AN OIL FREE AIR COMPRESSOR BASED ON THE STIRLING CYCLE

by

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ABSTRACT

The desirability of oil free compressed gas in certain industrial processes is so great that it is possible to contemplate a compressor design whose initial and/or running costs are inferior to the highly developed reciprocating compressor. Such a design is a Stirling engine modified so as to discharge its working fluid at a raised pressure rather than causing it to perform shaft work. To demonstrate the principle a machine has been built and tested. Electric heat was used for convenience but the machine would run on a primary fuel in practical form. Data has been obtained on the pressure ratio - discharge rate characteristic, which has also been obtained analytically. It was not possible to determine thermal efficiency from direct measurements though an estimate based on knowledge of component performance indicates that an overall thermal efficiency of about ten per cent could be expected. This is of the same order as that for a reciprocating compressor driven by a gasoline engine.

Sufficient information is presented on which an optimisation study may be based leading to a competitive design.

Thesis Supervisor: Samuel C. Collins
Title: Professor of Mechanical Engineering

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ACKNOWLEDGEMENT

The compression principle described herein was suggested to me by Professor Collins to whom I am also indebted for the circumstances in which to conduct the necessary work.

In the building and development of such a machine many frustrations were encountered and in this respect I have drawn heavily on the fund of good humor and understanding of Professor Collins and the staff of the Cryogenic Engineering Laboratory.

Over her objections I am bound to record the contributions of my wife Valerie whose encouragement and assistance extended far beyond the typing of this thesis.

A.P.M. Glassford
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1. INTRODUCTION

The excellence of an air compressor design may be judged on several criteria, the most common being the cost of manufacture and operation and to a lesser extent overall thermal efficiency. The reciprocating positive displacement compressor has achieved an almost impregnable position in these two respects. Another design requirement exists, however, in several major uses of compressed gases such as gas liquefaction and food processing. That is that the air delivered shall be oil free. The positive displacement compressor can only meet this requirement at great cost and maybe short working life. With this in mind an alternative device for supplying compressed air is proposed based on the hot air engine proposed by the Reverend Robert Stirling and first patented in 1816. This is described and illustrated in great detail by Bourne (1) in a handsome volume dedicated to His Excellency Kuo Sung-Tao, Minister of China at the courts of England and France . In principle this machine makes use of the fact that when air is displaced from a cold space to a hot space it will tend to expand. A piston moves in a cylinder connected with the cold space against which the expanding air may do work. By judiciously interconnecting the working piston and the device for moving the air a cycle may be performed producing net work whilst heat is supplied to the hot end and removed from the cold end. The outstanding feature of the design is the use of a regenerator to store heat from the air leaving the hot end and return it on the way back. With a

100% efficient regenerator and truly isothermal spaces the engine can be shown to have an efficiency equal to that of the Carnot engine.

Suppose now that the working piston were removed and the system kept at constant volume; movement of the air towards the hot end causes the system pressure to rise; movement towards the cold end causes it to fall. Figure 1 indicates how such a device could be made to deliver air. A cylinder contains a displacer which fits with a clearance just large enough not to require lubrication, but allowing negligible leakage. The spaces above and below the displacer are connected by an external passage containing a cooler, a regenerator and a heater. The heater and cooler are situated in the external circuit for practical reasons; it is not reasonable to heat a large volume of nearly stagnant air from its boundaries. Downward motion of the piston causes air to be displaced from cold to hot end; and vice versa. A certain maximum pressure P_{max} can be obtained when the displacer is at the cold end, being a function of the ratio of hot to cold end temperatures and of the ratio of unswept volume to swept volume. Air can be discharged at any pressure P_2 between P_1 and P_{max}. A mass of air equal to the discharged mass will be induced on the return stroke and a cycle completed. See figure 1. It is noted that if the spaces were isothermal then the device would effectively comprise a combined reversible engine and isothermal compressor. Even though many non-idealities will have to be built into a practical machine the principle is very sound.

2. PREVIOUS WORK AND AVAILABLE DATA

No reference has been found in the literature concerning such an application of the Stirling cycle. However, a great deal of useful literature is available on the design of Stirling engines and on regenerator analysis.

2.1 Work on the Stirling Cycle

The hot air engine as patented by Stirling was able to operate only very slowly as were all machines of that day. The hot air engine as a prime mover suffered from low mean effective pressure because of high void space and low temperature ratio. In spite of this its inherently high thermodynamic efficiency and the attention given to the regenerator lead to many successful engineering devices, among them Siemen's regenerative furnace, Jeffrey's breath warming instrument and Kirk's ice making machine, all mentioned in (1). The latter was a reversed Stirling engine. Ericson,also noted in (1), built his "Caloric Engine" around a regenerator which worked very well inspite of the fact that Ericson's explanation of its working brooked no interference from either the First or Second Laws of Thermodynamics. With the fast development of the steam engine and later the internal combustion engine the Stirling engine was neglected until recent decades during which the Philips Company of Eindhoven, Holland have paid it much attention. The only notable interest in the intervening years was the unsuccessful Malone engine (2) in which an attempt was made to use two co-existent phases as the working fluid in an attempt to reduce the external circuit pressure drop and to produce more nearly isothermal conditions in the hot space.

The work of the Philips Company was originally concerned with the development of a high speed air engine using the now established regenerator theory, advanced heat transfer knowledge and modern materials. In a series of papers, (3) (4) (5) and (6) the development of such an engine is traced from early design philosophy to the latest operating version. Problems have been encountered, though, mainly with lubrication and the hot air engine has not been a commercial possibility. However, it was found that when run in reverse the machine turned out to be a most effective heat pump and is now one of the commercially most successful machines in the field of liquid air production. This application has been described in (7) and (8).

Determination of thermal efficiency and power output of the Stirling engine is very difficult. All the papers by the Philips Company assume that the air temperature in each space is uniform and constant at T_h or T_c and avoid any derivations, presenting only formulae. A paper by Walker (9) which is concerned mainly with reporting experiments on different regenerator packings in a production model Philips refrigerator discusses these performance calculations. The traditional method assuming constant temperature spaces was proposed by Schmidt (10) in 1871. Only recently has a calculation method which allows variation of space temperature been published, that by Finkelstein (11). The analysis given in this thesis, Appendix A, allows for variation of space temperature but it has not been possible as yet to compare it with that of Finkelstein. It must be noted that the analysis for the air engine has several more variables than for the compressor application.

2.2 Data on Regenerator Analysis

General analysis of regenerator performance is amongst the most complex of problems, involving solution of simultaneous partial differential equations. In 1929 Hausen (12) published the first solutions to these equations. The answers are in the form of complicated series and are of little help in engineering design where reasonably rapid assessment is needed. Any particular design problem may be programmed for a computer and solved by finite difference methods. Several other noted German workers are mentioned by Walker. A useful paper in English on this German literature was published by Illiffe (13). The usefullness of such theory being limited by its complexity it is often possible to eliminate some terms in the governing equations by an order of magnitude analysis. Longitudinal conduction was omitted even by Hausen (though not by Schultz (14)). This approach is taken in Appendix B where the expression for regenerator efficiency is derived for this particular application from the governing equations. A paper by Schultz (15) is of particular interest in this respect. He deals with possible approximations when the regenerator period is short but not zero. Also of special interest is the work of Murray and others (17) which concerns regeneration with sinusoidally varying flow which obtains if a regular crank and connecting rod running gear are used. For a general discussion on regenerators Saunders and Smoleniec (16) is most useful.

3. DESIGN AND CONSTRUCTION OF A PROTOTYPE COMPRESSOR.

3.1. Compressor Body and Running Gear

Reference is made to figure 2. The dimensions given here are nominal. A stainless steel cylinder (a) 15 ins. long by 6.25 ins. inside diameter and o.140 wall thickness is mounted on a copper base plate (b) 0.5 ins. thick and 11 ins. outside diameter. Also mounted on the base plate is an outer case made from mild steel 0.125 ins. thick and having an inside diameter of about 7.75 ins. This leaves an annular space between case and cylinder. Access to this is by 0.25 ins. diameter holes drilled in the cylinder adjacent to the base plate at one end and by a clearance between the other end of the cylinder and the hot end plate (c). The hot end is made from 0.625 ins. mild steel and contains the heater element (d). A stainless steel displacer moves in the cylinder with about 0.015 ins. clearance all round, and is operated by a 0.625 ins. rod passing through an O-ring seal in the base plate to a double acting air cylinder. This form of motor is not very efficient but was very simple to build into the system and served its purpose well. Its maximum stroke was 6 ins. but it was not possible to use more than about 5.75 ins. due to the nature of the motor valve operation.

Reed type inlet and exhaust valves were originally designed and built for the compressor but as it became apparent that the machine would not run very fast these had to be replaced. Commercial check valves using a lightly spring-loaded poppet valve were fitted. These worked well but had a high pressure drop across them.

Asbestos gaskets were used throughout.

3.2. Heat Exchange System

This comprises the heater, regenerator and cooler all of which were situated in the annular space between the inner cylinder and the case.

3.2.1. Cooler The cooler was made from a strip of copper 1.5 ins. high and 0.020 ins. thick: dimples were made in it so that when wrapped around the cylinder annular passages about 0.025 ins. wide were formed. Strip was wound so as to fill the annulus. Radial holes were drilled in this element close to the bottom end so as to communicate with the holes in the cylinder. The lower edges were then soft soldered to the copper base plate. Air would enter the cooler by the holes in the cylinder and element and then turn 90° and pass up through the narrow annular spaces. Heat transferred to the copper strip would be conducted back along its length to the base plate. The base plate was cooled by soft soldering turns of 0.25 ins. outside diameter copper tubing in a 0.25 ins. deep groove on the outside to carry mains water.

An exact analysis of the heat exchange relationships obtaining would be very complicated in this case, especially when it is remembered that flow occurs in both directions. A gross assessment can be made very easily however. The annular gap is about 0.600 ins. wide. This admits about thirteen complete turns to be made or a total length of $\pi \times 13 \times 7.1 =$ 290 ins. The total surface area is thus $(290 \times 2 \times 1.5) + 144$

$= 6.04$ sq. ft.: the total cross sectional area is $\frac{290 \times 0.020}{144}$ $= 0.04$ sq. ft. For fully developed laminar flow between flat plates $\frac{hD_e}{k} = 7.6$ or $h \sim 30 \frac{\text{BTU.}}{\text{ft.}^2\text{hr.}^\circ\text{F.}}$. The surface resistance of the element is $\frac{1}{hA_s} = \frac{1}{181} \frac{^\circ\text{F.hr.}}{\text{BTU.}}$. The resistance of the entire fin length $L = \frac{L}{KA_x} = \frac{1.5}{12} \times \frac{1}{0.04 \times 225}$ $= \frac{1}{72} \frac{^\circ\text{F.hr.}}{\text{BTU.}}$. If the gross assumption is made that these two values be added to give the total resistance the very conservative value of $R_{TOT} = 0.0194 \frac{^\circ\text{F.hr.}}{\text{BTU.}}$. Now suppose that the entire contents of the cold space had to be cooled through 100°F. each stroke. The mass contained in the cold space $= \rho V_s$. It is found that the heat to be transferred is $15.3N \frac{\text{BTU.}}{\text{hr.}}$. So the average temperature difference required for $N = 30$ will be less than 9°F. In fact it will be very much less and for the range of cold space temperatures encountered will be more like one or two degrees.

3.2.2 Heater. For convenience it was decided to use an electric heater in this model. The design problem was very different from that of the cooler. Although the heater surface could be used near its melting point the structural members would have to work at much lower temperatures. Since the latter takes on the air temperature the net effect is that a large air to element temperature difference can be employed. For this application a heater must have much area in a small space. If an electric heater is to be used its electrical resistance is another variable. For instance if 2kw. is to be taken from the 115 volt line the resistance must be about 6.5 ohms. Resistance is given by $R = 6.5 = \frac{\rho . L}{A}$ where:-

ρ = electrical resistivity

L = length of conductor

A = cross sectional area of conductor.

Stainless steel has good high temperature properties. Its electrical resistance is $\rho = 0.0000276$ ohms. per inch cube. A conductor whose surface to volume ratio is high is desired and so strip was selected, 0.005 ins. thick. Then if its height is h ins.

$$6.5 = \frac{0.0000276 \times L}{0.005 \times h}$$

$$\frac{L}{h} = 1182 \qquad \text{(L and h in inches)}$$

$$\text{Surface area} = 2 \times \frac{L}{12} \times \frac{h}{12}$$

$$= 16.4\ h^2 \text{ sq. ft.}$$

h should be as large as possible but this means a very large L and space is limited in such an application. Temperature differences of the order of 300°F can be used for reasons previously given and by so designing the flow passages a heat transfer coefficient $30 \frac{BTU}{ft.^2 hr. {}^\circ F}$ seems reasonably conservative.

For a heat flux of 2 kw. or 6824 $\frac{\text{BTU.}}{\text{hr.}}$ this implies an area of

$\frac{6824}{30 \times 300}$ = 7.6 sq. ft. Thus h should be about 0.5 ins. and L about 44 ft.

Several elements were constructed, all sharing the same mounting shown in figure 3. The inner and outer walls were electrically insulated with strips of amber mica. Ceramic thermocouple insulator was broken into lengths of about 0.4 ins. which were again split into semi-circular cylinders. The flat sides were cemented to the top plate radially disposed so that air passages were formed as shown. Electrical leads were taken out of the plate via a ceramic insulated plug. Initially the whole element was insulated but later an earth return system was used so that a larger diameter conductor could be used in the plug to reduce heat generation therein.

The first element was made by winding stainless steel 0.25 ins. wide and 0.005 ins. thick on a former of suitable diameter having first smeared its surface with a mixture of petroleum jelly and graded sand of about 0.010 ins. diameter. The flat ring 0.25 ins. high thus made was fitted into the annular space. When in position the petroleum jelly was washed out with carbon tetrachloride leaving 0.010 ins. wide annular flow passages maintained by the sand. This element soon failed. About forty turns had been wound into the element so that with 120 volts applied the potential difference between adjacent turns was about 3 volts. There is a certain probability that somewhere in the element the successive coils may be close enough through bad construction or loss of sand for 3 volts to cause a spark. At this point the turns fuse together and short out one coil; this increases the potential difference between coils to a value at which more sparks could occur. The failure rate is exponential and in a matter of seconds enough coils short out to blow a fuse. A new element was built by bending the same size strip in the manner shown in figure 3. The effect was to reduce the potential difference between adjacent strips to something like 0.1 volts. Strips of mica were laboriously placed between each turn as shown to eliminate entirely the possibility of shorting: this proved to be its undoing as hot spots were promoted and at one of these points the element burned in two. When these mica strips were removed the design worked very well. The air passages turned out to vary between about 0.010 ins. and 0.030 ins. wide, the variation being bad workmanship and thermal deformation. The version of the heating element used in most of the runs was similar to this last design but using 0.50 ins. wide strip. The failure of this heater was due to failure of the insulation after several hours of use and stress. This was one of the reasons that wider strip was not used. Greater width increases the area to be insulated. Calculations for the last heater built show

Total surface area = 3.24 sq. ft.

If $\frac{hDe}{k} = 8.33$ for fully developed laminar flow between parallel with constant heat flux.

$$h = \frac{8.33 \times 12 \times 0.030}{0.040}$$

$$= 75 \quad \frac{\text{BTU.}}{\text{ft.}^2\text{hr.}^\circ\text{F.}}$$

For a power of 2 kw. $\Delta T = \dfrac{6824}{75 \times 3.24} = 28^\circ\text{F.}$

which is more than adequately low. Hot spots exist though where ΔT is very much greater than this.

3.2.3. <u>Regenerator</u> Qualitatively it may be seen that a regenerator packing should be made from a material having a high ratio of surface to volume. Also, if the regenerator is to be a component in a high speed machine low pressure drop per unit length is essential. Such a packing can be made by dimpling thin stainless steel strip 0.015 ins. thick and winding it around on itself so that annular flow passages are created about 0.015 ins. wide. Because the material was available in this form the regenerator was made from thirteen such coils each one inch in height and separated by wire mesh to preserve flatness. A continuous sheet could have been used, however,in this case as axial conduction is very small.

