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STOCHASTIC PROGRAMMING MODELS FOR STRATEGIC PLANNING: AN APPLICATION TO ELECTRIC UTILITIES*

by

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ABSTRACT

This paper reports on the application of stochastic programming with recourse models to strategic planning problems typical of those faced by an electric utility. A prototype model was constructed using realistic data, and optimized using Benders' decomposition method. The decomposition treats simultaneously stochastic programming and mixed integer programming structures arising naturally in strategic planning models.

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INTRODUCTION

In making its strategic plans, a company is often concerned with acquiring the resources that it will need to survive and prosper over the long term. In so doing, it must evaluate its options from two important perspectives. First, it must assess the potential benefits of proposed new resources when these resources are used in conjunction with existing resources. In short, the company must attempt to measure the effect over the long term that new resources would have on its resource allocation planning if they were to be acquired.

Second, the company must identify and assess the potential impact on its business of important uncertainties in the external environment. Included are uncertainties regarding demand, prices, technology, capital markets, government policies, and competition. In selecting new resources, the company should seek to develop long term strategies for hedging against these uncertainties. The strategies should also provide contingency plans to be put into effect as the uncertainties are revealed.

Stochastic programming with recourse models are ideally suited for analyzing strategic plans from both these perspectives. They combine deterministic mathematical programming models for allocating resources optimally with decision analysis models for determining optimal hedging strategies in an uncertain environment. A specific realization of such a model is presented in the following section. The reader is referred to Wagner [11] for an excellent introduction to stochastic programming; see also [6] and [10].

The practical application of stochastic programming with recourse models to strategic planning is largely dependent on recent advances in computational methods for generating and optimizing mathematical programming models, including the use of decomposition methods. Since the models will include a submodel describing each of a number of scenarios of the company's uncertain future, they can easily attain an enormous size. Thus, flexible generation methods are needed for constructing the models, and decomposition methods are needed for optimizing them, at least to a reasonable degree of accuracy, by breaking them down into manageable components.

The purpose of this paper is to report on the successful application of stochastic programming with recourse models to strategic planning problems faced by an electric utility. A prototype model was constructed using realistic data, and optimized using Benders' decomposition method. The decomposition approach is novel in its own right because it treats simultaneously stochastic programming and mixed integer programming decomposable structures.

STATEMENT OF THE MODEL AND DECOMPOSITION APPROACH

In this section, we present a simplified version of the electric utility strategic planning model that we implemented and optimized. The implemented model is discussed in detail in the following section. The simplifications assumed here allow us to more easily explain the form of the model and the decomposition approach that we devised for optimizing it.

The basic model is an extension of the standard two stage stochastic programming with recourse model; for example, see Wagner [11; pp 663-664]. Our model contains two types of decision variables; operational variables, and resource acquisition variables. In this context, operational refers to those variables characterizing resource allocation options such as the energy produced by existing or newly acquired plants, or the quantity of fuel consumed under an existing supply contract. Resource acquisition refers to options regarding capacity expansion, plant construction, or contracting for primary fuels.

The two stage stochastic model is most easily motivated by considering first a related single stage, deterministic model.

$$\max \sum_{j=1}^{n} c_j x_j - \sum_{r=1}^{R} \sum_{k=1}^{K_r} \{f_{rk}\delta_{rk} + v_{rk}y_{rk}\} \tag{1}$$

$$\text{s.t.} \sum_{j=1}^{n} a_{ij}x_j = b_i \text{ for } i = 1,2,\ldots,m \tag{2}$$

$$\sum_{j=1}^{n} a_{rj}x_j - \sum_{k=1}^{K_r} y_{rk} \leq w_r \text{ for } r = 1,\ldots,R \tag{3}$$

$$\left.\begin{aligned} y_{rk} - u_{rk}\delta_{rk} &\leq 0 \\ y_{rk} - \ell_{rk}\delta_{rk} &\geq 0 \end{aligned}\right\} \begin{aligned} &\text{for } k = 1,\ldots,K_r; \\ &\quad r = 1,\ldots,R \end{aligned} \tag{5}$$

$$x_j \geq 0,\ y_{rk} \geq 0,\ \delta_{rk} = 0 \text{ or } 1. \tag{6}$$

In this model, the constraints (2) reflect operating conditions dictated by fixed capacities and requirements. The constraints also include balance equations on transformation recipes for converting raw materials into finished products. The constraints (3), (4), (5) along with the objective function terms for the δ_{rk} and y_{rk} variables, model resource acquisition. In particular, w_r is the initial level of resource r. There are K_r alternatives for augmenting it, with the variable quantity

y_{rk} representing the decision under alternative k. The cost of acquiring y_{rk} is shown in Figure 1.

