

STUDY OF TWO-PHASE FLOW IN PIPE BENDS

by

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B. S., Agricultural and Mechanical College of Texas  
(1956)

SUBMITTED IN PARTIAL FULFILLMENT

OF THE REQUIREMENTS FOR THE

DEGREE OF MASTER OF

SCIENCE

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June. 1957

Signature of Author

Department of Mechanical Engineering  
May 20. 1957

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Thesis Supervisor

Accepted by

Chairman, Departmental Committee on Graduate  
Students



ACKNOWLEDGEMENT

The author wishes to express his gratitude to Professor Miguel A. Santalo for having devoted a great deal of his valuable time giving advice and encouragement during the development of this thesis.

The author is also grateful to Melvin Cohen for the use of his experimental results.

Eng. g. (m. e.) Oct. 17, 1957

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Submitted to the Department of Mechanical Engineering on May 20, 1957, in partial fulfillment of the requirements for the degree of Master of Science.

## ABSTRACT

An analysis has been made to determine the energy of rotation, due to the difference in densities, of a gas-liquid mixture as it goes through a horizontal bend. This energy, assumed dissipated due to frictional forces, has been supposed to account for part of the pressure drop of the mixture during the passage.

Both gas and liquid have been considered incompressible and the flow regimes analyzed have been separated and annular.

The flow has been assumed one dimensional along the pipe, and a one dimensional rotation has been superimposed in the bend.

The use of a rather large gas-liquid velocity ratio has shown values of pressure drop a great deal below those found experimentally. Consideration of the limiting case of equal velocity of gas and liquid yields values of pressure drop larger than experimental.

It has been concluded that although the energy dissipated during rotation must account for a great deal of the pressure drop, other phenomena taking place in the bend must also be sources of pressure losses.

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## TABLE OF CONTENTS

	Page
List of Illustrations	ii
Table of Significant Symbols	iii
I. Introduction	1
II. Literature Review	4
II. 1. Theoretical Work on Secondary Flow	4
II. 2. Experimental Work	11
III. Dynamic Analysis of Separated Flow Through a Bend	18
III. 1. Assumptions	18
III. 2. Mathematical Analysis	19
III. 3. Approximate Solution	23
IV. Energy Loss Evaluation	24
IV. 1. Pressure Drop Calculation for Separated Flow	24
IV. 2. Extension to Annular Flow	27
IV. 3. A Method to Determine the Gas-Liquid Velocity Ratio	29
V. Experimental Work	36
V. 1. Apparatus	36
V. 2. Performance for Water Only	37
V. 3. Performance for Two-Phase Flow	38
VI. Discussion of Results	39
Appendix	43
Bibliography	45

## LIST OF ILLUSTRATIONS

	Page
Fig. 1	Effect of relative radius on bend loss coefficients. 47
Fig. 2	Bend loss coefficients found by Beij and other investigators. 48
Fig. 3	Calculated and experimental values of fluid angular rotation for a pipe half filled with liquid of 0.750 in. I.D. and 0.375 ft. radius of curvature. 49
Fig. 4	Pressure drop correlation for a bend of $R/d = 6$ . 50
Fig. 5	Pressure drop correlation for a bend of $R/d = 4$ . 51
Fig. 6	Pressure drop correlation for a bend of $R/d = 1$ . 52
Fig. 7	Influence of relative radius on pressure drop through a bend. 53
Fig. 8	Experimental pressure drop correlation for bends of $R/d$ of 4 and 1. 54
Fig. 9	Total bend loss and secondary bend loss (water only) 55
Fig. 10	Comparison of predicted and experimental values of pressure drop for a bend of $R/d = 4$ . 56
Fig. 11	Comparison of predicted and experimental values of pressure drop for a bend of $R/d = 1$ . 57
Fig. 12	Schematic diagram of apparatus 58

TABLE OF SIGNIFICANT SYMBOLS

Symbol	Meaning
A	Area of fluid cross section
a	Distance between center of gravity of liquid part of fluid cross section and axis of pipe.
$\alpha$	Steady state angle of displacement about axis of pipe.
$C_1, C_2, C_3$	Constants, as defined in Section III
d	Diameter of pipe
$\gamma$	Half the angle subtended by chord formed by liquid interphase
g	Acceleration of gravity (standard value)
g	Acceleration of gravity (local value)
H	Constant, as defined in Section III
$I_o$	Moment of inertia of liquid part of cross section of fluid about axis of pipe
$K_o$	Radius of gyration of liquid part of fluid cross section about axis of pipe
L	Length of straight pipe
P	Static pressure of fluid
r	Radius of pipe
R	Radius of curvature of pipe
$R_g$	Ratio of area of part occupied by gaseous phase to total area of cross section of fluid
$\rho$	Mass density
$\Delta P_R$	Pressure drop of fluid due to rotation
$\Delta P_W$	Pressure drop of liquid phase only
$\Delta P_B$	Total pressure drop through bend
V	Velocity

$\bar{V}$	Root mean square velocity
$\bar{\bar{V}}$	Mean velocity
$w$	Angular velocity about axis of pipe
$\omega$	Mass rate of flow
$\Phi$	Variable, as defined in Section III
$\int$	Dummy variable of integration
$\varphi$	Angular displacement of fluid about axis of pipe
$v$	Specific volume
$x$	Quality
$x_s$	Static quality

#### Subscripts

$g$	Gas flow
$l$	Liquid flow

## I. Introduction

The 90° pipe bend is perhaps the most frequently used connection in piping systems. The pressure losses in such bends are therefore of considerable engineering importance.

A "pipe bend", as used in this paper will consist of a pipe of circular cross section bent to the form of a circular arc. Two straight pipes that connect to the ends of the bend will always lie in the plane of the bend, and be so long as to permit a steady, developed flow on both sides of the bends.

When a single fluid flows under pressure through a straight horizontal pipe, the pressure decreases in the direction of the flow as a consequence of the energy loss resulting from fluid friction. A thin layer of the fluid adheres to the wall and has zero velocity with respect to it. Thus, the velocity of the fluid at any cross section increases from zero to a maximum, which occurs approximately at the axis of the pipe. The loss of energy is brought about by the viscous resistance to sliding of concentric layers of the fluid over each other. For the case of turbulent flow the mixing of the fluid causes even greater losses.

In the case of two fluids (gas and liquid) flowing through the same pipe, considerably larger losses occur than in the case of a single fluid. This is mainly due

to the fact that, in general, the liquid and gaseous phases flow at different velocities, and account has to be taken of friction at the interphase and mass transfer between the liquid and the gas.

In going through a bend, a single fluid experiences a circulatory motion perpendicular to the direction of the flow. This motion is due to the non-uniform velocity of the fluid as it approaches the bend. It alters the character of the flow and causes a loss of energy in addition to that of friction. In the case of two fluids of different density going through a bend one additional aspect has to be considered since centrifugal acceleration causes the heavier fluid to flow towards the outer wall. The case of air and water as the two fluids has been considered numerically.

If the flow is assumed to be separated\* and the velocity uniform at the entrance of the bend, this motion can be idealized as a pure rotation imposed on the main flow.

The purpose of this thesis has been to determine the energy loss due to such motion and its correlation to the easily accessible flow variables.

The total energy loss of the bend has been assumed to be the energy loss due to the above mentioned rotation,

\*See Section III for definition of term

plus the energy loss that would have taken place due to secondary flow within each separate phase.

The pressure drop associated with this energy of rotation has been computed for the liquid phase and assumed to be equal to the pressure drop of the mixture.

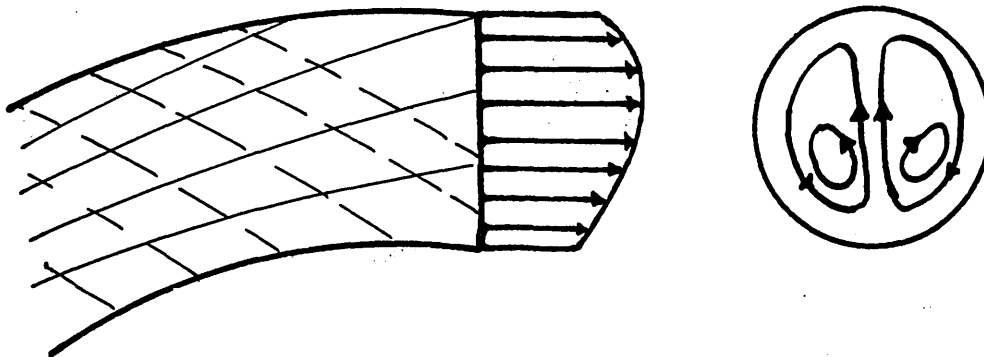
A non-dimensional plot of the energy loss through the bend, less frictional effects, has been made and compared with similar plots for experimental data obtained by Melvin Cohen in a yet unpublished work.

## II. Literature Review

The bulk of all the available literature on pressure drop through bends can be classified into two groups: analysis of secondary flow in the bend and investigation of its relation to the energy loss by the fluid in going through it, and finally, experimental work aimed to find "bend loss" coefficients.

### II. 1. Theoretical Work on Secondary Flow

Secondary flow occurs in curved pipes. This flow results from the fact that the fluid near the center of the cross section is moving at a higher velocity than at the sides due to the retarding effect of friction. This difference in velocities produces an outward flow in the central plane of the bend and a corresponding inward flow at each side due to greater centrifugal force on the faster moving liquid. (See sketch below)



Secondary flow formation in a bend

An exact determination of the secondary flow would involve the solution of the Navier-Stokes equations, together with an account of the effects of turbulence in the fluid as it enters a curved section of a pipe. Such a solution would be extremely complicated, if not impossible.

Most of the work on secondary flow has consisted of analyses and experiments with simplified models which have had in general, as an ultimate goal, the determination of the energy loss in the flow as a function of several variables of the system under consideration, such as radius of curvature of bend, diameter of pipe, etc. This type of approach has been quite extensively taken by many authors during the past two decades.

The first analytical approach to the problem of the motion of a fluid in a curved pipe was made by W. R. Dean (1) in 1927 and 1928. Even though his main purpose was to show the dependence of the pressure gradient along the central line of a curved pipe and rate of flow through a curved pipe on the curvature, he succeeded in presenting a first derivation of the nature of the flow along a curved pipe. As a main variable in the reduction of the rate of flow due to curvature, he introduces the variable  $K$ , equal to  $2 Re^2 \frac{r}{R}$ , where  $r$  is the radius of the pipe and  $R$  the radius of curvature. A modified version of this

variable is commonly known as Dean's number:

$$1/2 \text{ Re } \sqrt{r/R} = D.$$

Dean's analysis is confined to laminar flow in curved pipes of large bend radius as compared to the diameter of the pipe. The equations of motion obtained under these assumptions are solved by an expansion about the Reynolds number and restricted to values of it of 500 or less. Although his equations are limited and complex, and therefore represent little progress towards the derivation of a general theoretical expression for the flow in bends, his work paved the way for further investigations in the field.

His conclusions also became a great step in the study of secondary flow. He successfully explained the manner in which two symmetrical circulations are set up in the bend. Other investigations have come into a very close agreement with this result. He also comes to the conclusion that the most interesting effect of the curvature on the flow is that, in a curved pipe, part of the fluid is continually oscillating between the central part of the pipe, where the velocity is high, and the neighborhood of the boundary, where the velocity is low. This motion, due to the centrifugal force acting upon the fluid, causes a loss of energy that has no counterpart in a streamline motion through a straight pipe. Furthermore, turbulence is accompanied by a lateral movement of the fluid, which

also implies a loss of energy with no counterpart in steady motion.

Following Dean, several solutions have been obtained by making further simplifying assumptions in order to linearize his equations of motion.

