

III. PLASMA DYNAMICS

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A. MAGNETICALLY DRIVEN SHOCK TUBE STUDIES*

The steady-flow solution of shock waves, which is valid for pressure-driven shock tubes, is not applicable to magnetically driven tubes because the conditions of constant density and pressure behind the shock front and constant driving pressure are not met. A treatment that approximately satisfies all the conditions, however, is G. I. Taylor's blast-wave solution (1).

This report gives a one-dimensional solution that uses Taylor's method. It is assumed that there is an instantaneous release of energy, E , per unit area in an infinite plane at $x = 0$, $t = 0$. The boundary condition at $x = 0$ is such that the shock propagates only in the positive x -direction. A cylindrical section perpendicular to the plane of energy release may be taken as the shock tube with all heat conduction, viscosity, and radiation losses neglected. The gas is assumed to be polytropic (ideal with constant heat capacities). Strong shocks, with no ionization and with particle-isentropic flow behind the shock, are assumed.

The flow equations behind the shock are:

$$\text{Equation of motion} \quad \frac{du}{dt} + \frac{1}{\rho} \frac{dp}{dx} = 0 \quad (1)$$

$$\text{Equation of continuity} \quad \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u) = 0 \quad (2)$$

$$\text{Adiabatic equation} \quad \frac{dS}{dt} = \frac{d}{dt} \left(\frac{p}{\rho^\gamma} \right) = 0 \quad (3)$$

where u is the flow velocity, p is the pressure, ρ is the density, S is the entropy, and γ is the ratio of specific heats.

The solutions of these equations are known as "progressive waves" (2); the velocity, pressure, and density distributions do not change in shape, but only in scale, as the shock wave advances. We may then write

$$\frac{p}{p_0} = \frac{f_1(\eta)}{X} \quad (4)$$

$$\frac{\rho}{\rho_0} = \psi(\eta) \quad (5)$$

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$$u = \frac{\phi_1(\eta)}{X^{1/2}} \quad (6)$$

where $\eta = x/X$ is the normalized position, with X , the shock position. Substituting in the flow equations and separating X , we have

$$X^{1/2} \frac{dX}{dt} = \left(\frac{p_0 f_1'}{\rho_0 \psi} + \phi_1 \phi_1' \right) \left(\frac{\phi_1}{2} + \eta \phi_1' \right)^{-1} \quad (7)$$

$$X^{1/2} \frac{dX}{dt} = \frac{\phi_1 \psi' + \psi \phi_1'}{\eta \psi'} \quad (8)$$

$$X^{1/2} \frac{dX}{dt} = \frac{\phi_1}{\psi} \left(\gamma f_1 \psi' - \psi f_1' \right) \left(\frac{\gamma \eta f_1 \psi'}{\psi} - \eta f_1' - f_1 \right)^{-1} \quad (9)$$

We find that there is a solution if, and only if,

$$X^{1/2} \frac{dX}{dt} = A \quad (10)$$

where A is a constant. After integrating, we find that

$$X = \left(\frac{3}{2} A \right)^{2/3} t^{2/3} \quad (11)$$

The functions $f_1(\eta)$ and $\phi_1(\eta)$ may be made nondimensional. At the shock front, it is known that for strong shocks

$$\frac{p}{p_0} = \frac{2\gamma}{\gamma + 1} M^2 \quad (12)$$

$$\frac{\rho}{\rho_0} = \frac{\gamma + 1}{\gamma - 1} \quad (13)$$

$$\frac{u}{V_s} = \frac{2}{\gamma + 1} \quad (14)$$

where $V_s = dX/dt$, the shock velocity; and $M = V_s/a_0$, the shock Mach number, with a_0 , the velocity of sound ahead of the shock. Therefore, we can write

$$f_1(\eta) = \frac{A^2}{a_0^2} f(\eta) \quad (15)$$

$$\phi_1(\eta) = A \phi(\eta) \quad (16)$$

Equations 7, 8, and 9 can now be solved for the functions $f(\eta)$, $\psi(\eta)$, and $\phi(\eta)$. The result is

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$$\phi' = \frac{2f' - \gamma\psi\phi}{2(\eta-\phi)} \quad (17)$$

$$\psi' = \frac{\psi\phi'}{\eta-\phi} \quad (18)$$

$$f' = \frac{f\psi}{2} \frac{\gamma\phi + 2(\eta-\phi)}{f - \psi(\eta-\phi)^2} \quad (19)$$

These equations can be solved numerically with the use of boundary conditions (Eqs. 12-14):

$$f(1) = \frac{2\gamma}{\gamma+1} \quad (20)$$

$$\phi(1) = \frac{2}{\gamma+1} \quad (21)$$

$$\psi(1) = \frac{\gamma+1}{\gamma-1} \quad (22)$$

The solution is plotted in Fig. III-1 for $\gamma = 1.4$, the value for hydrogen gas.

The constant A can now be evaluated from the condition that the total energy behind the shock remains constant. Here, the original internal energy of the gas is neglected in the strong-shock approximation.

