

ELECTROHYDRODYNAMIC MODELS OF

WARM-CLOUD ELECTRIFICATION

by

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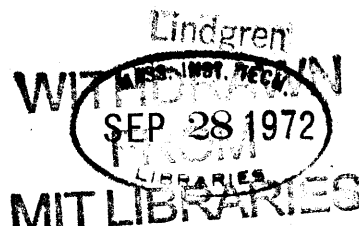
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ABSTRACT

ELECTROHYDRODYNAMIC MODELS OF WARM-CLOUD

ELECTRIFICATION

Robert F. D. Perret

Submitted to the Department of Meteorology on May , 1972, in partial fulfillment of the requirement for the degree of Doctor of Philosophy.

Electrification of non-precipitating warm clouds by convective transport of atmospheric space charge was studied by a one-dimensional steady state model. Three components of the system were analyzed; a sub-cloud region of steady vertical convection, a region of a monodisperse cloud of liquid droplets coupled to the sub-cloud layer, and the cloud environment parameterized as an electrical load. The model was restricted to an initial value problem in which initial fields are small to determine the conditions early in cloud development which might lead to the initiation of field-dependent charging currents in the cloud.

Three characteristic modes of electric structure in the cloud were predicted. Two modes were asymptotic modes in which electric fields are relatively small, but still large enough to initiate the field-dependent induction charging process. The third characteristic mode of cloud structure is not an equilibrium mode, but one which can lead to a large electric field in the cloud. The conductivity in the model cloud was found to be variable above cloud base because of changes in the ion balance between generation, recombination and droplet scavenging due to switching of the droplet charging modes in response to the changing electric field.

The operating point of the system was found to be sensitive to the sub-cloud layer depth and to the area of the load relative to the area of convection. For the same initial space charge, the shallower the sub-cloud layer, the more likely the system will be in a generating mode in which mechanical energy is converted to electrical energy. For a much deeper sub-cloud layer, the system is apparently in a brake or pump mode in which either both electrical and mechanical energy are dissipated (brake) or electrical energy is converted to mechanical energy (pump). It is not thought that this latter case is possible in the absence of other generating mechanisms. For the cases studied, it was found that the system could operate as a generator only if the area of the load was less than the convection area, suggesting that the electric perturbation of the cloud influences only a very localized region of its environment.

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CHAPTER I. A SYSTEMATIC APPROACH TO CLOUD ELECTRIFICATION

1.1 Introduction

The history of thunderstorm electrification studies is long and varied. Numerous observations and innovative experiments have delineated many of the probable mechanisms of the source of thunderstorm electricity. Yet, a survey of the previous work shows that, even so, there is no accepted mechanism principally responsible for the intense electrical activity observed. The conflicting theories and evidence are not only a tribute to the complexity of the problem as a whole, but also a remark on the methods employed in its solution.

1.1.1 Observations of Thunderstorm Electrification

Franklin [1752] made the first basic observation that the majority of thunderstorms have a net negative charge in the lower cloud region. Wilson [1921], on the basis of field measurements at the ground, enlarged on Franklin's observations and found that a net positive region persists above Franklin's lower net charge. Simpson and Serace [1937] and Simpson and Robinson [1941] made vertical soundings through thunderstorms and found that, in addition to the positive and negative regions of Wilson's dipole model, in many cases an additional region of positive charge could be detected below the lower negative region.

The relatively orderly picture of the macroscopic electrical structure of thunderstorms obtained by ground measurements was found to be nearly undetectable by observations made within the cloud proper. Inside the cloud, frequent and large positive and negative fields were observed by Gunn [1949, 1950], even though the position of the instrumented aircraft had changed little relative to the assumed macroscopic

structure determined from ground measurements. In fact, the existence of shielding charge in the cloud periphery, inferred from the rapid increase of electric field immediately upon penetration of the cloud [Gunn, 1957], casts a great deal of doubt on external field observations.

The observations of drop and droplet charge in clouds also has done little for the simple structure outlined from ground field measurements. Although highly charged hydrometeors are detected in electrically active storms, the data of Gunn [1947, 1949, 1950] show that, in most instances, it is rare that both signs of charge on the same size particles are not detected. In regions for which a unipolar drop charge was detected, field observations ensured that the opposite polarity was present in nearly the same amount, though residing on carriers not observed by the experiments [Gunn, 1950].

The implication of Gunn's in situ observations, and the orderly picture obtained from ground measurements, is that — while the modified dipole structure is probably qualitatively accurate — it is a consequence of an average structure obtained from a highly disordered background.

1.1.2 Proposed Electrification Mechanisms

The macroscopic structure has provided the guide for the proposed microscopic processes of charge generation in thunderstorms. Since the generation of a positive charge is always accompanied by a negative charge, the accumulation of charge within a region is accomplished only by a net flux of charge across the enclosing boundary. In terms of the average thunderstorm structure, this implies that, on the average, net positive charge must be transported into the upper cloud

regions and net negative charge to the lower. In nearly all mechanisms of charge separation, it is assumed that the transport agencies are small cloud droplets carried aloft by the convection and precipitation elements falling relative to cloud base. Thus, acceptable mechanisms under this restriction are those which cause a net positive charge to reside on small particles, and negative charge on the precipitation. The transport of charge by the cloud circulation [Grenet, 1947; Vonnegut, 1954] appears to be the single exception which purports to bring about the observed macroscopic structure without invoking gravitational separation of oppositely charged particles.

The possible mechanisms of thunderstorm electrification can be grouped according to several basic processes. Ion capture by cloud particles [Wilson, 1929, 1956; Gunn, 1954, 1955, 1956] and transport of atmospheric space charge by convection [Grenet, 1947; Vonnegut, 1954; Wilson, 1956; Phillips, 1967] are based on essentially the same motion that redistribution of atmospheric ions can lead to the observed electric fields. Influence mechanisms [Elster and Geitel, 1913; Muller-Hildebrand, 1954; Sartor, 1967] suggest that collisional charge exchange occurs between particles of different size in a polarizing electric field, and that the polarity of the charged drops is of the correct sign to enhance the polarizing field.

A third class of mechanisms involves the presence of the ice phase. While much of the experimental evidence indicates that the ice mechanisms are very important [Vonnegut, 1963; Wormell, 1953], observation of electrification in warm clouds with tops everywhere below the freezing level [Foster, 1950; Pietrowski, 1960; Moore et al, 1960; Sartor, 1964;

Rossby, 1966] suggests that the presence of the ice phase may not be necessary for the development of macroscopic fields. That most thunderstorms are observed to be glaciated may, in fact, be a consequence of the vigor of the convection necessary to give rise to active electrification. On the other hand, the laboratory evidence is compelling that when the ice phase is present, it can exert a considerable influence on the quantities of charge generated.

Each of the above mechanisms is subject to severe criticism. The observations of warm-cloud electrification effectively end the claim of universality by the ice theories. The influence theories rest heavily on the notion of the interacting particles colliding and rebounding. However, the separation probability varies inversely with the electric field, going from about 0.7 at no field to 0.05 at a field of 3300 V/m [Sartor, 1967], above which no further data are available. This field is considerably less than the 10^5 V/m commonly observed in thunderstorms [Gunn, 1955].

Mason (1953] and Wormell [1953] consider Wilson's ion capture mechanism inadequate on the argument that the maximum rate at which the separation process can proceed is dictated by the rate of ionization by cosmic rays, about $6 \text{ c-km}^{-3} \text{ -hr}^{-1}$. Taking account of the inefficient collection of ions, Mason calculated that the maximum charging rate by Wilson's mechanism would be about $.02 \text{ c-km}^{-3} \text{ -hr}^{-1}$. Comparing this with his estimate of the required separation rate for thunderstorms, about $50 \text{ c-km}^{-3} \text{ -hr}^{-1}$, Mason concluded that Wilson's mechanism was not important. This argument neglects the background ion density as a source of

ions for the drops, and ignores the consequences of ion transport by convection from outside the cloud.

Assuming the equilibrium ion density to be about 10^{-10} c-m⁻³, the convective contribution of space charge density to a region of height H is given by $10^{-10} U/H$ c-m⁻³ -sec⁻¹. If all of the charge is scavenged by droplets, then for a region 1 km high, and $U = 10$ m/sec, the convective contribution of space charge to cloud charge is about 3.6 c-km⁻³ -hr⁻¹. Allowing for Mason's ion-scavenging efficiency estimate, even two orders of magnitude increase in ionic number density is insufficient to allow Wilson's mechanism to be important in the charging currents of mature thunderstorms.

1.1.3 Statement of the Problem

On the basis of the above remarks, it is not at all clear which of the proposed generating processes should be considered the dominant one in developing the intense electric fields observed. It is probable that all are active to some extent in various regions of the cloud, and at different stages of the storm development. Without doubt, a great deal more data are urgently needed to further elucidate the important regions and stages of charge generation. In addition, more detailed study of the transport processes is necessary, since the charge generation itself is insufficient to develop macroscopic fields.

One of the major difficulties in designing experiments in thunderstorm electrification is the bewildering array of possibly important phenomena to be studied. Best results are often obtained from experiments the scope of which is sufficiently limited that the important parameters can be separated from the background and carefully observed.

This procedure does not seem to have been followed in most of the in situ observations in thunderstorm electrification. In fact, since it is not clear just exactly what important parameters should be measured for different situations, it is not surprising that the state of the data seems chaotic and has little general interpretation in terms of the proposed models of electrification.

An orderly approach toward the definition of an experimental program whose results can be interpreted in the light of a particular mechanism thus requires prior knowledge of the particularly important observations to be made. One approach, therefore, is to develop a model of a particular mechanism with the goal of determining the important parameters to be measured.

As a first step in such an approach, the purpose of the present work is to develop a mathematical model of the electrification of all-water clouds characteristic of the early stages of cumulus development. The model is designed to emphasize the influence of the convective electrification hypothesis in the sense that charging currents in the cloud are due to the convective transport of atmospheric space charge from the sub-cloud layer and bulk ionization by cosmic radiation. Although cloud particles are allowed to interact with the ionic charge via Wilson's impact charging mechanism, a collisionless cloud model is assumed, so that atmospheric space charge remains the sole source of cloud charge.

The justifications for the selection of the convective mechanism as the current source is based on Vonnegut's observations [1962a, 1962b] that small fair-weather cumulus clouds developing over regions of modified space charge develop electrical characteristics directly correlated

with the polarity of the emitted space charge at ground level. In fact, the polarity of the net charge in a particular cloud could be reversed by changing the polarity of the corona current at the ground. Restriction of the cloud models to the early stages of cumulus development has its basis in the desire to simplify the model, as well as in the notion that results of this work can be more accurately and safely evaluated by observations in these clouds rather than mature thunderstorms. Observations in small fair-weather clouds and others in their early growth stages have also the advantage that the kinds of charge generation possible are severely limited. Thus, interpretation of experimental results is simplified.

The development of the model differs appreciably from most of the past work in cloud electrification in several respects. First, charged particle momentum equations are included in a self-consistent fashion; that is, in addition to the particles giving rise to the field by their motion, the field also influences this motion. So conduction currents, modeled in the past by an assumed conductivity [Gunn, 1956; Sartor, 1967; Wormell, 1953] are calculated by the response of charged particles to the electric field. In this way, the variation of conductivity between clear air and cloud is accounted for in a consistent fashion.

Finally, a "systems" approach is taken in the model development. Regions of distinct properties and behavior are treated separately but consistently, so that coupling of one subsystem to any other is possible. In this way, a variety of different problems can be studied, and also, the results of each study can then be used in more comprehensive models later. Thus, the usefulness and results of the present work can be

considerably extended.

1.2 The Convective Electrification Mechanism

Observations of electrical phenomena in the atmosphere show considerable variability during gusty periods [Chalmers, 1957, p. 120], and the passage overhead of developing clouds many times coincides with surface electric field perturbations long before any liquid or solid precipitation has grown in the cloud.

Vonnegut and his coworkers [1962a] found conclusive evidence that local modification of the space charge near the surface could be traced over nine kilometers horizontally downwind, over which distance the atmospheric electric parameters were considerably perturbed from their average fair-weather state. In a subsequent study [Vonnegut et al, 1962b], it was determined that the net charge content of small growing fair-weather cumulus clouds which passed over a region of modified space charge could be directly related to the polarity of the space charge released near the ground. In fact, the polarity of the cloud charge, deduced by electric field measurements above cloud top, could be reversed by changing the polarity of the emitted charge at the ground.

The implication of these experiments would seem to be significant particularly to electrification theories which depend on the initial E field for their operation. Yet, no further experiments confined to basically electrical phenomena have been attempted.

Vonnegut's theory employs convection at the outset to transport the normal fair-weather positive space charge vertically from the clear sub-cloud region through the condensation level, where it is scavenged by the growing cloud droplets. Vertical convection subsequently carries

the droplets with net positive charge higher in the cloud to levels where the atmospheric conductivity external to the cloud is considerably greater than that near cloud base. Negative ions are attracted to the cloud periphery by the positive core, where they, in turn, are supposed to be captured by the cloud droplets. The much greater mass of the charged droplet effectively inhibits further migration of this relaxing charge to the core. According to Grenet and Vonnegut, downdrafts persist in the peripheral cloud region, which serve to transport the negative charge toward cloud base. Concurrent mixing of the periphery with the interior causes the cloud volume with negative charge to increase from cloud top toward cloud base. As the negative charge accumulates near cloud base, the normally negative surface E field increases through zero, and ultimately reaches a value such that positive corona ensues at the ground, providing additional positive space charge to participate in the convective transport. Figure 1.1 shows schematically the cloud charge distribution and the currents at this stage.

Phillips (1967a) considered the loss of ions from the air to cloud droplets by the mechanism of diffusion and an impact-charging mechanism which neglects the motion of the droplet relative to the air. His results indicated that, for small E fields, the diffusion loss of ions in a cloud with 1 g/m^3 liquid water in the form of $10 \text{ }\mu\text{m}$ particles would result in an equilibrium ion density two orders of magnitude less than that which would exist in clear air. When the field is increased, the predicted equilibrium value is smaller.

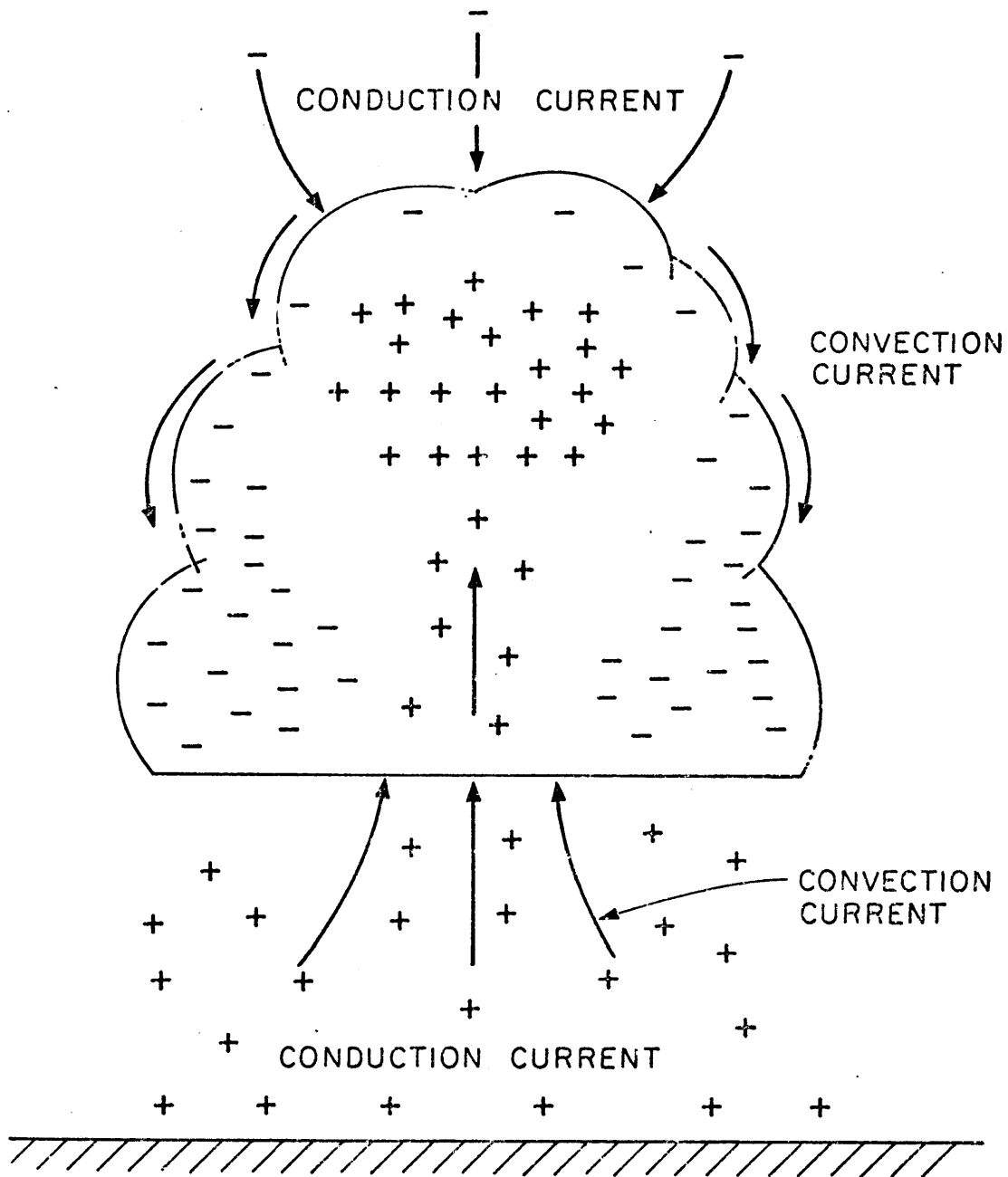


Fig. 1.1 The distribution of charge and transports according to Vonnegut's convective electrification theory

In a later paper Phillips, 1967b , these results were used to argue that convected charge near cloud base could be transported with little loss to the upper cloud regions. Here, the negative conduction current penetrates the upper regions of the cloud in response to the positive core to a distance of one or two kilometers, because of reduced cloud particle densities. Phillips assumes that precipitation forming near the top of the positive column becomes negatively charged by this conduction current and is carried by horizontal divergence of the air flow further into the more spacious region of net negative charge. The association of the negative droplets with the growing precipitation develops a gravitationally driven convection current of negative particles toward cloud base, which is roughly parallel to, but removed from the positive core. Thus, the macroscopic field growth in Phillips' model requires the onset of precipitation.

In Wilson's view (1956), the early stages of cloud growth are characterized by a flux of neutral but ionized air through cloud base. In the initial stages, the low values of the E field cause diffusional loss of ions to the droplets to considerably outweigh the impact-charging mechanisms employed by Vonnegut and Phillips. The early stage in Wilson's model, therefore, is characterized by a convecting system of cloud droplets in the ionized flow with the ion density reduced by diffusional loss to the droplets. According to Wilson's impact charging mechanism (1929, Appendix A), as long as the polar ion velocities relative to the air exceed the droplet terminal velocity, the droplets remain uncharged unless the ion mobilities differ. Thus, assuming the mobilities equal, the droplets in the early stages are not charged.

However, introduction of precipitation with terminal velocity exceeding the field-induced velocity of the ions causes the drop to accumulate net charge. If the field is initially determined by the fair-weather field, as in Wilson's model, the drops will accumulate net negative charge, leaving net positive space charge in the air. So, charging currents develop in the cloud to cause accumulation of negative charge below and positive above. As the field within the cloud grows, the ion velocities increase until they exceed the drop terminal speed and charging of the drop ceases. This critical field is dependent on the precipitation size and was taken to be about 50 V/cm in Wilson's model.

Saturation in the Wilson model guarantees that the selective ion capture mechanism alone cannot produce the intense electric fields of thunderstorms. Nevertheless, as an initial field source, this process has strong appeal. Moreover, as Wilson points out, ion capture by cloud droplets near the cloud-air interface is considerably enhanced with increasing field strength, and the processes of convective transport by the cloud circulation can augment the initial field in ways similar to Vonnegut's arguments.

The detailed structure of the convective electrification mechanisms is involved and highly speculative. Their formulation as physical models makes it difficult to assess their quantitative behavior without recourse to extreme approximations and assumptions. Nevertheless, they raise significant questions, not only about the basic electrification process, but also about the interaction of ions with cloud and precipitation and the role of atmospheric circulation in the distribution of ionic space charge.

The important distinctions between the theories are, first, Vonnegut's model predicts electrification before the onset of precipitation, whereas the Wilson and Phillips models require precipitation before appreciable fields develop. Second, Vonnegut and Phillips assume the ion density within the cloud is negligible, whereas a significant ion density is necessary for Wilson's theory, at least in the early stages.

When the cloud has developed a significant field (50-300 V/cm), the subsequent charging by all three mechanisms depends critically on scavenging of ions by the droplets near the cloud periphery. Also, all three mechanisms utilize the convective transport of space charge from the sub-cloud layer through the condensation level as the major source of thunderstorm charging currents.

1.3 Scope of the Model

In keeping with the desire to limit an experimental attack on cloud electrification to specific objectives, the model used to clarify processes and identify important parameters to be measured must be limited. The use of a systems approach to the problem, however, allows later refinements and additions. Thus, strict limitation in the present work does not necessarily affect the usefulness of the results later.

A program which seems to hold considerable promise of successful implementation because of its limited character, and which nonetheless incorporates the features common to all the above convective electrification mechanisms, is the study of the transformation of sub-cloud space charge into cloud charge and the accompanying convective generation of potential. Accepting this as a general goal, the pertinent regions or

subsystems of the problem are the clear sub-cloud region of convection, and the superposed regions of cloud. Since these two regions are coupled by their common environment, a third component of the system, the load, must be considered. Figure 1.2 shows schematically the basic units of the system to be considered.

Observations in and around clouds in modest scale experiments are plagued by severe restrictions on the simultaneity of measurements at different points of the system. Thus, the interpretation of results is usually restricted to steady or quasi-steady-state assumptions. The restriction of this model to the steady state finds its basis in the desire for simplification; however, in light of the experimental conditions, such an assumption may be justified. One measure of the validity of the steady state is that characteristic times of the problem be long relative to a typical charge relaxation time of the system. At cloud-forming heights, this period is less than ten minutes, which is short relative to the lifetime of small fair-weather clouds [Israel, 1970, p. 128].

A further simplification can be achieved by restricting the model to a one-dimensional analysis. This approximation requires that the vertical dimensions of the system be small relative to the horizontal dimensions of the convective core. The use of one-dimensional models in cloud electrification is common [Gunn, 1953; Sartor, 1967; Azad and Latham, 1969]. However, the application of the results must be restricted to consistent circumstances.

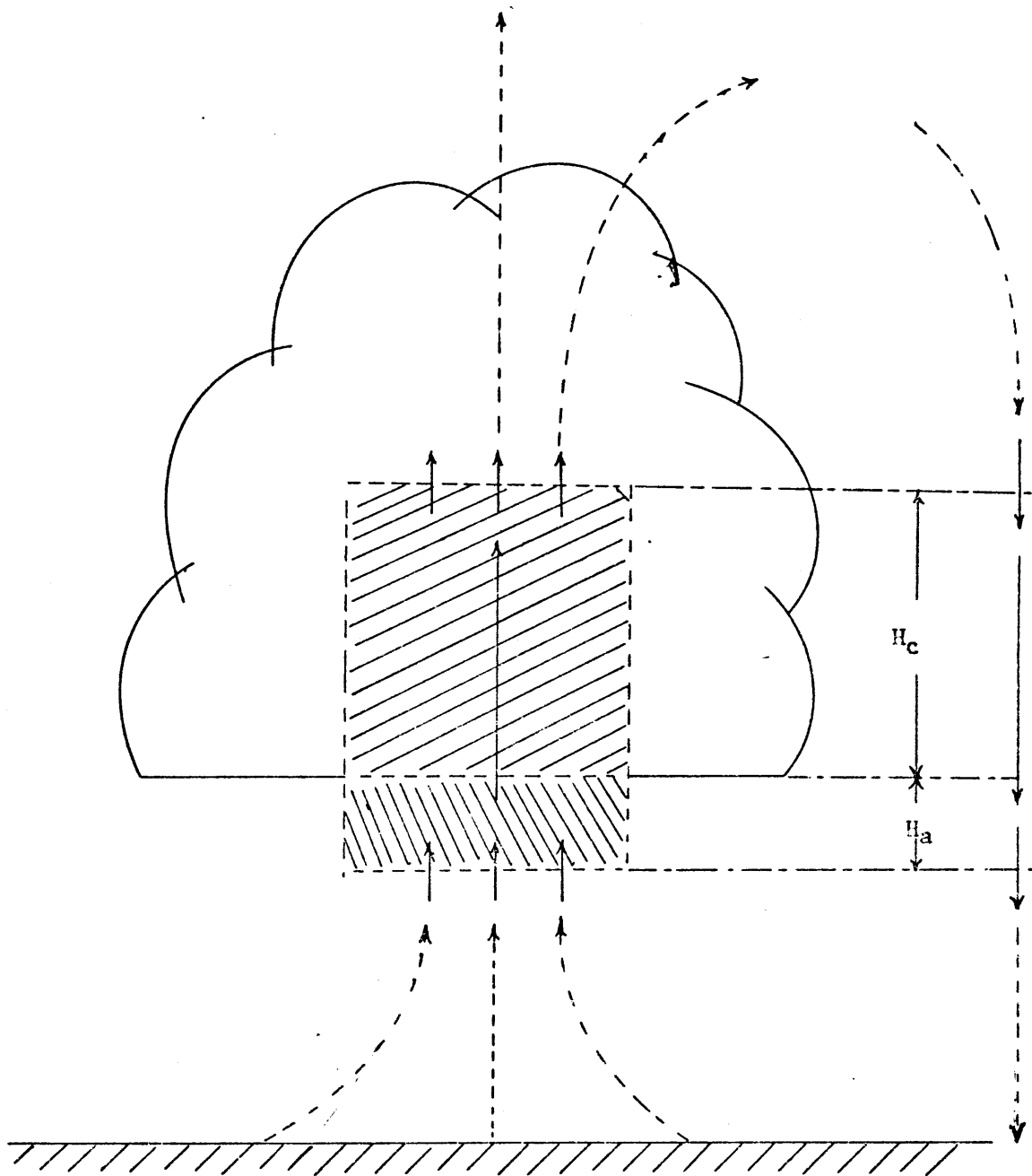


Fig. 1.2 Schematic representation of the model units

1.4 The Mathematical Model

The salient constituents of convective electrification discussed above are interacting systems of ions and charged cloud particles and a vertical flux of unstable air. The charged species, both ions and cloud particles, exhibit broad distributions of mass and charge in actual situations. The ions, although finding a common source from naturally occurring radioactivity and cosmic ray products, are formed by quick attachment to surrounding molecules and a variety of aerosol particles. For the smallest of these, the usual charge is but one elementary unit of charge, while larger pollutants may carry tens of unit charges.

Cloud droplets form at cloud base by condensation on a variety of nuclei. Because of differing nucleating properties and nucleant size and the actual unsteady and non-uniform character of the vertical flux at cloud base, the droplets do not form uniformly, nor do they necessarily grow at the same rate initially. This initially non-uniform droplet distribution is maintained above cloud base, in spite of the diffusional growth tendency toward a monodisperse distribution, probably because of mixing of particles with differing histories by the unsteady flow which persists in real clouds.

The charges residing on the cloud droplets are principally due to impact charging with atmospheric ions. Such charging can arise both from statistical, or random, motion of the ions [Gunn, 1954], and from ordered, field-induced conduction currents to the droplets [Wilson, 1929]. Unsteady flow of the neutral carrier gas will result in unordered fluctuations of background ion density, so that mixing within the rising

air will result not only in a droplet size distribution, but also in a complex distribution of droplet charges, even for particles of the same size.

To approach such a complex system analytically is a formidable task. However, with simplifications, the important features of electrification and the roles of the various constituents can be understood. Some of the simplifications will violate known properties of clouds and the growth of precipitation, but the results will yet be indicative of the processes in more realistic situations.

We will consider only situations for which the local air flow is steady and uniform in the horizontal. We assume the ions to be representative of an average of ion properties over mass and charge. Similarly, cloud particles will be represented by average properties, so that they will be uniform throughout the region of interest. Statistical growth and charging effects are neglected, with the result that particles of the same history have the same size and charge. In this way, the problem is reduced to the interaction of three distinct species; the positive and negative ions, cloud droplets, and the neutral convecting air.

The properties of each species are assumed to be "smeared" over a representative volume, so that each distinct family is treated as a continuum, rather than as a collection of discrete particles. This treatment averages out many statistical effects, but nevertheless retains the important features and considerably simplifies the analysis. Coupling between the representative "fluids" is accomplished through the body force densities of electric and gravitational fields for the

charged fluids, and a bulk viscous drag force and gravity between the neutral air and the charge carriers.

A complete mathematical treatment of the fluidization of particles in a fluid has been given by Murray [1965]. However, the relatively very low number density of cloud particles leads to large interparticle distances, so that the complicating effects of buoyancy force terms can be neglected. The interparticle distance in droplet radii for uniform spherical drops is given roughly by $\delta = (4\pi \times 10^3/3W)^{1/3}$ for W the liquid water content in kg/m^3 . Since W is almost always less than .01, the interparticle distance is larger than 10 radii for a given drop size. Such a large particle separation leads to the reasonable conclusion that the interaction of a given particle with the surrounding fluid can be considered independent of its nearest neighbors.

The large interparticle distance permits us to write the individual conservation equations of mass, charge, and momentum of each fluid independent of the others, with the provision that coupling terms be retained in a consistent fashion. Thus, the following equations are applicable to fluidized systems of particles with large particle spacing.

1.4.1 Space Charge Conservation Equation

We assume that the generation and destruction of ions is independent of height, and moreover, neglect the aerosol content in the formation of small ions. Thus, we assume that ionization products instantaneously attach to molecules, in the case of small ions, and to aerosols in the case of large ions. We assume a given region to be represented by either large or small ions, but not both. Under these

conditions, the conservation equation for ions becomes

$$\frac{\partial n_1}{\partial t} + \nabla \cdot (n_1 \vec{v}_1) = \alpha' - \beta' n_1 n_2 - s_1 \quad (1.4.1)$$

where subscript 1 refers to the positive ions, 2 to the negative; α' represents the rate of ion pairs generated per unit volume, β' the volume rate of recombination of n_1 and n_2 while s_1 represents all other ion losses. We have chosen to work with charge density, $\rho_1 = \pm e n_1$, for e the elementary unit of charge. Then the ion conservation equations become

$$\frac{\partial \rho_1}{\partial t} + \nabla \cdot (\rho_1 \vec{v}_1) = \alpha + \beta \rho_1 \rho_2 - S_1 \quad (1.4.2)$$

$$\frac{\partial \rho_2}{\partial t} + \nabla \cdot (\rho_2 \vec{v}_2) = -\alpha - \beta \rho_1 \rho_2 + S_2 \quad (1.4.3)$$

where $\alpha = \alpha'e$, $\beta = \beta'/e$ and S_1 are the generalized exchange terms, representing transformation of ionic space charge into charge density residing on drops or droplets.

The path followed by an ion in a gas under the influence of an electric field is a complicated random pattern with net displacement in the direction of the electric force on the particle. For our purposes here, the displacement of the ion due to collisions with air molecules in a direction different from the electric force is much too small to be of any consequence. Moreover, the impediment to forward motion of the ion due to molecular collisions in the fields and pressures normally encountered in the troposphere can be adequately modeled as a "viscous

drag" on the ion due to the extremely short time between collisions, relative to the pertinent times of our problems. For these reasons, there persists a balance between the accelerating electric force on the particle and the retarding effect of the collisions. This balance is written as

$$\frac{q_1 E}{m} - \nu_c \frac{v_1^2}{2} = 0$$

where q_1 and m are the charge and mass of the ion respectively, E the electric field, ν_c the effective collision frequency accounting for the rate of change of momentum in the direction of the field, and v_1 , termed the "drift velocity", or velocity of the ion relative to the surrounding gas due to the electric field. We write $|q|/m\nu_c = b_1$, the mobility of the ion, and note that over a wide range of electric fields, b is constant for given gas density and for a given ion species. Thus, the velocity relative to a fixed reference of an ion in a neutral gas moving with speed $\vec{U}$ under the influence of an electric field E can be written as

$$\vec{v}_1 = \vec{U} + b_1 E, \quad q_1 > 0. \quad (1.4.4)$$

The mobility of a small ion, that is an ionized molecule, is of the order of 10^{-4} m²/volt-sec. Thus, in the range of fields encountered in the atmosphere, say $1 \text{ V/m} < E < 10^6 \text{ V/m}$, the ion velocity varies within the range $10^{-4} \text{ m/sec} < v_1 < 100 \text{ m/sec}$. For nominal field strength and air speeds, the field-induced motion of ions is of the same order as that

due to the neutral convection. Thus, both conduction and convection are important in determining small ion transports in the atmosphere.

Large ions, on the other hand, have mobilities of the order of $10^{-8} \text{ m}^2/\text{sec volt}$. These particles, consisting of ionized molecules which have attached to atmospheric pollutants and hence are found in quantity near the surface, have essentially negligible field-induced velocity, and will therefore move with the neutral flow, finding little influence on their velocities due to the electric forces [Israel, 1970, p. 130].

1.4.2 Conservation Equations for Cloud Droplets

The condensation level is modeled as a boundary region between the clear convecting sub-cloud layer and the superposed cloudy region at which cloud droplets of uniform size and no charge are injected. The droplets are assumed to remain constant in size, so that over the height of the section considered, condensational growth and droplet collisions are neglected. Although the neglect of droplet collisions is justified in view of their small size, the region above cloud base is one of rapid growth of droplets by condensation. The neglect in the model may therefore not be justified, except for shallow horizontal sections of cloud. Since inclusion of condensational growth detracts from the primary purpose of the model, it was not included.

Under these constraints, the cloud droplet conservation equation within the cloud obeys

$$\frac{\partial N}{\partial t} + \nabla \cdot N\vec{V} = 0 \quad (1.4.5)$$

for N , the number density of cloud droplets and $\vec{V}$, their velocity.

The momentum equation for an ensemble of particles embedded in a gas with velocity U , and whose interparticle distance is large compared to their radius, is given by Murray [1965] as

$$\frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V} = \frac{q\vec{E}}{m} - v(\vec{V} - \vec{U}) - g\hat{i}_x \quad (1.5.6)$$

where q and m are the particle charge and mass, respectively, g is the acceleration of gravity, and v is a normalized drag coefficient with dimensions of a collisional frequency. (The viscous drag force on a spherical particle in air is customarily given as $\frac{\pi a^2}{m} \rho_a C_D |\vec{V} - \vec{U}|^2$ for a , the particle radius, ρ_a the air density, and where C_D , the drag coefficient, is a function of the Reynolds number of the flow around the particle. Thus, $v = \frac{\pi a^2}{2m} \rho_a C_D |\vec{V} - \vec{U}|$. For small particles in the Stokes drag regime, $v = 6\pi a \mu / m$.)

The maximum charge a drop of radius a can accrue by the impact charging mechanism is $12\pi\epsilon_0 a^2 E$ [Wilson, 1929], so that the ratio of electric to gravitational force in the model will always be less than $8.1 \times 10^{-15} E^2/a$. For droplets of radius $10 \mu\text{m}$, this ratio is unity in a field of nearly $3.5 \times 10^4 \text{ V/m}$.

Similarly, the maximum field-derived velocity of a droplet relative to the air, in equilibrium, is given by $12\pi\epsilon_0 a^2 E^2 / mv$. For small droplets in the Stokes region, this maximum relative speed is about $10^{-6} a E^2$. For 10μ droplets, the maximum relative velocity is only 10 cm/sec in a field of 10^5 V/m . Since fields of this magnitude are not expected in the early stages of cumulus development, the droplet velocity due to the field is expected always to be much smaller than the convection, U and can therefore be neglected in the charge flux calculation. Significantly, however,

Wilson's charging mechanism depends on the drop velocity relative to the air. It is possible that slight changes in the equilibrium drop velocity can cause switching of the charging regimes of Appendix A. The actual drop velocity in the charging calculations must therefore be monitored.

1.4.3 Conservation of Charge

Since there can be no unipolar charge generation, i.e., the generation of a positive unit of charge is always accompanied by a negative unit, the charge conservation equation is simply

$$\frac{\partial}{\partial t} \sum Q_i + \nabla \cdot \sum Q_i \vec{V}_i = 0 \quad (1.4.7)$$

where Q_i and V_i are the charge densities and velocities of the respective species. So, in our model,

$$\sum Q_i = \rho_1 + \rho_2 + Nq. \quad (1.4.8)$$

From Eqs. (1.4.2) and (1.4.3), Eqs. (1.4.7) and (1.4.8) give

$$\frac{\partial}{\partial t} (Nq) + \nabla \cdot (Nq\vec{V}) = S_1 - S_2. \quad (1.4.9)$$

Using Eq. (1.4.5), this result is equivalent to

$$\frac{\partial q}{\partial t} + \vec{V} \cdot \nabla q = \frac{S_1 - S_2}{N}. \quad (1.4.10)$$

The source terms S_1 , are discussed in Appendix A and in the development of the cloud system, Chapter V.

1.4.4 The Electrical Equations

The model we propose to study is characteristic of a much broader class of problems in which typical times of interest are much greater than the ratio L/c for L , a typical length, and c , the velocity

of light. In our case, moreover, the air conductivity is sufficiently low that large currents never occur. Hence, the appropriate approximations of Maxwell's equations are given by the quasistatic electric field system [Melcher and Woodson, 1968, p. B19]. In rationalized MKS units, therefore, the equations describing the electrical behavior of the model are given by Gauss's law,

$$\nabla \cdot \epsilon_0 \vec{E} = \sum Q_i, \quad (1.4.11)$$

Maxwell's displacement current

$$\nabla \cdot \left(\epsilon_0 \frac{\partial \vec{E}}{\partial t} + \sum Q_i \vec{V}_i \right) = 0, \quad (1.4.12)$$

the irrotational constraint on the electric field,

$$\nabla \times \vec{E} = 0, \quad (1.4.13)$$

and the charge conservation equation, (1.4.7).

1.4.5 The Kinematic Approximation

We will assume that the neutral flow is known, that is, U is independent of the presence of charge and field, and of the cloud droplets. According to this view, the electrical forces on the particles coupled to the flow by viscous drag are negligible compared to the buoyancy driving force for the air. This question was considered by Vonnegut [1963], who concluded that direct coupling of the electrical forces to the air flow was negligible. We will consider this calculation in Chapter III. If the air flow is independent of the other species present, the steady one-dimensional assumption requires that U be uniform in the vertical.

1.5 Organization of the Work

The systems approach allows each unit in the model to be attacked separately. This is wasteful in terms of pages before the final result, but it does provide a series of finite steps toward this goal, each step providing a check on the succeeding problem. For the reader whose time is limited, however, we note here the sequence of steps and their relevance to the whole.

Chapter II is devoted to a discussion of the parameters necessary to determine realistic boundary conditions for examples and case studies. In addition, some preceding results of observations of the space charge in clouds are discussed to provide a basis for arguing the model results. This chapter thus provides documentation for the numbers used in later studies.

Chapter III introduces convective generation from the standpoint of an electrogaskinematic generator. In this section, a unipolar generator model is discussed and the method of coupling system units is introduced and used to study the effect of a vertically varying ion mobility in a convection region. The static unipolar atmosphere solution is used as a load and compared to an ohmic conductivity load.

In Chapter IV, a bipolar generator is analyzed and a solution to the system equations in the absence of ionization and recombination is found. The operating characteristics of the bipolar generator are presented and compared to the unipolar results of Chapter III. The bipolar static atmosphere solution is used to compare the conduction processes in the atmosphere to those predicted by this simple model. Ionization and recombination effects are included in Sec. 4.4 and a

numerical model is formulated. The closed-form solution is used to verify the numerical model whose results are compared to the bipolar case without source terms to determine their importance in a convecting region.

Chapter V develops a numerical model of the cloudy region. The terms describing ion space charge loss to droplets are discussed and normalized to correspond to the normalization of the preceding models. Case studies of the vertical distribution of electrical properties in model clouds are given. Operating characteristics of a clear, sub-cloud region coupled to the cloud section are presented and self-consistent distributions of space charge and cloud charge are obtained, in terms of the sub-cloud boundary conditions and an assumed ohmic load. Transfer relations giving the transformation of sub-cloud space charge into cloud charge are given graphically, and the mechanism of convective electrification as a source of initial cloud electric field is discussed.

Chapter VI summarizes the results and conclusions of the study and includes some suggestions for further investigations.

CHAPTER II

RANGE OF THE MODEL PARAMETERS2.1 Introduction

The purpose of this chapter is two-fold. First, the models to be discussed in later sections are formalizations and idealizations of phenomena which are naturally complex. The virtue of the simplified model analysis lies in its ability to sort out trends and their relation to the basic mechanisms involved, rather than its strictly predictive capacity. However, to lend meteorological significance to the model results, it is necessary to use values of the involved parameters which bear at least some resemblance to the real atmosphere. Second, for the interested reader whose experience in atmospheric electricity is limited, this chapter provides some orientation and basic framework with which to better evaluate the models.

Observations in atmospheric electricity have tended, in the past, to be limited to the study of one or the other of the two extreme electrical situations represented by fair weather and mature thunderstorms. Since the study of thunderstorms has usually been mostly concerned with in-cloud processes, data describing the external storm environment are scanty. Although notable exceptions to these remarks exist, it is probably fair to say that detailed studies of the vertical electric structure of clear air are limited mainly to the benign fair weather conditions of stable lapse rate, clear skies, and negligible convection. Since the conditions imagined to exist in the model studies are considerably different from the fair-weather state, some caution must be

used when drawing conclusions based on the available data.

The important dependent variables in the sub-cloud region are seen to be the species space charge densities, ρ_1 , and the electric field. These quantities, with the neutral convection, serve to specify boundary conditions, or initial conditions, from which their vertical distributions are subsequently found. Looking ahead, we can use the model analyses to specify the important parameters to be discussed. Thus, the unipolar model is typified by the electric Reynolds number,

$$\text{Re} = \frac{\epsilon_0 U}{|\rho_0| b H} \quad (2.1.1)$$

for ρ_0 the boundary net space charge density, b the ionic mobility, U the convection speed and H , a typical length scale of the system. The bipolar model of Chapter IV is found to be characterized by the total density of charge carriers at the boundary, $\rho_1|_0 - \rho_2|_0$, in addition to Re . Gauss's law in a one-dimensional model relates the vertical distribution of net charge density to the behavior of the vertical electric field, while the conductivity is related to the local charge-carrier density. Since these are the most frequently measured quantities, we will use them to discuss the distribution of ρ_0 and $\rho_1|_0 - \rho_2|_0$ in Sec. 2.2.

The effect of bulk ionization and recombination in the bipolar model is examined in the last section of Chapter IV. The obviously important additional parameters in this model are the ionization rate and recombination coefficient. These are therefore considered in Sec. 2.3.

Finally, in Chapter V, the cloudy region is studied. In this model, the previously mentioned parameters appear, as well as the

interaction terms between cloud droplets and ions. Previous theoretical arguments as well as some recent observations of the vertical electric structure in clouds are therefore discussed in Sec. 2.4

2.2 Vertical Distribution of Electric Field and Conductivity in Clear Air

A complete history of the vertical observations of electric field and conductivity in the atmosphere goes far beyond the intent of the present section. A very good review can be found in Israel [1970, Chaps. III and IV]; for our purposes, we will consider only the general results of these observations.

2.2.1 Average Distribution of the Electric Field and Net Space Charge in Fair Weather

Restricting observations to periods of negligible convection and regions of low aerosol content, the vertical electric field is found to be nearly 130 V/m near the ground, and to decrease uniformly with altitude to a value near 10 V/m at six km. On the basis of many observations, von Schweidler [1929] compiled an empirical formula,

$$E = 90 e^{-3.5h} + 40 e^{-0.23h} \text{ V/m} \quad (2.2.1)$$

where the height, h , is measured in kilometers above the ground.

Gish [1944], measured the vertical conductivity, σ , and assuming a steady-state one-dimensional model, calculated the electric field

$$E = J/\sigma \quad (2.2.2)$$

With an assumed value of J corresponding to that normally observed in

fair weather, Gish's results were found to be in close agreement with those of von Schweidler.

The effect of aerosol contamination was included in an empirical formula for the average conductivity [Gish, 1944] giving a new approximate formula for the average electric field with the assumptions of Eq. (2.2.2). Since aerosol content decreases by nearly an order of magnitude per kilometer of altitude above the ground, Gish's derived result gives better agreement in the lower levels with the observed values of the electric field than does von Schweidler's. Gish's modified empirical field formula is given by

$$E = 81.8 e^{-4.52h} + 38.6 e^{-.375h} + 10.27 e^{-.121h} \text{ V/m.} \quad (2.2.3)$$

for h in kilometers.

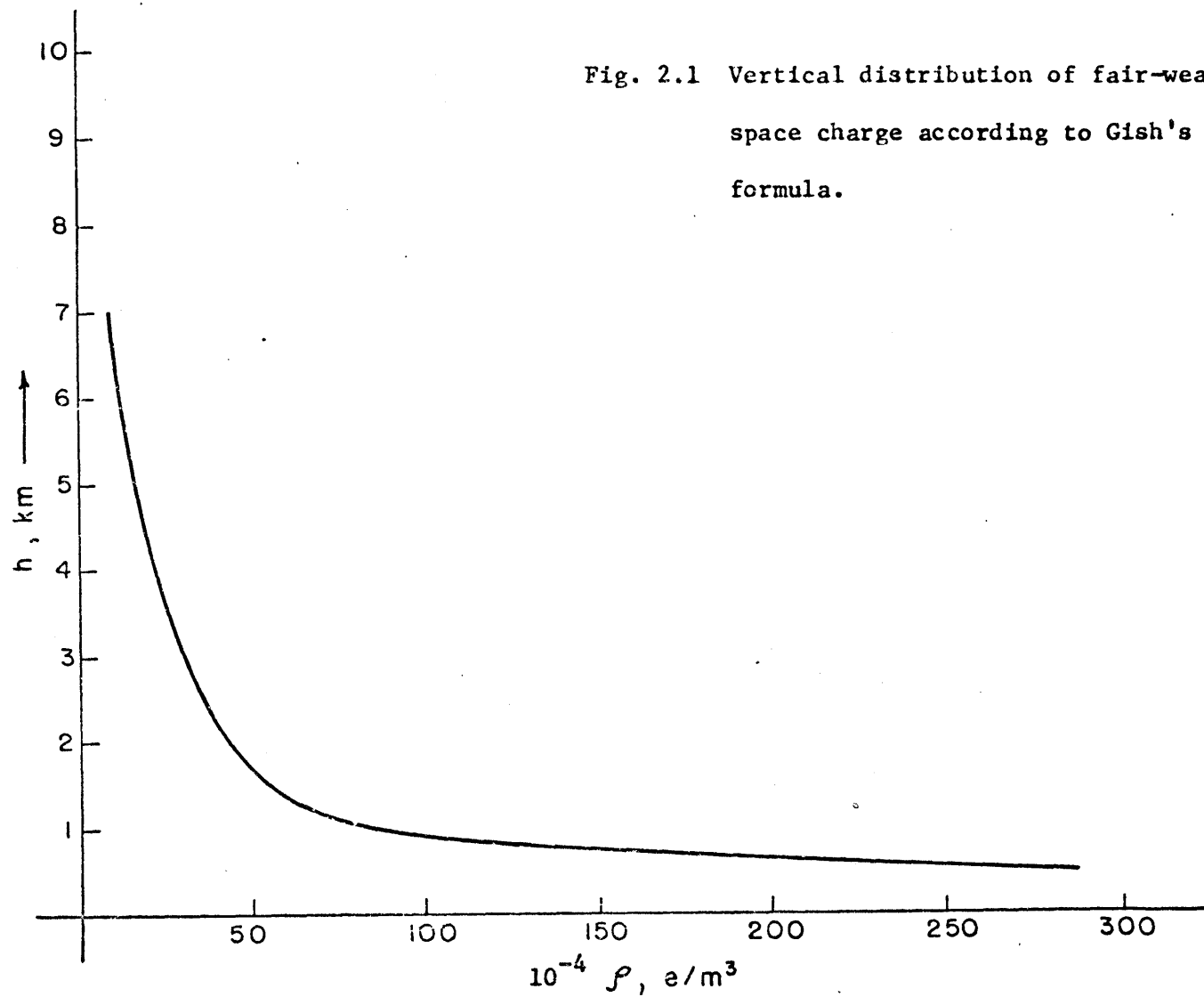
With the one-dimensional assumptions inherent in Eq. (2.2.3), the average net space charge distribution with height can be determined by Gauss's law,

$$\rho = \epsilon_0 \frac{dE}{dh} \times 10^{-3} \text{ c/m}^3 \quad (2.2.4)$$

This is shown in Fig. 2.1.

2.2.2 Observations of the Vertical Field in Disturbed Weather

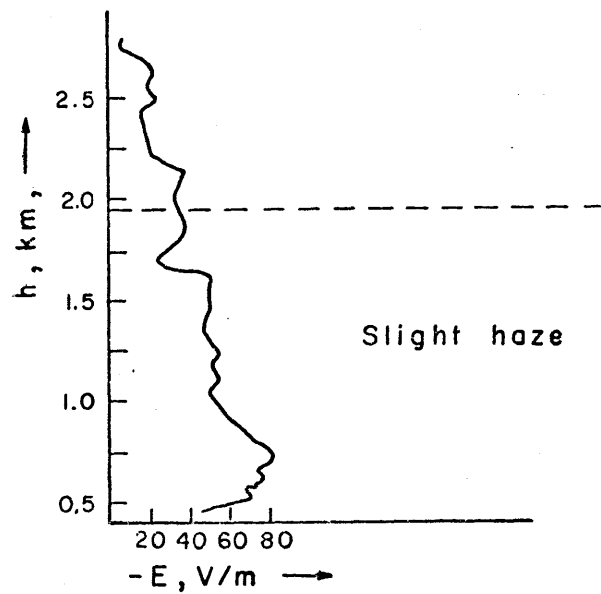
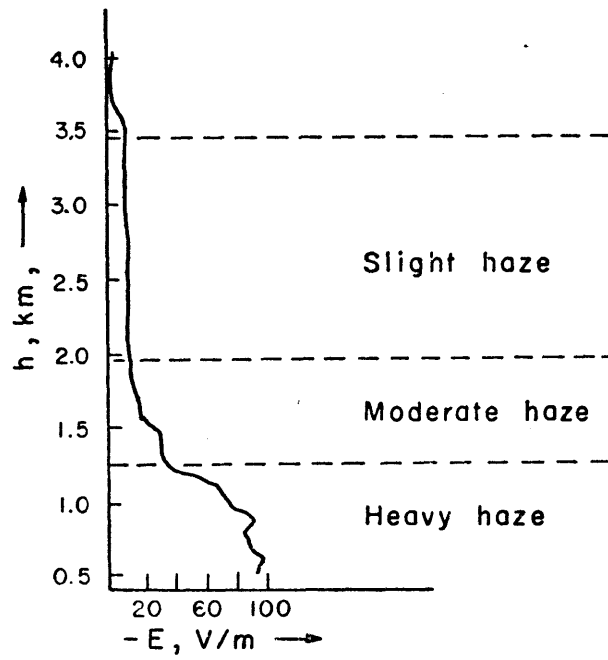
The condition under which Eqs. (2.2.1) and (2.2.4) can be expected to give reasonable estimates of the actual atmospheric electric field are severe. Ionic mobilities vary from about $10^{-4} \text{ m}^2/\text{V-sec}$ for small ions to less than $10^{-8} \text{ m}^2/\text{V-sec}$ for "large ions", or charged



aerosols. In the small fields of fair weather, charged particles will thus have velocities of the order of 0.2 cm/sec or less. Hence, even the slightest atmospheric circulation is likely to obviate the assumption of Eq. (2.2.2). Moreover, non-uniform distribution in the vertical of atmospheric contaminants such as occurs during a period of temperature inversion would be expected to cause significant local departures of the electric field from the average value.

Rossmann [1950] reported the results of many glider-borne experiments measuring the vertical distribution of electric field, conductivity, and the vertical temperature and relative humidity. In addition he noted the haze levels, condensation level, and cloud cover observed during his flights. Moore et al [1960] performed similar experiments using an instrumented powered aircraft from which they observed the electric field using a radioactive probe similar to that used by Rossmann. However, the conductivity was not observed in these later observations.

Figures 2.2 and 2.3 show sample results of Rossmann's flights to be compared with Figs. 2.4 and 2.5 from Moore et al, showing the evolution of the vertical structure from a stable, quiescent night-time state to a "very convective" state in the late morning. Since Rossmann's flights depended on vertical air motions, convection is clearly important in his observation. Apart from the magnitudes of the observed electric fields, the results of the electric field measurements by these two separate experiments show reasonable agreement when comparing Rossmann's results with the fully developed state in the experiments of Moore et al.



Figs. 2.2 and 2.3 Observed vertical electric field structure
glider flights [Rossmann, 1950]

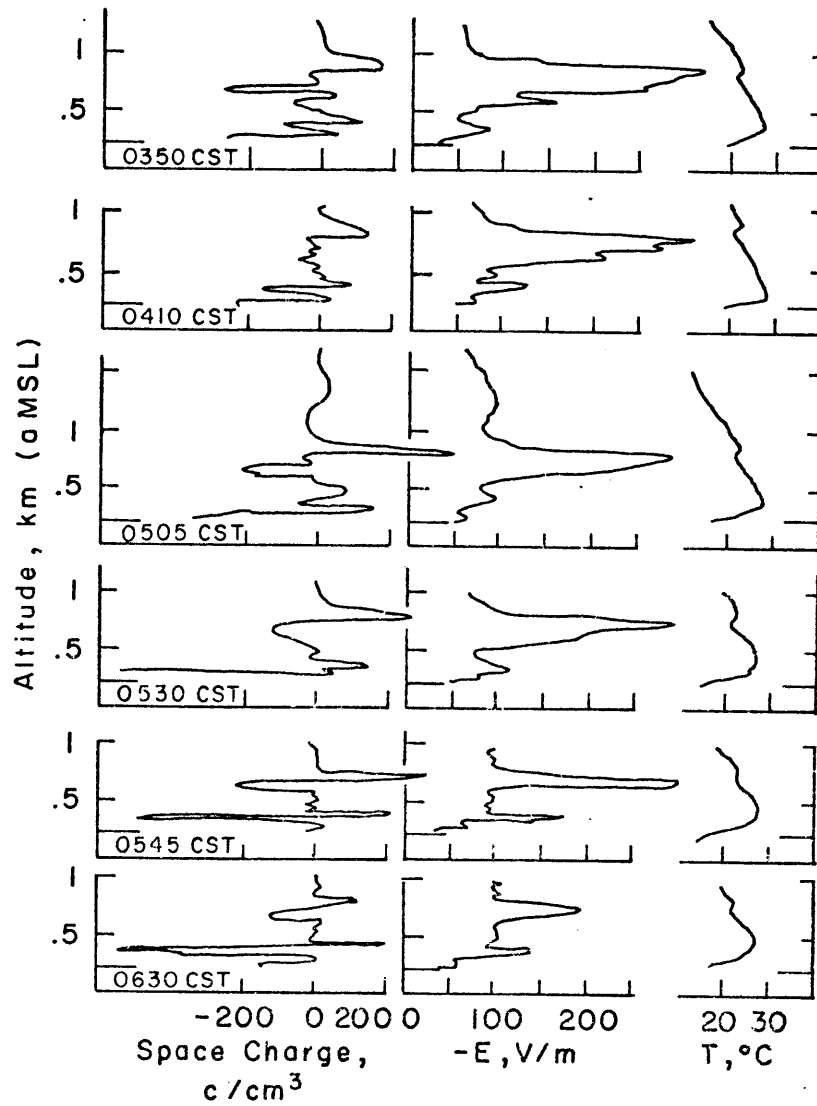


Fig. 2.4 Vertical electric field and associated space charge in a stable atmosphere [Moore, et al, 1960]

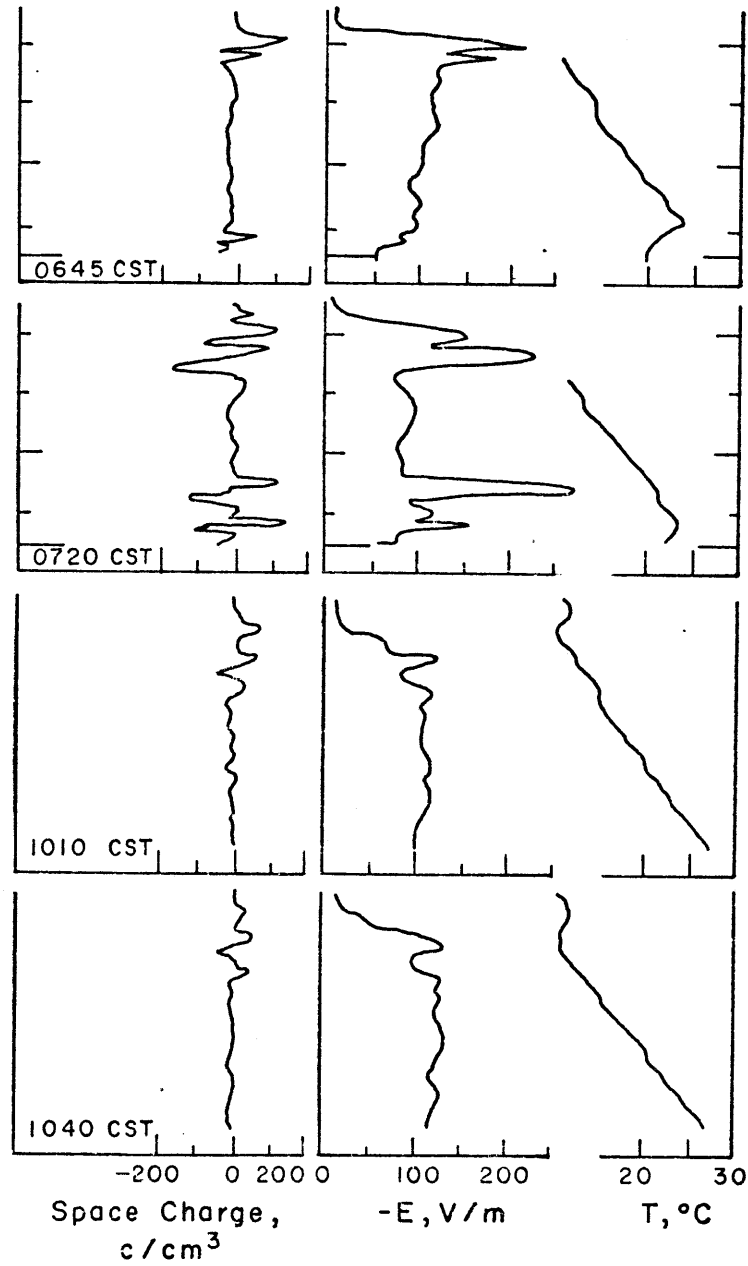


Fig. 2.5 Observed field and charge density during the transition from a stable to an unstable lower layer

Significantly, the regions of enhanced space charge density coincident with haze levels in stable air evident in Moore's results disappear as mixing progresses during the morning hours. As is evident from Fig. 2.2, haze levels above the first in Rossmann's flights made during significant convection have almost no effect on the field distribution, and hence the space charge. This contrasts markedly with the observations in quiet air shown in Fig. 2.4 in which convection current is negligible.

The exponential decrease of the average field results due to von Schweidler and Gish do not appear in the data of Rossmann and Moore et al. In fact, the lowest region, 0 - 2 km., is a region of maximum field gradient, according to the empirical field formulae, whereas the observations with haze layers and convection do not show this effect. The convective results show a tendency for the electric field to maintain a nearly uniform value over the depth of the exchange layer. Thus, although appreciable perturbations of the field in this region are evident, there is no strong tendency of the field to show a decrease with altitude until the level of the exchange layer is reached. The records of Moore et al, as well as most of Rossmann's results, showed the region of transition from haze to clear air to be a region of marked field change, hence also a region of significant space charge accumulation.

Whereas Gish's empirical formula suggests a uniformly decreasing space charge density from ground level, with about 10^{-13} c/m³ at about one km height, the observations during convective periods show the

exchange layer to be one of nearly uniform electric field, and thus negligible space charge until the height of the mixing layer is reached, usually between one and two kilometers. The field at this level shows a sharp decrease in magnitude, indicating that the top of the exchange layer is a region of noticeable charge accumulation of about 10^{-11} C/m^3 , or two orders of magnitude greater than the empirical models predict.

2.2.3 Vertical Variation of Conductivity in Fair Weather

Many workers in atmospheric electricity treat the local conductivity as an independent variable [see Israel, 1970, p. 83, for example], while others model electric dissipation according to a constant ohmic conduction model [Gunn, 1956]. Conduction in the ionized atmosphere is expressed by

$$J = (e \sum b_i n_i) E \quad (2.2.5)$$

where e is the elementary unit of charge, and $b_i n_i$ is the product of mobility and density of the various charged species. Assuming a bipolar model in which both charge carriers have mobility b ,

$$J = eb(n_1 + n_2)E \quad (2.2.6)$$

Defining the conductivity, σ , as the ratio J/E ,

$$\sigma = eb(n_1 + n_2) \quad (2.2.7)$$

In the static atmosphere, the total particle density is essentially determined by the ionization and annihilation processes which are independent of the local field strength. Thus, the assumption of independence of the conductivity in this case is probably valid. In other circumstances,

however, when the local particle density may depend appreciably on other sources such as vertical convection and convergence, this assumption may not be justified.

The average conductivity distribution in the vertical could be calculated from von Schweidler's result, Eq. (2.2.1), with the assumptions inherent in Eq. (2.2.2) and given a representative value for J . A similar calculation can be performed using Gish's average field formula, Eq. (2.2.3). Since Gish's result is based on observed conductivity, whereas von Schweidler measured the vertical field, we will use Gish's formula to show the average vertical conductivity in Fig. 2.6. The assumed value for J in this figure is 2.78×10^{-12} amp/m², which is very close to the uniform current density observed by Kraakevik [1961]. From Eq. (2.2.7) and the definitions of ρ_1 , the total charge density, $\rho_1 - \rho_2$, can be calculated from Fig. 2.6 for assumed values of the bipolar mobility,

$$\rho_1 - \rho_2 = \sigma/b \quad . \quad (2.2.8)$$

The average conductivity curve of Fig. 2.6 includes the effects of an assumed distribution of aerosol particles. In the higher levels, aerosols can be neglected and a two-species model more nearly approximates the true state of affairs. Stergis et al [1955] observed the vertical conductivity from about 3 km to nearly 26 km. Using their data from a parachute descent for comparison, they developed a theoretical model for the conductivity, using a bipolar model. The charged species in their model were determined from an assumed equilibrium between

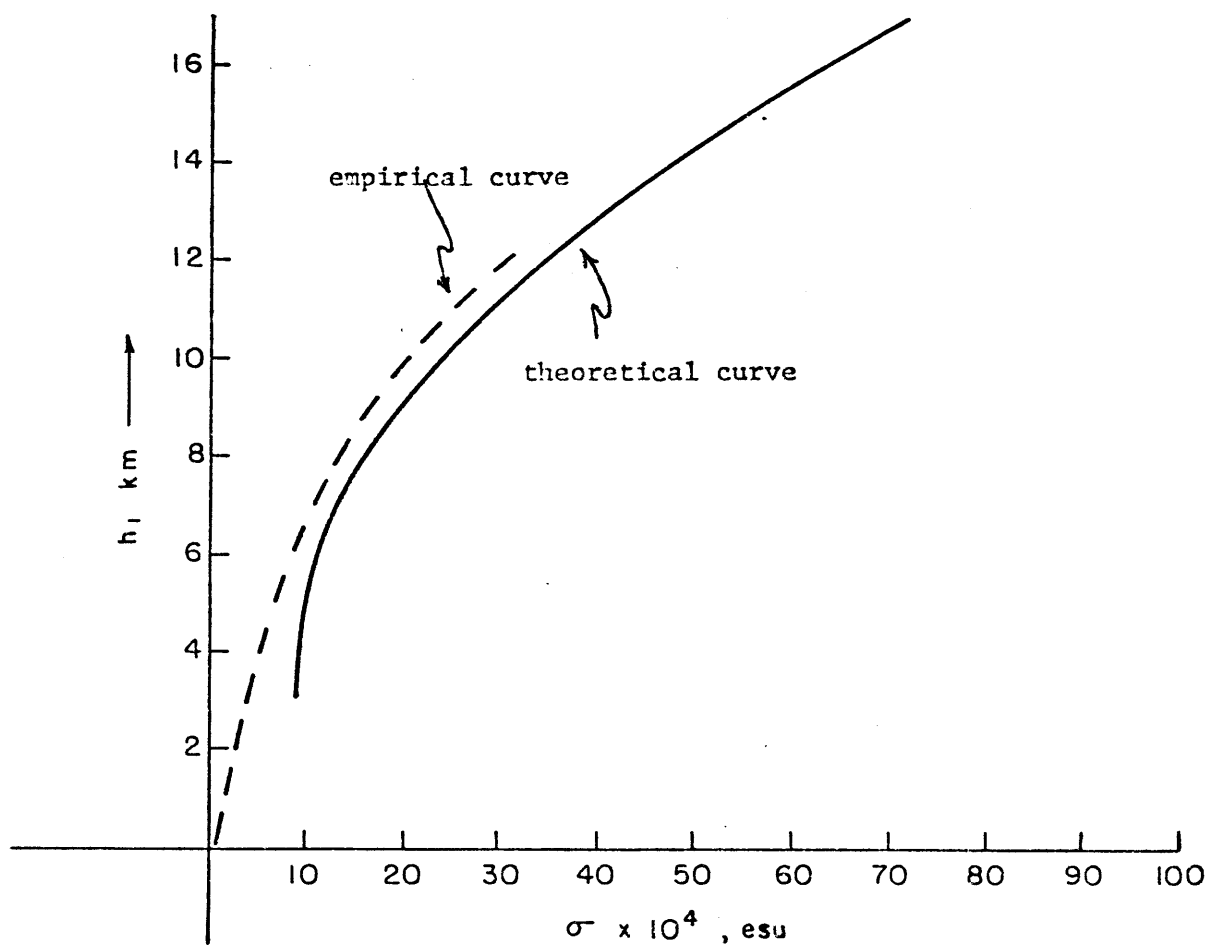


Fig. 2.6 Vertical conductivity according to the empirical formula of Gish [1944] and the theoretical model of Stergis et al [1955]

ionization by cosmic rays and mutual annihilation. Their vertical ionization rate was determined from the observations of Bowen et al, (1937) and the recombination coefficient was computed from Thomson's recombination theory [see Sec. 2.3]. The mobility of the bipolar species was assumed equal and was calculated assuming mobility to be inversely proportional to the ambient gas density. Their results for one flight are shown in Fig. 2.6. The particularly important observation to be made from these experiments is the important character of the ionization rate in determining the quiescent conductivity, at least in the higher levels of the atmosphere. Since their model neglects transport of the particles by an electric field or fluid motion, however, the application of their results in the vicinity of developing clouds may not be justified.

2.2.4 Observations of Disturbed Weather Conductivity

Sagalyn and Faucher [1954] studied the vertical distribution of large ions and conductivity by means of an instrumented airplane with which they made vertical soundings by repeated traverses at specific altitudes. The flights were restricted to fair weather; however, they were not restricted to regions in which convection currents could be neglected. Their results showed a clear division of the troposphere into two distinct regions of differing conductivity. The lowest region was found to correspond to the exchange layer in which significant vertical air motions persist, the large ion content is high, and the conductivity is low. Above the exchange layer, the temperature lapse rate corresponds to a more stable configuration, and consequently vertical air motions become insignificant. In this region, the large ion content

drops off sharply and the conductivity increases.. Comparing their observed conductivity to that predicted by a theory similar to Stergis et al, they found generally poor agreement below the exchange layer and fair agreement above. Moreover, they found an inverse relation between the large ion content and the conductivity, suggesting that in the lower level, the distribution of pollutants, controlled by convective transport, is a determining factor in the observed conductivity. The observed increase in conductivity from below to above the exchange layer was found to vary from a factor of about 1.5 to 6.