An analysis of the regenerator appropriate to this application is presented in Appendix B. From this analysis it is found that the regenerator efficiency η_r can be expressed as

$$\eta_r = \frac{\lambda}{2 + \lambda}$$

Where $\lambda = \dfrac{hAL}{wC_p}$

and A = surface area per unit length

L = length of regenerator

w = mass flow rate

For the configuration described

$$\frac{hDe}{k} = 8.33,$$

whence $h = 72.5 \ \frac{\text{BTU.}}{\text{ft.}^2\text{hr.}^\circ\text{F.}}$

Since $AL = 97$ sq. ft.

$C_p = 0.245 \ \frac{\text{BTU.}}{\text{lb.}^\circ\text{F.}}$

$w \sim V_s \times 2N \times 60 = 0.96N$ lb./hr.

then $$\lambda = \frac{72.5 \times 97}{0.245 \times .96N} \sim \frac{30{,}000}{N}$$

So $$\eta_r = \frac{1}{\frac{N}{15{,}000} + 1}$$

3.2.4. <u>System Pressure Drop</u> The pressure drop required to cause flow through the external circuit ΔP_r can be calculated from the well known equation applicable to laminar flow

$$\Delta P_r = \frac{32\mu LQ}{De^2Ag}$$

This is the relation for "bounded" channels - the flow passages in the regenerator being roughly triangular. Putting in the appropriate dimensions gives

$$\frac{\Delta P_r}{Q_r} = 0.0043 \ \frac{\text{lb/ins.}^2}{\text{ft.}^3/\text{min.}}$$

A pressure differential across the displacer will cause flow through the clearance space. From the laminar flow equation for flow between parallel plates it is found that

$$\Delta P_r = 0.0785 \ \frac{\text{lb./ins.}^2}{\text{ft.}^3\text{/min.}}$$

or the flow resistance of this clearance is about eighteen times that of the regenerator. In addition when the displacer moves Couette flow occurs between it and the stationary walls. If the stroke were 6 ins. the displacer speed would be Nft./min. The Couette discharge, which is in the opposite direction to Q_{cl}, is then given by

$$Q_{Co.} = \tfrac{1}{2} \times N \times \text{flow area}$$

$$\text{or} \quad Q_{Co.} = 0.0010N \ \text{ft.}^3\text{/min.}$$

If the volume displaced per stroke

$$= 0.1062 \times 2 \ \text{ft.}^3$$

$$\text{then } 2 \times 0.1062 \times N = Q_r + Q_{cl} - Q_{Co.}$$

$$= \Delta P_r \left[\frac{1}{0.0043} + \frac{1}{0.0785}\right] - 0.0010N$$

$$\text{or} \quad \Delta P_r = 0.000868N \ \text{lb./ins.}^2$$

N would have to be about 1000 before ΔP_r was as large as 1 lb./ins.^{2}

4 TEST PROCEDURE

The compressor was mounted hot end down in a stand and was supplied with water and electric power as shown in figure 4. Measuring equipment was arranged as indicated in this figure. Runs were made with and without insulation around the hot end. Cooling water was supplied from the mains via a pre-calibrated circuit fitted with a pressure gage so that flow rates could be read directly. Electrical power was supplied from the 115 volt mains through a Variac transformer. Voltage across and resistance of the heater were measured; the former with a regular a.c. voltmeter, the latter by using a resistance meter and comparing the reading with a standard resistance box. Air leaving the compressor flowed to a receiver the pressure of which was measured using a mercury manometer. A regular gas meter was used to measure air flow rates to and from the compressor and from the air motor. The motor stroke was a variable and had to be measured for each speed. This was done by attaching a pencil horizontally to a moving appendage of the piston and causing a line to be drawn on a piece of stiff paper.

Chromel-alumel thermocouples measured regenerator temperatures at four points along its length, also the gas temperatures entering and leaving the cold and hot spaces. Three sets were placed at 120° intervals around the compressor case. Other thermocouples were situated in the inlet and exhaust valve air passages. All the measured air temperatures were mean values over the cycle. The regenerator fluctuation was not observed. The space temperatures indicated changed only by one or two degrees. The air outlet temperature variation was most marked and was too violent to record at higher flow rates (this is discussed more fully in the next section). Other temperatures measured were those of cooling water at inlet and outlet and the outside centre of the hot end plate.

Performing a test run entailed turning on cooling water, starting the air motor and switching on the electric heater. The voltage was adjusted to give a suitable power input and after equilibrium had been reached a series of measurements as listed were taken:-

Compressor speed
Air flow to compressor
Air flow from compressor
Air flow from air motor
Water line pressure gage
Compressor stroke
Heater voltage
Heater resistance
Receiver pressure
Thermocouple readings as already noted

To find the pressure ratio - discharge relationship various receiver valve settings were used ranging between fully open and fully closed. Negligible changes in temperature levels were observed for the different valve settings.

Adequate results were obtained for all objectives except determination of thermal efficiency. The machine ran so slowly that the fixed heat losses swamped the heat interactions of the cycle and the calculation of thermal efficiency in section 5.2 is based largely on estimate.

The regenerator temperature distribution given by thermocouple readings with the heater on showed a hot end reading much higher than would be expected from a straight line passing through the other three points. This was deduced to be due to radiation to the thermocouple from the heater and so measurement of regenerator temperature distribution was made with the heater switched off, readings being taken rapidly before cooling was appreciable.

Power was increased in successive runs until the hot end temperature was such that the heater failed or the displacer seized. The upper limit reached was about 600°C. On several occasions the displacer seized near this temperature due to differential expansion aggravating the seizing potentialities of dirt particles etc. A device was constructed for cooling the displacer from the inside with a flow of air. This was successful but at a little higher temperature the heater failed. The heater failures were invariably due to shorting when the insulation had failed as the element expanded and found out the bad spots.

At this point it was decided to discontinue testing.

The displacer problem is one of mechanical design and the electric heater was used merely as a convenient device to be replaced by a combustion heater later. Neither of these problems is basic in the demonstration of this principle of compression.

Other data recorded consisted of pressure drop versus flow relations for the valves, the external circuit and the displacer clearance. These were routine tasks and need not be detailed here. The data for the valves is discussed in Appendix C whilst that for the external circuit and displacer clearance is presented in section 5.2

The main body of data is contained in table 2.

5. ANALYSIS OF RESULTS

5.1. Pressure ratio - discharge relationship

It is seen from Appendix A that P_2/P_1 and V_f/V_s are functions of geometry and a more or less complicated function of hot and cold end temperatures. To evaluate the performance of the compressor corresponding values of these parameters must be extracted from the data. The receiver pressure P_r was recorded and atmospheric pressure P_a assumed as 14.7 lb/ins.^{2}. Appendix C discusses in detail the relationship between P_r/P_a and P_2/P_1. Equations C1 and C2 show how to find the flow rate through either valve. The corresponding pressure drop is found from figure 9. Volume flow rates were recorded directly. By measuring compressor speed and stroke V_f/V_s can be calculated. Some difficulty occurs in sorting out suitable temperatures since without great sophistication it is not possible to measure the space temperatures at any other place than just inside them. On top of this the temperatures vary with time so only an average can be obtained. It is possible to calculate the actual air temperature if all the variables are known but this was out of the question here. However, it was noted that the end temperatures of the regenerator were always within several degrees of the measured air temperature, the difference being of the same order of magnitude as the variation between the similarly positioned thermocouples. Hence it may be suggested that for the purposes of continuity analysis an equation similar to A2 be used with T_h and T_c replaced by T_u

and T_l respectively where T_u and T_l refer to the average thermocouple readings. Hence the relation.

$$\frac{V_f}{V_s} = 1 + \frac{T_l}{T_r}\frac{V_r}{V_s} - \frac{P_2}{P_1}\left[\frac{T_l}{T_u} + \frac{T_l}{T_r}\frac{V_r}{V_s}\right] \qquad 1$$

Data was assembled for various P_a and V_f values for substantially constant values of T_u and T_l. Since the geometrical quantity V_r/V_s is known the experimental value of T_u/T_l can be substituted above and P_2/P_1 plotted against V_f/V_s. On the same graph can be plotted the experimental values of P_2/P_1 found from P_r/P_a and the corresponding value of V_f/V_s. Another difficulty lies in the fact that leakage from the compressor occurred and so there are two experimental numbers for V_f/V_s based on induced and exhausted air respectively. Two modes of operation may be analysed here. One is when the receiver pressure is zero lb./ins.^{2} gage. In this case a slight rise in pressure in the compressor is generated but the corresponding leakage would be negligible; hence the calculation of P_2/P_1 from P_r/P_a is made as previously described. The other mode of operation is when no air is exhausted. In this case air is induced to make up for the leakage. By continuity this mass of air which has leaked is identical to that which would have had to be exhausted if no leakage had occurred and the final pressure was the same. Hence P_2/P_1 can be found from P_r/P_a as indicated in Appendix C but allowing only for flow through the inlet valve.

A wide variety of results were taken for many different temperatures. These results are listed in table 2. The sets of readings for $T_u/T_l = 2.21$ and $T_u/T_l = 1.83$ have been analysed as suggested and the results are shown on figures 6 and 7 respectively. Referring to both figures curve 1 represents V_f/V_s based on air exhausted and curve 2 V_f/V_s based on air induced, both being plotted against P_r/P_a. Curve 3 is a suggested curve through the values of P_2/P_1 for the two modes of operation on curves 1 and 2 discussed in detail above. Curve 4 is the result of putting the appropriate value of T_u/T_l into equation 1. Good agreement is seen between 3 and 4 for low flow rates. At high flow rates the actual value of P_2/P_1 falls substantially below the predicted value. Since the measurement of P_2/P_1 and V_f/V_s is felt to be quite satisfactory this discrepancy can only be due to the actual value of T_u/T_l being less than the indicated value, the effect increasing with V_f/V_s. The cause of this lower value of T_u/T_l is simply heat transfer with the walls of the space. At the higher flow rates substantial mixing is promoted in the cold space by the flow of the induced change whereas at zero flow turbulence will be minimal. No such new charge enters the hot space but the same mixing effect will apply to a lesser degree. In both spaces heat transfer with the walls will decrease T_u/T_l. During the test runs it was noted that the exhaust valve got quite warm and the thermocouple in the exhaust passage read values much higher than the indicated space temperature T_l. Due to violent fluctuations in response to periodic discharge of air it was not possible to measure the high value accurately but for the case of zero receiver gage pressure the temperature reading was of the order of 50°C as compared with a cold space indicated temperature of 10°C. Consider the point Y on figure 6. This corresponds to the experimental point for P_a equal to zero lb./ins.^{2} gage. It will be found that if equation 1 is applied with the value of P_2/P_1 and V_f/V_s of point Y then T_u/T_1 is approximately equal to 1.9. The indicated value was 2.21 for $T_1 = 285°K$ and $T_u = 630°K$. The observed exhaust air temperature of $\sim$ 50°C was not typical of all the cold space air since that air which is eventually discharged has been in contact with the hotter parts of the space walls. It must be allowed though that the air passing close to the cold space thermocouples also passes close to the cold end plate and hence will be cooled before encountering the thermocouple. The same effect applies in reverse at the hot end. Therefore a reasonable estimate can be made that the cold space temperature is, say, 30°C hotter than that indicated and the hot space temperature 20°C colder than that indicated. This would make $T_u/T_1 = 610/315 = 1.93$. So this effect is sufficient to explain why the actual characteristic departs from the one based on T_u/T_1 indicated. A heat transfer calculation based on several gross assumptions and not worthy of reproduction shows that in order to transfer the amounts of heat implied in 20°C-30°C temperature rises demands a heat transfer coefficient of the order of 2 $\frac{\text{BTU s}}{\text{ft.}^2\text{hr.}°\text{F}}$

which is reasonable.

In summarizing it is seen that the experimental data can be made to agree with the predictions of equation 1 after adjustment has been made to allow for pressure drop through the valves and the fact that heat exchange with the space walls tends to make the indicated value of T_u/T_1 too high. Certainly if T_u/T_1 were raised to values nearer 3.5 as it has been in the Philips hot air engine (6) and the speed increased then the effect of heat transfer would become quite small relatively and the predictions of equation 1 would be more nearly realised.

5.2. Estimation of Overall Thermal Efficiency

Due to the very low running speed of the test compressor it was not possible to detect any variation in temperature readings when changes in pressure ratio were effected at constant power input. The heat extracted from the cooler was of the order of 2 to 5 watts when the input was from 100 to 1000 watts. Hence it was decided to consider the entire heat input as fixed heat losses to the surroundings. These values are plotted against T_u in figure 11. The running losses are to be calculated from various other noted data and a value for the overall thermal efficiency can be estimated.

Since the prototype compressor was obviously poorly designed in many respects perhaps more value could be attached to such an analysis if certain simple changes were supposed made in the specifications. A value of $T_u \sim 750°C$ was deemed quite workable in the Philips hot air engine (6). Let this value be used together with $T_1 = 17°C$. Also, to reduce the value of V_r/V_s, suppose that a 7.5 ins. long displacer with a 7.5 ins. stroke were used instead of the present one. As shown in table 1 this reduces V_r/V_s to 0.69. This specification enables a pressure ratio - discharge characteristic of the type curve 4 on figures 6 and 7 to be found. For any given value of T_u/T_1 and any constant speed N then η_t will be zero for V_f/V_s = zero and $P_2/P_1 = 1$. Somewhere between these values η_t will be a mamimum and any optimisation should determine the position of this for every running speed. As a reasonable estimate let the half way values of V_f/V_s and P_2/P_1 be used in the analysis. The relation to be solved is equation 1.

$$\frac{V_f}{V_s} = 1 + \frac{T_1}{T_r}\frac{V_r}{V_s} - \frac{P_2}{P_1}\left[\frac{T_1}{T_u} + \frac{T_1}{T_r}\frac{V_r}{V_s}\right]$$

Putting $V_r/V_s = 0.69$, $T_u/T_1 = (750 + 273)/(17 + 273) = 3.6$

$$\frac{V_f}{V_s} = 1.324 - 0.602\,\frac{P_2}{P_1}$$

$$\frac{V_f}{V_s}_{max} = 0.722$$

Half of this is 0.361 and the corresponding P_2/P_1 is 1.6. This will be the assumed operating point at all speeds.

Suppose now that the compressor is to driven by its own compressed air so that in computing the useful free air delivered the consumption of a suitable air motor would have to be subtracted. The required heat interactions can be found now, item by item.

(a) Heat requirement implied by the thermodynamic cycle.

If V_f/V_s is 0.36. and V_s is 0.133 ft.^{3} then the free air delivered is 0.0481N ft.^{3}/min. The work required to compress air through P_2/P_1 of 1.6 isothermally will be $P_1 \ln(P_2/P_1)/J$, or 1.28 BTU./ft.^{3} of free air. The ideal efficiency of the cycle is $(T_u-T_1)/T_u$, or 0.722. Suppose that due to non-ideal features this is reduced to 0.75 x 0.72 = 0.54. Thus the heat supplied = 1.28/0.54 = 2.37 BTU. per cubic ft. of free air. Using the above figure for the air delivery rate the isothermal compression work delivered is 0.0615N BTU./min. and the heat required is 0.114N BTU./min.

(b) Effect of regenerator inefficiency.

Assume that a mass of air equal to that contained in the hot space when V_u is equal to V_s is passed through the regenerator in each direction per cycle. This mass will be equal to

$$\frac{P_2V_s}{RT_u} = \frac{14.7 \times 1.6 \times 0.133 \times 144}{96 \times 1023} \text{ lbs./cycle}$$

$$= 4.58 \times 10^{-3} \text{ lbs./cycle}$$

During the course of one complete cycle this mass will transfer an amount of heat $Cp\left[T_u-T_1\right]\left[1-\eta_r\right]$ from the heater to the cooler by virtue of the inefficiency of the regenerator. From section 3.2.3.

$$\eta_r = \frac{1}{N/15000 + 1}$$

$$1-\eta_r = \frac{N}{N + 15000}$$

Suppose $N \ll 15000$, as it will be

$$1-\eta_r = \frac{N}{15000}$$

Thus with T_u = 750°C and T_1 = 17°C the above figures give the regenerator loss as $0.000097N^2$ BTU./min.