In measurements carried out by Adler (2) for values of  $R/r$  equal to 50, 100 and 200, he demonstrated the existence of a large increase in the resistance of the flow due to curvature for  $Re \sqrt{r/R} > 10^{1.2}$ . According to his calculations he is able to define a coefficient of resistance for a curved pipe given as:

$$\frac{\lambda}{\lambda_0} = 0.1064 \left[ Re \sqrt{r/R} \right]^{1/2}$$

where  $\lambda_0$  denotes the resistance of a straight pipe. Measurements have indicated, however, that the above equation is only valid for values of the parameter  $Re \sqrt{r/R}$  in excess of  $10^{2.8}$ . Later on, Prandtl (3) came out with an empirical formula which expressed Adler's results to a higher degree of precision:

$$\frac{\lambda}{\lambda_0} = 0.37 D^{0.36}$$

in the range:

$$2D = Re \sqrt{r/R} < 10^{3.0}$$

In 1949, Squire and Winter (4) showed that the secondary flow through a bend could occur as a result of the

non-uniform velocity profile of the fluid at the entrance of the bend. They suggested that a more general investigation to the rotational flow of a newtonian fluid in three dimensions could be made if attention was concentrated in the component of the vorticity the direction of the flow.

A similar approach has been made by Hawthorne (5), who derived the formula:

$$\left(\frac{\xi}{q}\right)_2 - \left(\frac{\xi}{q}\right)_1 = -2 \int_1^2 \text{grad} \left| \frac{P_0}{\rho} \right| \sin \phi \frac{d\theta}{q^2}$$

where:

$\xi$  = component of the vorticity in the direction of the flow.

$q$  = magnitude of the velocity vector.

$\phi$  = the angle between the directions of the principal normal and the normal to the Bernoulli surface.

$P_0$  = the stagnation pressure of the fluid, defined as  $\frac{P_0}{\rho} = \frac{P}{\rho} + \frac{q^2}{2}$

$\theta$  = the angle of displacement along the curved path.

With the help of the assumptions that the secondary flow occurs in planes which are normal to the direction of flow, that the secondary vorticity is normal to these planes and that the secondary flow may be treated as two dimensional, Hawthorne applies the above equation to the flow through a 90-degree bend. His results show that the fluid

in the bend will oscillate between the angles of 0 and  $\pi$  with a period for a complete oscillation approximately equal to  $2\pi/\sqrt{R/d}$  radians of turn.

This fact may perhaps be an explanation to Davis' results, who in 1910, in an attempt to plot bend loss coefficients  $\zeta$  against  $\sqrt{R/d}$  found an almost periodic variation of  $\zeta$  with  $\sqrt{R/d}$ . (See Figure 1).

Eichemberger (7) conducted investigations using Hawthorne's analysis. These experiments show that viscosity is of little influence for the phenomenon of secondary flow. He suggested the idea that viscous phenomena within the bend are of secondary importance and one therefore might suspect that the dissipation due to viscosity is not essentially higher in a bend than in a straight pipe.

His results also seem to show that the Kinetic energy contained in the secondary flow is small. He suggested that the main losses in the passage through a bend are not due to the dissipation of secondary velocities but to the displaced boundary of fluid.

His numbers, he attributes approximately one half of the total loss through a bend to dissipation within the bend itself, out of which approximately one fifth is due to secondary rotation. The other half is considered lost in the downstream tangent of the bend, due to the turbulence induced by the change in velocity profile.

Horlock (8) investigated the secondary vorticity in a flow going through a curved pipe which is subjected to repeated reversals of the radius of curvature. His differential equations for the fluid motion lead to elliptic functions, which are solved by numerical methods.

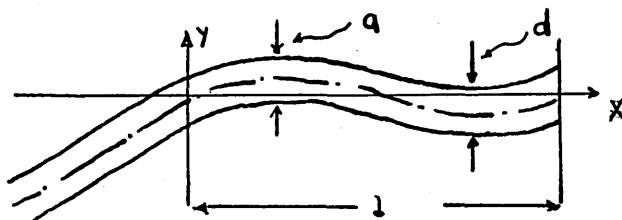
For a simple mathematical model, a sinusoidal pipe is analyzed. This type of pipe configuration yields an analytical solution for the secondary flow analogous to that of the motion of a ball pendulum in an alternating gravitational field.

$$\frac{d^2 \psi}{dz^2} = \frac{\bar{a}}{\bar{d}} \cos \psi \sin \psi$$

where  $z = 2\pi \bar{x} = 2\pi \frac{x}{l}$

$$a = \frac{\bar{a}}{l}, \quad \bar{d} = \frac{d}{l}$$

and  $\psi = a \sin \frac{2\pi x}{l}$



represents the equation for the rotation of the particle of the highest velocity.

Assuming small angles of rotation Horlock found that the rate of increase of the angle of rotation  $\psi$  increased with the parameter  $\frac{\bar{a}}{d}$  for entry conditions corresponding to  $\psi = 0$  and  $\frac{d\psi}{dt} = 0$  at  $z_0 = 0$  and  $z_0 = \pi/4$ .

For  $z_0 = \pi/2$  the solution suggested periodicity of the secondary flow for all values of the parameter  $\frac{\bar{a}}{d}$  but for  $z_0 = \frac{3\pi}{4}$  the solution suggested the possibility of a reversal of direction of secondary rotation, which would become sharper with increasing values of the parameter.

Experimentally, Horlock's results checked out pretty well. Instead of a sinusoidal pipe, a 90-degree pipe bend of circular cross section was cut into segments of 15-degrees each and bolted together. For this set up the rate of increase of the angle of rotation was found higher than predicted.

It was also found that viscosity damps the rotation as  $\psi$  approaches  $\pi/2$ . This was also found to be true by the author in a partially filled with liquid pipe.

In conclusion, Horlock's results seem to indicate that large velocities may be produced in bends in which the radius of curvature is reversed, and that these secondary velocities must be associated with the large pressure losses experienced in pipes.

## II. 2. Experimental Work

The most complete work on experimental data on

pressure loss through bends available to the author has been that published by Beij (9).

Beij investigated the pressure loss for several 4 in. ID steel, 90-degree bends of radii varying from 6 to 80 inches.

He considered the pressure drop through a bend as being made up of three parts: the pressure loss that would occur in a straight pipe of the same axial length as the bends, plus the excess loss in the bend, plus an excess loss in the downstream tangent.

$$\frac{P}{\rho} = H = H_S + H_B + H_T \quad (1)$$

where  $H_S$  = the head loss with characteristic velocity distribution in a straight pipe of axial length equal to the distance between the points of pressure measurement.

$H_B$  = the excess head loss in the bend.

$H_T$  = the excess head loss in the downstream tangent.

making  $H_B = \zeta \frac{V^2}{2g}$  ;  $H_T = \theta \frac{V^2}{2g}$  and  $H_S = \lambda_s \frac{1}{d} \frac{V^2}{2g}$

with the new symbols defined as

$V$  = the mean velocity, which is obtained by dividing the mass rate of flow by the cross sectional area of the pipe.

- $g$  = acceleration of gravity (standard value).  
 $s$  = the coefficient of resistance for a straight pipe with characteristic velocity distribution.  
 $l$  = length of the pipe.  
 $d$  = diameter of the pipe.  
 $\mathcal{J}$  = the deflection coefficient.  
 $\theta$  = the tangent coefficient.

we obtain: 
$$\frac{P}{\rho} = H = \lambda_s \frac{l}{d} \frac{v^2}{2g} + \mathcal{J} \frac{v^2}{2g} + \theta \frac{v^2}{2g} \quad (2)$$

- where:  $P$  = total pressure loss.  
 $\gamma$  = the specific weight of the fluid.  
 $H$  = the total head loss, (measured as the height of a column of the same fluid as that flowing through the pipe line.)

The last two terms of equation (2) can be combined to give:

$$H = H_s + \eta \frac{v^2}{2g} \quad (3)$$

where  $\eta = \mathcal{J} + \theta$  is the bend loss coefficient.

All these coefficients:  $\lambda_s$ ,  $\mathcal{J}$  and  $\theta$  have been assumed to be functions of the Reynolds number. The most difficult coefficient to measure proved to be  $\theta$ , the tangent loss coefficient. Beij defined a resistance coefficient  $\lambda_x$  by the relation:

$$\frac{dh}{dx} = \lambda_x \frac{1}{d} \frac{v^2}{2g}$$

where the left hand term represents the head loss gradient along the downstream portion of the bend.

For a section between the points  $x = a$  and  $x = b$ , by integration of the above equation it can be obtained:

$$h_a - h_b = \bar{\lambda}_x \left( \frac{b-a}{a} \right) \frac{v}{2g}$$

where  $\bar{\lambda}_x$  denotes a mean value of  $\lambda_x$  for the section in question. Experimentally Beij investigated this value for several sections downstream of the bend and found no variation of  $\lambda_x$  with Reynolds number. From the experimental data obtained, he attempted to derive  $\lambda_x$  as a function of

that would satisfy the conditions of having a maximum value at  $x = 0$  and steadily decreasing to  $\lambda_s$ , the straight pipe loss coefficient, as  $x$  goes to infinity. He assumed such function to be of the form:

$$\bar{\lambda}_x - \lambda_s = (\lambda_0 - \lambda_s) e^{-\beta \frac{x}{d}}$$

where  $\lambda_0$  and  $\beta$  were evaluated from the data.

For bends of relative radii less than  $R/d = 8$ , the excess loss in the downstream section of the bend was found to be independent of the relative radius, thereby simplifying calculations for such a range.

With the help of an extensive body of high quality Beij determined the coefficients  $\lambda_s$  and  $\eta$  as functions of the Reynolds number for several values of  $R/d$ .

The values of the straight pipe loss coefficients were obtained consistently the same in almost all tests performed. The bend loss coefficients were found independent of the Reynolds number within the range of all tests performed.

The bend loss coefficient, when plotted against  $R/d$  resembled qualitatively the shape obtained by Davis. Beij obtained a minimum for  $\eta$  in the neighborhood of  $R/d = 5$  and a maximum somewhere in the vicinity of  $R/d = 15$ . Other investigations have found the following results: Davis and Balch (10) obtained their minimum at  $R/d = 5$  to 7 and their maximum at  $R/d = 15$ , for higher values of  $R/d$  the values of the bend loss coefficient begin to decrease. Hofmann's (11) values indicate the beginning of a more gradual rise with corresponding lower values of the bend coefficients. Brightmore's (12) curve starts to rise but suddenly drops to low values in the neighborhood of Hofmann's. For all these different curves, the bend loss coefficient starts to drop in the vicinity of  $R/d = 20$ .

Although outside the range of his tests, Beij suggested the probability of the existence of a third region in which  $\eta$  decreases, and presumably approaches zero as the radius of curvature goes to infinity. He concluded, from comparing his data to that of the investigators mentioned above, that within the range of  $R/d = 5$  to  $R/d = 15$  or 20, the bend loss coefficient is not a function of the flow, the

relative radius or the roughness only. The irregularity of results obtained within this range by different observers, and sometimes the same observer in the same flow conditions, suggests the possibility of an irregularity or instability of the flow. This, Beij argues, justifies the drawing of the plot of  $\eta$  vs  $R/d$  through the highest points.

The deflection coefficients, when plotted against  $R/d$ , showed a minimum at  $R/D = 3$  or  $4$ . Values less than that yielded sharply rising coefficients as  $R/d$  goes to zero and gradually rising ones for values greater than  $3$  or  $4$ .

The minimum value of the deflection coefficient found by Beij was zero. In other words, the pressure loss in a bend of about  $3$  or  $4$  was found to be the same as in an equal length of straight pipe. Since the energy loss in the bend must undoubtedly be greater than that of a straight pipe, it must be concluded that the pressure loss does not give the total energy loss. Hence Beij states that a complete picture of the bend losses can be obtained only by determining velocity and pressure distributions in successive cross sections of the bends.

Since Beij's analysis constitutes an outline of most of the experimental work done on pipe bends, no other detailed account of any other work will be given here. Plots of bend loss coefficients against several flow variables

can be found in almost all textbooks on the subject of fluid mechanics. Further references on the subject may be found on the bibliography. Several of the curves mentioned above are shown in Figure 2.

### III. Dynamic Analysis of Separated Flow Through A Bend

In the following analysis the simultaneous flow of water and air through a pipe bend will be considered. By separated flow it is meant that type of flow in which the liquid phase flows in the lower portion of the pipe, and a clearly defined interphase separates it from the gaseous phase, which flows in the upper portion of the pipe.