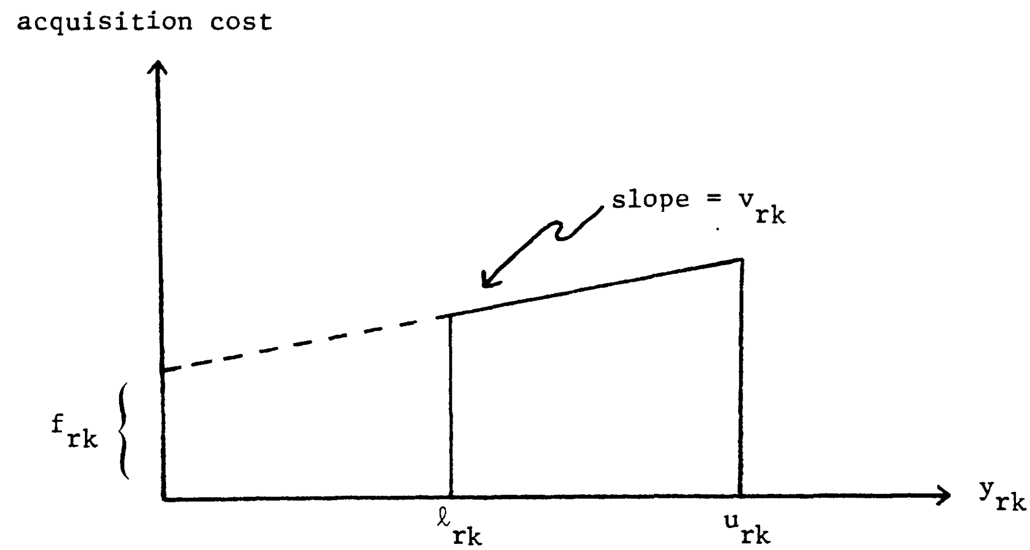


Figure 1

As we have depicted it, the quantity y_{rk} of resource r acquired under alternative k must lie between the lower bound ℓ_{rk} and upper bound u_{rk} associated with that alternative. The constraints (4) and (5) ensure that y_{rk} lies between its upper and lower bounds, if it is not zero, and that the fixed charge $f_{rk} + v_{rk}\ell_{rk}$ is occured whenever y_{rk} exceeds its conditional minimum ℓ_{rk}.

We emphasize that important new resources are typically acquired in a lumpy manner similar to the way we have described it in our model. It is not reasonable to assume, for example, that a new coal burning plant with a capacity less thatn 100MW, say, will be considered as an expansion option by a major electric utility. Moreover, there are definite fixed costs associated with construction; such as the cost of acquiring land, or of carrying out an environmental impact analysis. Similarly, suppliers of raw materials will offer competitive terms only if the buyer is willing to purchase a significant quantity.

A number of modeling extensions could easily be added to (1) - (6). The resource acquisition cost curve shown in Figure 1 could be generalized to an arbitrary piecewise linear curve exhibiting economies or diseconomies of scale. Capital budget constraints of the form

$$\sum_{r \in R'} \sum_{k=1}^{K_r} B_{r,k} \, \delta_{r,k} \leq B_0$$

could also be added, where B_0 is the capital available for acquiring the resources in the set R', and $B_{r,k}$ is the capital outlay for alternative k in resource category r. Finally, one could impose logical constraints on the resource acquisition variables, such as

$$\sum_{k=1}^{K_r} \delta_{r,k} \leq 1,$$

expressing the condition that no more than one of the alternatives for resource r can be selected. This might be the case if r referred to plant construction at a particular site and k referred to the generating capacity of the plant.

With this background, we can discuss how model (1) - (6) can be expanded into a two stage, stochastic model with recourse. We can view model (1) - (6) as describing the constrained decision options that must be selected <u>here and now</u>. Subsequently, the uncertainties will be revealed, and for each second stage scenario we will select optimal contingency plans. The decisions made here and now should take these contingency options into account, along with their associated probabilities of occurrence.

The following summarize the assumptions underlying construction of the two stage model:

1. The random second stage events occur with probabilities p_q, $q = 1, \ldots, Q$, that are independent of the first stage decisions.
2. There always exist feasible second stage contingency plans.
3. The overall objective is to maximize expected net benefits over the two stages.
4. The two stages of the planning horizon correspond to equal time periods.
5. Associated with each first stage resource acquisition option y_{rk}, and each scenario q, there is a random parameter β_{qrk} representing the proportion of y_{rk} available in the second stage. If the resource r is a plant to be constructed in the first stage, β_{qrk} may not have a large variance. In this case, the parameter reflects fixed capacity passed from the first stage to the second. If the resource r is a supply contract signed in the first stage, β_{qrk} can be a non-negative constant whose value depends on the details of the contract and the reliability of the supply source.
6. Also associated with each second stage scenario q are values of the parameters c_{qj}, f_{qrk}, v_{qrk}, b_{qi}, w_{qr}, a_{qrj}, u_{qrk}, ℓ_{qrk}.

The two stage model is

$$\max \sum_{j=1}^{n_1} c_j x_j - \sum_{r=1}^{R} \sum_{k=1}^{K_r^1} \left\{ f_{rk}\ \delta_{rk} + v_{rk}\ y_{rk} \right\}$$

$$+ \sum_{q=1}^{Q} p_q \left[\sum_{j=1}^{n_2} c_{qj} x_{qj} - \sum_{r=1}^{R} \sum_{k=1}^{K_r^2} \left\{ f_{qrk} \delta_{qrk} + v_{qrk} y_{qrk} \right\} \right] \tag{7}$$

$$\text{s.t.} \sum_{j=1}^{n_1} a_{ij} x_j = b_i \text{ for } i = 1,2,\ldots,m_1 \tag{8}$$

$$\sum_{j=1}^{n_1} a_{rj} x_j - \sum_{k=1}^{K_r^1} y_{rk} \leq w_r \text{ for } r = 1,\ldots,R \tag{9}$$

(First Stage)

$$y_{rk} - u_{rk} \delta_{rk} \leq 0 \quad \text{for } k = 1,\ldots,K_r^1; \tag{10}$$

$$y_{rk} - \ell_{rk} \delta_{rk} \geq 0 \quad r = 1,\ldots,R \tag{11}$$

$$x_j \geq 0,\ y_{rk} \geq 0,\ \delta_{rk} = 0 \text{ or } 1 \tag{12}$$

....

