IX. COMMUNICATION RESEARCH

A. MULTIPATH TRANSMISSION

Prof. L. B. Arguimbau
E. J. Baghdady
E. A. MacNair
E. E. Manna
R. D. Stuart
C. K. H. Tsao

1. Transatlantic Tests

Preparations are being made to carry on field tests during October and November.

L. B. Arguimbau

2. Narrow-Band Limiting

The problem of necessary limiter bandwidth for interference rejection in frequency-modulation reception, previously suggested in Technical Report No. 42, has now been worked out; and the effect of the selected limiter bandwidth upon the requirement in discriminator bandwidth has been determined. (See E. J. Baghdady: Master of Science Thesis, Department of Electrical Engineering, M.I.T., 1953.) After an experimental verification of the results, the investigation will be published in full detail as Technical Report No. 252.

The investigation has been carried out in terms of an ideal passband filter that follows a stage of ideal limiting. A thorough study of the spectrum of two carriers after limiting and of the properties exhibited by the spectral lines has led to a criterion for interference rejection when the limiter is followed by an ideal bandpass filter of arbitrary width. The two signals are assumed to have relative magnitudes, 1 and a (where $a < 1$) and constant or very slowly varying frequencies, p and $p + r$ radians/sec, respectively. In the linear sections of the receiver, the signals superimpose linearly, and the resultant spectrum is the sum of the individual spectra. The resultant signal undergoes instantaneous amplitude and frequency variations. The amplitude varies between maximum and minimum values of $1 + a$ and $1 - a$, respectively; whereas the frequency excursions range between $p + (ar)/(1 + a)$ and $p - (ar)/(1 - a)$, during a period of $2\pi/r$ sec. The average, over $2\pi/r$ sec, of the instantaneous frequency variations about the value p is zero, and the average frequency of the resultant signal is thus the frequency of the stronger signal. This average is maintained before, and immediately after, the limiting process. Thus, if the interference is to be suppressed at the output of the receiver, the bandwidths of the limiter-filter and the discriminator must be sufficient to preserve the average value of the frequency of the resultant signal, over $2\pi/r$ sec, at the value of p radians/sec.

Now, through the nonlinear action of the limiter, the instantaneous amplitude variations of the resultant signal are wiped out, and the interference spectrum is spread out, the individual lines being separated by the difference frequency r. The resultant signal at the output of the ideal limiter is

$$e(t) = \sum_{n=-\infty}^{\infty} A_n \cos (p - nr)\, t$$

From the properties exhibited by the spectral component amplitude, A_n, we have shown that if a passband filter follows the limiter, the average frequency of the resultant signal at the output of the filter will still be p radians/sec, if at all times,

$$\sum_{n=0}^{M} |A_n| > \sum_{n=1}^{N} |A_{-n}|$$

where M = number of A_n's passed, N = number of A_{-n}'s passed, and A_o = amplitude of the component having the frequency p.

The use of this criterion has enabled us to calculate the minimum bandwidths required after the limiter for preserving the interference rejection ability of the receiver for arbitrary values of a, the relative strength of the weaker signal. The results obtained indicate that

a. for $a \leq 0.863$, the minimum limiter bandwidth required is just one intermediate-frequency (i-f) bandwidth;

b. for $0.863 < a \leq 0.937$, the minimum limiter bandwidth required is twice the i-f bandwidth;

c. for $0.937 < a \leq 0.98$, the minimum limiter bandwidth is three times the i-f bandwidth.

The investigation has led to a better appreciation of the roles played by the limiter and discriminator bandwidths in making it possible for interference to be suppressed. Basically, the filter that follows the limiter must be capable of passing only those combinations of spectral components, from both sidebands, that will have a resultant whose average frequency is still that of the stronger signal. The discriminator must then have a detection characteristic that is linear over a bandwidth that will cover the whole range occupied by the instantaneous frequency deviations of the resultant signal from the frequency of the stronger signal. On the basis of this reasoning, several values of limiter bandwidth have been picked, and the corresponding required discriminator bandwidths calculated. The results show that the required discriminator bandwidths may be cut down considerably, below the values specified in Technical Report No. 42, by a proper choice of a narrow limiter bandwidth. In all of the important cases considered, it is found that when a limiter bandwidth of the order of the required minimum value is used, the required discriminator bandwidth is less than one-half the bandwidth required with a wideband limiter. For $a = 0.9$, for instance, with a limiter bandwidth of three to four times the i-f bandwidth, the required discriminator bandwidth is about seven times the i-f bandwidth. This is to be compared with the value required after a wideband limiter, namely, 19 times the i-f bandwidth.

Finally, it is reasonable to expect that the spectral components passed will continue to decrease, in relative magnitudes, after limiting, as they do with two-signal interference. Hence, the process of narrow-band limiting could be repeated often enough to result in the final reduction of the required discriminator bandwidth to that of the intermediate-frequency section. Furthermore, on account of the smaller jumps in the frequency of the resultant signal at the output of a narrow-band limiter, the quick-action requirements on the discriminator may be expected to be less stringent than with wideband limiters.

E. J. Baghdady

3. Thesis Work – Double-Heterodyne FM Receiver

Previously-reported work on the Paananen receiver indicated that the chief design difficulty for possible production purposes lies in the critical adjustments needed to keep the i-f filter in alignment. The requirements of flat response in the passband and sharp rejection at the sides put a heavy strain on coil and capacitor stability requirements.

Mr. Evan A. MacNair (E. A. MacNair: A Double Superheterodyne FM Receiver, Master of Science Thesis, Department of Electrical Engineering, M.I.T., Aug. 1953) has just constructed a receiver based on the principles of the earlier receiver but using a double-heterodyne circuit to lower the intermediate frequency and yet maintain good image rejection. This receiver gave adequate gain and limiting with only five tube envelopes and two diodes, exclusive of radiofrequency, audiofrequency, and rectifier tubes. The work indicated that a double-heterodyne circuit is an answer to the drift problem. It also gave partial confirmation of the narrow-band limiter results discussed in section 2. A take-over was obtained on a 3-db signal-to-noise threshold with fairly modest equipment.

L. B. Arguimbau

B. STATISTICAL THEORY OF COMMUNICATION

Prof. J. B. Wiesner
Prof. R. M. Fano
Prof. Y. W. Lee
Dr. B. Mandelbrot
Dr. F. A. Muller
R. M. Lerner
J. C. Stoddard
W. A. Youngblood

1. Pitchfinder

The instrument embodying the principles explained in the Quarterly Progress Report, July 15, 1953 has been completed. Fig. IX-1 shows a registration of the melody of the sentence: "The melody of vowels and liquids is recorded." Although the performance is sufficient for the purpose of registering the melody, the stray points have to be eliminated when a pitch indicating signal for a vocoder is desired. This elimination will be attempted by adding a circuit which considers only those consecutive points which differ less than a given small amount.