#### III. 1. Assumptions

In order to obtain workable mathematical expressions for the motion of the fluid as it goes through a bend, several simplifying assumptions will have to be made.

These are:

a) A uniform velocity for each phase will be assumed to exist through all points of the area of a cross section occupied by each phase. Since physically the velocity of the interphase must be the same for both air and water, this assumption implies the existence of an infinitesimal region where a velocity gradient exists.

The justification for this assumption is that the main feature under investigation is the rotation of the fluid due to centrifugal force and the effects of a velocity and/or pressure gradient are neglected.

b) The air-water interphase will be assumed to be a

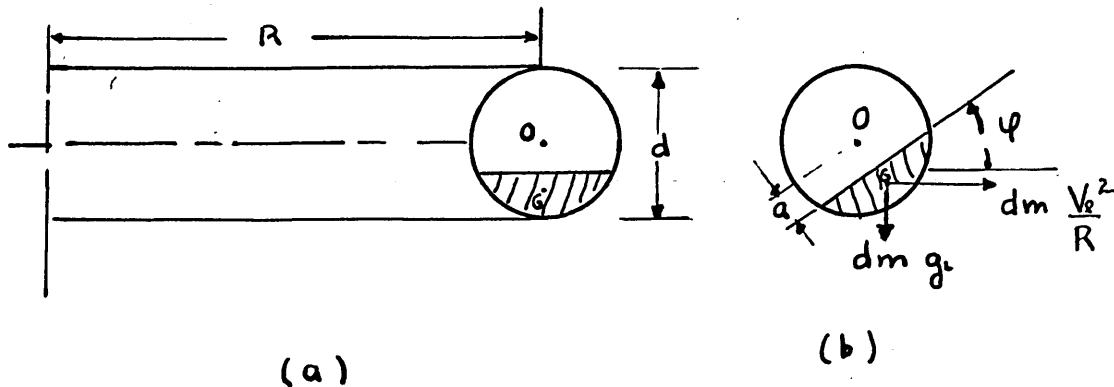
plane surface, and to remain plane during the fluid's passage through the bend.

c) The equations of motion will be applied to a section of the fluid of infinitesimal length along the direction of flow. Friction between this section and the walls and between this section and adjacent fluid will be assumed negligible.

All of these assumptions lead to a relatively simple problem of solid body rotation under the actions of gravity and centrifugal acceleration. Assumption (b) can be justified to a certain extent from experimental observation. In the experimental set up described later, visual observation indicated that the departure from the idealized condition stated is not very great, even at angular displacements of nearly 90-degrees.

### III. 2. Mathematical Analysis

Consider a cross section of the bend as shown in the sketch, and a section of fluid of area  $A$ , thickness  $Rd\theta$  and mass  $dm$  just entering the curved section of the pipe.



As the fluid enters the bend it is subjected to the action of a centrifugal force  $dm \frac{V_x^2}{R}$ , assumed to act through the center of gravity, which tilts it to the position shown in (b). Since the center of the pipe is the center of rotation, we obtain applying Newton's law to the liquid phase:

$$(1) \quad \sum M_o = I_o \frac{d^2\psi}{dt^2}$$

or, summing moments about O,

$$(2) \quad A_x \rho \frac{V_x^2}{R} a \cos\psi - A_x \rho g a \sin\psi = I_o \frac{d^2\psi}{dt^2}$$

which simplifies to:

$$(3) \quad \frac{d^2\psi}{dt^2} = \frac{V_x^2}{R K_o^2} a \cos\psi - \frac{g a}{K_o^2} \sin\psi$$

where  $K_o$  is the radius of gyration of the cross section of the liquid phase with respect to the geometric center O, of the total cross section.

Defining two constants  $C_1$  and  $C_2$  as:

$$C_1 = \frac{V_x^2}{R K_o^2} \quad \text{and} \quad C_2 = \frac{g a}{K_o^2}$$

Eq. (3) can be written as:

$$(4) \quad \frac{d^2\psi}{dt^2} = C_1 \cos\psi - C_2 \sin\psi$$

Defining two new constants,  $C_3$  and related to  $C_1$  and  $C_2$  by

$$C_3 = \sqrt{C_1^2 + C_2^2}$$

and

$$\alpha = \tan^{-1} \frac{C_1}{C_2}$$

Eq. (4) becomes:

$$(5) \quad \frac{d^2\psi}{dt^2} = -C_3 [\sin(\psi - \alpha)]$$

The solution of this differential equation is an elliptic integral of the first kind. To reduce it to standard form, we make the substitution

$$\bar{\phi} = \psi - \alpha$$

which transforms Eq. (5) into

$$(6) \quad \frac{d^2\bar{\phi}}{dt^2} = -C_3 \sin \bar{\phi}$$

Multiplying both sides by  $2\frac{d\bar{\phi}}{dt}$ , Eq.(6) gives:

$$(7) \quad \frac{d}{dt} \left( \frac{d\bar{\phi}}{dt} \right)^2 = -2 C_3 \sin \bar{\phi} \frac{d\bar{\phi}}{dt}$$

which, when integrated, gives

$$(8) \quad \left( \frac{d\bar{\phi}}{dt} \right)^2 = 2 C_3 \cos \bar{\phi} + H$$

where H is a constant of integration.

From the initial condition of  $\psi = 0$  when  $\frac{d\psi}{dt} = 0$ , we obtain a value for H of:

$$H = -2C_3 \cos \alpha ; \text{ hence eq. (8) can be written:}$$

$$(9) \quad \frac{d\Phi}{dt} = \sqrt{2C_3} \sqrt{\cos \Phi - \cos \alpha}$$

which gives us the value of the angular velocity as a function of the angular displacement at any point in the trajectory. To obtain the integral of eq. (9) the following substitution will be made:

$$\sin \frac{\Phi}{2} = \sin \frac{\alpha}{2} \sin \psi$$

which reduces Eq. (9) to:

$$(10) \quad t = \frac{1}{\sqrt{C_3}} \int_{\pi/2}^{\psi} \frac{d\psi}{\sqrt{1 - \sin^2 \frac{\alpha}{2} \sin^2 \psi}}$$

where  $\psi$ , if expressed as a function of the original variables, can be written as:

$$\psi = \sin^{-1} \left[ \frac{\sin (\psi - \alpha)/2}{\sin \alpha/2} \right]$$

Equation (10) gives a complete description of the motion under the assumed conditions. It represents an oscillatory motion about the equilibrium position, the steady state angle, and the maximum value of the angular displacement as given by this equation is twice the value of the steady state angle.

### III. 3. Approximate Solution

A linearized solution of the motion can be obtained by considering only very small angular displacements.

Letting  $\cos \varphi$  in equation (3) be equal to one and  $\sin \varphi$  equal to  $\varphi$ , we obtain:

$$(11) \quad \frac{d^2 \varphi}{dt^2} + \frac{g_L q}{K_0^2} \varphi = \frac{V_e^2 q}{R K_0^2}$$

which together with the boundary conditions  $\varphi$  and  $\frac{d\varphi}{dt}$  equal zero at  $t = 0$  yield:

$$(12) \quad \varphi = \frac{V_e^2}{R g_L} \left( 1 - \cos \sqrt{\frac{g_L q}{K_0^2}} t \right)$$

This equation, as it can be easily seen, gives  $\varphi$  a shape similar to the non-linear solution obtained before. Its maximum displacement corresponds to twice an angle of  $\frac{V_e^2}{Rg}$  radians, as compared to twice the steady state angle of equation (3). Also, for this linear case, we obtain a periodic motion whose period is independent of the value of the steady state angle, as opposed to the non-linear solution. However, for up to ten degrees of displacement around the bend, both equations yield similar values of angular displacements and velocities.

#### IV. Energy Loss Evaluation

##### IV. 1. Pressure Drop Calculation for Separated Flow

In the preceding section it has been shown how the motion of the fluid takes place under the assumed idealized conditions.

Since no heat or other forms of external energies have been transferred to the fluid, the total energy of the fluid as it rotates must remain constant if the losses associated with fluid friction are neglected.

Under these conditions, the energy relationships are expressed by the following generalized form of Bernoulli's equation:

$$(1) \quad \frac{P}{\rho} + \frac{V_1^2}{2} + \rho_1 z + \frac{\omega^2 K_0^2}{2} = \text{constant}$$

where the last term represents the Kinetic energy due to axial rotation of the fluid.

The continuity equation for steady flow is given by  $AV = \text{constant}$ . In the case of an incompressible fluid this reduces to  $AV = \text{constant}$ . It may be assumed that the area and, therefore, the average velocity in the direction of the flow remains constant.

Due to the angular velocity  $\omega$  the Kinetic energy of the fluid is increased. Furthermore, the elevation of the center of gravity of the fluid section is increased producing a corresponding increase in potential energy. Since

the velocity in the direction of flow has been assumed constant, the energy changes just described can be realized only if a corresponding decrease in the term  $\frac{P}{\rho_0}$  takes place. That is, there must be a drop in pressure across the bend if both the potential and the Kinetic energy of the fluid are to increase.

The oscillatory motion taken by the fluid would imply, in a frictionless fluid, an oscillatory variation of pressure, from a minimum, when the sum of the Kinetic energy of rotation and potential energy is a maximum, to a maximum when the center of gravity is at its lowest possible position and the angular velocity equals zero. In the case of a real fluid, transverse friction will dissipate all of this energy. Thus it can be assumed that the pressure drop would actually have to be associated with the maximum amount of Kinetic and potential energy acquired by the fluid in its motion.

To compute the value of the pressure drop, it will be assumed that had it not been for the damping effects of friction, the fluid would have attained velocities and displacements as evaluated in the preceding section and a corresponding maximum displacement of the center of gravity during the motion.

The point at which the sum of both the Kinetic energy of rotation and the potential energy become a maximum can be found by means of the following physical reasoning:

consider a centrifugal force field of magnitude  $\rho \frac{V^2}{R}$  to be set to act upon a section of fluid as soon as it comes into the bend. At this point such section has only a tangential velocity  $V$ , and its center of gravity is at the lowest possible position. At any subsequent instant, it will have an angular displacement  $\psi$ , an angular velocity  $\frac{d\psi}{dt}$ , and its center of gravity will be at a distance  $a(1-\cos\psi)$  higher than before. To obtain this position, work has been done against the above mentioned centrifugal force field and against gravity. In exchange, the fluid has now a rotational Kinetic energy of magnitude equal to the total work done.

If the initial level of energy is taken as zero, due to the conservation of energy we can write:

$$(2) \quad -\frac{V_s^2}{R} a \sin\psi + a g (1-\cos\psi) + \frac{K_e^2}{2} \left(\frac{d\psi}{dt}\right)^2 = 0$$

From this equation we can see that the maximum value of the sum of the last two terms is attained when the first one takes its maximum absolute value. In most cases this occurs when  $\psi$  is equal to  $90^\circ$  degrees. Thus the maximum energy loss will be considered to be  $\frac{V_s^2}{R} a$ , and the corresponding pressure drop:

$$(3) \quad \Delta P_R = \rho_s \frac{V_s^2}{R} a$$

where  $\Delta P_R$  denotes the drop in pressure through the bend due to rotation.

Although at the present moment this formula does not seem to have too much practical use due to the difficulties encountered in evaluating  $a$ , it will be shown later on how some of these obstacles can be overcome and " $a$ " conveniently approximated from a knowledge of the flow variables.