Rossmann's observations [1950] of conductivity also showed a general increase in conductivity as the regions of haze were surpassed . In addition, he observed several interesting cases of very large increases of conductivity when passing through ascending air currents of growing cumulus clouds. As the region of vertical motion was left, the conductivity fell back to its "normal" value. A sample of this observation is shown in Fig. 2.7, corresponding to the flight data of Fig. 2.3.

Other observations near the ground tend to support Rossmann's experiments. Gockel [1915] found that on days with thunderstorm activity, the conductivity was noticeably higher some considerable time before the storm development. Zegula [1936] found an increase associated with the passage of distinct stratocumuli and in the vicinity of thunderstorms without precipitation. Kähler [1929] associated the presence of low clouds with an increase in conductivity.

Israel [1970, p. 126] suggests the cause of this increase to be due to the vertical transport of air whose ions are formed by radioactivity

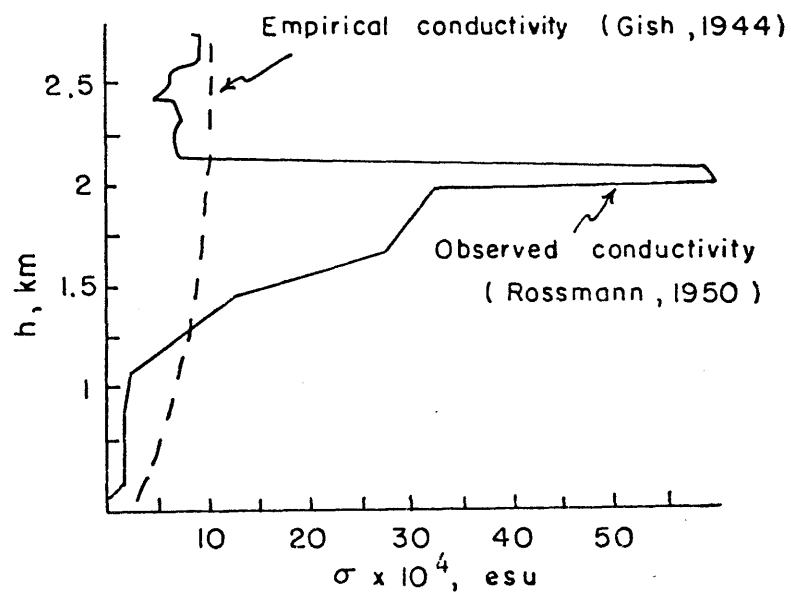


Fig. 2.7 Increased conductivity in ascending air currents

at the ground, reasoning that the main source of ionization in the lowest region can be attributed to this process. However, this is precisely the condition under which large ion contents of the air will be enhanced, so that the observations of Sagalyn and Faucher would suggest a lower conductivity. Another possible explanation is that these ascending currents have somehow been contaminated with air from above the exchange layer, the low aerosol content of which is reflected by a higher conductivity.

2.3 Vertical Variation of Ionization and Recombination

Charged particles of the clear atmosphere are the end-products of a perhaps complicated sequence of events, beginning with the generation of a positive ion and free electron by the primary ionization event, subsequent attachment of the electron to a neutral gas molecule, usually O_2 , and finally the migration of the positive-negative small ions thus formed either to the variety of atmospheric contaminants usually present to form large ions or annihilation of the charge by mutual recombination of small ions, or recombination of a small ion with an oppositely charged large ion.

Since the mobility of large ions is usually more than three orders of magnitude less than that of the small ions, this latter family of charged species is the principal agent contributing to the observed conductivity. Moreover, the great simplification afforded by neglect of the aerosol content outweighs the loss of generality by exclusion of this admittedly important constituent of the atmospheric charge. Strictly speaking, the results which neglect the large ion content

should be restricted to regions above the exchange layer where the aerosol content drops off sharply.

2.3.1 The Ionization Rate

Ionization near the earth surface in addition to cosmic rays is a consequence of the radioactive sources of α , β and γ radiation, radium, thorium and actinium and their gaseous intermediate products radon, thoron and actinon, which are sources of α radiation. The relatively short ionizing path lengths of α - and β - particles allow neglect of these components from earthbound materials, while their γ - radiation is influential for tens of meters above the surface. Thus, radioactive sources of ionization in the atmosphere can be limited to the consideration of γ - radiation from the surface, α particles emitted by decaying radioactive gases in the atmosphere, and cosmic ray ionization.

The absorption of γ - radiation with path length follows an exponential decay whose characteristic decay length is approximately 200 meters [Israel, 1970, p. 191]. Thus, at a height of 1 km, the ionization rate due to γ - radiation is reduced to about 0.1% of its value at the surface. The contribution of the gaseous products certainly diminishes from the value near the surface, which corresponds to the source. However, the transport of these components is accomplished by diffusion, turbulence and convection, so that a simple decay expression cannot be given.

Indeed, the ionization rate due to radon and thoron in the lower levels can be expected to vary considerably with the meteorological conditions. As a rule of thumb, the contribution to the atmospheric ionization within the exchange layer due to radioactive gases might be

expected to be nearly constant with altitude in accordance with the nearly uniform properties of other atmospheric contaminants in this region. However, this reasoning is complicated by the relatively short half-lives of thoron and actinon, 55 sec and 3.9 sec respectively, compared to 3.8 days for radon. This disparity in half-life causes radon to be the most important contributor of the radioactive substances to the ionization in regions distant from the surface.

Ionization by the highly energetic particles of cosmic rays which penetrate the atmosphere from above varies considerably with height. Their interaction with atmospheric molecules in the absence of a direct nuclear collision is negligible. As a consequence of their high energy, such a collision results in the generation of secondary particles of ionizing energies which, in conjunction with the primary particles, generate the so-called cosmic ray "showers" of ionized molecules. Due to the lower probability of collisions in regions of lower density, the ionization increases with decreasing height until a maximum is reached when the increasing collision probability is counteracted by the depletion of primary particles due to collisions higher up. This maximum is found above 10 km altitude, with 12 km reported by Bowen et al [1937].

Cosmic radiation particles whose paths intersect the earth's magnetic field are deflected to an extent determined by the angle of intersection and the energy of the incident particles. The higher the energy, the less influential is the curvature of the path. Because of this effect, there is a noticeable latitudinal variation of the intensity of cosmic ray ionization at the surface, with minimum values in the equatorial region (~ 2 ionization events per cm^3 per sec near 3°N) and

maximum near the poles (~ 3 events per cm^3 -sec near 51°N) [Gish, 1951].

The contributions to the atmospheric ionization by the various agencies near the ground is shown in Table 2.1, after Israel [1970, p. 194], where j is the volume rate of production of ion pairs. The data for higher altitudes were taken from the measured ionization rate vs altitude by Gangnes et al [1949] and converted to the indicated height by means of the 1962 Standard Atmosphere. Cut-off of γ - radiation from the surface is assumed to occur before an altitude of 1 km is attained. The radioactive gas contribution is assumed to be negligible above 2 km, reflecting both effects of diffusion and an assumed exchange layer near 2 km. Thus, above 2 km, the ionization is assumed to be wholly due to cosmic rays. These data are merely meant to be indicative of a trend. Clearly, the actual ionization rate will depend on meteorological conditions near the surface as well as the local relative quantities of radioactive materials in the earth surface soil.

Table 2.1

Height, km	j , ion pairs/ cm^3 -sec
0	$\gamma \sim 4.0$
	$\alpha, \beta \sim 4.6$
	cosmic ray $\sim \underline{2.1}$
	total ~ 10.7
2.5	11.6
5.0	18.9
7.5	24.8
10.0	62.8
12.5	82.5
15.0	64.8
17.5	44.0
20.0	27.6

2.3.2 The Mutual Recombination Coefficient

The classical theory of volume recombination of small ions subdivides this process into four steps [Loeb, 1955, p. 516]. These are:

- 1) The diffusive-approach period during which ions are separated by distances large compared to their electrical interaction distance. In this stage, ions move randomly according to their thermal energies until two oppositely charged ions by chance find themselves moving under their combined electric fields.

- 2) The period of active attraction then follows during which the particle velocities have an average drift component toward each other superposed on their background thermal velocities.

The sphere of active attraction is characterized by the distance between the ions, R , at which the electrical potential energy of the particles equals their thermal energy,

$$\frac{e^2}{4\pi\epsilon_0 R} = \frac{3}{2} kT \quad (2.3.1)$$

for e , the ion charge, and $3/2 kT$, their thermal energy in electron-volts.

- 3) As the ions approach one another under their mutual central force fields, two possibilities of subsequent motion exist. If the neutral particle density is high, the retarding collisions experienced by the ions can be thought of as an effective damping term, and ultimate collision of the mutually attractive particles is assured. On the other hand, if there

are few collisions within the sphere of attraction, the ions move about their common charge center in orbits which may be open or closed, depending on the relative magnitudes of the kinetic energy and the electric potential energy of the interaction. This is called the period of orbital encounter.

- 4) The period of charge transfer is one of some mystery to this writer. It is perhaps sufficient simply to say that the exchange of charge between positive and negative particles in close proximity is determined by circumstances whose likelihood of being met is increased the longer the particles remain close. With this rather vague statement as a guide, one expects closed orbits to be more likely to allow charge transfer than open orbits.

The two main theories of volume recombination are attributed to Langevin [1903] and Thomson [1924]. Langevin assumed the ionic mean free path much less than the radius of the sphere of attraction, R . Thus, his theory incorporates only the first two of the four steps above, since the damping due to collisions within R guarantees ultimate collision of the ions. According to Langevin's theory, the number of positive ions approaching a negative ion is given by the flux of positive ions into the sphere of attraction. Neglecting thermal motion of the ions, this flux is given by $n_1 4\pi R^2 \bar{v}$, where $\bar{v} = (b_1 + b_2)e / (4\pi\epsilon_0 R^2)$ is the relative velocity of the particles. Assuming each collision results in recombination, Langevin's theory gives the volume rate of destruction of n_1 as

$$\frac{dn_1}{dt} = -n_1 n_2 [b_1 + b_2] e/\epsilon_0 \quad (2.3.2)$$

Thus, Langevin's recombination coefficient is given by $(b_1 + b_2)e/\epsilon_0$. The basic assumption of his recombination is that the mean free path must be small relative to R . This suggests that his theory would be most applicable at high pressure, or for the recombination of small ions with highly charged atmospheric aerosols. The applicability of Langevin recombination at high pressure in air (2-20 atm) has in fact been verified [Loeb, 1955, p. 556].

Thomson [1924] reasoned that the flux of ions into the sphere of attraction is due to the thermal motion, and not the drift velocity, as calculated by Langevin. Moreover, for lower pressures, the number of ion-neutral collisions within R can be quite small, so that simple approach of ions to the critical distance R is insufficient to guarantee recombination. Assuming that at least one collision within the sphere of attraction would guarantee a closed orbit, and hence transfer of charge, Thomson derived the probability that such a collision would occur. Then, the number of recombination events is given by the flux of ions into the sphere of attraction which subsequently impact a third, neutral, particle.

Assuming the differential thermal speeds of the ions can be given by a suitable average, $\bar{v}$, the flux of positive ions into the interaction region of a negative ion is given by the particle flux, $n_1 \bar{v}$, times the interaction cross section, πR^2 , for R given by Eq. (2.3.1). If $f(x)$ is the probability that at least one ion collision will occur in the interaction region, the rate of recombination events is given by

$$\frac{dn_1}{dt} = -n_1 n_2 R^2 \bar{v} f(x) \quad . \quad (2.3.3)$$

The expression $\pi R^2 \bar{v} f(x)$ is derived in terms of the molecular species, absolute temperature and pressure by Loeb [1955, p. 548]. For our purposes, we merely note that x is twice the ratio of the interaction distance, R , to the mean free path for ions, and is proportional to the gas density. The product $R^2 \bar{v}$, however, is dependent on temperature alone, and varies as $T^{-3/2}$. Thus, the probability of a three-body collision decreases with altitude, whereas the flux of particles into the interaction region increases with decreasing temperature. These two effects conspire to make the recombination coefficient of Thomson remain nearly constant, at least in the lower atmosphere. Using data from the NACA Standard Atmosphere, the recombination coefficient at ground level was found to be 0.92 times the value at 10 km.

This contrasts markedly with a similar computation using Langevin recombination in which $b_1 = b_2$ and we assume the mobility to vary inversely with density. In that case, the ratio of the ground level to the 10 km level recombination coefficient is about 0.169.

Thomson's theory was verified for clean dry air for pressures less than atmospheric by Sayers [1938]. Loeb [1955] suggests that the general mechanism is valid over the pressure range 10^{-2} mm Hg $< P < 760$ mm Hg. Although it is likely that the presence of water vapor and solid contaminants make noticeable differences in the recombination coefficient, we will neglect these effects. Thus, on the basis of the very small change in the recombination coefficient in the lower atmosphere predicted by Thomson's

theory, we will assume a constant value in our calculations. The generally accepted value at normal temperature and pressure is given by Israel [1970, p. 25] and corresponds to that used by Stergis et al [1955] in their vertical conductivity model. We will use this value in subsequent computations,

$$\beta' = 1.6 \times 10^{-6} \text{ cm}^3 \text{ sec}^{-1} \quad (2.3.4)$$

or

$$\beta = 10^7 \text{ c}^{-1} \text{ m}^3 \text{ sec}^{-1} \quad (2.3.5)$$

2.4 Effect of Clouds on the Vertical Electric Structure

In this section, we examine briefly some of the experiments and theories about the microscopic interactions of cloud droplets and space charge and the resulting macroscopic electric structure. The typical microscopic interactions considered are ionic diffusion to the droplet surface [Gunn, 1954; Wilson, 1956; Phillips, 1967a] and ionic conduction currents to the droplet in a polarizing electric field [Wilson, 1929; Gunn, 1956].

There is a wide divergence of opinion concerning the influence of cloud on the local conductivity leading to serious dispute about the microscopic ion-droplet interaction theories. In addition, there are notable contradictions in the experimental observations of cloudy conductivity. The approximations in the theories and experimental limitations are therefore worth considering.

2.4.1 Ionic Diffusion and Conduction to Cloud Droplets

Gunn [1954] considered an isolated static drop in an ionized atmosphere in which the electric field is due solely to the net charge on the drop. Assuming ion velocities could be represented by two

independent parts, the thermal velocity and the mobility-governed drift velocity, Gunn calculated the polar currents to the drop and the equilibrium drop charge assuming the ion density far from the drop was constant. Under these circumstances, Gunn found the polar currents to the drop (in esu) to be

$$I_{\pm} = \pm \frac{4\pi q b_1 \rho_1}{\exp\left(\pm \frac{qe}{akT}\right) - 1} \quad (2.4.1)$$

The equilibrium charge was

$$q = \frac{akT}{e} \ln \frac{\rho_2 b_2}{\rho_1 b_1}, \quad (2.4.2)$$

where q is the droplet charge, a the radius, e the ion charge and kT the ion thermal energy.

Assuming the effect of flow of ionized air around the droplet could be represented by a "ventilation factor", similar to that used in condensational growth and evaporation of falling drops, Phillips and Gunn [1954] observed the electrification of suspended conducting spheres in an ionized flow and found essential agreement with the predicted equilibrium charge of Eq. (2.4.2).

A significant limitation of Gunn's theory is its restriction to a spherically symmetric electric field. Thus, his results may not be applicable in situations for which an external electric field, E_0 , exists because of polarization charges induced on the droplet surface.

Wilson [1929] and Gunn [1956] considered the effect of these induced surface charges and the consequent field distribution around a

conducting drop in an ionized medium. Appendix A discusses this situation in detail. Here, it is sufficient to note that, neglecting space charge and diffusion, the ion conservation equations are

$$\frac{\partial \rho_1}{\partial t} + (\vec{U}_d \pm b_1 \vec{E}) \cdot \nabla \rho_1 = 0 \quad (2.4.3)$$

where $\nabla \cdot \vec{U}_d = 0$, so that the charge density ρ_1 is constant along characteristics defined by

$$\frac{d\vec{r}}{dt} = \vec{U}_d \pm b_1 \vec{E} \quad (2.4.4)$$

Assuming irrotational flow around the drop, a stream function of ion flux can be derived from which determination of streamlines which intersect the drop surface gives the drop charging rate.

In Gunn's model, which neglects the flow of air around the drop, $\vec{U}_d$, the drop charges in regimes determined by the surface charge density due to the uniform electric field and the drop charge. Charging regimes in Wilson's model, however, are determined by the drop surface charge density and the origin of the characteristics followed by ions which impact the drop surface. Thus, since $U_d = 0$ in Gunn's model, ions always approach the drop on the side of the charge-accepting region, whereas in Wilson's model convection can carry ions against the electric field, causing them to approach the drop from the side opposite the charge-accepting region. For details, the reader is referred to Appendix A.

The influence of convection can have a marked effect on the drop equilibrium charge. For example, in a neutral ionized atmosphere in which

$b_1 = b_2$, the equilibrium drop charge in Gunn's model is zero, but in Wilson's, the equilibrium drop charge is non-zero when $U_d > |bE_0|$. On the other hand, the polar charging currents to the drop do not differ greatly in equilibrium, suggesting that the total loss of atmospheric ions to the drop may be nearly the same for both models.

Phillips [1967a] considered the effects of both diffusion and conduction currents to drops on the equilibrium ion density inside clouds. The flux of ions to the drops by diffusion was given by the term $4\pi a D N n$ for a , the drop radius, D the diffusion coefficient, N the number density of drops, and n , the ion density far from the drop surface. Using the Einstein relation $D/b = kT/e$, [McDaniel, 1964], this diffusion term is seen to correspond to Gunn's diffusion current, Eq. (2.4.1), when $q \rightarrow 0$. Conduction currents to the droplets were calculated by Gunn's model of ion-drop impact charging in which the droplet is assumed to be stationary relative to the air.

Phillips argued that the diffusion term could be considered independent of the impact-charging mechanism, since for small droplets, diffusion currents dominate for low electric fields ($\lesssim 60$ V/cm for 10μ droplets), while conduction currents to the droplets are predominant for high electric fields. The diffusion term, however, is derived for a spherically symmetric electric field, so that his argument seems questionable.

Neglecting convective transport of ions as well as the flux of ions due to the ambient electric field, Phillips calculated the equilibrium ion density as a function of applied field and liquid water content for clouds of uniform particles. In general, he found that the greater the number of droplets, the lower the equilibrium ion density. For a typical cloud of 1 g/m^3 water content, and radius 10μ , his results showed a

decrease in ion density of nearly two orders of magnitude for zero field strength and a greater depletion of ions for increasing field.

Phillips' results differ sharply with the observations of Freier [1962] and Evans [1969] who both noted a large increase in conductivity within clouds in their experiments. However, Freier measured field recovery rates at the ground following lightning discharges, so that his determination of cloudy conductivity seems questionable, in view of the effects of screening charges at the cloud-air interface.

Evans' observations were made within the cloud proper. However, he neglected the convection current in his analysis so that his observed conductivities are expected to be too high by an unknown amount. Finally, Allee and Phillips [1959] made in situ observations of cloud conductivity and found a diminished conductivity within the cloud. They observed cloudy conductivities nearly an order of magnitude less than the clear air value at the same altitude.

The observations of Allee and Phillips were made at a mountain-based observatory which was obscured by low clouds. Their measurements were therefore near a solid boundary and in a region of little organized convection. This contrasts with Evans' results which were made from a probe dropped through the cloud, and hence through regions of possibly significant convection. Whether the convective transport of ionic change can explain the differences in the predicted and observed conductivity awaits more detailed measurements. However, our model will allow some insight into the possibility.

2.5 Summary

The empirical formula for the fair-weather electric field allows an estimate of the vertical distribution of fair-weather space charge. In the lower regions, say between 0.5 and 3 km above the ground, this formula predicts a low net space charge density of between 10^{-13} and 10^{-14} c/m³. The observations of the field distributions on a given day show notable departures from the average structure depicted by the empirical formula, with particularly important variations associated with atmospheric strata and regions of increased aerosol content. These regions have associated charge densities orders of magnitude greater than the average structure would lead us to believe. The region of transition between the exchange layer near the ground and the upper, more stable, layer shows charge densities of the order of 10^{-11} c/m³ and higher.

Assuming an average mobility of 10^{-4} m /V-sec can be associated with this charge, an estimated electric Reynolds number for different situations is

$$Re \approx \frac{\epsilon_0 U}{\rho_0 bH} \approx \frac{U}{bH} \approx 10^4 \frac{U}{H} \quad . \quad (2.5.1)$$

For a length scale of one km, and for U between band 10 m/sec, the above remarks thus estimate Re to be between 10 and 100. For particles whose mobility is significantly less than the small ion mobility, Re becomes very large.

The empirical conductivity results of Gish and, higher up, the

theoretical model of Stergis et al, give an estimate of the total ion content to be expected in quiet conditions. These vary from about 5×10^{-10} to 5×10^{-9} c/m³ in the height range one to three km, assuming a small-ion mobility of 10^{-4} m²/V-sec. The glider flights of Rossmann showed low level enhancement of conductivity in the rising thermals of large cumulus congestus with associated ion densities of about 5×10^{-9} c/m³.

An important parameter of the bipolar model is given by the ratio ρ / ρ_T , for ρ_T the total ion density. Thus, the range of this parameter can be expected to be between about .001 and 0.1. The alternative expression of ρ / ρ_T is the ratio of the polar ion densities, $\rho_2 / \rho_1 = B_1$,

$$\rho / \rho_T = \frac{1 + B_1}{1 - B_1} \quad (2.5.2)$$

For the values above, B_1 varies between -0.8 and -0.999.

The vertical variation in the recombination coefficient is found to be negligible in the lower 10 km, and the value $\beta = 10^7$ m³/c-sec is assumed in the study. The vertical variation in ionization rate is not necessarily ignorable, but this effect will not be considered. From Table 2.1, in the lower regions α' varies between 10 and 60 ion pairs per cm³-sec, or $\alpha \sim 10^{-11}$ - 10^{-12} cm³-sec.

Finally, there is considerable disagreement among observations

of conductivity in clouds as well as the expected conductivity according to theories of ion scavenging by drops. Of particular interest are the observations of Rossmann which show regions of enhanced conductivity in rising air associated with cumulus clouds and the in-cloud observations of highly variable conductivity noticed by Evans. These observations are discussed later in terms of the models considered.

CHAPTER III. UNIPOLAR CONVECTIVE GENERATION

3.1 Introduction

The generation of potential in its simplest form requires the presence of free charge and some means of transporting this charge against the retarding force of an adverse potential gradient. As is clear from the preceding chapter, free charges in the atmosphere are always present in varying amounts, while agencies of transport can be found in fluid convection or falling charged precipitation. The purpose of this chapter is to investigate the behavior of a very simple model of a generator using the former of these two transports to maintain a potential difference in the generating region which differs from that in the undisturbed environment.

All observations have shown that at all levels of the atmosphere, there exists a multitude of charge carriers, both positive and negative, whose motions under the action of an existing electric field depend on the size and resident charge of the particles. Thus, a complete model of the natural situation would require as many equations as would be necessary to describe the dependent variables governing the distribution, charge, and velocity of each carrier, as well as the local electric field and air speed U . The difficulty and tedium of such an approach would detract from the goals of this work, so reasonable approximations which retain the important aspects of generation must be made. Thus, the model in this chapter is a unipolar electrogaskinematic generator in which a single ion species of finite mobility interacts with a specified neutral flow configuration and the self-consistent electric field to maintain a potential difference between boundaries transverse to the vertical flow. (See Fig. 3.1). In the unipolar model, bulk generation and annihilation

of ions are neglected; the ion source is external and assumed to provide ions at the boundary of the generating region.

3.1.1 Ion-drag Pumps and Generators

That the interaction of charged particles and a macroscopic electric field in a neutral flow can lead to significant effects has been demonstrated by many authors. For example, the ion-drag pump analyzed by Stuetzer [1960] utilizes the viscous coupling between a liquid and entrained charged particles which are accelerated by an electric field to impart momentum to the fluid. A more graphic illustration of this effect in air is shown in Woodson and Melcher [1968], in which a balloon is levitated by the "corona wind" arising from air accelerated by this means through a grid of corona points. The configuration in these devices is such as to cause a pressure rise along the flow axis, and electrical energy is converted to mechanical form.

Alternative configurations are evident in the generating devices of Pauthenier and Moreau-Hanot [1935], Marks, Barreto and Chu [1964] and Gourdine, Barreto and Khan [1965], in which the energy conversion is reversed. The charged particles are then carried by the fluid against the potential gradient, giving rise to high potentials across the terminals of the flow channel. This is the basis for the convective generating mechanisms in thunderstorms of Vonnegut [1963] and others.

3.1.2 The Atmospheric Analogy

A piece-wise continuous model of convective generation in the atmosphere can be constructed by analogy with a laboratory-like generating device shown in Fig. 3.1a. In this machine, a corona source provides charged particles at the inlet which are carried through a

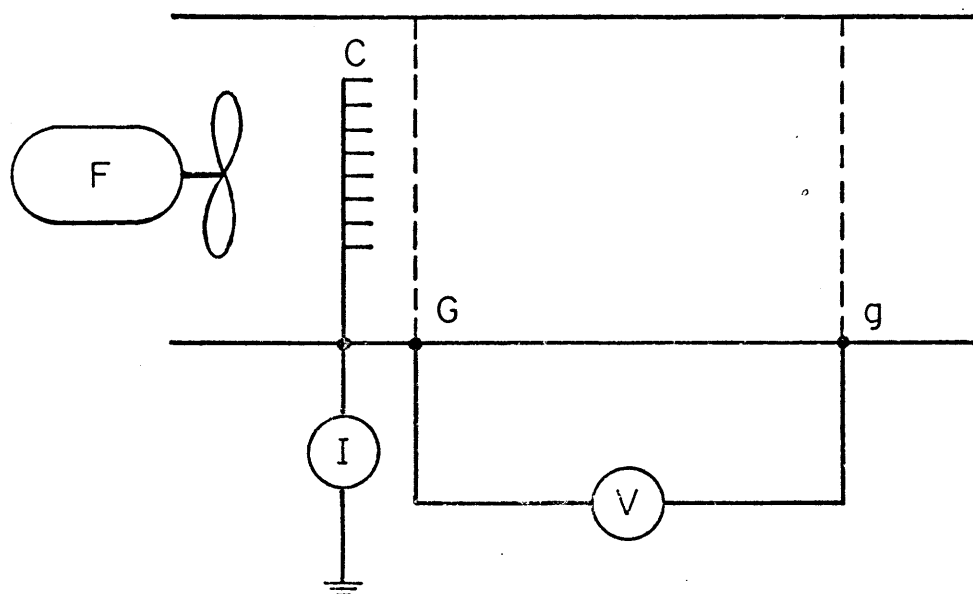


Fig. 3.1a An electrogaskinematic generator. A current source, I, provides ions through corona at C. The fan, F, supplies convection which carries ions through the grid, G, and into the generating section where the ions move according to the dual forces of drag and electric field. The charged particles are captured at grid g and returned through the load, V, completing the circuit.

conducting grid into the enclosed generating section by the air motion, induced perhaps by a fan. Another conducting grid at the far end serves to collect the charge, and a consequent potential difference is maintained between the two grids by a load depicted schematically as V in the figure.

The mode of operation of the device is determined by the electrical and mechanical energy conversion which occurs. If we assume the convecting fluid incompressible, and treat the one-dimensional steady state, then the convection speed is uniform throughout the section. Thus, the mechanical energy input is positive or negative, depending on whether the pressure difference between the longitudinal boundaries is negative or positive. The electrical power input is given by $-JAV$ for A the cross-sectional area, J the uniform current density, and V the potential difference between the longitudinal boundaries. Considering positive J , the system is a generator when V is positive and the pressure difference negative; that is, mechanical power is supplied and converted by the interaction to electrical power. On the other hand, when V is negative and ΔP positive, electrical energy is converted to mechanical form and the system acts like a pump. If both V and ΔP are negative, both electrical and mechanical energy are supplied to the system and are converted to heat energy in what is termed the "brake" mode of operation.

A schematic picture of the atmospheric analogy to the generator of Fig. 3.1a is shown in Fig. 3.1b. In the analog, vertical convection of unstable air takes the place of the fan, and the analysis is limited to local regions of the systems. An example might be relatively limited areas of buoyant plumes so that the one-dimensionality assumption can be employed. The source of ions assumed could be attributed to corona at

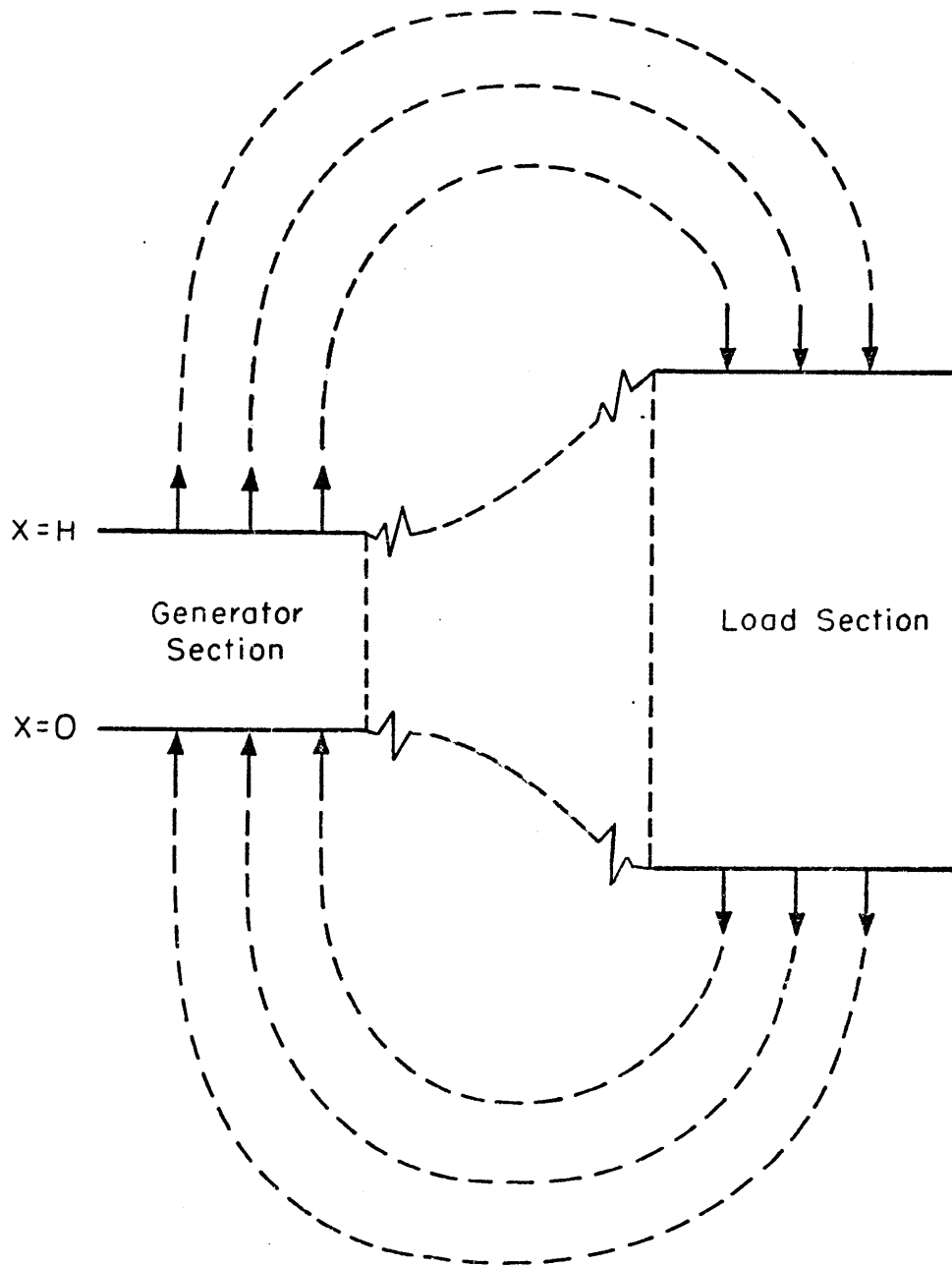


Fig. 3.1b Atmospheric analog to the electrogaskinematic generator of Fig. 3.1a. Ions at the boundary of the generator region move vertically, according to the convection speed and local electric field. The generating region is coupled at equipotential lines to a load which, in this configuration, serves as a current return path.

the ground, ionization external to the generating section, or horizontal exchange with areas removed from the vertical air flux. The modeling of the conducting wires of the laboratory model by unspecified horizontal exchange mechanisms typifies this approach to electrification. Thus, the current return path or load in the atmospheric model is conceived as a distant region coupled to the convecting region by these exchange mechanisms. The constraints on the load are that it provide a return path for the current which penetrates the generating section, and that it be coupled to the generator at common equipotential levels. That is, the upper boundary of the load section is designated by the distant level of the equipotential of the upper boundary of the generator, and the same holds for the lower boundaries of the two regions.

Confining the study to distinct regions allows the immense simplifications of the one-dimensional assumption and a "systems" approach to the overall electrification dynamics. For purposes of illustration, the unipolar generator is constrained by a single simple loading region. However, the results and general approach can be used with the generating section described here, in general interacting with a complex system of "loads" which might even include other generating regions.

After the derivation of the generator model in Sec. 3.2, two load models are presented in Secs. 3.3 and 3.4.

3.2 The Unipolar Convective Generator

The steady-state one-dimensional behavior of the system shown in Fig. 3.1 can be investigated by the use of the equations introduced in Chapter 1. The pertinent equations for the ions are conservation of charge, Gauss's law for the electric field, and the mobility law for the

ion velocity. In this case, neglect of bulk ion sources and the steady-state criterion make the charge conservation expression simply

$$\frac{dJ}{dx} = 0 \quad (3.2.1)$$

where

$$J = \rho(U + bE) \quad (3.2.2)$$

and

$$\epsilon_0 \frac{dE}{dx} = \rho \quad (3.2.3)$$

Finally, the solenoidal character of the E field allows us to write the equation for the potential,

$$\frac{d\Phi}{dx} = -E \quad (3.2.4)$$

The above equations are written for a positive ion species. For the case of a negative carrier, ρ and b must be negative quantities. This departs from our customary definition of $J_1 = \rho_1(U \pm b_1 E)$, where both b_1 and b_2 are positive quantities. However, in this case the simplification allows us to drop $\pm$ and merely substitute the proper sign in the final result.

The solutions to Eqs. (3.2.1)-(3.2.4) illustrated in the following have been presented earlier in the context of electrogasdynamic generators by Steutzer [1959] and in texts such as Melcher and Woodson [1968].

Equations (3.2.1) - (3.2.4) can be written in dimensionless form by the following definitions. Let primes indicate the dimensionless variables. Then, defining $J' = J/\rho_0 U$ where ρ_0 is the boundary value ion density, Eq. (3.2.2) becomes

$$J' = \rho'(1 + E') \quad (3.2.5)$$

where

$$\rho' = \rho/\rho_0 \quad (3.2.6)$$

and

$$E' = bE/U \quad (3.2.7)$$

is the ratio of field-induced velocity to the convection speed. Using these definitions in Eq. (3.2.3), we find

$$\frac{dE'}{dx'} = \frac{1}{Re} \rho' \quad (3.2.8)$$

where $x' = x/H$, for H a length scale of the system, and where

$$Re = \frac{\epsilon_0 U}{\rho_0 b H} \quad (3.2.9)$$

is the "electric Reynolds number" for the system. The electric Reynolds number is better understood, perhaps, by multiplying the top and bottom of Eq. (3.2.9) by ρ_0 . Then,

$$Re = \rho_0 U \left(\frac{\epsilon_0}{b \rho_0 H} \right)$$

The first term is clearly the current density due to the convection, i.e., the convection current. The second term (in brackets) is the inverse of a measure of the conduction current density due to the action of the self-induced, or space-charge-induced, electric field. In this sense, the Reynolds number is a measure of the ratio of convection current to pure conduction current, with high Re corresponding to vigorous mechanical transport of charge, and low Re referring to situations in which the conduction process dominates.

Equation (3.2.4) is written

$$\frac{d\phi}{dx'} = - \frac{UH}{b} E'$$

or, defining $\phi' = b\phi/UH$,

$$\frac{d\phi'}{dx'} = -E' \quad . \quad (3.2.10)$$

For convenience, Table 3.1 gives the dimensionless variables and their definitions.

Table 3.1

Dimensionless Variables for Unipolar Generator Model

<u>Dimensionless Variable</u>	<u>Definition</u>
$\bar{J}'$	$J/\rho_o U$
ρ'	ρ/ρ_o
E'	bE/U
Re	$\epsilon_o U/\rho_o bH$
x'	x/H
v'	bV/UH

3.2.1 The Terminal Relations

The steady-state solution to Eqs. (3.2.5) - (3.2.10) is found by recognizing from Eq. (3.2.1) that $J' = \text{constant}$. Then, Eqs. (3.2.5) and (3.2.8) can be combined by eliminating E' to obtain

$$\frac{d}{dx'}\left(\frac{1}{\rho'^2}\right) - \frac{2}{ReJ'} = 0 \quad . \quad (3.2.11)$$

Further progress requires that we specify the boundary at which the ion density is known in order to determine how the integration of Eq. (3.2.11) proceeds. Specifying for this problem that H be equivalent to the height of the generating section, the question of whether the boundary value obtains at the upper or lower boundary resolves itself into whether the integration proceeds from x' to 1, or from 0 to x' . Thus, (3.2.11)

integrates to the result

$$\frac{1}{\rho'^2} = 1 + \frac{2x'}{\text{Re } J'}$$

for $\rho'(0) = 1$, and

$$\frac{1}{\rho'^2} = 1 - \frac{2(1-x')}{\text{Re } J'}$$

when the boundary value ρ_0 is fixed at the upper boundary so $\rho' = 1$ at $x' = 1$. Thus,

$$\rho' = \left(1 + \frac{2x'}{\text{Re } J'}\right)^{-1/2}, \quad \rho'(0) = 1 \quad (3.2.12)$$

and

$$\rho' = \left(1 - \frac{2(1-x')}{\text{Re } J'}\right)^{-1/2}, \quad \rho'(1) = 1 \quad (3.2.13)$$

These expressions for ρ' can now be substituted into Gauss's law, Eq. (3.2.8), which then gives E' required to evaluate Eq. (3.2.10). Hence, two further integrations give the normalized electric field, E' , and potential, Φ' , against x' . Alternatively, the definition of normalized current density, Eq. (3.2.5), can be solved for the normalized electric field and the expression for ρ' , Eqs. (3.2.12) or (3.2.13) can be substituted in the result. This gives directly the field distribution

$$E'(x') = J' \left(1 + \frac{2x'}{\text{Re } J'}\right)^{1/2} - 1 \quad (3.2.14)$$

$$\rho'(0) \equiv 1$$

Integrating the negative of Eq. (3.2.14) gives the normalized potential distribution

$$\phi'(x') = x' - \frac{\text{Re}J'^2}{3} \left[\left(1 + \frac{2x'}{\text{Re}J'} \right)^{3/2} - 1 \right] \quad (3.2.15)$$

$$\rho'(0) \equiv 1$$

where we have taken $\phi(0) = 0$ without loss of generality. The alternative expressions for E' and ϕ' when $\rho'(1) \equiv 1$ are

$$E'(x') = J' \left(1 - \frac{2(1-x')}{\text{Re}J'} \right)^{3/2} - 1 \quad (3.2.16)$$

and

$$\phi'(x') - \phi'(1) = x' - 1 - \frac{\text{Re}J'^2}{3} \left[\left(1 - \frac{2(1-x')}{\text{Re}J'} \right)^{3/2} - 1 \right]. \quad (3.2.17)$$

In Eqs. (3.2.16 and (3.2.17), Re is evaluated at $x' = 1$.

These equations, (3.2.12) - (3.2.17), give the electrical characteristics of the model under discussion. Under the definition of H given above, we define the variable V' to be the normalized potential of the upper level relative to the lower; i.e., $V' = \phi'(1) - \phi'(0)$. Then, for $\rho'(0) = 1$, V' is found from Eq. (3.2.15) evaluated at $x' = 1$, or

$$V' = 1 - \frac{\text{Re}(J')^2}{3} \left[\left(1 + \frac{2}{\text{Re}J'} \right)^{3/2} - 1 \right] \quad (3.2.18)$$

and when $\rho'(1) = 1$, V' is calculated from (3.2.17) for $x' = 0$,

$$V' = 1 + \frac{\text{Re}}{3} (J')^2 \left[\left(1 - \frac{2}{\text{Re}J'} \right)^{3/2} - 1 \right]. \quad (3.2.19)$$

The normalized current-voltage operating characteristics of the monopolar generating section calculated from Eq. (3.2.18) are shown in Fig. 3.2. by the solid lines. The region $V' > 0$ is pertinent to the atmospheric analogy and is shown in greater detail in Fig. 3.6.

Figures (3.3) and (3.4) are sample calculations of the vertical distribution of normalized space charge and electric field, as determined by the electric Reynolds number. From Eq. (3.2.12), the higher Re , the

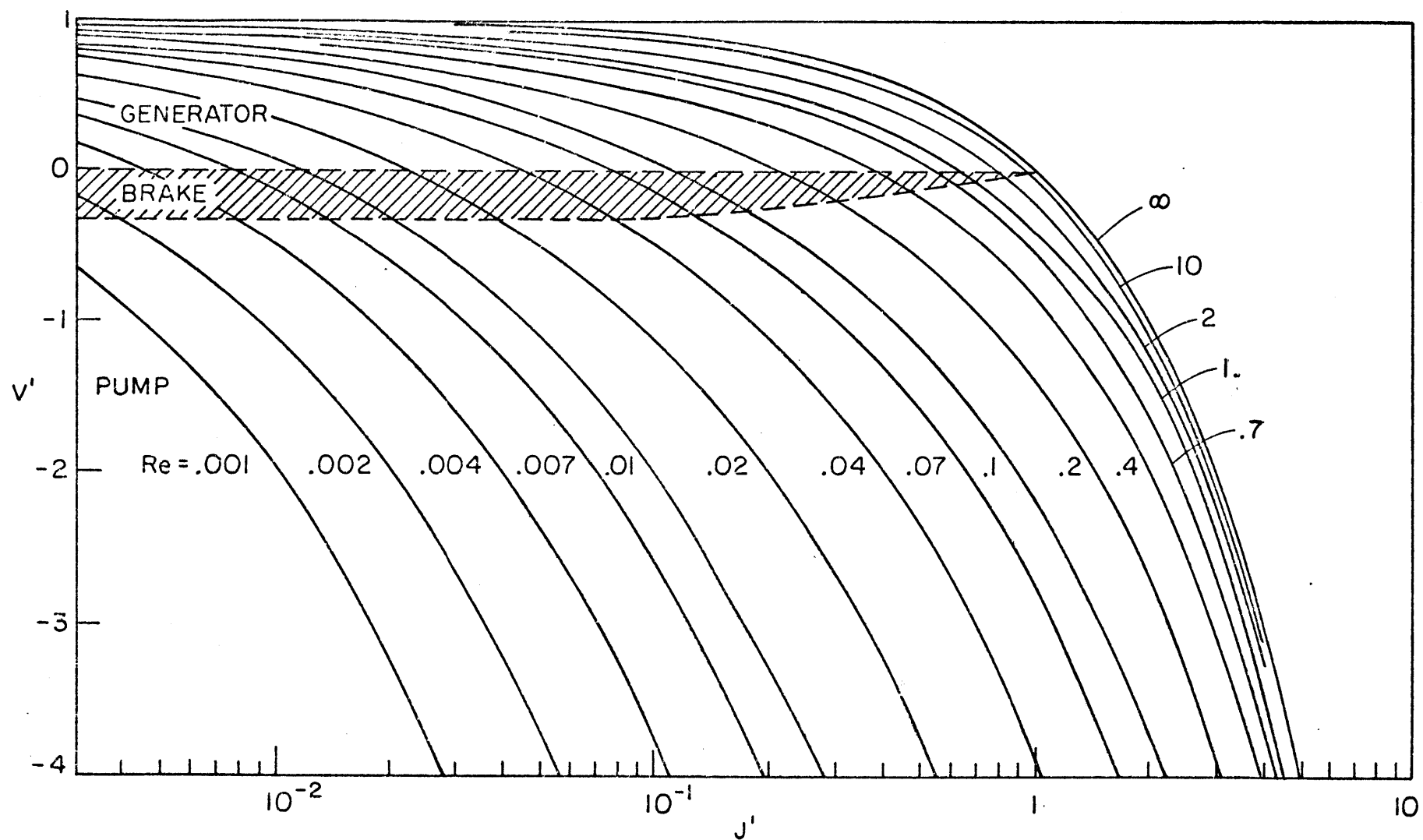


Fig. 3.2 Terminal relation for a unipolar generator showing the three modes of operation

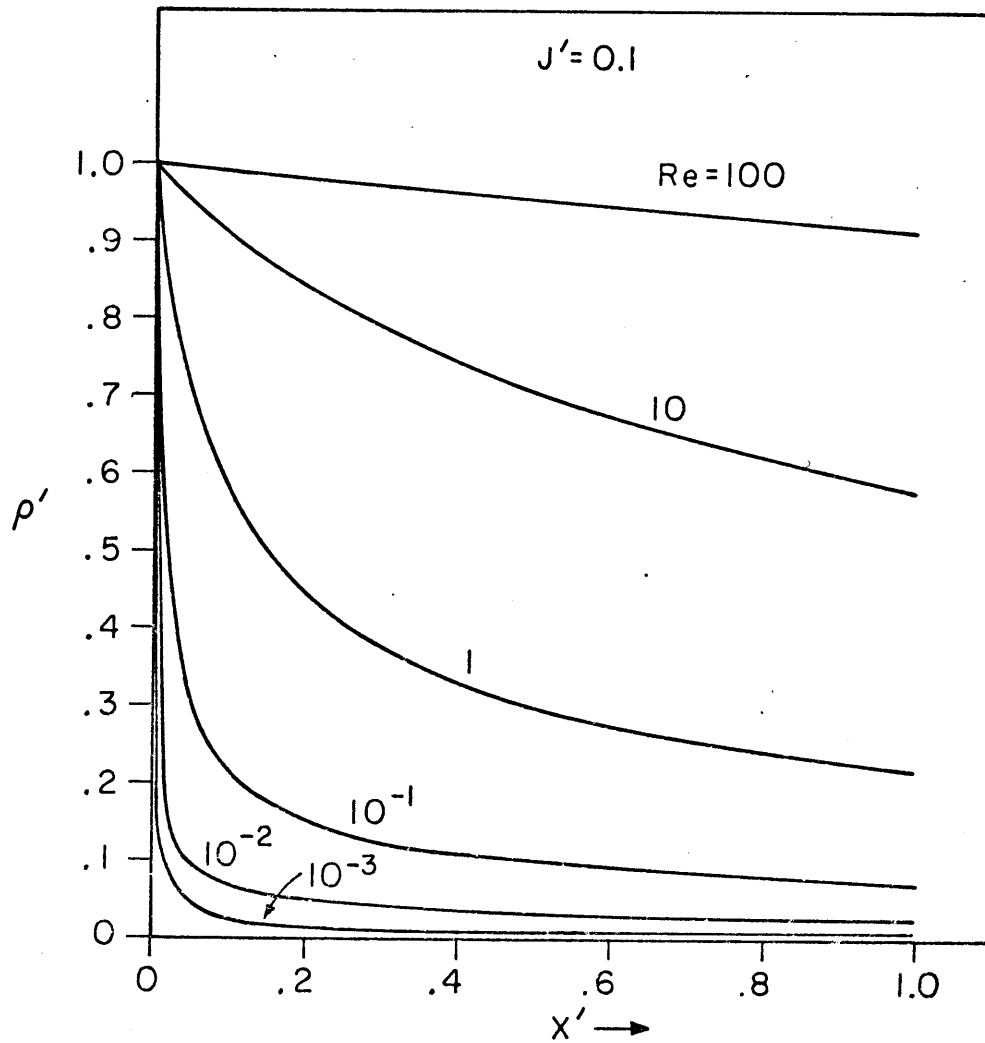


Fig. 3.3 Sample distribution of normalized charge density
for the indicated electric Reynolds number
when $J' = 0.1$

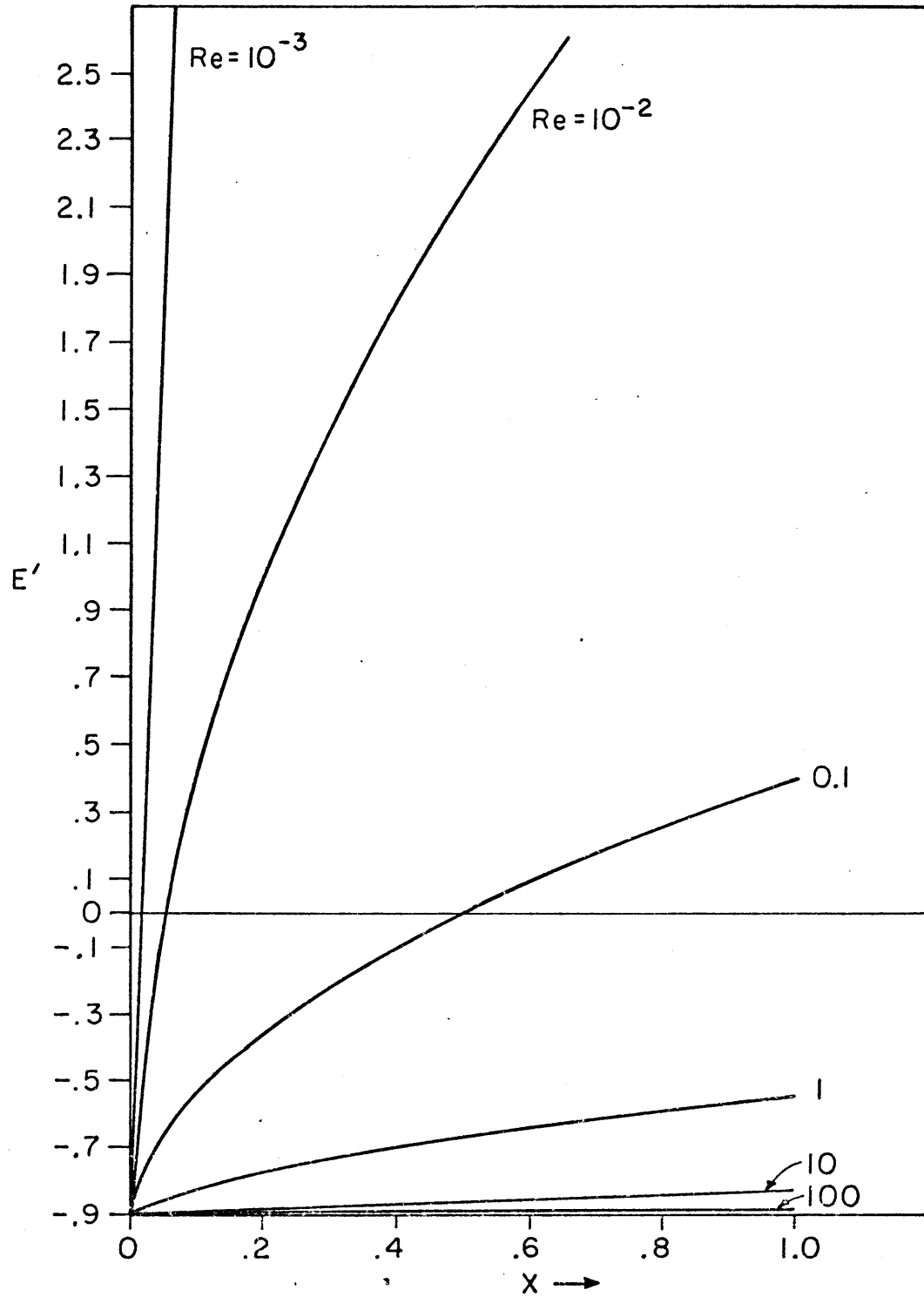


Fig. 3.4 Sample distribution of the normalized electric field for the indicated electric Reynolds number corresponding to the case study of Fig. 3.3

more nearly uniform is the distribution of charge within the convecting region. The lower electric Reynolds number case, corresponding to relatively more charge transport by conduction and less by convection, is characterized by a less uniform charge distribution in the convecting region.

3.2.2 Energy Conversion Regimes

To emphasize the electromechanical energy conversion process, consider the reaction of the electrical forces on the neutral gas. Our calculation should show that this reciprocal effect is small, for otherwise the kinematic approximation is not valid. The most efficient coupling between carrier and electrical forces occurs when the charged particles are rigidly fixed to the fluid, i.e., no slip between vertical carrier and the charged species is possible. In that event, all electrical forces are transmitted to the bulk of the fluid with no losses. Then the momentum balance for an element of fluid of mass density ρ_a carrying ρ coulombs per cubic meter is given by

$$\frac{dU}{dt} = - \frac{1}{\rho_a} \frac{\partial P}{\partial x} - g + \frac{\rho E}{\rho_a} \quad (3.2.20)$$

Since an upper bound on the electric field strength is probably about 10^5 V/m, and since ρ_a is of the order of 1 kg/m^3 , an upper bound on the electrical force is of the order $10^5 \rho$. Assuming 10^{10} m^{-3} elementary particles [Vonnegut, 1963], $\rho \sim 10^{-9} \text{ c/m}^3$, and the upper bound on the electrical acceleration is of order 10^{-4} m/sec^2 . As Vonnegut has noted, this is several orders of magnitude less than the typical acceleration of an air parcel whose temperature differs from the environment by only 1°C .

Assuming the carrier fluid to be incompressible, the steady-state pressure gradient due to the electrical interaction in a system in which the charged particles slip relative to the carrier gas is given by

$$\frac{dP}{dx} = \frac{\rho}{b} (v - U) \quad . \quad (3.2.21)$$

Using Gauss's law and the ion velocity constitutive relation, $v = U + bE$, Eq. (3.2.21) gives

$$\frac{dP}{dx} = \frac{\epsilon_0}{2} \frac{d}{dx} E^2 \quad (3.2.22)$$

so that

$$\Delta P = \frac{\epsilon_0}{2} (E^2 - E_0^2) \quad (3.2.23)$$

is the pressure difference between the boundaries due to the electrical forces. Thus, if the difference in boundary field strengths in Eq. (3.2.23) is of the order of (10^5 V/m) , then $\Delta P \approx .004 \text{ mb}$, which is certainly small relative to the pressure variations observed in thunderstorms. Using the terminal relation, Eq. (3.2.14), the above pressure difference can be calculated in terms of the normalized current density and electric Reynolds number. The line of zero pressure difference delineates the regimes of pumping action and braking action, while the line of zero potential difference separates the generation from the brake regime. These regions are shown in Fig. 3.2, with the braking regime indicated by the shaded area. The upper boundary is the line of zero potential difference, while the lower is the line of zero pressure difference found from Eqs. (3.2.22) and (3.2.14) to be given by

$$Re = \frac{J/\rho/oU}{2(1 - J/\rho/oU)} \quad . \quad (3.2.23)$$

3.2.3 Total Charge in the Generating Region

Gauss's law relates the normal field components at the surface of a closed region to the total charge within the enclosed volume. Since our model is one-dimensional, the only normal field components are to be found at $x' = 0$ and $x' = 1$. Defining Q' to be the normalized charge in the generating region,

$$Q' = \int_0^1 \rho' dx' \quad (3.2.24)$$

Then, from Gauss's law, Eq. (3.2.8),

$$Q' = \operatorname{Re} \int_0^1 \frac{dE'}{dx'} dx' \quad (3.2.25)$$

or

$$Q' = \operatorname{Re} [E'(1) - E'(0)]. \quad (3.2.26)$$

Then, for $\rho'(0) \equiv 1$, Eq. (3.2.14) defines E' to substitute in (3.2.26):

$$Q' = \operatorname{Re} J' \left[\left(1 + \frac{2}{\operatorname{Re} J'} \right)^{1/2} - 1 \right] \quad (3.2.27)$$

The relation between the normalized charge and actual charge can be found from Eq. (3.2.24),

$$Q' = \frac{Q}{\rho_0 AH} \quad (3.2.28)$$

where A is the horizontal area of the generating region. Thus,

$$Q = (\rho_0 AH) \operatorname{Re} J' \left[\left(1 + \frac{2}{\operatorname{Re} J'} \right)^{1/2} - 1 \right] \quad (3.2.29)$$

defines the generator charge content.

The normalized charge in the generating region is shown against $\operatorname{Re} J'$ in Fig. 3.5.

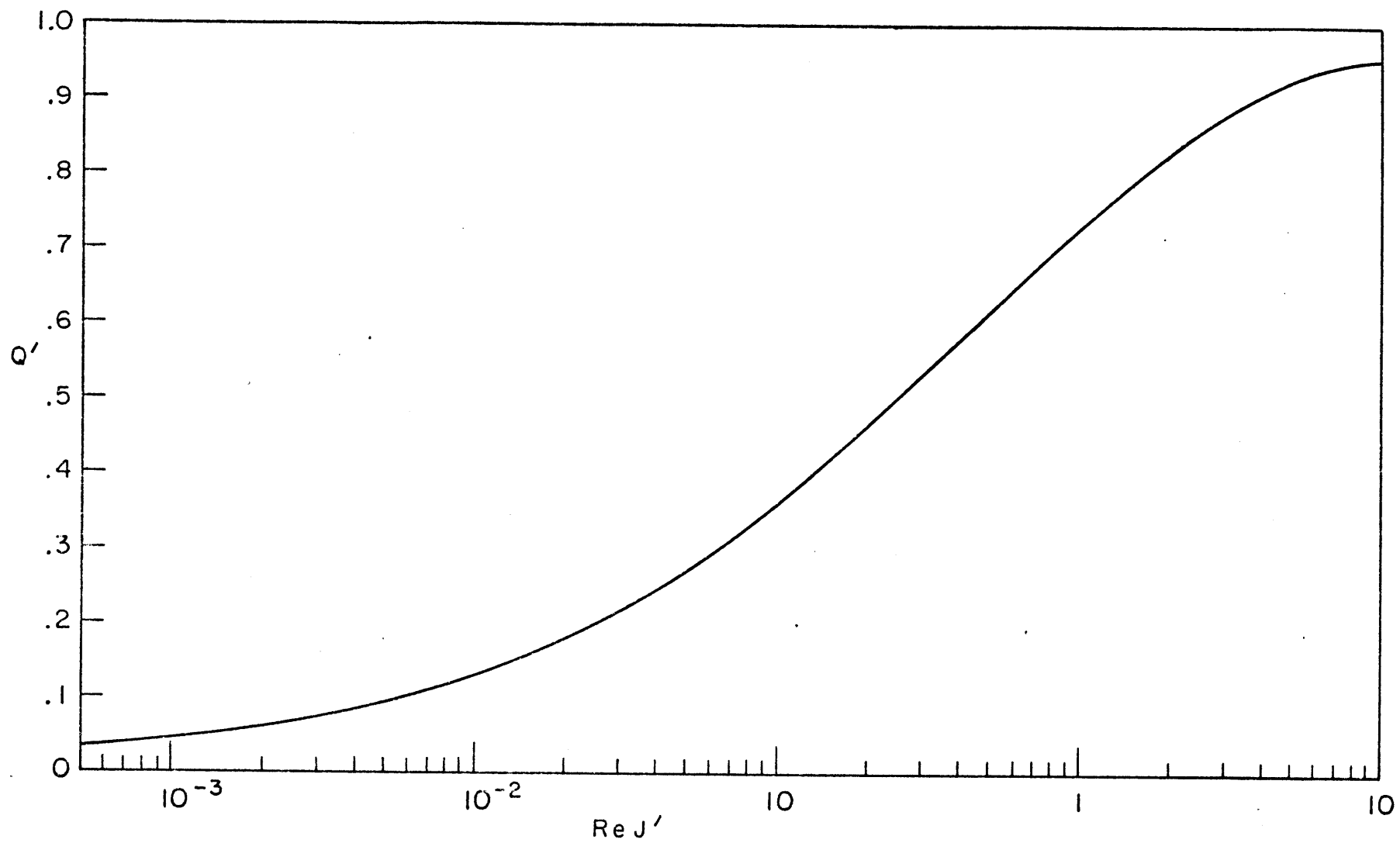


Fig. 3.5 Normalized generator charge content

3.3 The Unipolar Load

The calculations in Fig. 3.2 are independent of the manner in which the steady state is assumed by the return path, shown schematically in Fig. 3.1b. These curves can be used to represent the generator as one part of a total, perhaps complex, system. From the point of view of the generator, the remainder of the system appears as a load. Given "terminal" characteristics of the load, we can find the generated potential in terms of the electric Reynolds number alone.

The importance of the load is emphasized by noting that the coupling of the generator to a load of known characteristics determines uniquely the normalized current density, J' , associated with a given Reynolds number. Thus, the normalized charge within the generating region given by Eq. (3.2.27) is a determined quantity. This contrasts with the gravitational generator model of Gunn [1954] in which he analyzed the steady-state distribution of charge and field in a system of falling charged raindrops in a conducting medium. In Gunn's model, the load was not specified and results were given in terms of an assumed boundary electric field strength. This assumption is not necessary in a closed system; in fact, it predetermines the quantity of charge in the generating region.

In this section, we consider a simple model for a load, and exemplify the application of the results for the unipolar generator having characteristics summarized by Fig. 3.2. In the following discussion, we will assume the convection speed in the load vanishes, but it should be clear that, in general, the load convection could be arbitrary.

The localized nature of vertical convection with which we are concerned suggests that the area of the load could be considerably larger

than that of the generator. The extreme case is seen in situations for which the convection is so intense that coupling of the ground equipotential to the equalizing layer at about 50 km is accomplished through the agency of convective transport. In this case, the area of the load is given essentially by the remainder of the global atmospheric circuit.

3.3.1 The Static Atmosphere Solution

The unipolar load, as depicted in Fig. 3.1b, is visualized as a region similar to the generating region, except that the convection speed vanishes. Since the area of the load can be different in general from that of the generating section, the current density in the load may differ from that in the generator, although the total current through both regions must balance. Also, it is not necessary that the charge carriers in the load have the same characteristics as those in the generator, nor should the boundary charge densities in the two regions necessarily be the same. For these reasons, these quantities in the load are identified with subscript L to distinguish them from their counterparts in the generator.

The equations for the potential in a static atmosphere can be obtained from Eqs. (3.2.18) and (3.2.19). To do this, expand the normalized current density J' and the electric Reynolds number, Re , into their components. Then solve for the electric potential $V = (UH/b)V'$ and take the limit as U goes to zero. Then the unipolar static atmosphere equations for the potential are

$$V_L = - \frac{\epsilon_o J_L^2}{3b_L^2 \rho_L^3(0)} \left[\left(1 + \frac{2b_L H_L \rho_L^2(0)}{\epsilon_o J_L} \right)^{3/2} - 1 \right] \quad (3.3.1)$$

for boundary values $\rho_L(0)$ fixed at $x' = 0$, and

$$V_L = \frac{\epsilon_o J_L^2}{3b_L^2 \rho_L^3(1)} \left[\left(1 - 2 \frac{b_L H_L \rho_L^2(1)}{\epsilon_o J_L} \right)^{3/2} - 1 \right] \quad (3.3.2)$$

when the charge density boundary value $\rho_L(1)$ is determined at $x' = 1$.

3.3.2 Coupling of the Unipolar Load and Generator

The current density in the load, J_L , is related to that in the generator by

$$J_L = -fJ \quad (3.3.3)$$

where f is the ratio of the generator area to the load area. The static atmosphere solutions can be normalized according to the same normalization as in the generator. Thus, multiplying Eq. (3.3.1) and (3.3.2) by the normalizing factor, b/UH , and after some manipulations, the normalized load potential in terms of the load characteristics and the generator parameters is given by

$$V'_L = -\frac{f^2}{3} \left(\frac{b}{b_L} \right)^2 \left(\frac{\rho_o}{\rho_L(0)} \right)^3 \operatorname{Re} J'^2 \left\{ \left[1 - \frac{2}{f} \frac{H_L}{H} \frac{b}{b_L} \left(\frac{\rho_L(0)}{\rho_o} \right)^2 \frac{1}{\operatorname{Re} J'} \right]^{3/2} - 1 \right\} \quad (3.3.4)$$

for load boundary values at $x' = 0$, and

$$V'_L = \frac{f}{3} \left(\frac{b}{b_L} \right) \left(\frac{\rho_o}{\rho_L(1)} \right)^3 \operatorname{Re} J'^2 \left\{ \left[1 + \frac{2}{f} \frac{H_L}{H} \frac{b}{b_L} \left(\frac{\rho_L(1)}{\rho_o} \right)^2 \frac{1}{\operatorname{Re} J'} \right]^3 - 1 \right\} \quad (3.3.5)$$

when load boundary conditions are imposed at $x' = 1$. Note that in these expressions, it is immaterial at which boundary the generator ion density is determined.