F. A. Muller

2. Spectral Analysis of Signals

A new method of analysis of signals has been devised and tested provisionally. The principle is closely related to autocorrelation analysis. When the filters F in the block diagram, Fig. IX-2, are identical purely delaying filters (transfer function exp $(-\tau_o s)$), the instrument is an autocorrelator giving equidistant points of the autocorrelation curve. The output of the mth terminal C_m is, in this case, related to the energy spectrum $E(\omega)$ by the relation of Wiener-Khinchine:

$$C_m = \phi(m\tau_o) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} E(\omega)\cos(\omega m\tau_o)\, d\omega$$

A similar relation still holds when the filters F are identical filters of the general allpass type, characterized by the phase $\phi(\omega)$:

$$C_m = \frac{1}{2\pi}\int_{-\infty}^{+\infty} E(\omega)\cos\left[m\phi(\omega)\right] d\omega = \frac{1}{2\pi}\int_{\phi(-\infty)}^{\phi(+\infty)} E(\phi)\cos m\phi\, d\phi$$

where $E(\phi) = E\left[\omega(\phi)\right].\,(d\omega)/(d\phi)$ is the energy spectrum transformed on the ϕ scale. The output results C_m are recognized as Fourier coefficients of this transformed energy spectrum. These coefficients allow description of the energy distribution in an interval $n\pi<\phi<(n+1)\pi$ (n is an integer) when no energy is present outside this interval. This interval spans the whole frequency scale when the filters F have the transfer function $(1 - s/s_o)/(1 + s/s_o)$ and therefore $\phi = 2\tan^{-1}\omega/s_o$. This choice is the simplest and most interesting possibility.

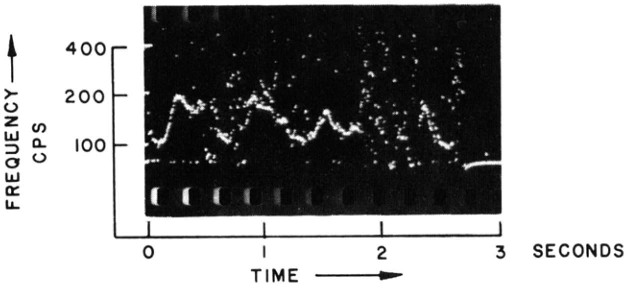

Fig. IX-1

Melody of a sentence registered by the pitchfinder.

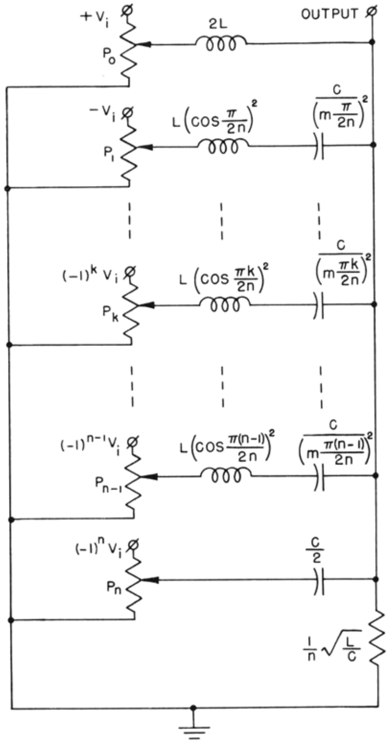

Fig. IX-3

Controllable filter.

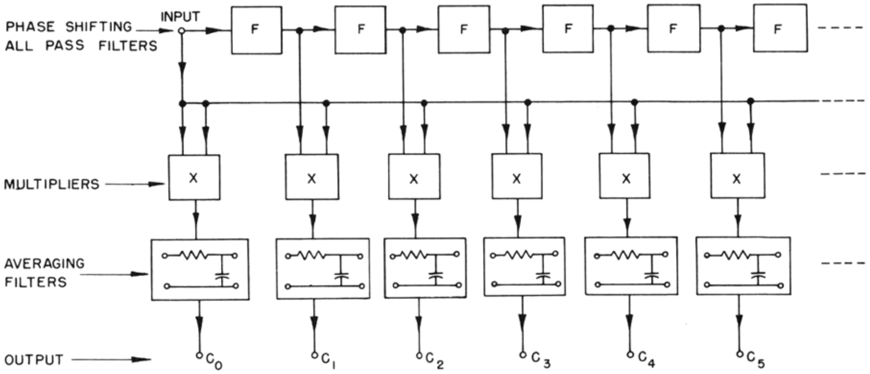

Fig. IX-2

Signal analyzer.

Fourier analysis of transfer functions on this tangent scale in connection with the use of chains of these allpass networks has been used for filter synthesis purposes by Lee and Wiener (1, 2).

Possible applications of the method include the analysis of waveforms in electroencephalography and the analysis of speech waves. This latter application is interesting because the transformed frequency scale shows a strong resemblance to the Mel scale when a suitable value of s_o is chosen.

F. A. Muller

3. Controllable Filter

The filter shown in Fig. IX-3 allows variation of the amplitude characteristic without change of the phase characteristic. A different structure with these properties has been described by Lee and Wiener (3).

In the present filter, the position of potentiometer P_k ($k = 0, 1, 2, \ldots, n$) alone determines the amplitude at the corresponding resonance frequency $\omega_k = (LC)^{-1/2} \tan\left[(k\pi)/(2n)\right]$. (It is assumed that the loss in the coils and the resistance of the potentiometers may be neglected.) The amplitude characteristic flows through the points thus fixed in such a way that the Fourier analysis of the amplitude (absolute value of the transfer function with the appropriate sign) as a function of $\phi = 2 \tan^{-1} \omega(LC)^{1/2}$ contains no higher components than $\cos n\phi$. The crossover behavior between the fixed points is, therefore, under rigorous control. This method of interpolating values of a continuous function between sample points is closely related to the well-known use of the function $(\sin x)/x$.

The filter can be used for speech synthesis when the potentiometers are replaced by modulators. When these modulators are driven by functions which are derived from an analyzer as described above, a version of Dudley's Vocoder is obtained which shows a uniform accuracy of approximation throughout the spectrum. This might lead to an improved performance.

F. A. Muller

References

1. Y. W. Lee: Doctoral Thesis, Department of Electrical Engineering, M.I.T., 1930
2. Y. W. Lee, N. Wiener: U. S. Patent No. 2, 024, 900, Dec. 17, 1935
3. Y. W. Lee, N. Wiener: U. S. Patent No. 2, 128, 257, Aug. 30, 1938

C. HUMAN COMMUNICATION SYSTEMS

Dr. L. S. Christie
Dr. R. D. Luce
Dr. G. O. Rogge
J. Macy, Jr.
S. C. Bedard

Analysis of the Pattern Change experiment (2, 3, 5) was completed in a form which yields results that are generalizations of those obtained in the action-quantized number experiment (1, 4). These results will be issued as Technical Report No. 264.

Analysis of the experiment on semantic confusion has not been completed except for the questionnaire results (5), which will be issued as Technical Report No. 265.

The Whirlwind computer solution of the stochastic model for the action-quantized communication situation remains in the state described in the Quarterly Progress Report, April 15, 1953. It is hoped that this problem will be brought to a conclusion this fall under the direction of Macy.