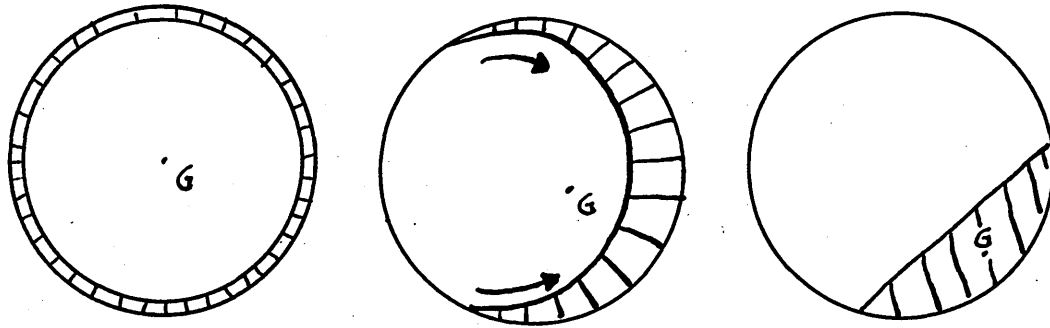
#### IV. 2. Extension to Annular Flow

Since the previous analysis has been made under the assumption of separated flow, it is expected that formula (3) would only apply in such cases. However, with a slight assumption, this restriction can be lifted to comprise also annular flow. This would indicate a much wider range of use, since a great percentage of two phase flow cases found in practice are either annular or separated.

To extend the analysis to the case of annular flow it will be imagined that as annular flow enters the bend the portion of fluid next to the interior wall will try to move outward as a consequence of the centrifugal force acting upon it.

For this motion to take place without producing discontinuities in the flow, it will be assumed that such motion will be realized in the form of a rotation over the walls until a final position, as shown in the graph, is achieved. Again, under the assumption of no friction, this

final position can be analyzed like in the case of separated flow.



a) before reaching the bend

b) once motion is started

c) final position

However, in this case there has been a decrease in potential energy, since the center of gravity of the liquid phase, which was originally assumed at the center of the pipe, is now a distance  $a \cos \varphi$  lower. Hence equation (2) should now read:

$$-\frac{V_L^2}{R} a \sin \varphi - a g_c \cos \varphi + \frac{K_e^2}{2} \left( \frac{d\varphi}{dt} \right)^2 = 0$$

In this case we must obtain a maximum for the difference between the Kinetic energy gained and the potential energy lost. Again, this maximum must occur at the maximum value of  $\varphi$ , which yields exactly the same results as in

separated flow.

#### IV. 3. A Method to Determine the Gas-Liquid Velocity Ratio

One of the basic characteristics of two-phase flow is the fact that, in general, the liquid and the gas flow at different velocities. The ratio of these velocities is a very difficult quantity to measure in practice. Although various attempts have been made to simplify its determination, success has been much less than desired. On the other hand, this flow variable bears such importance that even a rough approximation of its value would clarify many a problem. In our case, the value of this ratio will enable us to compute the ratio of the area of the gaseous phase at any cross section of the pipe, to the area of the liquid phase.

This variable, in turn, will link the geometry of the flow to the continuity requirements, and therefore relate a pressure drop for any given quality, where by quality it is meant the weight of gas flowing through the pipe in relation to the weight of the total gas-liquid mixture.

Before making any attempt to determine  $V_g/V_l$  let us define some of the variables involved.

If the "static quality" of the flow is defined as the fraction by weight of gas present within the pipe at any given instant, we can write:

$$(6) \quad X_s = \frac{\frac{A_l}{v_l}}{\frac{A_l}{v_l} + \frac{A_g}{v_g}}$$

where  $X_s$  denotes the static quality.

Substitution from the relation:

$$\omega_e = \omega(1-x) = \frac{V_e A_e}{v_e}$$

leads to:

$$(7) \quad \frac{X}{1-X} = \left( \frac{V_e}{V_g} \right) \frac{X_s}{1-X_s}$$

where  $X$  represents the quality of the flow as defined above.

Equation (7) shows a way of computing  $\frac{V_g}{V_e}$  provided both qualities are known.

Unfortunately,  $X_s$  is not a variable easily measured experimentally, and the problem of obtaining it is just as cumbersome as solving for the velocity ratio itself.

In an attempt to solve this problem, we shall apply the so-called "minimum energy" principle to the flow of gas and liquid through a pipe. This method was suggested to the author by Professor M. A. Santalo. Its justification for the case under consideration is based only on physical intuition, therefore no proof of its validity will be given here.

Experimentally it has been found to give approximately correct answers in several cases, particularly at high qualities. At low qualities the velocity ratio given by this expression appears, at present, to be too high. From an optimistic viewpoint it can be argued that discrepancies found in those cases are due not to the inexactness of the

assumption, but to the simplifications that must be made, to simplify the mathematical operations concerned with their application.

In the case of two phase flow it is reasonable to expect that the velocities and areas of the total cross section occupied by the fluids will adjust themselves in such a way as to result in minimum energy loss and hence a minimum energy to drive the fluids through the pipe will be necessary.

Defining the "total energy" of the flow as the sum of the enthalpy and velocity terms of the liquid and gaseous phases, we can write:

$$(8) \quad H_o = \omega_g h_g + \omega_x h_x + \omega_g \frac{\bar{V}_g^2}{2g_c} + \omega_x \frac{\bar{V}_x^2}{2g_c}$$

Instead of the usual symbols  $V_g$  and  $V_x$ ,  $\bar{V}_g$  and  $\bar{V}_x$  have been used to indicate that these values are the root mean square values of velocity, for in the calculation of the Kinetic energy of the flow a process of integration has to be carried out due to the existence of a velocity gradient.

Both enthalpies are fixed by pressure and temperature at any point, and since the mass rate of flow of both air and water is supposed to be known, equation (8) can be reduced to a function of  $\bar{V}_g$  and  $\bar{V}_x$ , where these variables are not independent, for if one of them is known, from the mass rate of flow, the area occupied by the phase with that

velocity can be calculated. This enables us to determine the other fluid's area and hence its velocity.

The assumption of minimum energy tells us that if we arbitrarily select a velocity, determine the other one and plot  $H_0$  by successive selections of compatible velocities, that which renders  $H_0$  stationary with respect to neighboring positions represents the true value of  $H_0$ . To do this mathematically we use the relations:

$$(9) \quad \begin{aligned} \omega_3 &= \bar{V}_3 A_3 \rho_3 \\ \omega_2 &= \bar{V}_2 A_2 \rho_2 \\ A &= A_3 + A_2 \end{aligned}$$

which are obtained from continuity and geometrical considerations. The double bar on the velocities indicates that unlike in equation (8), these velocities are mean velocities and, in general, not equal to the mean square root velocities.

We now make the assumption that they are equal, what is to say, a uniform velocity profile exists on both phases. This represents a fair approximation for low velocities only, for other cases we should expect to obtain results of at least the same order of magnitude.

Substituting the values of eq. (9) in eq. (8), we obtain:

$$(10) \quad H_0 = \omega_3 h_3 + \omega_2 h_2 + \frac{V_3^3 A_3 \rho_3}{2g_c} + \frac{V_2^3 (A - A_3) \rho_2}{2g_c}$$

Now, instead of plotting  $H_o$  to determine its minimum, we can minimize  $H_o$  by simply differentiating with respect to  $A_g$  and equating to zero. This operation yields  $H_o$  a minimum for:

$$(11) \quad \frac{V_2}{V_1} = \left( \frac{P_2}{P_3} \right)^{\frac{1}{3}}$$

This will be the value of the velocity ratio used in relation to pressure drop computations later on.

Although not obvious, the fact remains that in addition to the assumptions made before, one more has been made and not mentioned as yet. This is the fact that in defining the term "total energy" only the main contributing factors to the energy of the flow were taken into account, and terms such as surface tension were neglected. This seems perfectly reasonable, except for the fact that in differentiating  $H_o$  with respect to  $A_g$  the value of the derivatives of such terms were also considered negligible. Hoping that this is the case, we shall now proceed to the next section.

#### IV. 4.

As defined before, "a" is the distance from the center of gravity of the liquid section of the pipe to the axis of rotation, at the center of it. From purely geometrical considerations a formula for "a" can be given as:

$$(12) \quad a = \frac{2}{3} r \frac{\sin^3 \gamma}{\gamma - \sin \gamma \cos \gamma}$$

where  $r$  is the radius of the pipe and  $\gamma$  is half the angle subtended by the chord formed by the liquid interphase.

Also,  $R_g$ , or the ratio of the area occupied by the gas to the total area of the cross section can be written as:

$$(13) \quad R_g = \frac{A_g}{A} = \frac{\pi - \gamma + \frac{1}{2} \sin 2\gamma}{\pi}$$

or

$$(14) \quad 2\gamma - \sin 2\gamma - 2(\pi - R_g) = 0$$

Combining formulas (12) and (14) a tables can be built to give values of "a" as a function of  $R_g$ . In turn,  $R_g$  can be correlated with other variables in the following manner:

From the definition of quality:

$$x = \frac{w_g}{w_g + w_l} = \frac{\rho_g V_g A_g}{\rho_g V_g A_g + \rho_l V_l A_l}$$

which can also be written as:

$$(15) \quad x = \frac{R_g \left( \frac{V_g}{V_l} \right)}{R_g \left( \frac{V_g}{V_l} \right) + \frac{\rho_l}{\rho_g} (1 - R_g)}$$

Taking the value of  $\frac{V_g}{V_l}$  calculated in the preceding section we can obtain values of  $R_g$  and hence of "a" as function of the quality of the flow.

Using equation (3) a pressure drop computation has been made for velocities of 5, 10 and 15 ft/sec and bends of relative bend radii of 6, 4 and 1. The results have been

plotted against quality and  $R/d$ . (See figures 4 to 7).

The parameter used to correlate the pressure drop has been the pressure drop that would occur in flow of only liquid through the bend, with a mass rate of flow equal to that of the two-phase flow case.

Curves for these values were found experimentally as described in the next section.

## V. Experimental Work

In order to carry out experimental investigation on the pressure drop of two-phase flow in bends, Cohen (13) has constructed the apparatus shown diagrammatically in figure 12.

### V. 1. Apparatus

The apparatus consists of four sections: mixing, entry, test and exit.

In the mixing section the water and air are brought together from their sources, their mass rates of flow measured by nozzles installed in the individual pipes, and are finally mixed and introduced into the entry pipe. Mixing was accomplished in a jet-type steam ejector in which a jet of air was introduced into the center of an annular flow of water. This type of mixer was used because it facilitates obtaining a fairly steady separated flow, relatively free from disturbances.

The entry section consists of 4 1/2 feet of 1/16 " thick, 3/4 " ID clear lucite pipe attached at the mixer at one end and to the test section at the other by plexiglass flanges bonded to the pipe. The purpose of this pipe was to permit the flow to fully develop before entering the test section.

Pressure taps of 0.040" diameter were placed every 12" along the length of the straight pipe.

The test section consisted of a removable 90° bend. Bends of relative radii of 6, 4 and 1 were used to insure a wide spread of investigation.

Three feet of straight pipe fitted with two more pressure taps comprised the test section. It is at this region that disturbances of the flow may be damped out before a final reading of the pressure is taken. At the outlet of the pipe a gate valve was installed for the purpose of controlling the pressure in the entire apparatus. Extreme care was taken to keep the entire set up on a horizontal plane.

#### V. 2. Performance for Water Only

The total pressure drop through the bend was assumed to be the sum of the frictional loss that would occur in a straight pipe of the same axial length as the bend and the loss due to factors other than axial friction. These losses have been called by Cohen secondary losses and in this paper are denoted by the symbol  $(\Delta P)_w$ .

The total bend loss  $(\Delta P)_b$  was found by plotting the pressure drop from tap 1 to each of the other taps against distance along the straight pipe. A line representing  $\frac{\Delta P}{\Delta L}$  was drawn through points three and four and extrapolated to the beginning of the bend. Similarly, a line with the same slope was put through the point representing  $\Delta P_{i,c}$  and extrapolated back to the bend. The vertical distance between

these points represents total bend pressure loss. The secondary loss was found by subtracting  $\Delta P / \Delta L$  times the axial length of the bend, from the total pressure loss.

### V. 3. Performance for Two-Phase Flow

An attempt was made to verify Martinelli's (14) correlation with this set up. Investigations were carried out for qualities up to 0.1 with air pressures varying from forty to fifty psig and water pressure varying from fifty to sixty psig.

All of the data taken corresponds to either annular or separated flow, and it was correlated in the manner shown in figure 8. The curve for the bend of relative radius of G was not shown because the accuracy of the apparatus did not permit a careful measurement of the pressure drop for this case. The values used by the author for  $(\Delta P)_w$  in figure 4 were actually computed by assuming that the ratio of the total bend losses to secondary losses within this range was equal to 1-5. This result was experimentally found by Cohen.