A case study is presented in Figs. 3.6 and 3.7 in which positive ions are injected at $x' = 0$ in the generator, and returned through a load whose characteristics are described by $b_L/b = 1$, $H_L/H = 3$, $\rho_L(1)/\rho_o = 5$ and $f = 0.1$. Then Eq. (3.3.5) is used to generate the normalized load

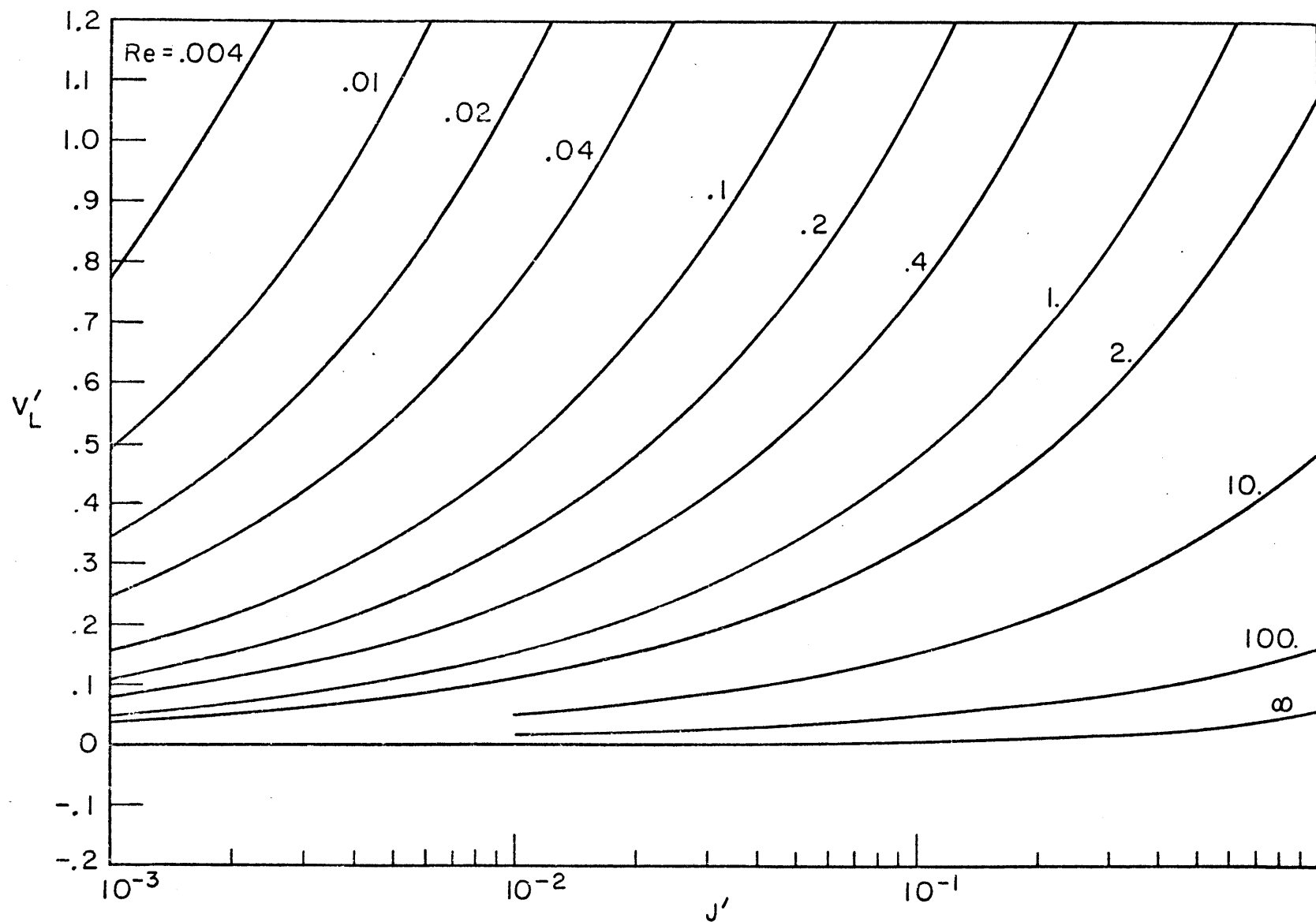


Fig. 3.6 Sample calculation of load operating characteristics computed for the parameters $b_L/b = 1$, $H_L/H = 3$, $\rho_L(1)/\rho(0) = 5$ and $f = 0.1$

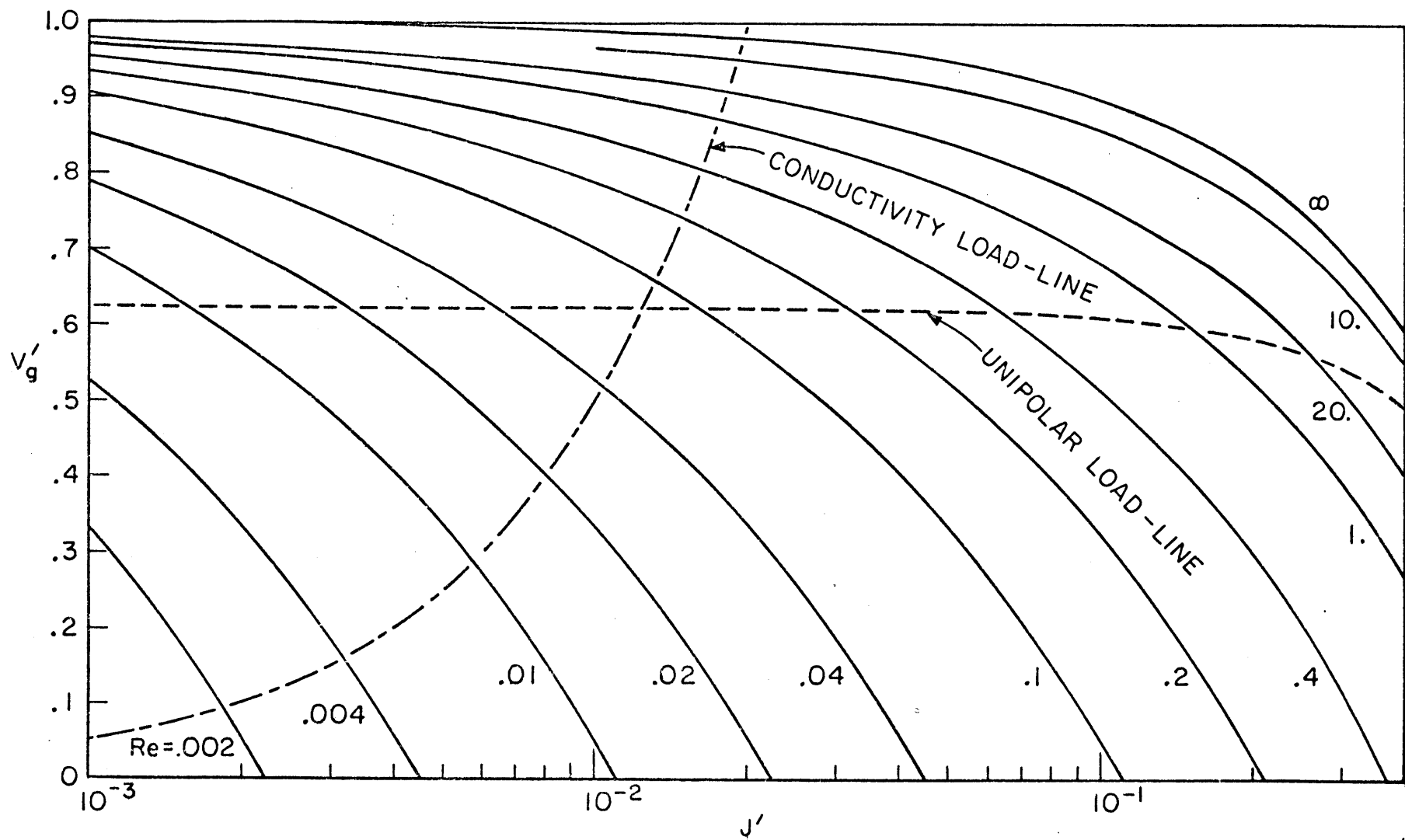


Fig. 3.7 Terminal relations of Fig. 3.2 expanded for values of $V' > 0$ with superposed load-lines of a conductivity load and the unipolar load of Fig. 3.6

potential curves in Fig. 3.6 and Eq. (3.2.18). gives the generator curves in Fig. 3.7. Since $V' = V'_L$ when the two sections are coupled, overlapping the two sets of curves and marking the coincident solutions gives a mapping of the normalized current density in the generator against electric Reynolds number, along with the steady-state potential established across the generating and loading section by virtue of the convective transport. The load line corresponding to the load shown in Fig. 3.6 is indicated in Fig. 3.7 as the nearly straight dashed line.

Notice that the steady-state normalized potential in this case is constant over a large range of Re , showing that for $Re < 1$, the potential established is independent of ion density, and dependent on the section height, ion mobility, and convection speed. Thus, for $Re < 1$,

$$V \propto \frac{UH}{b} \quad (3.3.6)$$

This result is explained by noting that, when

$$\left| \frac{2}{f} \frac{H_L}{H} \frac{b_L}{b} \frac{\rho_L^2(1)}{o_o^2} \frac{1}{Re J'} \right| \gg 1 \quad (3.3.7)$$

Eq. (3.3.5) is approximately

$$V'_L \approx \left[\frac{8f}{9} \frac{b}{b_L} \left(\frac{H_L}{H} \right)^3 \frac{J'}{Re} \right]^{1/2} \quad (3.3.8)$$

For $Re \ll 1$, Eq. (3.2.18) is approximately

$$V' = 1 - \left(\frac{8}{9} \frac{Re}{J'} \right)^{1/2} \quad (3.3.9)$$

Setting $V'_L = V'$ in Eqs. (3.3.8) and (3.3.9) gives the constant of proportionality in Eq. (3.3.6),

$$V = \left[\frac{\left[f \left(\frac{H_L}{H} \right)^3 \frac{b}{b_L} \right]^{1/2}}{1 + \left[f \left(\frac{H_L}{H} \right)^3 \frac{b}{b_L} \right]^{1/2}} \right] \frac{UH}{b} \quad (3.3.10)$$

Moreover, in this region the relation of the normalized current density to Re can be found from Eqs. (3.3.8) and (3.3.9):

$$J' = \frac{9 \text{ Re}}{8 \left\{ 1 + \left[f \left(\frac{H_L}{H} \right)^3 \frac{b}{b_L} \right]^{1/2} \right\}^2} \quad (3.3.11)$$

In the limit as $f \rightarrow 0$, $V = 0$ and $\text{Re} = \frac{8}{9} J/\rho_0 U$, corresponding to the short-circuit load approximation. The curves in Fig. 3.8 are given for Re small and demonstrate the influence of changing the parameter $f(H_L^3/H^3)(b/b_L)$.

The large Re limit is easily analyzed by noting that for large Re, Gauss's law of Eq. (3.2.8) is effectively homogeneous. Then in the generator,

$$V' \approx 1 - J' \quad (3.3.12)$$

Since Eq. (3.3.7) still holds, so long as f is made small enough, we can combine these two equations to obtain

$$V' = \frac{1}{2} [(A^2 + 4A)^{1/2} - A] \quad (3.3.13)$$

where

$$A = \frac{8/9 f H_L^3/H^3 b/b_L}{\text{Re}} \quad (3.3.14)$$

This approximation is valid only if the product $f \text{ Re} \ll 1$.

Figure 3.9 shows the logical extension of Fig. 3.8 as Re increases. Notice that the curves are identified by the parameter $f(H_L^3/H^3)(b/b_L)$ for $\text{Re} < 1$ and for larger Reynolds numbers the pertinent parameter is $(1/f)(H/H_L)(b_L/b)[\rho_L^2(1)/\rho_0^2]$ since in this region the ratio of boundary

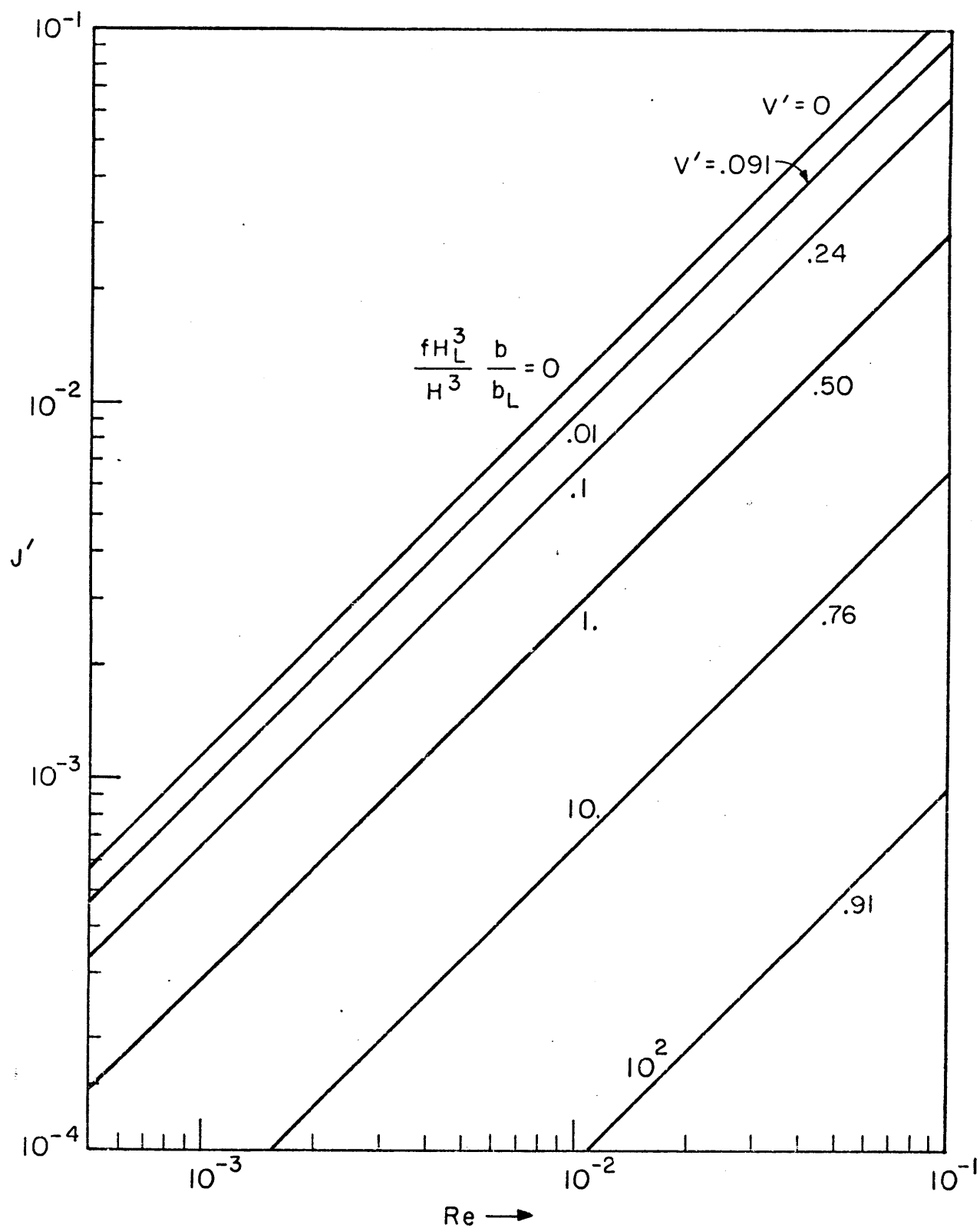


Fig. 3.8 Low electric Reynolds number operation of a unipolar generator coupled to a unipolar load. Normalized voltage is constant for each load characterized over the range of Re of the plot.

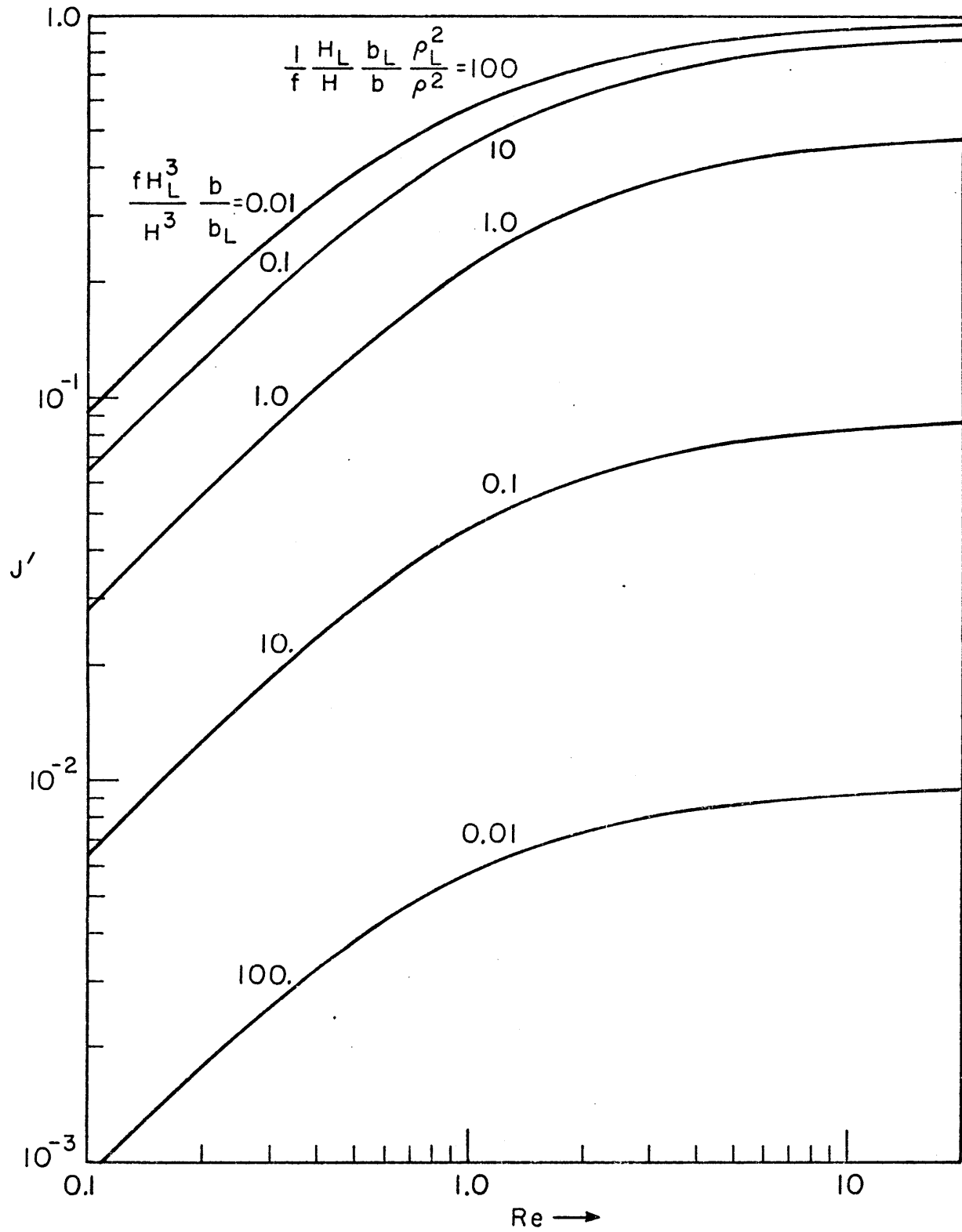


Fig. 3.9 Transition from low to high electric Reynolds number for a unipolar generator coupled to a unipolar load; note the change in parameter between $Re \sim 0.1$ and $Re > 1$.

charge densities for the generator and load sections becomes important. The curves in Fig. 3.8 were calculated directly from Eq. (3.3.11), whereas those of Fig. 3.9 were obtained numerically by using Newton's method to solve simultaneously Eqs. (3.3.1) and (3.3.5). The success of both approaches is suggested by the coincident solutions at $Re = 0.1$.

3.4 An Empirical Ohmic-Conductivity Load

According to most of the past work in atmospheric electricity, the current-field relation in a static atmosphere can be expressed by a conductivity model in which the current density is linearly proportional to the electric field,

$$J_L = \sigma E \quad (3.4.1)$$

where σ is the local conductivity. The major distinction between the conductivity and unipolar mobility models of current flux is that, in a conductivity model, there can persist a current density in the bulk in the absence of a flux of net charge, whereas in the unipolar model, the current density is a consequence of the presence of net charge. In addition, the conductivity is independent of the electric field, whereas in a mobility model, the space charge and electric field are related by Gauss's law.

3.4.1 The Integrated Resistivity

Integrating Eq. (3.4.1) for the steady state,

$$V_L = - J_L \int_0^{H_L} \frac{dx}{\sigma} \quad (3.4.2)$$

The inverse of the conductivity is termed the "resistivity", w , and is given by Israel [1970] against height above the ground, as

$$w = (2.94 e^{-4.52h} + 1.39 e^{-0.375h} + 0.369 e^{-0.121h}) \times 10^{13} \text{ ohm-m} \quad (3.4.3)$$

where h is given in kilometers above ground level. Table 3.2 gives values of the integrated resistivity, w , as a function of height above the ground. The third column of Table 3.2, $w_1(H_L)$, gives the integrated resistivity between the one-kilometer level and levels H_L km higher. Clearly, the 1 km level chosen is arbitrary, and the tabulated data provide the necessary information to choose any other starting point desired.

Table 3.2

h , km	$w \times 10^{-16}$, ohm-m ²	$w_1(H_L) \times 10^{-16}$ ohm-m ²	H_L , km
.5	1.3		
1.0	2.3		
1.5	2.9		
2.0	3.4	1.1	1
2.5	3.7		
3.0	4.0	1.7	2
3.5	4.2		
4.0	4.4	2.1	3
4.5	4.5		
5.0	4.7	2.4	4
5.5	4.75		
6.0	4.8	2.5	5
6.5	4.85		
7.0	4.9	2.6	6
7.5	5.0		
8.0	5.1	2.8	7
8.5	5.15		
9.0	5.2	2.9	8
9.5	5.25		
10.0	5.3	3.0	9

3.4.2 Coupling of the Unipolar Generator and Conductivity Load

Using the integrated resistivity values given in Table 3.2, the load current density of Eq. (3.4.1) is written

$$V_L = - J_L w_k(H_L) \quad (3.4.4)$$

where the subscript, k , refers to the reference level at which the integration of Eq. (3.4.2) is initiated. In the following, we take $k = 1$ for purposes of illustration.

In order to find the consistent solutions, $V_L = V$, we normalize V_L in Eq. (3.4.4) by the generator potential normalizing factor, b/UH , and making use of the constraint

$$J_L = - fJ \quad , \quad (3.4.5)$$

we write for the load-normalized potential,

$$V'_L = \frac{bf\rho_0}{H} w_1(H_L) J' \quad . \quad (3.4.6)$$

In the unipolar load, we used the parameters $f = 0.1$, $H_L/H = 3$. For purposes of comparison of the two load results, we choose $H_L = 3$ km and $H = 1$ km, $f = 0.1$. Then, $w_1(3) = 2.1 \times 10^{16}$ from Table 3.2, and for $b = 10^{-4}$,

$$V'_L = 2.1 \times 10^8 \rho_0 J' \quad (3.4.7)$$

where ρ_0 is the boundary ion density in the generator. Thus, a variety of load-lines can be constructed on Fig. 3.7, depending on the boundary ion density in the generator.

Choosing $\rho_0 \approx 2.5 \times 10^{-7}$ c/m³, which is perhaps high for fair-weather conditions, but may be attained in disturbed conditions, results in the load-line shown in Fig. 3.7.

In Fig. 3.10 is plotted the normalized current density against electric Reynolds number for this value of boundary space-charge density. Since the boundary ion density is a fixed quantity for this curve, the abscissa, instead of electric Reynolds number, is really a measure of the ratio of convection speed to the height of the generator, U/H .

3.4.3 Comparison of Theoretical and Empirical Load Results

Also shown for comparison in Fig. 3.10 are the dashed lines derived from the theoretical load lines of Figs. 3.6 and 3.7, in which the current return path is an identical unipolar model whose convective speed is zero. The difference between the two curves is in the ratio of generator and load mobility values. In the lower of the two curves, the mobility ratio is unity, signifying that the charged particles in both load and generator sections have a common origin. The upper of these two curves is calculated for a generator mobility three orders of magnitude less than that in the load, and could be representative of situations in which convective roots extend below the exchange layer. In this region, the aerosol density is high and more than half the charge carriers are found to be large ions whose mobility is nearly 10^{-3} times the mobility of the small ions found in the cleaner upper levels. The generated potential for given Re is greater in the case for which $b_L = b$ than when $b_L = 10^3 b$. For example, choosing $Re = 1$, the former of the two cases gives $V' = 0.59$, whereas the latter is $V' = 0.04$. Thus, the voltage across the load necessary to return the current is much less in the second case, because of the higher load particle mobility implied by $b_L = 10^3 b$. On the other hand, since the product $Re J'$ is always greater for the case $b_L > b$ than it is when $b = b_L$, the total charge contained in the generating

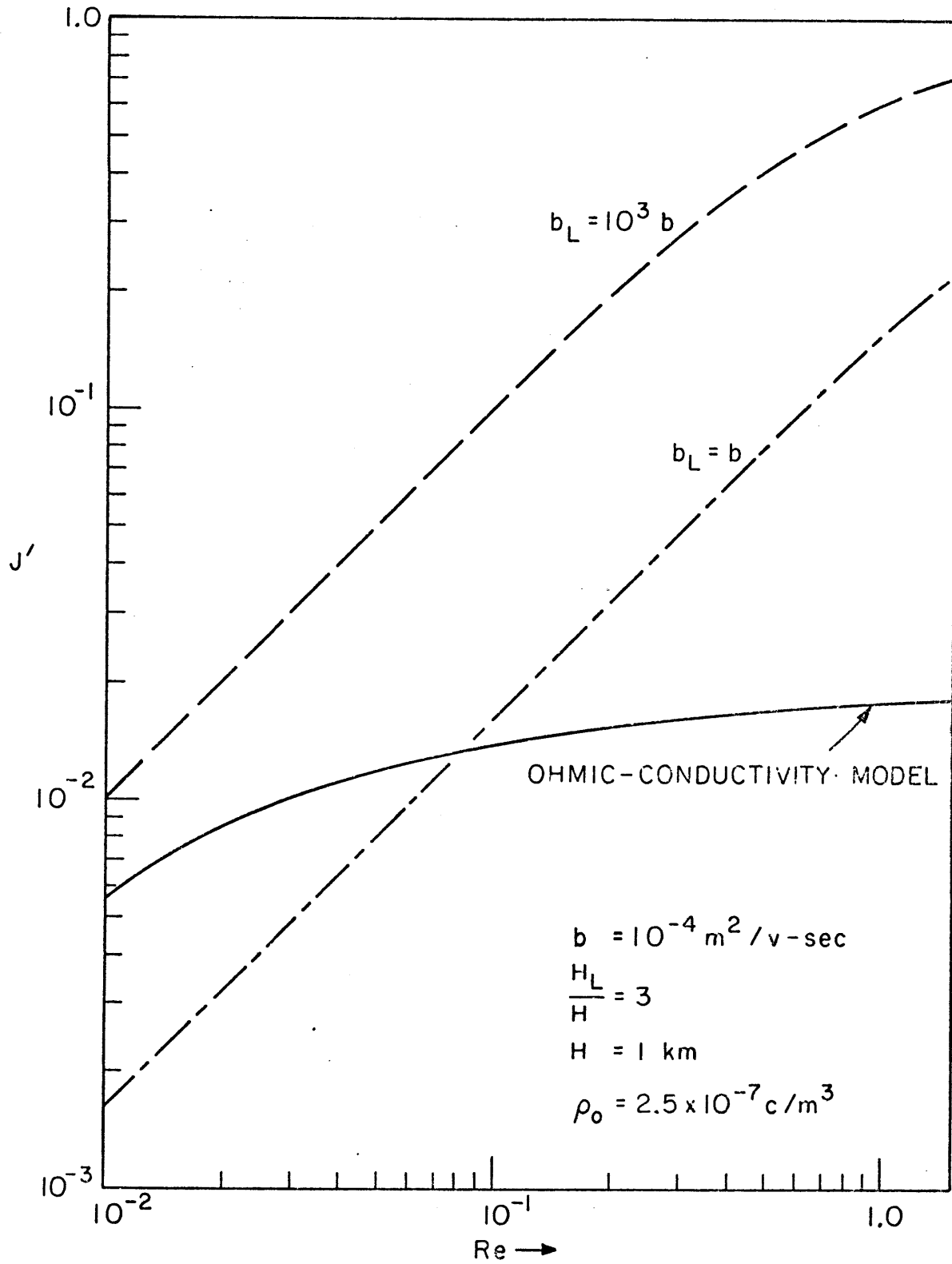


Fig. 3.10 Case studies of the unipolar generator coupled to a conductivity load (solid line) and two different unipolar loads (dashed lines)

region is greater, as can be seen from Fig. 3.5. The general result of this study is that, when the load particle mobility is greater than the generator particle mobility, the generated potential is less, but the generator charge content is greater.

Since only the ratio b_L/b appears in the terminal relations for V' and V'_L , the two case studies can also be used to investigate the effect of changing the generator particle mobility while maintaining the load particle mobility constant. The comparison in this case, however, must be done for different Re in the two cases, since keeping U and ρ_o constant while changing b changes Re . Thus, selecting $Re = 10$ in the case $b = b_L/10^3$, then the same initial conditions exist in the case $b = b_L$ when $Re = 10^{-2}$. In the former case, $J' = 0.95$ and $V' \approx 0$ for $Re = 10$, whereas in the latter case, $J' = .0016$ and $V' \approx 0.62$. Thus, increasing the generator particle mobility, for the same load terminal characteristics and the same convection speed and boundary ion density, gives an increased generated potential, but a lower charge content in the generator.

Of some considerable interest is the demonstrated difference in behavior between the theoretical and empirical load behavior shown in Fig. 3.10. On the one hand, much of the past work in atmospheric electricity is based on a conductivity model for current-field relationships, while on the other hand, many order-of-magnitude calculations are made using the unipolar atmosphere concept as, for example, Vonnegut's calculations of the convective electrification effect. Such a contrast in results using the two models suggests that a somewhat more critical view of their use is advisable.

That the conductivity model is probably the more realistic of the two studies is suggested by considering the difference between a unipolar and a bipolar atmosphere. There is no question that the atmosphere is bipolar, and the approximation by a unipolar model introduces some error. If we define a new variable, ρ_T , to represent the total charge carrier density, while retaining the definition of ρ for the net charge density, then the current density expression Eq. (3.2.2) becomes

$$J = \rho U + b \rho_T E \quad . \quad (3.4.8)$$

Thus, in a bipolar atmosphere, the net space charge couples to the convection, whereas it is the total charge carrier population which participates in the conduction current. Rewriting Eq. (3.4.8) in the form of a unipolar model,

$$J = \rho \left[U + b \frac{\rho_T}{\rho} E \right] \quad (3.4.9)$$

where the magnitude of the correction factor, ρ_T/ρ , is always greater than one, and usually very much so. In the load, the convection vanishes and hence the unipolar current calculation is wrong by this correction factor. The implicit inclusion of the bipolar effect in the empirical conductivity model suggests that these results may be more applicable to atmospheric problems.

When the convection current dominates, the unipolar model is likely to be more applicable, since then the bipolar effects in the conduction current are not so important. However, in the static atmosphere load, the conduction current dominates, so that the results of the unipolar load coupled with the unipolar generator are probably not realistic.

3.5 A Piece-wise Continuous Model with Variable Mobility

The general application of the terminal relations, Eqs. (3.2.18) and (3.2.19) to any horizontal section in which the model constraints are valid provides a useful technique to examine less restrictive situations which are more representative of actual atmospheric conditions. For example, two regions with differing properties which are coupled at a common horizontal boundary can be treated by separate application of the terminal relations, provided the model constraints are valid in each region, and consistent boundary conditions coupling the two sections can be found.

A particular example of this method is one in which the mobility varies from one section to the next. Since ionic mobility is a sensitive function of the ambient gas density, and since atmospheric pressure and temperature vary with height, mobility is also a function of altitude. Thus, the constant mobility constraint in the model is not obeyed in the atmosphere, particularly for horizontal slabs whose depths are of the order of a kilometer or greater. A means of approximating the variable mobility effect is to construct a piece-wise continuous model in which the generating section is subdivided into numerous thin slices in each of which the mobility varies only slightly. In each subsection, the mobility is treated as constant, and given by the average value over the thin slice. The terminal variables for that section can then be calculated and used along with the steady-state assumption to provide boundary conditions for the next section.

3.5.1 Calculation of the Vertically Averaged Mobility

Israel [1970] states that small ion mobility is given by

$$b(p,T) = b(p_o T_o) \frac{p_o T}{T_o p} \quad (3.5.1)$$

Assuming the vertical temperature and pressure conform to a dry adiabatic atmosphere, Eq. (3.5.1) can be expressed as

$$b(p,T) = b(p_o T_o) \left(\frac{T}{T_o} \right)^{1/1-\gamma} \quad (3.5.2)$$

where the ratio of specific heats of dry air is $\gamma = 1.403$. According to Brunt [1952], rising dry air tends to assume a vertical temperature gradient given by the dry adiabatic lapse rate, $\Gamma = 9.86 \times 10^{-3} \text{ }^\circ\text{K/m}$, so that within the convecting region,

$$b(x) = b(0) \left(1 - \frac{\Gamma x}{T_o} \right)^{1/1-\gamma} \quad (3.5.3)$$

The average mobility in a layer of thickness $\Delta_i = x_i - x_{i-1}$ is

$$\bar{b}_i = \frac{1}{\Delta_i} \int_{x_{i-1}}^{x_i} b(x) dx \quad (3.5.4)$$

or, From (3.5.3),

$$\bar{b}_i = \frac{b_o}{\Delta_i} \int_{x_{i-1}}^{x_i} \left(1 - \frac{\Gamma x}{T_o} \right)^{1/1-\gamma} dx \quad (3.5.5)$$

where b_o and T_o are the mobility and temperature at the lower boundary of the entire horizontal section. Thus, the average mobility in the i^{th} subdivision is

$$\bar{b}_i = \frac{b_o T_o}{\Gamma \Delta_i} \left(\frac{1-\gamma}{2-\gamma} \right) \left[\left(1 - \frac{\Gamma x_{i-1}}{T_o} \right)^{\frac{2-\gamma}{1-\gamma}} - \left(1 - \frac{\Gamma x_i}{T_o} \right)^{\frac{2-\gamma}{1-\gamma}} \right] \quad (3.5.6)$$

3.5.2 Variable Mobility Case Study

Table 3.3 gives calculated values of $\bar{b}_i$ from Eq. (3.5.6)

assuming that $b_o = 10^{-4} \text{ m}^2/\text{volt-sec}$ and $T_o = 23 \text{ }^\circ\text{C}$. The two cases pre-

sented are for one- and five-kilometer sections, each divided into 10 subsections of 100 and 500 meter thicknesses, respectively. Calculation of the curves of Figs. 3.11 and 3.12 corresponding to the above two cases proceeds as follows. Choose Re and J' and calculate $Re_{i=1}$ according to the relation

$$Re_{i=1} = \frac{Re H}{\Delta_1} \quad (3.5.7)$$

Table 3.3

x'	$H = 1 \text{ km}$	$H = 5 \text{ km}$
	$\bar{b}_i \times 10^3$	$\bar{b}_i \times 10^3$
0.1	.100	.102
0.2	.101	.106
0.3	.102	.111
0.4	.103	.116
0.5	.104	.121
0.6	.105	.127
0.7	.106	.132
0.8	.106	.139
0.9	.107	.145
1.0	.108	.152
	$\bar{b} = .104$	$\bar{b} = .125$

where H/Δ_1 is the ratio of the entire depth of the section to the depth of the first subdivision. The normalized potential across the first subsection, $V'_{i=1}$, is calculated via the terminal relation (3.2.18), and the ratio of ion densities at the upper and lower boundaries of this section, $\rho'_{i=1}$, is calculated from (3.2.12) for $x' = 1$.

Calculation of the boundary values Re and J' for the next subsection can then be accomplished by virtue of the steady-state uniformity of the current density J and continuity of the electric field across the boundary. Thus, for the next subsection,

$$J'_{i+1} = \frac{J}{\rho_o|_{i+1} U} = 1 + \frac{\bar{b}_{i+1}}{\bar{b}_i} \left[\frac{J'_i}{\rho'_i} - 1 \right]. \quad (3.5.8)$$

The electric Reynolds number in the next section is given by

$$Re_{i+1} = \frac{\frac{\bar{b}_i}{\bar{b}_{i+1}} - 1 + \frac{J'_i}{\rho'_i}}{J'_i} Re_i \quad (3.5.9)$$

and calculation of V'_{i+1} and ρ'_{i+1} proceeds as before. Then the total potential normalized to the depth H , and overall average mobility $\bar{b} = \frac{1}{H} \int_0^H b(x) dx$ is given by

$$V' = \frac{\bar{b}}{H} \sum_{i=1}^N \frac{\Delta_i}{\bar{b}_i} V'_i \quad (3.5.10)$$

and can be compared to the value obtained for the same Re and J' , assuming $b = \bar{b}$ throughout the section. In principle, these calculations can be carried out with the curves of Figs. 3.2 and Eq. (3.2.12). In practice, however, it was found that the errors incurred even with interpolation become too great for practicable results, because of the small differences from one section to the next. Consequently, the above procedure was followed using the computer and the results are shown by the solid lines of Figs. 3.11 and 3.12 for the one- and five-kilometer sections, respectively. The dashed lines are conductivity model load-lines identical to the ohmic

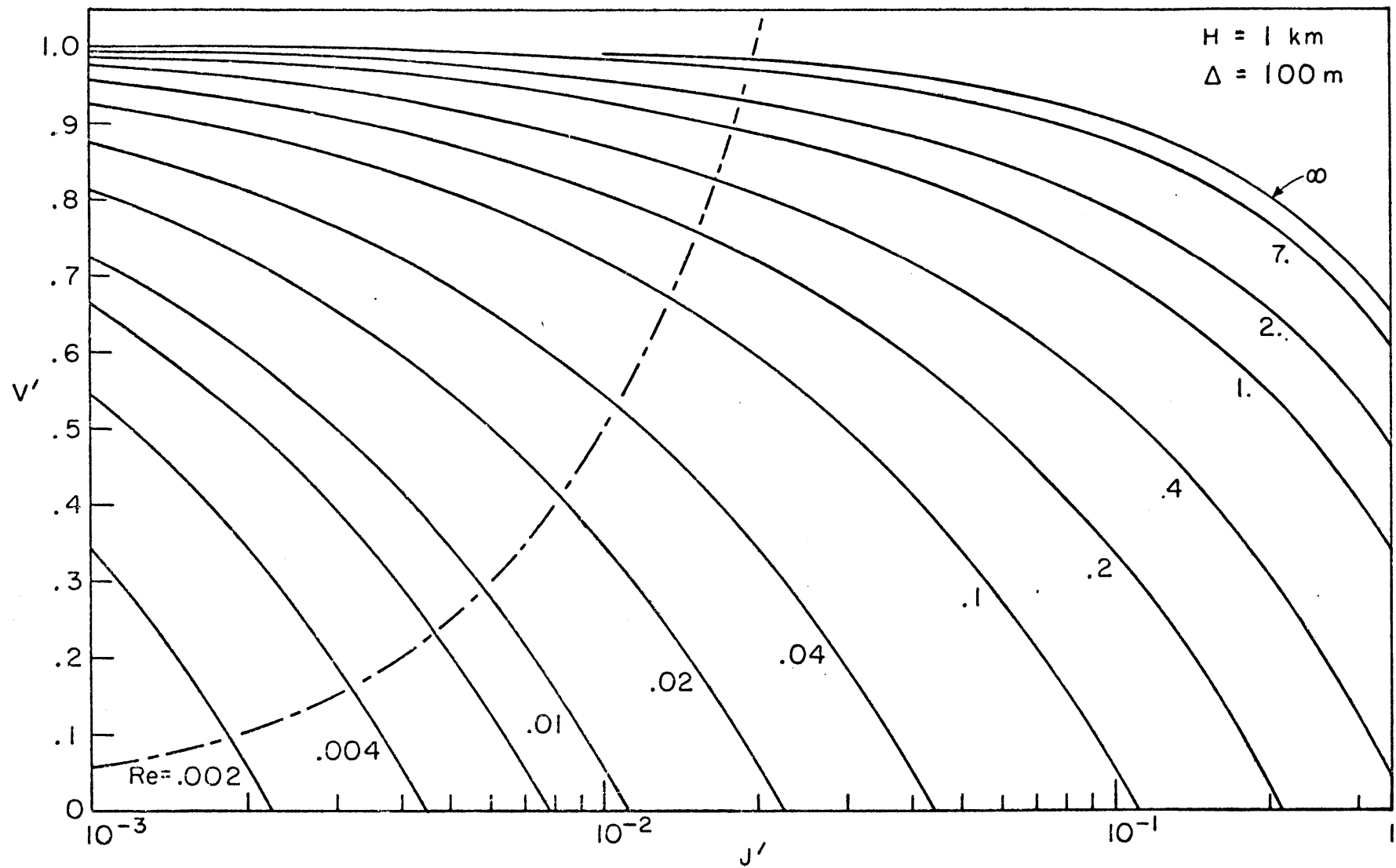


Fig. 3.11 Terminal relations for a 1 km generating region with variable mobility

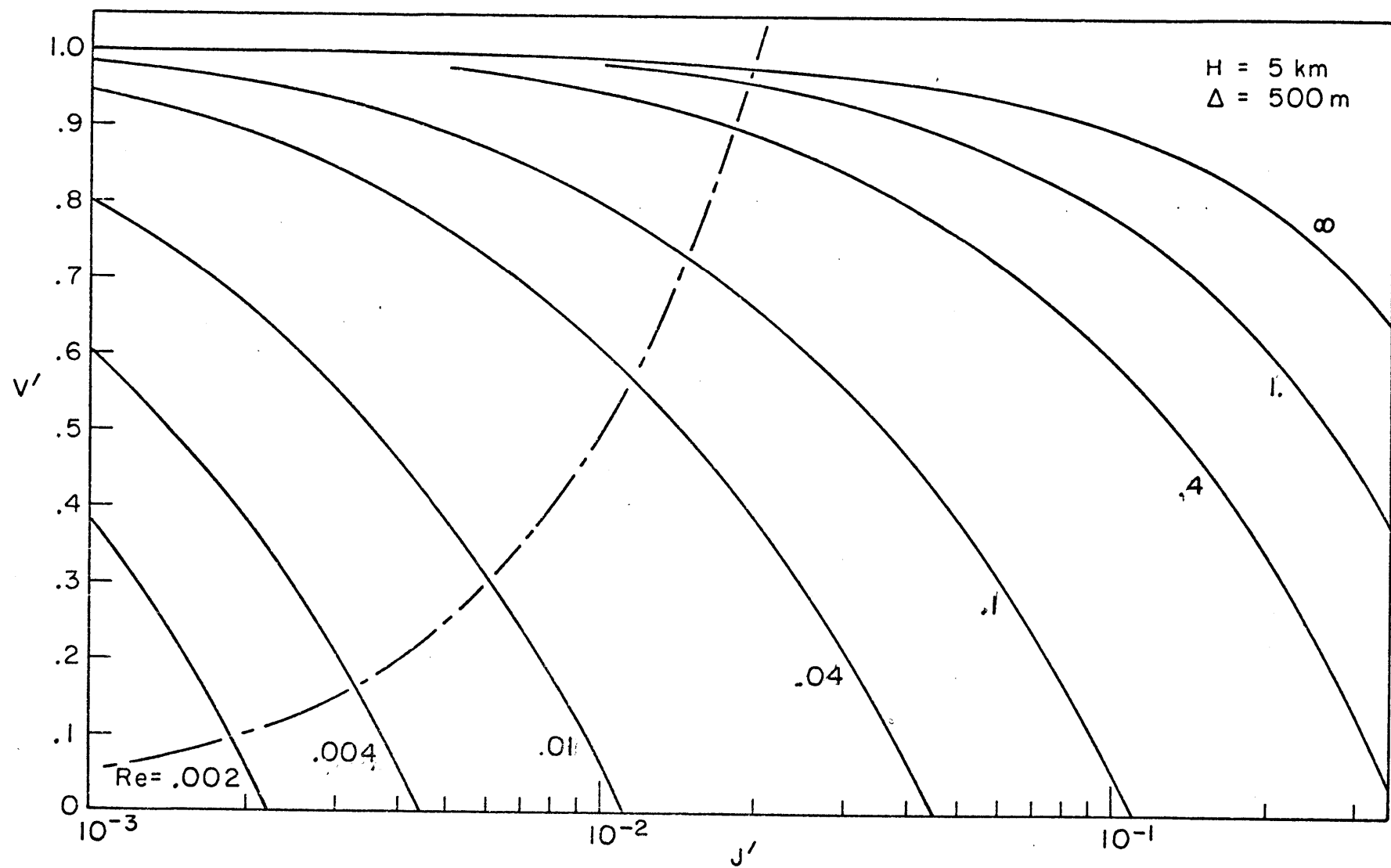


Fig. 3.12 Terminal relations for a 5 km generating region with variable mobility

load-line of Fig. 3.6.

The coincident solutions for these two cases are shown in Fig. 3.13 giving the normalized current density against the electric Reynolds number as in Fig. 3.10. It is immediately apparent that there is little difference between the two curves, although there is considerable difference in the range of mobility variation for the two cases. Overlap of Fig. 3.10 and Fig. 3.13 demonstrated further that the normalized boundary field strength is not very sensitive to a variable mobility, at least for an ohmic load. A further examination showed that the predicted values of normalized potential in the two cases differed by amounts less than 10 percent.

It appears that the assumption of a uniform mobility in the unipolar model is acceptable, at least for slab thicknesses of five km or less. It is nevertheless important to use a representative mobility corresponding to particular applications when descaling the parameters V' and E' , since a 50 percent error in mobility results in a 50 percent error in the calculated potential and electric field.

3.6 Summary

The results of this chapter are to be found principally in the development and exemplified use of a methodology for treating individually parts of a complex system, and coupling these sections to better understand their relationships and predict their behavior. It is not expected that the results of the case studies presented are likely to have explicit value in application to the natural atmosphere, since the unipolar approximation is probably too gross for very realistic quantitative results. However, some generally applicable conclusions can be reached from the work.

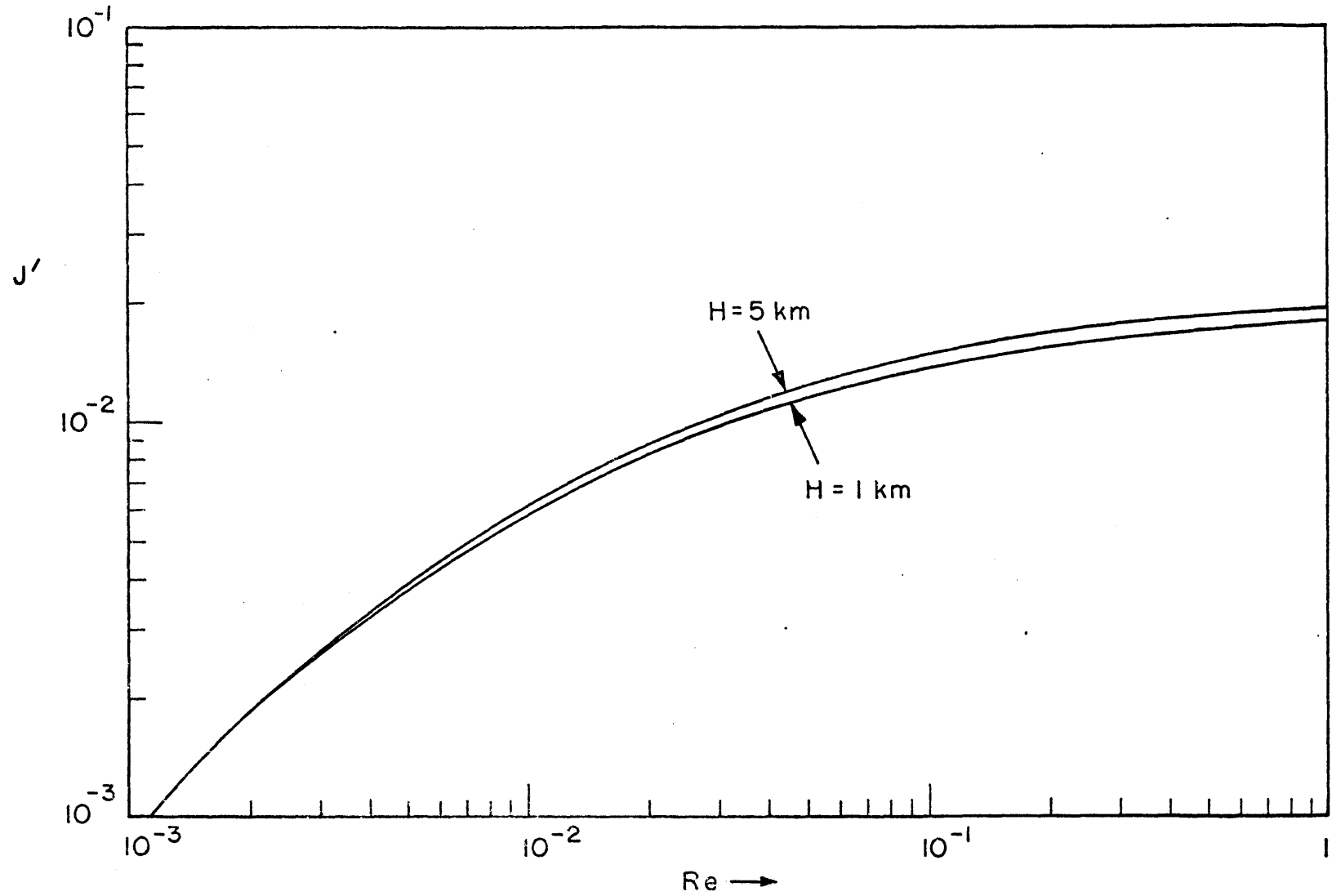


Fig. 3.13 Operating characteristics of 1 and 5 km generating sections with variable mobility

First, it was demonstrated in § 3.4.3 that a serious discrepancy exists between the predicted behavior of a generator when coupled to a conductivity load and a unipolar model. Reasons were given for believing that the conductivity load model is more desirable than the unipolar load, and it is concluded that other calculations based on a unipolar concept may be in serious error. This conclusion leads naturally to the topic of the next chapter in which the bipolar convective generation is analyzed.

The second main conclusion of this study is that the model results are not very sensitive to a variable mobility, at least over the range of small ion mobilities in the lower atmosphere. As was noted in § 3.5.2, however, the assumed average value of the mobility used in the calculation is crucial to obtaining true magnitudes of the dependent variables from the normalized results of Fig. 3.2 and others. On the basis of this conclusion, the effect of variable mobility will henceforth be ignored.

CHAPTER IV. BIPOLAR CONVECTIVE GENERATION

4.1 Introduction

Charge transport in the atmosphere is accomplished by the motion of charged particles of diverse mobilities and both polarities, whereas the unipolar model of the previous chapter treats the transport by the motion of a single charge carrier family representing the net space charge density. The convection current of a bipolar atmosphere is given by the transport by neutral flow of the net space charge, and in this respect, the unipolar model resembles the atmosphere. However, the bipolar conduction current is given by the response of the total charge density to the electric field instead of the net charge density, and thus may differ considerably from the unipolar model. The error committed by the unipolar model is such as to cause the generated potentials to be too large for realistic boundary conditions on the space charge density and electric field.

To determine the validity of the unipolar model, and to more realistically predict the action of convective generation, it is desirable to investigate a bipolar convective generator. In addition, since the ions come about by virtue of bulk generation and annihilation processes, it is of interest to include these source terms to determine their effect on the generation mechanism. Generation and annihilation processes demand that we recognize the existence of both polarities of charge.

4.1.1 Previous Work in Bipolar Convective Charge Transport

Workers in electro-fluid pumping and generation have generally ignored the consequences of dual polarity in their analyses because of the specialized circumstances accompanying the engineering design and

operation of these devices. To minimize particle slip (relative to the convecting fluid) it is important to utilize massive charge carriers relative to the size of molecular ions and clusters. Thus, in the design of pumps and generators, particles are introduced and charged with a single polarity at the boundary of the interaction region, and care is taken to minimize the introduction of neutralizing charges of the opposite polarity.

Quite the contrary case obtains in the atmosphere, where charges of both polarities participate in the transport and, indeed, are generated throughout the bulk of the interaction region. Thus, although the boundary injection of net space charge is crucial to the convective generation process, the surplus of charge of either polarity at the boundary may yet be small relative to the total number of charged particles present and bipolar effects must be considered.

Since so little has been done in atmospheric electricity on the convective transport of charge, there appear to be no earlier papers applicable to the present work. Two authors have approached a somewhat similar problem of a bipolar atmosphere, however, in the context of studying the "electrode effect", or the alleged space-charge sheath at the earth boundary. Chalmers [1966] and subsequently Hoppel [1966] dealt with this problem by treating a bipolar region with ionization and recombination bounded below by an equipotential plane with constant surface charge density. Their problem differs from that of this chapter in their neglect of air motions and the presence of a solid conducting boundary with constant surface charge. In our approach, convection is, of course, the essence and, moreover, the boundary electric field is a

consequence of the solution of the closed system, rather than an arbitrary boundary condition.

4.1.2 The Bipolar Model

The major distinction between the unipolar and bipolar models can be summarized by the current density expressions for the unipolar case,

$$J = \rho(U + bE) \quad (4.1.1)$$

and for the bipolar model, when ρ_1 and ρ_2 are the positive and negative charge densities, respectively,

$$J = (\rho_1 + \rho_2)U + E(b_1 \rho_1 - b_2 \rho_2) \quad (4.1.2)$$

Defining the net charge density as

$$\rho = \rho_1 + \rho_2 \quad (4.1.3)$$

and the total charge density as

$$\rho_T = \rho_1 - \rho_2 \quad (4.1.4)$$

Eq. (4.1.2) can be rewritten as

$$J = \rho(U + bE \frac{\rho_T}{\rho}) \quad (4.1.5)$$

when $b_1 = b_2$.

The bipolar conduction current expression has a correction factor ρ_T/ρ not present in the conduction current part of the unipolar current density. Since a typical space-charge density in the atmosphere may be nearly 10^{-12} c/m^3 , [Gish, 1944], whereas typical charged particle densities are of the order $10^{10}/\text{m}^3$ [Israel, 1970] (whence $\rho_T \sim 10^{-9} \text{ c/m}^3$), the

correction factor may be nearly 10^3 , so that the unipolar current density expression is likely to be in considerable error when used to describe a bipolar atmosphere. For large Re , corresponding to dominantly convective transport, it is possible that the unipolar model is adequate. However, for decreasing electric Reynolds number, the error becomes more significant.

The equations governing the steady-state one-dimensional system with two ion families are similar to those given in Eqs. (3.2.1) - (3.2.4), with one important difference. In the unipolar model, conservation of charge is synonymous with particle conservation, there being one unit of charge of the same polarity associated with each particle. Thus, a single conservation equation suffices. The bipolar model requires two conservation expressions because of the presence of two independent charge carriers.

Two equivalent approaches to the bipolar problem are examined in the following. The first accounts for particle and charge conservation by dealing explicitly with each ion species. That is, a current density, J_1 , is associated with each ion family and individual species conservation equations are stated. This approach is useful, since it allows closed-form solutions to the vertical particle-field distribution when the bulk source terms of ionization and recombination are neglected. These closed-form solutions are particularly useful later by providing verification of the numerical integration necessary when the bulk source terms are included.

This latter case is treated by forming the two independent variables, space charge density, $\rho = \rho_1 + \rho_2$, and charge carrier density,

$\rho_T = \rho_1 - \rho_2$, and writing the conservation equations explicitly in terms of space charge and particle density. Thus, when bulk ionization is included, conservation of charge is no longer equivalent to particle conservation, since each ionization event generates a positive and a negative carrier, simultaneously. Hence, the net charge is not affected by the event, but the total charge carrier population is increased.

The physical picture of the model is the same as that given previously in Fig. 3.1b. A source of charge carriers is assumed to provide ions at the boundary of injection for each species. In the generating region, a flux of neutral air exists with constant and uniform vertical velocity, U . In the steady state, ions of both polarities move according to the neutral convection and the self-consistent electric field. As for the development of the unipolar model, general terminal relations for the bipolar generator are obtained and presented in graphical form. These curves, in conjunction with terminal characteristics of an arbitrary load, serve to define the generated potential in terms of the boundary space charge and the electric Reynolds number.

The bipolar mathematical model is analyzed in Sec. 4.2 at the end of which the bipolar terminal characteristics are presented. In Sec. 4.3, a bipolar load with $U = 0$ is studied. In this section, the bipolar terminal relations are cast in the form of a resistivity function, and these results are compared to the empirical resistivity formula discussed in Sec. 3.4. With these results, the bipolar and unipolar generators coupled to a common load are compared in Sec. 4.3.3.

In the final section, the model is altered to include the effects of bulk ionization and recombination on the generator characteristics. Numerical solutions to the steady-state equations are presented and

compared to the source-free model, to ascertain the importance of these effects in the convective generation process.

4.1.3 Remarks on Notation

In the previous expressions for ρ and ρ_T , and henceforth, subscript 1 refers to the positive species and subscript 2 to the negative. Thus, ρ_1 is always a positive quantity and ρ_2 always negative. Then, ρ can be positive or negative, depending on the relative magnitudes of ρ_1 and ρ_2 , while ρ_T is always positive.

The bipolar equations in Sec. 4.2 are written for the individual ion species in the form

$$J_{\frac{1}{2}} = \rho_{\frac{1}{2}} (U \pm b_{\frac{1}{2}} E) \quad . \quad (4.1.6)$$

That is, when the positive current density is desired, select variables with subscript 1 and use the upper sign,

$$J_1 = \rho_1 (U + b_1 E) \quad . \quad (4.1.7)$$

For the negative species, we choose subscript 2 and the lower sign,

$$J_2 = \rho_2 (U - b_2 E) \quad . \quad (4.1.8)$$

In order to simplify the subscript notation, we will write for Eq. (4.1.6),

$$J_m = \rho_m (U \pm b_m E) \quad (4.1.9)$$

where m can take on the values 1 or 2; i.e., $m \equiv \frac{1}{2}$, with the upper sign associated with $m = 1$, and the lower with $m = 2$. Notice that Eq. (4.1.6) can be written equivalently,

$$J_{\frac{1}{2}} = \rho_{\frac{1}{2}} (U \mp b_{\frac{1}{2}} E) \quad . \quad (4.1.10)$$

In this case, we write n for the inverse of m , i.e., $n \equiv \frac{2}{1}$, that is

$$J_n = \rho_n (U \mp b_n E) \quad (4.1.11)$$

where the upper sign is associated with $n = 2$ and the lower with $n = 1$.

For example, the parameter $A_{\frac{1}{2}} \equiv A_m$ is

$$A_{\frac{1}{2}} = - \frac{b_2 \rho_2 \big|_0 J_2}{b_1 \rho_1 \big|_0 J_1} \quad (4.1.12)$$

and in our notation,

$$A_m = - \frac{b_n \rho_n \big|_0 J_m}{b_m \rho_m \big|_0 J_n} \quad (4.1.13)$$

In this case,

$$A_m = \frac{1}{A_n} \quad (4.1.14)$$

and, since only one sign is associated with Eq. (4.1.13), no choice of sign is necessary.

4.1.4 Boundary Values and the Steady State

Similar to the unipolar model, the bipolar ion distributions can be determined in terms of fixing boundary values at $x' = 0$ or $x' = 1$. However, unlike the unipolar model, there are situations in the bipolar case for which one ion family is injected at $x' = 0$ while the other is injected at $x' = 1$. Since in the following we will fix boundary values at a single boundary for both ion families, the solution in the case of counter-streaming particles at the boundary is found by "working backward" from a boundary condition at $x' = 0$ to determine the self-consistent boundary conditions at $x' = 1$. That we are guaranteed such a solution in the steady state is a consequence of the one-to-one mapping of boundary

values at one boundary into those at the opposite boundary, with the transformation given by the differential equations of the system.

To see that the unique mapping exists in the steady state, consider the bipolar conservation equations in the form,

$$\frac{\partial \rho_1}{\partial t} + (U \pm bE) \frac{d\rho_1}{dx} = - \frac{b_1 \rho_1}{\epsilon_0} (\rho_1 + \rho_2) \quad (4.1.15)$$

in which we have used $dU/dx = 0$ and Gauss's law. Then, along the characteristic curves, C_1 , in x - t space defined by

$$C_1 = \frac{dx}{dt_1} = U \pm b_1 E \quad (4.1.16)$$

the species charge densities obey

$$d\rho_1 = - \frac{b_1 \rho_1}{\epsilon_0} (\rho_1 + \rho_2) dt_1 \quad (4.1.17)$$

Since both U and E are single valued, C_1 are single valued. Moreover, since in the steady state, $U \pm b_1 E \neq 0$, $dx/dt_1 \neq 0$. Hence, all characteristics with origin on one boundary are guaranteed of terminating on the opposite boundary in the steady state.

Figure 4.0 illustrates a possible case when $E_0 > 0$, $U \pm bE_0 > 0$. At time τ , species 1 particles are injected at $x' = 0$ and species 2 at $x' = 1$. The characteristic curves for particles injected at $t = \tau$ are given by $C_1(\tau)$. Since, in the steady state $C_1(t) \equiv C_1(\tau)$, the characteristic $C_2(\tau - \Delta)$, corresponding to species 2 particles which reach $x' = 0$ at $t = \tau$, is equivalent to $C_2(\tau)$. Thus, the integration of Eq. (4.1.17) along $C_1(\tau)$ is equivalent to following $C_1(\tau)$, $C_2(\tau - \Delta)$.

The crucial issue in the counter-streaming ion case is that the

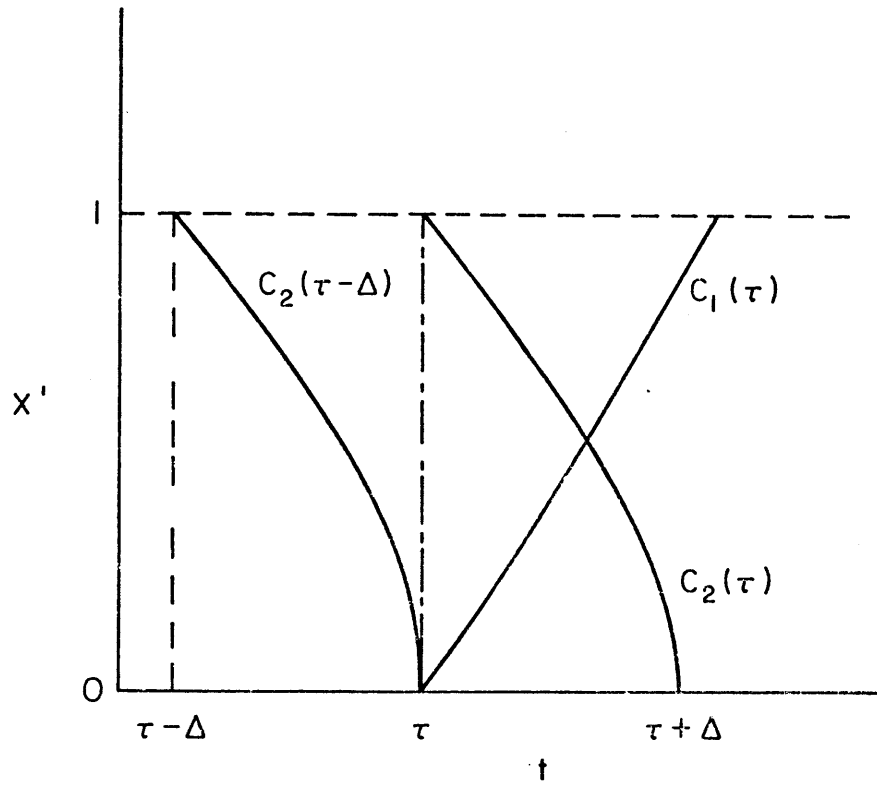


Fig. 4.0 Schematic characteristic curves when
 $E > 0$, $U \pm bE \gtrless 0$

boundary conditions imposed at a common boundary be consistent with the steady state. Thus, it is possible to choose boundary values such that one of the characteristics C_1 never reaches the opposite boundary. Although it is not clear what physical meaning is to be attached to such an event, it is clear that the steady-state assumption is violated. So we term boundary values leading to vanishing dx/dt inconsistent with the steady state.

It is a simple matter to determine regimes of boundary conditions on charge density which guarantee a steady-state solution when ionization and recombination are neglected. Thus, when both species are injected at the same boundary, the possible space charge boundary values are unlimited, since the solution marches forward in time for both characteristics. On the other hand, when the particles counter-stream at the boundary, the boundary electric field is of such magnitude that one particle moves counter to the neutral convection. To insure that this condition obtains for all x' , we choose space charge boundary values such that the magnitude of the electric field increases for all x' . From Eq. (4.1.17), if the net space charge vanishes along a characteristic, it remains zero thereafter. Thus, the space charge never changes sign. Hence, if the particles counter-stream and $E_0 < 0$, a negative boundary space charge insures that $E(x') < E_0$ and if $E_0 > 0$, $\rho_0 > 0$ is the consistent space charge boundary value. The consistent space charge boundary values are illustrated in Table 4.0.

Table 4.0

	$\rho_o > 0$	$\rho_o < 0$
Co-streaming	$U \pm b_1 E_o > 0$	$U \pm b_1 E_o > 0$
Counter-streaming	$U \pm b_1 E_o \geq 0$	$U \pm b_1 E_o \leq 0$

Since $U > 0$, the case $U \pm b_1 E_o < 0$ is impossible.

4.2 The Bipolar Generator in the Absence of Bulk Ionization

This section is devoted to determining the terminal relations for a bipolar generator with no source terms. In this approach, we write the conservation equations in terms of each species of ion. Thus, the charge conservation equation for each ion species also serves as an equivalent particle conservation statement for the charge carriers, since there are no sources or sinks of charge or particles in the volume. In Sec. 4.2.1, the governing differential equation for the system is obtained. Section 4.2.2 deals with the special case for which one ion species has vanishing mobility which could correspond to an electrodynamic generator in which some neutralizing charge escapes into the interaction region in addition to the charged particles specifically injected. Section 4.2.3 presents the terminal relation for the case in which both ion families possess finite mobility.

4.2.1 The Mathematical Model

The equations governing the steady-state, one-dimensional system with two ion species are similar to those given in Eqs. (3.2.1) - (3.2.4). This case differs by introducing a second oppositely charged family of ions whose motion is also governed by the electric field and neutral convection. The electric field is now calculated from the net space charge, $\rho_1 + \rho_2$, so that, in effect, the ion families are coupled to one another through the electric field.

The equations are therefore given by the definition of species current density,

$$J_m = \rho_m (U \pm b_m E) , \quad (4.2.1)$$

Gauss's law,

$$\frac{dE}{dx} = \frac{\rho_1 + \rho_2}{\epsilon_0} \quad (4.2.2)$$

and the charge conservation equations, which in the absence of bulk sources of ions are:

$$\frac{dJ_m}{dx} = 0 \quad (4.2.3)$$

The potential function, Φ , is as usual defined by

$$\frac{d\Phi}{dx} = -E \quad (4.2.4)$$

The vertical uniformity of U and J_m in the steady one-dimensional model is the crux of the bipolar solution. Thus, from the definition of species current density, Eq. (4.2.1),

$$\pm E = \frac{1}{b_m} \left(\frac{J_m}{\rho_m} - U \right) \quad (4.2.5)$$

Then, taking the derivative of Eq. (4.2.5) and using (4.2.3), we obtain

$$\pm \frac{dE}{dx} = \frac{J_m}{b_m} \frac{d}{dx} \frac{1}{\rho_m} \quad (4.2.6)$$

since

$$\frac{dU}{dx} = 0 \quad (4.2.7)$$

Addition of the two equations implied by (4.2.6) gives

$$\frac{J_1}{b_1} \frac{d}{dx} \left(\frac{1}{\rho_1} \right) = - \frac{J_2}{b_2} \frac{d}{dx} \left(\frac{1}{\rho_2} \right) \quad (4.2.8)$$

Thus,

$$\frac{J_m}{b_m} \frac{d}{dx} \frac{1}{\rho_m} = - \frac{J_n}{b_n} \frac{d}{dx} \frac{1}{\rho_n} . \quad (4.2.9)$$

Then, defining the inverse normalized species space charge as

$$Z_m = \frac{\rho_m|_0}{\rho_m} \quad (4.2.10)$$

where $\rho_m|_0$ are the species boundary values at a common boundary, Eq. (4.2.9) is

$$\frac{J_m}{b_m \rho_m|_0} \frac{d}{dx} Z_m = - \frac{J_n}{b_n \rho_n|_0} \frac{d}{dx} Z_n . \quad (4.2.11)$$

Thus, Eq. (4.2.11) defines the relation of one ion family to the other in terms of the boundary conditions so long as the ion boundary values are determined at a common boundary. Equation (4.2.11) is valid regardless of this condition, but it is the integral result which defines one species density in terms of the other, and the integration must proceed from a given boundary.