For $q = 1,\ldots,Q$

$$\sum_{j=1}^{n_1} a_{qij} x_j + \sum_{j=1}^{n_2} a_{qij} x_{qj} = b_{qi} \quad \text{for } i = 1,\ldots,m_2 \tag{13}$$

$$\sum_{j=1}^{n_1} a_{qrj} x_j + \sum_{j=1}^{n_2} a_{qrj} x_{qj} - \sum_{k=1}^{K_r^1} \beta_{qrk} y_{rk} - \sum_{k=1}^{K_r^2} y_{qrk} \leq w_{rq} \quad \text{for } r = 1,\ldots,R \tag{14}$$

(Second Stage)

$$y_{qrk} - u_{qrk} \delta_{qrk} \leq 0 \quad \text{for } k = 1,\ldots,K_r^2; \tag{15}$$

$$y_{qrk} - \ell_{qrk\ qrk} \geq 0 \quad r = 1,\ldots,R \tag{16}$$

$$x_{qj} \geq 0,\ y_{qrk} \geq 0,\ \delta_{qrk} = 0 \text{ or } 1 \tag{17}$$

The two stage model (7) - (17) is somewhat formidable. The number of variables and constraints can clearly become very large as Q, the number of scenarios, grows. Benders' decomposition has been proposed as a resource directed method for breaking the model down into small components that can be separately analyzed. It has also been proposed as a method for dividing mixed integer programming models into their integer and continuous components. These two applications of resource directed decomposition combine perfectly when applied to the two stage model.

We do not intend to give precise details describing the application of Benders' decomposition to the two stage model (7) - (17). These details are well known; for example, see Shapiro [7]. Instead, in Figure 2, we provide the reader with a scheme depicting the block structure of our model. On the left, we have the matricies corresponding to the first stage variables. These variables are concerned with and enter into constraints on first stage resource allocation and resource acquisition. They also enter into constraints on resource allocation and acquisition for every second stage scenario. The submodel for each second stage scenario also has variables and constraints describing resource allocation and acquisition.

Figure 3 depicts how the two stage model has been rearranged to exploit its structure via decomposition. As we have depicted it, the master model consists of all the first stage variables and constraints. In addition, it contains Benders' cuts written with respect to the first stage variables plus resource acquisition variables (that is, the y_{qrk} and the δ_{qrk}) for all the second stage scenarios. These Benders' cuts