L. S. Christie, R. D. Luce, J. Macy, Jr., G. O. Rogge

References

1. Lee S. Christie, R. Duncan Luce, Josiah Macy, Jr.: Communication and Learning in Task-Oriented Groups, Technical Report No. 231, Research Laboratory of Electronics, M.I.T., May 1952
2. Quarterly Progress Report, Research Laboratory of Electronics, M.I.T., Jan. 15, 1952, pp. 56-58
3. Quarterly Progress Report, Research Laboratory of Electronics, M.I.T., April 15, 1952, pp. 62-64
4. Quarterly Progress Report, Research Laboratory of Electronics, M.I.T., Oct. 15, 1952, pp. 42-43
5. Quarterly Progress Report, Research Laboratory of Electronics, M.I.T., Jan. 15, 1953, pp. 32-33

D. SPEECH ANALYSIS[*]

Prof. M. Halle
G. W. Hughes

Equipment has been assembled for further study of phonemes of 5-150 msec duration. The energy spectrum (energy vs frequency) is desired. The center frequency and the bandwidth of the incorporated filter are continuously variable over the ranges of 50-10,000 cps and 60-290 cps, respectively.

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[*] This work was supported in part by the Carnegie Foundation.

E. MECHANICAL TRANSLATION

If translation of technical articles from one language to another is ever to be done by machine, it is obvious that one operation such a machine would have to perform is that of consulting a dictionary. Such an operation would be rather easy to mechanize in a special purpose machine, or to program in existing digital computers. It is also clear that other operations will have to be mechanized in order to obtain a good translation. The nature of these other operations is, at present, not completely clear.

An experiment has been performed to shed some light on these other needed operations and to determine what kind of "translation" would be produced by a strict dictionary search and word substitution. In this experiment, a sample word-for-word first approximation or partial translation from German to English was prepared. Grammatical words such as "der", "sein", and "auf" were not translated, the original German word being retained. Furthermore, some of the German endings were retained in an attempt not to obscure the grammatical relations of the original sentence. Thus "beiden" becomes "BOTHen", but "Aussagen" becomes "EXPRESSIONS" because here the ending "en" uniquely means the plural and can be replaced by the English "S".

Several conclusions can be drawn from the result of the experiment. A person who is familiar with German grammar, but not with the specialized vocabulary, can read the material with fair facility. For him, the grammatical relations in the sentences are clear. His reading speed is fairly high because only one equivalent was used for each word of the original text, and the resulting words, whether English, German, or hybrid, are pronounceable. On the other hand, some passages are still obscure. The word "sicher" was translated as "SAFE" when in this context it should have been "CERTAIN", and "Würfels" appeared as "CUBEs" when, from the context, it should have been "DICEs". In a partial translation of this sort, no information is lost, since the reader can recreate the original German text by means of an appended glossary and can puzzle out the correct meaning by standard procedures. However, he first has to be able to recognize that the meaning is obscure. Some words may have the appearance of being the correct English equivalent when in reality they are not. In this respect, systems which supply alternate translations of some words are better, though reading and comprehension speed suffer sharply. A person who does not know German can, in most cases, gather the drift of this partial translation though it costs him quite a bit of effort. He is frequently at a loss to understand exactly what is said, but probably can get enough to determine whether or not the article contains anything that would interest him. It is thought that with a minimum of study he could learn enough grammar to be able to read such "translations".

Look-up time on Whirlwind I turns out to be under 50 msec per word. Machine time would be distributed about equally between input using the photoelectric tape reader, look-up from a vocabulary of about 10,000 words, and delayed output via magnetic tape. Whirlwind I could do such partial translations at the rate of 20,000 words per hour of machine time (about 50 hours to "partially translate" an entire year of Zeitschrift für Physik).

V. H. Yngve

F. TRANSIENT THEORIES

Prof. E. A. Guillemin
Dr. M. V. Cerrillo
Dr. F. M. Reza
E. F. Bolinder

1. Transient Synthesis

During the most recent period, work was concentrated on the design and testing of a three-channel audiofrequency amplifier for the reproduction of music. The design was carried out to get some qualitative experience with the time-domain synthesis procedure reported previously.

The design of this multichannel amplifier was based on a set of time-domain properties that exist in connection with the transmission of signals. The basic theoretical ideas will appear in Technical Report 270, "On the Basic Existence Theorem in Network Synthesis, Part 5."

The system proved to be very suitable for the reproduction of sound, having unusual tonal qualities and high fidelity; however, quantitative measurements proved to be difficult. For this reason, we are planning to make sample tests of the theory, using existing delay lines to achieve separation of signals.

M. V. Cerrillo

2. Error-Fresnel Solids

Some years ago (in an unpublished report), D. M. Powers, at this laboratory, calculated by series expansions the first quadrant of the Error-Fresnel Solid (1), defined by

$$G(z) = |G(z)| \exp\psi(z) = \frac{1}{2} + \frac{\exp(i\pi/4)}{\pi^{1/2}} \int_0^{z[\exp(-i\pi/4)]} \exp(-it^2)\, dt$$

or, substituting $v = t \exp(i\pi/4)$,

$$G(z) = \frac{1}{2}(1 + \operatorname{erf} z)$$

where

$$\operatorname{erf} z = \frac{2}{\pi^{1/2}} \int_0^z \exp(-v^2)\, dv$$

and

$$z = x + iy = |z| \exp(i\phi)$$

Powers' results have been carefully checked and extended by means of tables of integrals associated with the error function of a complex variable (2). Using the following functions defined in the tables

$$\phi_2(a, b) = \frac{1}{(2\pi)^{1/2}} \int_0^a \exp\left(\frac{-t^2}{2}\right) \cos bt\, dt$$

and

$$\phi_3(a, b) = \frac{1}{(2\pi)^{1/2}} \int_a^\infty \exp\left(\frac{-t^2}{2}\right) \sin bt\, dt$$

the error function of a complex variable may be written

$$\operatorname{erf} z = 2 \exp\left(\frac{b^2}{2}\right) \left[\phi_2(a, b) + i\phi_3(a, b)\right]$$

where

$$z = \frac{a + ib}{2^{1/2}}$$

Thus we obtain the following expressions for the Error-Fresnel Solids for the amplitude function $|G(z)|$ and the phase function $\psi(z)$

$$|G(z)| = \frac{1}{2}\left\{\left[1 + 2 \exp\left(\frac{b^2}{2}\right) \phi_2(a, b)\right]^2 + 4 \exp b^2 \cdot \phi_3^2\,(a, b)\right\}^{1/2}$$

and

$$\psi(z) = \tan^{-1} \frac{\phi_3(a, b)}{\phi_2(a, b) + \frac{1}{2} \exp(-b^2/2)}$$

The results of the calculations have been drawn as isometric plots for $0 \leqslant z \leqslant 3$ and $\phi = 0°, 10°, 20°, 30°, 40°, 45°, 50°, 60°, 70°, 80°$, and $90°$. See Figs. IX-4 and IX-5. The amplitude solid is symmetric around the x axis and the phase solid is antisymmetric around the x axis. Considerable simplification occurs for certain values of the angle ϕ. Thus

a. for $\phi = 0°$,

$$|G(z)| = \frac{1}{2} + \phi_2(a, o)$$

and

$$\psi(z) = 0$$

b. for $\phi = 45°$, $a = b$, the solids can be shown (3) to be

$$|G(z)| = \frac{1}{2^{1/2}} \left\{\left[\frac{1}{2} + C\left(a\left(\frac{2}{\pi}\right)^{1/2}\right)\right]^2 + \left[\frac{1}{2} + S\left(a\left(\frac{2}{\pi}\right)^{1/2}\right)\right]^2\right\}^{1/2}$$

