

ELECTROMAGNETIC PROBING OF THE LUNAR INTERIOR
IN THE FREQUENCY REGION FROM ONE KILOHERTZ TO
TEN KILOHERTZ .

by

Albert Colbert Reisz

S. B., Physics, Massachusetts Institute of Technology
(1970)

SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE
DEGREE OF MASTER OF
SCIENCE
at the
MASSACHUSETTS INSTITUTE OF
TECHNOLOGY
September 1970

Signature of Author _____
Department of Earth and Planetary Sciences, September, 1970

Certified by _____
Thesis Supervisor

Accepted by _____
Chairman, Departmental Committee on Graduate Students



ELECTROMAGNETIC PROBING OF THE LUNAR INTERIOR
IN THE FREQUENCY REGION FROM ONE KILOHERTZ TO
TEN KILOHERTZ

by

Albert Colbert Reisz

Submitted to the Department of Earth and Planetary Sciences,
Massachusetts Institute of Technology, September 1970, in
partial fulfillment of the requirements for the degree
of Master of Science

Abstract

For probable values of the lunar electromagnetic properties, a transverse magnetic surface wave may be propagated along the solar wind plasma-moon interface in the frequency range from about one kilohertz to about ten kilohertz. The horizontal propagation characteristics of this surface wave as a function of frequency may be used to probe the lunar electromagnetic properties to a probable maximum depth of about fifty kilometers.

The electromagnetic fields of idealized dipole sources located within a few meters of the lunar surface and immersed in the solar wind plasma are considered. For a vertical electric dipole, or broadside to a horizontal magnetic dipole, the surface fields at radial distances greater than about ten kilometers are found to be dominated by the surface wave. These sources may also excite transverse magnetic wave guide modes confined between the plasma and a lunar conducting basement.

If a wet model of the moon is adopted, only the surface wave mode will be present, and it will be sensitive to the depth of the ice to water transition, expected at about one kilometer, and to lunar dielectric properties between the transition depth and the surface. If the moon is dry, a thermally activated conductive basement is expected at about one hundred kilometers, and the surface wave will be sensitive to dielectric properties to a probable maximum depth of about fifty kilometers. For the dry model, the fields due to the wave guide modes become important below about two kilohertz for the horizontal magnetic dipole source.

Thesis Supervisor: Theodore R. Madden

Title: Professor of Geophysics

TABLE OF CONTENTS

	Page
Abstract	ii
Table of Contents	iv
Acknowledgements	v
I. Introduction	1
II. Formulation and Solution of the Dipole Excitation Problem	
A. Modeling of the electromagnetic properties of the lunar interior and the solar wind plasma	3
B. Electromagnetic fields of oscillating dipoles located a distance h above the horizontally stratified lunar model	4
C. Poles of the plane wave reflection coefficient for parallel incidence in simplified cases	12
D. Approximate evaluations of the electromagnetic fields along the lunar surface	14
E. Discussion of results	20
III. Appendices	
A. Electromagnetic wave propagation	29
B. Reflection of electromagnetic plane waves from a horizontally stratified medium consisting of N electromagnetically linear, isotropic, homogeneous layers	33
IV. References	37

ACKNOWLEDGEMENTS

The author wishes to acknowledge the help provided by his thesis advisor, Prof. Madden, who suggested the area of this investigation and provided invaluable assistance and patience throughout. The author also wishes to acknowledge the help of Prof. Simmons, who provided support, and initiated the consideration of measurement possibilities on the moon.

Support for this investigation was provided under NASA contract NGR 22-009-329, "Lunar Traverser, Geophysics"

I. INTRODUCTION

For radio transmission on the earth in the very low frequency band (~ 3 to 30 kilohertz), the ground and the ionospheric plasma are effectively good electrical conductors, and the transmitted wave energy is confined to the earth-ionosphere cavity. The waveguide nature of this propagation has been thoroughly investigated and has been exploited as a means of probing the ionosphere.

Studies of the electrical properties of the Apollo lunar samples, and of similar materials of terrestrial origin under expected lunar conditions, show that in the frequency range above about one hundred hertz the outer region of the moon may be considered as a low loss dielectric. If water is present in the subsurface lunar material, a conductive basement at about one kilometer is expected. If no water is present, as now seems more probable, a thermally activated conductive basement at about one hundred kilometers would be expected. For frequencies in the range from one kilohertz to the electron plasma frequency ($\sim 28 \times 10^3$ hz), the solar wind plasma is considered to be isotropically in cutoff. On the sunward portion of the moon, where the solar wind is directly incident upon the lunar surface, this cutoff frequency region may be exploited to propagate a transverse magnetic surface wave along the plasma-dielectric interface. The depth attenuation of such a surface wave is frequency dependent, and thus its radial propagation characteristics are sensitive to possible depth

variations in the lunar electromagnetic properties. In addition, wave guide modes contained within the plasma-conductive basement cavity may be excited.

The purpose of this investigation is to examine the excitation and propagation properties of the surface wave and wave guide modes, and to determine to what extent they may be used to probe the electromagnetic properties of the lunar interior.

II. FORMULATION AND SOLUTION OF THE DIPOLE EXCITATION PROBLEM

II. A. MODELING OF THE ELECTROMAGNETIC PROPERTIES OF THE LUNAR INTERIOR AND THE SOLAR WIND PLASMA

The electromagnetic properties of the moon on a horizontal scale of about one hundred kilometers, and on a slightly greater depth scale, are approximated by a horizontally stratified medium consisting of N electromagnetically linear, isotropic, homogeneous layers. As shown in Appendix A, for oscillating fields the electromagnetic properties of each layer are contained in the dimensionless parameters K_{mag} and $\overset{\vee}{K}$, which are, respectively, the relative permeability and the complex relative dielectric constant.

Two major models of lunar depth variation in $\overset{\vee}{K}$ are adopted. The first is a wet model in which a transition from ice to water is expected at about one kilometer¹. The values of conductivity and dielectric constant are taken from frequency and temperature measurements performed on water contaminated lunar samples² and terrestrial samples of similar composition^{2,3,4}. The second model assumes the lunar material devoid of all water and the values adopted for the electromagnetic parameters are typical of measurements made on uncontaminated lunar samples⁵ and dry geologic samples^{3,6}. The dry model is expected to have a thermally activated conductive basement at about one hundred kilometers¹.

The electromagnetic propagation properties of the solar wind plasma at the earth's orbit⁷ are adequately described above a few hertz by the tensor form of $\underline{\underline{K}}$, obtained in Appendix A by neglecting collisions and ion motion. In the limit $\nu \gg$ electron cyclotron frequency (one hundred hertz), the influence of the solar wind magnetic field becomes negligible and propagation becomes isotropic. On the sunward portion of the moon the solar wind plasma impinges directly upon the lunar surface⁸, and in the frequency range above one kilohertz is considered as an isotropic half-space overlaying the stratified model of the moon.