Carrying out this integration, since $Z_m|_0 \equiv 1$, the relation of the polar ion densities is given by

$$Z_m = 1 + A_n (Z_n - 1) \quad (4.2.12)$$

where

$$A_n = - \frac{J_n b_m \rho_m|_0}{J_m b_n \rho_n|_0} . \quad (4.2.13)$$

Gauss's law, Eq. (4.2.2), in terms of Z_m is found by multiplying the right-hand side of Eq. (4.2.2) by $\rho_m|_0$ and defining

$$B_m = \frac{\rho_n|_0}{\rho_m|_0} \quad (4.2.14)$$

$$\frac{dE}{dx} = \frac{\rho_m|_0}{\epsilon_0} \left(\frac{1}{Z_m} + \frac{B_m}{Z_n} \right) \quad (4.2.15)$$

Solving Eq. (4.2.12) for Z_n and substituting the result into (4.2.15),

$$\frac{dE}{dx} = \frac{\rho_m|_0}{\epsilon_0} \left[\frac{1 + Z_m \left(\frac{A_m + B_m}{1 - A_m} \right)}{Z_m + \frac{A_m}{1 - A_m} Z_m^2} \right] \quad (4.2.16)$$

Equation (4.2.6) written in terms of the inverse normalized polar space charge, Z_m , is

$$\frac{dE}{dx} = \pm \frac{J_m}{\rho_m|_0 b_m} \frac{dZ_m}{dx} \quad (4.2.17)$$

Equations (4.2.16) and (4.2.17) when combined, give:

$$\frac{dZ_m}{dx} = \pm \frac{\rho_m^2|_0 b_m}{\epsilon_0 J_m} \left[\frac{1 + Z_m D_m}{Z_m + \frac{A_m}{1 - A_m} Z_m^2} \right], \quad (4.2.18)$$

where we have defined

$$D_m = \frac{A_m + B_m}{1 - A_m} \quad (4.2.19)$$

Defining the normalized polar current density by

$$J_m' = \frac{J_m}{\rho_m|_0 U} \quad (4.2.20)$$

and the polar electric Reynolds number by

$$Re_m = \pm \frac{\epsilon_0 U}{\rho_m|_0 b_m H} \quad (4.2.21)$$

Eq. (4.2.18) is then written

$$\frac{Z_m + \frac{A_m}{1 - A_m} Z_m^2}{1 + D_m Z_m} dZ_m = \frac{dx'}{\text{Re } J_m'} \quad (4.2.22)$$

where

$$x' = \frac{x}{H} \quad (4.2.23)$$

The normalized current density, J_m' , is related to the electric field and space charge by

$$J_m' = \frac{1 \pm E_m'}{Z_m} \quad (4.2.24)$$

where the bipolar analogy to the unipolar normalized electric field is

$$E_m' = \frac{b_m E}{U} \quad (4.2.25)$$

Equation (4.2.22) is the fundamental differential equation describing the spatial ion distribution in a bipolar convecting region. In its integrated form, it gives a closed but transcendental expression of Z_m against x' , so that a simple expression for Z_m is not found. Nevertheless, mappings of Z_m against x' can be computed accurately and these provide the necessary information to calculate the potential and field distributions for given situations.

Table 4.1 is included to summarize the normalized variables and their definitions introduced in this section.

Table 4.1

<u>Dimensionless Parameter</u>	<u>Definition</u>
Z_m	$\frac{\rho_m _0}{\rho_m}$
$B_m = \frac{1}{B_n}$	$\frac{\rho_n _0}{\rho_m _0}$
$Re_m = -\frac{b_n}{b_m} B_m Re_n$	$\pm \frac{\epsilon_0 U}{b_m \rho_m _0 H}$
$J_m' = 1 - \frac{b_m}{b_n} (J_n' - 1)$	$\frac{J_m}{\rho_m _0 U}$
$A_m = \frac{1}{A_n}$	$-\frac{b_n J_m'}{b_m J_n'}$
$D_m = -B_m D_n$	$\frac{A_m + B_m}{1 - A_m}$
$E_m' = \frac{b_m}{b_n} E_n'$	$\frac{b_m E}{U}$
$V_m' = \frac{b_m}{b_n} V_n'$	$\frac{b_m V}{U H}$

4.2.2 The Immobile Ion Case

Two special cases of Eq. (4.2.22) are of some interest. The first case occurs for vanishing ion density for one species resulting in the unipolar equation (3.2.11) in somewhat different form. The second is the case for which one ion has sufficiently low mobility that it can be thought of as stationary in the electric field. Since the first case is a special case of the second, we consider the immobile ion case first.

If $b_1 = 0$ or $b_2 = 0$, then $A_2 = 0$ or $A_1 = 0$, so that Eq. (4.2.22)

becomes

$$\frac{Z_m}{B_m Z_m + 1} dZ_m = \frac{dx'}{\text{Re}_m J_m'} \quad (4.2.26)$$

$$b_n = 0$$

The interpretation of this case is that if, for example, species 1 has vanishing mobility, then the distribution of Z_1 is uniform in the vertical and we use (4.2.26) to determine the distribution of the mobile ion species 2. If, in addition, we specify $\rho_1|_0 = 0$ in Eq. (4.2.26), then $B_2 = 0$ and we reproduce the unipolar equation (3.2.11) in somewhat different form:

$$Z_2 dZ_2 = \frac{dx'}{\text{Re}_2' J_2'} \quad (4.2.27)$$

The integration of (4.2.26) gives the distribution of the ion species in the vertical with the form of the right side dictated by the level at which the boundary conditions are specified. Thus, if boundary values are chosen at $x' = 0$,

$$Z_m - \frac{1}{B_m} \ln \left(\frac{1 + B_m Z_m}{1 + B_m} \right) = 1 + \frac{B_m x'}{\text{Re}_m J_m'} \quad (4.2.28)$$

where $b_n = 0$. If boundary values are fixed at $x' = 1$, then this result is:

$$Z_m - \frac{1}{B_m} \ln \left(\frac{1 + B_m Z_m}{1 + B_m} \right) = 1 + \frac{B_m (1 - x')}{\text{Re}_m J_m'} \quad (4.2.29)$$

When ion densities are equal at the boundary, $B_1 = -1$, and the solutions in Eqs. (4.2.28) and (4.2.29) are not valid. As would be expected, the

consistent solution to the steady-state situation when $B_m = -1$ corresponds to the trivial case, $Z_1 = Z_2 = 1$, everywhere. Then, $d\phi/dx = 0$ and $E = E_0$. This case corresponds to no generation at all by the neutral convection. The uniform field E_0 is an externally maintained field, independent of the presence of space charge.

To determine the nature of the charge distribution, we note that the first derivative of Z_m given by Eq. (4.2.26)

$$\frac{dZ_m}{dx'} = \frac{1 + B_m Z_m}{\text{Re } J_m' Z_m} \quad (4.2.30)$$

has a critical point at

$$Z_m = -\frac{1}{B_m} \quad (4.2.31)$$

Calculating the second derivative,

$$\frac{d^2 Z_m}{dx'^2} = -\left(\frac{B_m}{\text{Re } J_m'}\right)^2 \frac{B_m (1 + B_m Z_m)}{(B_m Z_m)^3} \quad (4.2.32)$$

we see that the second derivative vanishes when $Z_m = -1/B_m$. Since, from Eq. (4.2.28), $Z_m \rightarrow -1/B_m$ implies $x' \rightarrow \infty$, we conclude that $-1/B_m$ is an asymptote of Z_m .

More general remarks concerning the ion distribution can be made for this case of $b_n = 0$. By definition, $Z_m|_0 = 1$. Considering the case for which $J_m' > 0$, it is apparent that for $|B_m| > 1$, corresponding to the case of the immobile ion density being greater than that of the mobile ion density at the boundary, $dZ_m/dx' < 0$ everywhere. Since Z_m is the inverse of the ion density, the mobile ion density increases throughout the section, tending monotonically to the value of the immobile ion. Similarly, when $|B_m| < 1$, $dZ_m/dx' > 0$ everywhere, and the mobile ion density decreases toward the uniform value of the immobile ion density.

When $J_m' < 0$, however, the boundary values imposed at the common boundary, $x' = 0$, may be inconsistent, since the mobile species is injected at $x' = 1$. In this case, $dZ_m/dx' < 0$ when $(1+B_m) > 0$, i.e., when the boundary space charge is positive. Thus, Z_m decreases and the asymptote $-1/B_m > 1$ is never reached. Ultimately, the mobile ion velocity vanishes and crossing of characteristics occurs. Whether Z_m reaches 0 before $x' = 1$ is determined by the product $Re_m J_m'$. In general, however, when $J_m' < 0$, the boundary condition $(1 + B_m) > 0$ is dangerous, since it may be inconsistent with the steady state. These remarks can be seen to agree with the consistent boundary values noted in Table 4.0.

The distance over which the ion densities of the two species remain significantly different for given current density is determined by the electric Reynolds number of the mobile species. Thus, for large Re_m , the dominant current is convection current and $|dZ_m/dx'|$ is small, so that an increase in path length x' is necessary before the mobile ion density approaches the immobile density. On the contrary, small Re_m corresponds to predominantly conductive transport of the mobile species in response to the space charge field of the immobile species. Thus, the term $B_m/Re_m J_m'$ can be rewritten as $\pm \rho_m|_0 b_m (\rho_n|_0 H/\epsilon_0)/J_m$. The numerator is the current density of the mobile ion due to the space charge field of the immobile ion, $\rho_n|_0 H/\epsilon_0$, while the denominator is the total current density, conductive and convective, of the mobile species. When this ratio is large, i.e., Re_m small, $|dZ_m/dx'|$ is large and the mobile ion responds rapidly to the space charge field of the immobile species.

As a rule of thumb the space charge density decays with a charac-

teristic length,

$$\ell' = \frac{Z_m|_0}{\frac{dZ_m}{dx'}|_0} = \frac{\text{Re } J_m'}{1 + B_m} \quad (4.2.33)$$

An example of this behavior is shown in Fig. 4.1, which gives the distribution of $\rho_1' = 1/Z_1$ against x' from Eq. (4.2.28). In this figure, the negative ion is immobile and at the boundary its charge density is one-tenth the boundary value of the mobile positive ion species. The curves are parameterized in the term $\text{Re}_1 J_1'$. Clearly, the net space charge disappears rapidly for the low Reynolds number case, so that conduction current dominates over most of the channel length.

The electric field structure can be derived from the distribution of Z_1 used to draw Fig. 4.1 from the definition of the normalized current density, Eq. (4.2.24). Thus, for $b_2 = 0$, the normalized electric field is given by

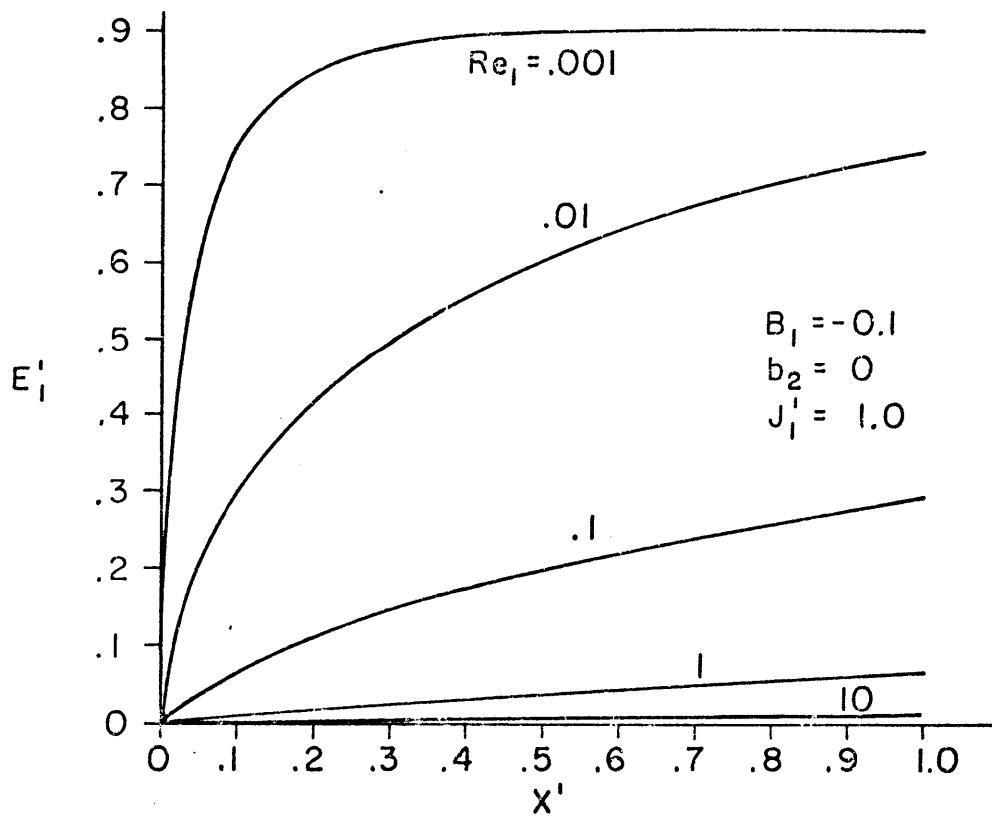
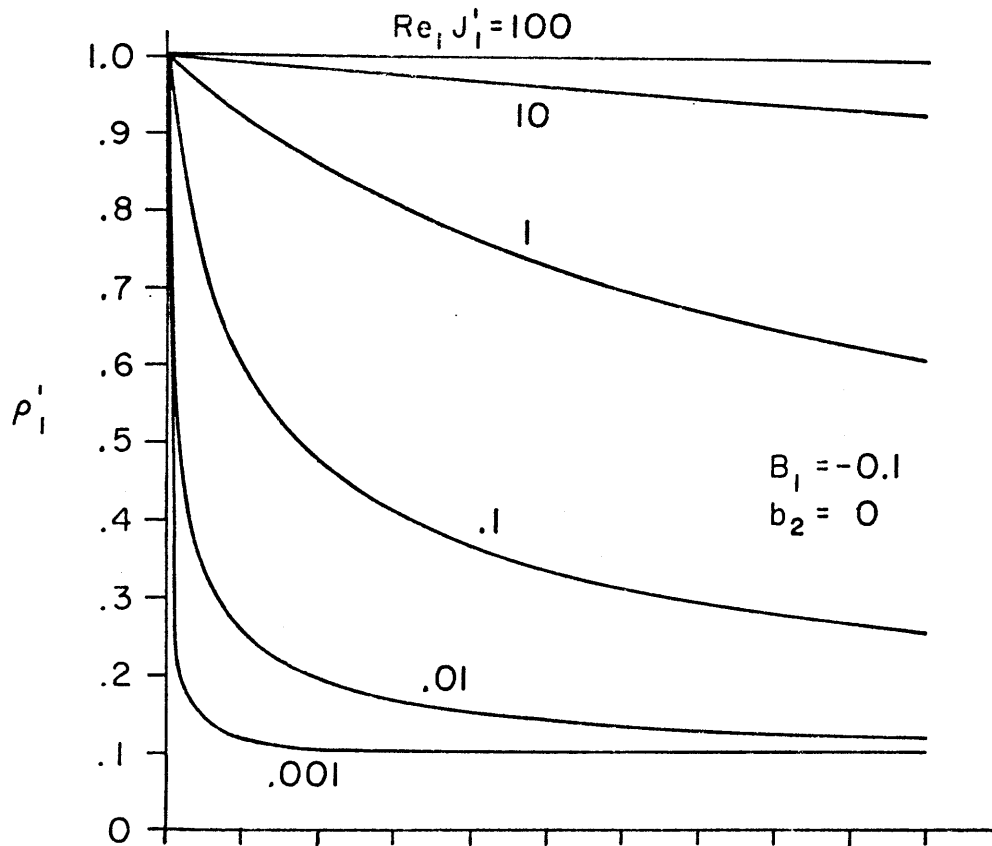
$$E_1' = J_1' Z_1 - 1 \quad (4.2.34)$$

For purposes of illustration in Fig. 4.2, we choose $J_1' = 1$, corresponding to vanishing electric field at the boundary. Then the distribution of normalized field follows immediately from Eq. (4.2.34).

For values of J_m' or Re_m such that $J_m' \text{Re}_m$ is sufficiently small so that the asymptote $Z_m = -1/B_m$ is attained within the channel, the final electric field is given by

$$E_m'|_f = \mp \left(\frac{J_m'}{B_m} + 1 \right) \quad (4.2.35)$$

In Fig. 4.2, $\text{Re} = .001$ causes $E_m'|_f$ to be attained within the range



Figs. 4.1

and 4.2

Charge and field for the immobile ion case

$x' < 1$.

A comparison between Figs. 4.1 and 4.2 on the one hand, and Figs. 3.3 and 3.4 on the other, confirms the earlier remarks to the effect that the higher the electric Reynolds number, the more nearly correct is the unipolar model. To compare the ion density curves, recall that the curves in Fig. 4.1 are given for the parameter $Re_1 J_1'$, whereas those of the unipolar model in Fig. 3.3 are given for $J' = 0.1$ and the electric Reynolds numbers specified. For example, we compare the $Re = 100$ curve in Fig. 3.3 with the curve $Re_1 J_1' = (0.1)(100) = 10$ in Fig. 4.1. The mobile particle distributions in both cases are not notably different for $Re > 1$, whereas significant difference is found for smaller Re .

It should be kept in mind that the normalized charge density in Fig. 3.3 represents the whole of the space charge, whereas in Fig. 4.1, the net space charge is given by $(\rho_1' - 0.1)$, due to the presence of the opposite polarity immobile space charge. It is this screening charge which causes the marked difference in electric field distribution between the unipolar and bipolar cases at small Re .

The terminal relation giving the normalized potential for the immobile ion case is found from Eq. (4.2.4), and the electric field determined by Eq. (4.2.24). Thus,

$$\frac{d\phi_m'}{dx'} = \mp (J_m' Z_m - 1) \quad (4.2.36)$$

and the terminal potential is therefore

$$V_m' = \mp \left[J_m' \int_0^1 Z_m dx' - 1 \right] \quad (4.2.37)$$

where $Z_m(x')$ is defined by Eqs. (4.2.28) and (4.2.29). Figures 4.3 and 4.4 show the general terminal relation in the form of the ordinate $\mp (V_m' \mp 1)/J_m'$ plotted against abscissas $J_m' Re_m$, with parameter B_m . From this plot can be generated the voltage-current relation for a generator of this type.

The curves shown in Figs. 4.3 and 4.4 were obtained by calculating the distribution of Z_m against x' from Eq. (4.2.28) and using a simple trapezoidal numerical quadrature to obtain the integral result of Eq. (4.2.37). The step-size in the independent variable was maintained less than .01 so that the error for each step is of order 10^{-6} or less. With the possibility of error accumulation, it is estimated that the overall error in the final result is less than .001, since the maximum number of steps was always less than 10^3 .

The asymptotes of the curves are explained by noting that for very large values of the parameter $J_m' Re_m$, the ion densities change very little with x' , remaining near the inlet value of unity. Thus, from Eq. (4.2.37),

$$V_m' \approx \mp [J_m' - 1] \quad (4.2.38)$$

and hence the parameter $\mp (V_m' \mp 1)/J_m' \approx 1$ for large $Re J_m'$. When $Re J_m' \ll 1$, Z_m very quickly attains its asymptotic value of $-1/B_m$, and again from Eq. (4.2.37),

$$V_m' \approx \pm [J_m' \frac{1}{B_m} + 1] \quad (4.2.39)$$

whence $\mp (V_m' \mp 1)/J_m' \approx -1/B_m$ for very small $Re J_m'$.

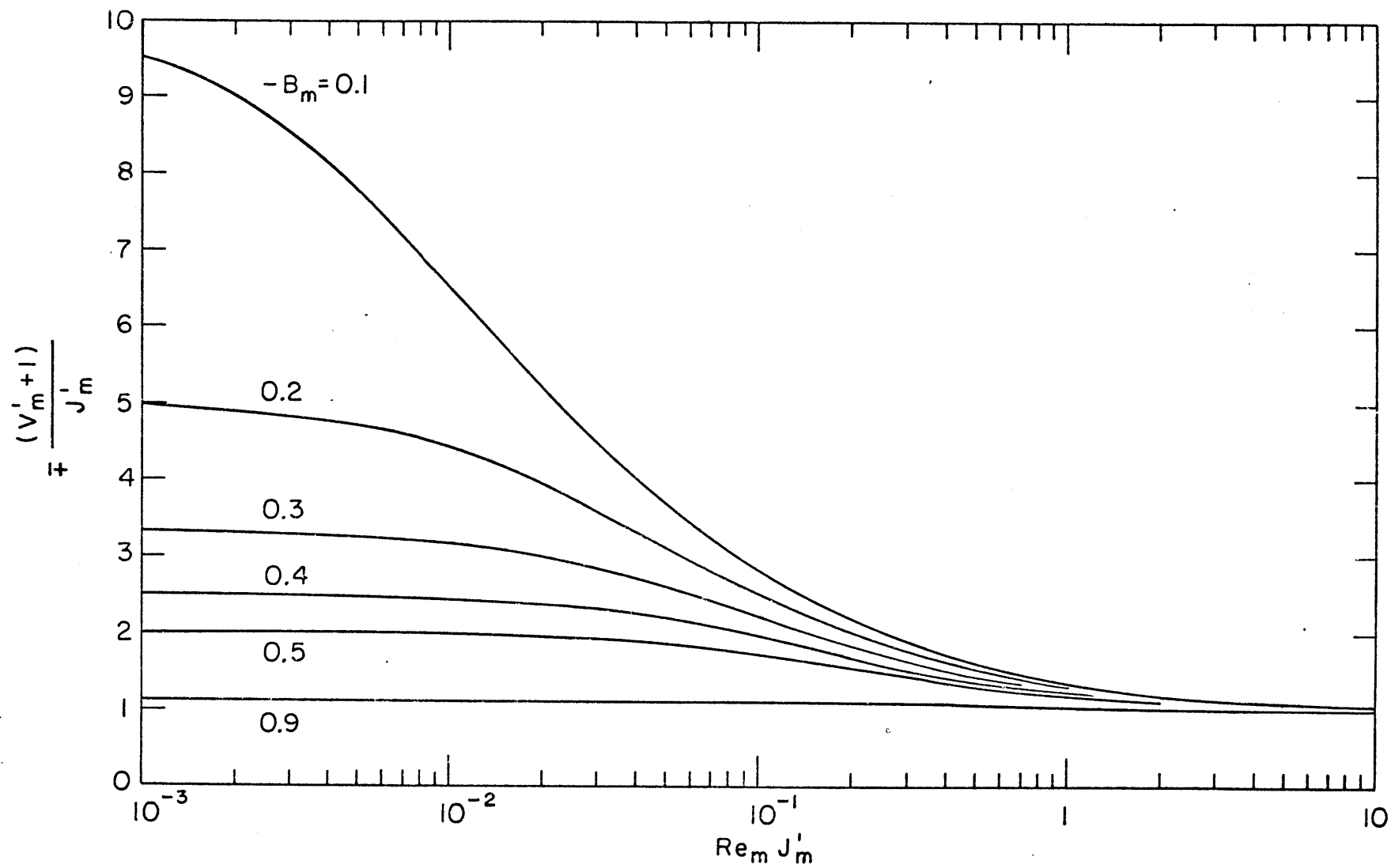


Fig. 4.3 Generalized terminal relation for an immobile ion generator, $-B_m < 1$

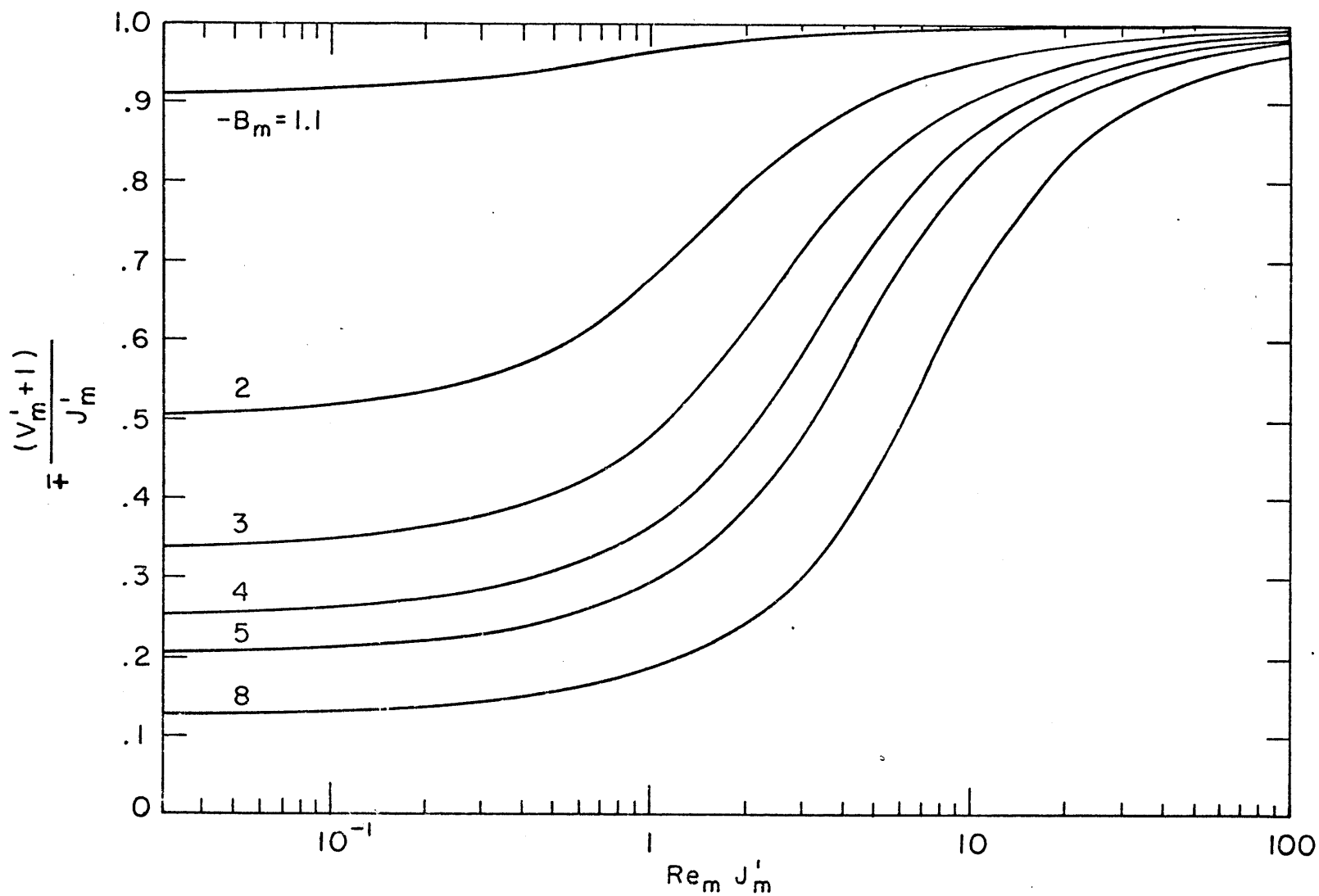


Fig. 4.4 Generalized terminal relation for an immobile ion generator, $-B_m > 1$

The immobile ion case is useful primarily for the insight it provides into the particle-field distributions away from a boundary where they are known. Although relatively immobile charged particles do co-exist with mobile space charge in electrified clouds, it is almost certain that the mobile charge is itself bipolar, so that the bipolar immobile particle case simply does not apply. However, it is possible that this case applies in some electrogasdynamic devices whose charging sections allow leakage of mobile space charge into the generating region along with the primary relatively immobile charge carriers.

Table 4.2

Summary of Results for the Immobile Ion Case: $b_2 = 0$.

Species charge density distributions:

$$Z_m - \frac{1}{B_m} \ln \left(\frac{1 + B_m Z_m}{1 + B_m} \right) = 1 + \frac{B_m x'}{\text{Re } J_m'} \quad Z_m(0) \equiv 1$$

$$Z_m - \frac{1}{B_m} \ln \left(\frac{1 + B_m Z_m}{1 + B_m} \right) = 1 + \frac{B_m (1 - x')}{\text{Re } J_m'} \quad Z_m(1) \equiv 1$$

Electric field distribution:

$$E_m' = \pm (J_m' Z_m - 1)$$

Terminal relation:

$$V_m' = \mp \left[J_m' \int_0^1 Z_m dx' - 1 \right] \quad (\text{See Figs. 4.3 and 4.4}).$$

Asymptotes of the system as $x' \rightarrow \infty$ for $J_m' > 0$:

$$Z_m \rightarrow -1/B_m$$

$$E_m' \rightarrow \mp \left(\frac{J_m'}{B_m} + 1 \right)$$

Limits:

$$V_m' \approx \mp (J_m' - 1), \quad \text{Re } J_m' \gg 1$$

$$V_m' \approx \pm \left[\frac{J_m'}{B_m} + 1 \right], \quad \text{Re } J_m' \ll 1$$

4.2.3 Finite Mobility in Both Ion Species

When both ion families have finite mobility, the integration of Eq. (4.2.22) gives the ion distribution as the transcendental equation,

$$\begin{aligned} \frac{D_m A_m}{2(1-A_m)} Z_m^2 + \left(D_m - \frac{A_m}{1-A_m} \right) Z_m + \left(\frac{A_m}{D_m(1-A_m)} - 1 \right) \ln \left(\frac{1 + D_m Z_m}{1 + D_m} \right) \\ = D_m \left(1 + \frac{A_m}{2(1-A_m)} \right) - \frac{A_m}{1-A_m} + \frac{D_m^2 x'}{\text{Re } J_m'} \end{aligned} \quad (4.2.40)$$

when $Z_m(0) \equiv 1$. The alternative, $Z_m(1) \equiv 1$ is the same as Eq. (4.2.40) with x' replaced by $(1-x')$.

There are three singular points in Eq. (4.2.40), $A_m = 1$, $D_m = -1$, and $Z_m = -1/D_m$. From the definition of A_m in Table 4.1,

$$A_m = - \frac{b_n}{b_m} \frac{J_m'}{J_n'} \quad (4.2.41)$$

The ratio of J_m'/J_n' is the ratio of ion velocities at the boundary.

So, if $U \neq 0$,

$$A_m = - \frac{b_n}{b_m} \left(\frac{1 \pm E_m'|_0}{1 \mp E_n'|_0} \right) \quad (4.2.42)$$

Since

$$\frac{b_n}{b_m} E_m' \big|_0 = E_n' \big|_0 \quad (4.2.43)$$

from Table 4.1, $A_m = 1$ requires

$$-\frac{b_n}{b_m} \mp E_n' \big|_0 = 1 \mp E_n' \big|_0 \quad (4.2.44)$$

or

$$\frac{b_n}{b_m} = -1 \quad (4.2.45)$$

Since b_1 and b_2 are positive quantities, $A_m \neq 1$ so long as $U \neq 0$. On the other hand, $D_m = -1$ signifies that $B_m = -1$, or the magnitudes of the two ion densities at the boundary are the same. This is analogous to the immobile ion case for which $B_m = -1$, in which it was stated that $Z_m = Z_n$ everywhere and the electric field is uniform.

The case $Z_m = -1/D_m$ is investigated by examining the first and second derivatives of Z_m with respect to x' . Thus, from Eq. (4.2.22),

$$\frac{dZ_m}{dx'} = \frac{1}{\text{Re}_m J_m'} \frac{(1-A_m)(1+D_m Z_m)}{Z_m(1-A_m + A_m Z_m)} \quad (4.2.46)$$

$$Z_m \neq \frac{A_m - 1}{A_m} \quad .$$

The critical point in this case is seen to occur for $Z_m = -1/D_m$. The second derivative also vanishes at the critical point, so that again we suspect $-1/D_m$ to be an asymptote of Z_m as $x' \rightarrow \infty$. Using the definition of Z_n in terms of Z_m from Table 4.1, we find that if $Z_m = (A_m - 1)/A_m$, then $Z_n = 0$; that is, the ion density of one species becomes infinite. This corresponds to the case of vanishing velocity for that species.

Investigating further, the initial change of Z_m with x' at the boundary is

$$\left. \frac{dZ_m}{dx'} \right|_0 = \frac{1 + B_m}{Re_m J_m'} \quad (4.2.47)$$

which can be found from Eq. (4.2.46) by setting $Z_m = 1$ and using the definition of D_m in the result. When both species have positive velocity, $J_m'/J_n' > 0$ and thus $A_m < 0$. Then the asymptote is given by

$$- \frac{1}{D_m} = \frac{1 + |A_m|}{|B_m| + |A_m|} \quad (4.2.48)$$

and hence $-1/D_m \geq 1$ when $1 + B_m \geq 0$. Since $1 + B_m$ determines the slope, dZ_m/dx' in Eq. (4.2.47), the monotonic behavior of Z_m with x' shows that Z_m approaches the asymptote $-1/D_m$ when both species have positive velocity at $x' = 0$. We note that when $A_m < 0$, $(A_m - 1)/A_m > 1$ and since also $(A_m - 1)/A_m > -1/D_m$, this forbidden value is never reached when both species have positive velocity at $x' = 0$.

When the particles counterstream relative to the boundary, it is possible to use similar methods to show that the boundary values which are consistent with the steady state are identical to those outlined in Table 4.0. From those results, the consistent boundary values are that when $E_0 > 0$, $(1 + B_1) > 0$ and when $E_0 < 0$, $(1 + B_1) < 0$.

To this point, boundary conditions and parameters which describe the charge density distribution have been given for simplicity in terms of the individual ion species parameters, J_m' and Re_m . In the usual situation, the total current density and the electric Reynolds number based on the net space charge are the pertinent parameters. Thus, we

give here the transformations which allow the individual species parameters to be found from the normalized total current density and the general electric Reynolds number.

The electric Reynolds number based on the net charge density is given by

$$\text{Re} = \frac{\epsilon_0 U}{|\rho_1|_0 + |\rho_2|_0 |bH|} \quad (4.2.49)$$

where the average particle mobility is

$$\bar{b} = \frac{b_1 + b_2}{2} \quad (4.2.50)$$

Equation (4.2.49) can be rewritten as

$$\text{Re} = \frac{2 \epsilon_0 U b_1}{|\rho_1|_0 b_1 H \left| 1 + \frac{|\rho_2|_0}{|\rho_1|_0} \right| (b_1 + b_2)} \quad (4.2.51)$$

or

$$\text{Re} = \frac{2 \text{Re}_1 b_1}{|1 + B_1| (b_1 + b_2)} \quad (4.2.52)$$

Similarly, Re can be found in terms of Re_2 and the general transformation is

$$\text{Re}_m = \frac{|1 + B_m|}{b_m} \bar{b} \text{Re} \quad (4.2.53)$$

The total normalized current density is

$$J' = \frac{J_1 + J_2}{(|\rho_1|_0 + |\rho_2|_0) U} \quad (4.2.54)$$

which can be written in the form

$$J' = \frac{J_1'}{1 + B_1} + \frac{J_2'}{1 + B_2} \quad (4.2.55)$$

Since $B_1 = 1/B_2$,

$$J' = \frac{J_1' + B_1 J_2'}{1 + B_1} \quad (4.2.56)$$

From Table 4.1,

$$J_2' = 1 - \frac{b_2}{b_1} (J_1' - 1) \quad (4.2.57)$$

so that Eq. (4.2.56) gives

$$J' = \frac{J_1' (1 - B_1 \frac{b_2}{b_1}) + B_1 (1 + \frac{b_2}{b_1})}{1 + B_1} \quad (4.2.58)$$

Thus, the normalized species current densities in terms of the total normalized current density are

$$J_m' = \frac{J' (1 + B_m) - B_m (1 + \frac{b_n}{b_m})}{1 - B_m \frac{b_n}{b_m}} \quad (4.2.59)$$

The parameter A_m given in Table 4.1 can be expressed in terms of the variables J' and B_m by using Eq. (4.2.59) and the relation $B_m = -1/B_n$.

Thus,

$$A_m = \frac{J' (1 + B_m) - B_m (1 + \frac{b_n}{b_m})}{J' (1 + B_m) - (1 + \frac{b_m}{b_n})} \quad (4.2.60)$$

Finally, the asymptote, D_m , can be found with the definition of D_m in Table 4.1 and Eq. (4.2.60) as:

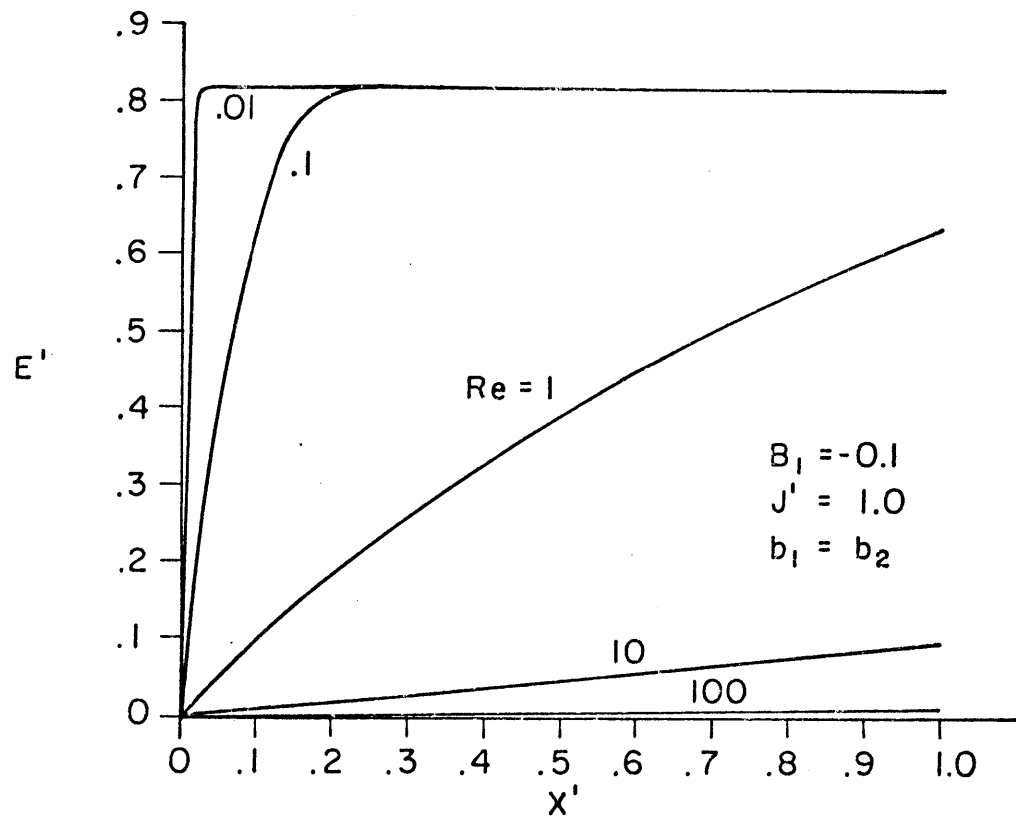
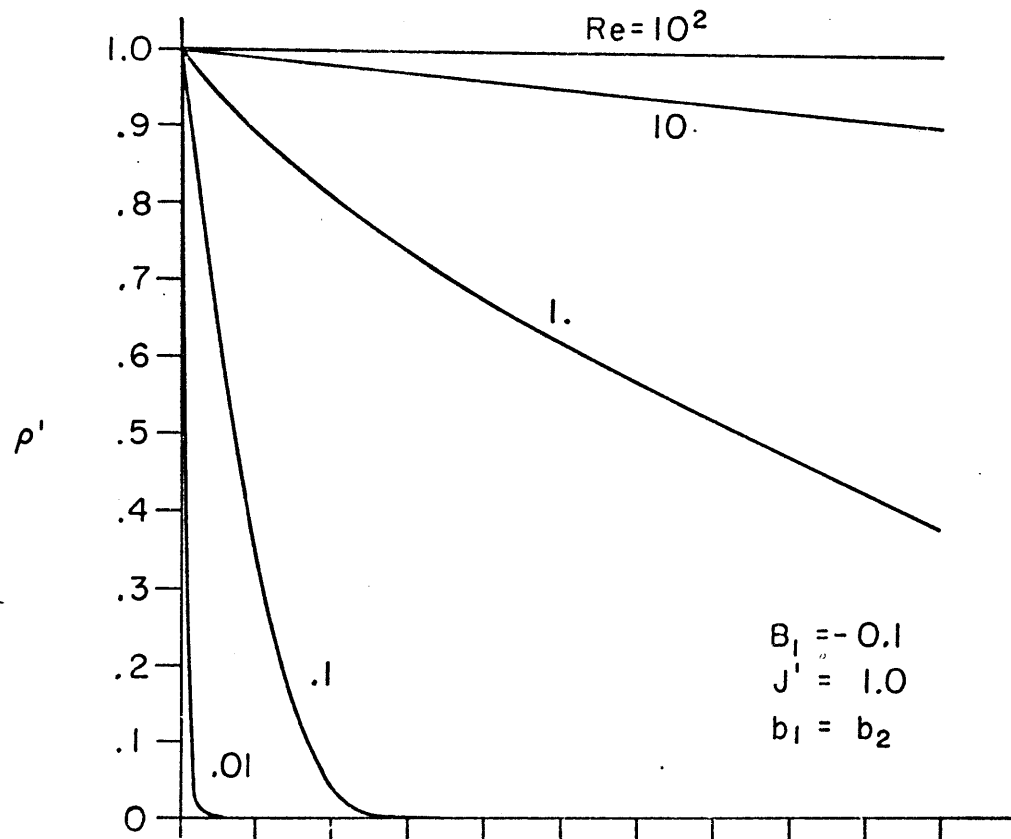
$$D_m = \frac{J' (1 + B_m)^2 - B_m (\frac{b_n}{b_m} + \frac{b_m}{b_n} + 2)}{(1 + \frac{b_m}{b_n}) (\frac{b_n}{b_m} B_m - 1)} \quad (4.2.61)$$

With the aid of Eqs. (4.2.53) and (4.2.59)-(4.2.61), the distribution of Z_m with x' is determined by the parameters Re , J' , and B_m . The total normalized space charge distribution can be found by calculating the distribution of Z_m and using the transformation implied in Eq.(4.2.16)

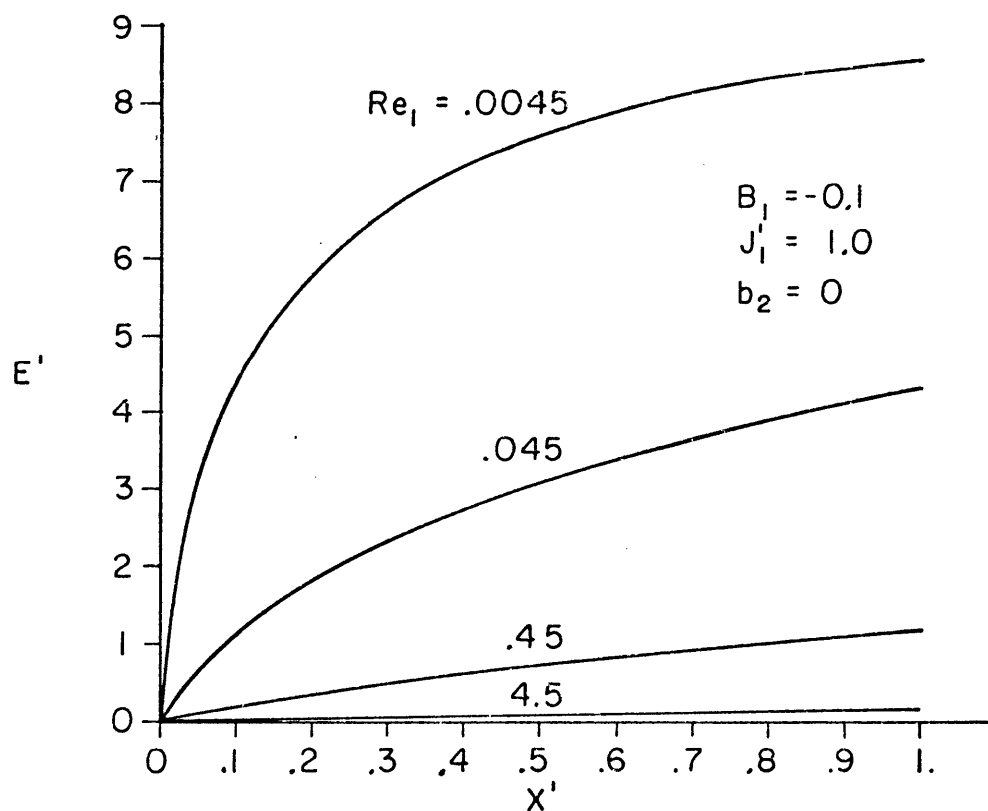
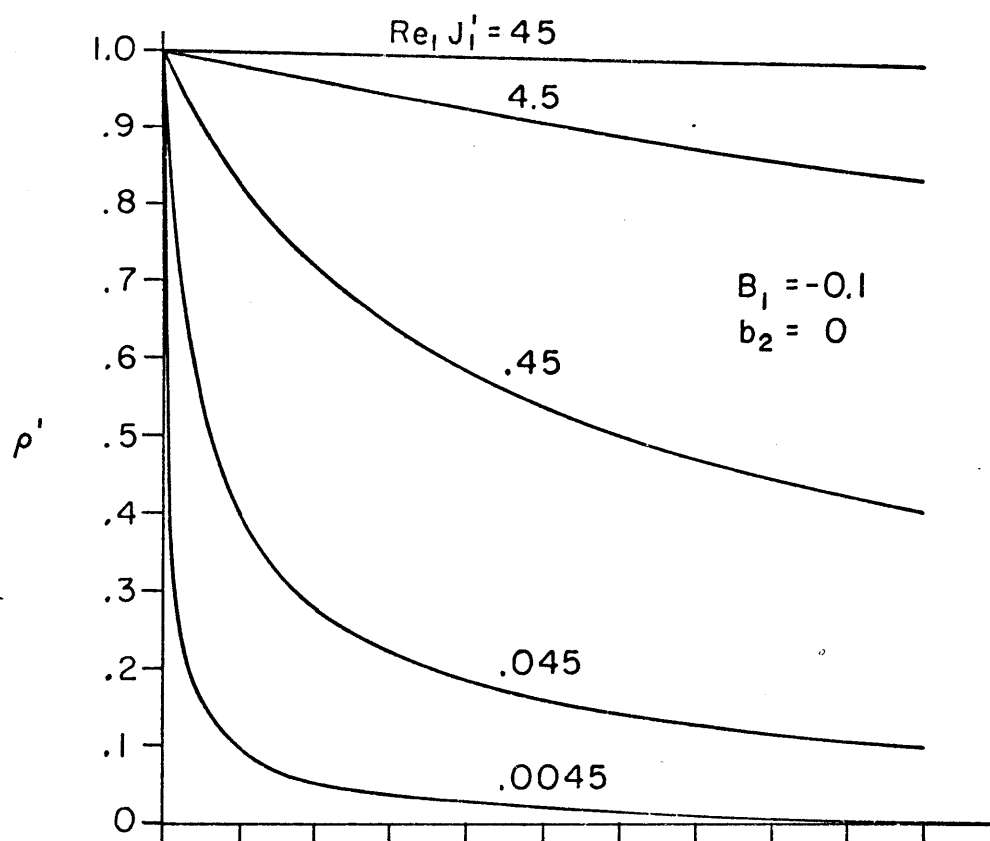
$$\rho' = \frac{1}{1 + B_m} \left[\frac{1 + Z_m B_m}{Z_m + \frac{A_m}{1-A_m} Z_m^2} \right] \quad (4.2.62)$$

Figures 4.5 and 4.6 show sample calculations of the normalized space charge and electric field distribution for $J' = 1$. For comparison with the immobile ion case, Figs. 4.7 and 4.8 show the same parameters with identical boundary conditions; for comparison, the bipolar Re should be multiplied by 0.45 to find the analogous curve for the immobile ion case. It is noteworthy that the bipolar model predicts vanishing of the net space charge much more rapidly than the immobile ion case, although the characteristic length for charge decay is the same for both cases [Eq. (4.2.33)]. The reason for the faster decay in the bipolar model, Figs. 4.5 and 4.6, is that both ion species are free to move with the field lines instead of the single mobile ion of Figs. 4.7 and 4.8.

Table 4.3 displays, for convenience, the species parameters in Eq. (4.2.40) and their definitions in terms of the basic variables, B_m , Re , and J' .



Figs. 4.5 and 4.6 Charge and field for a bipolar generator



Figs. 4.7 and 4.8 Immobile ion case for comparison with Figs. 4.5 and 4.6

Table 4.3

<u>Species Parameter</u>	<u>Definition</u>
Re_m	$\frac{ 1 + B_m \bar{b}}{b_m} \quad Re$
J_m'	$\frac{J'(1 + B_m) - B_m(1 + \frac{b_n}{b_m})}{1 - B_m \frac{b_n}{b_m}}$
A_m	$\frac{J'(1 + B_m) - B_m(1 + \frac{b_n}{b_m})}{J'(1 + B_m) - (1 + \frac{b_m}{b_n})}$
D_m	$\frac{J'(1 + B_m)^2 - B_m(\frac{b_n}{b_m} + \frac{b_m}{b_n} + 2)}{(1 + \frac{b_m}{b_n})(\frac{b_n}{b_m} R_m - 1)}$

4.2.4 Terminal Relations for the Bipolar Generator

The procedure for calculating the terminal relations for the bipolar generator is the same as that outlined for the immobile ion case, except that here the curves are calculated for given J' and Re instead of J_m' , Re_m . For given B_m , Re , and J' , the species parameters J_m' , Re_m are calculated with the aid of the definitions of Table 4.3. Then Eq. (4.2.40) is used to generate a mapping of Z_m against x' . The terminal relation

$$V_m' = \mp \left[J_m' \int_0^1 Z_m dx' - 1 \right] \quad (4.2.63)$$

is then calculated using a trapezoidal numerical integration. Thus, the terminal characteristics of the bipolar generator are given against J' for parameter Re and the boundary ion ratio B_1 in Figs. 4.9a - 4.9d.

The rationale for the choice of $J' > 0$, $(1+B_1) > 0$ lies in the desire to apply these results to the conditions obtaining in the atmosphere during the early stages of cloud electrification when it is to be expected that the ambient field is weak and hence both species are injected at $x' = 0$. This also avoids the embarrassing choice of inconsistent boundary values. Since the fair-weather space charge is positive, $1 + B_1 > 0$ is the proper choice. From Eq. (4.2.59), both ion species are injected at $x' = 0$ when, for $(1 + B_1) > 0$,

$$\frac{2B_1}{1 + B_1} < J' < \frac{2}{1 + B_1}$$

It is interesting to note that if $b_1 = b_2$

$$V_1'(B_1) = V_2'(B_2) \quad . \quad (4.2.64)$$

On the other hand, if we choose $B_1 = \alpha$, then

$$V_1'(\alpha) = -V_2'(\alpha) \quad , \quad (4.2.65)$$

so that

$$V_1'(B_1) = -V_1'(1/B_1) \quad . \quad (4.2.66)$$

Thus, calculation for $-1 < B_1 < 0$ also gives the results for $B_1 < -1$.

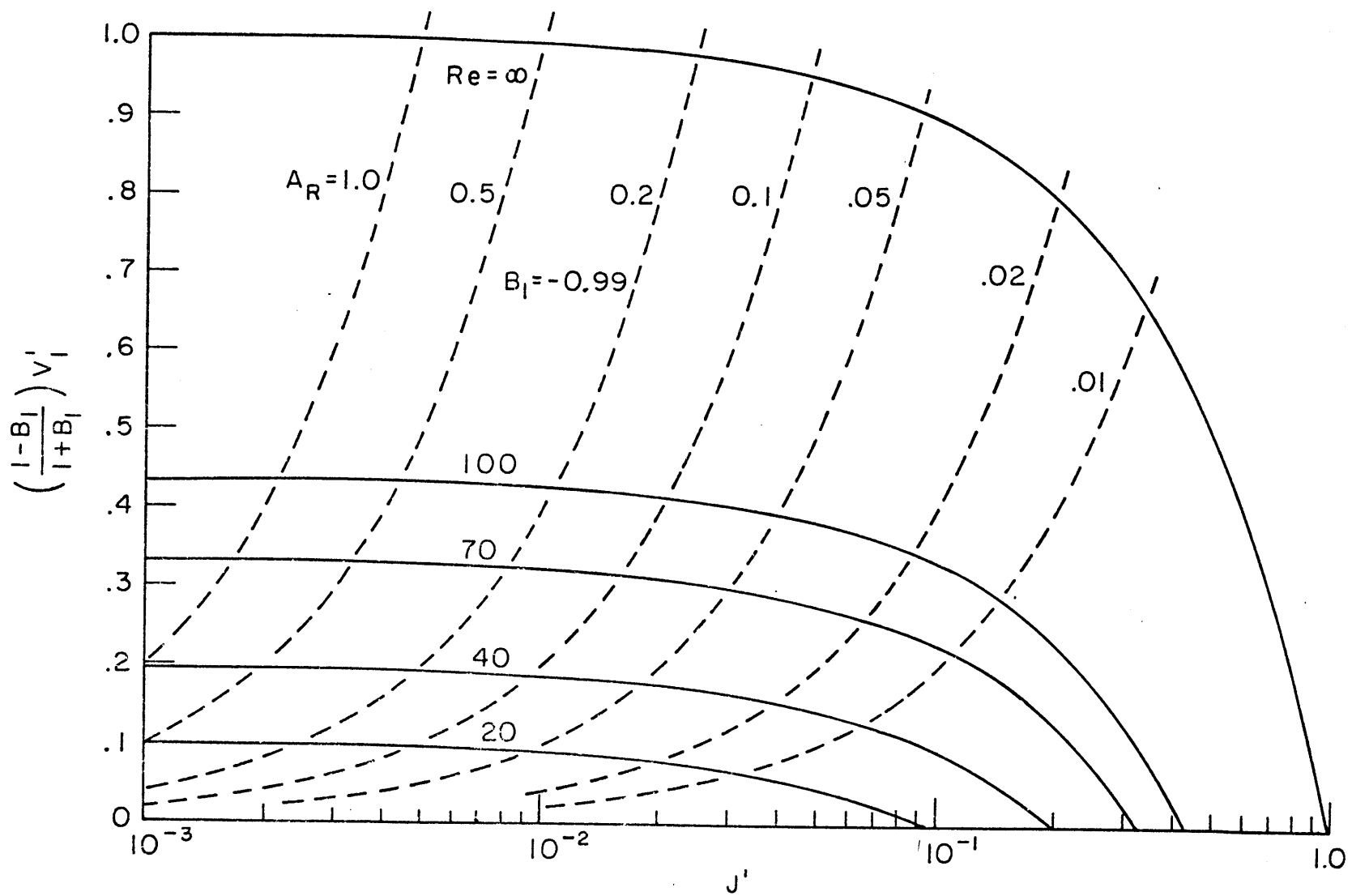


Fig.4.9a Bipolar terminal voltage for small initial space charge

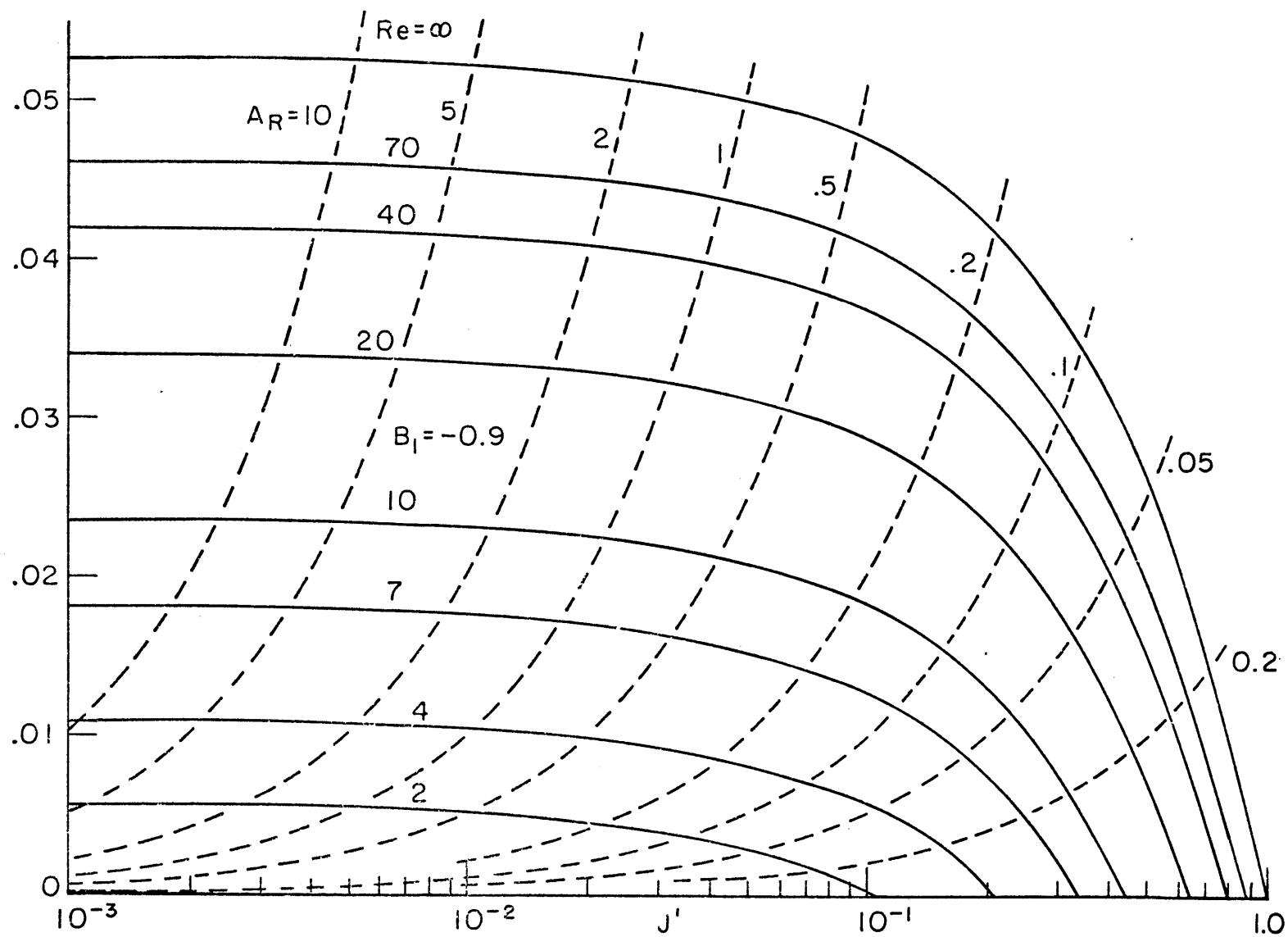


Fig. 4.9b Bipolar terminal voltage for moderate initial space charge

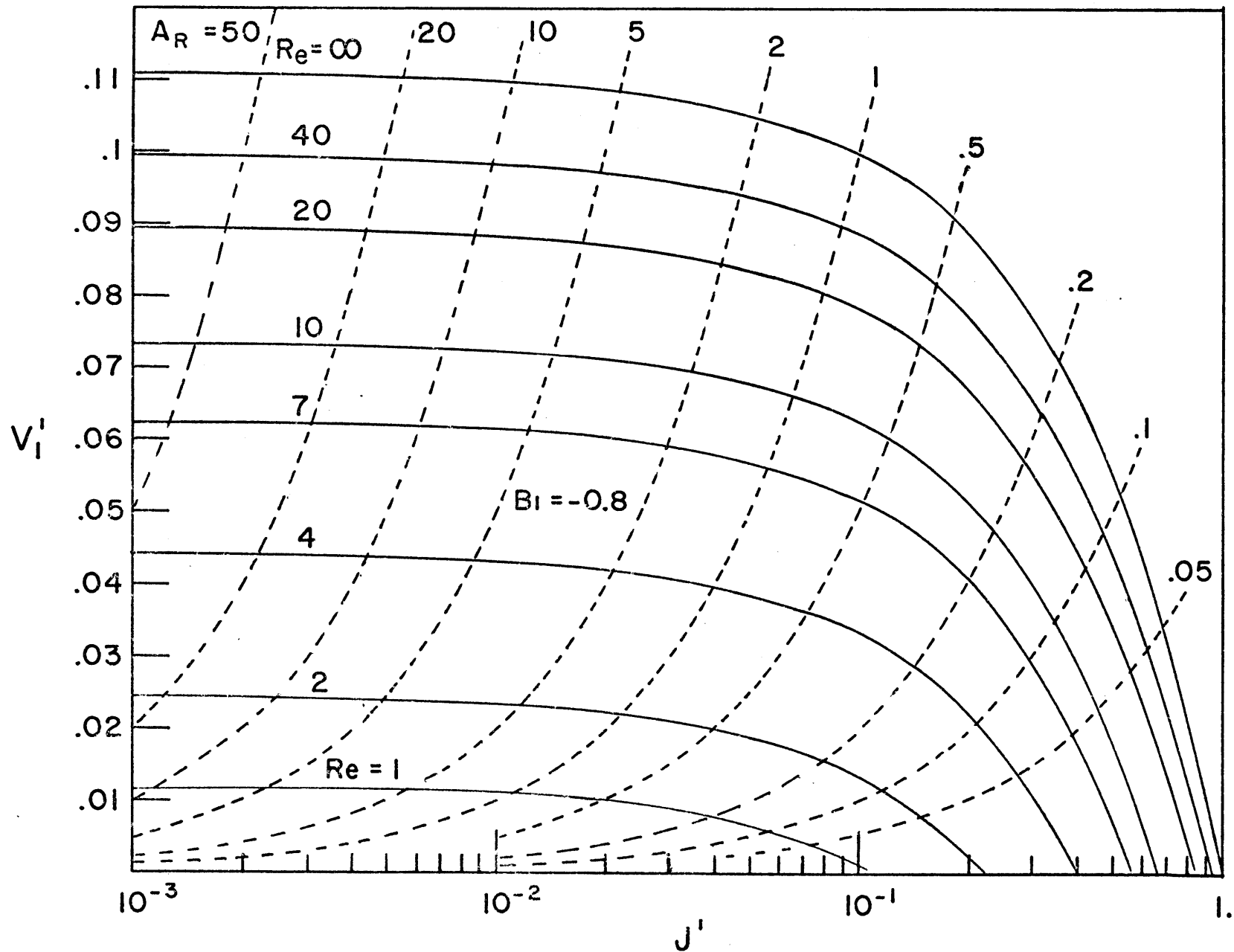


Fig. 4.9c Bipolar terminal voltage for large initial space charge

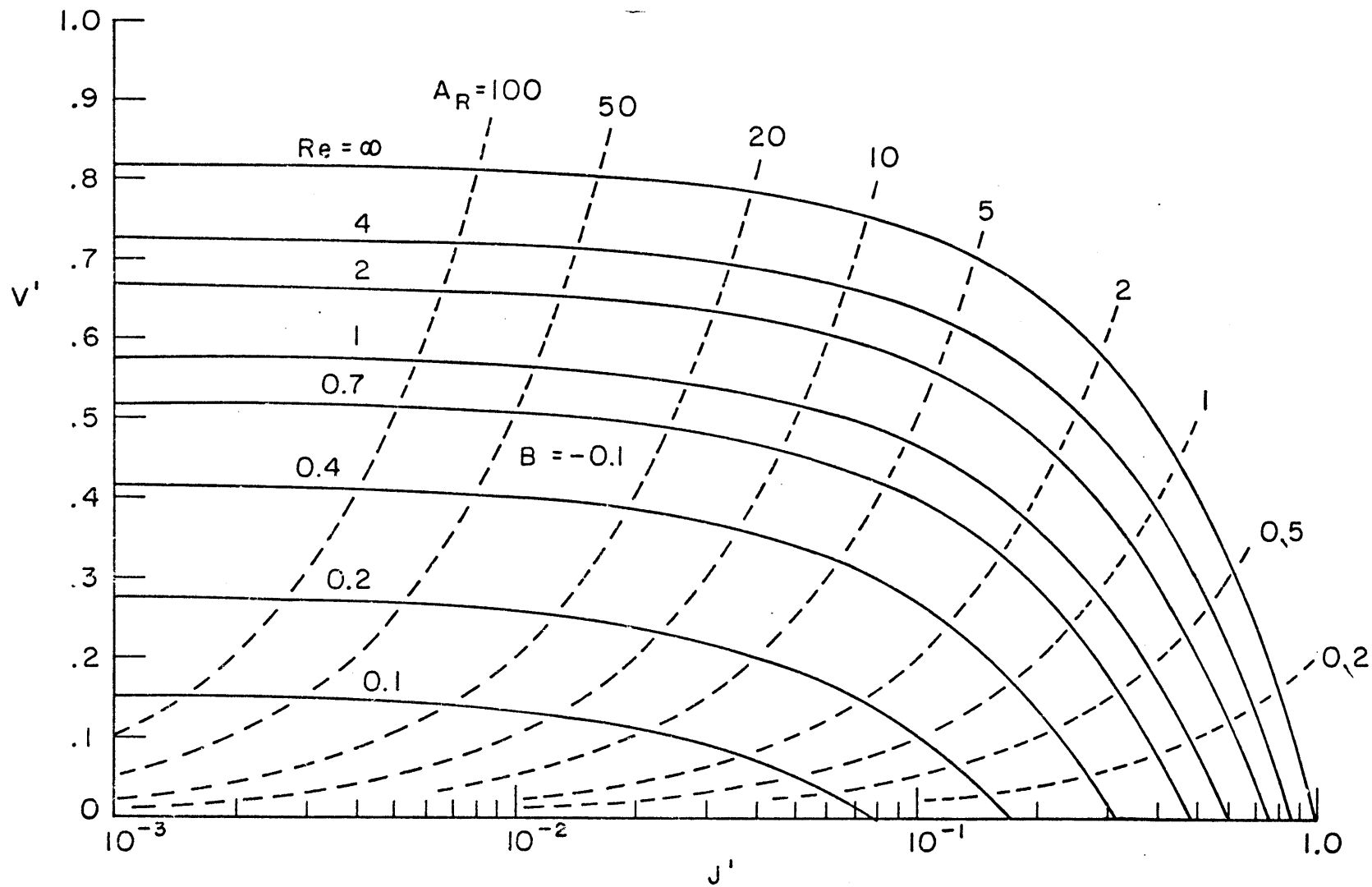


Fig. 4.9d Bipolar approximation of the unipolar generator

The $Re \rightarrow \infty$ curve bounds the potential in the terminal relation. Since for $Re \rightarrow \infty$, $Z_1 \approx 1$, the limiting curve can be calculated from Eq. (4.2.63) and the transformation, Eq. (4.2.59),

$$V_m' = \left(\frac{1 + B_m}{1 - B_m \frac{b_n}{b_m}} \right) (1 - J') \quad (4.2.67)$$

Thus, the regions of generation are bounded by $J' < 1$ and $B_m > -1$, with the maximum normalized potential given by

$$V' = \frac{1 + B_m}{1 - B_m \frac{b_n}{b_m}} \quad (4.2.68)$$

Table 4.4 summarizes the results of the bipolar convective generator.

4.2.5 Summary

Section 4.2 gives a relatively complete analysis of convective charge transport in a bipolar atmosphere with uniform convection. The essential points of the results are found in the comparison with the unipolar charge and field distribution of Chapter II. The facility for self-consistent charge relaxation is of considerable importance unless the electric Reynolds number is high.

Comparison of the terminal relations of the two cases shows that the predicted potential in the bipolar model is always less than that of the unipolar case, with the difference at $Re \rightarrow \infty$ given by the ratio $(1 + B_1)/(1 - B_1)$. That is, the more nearly equal the polar ion densities, the greater the difference between the bipolar and unipolar results.

It is of some interest to note that the vertical "conductivity", $b_1 \rho_1 - b_2 \rho_2$, can be determined from the bipolar results. When $Re \ll 1$,

the ion densities reach their asymptotic values before $x' = 1$, and the ratio of conductivity at $x' = 1$ to that at $x' = 0$ can be computed.

Defining the normalized conductivity by

$$\sigma' = \frac{\sigma}{b_1 \rho_1|_0} = \frac{1}{z_1} - \frac{b_2}{b_1} \frac{B_1}{z_2} \quad (4.2.69)$$

and defining $\sigma'|_f = \sigma'(x'=1)$, $\sigma'|_0 = \sigma'(x'=0)$, then, when $Re \ll 1$,

$$\frac{\sigma'|_f}{\sigma'|_0} = \frac{-D_1(1 + \frac{b_2}{b_1})}{(1 - \frac{b_2}{b_1} B_1)} \quad (4.2.70)$$

From the definition of D_1 in Table 4.3, assuming $b_1 = b_2$, Eq. (4.2.70) gives

$$\frac{\sigma'|_f}{\sigma'|_0} = \frac{J'(1 + B_1)^2 - 4B_1}{(1 - B_1)^2} \quad (4.2.71)$$

When $J' = 1$, corresponding to negligible electric field at $x' = 1$, the above ratio is unity. For values of J' in the range $0 < J' < 1$, $\sigma'|_f/\sigma'|_0 < 1$. The bipolar model, therefore, cannot explain the large increase in conductivity seen by Rossmann in his glider flights, discussed in Chapter II.