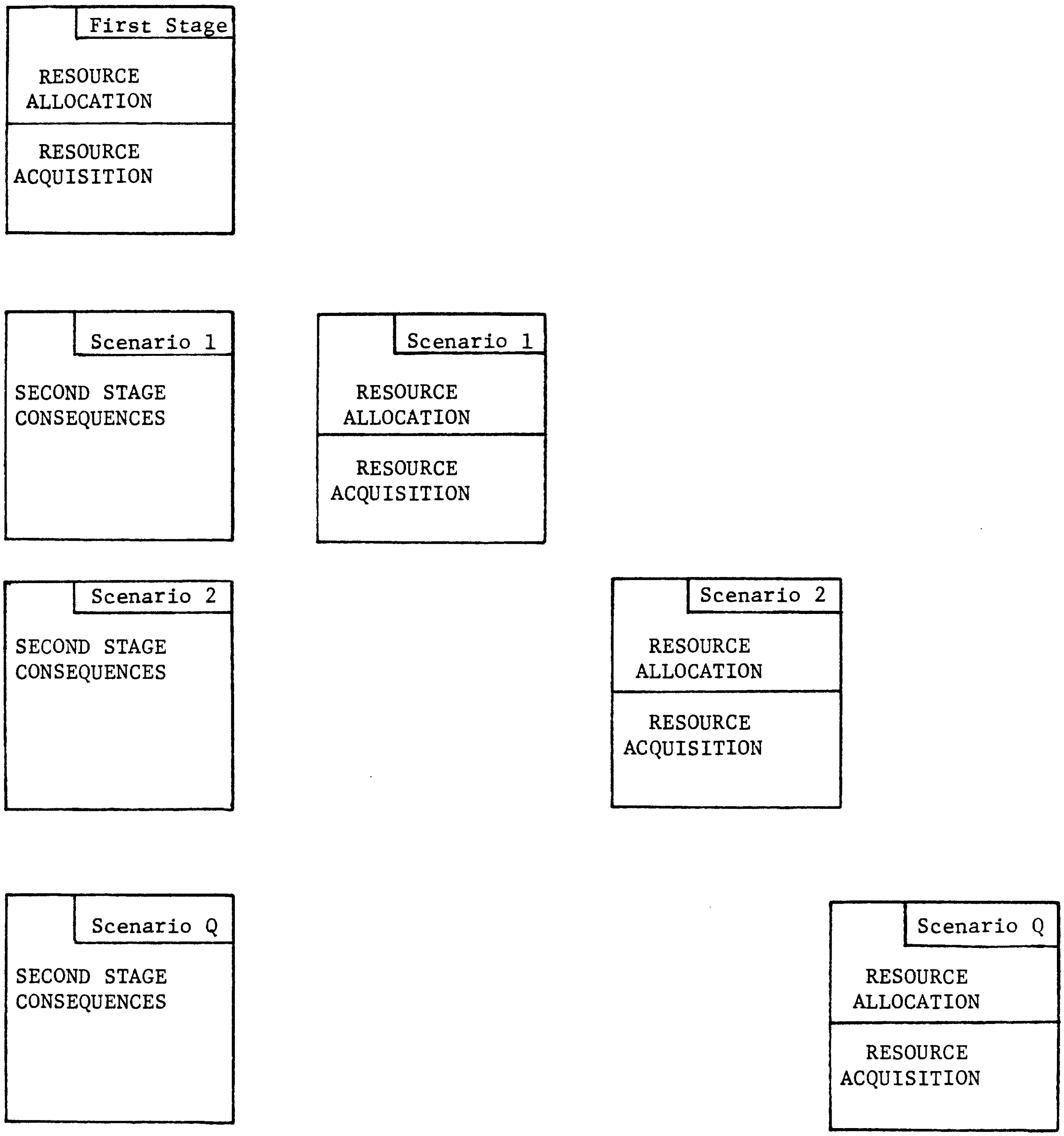
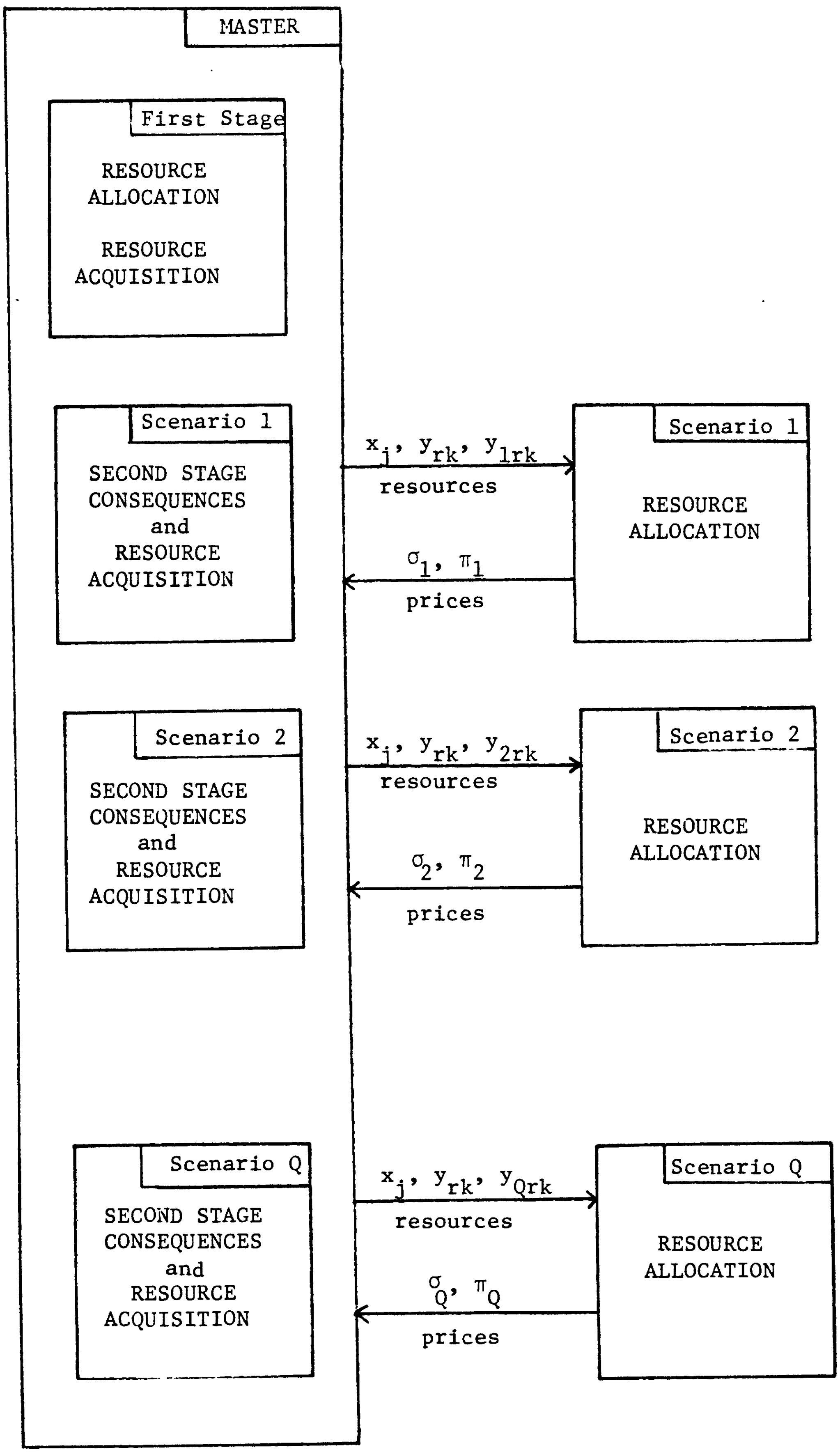