II. B. ELECTROMAGNETIC FIELDS OF OSCILLATING DIPOLES LOCATED A DISTANCE h ABOVE THE HORIZONTALLY STRATIFIED LUNAR MODEL⁹

In a homogeneous region in which the source of the electromagnetic fields lies exterior to the region, the Maxwell equations III.A.15 to 18 may be used to resolve the oscillating fields into two partial fields¹⁰, the one derived from the oscillating cartesian electric Hertz vector $\underline{\underline{\Pi}}$:

$$\left(\nabla^2 + \frac{\omega^2}{c^2} K_{\text{mag}} \underline{\underline{K}} \right) \underline{\underline{\Pi}} = 0 \quad \text{II.B.1}$$

where

$$\underline{\underline{H}} = i\omega\epsilon_0 \underline{\underline{K}} \nabla \wedge \underline{\underline{\Pi}} \quad \text{II.B.2}$$

$$\underline{\underline{E}} = \nabla \nabla \cdot \underline{\underline{\Pi}} + \frac{\omega^2}{c^2} K_{\text{mag}} \check{K} \underline{\underline{\Pi}} \quad \text{II.B.3}$$

and the other derived from the oscillating cartesian magnetic Hertz vector $\underline{\underline{\Pi}}^*$:

$$\left(\nabla^2 + \frac{\omega^2}{c^2} K_{\text{mag}} \check{K} \right) \underline{\underline{\Pi}}^* = 0 \quad \text{II.B.4}$$

where

$$\underline{\underline{H}} = \nabla \nabla \cdot \underline{\underline{\Pi}}^* + \frac{\omega^2}{c^2} K_{\text{mag}} \check{K} \underline{\underline{\Pi}}^* \quad \text{II.B.5}$$

$$\underline{\underline{E}} = -i\omega\mu_0 \nabla \wedge \underline{\underline{\Pi}}^* \quad \text{II.B.6}$$

If the entire source of the electromagnetic fields in a region is due to dipole oscillators activated by external power sources, in this region equations II.B.1 and II.B.4 are replaced by the inhomogeneous Helmholtz equations¹⁰

$$\left(\nabla^2 + \frac{\omega^2}{c^2} K_{\text{mag}} \check{K} \right) \underline{\underline{\Pi}} = -\frac{\underline{\underline{P}}_0}{\epsilon_0 \check{K}} \quad \text{II.B.7}$$

and

$$\left(\nabla^2 + \frac{\omega^2}{c^2} K_{\text{mag}} \check{K} \right) \underline{\underline{\Pi}}^* = -\underline{\underline{M}}_0 \quad \text{II.B.8}$$

where $\underline{\underline{P}}_0$ and $\underline{\underline{M}}_0$ are the prescribed electric and magnetic polarizations. In terms of the prescribed current distribution $\underline{\underline{J}}_0$, we may write II.B.7 as:

$$\left(\nabla^2 + \frac{\omega^2}{c^2} K_{\text{mag}} \check{K} \right) \underline{\underline{\Pi}} = -\frac{\underline{\underline{J}}_0}{i\omega\epsilon_0 \check{K}} \quad \text{II.B.9}$$

As the source of electric polarization we consider a line of length Δl carrying the current $\hat{\zeta}I$ with time dependence $\exp(i\omega t)$. As the source of magnetic polarization we take a circular loop of area Δa with positive normal $\hat{\zeta}$, carrying the current I , again with the time dependence $\exp(i\omega t)$. Under the assumption that the source dimensions are small compared to a wavelength, the amplitudes of the currents may be assumed constant. Under the further approximation that the source dimensions may be considered infinitesimal, we may write for the oscillating electric dipole:

$$\underline{J}_0 = \hat{\zeta} I \Delta l \delta(r) \quad \text{II.B.10}$$

and for the oscillating magnetic dipoles:

$$\underline{M}_0 = \hat{\zeta} I \Delta a \delta(r) \quad \text{II.B.11}$$

where $\delta(r)$ is the Dirac delta function and the sources are located at $r = 0$, on the z axis at a height h above the stratified lunar model (Figure 1).

We first consider the electric dipole source II.B.10 with $\hat{\zeta} = \hat{z}$. The primary fields in the upper region may then be written in terms of an electric type Hertz vector with only a z component:

$$\Pi_{0,z}^P = \frac{I \Delta l}{4 \pi i \epsilon_0 \omega K_0} \frac{\exp(-ik_0 r)}{r} \quad \text{II.B.12}$$

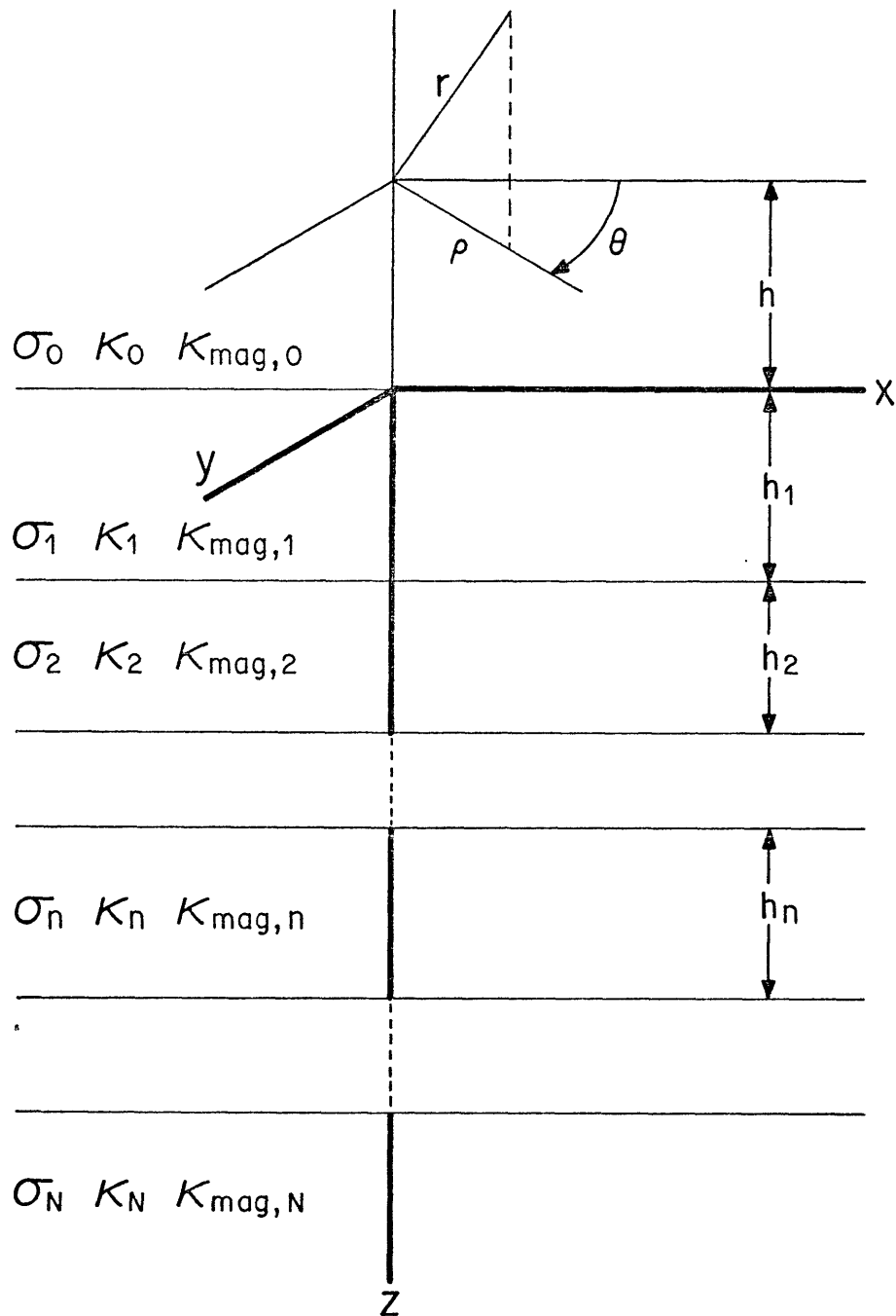


Figure 1. Coordinate system for treating dipole sources a height h above a horizontally layered lunar model

where

$$r = \left[\rho^2 + (z+h)^2 \right]^{\frac{1}{2}}$$

and

$$k_o^2 = \frac{\omega^2}{c^2} K_{o, \text{mag}} K_o^u$$

The secondary fields in each layer may then be expressed in terms of an electric Hertz vector with only a z component which must satisfy the homogeneous Helmholtz equation:

$$\left(\nabla^2 + k_n^2 \right) \Pi_{n,z}^s = 0 \quad n = 0, 1, \dots, N \quad \text{II.B.13}$$