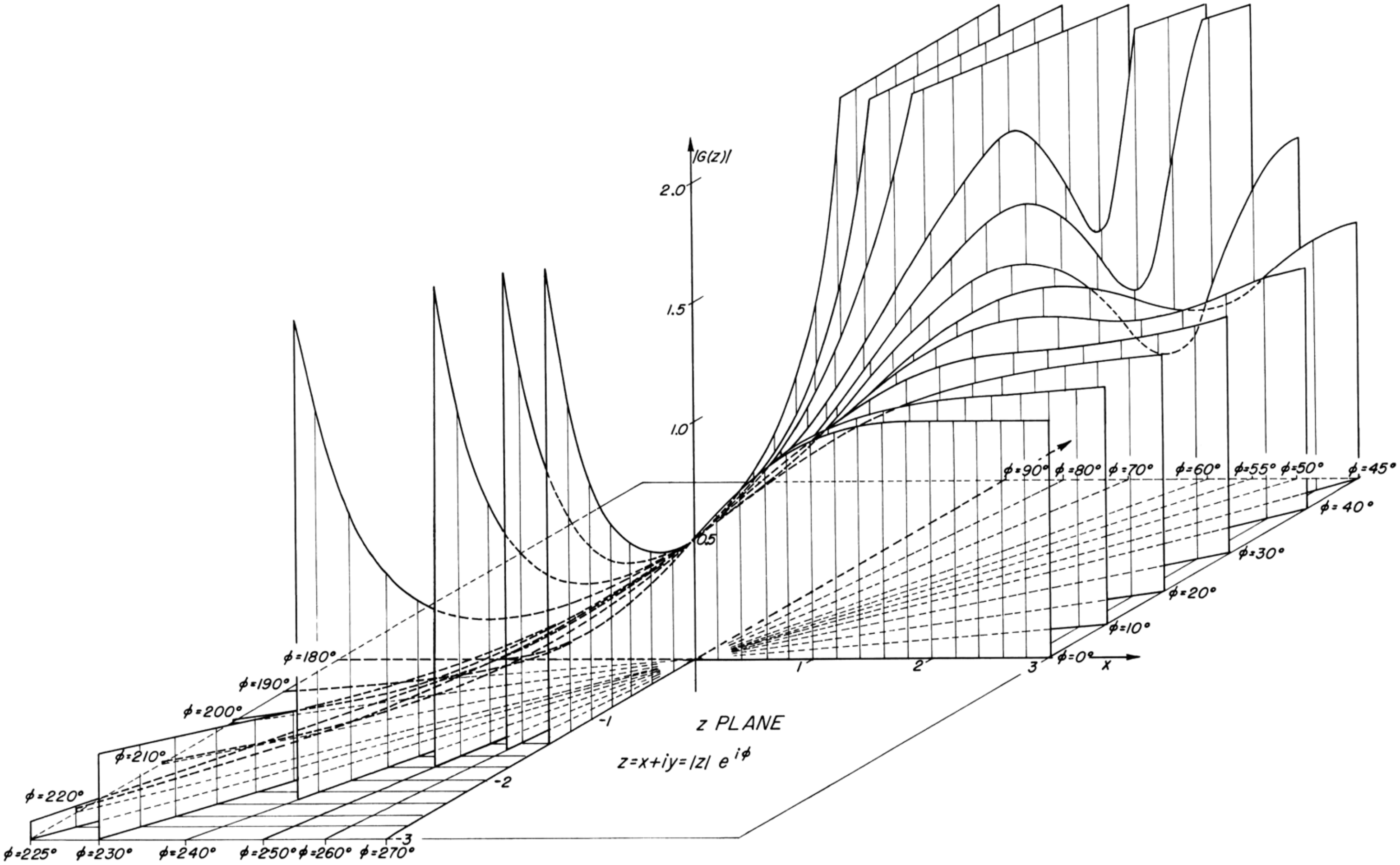

Fig. IX-4

Error-Fresnel amplitude solid.

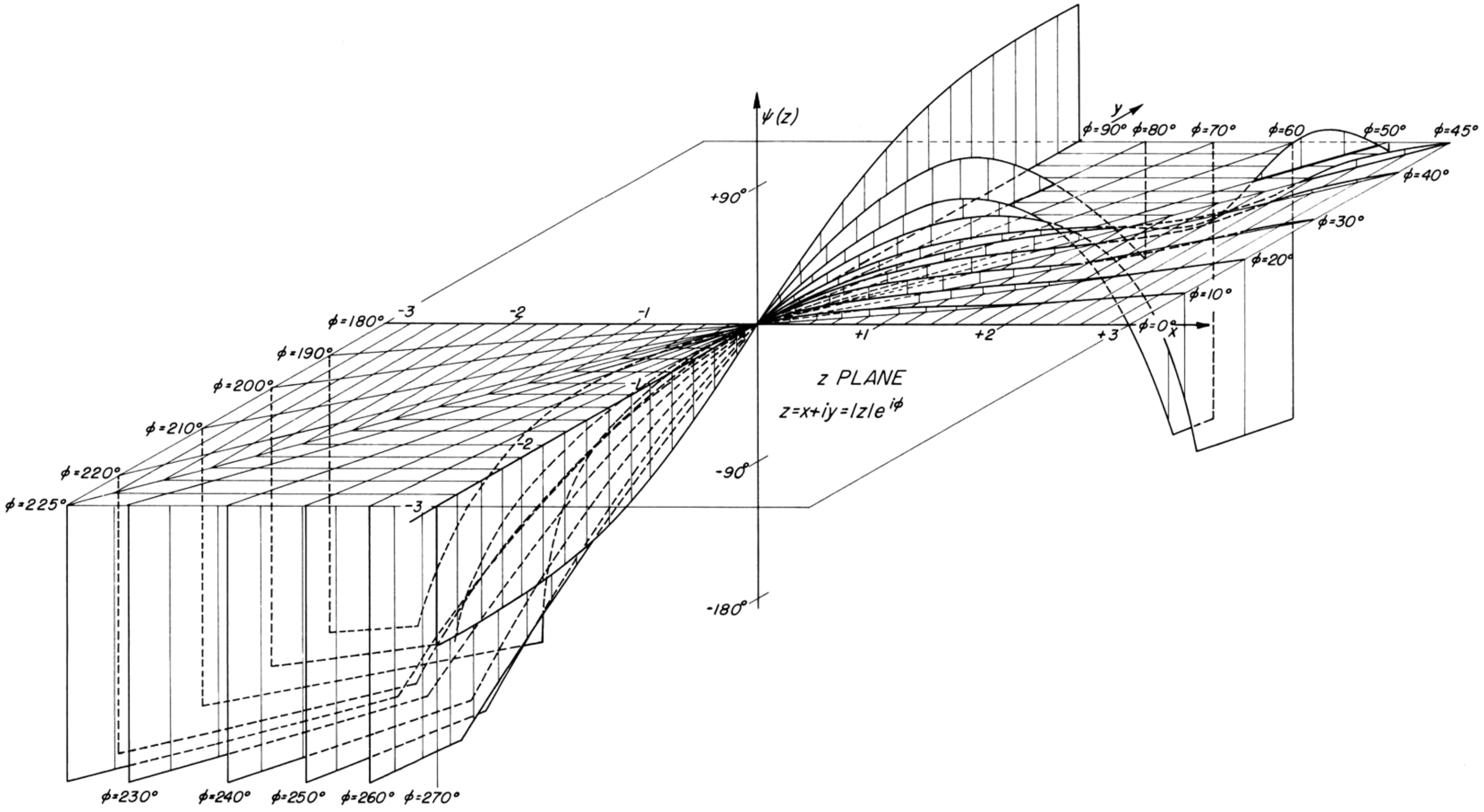

Fig. IX-5

Error-Fresnel phase solid.

