P_L} = 1 + \epsilon^2 T_n^2(x) = 1 + \epsilon^2 \cos^2(n \cos^{-1} x) \qquad (21)$$

where $T_n(x)$ is a Tschebyscheff polynomial of the first kind and order n, and ϵ^2 is a positive constant. The function $T_n^2(x)$ is equal to unity for $x = \pm 1$, oscillates n times between zero and one for $-1 < x < 1$, and tends monotonically to infinity as $2^{n-1}x^{2n}$ for $|x| > 1$. It follows that the maximum value of the power-loss ratio in the pass band is $1 + \epsilon^2$, while the sharpness of cut-off is proportional to ϵ^2. The choice of ϵ^2 depends, therefore, on a compromise between pass-band loss and sharpness of cut-off. The T_n polynomials of order 1 to 4 are given below with a recurrence formula from which higher order polynomials may be derived.

$$\begin{aligned}
T_1(x) &= x \\
T_2(x) &= 2x^2 - 1 \\
T_3(x) &= 4x^3 - 3x \\
T_4(x) &= 8x^4 - 8x^2 + 1 \\
T_{n+1}(x) &= 2xT_n - T_{n-1}
\end{aligned} \tag{22}$$

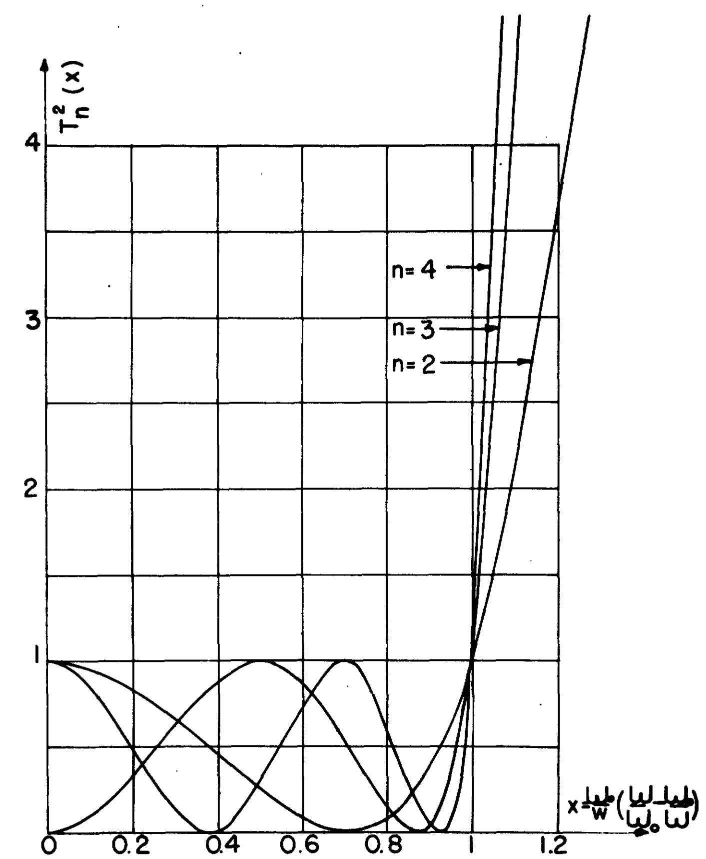


FIG. 14

The functions, $T_2^2(x)$, $T_3^2(x)$ and $T_4^2(x)$ are plotted in Figure 14. Design equations for n = 2 and n = 3 can be obtained without difficulty by matching the coefficients of the powers of x in the expression for the power-loss ratio with the corresponding coefficients in $T_2^2(x)$ and $T_3^2(x)$. With reference to Figure 3, one obtains for n = 2:

$$\epsilon = \frac{1 - R}{2\sqrt{R}} \tag{23}$$

$$L_2 = C_1 R = \frac{\sqrt{2R(1 - R)}}{w} \tag{24}$$

and for n = 3:

$$\epsilon = \frac{(C_1 w)^3}{3(C_1 w)^2 + 4} \tag{25}$$

$$Lw = \frac{8\epsilon}{(C_1 w)^2} \tag{26}$$

$$R = 1 \,;\; C_1 = C_3 \tag{27}$$

Note that Equations 25, 26, and 27 are satisfied when $C_1 = C_2 = C_3$, if $\epsilon^2 = \frac{1}{27}$. It follows that a filter employing three identical resonant elements is an optimum design for this value of ϵ^2.

When the pass band characteristics of a filter are to be held within very strict tolerances, the design procedure must be modified to take inte account the effect of incidental dissipation. This modification may consist simply, as pointed out by Darlington[3], of an appropriate predistortion of the desired characteristics. One must remember in this connection that in a dissipative network the power reflected is not necessarily zero when the power transmitted is a maximum so that the reflection and the transmission characteristics cannot be optimum simultaneously.

