XIX. ADVANCED COMPUTATION RESEARCH[*]

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A. NETWORK ANALYSIS BY DIGITAL COMPUTER

The amount of work required to analyze an electrical network grows rapidly as a function of the number of network nodes, N. The digital computer is a tool that can reduce this growth rate if the analysis is done in a proper manner. A study of analysis procedures that are applicable to digital-computer automation can be divided into two parts: topological and matrix. Such a study shows that the computation time required by topological methods grows as N!, while that of matrix methods can be made to grow more slowly. The use of matrix methods requires a set of independent Kirchhoff voltage or current equations. Further evaluation shows that node-to-datum voltage equations require a minimal amount of equation setup time. These equations are linear in the node-to-datum voltage variables but have polynomial coefficients of the form:

$$Y_{ij} = C_{ij}D^{+1} + G_{ij}D^{+0} + K_{ij}D^{-1},$$

where the C's, G's, and K's are either numerical or symbolic literals, and the D's are either time-domain operators or frequency functions. Because of this generality of coefficient and because most practical electrical networks have few interconnected node pairs, Cramer's Rule was chosen as the procedure to solve these equations. Cramer's Rule requires the expansion of two determinants.

The work for expanding a determinant by the straightforward application of the classical Laplace method grows factorially. The topology of the determinant expansion reveals a treelike structure, referred to as the determinant tree.

$$\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$= a(ei-fh) - b(di-fg) + c(dh-eg)$$

The Determinant
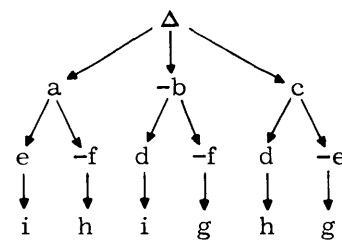
The Determinant Tree

If the tree is pruned from bottom to top, the order of the simplification is the same as that of evaluating the determinantal equation, starting on the inside of the parentheses and working outward. Thus, the determinant tree indicates the minimum expansion

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work procedure. The structure of the tree can be pruned as the expansion progresses, and the memory space required to store the tree grows only as N^3. The work for expanding the determinant of a general network by the tree algorithm grows as $(N-1)!$. However, the construction rules of the determinant tree allow the computer to greatly reduce the expansion work for networks having few interconnected node pairs. For example, experimental results show that the work required for analyzing a ladder network grows geometrically at approximately 1.5 per node instead of factorially.

A program has been written in the MAD language and tested on sample networks. The input is a list of network elements and their terminal nodes, the desired unknown voltage, and the specified datum node. The present state of the loading routine allows for linear, passive nonmutual elements and for independent, current-controlled, and voltage-controlled current sources. Also, one symbolic circuit element may be included to allow the study of the circuit behavior as a function of this element. The output of the program is the desired unknown voltage as a ratio of polynomials. By using a unity current input, the input admittance or any transfer admittance may be found. The loading routine can easily be modified to handle any other linear circuit elements and more symbolic elements. Also, the expansion subroutine can be generalized for use with determinants with general polynomial entries.

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