XV. NETWORK SYNTHESIS

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VOLTAGE TRANSFER SYNTHESIS – RLC LATTICE

The work on voltage transfer synthesis has been extended to cover the RLC lattice. (The RC lattice was discussed in the Quarterly Progress Report, April 15, 1954.) The necessary and sufficient conditions for a transfer function to be realizable as a lattice have been found, and methods of synthesis have been developed.

1. Theorem

The necessary and sufficient conditions that a voltage transfer function

$$A = \frac{E_2}{E_1} = \frac{K\left(S^n + a_{n-1} S^{n-1} + \ldots + a_1 S + a_0\right)}{S^m + b_{m-1} S^{m-1} + \ldots + b_1 S + b_0} = \frac{KN}{D}$$

be realizable as an RLC lattice are: 1. D is a Hurwitz polynomial; 2. D + KN and D - KN are Hurwitz polynomials. The order of these two polynomials can differ by no more than two. (This condition specifies the maximum value of the parameter K.)

Proof:

(a) The proof of condition 1 is well known and will not be repeated here.

(b) Consider the lattice shown in Fig. XV-1.

The voltage transfer function is

$$A = \frac{KN}{D} = \frac{Z_b - Z_a}{Z_b + Z_a} = \frac{1 - (Z_a/Z_b)}{1 + (Z_a/Z_b)}$$

Solving this equation

$$\frac{Z_a}{Z_b} = \frac{1 - A}{1 + A} = \frac{D - KN}{D + KN}$$

Since the ratio of two positive real impedances must have Hurwitz polynomials in both its numerator and denominator, the theorem is proved.

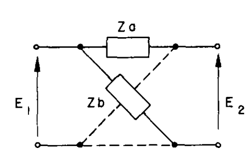

Fig. XV-1
An RLC lattice.

2. Synthesis

Given a voltage transfer function which satisfies the above conditions, one can immediately write

$$\frac{Z_a}{Z_b} = \frac{1 - A}{1 + A} = \frac{D - KN}{D + KN}$$

The problem is then to split Z_a/Z_b into Z_a and Z_b. Three different ways of doing this are suggested.

1. Enough surplus factors can always be added so that Z_a/Z_b can be immediately split into Z_a and Z_b. However, after this is done, the methods of Brune or Bott and Duffin must be used to synthesize the resulting networks. This direct approach may therefore be impractical.

2. If either the odd or the even parts of D + KN and D - KN are proportional, a particularly easy synthesis is possible. The details will be presented for the odd parts proportional. The extension to the other case is obvious.

If

$$\frac{Z_a}{Z_b} = \frac{D - KN}{D + KN} = \frac{m_1 + n_1}{m_2 + kn_1}$$

where m and n are the even and odd parts of the polynomials, then we can write

$$\frac{Z_a}{Z_b} = \frac{1 + (m_1/n_1)}{k + (m_2/n_1)}$$

$$Z_a = 1 + \frac{m_1}{n_1} \quad \text{and} \quad Z_b = k + \frac{m_2}{n_1}$$

Then Z_a and Z_b are realized as lossless networks in series with resistors. Part of this resistance can be removed from the lattice to give a series resistor. If a parallel resistance termination is desired, the function can be split into

$$Z_a = \frac{1}{k + (m_2/n_1)} \quad \text{and} \quad Z_b = \frac{1}{1 + (m_1/n_1)}$$

This type of synthesis will be possible when N is an even or odd polynomial, when D is of the second order, and in certain other particular cases.

3. If the maximum possible gain is not required, a synthesis in terms of partial fractions is possible. Split Z_a/Z_b into

$$Z_a = 1 - A \quad \text{and} \quad Z_b = 1 + A$$

$$\left(\text{or} \quad Z_a = \frac{1}{1 + A} \quad \text{and} \quad Z_b = \frac{1}{1 - A}\right)$$

Expand A in partial fractions. The resulting terms may or may not be realizable as simple RLC circuits. However, those terms that are not, can be made realizable by the addition of a suitable resistance. Consider the 1 in the expressions $1 - A$ and $1 + A$ as a one-ohm resistance reservoir that can be used for this purpose. Pick K small enough so that the one ohm is sufficient to make all the terms realizable. The resulting RLC partial fraction canonic forms are well known.

If K is made smaller than is required, some extra resistance will be left over to remove as a series or parallel resistor. Incidentally, the input impedance of this lattice is

$$Z_{in} = \frac{1}{2}(Z_a + Z_b) = \frac{(1-A) + (1+A)}{2} = 1 \text{ ohm}$$

This is particularly convenient for some applications.

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