and

$$\psi(z) = \tan^{-1} \frac{\left[\frac{1}{2} + C\left(a\left(\frac{2}{\pi}\right)^{1/2}\right)\right] - \left[\frac{1}{2} + S\left(a\left(\frac{2}{\pi}\right)^{1/2}\right)\right]}{\left[\frac{1}{2} + C\left(a\left(\frac{2}{\pi}\right)^{1/2}\right)\right] + \left[\frac{1}{2} + S\left(a\left(\frac{2}{\pi}\right)^{1/2}\right)\right]}$$

where

$$C(k) - iS(k) = \int_0^k \exp\left(-i\,\frac{\pi}{2}\,u^2\right) du$$

defines the Fresnel functions

c. for $\phi = 90°$,

$$|G(z)| = \frac{1}{2}\left[1 + 4 \exp b^2 \cdot \phi_3^2(o, b)\right]^{1/2}$$

and

$$\psi(z) = \tan^{-1}\left[\phi_3(o, b)\ 2 \exp \frac{b^2}{2}\right]$$

E. F. Bolinder

References

1. M. V. Cerrillo: Technical Report No. 55, Research Laboratory of Electronics, M.I.T., (to be published)

2. C. Hastings, Jr., J. I. Marcum: Tables of Integrals Associated with the Error Function of a Complex Variable, USAF Project Rand Research, Memorandum RM-50, Aug. 1, 1948

3. M. V. Cerrillo: Transients in Waveguides, Technical Report No. 33, Research Laboratory of Electronics, M.I.T., 1948

3. Certain Restrictions on the Driving-Point Impedance Functions

Consider a driving-point impedance function Z(s):

$$Z(s) = Z(j\omega) = U(\omega) + jV(\omega) \tag{1}$$

The average active power dissipated in the corresponding network is proportional to $U(\omega)$. It is physically clear that this average active power must have a limited range of variation over the whole frequency range. This information conveys that the analytic function Z(s), regular in the right half-plane, also has a bounded real part in that region. A number of interesting properties of these functions can be obtained by application of principles of conformal mapping. We present here a few examples of such properties.

Let $f(\lambda)$ be a function of the complex variable λ, regular in the unit circle and such that

$$-1 \leqslant \mathrm{Re}\left[f(\lambda)\right] \leqslant 1 \quad \text{in} \quad |\lambda| < 1 \tag{2}$$

By a proper application of the principle of subordination one may find (ref. 1) that

$$|f'(0)| \leqslant \frac{4}{\pi} \cos\left\{\frac{\pi}{2} \mathrm{Re}\left[f(0)\right]\right\} \quad \text{in} \quad |\lambda| < 1 \tag{3}$$

In order to obtain a similar relationship for positive real functions, one may use the transformations in Eqs. 4 and 5.

$$F(s) = \frac{1 - Z(s)}{1 + Z(s)} \qquad Z(s) = \frac{1 - F(s)}{1 + F(s)} \tag{4}$$

$$s = \frac{1 - \lambda}{1 + \lambda} \qquad \lambda = \frac{1 - s}{1 + s} \tag{5}$$

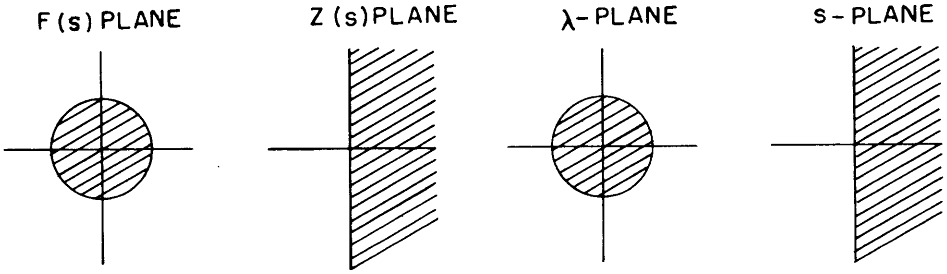

Fig. IX-6

Any point of the unit circle of the λ plane is transformed in a point of the right half-plane in the s-plane, leading to a point of the unit circle in $F(s)$ or $F\left[s(\lambda)\right] = f(\lambda)$ plane.

$$f(\lambda) = F\left[s(\lambda)\right] = \frac{1 - Z\left[s(\lambda)\right]}{1 + Z\left[s(\lambda)\right]} \tag{6}$$

Taking the derivative of Eq. 6, one finds

$$f'(\lambda) = \frac{4Z'(s)}{(1+\lambda)^2 \left\{1 + Z\left[s(\lambda)\right]\right\}^2} \tag{7}$$

and

$$f'(0) = \frac{4Z'(1)}{\left[1 + Z(1)\right]^2} \tag{8}$$

Using Eqs. 8 and 3, we find

$$Z'(1) \leqslant \frac{\left[1 + Z(1)\right]^2}{\pi} \cos \left\{\frac{\pi}{2} \frac{1 - Z(1)}{1 + Z(1)}\right\} \tag{9}$$

This is an interesting restriction on positive real functions. It states that the driving-point impedance function Z(s), considered as a function of positive real variable s, has a limited range of growth at each point. In a particular case, when the driving-point impedance is normalized at $Z(1) = 1$, the inequality (Eq. 9) leads to

$$Z'(1) \leqslant \frac{4}{\pi} \tag{10}$$

This result is not sharp. In fact, all values of $|f(\lambda)|$ corresponding to a point in or on the unit circle are confined to the unit circle, thus the Schwarz lemma could be directly employed. For this application we let $f(0) = 0$ or $Z(1) = 1$ and find $|f(\lambda)| \leqslant |\lambda|$ in the unit circle. Now one may expand $f(\lambda)$ about the origin

$$f(\lambda) = f'(0)\, \lambda + \frac{f''(0)}{2!} \lambda^2 + \ldots \tag{11}$$

which leads to the classical result

$$|f'(0)| \leqslant 1 \tag{12}$$

and finally

$$|Z'(1)| \leqslant 1 \tag{13}$$

Another interesting property of positive real functions could be obtained by applying the elegant theorem of E. Landau, discovered in 1906 (2), (3). The theorem states that the derivative of a function $f(\lambda)$, regular in the unit circle is bound to the following inequality:

$$|f'(0)| \leqslant \frac{2}{\pi} M \tag{14}$$

where M is the oscillation of Re $f(\lambda)$ on the unit circle.