Table 4.4

Summary of Results for the Bipolar MobileIon Case

Species charge density distribution:

$$\begin{aligned} \frac{D_m A_m}{2(1-A_m)} Z_m^2 + \left(D_m - \frac{A_m}{(1-A_m)} \right) Z_m + \left(\frac{A_m}{D_m(1-A_m)} - 1 \right) \ln \left(\frac{1 + D_m Z_m}{1 + D_m} \right) \\ = D_m \left(1 + \frac{A_m}{2(1-A_m)} \right) - \frac{A_m}{1-A_m} + \frac{D_m^2 x'}{Re_m J_m'} \end{aligned}$$

$$Z(0) \equiv 1$$

Electric field distribution:

$$E_m' = \pm \left[\frac{J'(1+B_m) - B_m(1 + \frac{b_n}{b_m})}{1 - B_m \frac{b_n}{b_m}} Z_m - 1 \right]$$

Terminal relation:

$$V_m' = \mp \left[\frac{J'(1+B_m) - B_m(1 + \frac{b_n}{b_m})}{1 - B_m \frac{b_n}{b_m}} \int_0^1 Z_m dx' - 1 \right] \quad (\text{See Figs. 4.9a - 4.9d})$$

Asymptotes as $x' \rightarrow \infty$:

$$\begin{aligned} Z_m \rightarrow \frac{\frac{b_m}{b_n} - B_m}{J'(1+B_m) - B_m(1 + \frac{b_m}{b_n})} \\ E_m' \rightarrow \mp \left[\frac{\frac{b_n}{b_m} \frac{J'(1+B_m) - B_m(1 + \frac{b_n}{b_m})}{J'(1+B_m) - B_m(1 + \frac{b_m}{b_n})} - 1 \right] \end{aligned}$$

Limit as $Re \rightarrow \infty$:

$$V_m' = \frac{1 + B_m}{1 - B_m \frac{b_n}{b_m}} (1 - J')$$

4.3 The Bipolar Load

The unipolar load discussed in Sec. 3.3 was found to be inadequate for describing a bipolar atmosphere because the only charge available for participation in the conduction current in such a model is the net space charge, whereas the bipolar conduction current arises from the motions of the total charge carrier population. Using the bipolar model formulated in Sec. 4.2, we can determine the bipolar terminal characteristics of a static load which should give more representative results when coupled to the generating section. This is done in Sec. 4.3.1.

The fair-weather, air-earth current density is reasonably steady and remarkably uniform in the vertical [Kraakevik, 1961]. Since the electric field varies appreciably near the ground and perceptibly at higher levels, the usual model of the fair-weather, earth-ionosphere circuit is a current source in series with a conducting column whose resistivity varies with distance above the ground. In models such as those we are considering, any part of this conducting column may be considered a portion of a more complex system, so that the terminal characteristics of the columnar resistance are desirable. These can be found from the empirical formula given in Sec. 3.4.1, so long as the operating characteristics of the columnar resistance are independent of circumstances such as increased particle and current densities and enhanced electric field due to nearby generators. The empirical nature of the resistivity formula of Eq. (3.4.3), however, and the restriction to clear undisturbed weather patterns of the

data used to compile the formula in no way guarantees that the same result should be applied in situations of a wholly different nature.

The bipolar load, on the other hand, is formulated in terms of boundary values which can be tailored to meet the different conditions which may obtain in disturbed conditions and thus may be of more general value. One test of confidence in the bipolar load is its ability to reproduce the empirical results in fair-weather conditions. This question is examined in Sec. 4.3.2.

Finally, Sec. 4.3.3 examines the coupled generator and load for comparison with the unipolar model of Chapter III.

4.3.1 The Bipolar Static Atmosphere Solution

The polar current densities in a bipolar atmosphere in which convection is absent are given by

$$J_m = \pm b_{m0} E \quad (4.3.1)$$

Thus,

$$\frac{J_1}{\rho_1 |_0 b_1} = - \frac{J_2}{\rho_2 |_0 b_2} \quad (4.3.2)$$

in the steady state. Hence, the term

$$A_m = \frac{- J_m b_n \rho_n |_0}{J_n b_m \rho_m |_0} \quad (4.3.3)$$

is unity when $U = 0$. From Eq. (4.2.19), when $A_m = 1$,

$$Z_m = Z_n \quad (4.3.4)$$

and the distribution of Z_m with x' can be found from Eq. (4.2.22)

with $A_m = 1$,

$$\frac{d}{dx'} Z_m^2 = \pm 2(1 + B_m) \frac{b_{m0}^2 |_{OH}}{\epsilon_0 J_m} \quad (4.3.5)$$

whence

$$Z_m = \left[1 \pm 2(1 + B_m) \frac{b_{m0}^2 |_{OH}}{\epsilon_0 J_m} x' \right]^{1/2} \quad (4.3.6)$$

$$Z_m(0) \equiv 1$$

or

$$Z_m = \left[1 \mp 2(1 + B_m) \frac{b_{m0}^2 |_{OH}}{\epsilon_0 J_m} (1 - x') \right]^{1/2} \quad (4.3.7)$$

$$Z_m(1) \equiv 1 .$$

From Eq. (4.3.1) and the definition of Z_m , the electric field distribution is

$$E = \pm \frac{J_m}{b_{m0} |_{OH}} Z_m \quad (4.3.8)$$

and since

$$\frac{1}{H} \frac{d\Phi}{dx'} = -E , \quad (4.3.9)$$

the terminal characteristics of a bipolar static load are given by

$$V = \mp \frac{J_m H}{b_{m0} |_{OH}} \int_0^1 Z_m dx' . \quad (4.3.10)$$

$$Z_m(0) \equiv 1 .$$

Thus,

$$V = - \frac{\epsilon_0 J_m^2}{3b_{m0}^2 |_{OH}} (1+B_m) \left[\left(1 \pm 2 \frac{(1+B_m)b_{m0}^2 |_{OH}}{\epsilon_0 J_m} \right)^{3/2} - 1 \right] \quad (4.3.11)$$

$$Z_m(0) \equiv 1 .$$

or

$$V = \frac{\epsilon_0 J_m^2}{3b_m^2 \rho_m^3 |_0} (1+B_m) \left[\left(1 + 2 \frac{(1+B_m)b_m \rho_m^2 |_0^H}{\epsilon_0 J_m} \right)^{3/2} - 1 \right] \quad (4.3.12)$$

$$Z_m(1) \equiv 1 \quad .$$

Equations (4.3.11) and (4.3.12) are identical to the terminal relations for a unipolar atmosphere, Eqs. (3.3.1) and (3.3.2) with the exception of the term $(1+B_m)$ in the bipolar solution. Also, J_m represents only a part of the total current density, $J = J_1 + J_2$, whereas J_L in the unipolar solution signifies the total current density. The transformation necessary to replace J_m by the total current density, J , can be found from Eq. (4.3.2) and the definition of B_m . Thus,

$$J = J_1 + J_2 = J_m (1 - [b_n/b_m] B_m) \quad (4.3.13)$$

or

$$J_m = \frac{J}{(1 - \frac{b_n}{b_m} B_m)} \quad (4.3.14)$$

4.3.2 Comparison with an Empirical Conduction Model

As was noted in Sec. 3.4, one of the major differences between an ohmic conduction model and a unipolar model is that conduction in the former can take place in the absence of net charge, in contrast to the unipolar case. To see this more clearly, consider a steady-state current in a medium of conductivity σ ,

$$\vec{J} = \sigma \vec{E} \quad (4.3.15)$$

Since $\nabla \cdot \vec{J} = 0$ in the steady state,

$$\vec{E} \cdot \nabla \sigma + \sigma \nabla \cdot \vec{E} = 0 \quad (4.3.16)$$

and when σ is uniform, therefore

$$\nabla \cdot \vec{E} = \rho / \epsilon_0 = 0. \quad (4.3.17)$$

On the contrary, when $\nabla \sigma \neq 0$, net charge must exist whose distribution is

$$\rho = - \frac{\epsilon_0 \vec{E} \cdot \nabla \sigma}{\sigma}. \quad (4.3.18)$$

Similar remarks apply in the bipolar mobility conduction model, since from Eq. (4.3.5) when $B_m = -1$, i.e., $\rho_1|_0 + \rho_2|_0 = 0$, the polar space charge is uniform and zero within the load. Thus, the uniform conductivity model is reproduced by the bipolar model when $B_m = -1$ with the current density given from Eqs. (4.3.1) and (4.3.14),

$$J = \pm (b_1 + b_2) \rho_m|_0 E \quad (4.3.19)$$

and the uniform conductivity is thus

$$\sigma = \pm (b_1 + b_2) \rho_m|_0. \quad (4.3.20)$$

On the other hand, when $B_m \neq -1$, net space charge exists at the boundary, and, at least to some extent within the region of conduction. In this case, the current density expression is

$$J = \pm (b_m - b_n B_m) \frac{\rho_m|_0}{Z_m} E \quad (4.3.21)$$

or

$$J = \pm \frac{(b_m \rho_m|_0 - b_n \rho_n|_0)}{Z_m} E. \quad (4.3.22)$$

Thus, the nonuniform conductivity is given by

$$\sigma(x') = \pm \frac{(b_m \rho_m|_0 - b_n \rho_n|_0)}{Z_m} \quad (4.3.23)$$

where Z_m is given by Eq. (4.3.6) or (4.3.7). When $B_m = -1$, $Z_m = 1$, and thus $\rho_1 + \rho_2 = 0$ everywhere. Equation (4.3.23) shows in this case that the current in the uniform bipolar and conductivity models exists by virtue of a net flux of charge, rather than a flux of net charge.

The resistivity in the bipolar model, W_b , is given by the inverse of the conductivity,

$$W_b = \pm \frac{Z_m}{b_m \rho_m|_0 - b_n \rho_n|_0} \quad (4.3.24)$$

The behavior of W_b with height can be compared to the empirical formula of Gish [1944], given earlier and repeated here:

$$W = (2.94 e^{-4.52h} + 1.39 e^{-0.375h} + 0.369 e^{-0.121h}) \times 10^{13} \text{ ohm-m} \quad (4.3.25)$$

provided suitable boundary values in the bipolar model can be found. Since Gish's formula is restricted to fair-weather conditions, the proper comparison must be made with these conditions obtaining in the calculation of W_b .

The fair-weather current density is negative, and since $U = 0$, injection of the negative species occurs at $x' = 0$. Thus, substituting J from Eq. (4.3.14) for J_2 in Eq. (4.3.6),

$$Z_2 = \left[1 - \frac{2b_2(1 - \frac{b_1}{b_2} B_2)(1+B_2)\rho_2^2|_0 H x'}{\epsilon_0 J} \right]^{1/2} \quad (4.3.26)$$

Expanding $B_2 = \rho_1|_0 / \rho_2|_0$ and multiplying through by b_2 and $\rho_2^2|_0$,

$$Z_2 = \left[1 - \frac{2(b_2 \rho_2|_0 - b_1 \rho_1|_0)(\rho_1|_0 + \rho_2|_0) H x'}{\epsilon_0 J} \right]^{1/2} \quad (4.3.27)$$

Defining

$$b\rho_T|_0 = b_1\rho_1|_0 - b_2\rho_2|_0 \quad (4.3.28)$$

and since $\rho_0 = \rho_1|_0 + \rho_2|_0$, Eqs.(4.3.27) and (4.3.28) with Eq. (4.3.24) give

$$W_b = \frac{1}{b\rho_T|_0} \left[1 + \frac{2b\rho_T|_0\rho_0}{\epsilon_0 J} H x' \right]^{\frac{1}{2}}. \quad (4.3.29)$$

One observation is immediately apparent. The observed resistivity decreases with altitude, probably as a consequence of increasing particle mobility and higher concentrations of charged particles due to an increase in the ionization rate. Since J is negative and the observed atmospheric space charge is positive in fair-weather conditions, the theoretical resistivity also decreases with height. However, Eq. (4.3.29) is not valid for arbitrary H , since the argument under the square root must be positive. This is so only if H is bounded and hence the bipolar load can fit the observed fair-weather conduction for limited regions only.

To make further comparison between the theoretical and observed resistivities of Eqs. (4.3.29) and (4.3.25), it is necessary to determine the boundary values, $b\rho_T|_0$ and ρ_0 to use in the calculation of W_b . In our determination, we will choose these values so that $W = W_b$ at $x' = 0$. Thus, the boundary condition $b\rho_T|_0$ is found from Eq. (4.3.29) to be:

$$b\rho_T|_0 = \frac{1}{W(h)} \quad (4.3.30)$$

where $W(h)$ is the observed resistivity of Eq. (4.3.25) at the height, h ,

of the $x' = 0$ level.

The space charge boundary value is found from the empirical conduction equation,

$$E = JW \quad . \quad (4.3.31)$$

where, since J is constant, the empirical space charge distribution is given by Gauss's law,

$$\rho(h) = (\epsilon_0 J) \frac{dW}{dh} \times 10^{-3} \text{ c/m}^3 \quad (4.3.32)$$

where the factor 10^{-3} arises since h is measured in kilometers. The boundary value $\rho_0/\epsilon_0 J$ is therefore

$$\frac{\rho_0}{\epsilon_0 J} = - (13.3 e^{-4.25h} + 0.521 e^{-0.375h} + .045 e^{-0.121h}) \times 10^{10} \quad (4.3.33)$$

Equations (4.3.33) and (4.3.30) thus provide boundary values of $b\rho_T|_0$ and $\rho_0/\epsilon_0 J$ to calculate W_b from Eq. (4.3.29) which are consistent with the conditions obtaining in the empirical formula.

To assess the different results between the theoretical model and the empirical results, the influence of the scale height H was first deduced by comparing the predicted columnar resistance, R_b ,

$$R_b(h, H) = \int_h^{h+H} W_b(x-h) dx \quad (4.3.34)$$

with the empirical columnar resistance,

$$R(h, H) = \int_h^{h+H} W(h) dh \quad . \quad (4.3.35)$$

Equations (4.3.34) and (4.3.35) give the predicted and observed columnar resistance in ohm-m^2 for a column whose base is at h km above the ground and whose top is H km above the base. Figure 4.10 shows the fractional error in the columnar resistance for $H = 2, 5, 10$ km, and $0 \leq h \leq 5$ km.

The fractional error is given by

$$\frac{R_b(h, H) - R(h, H)}{R(h, H)} \quad (4.3.36)$$

The results of the columnar resistance study are two-fold. First and foremost, the error incurred by the bipolar model becomes very large even for small H , when the integration proceeds from ground level. In fact, this is not surprising, since the empirical model includes the effects of much higher aerosol content below the exchange layer which is not included in the bipolar model. Significantly, the error drops off rapidly when the column base is moved upward to 1 km height above the ground, and falls off only slowly thereafter as h is further increased. Since 1 km is near the average level of the exchange layer, it is reasonable to attribute the large error for $h < 1$ km to that effect.

The second conclusion has to do with the effect of the scale height H on the results. Clearly, $H = 10$ km involves a significant error, and for $H = 5$ km, the average error is yet of the order of 50%. When H is further reduced to 2 km, however, the error decreases to about 10% for $h = 1$ km and further decreases to near 6% as h increases. Thus, the columnar resistance of a 2 km section whose base is above the exchange layer is predicted with fair accuracy by the bipolar model.

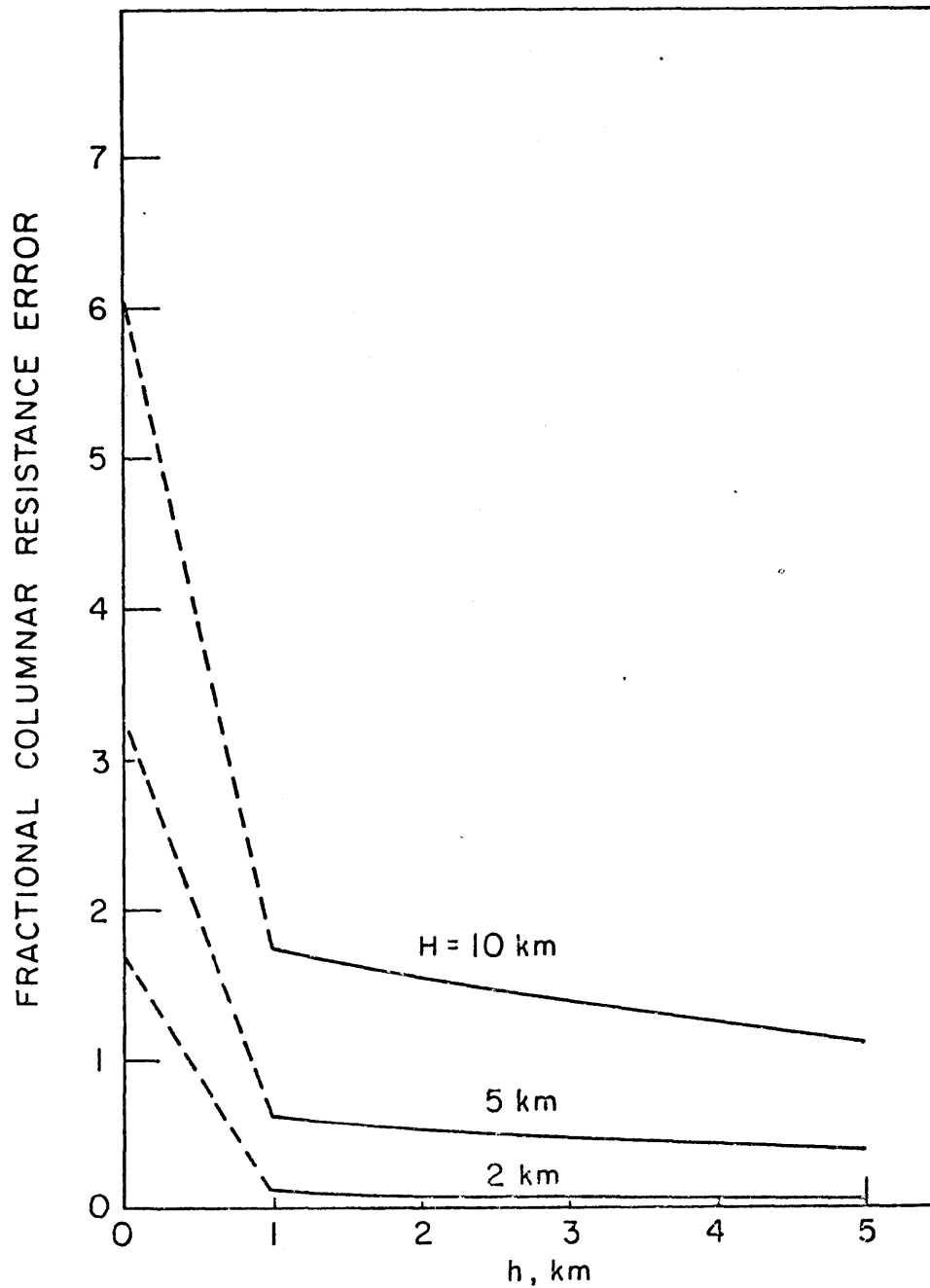


Fig. 4.10 Comparison of the columnar resistance of a static section of height, H , predicted by the bipolar model, to that of the empirical formula due to Gish [1944]

Figure 4.11 shows the error distribution within a load section of heights 5, 2, and 1 km with base at $h = 1$ km. In these results, the fractional resistivity error is given by

$$\frac{W_b(x', h) - W(h + x'H)}{W(h + x'H)} \quad (4.3.37)$$

The theoretical and empirical results fit exactly at $x' = 0$ because of the method of choosing boundary values for the bipolar model. The error increases with x' until it reaches a peak, and decreases somewhat thereafter. The cause of this is examined below. It is noteworthy that, for $H = 1$ km, the error maximum is not much over 5% when the base of the region is at 2 km.

Figures 4.12 and 4.13 show examples of the empirical and theoretical resistivity and columnar resistance for a 2 km section with base at 2 km. Examination of the resistivity results shows the cause of the error peak in Fig. 4.11. Note that $d^2W_b/dx^2 < 0$, while $d^2W/dx^2 > 0$. Since

$$\epsilon_0 J \frac{d^2W}{dx^2} = \frac{d\rho}{dx} \quad (4.3.38)$$

from Eq. (4.3.32), negative J implies that $d\rho/dx > 0$ in the theoretical model and $d\rho/dx < 0$ in the empirical result. To understand this result, consider Eq. (4.3.24) in which we replace Z_m by the normalized space charge,

$$\frac{\rho}{\rho_1|_0 + \rho_2|_0} = \frac{1}{(1+B_1)Z_1} + \frac{1}{(1+B_2)Z_2} \quad (4.3.39)$$

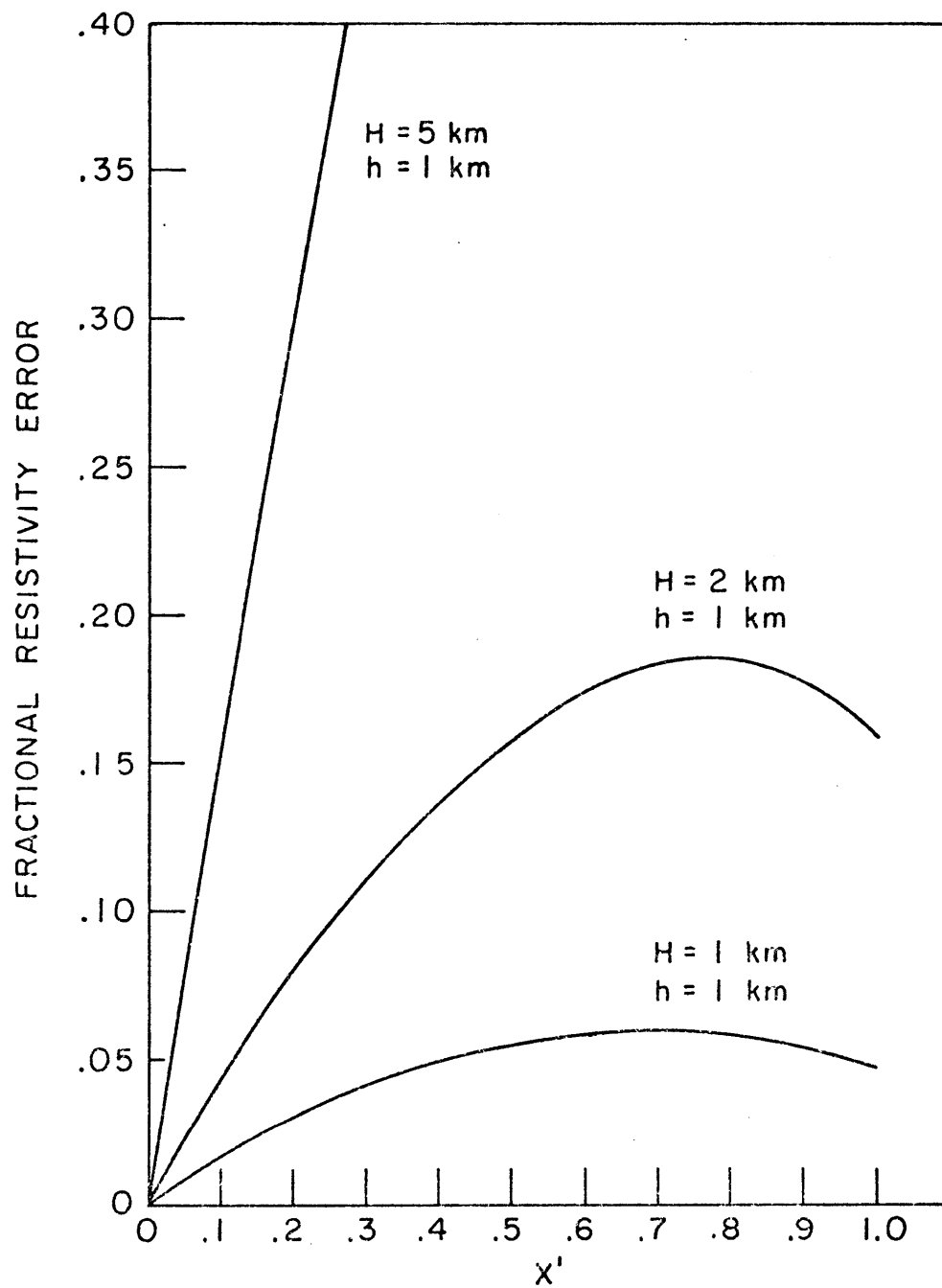


Fig. 4.11 Distribution of the resistivity error within a section of height H whose base is at 1 km from comparison of the bipolar model results with Gish's formula [1944]

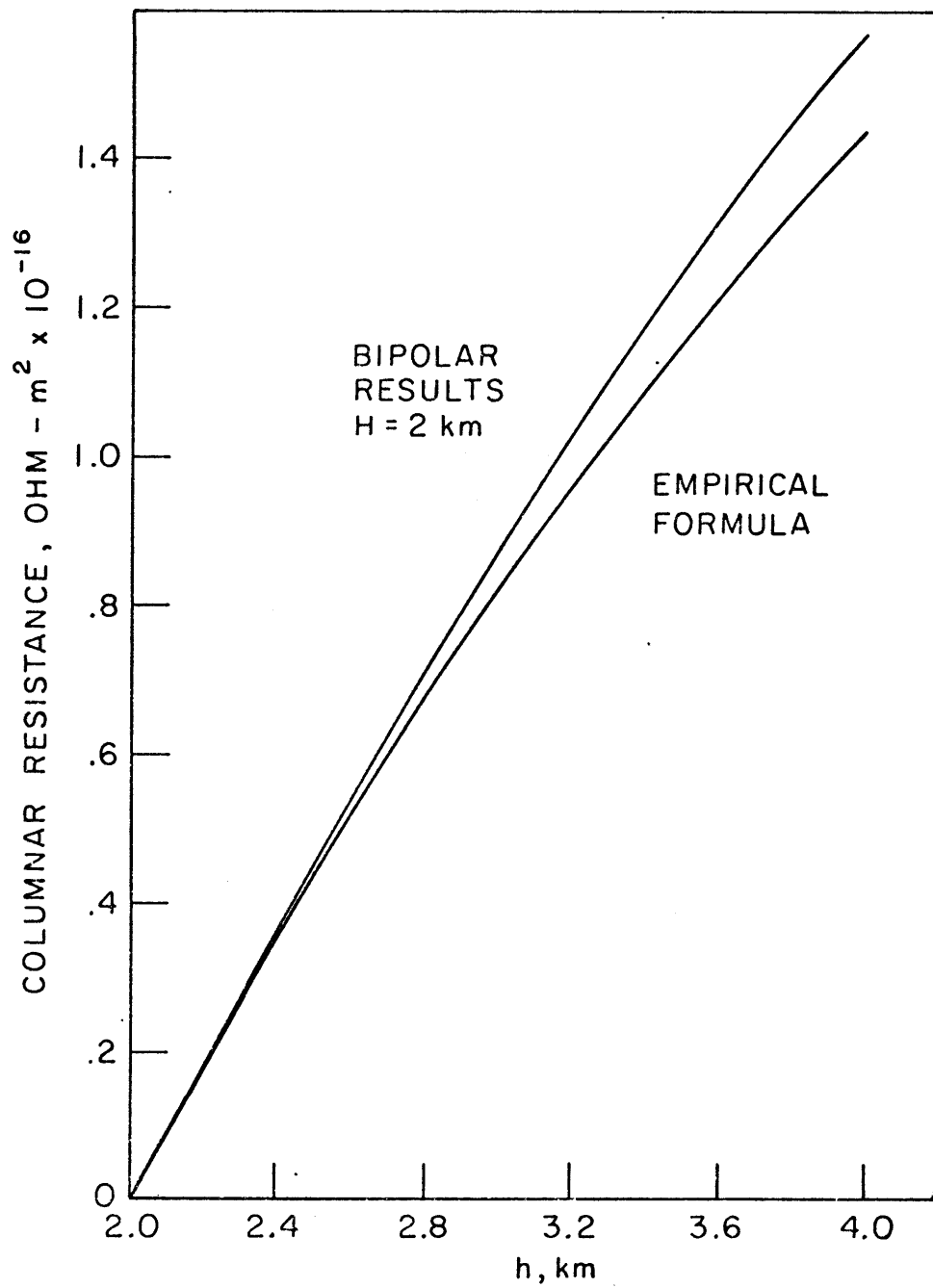


Fig. 4.12 Columnar resistance in a section with base at 2 km predicted by the bipolar model and the empirical formula of Gish

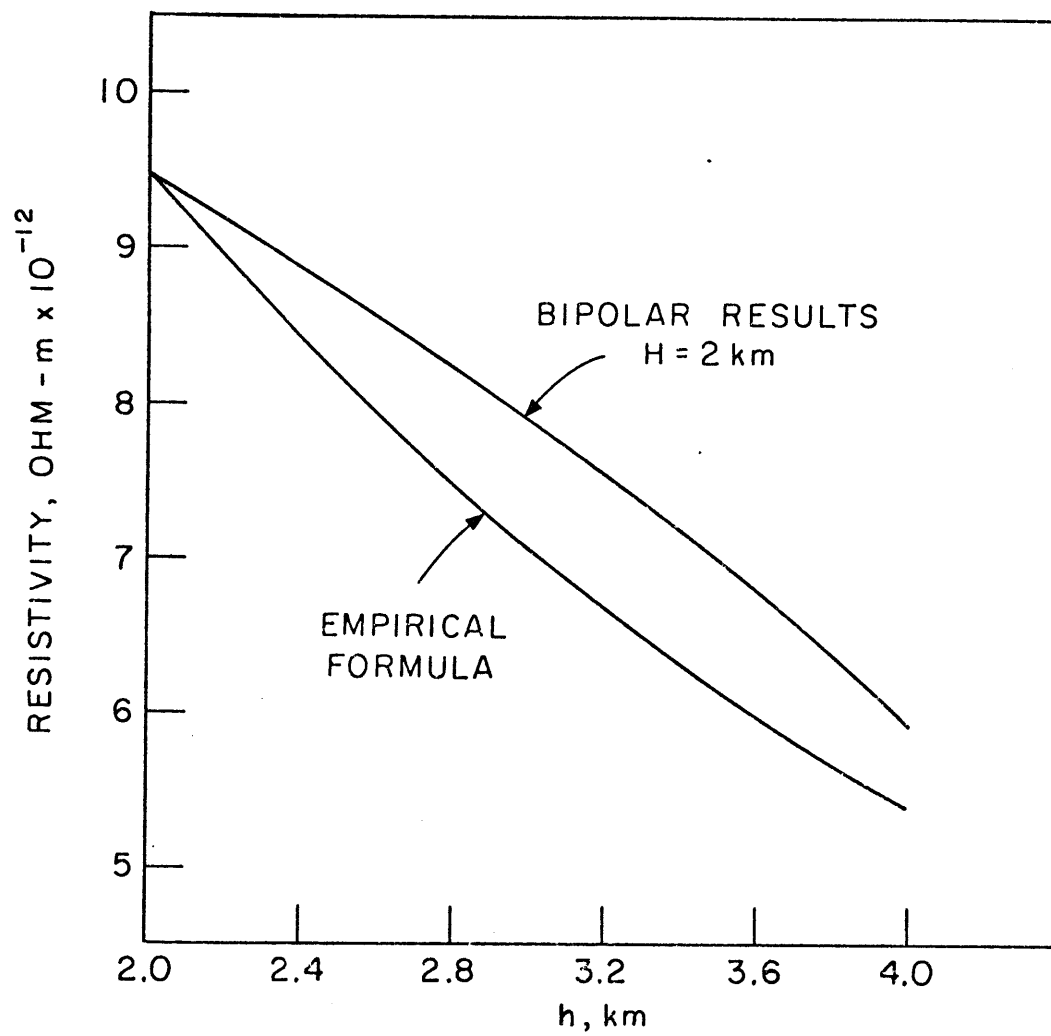


Fig. 4.13 Empirical and predicted resistivity in a section of height 2 km with base at 2 km

or, since $B_1 = 1/B_2$ and $Z_1 = Z_2$ in the bipolar load,

$$\rho' = \frac{1}{Z_m} \quad . \quad (4.3.39a)$$

Hence, Eq. (4.3.24) becomes

$$W_b = \frac{1}{b\rho_T|_0 \rho'} \quad . \quad (4.3.39b)$$

Since $b\rho_T|_0$ is a boundary value, and therefore constant, the only way that W_b can decrease with altitude is for ρ' to increase. In the atmosphere, however, $b\rho_T$ can also increase with altitude by virtue of increasing b_m with decreasing pressure and increasing total particle densities because of increasing ionization.

4.3.3 Coupling of the Bipolar Load and Generator

In principal, the bipolar load and generator can be coupled in the same way as was done for the unipolar load and generator in Sec. 3.3.2. In fact, those results are completely analogous, so long as care is taken to include the $(1+B_m)$ terms and it is recognized that the normalized current density and electric Reynolds number are written in their polar forms. Thus, in Eq. (3.3.4), the term $\rho_0/\rho_L(0)$ is written $\rho_m|_0/\rho_{Lm}(0)$ in the unipolar coupled equation, and the unipolar term $\text{Re}J'$ is written $\text{Re}J'_m$ in the bipolar case.

Thus, the coupled-load normalized potential from Eqs. (4.3.11) and (4.3.12), using (3.3.4) and (3.3.5) as guides is:

$$V_L' = -\frac{f^2}{3} \left(\frac{b}{b_L}\right)^2 \left(\frac{\rho_m|_0}{\rho_{Lm}(0)}\right)^3 \frac{\text{Re } J_m'^2}{(1+B_m)} \times \left\{ \left[1 - \frac{2}{f} \frac{H_L}{H} \frac{b}{b_L} \left(\frac{\rho_{Lm}(0)}{\rho_m|_0}\right)^2 \frac{1+B_{Lm}}{\text{Re } J_m'} \right]^{3/2} - 1 \right\} \quad (4.3.40)$$

for load boundary values at $x' = 0$, and

$$V_L' = \frac{f^2}{3} \left(\frac{b}{b_L}\right)^2 \left(\frac{\rho_m|_0}{\rho_{Lm}(1)}\right)^3 \frac{\text{Re } J_m'^2}{1+B_m} \times \left\{ \left[1 + \frac{2}{f} \frac{H_L}{H} \frac{b}{b_L} \left(\frac{\rho_{Lm}(1)}{\rho_m|_0}\right)^2 \frac{1+B_{Lm}}{\text{Re } J_m'} \right]^{3/2} - 1 \right\} \quad (4.3.41)$$

for load boundary values imposed at $x' = 1$.

Our primary interest is comparing the operating characteristics of the bipolar generator with those of the unipolar generator of Chapter III when coupled to a similar load. Moreover, the results of Sec. 4.3.2 indicate that the bipolar load compares favorably with the observational data when the load base is at least a kilometer above the ground. Thus, instead of generating load curves from Eq. (4.3.41) to overlay the generator curves of Fig. 4.9, it was decided that a reasonable comparison could be had by comparing the two generators when coupled to the empirical load discussed in Sec. 4.3.2.

Using the columnar resistance, $R(h, H)$, the load potential is

$$V_L = -J_L R(h, H_L) \quad (4.3.42)$$

Normalizing by the generator potential scaling factor,

$$V_L' = - \frac{b_m}{UH} J_L R(h, H_L) \quad (4.3.43)$$

Since

$$J_L = - fJ \quad (4.3.44)$$

$$V_L' = \frac{b_m f \rho_m|_O (1 + B_m)}{H} J' R(h, H_L) \quad (4.3.45)$$

where the net boundary space charge is

$$\rho_O = \rho_m|_O (1 + B_m) \quad (4.3.46)$$

The load-lines of Eq. (4.3.45) are plotted as dashed lines on the bipolar curves of Figs. 4.9a - 4.9d with parameter

$$A_R = \frac{b_m f \rho_m|_O (1 + B_m) R(h, H_L)}{H} \quad (4.3.47)$$

Table 4.5 compares the unipolar predicted potentials when the load-lines of Eq. (4.3.45) with $B_m = 0$ are used in conjunction with the generator curves of Fig. 3.6.

Table 4.5

A_R	Unipolar	$V'(Re = 0.1)$		$V'(Re = 1.0)$	
		Bipolar $B_1 = -0.1$	Unipolar	Bipolar	
				$B_1 = -.1$	$B_1 = -.9$
100	.74	.15	.91	.57	.003
50	.66	.15	.88	.56	.003
20	.52	.14	.80	.54	.003
10	.40	.13	.74	.51	.003
5	.28	.11		.47	
2	.15	.08		.37	
1	.09	.05		.28	

When $B_1 = -0.99$, corresponding to nearly equal numbers of positive and negative carriers, the bipolar model predicts $V' \sim 9.5 \times 10^{-5}$ for $Re = 4$.

In making the comparison shown in Table 4.5, it is only fair to point out that the conditions at the boundaries of the two models are not identical in all respects. In particular, we have compared the two models when the boundary space charge and total current are the same in each case. The bipolar normalized current density is

$$J' = 1 + \frac{1 - B_m}{1 + B_m} E'_m \Big|_0 \quad (4.3.48)$$

and the unipolar current density is

$$J' = 1 + E' \Big|_0 . \quad (4.3.49)$$

Thus, under the conditions of the comparison,

$$E' \Big|_0 = \frac{1 - B_m}{1 + B_m} E'_m \Big|_0 . \quad (4.3.50)$$

4.3.4 Summary

The value of a bipolar model over the unipolar model of Chapter II is seen from the large difference in predicted potential by the two cases shown in Table 4.5. Moreover, there is even greater difference in the vertical distribution of space charge and electric field because of the relaxation of net space charge in the bipolar model which is not allowed in the unipolar case.

The close agreement between the static bipolar model and the empirical resistivity and columnar resistance examined in Sec. 4.3.2

is of no great significance, since a straight-line fit to the empirical data of Figs. 4.12 and 4.13 would do just as well as the theory. What is significant, however, is the difference between the observed and predicted values, since it shows qualitatively how the bipolar conduction model departs from the conduction processes in the atmosphere. Thus, the variation of ion mobility with density and the vertical equilibrium ion density determined by ionization and recombination are essential ingredients to the maintenance of a uniform vertical air-earth current in a region of positive space charge density, as observed.

The bipolar load terminal equations derived in Sec. 4.3.3, Eqs. (4.3.40) and (4.3.41), are of little use at present, since experimental observations are necessary to establish the notion of a localized load. The coupling of load and generator is accomplished at equipotential levels, and it is assumed that insignificant current flows through the regions of transition of the generator equipotential level, H , to the load equipotential level, H_L . This is likely to be a poor assumption, since it is the horizontal variation of current density which characterizes the transition region. That is, the current density is greater in the transition region than in the load. Thus, determination of the relative areas of the load and transition region are necessary to estimate the current flux through each. Nevertheless, the terminal relations give the important parameters to be determined in an experiment designed to investigate the relation of local regions of convective charge transport to the surroundings.

4.4 Volume Ionization and Recombination Effects

The space charge in the atmosphere which participates in the convective generation process is a consequence of ionization from radioactivity near the ground, radioactive gases emitted from the surface and distributed through the lower atmosphere, and ionization from the high energy particles of cosmic rays and their collision products. At cloud-forming altitudes, it is this latter source of ionization which is important [Callahan, et al, 1951].

In ordinary air, there is a complicated charge exchange process between small ions and the large ions and neutral aerosol particles. To account for this process, a multi-species ion-balance equation would have to be written. Therefore, to retain the bipolar model in which only two kinds of particles are present, we will make the approximation that recombination is limited to mutual annihilation of a positive and a negative carrier.

Moreover, we will assume that the ionization rate and recombination coefficient are independent of height. In the case of the recombination coefficient, this assumption is not unreasonable, since the value of this quantity at sea level is 0.92 times its value at 10 km. On the other hand, the ionization rate is a strong function of altitude, increasing by nearly two orders of magnitude from ground level to 10 km. For these reasons, as well as the restriction to only two charged species, the results of this section should be viewed in a qualitative light.

4.4.1 Effect of Convection on the Ion-balance Equation Without Space Charge

Under the conditions of isotropic ionization and recombination,

the small ion distribution follows closely the idealized ion-balance equation [Loeb, 1955]:

$$\frac{dn_2}{dt} = \alpha' - \beta' n_1 n_2 \quad (4.4.1)$$

where n_1 are the polar ion number densities, α' is the volume rate of ionization, and β' is the mutual recombination coefficient. In Eq. (4.4.1), it is implicit that motion of the surrounding gas, presence of electric fields, and processes of diffusion are neglected. Under these conditions,

$$\frac{d}{dt} (n_1 - n_2) = 0 \quad (4.4.2)$$

so that ionization and recombination do not of themselves alter the initial differences in the local particle population. This is the basis for finding the most common solution to the evolution in time of n_2 as described by Eq. (4.4.1). Thus, if particle densities are initially equal, characteristic of the case when only bulk sources are allowed, then $n_1 = n_2$ for all t . Equation (4.4.1) is therefore

$$\frac{dn}{dt} = \alpha' - \beta' n^2 \quad (4.4.3)$$

the solution of which is well known:

$$n = n_\infty \left[\frac{B \exp(At) - 1}{B \exp(At) + 1} \right] \quad (4.4.4)$$

where

$$n_\infty = (\alpha'/\beta')^{1/2} \quad (4.4.5)$$

$$A = 2(\alpha'\beta')^{1/2} \quad (4.4.6)$$

$$B = \frac{n_{\infty} + n_0}{n_{\infty} - n_0} \quad (4.4.7)$$

The effect of convection in the absence of diffusion and space charge effects can be inferred from Eqs. (4.4.1) and (4.4.4) as long as both particles move with the same velocity in addition to having equal densities at the boundary. For example, imagine both species injected at $x = 0$ with constant and uniform velocity, v . At time $t = 0$, let the prime coordinate frame moving with the convection be coincident with the stationary frame. Then following the particle motion, i.e., in the moving coordinate frame, Eq. (4.4.4) gives

$$n(x', t') = n_{\infty} \left[\frac{B(x', t') \exp(At') - 1}{B(x', t') \exp(At') + 1} \right] \quad (4.4.8)$$

where, now

$$B(x', t') = \frac{n_{\infty} + n(x', 0)}{n_{\infty} - n(x', 0)} \quad (4.4.9)$$

Then with the coordinate transformation,

$$x' = x - vt \quad (4.4.10)$$

$$\text{and} \quad t' = t \quad (4.4.11)$$

the particle distribution is

$$n(x, t) = n_{\infty} \left[\frac{B(x-vt) \exp(At) - 1}{B(x-vt) \exp(At) + 1} \right] \quad (4.4.12)$$

where

$$B(x - vt) = \frac{n_{\infty} + n(x - vt)}{n_{\infty} - n(x - vt)} \quad (4.4.13)$$

$$x - vt > 0$$

and

$$B(x - vt) = \frac{n_{\infty} + n_0}{n_{\infty} - n_0} \quad (4.4.14)$$

$$x - vt \leq 0$$

If t is large enough that $x - vt \leq 0$ for all pertinent x , the steady-state distribution is found from Eqs. (4.4.14) and (4.4.12),

$$n(x) = n_{\infty} \left[\frac{B \exp(Ax/v) - 1}{B \exp(Ax/v) + 1} \right] \quad (4.4.15)$$

A similar steady-state solution can be found when the particle fluxes are unequal at the boundary. For example, suppose $v_1 = U \pm b_1 E$. Then, again neglecting space charge, Eq. (4.4.1) in the steady state gives

$$v_1 \frac{dn_1}{dx} = \alpha' - \beta' n_1 n_2 \quad (4.4.16)$$

where

$$v_1 \frac{dn_1}{dx} = v_2 \frac{dn_2}{dx} \quad (4.4.17)$$

It is clear from Eq. (4.4.17), as well as from the implication of Eq. (4.4.2), that inequities in particle flux at the boundary are not influenced by ionization and recombination. Defining the normalized particle densities,

$$n_1' = n_1/n_1|_0, \quad (4.4.18)$$

integration of (4.4.17) gives

$$n_1' = 1 + Q(n_2' - 1) \quad (4.4.19)$$

where

$$Q = \frac{n_2|_0 v_2}{n_1|_0 v_1} \quad (4.4.20)$$

is the ratio of particle fluxes at the boundary.

Substituting Eq. (4.4.19) into (4.4.16) gives

$$\frac{dn_2'}{dx} = -\beta' \frac{n_2|_0}{v_1} \left[n_2'^2 + \left(\frac{1-Q}{Q}\right) n_2' - \frac{\alpha'}{\beta' n_1|_0 n_2|_0 Q} \right] \quad (4.4.21)$$

the solution to which is

$$n_2' = \frac{n_2'|_\infty B_2 \exp \left[\frac{\beta' n_2|_0}{v_1} (n_2'|_\infty + \eta)x \right] - \eta}{B_2 \exp \left[\frac{\beta' n_2|_0}{v_1} (n_2'|_\infty + \eta)x \right] + 1} \quad (4.4.22)$$

where

$$B_2 = \frac{\eta + 1}{n_2'|_\infty - 1} \quad (4.4.23)$$

$$\eta = n_2'|_\infty + \frac{1-Q}{Q} \quad (4.4.24)$$

and

$$n_2'|_\infty = \frac{1}{2} \left(\frac{1-Q}{Q} \right) \left\{ -1 \pm \left[1 + \frac{4 \alpha' Q}{\beta' n_1|_0 n_2|_0 (1-Q)^2} \right]^{\frac{1}{2}} \right\} \quad (4.4.25)$$

where the positive root is selected when $Q \leq 1$ and the negative root is for $Q > 1$.

When $Q = 1$ and $v_1 = v_2$, the conditions of (4.4.15) obtain. In this case, Eq. (4.4.22) can be shown to be equivalent to Eq. (4.4.19). If, on the other hand, the particle fluxes are equal but $v_1 \neq v_2$, Eq. (4.4.22) gives

$$n_2 = n_2|_\infty \frac{\left[B_2 \exp \left(\frac{A_2 x}{v_1} \right) - 1 \right]}{\left[B_2 \exp \left(\frac{A_2 x}{v_1} \right) + 1 \right]} \quad (4.4.26)$$

where

$$B_2 = \frac{n_2|_\infty + n_2|_0}{n_2|_\infty - n_2|_0} \quad (4.4.27)$$

and

$$A_2 = 2(\alpha'\beta' n_2|_0/n_1|_0)^{1/2} \quad (4.4.28)$$

while $n_2|_\infty$ is found from Eq. (4.4.25) with $Q = 1$ to be

$$n_2|_\infty = (\alpha'n_2|_0/\beta'n_1|_0)^{1/2} \quad (4.4.29)$$

4.4.2 The Mathematical Model with Space Charge

When space charge effects are included, Eq. (4.4.1) no longer describes the particle conservation equations. Since space charge is influential in the remainder of this section, we want to express the conservation equations in terms of the space charge densities ρ_1 . To do this, we first write the conservation equations in terms of n_1 , as

$$\frac{\partial n_1}{\partial t} + \nabla \cdot n_1 v_1 = \alpha' - \beta' n_1 n_2 \quad (4.4.30)$$

Then, multiplying by $\pm e$, for e the elementary unit of charge, and setting $\rho_1 = \pm e n_1$, Eq. (4.4.30) becomes

$$\frac{\partial \rho_1}{\partial t} + \nabla \cdot \rho_1 v_1 = \pm(\alpha + \beta \rho_1 \rho_2) \quad (4.4.31)$$

where

$$\alpha = \alpha' e \quad (4.4.32)$$

and

$$\beta = \beta' / e \quad (4.4.33)$$

Thus, since $J_1 = \rho_1 v_1$, the pertinent equations for the steady-state convecting system are identical to those of Sec. 4.2 [Eqs. (4.2.1) - (4.2.4)] except that the species charge conservation equations are no longer homogeneous.

Since a numerical integration is necessary to obtain the terminal relations when α and β are finite, no advantage is gained by developing the model in terms of the polar species. Instead, we form immediately the independent variables,

$$\rho = \rho_1 + \rho_2 \quad (4.4.34)$$

and

$$\rho_T = \rho_1 - \rho_2 \quad (4.4.35)$$

The total current density is

$$J = J_1 + J_2 \quad (4.4.36)$$

whence, from Eq. (4.2.1) and Eqs. (4.4.34) and (4.4.35), we get

$$J = \rho(U + \bar{b}\gamma E) + \bar{b}\rho_T E \quad (4.4.37)$$

$$\text{where } \bar{b} = \frac{b_1 + b_2}{2} \quad (4.4.38)$$

$$\text{and } \gamma = \frac{b_1 - b_2}{b_1 + b_2} \quad (4.4.39)$$

Similarly, the charge carrier flux is defined by

$$J_p = J_1 - J_2 \quad (4.4.40)$$

so that

$$J_p = \rho_T(U + \bar{b}\gamma E) + \bar{b}\rho E \quad (4.4.41)$$

In terms of these new variables, the steady, one-dimensional system is described by the normalized equations,

$$\frac{dJ'}{dx'} = 0 \quad (4.4.42)$$

$$\frac{dE'}{dx'} = \frac{R}{|R|} \frac{\rho'}{Re} \quad (4.4.43)$$

$$\frac{d\Phi'}{dx'} = -E' \quad (4.4.44)$$

and

$$\frac{dJ_p'}{dx'} = \frac{2\alpha H}{\rho_T|_0 U} + \frac{\beta H \rho|_0^2}{2\rho_T|_0 U} \left(\rho'^2 - \frac{\rho_T'^2}{R^2} \right) \quad (4.4.45)$$

where

$$\rho' = \rho/\rho|_0 \quad (4.4.46)$$

$$\rho_T' = \rho_T/\rho_T|_0 \quad (4.4.47)$$

$$R = \rho|_0/\rho_T|_0 \quad (4.4.48)$$

and

$$J' = \frac{J}{\rho|_0 U} = \rho'(1 + \gamma E') + \frac{\rho_T' E'}{R} \quad (4.4.49)$$

$$J_p' = \frac{J_p}{\rho_T|_0 U} = \rho_T'(1 + \gamma E') + R \rho' E' \quad (4.4.50)$$

4.4.3 An Approximate Solution for $|R| \ll 1$

When both particles are injected at $x = 0$, and space charge effects are neglected, Eq. (4.4.22) gives the distribution of n_2 with x . From Eq. (4.4.19), n_1' is determined by n_2' , hence the space charge and total carrier density distributions are found. A similar situation appears to exist when, in Eqs. (4.4.42 - 4.4.50), $R \ll 1$ and the particles have positive velocity at $x' = 0$. Assuming $\gamma = 0$, i.e., $b_1 = b_2$, Eqs. (4.4.49) and (4.4.50) are approximately

$$J' \approx \frac{\rho_T' E'}{R} \quad (4.4.51)$$

and

$$J_p' \approx \rho_T' \quad (4.4.52)$$

Assuming $\rho_0 > 0$ for purposes of illustration, $R/|R| = 1$ and so long as Eqs. (4.4.51) and (4.4.52) obtain, the charge conservation equation, (4.4.42), gives from Eq. (4.4.51)

$$\frac{J'}{R} \frac{d\rho_T'}{dx'} + \frac{\rho_T'^2 d}{R Re} = 0 \quad (4.4.53)$$

and the particle conservation equation (4.4.45) with (4.4.52) gives

$$\frac{d\rho_T'}{dx'} = \frac{2 \alpha H}{\rho_T|_0 U} - \frac{\beta H}{2 \rho_T|_0 U} \rho|_0^2 \frac{\rho_T'^2}{R^2} \quad (4.4.54)$$

where we have neglected ρ'^2 relative to $\rho_T'^2/R^2$. In their unnormalized forms, Eqs. (4.4.53) and (4.4.54) are

$$\frac{\epsilon_0 J}{b} \frac{d\rho_T}{dx} + \rho_T^2 \rho = 0 \quad (4.4.55)$$

and

$$\frac{d\rho_T}{dx} = -\frac{2\alpha}{U} - \frac{\beta}{2U} \rho_T^2 \quad (4.4.56)$$

Thus, from the results of Sec. 4.4.1,

$$\rho_T = \rho_T|_\infty \left[\frac{B \exp(Ax/U) - 1}{B \exp(Ax/U) + 1} \right] \quad (4.4.57)$$

where

$$B = \frac{\rho_T|_\infty + \rho_T|_0}{\rho_T|_\infty - \rho_T|_0} \quad (4.4.58)$$

$$A = 2(\alpha\beta)^{1/2} \quad (4.4.59)$$

and

$$\rho_T|_\infty = 2(\alpha/\beta)^{1/2} \quad (4.4.60)$$

Thus, from Eqs. (4.4.55) and (4.4.51),

$$\rho(x) = \frac{\epsilon_0 J}{\bar{b}U} \frac{[2\alpha - \beta/2 \rho_T^2]}{\rho_T^2} \quad (4.4.61)$$

and

$$E(x) = \frac{J}{\bar{b}\rho_T} \quad (4.4.62)$$

where ρ_T is given by Eq. (4.4.57).

Comparing Eq. (4.4.57) with the results of Sec. 4.4.1, it is seen that the approximations in Eqs. (4.4.51) and (4.4.52) are equivalent to determining $Q = 1$ and $v_1 = v_2 = U$ in Eq. (4.4.26). Thus, the approximations, (4.4.51) and (4.4.52), not only neglect space charge, but also neglect the field-derived difference in particle velocities at the boundary.

4.4.4 The Numerical Model

Terminal relations can be found by a numerical integration of Eqs. (4.4.42) - (4.4.45) using the definitions of Eqs. (4.4.46) - (4.4.50). The addition of volume ionization and recombination effects introduces a new parameter. In the bipolar model without ionization, the normalization allowed determination of the normalized potential in terms of the parameters, J' , Re and the ratio of the polar ion densities at the boundary. Consideration of Eq. (4.4.45) shows that inclusion of the ionization rate requires an additional boundary value.

The coefficient of the second term on the right-hand side of Eq. (4.4.45) can be normalized,

$$\frac{\beta H \rho|_0^2}{2\rho_T|_0 U} = \frac{\bar{b}H|\rho|_0|}{\epsilon_0 U} \frac{\epsilon_0 \beta |R|}{2\bar{b}} \quad (4.4.63)$$

Using the definition of Re , this term is

$$\frac{\beta H \rho|_0^2}{2 \rho_T|_0 U} = \frac{\epsilon_0 |R|}{2b \text{ Re}} \beta . \quad (4.4.64)$$

Since

$$R = \pm \frac{1 + B_m}{1 - B_m} \quad (4.4.65)$$

this second term normalizes to the parameters Re and B_m .

The recombination rate is dependent on the spatial distribution of ρ and ρ_T and hence the normalization of this term reflects this dependence. However, the ionization rate arises from a wholly independent mechanism and thus does not normalize in the same way as the recombination mechanism.

The degree of influence of the ionization rate is determined by the ratio of the local generation of ion pairs by ionization, 2α , to the flux of ions into the region, $\rho_T|_0 U / H$, by convection. Defining the normalized ionization rate, g , as this ratio, Eq. (4.4.45) becomes

$$\frac{dJ_p'}{dx'} = g + \frac{\epsilon_0 |R|}{2b \text{ Re}} \beta \left[\rho'^2 - \frac{\rho_T'^2}{R^2} \right] \quad (4.4.66)$$

where

$$g = \frac{2\alpha H}{\rho_T|_0 U} \quad (4.4.67)$$

Using the definitions of J' and J_p' , Eqs. (4.4.49) and (4.4.50) in (4.4.42) and (4.4.45), we obtain the two simultaneous differential equations,

$$(1 + \gamma E') \frac{d\rho'}{dx'} + \frac{E'}{R} \frac{d\rho_T'}{dx'} + (\gamma \rho' + \frac{\rho_T'}{R}) \frac{\rho'}{\text{Re}} \frac{R}{|R|} = 0 \quad (4.4.68)$$

$$\text{Re}' \frac{d\rho'}{dx'} + (1 + \gamma E') \frac{d\rho_T'}{dx'} + (R\rho' + \gamma \rho_T') \frac{\rho'}{\text{Re}} \frac{R}{|R|} = G \quad (4.4.69)$$

where the normalized source term, G , is defined by

$$G = g + \frac{\epsilon_0 |R|}{2b \text{ Re}} \beta \left[\rho'^2 - \frac{\rho_T'^2}{R^2} \right]. \quad (4.4.70)$$

Then, multiplying Eq. (4.4.68) by $(1 + \gamma E')$ and Eq. (4.4.69) by E'/R and subtracting, we get

$$\begin{aligned} [(1 + \gamma E')^2 - E'^2] \frac{d\rho'}{dx'} + \frac{\rho'}{\text{Re}} \frac{R}{|R|} \left[(1 + \gamma E') \left(\gamma \rho' + \frac{\rho_T'}{R} \right) \right. \\ \left. - \frac{E'}{R} (R\rho' + \gamma \rho_T') \right] = - \frac{GE'}{R}. \end{aligned} \quad (4.4.71)$$

Similarly,

$$\begin{aligned} [(1 + \gamma E')^2 - E'^2] \frac{d\rho_T'}{dx'} + \frac{\rho_T'}{\text{Re}} \frac{R}{|R|} \left[(1 + \gamma E') (R\rho' + \gamma \rho_T') - RE' \left(\gamma \rho' + \frac{\rho_T'}{R} \right) \right] \\ = (1 + \gamma E')G. \end{aligned} \quad (4.4.72)$$

Equations (4.4.43), (4.4.71) and (4.4.72) define a closed set of ordinary differential equations provided the term $[(1 + \gamma E')^2 - E'^2] \neq 0$. This can be shown to be the condition that neither ion velocity vanish.

So, provided boundary conditions compatible with the steady state in a region of height H are chosen, the vertical distribution of particle density, space charge, and field can be obtained by numerical integration.

The numerical integration is accomplished through a fourth-order Runge-Kutta method described by Hildebrand [1956, pp. 233-239] in which the error for each step is of order h^5 for a first-order system, where h is the step-size. A simple expression for the error in higher order systems is not known. However, Hildebrand gives a method due to

Gill [1951] for estimating the error by comparing the truncation errors incurred over the interval $2h$ when computed in a single increment of $2h$, and two steps of interval h . According to Gill, if the true functional value is F_{n+1} , and $f_{n+1}^{(h)}$ is the computed value with step-size h , then the error is approximately

$$F_{n+1} - f_{n+1}^{(h)} \approx \frac{f_{n+1}^{(h)} - f_{n+1}^{(2h)}}{2^r - 1} \quad (4.4.73)$$

where r is the order of the Runge-Kutta method.

In our approach, we have arbitrarily weighted equally the errors in each dependent variable, ρ' , ρ_T' , and E' . Then an approximate test of the accuracy of the solution for each step is given by

$$\sum \left| f_{n+1}^{(h)} - f_{n+1}^{(2h)} \right|_i < 15 \delta \quad (4.4.74)$$

where $\left| f_{n+1}^{(h)} - f_{n+1}^{(2h)} \right|_i$ is the truncation error for the i^{th} dependent variable, and δ is the maximum permissible error per step for any dependent variable. Assuming ρ' , ρ_T' and E' are of order 1, we selected $\delta \approx 10^{-5}$. As a check on the numerical method, the normalized potential, V' , computed by the numerical method with $\alpha = \beta = 0$ was compared with the exact solution from the results of Sec. 4.2. This comparison is shown in Table 4.6. As can be seen, the numerical solution gives excellent results in this case.

TABLE 4.6

Normalized Potential Computed for $B_1 = -0.5$, $Re = 1.0$ by a Numerical Integration and an Exact Solution

v'		
J'	Numerical Solution	Exact Solution
.001	.11180	.11179
.002	.11144	.11143
.004	.11072	.11072
.007	.10964	.10964
.01	.10857	.10856
.02	.10498	.10497
.04	.097805	.097797
.07	.087076	.087069
.1	.076381	.076373
.2	.040962	.040954
.4	- .028841	- .028849
.7	- .13107	- .13108
1.0	- .23039	- .23039

4.4.5 Asymptotic Values

The inclusion of the additional terms of ionization and recombination requires the specification of an additional parameter, g . Thus, model solutions are of less general value unless g can be specified for all situations. In general, this cannot be done and we must be satisfied with a study of the effect of changing g and understand that, for a given situation, g must be specified.

A means of appreciating the physical significance of g is found by determining the asymptotic behavior of particle density and electric field. Restricting the following remarks to the region $0 < J' < 1$, so that particle velocities are positive at the boundary, we find the asymptotic values by requiring that all spatial derivatives vanish. From Gauss's law, this means that $\rho' = 0$, also. So, from Eqs. (4.4.68) and (4.4.69), the asymptotic particle density is given by

$$G = 0 \quad (4.4.75)$$

or

$$\rho_T' |_{\infty}^2 = \frac{2b \operatorname{Re}|R|}{\epsilon_0 \beta} g \quad (4.4.76)$$

Expansion of Eq. (4.4.76) gives the expected result that

$$\rho_T |_{\infty} = 2 \sqrt{\alpha/\beta} \quad (4.4.77)$$

However, in the form (4.4.76), the expression is particularly useful in determining regions of boundary conditions compatible with the steady-state requirement that $[(1+\gamma E')^2 - E'^2] \neq 0$. For example, assuming $\gamma = 0$, Eqs. (4.4.49) and (4.4.42) give the asymptotic electric field,

$$E'|_{\infty} = \frac{J'R}{\rho_T'^2|_{\infty}} \quad (4.4.78)$$

When $\gamma = 0$, the steady-state condition is, therefore,

$$1 - \frac{J'^2 R^2}{\rho_T'^2|_{\infty}} > 0 \quad (4.4.79)$$

From Eq. (4.4.76), the consistent boundary conditions, Re , R , g and J' must therefore obey the relation

$$\frac{\epsilon_0 \beta}{2 \bar{b}} \frac{|R| J'^2}{Re g} < 1 \quad (4.4.80)$$

A further interpretation of g is possible from Eq. (4.4.76). $\rho_T'|_{\infty}$ is the ratio of the asymptotic particle density to the boundary particle density. If α and β are constant, the left-hand side of (4.4.76) is then a measure of the departure of the boundary particle density from its equilibrium value. Assuming the boundary particle density is in equilibrium, it follows that

$$g_e = \frac{\epsilon_0 \beta}{2 \bar{b} Re |R|} \quad (4.4.81)$$

In the examples which follow, we will specify the departure of the boundary particle density from its equilibrium value by the parameter,

$$r = \rho_T'^2|_{\infty} = g/g_e \quad (4.4.82)$$

The steady-state requirement is now given by

$$\frac{g_e}{g} R^2 J'^2 < 1 \quad (4.4.83)$$

When $g_e/g \gg 1$, $\rho_T'|_\infty \ll 1$ from Eq. (4.4.82). Then in the limit $\rho' = 0$, so that

$$RJ' = \rho_T'|_\infty E'|_\infty \quad . \quad (4.4.84)$$

The steady-state condition is thus seen to be the requirement that the independent processes of ionization and recombination do not so deplete the charge carrier population that particle reversal occurs to maintain a uniform current density.

4.4.6 Sample Calculations and Discussion

Figures 4.14 and 4.15 show sample calculations of the space charge distribution for $Re = 1$ and $Re = 10$ and values of $g = 0.1, 1, 10$. The case $\alpha = \beta = 0$ is indicated in Fig. 4.14, and nearly coincides with the $g = 0.1$ curve in Fig. 4.15. The large depletion of net space charge density found for $g = 10$ in these cases is understood by considering the distribution of total carrier population shown in Fig. 4.16.

The ratio $r = g/g_e$ is given for these studies by

$$r = 1.85 Re g \quad . \quad (4.4.85)$$

Thus, for $g = 10$ in Fig. 4.14, $r = 18.5$. This corresponds to a boundary value of ρ_T much less than the equilibrium value, $\rho_T|_\infty$. Thus, as time progresses for a parcel of air injected at the boundary, the total particle density, hence the conductivity, increases. This increase in conductivity allows faster relaxation of the net space charge, as shown. For extreme cases, of large g and small Re , the charge relaxation overshoots and a negative space charge develops within the region. For

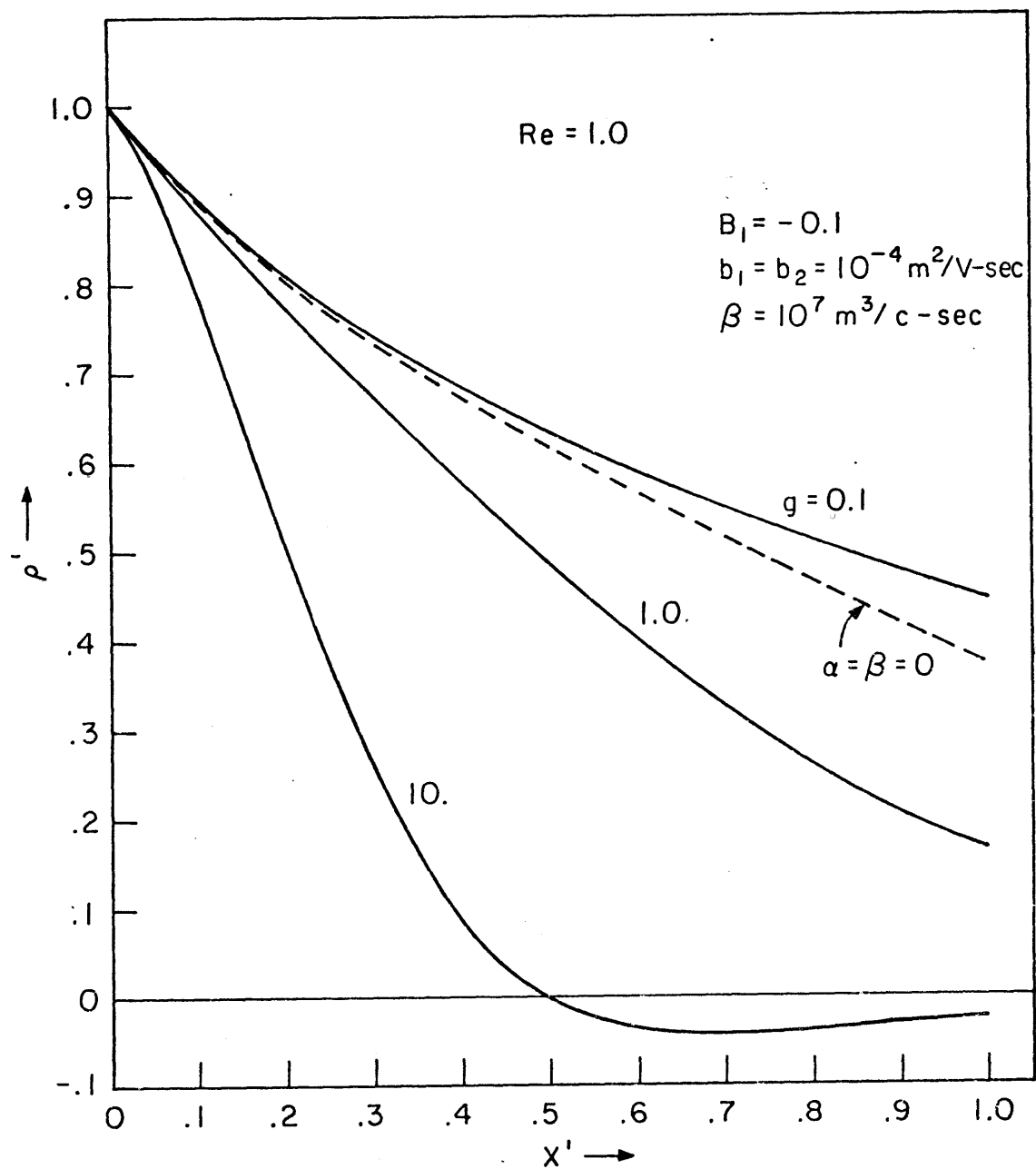


Fig. 4.14 Space charge distribution for $Re = 1.0$, $r = 1.85 \text{ g}$.