First Stage
RESOURCE ALLOCATION
RESOURCE ACQUISITION
Scenario 1
SECOND STAGE CONSEQUENCES
Scenario 1
RESOURCE ALLOCATION
RESOURCE ACQUISITION
Scenario 2
SECOND STAGE CONSEQUENCES
Scenario 2
RESOURCE ALLOCATION
RESOURCE ACQUISITION
Scenario Q
SECOND STAGE CONSEQUENCES
Scenario Q
RESOURCE ALLOCATION
RESOURCE ACQUISITION

Figure 2

Figure 3


MASTER
First Stage
RESOURCE ALLOCATION
RESOURCE ACQUISITION
Scenario 1
SECOND STAGE CONSEQUENCES and RESOURCE ACQUISITION
x_j, y_{rk}, y_{1rk}
resources
σ_1, π_1
prices
Scenario 1
RESOURCE ALLOCATION
Scenario 2
SECOND STAGE CONSEQUENCES and RESOURCE ACQUISITION
x_j, y_{rk}, y_{2rk}
resources
σ_2, π_2
prices
Scenario 2
RESOURCE ALLOCATION
Scenario Q
SECOND STAGE CONSEQUENCES and RESOURCE ACQUISITION
x_j, y_{rk}, y_{Qrk}
resources
σ_Q, π_Q
prices
Scenario Q
RESOURCE ALLOCATION

are written with respect to optimal shadow prices σ_q and π_q on the constraints (13) and (14) for the corresponding right hand sides in these constraints.

Thus, the master model in the resource directed decomposition is a mixed integer programming model. It determines trial values for the first stage resource allocation and acquisition variables, plus trial values for all the second stage resource acquisition variables. These trial values are used to generate the individual second stage resource allocation planning models, one for each scenario, in which optimal values for the resource allocation variables x_{qj} are selected. At the same time, optimal shadow prices on the resource allocation and acquisition constraints (13) and (14) for each scenario are determined. These are sent back to the master model so that, in later iterations, it can make better determinations of the x_j, y_{rk}, and y_{qrk} variables.

DETAILS OF THE IMPLEMENTED MODEL

The traditional utility expansion models of the type surveyed by Anderson [1] served as the point of departure for our implementation. These models are multi-period linear programming models in which the discounted sum of investment and operating costs required to meet forecasted demand are minimized. Our extensions include variables and constraints describing fixed and nonlinear costs for adding plants, contracts and pollution legislation limits. Mixed integer programming constructs are required to model the first two extensions.

Moreover, we extended the traditional deterministic models to two stage stochastic programming with recourse. Specifically, our model consists of 5 two-year periods, with decisions taken here-and-now affecting the first 4 years (that is, the first 2 periods). Uncertainties regarding growth in electricity demand, environmental restrictions, and primary fuel costs are explicitly modeled. These uncertainties are assumed to be revealed after 4 years, after which time the decision maker selects optimal decisions for the remaining 6 years of the planning horizon.

Our objective function is the minimization of total expected discounted cost over the ten years. Included are construction, operating and fuel purchase costs. Expected salvage values for useful plants still in operation at the end of the ten years are credited to the total cost. Costs are measured in constant 1983 dollars and discounted at 5% per year.

Plant Additions

In each period, we consider five types of plant additions: coal,

coal with flue gas desulphurization (fgd), oil, gas and peaker. For all types we assume a cost curve of the form shown in Figure 3. In this figure, b

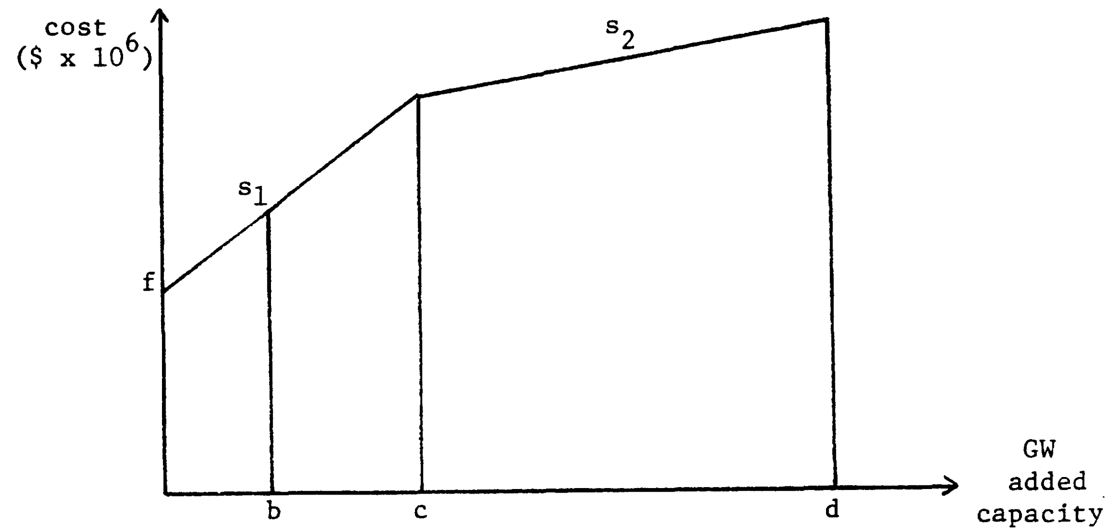


Figure 3

represents the conditional minimal capacity addition; that is, the smallest positive addition. Of course, no addition with zero cost is allowed. The parameter d is the largest capacity addition allowed. The parameter f is the fixed cost, s_1 is the slope of the left hand linear segment, and s_2 is the slope of the right hand linear segment. The data we used in our model is shown in Table 1.