In the upper region we may combine II.B.12 and II.B.13 and write the total electric Hertz vector as:

$$\Pi_{o,z} = \Pi_{o,z}^p + \Pi_{o,z}^s \quad \text{II.B.14}$$

where

$$\left(\nabla^2 + k_o^2 \right) \Pi_{o,z} = 0 \quad \text{II.B.15}$$

except at the source. In cylindrical coordinates we may write the solutions of II.B.13 and II.B.15 in terms of linear combinations of

$$\begin{aligned} \exp(\pm i k_{n,z} z) H_o^1(k_\rho \rho) \\ \exp(\pm i k_{n,z} z) H_o^2(k_\rho \rho) \end{aligned} \quad \text{II.B.16}$$

where H_o^1 and H_o^2 are the Hankel functions of the first and second kind, $k_n^2 = k_\rho^2 + k_{n,z}^2$, and $k_{n,z}$ and k_ρ are defined in the fourth quadrant of the complex plane. The solutions involving $H_o^1(k_\rho \rho)$ may be rejected as they represent incoming waves at infinity. The general solution for $\Pi_{n,z}$ may then be written:

$$\Pi_{n,z} = \int_{-\infty}^{\infty} \left[a_n \exp(-i k_{n,z} z) + b_n \exp(i k_{n,z} z) \right] H_0^z(k_p \rho) d k_p \quad \text{II.B.17}$$

Writing II.B.12 as:

$$\Pi_{0,z}^P = \frac{I \Delta l}{8 \pi i \omega \epsilon_0 \check{K}_0} \int_{-\infty}^{\infty} \frac{H_0^z(k_p \rho) k_p \exp[-|z+h| \sqrt{k_p^2 - k_0^2}]}{(k_p^2 - k_0^2)^{\frac{1}{2}}} d k_p \quad \text{II.B.18}$$

the solution in the upper region may then be written:

$$\Pi_{0,z} = \frac{1}{2} \int_{-\infty}^{\infty} \left[\frac{a_0 \exp(-i k_{0,z} |z+h|)}{(k_p^2 - k_0^2)^{\frac{1}{2}}} + \frac{b_0 \exp(i k_{0,z} (z-h))}{(k_p^2 - k_0^2)^{\frac{1}{2}}} \right] H_0^z(k_p \rho) k_p d k_p \quad \text{II.B.19}$$

where

$$a_0 = \frac{I \Delta l}{4 \pi i \omega \epsilon_0 \check{K}_0}$$

The boundary conditions requiring continuity of tangential \underline{E} and \underline{H} at the interfaces are obtained from II.B.2 and 3. In terms of the electric Hertz vector, they require that at the interface between the n^{th} and $(n-1)^{\text{st}}$ media:

$$\frac{\partial \Pi_{n,z}}{\partial z} = \frac{\partial \Pi_{n-1,z}}{\partial z} \quad \text{II.B.20}$$

and

$$\check{K}_n \Pi_{n,z} = \check{K}_{n-1} \Pi_{n-1,z} \quad \text{II.B.21}$$

Substitution of II.B.17 and II.B.19 into II.B.20 and II.B.21 leads to $2(N-1)$ equations for the $2(N-1)$ unknown coefficients, and the solution is obtained by analogy with the plane wave solution for parallel incidence of Appendix B.

In the upper region the fields are obtained, using II.B.2 and 3, from the electric Hertz vector:

$$\Pi_{0,z} = \frac{I \Delta l}{4\pi i \omega \epsilon_0 \check{K}_0} \left[\frac{\exp(-i k_0 r)}{r} + \frac{1}{2} \int_{-\infty}^{\infty} \frac{R_{\parallel} \exp(i k_{0,z}(z-h)) H_0^2(k_{\rho} \rho) k_{\rho} d k_{\rho}}{(k_{\rho}^2 - k_0^2)^{\frac{1}{2}}} \right] \quad \text{II.B.22}$$

where R_{\parallel} is the plane wave reflection coefficient for parallel incidence.

Considering now the magnetic dipole source II.B.11 with $\hat{\zeta} = \hat{y}$, the fields in the x-z plane will be transverse magnetic and are determined from a magnetic Hertz vector with only a y component, $\Pi_{n,y}^*$, which in the plane is symmetric with respect to y. In analogy with the vertical electric dipole case, the solution in the x-z plane for the upper region may be written:

$$\Pi_{0,y}^* = \frac{I \Delta a}{4\pi} \left[\frac{\exp(-i k_0 r)}{r} + \frac{1}{2} \int_{-\infty}^{\infty} \frac{R_{\parallel} \exp(i k_{0,z}(z-h)) H_0^2(k_{\rho} \rho) k_{\rho} d k_{\rho}}{(k_{\rho}^2 - k_0^2)^{\frac{1}{2}}} \right] \quad \text{II.B.23}$$

where the fields are now determined using II.B.5 and 6.

Considering the source to be the magnetic dipole of II.B.11 with $\hat{\zeta} = \hat{z}$, we may write the fields in terms of a magnetic Hertz vector with only a z component. Proceeding as before, the magnetic Hertz vector in the upper region is written:

$$\Pi_{0,z}^* = \frac{I \Delta a}{4\pi} \left[\frac{\exp(-i k_0 r)}{r} + \frac{1}{2} \int_{-\infty}^{\infty} \frac{R_{\perp} \exp(i k_{0,z}(z-h)) H_0^2(k_{\rho} \rho) k_{\rho} d k_{\rho}}{(k_{\rho}^2 - k_0^2)^{\frac{1}{2}}} \right]$$

where R_{\perp} is the plane wave reflection coefficient for perpendicular incidence, and the fields are determined from II.B.5 and 6.

II. C. POLES OF THE TRANSVERSE MAGNETIC PLANE WAVE REFLECTION
COEFFICIENT IN SIMPLIFIED CASES

As shown in II.B., the formal solutions of the excitation problems are obtained as contour integrals involving either the parallel or the perpendicular plane wave reflection coefficients, as obtained in Appendix B. To determine the physical significance of the poles of the reflection coefficients, we first examine the parallel reflection coefficient for a half space overlaying a two layer medium in which medium one is a perfect dielectric of depth h_1 and medium two is a terminating half space of infinite conductivity. The condition that the reflection coefficient for parallel incidence, $R_{||}$, have a pole, then simplifies to:

$$\frac{k_{0,z}}{\epsilon_0 \omega K_0} + \frac{k_{1,z}}{\epsilon_0 \omega K_1} \tanh(i k_{1,z} h_1) = 0 \quad \text{II.C.1}$$

where $k_{n,z} = a'_n - i b'_n$ with $a'_n, b'_n \geq 0$ and $k_n^2 = k_{n,z}^2 + k_\rho^2$

If we choose k_ρ as positive real with $k_\rho < k_1$, $k_{1,z}$ will be positive real. In the limit that the upper region approaches infinite conductivity, each term in II.C.1 approaches zero, and the condition for a pole becomes:

$$\frac{-i a'_1}{\epsilon_0 \omega K_1} \tan(-a'_1 h_1) = 0 \quad \text{II.C.2}$$

and is satisfied if

$$a'_{1,m} h_1 = m\pi \quad m = 0, 1, \dots, \infty$$

where a'_1 may not exceed $\frac{\omega}{c} K_1$. These poles then represent the transverse magnetic wave guide modes.