(c) Pumping Power

The pressure drop versus discharge characteristic for the regenerator passage was calculated and found experimentally. The latter is displayed on figure 8 where also is shown the displacer gap characteristic. The flow past the displacer is less than one eighteenth of that through the external circuit and will be ignored. The equation of the line is

$$\frac{\Delta P}{Q} = 0.00375 \ \frac{\text{lb./ins.}^2}{\text{ft.}^3\text{/min.}}$$

Pumping work = Pressure drop x volume flow rate.

= $0.00375Q \times 144 \times Q$ ft.lbs./min

$Q = 2V_sN$ = $0.266N$ ft.^{3}/min.

Pumping work = $\frac{0.00375 \times 144}{778} \times (0.266)^2 N^2$ BTU./min.

= $0.0000492N^2$ BTU./min.

The work done by 1 lb. of air in expanding adiabatically from P = 14.7 x 1.6 lb./ins.^{2} to P = 14.7 lb./ins.^{2} is $Cp[T_2-T_1]$

= $0.24 \times 1.8 \times 290 [1.6^{0.286} - 1]$

= 18 BTU./lb.

Since the free air density is 14.7 x 144/96 x 290 = 0.076 lb./ft.^{3} the work done per cubic foot of free air

= 1.37 BTU.

Suppose the ratio $\frac{\text{actual pumping work done in the compressor.}}{\text{adiabatic work done in the air motor.}}$

equals 0.5 to allow for friction and non - adiabatic working.

The air motor free air consumption will be therefore

$$= \frac{0.0000492N^2}{1.37 \times 0.5} \text{ ft.}^3\text{/min.}$$

$$= 0.0000718N^2 \text{ ft.}^3\text{/min.}$$

The loss of useful isothermal work

$$= 0.0000718N^2 \times 1.28 \text{ BTU./min.}$$

$$= 0.0000918N^2 \text{ BTU./min}$$

(d) Fixed Heat Losses

For a nominal hot end temperature of 750°C the fixed heat losses are given by figure 11 as 1.120 kw.

$$= 63.6 \text{ BTU./min.}$$

(e) Overall Thermal Efficiency

The overall thermal efficiency η_t is defined as

$$\eta_t = \frac{\text{total compressive work - pumping load.}}{\text{fixed heat losses + regenerator losses + cycle power.}}$$

$$\eta_t = \frac{0.0615N - 0.0000918N^2}{63.6 + 0.000097N^2 + 0.114N}$$

η_t is plotted versus N on figure 12. The maximum value reached is about 9.8% near 300 strokes/min. This is a very approximate estimate of η_t but it was based on a far from optimum design. Valve pressure drop has been ignored as it was felt that the ones fitted were very poor in this respect and a practical design could reduce this pressure drop to a very small value.

A comparison may be made with a gasoline engine - driven reciprocating compressor. The data is taken from (18)

For a gasoline engine of the type used a specific fuel consumption of 0.75 lbs./B.H.P. hr. is given. The calorific value of the fuel is about 19000 BTU./lb. This represents an overall thermal efficiency for the engine of 0.18. Also stated in (18) is the fact that to compress 100 ft.^{3} of air per minute to 40 lb./ins.^{2} gage 12.8 B.H.P. is needed. This means a compressor efficiency of 0.65 so the system overall thermal efficiency is 0.18 x 0.65 = 0.12. So the experimental compressor could be modified a reasonably small amount and when run at 300 strokes/min.should compete with the conventional machines. Of course the output per unit weight of machine would be very much lower for the Stirling compressor at this speed.

5.3 Regenerator Performance.

In the analysis of regenerator design, section 3.2.3., it was shown that in this case the expression for regenerator efficiency η_r given in Appendix B can be reduced to

$$\eta_r = \frac{1}{1 + N/15000}$$

From recorded data an attempt can be made to verify this result. Readings of regenerator temperatures and cold end space temperatures were taken for a typical case. The thermocouple readings were taken immediately after switching off the heater to avoid radiation to the top thermocouple as mentioned earlier. These temperatures are shown plotted in figure 10 as a function of position. A straight line is obtained. This line has been extrapolated in both directions to obtain the temperatures right at the ends. Since the cooler is very effective the assumption is made that the air entering the regenerator at that end has the temperature of the cold sink. This is a conservative estimate from the point of view of the regenerator. In this case the cold sink temperature was 12.5°C and from figure 10 the cold end of the regenerator is seen to be about 20°C. Thus the cold end regenerator-to-air temperature difference is 7.5°C. The hot end of the regenerator is 380°C. If there is a constant difference in temperature between air and regenerator then the regenerator efficiency can be expressed as $\eta_r = \frac{(380 - 7.5) - (20 - 7.5)}{(380 + 7.5) - (20 - 7.5)}$

$= 0.96.$

The results were taken for N = 28 strokes/min. so that the predicted value is

$$\eta_r = \frac{1}{1 + 0.002}$$

$$= 0.998.$$

The experimental value is lower than predicted. However, it is very difficult to evaluate inefficiency experimentally when it becomes so small and in any case minute end effects may seriously affect the usefulness of the data on regenerator temperature distribution presented. This being so the agreement between actual and predicted performance is quite acceptable.

6. SUMMARY.

Although the compressor built and tested was plainly far from competitive in performance the data, practical experience and theoretical analysis presented here are sufficient foundation for an optimization study. The performance of this configuration may be expressed as a function of geometrical and thermal parameters using the component test results and the analysis of Appendix A. However, several other configurations for the Stirling engine exist and any further work should include a study of these. They will be in main mechanical variations and it is anticipated that the thermodynamic analyses will be nearly identical.

The regenerator analysis presented is a particular one strictly applicable only for the numerical values obtaining. It is noted that for a laminar flow regenerator both pumping work and regenerator loss are functions of $(\text{speed})^2$ and a balance between these quantities can be obtained intuitively.

The testing of the machine was made using relatively unsophisticated instruments. If a serious study of these machines is to be made then a resistance wire grid and oscilloscope would be more suitable for gas temperature measurement and a pressure transducer and oscilloscope likewise for cylinder pressure.

Since the writing of the body of the thesis the heat transfer analysis of the cooler as a combined fin and heat exchanger has been performed by the author as a term paper for the M.I.T. Heat Transfer Course 2.522. It appears from this analysis that the same efficiency is obtained for a given sink and gas inlet temperature with flow in either direction.

Finally attention is drawn to the work of Dr. Theodore Finkelstein of Battelle Memorial Institute. He has done much work on the subject, including demonstrating how real hot and cold space conditions (model (1)) may be analysed using digital computers. He is reputed to have a book on Stirling engines in print, and it would appear as though Battelle Memorial Institute may become as considerable a source of information as the Philips Company.

7 REFERENCES.

1. Bourne, J., Steam, Air and Gas Engines, Longmans, Green and Co., 1878.
2. Malone, J.F.J., A New Prime Mover, Jour. Roy. Soc. Arts, Vol. LXX1X, 1931.
3. Rinia, H. and du Pre, F.K., Air Engines, Philips Technical Review, Vol. 8, No. 5, 1946.
4. de Brey, H., Rinia, H. and van Weenan, F.L., Fundamentals for the Development of the Philips Hot Air Engine, Philips Technical Review, Vol. 9, No. 4, 1947/48.
5. van Weenan, F.L., The Construction of the Philips Air Engine, Philips Technical Review, Vol. 9, No. 5, 1947/48
6. Meijer, R.J., The Philips Hot Gas Engine with Rhombic Drive Mechanism, Philips Technical Review, Vol. 20, No. 9, 1958/59.
7. Kohler, J.W.L., and Jonkers, C.O., Fundamentals of the Gas Refrigerating Machine, Philips Technical Review, Vol 16, No.4, 1954/55.
8. Kohler, J.W.L., and Jonkers C.O., Construction of a Gas Refrigerating Machine, Philips Technical Review, Vol. 16, No. 4, 1954/55.

9. Walker, G., Operations Cycle of the Stirling Engine with particular reference to the function of the Regenerator, Jour. Mech. Eng. Science, Vol. 3, No. 4, 1961

10. Schmidt, G., Theorie der Lehmann'schen calorischen Maschine, Z. Ver. dtsch. Ing., Vol. 15, No. 1,

11. Finkelstein, T., Generalized Thermodynamic Analysis of Stirling Engines, S.A.E. Annual Meeting, N.Y., 1960

12. Hausen, H., Uber die Theorie des Warmeaustausches in Regeneratoren, Z. Angew. Math. Mech., Vol. 9, 1929

13. Illiffe, C., Thermal Analysis of the Contra-flow Regenerator Heat Exchanger, Proc. Instn. Mech. Eng., London, Vol. 159, 1948

14. Shultz, B.M., Regenerators with Longitudal Heat Conduction, Proc. Gen. Disc. Heat Trans. 1951, I. Mech. E. London-A.S.M.E.

15. Shultz, B.M., Approximative Formulae in the Theory of Thermal Regenerators, Applied Science Review, Vol. 3, 1952.

16. Saunders, O., and Smoleniec, S., Heat Transfer in Regenerators, Proc. Gen. Disc. Heat Trans. 1951, I. Mech. E. London-A.S.M.E.

17. Murray, J.A., Martin, B.W., Bayley, F.J., and Rapley, C.W., Performance of Thermal Regenerators under Sinusoidal Flow Conditions, 1961 Int. Heat Trans. Conf., I. Mech. E.-A.S.M.E.

18. Feller, E.W.F., Air Compressors, McGraw-Hill, 1944.

19. Keenan, J.H., Thermodynamics, Wiley, 1941.

8. NOMENCLATURE.

P_1	Lowest pressure inside the compressor, occurs between states 4 and 5.
P_2	Highest pressure inside the compressor, occurs between states 2 and 3.
P_r	Receiver pressure.
P_a	Atmospheric pressure.
ΔP	Difference of pressure (specified in context).
T_h	Air temperature leaving the heater in either direction; also assumed equal to the time mean hot end temperature of the regenerator.
T_c	Air temperature leaving the cooler in either direction; also assumed equal to the time mean cold end temperature of the regenerator.
T_u	Mean or nominal air temperature in the hot space.
T_l	Mean or nominal air temperature in the cold space.
T_r	Characteristic temperature of air in the regenerator.
V_s	Volume swept out by the displacer.
V_u	Hot space volume ($0 < V_u < V_s$).
V_l	Cold space volume ($0 < V_l < V_s$). (Note $V_s = V_l + V_u$).
V_r	Total unswept volume in the system.
V_f	Free air discharged per completed cycle.
N	Number of strokes per minute.
Q	Volume rate of flow (specified in context)

q	Heat flux per unit mass of air delivered.
$\bar{q}$	Heat flux per stroke
	Subscript to the q's of r, c and h refer to regenerator, cooler and heater.
M_l	Mass of air in the cold space.
M_u	Mass of air in the hot space.
M_r	Mass of air in the unswept volume.
M_d	Mass of air discharged per stroke.
η_t	Overall thermal efficiency.
η_r	Regenerator efficiency.
λ	See Appendix B.
R	Universal gas constant.
Cp	Specific heat at constant pressure.
Cv	Specific heat at constant volume.
k	Cp/Cv
n	(k-1)/k
Ie, Id	defined in Appendix D.
V_e, V_d	defined in Appendix D.
Se, Sd	defined in Appendix D.
a, b	defined in Appendix D.
J	Mechanical Equivalent of Heat
r	P/P_1 ; $P_1 < P < P_2$
r_a	P/P_2 ; $P_1 < P < P_2$

APPENDIX A

Part 1. Analysis of the Pressure-Discharge Relationships

The relation between pressure ratio and discharge can be derived for any given description of compressor operation. In general the more ideal the assumed mode of operation the less complicated the analysis. Two analyses are presented here, the first of which represents an ideal cycle, the second being closer to reality but showing such an increase in complexity that any further refining of the model would lead to an analysis of (in this context) unmanageable length.

The following assumptions have been made in both cases:-

(a) The distribution of the air in the passage between hot and cold spaces is linear between T_c and T_h. This linearity has been verified for the regenerator (see figure 10 and Appendix B) which accounts for most of the free space. In addition to the regenerator void there will be undisplaced air at the hot and cold ends due to the heater, cooler and insufficient displacer travel. It is proposed to account for these spaces by lumping them with the regenerator void. A consequence of this is that a characteristic temperature T_r dependent only on T_h and T_c and applicable strictly only to the regenerator can be assigned to all the unswept volume V_r. Suppose that all of V_r was in the regenerator and contained air with temperature $T\left(\frac{x}{L}\right) = T_c + (T_h - T_c)\frac{x}{L}$ where x is the distance from the cold end and L is the total regenerator length.

Consider a short length of void dx. The mass of air contained in this element of void

$$= \frac{P}{RT} \cdot \frac{V_r \cdot dx}{L}$$

$$M_r = \int_0^1 \frac{PV_r \, d\left(\frac{x}{L}\right)}{RT\left(\frac{x}{L}\right)}$$

$$= \frac{PV_r}{R} \cdot \frac{\ln(T_h/T_c)}{T_h - T_c}$$

Hence $$T_r = \frac{T_h - T_c}{\ln(T_h/T_c)}$$

(b) No leakage occurs through the clearance between displacer and cylinder. Experiment and calculation show that the ratio of flow resistance in this passage to that of the regenerator is about eighteen to one so this assumption is nearly the truth.

(c) The volume of the displacer rod is zero. In fact its volume is about 1% of the swept volume so that the maximum cold space volume is about 1% less than the maximum hot space volume.

(d) The pressure difference across the passage and displacer ΔP_r is zero. It has been shown earlier that ΔP_r is equal to 0.00087N lb./ins.^{2}. In the reported tests N never exceeded 30 so $\Delta P_r \sim 0.03$ lb./ins.^{2} so again the assumption is nearly true.

(e) Air leaving the heater does so at T_h and air leaving the cooler does so at T_c. A consequence of this is that the hot end of the regenerator is at T_h and the cold end is at T_c. Hence air entering the hot space does so at T_h; air entering the cold space does so at T_c.

(f) Air is a perfect gas in the range of properties considered.

Models may now be proposed describing events in the hot and cold spaces in at least three different ways. These are:-

Model 1. Air enters the hot space at T_h and mixes partially with air already in there. Some heat transfer takes place with the walls. Air enters the cold space at T_c from the cooler and T_a from the atmosphere. Air is exhausted from the cold space at some temperature resulting from partial mixing. This is the true state of affairs.

Model 2. As model 1 but the spaces are to be considered adiabatic and the temperature of the air therein is uniform.

Model 3. The air in the hot space is uniform and constant at T_h; that in the cold space is uniform and constant at T_c. The atmospheric temperature is T_c.

Although model 1 represents the truth its analysis is too complex. Models 2 and 3 will be analysed here.

The following points in the cycle are identified by suffices:- (see figure 1)

1. Displacer is at hot end. V_u is zero. Pressure is P_1.

2. Displacer is part way towards cold end. Pressure is P_2.

3. Displacer is at cold end. V_1 is zero. Pressure is P_2 and some air has been discharged at constant pressure P_2.

4. Displacer is part way towards hot end. Pressure is P_1

5. Displacer is at hot end. V_u is zero. Pressure is P_1 and some air has been induced at constant pressure P_1.

Analysis of Model 3 - Isothermal Spaces. This case can be completely solved by applying continuity between 1. and 3. That is:-

(Mass of air in compressor at 1.)