Coal Spot Purchases and Long Term Contracts

We assume that there are two types of coal, high and low sulphur, which could be bought in limited quantities on the spot market in each period. The unit prices are $\$1.61/10^6$ BTU and $\$1.84/10^6$ BTU, respectively. In addition, two long term contracts are available for ensuring

PLANT TYPE	b GW	c GW	d GW	f $\$x10^6$	s_1 $\frac{\$x10^6}{GW}$	s_2 $\frac{\$x10^6}{GW}$
Coal	.10	.65	1.20	76.7	933	667
Coal & fgd	.10	.80	1.50	82.0	940	667
Oil	.10	.45	.80	37.5	675	600
Gas	.10	.50	.90	22.5	675	600
Peaker	.025	.10	.10	5.5	200	200

Plant Addition Costs

Table 1

supply of low sulphur coal in significant quantities. One contract permits the utility to purchase approximately 225×10^2 BTU's at a cost of $\$1.97/10^6$ BTU. However, the utility is constrained to purchase in each period a fraction between .18 and .22 of this total. The second contract permits the utility to purchase approximately 300×10^2 BTU's of low sulphur coal at a cost of $\$1.93/10^6$ BTU. The same constraints on each period's purchases is in effect for this contract.

Plant Utilization and Operating Costs

The model contains 11 existing plants in addition to the 5 capacity expansion options discussed above. The load duration curve describing yearly electricity demand was broken down into three components:

base	8760 hours/year
intermediate	6132 hours/year
peak	1314 hours/year

Letting p denote the index for block, and w the index for plant type, plant utilization is constrained in each period by

$$\sum_{p=1}^{3} \frac{1}{A(w,p)} U_{wpt} \leq \text{power of plant w in year t}$$

where A(w,p) is the availability of plant w in load p, U_{wpt} is the generation of plant w in block p in year t. The specific numbers used in the model are given in Table 2. The parameters A(w,p) were set equal to AV(w,p) - loss(w) for all 16 plants. The underlined plants are the capacity addition options.

Table 3 also gives the variable costs at the start of the planning horizon associated with the different plants. These costs include oper-

PLANTS	COAL				COAL + FGD			OIL			GAS	PEAKERS				NUCLEAR
	A	B	C	E	J	K	L	O	P	R	G	S	T	X	Z	N
p = 1	.6	.6	.6	.6	.6	.6	.6	.6	.6	.6	.6	.7	.7	.7	.7	.6
AV p = 2	.7	.7	.7	.7	.7	.7	.7	.7	.7	.7	.7	.8	.8	.8	.8	.7
p = 3	.8	.8	.8	.8	.8	.8	.8	.8	.8	.8	.8	.9	.9	.9	.9	.8
LOSS	0	.10	.12	0	0	0	0	0	.15	0	0	0	0	.10	0	0
CAPACITY IN YEAR 0 (GW)	.85	.40	.55	0	.50	.50	0	.60	.40	0	0	.10	.05	.09	0	.80
VARIABLE COST IN YEAR 0 ($\$x10^6$/GWh)	1.9	1.9	1.9	1.9	2.1	2.1	2.1	1.05	1.05	1.05	.95	1.5	1.5	1.5	1.5	3.0

TABLE 2

ating cost and fuel cost (for plants other than coal plants for which fuel costs are computed separately). The fuel cost component measured in $\$ \times 10^6$/GWh is based on the following figures which are assumed to increase at a rate of 2% per year:

oil	.051
gas	.037
peaking	.09
nuclear	.009

Demand

Total demand in year one was taken to be 9000 GWh; it was inflated for each subsequent year at a rate of 2%. In writing constraints on plant operations in each period, this energy demand was converted to power to be met in each block. In year one, the figures are

base	0.563
intermediate	1.109
peak	1.656

The reserve margin in each period equaled (1.30) times the demand in the peak block.

Pollution Constraints

Pollution constraints were imposed on the operations of the coal plants as follows. In each period, a limit was imposed on the total sulphur emissions allowed from all coal plants. Typical limits used ran from .7% in the first two-year period to .4% in the fifth two-year period. Table 4 shows the emissions produced by individual plants.

Uncertainties

The two stage stochastic programming with recourse models we implemented and optimized addressed uncertainties in demand, air pollution control limits, the time of completion of a slurry pipeline, and fuel

	High Sulphur Coal	Low Sulphur Coal
Coal plant without fgd	.020	.005
Coal plant with fgd	.005	.001

Sulphur Emissions

Table 4

prices. A number of runs were made with varying combinations of uncertain factors. In the following section, we report in detail on the results of one of these runs.