If the upper region represents the solar wind plasma in cutoff, the first term in II.C.1 becomes positive imaginary, and a smaller value of $a'_{1,m}$ is required to satisfy each mode condition. Thus the zero order mode vanishes, and its disappearance coincides with the emergence of an independent surface wave solution, for which $k_\rho > k_1$ and $k_{1,z} = -ib'_1$. The condition for this surface wave pole is given by:

$$S_0 - \frac{ib'_1}{\epsilon_0 \omega k_1} \tanh(b'_1 h_1) = 0 \quad \text{II.C.3}$$

This surface wave solution remains when the conducting basement is removed and the dielectric is allowed to extend to infinity.

No surface wave is present in the case of the plane wave reflection coefficient for perpendicular incidence, R_\perp , and hence we restrict our further attention to the vertical electric and horizontal magnetic dipole sources.

II. D. APPROXIMATE EVALUATIONS OF THE ELECTROMAGNETIC FIELDS ALONG THE LUNAR SURFACE

The integral in II.B.22 and 23 is now evaluated, using the contour of Figure 2 as R goes to infinity, where the modes are now considered leaky in order to handle losses in the lunar models. In the leaky mode approximation, the poles of the plane wave reflection coefficient for parallel incidence are determined numerically from III.B.7 by allowing k_ρ to acquire a negative imaginary part while restricting k_ρ and $k_{n,z}$ to the fourth quadrant of the complex plane.

In addition to the wave mode poles, the integrand possesses branch points at the origin and infinity due to the Hankel function, and at $k_\rho = k_n$. The branch points due to the Hankel function and to the dielectric layers, for which k_n is pure real, may be connected through infinity in the upper half plane, and do not affect the integration. The branch points corresponding to the conducting basement and the cutoff plasma are characterized by k_n having a large imaginary part. These branch points are connected through infinity as shown for $k_\rho = \pm k_0$ in Figure 2. The contributions to the contour integral from these branch cut integrations are seen to be heavily damped, and may be neglected for distances at which the primary fields become insignificant.

Under these conditions II.B.22 and 23 may be written as residue sums over the leaky modes, $k_{\rho,j}$:

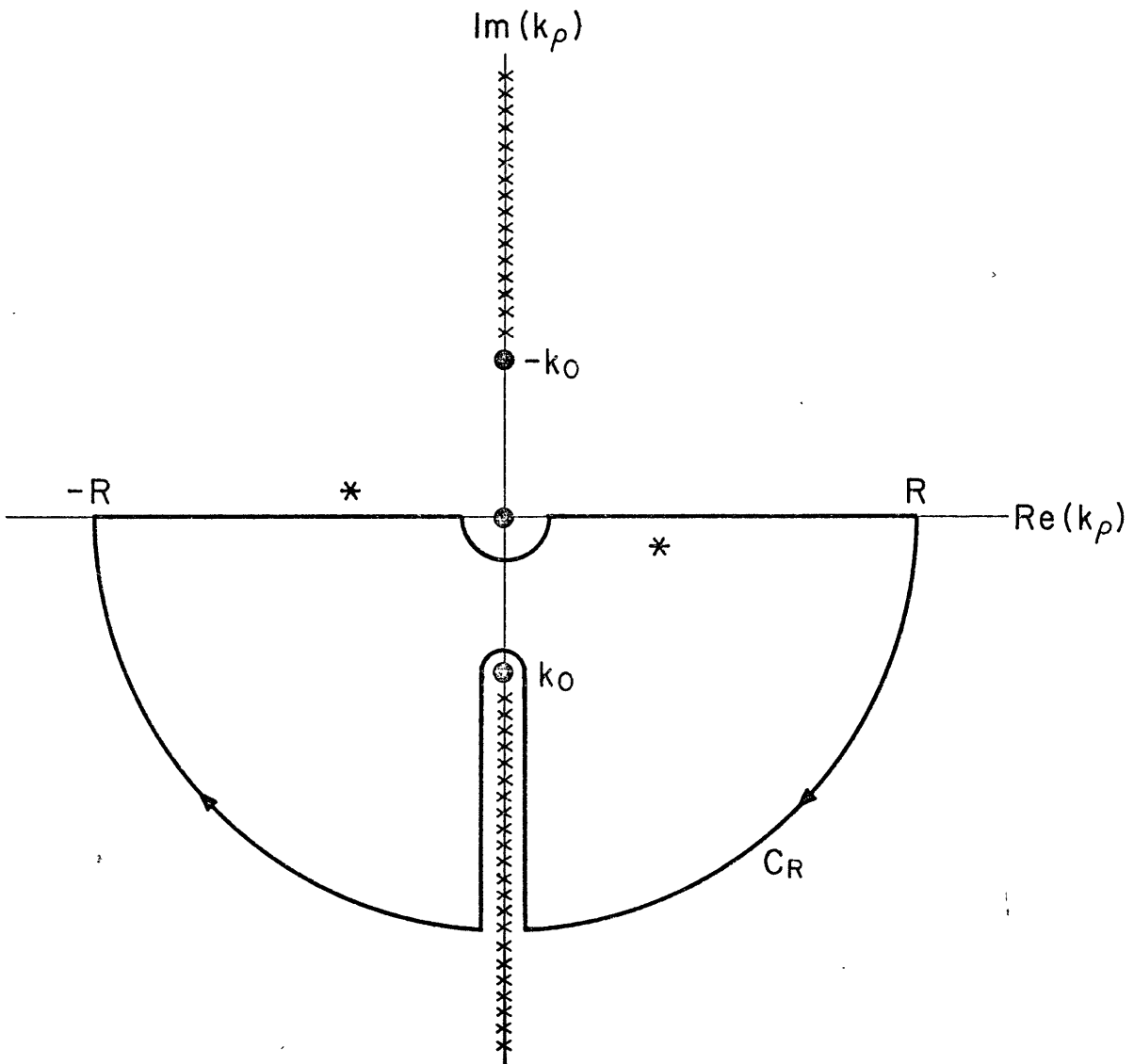


Figure 2. Integration contour in the complex k_ρ plane

$$\begin{pmatrix} \Pi_{0,z} \\ \Pi_{0,y}^* \end{pmatrix} \approx \begin{pmatrix} \frac{I \Delta \rho}{4\pi i \omega \epsilon_0 K_0} \\ \frac{I \Delta a}{4\pi} \end{pmatrix} \sum_j -i\pi \frac{(S_0 - Z_1) \exp(i k_{0,z}(z-h)) H_0^2(k_p \rho) k_p}{\frac{d}{dk_p} (S_0 + Z_1) (k_p^2 - k_0^2)^{\frac{1}{2}}} \Bigg|_{k_p = k_{p,j}}$$

II.D.1

Source elevations h , or receiver elevations z , on the order of lunar orbital heights are seen to be impractical for examining the modes because of the severe mode field attenuation due to the term $\exp(ik_{0,z}(z-h))$. We therefore confine further examination of the fields to sources and receivers with elevations above the lunar surface on the order of meters, for which this attenuation is negligible.

In Figure 3 the magnitude of the transverse magnetic field of the surface wave and of the primary field are plotted as a function of frequency and radial distance for a vertical electric dipole source of unit strength (one ampere meter), where the moon is modeled as a dielectric with $K=9$. It is seen that for radial distances greater than about ten kilometers, the approximation made in writing the fields as a mode sum is valid in the frequency range where the surface wave is present.

The addition of a conducting basement confines the low frequency surface wave fields, and may introduce wave guide mode fields. For the wet lunar model, the depth of the basement is too shallow for the wave guide modes to be excited in the frequency range over which the surface wave is present.