$$M = \underset{|\lambda_1|=|\lambda_2|=1}{\text{Maximum}} \left|\text{Re } f(\lambda_1) - \text{Re } f(\lambda_2)\right| \tag{15}$$

The relation (Eq. 14) exemplifies an implicit restriction of the power consumption in a one terminal-pair network. In fact, as the real frequency $s = j\omega$ is varied from 0 to $+\infty$, the corresponding λ point moves along a half of the unit circle. Using Eqs. 1 and 6, we find

$$f(\lambda) = \frac{1 - U^2 - V^2}{(1+U)^2 + V^2} - j2 \frac{V}{(1+U)^2 + V^2} \tag{16}$$

Thus, the oscillation of Re f(λ) could be determined as a function of frequency ω and used in conjunction with Eq. 14.

$$|Z'(1)| \leqslant \frac{\pi}{2}\left[1 + Z(1)\right]^2 M \tag{17}$$

F. M. Reza

References

1. Z. Nehari: Conformal Mapping, McGraw-Hill, New York, 1952, Chap. 13
2. E. Landau: Arch. Math. Phys. 3, 31-36, 1906
3. M. S. Robertson: On the Coefficients of a Typically-Real Function, Bull. Am. Math. Soc., 565-572, 1935

G. COMMUNICATION BIOPHYSICS

Prof. W. A. Rosenblith	L. S. Frishkopf	K. Putter
R. M. Brown		W. T. Rusch

1. Electrophysiological Studies of the Auditory Nervous System

Acoustic click stimuli evoke characteristic electrical responses at all levels of the auditory nervous system of anesthetized cats. The results reported here are concerned with responses recorded by wire electrodes at locations that emphasize the contributions of the more peripheral neural structures of the auditory system.

A typical response to a click stimulus is composed of so-called microphonic and neural (action potential) components. (See Figs. IX-7 and IX-8.) The microphonic components are electrical events that start in the inner ear of the animal as soon as the stimulus arrives at the eardrum; they reverse polarity if a rarefaction click is substituted for a condensation click. Neural components occur with definite delays after delivery of the stimulus and have been identified on the basis of their latencies; thus N_1, the earliest neural component of the complex response, originates presumably in the most peripheral structures. The neural component, N_2, has been variously interpreted as an electrical event produced in higher neural centers or more recently (by Tasaki) as

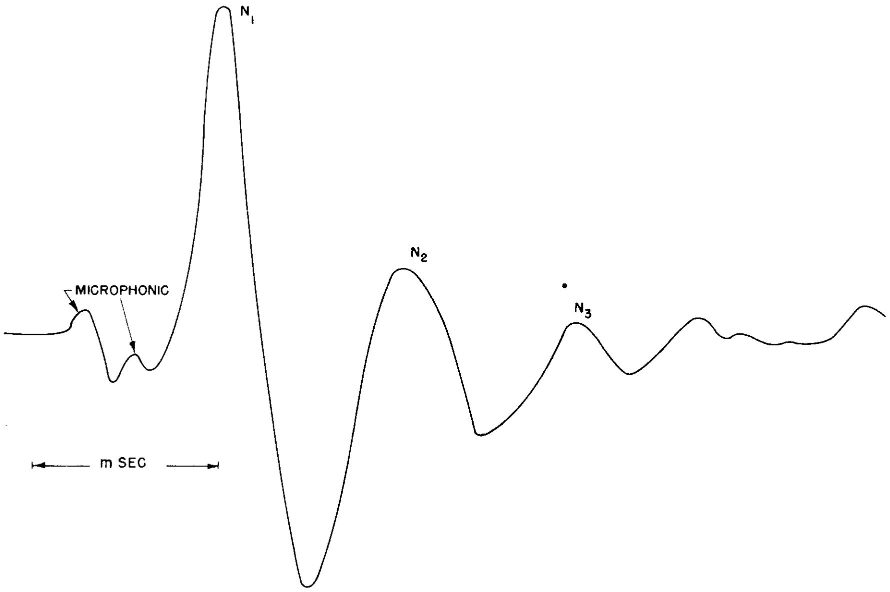

Fig. IX-7

Typical peripheral response to a condensation click. Very little microphonic content.

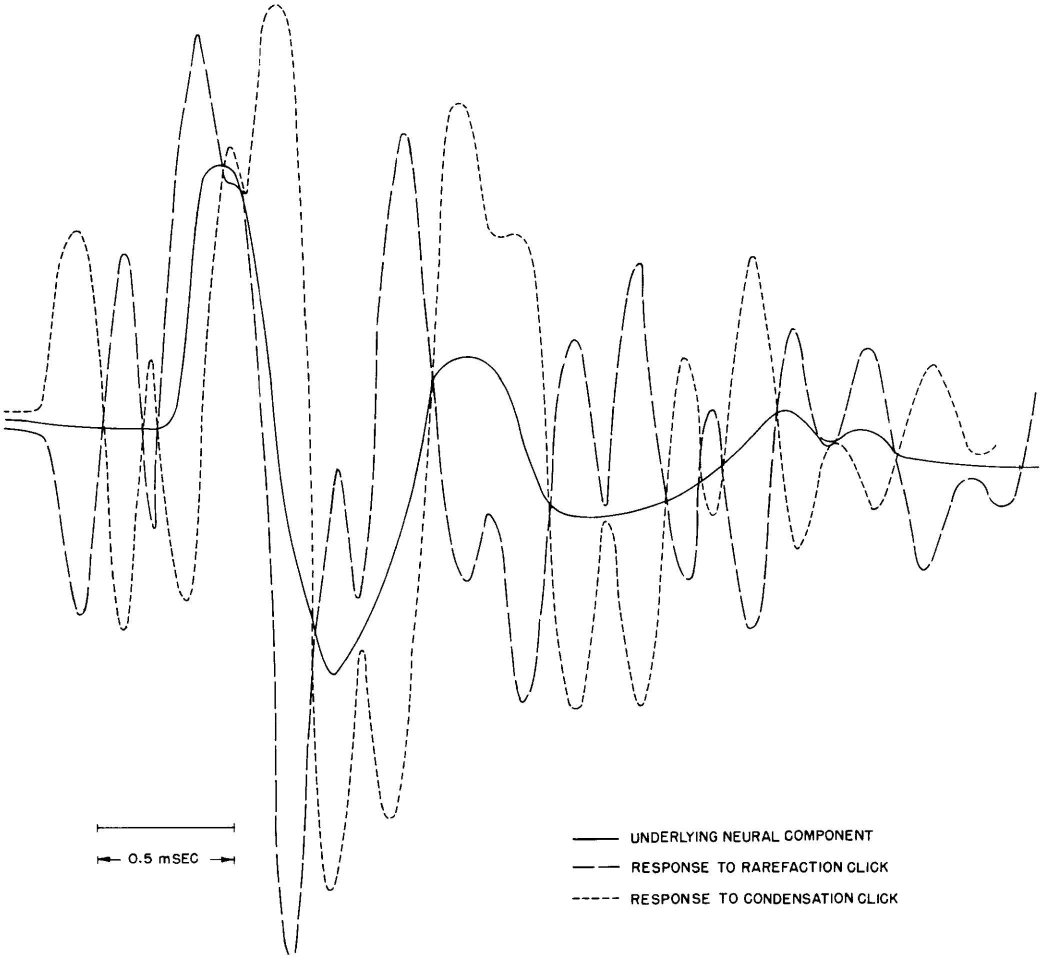

Fig. IX-8

Round-window responses with high microphonic content. The underlying neural component is obtained from condensation and rarefaction responses by averaging the two to eliminate the oppositely phased microphonic component.

an expression of repetitive activity of the neural elements responsible for N_1. Both microphonic and neural components are intensity-sensitive: over a range of nearly 80 db, the microphonic components increase monotonically with stimulus intensity. The precise form of the intensity function for the neural components is discussed below. We can say here that their latencies decrease to a minimum value as the click intensity is increased.