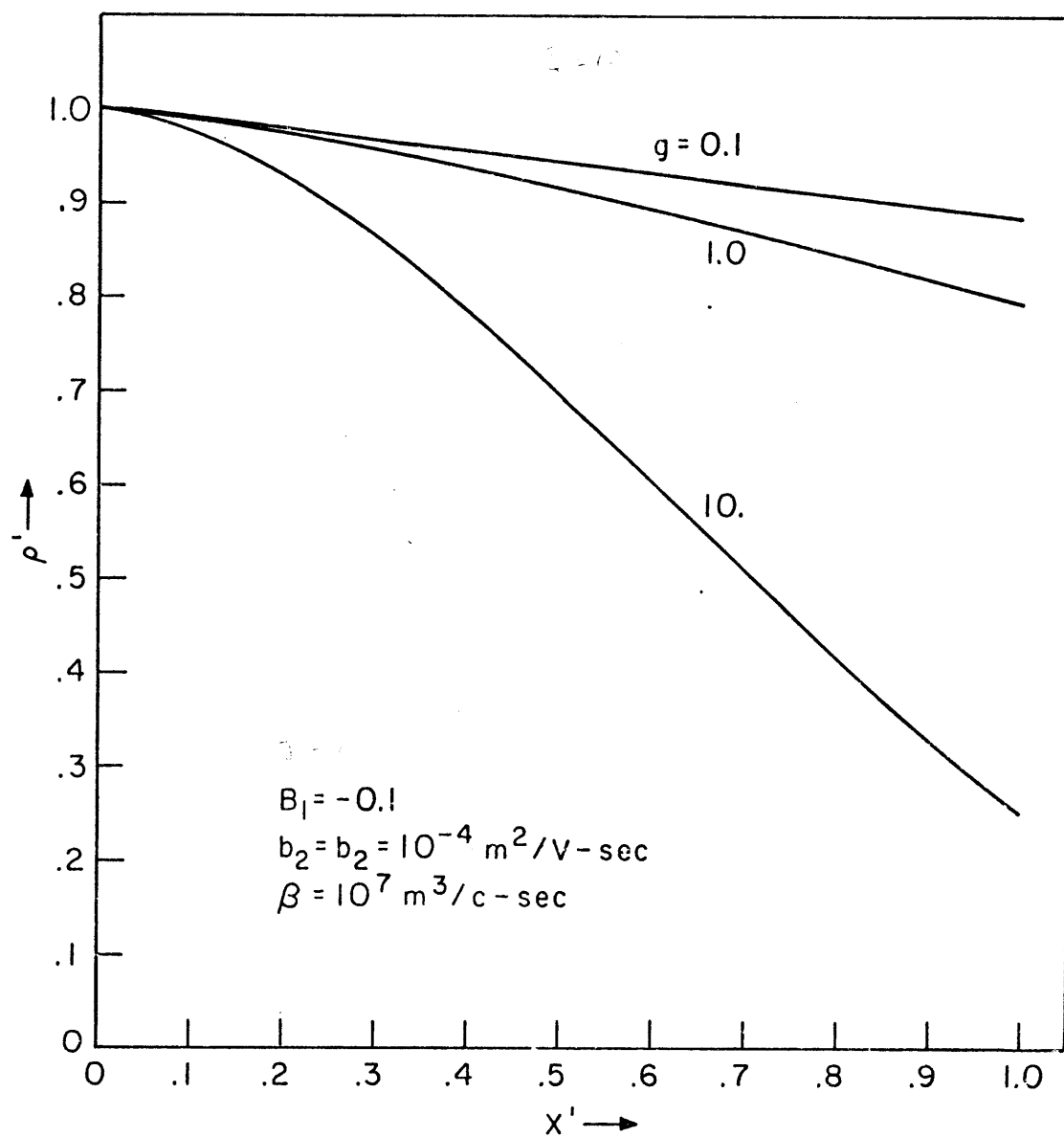
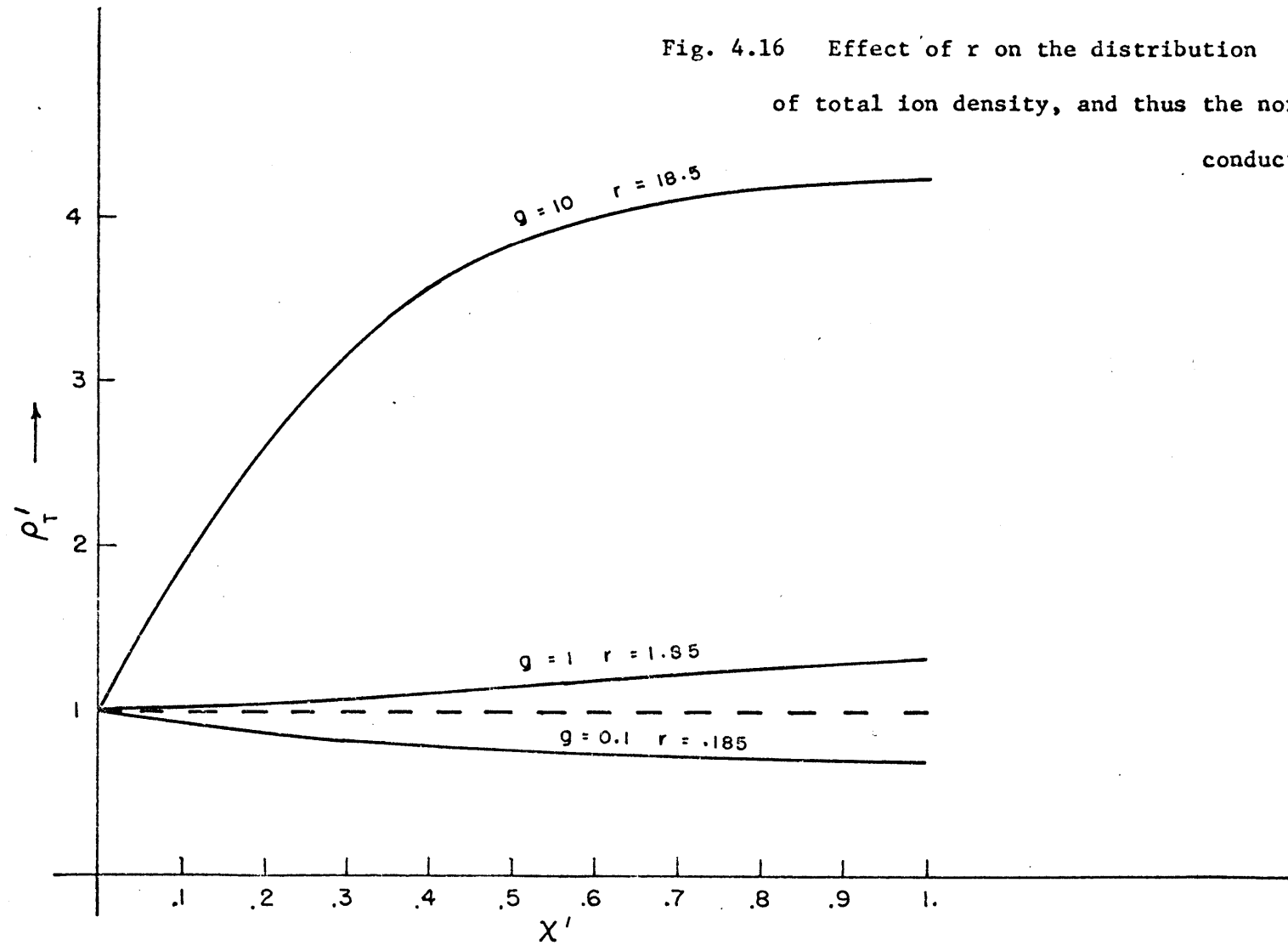


Fig. 4.15 Space charge distribution for $Re = 10.$,

$r = 18.5 \text{ g}$

Fig. 4.16 Effect of r on the distribution
of total ion density, and thus the normalized
conductivity



larger x' , however, the approach to the asymptotic value $\rho' = 0$ ensues.

Whether the increases of conductivity noted by Rossmann [1950] can be explained on the basis of the transport of air with relatively large radioactive gas content which has not equilibrated is a moot point. Nevertheless, the present model provides some insight into such a mechanism.

Figure 4.17 shows sample calculations of the terminal characteristics in the model with ionization and recombination. Comparison of Fig. 4.17 with the terminal relations in Fig. 4.9 shows that for small g , the predicted normalized potential is not much different from that found when $\alpha = \beta = 0$. From Eq. (4.4.66), when $g \ll 1$, the particle conservation equation for the parameters used in the studies is approximately

$$\frac{dJ_p'}{dx'} \sim \frac{\rho_T'^2}{Re|R|} \quad (4.4.86)$$

Thus, for large Re , recombination effects are not important, unless $B_1 \sim -1$, whereupon $|R|$ is very small.

A comparison of the predicted potentials with and without ion source terms showed that, for $Re > 100$ and the boundary ion density near its equilibrium value, the bipolar model of Sec. 4.2 without source terms gave results within one percent of that predicted when source terms are included. When the boundary ion density was significantly different from its asymptotic value, when $Re > 100$, the error increased, reaching nearly a factor of two when $B_1 = -0.99$, or $R \sim .005$. For

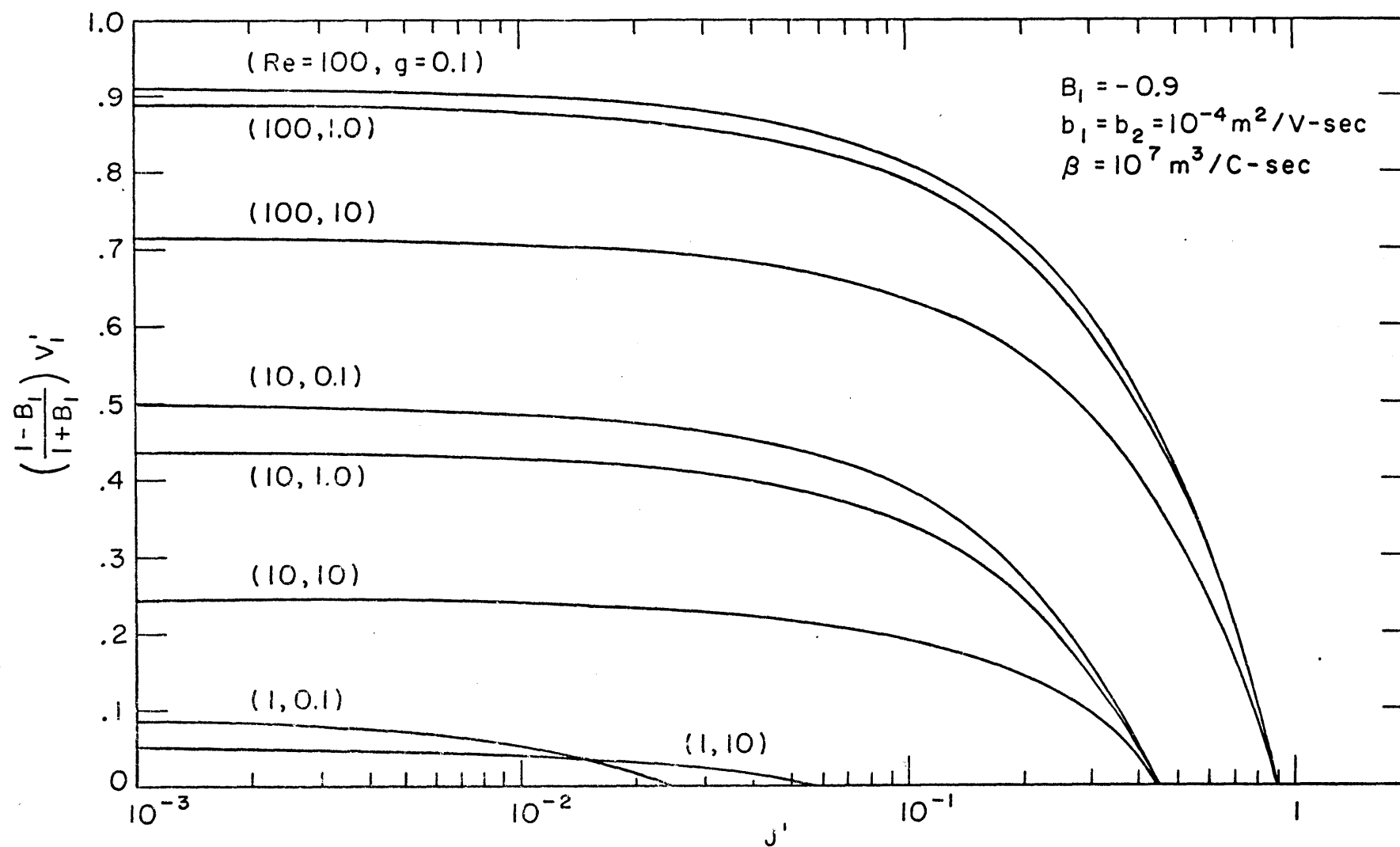


Fig. 4.17 Terminal characteristics of a bipolar generator with source terms, $r = 1.19 \text{ Re } g$.

many applications, a factor of two is quite satisfactory, so that the model results of Sec. 4.2 can provide reasonably good estimates of the potential generated without recourse to the less general model which includes the source terms.

4.5 Summary

A transcendental solution for a bipolar convective generator was found in Sec. 4.2, and the distribution of ion densities within the generating region was discussed and compared with the unipolar model. The characteristic decay length for net space charge in the unipolar model is, from Eq. (3.2.11),

$$\ell' \sim \frac{1}{\text{Re} J'} \quad (4.5.1)$$

whereas in the bipolar model, this characteristic length is

$$\ell' \sim \frac{1 + B_m}{\text{Re} J'_m} \quad (4.5.2)$$

For situations wherein $1 + B_m \ll 1$, the bipolar model decay length is considerably less than the unipolar decay length. An even greater disparity between the two models is seen in the vertical electric field distributions because of the presence of screening charges in the bipolar model.

The asymptotic values of the bipolar ion densities and electric field were found for the initial value problem. These are given in Table 4.4. Terminal relations for a bipolar atmosphere without source terms were presented for four values of the parameter $B_2 = \rho_2|_0 / \rho_1|_0$ in Figs. 4.9a - 4.9d.

The bipolar static atmosphere solution was discussed in Sec. 4.3 and a comparison of the model predictions with an empirical formula showed the importance of increasing mobility and increasing equilibrium ion density in the vertical to the fair-weather conduction processes in the atmosphere. Using the empirical formula, the bipolar and unipolar generators coupled to a common load were compared. The very large disparity in the two model results underscores the importance of charge relaxation and screening effects inherent in the bipolar atmosphere.

The effects of ionization and recombination were discussed in Sec. 4.5. An approximate solution for $R \ll 1$ was found to neglect velocity differences at the boundary, in addition to the neglect of space charge. However, by treating the ions separately, a solution was found for the case of different particle velocities at the boundary when space charge effects are negligible. A numerical model of convective generation including ionization and recombination was developed in Sec. 4.4.4 and sample calculations of the net space charge and ion density distributions were given, as well as a sample calculation of the terminal relation for this case. A comparison with the bipolar model without source terms showed that, for most circumstances, when $Re > 100$, the terminal relations of Sec. 4.2 are adequate.

It is difficult to present general results in a model of this sort, simply because of the variety of initial conditions which must be specified. However, one general observation about the bipolar (and indeed unipolar) model behavior is that the electric Reynolds number

can be thought of as the determining influence in the character of the transfer function of boundary charge density and electric field, at least when source terms are such that initial values do not deviate considerably from final values. A brief summary of this character is simply that for large Re , the dependent variables at the section exit are not greatly different from their initial values, the difference decreasing with increasing Re . This underscores the true character of Re in the bipolar model with source terms; specification of B_I and r is equivalent to specifying ρ/o so that Re in this case is simply an inverse measure of the height of the generating section. The departure from this behavior when source terms become important is illustrated in Fig. 4.14 and 4.15, in which initial values of total ion density much less than the equilibrium value cause a large difference between initial and final values of space charge, with the difference increasing with increasing r .

CHAPTER VCONVECTIVE GENERATION WITHIN A MODEL CLOUD5.1 Introduction

The convective electrification mechanisms discussed in Chapter I incorporate the features of the bipolar convective generator discussed in Chapter IV with the important addition of charge exchange between the ionic space charge and cloud droplet families. The space charge, consisting of mobile ions, moves with an important velocity component determined by the electric field. Charge associated with cloud droplets, however, moves according to the difference between the convection speed and the droplet terminal velocity. Thus, an important part of the space-charge current is a conduction current which tends to relax the macroscopic electric field. The charged droplet current, however, is independent of the field distribution and hence may be generative or dissipative, depending on the disposition of the cloud charge centers giving rise to the macroscopic electric field relative to the cloud circulation. As Wilson [1956] puts it, the essence of the convective electrification mechanism is the transformation of external dissipation currents into internal generation currents via the transformation of ionic space charge into cloud charge.

On the basis of approximate calculations, Wilson [1956], Vonnegut [1963] and Phillips [1967a] assume that the ionic space charge in the cloud is quickly absorbed by surrounding droplets, thereby charging the droplets and reducing the cloud "conductivity" to negligible values. Independent observations of cloudy conductivity, however, show considerable disagreement with this assertion [Freir, 1962; Evans, 1969; Allee and Phillips, 1959]. Thus, the theory of ion-droplet interaction is cast in some doubt.

However, Wilson's impact charging mechanism has been experimentally verified by Gott [1933] and by numerous applications in the area of industrial pollution control [Melcher and Sachar, 1971]; moreover, a special case of impact-charging, Gunn's hyperelectrification theory [1956] and the diffusion of ions to droplets [Gunn, 1954], has been independently verified [Phillips and Gunn, 1954] and suggested as the basis for the observed layers of screening charge found near the periphery of active thunderstorms [Gunn, 1957].

A probable cause for the disparity between observed and predicted ion densities within clouds is suggested by the neglect of convective transport in the calculations of Wilson, Vonnegut and Phillips. For example, Phillips [1967a] computes an approximate thickness of the region of transition from space charge to cloud charge by the distance traveled by an ion due to the electric field in a time typical of the depletion of ions by the droplets. If convection is included in his calculation, the thickness can increase by more than an order of magnitude, depending on the ratio of convection speed to the field-induced ion velocity. For a model cloud region of 3μ droplets with a liquid water content of 0.1 g/m^3 , this region can increase from 100 meters to over a kilometer due to convection.

Sartor [1967] assumed the cloud conductivity to be reduced by a factor of 20, and on the basis of an assumed steady state and uniform vertical current density, he suggested that the field inside the cloud would be up to 20 times the clear-air field at the same level. This high field served to initiate the field-dependent induction charging process. Although he neglected convection in his calculation, the suggestion that convective

generation might provide the boot strap necessary to initiate field-dependent thunderstorm charging currents is an important possibility.

The only available estimates of the convective electrification mechanism as an initial field-growth mechanism are due to Vonnegut [1954] in which bipolar effects and field-induced ion velocities were neglected. In view of the demonstrated inadequacy of the unipolar model as well as the significant field effect on the total current density, the cloud electric field generated by the convective mechanism should be re-examined.

The earlier studies of the effect of cloud on equilibrium ion density by Wilson and Phillips showed that a considerable weight must be given to the ionization and recombination within the cloud. In the absence of ionization, according to their results, the ion density very quickly vanishes. Anticipating this effect, therefore, we will from the outset include these terms in the model. The sub-cloud region thus corresponds to the model discussed in Sec. 4.4. Although facility for a variable ionization rate could be made in the numerical study, it is not felt that such an effect will significantly alter the results, particularly in view of other simplifications of the model.

The cloud is assumed to consist of uniform droplets whose size is constant, at least over the range of significant interaction. The droplets are assumed to be formed at cloud base with initial size as a parameter of the model, and constant velocity dictated by the convection speed and equilibrium terminal speed of the droplet. Since cloud base is a region of highly variable particle number density, and since the region of maximum droplet growth by condensation is near cloud base, the assumptions of constant droplet size and number density are particularly weak.

The techniques of coupling two regions with distinct properties were discussed in Chapter III. Examples of coupling the region of active convection with a static load to find a self-consistent solution were given in Chapters III and IV. These techniques will be used in this chapter to develop case studies of the transformation of sub-cloud ionic space charge into cloud charge, i.e., charge residing on the droplets. In addition, the development of a self-consistent electric field within the cloud can be determined. The magnitude of the field generated by the steady transport of charge into the cloud is central to the initiation of other field dependent thunderstorm charging currents, and therefore must be considered a prime objective of the study.

The additional terms in the mathematical model are the droplet charge and the annihilation terms describing ion flux to the droplets. The interaction of space charge with droplets via Wilson's impact-charging mechanism is discussed in Appendix A. Inclusion of these terms in the model, as well as Gunn's diffusion equation is discussed in Sec. 5.2. Since calculation of droplet charging currents in both the impact-charging mechanism and the diffusion model neglect space-charge effects, their use in a model in which space charge plays a dominant role must be examined. This question is considered in Appendix B.

Normalization of the charge exchange equations is discussed in Sec. 5.3. With these expressions, the cloud model is formulated in terms of the same parameters as the bipolar model of Sec. 4.4. Examples of the space charge, droplet charge, and the electric field distribution within the cloud are given

in Sec. 5.4. Terminal relations for the coupled system of the sub-cloud region and cloudy region are developed in Sec. 5.5. Coupling with an assumed load is developed to find the steady-state operating characteristics of the coupled system. The self-consistent electric field and charge distributions corresponding to the determined operating points are then examined.

5.2 The Mathematical Model

The development of the cloud model is similar to the bipolar model with ionization and recombination given in Sec. 4.4. The additional dependent variable, drop charge q , however, and the ion-droplet charge exchange processes, must be included. The current density expression of Sec. 4.4 with droplet convection current from Eq. (4.4.49) becomes

$$J' = \rho'(1 + \gamma E') + \frac{\rho_T' E'}{R} + \frac{NqV}{\rho_o U} \quad (5.2.1)$$

The equilibrium droplet velocity is given by Eq. (1.4.6), as

$$V = U - g/v \quad (5.2.2)$$

where g/v is the droplet terminal speed relative to the air, and where we have neglected electric forces on the droplet. Defining the normalized droplet charge density in a region containing many droplets by

$$q' = \frac{Nq}{\rho|_0} , \quad (5.2.3)$$

the normalized current density becomes

$$J' = \rho'(1 + \gamma E') + \frac{\rho_T' E'}{R} + V' q' \quad (5.2.4)$$

where $V' = (1 - g/vU)$ is the normalized droplet velocity. For small droplets, this quantity is nearly unity. For droplets in the Stokes drag region, the droplet normalized velocity is approximately

$$V' = 1.3 \times 10^8 \frac{a^2}{U} + 1 \quad (5.2.5)$$

where a is the droplet radius in meters. Since the Stokes drag region is valid for particles less than about 10μ radius, the normalized terminal velocity will be less than $.013/U$. For $U \sim 10$ m/sec, $V' \sim 1$, therefore.

The total ion flux, J_p , given in normalized form by Eq. (4.4.50), is not changed by inclusion of droplets in the model. Repeating here for convenience,

$$J_p' = \rho_T'(1 + \gamma E') + R\rho' E' . \quad (5.2.6)$$

The conservation of charge, Eq. (4.4.42) with J' given by (5.2.4) is as before,

$$\frac{dJ'}{dx'} = 0 \quad (5.2.7)$$

reflecting the absence of volume sources of net charge. However, the continuity expression for ions must account for the flux of ions to the droplets. From Eqs. (1.4.2) and (1.4.3), and the definition of J_p in Eq. (4.4.40), this new ion conservation law is

$$\frac{dJ_p}{dx} = 2(\alpha + \beta\rho_1\rho_2) - (S_1 + S_2) \quad (5.2.8)$$

where S_1 represent all volume rates of ion flux to the droplets. Since J_p' is unchanged in the cloud model, a new source term, G_c , can be defined as in Eq. (4.4.70),

$$\frac{dJ_p'}{dx'} = G_c \quad (5.2.9)$$

where

$$G_c = g + \frac{\epsilon_0 |R|}{2b \text{ Re}} \beta [\rho'^2 - \frac{\rho_T'^2}{R^2}] - R \frac{H}{\rho|_0 U} (S_1 + S_2) . \quad (5.2.10)$$

Gauss's law must include the effect of droplet space charge; using the definition of normalized droplet charge in Eq. (5.2.3), the normalized form of Gauss's law in the cloud becomes

$$\frac{dE'}{dx'} = \frac{R}{|R|} \frac{\rho' + q'}{\text{Re}} . \quad (5.2.11)$$

Finally, Eq. (1.4.10) gives the conservation of droplet charge from the general charge continuity expression and the polar ion conservation equations. In normalized form, this result is written

$$v' \frac{dq'}{dx'} = \frac{H}{\rho|_0 U} (S_1 - S_2) . \quad (5.2.12)$$

The system of coupled ordinary differential equations described by Eqs. (5.2.4) - (5.2.12) can be obtained from the results in Sec. 4.4. The conservation of ions is given by Eq. (4.4.69) with G_c of Eq. (5.2.10) substituted for G and ρ' replaced by $(\rho' + q')$ in the term accounting for Gauss's law. The charge conservation equation, (4.4.68), is treated by replacing ρ' with $(\rho' + q')$ in Gauss's law and accounting for the droplet charge density by (5.2.12). Carrying out these operations, we get the following set of coupled ordinary differential equations,

Conservation of charge

$$(1 + \gamma E') \frac{d\rho'}{dx'} + \frac{E'}{R} \frac{d\rho_T'}{dx'} + \left(\gamma \rho' + \frac{\rho_T'}{R} \right) (\rho' + q') \frac{R}{|R| Re} = - v' \frac{dq'}{dx'} \quad (5.2.13)$$

Conservation of ions

$$RE' \frac{d\rho'}{dx'} + (1 + \gamma E') \frac{d\rho_T'}{dx'} + (R\rho' + \gamma \rho_T') (\rho' + q') \frac{R}{|R| Re} = G_c \quad (5.2.14)$$

Conservation of drop charge

$$v' \frac{dq'}{dx'} = \frac{H}{\rho|_0 U} (S_1 - S_2) \quad (5.2.15)$$

Gauss's law

$$\frac{dE'}{dx'} = \frac{R}{|R|} \frac{(\rho' + q')}{Re} \quad (5.2.16)$$

Equations (5.2.13) and (5.2.14) can be used to find the differential equations for ρ' and ρ_T' . Using (4.4.71) and (4.4.72) as guides, these results are

$$[(1 + \gamma E')^2 - E'^2] \frac{d\rho'}{dx'} + (\rho' + q') \frac{1}{|R|Re} [(1 + \gamma E')(\gamma R\rho' + \rho_T')] - E'(R\rho' + \gamma\rho_T')] = -\frac{G_c E'}{R} - (1 + \gamma E') V' \frac{dq'}{dx'} \quad (5.2.17)$$

and

$$[(1 + \gamma E')^2 - E'^2] \frac{d\rho_T'}{dx'} + (\rho' + q') \frac{R}{|R|Re} [(1 + \gamma E')(R\rho' + \gamma\rho_T')] - E'(R\gamma\rho' + \rho_T')] = (1 + \gamma E')G_c + RE' V' \frac{dq'}{dx'} \quad (5.2.18)$$

Equations (5.2.15) - (5.2.18) can be solved by the same technique as discussed in Sec. 4.4, so long as the term $[(1 + \gamma E')^2 - E'^2]$ is non-zero everywhere in the region $0 \leq x' \leq 1$. This is the same condition as before, that ion velocities must remain non-zero for the steady-state operation.

5.3 Normalized Charge Exchange Expressions

The terms S_2 refer to the volume rate of flux of positive and negative ions to the droplets. Since the rate of change of droplet charge following its motion is

$$\frac{dq}{dt} = i_1 - i_2 \quad (5.3.1)$$

for i_1 and i_2 the currents of positive and negative charge to the droplet, Eq. (1.4.10) gives

$$S_2 = N i_2 \quad (5.3.2)$$

The droplet currents due to Wilson's impact charging are $I_2^{(1)}$ where the subscript refers to the ion polarity, and the superscript denotes the charging regime. Charging regimes are given in Fig. A.5 of Appendix A for a droplet in a bipolar atmosphere; the charging rate is determined

by the droplet charge and the ion velocities relative to the drop in the unperturbed region far from the droplet surface. When the drop surface charge is uniformly one sign, the flux to the surface of ions of that polarity vanishes. This regime is encountered when

$$|q| > 12\pi\epsilon_0 a^2 |E| . \quad (5.3.3)$$

That is, when (5.3.3) holds, negative ion flux to the droplet is zero for negative q , and positive ion charging stops when q is positive. This is referred to as the 0 regime in the model. Other regimes of charging are determined by the orientation of the charge-accepting region of the droplet surface relative to the direction of approach of the ions. When ions far from the droplet approach on the same side as the charge-accepting region, the charging regime is termed the 1 mode, while approach of ions from the opposite side of the charge-accepting region leads to the 2 mode. These regimes are selected by the sign of the electric field and the velocity of the ion relative to the droplet,

$$\begin{aligned} g/v \pm b_1 E & \begin{matrix} > \\ < \end{matrix} 0 \\ E & \begin{matrix} > \\ < \end{matrix} 0. \end{aligned} \quad (5.3.4)$$

5.3.1 Normalized Impact-charging Current

The droplet currents in Wilson's model are given by Eqs. (A.32) and (A.33). In normalized form, The 1 mode becomes

$$I_2^{(1)} = \pm \frac{b_1 \rho|_0}{8 \epsilon_0 R} (R \rho' \pm \rho_T') \frac{(|q_c| \mp q)^2}{|q_c|} \quad (5.3.5)$$

where $|q_c| = 12 \pi \epsilon_0 a^2 |E|$. Defining the parameter A_s ,

$$A_s = 12 \pi a^2 N H, \quad (5.3.6)$$

$|q_c|$ can be written

$$|q_c| = \frac{\epsilon_0 A_s}{N H} |E|. \quad (5.3.7)$$

From the definition of the electric Reynolds number, (5.3.7) becomes

$$|q_c| = \frac{|\rho|_0}{N} A_s \operatorname{Re} |E'|. \quad (5.3.8)$$

Thus the mode 1 charging currents to the drop are

$$I_2^{(1)} = \pm \frac{1}{|R|} \frac{b_1 \rho|_0^2}{8 \epsilon_0 N} (R \rho' \pm \rho_T') \frac{(A_s \operatorname{Re} |E'| \mp \frac{R}{|R|} |q'|)}{A_s \operatorname{Re} |E'|}. \quad (5.3.9)$$

Similarly from Eq. (A.33),

$$I_2^{(2)} = - \frac{b_1 \rho|_0^2}{2 \epsilon_0 R N} (R \rho' \pm \rho_T') q'. \quad (5.3.10)$$

Using the definition of the electric Reynolds number in Eq. (5.2.12) with Eqs. (5.3.2), (5.3.9) and (5.3.10), the currents to the droplet are given by

$$I_2^{(1)} = \frac{\rho |e| U}{NH} I_2^{(1)'} \quad (5.3.11)$$

where

$$I_2^{(1)'} = \pm \frac{b_2}{b_R} (R\rho' \pm \rho_T') \frac{(A_S \text{Re} |E'| \mp \frac{R}{|R|} q')^2}{8A_S \text{Re} |E'|} \quad (5.3.12)$$

and

$$I_2^{(2)'} = - \frac{b_2}{2b |R|} (R\rho' \pm \rho_T') q' . \quad (5.3.13)$$

Thus, Eq. (5.2.12) is in normalized form,

$$V' \frac{dq'}{dx'} = \frac{1}{\text{Re}} (I_1^{(1)'} - I_2^{(1)'}) \quad (5.3.14)$$

where the terms in brackets are given by Eqs. (5.3.12) or (5.3.13), depending on the charging regime.

The normalized charging current in Eq. (5.3.14) is a measure of the total flux of ions to the cloud droplets in a region, instead of the current to a single drop. Significantly, the magnitudes of the Wilson charging currents are determined by the same parameters which govern the distribution of charge in the bipolar model with the additional parameter A_s . A_s can be cast in the form of a ratio of cloud particle surface area to the cross-sectional area of the convecting region. For given initial conditions, the product $A_s Re$ is constant; the charging currents, Eq. (5.3.11), vary directly with A_s and inversely with Re . Thus, the higher the liquid water content of a cloud of given size droplets, the greater the depletion of ionic space charge.

5.3.2 Charging Regimes

Although the magnitudes of the polar currents to the droplet in a given regime are independent of droplet size, the regime is determined by the droplet charge and the ion velocity relative to the drop in the unperturbed region of flux. Phillips [1967a] neglected the relative velocity altogether, so that the charging currents in his model are always $I_1^{(1)} - I_2^{(1)}$. This approach is consistent when space charge and droplet terminal speeds in the macroscopic model are negligible. However, his results include larger droplets whose fall speeds are hardly negligible. It is unlikely that his results in these cases are valid.

In our model, relative drop motion and space charge are included. Although the drop terminal speed relative to the air is assumed constant, the self-consistent electric field is not. That is, as the space charge density changes due to relaxation and interaction with the drops, the macroscopic electric field of Eq. (5.2.11) changes. The ion velocity due to the macroscopic field therefore changes and switching of the charging regimes of Fig. A.5 is possible.

To determine the charging regime, we assume the local electric field to be nearly uniform over several droplet radii (this assumption is considered in Appendix B). Then, at a point above cloud base, the charging regime is determined from Fig. A.5 by the droplet size, charge, and electric field at that point. To determine the charging regime in terms of the normalized variables given earlier, the ion-drop relative velocity of Fig. A.5 is given by

$$U_d \pm b_1 E_o = U[1 - V' \pm \frac{b_1}{b} E'] \quad (5.3.15)$$

Thus, $U_d \pm b_1 E_o \gtrless 0$ is given, for positive U , by

$$1 - V' \pm \frac{b_1}{b} E' \gtrless 0 \quad (5.3.16)$$

Similarly, the relation of q to $|q_c|$ is found in normalized form from Eqs. (5.2.2) and (5.3.8) with the definition of q' in Eq. (5.2.3), as

$$- |q_c| \leq q \leq |q_c| \leftrightarrow - A_s \text{Re} |E'| \leq \frac{R}{|R|} q' \leq A_s \text{Re} |E'| \quad (5.3.17)$$

Then, with the aid of Eqs. (5.3.16) and (5.3.17), Fig. A.5 can be interpreted to determine which of Eqs. (5.3.12) and (5.3.13) is to be used in calculating the drop charging rates in the normalized model.

5.3.4 The Normalized System of Equations

The results of Secs. 5.2 and 5.3 are summarized here for convenience by writing the system of equations in normalized form. Thus, from Eqs. (5.2.17) and (5.3.14),

$$\begin{aligned} [(1+\gamma E')^2 - E'^2] \frac{d\rho'}{dx'} + \frac{(\rho' + q')}{|R| \text{Re}} [(1+\gamma E')(\gamma R \rho' + \rho_T')] \\ - E'(R \rho' + \gamma \rho_T')] = - \frac{G_c E'}{R} - \frac{(1+\gamma E')}{\text{Re}} (I_1^{(1/2)'} - I_2^{(1/2)'}). \end{aligned} \quad (5.3.18)$$

Similarly, Eqs. (5.2.18) and (5.3.14) give

$$\begin{aligned} [(1+\gamma E')^2 - E'^2] \frac{d\rho_T'}{dx'} + \frac{R(\rho' + q')}{|R| \text{Re}} [(1+\gamma E')(R \rho' + \gamma \rho_T')] \\ - E'(R \gamma \rho' + \rho_T')] = (1+\gamma E')G_c + \frac{R E'}{\text{Re}} (I_1^{(1/2)'} - I_2^{(1/2)'}) . \end{aligned} \quad (5.3.19)$$

The source term, G_c , is found from Eq. (5.2.10) in the form

$$G_c = g + \frac{\epsilon_c \beta}{2b \text{Re} |R|} [R^2 \rho'^2 - \rho_T'^2] - \frac{R}{\text{Re}} (I_1^{(1/2)'} + I_2^{(1/2)'}) \quad (5.3.20)$$

and, repeating Eqs. (5.3.14) and (5.2.16), the normalized equations are completed by

$$V' \frac{dq'}{dx'} = \frac{1}{\text{Re}} (I_1^{(1/2)'} - I_2^{(1/2)'}) , \quad (5.3.21)$$

$$\frac{dE'}{dx'} = \frac{R}{|R|} \frac{\rho' + q'}{\text{Re}} . \quad (5.3.22)$$

In Eq. (5.3.18), the second term on the left represents the relaxation effects of a bipolar model and is identical to the same term in the

sub-cloud model, Eq. (4.4.71). The first term on the right-hand-side of (5.3.18) represents changes in the net space charge due to the oppositely directed, field-induced velocity of positive and negative ions in a region of total ion density gradient. The equivalent statement of this effect is seen to be the term $(E'/R)(d\rho_T'/dx')$ in Eq. (5.2.13). When ρ_T' is uniform, relative motion of the positive and negative species does not affect the net space charge density, whereas in a region of non-zero $d\rho_T'/dx'$, the response of the polar species to the electric field causes separation of charge, which tends to reduce the local field strength. This term in Eq. (5.3.18) is the same as G in (4.4.71), except for the additional effect of the droplets in G_c which act as sites of increased recombination of ions in a mode determined by the charging regime.

The final term on the right of Eq. (5.3.18) accounts for the transformation of ionic space charge into droplet charge. Its origin is the term $-V' dq'/dx'$ in Eq. (5.2.13). Thus, apart from relaxation and field-induced transport, the net charge density composed of ionic space charge and droplet charge does not change; an increase in droplet charge is balanced by the deficit of positive ions necessary to bring about the droplet charge increase.

In the cloud-free bipolar model, the distribution of net space charge, and hence electric field, is determined by field-induced transport and relaxation, and characterized by the electric Reynolds number. In the cloud, the distribution of net space charge is similarly determined, with the important difference in the source terms, G and G_c . Thus, the field distribution in the cloud should be similar to the sub-cloud behavior unless ion depletion by the droplets gives the term $-G_c R'/R$ a controlling character in Eq. (5.3.18). Thus, when charge relaxation

dominates the field structure, the characteristic distance for the interaction is the same as for the cloud-free case. From Eq. (4.2.33), and the definitions of J_m' and Re_m in Table 4.3, this normalized characteristic length, when $b_1 = b_2$, is

$$\ell' = Re[1 - R(1 - J')] \quad . \quad (5.3.23)$$

The numerical procedure used to find the solution to this set of coupled, ordinary differential equations is the same as that discussed in Sec. 4.4.4. The additional dependent variable, q' , and the charge exchange, are included by determining the charging regime from the relations (5.3.16) and (5.3.17) with the aid of the charging regimes in Fig. A.5. Once the charging regime is established for a point, the terms in $I_{1/2}^{(1/2)'} are computed from Eqs. (5.3.12) and (5.3.13).$

5.4 Asymptotic Values and Sample Distributions

The analysis of the bipolar model showed that, for the initial value problem, i.e., when both species are injected at $x' = 0$, the species ion densities tend toward an asymptotic value such that the net space charge vanishes and an equilibrium solution, all the spatial derivatives of which are zero, persists thereafter. When ionization and recombination are included, a similar equilibrium state can be found in which a balance between the ionization and annihilation terms maintains constant ρ_T' with $\rho' = 0$, Eq. (4.4.76). In this case, an additional condition for the steady state at infinity must be satisfied, Eq. (4.4.79), which maintains a sufficiently high carrier density that the uniform current density can be maintained at infinity without a change in direction of one polar

ion velocity. In a system whose characteristic length is small compared to the "turn-around" distance, however, the steady state can be considered even when ion reversal is predicted, since we need not specify conditions external to the generating region.

The asymptotic solutions in the cloud model are expected to be similar to the model with ionization and recombination, since the cloud particles can be thought of as sites of ion annihilation. However, distinct from the bipolar case, the possibility of finite droplet charge in the asymptotic solution allows non-zero ionic space charge density in equilibrium with the droplets. Too, the determination of the equilibrium state is complicated by the ion-droplet interaction terms and the existence of different charging regimes for the different equilibrium states.

5.4.1 Asymptotic Solutions Neglecting Diffusion

To find the asymptotic behavior, we set all spatial derivatives to zero in Eqs. (5.3.18) to (5.3.22). Then the necessary conditions for the asymptotic solution are that

$$G_c = 0 \quad (5.4.1)$$

$$\rho' + q' = 0 \quad (5.4.2)$$

and

$$I_1 \left(\frac{1}{2} \right)' = I_2 \left(\frac{1}{2} \right)' \quad (5.4.3)$$

where in the remainder of this section, all dependent variables are to be interpreted to be their asymptotic values.

To determine the asymptotic behavior dictated by Eqs. (5.4.1) - (5.4.3) it is necessary to specify the droplet charging regimes. The possible equilibrium modes are determined by the magnitude of the asymptotic elec-

tric field relative to the droplet terminal velocity:

- 1) $|E'| < 1 - V'$. In this case, both ion species approach the droplet from the same side, and the charging currents to it are $I_1^{(1)} - I_2^{(2)}$ when $E' > 0$ [the (1,2) mode], and $I_1^{(2)} - I_2^{(1)}$ when $E' < 0$ [the (2,1) mode].
- 2) $|E'| > 1 - V'$. The field-induced ion velocities in this mode are greater than the droplet terminal speed, hence both ion species approach the drop on the same side as their respective charging windows. The current to the drop is thus $I_1^{(1)} - I_2^{(1)}$ for both positive and negative equilibrium fields [the (1,1) mode].

From Fig. A.5, the other possible charging regimes are designated by the (1,0), (0,1) (2,0) and (0,2) modes. These can never be equilibrium modes, since from Eq. (5.4.3), both polar currents to the droplet would have to vanish, which would imply both $\rho' = 0$ and $\rho_T' = 0$. Because of the local generation of ion pairs by cosmic radiation, however, ρ_T' may be small, but never zero, and hence the charging regime will change, however slowly to either the (1,2) or (2,1) mode.

5.4.2 The (1,1) Equilibrium Mode

The normalized charge exchange equations, (5.3.12) and (5.3.13), show that when $\gamma = 0$, the polar currents to the droplet are equal in the (1,1) mode only when $q' = 0$. Thus, using the definition of g_e and r from Eqs. (4.4.81) and (4.4.82), and G_c from (5.3.25), the equilibrium condition, $G_c = 0$, gives

$$r - \frac{2R}{Re} \frac{I_1^{(1)}}{g_e} - \rho_T'^2 = 0 \quad (5.4.4)$$

Using $I_2^{(1)'} from Eq. (5.3.12) in the above, and rearranging terms, the (1,1) equilibrium ion density is$

$$\rho_T'^2 + \frac{A_s \text{Re}}{4g_e} \frac{\rho_T' |E'|}{R} - r = 0 \quad (5.4.5)$$

The vertical uniformity of current density and the (1,1) equilibrium restriction that $q' = \rho' = 0$ gives, from Eq. (5.2.4),

$$|J'| = \rho_T' \frac{|E'|}{|R|} \quad (5.4.6)$$

so that the asymptotic total ion density is

$$\rho_T'^2 = r - \frac{A_s |R|}{4 g_e} |J'| \quad (5.4.7)$$

or, from the definition of g_e , Eq. (4.4.81)

$$\rho_T'^2 = r - \frac{\bar{b} A_s \text{Re} R^2}{2 \epsilon_0 \beta} |J'| \quad (5.4.8)$$

In this result, r is the square of the departure of the total ion density at the boundary from the equilibrium value given by ionization and recombination in the absence of cloud. The effect of cloud in the (1,1) mode is therefore the expected result of reducing the equilibrium ion density by an amount proportional to the projected area of the cloud droplets.

When $U \rightarrow 0$, Eq. (5.4.8) can be shown to be equivalent to the result found by Phillips [1967a] when diffusion terms are neglected. Phillips did not consider the effects of convection and space charge, so that his solution was valid for all A_s . In our case, these effects place a

restriction on the boundary values to insure positive equilibrium ρ_T' . That is, a necessary condition for the (1,1) mode equilibrium is that

$$r - \frac{\bar{b}_s \text{Re } R^2}{2 \epsilon_0 \beta} |J'| > 0 \quad . \quad (5.4.9)$$

We will return to this restriction later.

From the equilibrium current density expression, the sign of the equilibrium electric field is given by the sign of the product RJ' . Thus, when the boundary net space charge is positive, and the net current density is determined by convection, i.e., $0 < J' < 1$, the (1,1) mode equilibrium field is positive, and given by

$$E' = RJ' \left[r - \frac{\bar{b}_s \text{Re } R^2}{2 \epsilon_0 \beta} J' \right]^{-1/2} \quad . \quad (5.4.10)$$

5.4.3 The (1,2) Mode Asymptotic Solution

When $R > 0$, corresponding to a positive net space charge at the boundary, values of $J' < 1$ correspond to a negative electric field at cloud base. As the electric field increases within the cloud in response to the space charge and droplet charge, various charging regimes are encountered which are determined by the boundary conditions. Before the (1,1) mode equilibrium is reached, the (1,2) mode must be considered, since this latter regime is encountered for $0 < E' < 1 - V'$. Whether the (1,1) mode asymptote is attained thus depends on the system passing through the (1,2) mode without attaining equilibrium.

The equilibrium charge in the (2,1) mode corresponding to negative E' and the (1,2) mode for positive equilibrium field is given by Eqs. (A.39)

and (A.40). Defining the new variable,

$$y = \frac{Rq'}{\rho_T'} \quad (5.4.11)$$

and using the equilibrium condition that $\rho' + q' = 0$ and the definition of $|q_c|$ in Eq. (5.3.8), Eqs. (A.39) and (A.40) define the equilibrium relation,

$$\frac{Rq'}{A_s Re|RE'|} = \pm \left[\frac{\pm 3 - y + (8 \mp y)^{1/2}}{\mp 1 - y} \right] \equiv f(y) \quad (5.4.12)$$

where the selection of the proper sign is made according as the equilibrium field is negative or positive, $E \lesseqgtr 0$. When $0 < J' < 1$, this choice is made by the sign of the boundary charge density, $R \lesseqgtr 0$.

The equilibrium condition $G_c = 0$ from Eq. (5.3.25) and (5.3.13) for $I_1^{(2)'} \quad$ gives the equilibrium ion density in terms of the variable, y , as

$$\frac{\rho_T'}{r^{1/2}} = \left[y^2 \left(\frac{2\bar{b}}{\epsilon_0 \beta} - 1 \right) \mp \frac{2\bar{b}}{\epsilon_0 \beta} y + 1 \right]^{1/2} \equiv g(y) \quad (5.4.13)$$

in which, again, the choice of sign is determined by $R \lesseqgtr 0$. Dividing Eq. (5.4.12) by (5.4.13) and rearranging gives the equilibrium electric field,

$$\frac{A_s Re|RE'|}{r^{1/2}} = \frac{y g(y)}{f(y)} \quad (5.4.14)$$

From Eqs. (5.4.12) and (5.4.14), therefore, the equilibrium droplet charge, when $|E'| < 1 - V'$, is given by

$$Rq' = r^{1/2} y g(y) \quad (5.4.15)$$

Therefore, when $R > 0$, Eqs. (5.4.12) - (5.4.15) define the asymptotic state in the (1,2) mode provided the proper choice of y can be made. Since $\rho' = -q'$, the current density in the (1,2) mode is

$$J' = \frac{\rho_T' E'}{R} - q'(1-V') \quad , \quad (5.4.16)$$

or

$$RJ' = r^{1/2} \frac{1}{A_s ReR} \frac{g(y)}{f(y)} - (1 - V') y g(y) \quad . \quad (5.4.17)$$

Thus, for given boundary conditions, the proper choice of y is found by that value which satisfies Eq. (5.4.17).

The transition between the (1,2) mode and the (1,1) mode occurs when $E' = (1-V')$. Thus, from Eq. (5.4.14), the transition value of y , y_T , is given by

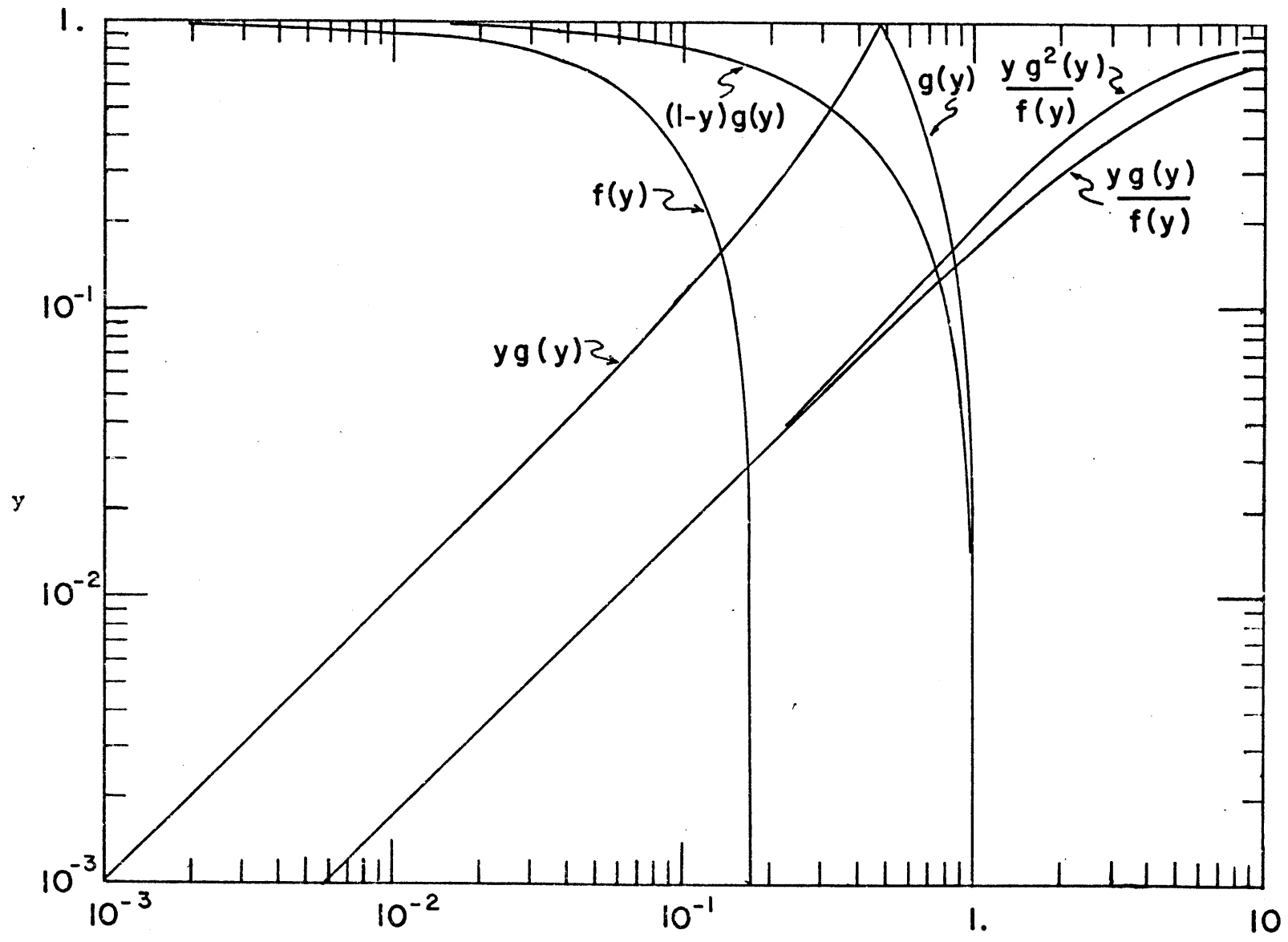
$$y_T \frac{g(y_T)}{f(y_T)} = \frac{A_s ReR(1 - V')}{r^{1/2}} \quad (5.4.18)$$

and for a given set of boundary values, the transition occurs for

$$J_T' = \frac{r(1 - V')}{R} [(1 - y_T) g(y_T)] \quad . \quad (5.4.19)$$

For the positive field case, $R > 0$, values of y range between 0 and 1. So Fig. 5.1 gives the functions $f(y)$, $g(y)$, $yg(y)$, $yg(y)/f(y)$, $(1-y)$ and $y \frac{g^2(y)}{f(y)}$ for values of $2\bar{b}/\epsilon_0 \beta = 2.26$, i.e., $\bar{b} \approx 10^{-4} \text{ m}^2/\text{V-sec.}$, $\beta = 10^7 \text{ m}^3/\text{c-sec.}$ These curves provide all the necessary information to analyze the (1,2) mode equilibrium state. Since in some circumstances the desired information involves small differences, graphical read-out error can be

Fig. 5.1 (1,2) equilibrium parameters



serious. For this reason Table 5.1 gives a listing of the above functions from which more precise calculations can be made.

Table 5.1

y	f(y)	g(y)	y g(y)	yg(y)/f(y)	(1-y)g(y)	$\frac{yg^2(y)}{f(y)}$
.01	.16916	.98883	.00988	.058456	.97894	.057804
.02	.16676	.97791	.01956	.11728	.95836	.11469
.03	.16439	.96723	.02902	.17651	.93822	.17073
.04	.16204	.95679	.03827	.23618	.91852	.22598
.05	.15971	.94656	.04733	.29634	.89924	.28050
.06	.15740	.93656	.05619	.35701	.88037	.33436
.07	.15511	.92676	.06487	.41824	.86189	.38761
.08	.15284	.91717	.07337	.48007	.84380	.44031
.09	.15059	.90778	.08170	.54254	.82608	.49251
.10	.14836	.89858	.08986	.60569	.80872	.54426
.15	.13747	.85525	.12929	.93320	.72696	.79811
.20	.12702	.81592	.16318	1.2847	.65273	1.0482
.25	.11696	.78006	.19502	1.6673	.58505	1.3006
.30	.10728	.74724	.22417	2.0896	.52306	1.5614
.35	.09795	.71707	.25097	2.5624	.46609	1.8374
.40	.08893	.68925	.27570	3.1001	.41355	2.1367
.50	.07180	.63964	.31982	4.4545	.31982	2.8493
.60	.05573	.59671	.35803	6.4245	.23868	3.8335
.70	.04061	.55919	.39143	9.6394	.16776	5.3902
.80	.02633	.52611	.42089	15.983	.10522	8.4088
.90	.01282	.49674	.44707	34.865	.04967	17.319
.99	.00125	.47298	.46825	373.66	.00473	176.73

When $y < 10^{-2}$, we neglect $R\rho'$ terms relative to ρ_T' , i.e., $y \sim 0$ in Eq. (5.4.11). Then, $q \approx |q_c|$, or

$$Rq' \approx 0.1716 R A_s Re E' \quad . \quad (5.4.20)$$

Neglecting second-order terms in y in Eq. (5.4.15), we have

$$\rho_T'^2 + \frac{2\bar{b}}{\epsilon_0 \beta} Rq' \rho_T' - r = 0 \quad . \quad (5.4.21)$$

The normalized current density in the (1,2) mode equilibrium is

$$J' = \frac{\rho_T' E'}{R} - q'(1 - V') \quad . \quad (5.4.23)$$

When the asymptotic droplet convection current is negligible, Eq. (5.4.21) gives

$$\rho_T'^2 = r - \frac{0.1716 A_s R}{g_e} J' \quad , \quad (5.4.24)$$

which compares with the (1,1) mode solution, Eq. (5.4.7),

$$\rho_T'^2 = r - \frac{0.25 A_s R}{g_e} J' \quad .$$

The current density in the (1,2) mode is

$$RJ' = Rq' \left[\frac{E'}{y} - (1 - V') \right] \quad (5.4.25)$$

so that, unless $E' \ll y$, where $y \ll 1$, neglect of the droplet convection current is a good approximation.

5.4.4 Examples of the Equilibrium Modes

Figures 5.2 through 5.4 illustrate the vertical structure in a cloud characterized by $A_s = 9750$, $1 - V' = 0.01$ for two cases, $J' = 0.08$ (Figs. 5.2 and 5.3) and $J' = 0.1$, Fig. 5.4. A small electric Reynolds number, $Re = 0.5$, is used in these case studies in order to show the entire model behavior within the range $0 < x' < 1$.

If we define L as the liquid water content in kg/m^3 ,

$$A_s = \frac{9H}{a} \times 10^{-3} L \quad (5.4.26)$$

For small droplets, from Eq. (5.2.5),

$$A_s (1-V') \approx 1.02 \times 10^6 \frac{L Ha}{U} \quad (5.4.27)$$

Thus, the cases shown in Figs. 5.2 - 5.4 are characterized by

$$\frac{LHa}{U} \approx 9.5 \times 10^{-5} \quad (5.4.28)$$

For a liquid water content of 10^{-3} kg/m^3 , scale height of about 10km, and a convection speed of 1 m/sec, the model cloud droplet radius is about 9.5 μm .

Choosing $\bar{b} \approx 10^{-4} \text{ m}^2/\text{V-sec}$, $\beta = 10^7 \text{ m}^3/\text{c-sec}$, $R = .0526$ and $r = 1$, Eq. (5.4.9) gives the maximum J' for which the (1,1) equilibrium mode is possible, $J'_{\max} = 0.13$. From Eq. (5.4.18) and Fig. 5.1, the transition from the (1,2) to the (1,1) mode occurs for $y_T = 0.36$ and from Eq. (5.4.19), the normalized current density at the transition is $J'_T = 0.087$. Thus, the system is characterized by a (1,2) mode equilibrium for $0 < J' < .087$

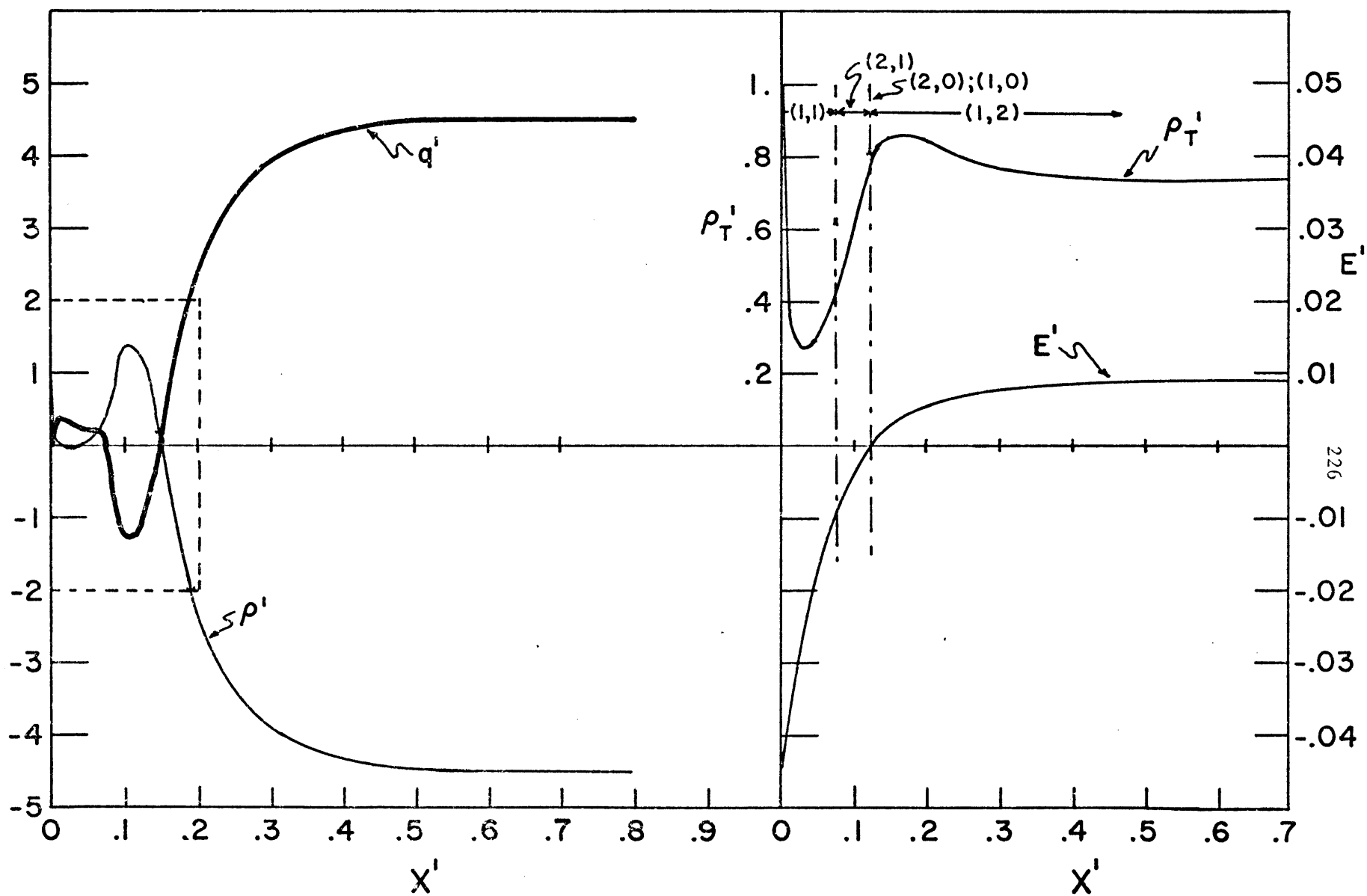


Fig. 5.2 (1,2) mode electric structure

a (1,1) mode equilibrium for $0.087 < J' < 0.13$, and a non-equilibrium mode behavior to be discussed later for $J' > 0.13$.

Figures 5.2 and 5.3 are shown for $J' = 0.08$, giving a sample calculation of the (1,2) mode equilibrium. Figure 5.2 shows the general distribution in the region $0 < x' < 1$, while Fig. 5.3 shows an expanded view in the region $0 < x' < 0.2$.

The asymptotic values in the (1,2) mode equilibrium are given by Eqs. (5.4.13) - (5.4.15) which are evaluated by determining the proper choice of y from Eq. (5.4.17). The (1,2) mode asymptotic values therefore require a graphical solution which must be carried out for each particular case. When $J' = .08$, $RJ' = .00421$, and a graphical solution of (5.4.17) gives $y \approx 0.325$. Thus, Eq. (5.4.15) from Table 5.1 gives the equilibrium droplet charge, $q' = 4.51$; the asymptotic ion density is predicted to be $\rho_T' = 0.733$, and the field is $E' = 0.009$. The numerical integration predicted these quantities to be $q' = 4.50$, $\rho_T' = 0.733$ and $E' = 0.00898$.

Figure 5.3 shows the initial stages of interaction between ionic space charge and the droplet cloud for an expanded x' . This shows more clearly the switching of droplet charging regimes and the contrary behavior of ionic space charge and droplet charge. In the initial phase, the charging regime is in the (1,1) mode and droplet charge increases in response to the positive ionic space charge at the boundary, since $R > 0$. When the ionic space charge is depleted, droplet charging becomes insignificant and the charge remains nearly constant throughout the remainder of the (1,1) mode charging region, even though the total ion density, hence conductivity, is significant. This effect is attributed to the large disparity between the magnitudes of the terms $A_s \text{Re}|E'|$ and $\frac{R}{|R|} q'$ in the

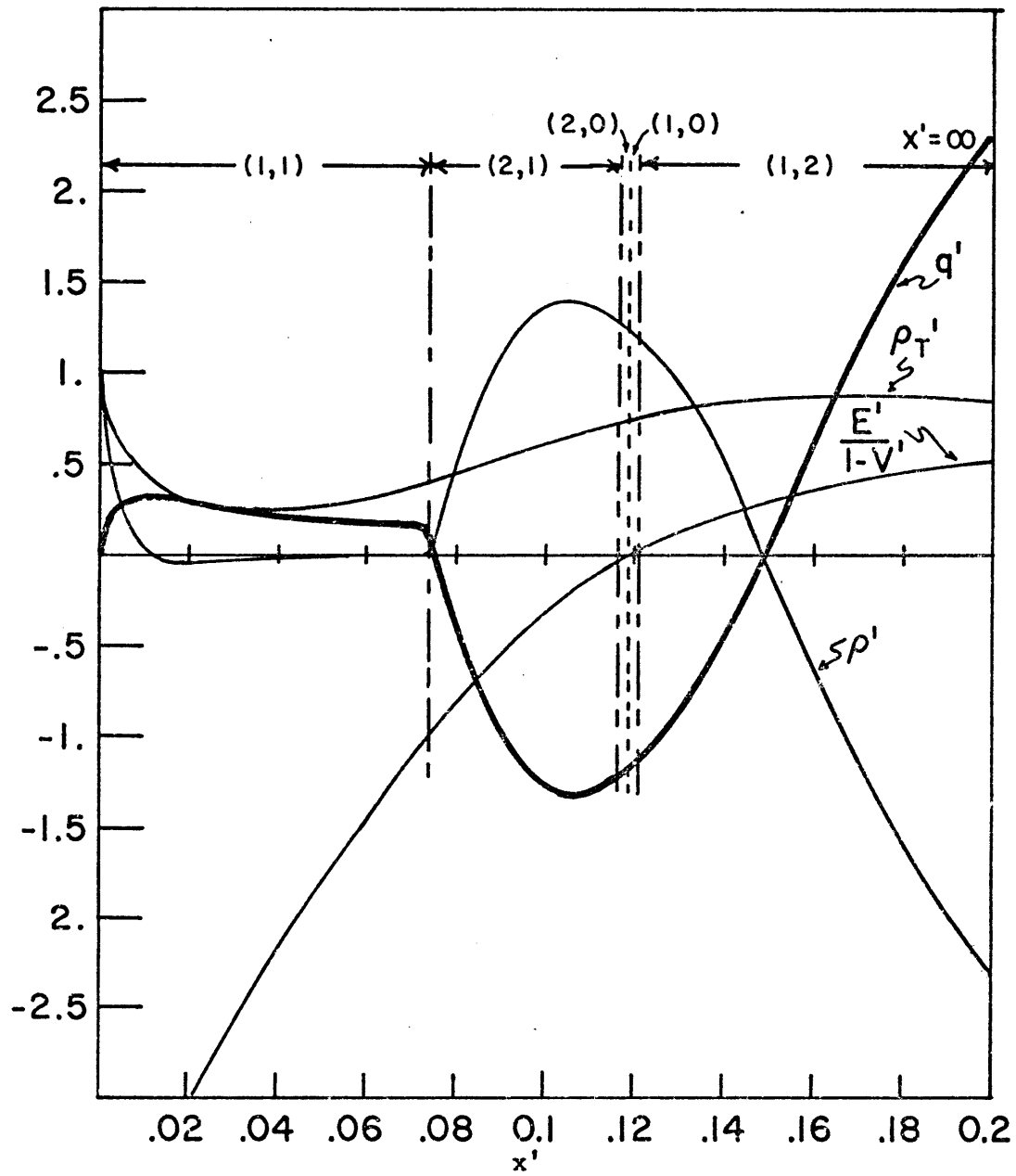


Fig. 5.3 Expanded scale (1,2) mode example

normalized current expression, Eq. (5.3.12). The droplet charging current is proportional to $I_1^{(1)'} - I_2^{(1)'}$. When $A_s \text{Re}|E'| \gg R/|R| (q')$,

$$V' \frac{dq'}{dx'} \approx \frac{A_s |E'|}{8R} [(R\rho' + \rho_T') + (R\rho' - \rho_T')]$$

or

$$V' \frac{dq'}{dx'} \approx \frac{A_s |E'|}{4} \rho' \quad (5.4.29)$$

Thus, in the (1,1) mode, when $A_s \text{Re}|E'| \gg R/|R| (q')$, droplet charging is determined by the net ionic space charge, regardless of the local conductivity.