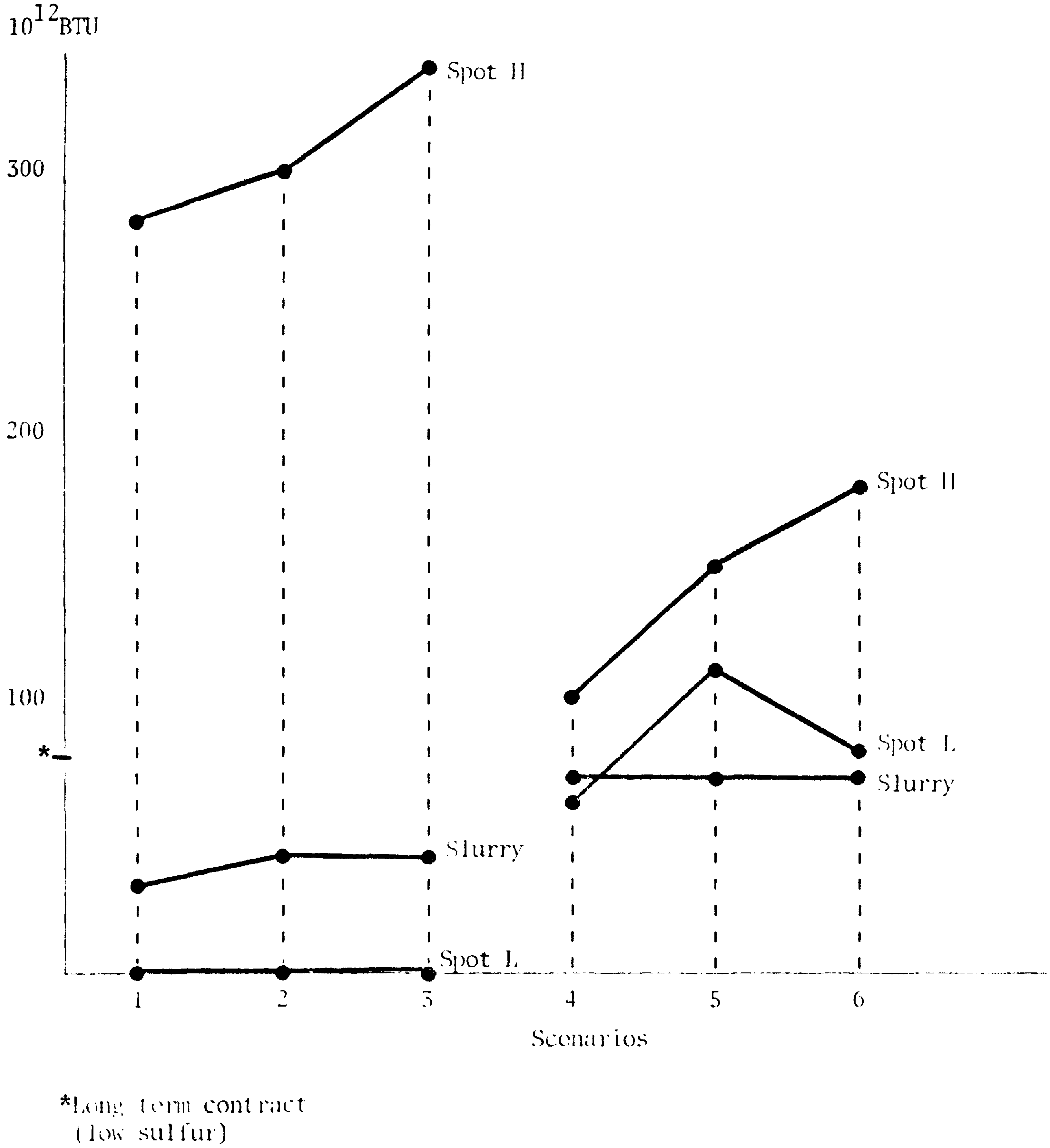
RESULTS

In this section, we discuss the application of two stage, stochastic programming with recourse to a specific utility capacity expansion problem. The data used in constructing our model was discussed in the previous section. The following summarizes the main features of the problem:

- energy demand in year 1: 9.00 GWH
- power capacity in year 1 without additions: 4.74 GW
  - 57% coal
  - 21% oil
  - 17% nuclear
  - 5% peaking
- possible investments
  - coal plant 1
  - coal plant 2 (with flue gas desulphurization)
  - gas plant
  - oil plant
  - peaking plant
  - 2 long-term coal contracts
  - slurry pipeline
- second stage uncertainties (6 scenarios)
  - 3 demand cases
    - 1% growth per year (30%)
    - 2% growth per year (40%)
    - 4% growth per year (30%)
  - 2 environmental cases
    - non-stringent .7% standard (70%)
    - stringent .4% standard (30%)

Recall that year 1 of the planning horizon is 1983, and that our planning horizon consists of five 2-year periods.