The relative size of the microphonic and neural components depends critically upon the location of the recording electrode. The classical round-window location emphasizes the microphonic components, especially for intense clicks. Rosenblith and Rosenzweig have shown that there are locations outside the cat's bulla that minimize microphonic contributions relative to the events that represent action potentials. Since a solution of the field equations for the cat is not available, empirical proof is required that events that are recordable at the same instant in time at various locations have a common physical origin.

a. The intensity function

If N_1 indeed represents the summated activity of one or several populations located near the end organ, the shape of the function relating amplitude of the N_1 deflection to click intensity becomes a key relation in a statistical interpretation of neural activity.

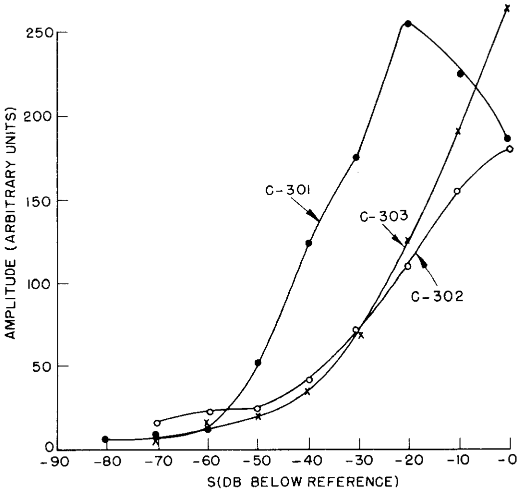

Fig. IX-9

Intensity functions, taken at the round windows of three animals, showing normal growth and bending over. Each point is based upon the average value of 10 to 20 responses to identical stimuli. A given curve is reproducible in a given animal over a period of several hours, provided that the animal is not stimulated at a rate and at intensity values that produce irreversible deterioration. (The absolute amplitude scale is different for each curve.)

We would at present hesitate to specify a generalized shape of this intensity function; Fig. IX-9 shows several of the functions that we have obtained from different animals. While there is a general upward trend for intensities near threshold and throughout the middle range, the behavior of the curves for high stimulus intensities prevents us from describing it as monotonically increasing. There is a tendency towards flattening, and in one animal, even towards "bending over," that has also been observed in some previous instances.

b. The two-click paradigm

A classical experiment for physiological systems consists in presenting a "conditioning" stimulus whose function it is to put the system into a state that can then (after an interval $\Delta\tau$) be explored by means of a testing stimulus. In our experiments the N_1 response to the second click can then be compared to the resting N_1 response, that is, to the unconditioned response to a stimulus of the same intensity. At this point it becomes convenient to introduce the dimensionless parameter "relative amplitude" (R. A.) which we define as the ratio of the conditioned-response amplitude to the resting-response amplitude for a given neural component; R. A. as a function of the interval $\Delta\tau$ is called the recovery function.

We have, in general, confirmed the findings of McGill and Rosenblith, McGill, and of Rosenzweig and Rosenblith (see below); we have however, extended the range of their observations by a systematic investigation of the effect of S_2 (intensity of the second click stimulus) upon the shape of the recovery function. In one animal our observations contradict their findings: in this animal (C-301) we find that there is a region of stimulus intensities and time intervals for which the recovery function becomes "supernormal".

We note that the unusual behavior of the N_1 component (supernormality) occurs in the same animal whose normal intensity function exhibited "bending over". The following microphonic effects may be significantly related to these neural observations: (a) at high click intensities the intensity function of the microphonics exhibits a slight bending over; (b) in C-301 the microphonic components of the response to a click were affected by a strong "conditioning" click. This consititues a phenomenon heretofore not encountered or described in the literature.

c. Electrode locations and "late neurals"

In continuing the search for electrode locations that emphasize differentially various components of the complex click response the following observations have been made:

Figure IX-8 showed some indication of the occurrence of "late neurals", that is, action potentials that occur after N_2. The deflections were, however, too small to permit reliable measurement. There exist locations (outside the bulla and along the cat's nuchal ridge) at which at least partial intensity-functions for these late neurals can be

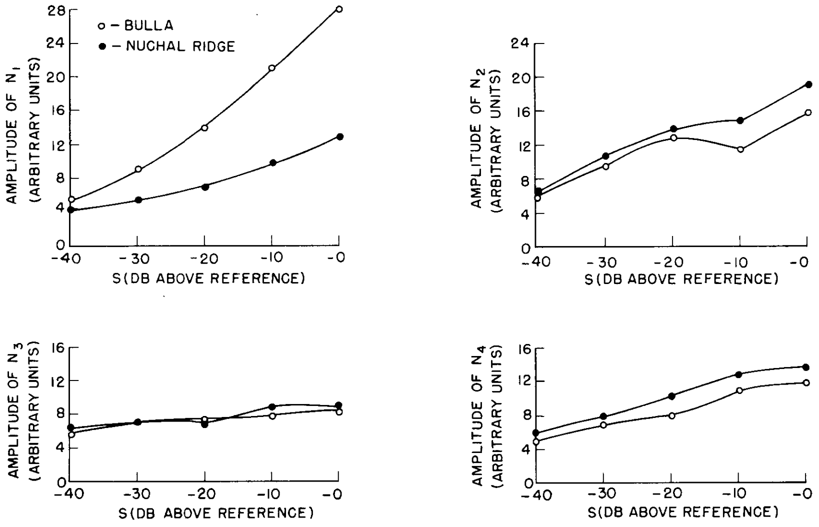

Fig. IX-10

Intensity functions of N_1, N_2, N_3, N_4 from responses taken at the bulla and on the nuchal ridge, using condensation clicks. Each point is based upon the average value of about 10 responses to identical stimuli. (A single amplitude scale has been used in all the graphs.)

plotted. Figure IX-10 illustrates the degree to which the intensity-functions for the various neural components differ. If the electrode is moved from the bullar location to the nuchal ridge, the size of, and the intensity-functions for, N_2, N_3, and N_4 remain rather constant, while there is a considerable change in the behavior of N_1. These facts could be interpreted to mean that the populations responsible for N_1 are relatively close to the bullar location while the sources of the late neurals are relatively more distant from both electrode locations. McGill has previously shown that the amplitude ratio N_1/N_2 at the round window is different from that observed at the bulla. Present evidence indicates that the N_1 and N_2 intensity functions differ in shape.