= (mass of air in compressor at 3) + M_d.

$$M_{u1} + M_{r1} + M_{11} = M_{u3} + M_{r3} + M_d + M_{13}$$

$$M_{13} = M_{u1} = 0$$

$$M_{r1} = \frac{P_1 V_r}{RT_r}$$

$$M_{r3} = \frac{P_2 V_r}{RT_r}$$

$$M_{11} = \frac{P_1 V_s}{RT_c}$$

$$M_{u3} = \frac{P_2 V_s}{RT_h}$$

So

$$M_d = \frac{P_1 V_s}{RT_c}\left[1 - \frac{P_2 . T_c}{P_1 \; T_h}\right] - \frac{(P_2 - P_1)V_r}{RT_r} \qquad \text{(A1}$$

If V_f = the volume of M_d at P_1 and T_c, known as the "free air delivered"

$$\frac{V_f}{V_s} = 1 + \frac{T_c}{T_r}\cdot\frac{V_r}{V_s} - \frac{P_2}{P_1}\left[\frac{T_c}{T_h} + \frac{T_c}{T_r}\cdot\frac{V_r}{V_s}\right] \quad \text{(A2}$$

For a given value of T_h/T_c and V_r/V_s this is a linear relationship between V_f/V_s and P_2/P_1. The effect of void space V_r/V_s in reducing V_f/V_s for a given P_2/P_1 can be seen.

<u>Analysis of Model 2 - Adiabatic Spaces.</u> As before continuity can be applied between 1. and 3.

$$M_{r1} + M_{1_1} = M_{u3} + M_d + M_{r3}$$

$$M_{r1} = \frac{P_1 V_r}{RT_r}$$

$$M_{r3} = \frac{P_2 V_r}{RT_r}$$

$$M_{1_1} = \frac{P_1 V_s}{RT_{1_1}}$$

$$M_{u3} = \frac{P_2 V_s}{RT_{u3}}$$

$$M_d = \frac{P_1 V_s}{RT_{1_1}}\left[1 - \frac{P_2}{P_1}\cdot\frac{T_{1_1}}{T_{u3}}\right] - \frac{(P_2 - P_1)V_r}{RT_r} \quad \text{(A3}$$

M_d is discharged at T_{12} but it is more useful to consider the free air delivered at the atmospheric temperature T_a

$$M_d = \frac{P_1 V_f}{RT_a}$$

So $\frac{V_f}{V_s} = \frac{T_a}{T_{l_1}} + \frac{T_a}{T_r} \cdot \frac{V_r}{V_s} - \frac{P_2}{P_1}\left[\frac{T_a}{T_{u3}} + \frac{T_a}{T_r} \cdot \frac{V_r}{V_s}\right]$ (A4

Here T_{l_1} and T_{u3} are as yet unknown. The relation does not depend on T_h or T_c. In order to obtain more information about the process the First Law of Thermodynamics as defined for open systems (19) equation 5 can be applied to the hot and cold spaces. Control volumes have been defined in figure 5b. It is noted again that air enters the hot space from the heater at T_h and leaves at T_u: air enters the cold space at T_c from the cooler and T_a from the atmosphere and leaves at T_l. Application of equation 5, (19), for process 1. to 2. gives for the hot end, control volume A:-

$$0 = PdV_u + Cv\left[T_u dM_u + M_u dT_u\right] - CpT_h dM_u \quad \text{(A5}$$

and for the cold end control volume B:-

$$0 = PdV_l + Cv\left[T_l dM_l + M_l dT_l\right] - CpT_l dM_l \quad \text{(A6}$$

For a perfect gas

$$PV = MRT$$

So $PdV_u + V_u dP = (Cp - Cv)(T_u dM_u + M_u dT_u)$ (A7

and $PdV_l + V_l dP = (Cp - Cv)(T_l dM_l + M_l dT_l)$ (A8

Add equations A6 and A8

$$V_l dP = CpM_l dT_l \quad \text{(A9}$$

$$\frac{dP}{P} = \frac{k}{k-1} \frac{dT_l}{T_l} \quad \text{(A10}$$

$$\frac{P}{P_1} = \left(\frac{T}{T_{l_1}}\right)^{k/k-1} \quad \text{(A11}$$

The familiar adiabatic relationship thus applies for the cold space when air is leaving it.

Multiply A7 plus A8 by -Cv/(Cp - Cv) and add to A5 plus A6

Note $\qquad V_1 + V_u = V_s$

So $\qquad dV_u = -dV_1$

$$\frac{V_s dP}{k-1} = CpT_h dM_u + CpT_1 dM_1 \qquad \text{(A12}$$

From 1 to 2

$$M_1 + M_r + M_u = \text{constant}$$

or

$$dM_u = -\left[dM_1 + dM_r\right] \qquad \text{(A13}$$

also

$$dM_r = \frac{V_r dP}{RT_r} \qquad \text{(A14}$$

Put A13 and A14 in A12 and simplify to

$$\left[V_s + \frac{kV_r T_h}{T_r}\right]\frac{dP}{k-1} = -CpdM_1\left[T_h - T_1\right] \qquad \text{(A15}$$

Let

$$V_e = V_s + \frac{kV_r T_h}{T_r}$$

Note equations A10 and A11. Put :-

$$\frac{P}{P_1} = r$$

and

$$\frac{k-1}{k} = n$$

A15 can be reduced to:-

$$\frac{V_e P_1}{kRT_h} \frac{dr}{1 - \frac{T_{1_1}}{T_h} r^n} = -dM_1 \qquad \text{(A16}$$

The integral $\int \frac{dr}{1-ar^n}$ where a equals T_{1_1}/T_h is discussed in Appendix D and is given the symbol Ie. Equation A16 integrates to

$$M_{1_1} - M_{12} = \frac{P_1 V_e}{RT_h} \cdot \frac{Ie}{k}$$

and

$$M_{12} = \frac{P_1 V_s}{RT_{1_1}} \left[1 - \frac{V_e}{V_s} \cdot \frac{T_{1_1}}{T_h} \cdot \frac{I_e}{k} \right] \qquad \text{(A17}$$

For the process 2. to 3. equations A5 and A7 still are true. Multiply A7 by Cv/(Cp - Cv) and subtract A5

$$\frac{k}{k-1} P_2 dV_u = C_p T_h dM_u$$

$$P_2 \left[V_s - V_{u2} \right] = RT_h \left[M_{u3} - M_{u2} \right] \qquad \text{(A18}$$

By continuity between 2. and 3.

$$M_{u3} - M_{u2} = M_{12} - M_d$$

and

$$P_2 \left[V_s - V_{u2} \right] = P_2 V_{12}$$

$$= RT_{12} M_{12}$$

M_{12} is given by A17

$$M_d = \frac{P_1 V_f}{RT_a}$$

Equation A18 reduces to

$$RT_{12} M_{12} = RT_h \left[M_{12} - M_d \right]$$

$$\frac{V_f}{V_s} = \frac{T_a}{T_h} \left[1 - r^n \frac{T_{1_1}}{T_h} \right] \left[1 - \frac{V_e}{V_s} \cdot \frac{T_{1_1}}{T_h} \cdot \frac{I_e}{k} \right] \qquad \text{(A19}$$

Between equations A4 and A19 either T_{1_1} or T_{u3} may be eliminated. One more equation must be found in order to express V_f/V_s as a function solely of T_h, T_c and P_2/P_1. This can be obtained by following the process through from 3. to 4. to 5. in just the same manner as 1. to 2. to 3.. The First Law equations will now be, for process 3. to 4.

$$0 = PdV_u + Cv\left[T_u dM_u + M_u dT_u\right] - CpT_u dM_u \qquad (A20$$

and

$$0 = PdV_1 + Cv\left[T_1 dM_1 + M_1 dT_1\right] - CpT_c dM_1 \qquad (A21$$

From A20 and A7 may be deduced that

$$\frac{dP}{P} = \frac{k}{k-1}\,\frac{dT_u}{T_u} \qquad (A22$$

$$\frac{P}{P_2} = \left(\frac{T_u}{T_{u3}}\right)^{\frac{k}{k-1}} \qquad (A23$$

From A21 and A8 can be found

$$\frac{V_s dP}{k-1} = CpT_u dM_u + CpT_c dM_1$$

$$\frac{V_d dP}{k-1} = CpdM_u\left[T_u - T_c\right]$$

where

$$V_d = V_s + \frac{kV_r T_c}{T_r}$$

if

$$r_a = \frac{P}{P_2}$$

$$\frac{V_d P_2}{kRT_{u3}}\;\frac{r_a^{-n}\,dr_a}{1-\frac{T_c}{T_{u3}}\,r_a^{-n}} = dM_u \qquad (A24$$

The integral $\int \frac{r_a^{-n} dr_a}{1-br_a^{-n}}$ where b is equal to $\frac{T_{11}}{T_h}$ is discussed in Appendix D and is given the symbol Id. Equation A24 integrates to

$$M_{u4} - M_{u3} = \frac{P_2 V_d Id}{kRT_{u3}}$$

$$M_{u4} = \frac{P_2 V_s}{RT_{u3}}\left[1 + \frac{V_d}{V_s}\cdot\frac{Id}{k}\right] \qquad \text{(A25}$$

For process 4. to 5. the First Law equation for the cold space is:-

$$0 = PdV_1 + Cv\left[T_1 \, dM_1 + M_1 \, dT_1\right] - CpT_c\left[dM_1 - dM_i\right] - CpT_a dM_i \qquad \text{....(A26}$$

where M_i = mass induced at temperature T_a.

Combined with A8 this gives:- ($dP = 0$)

$$\frac{k}{k-1} P_1 dV_1 = CpT_c dM_1 + Cp\left[T_c - T_a\right]dM_i$$

$$\text{or } P_1\left[V_s - V_{14}\right] = R\, T_c(M_{15} - M_{14}) + (T_a - T_c)M_i \qquad \text{(A27}$$

This can be simplified using A25 and the relation

$$M_i = M_d = \frac{P_1 V_f}{RT_a}$$

$$\frac{V_f}{V_s} = \frac{P_2}{P_1}\left[r_a^{\,n} - \frac{T_c}{T_{u3}}\right]\left[1 + \frac{V_d}{V_s}\circ\frac{Id}{k}\right] \qquad \text{(A28}$$

Equations A4, A19 and A28 involve four variables T_{u3}, T_{1_1}, P_2/P_1 and V_f/V_s for any fixed geometry and values of T_c and T_h. Using two of them T_{u3} and T_{1_1} can be eliminated, leaving a relation between P_2/P_1 and V_f/V_s which is the desired result.

No explicit analytical solution is possible, however. T_{u3} and T_{l1} have to be eliminated by iteration of a numerical solution.

Part 2. Analysis of Heat Interactions

The processes described in Part 1. concern only the space conditions. Certain heat interactions are implied in each case and although these will not be needed to analyse the particular experimental data it is useful to compare the efficiencies of the two models.

Analysis of Model 3. Since this model is ideal and air is compressed isothermally the heat supplied per pound of air delivered

$$q_h = \frac{T_h}{T_h - T_c} \cdot RT_c \ln(P_2/P_1) \quad \text{(A29}$$

and heat rejected per pound of air delivered

$$-q_c = \frac{+T_c}{T_h - T_c} \cdot RT_c \ln(P_2/P_1) \quad \text{(A30}$$

The regenerator may be analysed using the First Law of Thermodynamics for an open system.

$$\begin{aligned} d\bar{q}_r &= 0 + CpT_h dM_u + CpT_c\left[-dM_u - dM_r\right] + CvT_r dM_r \\ &= Cp\left[T_h - T_c\right]dM_u + \left[CvT_r - CpT_c\right]dM_r \quad \text{(A31} \end{aligned}$$

Integrating between 1. and 3. gives the heat supplied per stroke.

$$\bar{q}_r = \frac{P_2V_sCp}{R}\left[1 - \frac{T_c}{T_h}\right] + \frac{(P_2-P_1)V_r}{R}\left[Cv - \frac{CpT_c}{T_r}\right]$$

Dividing this by equation A1 gives the heat stored in the regenerator per pound of air delivered.

$$q_r = CpT_c\left[\frac{\left(1 - \frac{T_c}{T_h}\right) + \left(1 - \frac{P_1}{P_2}\right)\left(\frac{1}{k} - \frac{T_c}{T_r}\frac{V_r}{V_s}\right)}{\frac{P_1}{P_2} - \frac{T_c}{T_h} - \left(1 - \frac{P_1}{P_2}\right)\left(\frac{V_r}{V_s}\cdot\frac{T_c}{T_r}\right)}\right] \quad \text{(A32}$$

If the regenerator has an efficiency $_r$ then the additional heat transferred directly from heater to cooler per pound of air delivered is $\left[1 - \eta_r\right]q_r$

So thermal efficiency

$$\eta_t = \frac{RT_c \ln(P_2/P_1)}{(1 - \eta_r)q_r + q_h} \quad \text{(A33}$$

Analysis of Model 2. Applying the First Law of Thermodynamics for open systems to the cooler gives for 1. to 3.

$$d\bar{q}_c = Cp\left[T_1 - T_c\right]\left[dM_1 + dM_d\right] \quad \text{(A34}$$

From 1. to 2. dM_d is equal to 0. Note equations A11 and A16

$$\bar{q}_{c12} = \frac{-P_1V_e}{k-1}\int_1^2 \frac{\frac{T_{1_1}}{T_h} r^n - \frac{T_c}{T_h}}{1 - \frac{T_{1_1}}{T_h} r^n}\, dr$$

$$\bar{q}_{c12} = \frac{-P_1V_e}{k-1}\left[\left(1-\frac{T_c}{T_h}\right)Ie - \left(\frac{P_2}{P_1} - 1\right)\right] \quad \text{(A35}$$

From 2. to 3. T_1 is constant, equal to $T_{1_1}r^n$

$$\bar{q}_{c23} = -Cp\left[T_1 r^n - T_c\right]\left[M_{12} - M_d\right]$$

$$\bar{q}_{c23} = -\left[T_{1_1} r^n - T_c\right]\left[k + \frac{V_e}{V_s}\cdot\frac{T_{1_1}}{T_h}\cdot\frac{Ie}{}\right]\frac{P_1 V_s \cdot r^n}{k-1 T_h} \quad \text{(A36}$$

During the process 3. to 5. the heat transferred to the cooler is the consequence of regenerator inefficiency

$$\bar{q}_{35} = -\left[1 - \eta_r\right]\bar{q}_r \quad \text{(A37}$$

For the heater between 1. and 3. the heat transfer is:-

$$\bar{q}_{113} = \left[1 - \eta_r\right]\bar{q}_r \quad \text{(A38}$$

Between 3. and 5. the First Law of Thermodynamics gives

$$d\bar{q}_h = -Cp\left[T_h - T_u\right]dM_u$$

Note equations A23 and A24

$$\bar{q}_{h34} = \frac{-V_d P_2}{k-1}\int_3^4 \frac{\frac{T_h}{T_{u3}} r_a^{-n} - 1}{1 - \frac{T_c}{T_{u3}}\cdot r_a^{-n}}\cdot dr_a$$

$$\bar{q}_{h34} = \frac{-V_d P_2}{k-1}\left[\left(\frac{T_h - T_c}{T_{u3}}\right) Id + \left(\frac{P_2}{P_1} - 1\right)\right] \quad \text{(A39}$$

from 4. to 5. T_u is constant, equal to $T_{u3} r_a^{\,n}$

$$\bar{q}_{h45} = +Cp\left[T_h - T_{u3} r_a^{\,n}\right]M_{u4}$$

Note equation A25

$$\bar{q}_{h45} = \frac{kP_2 V_s}{k-1}\left[\frac{T_h}{T_{u3}} - r_a^{\,n}\right]\left[1 + \frac{V_d}{V_s}\cdot\frac{I_d}{k}\right] \quad \text{(A40}$$

The heat stored in the regenerator is the same as for model 3.

given in equation A32.

M_d is given by equation A3. The heat supplied per pound of air delivered is equal to

$$\frac{1}{M_d}\left[\bar{q}_{h34} + \bar{q}_{h45}\right] + \left[1 - \eta_r\right]q_r \qquad \text{(A41}$$

Useful work done per pound of air delivered

$$= \quad RT_a \ln(P_2/P_1)$$

So the overall thermal efficiency is

$$\eta_t = \frac{RT_a \ln(P_2/P_1)}{\frac{\bar{q}_{h35}}{M_d} + \left[1-\eta_r\right]q_r} \qquad \text{(A42}$$

APPENDIX B

Analytical Determination of Regenerator Efficiency.

In section 2.2. it was stated that for certain applications some of the terms in the governing differential equations for regenerators could be neglected. Some of these terms may be dropped by intuition, some by more rigorous methods. By the first method it is decided to assume that the metal temperature at any section perpendicular to the flow is uniform. That is to say thermal conductivity is infinite in this plane. Also it is assumed that at any such cross-section the air temperature is constant during any one pass. Several other terms can be neglected by virtue of the analysis below.