## VI. Discussion of Results

To verify the validity of the equations obtained using the simplifying assumptions listed, observation of the angular rotation of the fluid was made.

The assumption of a plane interphase appeared valid at low velocities. Even at angular displacements close to 90 degrees it could be visually observed that the interphase was nearly plane.

Comparison of the predicted values of angular displacements with those found experimentally showed a large amount of damping after approximately 30 degrees of fluid displacement along the bend. The amount of damping seemed to vary according to some power of the angular velocity and of the steady state, that is to say: nonlinearly.

The amount the maximum angular displacement exceeded the steady state angle seemed to decrease as the steady state angle approached 90 degrees. In no case did the maximum angular displacement exceed 90 degrees.

Even though there isn't enough data to make a general statement in this respect, it can be supposed that the system had a damping below critical, and that critical damping is the limiting case as the steady state angle approaches 90 degrees. (See figure 3)

The predicted curves of pressure drop are independent of velocity, extending in this way to bends Martinelli's

results for a straight pipe.

A comparison of the predicted pressure drop curves with those found experimentally shows a close agreement as to their shape.

Quantitatively, the predicted values fall a great deal below those found experimentally, except in the case of assumption of a gas-liquid velocity ratio of unity, when the calculated values of pressure drop lie above the experimental. This suggests one of two things: (a) either the calculated qualities are not in close agreement with the actual qualities, or (b) other phenomena taking place in the bend, besides rotation, give rise to a large amount of pressure drop.

The relation between the geometry and continuity of the flow was obtained by deriving, under simplifying assumptions, the equation  $\frac{V_3}{V_2} = \left(\frac{P_2}{P_3}\right)^{1/3}$ . Hence the calculated qualities are as close to the actual qualities as the above relation is to the actual velocity ratio. It is the belief of the author, though, that besides rotation in the bend, relative motion of the fluid particles have great influence on the energy loss, because of resulting turbulence and change in velocity profile. But whatever factors, besides rotation, may influence the loss of energy in the bend, the similarity of the predicted and experimental curves seem to indicate certain proportionality between these factors and rotation of the fluid in the bend.

A plot of pressure drop against relative radius indicated a minimum loss for an  $R/d$  of 4, for all qualities under investigation. This startling fact has not been analytically proven by the author due to the extreme difficulty, if not impossibility, of obtaining a formula for pressure drop a function of  $R/d$  alone. Nevertheless, the predicted results have been corroborated in the experimental work of Cohen.

The plot (shown in figure 7), shows a decrease in the pressure drop ratio  $\frac{\Delta P_R + \Delta P_w}{\Delta P_w}$  as  $R/d$  approaches 4, then it increases again, presumably in a steady manner to an  $R/d$  of 6. From there on it is impossible to determine what will happen because bends of relative radii larger than 6 were not investigated. The assumption can be made though, that the pressure drop ratio continues to rise for a while longer until reaching a maximum, and then it decreases as  $R/d$  goes to infinity.

This result can be referred back to Davis's result, (figure 1) where the bend loss coefficient is seen to decrease from an initial maximum at  $R/d = 1$  to a minimum of  $R/d = 4$ . Although he only considers a single phase, the recurrence of the same phenomenon in the case of two-phase flow is significant.

As a final conclusion, it can be said that the accurate prediction of the relative positions of the pressure drop

curves for  $R/d$  of 1, 4 and 6, from rotational losses alone, indicates a definite influence of the latter in the total pressure drop of separated and annular two-phase flow through a bend.

APPENDIX

"Pressure Drop When Gas and Liquid Flow at  
the Same Velocity"

The gas-liquid velocity ratio calculated in Section IV shows close agreement when compared to experimental values, at high qualities. As the quality decreases, the discrepancy between the calculated and actual value becomes larger and at very low qualities only the order of magnitude is preserved.

Since all the pressure drop investigations were carried out at relatively low qualities, it is expected that the actual velocity ratio was smaller than predicted. Thus, the predicted curves fall more than what they actually should below the experimental. An idea as to what would have happened, should the velocity ratio had been smaller, can be obtained by considering the limiting case of equal velocity of both phases.

Figures **10 and 11** show the predicted pressure drop curves for bends of  $R/d$  of 4 and 1. As it can be seen, the new values fall much closer to the experimental values for very low qualities. As the quality increases the predicted values become larger than the experimental.

The only conclusion that we can derive from this result is that even though there is no reason to believe that

the actual gas-liquid velocity ratio is closer to unity than to the previously calculated value, one can assume that if its actual <sup>value</sup> had been used to compute the pressure drop, the curves would have ~~laid~~ somewhat closer to the experimental values.

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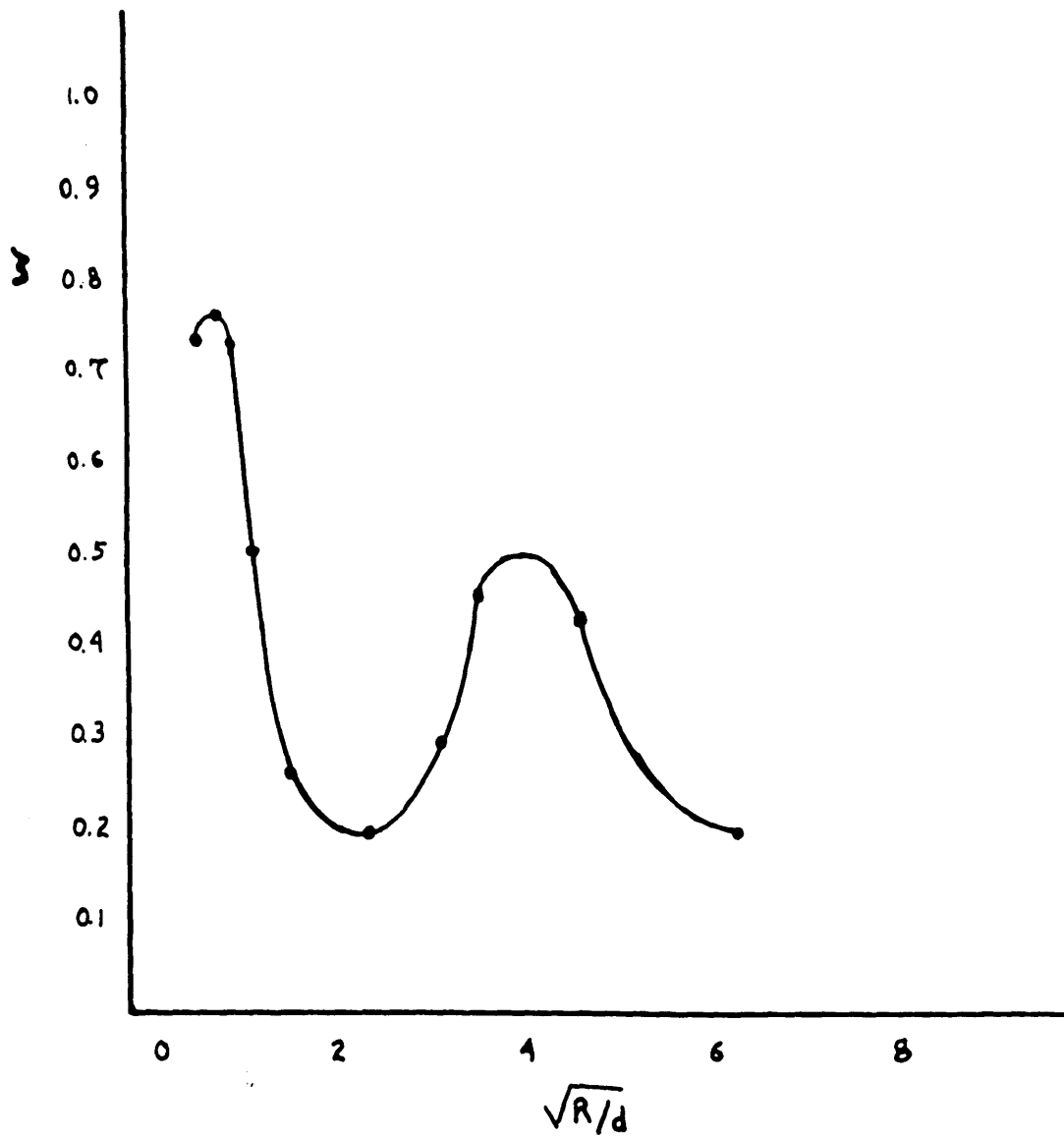


Fig 1. - Effect of relative radius on bend loss coefficients. (After Davis)