Another difficulty arising when close tolerances are to be held concerns the effects of the approximations made in transforming lumped element structures into microwave structures. It is difficult to determine these effects in the general case. As far as the frequency dependence of the electrical length of the coupling line is concerned, we may get an estimate of its effect by considering the particular case of a filter consisting of identical elements. One obtains for the power-loss ratio

$$\left(\frac{P_a}{P_L}\right)_n = 1 + x^2 U_n^2(kx) \tag{28}$$

where

$$k = 1 + \frac{\pi}{4} \frac{1}{Q_L} \left(\frac{\lambda_g}{\lambda}\right)^2 \quad . \tag{29}$$

A comparison of this equation with Equation 12 shows that, to a first approximation, the bandwidth of the filter is reduced by a factor equal to k. The pass-band tolerance is also reduced since x^2 is inversely proportional to k^2 for any given value of U_n^2. In the particular case of n = 3, Equation 27 and Equation 12 can be written in the form of Equation 19 by means of the change of variable $x' = \sqrt{3}x.$ This substitution yields for Equation 12:

$$\left(\frac{P_o}{P_L}\right)_3 = 1 + \frac{1}{27} T_3^2(x') \qquad (30)$$

and for Equation 27:

$$\left(\frac{P_o}{P_L}\right)_3 = 1 + \frac{1}{27k^2} T_3^2(kx') \qquad (31)$$

A comparison of these two equations shows that both the bandwidth and the tolerance ϵ are reduced by a factor equal to k.

Very little is known about the effect of the reactances X_1 and X_2 in the equivalent circuit of a cavity resonator. However, this effect is believed to be negligible when the loaded Q is larger than about 10.

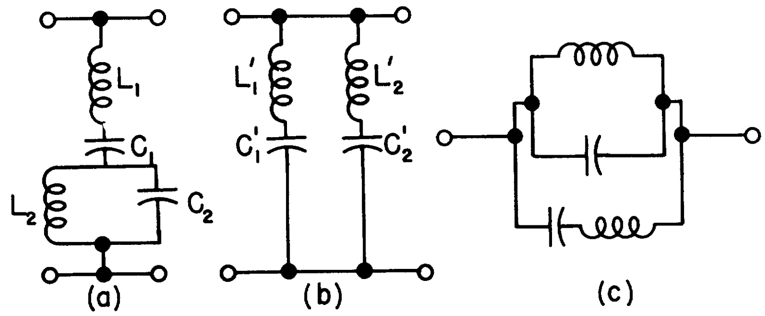
Filters of the "m derived" Type and Band Rejection Filters

Filters of the m derived type are used when fast rising attenuation functions are required(1,3). No quarter-wave filter of this type has yet been built by the authors, but the design procedure is straight forward. The only difference between the simple band-pass ladder structure and the m derived structure is the presence of shunt branches of the type shown in Figure 15a or of series branches of the type shown in Figure 15c. Since these two types of branches are equivalent when quarter wave coupling is employed, only the first type will be considered in detail. Foster's reactance theorem(1,2) permits the transformation of the branch of Figure 15a into the network shown in Figure 15b, in which the two resonant circuits are tuned at the frequencies $\omega_{\infty 1}$ and $\omega_{\infty 2}$. These frequencies correspond to peaks of infinite attenuation in the characteristics of the filter. A microwave realization of the network of Figure 15b is readily obtained, as shown in Figure 16a. One may also consider the network of Figure 15a as a two-element ladder structure with its output terminals open circuited. (See Figure 16b). Its microwave equivalent shown in Figure 16c is then obtained by quarter-wave coupling two cavities tuned at the mean frequency ω_0 of the filter. In this case the separation of the frequencies $\omega_{\infty 1}$ and $\omega_{\infty 2}$ is controlled by the ratio of the loaded Q's of the two cavities. A microwave realization of an m-derived section is shown in Figure 17. In the case of waveguide filters the same design technique is employed, although the appearance of the final structure might be somewhat different.

Band elimination filters can be designed in the same manner since the lumped-element ladder structure of this type of filter can be obtained from the band-pass structure by the simple process of substituting a series tuned circuit for any parallel tuned circuit and vice versa(1). A waveguide realization of a band elimination filter is shown schematically in Figure 18. Note in this respect that the cavities behave like series tuned circuits in shunt to the line, since their input terminals are effectively spaced one-quarter wave from the guide. In the case of broad band filters it is convenient to use rejection resonant irises which behave like series tuned circuits. The simplest type of rejection iris consists of a screw parallel to the narrow edge of the guide, and approximately $\lambda/4$ long.

