

XIV. ELECTRODYNAMICS OF MOVING MEDIA*

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RESEARCH OBJECTIVES

We are developing the fundamental equations of electrodynamics (for both electromagnetism and dynamics) of moving and deforming polarizable and magnetizable material. Our ultimate goal is the determination of the macroscopic force acting on a continuum, and the distribution of that force as a function of space and time. The fundamental equations must be consistent with (a) the special theory of relativity; (b) electromagnetism of free space; (c) electromagnetism of stationary bodies; (d) relativistic continuum mechanics of solids or fluids without electromagnetic fields; (e) thermodynamic measurements of the various kinds of power flow; and (f) nonlinear material properties.

Thus far, we have developed the relativistic thermodynamic treatment of irreversible processes in nonlinear, nondispersive electromagnetic materials in the presence of heat conduction and ohmic losses. The nonrelativistic treatment of space dispersive media (e.g., media supporting spin waves) has been completed. The generalization to include relativistic effects occurring in such media is now under investigation. The experience gained is proving useful in the treatment of nonlinear media under the influence of intense electromagnetic radiation. Further investigation of such phenomena is planned.

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A. HIGHER ORDER TRAPPED LIGHT-BEAM SOLUTIONS

In their classic paper Chiao, Garmire, and Townes¹ have shown that a light beam of finite radial extent can propagate in a nonlinear medium and preserve its cross section. The diffraction is counteracted by the increase in the dielectric constant produced by the high-intensity field. They showed one beam profile that they had obtained from a computer solution of the normalized equation for the normalized electric field E^* :

$$\frac{d^2}{dr^{*2}} E^*(r^*) + \frac{1}{r^*} \frac{dE^*(r^*)}{dr^*} - E^*(r^*) + E^{*3}(r^*) = 0. \quad (1)$$

The transverse dimensions of the trapped-beam solution are arbitrary because the normalization parameter $\Gamma(r^* = \Gamma r$, where r is the radial coordinate) expressing the deviation of the longitudinal propagation constant from the propagation constant of an infinite parallel plane wave in the linear medium may be chosen at the outset. The solution is cylindrically symmetric. The profile of this solution with a single extremum is shown in Chiao's paper.² The power P carried by this solution is uniquely fixed even

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though the diameter of the beam is not. It has been shown³ that

$$P = 5.763 \lambda^2 c n_{\text{eff}} \frac{1}{16\pi^3 n_2 n_0} \tag{2}$$

Here, for Chiao's solution $\int (E^*(r^*))^2 r dr = \frac{5.763}{\pi}$. It is of interest to ask whether (1) allows for solutions with a single extremum which would carry an amount of power different from the solution derived and quoted by Chiao.¹ In this report we give the answer to this question, as well as other computer solutions of (1).

Equation 1 can be considered an analog of a particle moving in a nonparabolic well. The general shape of the well is shown in Fig. XIV-1. One identifies the normalized

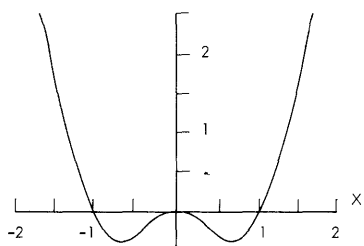


Fig. XIV-1. The potential well.

electric field E^* with the position X of the particle and the normalized radius r^* with time t . Then (1) stands for the equation of motion of a particle in the well of Fig. XIV-1 under the influence of a viscous force that depends upon time as $1/t$. Initially, the particle is launched from some position X_0 (see Fig. XIV-1). The particle experiences decelerations, because of the viscous forces, and, therefore, in order to reach the origin, it has to be launched from a height higher than that of the origin. A solution yielding bounded electromag-

netic fields (that is, a position X that vanishes as t goes to infinity) is that solution for which the launching height of the particle is adjusted just so that the particle reaches the origin and stays there for all time. The solution quoted by Chiao et al.¹ corresponds to a launching position of 2.21. For $X_0 < 2.21$, the particle does not reach the origin. The question then arises whether for $X_0 > 2.21$, the greater potential energy is offset by the greater viscous dissipation so that the particle can still reach the origin with zero velocity upon first passage. A computer evaluation of the velocity of first passage shows that it is a monotonically increasing function of the launching height. This finding excludes the possibility of solutions with one single extremum different from theirs. As one might expect, one finds other solutions in which the particle passes the origin once, twice, and so on, and upon the final return to the origin reaches it with zero velocity. Translated into the electromagnetic field problem, this finding indicates that there are solutions in which the electromagnetic power and energy is concentrated in one or more rings around the center beam. Computer solutions thus obtained, up to four rings are shown in Fig. XIV-2. The power flow in these higher order solutions expressed by $\int (E^*(r^*))^2 r^* dr^*$ is shown in tabular form below.