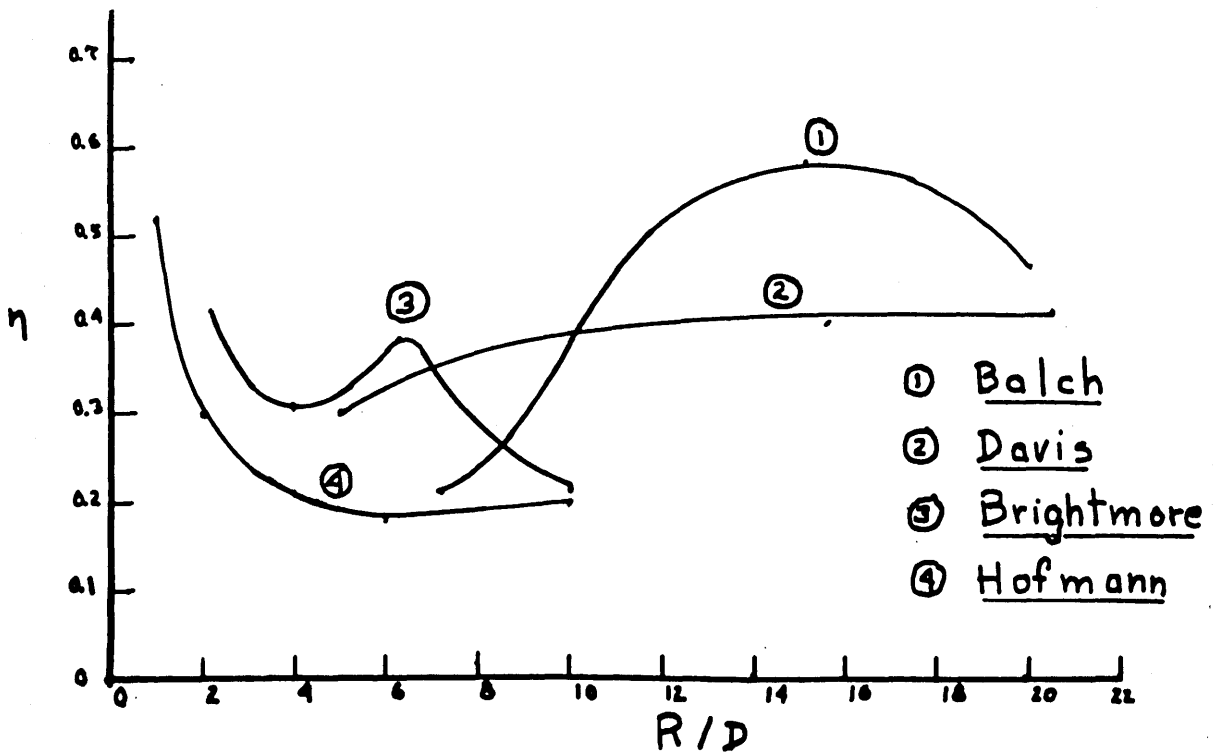
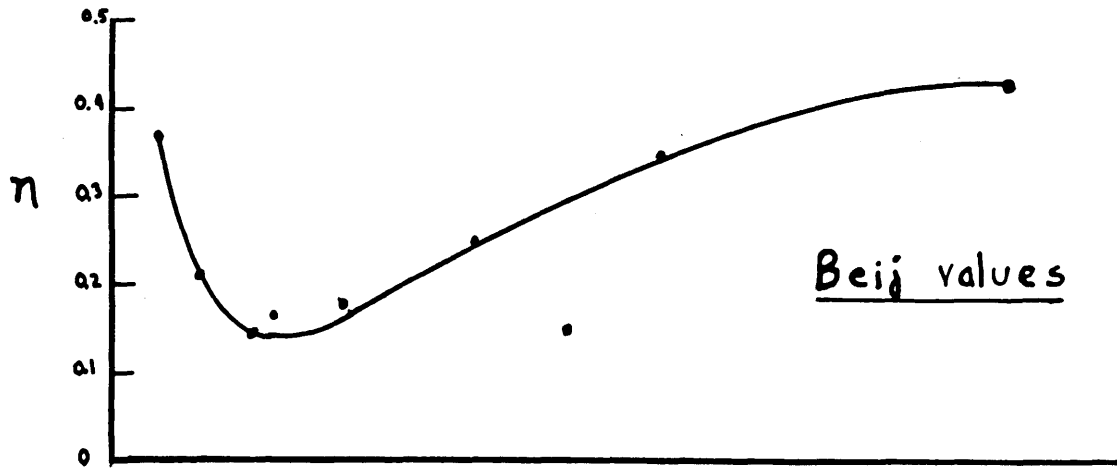
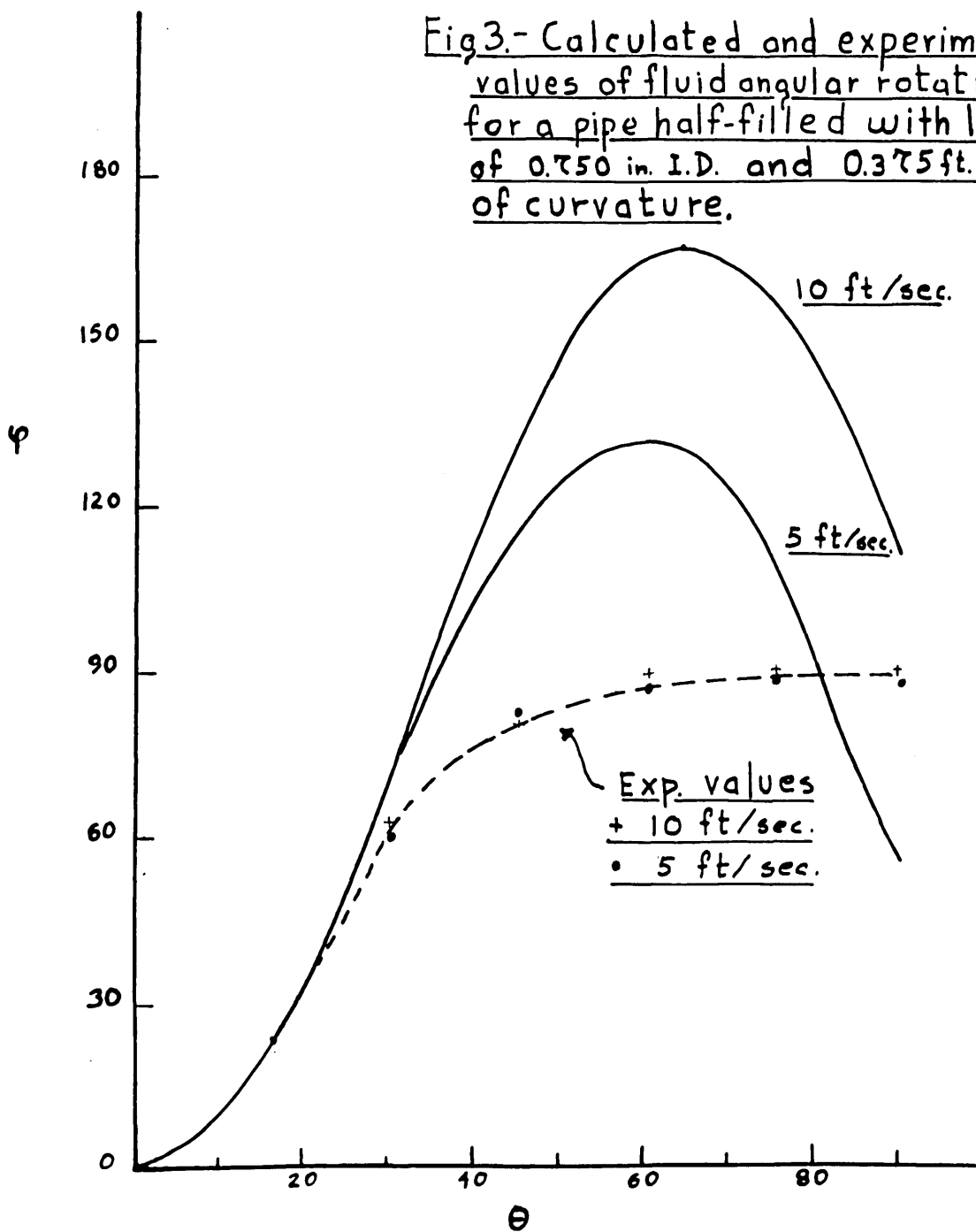


Fig. 2.- Bend coefficients found by Beij and other investigators.

Fig3.- Calculated and experimental values of fluid angular rotation for a pipe half-filled with liquid, of 0.750 in. I.D. and 0.375 ft. radius of curvature.