The total ion density shows an initial decrease to a minimum and a slow increase thereafter in the (1,1) charging regime. In the cloud-free region, for $r = 1$, changes in ρ_T' are insignificant, so that this effect can be attributed to the droplets. Considering, therefore, the droplet terms in the total ion density equation, (5.3.19),

$$(1-E'^2) \frac{d\rho_T'}{dx'} \sim r - \frac{R}{\text{Re}} (I_1^{(1)'} + I_2^{(1)'}) + \frac{RE'}{\text{Re}} (I_1^{(1)'} - I_2^{(1)'}).$$

Since $A_s \text{Re}|E'| \gg R/|R| (q')$, this result is

$$(1-E'^2) \frac{d\rho_T'}{dx'} \sim r + \frac{A_s |E'|}{4 \text{Re}} [RE'\rho' - \rho_T'].$$

For small R , the first term in brackets can be neglected relative to ρ_T' , and the distribution of ρ_T' is determined by the magnitude of the term

$\frac{A_s |E'|}{4\text{Re}} \rho_T'$ relative to the ion source term r . As the field increases

from its initial negative value, the droplet depletion of ions becomes

less important, until the source term r just balances the droplet depletion term. Thereafter in the (1,1) mode, the decreasing magnitude of the field causes droplet depletion of ions to be less than the ionization and ρ_r' increases.

When E' reaches the range $-E' < 1 - V'$, the charging regime switches to the (2,1) mode. Since throughout most of this regime $A_s \text{Re}|E'| \gg q'$ as before, the principal current to the droplet is $I_2^{(1) '}$ and the droplet charge consequently becomes negative with a corresponding increase in the positive ion space charge. As the field increases toward zero, the droplet charge becomes more influential, until finally $I_1^{(2) '} = I_2^{(1) '}$ and the droplet charge reaches a maximum value for this regime. Past the point of maximum charge, the positive ion current to the drop exceeds the negative current and droplet charge again increases. Characteristically throughout this regime, the field-dependent depletion of total ion density by the droplets is less than the ionization, so that ρ_T' increases throughout.

The (2,0) and (1,0) regimes are reached when the electric field nears zero, so that $-q > |q_c|$, or

$$|q'| > A_s \text{Re} |E'|$$

from Eq. (5.3.17). These two regions are characteristically narrow bands of transition from the (2,1) to the (1,2) modes of charging and are not very important to the final state.

In the early stage of the (1,2) mode, the droplet charge dominates the charge flux to the droplets. Since the droplet is negative in this

initial phase, $I_1^{(1) '}$ dominates and the droplet charge increases through zero. The electric field continues to increase, however, and as the droplet charge becomes positive, the term $A_s \text{Re} |E'|$ becomes dominant so that the droplet charge continues to increase although at a slower pace in response to the increasingly negative net ionic space charge and the positive droplet charge. Charge relaxation has been active throughout the various charging regimes and as a consequence, the electric field approaches its asymptotic value. When the final normalized equilibrium field is less than the normalized droplet terminal speed, the (1,2) equilibrium is established in which the equilibrium droplet charge represents a balance between the flux of positive ions (determined by the equilibrium electric field and the difference between the equilibrium values of total ion density and droplet charge), and the flux of negative ions (determined by the droplet charge and the difference between the equilibrium values of total ion density and net ionic space charge).

When the final normalized field exceeds $1 - V'$, the (1,1) mode shown in Fig. 5.4 is attained. Up to this point, the previous discussion is the same. However, on switching to the (1,1) mode, for large $A_s \text{Re} |E'|$ the droplet charging rate is essentially determined by the net ionic space charge, Eq. (5.4.29). This being negative, the droplet charge quickly approaches zero with a corresponding increase in ρ' toward zero. As both q' and ρ' approach zero, disparities between the positive and negative ion fluxes to the droplet become less significant, until finally,

$$I_2^{(1) '} = \frac{A_s \text{Re} |E'|}{8 R} \rho_T' \quad . \quad (5.4.30)$$

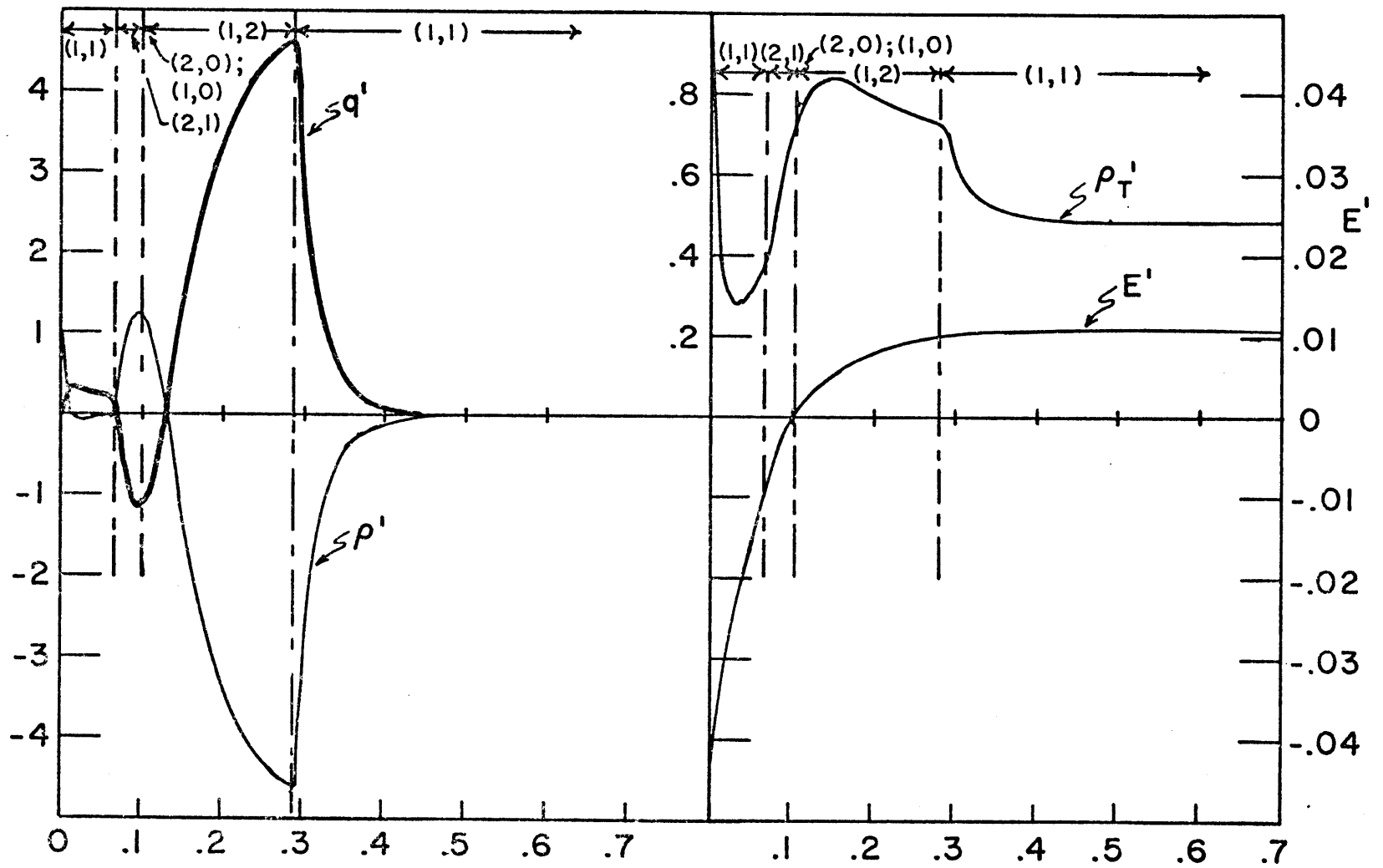


Fig. 5.4 (1,1) mode electric structure

and droplet charge and ionic space charge remain zero thereafter.

The significant differences between the possible equilibrium states are obvious; the (1,2) mode equilibrium is characterized by significant droplet charge density in equilibrium with a negative ionic space charge density of the same magnitude. The (1,2) equilibrium field is bounded by the normalized droplet terminal velocity, $1 - V'$, and the equilibrium total ion density ρ_T' (or conductivity) is relatively high. The (1,1) mode equilibrium state is characterized by zero droplet charge and ionic space charge, while the equilibrium electric field is greater than $1 - V'$ and the conductivity may be appreciably reduced, depending on the magnitude of the final field. The equilibrium field, however, is bounded by the requirement that $\rho_T' > 0$, and the steady-state condition that $E' < 1$. The first of these conditions is given by Eq. (5.4.9) for positive J', R ,

$$r - \frac{\bar{b}_A \text{Re} R^2}{2 \epsilon_0 \beta} J' > 0 \quad (5.4.31)$$

whereas the second is

$$r - \frac{\frac{(\text{RJ}')^2}{\bar{b}_A \text{Re} R^2 J'}}{\frac{2 \epsilon_0 \beta}{s}} < 1 \quad (5.4.32)$$

For large values of the parameter $\frac{\bar{b}_A \text{Re} R}{2 \epsilon_0 \beta}$ relative to r , the conditions above are equivalent. When this parameter is somewhat smaller, however, the controlling condition is given by Eq. (5.4.32). Thus, the maximum field in the (1,1) mode equilibrium is $E' \sim 1$, and occurs for

$$J' = \frac{\bar{b}_A \text{Re}}{4 \epsilon_0 \beta} \left[\left(4r \left(\frac{2 \epsilon_0 \beta}{\bar{b}_A \text{Re} R} \right)^2 + 1 \right)^{1/2} - 1 \right] \quad (5.4.33)$$

Since the maximum field is reached only when $x' \rightarrow \infty$, Eq. (5.4.33) does not give the maximum field in the region $0 < x' < 1$, and hence is principally useful in delineating the boundary values separating the equilibrium regime from the non-equilibrium regime in which particle reversal occurs for some x' .

The characteristic length associated with the cases shown in Figs. 5.2 - 5.4 is given by Eq. (5.3.23), $\ell' \approx 0.48$. This is seen to be a good estimate of the equalizing distance when boundary conditions leading to an equilibrium distribution are imposed. However, as the transition from the equilibrium to the non-equilibrium conditions are approached, the characteristic length increases markedly, and is no longer given by Eq. (5.2.23).

5.4.5 The Non-equilibrium Mode

Eq. (5.4.33) defines the transition from the equilibrium mode to one in which for some x' , one polar ion species velocity approaches zero. Thus for J' greater than that given by Eq. (5.4.33) the dependent variables do not approach an asymptotic value and no equilibrium electric field is established in which ion generation, recombination and droplet depletion balance in a region of zero net charge density. The critical distance at which $E' = 1$ depends on the cloud characteristics, the electric Reynolds number and the boundary values of electric field and net space charge. For many cases this critical distance is much greater than the scale height ($x' \gg 1$) so that it is possible for the non-equilibrium state to exist in the cloud in the steady state provided some external mechanism can be called upon to maintain the steady uniform current outside the generating region.

The (1,1) equilibrium is characterized by $\rho' = q' = 0$ so that the current in this regime is a consequence of the response of the total ion density to the electric field. The magnitude of the equilibrium ion density is found from the equilibrium condition, $G_c = 0$, in which a balance exists between generation and annihilation of ions. Ion scavenging by droplets in the (1,1) mode equilibrium is found by the product $\rho_T' |E'|$ since $\rho' = 0$ so that a critical field exists above which the field-independent processes of ionization and recombination cannot maintain the balance $G_c = 0$. For example, when

ρ' and q' are small, Eq. (5.3.19) is approximately

$$(1-E'^2) \frac{d\rho_T'}{dx'} = -g_e (\rho_T')^2 + \frac{\bar{b}A_S \text{Re } R}{2\epsilon_0 \beta} \rho_T' E' - r \quad (5.4.34)$$

so that for large positive field, the terms in the brackets give a positive resultant and ρ_T' decreases slowly to a value approximated by

$$\rho_T' E' = \frac{2\epsilon_0 \beta r}{\bar{b}A_S \text{Re } |R|}, \quad E' < 1. \quad (5.4.35)$$

When $r = 1$, i.e., the initial particle density is given by a balance between ionization and mutual recombination,

$$\rho_T' E' = \frac{\alpha \beta}{3\pi a^2 N U} \quad (5.4.36)$$

or

$$\rho_T E = \frac{2\alpha}{3\pi a^2 N b} \quad (5.4.37)$$

In the discussion of the (1,1) mode, droplet charging is determined essentially by the net space charge when

$$A_S \text{Re } |E'| > R/|R|q'.$$

Amore precise statement of this behavior is

$$V' \frac{dq'}{dx'} \approx \frac{1}{Re} \left(\frac{A_S Re E'}{4} \rho' - \frac{\rho_T' q'}{2|R|} \right) \quad (5.4.38)$$

Thus when $A_S Re E'$ is large, the net space charge is rapidly depleted and thereafter the droplet discharges in a conductivity mode.

The transition from the (1,1) equilibrium for the cases given in Figs. 5.2-5.4 occurs for $J' = 0.13$ from Eq. (5.4.33). Fig. 5.5 shows the distribution for $J' = 0.2$, well within the non-equilibrium regime. In this mode, a non-zero droplet charge persists in the upper cloudy region, and the ionic net space charge is reduced to negligibly small values. The non-zero droplet charge which persists in the (1,1) mode after the ionic space charge tends to zero is a consequence of the (1,1) charging regime being reached before charge relaxation has reduced the net charge density to negligible values. This reflects the consistent charge exchange between ionic space charge and the droplet charge which maintains a net charge density which can change only by relaxation. Because of the very low mobility of charged droplets, relaxation can occur only by the relatively slow process of conduction to the droplet surface.

Although no true equilibrium values of the dependent variables exist in the non-equilibrium mode, the drop charge decreases very slowly with distance and the ionic space charge is nearly zero. The current density in the non-equilibrium regime is therefore approximately

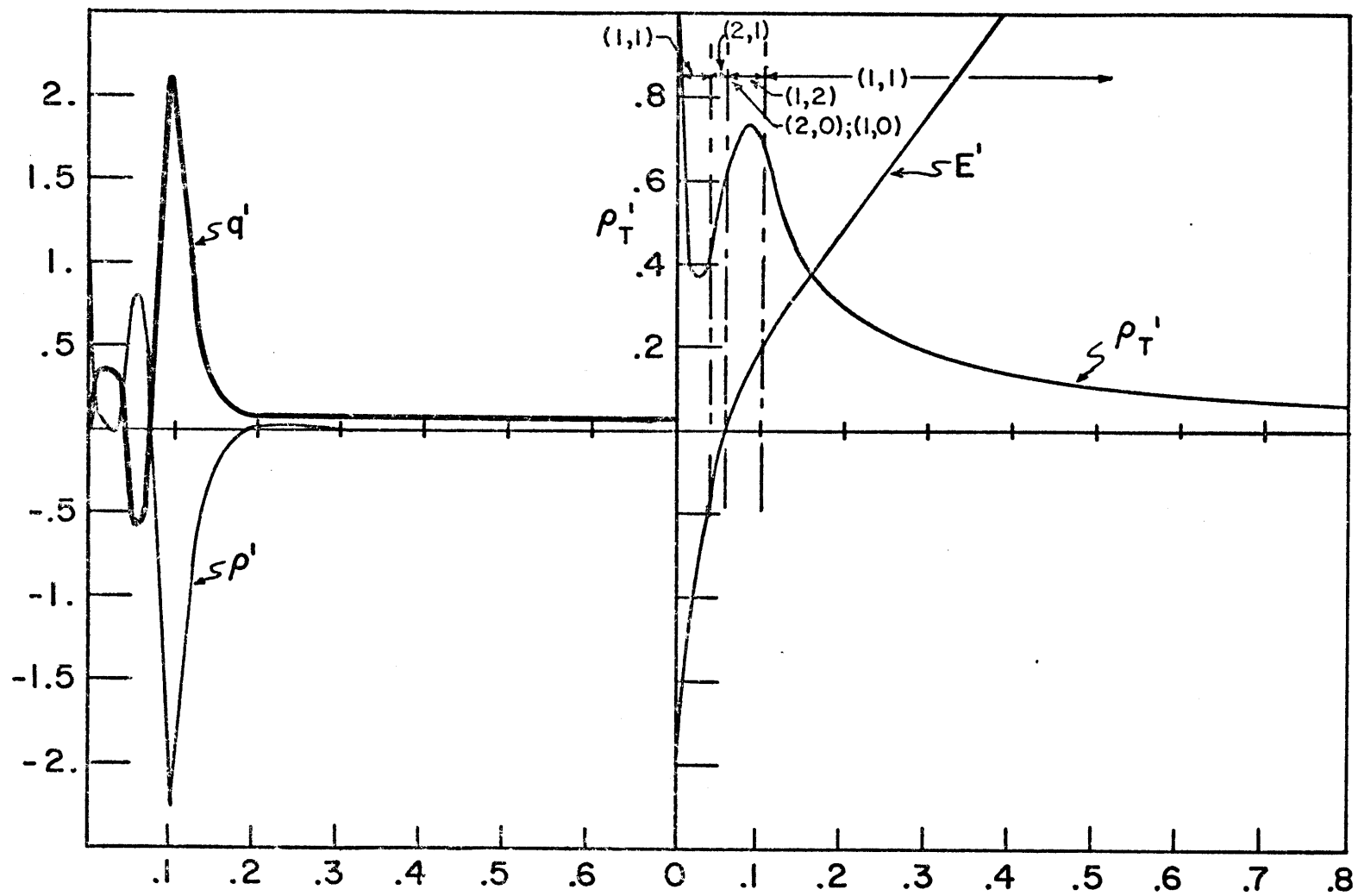


Fig. 5.5 Non-equilibrium mode electric structure

$$J' = \frac{\rho_T' E'}{R} + q' V' \quad (5.4.39)$$

so that from Eq. (5.4.35), the drop charge in this region is approximately,

$$q' = \frac{1}{V'} \left(J' - \frac{2\epsilon_0 \beta r}{b A_S R e R^2} \right) \quad (5.4.40)$$

The electric field in this region grows linearly with distance since the droplet charge is not masked by ionic space charge. The drop charge in the (1,1) region of the case study of Fig. 5.5 is predicted by the numerical model to be 0.0692, while Eq. (5.4.40) predicts this value to be 0.0694.

In the initial (1,1) regime of Fig. 5.5, the boundary charge density is transformed to droplet charge. In this region, the negative ionic space charge is reduced somewhat by relaxation, and the drop charge drops by conduction. Transition to the (2,1) regime reverses the polarity of the ion-droplet charge densities, but relaxation continues in this region. Since E' is approaching zero in this stage the field-dependent currents to the droplet become smaller and the droplet charge in the (2,1) region is somewhat smaller than in the (1,2) region where the field is increasing through positive values. When the (1,1) mode is reached, a slight positive net charge still exists and the droplet charge tends rapidly to a small but positive value which changes only slowly thereafter because of the non-zero

conductivity. Fig. 5.6 shows a slightly more extreme case of the non-equilibrium mode with an expanded view of the early stages in which the differences in droplet and ionic charge densities is more evident. Since the initial field is greater in this case than that for Fig. 5.5, the electric field exceeds $1 - V'$ sooner and relaxation of the initial space charge is less so that the droplet charge density is greater than in the previous case.

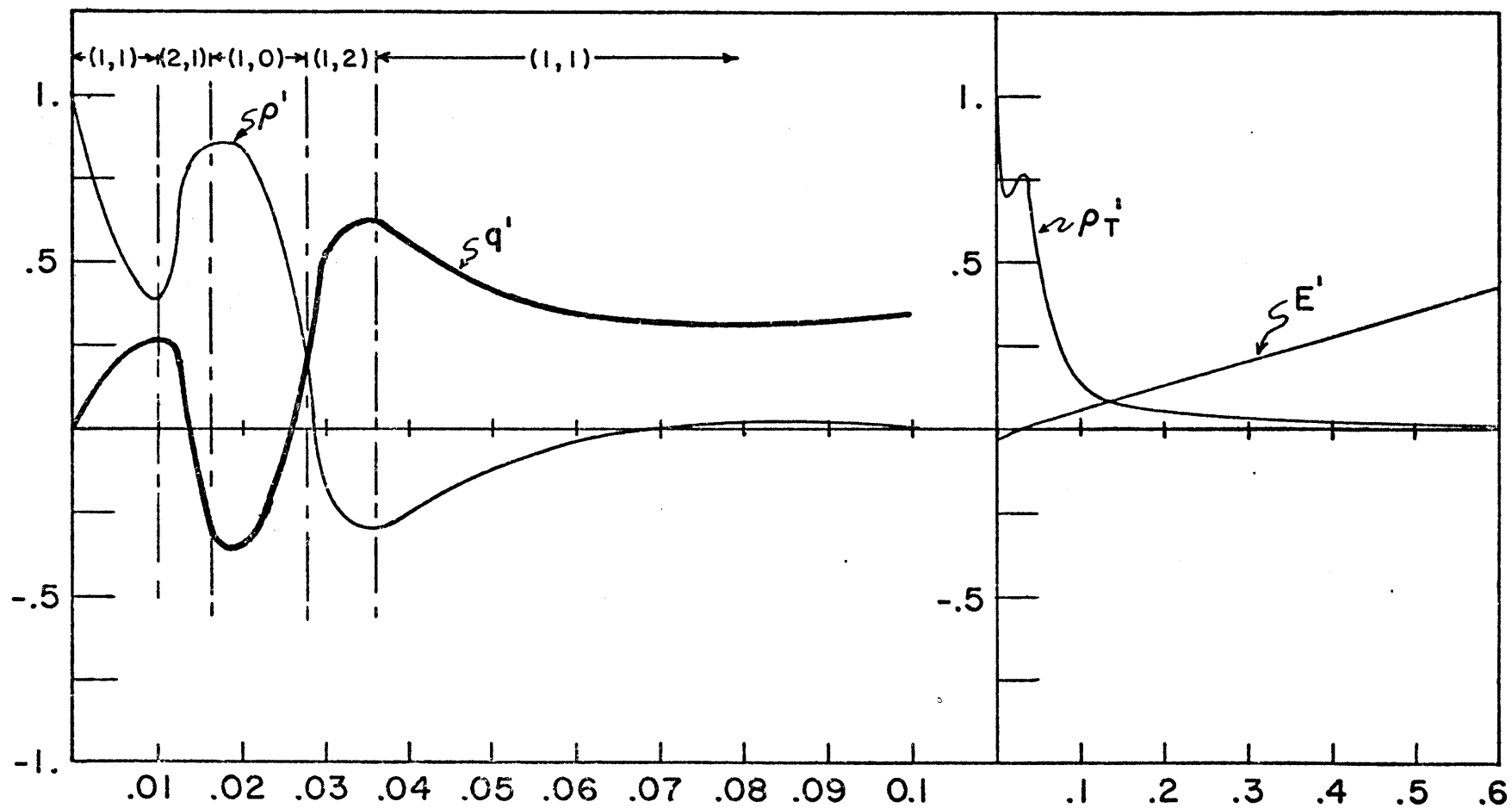


Fig. 5.6 Expanded scale non-equilibrium mode

5.4.6 Summary

Three characteristic modes of cloud electric structure are possible. The modes encountered in order of increasing normalized current density are termed the (1,2) equilibrium mode, the (1,1) equilibrium mode, and the non-equilibrium mode. Table 5.2 lists the range of normalized current density associated with each mode and the values of the dependent variable for large x' .

The (1,2) mode leads to a relatively small cloud electric field, depending on the terminal speed of the cloud particles. However, the equilibrium droplet charge is non-zero and later collection of these droplets by growing precipitation could lead to large rain-drop charges. The total ion density in the (1,2) mode is not greatly different from the initial value, so that conversion of sub-cloud ionic space charge into cloud charges is not very efficient in this regime. This mode occurs for relatively small J' and is therefore associated with relatively large, but negative electric fields at cloud base.

The (1,1) mode can lead to larger electric fields in the cloud. This mode is characterized by vanishing drop charge and ionic space charge. The conductivity in the cloud can be quite low, approaching zero as the normalized current density approaches its upper bound for this mode. The boundary electric field for the (1,1) mode is negative, but smaller in magnitude than that for the (1,2) mode.

The non-equilibrium mode is encountered as the boundary electric field approaches zero through negative values. This

TABLE 5.2(1,2) equilibrium:

$$0 < J' < \frac{r(1 - V')}{R} \quad (1 - y_T)g(y_T)$$

where

$$\frac{y_T g(y_T)}{f(y_T)} = \frac{A_s ReR(1 - V')}{r^{\frac{1}{2}}}$$

$$\rho_T' = r^{\frac{1}{2}}g(y)$$

where

$$RJ' = \frac{r^{\frac{1}{2}}}{A_s ReR} \frac{g(y)}{f(y)} - (1 - V')yg(y)$$

$$\rho' = -\frac{r^{\frac{1}{2}}}{R} yg(y) = -q'$$

$$0 < E' < (1 - V')$$

$$E' = \frac{r^{\frac{1}{2}}}{A_s ReR} \frac{y_T g(y)}{f(y)}$$

TABLE 5.2 (continued)(1,1) equilibrium:

$$\frac{r(1 - V')}{R} \quad (1 - y_T)g(y_T) < J' < \frac{2\epsilon_0\beta r}{bA_s ReR^2}$$

$$\rho_T' = \left(r - \frac{bA_s ReR^2 J'}{2\epsilon_0\beta} \right)^{1/2}$$

$$\rho' = q' = 0$$

$$1 - V' < E' < 1$$

$$E' = \frac{RJ'}{\rho_T'}$$

Non-equilibrium mode:

$$J' > \frac{2\epsilon_0\beta r}{bA_s ReR^2}$$

$$\rho_T' = \frac{2\epsilon_0\beta r}{bA_s ReR}$$

$$\rho' \approx 0$$

$$q' \approx \frac{1}{V'} \left(J' - \frac{2\epsilon_0\beta r}{bA_s ReR^2} \right)$$

mode is distinct from the previous kinds of behavior since it is not an asymptotic solution, and the electric field grows linearly with height in the cloud. In this mode the ionic space charge vanishes, the conductivity approaches zero, and the droplet charge is positive and nearly constant with height. The non-equilibrium mode is noteworthy because relatively large values of electric field can develop for small x' , contrary to the behavior of the equilibrium modes.

5.5 Transfer Characteristics and Terminal Relations

Just which of the three characteristic structures should be expected to obtain in a given cloud is determined by the cloud characteristics A_s and $(1-V')$, and the initial conditions at cloud base, r , R , J' and Re . Because Re is an inverse length scale in the bipolar model, small values of electric Reynolds number at cloud base ($Re < 1$) are necessary for the equilibrium structures to be attained within the cloudy section. That is, the predicted structure is attained only if the cloud height is sufficient to allow relaxation to the equilibrium before the section top is reached. The non-equilibrium case obtains when the (1,1) charging regime is reached before relaxation has reduced the net charge density to negligible values. The non-equilibrium case therefore depends on both Re being small and the initial field at cloud base being a small negative value.

The initial conditions at cloud base are determined by the conditions imposed at the lower boundary of the clear section, the transfer characteristics of this region, and a geometric factor, the ratio of the height of the cloudy section to that of the clear section. The results of the bipolar model showed that large Re in the clear region allows little relaxation through the generating section so that the initial values of the cloud are nearly the same as those for the sub-cloud region. On the other hand, small Re allows significant relaxation and the cloud initial values are

therefore wholly different from those of the clear section.

Figures 5.7 through 5.9 show the transfer characteristics of the clear region, parameterized in terms of R (or equivalently, $B_1 = \rho_2 / \rho_1$). These curves provide cloud initial conditions in terms of the sub-cloud values of Re and R . The transfer characteristics were computed for $J' = 0.1$; however extreme values, $J' = 0.001$ and $J' = 1.0$ were tested. The maximum departure of the transfer characteristics over the range of J' above from that for $J' = 0.1$ was small in all cases. As expected, small Re in the sub-cloud section leads to relaxation of the initial net charge density so that the normalized initial conditions at cloud base are very much different from those at the sub-cloud boundary; large Re leads to essentially the same initial values for both sections.

The curves of Figure 5.7 showing the transfer characteristics for Re are significant since these in conjunction with the ratio of heights of the clear and cloudy regions determine whether the equilibrium regimes within the cloud can be attained. If we write H_a for the height of the clear section and H_c for that of the cloudy section, then the cloud electric Reynolds number is

$$Re = Re_c \frac{H_a}{H_c} \quad (5.5.1)$$

Thus for a model with initial conditions $-B_1 > 0.8$ corresponding to the values of net and total space charge discussed in

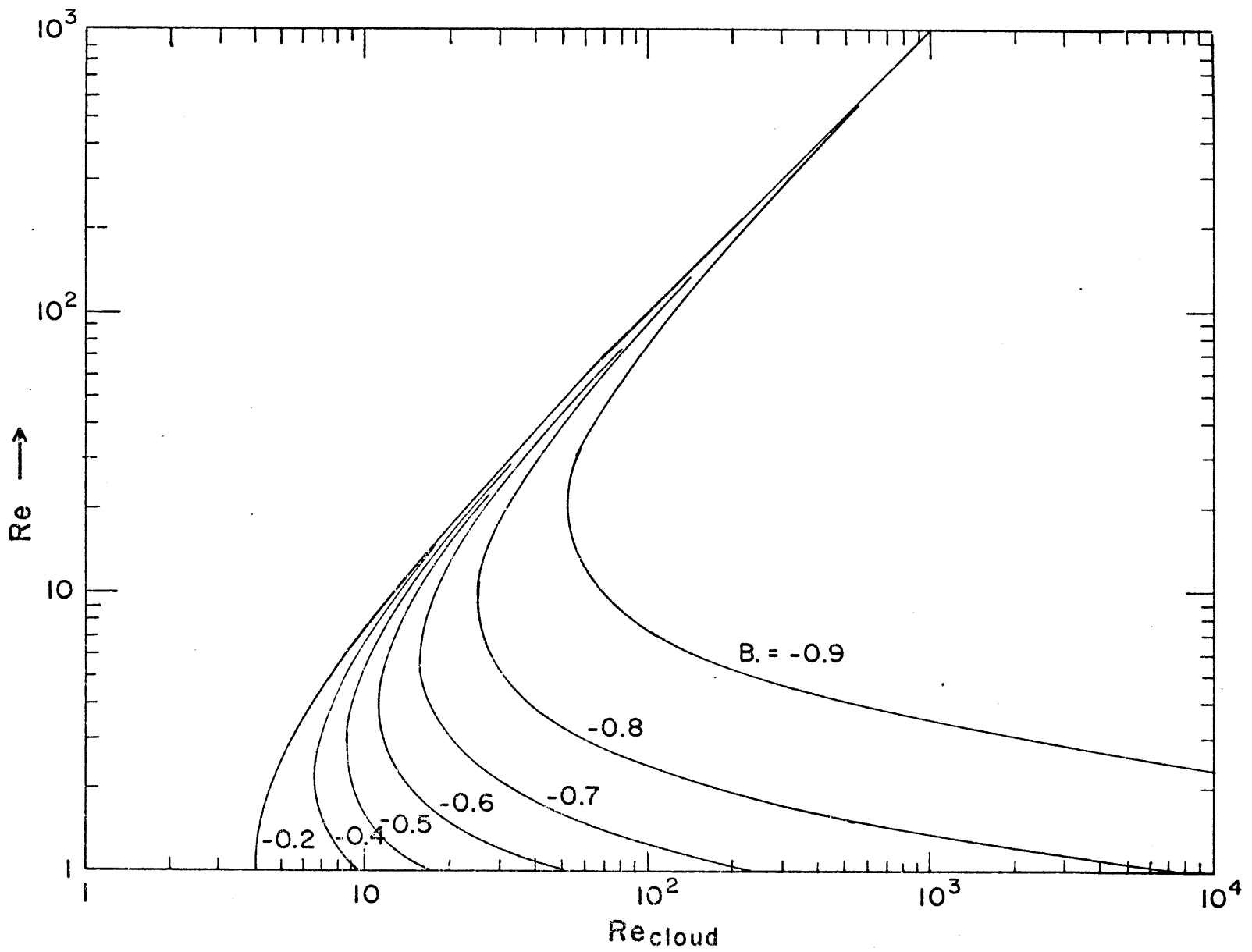


Fig. 5.7 Transfer characteristics for Re

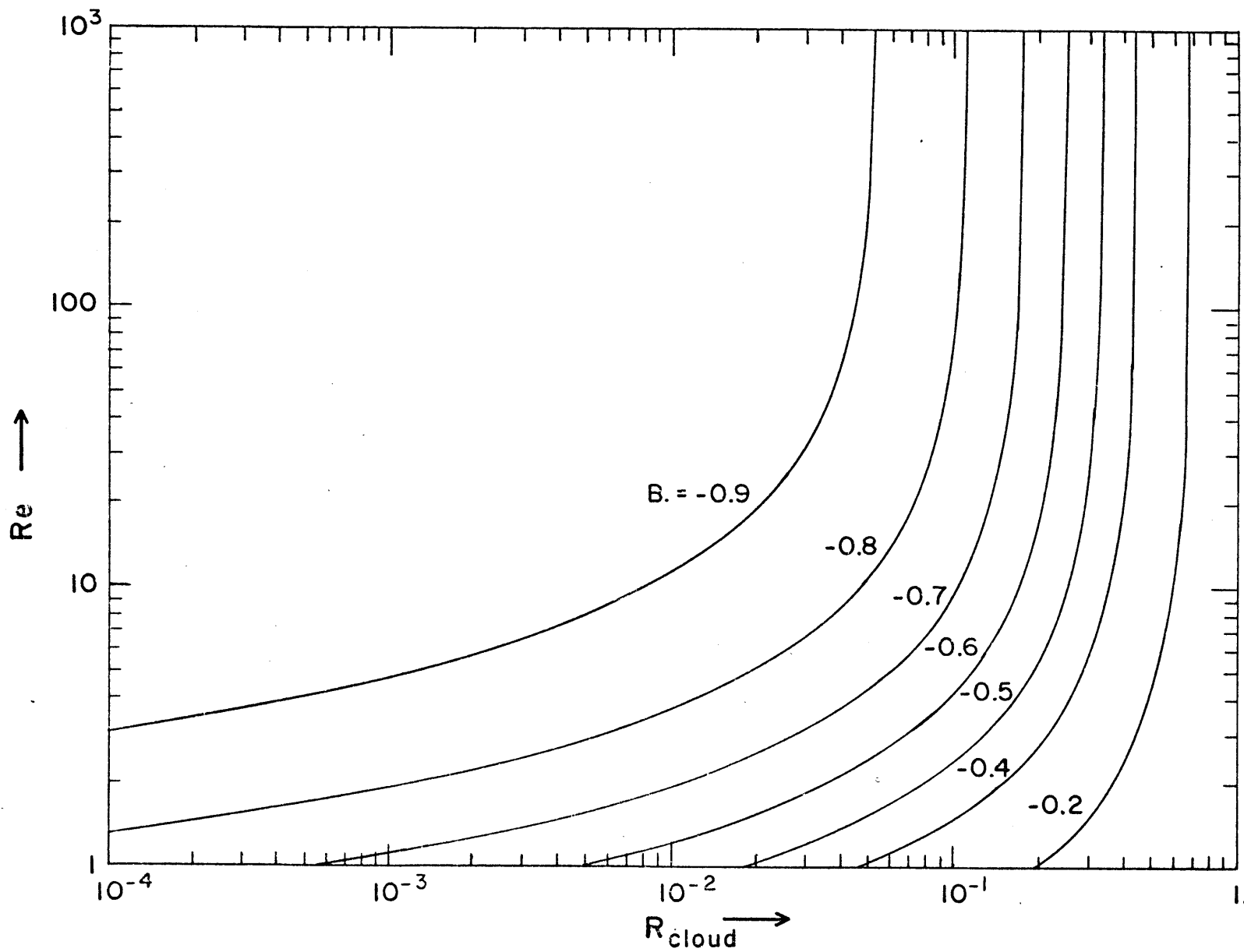


Fig. 5.8 Transfer characteristics for R

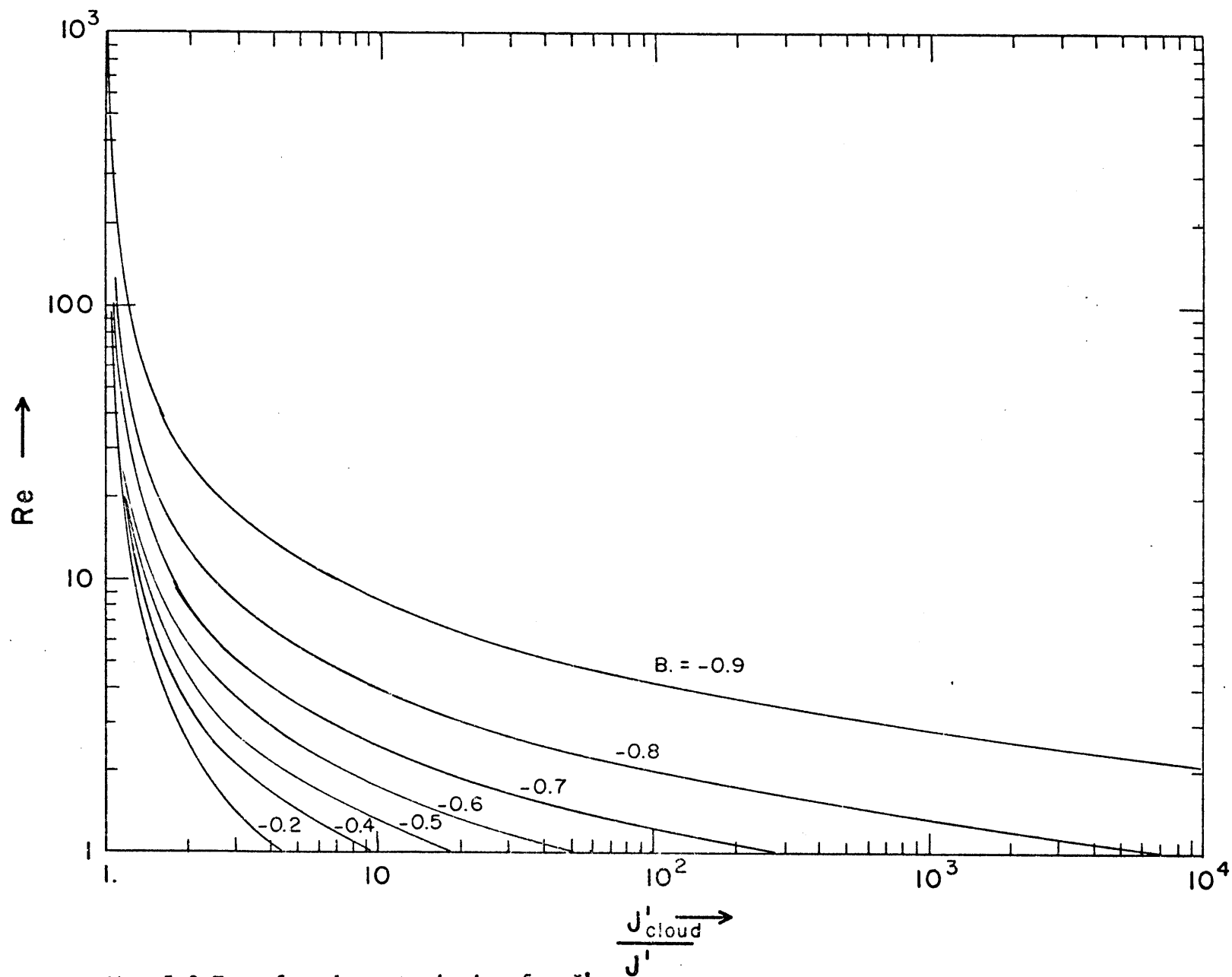


Fig. 5.9 Transfer characteristics for J'

Chapter 2, the height of the cloudy section must be at least 25 times that of the clear section in order for the equilibrium regimes to exist within the cloud. as B_1 becomes smaller, this condition becomes less stringent; for example, if $-B_1 = 0.5$, the ratio $\frac{H_a}{H_c}$ must be about 1/8.5 for Re in the cloud to be near unity. This value of R corresponds to a net space charge density at the lower boundary of the clear section of about $2 \times 10^{-10} \text{ c/m}^3$ for the values of ρ and $\rho_{T/O}$ indicated in Chapter 2.

The height ratio places considerable restriction on the usefulness of the cloud model results since if the cloud electric Reynolds number is large, values of the dependent variables can be obtained only by numerical integration. If Re in the cloud is small, characteristic values of these quantities can be calculated easily for the upper regions of the cloud section.

The lower sub-cloud region is approximated in the model by the region between the mixed layer or exchange level and cloud base. Thus a measure of the height ratio is given by the ratio of this intervening region and cloud top height. Byers and Hall (1955) observed cloud top heights over the ocean near Puerto Rico for clouds whose bases were between 2000 and 2300 feet above sea level. Bunker et al (1949) observed the depth of the mixed layer over the Caribbean and found that the depth of the layer from the top

of the mixed region to cloud base ranged from 100 to 300 meters. A summary of these results is shown in Table 5.3. Notice that the height ratios are computed for the height extremes of the region between cloud base and the mixed layer and for the cloud height. The actual ratio used in the model must be some fraction of the values shown since the one-dimensional restriction is valid only for cloud heights small compared to cloud diameter. In the table, values were calculated assuming all cloud bases were at 2000 feet above sea level. The values of Table 5.3 are considered typical of oceanic cumuli over the Caribbean.

Battan and Braham (1956) observed top heights over the north central United States. They did not report heights of cloud base or the depth of the mixed region. They stated that the height of cloud base above ground was variable with the average height about 1.37 km. Sagalyn and Faucher (1954) observed the height of the mixed region over the north-east U.S., but did not report cloud base height. These results showed that the exchange layer depth varied between about 300 and 3000 meters with an average about 1830 meters. No observations of both cloud base height and exchange layer depth over continents have been found. Since these parameters are highly variable, it is difficult to determine reasonable values of the height of the clear section for a continental-type cloud. With no further information, a height of 400 m was assumed for the clear section, and the results reported by Battan and Braham were used.

TABLE 5.3

Height ratios for oceanic cumuli:

No. observations	Cloud height, km	H_c/H_a	
		$H_a = 100 \text{ m}$	$H_c = 300 \text{ m}$
10	.92 - 1.07	9 - 11	3
19	1.07 - 1.222	12	4
53	1.22 - 1.37	14	4.7
51	1.37 - 1.52	15	5
57	1.52 - 1.68	17	5.7
62	1.68 - 1.83	18	6
55	1.83 - 1.98	20	6.7
39	1.98 - 2.14	21	7
47	2.14 - 2.29	23	7.7
35	2.29 - 2.44	24	8
22	2.44 - 2.59	26	8.7
19	2.59 - 2.74	27	9
6	2.74 - 2.90	29	9.7
20	> 2.90	> 29	> 9.7

to calculate the height ratios shown in Table 5.4. Observations by sail-plane pilots seem to indicate that the region between cloud base and the upper level of the mixed layer can be large, exceeding 1000 m in some cases (John Hart, personal communication, 1972). In these cases, the sub-cloud Re will be quite small leading to essential neutrality of the air penetrating cloud base so that the current density at cloud base will be primarily conduction current and the system will be in the brake or pump mode of operation instead of the generating mode.

Several case studies were prepared using the data reported in Tables 5.3 and 5.4 to determine terminal relations for the combined clear and cloudy section generator. In the case studies, a cloudy section height of 2 km for oceanic cumuli and 3.2 km for continental cumuli was assumed. These assumed heights are reasonable provided the cloud diameter is greater than the cloud height and heights exceed 2 and 3 km for the oceanic and continental clouds respectively. Two case studies for oceanic clouds were prepared, one assuming a sub-cloud depth of 100 m, and thus a height ratio $H_a/H_c = .05$, and the other for a sub-cloud layer of 300 m thickness, $H_a/H_c = 0.125$.

A droplet radius typical of oceanic cumuli was chosen to be about $10 \mu m$, while a continental cloud droplet radius of $5 \mu m$ was chosen to reflect the greater density of condensation nuclei over continental regions. Each case study was computed for two liquid water contents, $0.5 g/m^3$ and $1 g/m^3$. These correspond to

TABLE 5.4

Height ratios for continental cumuli

No. observations	Cloud height, km	H_c/H_a
		$H_a = 400 \text{ m}$
53	1.89 - 3.11	4.7 - 7.8
99	3.11 - 4.33	10.8
48	4.33 - 5.55	13.9
9	> 5.55	> 13.9

$A_s = 900$ and 1800 for the oceanic clouds studied and $A_s = 2880$ and 5760 for the continental case. Initial values of R and Re in the sub-cloud layer are given for each case studied for values of B_1 between -0.4 and -0.9 . Using the transfer characteristics of the sub-cloud layer, cloud initial values R_c and Re_c were determined and the normalized current density at the lower boundary of the sub-cloud section corresponding to transitions within the cloud from the $(1,2)$ to the $(1,1)$ equilibrium modes and the $(1,1)$ equilibrium to the non-equilibrium mode were calculated.

Table 5.5 shows the essential calculations for an oceanic cloud of depth 2 km with a sub-cloud layer of depth 100 m. Significantly, a cloud electric Reynolds number of about 1 is predicted for $B_1 \approx -0.7$, or $R \approx 0.1765$. Assuming about $4 \times 10^{-10} \text{ c/m}^3$ for ρ_T/o , this initial value of ionic space charge density is not excessive, although large compared to that predicted by the empirical field structure discussed in Chapter 2. Larger initial values of ionic space charge in this case lead to $Re_c < 1$ so that terminal values of the space charge, conductivity, electric field and droplet charge can be determined in terms of J' by the results given in Table 5.2.

Table 5.6 shows the results of the oceanic cloud case when the sub-cloud layer is 300 m thick. In this case, Re_c is greater than one unless the initial space charge is quite large, about

TABLE 5.5

Model parameters and transition values of J' for a model cloud of 10 micron droplets of height 2 km with a 100 m sub-cloud layer. Calculations are carried out for 0.5 and 1 g/m³ liquid water content ($A_s = 900$ and 1800).

B_1	R	Re	R_c	Re_c	$A_s=900$				$A_s=1800$			
					(1,2)	(1,1)	(1,1)	NE	(1,2)	(1,1)	(1,1)	NE
-.9	.0526	42	.0335	3.3	.1360			.3375	.1020			.1688
-.8	.111 1	20	.0710	1.55	.0639			.1592	.0481			.0796
-.7	.1765	12.5	.1135	1.0	.0397			.0988	.0299			.0494
-.6	.2500	8.8	.1565	.7	.0286			.0711	.0215			.0356
-.5	.3333	6.6	.2053	.55	.0214			.0533	.0161			.0267
-.4	.4286	5.2	.2600	.42	.0166			.0414	.0125			.0207

TABLE 5.6

Model parameters and transition values of J' for a model cloud of 10 micron droplets of height 2 km with a 300 m sub-cloud layer. Calculations are carried out for 0.5 and 1 g/m³ liquid water content ($A_s = 900$ and 1800).

$A_s = 900$								$A_s = 1800$			
B_1	R	Re	R_c	Re_c	(1,2)	(1,1)	(1,1) NE	(1,2)	(1,1)	(1,1) NE	
-.9	.0526	14	.0135	8.4	.1360		.3384	.1023		.1692	
-.8	.1111	6.6	.0284	4.05	.0643		.1602	.0484		.0801	
-.7	.1765	4.18	.0457	2.44	.0405		.1008	.0305		.0504	
-.6	.2500	2.95	.0650	1.72	.0286		.0711	.0215		.0356	
-.5	.3333	2.2	.0863	1.29	.0214		.0534	.0161		.0267	
-.4	.4286	1.73	.1135	.975	.0167		.0415	.0125		.0208	

TABLE 5.7

Model parameters and transition values of J' for a model cloud of 5 micron droplets of height 3.2 km with a 400 m sub-cloud layer. Calculations are carried out for 0.5 and 1 g/m³ liquid water content ($A_s = 2880$ and 5760).

B_1	R	Re	R_c	Re_c	$A_s = 2880$				$A_s = 5760$		
					(1,2)	(1,1)	(1,1)	NE	(1,2)	(1,1)	Ne
-.9	.0526	10.5	.0086	8.5		.0462		.1689	.0382		.0845
-.8	.1111	5.	.0184	3.88		.0219		.0800	.0181		.0400
-.7	.1765	3.14	.0293	2.38		.0138		.0504	.0114		.0252
-.6	.2500	2.21	.0418	1.68		.0097		.0356	.0080		.0178
-.5	.3333	1.65	.0563	1.2		.0073		.0267	.0060		.0133
-.4	.4286	1.29	.0746	.9		.0057		.0207	.0047		.0104

$1.3 \times 10^{-10} \text{ c/m}^3$. The continental case is similar showing the controlling nature of the sub-cloud section in determining whether the characteristic modes are attained within the cloud section. An increase in cloud height of at least a factor of two is necessary in these cases before the asymptotes of the previous section are attained for nominal values of space charge at the lower boundary.

Terminal relations were computed for the above case studies to be used with an assumed load to find their respective operating points. The terminal voltage normalized by the product $\left(\frac{1 - B_1}{B_1 + 1} \right) \frac{E_0}{UH}$ is shown against J' in Figures 5.10 - 5.13. It is of some interest to note that the generating region of all cases is found for J' less than about 0.1. Since the initial electric field is given by $R(J' - 1)$, the boundary field strength in all cases is less than about -473 V/m. This value compares to the observed fields below fair-weather cumuli of about -200 V/m. Increasing Re leads to larger range of J' for which generating action is possible, with the maximum $0 < J' < 1$ approached as $Re \rightarrow \infty$. The large initial field magnitudes are therefore attributed to either excessive initial space charge, or excessive generator height ($H_a + H_c$).

The actual operating point of the model is determined by the simultaneous solution of the generator coupled to a load. In the case studies, the initial field determined by the load is nearly independent of the load, with value ranging between $-R < E'/o < -0.9 R$. The generated potential however is sensitive

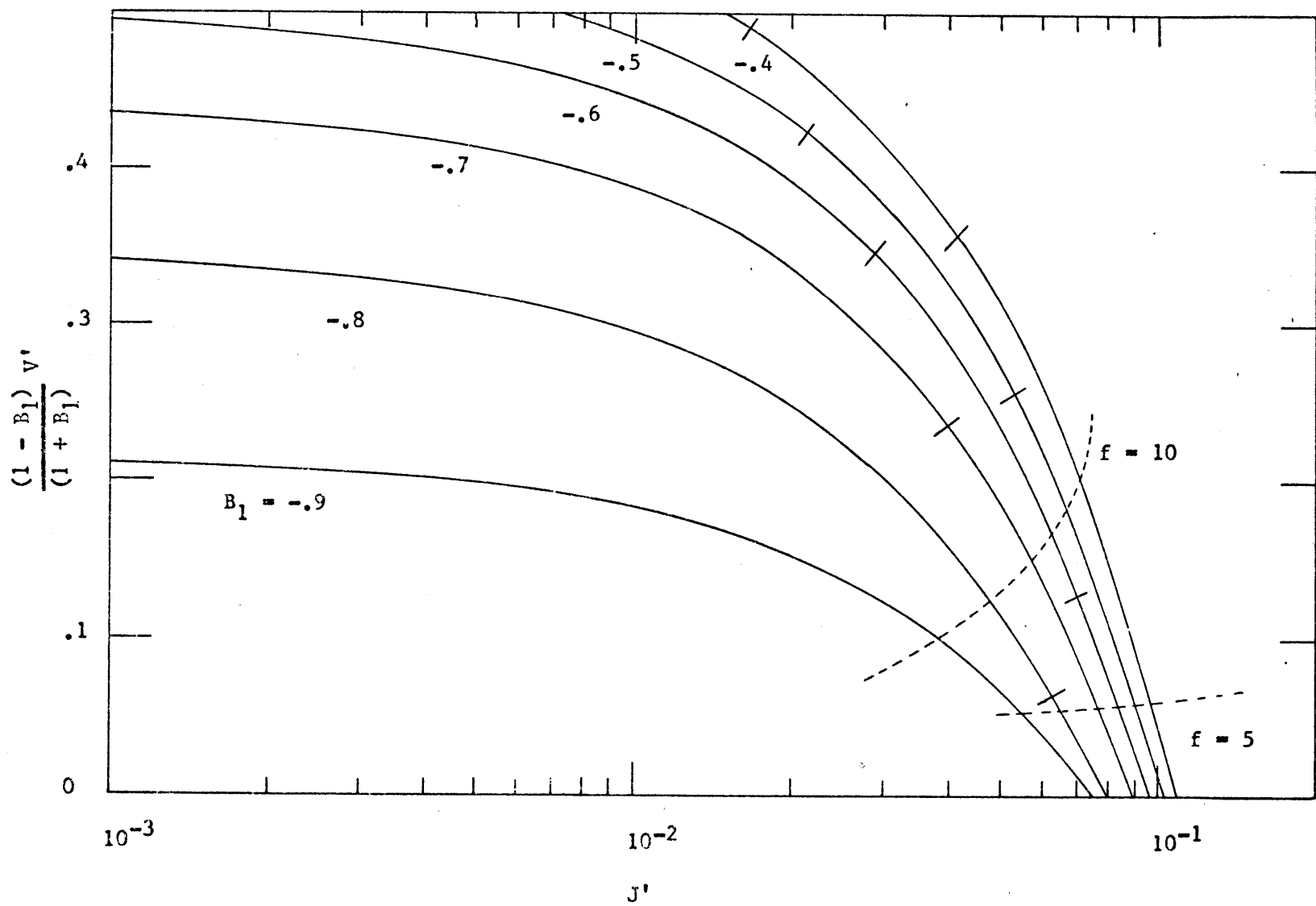


Fig. 5.10 Terminal plot for oceanic clouds of Table 5.5 , .5 g/m³ liquid water

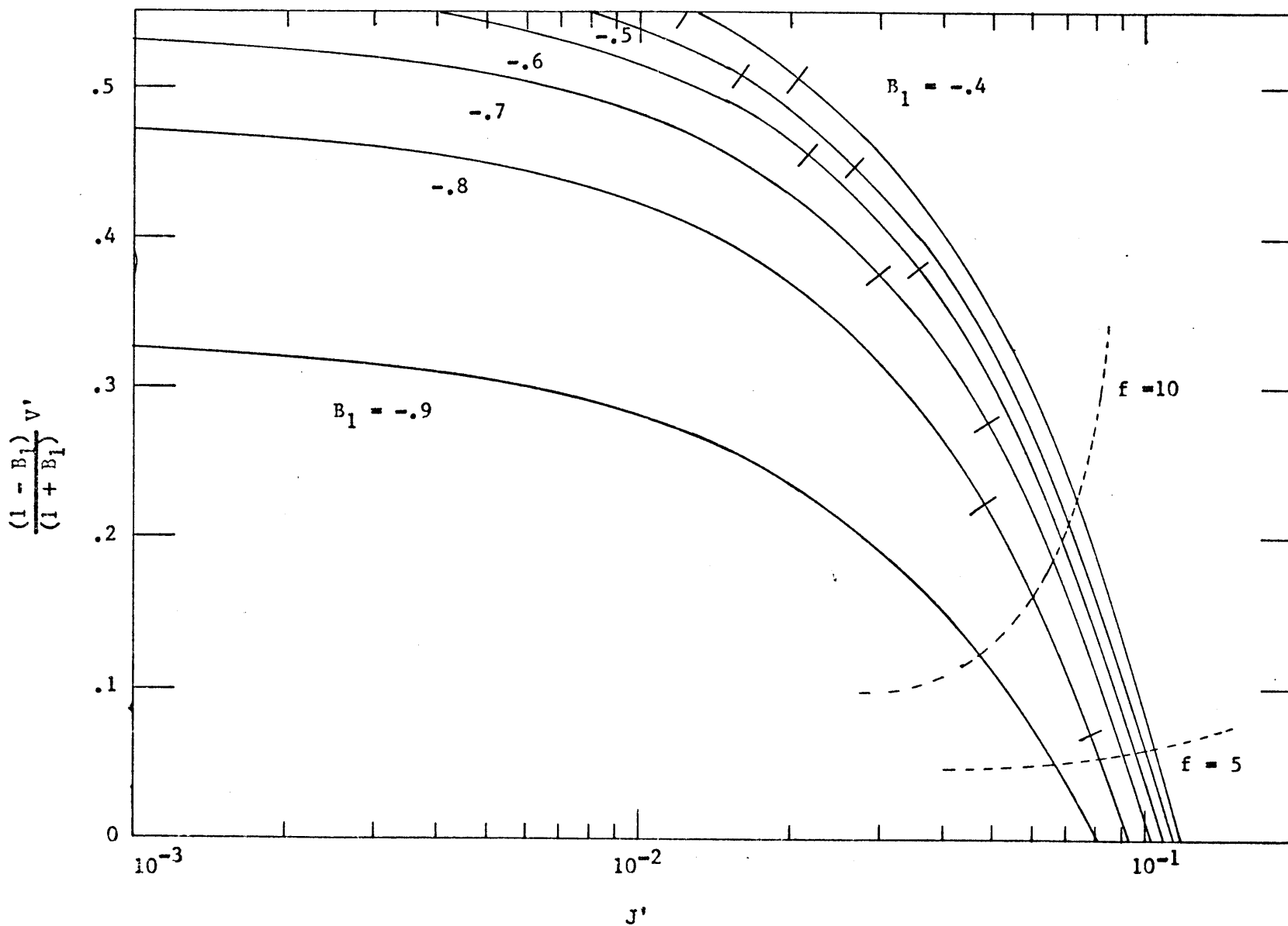


Fig. 5.11 Terminal plot for clouds of Table 5.5, 1g/m^3 liquid water

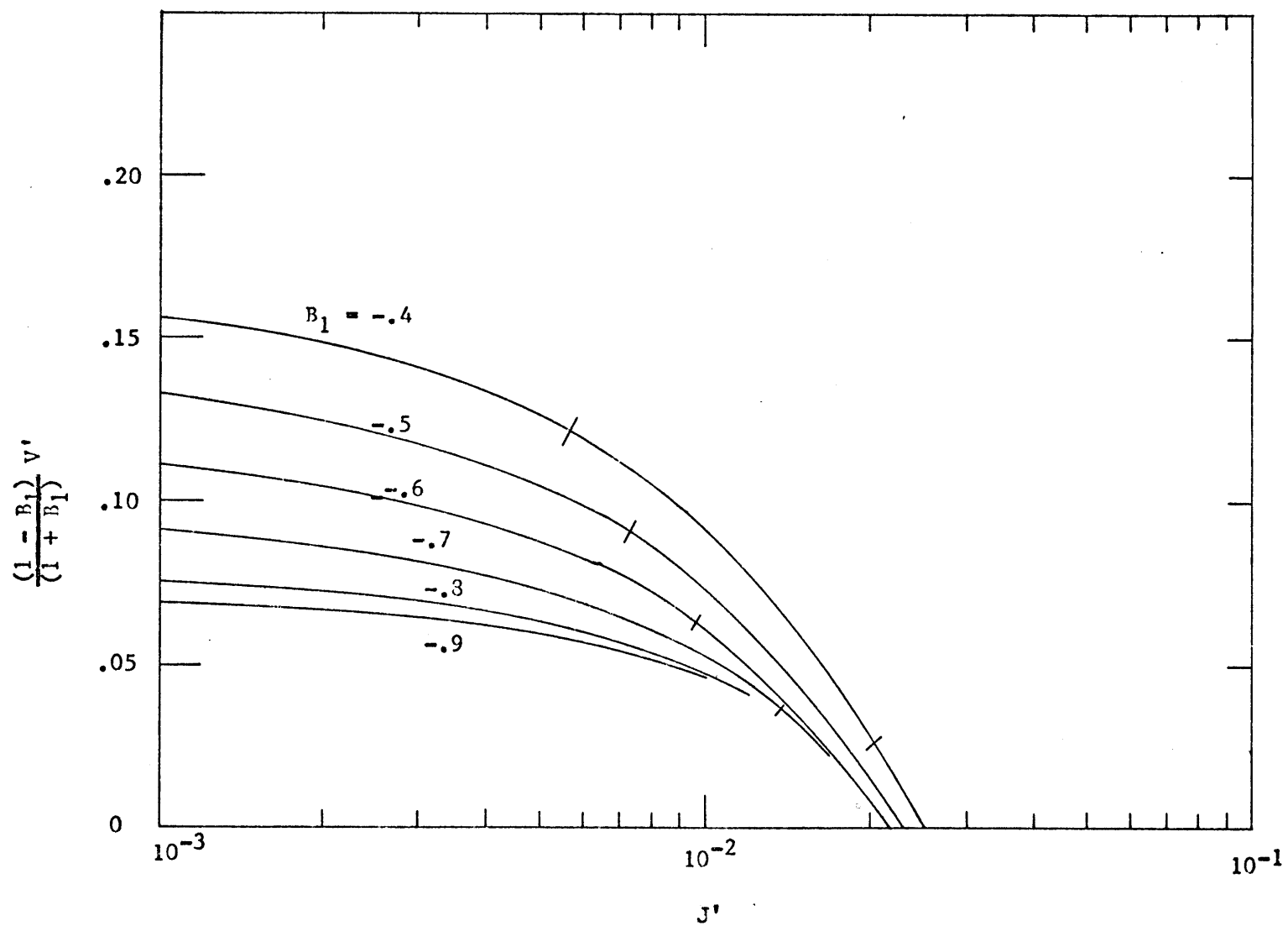


Fig. 5.12 Terminal plot for continental clouds of Table 5.7, $.5\text{g/m}^3$ liquid water

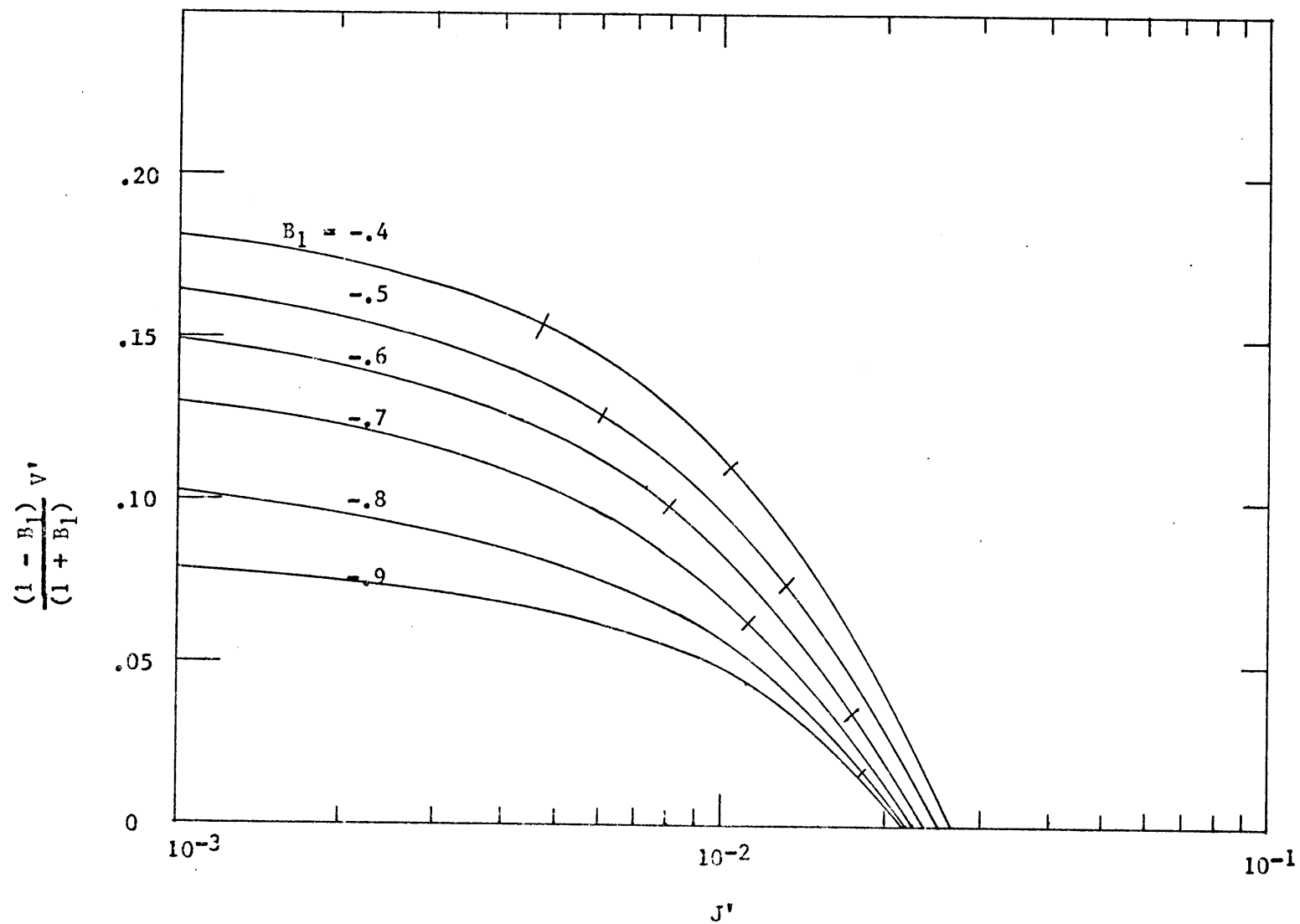


Fig. 5.13 Terminal plot for continental clouds of Table 5.7, 1 g/m^3 liquid water

to the load-line as is the characteristic cloud structure as indicated on the terminal plots.

The load is modeled by an external region of vanishing convection which is coupled to the generating region at equipotential levels, providing a closed circuit for the system. Assuming Gish's empirical conductivity expression is approximately correct in the load and that the load is essentially an undisturbed region of the cloud carrying the generator current in addition to the fair weather current, then the load line is given by

$$V_L' = \frac{b\rho_T|_0}{(H_a + H_c)} (RfJ' + \frac{2.78 \times 10^{-12}}{a/m^2}) R(h, H_L) \quad (5.5.2)$$

where h and H_L are the lower and upper equipotential levels, f is the ratio of generator to load areas and the fair weather current density is -2.78×10^{-12} a/m². For the conditions of the case studies,

$$V_L' = \frac{b\rho_T|_0}{(H_a + H_c)} (RfJ' + 0.00695) R(h, H_L) \quad (5.5.3)$$

A fundamental difficulty with the one-dimensional assumption is that there is no analytic basis for the choice of f , h and H_L , so that the load line is undetermined. However, reasonable assumptions at least give an upper bond on the potential. We couple load

and generator at the surface which is at least one equipotential, and assume that the mixed or homogeneous layer between the surface and boundary of the sub-cloud layer is one of uniform field which is therefore $-R(1-J')$. Since the lowest 2 km of the atmosphere contributes most to the columnar resistance, the height of the load is not a determining factor unless it becomes very large. Then the load potential for the case studies is

$$V'_L = \frac{b\rho_T|_0}{H_a + H_c} (RfJ' + .00695)R(0, H_a + H_c) - R(1-J')x'_s \quad (5.5.4)$$

where x'_s is the height of the mixed layer normalized by the generator height. x'_s varies between about 0.26 for oceanic clouds and 0.28 for continental clouds.