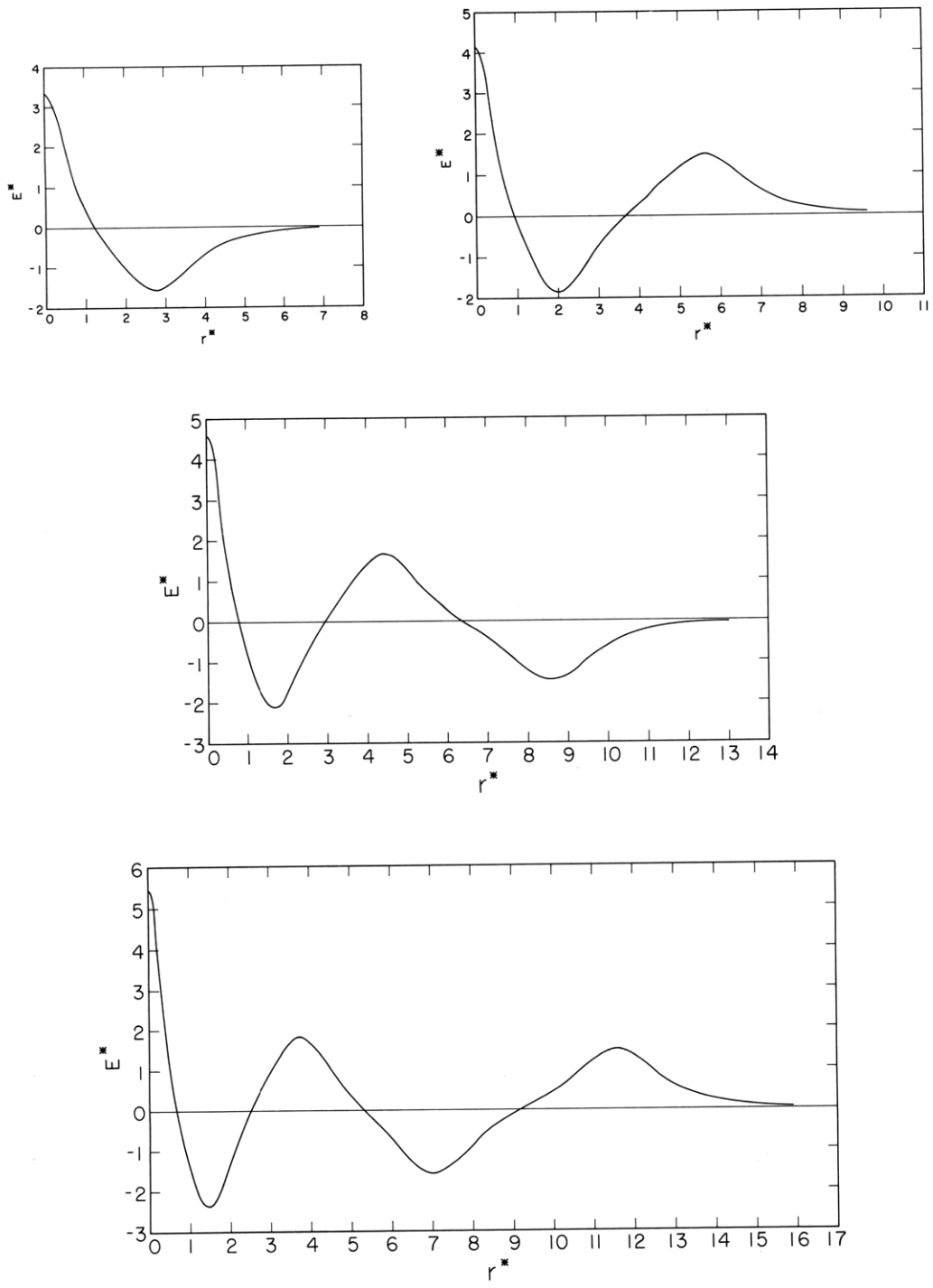


Fig. XIV-2. Computer solutions.

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$\int E^2 r^* dr^*$	Mode Number
1.86	1*
12.25	2
31.26	3
58.57	4
94.23	5

*This number differs approximately 1.4% from the number quoted by Chiao, Garmire, and Townes.¹

The author gratefully acknowledges the help of Miss Martha Pennell, who carried out the computations. The use of the Project MAC computational facility is also gratefully acknowledged.

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References

1. R. Y. Chiao, E. Garmire, C. H. Townes, Phys. Rev. Letters 13, 479 (1964).
2. Ibid., see Figure 1.
3. We have corrected the formula by a factor 1/2 which was missing in their work.