Additional Nomenclature:-

V = regenerator packing volume per unit length.

A = regenerator packing surface area per unit length.

r_b = V/A

T_g = Air temperature, function of x.

T = Regenerator temperature, function of x and τ.

W = Mass flow rate through the regenerator.

t = Thickness of regenerator strip.

τ = Time since the pass began.

x = Distance from the cold end of the regenerator.

d = Total length of strip, including all turns.

Subscript m on properties refers to the regenerator material.

Shown in figure 5a is an element of regenerator packing of thickness t and length Δx in the direction of flow. Air passages on either side of the element. Heat is transferred to the element by conduction and convection. Consider the two control volumes A and B and let them include all the strip at one level, length d.

For control volume A

$$\rho_m V C p_m . \frac{\partial T}{\partial \tau} . \Delta x = hA\left[T_g - T\right]\Delta x + tdk_m . \frac{\partial^2 T}{\partial x^2} . \Delta x \qquad \text{(B1}$$

For control volume B

$$hA\left[T_g - T\right]\Delta x = -WCp . \frac{\partial T_g}{\partial x} \Delta x \qquad \text{(B2}$$

Let dimensionless temperatures be defined thus:—

$$T' = \frac{T}{T_h - T_c}$$

$$T'_g = \frac{T_g}{T_h - T_c}$$

Also, dimensionless time is

$$\tau' = \frac{\tau}{\tau_0}$$

where τ_0 is the time between flow reversals: in this case τ_0 is equal to 1/2N mins., and dimensionless length

$$x' = \frac{x}{L}$$

where L is the total regenerator length.

Noting that td is equal to V and that V/A is equal to r_b equations B1 and B2 can be rewritten in dimensionless form

$$\frac{\rho_m r_b C p_m}{h \tau_0} . \frac{\partial T'}{\partial \tau'} = T'_g - T' + \frac{k_m r_b}{hL^2} . \frac{\partial^2 T'}{\partial x'^2} \qquad \text{(B3}$$

$$T'_g - T' = \frac{-WCp}{hAL} \cdot \frac{\partial T'_g}{\partial x'} \qquad (B4$$

Let us put $\theta = \dfrac{h\tau_0}{\rho_m V C p_m}$

$$\beta = \frac{hL^2}{k_m r_b}$$

$$\lambda = \frac{hAL}{WCp}$$

Substituting B4 in B3 gives

$$\frac{\lambda}{\theta} \cdot \frac{\partial T'}{\partial \tau'} = \frac{-\partial T'_g}{\partial x'} + \frac{\lambda}{\beta} \cdot \frac{\partial^2 T'}{\partial x'^2} \qquad (B5$$

But $\dfrac{\partial T'}{\partial x'} \sim \dfrac{\partial T'_g}{\partial x'}$

and since

$$x' \sim 1$$

$$\frac{\partial^2 T'}{\partial x'^2} \sim \frac{\partial T'_g}{\partial x'}$$

Putting in numerical values given elsewhere in the thesis the dimensionless quantities become:-

$$\lambda \sim 100$$

$$\theta \sim 0.2$$

$$\beta \sim 10^4$$

Hence $\lambda/\beta \sim 10^{-2}$ and the last term in B5 can be neglected with respect to the second. So $-\partial T'_g/\partial x' \sim \lambda/\theta \; \partial T'/\partial \tau'$ for B5 to hold. As $\lambda/\theta \sim 500$, $\partial T'/\partial \tau'$ must be very small with respect to $\partial T'_g/\partial x'$ and can be neglected. Thus each element can be considered to be an isothermal heat sink exchanging heat only with the passing air. An assumption has to be made now about the regenerator metal temperature distribution $T(x/L)$. It is assumed linear for two reasons. One is the necessary symmetry of the equations applied for both passes; this could readily be shown but is not necessary since experiment has proved this linearity as shown in figure 10.

So

$$T\left(\frac{x}{L}\right) = T_c + (T_h - T_c)\frac{x}{L} \qquad \text{(B6}$$

Substitute this in B2

$$T_g - \left[T_c + (T_h - T_c)\frac{x}{L}\right] = -\frac{1}{\lambda}\frac{\partial T_g}{\partial\left(\frac{x}{L}\right)} \qquad \text{(B7}$$

When B7 is integrated and B6 reintroduced

$$(T - T_g) = (T_h - T_c)\frac{1}{\lambda} + K_1 \exp(-x\lambda/L) \qquad \text{(B8}$$

This equation says that whatever temperature air enters the regenerator $(T - T_g)$ will tend to $(T_h - T_c)1/\lambda$ with a short distance through the matrix. By symmetry again it is known that the same equations must hold for both passes and if λ is the same for both passes then the same temperature difference will hold at each point for the heat stored to be equal to the heat given up. Hence the regenerator temperature distribution will adjust so that K_1 in equation B8 is identically zero for equilibrium.

$$(T - Tg) = (T_h - T_c)\frac{1}{\lambda}$$

$$= \text{a constant.}$$

Regenerator efficiency is defined as

$$\eta_r = \frac{\text{actual heat stored in the regenerator}}{\text{maximum - minimum enthalpies of the air}}$$

If the air is a perfect gas

$$\eta_r = \frac{Cp\left[T_h - T_c\right]}{Cp\left[T_h + \frac{T_h - T_c}{\lambda}\right] - Cp\left[T_c - \frac{T_h - T_c}{\lambda}\right]}$$

Finally

$$\eta_r = \frac{\lambda}{2 + \lambda} \qquad \text{(B9}$$

APPENDIX C

Flow Rate and Pressure Drop through the Valves

The analysis of Appendix A discusses the discharge rate per stroke. It is apparent that the calculation of flow rate per unit time through the valves involves the additional knowledge of strokes per unit time and also the fraction of the stroke during which the discharged or induced air passes through the respective valves. In the case of the prototype compressor the motor speed was almost constant with very high accelerations at each stroke end. Thus if the time of one stroke is 1/N mins. the compression stroke takes 1/2N mins. and as air is dicharged whilst V_1 goes from V_{12} to V_{13}(zero) then the duration of discharge is $V_{12}/2NV_s$. If the simplified equation 1 given in the analysis of results is used then application of continuity between 2. and 3. gives

$$\frac{P_2\left[V_s - V_{12}\right]}{RT_u} + \frac{P_2V_{12}}{RT_1} = \frac{P_2V_s}{RT_u} + M_d$$

$$\frac{V_{11}}{V_s} = \frac{RM_d}{P_2V_s}\left[\frac{T_1 \cdot T_u}{T_u - T_1}\right]$$

Volume discharged $= \dfrac{M_dRT_1}{P_2}$

$$Q_{ex} = \frac{M_dRT_1}{P_2} \cdot \frac{2V_sN}{V_{12}}$$

or $$Q_{ex} = 2V_sN\left[1 - \frac{T_1}{T_u}\right]$$

The discharge flow rate is therefore independent of pressure ratio. The same mass of air is induced and its volume is $\frac{M_dRT_1}{P_1}$. This is induced whilst V_u goes from V_{u4} to V_{u5}(zero).

Applying the continuity equation as before between 4. and 5.

$$M_d + \frac{P_1\left[V_s - V_{u3}\right]}{RT_1} + \frac{P_1V_{u3}}{RT_u} = \frac{P_1V_s}{RT_1}$$

$$\frac{V_{u3}}{V_s} = \frac{M_dR}{P_1V_s}\left[\frac{T_u \cdot T_1}{T_u - T_1}\right]$$

$$Q_{in} = \frac{M_dRT_1}{P_1} \cdot \frac{2V_sN}{V_{u3}}$$

or $$Q_{ex} = Q_{in} = 2V_s\left[1-\frac{T_1}{T_u}\right]N \qquad \text{(C1}$$

A pressure drop versus flow rate test has been performed on the valves (which are identical). The characteristic is displayed on figure 9. For any value of T_u/T_1, Q_{in} and Q_{ex} can be found from equation C1. From figure 9. the corresponding pressure drop can be found. The recorded data give values for receiver pressure P_r. Hence if ΔP is the appropriate pressure drop then

$$P_2 = P_r + \Delta P$$

and $$P_1 = P_a - \Delta P$$

Hence $$P_2 = \frac{\frac{P_r}{P_a} + \frac{\Delta P}{P_a}}{1 - \frac{\Delta P}{P_a}} \qquad \text{(C2}$$

APPENDIX D

Evaluation of the Integrals Ie and Id.

The analysis of the adiabatic model entails the evaluation of the following integrals:-

1. $$Ie = \int_1^2 \frac{dr}{1 - ar^n}$$

2. $$Id = \int_3^4 \frac{r_a^{-n} dr_a}{1 - br_a^{-n}}$$

where $r = \frac{P}{P_1}$

$a = \frac{T_{l_1}}{T_h}$

$r_a = \frac{P}{P_2}$

$b = \frac{T_c}{T_{u3}}$

$n = \frac{k-1}{k}$

Evaluation of Ie. ar^n equals $T_{l_1} r^n / T_h$ equals T_l / T_h. The maximum value of this is unity at which point air enters the cooler with the same temperature at which it leaves the heater and no rise in pressure would result. Hence ar^n is always less than unity. The integral can be expanded as a power series.

$$Ie = \int_1^2 \left[1 + ar^n + (ar^n)^2 + \ldots\ldots\ldots (ar^n)^m \ldots\ldots \right] dr$$

$$= \left[r + \frac{ar^{n+1}}{n+1} + \frac{a^2 r^{2n+1}}{2n+1} + \cdots\cdots \right]_1^2$$

$$= r\left[1 + \frac{ar^n}{n+1} + \frac{(ar^n)^2}{2n+1} + \frac{(ar^n)^3}{3n+1} + \cdots\cdots \right]_1^2$$

The series converges for all $ar^n < 1$. Let the series in the bracket be Se/r. This is a function only of ar^n and hence can be plotted against ar^n as shown on figure 13 for k equal to 1.4 or n equal to .286. Hence

$$Ie = r_2\left(\frac{Se}{r}\right)_2 - r_1\left(\frac{Se}{r}\right)_1$$

where $\left(\frac{Se}{r}\right)_{1,2}$ are found for $(ar^n)_{1,2}$.

Evaluation of Id. br_a^{-n} equals T_c/T_u. By a similar line of reasoning as for ar^n it is found that br_a^{-n} is always less than unity. Id can therefore be expanded as a power series.

$$Id = \int_3^4 \left[\frac{1}{r_a^{\,n}} + \frac{b}{r_a^{\,2n}} + \frac{b^2}{r_a^{\,3n}} \cdots\cdots\cdots + \frac{b^m}{r_a^{\,(m+1)n}} + \cdots \right] dr.$$

$$Id = \left[\frac{r_a^{\,1-n}}{1-n} + \frac{br_a^{\,1-2n}}{1-2n} + \frac{b^2 r^{1-3n}}{1-3n} + \cdots\cdots\cdots \right]_3^4$$

$$Id = \left[r_a^{\,1-n} \frac{1}{1-n} + \frac{br_a^{\,-n}}{1-2n} + \frac{(br_a^{\,-n})^2}{1-3n} + \cdots\cdots\cdots \right]_3^4$$

The series converges for all $br_a^{-n} < 1$. Let the series in the bracket be Sd/r_a^{1-n}. This is a function only of br_a^{-n} and hence can be plotted against br_a^{-n} as shown on figure 13 for k equal to 1.4. Hence

$$Id = r_{a4}^{1-n}\left(\frac{Sd}{r_a^{1-n}}\right)_4 - r_{a3}^{1-n}\left(\frac{Sd}{r_a^{1-n}}\right)_3$$

where $\left(\frac{Sd}{r_a^{1-n}}\right)_{3,4}$ are found for $(br_a^{-n})_{3,4}$

TABLE 1

Listing of Volumes

a) Volumes pertinent to the compressor constructed.

1.	Total free space in the external circuit		= 133.5	ins.^{3}
2.	Gross free volume of the cylinder		= 476.5	ins.^{3}
3.	Volume of displacer		= 272.0	ins.^{3}
4.	Net free volume of the cylinder = (2)-(3)		= 204.5	ins.^{3}
5.	Area of cross section of cylinder		= 30.5	ins.^{2}
6.	Swept volume V_s if stroke is S ins., = (5)xS		= 30.5S	ins.^{3}
7.	Unswept cylinder volume (4)-(6)		= 204.5-30.5S	ins.^{3}
8.	Total unswept free space (1) + (7)	V_r	= 338.0-30.5S	ins.^{3}
For most test runs S was about 5.75 ins. so		V_s	= 175.2	ins.^{3}
		V_r	= 162.8	ins.^{3}

$$\frac{V_r}{V_s} = 0.93$$

b) Volumes pertinent to the 7.5 ins. displacer with 7.5 ins. stroke

3(b).	Volume of displacer		= 227.0	ins.^{3}
4(b).	Net free volume of cylinder (2)-(3)(b)		= 249.5	ins.^{3}
6(b).	Swept Volume if stroke is 7.5 ins.	V_s	= 227.0	ins.^{3}
7(b).	Unswept cylinder volume(4b)-(6b)		= 22.5	ins.^{3}
8(b).	Total free space unswept (1)+(7b)	V_r	= 156.0	ins.^{3}

$$\frac{V_r}{V_s} = 0.69$$

TABLE 2

Experimental Data

In the following tables the important recorded data for the series of tests is listed. Insulation was not used around the compressor body on runs 1 through 23 as it was for runs 24 through 36.

Regenerator temperatures were taken for each run. Each set demonstrated linearity between the hot and cold ends. One series has been presented on figure 10.

Data for the valve and internal pressure drop-flow rate relationships are presented on figures 8 and 9.

Heater failures occurred after runs 23 and 36 on the respective series of runs.