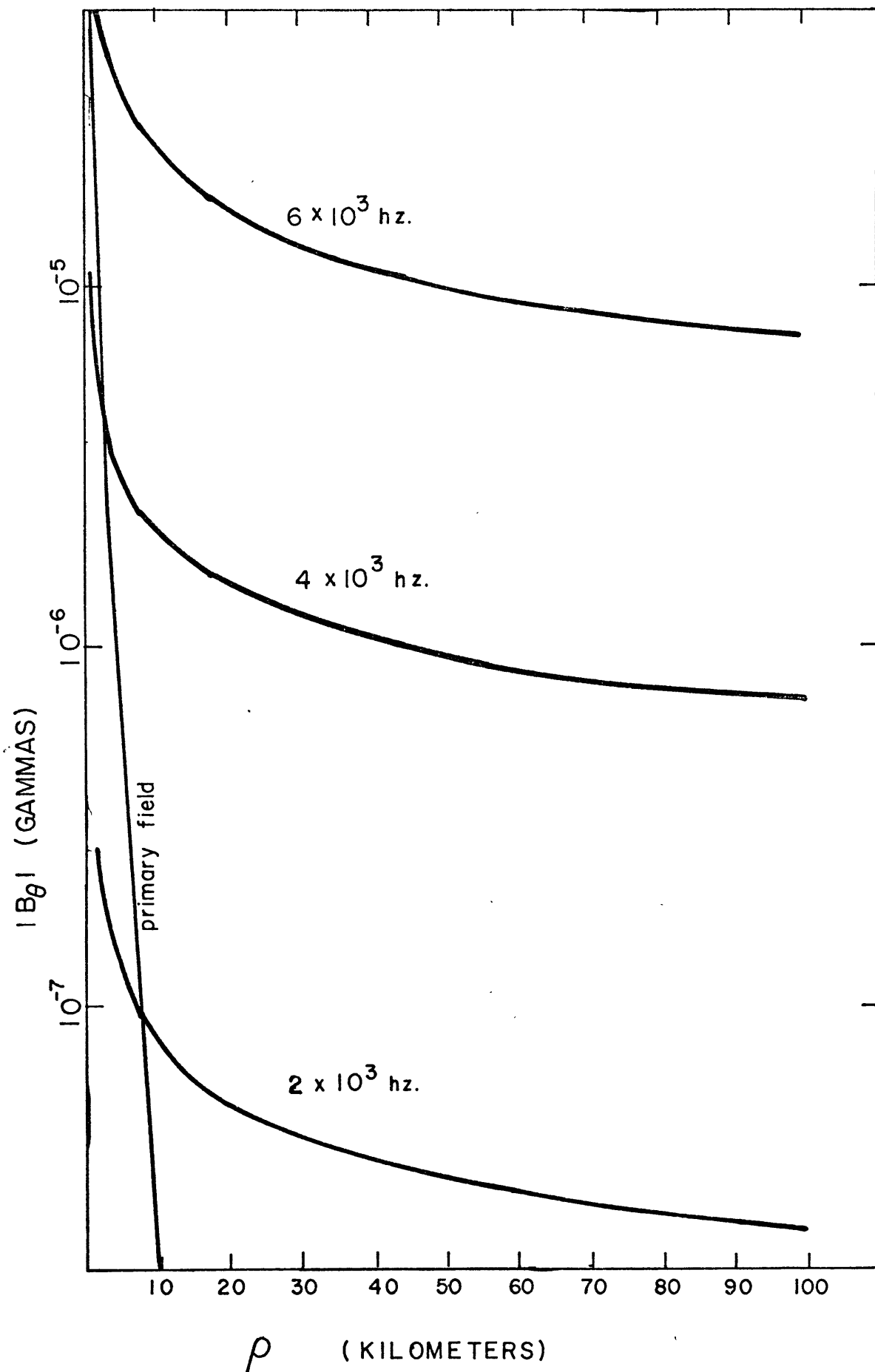


Figure 3. Magnitude of the TM fields of a unit vertical electric dipole. $K_1=9$, $h_1 = \text{infinity}$

For the dry lunar model, the basement depth is greater than the penetration depth expected for the surface wave above one kilohertz, and thus the surface wave will be confined more than the wave guide modes.

In Figure 4 a conducting basement is introduced into the previous example at a depth of one hundred kilometers, and the surface wave and wave guide mode fields compared at four kilohertz. The fields due to the wave guide modes are found negligible above one kilohertz for the vertical electric dipole, and above two kilohertz for the horizontal magnetic dipole, compared to the respective surface wave fields.

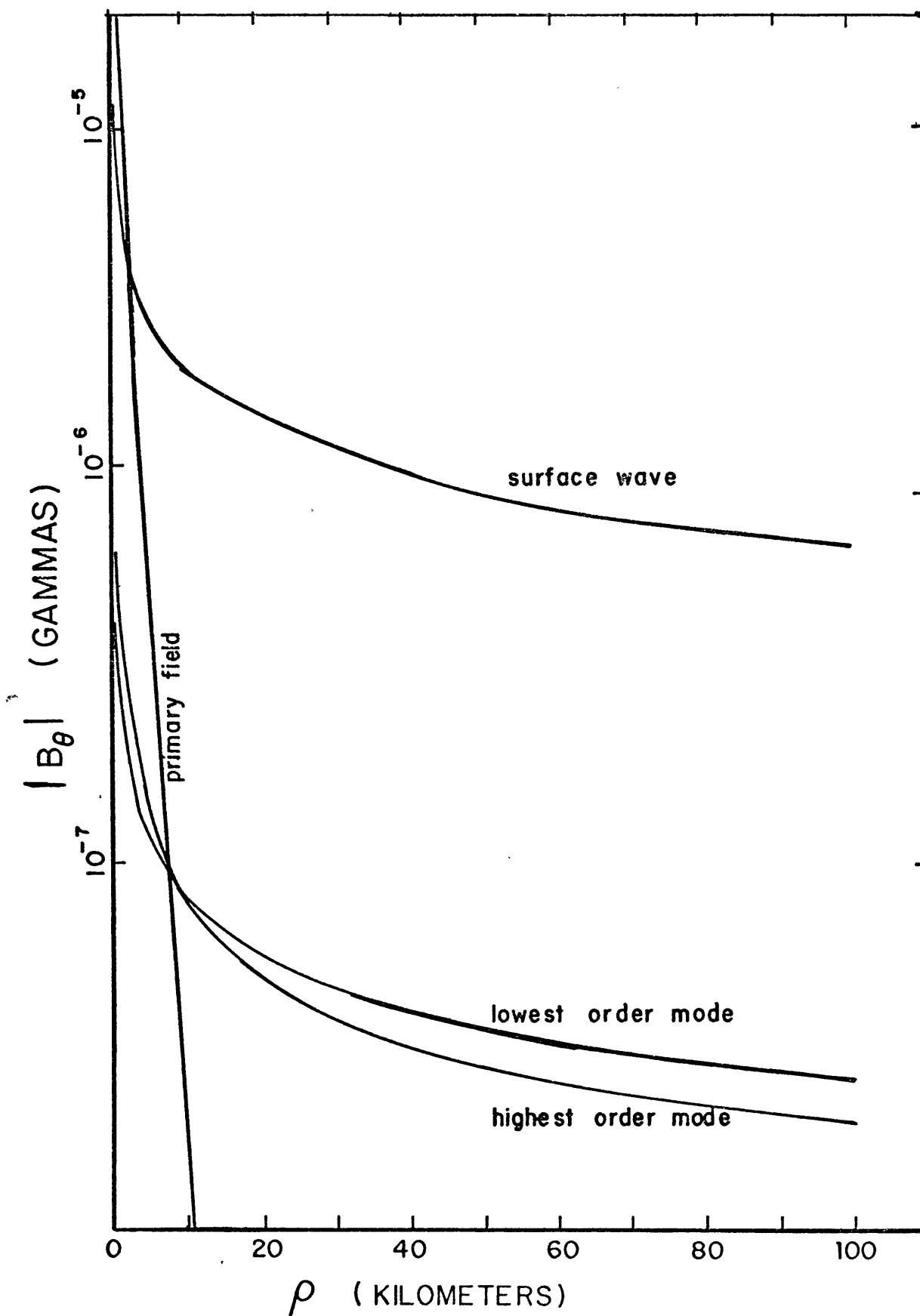


Figure 4. Magnitude of the transverse magnetic fields of a unit vertical electric dipole near the lunar surface $K_1=9$, $\nu=4 \times 10^3$ Hertz, $h_1 = \text{infinity}$