All these findings render unlikely the hypothesis according to which N_2 represents, in the main, the repetitive firing of primary auditory neurons.

A final observation on "late neurals": in several two-click experiments the recovery function for later neurals exhibited some supernormality; in almost every instance this supernormality was accompanied by a "bending-over" of the intensity-function for the given neural.

L. S. Frishkopf, W. A. Rosenblith, R. M. Brown
(With the collaboration of J. R. Hughes, Harvard University.)

2. Note on a Statistical Model for the Behavior of the N_1 Component

The neural system whose properties are under investigation is known to consist of many thousands of nerve cells. An attempt has been made to predict the observed behavior of neural responses from the postulated properties of assumed elementary "neural units". The elementary unit would have to correspond closely to the 'real' underlying entity, the single nerve cell. With this in mind and considering the results of single-cell and gross-response experiments, we have postulated a neural unit with a rapidly fluctuating threshold which can be described by a few statistical parameters. These parameters characterize an entire population consisting of many such units. The firing of an elementary unit obeys the all-or-none law; having fired, the threshold distribution is altered but recovers in time. These are the rough essential features of the model; the exact nature of the threshold distribution has been left unspecified, although a normal distribution has been tried with some success. It is probable that the existence of several distinct populations will have to be assumed (see W. J. McGill: A Statistical Description of Neural Responses to Clicks Recorded at the Round Window of the Cat, Doctoral Thesis, Department of Psychology, Harvard University, 1952). The nature of threshold recovery is still in the process of investigation. An entire field is left untouched by the absence of any specification, as yet, for interaction among these units. For it seems likely that in a realistic theory the probabilities for the firing of different units sequentially in time cannot be entirely independent. A first attempt in this direction via inhibitory properties is being attempted. Such an explanatory scheme could tentatively account for the supernormal behavior of N_1 in C-301 and for the accompanying "bending over" in the intensity function. We might postulate the existence of two neural populations (each characterized by a single set of threshold parameters), a sensitive one and a relatively insensitive one. The latter population might be assumed to exert a partially inhibitory effect on the more sensitive population. The existence of two such populations is neither anatomically nor physiologically unreasonable.

L. S. Frishkopf

3. The Time-Gated Amplitude Quantizer (TGAQ)

Tests checking the specifications of the time-gated amplitude quantizer have been completed. While the equipment in its final form has essentially the same block diagram and timing sequency as given in the Quarterly Progress Report, July 15, 1952, the circuitry has been altered in order to extend the original specifications.

As revised, the TGAQ is capable of handling responses at nominal repetition rates of 10, 5, 2, 1, 1/2, 1/5, and 1/7 per second. The portion of the response to be examined may lag the stimulus by 1 to 10 msec and may have a duration of 1 to 10 msec.

The highest frequency accepted without loss of accuracy is 2000 cps, whereas the low frequency response is limited only by the gatewidth selected. (These specifications are adequate for most peripheral data and some cortical data. If lower frequency phenomena are to be quantized, the gatewidth may be increased with an ensuing loss of accuracy at the high-frequency end.) Under the worst conditions, the over-all accuracy of quantization is 2.5 percent, corresponding to a maximum of forty levels. Provisions have been made to quantize into 20, 30, or 40 levels, as desired. The accuracy becomes better than 2.5 percent as the repetition rate is increased and as the frequency is decreased, so that under special conditions the number of levels may be increased. The output consists of illuminated numbers which may be photographed at all speeds or read visually at the lower repetition rates. The amplifier driving the TGAQ must be capable of delivering 60 volts with a maximum internal impedance of 500 ohms and should, of course, be flat to 2000 cps.

Since the peak-to-peak responses the TGAQ is designed to measure are normally riding on some other electrical activity, the actual potentials of the peaks may vary widely over several responses. A clamp circuit has been incorporated into the input circuitry. Its function is to establish a reference voltage at an earlier deflection than the one under study. The inherent assumption of constant latency of the reference peak, and the difficulty of adequately synthesizing the animal's response electronically, make it impossible to include the effect of the clamp circuit on the accuracy of quantization until the TGAQ is used in actual experimental conditions.

K. Putter

4. Responses to Pairs of Acoustic Clicks at the Round Window of the Cochlea and at the Auditory Cortex

(This is the abstract of a paper presented to the Third International Congress of Electroencephalography and Clinical Neurophysiology, held in Cambridge, Mass., August 17-22, 1953. A report of this research carried out at the Psycho-Acoustic Laboratory is scheduled for early publication in the Psychological Monographs.)

Electrical responses to single and paired clicks were recorded with gross electrodes at the round window and at the auditory cortex of over 30 anesthetized cats. The behavior of these responses is described as a function of the intensities of the stimuli, the time interval between paired stimuli, and the physiological state of the animal. At both locations the response to the second of a pair of clicks differs for considerable time intervals from the resting response.

Round Window. Recovery curves for the so-called first neural component ($R_2/R_0 = f(t)$) show that (a) recovery is monotonic and there is no evidence of supernormality; (b) the recovery curves are negatively accelerated with the shape of a particular curve depending upon the intensities of both clicks, but it is possible to predict the relative position of a family of parametric curves; (c) reference is made to a mathematical treatment developed elsewhere that tries to account for the data on the basis of a probability model.

Auditory cortex. Here recovery curves can all be interpreted as having a monotonic component around which cyclical variations occur. The monotonic and cyclical components are affected differentially by varying the intensity of the conditioning stimulus, the level of anesthesia, and the body temperature. The monotonic component may be a simple (perhaps even linear) function of the logarithm of the time interval. Depression of responsiveness is roughly proportional to the amplitude of the response to the first stimulus, at least for the lower intensities; maximum depression develops within a few milliseconds after the first response; the rate of recovery varies with the intensity of the first stimulus and with physiological state. The cyclical component behaves roughly like a negative sine function starting at a time interval of 0; it becomes more prominent and its period is somewhat reduced as the first stimulus becomes more intense. Cortical after-activity behaves somewhat like the cyclical component, but it may be slight or even absent when the cyclical component is prominent.

M. R. Rosenzweig (Univ. of Calif., Berkeley), W. A. Rosenblith

5. An Electro-Mechanical Audiofrequency Vibrato

The purpose of this Master of Science thesis was to design, construct, and test a system capable of introducing a frequency vibrato into musical notes of any frequency. It was hoped that such an "artificial" vibrato might be used to change, and perhaps improve, the quality of certain types of music. Economic and time considerations necessitated an engineering compromise between the physical system and the psychophysical criteria it should satisfy to produce a perfectly natural sounding vibrato.

An electro-mechanical system which was compatible with the compromises and which solved the specific problem was developed. It consists of electronic phase inverters and mixers and two speed-controlled motor-driven potentiometers. This system did introduce a vibrato into the production of the several musical instruments tested. Further refinements can be added to this basic system to permit closer compliance with the psychophysical criteria.

W. T. Rusch