TABLE 2 - continued

Power watts	915	915	915	661	661	661	661	870	870	870
Comp. speed strokes/min.	28.0	28.0	28.0	26.5	28.0	26.7	26.6	27.1	28.4	27.7
Stroke inches	5.69	5.69	5.69	5.69	5.69	5.69	5.69	5.63	5.63	5.63
Inlet flow ft.^{3}/min.										
Disch. flow ft.^{3}/min.	0	0.323	0.741	0	0.122	0.375	0.511	0	0.59	0.316
Recvr. press. cm. Hg.	22.6	10.5	1.2	17.5	13.6	6.7	1.2	20.5	1.2	9.0
Cold space temp. °C.	11.5	11.5	9.5	9.5	11.0	11.0	11.0	9.0	10.0	10.0
Hot space temp. °C.	299.0	301.0	298.0	251.0	258.0	255.0	257.0	292.0	289.5	289.5
Air inlet temp. °C.										
Air exhaust temp. °C.										
Water flow lbs./min										
Water inlet temp. °C.										
Water outlet temp °C.										
Test No.	1	2	3	4	5	6	7	8	9	10

TABLE 2 - continued

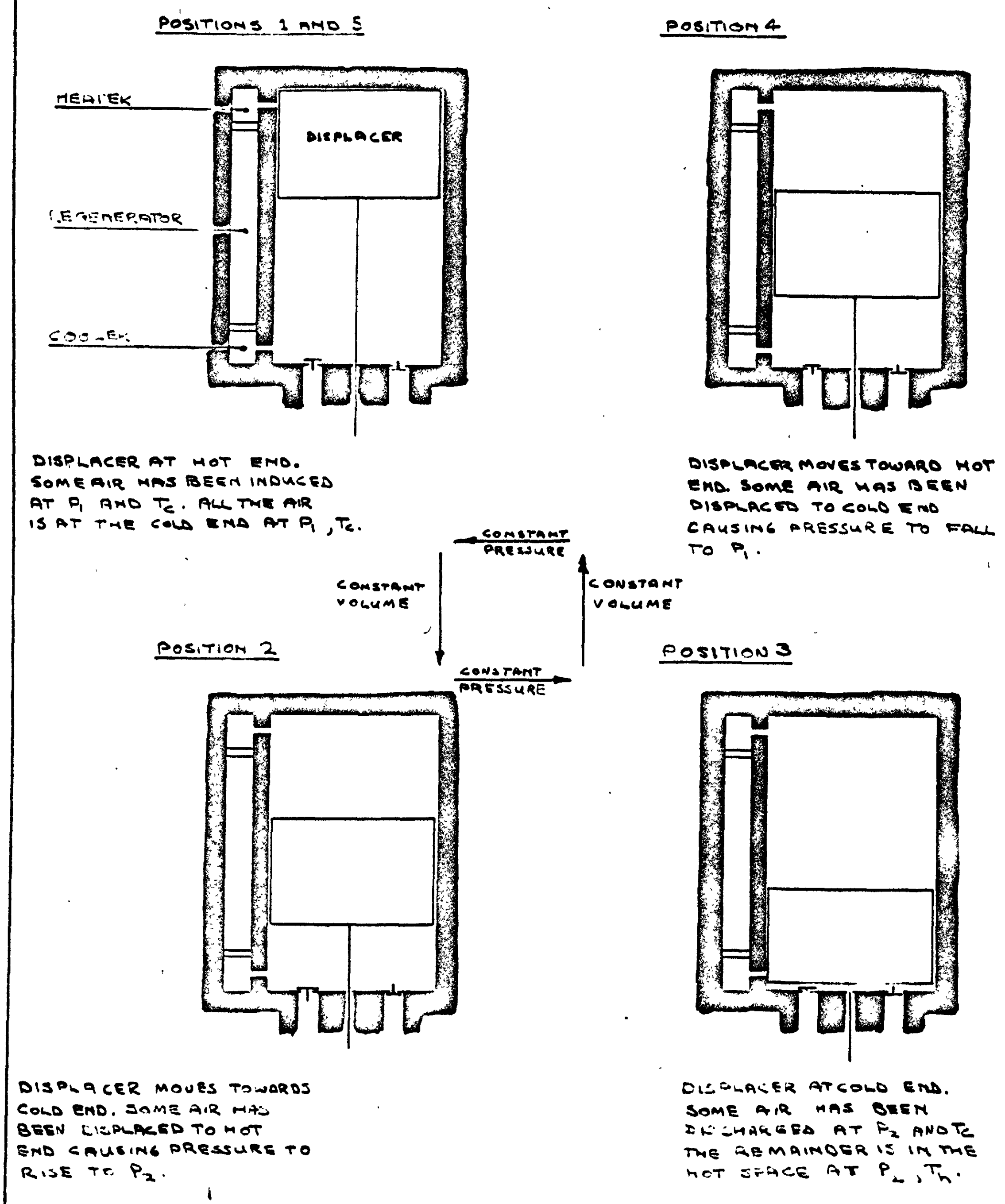
Power watts	870	1095	1095	1480	1480	1480	1480	1750	1750	1750
Comp. speed strokes/min.	27.7	26.8	28.1	27.8	29.5	29.4	28.5	27.8	27.8	27.1
Stroke inches	5.63	5.65	5.65	5.73	5.73	5.73	5.73	5.73	5.73	5.73
Inlet flow ft.^{3}/min.										
Disch. flow ft.^{3}/min.	0.160	0	0.635	0	0.734	0.464	0.316	0	0.800	0.586
Recvr. press. cm. Hg.	14.6	23.0	1.8	28.6	2.4	10.8	16.6	30.6	2.0	7.0
Cold space temp. °C.	10.0	9.0	9.0	8.5	8.5	8.5	8.5	12.0	12.0	12.0
Hot space temp. °C.	289.5	330.0	329.0	404.0	403.0	403.0	403.0	449.0	449.0	449.0
Air inlet temp. °C.										
Air exhaust temp °C.										
Water flow lbs./min										
Water inlet temp. °C.										
Water outlet temp °C.										
Test No.	11	12	13	14	15	16	17	18	19	20

TABLE 2 - continued

Power watts	1750	1750	2000	230	230	230	230	351	351	351
Comp. speed strokes/min.	27.1	27.5	27.5	27.5	30.6	30.6	30.6	29.2	29.2	29.2
Stroke inches	5.73	5.73	5.73	5.75	5.63	5.63	5.63	5.63	5.63	5.63
Inlet flow ft.^{3}/min				.319	.728	.556	.441	.355	.861	.491
Disch. flow ft.^{3}/min.	0.445	0.296	0	0	0.728	0.430	0.217	0	0.839	0.239
Recvr. press. cm. Hg.	11.5	17.0	31.3	15.2	0.7	5.0	9.2	22.2	1.4	14.8
Cold space temp. °C.	12.0	12.0	10.0	11.0	11.0	11.0	11.0	8.5	8.5	8.5
Hot space temp °C.	449.0	449.0	486.0	247.0	253.0	257.0	257.0	359.0	357.0	357.0
Air inlet temp °C.				12.5	13.5	13.0	13.0	11.1	13.0	11.4
Air exhaust temp °C.				16.5	30.0	25.0	20.0	20.5	43.0	32.0
Water flow lbs./min.				6.0	6.0	6.0	6.0	6.4	6.4	6.4
Water inlet temp. °C.				5.5	5.2	5.2	5.2	4.0	4.3	4.3
Water outlet temp. °C.				7.0	6.5	6.5	6.5	6.0	6.0	6.0
Test No.	21	22	23	24	25	26	27	28	29	30

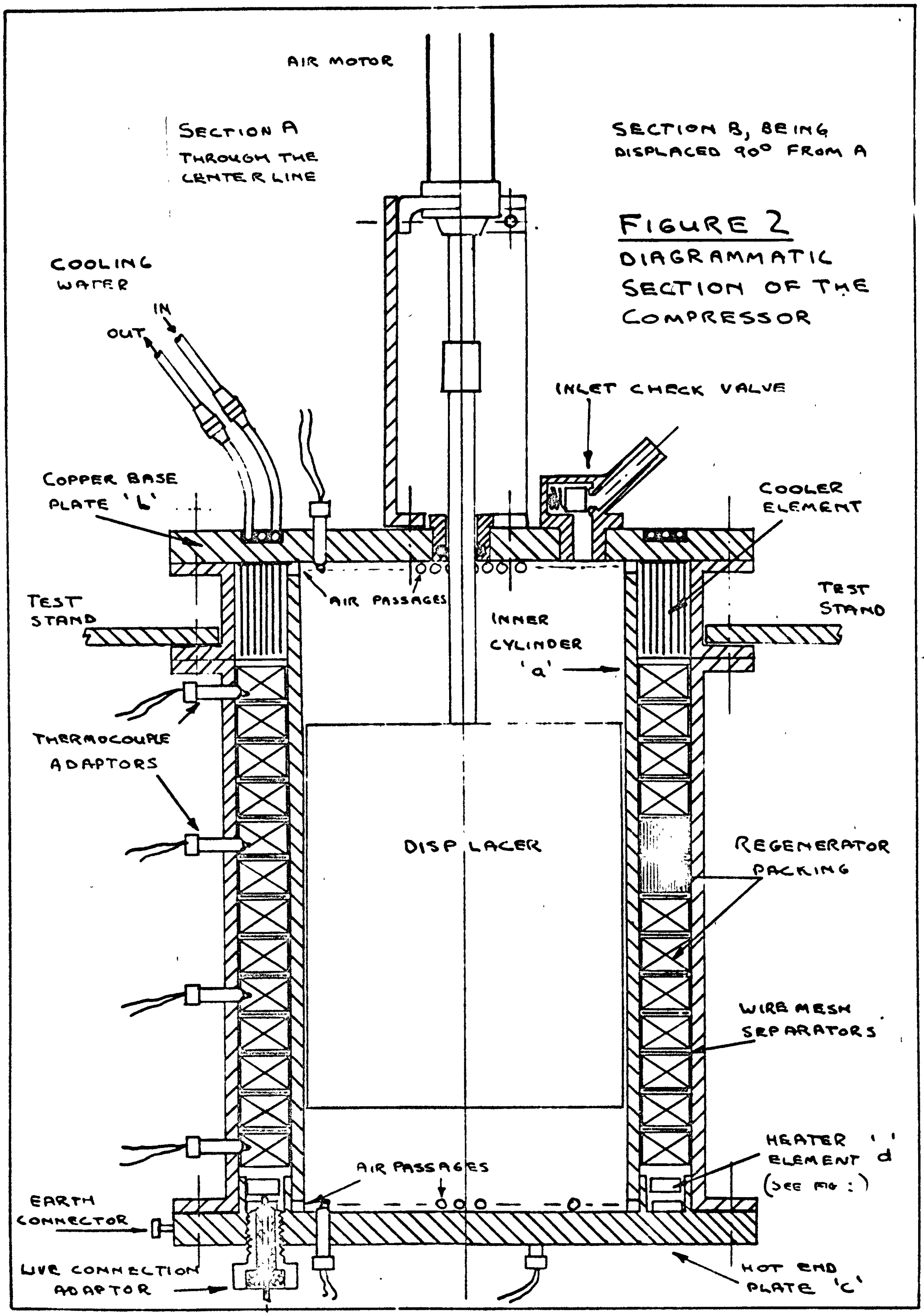
TABLE 2 - continued

Power watts	351	365	365	365	365	670
Comp. speed strokes/min.	29.2	23.5	23.5	23.5	23.5	
Stroke inches	5.63	5.83	5.83	5.83	5.83	5.83
Inlet flow ft.^{3}/min.	0.671	0.690	0.078	0.373	0.192	
Disch. flow ft.^{3}.min.	0.560	0.664	0			0
Recvr. press. cm. Hg.	7.2	1.0	31.8	14.4	24.2	36.5
Cold space temp.°C.	8.5	18.5	16.0	16.0	16.0	16.0
Hot space temp. °C.	358.0	378.0	378.0	378.0	378.0	513.0
Air inlet temp. °C.	11.4	11.8	11.8	11.8	11.8	
Air exhaust temp. °C.	43.0	41.0				
Water flow lbs./min.	6.4	4.1	4.1	4.1	4.1	
Water inlet temp. °C.	4.3	3.2	3.2	3.2	3.2	
Water outlet temp. °C.	6.0	5.9	5.9	5.8	5.7	
Test No.	31	32	33	34	35	36



DIAGRAMMATIC DESCRIPTION OF THE COMPRESSOR CYCLE

FIGURE 1.


AIR MOTOR
SECTION A THROUGH THE CENTER LINE
SECTION B, BEING DISPLACED 90° FROM A
FIGURE 2
DIAGRAMMATIC SECTION OF THE COMPRESSOR
COOLING WATER
IN
OUT
INLET CHECK VALVE
COPPER BASE PLATE 'L'
COOLER ELEMENT
TEST STAND
AIR PASSAGES
INNER CYLINDER 'a'
TEST STAND
THERMOCOUPLE ADAPTORS
DISPLACER
REGENERATOR PACKING
WIRE MESH SEPARATORS
HEATER ELEMENT 'd'
AIR PASSAGES
EARTH CONNECTOR
LIVE CONNECTION ADAPTOR
HOT END PLATE 'c'