FIGURE 4

II.E DISCUSSION OF RESULTS

From section II.D we expect both major models of the lunar electromagnetic properties to admit a surface wave solution between one and ten kilohertz, the fields of which dominate the surface fields of vertical electric and horizontal magnetic dipoles located within meters of the lunar surface, at distances greater than about ten kilometers.

The horizontal propagation characteristics of these surface waves for varying lunar electromagnetic properties are shown in Figures 5 through 10. In each case the plasma frequency is taken as 27×10^3 Hertz.

In Figures 5 and 6, the phase and group velocities of the surface wave are shown for a dielectric moon, with K a constant with depth. The dielectric constant of the lunar material should be on the order of nine, and thus a surface wave is expected for frequencies below about ten kilohertz.

In Figure 7, a change in the dielectric constant from $K=$ nine to twelve is introduced at different depths. This change is typical of a change in dielectric properties that might be associated with a change in rock type. It is seen that a variation of this type is detectable to a maximum probable depth of about fifty kilometers for frequencies above one kilohertz.

In figure 8, a basement with a conductivity of 10^{-3} mhos/meter is introduced at different depths, and the associated attenuation coefficient, $\text{Im}(k_\rho)$, is shown in Figure 9. As in

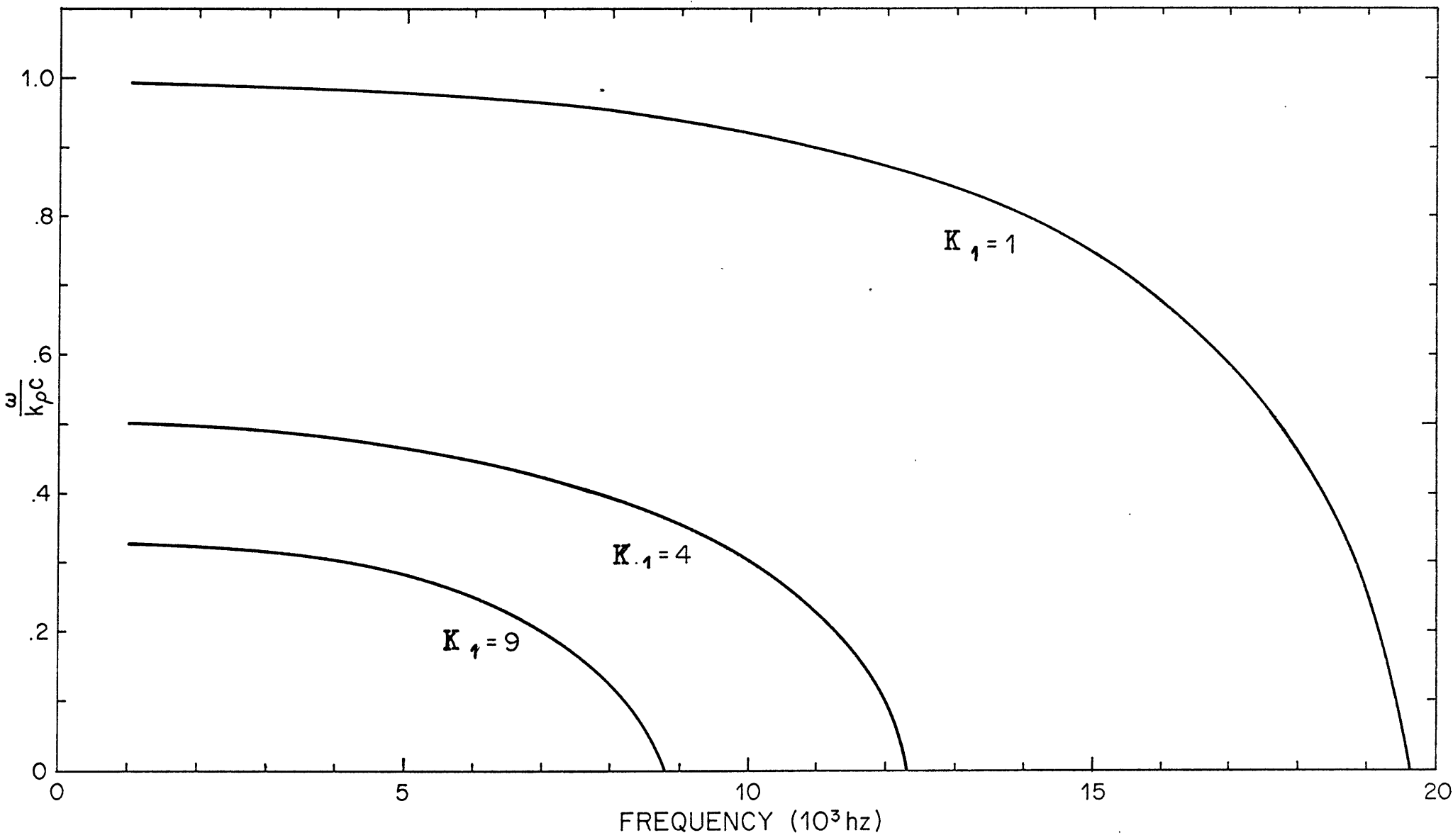


Figure 5. Phase velocity of the surface wave as a function of lunar dielectric constant (normalized to the velocity of light) $h = \infty$