A two-stage stochastic programming with recourse model was generated to study this and related probelms. The models were optimized



Total Coal Consumption

Figure 4

DEMAND

	LOW	MEDIUM	HIGH
Non-Stringent Pollution Control	1 Coal Plant W/O FGD In Period 5 111MW Slurry Use: 32×10^{12} BTU p=.09	2 Coal Plant W/O FGD In Period 5 1000MW Slurry Use: 46×10^{12} BTU p=.12	3 Gas Plant In Period 4 478MW Slurry Use: 48×10^{12} BTU p=.09
Stringent Pollution Control	4 No Plants Slurry Use: 77×10^{12} BTU p=.21	5 Coal Plant W/O FGD In Period 5 1000MW Slurry Use: 78×10^{12} BTU p=.28	6 Gas Plant In Period 4 600MW Slurry Use: 79×10^{12} BTU p=.21

Second Stage Contingency Plans

Table 5

using the decomposition scheme described in an earlier section; see Figure 3. The master mixed integer programming model in these applications typically consisted of around 500 constraints, with 165 integer variables describing the first and second stage investment options, and 600 to 1000 continuous variables. The second stage linear programming models, one for each scenario consisted of around 125 constraints and 200 variables.

The following summarizes the results of our two stage optimization of the planning problem described above. In the first stage, the model selected:

- larger long term coal contract (300 x 10^{12} BTU of low sulphur coal) for the entire five year planning horizon
- slurry pipeline for use beginning in period 3
- construction of a 795 coal plant with flue gas desulphurization in period 1

Table 5 summarizes the optimal contingency plans to be put into effect in the second stage after the uncertainties in demand and pollution control are revealed. The numbers in the lower right hand corner of each scenario is the associated probability of occurrence.

The hedging and contingency plans for coal consumption developed by the model are graphically depicted in Figure 4. Under scenarios 1, 2, 3 when the pollution controls are less stringent, high sulphur coal is bought in large quantities on the spot market. Under scenarios 4, 5, 6 when the pollution controls are more stringent, less high sulphur coal is bought on the spot market. This reduction is offset by greater purchases of low sulphur coal from the slurry pipeline and the spot market. However, in the highest demand case, these purchases are reduced because it becomes economic to build and operate a gas fired plant in period 4.

We experimented with deterministic versions of the model to assess the differences in optimal strategies it would produce. Generally speaking, the main difference is that deterministic analysis suggests building a larger number of plants in earlier periods, and contracting for less low-sulphur coal. This result is consistent with what we would expect; namely, the stochastic model takes a more pronounced wait-and-see attitude about plant construction, and hedges against the environmental uncertainties by committing more heavily to the acquisition of coal that can be burned in any type of plant.

Space does not permit an extensive discussion of some of the other planning questions we analyzed. One set of runs examined the consequences of uncertainties in fuel cost. Another run addressed a presumed uncertainty in completion of the slurry pipeline. Specifically, the model was used to measure the impact on expansion strategies if there was a positive probability that the pipeline's completion would be delayed by two years.

We conclude this section with a few comments about the implementation of the decomposition scheme. The implementation was performed on a PRIME 800 computer using the LINDO package [7]. LINDO is a FORTRAN callable program that was designed to be used in the flexible manner required by a decomposition method. The frequently observed deficiency of Benders' decomposition that it can take a long time to converge was overcome by preliminary optimization of the submodels to produce an effective set of cuts for the master model. In addition, exact optimality of the master model was not required in determining new investment strategies to send to the second stage submodels. As a result of using these solution strategies, the models were typically optimized to within a fraction of 1% of optimality in less than 10 interations between the master and the submodels.

CONCLUSIONS

In this paper, we have reported on the successful application of stochastic programming with recourse models to long term electric utility planning problems. The prototype models that we constructed incorporated capacity expansion and long term supply decisions that are representative of those faced by utility planners. Optimizing the models illustrated the nature of hedging strategies and contingency plans that such models are capable of identifying. In suitably modified form, the models would be appropriate for analyzing long term planning problems faced by companies in many other industries.

The resource directed decomposition methods we implemented proved efficient and stable, indicating that it should be possible, in a reasonable amount of computer time, to construct and solve larger models of actual planning problems. These methods exploit decomposable structures arising from both stochastic programming and mixed integer programming modeling of uncertainties, fixed costs and returns to scale inherent in long term planning. Our computational experiments also showed that convergence of the decomposition schemes can occur rapidly, at least to a reasonable degree of approximation, when human judgment is used in setting up the initial master model.

A number of areas of future research remain to be investigated. One obvious and important extension of the model presented here is to those with more than one recourse stage. The decomposition methods developed for the two stage model can be readily nested to handle multiple stages, but the computational burden can grow rapidly. In any event, this extension, and approaches for dealing with problem size, should be carried out in the context of specific applications.

A second area of research arises in the case when uncertainties are characterized by forecasts of continuous parameters, such as sales of a company's products. Approximation is clearly involved in going from these continuous forecasts to the finite number of scenarios. In general, we would expect that the greater the number of scenarios, the more accurate will be the contingency plans and hedging strategies determined by the model. A thorough investigation of properties of the approximations would undoubtedly lead to methods for controlling them.

Finally, an important model extension is to replace the linear objective function used here in minimizing expected cost by a nonlinear one corresponding to the maximization of expected utility. Maximizing utility is especially important when the amount of money involved in the company's long range plans is a significant percentage of its total assets. One approach would be to use resource directed decomposition methods to compute piecewise linear approximations to the utility function. In this way, the modeling and optimization approaches developed in this paper could still be directly applied.

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