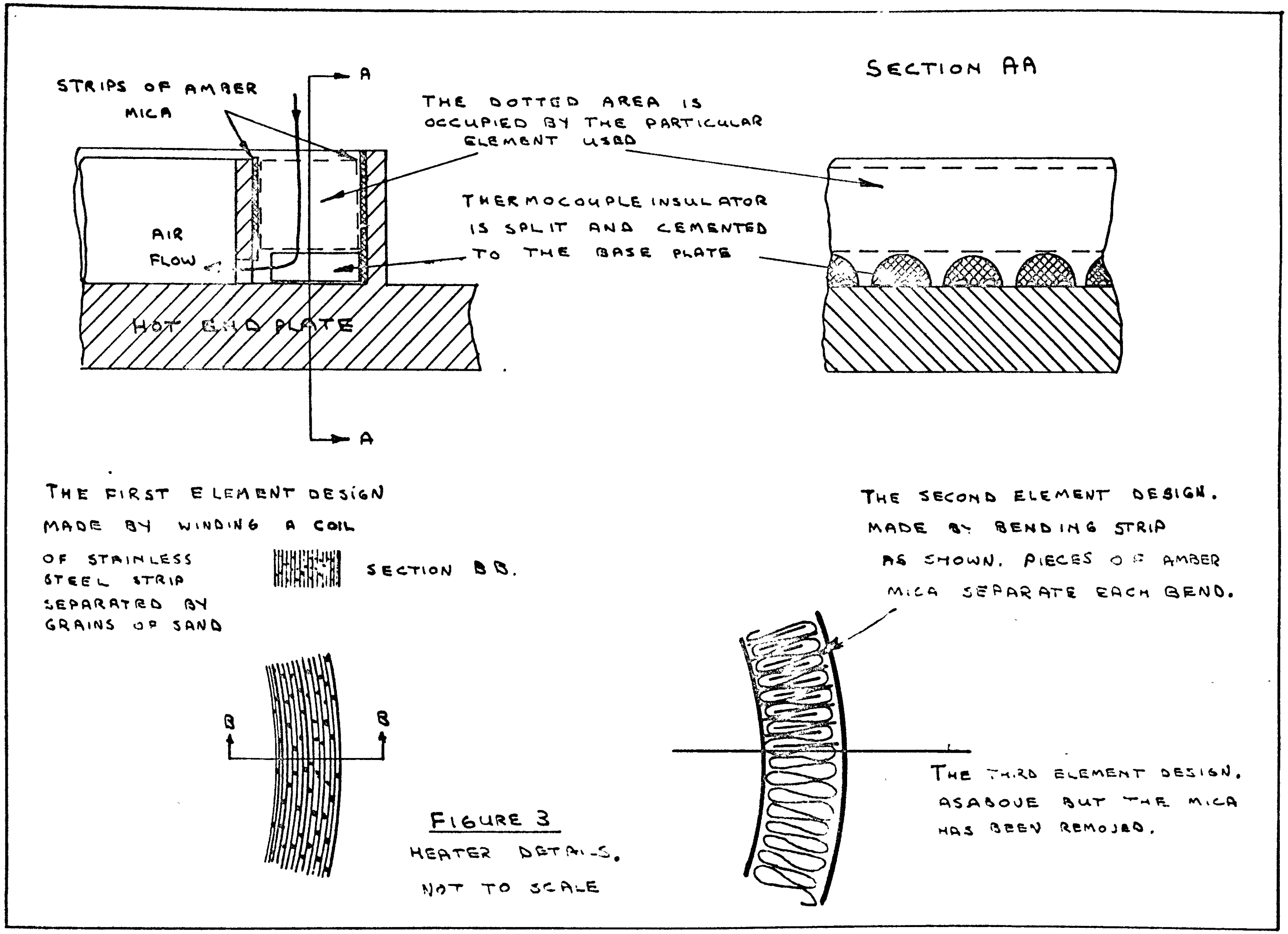


FIGURE 3

HEATER DETAILS,
NOT TO SCALE

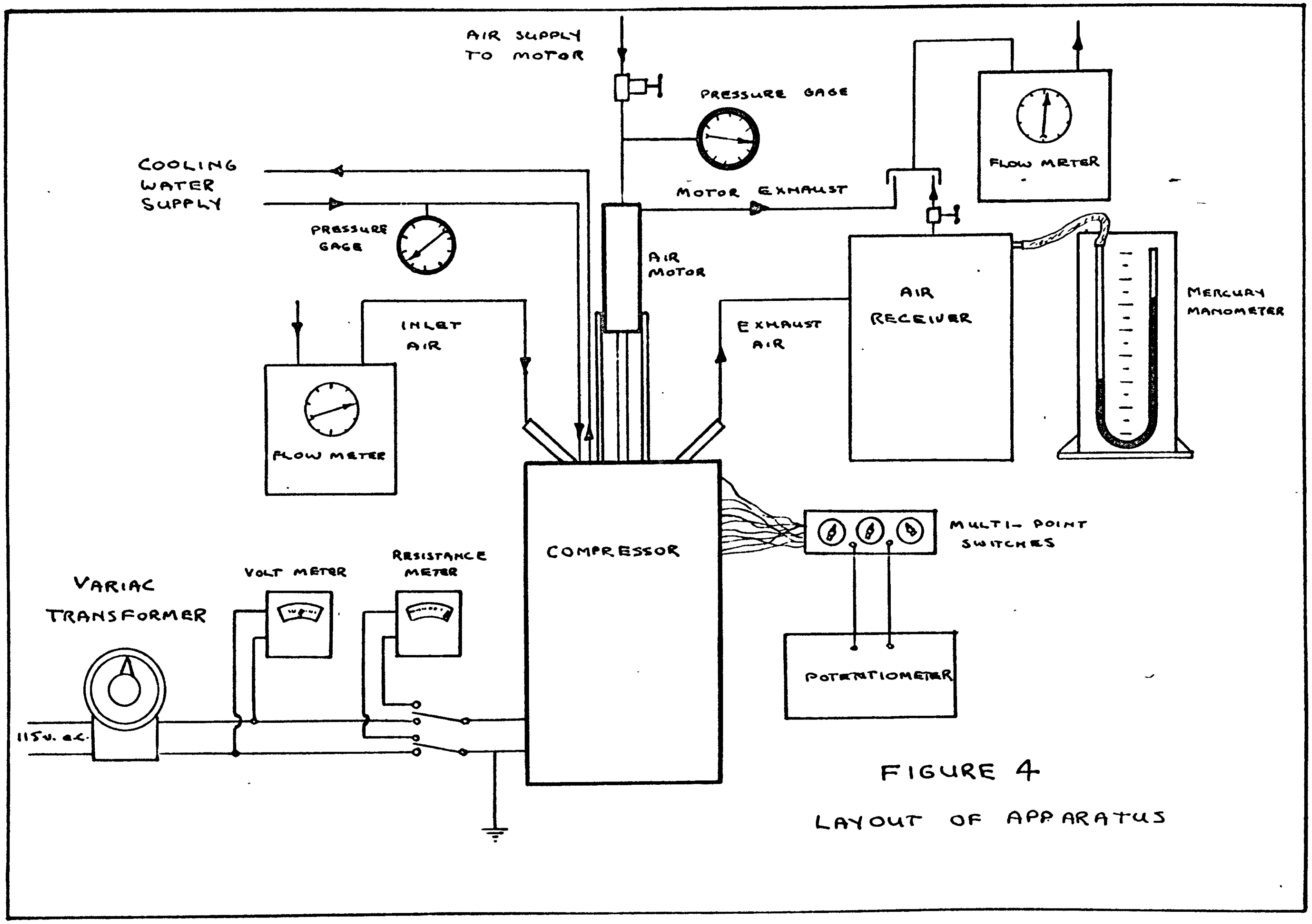


FIGURE 4

LAYOUT OF APPARATUS

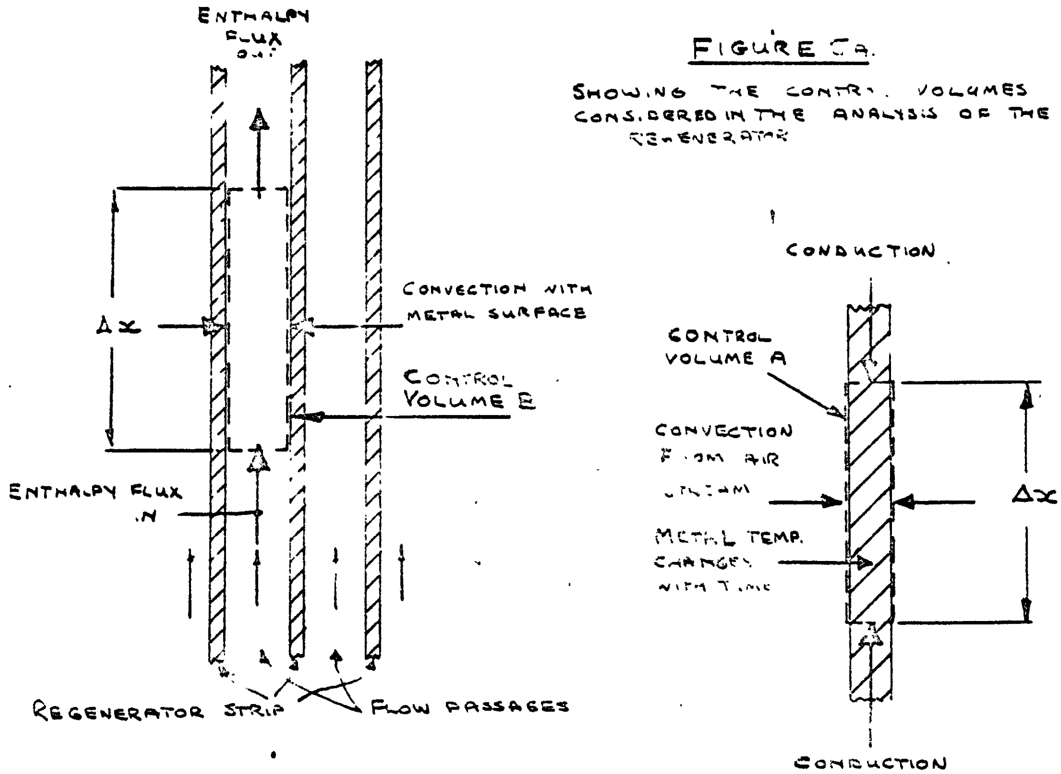


FIGURE 5A.
SHOWING THE CONTROL VOLUMES CONSIDERED IN THE ANALYSIS OF THE REGENERATOR

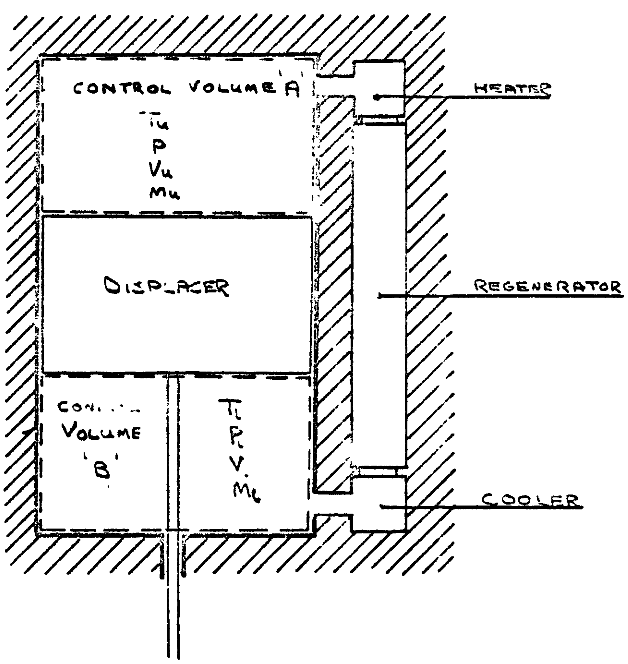


FIGURE 5B
SHOWING THE CONTROL VOLUMES CONSIDERED IN THE THERMODYNAMIC ANALYSIS

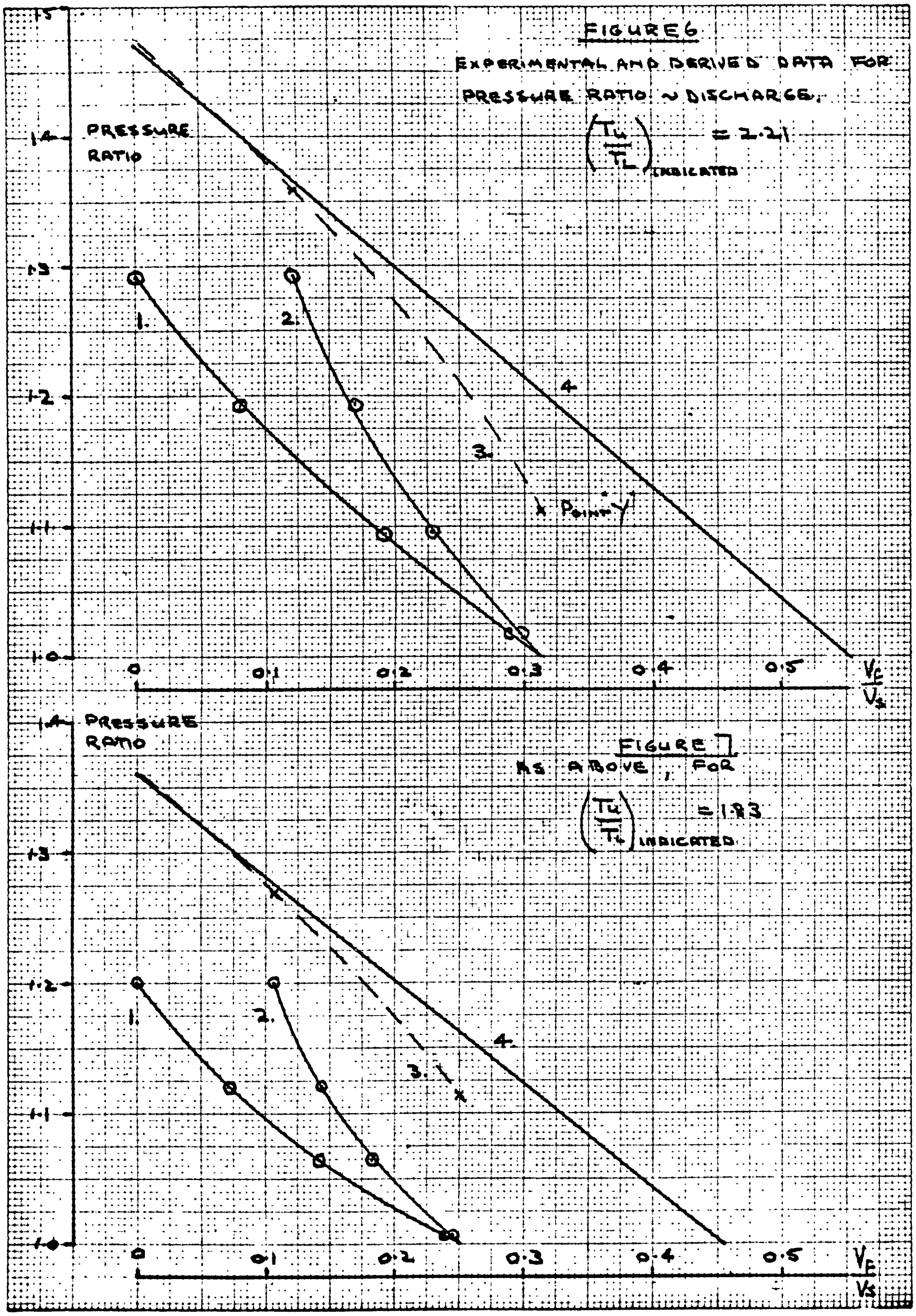


FIGURE 6
EXPERIMENTAL AND DERIVED DATA FOR PRESSURE RATIO ~ DISCHARGE.
$\left(\frac{T_u}{T_L}\right)_{INDICATED} = 2.21$
PRESSURE RATIO
Point Y
FIGURE 7
AS ABOVE, FOR
$\left(\frac{T_u}{T_L}\right)_{INDICATED} = 1.83$
PRESSURE RATIO
$\frac{V_F}{V_S}$

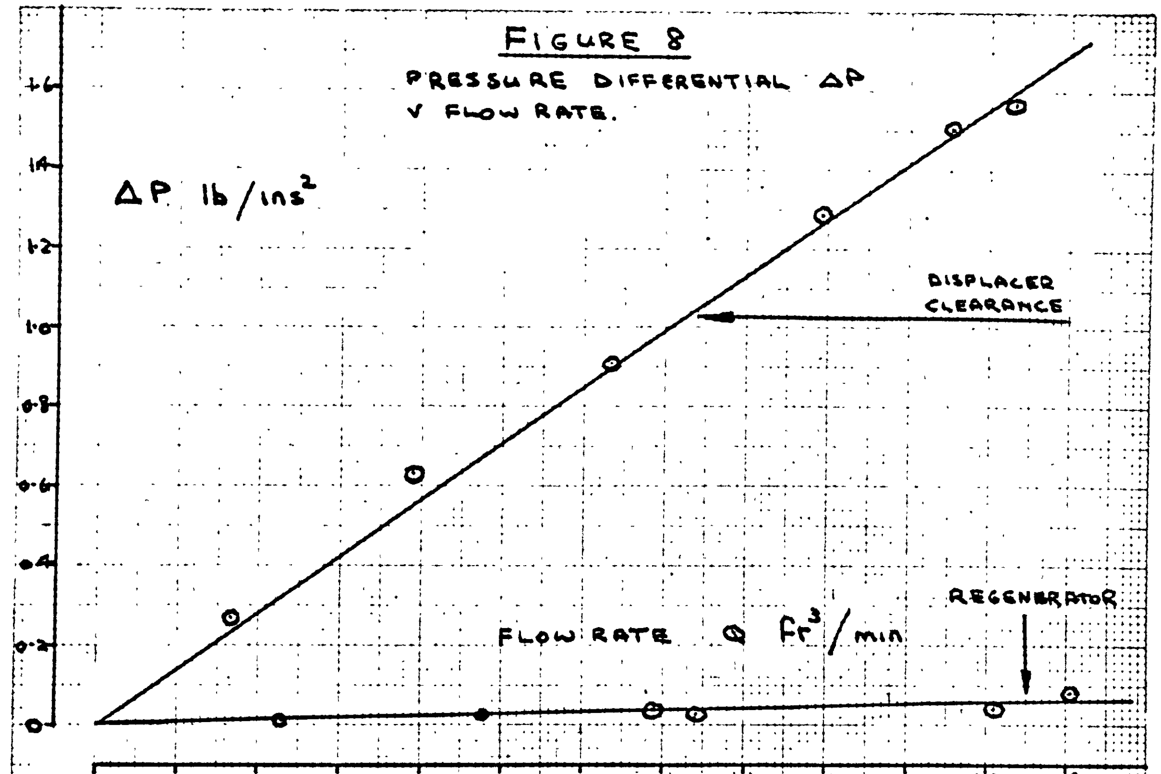
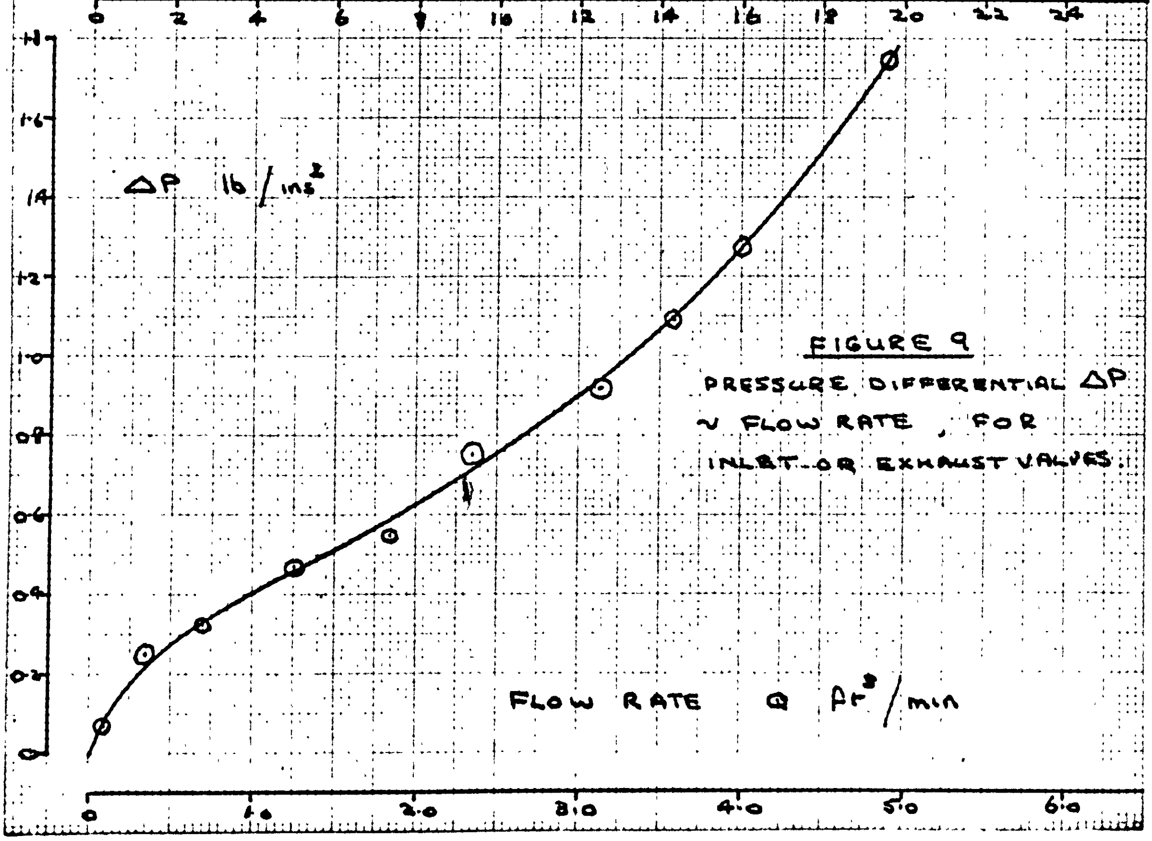