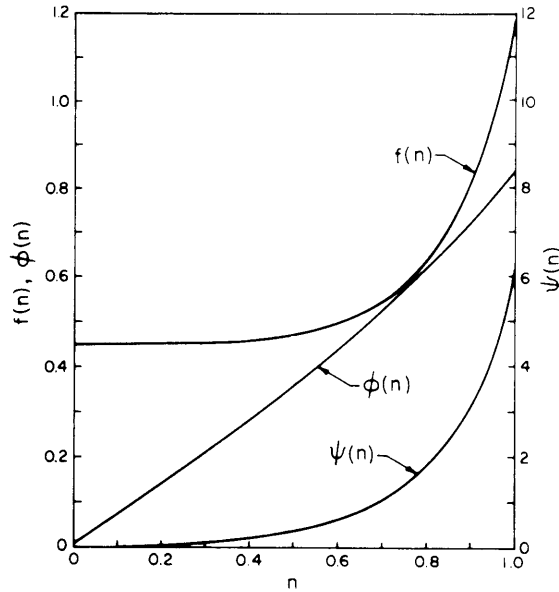


Fig. III-1. Normalized pressure, velocity, and density distributions for $\gamma = 1.4$ in an impulse-driven shock tube.

$$E = \int_0^X \rho \left(C_V T + \frac{1}{2} u^2 \right) dx = \int_0^X \left(\frac{p}{\gamma - 1} + \frac{1}{2} \rho u^2 \right) dx \quad (23)$$

Substituting Eqs. 4, 5, 6, 15 and 16 in Eq. 23, we obtain

$$E = \rho_0 A^2 \int_0^1 \left(\frac{\gamma}{\gamma - 1} f(\eta) + \frac{\psi(\eta) \phi^2(\eta)}{2} \right) d\eta \quad (24)$$

or $E = C_0 \rho_0 A^2$, where C_0 is the integral that depends on γ alone. C_0 can be evaluated by numerical integration and is found to be 2.04. Equation 11 then becomes

$$X = \left(\frac{E}{\rho_0} \right)^{1/3} t^{2/3} \quad (25)$$

and

$$V_s = \frac{2}{3} \left(\frac{E}{\rho_0 t} \right)^{1/3} \quad (26)$$

The behavior of the shock as a function of these variables may be determined experimentally. Our treatment also gives the distributions of pressure, density, and temperature behind the shock which may be taken as approximately correct for those regions in which the assumptions are valid; that is, immediately behind the shock.

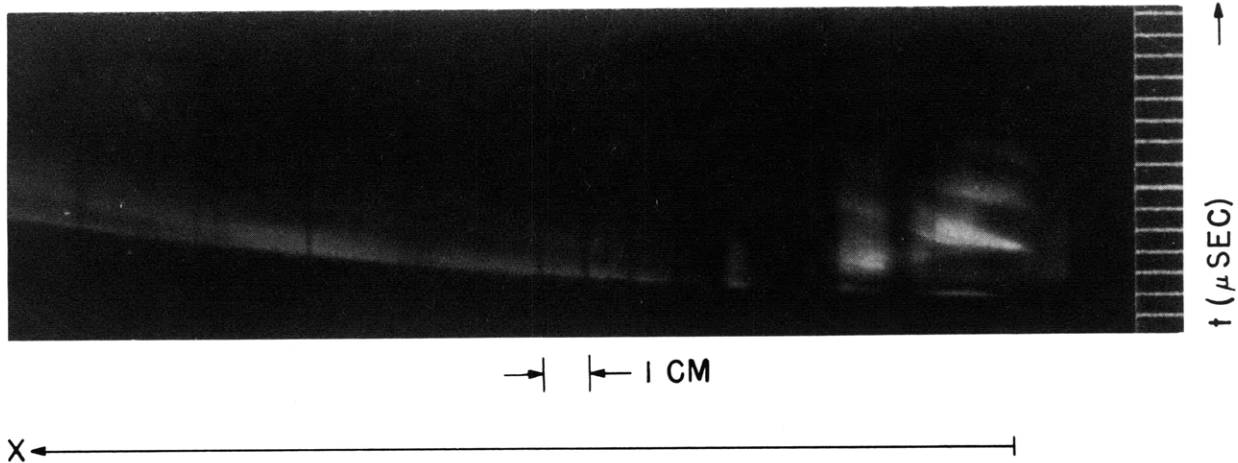


Fig. III-2. Smear camera photograph of shock wave from 25-kv discharge into hydrogen at pressure of 1 mm Hg.