If $f = 1$, the contribution to the load potential by the generator current becomes significant for $J' \sim 0.00695/R$. For all cases, this occurs for $J' > 0.01$, and for nominal space charge, i.e., $B_1 = -.9$ to $-.8$, J' must be at least 0.04 to 0.07. For smaller J' , the load potential is determined principally by the fair-weather current unless $f > 1$. Fig. 5.14 shows the load potential given by Eq. (5.5.4) for H_L given by a generator height of 2.1 km and $R = 0.0526$ ($B_1 = -.9$). This is representative of the load for the oceanic cloud case with $H_a = 100m$. The surprising result is that the generator is in the generating mode only if

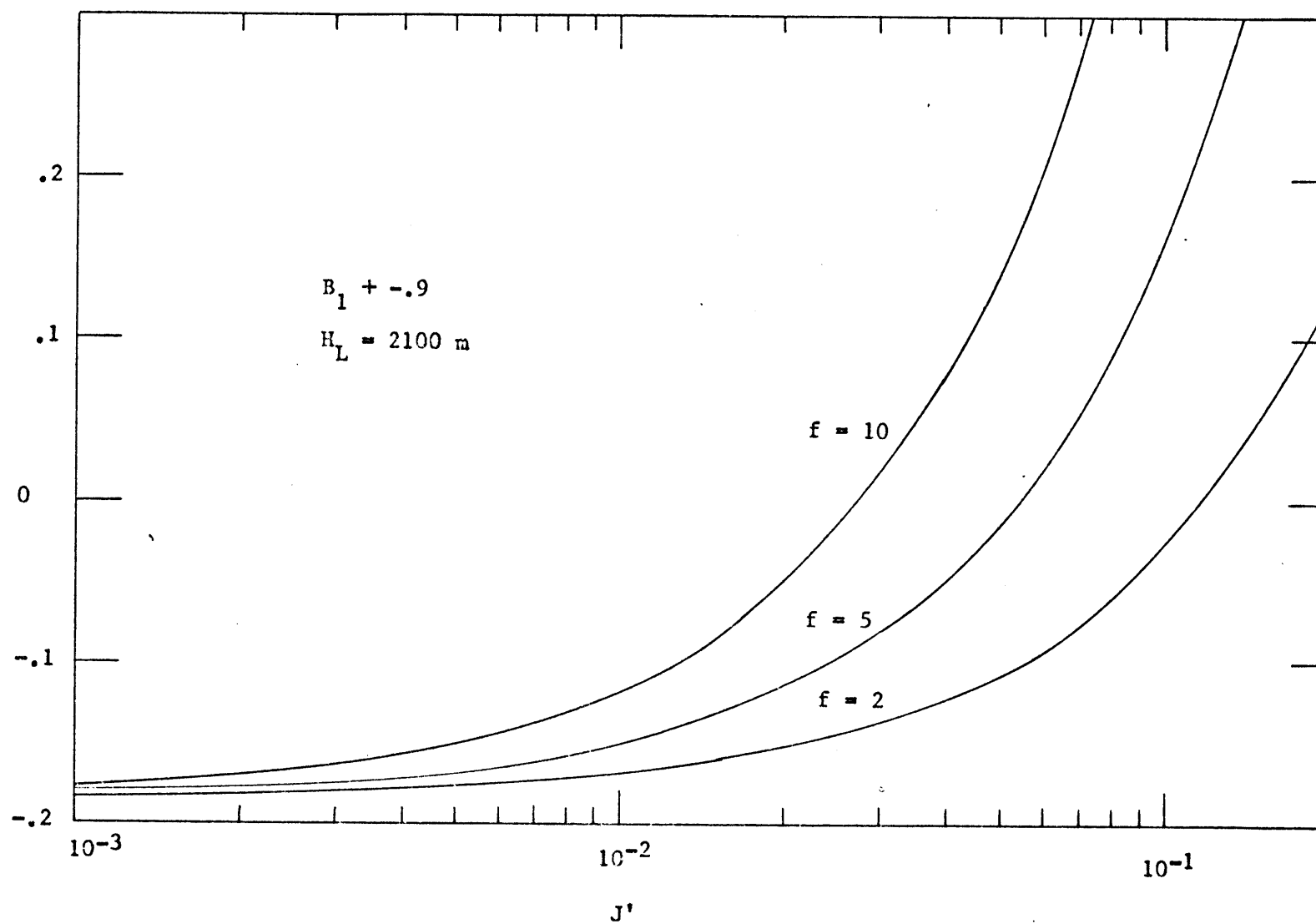


Fig. 5.14 Load potential example for maritime clouds of Table 5.5

$f > 1$ because of the large potential across the mixed layer. The same general result holds for the other case studies. Israel (1970, p. 128) noted that around growing cumulus clouds, the fair-weather structure seems unperturbed by the convective flux of charge into the cloud. The model results seem to confirm this view in one of two ways. Either the cloud is in the generating mode, whereupon the load must be a very localized region of return current, or the system is in the brake or pump mode, and the environment is unperturbed simply because the generator current times f is small compared to the fair-weather current. The load-lines in the terminal plots of Figs. 5.10-5.11 were computed by Eq. (5.5.4) with the load height given by the generator height in each case. It was assumed that the system would be in the generating mode so that values of $f > 1$ were used. Since terminal voltage becomes negative for J' about a factor of four less in the continental cloud cases of Figs 5.12 and 5.13, the area ratio for these cases had to be nearly a factor of four larger than for the maritime clouds with shallow sub-cloud layers. Similar behavior occurs for the maritime clouds with deeper sub-cloud layers.

5.6 Summary

Convective electrification of a region of uniform cloud particles was analyzed and coupled to the sub-cloud generator through an empirical load. The main features of electrification in the cloud for small cloud electric Reynolds number are a non-equilibrium regime and two equilibrium regimes. The equilibrium regimes are characterized by relatively small electric fields and a balance between ionic generation and mutual recombination of ions on one hand and ion-scavenging by droplets on the other.

The first equilibrium mode occurs for electric fields in the cloud less than the droplet terminal speed divided by ionic mobility, or about $+100 \text{ v/m}$ in a cloud of $10 \mu\text{m}$ droplets and about $+30 \text{ v/m}$ in a cloud of $5 \mu\text{m}$ droplets. This regime shows typical values of ionic conductivity in the cloud to be nearly the same as that at cloud base, and therefore does not transform sub-cloud space charge into cloud charge efficiently. In this mode, the equilibrium droplet charge is positive, but is masked by an opposite polarity ionic space charge. Since the droplets are small in the case studies, their velocity does not differ significantly from the air speed so that the primary current in the equilibrium is conduction current. Significantly, the case studies show that small net space charge at the sub-cloud boundary is most likely to lead to this kind of equilibrium in clouds whose heights are sufficiently large. In addition,

the greater the load resistance, the more likely the (1,2) equilibrium mode will be established for all values of initial space charge.

For smaller load resistance, or for larger sub-cloud space charge, the (1,1) equilibrium is established in the cloud. In this case, both ionic space charge and droplet charge vanish higher in the cloud. The conductivity in the cloud is reduced considerably, but the current is still carried by conduction current in the equilibrium. The electric field is larger than for the (1,2) mode, but still bounded by the constraint on non-zero conductivity to account for the conduction current and the steady-state requirement that $E' < 1$ in the model.

If the load resistance is decreased further, or the space charge initial value becomes larger, the cloud switches to the non-equilibrium mode in which a net positive drop charge persists throughout the section and the conductivity is reduced to small values. In this mode the primary current is convection current due to droplet charge so that it is this mode which most efficiently transforms the sub-cloud space charge into cloud charge.

The transfer characteristics of the sub-cloud layer were calculated to allow determination of boundary conditions at cloud base. These results show that for all cases, the cloud height must be much greater than the depth of the sub-cloud layer in order for the equilibrium states to be established in the cloud. Data

for oceanic cumuli show the greatest promise in satisfying this restriction, although a strong case for continental clouds has not been made.

CHAPTER 6

Conclusions and Recommendations for Further Work6.1 Recapitulation

The motivation for this study was basically a desire to quantify the mechanism of convective electrification of warm clouds in terms of a simple model allowing identification of important parameters and regions of the natural process. It is felt that this approach, which hopefully restricts the variety of possible electrification mechanisms, can lead to the development of a more precise experimental program to evaluate this particular process of cloud electrification. Because of the great simplification, the model should not be viewed as a simulation of convective electrification, but rather as a guide to the pertinent measurements, regions of observation and the kinds of situations which are most likely to lead to a meaningful or definitive experiment. In this sense, the results of the study are gratifying.

Three main regions of a convective system were studied, the sub-cloud region of organized convection, a cloudy region of vertical motion and an external non-convecting region serving as a load or current return path. The model was restricted from the outset to the initial stages of electrification when external electric fields are small (less than U/b , or about 100 V/cm) in order to determine the effectiveness of the convective generator as a "boot-strap" process in developing fields in the cloud sufficient to initiate field-dependent mechanisms of charge separation in the cloud. This

restriction also allows the model to be treated as an initial value problem instead of a boundary value problem since initial conditions can be fixed at a common boundary, the lower level of the sub-cloud layer.

The sub-cloud region was studied as both a unipolar and bipolar generator with the expected result that, at least for small Re , the unipolar model leads to serious error. The masking effects on net space charge by the presence of opposite polarity charge carriers cause the electric field to reach its equilibrium value in a bipolar atmosphere much faster than is predicted by the unipolar model. Since positive and negative ions in the atmosphere are found in comparable quantities, the unipolar model predicts terminal fields and voltages up to two orders of magnitude greater than should be expected. Order of magnitude estimates of convective generation based on a unipolar concept are therefore highly suspect. In terms of experimental design, it must be concluded that simple observation of net space charge (or equivalently, vertical electric field) is insufficient. Either net space charge and conductivity or densities of both positive and negative ions must be observed simultaneously. Since previous measurements of electric field and conductivity have shown that these variables can deviate significantly from their average values, the assumption of a mean conductivity, or a fair-weather field can lead to serious error in convective situations.

The cloud model results are very significant to previous theories of convective electrification. First, the notion that ions are scavenged very near cloud base, as is assumed in both the models of Vonnegut and Phillips, is shown to be true only in exceptional circumstances, at least when diffusion of ions to droplets is neglected; the net space charges at cloud base must be large, and the load resistance small for the non-equilibrium mode to obtain in the cloud. Since cloud droplet scavenging is sensitive to the electric field, a cloud in which other generating currents serve to enhance the cloud base field will be efficient in reducing the conductivity, even when neutral air penetrates cloud base. This condition is not met, however, in the electrification schemes as proposed by Vonnegut and Phillips. Their models require an accumulation of positive charge aloft in the cloud; this can happen only in the non-equilibrium mode. Similar remarks apply to Wilson's model. However, in his case, he assumed, and apparently correctly, that ion densities and cloud electric fields remain near their values below the cloud until precipitation develops, where upon charge accumulation above and below ensues, leading to the transformation of sub-cloud space charge into cloud charge. On the other hand, it is possible for this condition to obtain before precipitation develops, if space charge initial values are high enough and the load area is smaller than the generator area.

Sartor's (1967) notion that induction charging can proceed in clouds whose electric fields are enhanced by a drop in cloud conductivity do not seem correct in view of the convection in the cloud. The induction charging process tends to enhance a pre-existing electric field. A steady field established in a layer of decreased conductivity will be larger than, but in the same direction as the external field. Since Sartor assumes that the field in the cloud is developed by the fair-weather field, it will be of the wrong sign to generate the observed thunderstorm field. If, on the other hand, the field is presumed to be developed by convective charge transport, the equilibrium cloud field will be of the correct polarity, and of proper magnitude to allow induction charging to proceed. For example, the equilibrium field in a cloud of 10 micron droplets in the (1,2) equilibrium will be about +1 V/cm, which is the initial field assumed by Sartor in his calculations. The same cloud in the (1,1) equilibrium or in the non-equilibrium mode will have larger internal fields.

The dropsonde observations of cloud conductivity made by Evans showed variable conductivity above cloud base, and sometimes enhanced conductivity over the clear air values at the same level. The simplified model predicts at least some variability; whenever droplets are in the (1,1) mode for $E \lesssim 0$, ion depletion by droplets exceeds the generation rate assumed, whereas the other modes of ion scavenging result in the balance reverting to generation and

mutual recombination. Thus as drop charging modes switch, the total ion density can increase or decrease in the cloud proper. This does not explain enhancement of cloudy conductivity above clear-air values however.

There is a distinct tendency for droplet charge density within the cloud to be masked by opposite ionic charge density. This behavior is much like that noted by Gunn (1957), in which regions of net drop charge were at least partially masked by opposite charge of droplets and ions. Observations in clouds should be so designed that all charged species can be measured to determine which species contribute to charging currents, and which to dissipating currents.

The transfer characteristics of the sub-cloud layer showed the controlling influence of the cloud environment on the character of charge distribution within the cloud. Unless cloud heights are very much greater than the sub-cloud layer between the mixed layer and cloud base, there is only a small band of initial conditions which can lead to the establishment of one of the characteristic modes of cloud electric structure. For the expected range of local net space charge densities, this band is characterized by sub-cloud electric Reynolds numbers in the range from about 1 to 20. Even in this range, cloud heights must be between 5 and 25 times the height of the sub-cloud layer. This result, which perhaps could have been anticipated, was quite surprising and makes clear the

importance of either making observations at cloud base to establish initial conditions, or else determining for each individual case, initial conditions within the region between the upper level of the mixed layer and cloud base. In this second case, the distance between cloud base and the observations must be determined in order to calculate transfer relations for the cloud initial conditions. As a rule of thumb, a cloud of given height will be more likely to exhibit an equilibrium state in its interior for the same mixed layer condition, the nearer the cloud base is to the homogeneous layer.

The data which have been examined seem to indicate that cumulus clouds over the Caribbean are most likely to develop under conditions conducive to the establishment of electrical equilibrium within the cloud. In these regions, relatively uniform cloud base height is observed, and the sub-cloud layer depth is not as variable as is apparently the case in continental clouds. It must be admitted however, that the continental cloud case has not been well documented, and a much more extensive search must be made before any firm conclusions about these clouds can be formed.

The final region of the model which deserves comment is the load whose operating characteristics serve to fix the operating point of the generator system. The load is perhaps the most crucial part of the system, and, in a one-dimensional model, least amenable to analysis.

The one-dimensional model simply assumes that the load and generator are coupled at common equipotential levels. Without at least a two-dimensional model which determines both the effective coupling level and the region of primary current return in terms of the properties of the generator and load regions, these two parameters are indeterminate. Initially, it was not thought that this would be a problem, the notion being that the load could be approximated by a large area compared to the generator area, and that estimates of the load-generator area ratio could be obtained from estimates of the area of clear regions between clouds. This would leave only one parameter, the coupling level, as a free parameter. The very surprising result of the case studies that the clouds could be in the generating mode only if the area ratio, f , was large compared to one, made the planned procedure untenable. This result is thought to be primarily due to the one-dimensional assumption of a uniform electric field between the surface and the boundary of the sub-cloud layer. Since the case studies indicated that the generating mode was restricted to small J' , for those situations the sub-cloud electric field is a large negative value, determined basically by the positive charge aloft in the generator system. This large negative field leads to a potential difference between the surface and the lower boundary of the generator which is greater than the fair-weather potential drop between the surface and the ionosphere. If the system is actually in the generating mode, therefore, the load must

be a very localized region of the cloud environment. The alternative is that the system is not in the generating mode after all, and that, in effect, the fair weather potential drop between the surface and the ionosphere is driving the cloud system as an electrohydrodynamic pump or brake. Until a two- or three-dimensional model is analyzed, this question cannot be resolved analytically. However if the system indeed is steady, then small f and small H_L lead to the brake or pump mode, while large f and large H_L lead to the generator mode.

6.2 Critique of the Model

The model results are restricted to one-dimensional steady state regions of vertical convection and to clouds of uniform droplets. None of these conditions are likely to be met in nature. The updraft speed in convective clouds is unsteady and non-uniform, with some regions of cloud base exhibiting positive and negative velocities. Moreover, the mean vertical velocity in clouds is non-uniform, increasing in the lower levels to a maximum near cloud top. So long as the model is restricted to regions below the peak vertical velocity, it is not felt that the slowly varying updraft is critical to the results. However, the unsteady part of the velocity field can cause significant fluctuations in the local ion density and space charge, so that actual droplet charge can be expected to be variable in response.

The conductivity predictions neglect mixing or entrainment of environmental air with the cloudy air. It is suspected that the

highly variable and enhanced conductivity measurements made by Evans in thunderstorms can be explained partially on this basis. Although droplet charges might be slightly smaller because of this effect, the selective charging process probably leads to essentially the same characteristic structure if the mixing is homogeneous. However, the local structure is probably quite variable, partly as a consequence of mixing and partly as a consequence of velocity fluctuations, so that entrainment also leads to a more variable droplet charge within the cloud.

The uniform droplet assumption is particularly poor near cloud base. It is in this region that droplets grow most rapidly, partly because the supersaturation tends to be higher near cloud base and partly because the small droplets in this region grow faster than larger ones aloft. Calculations of droplet growth in rising moist air show the period of fastest growth to be found within the first 100 - 200 meters of the cloud. Above this level, droplets grow only slowly by condensation, so that constant droplet radius in a steady updraft may be reasonable provided the cloud section height is large compared to this shallow layer of rapid growth. Of course, entrainment and fluctuations in vertical velocity as well as a broad condensation nucleus spectrum will lead to a droplet spectrum within the cloud, even well away from cloud base. Since the parameter A_s is proportional to $N a^2$, the effect of a droplet distribution will be small so long as the number density of large droplets falls off

rapidly. However, droplet spectra should be observed in clouds which are part of an experiment to investigate the convective electrification process, if simply to establish a mean droplet size.

The steady state assumption requires a current return path so that charge accumulation in the model does not occur. The steady state is perhaps not unreasonable provided the duration of the updraft is long relative to a typical relaxation time for the cloud environment. Unfortunately, the typical fair-weather cumulus lifetime is of the same order as the relaxation time, around 10 minutes. Whether the results are significantly altered awaits a more sophisticated model. However, the steady one-dimensional character of the load, or current return path, disallows determination of the generator operating point except in terms of the undetermined parameters, f and H_L . The assumption that the load is basically a conducting region of no convection causes the generator mode to be very sensitive to the choice of f and H_L . In fact, to be in the generating mode, f must be large; that is the load must be a very localized region of the cloud environment suggesting that a one-dimensional assumption is unwarranted.

Finally, the charge exchange equations between droplets and ions neglected ionic diffusion to the droplets. For small fields and small droplets, this is not justified. Indeed, diffusion of ions to droplets is likely to be as important as impact-charging for the conditions

which obtain in the case studies. It is a simple matter to include diffusion in the droplet charging equations so long as the external field is neglected. However, diffusion to a droplet in an asymmetric field is not simple; moreover, it is not clear how one justifies the use of the impact-charging theory of Wilson when the thermal or random motion of ions is as important as the field-derived motion. If positive and negative ion mobilities are equal, the most important effect of diffusion is to lower the ion density or conductivity below that predicted by Wilson charging alone. Since entrainment is likely to enhance the conductivity, it is at least possible that neglect of these two effects is somewhat offsetting.

6.3 Suggested Further Work

The possible avenues of research suggested by these results can be broken down into basically three areas; field observations of convective clouds, the development of more sophisticated mathematical models and laboratory experiments.

The question of the effect of diffusion of ions to droplets in an external electric field needs to be examined both on an experimental and a theoretical basis to better establish realistic exchange relations between drops and ions in the air. One major obstacle in the theory must be overcome to compare diffusion currents to conduction currents. The usual boundary condition in diffusion is that the ion density vanish at the droplet surface. On the contrary, the conduction current to the droplet surface is given by $(\rho_1 + \rho_2)E$

where the space charge densities and electric field are evaluated at the droplet surface. It is not clear yet how these two boundary conditions can be made consistent. The last reported measurements of Wilson charging were made by Gott (1933); his results were for larger drops and diffusion was not important. A new set of observations with smaller drops would help delineate the regimes of impact-charging and diffusion of ions to droplets.

The unfortunate indeterminacy in the model caused by the one-dimensional assumption cannot be allowed to stand. It seems that the crucial question of generator versus brake or pump behavior of the convective system cannot be answered until at least a two-dimensional model is developed. The framework of such a model could be an axisymmetric kinematic circulation with a superposed cloud. Boundary values of potential could be fixed by assuming a constant potential between equipotentials defined by the surface and the high equalizing layer. Far from the cloud an assumed vertical fair-weather field could be imposed. Initial conditions of space charge, electric field and conductivity could be determined in terms of the fair weather structure. Vanishing net space charge at the equalizing layer, and vanishing positive or negative space charge at the boundary for a positive or negative surface electric field complete the boundary conditions. The evolution in time of the two-dimensional model to a steady state would allow contours of

potential, space charge and conductivity to be presented as well as the region of primary return current in the cloud environment.

One promising area of research is the development of a field program whose objectives are to evaluate the model predictions and accumulate a set of characteristics describing the nature of the main regions surrounding small cumulus clouds. In particular, vertical soundings of electric field, conductivity and vertical wind speed would provide sufficient data to test the model predictions of the sub-cloud transfer characteristics. Statistics should be generated giving the relation of the depth of the homogeneous layer to the height of cloud base, particularly for high continental clouds. If the observed transfer characteristics compare favorably with the model results, then the sub-cloud layer depth statistics will permit determination of conditions which are favorable to the establishment of equilibrium states within clouds as a function of cloud height. A more ambitious program of in-cloud observations might then be indicated.

The electrical environment of small clouds needs to be examined much more closely than has been attempted in the past. The possibility that small clouds develop their electrical character in essentially a self-contained fashion as suggested by Israel (1970, p. 128) seems indicated by the requirement $f > 1$. Since the convection current is usually small compared to the fair-weather current, however, it may be difficult to distinguish the transition from the cloud environment

to the fair-weather structure.

Recently reported measurements of raindrop size and charge from maritime clouds over Hawaii (Takahashi and Fullerton, 1972) showed that drops larger than about 700 - 800 microns radius typically were positively charged on the average, while smaller drops were negative on the average. Assuming the raindrops in the cloud do not seriously disturb the equilibrium established by the droplets, it is possible to calculate the charge of a drop at cloud base in terms of the depth of cloud through which it falls as a function of initial drop charge and size. Approximate calculations were carried out assuming a cloud of 10 micron droplets of liquid water content 0.1 g/m^3 was in the (1,2) mode equilibrium. Drops of various size and initial charge were allowed to fall through the cloud and charge by collection of charged droplets and ionic conduction currents. The results of the calculations showed the polarity of drop charge at cloud base to be insensitive to initial charge, but sensitive to terminal speed of the drop. In the upper regions of the cloud, the field is positive, droplets are positive and the ionic space charge is negative. In this region, a drop will charge positively by collection and will discharge by conduction in the (1,2) or (0,2) modes depending on the drop charge. Because of the inefficient charging by negative ions in these regimes, drop charge through the (1,2) droplet regime will tend to be positive regardless of their initial charge. When the drop passes the zero field

level and encounters negative fields, the droplet regime enters the (2,1) mode, droplet charges become negative and ionic space charge becomes positive. However, here the drop charges negatively by collection and by conduction in the (2,1) mode. There is therefore efficient collection of negative ions in this region. As the droplet regime switches to the (1,1) regime near cloud base, droplet charges become positive and ionic space charge becomes negative. Here, the drop is charged positively by collection, but still scavenges negative ions efficiently in the (2,1) mode.

The apparent tendency in general is for drop charges to be positive in the droplet (1,2) regime because of collection of positive droplets and inefficient scavenging of negative ions by drops in the (1,2) or (0,2) mode. In the layer near cloud base of droplets in the (1,1) mode, there is a tendency for drop charge to remain about the same by a balance between positive droplet collection and efficient scavenging of negative space charge. In the droplet (2,1) regime, however, the drop accumulates negative charge both by collection of negative droplets and by conduction currents in the (2,1) or (0,1) mode. The drop polarity at cloud base is apparently determined by how fast the drop penetrates the (2,1) droplet regime; larger drops with large terminal speeds tend to retain their positive charge through this regime, whereas smaller drops become negatively charged here and exit cloud base with negative charge.

The transition at cloud base from negative to positive drops occurred for drop radii between 400 and 600 microns when the initial drop charge was varied from large positive values to zero. Since below cloud base, the drop will continue to collect negative ions efficiently in the (2,1) mode, the transition radius at the ground is expected to be larger than the model predicts at cloud base. Although the approximations in the calculations are rather gross, the encouraging qualitative agreement between the model results and the observations suggests that further efforts in this direction be undertaken.

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BIOGRAPHICAL SKETCH

Robert F.D. Perret was born on May 24, 1940 in Vicksburg, Mississippi. He graduated from Highland High School, Albuquerque, New Mexico in 1958. He received the S.B. in mathematics from the University of New Mexico in 1964.

He entered the Massachusetts Institute of Technology in 1964. During 1964-1966 and 1969-1970 he was supported by research assistantships in the Department of Geology and Geophysics and the Department of Meteorology. From 1966 to 1969, he was a National Science Foundation Fellow.

He was married in 1961 to Virginia Branch of Houston, Texas. They have two sons and a daughter, Christopher E., William R., and Jennifer L.

APPENDIX A: The Field-dependent Charge Exchange Equations

Ion-drop impact charging is the term we use to designate the charge exchange which occurs between a family of positive and negative ions and a distribution of charged or neutral drops in an electric field. The term "exchange", rather than "generation" or "separation", is chosen because the charge generation occurs by ionization within the neutral fluid. Subsequent separation comes about by virtue of the differential velocity of charged drops and droplets influenced by gravity and the neutral flow and that of the uncaptured ions driven by the electric field and the neutral flow.

There are two fundamentally distinct types of impact charging. The first is statistical in nature, and comes about because of thermal agitation of molecular ions surrounding the drop. This results in occasional ion-drop collisions which, because of the strong image forces between a conducting surface and a point charge, always result in capture of the ion. It is important to note that these same image forces are sufficiently strong that no charge can escape the surface, so that the drops are diodes in the sense that they can capture, but not release, charge. If the respective densities, thermal energies and diffusivities of the polar ion species present do not differ appreciably, then no significant net average charge will accumulate on the drop by this mechanism.

On the other hand, significant difference in these properties gives rise initially to more frequent impacts of the favored ion species. Under these circumstances, an equilibrium is established in which the drop accrues net charge whose sign causes more frequent impacts by the less

favorable ion species. The magnitude of the equilibrium charge is such that the two polar species impact the drop with equal frequency. An uncharged drop in a field-free environment is subject to the diffusional charging above, usually attributed to Gunn [1954].

The second basic kind of impact charging is an ordered process which exists because of the tendency of ions to follow lines of force of an electric field. Thus, a charged or neutral drop immersed in an electric field gives rise to field lines, some of which may terminate and some originate on the drop surface. Ions which follow intercepted field lines may thus impact the drop surface and be captured. In the case for which the flow of neutral gas around the drop is absent or negligible, the ions follow the field lines exactly, and the drop charge accumulates as described by Moreau-Hanot and Pauthenier [1932], and later by Gunn [1956]. In the case of significant flow, the ions follow trajectories described by the vector sum of the neutral flow and the velocity imparted to the ion by the local electric field. This case was first suggested qualitatively by C. T. R. Wilson [1929], and later considered in more quantitative detail by Whipple and Chalmers [1944]. Melcher [1971] considered Wilson's impact charging mechanism in a more general fashion, allowing a more organized presentation of the charging regimes. Thus, the elaboration of the impact-charging mechanism presented here is due wholly to his insight.

The current, I , to a drop surrounded by ions can be expressed by

$$\frac{dq}{dt} = I = - \iint_S (\vec{J}_1 + \vec{J}_2) \cdot \hat{n} dS \quad (A.1)$$

for J_2 , the current densities of the positive and negative ion species evaluated at the drop surface, S . The current density in a mobility model is given by

$$J_2 = \rho_2(\vec{U}_d \pm b_2\vec{E}) \quad (\text{A.2})$$

for ρ_2 the charge density of the respective ion species, E the local field strength, U_d the local velocity of the streaming neutral gas relative to the drop, and b_2 the mobility of the ion species (the upper sign is associated with the positive ion species). Substituting (A.2) into (A.1) gives

$$I = -\iint \Sigma \rho_i(\vec{U}_d \pm b_i\vec{E}) \cdot \hat{n} dS \quad (\text{A.3})$$

Since we know the electric field and neutral flow at the drop surface, the only obstacle to immediately writing down the solution is knowledge of the polar ion densities immediately outside the drop surface. Since we have no a priori knowledge of this parameter, we must look for another method.

The solution is found by recognizing the significance of neglecting space charge in calculating the field strength and assuming the neutral flow to be two-dimensional and incompressible. Under these circumstances, the flux of ions to the drop surface can be calculated using the ion densities far from the perturbing influence of the drop, i.e., several drop diameters removed. To see this, we write down the expression for conservation of ions; using Eq. (A.2) we obtain

$$\frac{\partial \rho_2}{\partial t} + \nabla \cdot \rho_2(\vec{U}_d \pm b_2\vec{E}) = 0 \quad (\text{A.4})$$

which reduces, by virtue of the above assumptions, to

$$\frac{\partial \rho_1}{\partial t} + (\vec{U}_d \pm b_1 \vec{E}) \cdot \nabla \rho_1 = 0 \quad . \quad (A.5)$$

Thus, along characteristics given by

$$\frac{d\vec{r}}{dt} = \vec{U}_d \pm b_1 \vec{E} \quad (A.6)$$

the ion density is constant. Since we can calculate $\vec{U}_d$ and $\vec{E}$ from the above assumptions, we can find the locus of characteristics which intercept the drop surface. Far from the drop, where $\vec{U}_d$ and $\vec{E}$ assume their uniform values, the ion density is known, so that it remains only to determine the density of intercepting characteristics in the uniform region, relative to that at the drop surface.

This calculation is facilitated by the existence of stream functions implied by the assumptions about the neutral flow and the solenoidal character of $\vec{E}$.

The significance of flow which can be described by a stream function is that the flux through a curve joining two points is a function only of the position of the points, so that the shape of the curve is irrelevant. Then, since there is no flux across a streamline, it is necessary merely to construct a surface which is determined by the two limiting streamlines which just intercept the drop surface, and the intervening portion of the drop. If the two limiting trajectories are extended far from the drop, where the ion density is known, then the flux to the drop surface can be calculated as the flux through the area generated by rotating the line joining the limiting trajectories about the ion flow axis.

According to the coordinate frame shown in Fig. A.1, the stream

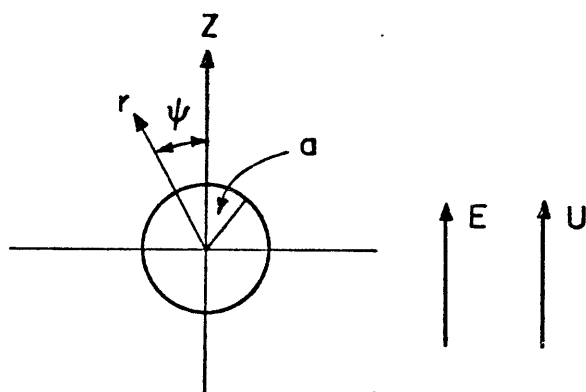


Fig. A.1 Coordinate system for impact-charging analysis

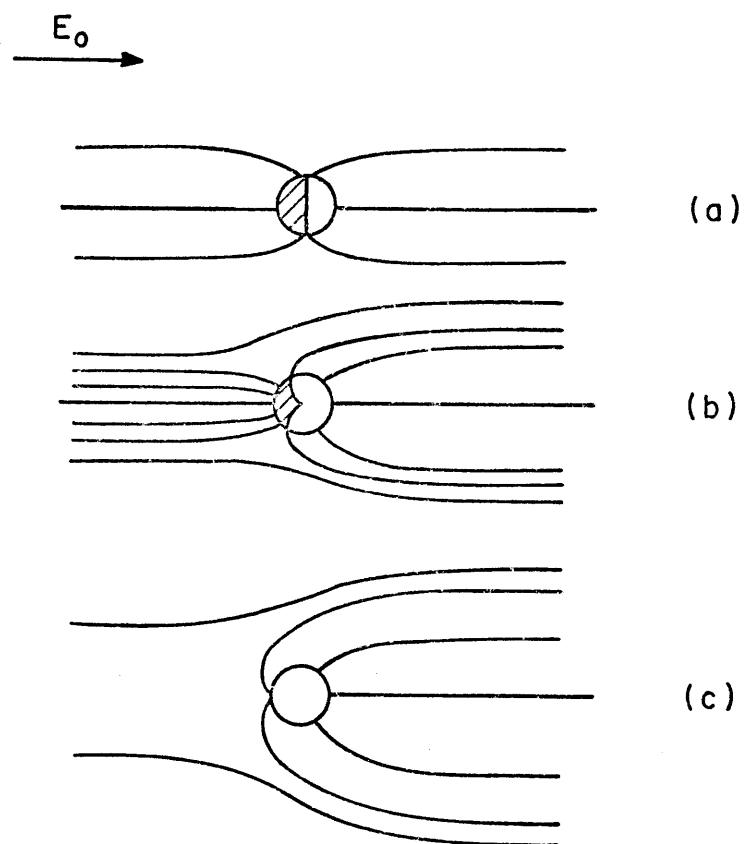


Fig. A.2 Impact-charging of a drop in a positive ion atmosphere when $U_d = 0$. Shaded regions denote regions of negative surface charge.

functions Σ and ϕ define $\vec{E}$ and $\vec{U}_d$ according to

$$\begin{Bmatrix} b_1 \vec{E} \\ b_2 \vec{U}_d \end{Bmatrix} = -\nabla \times \frac{\hat{i}_\theta}{r \sin \psi} \begin{Bmatrix} \pm b_1 \Sigma \\ \phi \end{Bmatrix} \quad (\text{A.7})$$

and were given by Melcher [1971] for a charge conducting sphere in a uniform electric field, under the conditions of Stokes flow, as

$$\Sigma = -E_0 a^2 \sin^2 \psi \left[\frac{a}{r} + \frac{1}{2} \left(\frac{r}{a} \right)^2 \right] + q \frac{\cos \psi}{4\pi\epsilon_0} \quad (\text{A.8})$$

and

$$\phi = -\frac{U_d a^2}{2} \left[\left(\frac{r}{a} \right)^2 + \frac{1}{2} \frac{a}{r} - \frac{3}{2} \frac{r}{a} \right] \sin^2 \psi \quad (\text{A.9})$$

where a is the sphere radius, E_0 the uniform electric field, q the drop charge, and U_d the uniform neutral flow relative to the drop charge.

Streamlines of the ion flow are given by lines of constant stream function,

$$\phi \pm b_1 \Sigma = C \quad (\text{A.10})$$

or

$$\begin{aligned} & -\frac{U_d a^2}{2} \sin^2 \psi \left[\left(\frac{r}{a} \right)^2 + \frac{1}{2} \frac{a}{r} - \frac{3}{2} \frac{r}{a} \right] \pm b_1 \frac{E_0 a^2}{2} \left[\frac{2a}{r} + \left(\frac{r}{a} \right)^2 \right] \sin^2 \psi \\ & \pm \frac{b_1 q}{4\pi\epsilon_0} \cos \psi = C. \end{aligned} \quad (\text{A.11})$$

Just what constant charge should be associated with the characteristic lines is determined by a single boundary condition imposed at the drop surface, or far from the drop (nominally $\pm \infty$). Boundary conditions on ion density can be fixed only when the flux of ions is into the volume

of interest [Melcher, 1969]; that is, when

$$\hat{n} \cdot (\vec{U}_d \pm b_1 \vec{E}) < 0 \quad (\text{A.12})$$

for $\hat{n}$, the unit normal vector to a surface, reckoned positive when directed out of the volume enclosed by the surface. Recall that the volume which interests us is the intervening space between the drop surface and an imaginary surface at $\pm \infty$. Then, suppose the field and neutral flow are such that at $+\infty$,

$$\vec{U}_d + b_1 \vec{E} < 0 .$$

If the ion density at $+\infty$ is ρ_0 , this is the appropriate density to impose on the characteristic lines given in Eq. (A.6). If, on the other hand, the above quantity is positive, the appropriate boundary condition for the characteristic is either the ion density at $-\infty$ or at the drop surface, depending on the origin of the particular characteristic. Due to the aforementioned diode character of the drop surface, characteristics which originate on the drop surface, and for which

$$\hat{n} \cdot (\vec{U}_d \pm b_1 \vec{E}) > 0 ,$$

are loci of zero positive ion density.

The normal components of the neutral velocity must vanish at the drop surface. Thus, whether a given polarity ion can be accepted at a point on the surface is wholly determined by the value of $\pm b_1 \vec{E} \cdot \hat{n}$ at that point. In Fig. A.2 are shown three stages of charge accumulation by a drop in an atmosphere of a single ion species. In the first stage, the drop has zero charge and can accept positive ions over one hemisphere,

denoted by characteristic lines terminating on the drop surface; negative ions could be collected on the opposite hemisphere. The second stage is shown after the drop has accumulated some charge. Notice that the area which can accept the ions has decreased. The final stage shows the situation at positive ion cutoff, when the drop has accumulated the critical charge, q_c , which causes the characteristic force lines to point out from the drop surface everywhere. No further charging by positive ions is possible.

The understanding and important parameters which are found from considering the simple case of a stationary drop in a unipolar ion atmosphere makes this problem worthy of discussion. From Eq. (A.8), the normal component of the electric field evaluated at the drop surface is given by

$$E_r = 3 E_0 \cos \psi + \frac{q}{4\pi\epsilon_0 a^2} \quad (A.13)$$

Charging of a drop in a $\begin{bmatrix} \text{positive} \\ \text{negative} \end{bmatrix}$ ion atmosphere can occur, provided E_r of Eq. (A.13) is $\begin{bmatrix} \text{negative} \\ \text{positive} \end{bmatrix}$ somewhere on the drop surface. Thus, the area of the drop surface defined by

$$\cos \psi \leq - \frac{q}{12\pi\epsilon_0 a^2 E} \quad (A.14)$$

can accept $\begin{bmatrix} \text{positive} \\ \text{negative} \end{bmatrix}$ ions for positive E . When $\cos \psi = \mp 1$, or $\psi_c = (\frac{\pi}{0})$ radians, the entire drop surface has $\begin{bmatrix} \text{positive} \\ \text{negative} \end{bmatrix}$ surface charge and no longer accepts $\begin{bmatrix} \text{positive} \\ \text{negative} \end{bmatrix}$ ions. This occurs for the critical charge,

$$q_c = \pm 12\pi\epsilon_0 a^2 |E_0| \quad (A.15)$$

The polarity of the accumulated charge is independent of the field direction, and depends on the polarity of the ionized environment.

The charging rate is found from the flux of ions through the envelope defined by the two limiting characteristics which just intercept the drop surface. The limiting streamline is given by using Eq. (A.14) in (A.11) with $U_d = 0$ and $r = a$. Thus, the constant C of Eq. (A.11) corresponding to the limiting trajectory, is found to be

$$C_1 = \mp \frac{3}{2} a^2 b_1 E_0 [1 + (q/q_c)^2] \quad . \quad (A.16)$$

Our choice of coordinates with E_0 reckoned positively directed from $-\infty$ to $+\infty$ and the absence of convection dictates that positive ions which impact the drop must be at $-\infty$. Thus, tracing the limiting trajectory for this ion species from the drop surface must be done in the $-\infty$ direction. If we take the limit of Eq. (A.11) as $z \rightarrow -\infty$, and C as given in Eq. (A.16), we find for $U_d = 0$,

$$\frac{1}{a^2} \lim_{z \rightarrow -\infty} r^2 \sin^2 \psi - \frac{6q}{|q_c|} \lim_{z \rightarrow -\infty} \cos \psi = 3[1 + (q/q_c)^2]$$

so that for

$$Y^{*2} = \lim_{z \rightarrow \pm \infty} r^2 \sin^2 \psi,$$

$$\frac{Y^{*2}}{a^2} = 3(1 - q/|q_c|)^2 \quad . \quad (A.17)$$

Y^* in Eq. (A.17) is half the distance between the two limiting trajectories far from the perturbing influence of the drop. Thus, the flux of ions through the area of radius Y^* oriented normal to the flow at $-\infty$ gives the rate of charge accumulation. This charging current is

$$i_1^{(1)} = b_1 \rho_1 |E_0| (\pi Y^*{}^2) = \frac{b_1 \rho_1}{4\epsilon_0} |q_c| (1 - q/|q_c|)^2 \quad (\text{A.18})$$

where $i_1^{(1)}$ signifies the charging due to positive ions in the Gunn mode of neglecting the neutral flow and $|q| < |q_c|$. As should be clear, positive charging can proceed whenever $q < |q_c|$, but $i_1^{(1)} = 0$ for $q \geq |q_c|$.

In the same way, charging currents due to negative ions can be calculated to be

$$i_2^{(1)} = - \frac{b_2 \rho_2}{\epsilon_0} |q_c| (1 + q/|q_c|)^2 \quad (\text{A.19})$$

where, if $q < -|q_c|$, $i_2^{(1)} = 0$.

Equations (A.18) and (A.19) are valid for their stated regions, so long as the drops are situated in a unipolar ion atmosphere of the same polarity as their charge. If, however, a positive or negative drop is exposed to an atmosphere containing ions of the opposite polarity, a different charging rate is possible, provided the drop carries at least the critical charge, q_c . If this is the case, then the negative ion limiting trajectory intersects the drop surface at π radians for a positive drop with negative ions, and at $\psi = 0$ for a negative drop with positive ions, both in a positive field. Then

$$C_2 = \mp 3 b_1 a^2 E_0 q/q_c \quad (\text{A.20})$$

from Eq. (A.11). If the proper limits of (A.11) are taken, the half-distance between limiting trajectories for the cases of $|q| > |q_c|$ when exposed to ions of the opposite polarity is

$$Y^*{}^2 = 12 a^2 |q/q_c|. \quad (\text{A.21})$$

In finding this result, recall that for positive field, the limit is taken for $z \rightarrow +\infty$ when $q > 0$, and $z \rightarrow -\infty$ when $q < 0$. Thus, the current to the drop in this mode is given by

$$\stackrel{(2)}{i_1} = - \frac{\rho_1 b_1 q}{\epsilon_0} \quad . \quad (A.22)$$

Thus, the current in this regime is always dissipative, since q tends to zero.

The neglect of space charge in formulating the model allows superposition of Eqs. (A.18) and (A.19) to calculate the drop charging rate in a bipolar atmosphere,

$$\frac{dq}{dt} = \stackrel{(1)}{i_1} - \stackrel{(1)}{i_2} = \frac{|q_c|}{4\epsilon_0} [b_1 \rho_1 (1 - q/|q_c|)^2 + b_2 \rho_2 (1 + q/|q_c|)^2] \quad (A.23)$$

when $-|q_c| \leq q \leq |q_c|$

and

$$\frac{dq}{dt} = \pm \stackrel{(2)}{i_1} = \begin{cases} - \frac{b_1 \rho_1 q}{\epsilon_0} , & q < -|q_c| \\ \frac{b_2 \rho_2 q}{\epsilon_0} , & q > |q_c| \end{cases} \quad (A.24)$$

It is important to note that this special case of Wilson charging can result in no equilibrium net charge on a drop, unless there persists some systematic difference in the polar ion species properties. Thus, for example, if the ion densities are given by $\pm \rho_0$, say, and the mobilities are b , then Eqs. (A.23) and (A.24) give

$$\frac{dq}{dt} = - \frac{b |\rho_0| |q|}{\epsilon_0} \quad . \quad (A.25)$$

That is, in a neutral, ionized atmosphere of ions of the same mobility, the total charging current to a drop is always dissipative and the drop charge tends to zero when the neutral flow is neglected.

The equilibrium charge on the drop is reached when $dq/dt = 0$. For drops in a unipolar atmosphere, this occurs for $q_e = \pm |q_c|$, with the sign given by the ion polarity. In an ionized atmosphere, the equilibrium charge is found by setting $dq/dt = 0$ in Eq. (A.23).

Defining the total charge carrier density, ρ_T , and net space charge density, ρ , according to

$$\rho_T = \rho_1 - \rho_2 \quad (\text{A.26})$$

and

$$\rho = \rho_1 + \rho_2, \quad (\text{A.27})$$

the equilibrium charge is given by

$$q_e = |q_c| \left\{ \frac{\gamma\rho + \rho_T}{\rho + \gamma\rho_T} \pm \left[\left(\frac{\gamma\rho + \rho_T}{\rho + \gamma\rho_T} \right)^2 - 1 \right]^{1/2} \right\}$$

where

$$\gamma = \frac{b_1 - b_2}{b_1 + b_2}, \quad (\text{A.29})$$

and the choice of root is determined by the net space charge polarity.

That is, since $|q_e| < |q_c|$, if $\rho > 0$, we choose the negative root, and if $\rho < 0$, we choose the positive root. Equation (A.28) is valid provided $\gamma\rho + \rho_T \neq 0$, i.e., $b_1\rho_1 + b_2\rho_2 \neq 0$. From Eq. (A.28), if $b_1\rho_1 + b_2\rho_2 = 0$, $q_e = 0$. The results of the $U_d = 0$ solution are summarized in Table A.1.

TABLE A.1

Drop Charge	Drop Charging Rate		
	Positive Atmosphere	Negative Atmosphere	Ionized Atmosphere
$q \geq q_c $	0	$-i_2^{(2)}$	$-i_2^{(2)}$
$- q_c < q < q_c $	$i_1^{(1)}$	$-i_2^{(1)}$	$i_1^{(1)} - i_2^{(1)}$
$q \leq - q_c $	$i_1^{(2)}$	0	$i_1^{(2)}$
Equilibrium Charge $ q_c $		$- q_c $	Eq. (A.28)

The charging of a stationary drop in a unipolar atmosphere is characterized by the closing of the charge-accepting area on the same side of the drop approached by ions whose motion is determined wholly by the polarizing electric field. Charging ceases when the drop surface charge is of the same polarity as the unipolar environment. The more general case of finite gas velocity relative to the drop is distinguished from the Gunn mode by the possibility of ions approaching the drop from the opposite side of the charge-accepting area on the drop surface. Thus, if the gas velocity exceeds the field-induced ion velocity, boundary conditions on both ions are imposed at $-\infty$ for positive U_d , regardless of the direction of the electric field. In this case, charging currents to the drop in a unipolar atmosphere may vanish, even though the drop surface charge is not uniformly of the same polarity as the ions.

When the ion flux is in the same direction as the electric field, and $|q| < |q_c|$, evaluation of the stream function, Eq. (A.11), using the definition of $\cos \psi$ in (A.14) with $r = a$ define the limiting streamline,

$$C_1 = 3/2 a^2 b_1 E_0 [1 + (q/q_c)^2]. \quad (A.30)$$

Comparing Eq. (A.30) with (A.16) shows that in this case the convection does not influence the limiting streamline. Taking the limit as $z \rightarrow \pm\infty$ of Eq. (A.11) with C_1 , above, we get

$$Y^{*2} \left(1 \pm \frac{U_d}{b_1 E_0} \right) = 3a^2 (1 + q/|q_c|)^2 \quad (A.31)$$

whence the ion flux to the drop is

$$I_2^{(1)} = \pi J_1 Y^{*2} = \pm \frac{\rho_1 b_1}{4\epsilon_0} |q_c| (1 \mp q/|q_c|)^2. \quad (A.32)$$

Thus, the convection does not influence the charging rate when the ion flux is in the same direction as the electric field.

Similarly, when $|q| > |q_c|$, the critical angle $\cos \psi = \pm 1$ and C_2 in the case of convection is given by Eq. (A.20), and the current to the drop is

$$I_2^{(2)} = J_1 \pi Y^{*2} = - \frac{\rho_1 b_1}{\epsilon_0} q. \quad (A.33)$$

For the case of ions approaching the drop from the opposite side of the charge-accepting region, a new situation prevails. Consider the case of positive ions with positive velocity in a negative field. Since $U + bE_0 > 0$ and $E_0 < 0$, the ions are injected at $-\infty$, whereas the charging region of the drop is on the $+\infty$ side. To determine the limiting

characteristic with origin at $-\infty$ which just intercepts the drop surface, we note that since crossing of characteristics is prohibited, the limiting characteristic will be that one coincident with the z -axis at $-\infty$. Thus, evaluating Eq. (A.11) with $\psi = \pi$ gives the constant associated with the limiting characteristic,

$$C_L = \mp \frac{b_1 q}{4\pi\epsilon_0} \quad (\text{A.34})$$

Thus, particles in this regime have charging currents given by Eq. (A.33) so long as the limiting characteristic intercepts the drop surface. Using Eq. (A.34) in (A.11), with $r = a$, an expression is found which gives the angular position of interception of the limiting characteristic,

$$\sin^2 \psi = \frac{2q}{12\pi\epsilon_0 a^2 E_0} (1 + \cos \psi) . \quad (\text{A.35})$$

Since $\sin^2 \psi > 0$ and $(1 + \cos \psi) > 0$ for all ψ , the characteristics intercept the drop surface only when $q/E_0 > 0$. For positive ions in a negative field, therefore, Eq. (A.33) describes the current to the drop so long as $q < 0$. For $q > 0$ in this case, the drop current is zero. Similarly, for negative ions in a positive field such that $U - b_1 E_0 > 0$, the drop charges according to Eq. (A.33) so long as $q > 0$. The current vanishes when $q \leq 0$.

The charging regimes discussed above are illustrated in Figs. A.3 and A.4, for positive and negative ions respectively, after Melcher [1971]. The regimes are distinguished by the drop charge and the ion velocity, $U \pm b_1 E_0$. Regimes (e) and (h) in both figures represent the cases discussed above for which ions approach the drops from the side

POSITIVE IONS

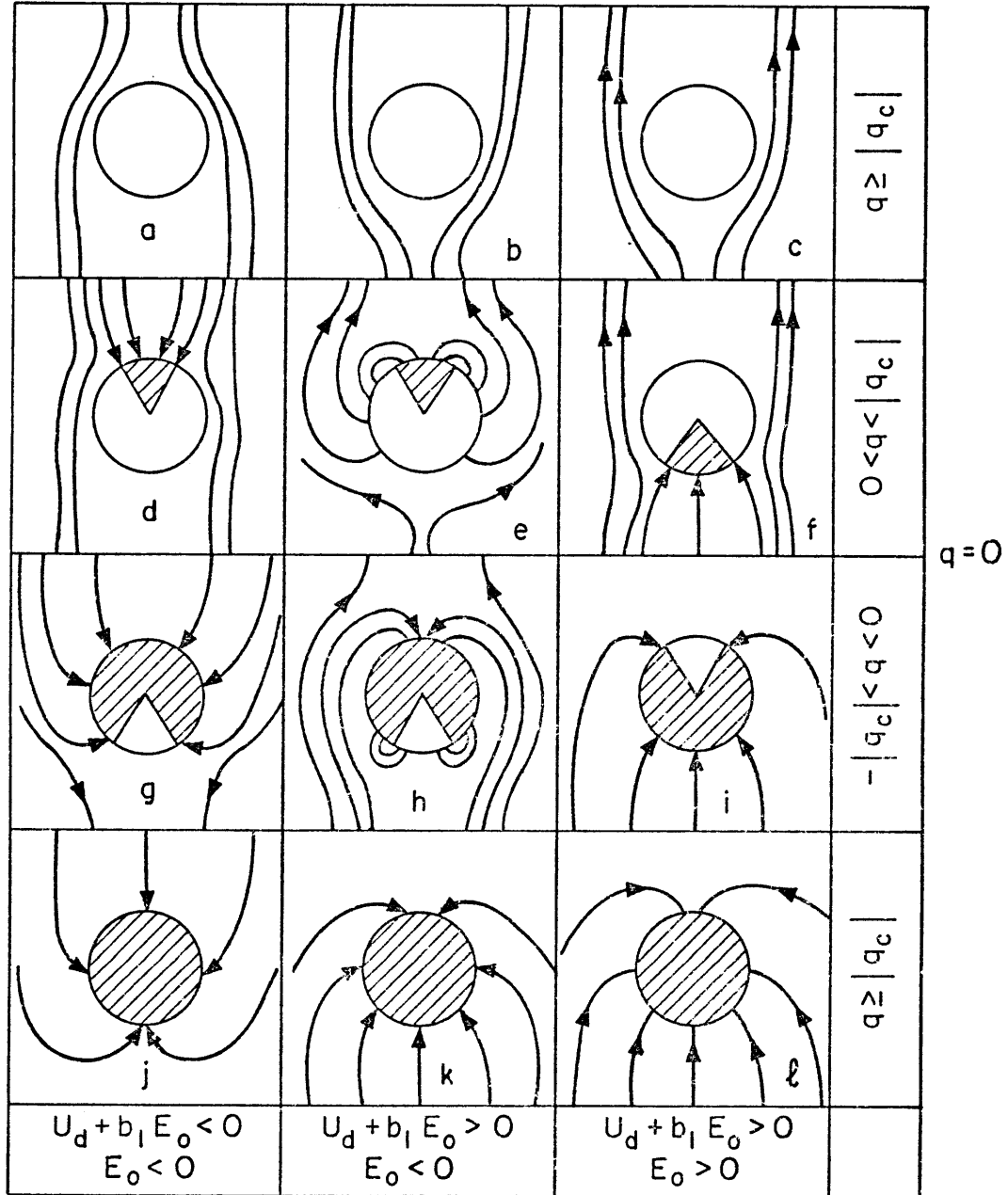


Fig. A.3 Charging regimes and schematic ion trajectories for a drop in a positive ion atmosphere (after Melcher, 1971)

NEGATIVE IONS

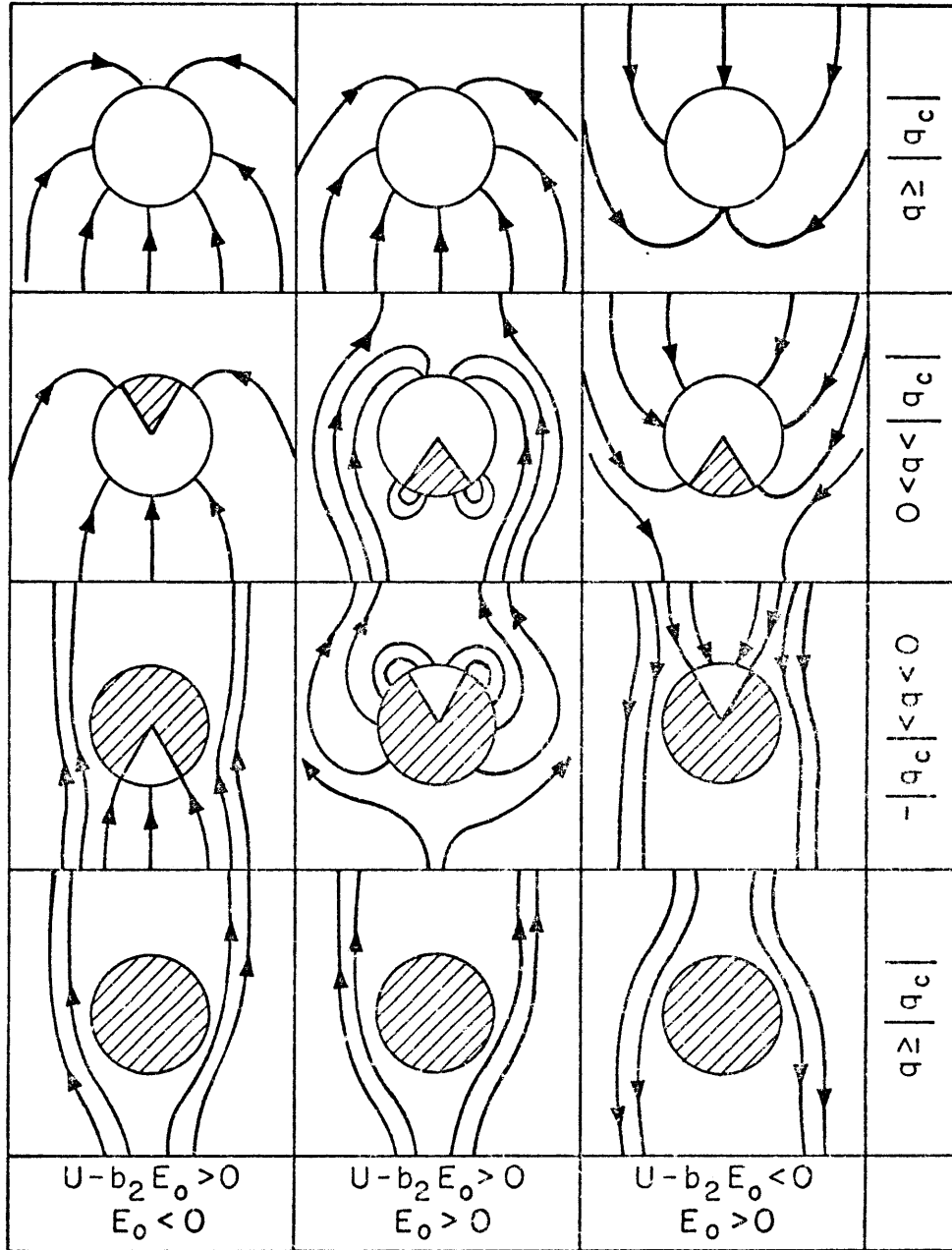


Fig. A.4 Charging regimes and schematic ion trajectories for a drop in a negative ion atmosphere (after Melcher, 1971)

opposite the charging window. All other regimes shown are also characteristic of the Gunn mode of neglecting the motion of the particle relative to the surrounding gas. In the figures, particle trajectories are shown schematically. Regions of positive surface charge are indicated by no shading, while regions of negative surface charge are shaded.

Superposition of the positive and negative ion results follows from our neglect of space charge and gives the charging equations for a bipolar atmosphere. Figure A.5, after Melcher, shows the superposed results in terms of the currents to the drop as a function of drop charge and the ion velocity relative to the drop at $\pm \infty$.

The equilibrium charge for counter-streaming ion velocities, i.e., $U_d \pm b_1 E_o \gtrless 0$ or $U_d \pm b_2 E_o \gtrless 0$ are given by Eq. (A.28), since in these cases drop currents are the same as in the Gunn mode. When $U_d \pm b_1 E_o > 0$, however, the drop current balance is

$$I_1^{(2)} - I_2^{(1)} = 0, \quad E_o < 0 \quad (\text{A.36})$$

and

$$I_1^{(1)} - I_2^{(2)} = 0, \quad E_o > 0 \quad (\text{A.37})$$

From Eqs. (A.32) and (A.33), the equilibrium charge is found to be

$$q_e^{(2)} = \pm |q_c| \left\{ 1 - \frac{2\rho_2 b_2}{\rho_1 b_1} - \left[\left(1 - \frac{2\rho_2 b_2}{\rho_1 b_1} \right)^2 - 1 \right]^{1/2} \right\} \quad (\text{A.38})$$

according as $E_o \gtrless 0$.

In terms of the net space charge and total particle density, Eq. (A.38) is

$q \geq q_c $	$-I_2^{(2)}$ ↓	$-I_2^{(2)}$ ↓	$-I_2^{(2)}$ ↓	$-I_2^{(2)}$ ↓
$0 < q < q_c $	$I_1^{(1)} - I_2^{(1)}$ ↓	$I_2^{(1)}$ ↓	$I_1^{(1)} - I_2^{(2)}$ ↓ $q_e^{(2)}$	$I_1^{(1)} - I_2^{(1)}$ ↓
$- q_c < q < 0$	$I_1^{(1)} - I_2^{(1)}$ ↑	$I_1^{(2)} - I_2^{(1)}$ ↑ $q_e^{(2)}$	$I_1^{(1)}$ ↑	$I_1^{(1)} - I_2^{(1)}$ ↑
$q \leq - q_c $	$I_1^{(2)}$ ↑	$I_1^{(2)}$ ↑	$I_1^{(2)}$ ↑	$I_1^{(2)}$ ↑
	$U_d \pm b_2' E_0 \leq 0$ $E_0 < 0$	$U_d \pm b_2' E_0 > 0$ $E_0 < 0$	$U_d \pm b_2' E_0 > 0$ $E_0 < 0$	$U_d \pm b_2' E_0 \leq 0$ $E_0 < 0$

Fig. A.5 Charging regimes for a drop in a bipolar atmosphere

$$q_e^{(2)} = |q_c| \left\{ \frac{2b_1}{b_2} \left(\frac{\rho + \rho_T}{\rho - \rho_T} \right) - 1 + \left[\left(\frac{2b_1}{b_2} \cdot \frac{\rho + \rho_T}{\rho - \rho_T} - 1 \right)^2 - 1 \right]^{\frac{1}{2}} \right\} \quad (\text{A.39})$$

when $E_o < 0$, and

$$q_e^{(2)} = - |q_c| \left\{ \frac{2b_2}{b_1} \left(\frac{\rho - \rho_T}{\rho + \rho_T} \right) - 1 + \left[\frac{2b_2}{b_1} \left(\frac{\rho - \rho_T}{\rho + \rho_T} - 1 \right)^2 - 1 \right]^{\frac{1}{2}} \right\} \quad (\text{A.40})$$

When $b_1 = b_2$, $\rho_1 = |\rho_2|$ i.e., $\rho = 0$,

$$q_e^{(2)} = \pm 0.176 |q_c| \quad (\text{A.41})$$

$$E_o \gtrless 0.$$

APPENDIX B: Justification of Wilson-charging Currents in a Macroscopic Model

The particle charging mechanism conceived by Wilson [1929] and treated analytically by Whipple and Chalmers [1944] can be used as a source term in a macroscopic model designed to investigate the distribution of charges and fields within a convecting system of polar ions and charged drops. However, since the quasi-steady charging currents to the drops were calculated under circumstances and assumptions possibly distinct from the conditions obtaining within the cloud model, it is necessary to examine these assumptions for an inconsistency or incompatibility with the large-scale continuum model.

The conditions under which the particle charging currents are calculated require that first the particles be widely separated in a uniform, externally maintained electric field. The distance between particles must be of the order of several particle diameters in order to calculate particle currents independent of nearest neighbors. Moreover, since the currents are calculated according to a field distribution corresponding to a charged conducting drop in a uniform field, it is apparent that space charge effects must be ignored in this calculation. However, another way of stating this assumption is to require the field to be sensibly constant over several particle radii. In this way, the field calculated according to the above prescription will not differ appreciably from that which exists in a region of very small gradient.

The second condition which must be satisfied is the requirement

that local field changes must occur slowly relative to the time required for an ion to come under the influence of a particular drop and subsequently impact its surface. This essentially reflects the quasi-steady nature of the problem. The calculation of charging currents is accomplished by finding the distribution of all ion trajectories which intersect the drop surface with origin at $\pm \infty$. Since the ion population far from the drop is known and uniform, it is an easy process to calculate the charge flux to the drop without recourse to the ion distribution in its immediate vicinity. If, however, the field, and hence the trajectories, change significantly in the time required for the ion to follow its prescribed path, an obvious error will result. There are two conditions which must be obeyed in order that the quasi-steady character of the charging mechanism be preserved. The first is a microscopic property, and requires that a typical charging time, τ_c , be large relative to a typical ion transit time, τ_t . The second is a macroscopic prescription and requires that field changes due to translation of the ion-drop system through a region of field gradient must occur slowly, relative to the ion transit time.

One major source of conceptual difficulty in justifying the procedure is the mixing of two scales of interest in the same problem. On the microscopic scale, we imagine a single drop immersed in a continuum of ions all under the influence of an external uniform field. On the basis of this model, the currents to the drop are calculated, and particle charges arise from differences in the polar ion behavior. In the macroscopic problem, we deal with scales sufficiently large that the drops may also be viewed as a continuum and now we must ask

if the assumptions which were in effect in the microscopic calculations preclude new assumptions on the macroscopic scale. The problem with which we are particularly concerned is whether one can, with consistency in the model, neglect space charge effects on the microscopic scale, but nevertheless include space charge in the macroscopic model. To understand the source of the difficulty, it is worthwhile to write down Gauss's law in dimensionless form.

In the following, we let L be a characteristic dimension, ρ_0 a characteristic space charge ion density. E_0 is a measure of the electric field intensity, while e and ϵ_0 are the elementary electronic charge and permittivity of free space, respectively. N is characteristic of the number density of drops of radius a . Then, if ρ_1, ρ_2 are the positive and negative space charge densities, respectively, and q the drop charge, the scaled parameters are given by

$$\rho_1' = \rho_1 / \rho_0$$

$$\rho_2' = \rho_2 / \rho_0$$

$$q' = q / 12\pi\epsilon_0 a^2 E_0 .$$

Since the effect of cloud, in the absence of ionizing processes due to intense field, is to cause a considerable drop in ion density, selection of the clear-air, steady-state ion density for ρ_0 assures us $\rho_1' \leq 1$. Moreover, since the maximum charge accrued by a drop through the Wilson charging mechanism is $12\pi\epsilon_0 a^2 E_0$, q' is also less than unity. Then, Gauss's law is written as

$$\frac{dE'}{dx'} = \frac{\rho_0 L}{\epsilon_0 E_0} (\rho_1' + \rho_2') + 12 \pi a^2 N L q' \quad (B.1)$$

where the dependent variables, E' , ρ_1' , and q' are all of order unity or less.

Notice that the spatial dimension L appears symmetrically in both the space charge and drop charge terms. This appears to demand that neglect of space charge on the microscopic scale requires that it also be neglected for the large-scale model. Since, for typical values of the parameters, the space charge coefficient is of order $10^2 L/E_0$, or nearly L , while the drop charge coefficient is nearly $L/10$, it seems that the space charge is at least as important as the drop charge in the macroscopic model.

It is important to remember, however, that Eq. (B.1) is strictly valid only if L is many multiples of the average inter-particle distance. Gauss's law for computing charging currents via Wilson's mechanism, on the other hand, is meant to be applied on a scale of the order of several drop radii. There is, therefore, some basis for re-examining the foundations for the neglect of space charge in the charging calculation.

As shown by Melcher [1971], the Whipple-Chalmers relation for charging currents is directly attributable to the assumed solenoidal character of the neutral flow, $\bar{U}$, and the electric field, E . Under these circumstances, the conservation equation for the ions is written

$$\frac{\partial \rho_1}{\partial t} + (\bar{U} \pm b_1 \vec{E}) \cdot \nabla \rho_1 = 0 \quad (B.2)$$

The implication of (B.2) is that

$$\rho_2 \nabla \cdot b_2 \vec{E} \ll (\vec{U} \pm b_2 \vec{E}) \cdot \nabla \rho_2, \quad (B.3)$$

or, for a characteristic length like the particle diameter, a ,

$$\frac{b_2 \rho_0 a}{\epsilon_0 (U \pm b_2 E)} \ll 1. \quad (B.4)$$

Notice that, since this result was derived independent of the presence of other drops, the neglect of space charge in the charging calculation is not in any way related to a comparison of the relative weights of space charge and drop charge in Gauss's law for the macroscopic model. In fact, for typical values of the parameters in Eq. (B.4), we find

$$\frac{b_2 \rho_0 a}{\epsilon_0 (\pm b_2 E + U)} \approx 10^{-2} a \quad (B.5)$$

so that condition (B.3) is not valid for a on the order of a hundred meters.

It is worthwhile to note in passing that satisfaction of Eq. (B.4) also assures us that even in a region of only one ion species, the field departs insignificantly from a uniform field, over several drop radii, so that it is not necessary to resort to calculation of the field distribution around a conducting sphere within a gradient electric field.

Melcher [1970] showed that neglect of space charge as in Eq. (B.2) is equivalent to the requirement that the transit time be short relative to a typical charging period. Thus, when Eq. (B.4) holds, the quasi-static character of the problem is also retained.

The final requirement for the quasi-steady calculation demands that the local field change due to translation in a gradient field must be small

during the course of a typical transit time. One measure of this can be expressed by the statement

$$\frac{a}{bE} \ll \frac{L}{U} \quad (\text{B.6})$$

for L a typical macroscopic length scale, and a/bE , characteristic of an ion transit period. Then

$$U \ll \frac{bEL}{a} \quad (\text{B.7})$$

For typical values of L/a , the above requires $U \ll 10^7 bE$, which is almost always true. A better expression, however, of the limitation on U can be given by considering the field change experienced by a translating particle in a steady-state system,

$$\frac{dE}{dt} = U \frac{dE}{dx} \approx \frac{\rho_o U}{\epsilon_o} \quad (\text{B.8})$$

Thus, a characteristic time for field change due to convection is given by

$$\tau_E = \frac{\epsilon_o E_o}{\rho_o U} \quad (\text{B.9})$$

Then,

$$\frac{\tau_t}{\tau_E} = \frac{\rho_o U a}{b \epsilon_o E_o^2} \ll 1 \quad (\text{B.10})$$

requires

$$U \ll \frac{\epsilon_o b E_o^2}{\rho_o a} \quad (\text{B.11})$$

For typical values, the above requires the convection to be small compared to about $.1 E_o^2$. Since typical convection speeds vary from one to the order of 10 or 20 m/sec, E_o must be of the order of 10-100 v/m.

Actually, in the smaller field strength cases, the ions interact very little with the drops, and any gradients established within the cloud are small; much smaller, in fact, than estimated by the above process. Thus, even though E_0 is relatively small, the conditions will be satisfied unless the convection speed is exceptionally high.