FIGURE 8
PRESSURE DIFFERENTIAL ΔP
V FLOW RATE.
ΔP lb/ins²
DISPLACER CLEARANCE
REGENERATOR
FLOW RATE Q ft³/min
0 2 4 6 8 10 12 14 16 18 20 22 24
0 ·2 ·4 ·6 ·8 1·0 1·2 1·4 1·6


ΔP lb/ins²
FIGURE 9
PRESSURE DIFFERENTIAL ΔP
~ FLOW RATE, FOR
INLET OR EXHAUST VALVES.
FLOW RATE Q ft³/min
0 1·0 2·0 3·0 4·0 5·0 6·0
0 0·2 0·4 0·6 0·8 1·0 1·2 1·4 1·6 1·8

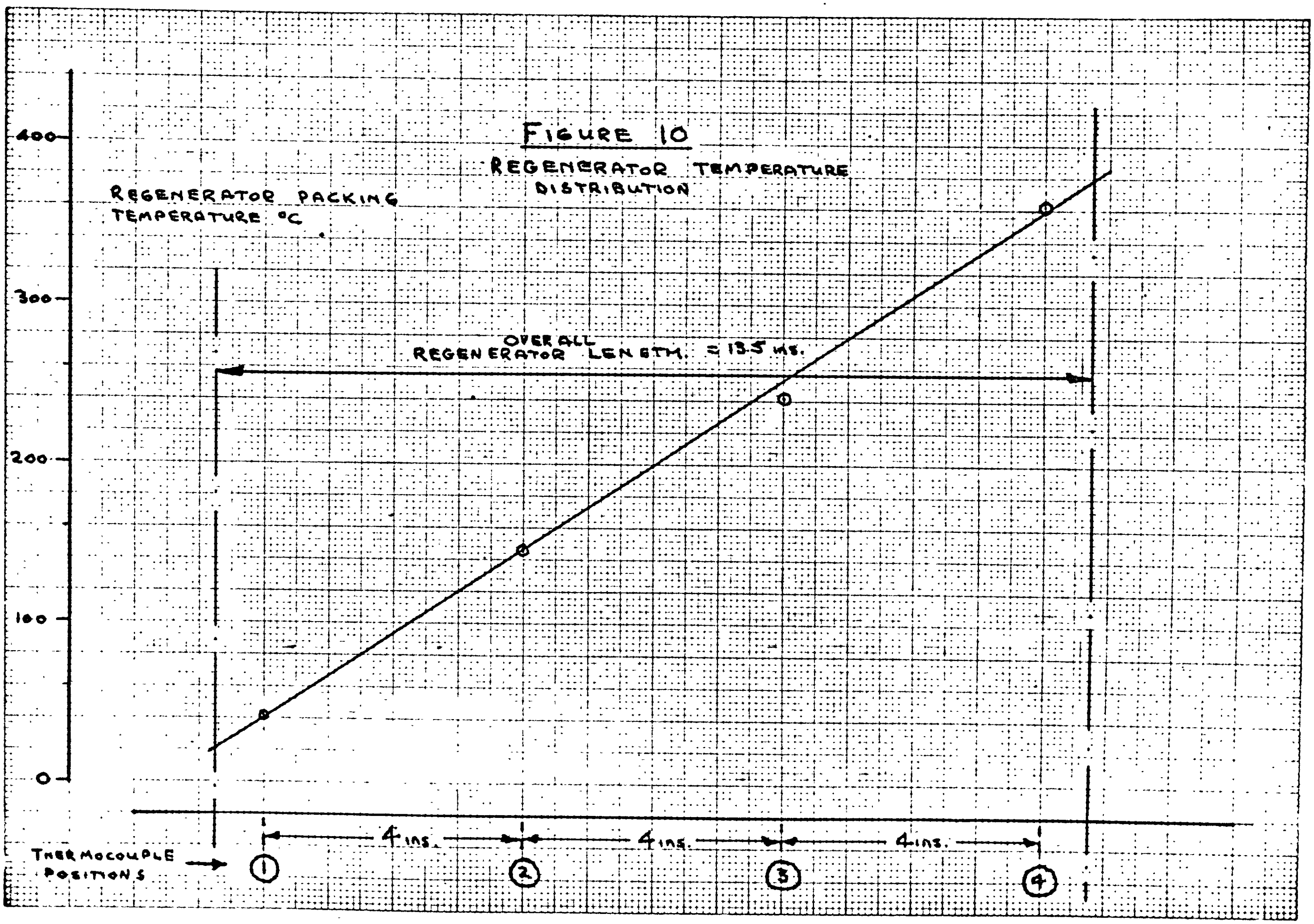
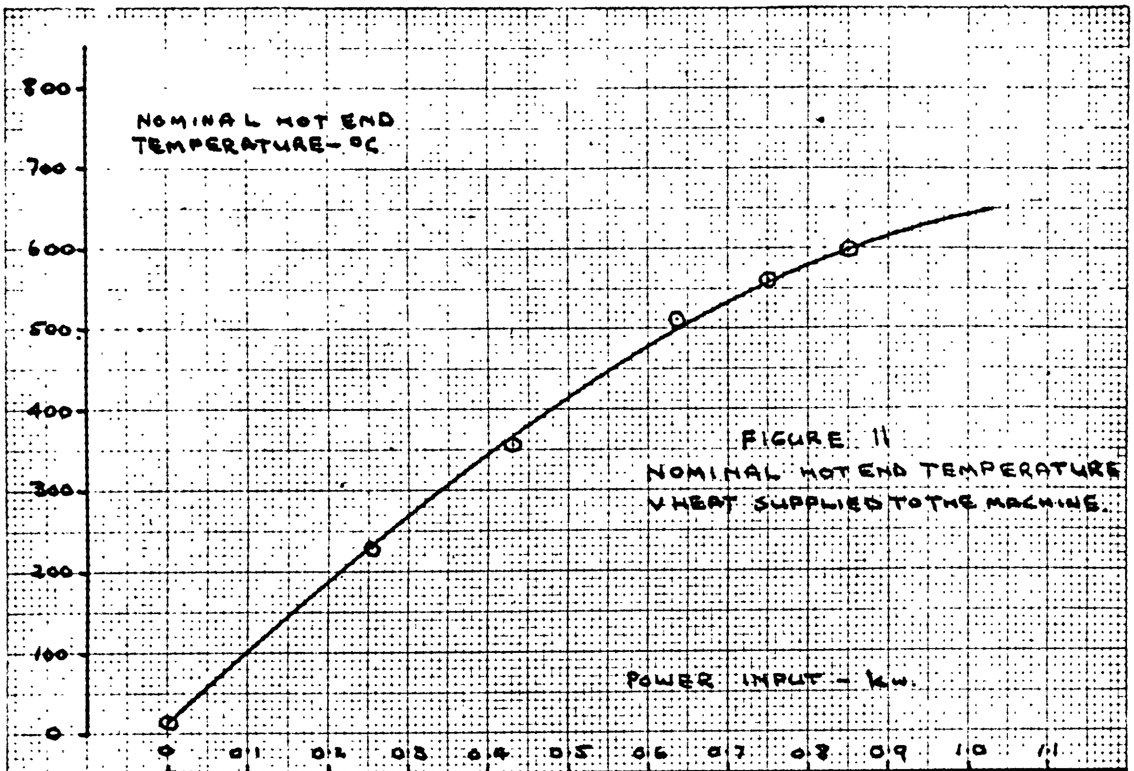


FIGURE 10
REGENERATOR TEMPERATURE DISTRIBUTION


NOMINAL HOT END TEMPERATURE — °C
FIGURE 11
NOMINAL HOT END TEMPERATURE V HEAT SUPPLIED TO THE MACHINE.
POWER INPUT — KW.
800
700
600
500
400
300
200
100
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1

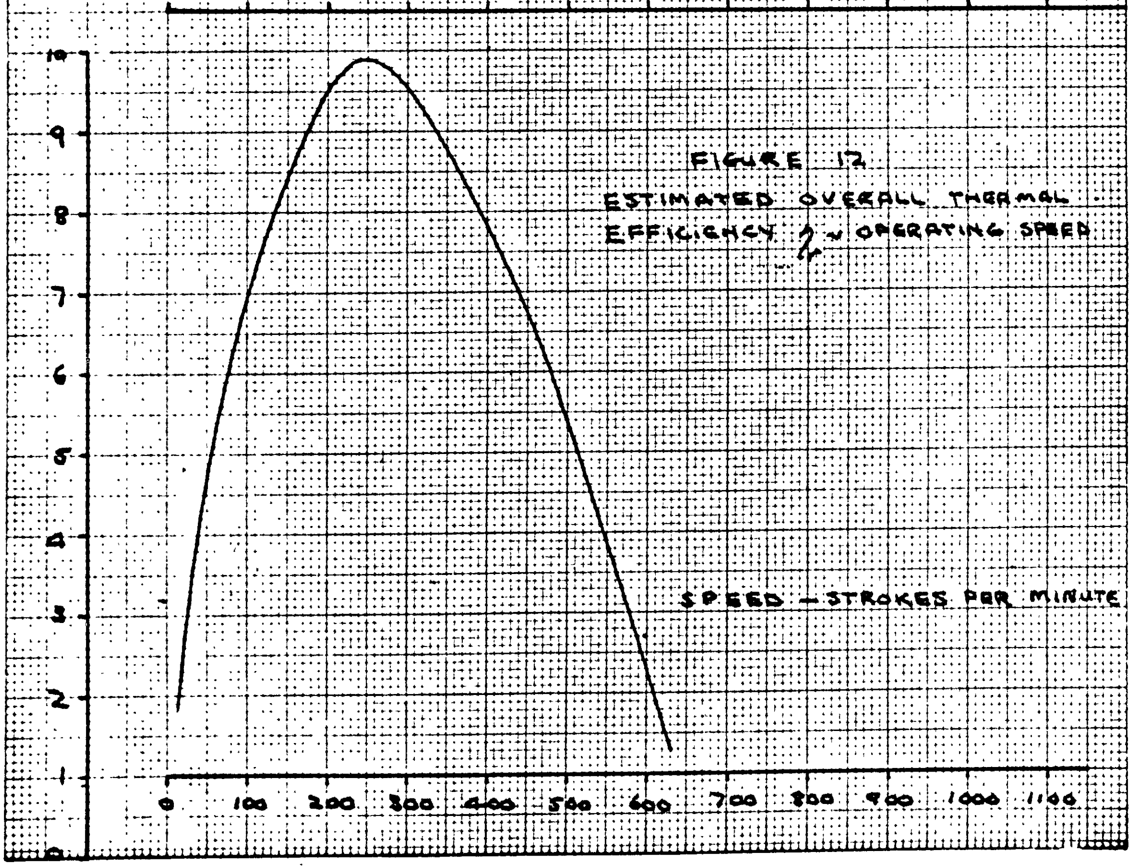


FIGURE 12
ESTIMATED OVERALL THERMAL EFFICIENCY η ~ OPERATING SPEED
SPEED — STROKES PER MINUTE
10
9
8
7
6
5
4
3
2
1
0
0 100 200 300 400 500 600 700 800 900 1000 1100

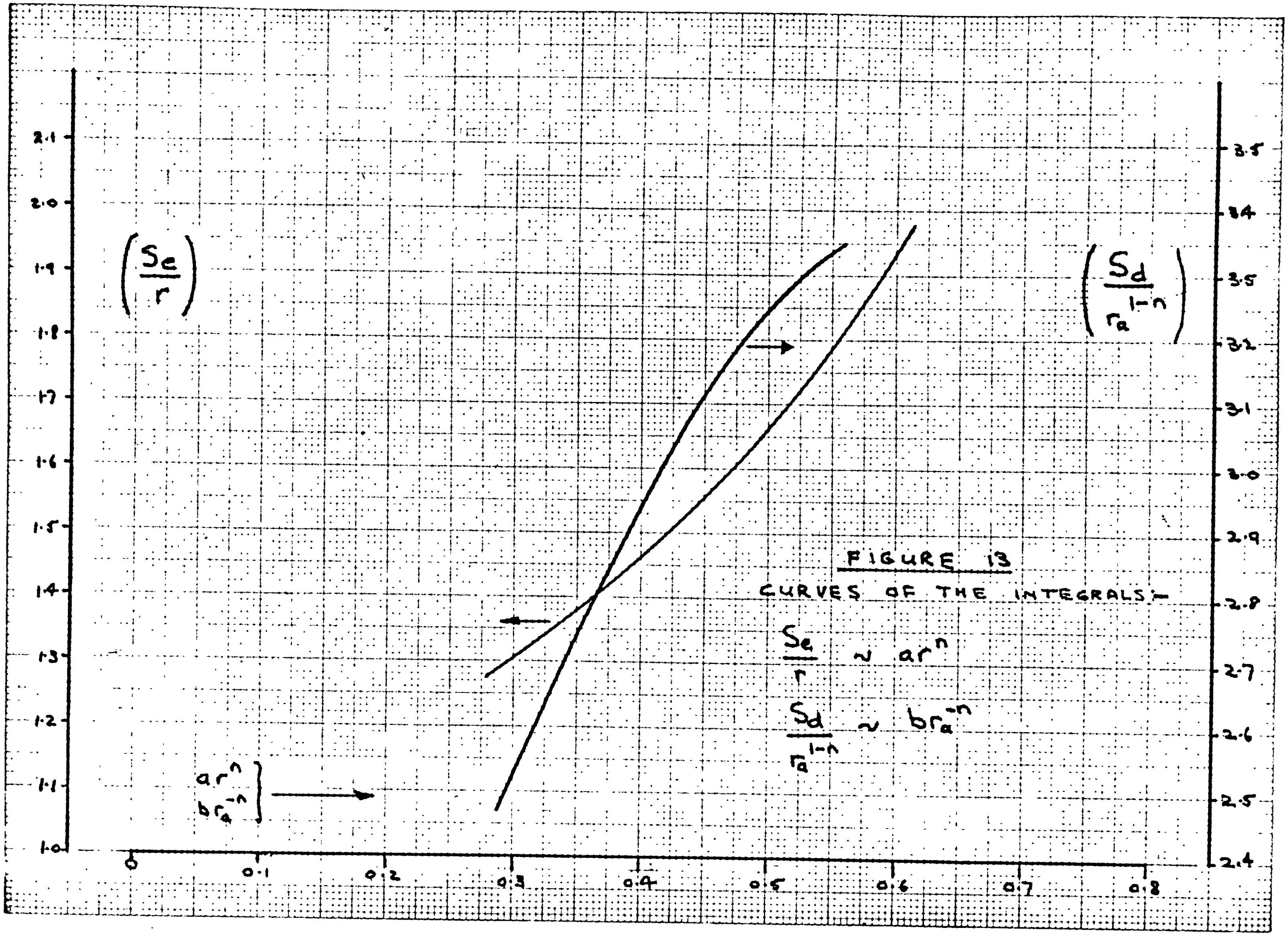


FIGURE 13

CURVES OF THE INTEGRALS:–

$$\frac{S_e}{r} \sim ar^n$$

$$\frac{S_d}{r_a^{1-n}} \sim br_a^{-n}$$