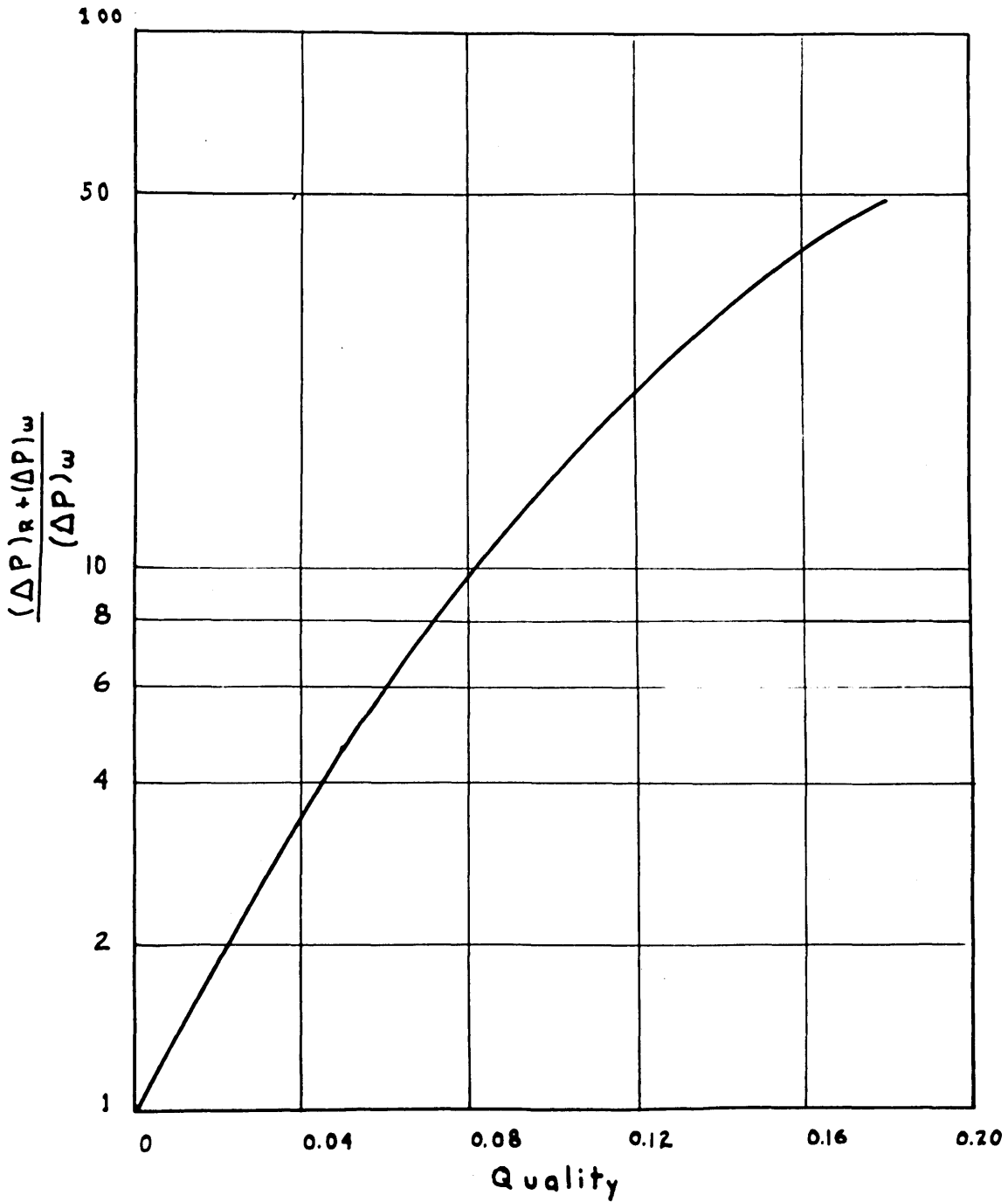


Fig 4.- Pressure Drop Correlation for a Bend of  $\frac{R}{d} = 6$

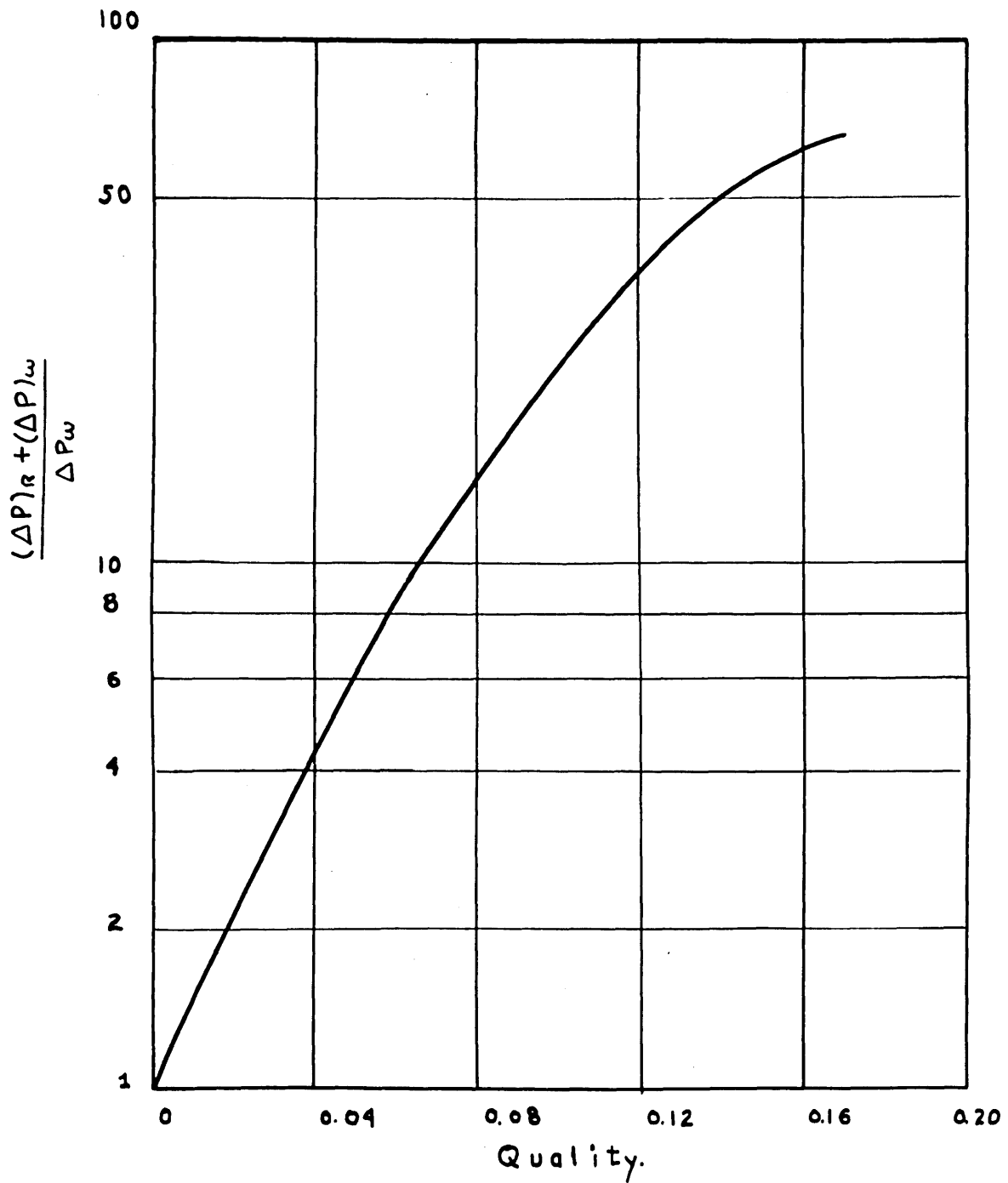


Fig 5.- Pressure Drop Correlation for a Bend of  $\frac{R}{d} = 4$

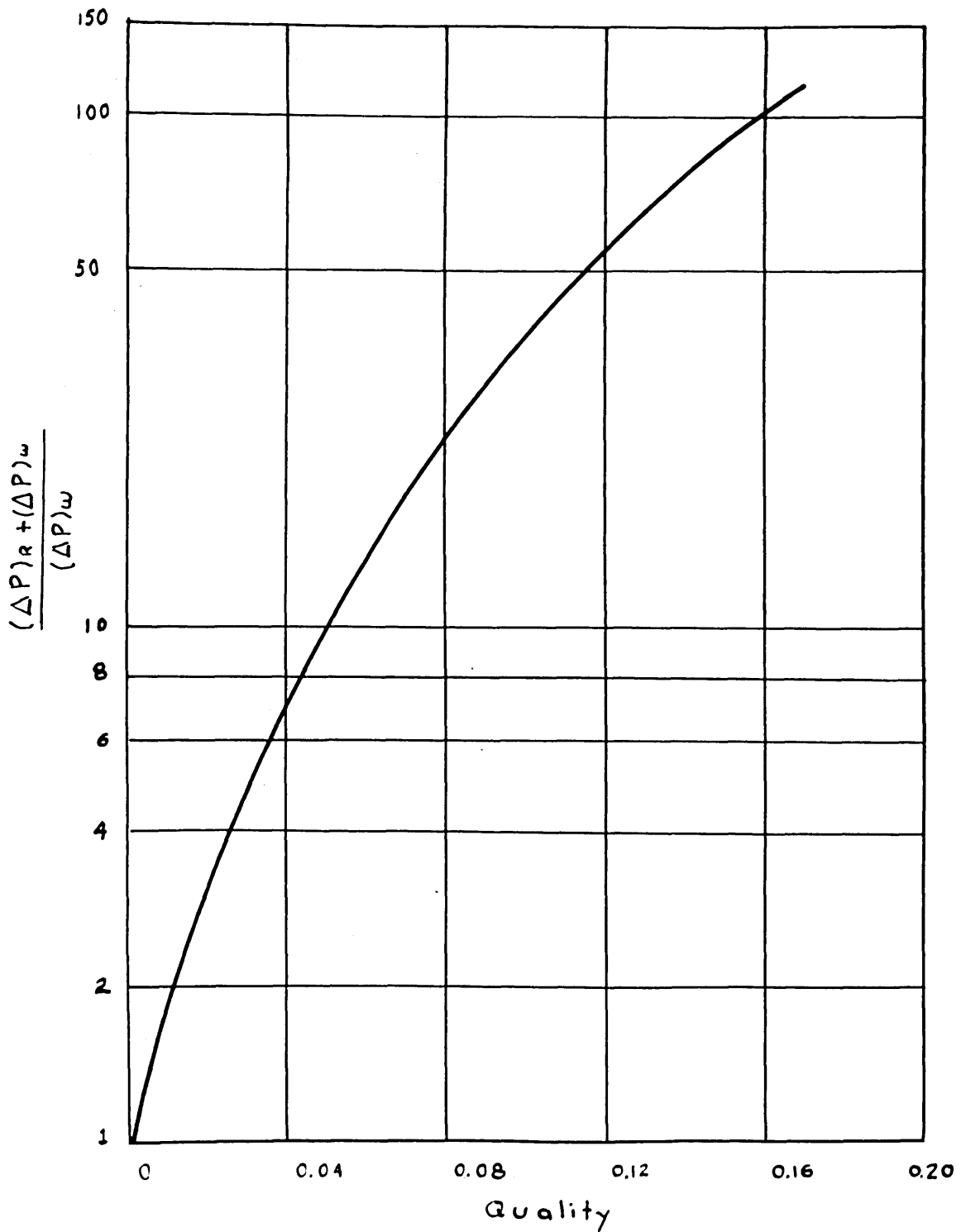


Fig 6.- Pressure Drop Correlation for a Bend of  $R/d = 1$

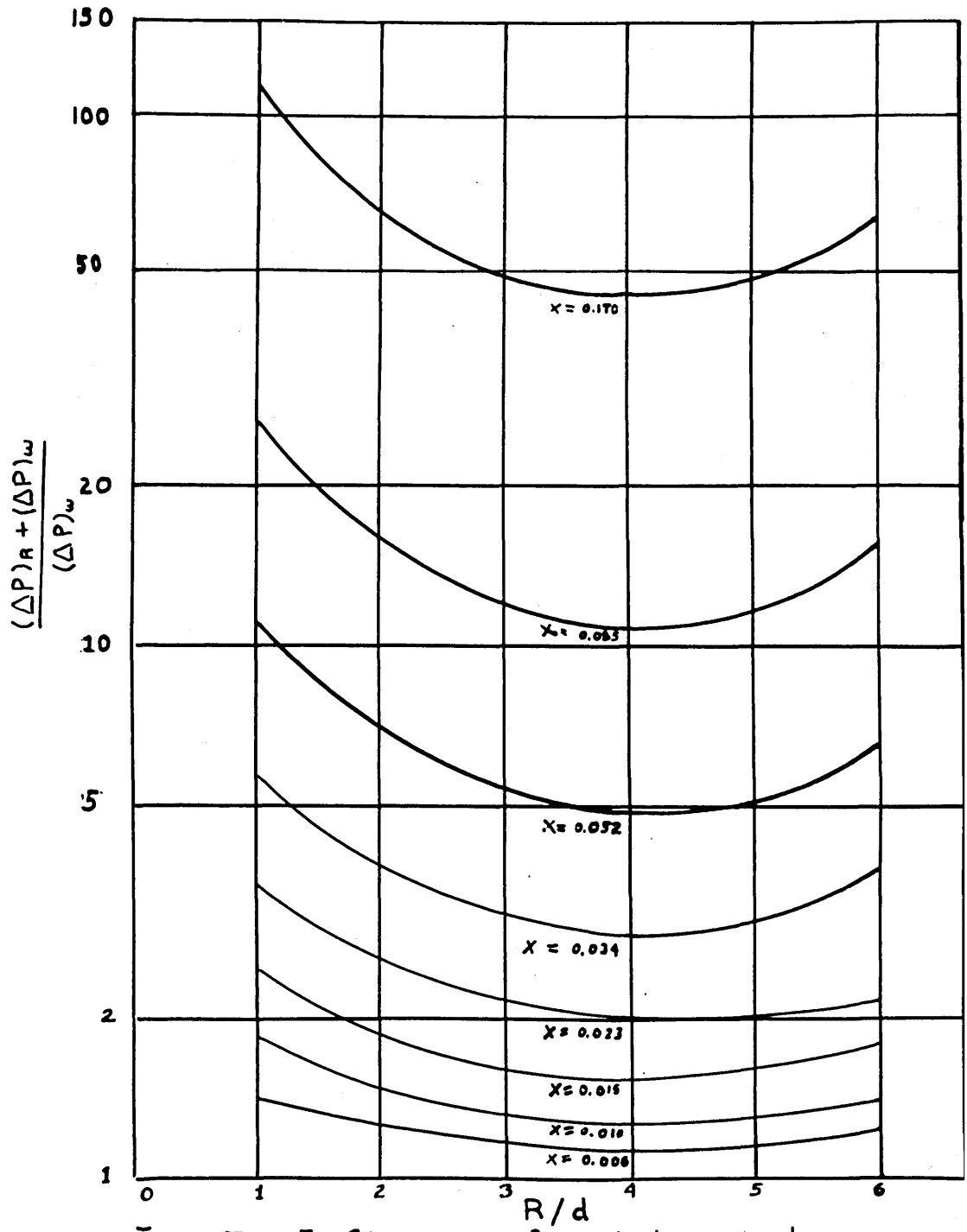


Fig.-7 . Influence of relative radius on pressure drop through a bend.