Conclusions

The quarter-wave coupled structure has been shown to be a microwave



$$L_1 C_1 = L_2 C_2 = \frac{1}{\omega_0} \qquad L'_2 C'_2 = \frac{1}{\omega^2_{\infty 2}}$$

$$L'_2 C'_2 = \frac{1}{\omega^2_{\infty 1}} \qquad \omega_{\infty 1}\ \omega_{\infty 2} = \omega_0^2$$

FIG. 15

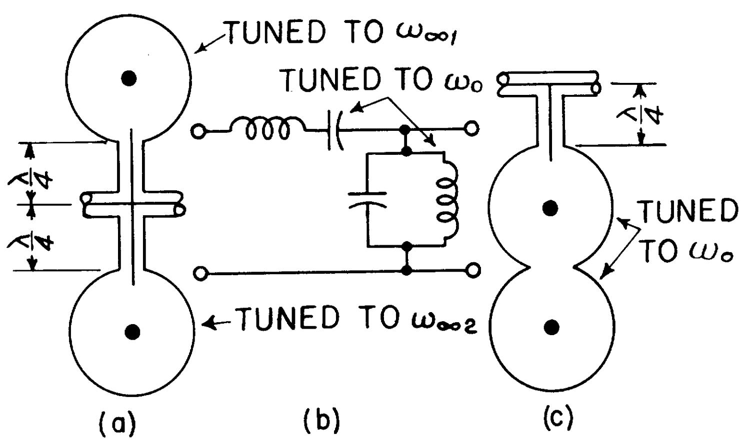


FIG. 16

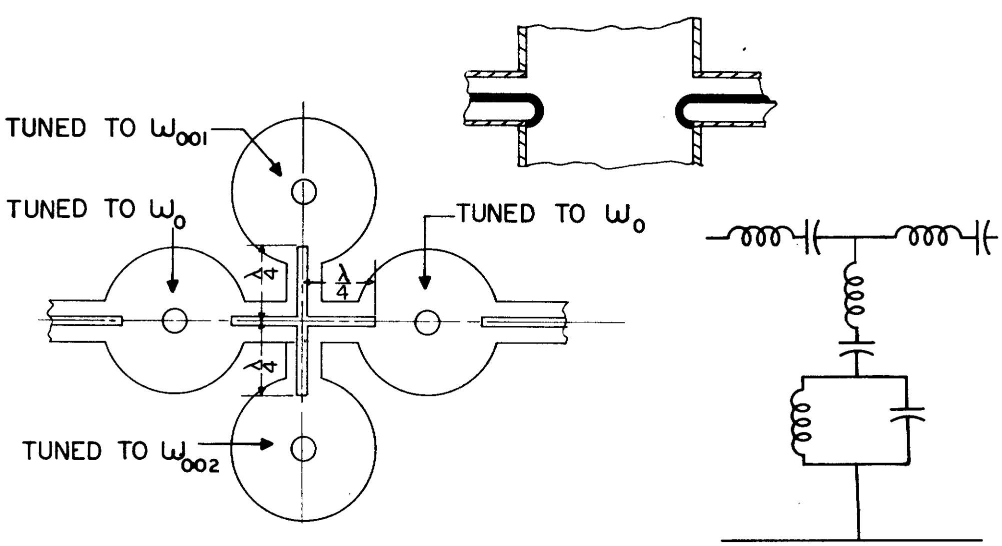


FIG. 17

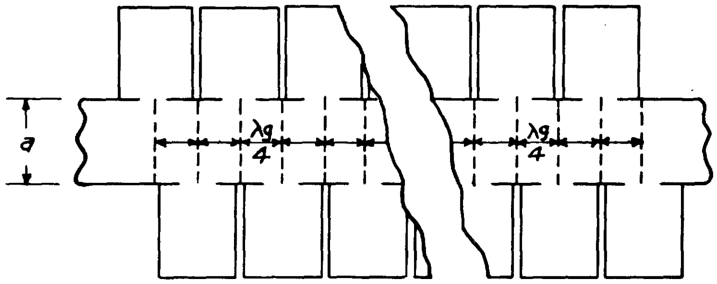


FIG. 18

equivalent of the lumped-element ladder structure for the purpose of designing band-pass and band elimination filters. This structure provides physical spacing between the resonant elements, and at the same time permits the construction and test of these elements as separate units which can be easily assembled afterwards. The method of design discussed in this paper, however, is not necessarily the best method in all cases, For instance, filters consisting of directly coupled cavities may be preferable when space is at a premium. These filters can also be designed by properly transforming lumped-element ladder structures but they may present serious construction problems when more than two cavities are used. Therefore, in some cases the two methods of design may be effectively combined to satisfy space requirements without introducing excessive construction difficulties. In such instances one may also use special cavities which alone behave like two or three element filters. These cavities will be the subject of a future paper.