The T tube described in Quarterly Progress Report No. 51, page 28, has been used with a smear camera in an attempt to verify Eqs. 25 and 26. A typical smear photograph of a 25-kv discharge in hydrogen at a pressure of 1 mm Hg is shown in Fig. III-2. The dark markings are 1 cm apart, and the light scale is in microseconds. These curves

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are plotted with $\log X$ versus $\log t$ in Fig. III-3 so that the slope is the exponent n in the relation $X \sim t^n$. This is found to be $n = 0.72$, which indicates that the shock is faster than the predicted $n = 0.666$. The only explanation for this is a continued driving of the

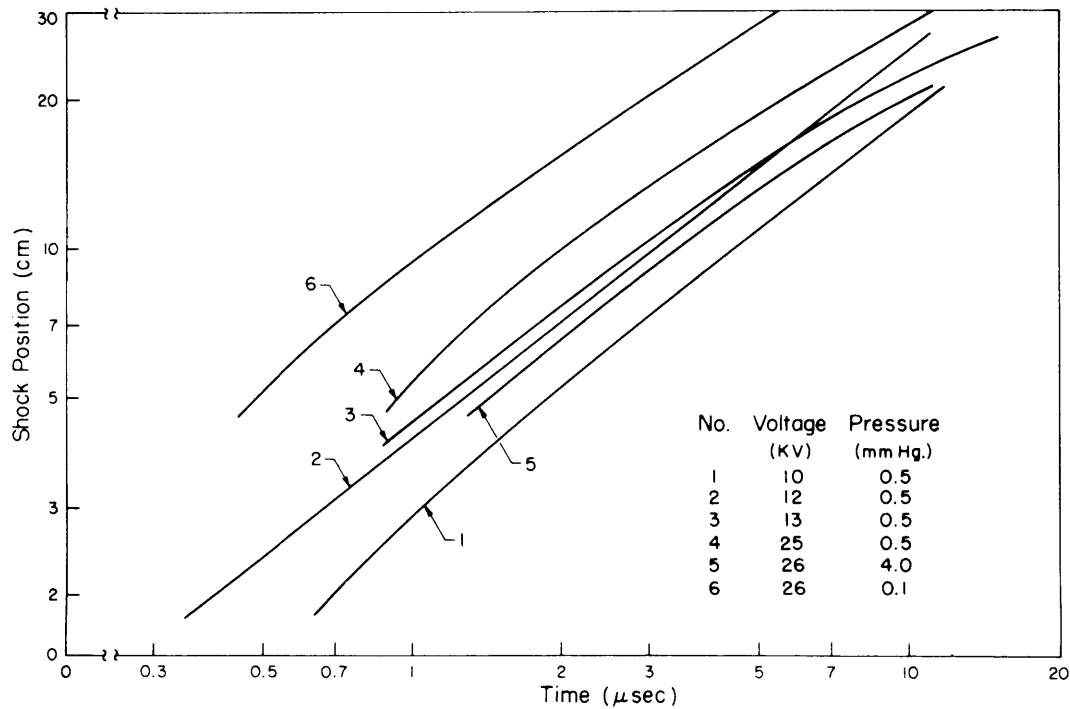


Fig. III-3. X plotted against t for several smear photographs.

shock for the first half-cycle of the capacitor discharge; that is, energy is not put in impulsively.

An attempt was made to check the dependence of shock velocity on energy and density to verify that

$$V_s \sim \left(\frac{E}{\rho}\right)^{1/2} \quad (27)$$

at a given position along the tube. The errors in measured quantities are too large, however, to provide a check. Also, as the density is changed, the amount of energy actually delivered to the gas is changed, and no way has been found to actually measure it.

Equation 25 does give an estimate of the efficiency ξ of energy transfer from the capacitor bank to the shock wave. With $\pi R^2 E$ representing the total energy delivered to the shock, and W , the stored energy in the capacitors,

$$\xi = \frac{\pi R^2 E}{W} = \frac{2\pi R^2 X^3 \rho_0}{C V^2 t^2} \quad (28)$$

The value was found to be between 0.10 and 0.20 for most shocks.

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References

1. G. I. Taylor, The formation of a blast wave by a very intense explosion, Proc. Roy. Soc. (London) A201, 159-186 (1950).
2. R. Courant and K. O. Friedrichs, Supersonic Flow and Shock Waves (Interscience Publishers, Inc., New York, 1948), p. 419.

B. CHANNELING AND COMPRESSION OF A PLASMA BEAM

Work on this problem has been concentrated on analysis of the modifying effects of collisions on the results previously obtained for the rate of growth of the boundary layer between the plasma beam and the magnetic field. Under the circumstances of the experiment, the most important type of collision is that between the ions in the beam and the stationary neutral particles in the vacuum tube. Collisions involving protons will always be of negligible importance at the densities involved in these experiments, but the plasma beam also contains H_2^+ ions which have a much higher collision cross section. The analysis indicates that collisions between H_2^+ ions and neutral molecules will be the main cause of growth of the boundary layer at base pressures greater than approximately 3×10^{-4} mm Hg, but will be negligible at the lowest pressures used in the experiments. By adjusting the base pressure, it will thus be possible to isolate the effects of collisions and to make an experimental test of the theory.

In addition, the integral equation for the boundary-layer thickness has been solved for the case in which the angular spread of the particles in the plasma beam is very small. Explicit results have been obtained for the rate of growth of the boundary layer and the maximum compression obtainable under the experimental conditions.

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