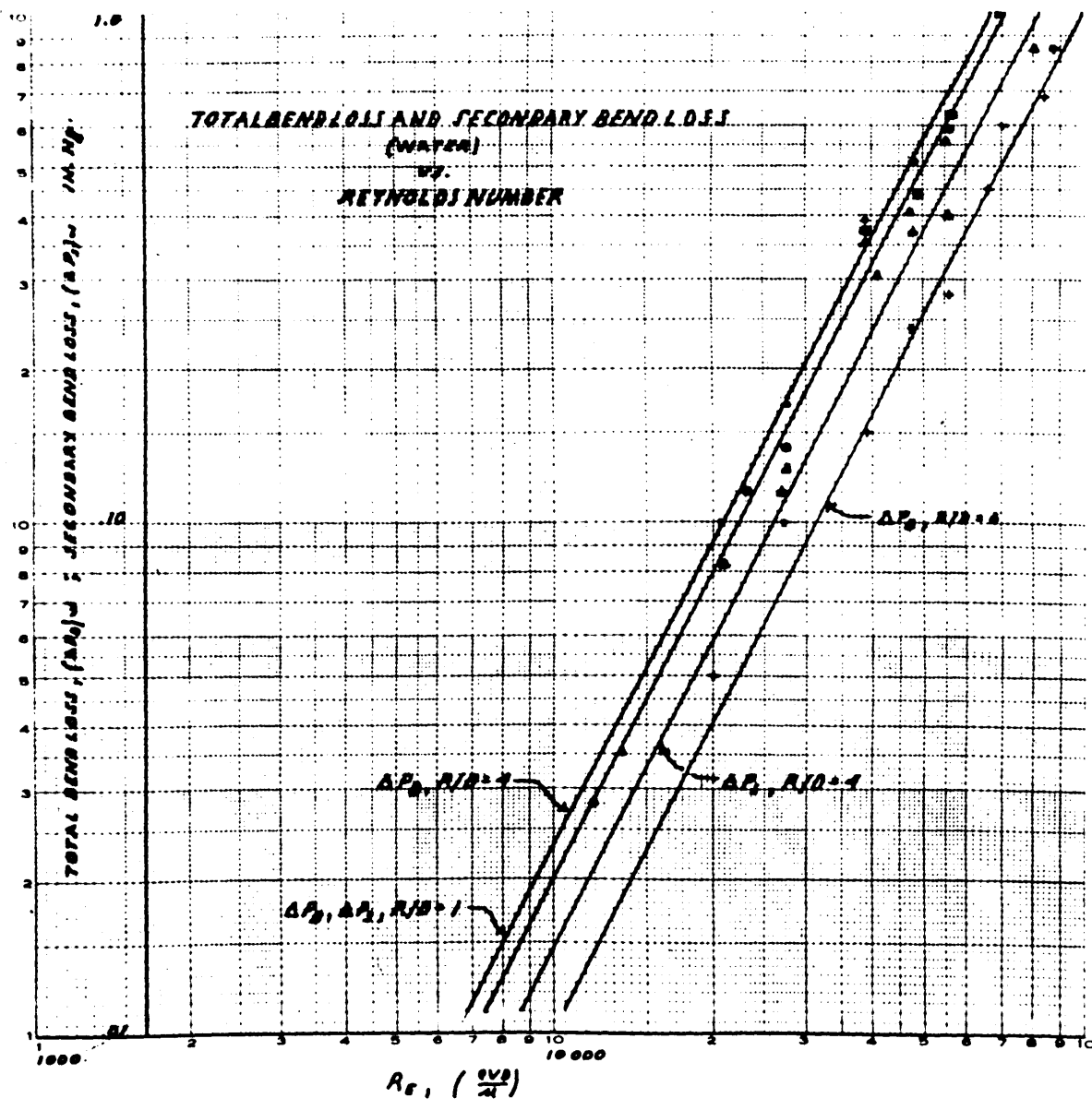


FIGURE 9

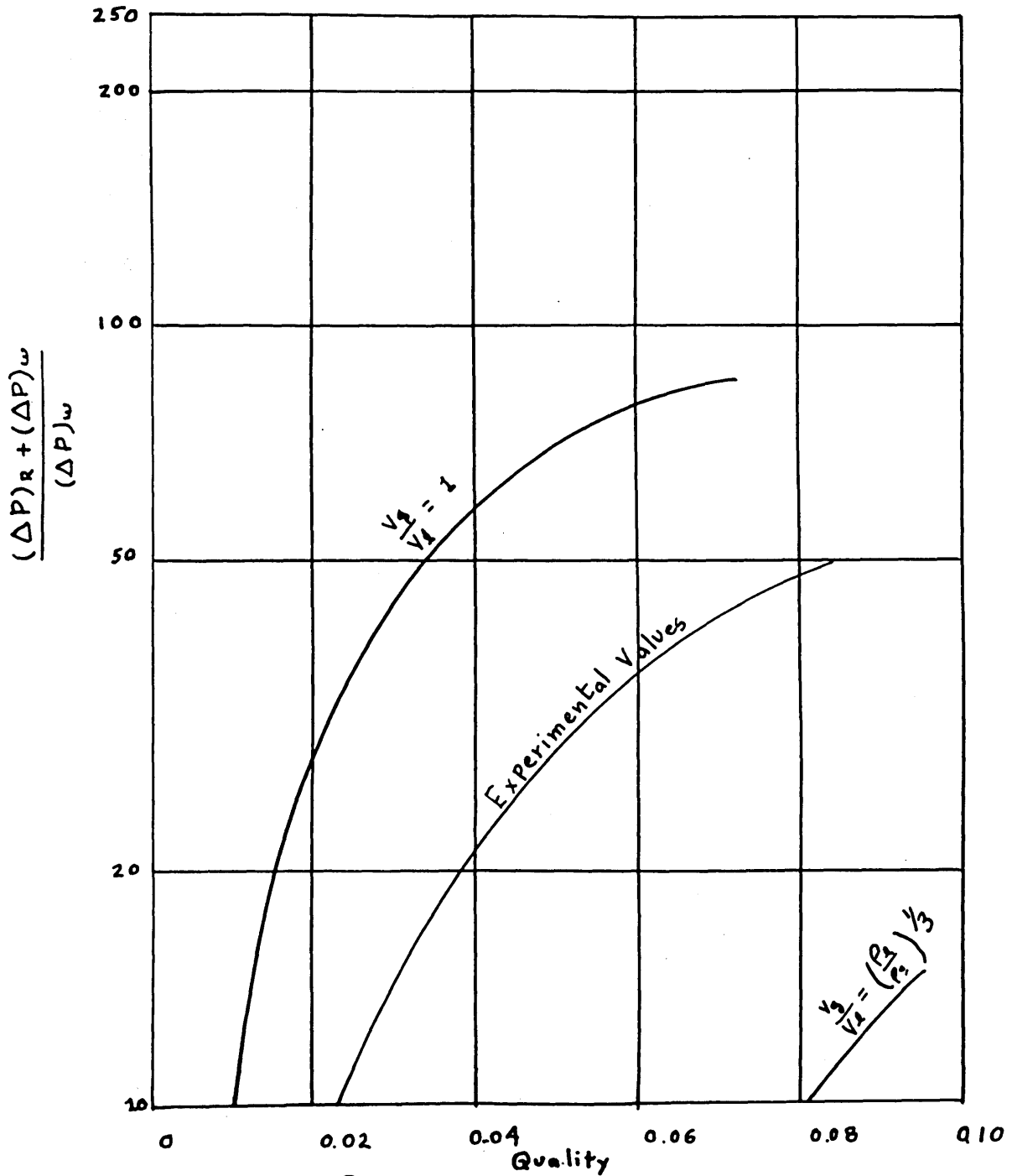


Fig.10.- Comparison of predicted and experimental values of pressure drop for a bend of  $R/d=4$

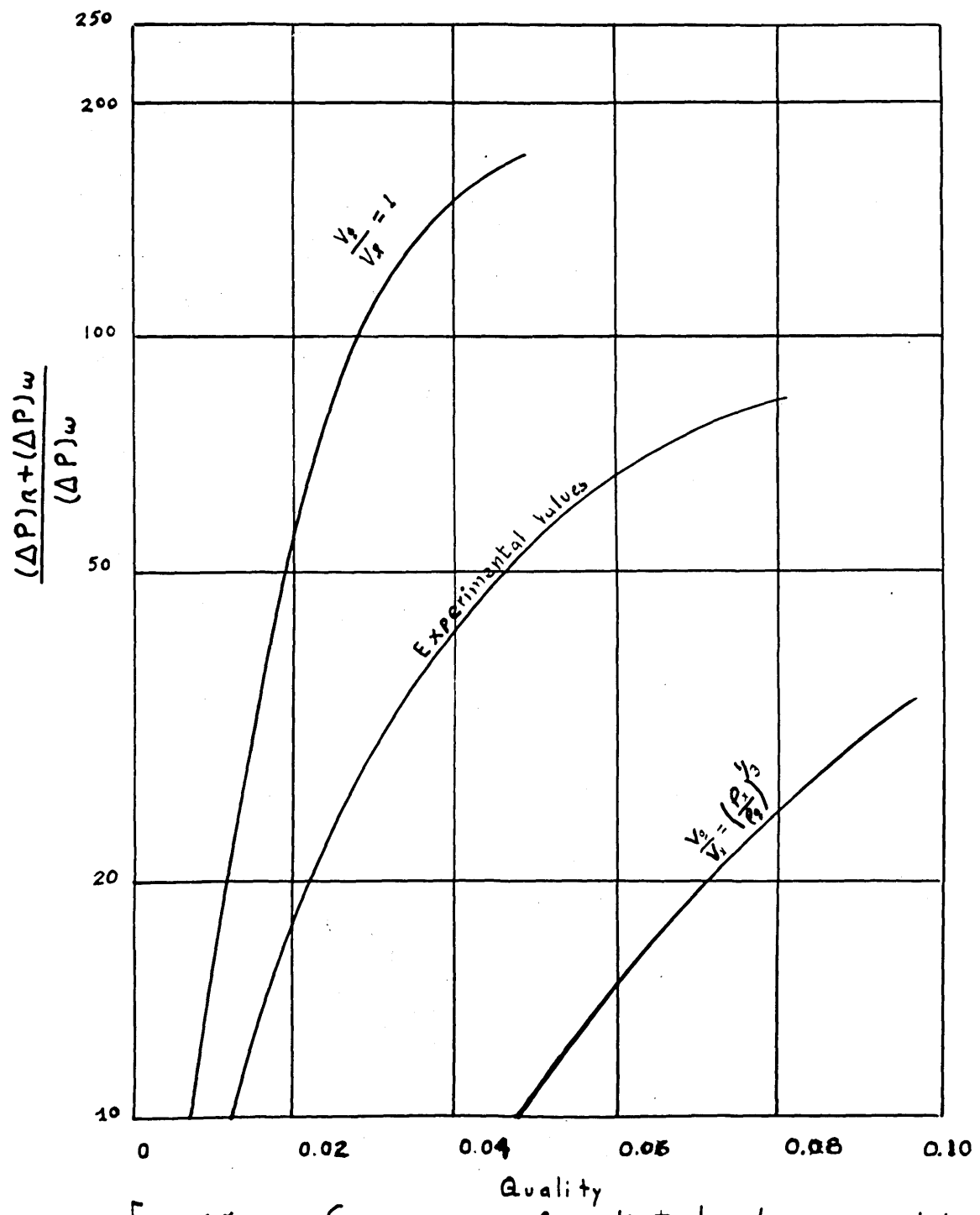
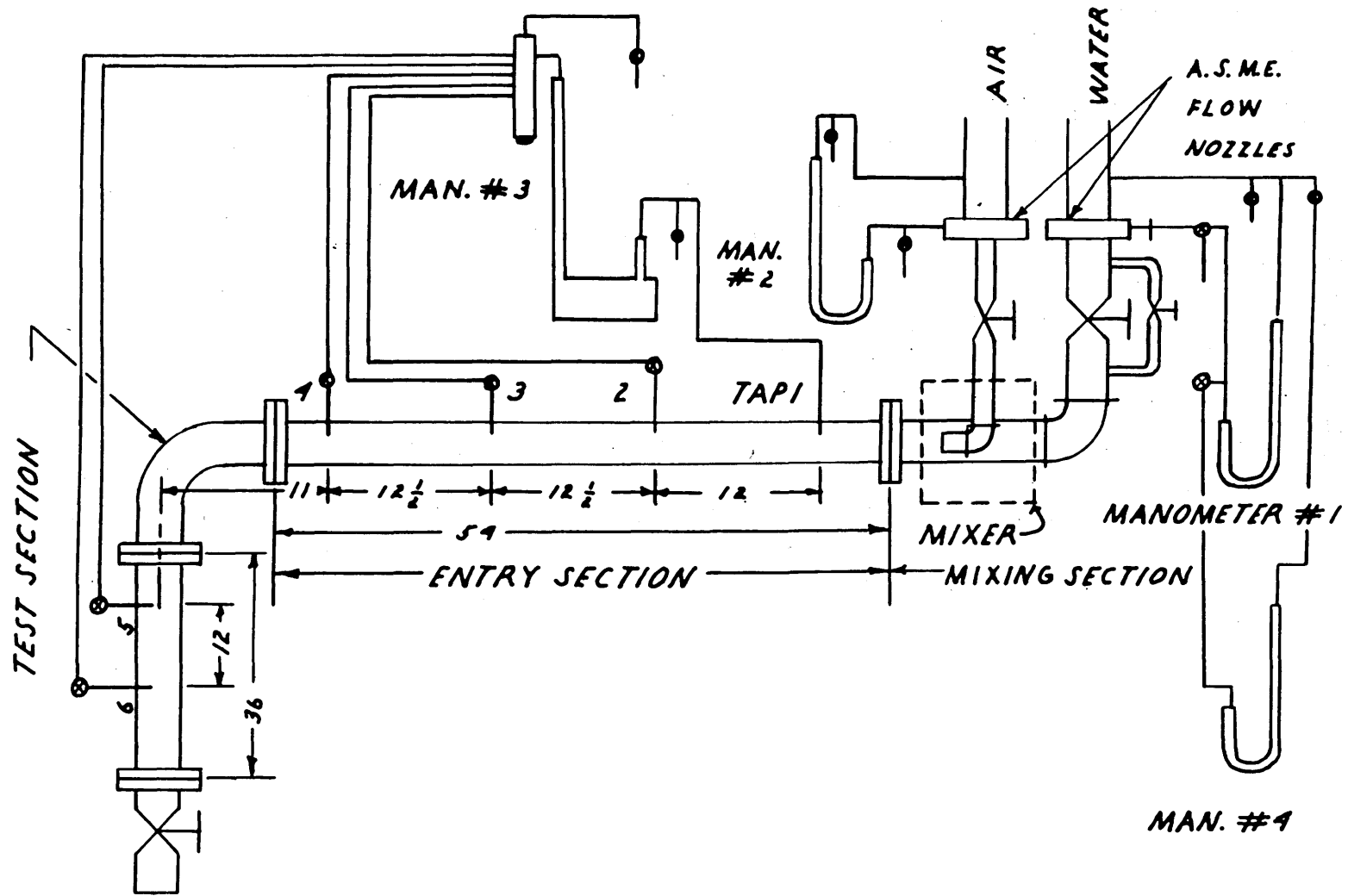


Fig. 11 .- Comparison of predicted and experimental values of pressure drop for a bend of  $R/d=1$



SCHMATIC DIAGRAM OF APPARATUS

FIGURE 12