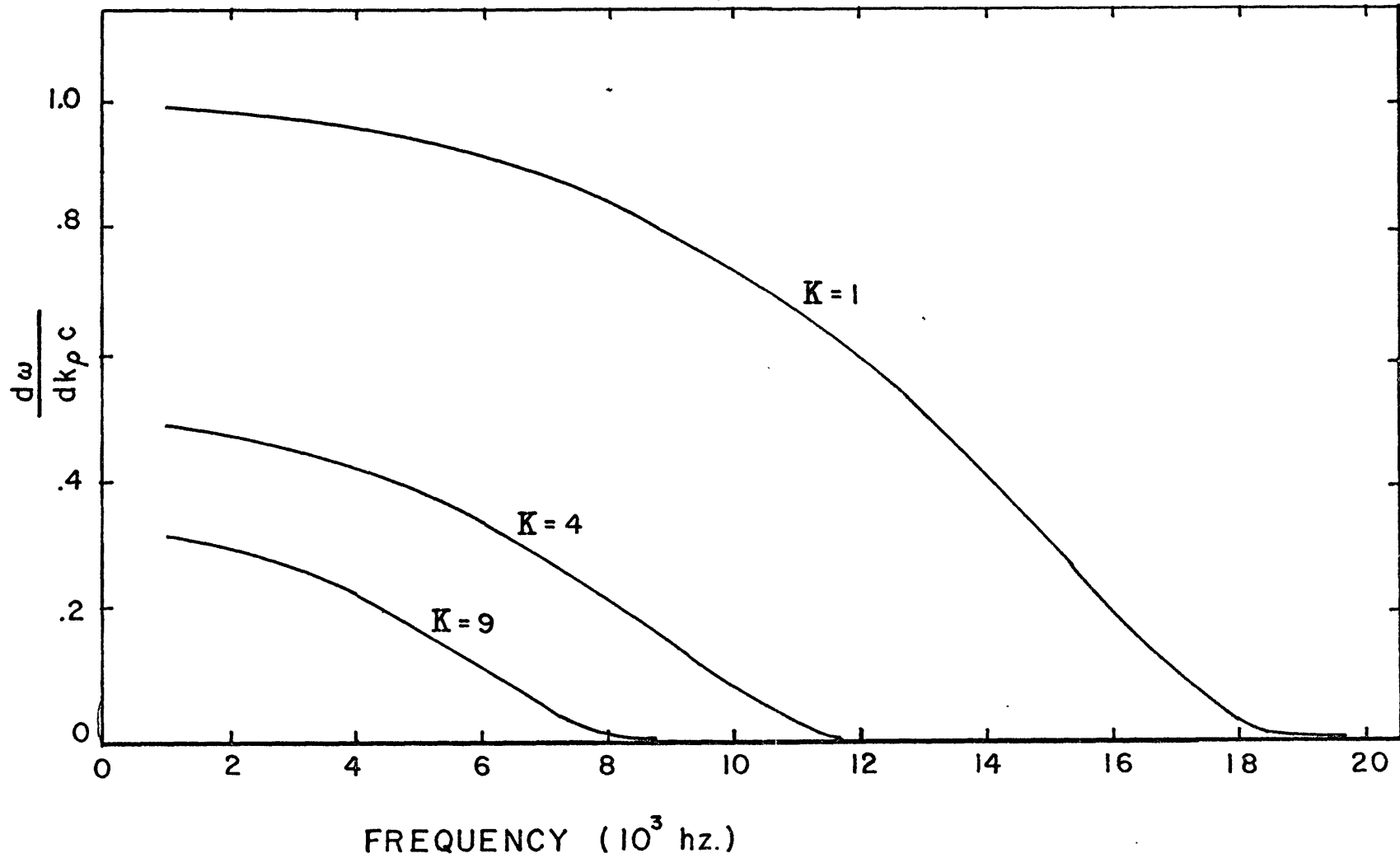


Figure 6. Group velocity of the surface wave as a function of lunar dielectric constant (normalized to the velocity of light) $h_1 = \infty$

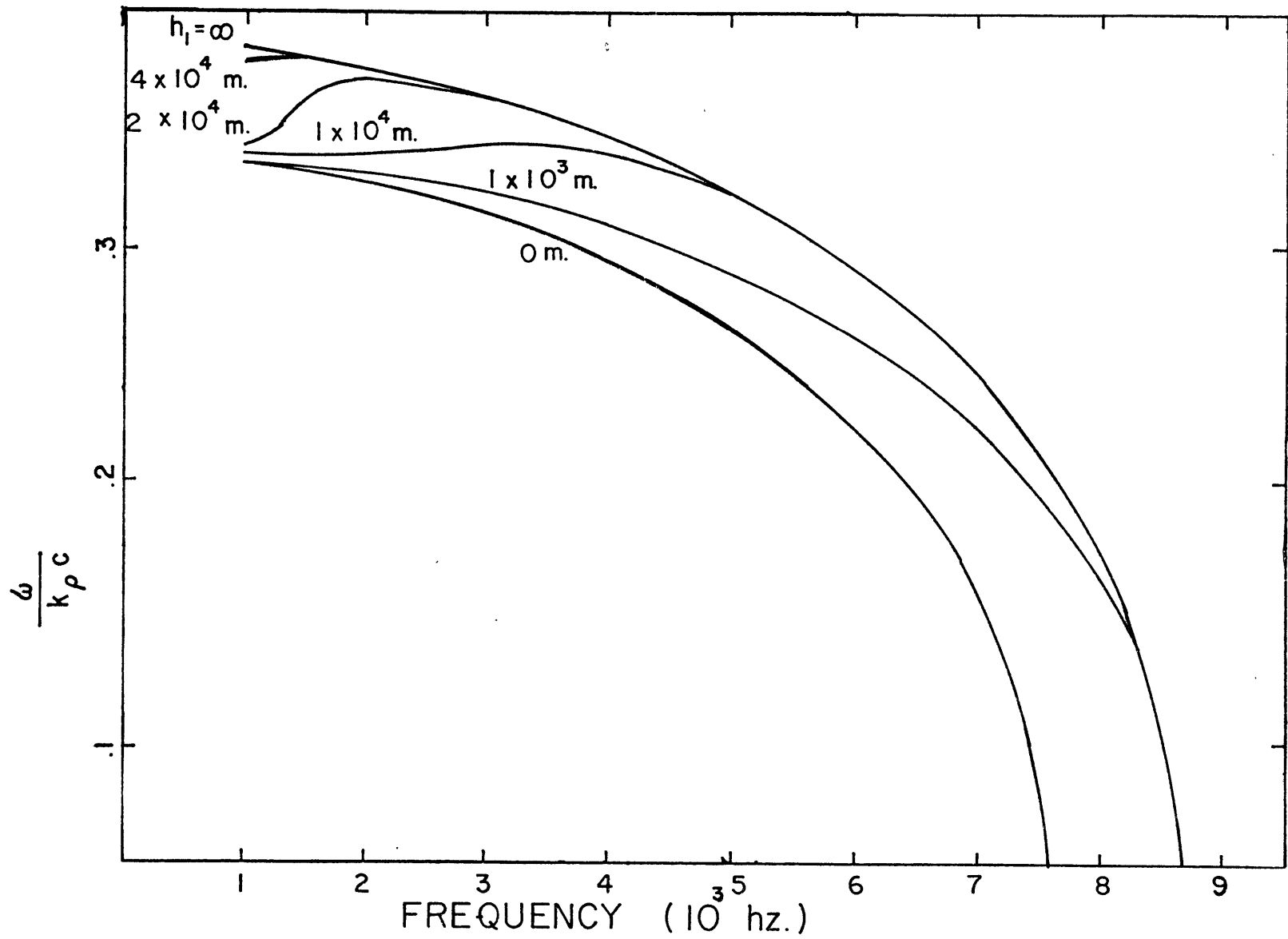


Figure 7. Relative phase velocity of the surface wave for $K_1=9$, $K_2=12$ and the transition occurs for $z=h_1$

FIGURE 7

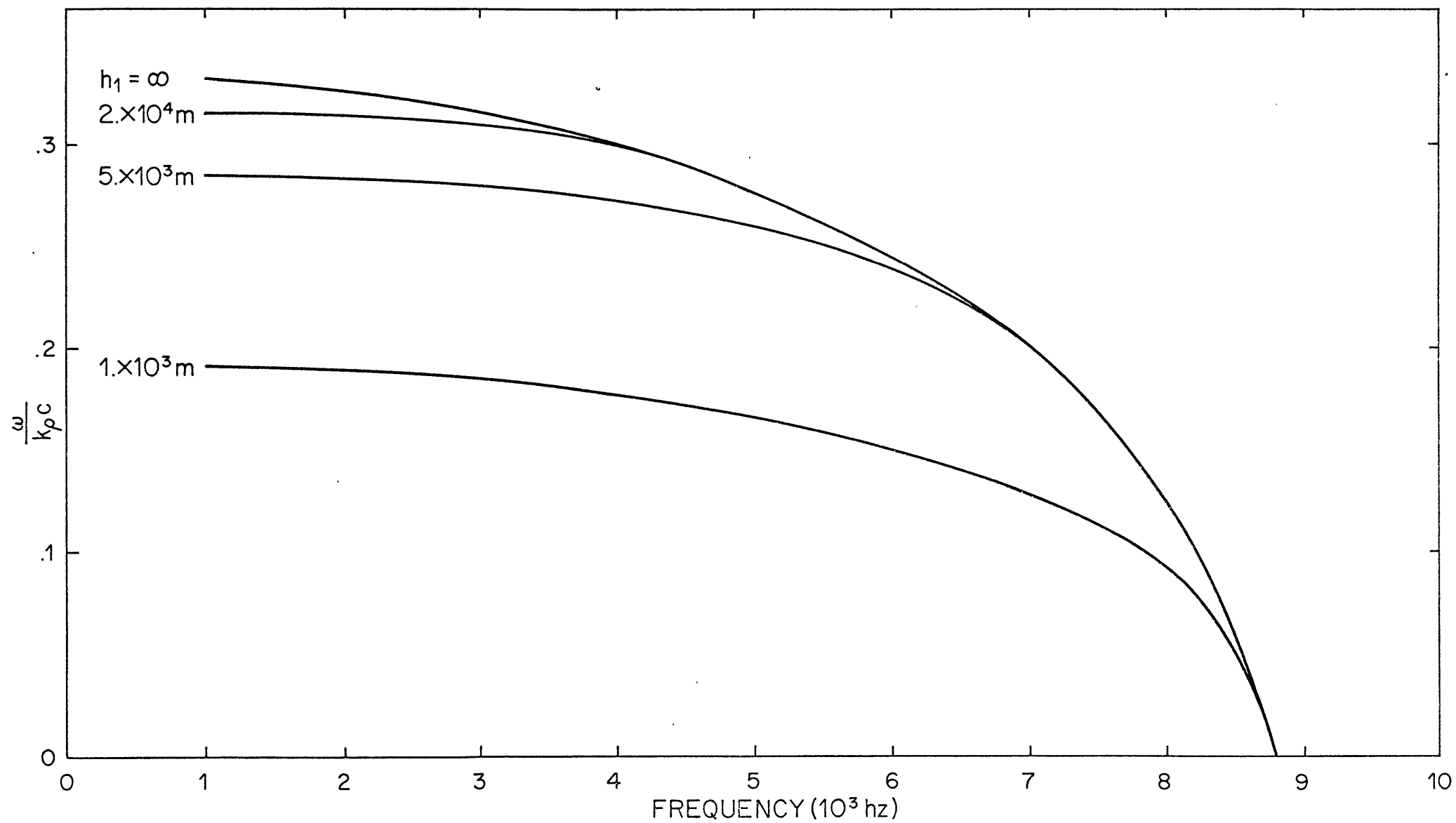


Figure 8. Relative phase velocity of the surface wave for $K_1=9$ and a conducting basement at $z=h_1$

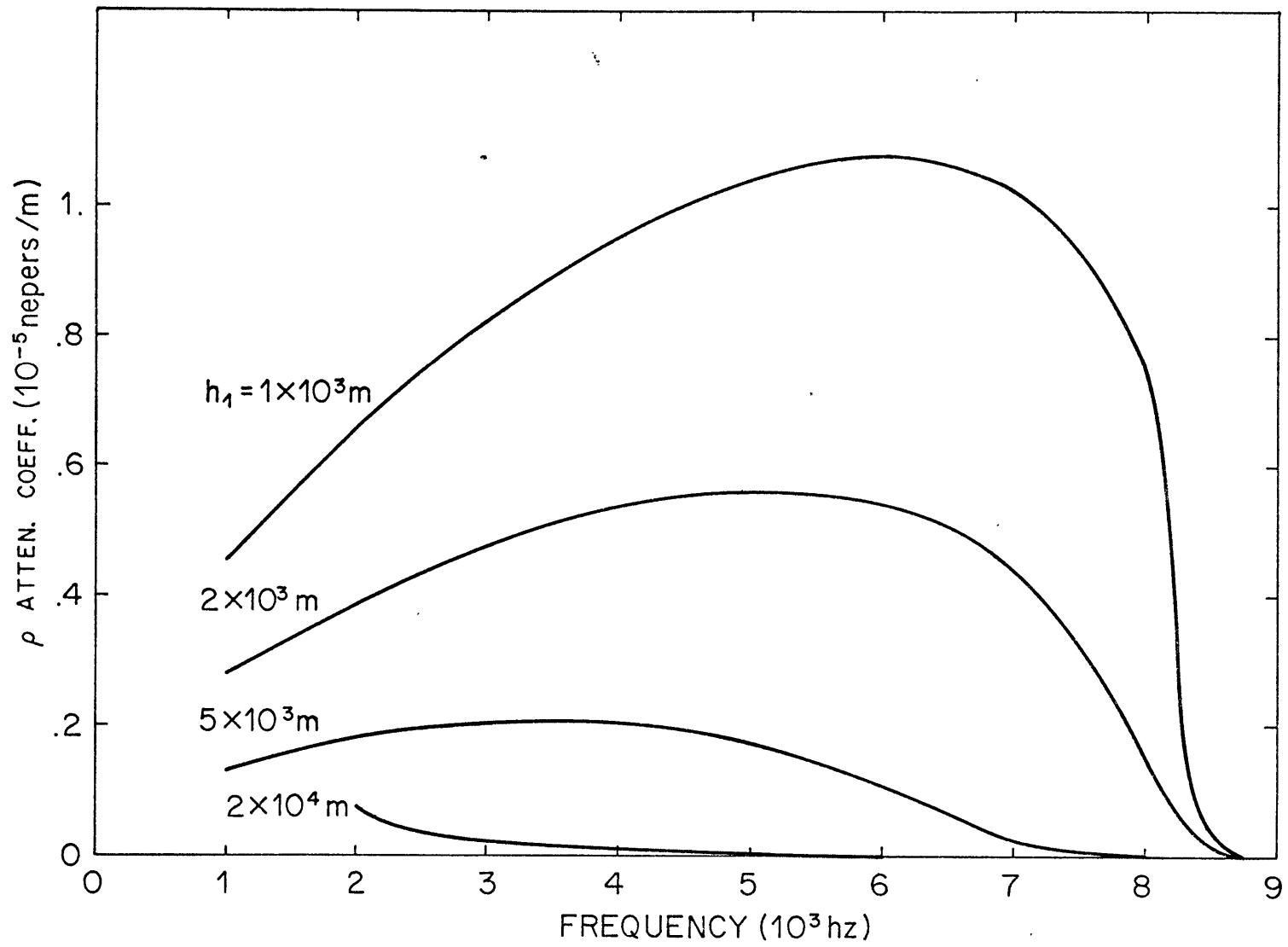


Figure 9. Attenuation coefficient for $\sigma_2=10^{-3}$ mhos/meter
 $K_1=9$, and the conducting basement starts at $z=h_1$

FIGURE 9

the case of variations in the dielectric properties, conductive basements below about fifty kilometers are not probed by the surface wave above one kilohertz. In Figure 10, the attenuation coefficient for the wet model is presented for different values of the basement conductivity.

In addition to variations in the electromagnetic properties of the moon with depth, the propagation characteristics of the surface wave are sensitive to variations in the electromagnetic properties of the solar wind plasma, which may be in time. In the higher frequency region of the surface wave propagation, the surface wave propagation is sensitive to the electron density, as shown in Figure 11. In the low frequency region, the direction and magnitude of the solar wind magnetic field become important as ω_c/ω becomes non negligible with respect to unity. To avoid the complexities in this frequency region, the present analysis has been restricted to frequencies above one kilohertz.

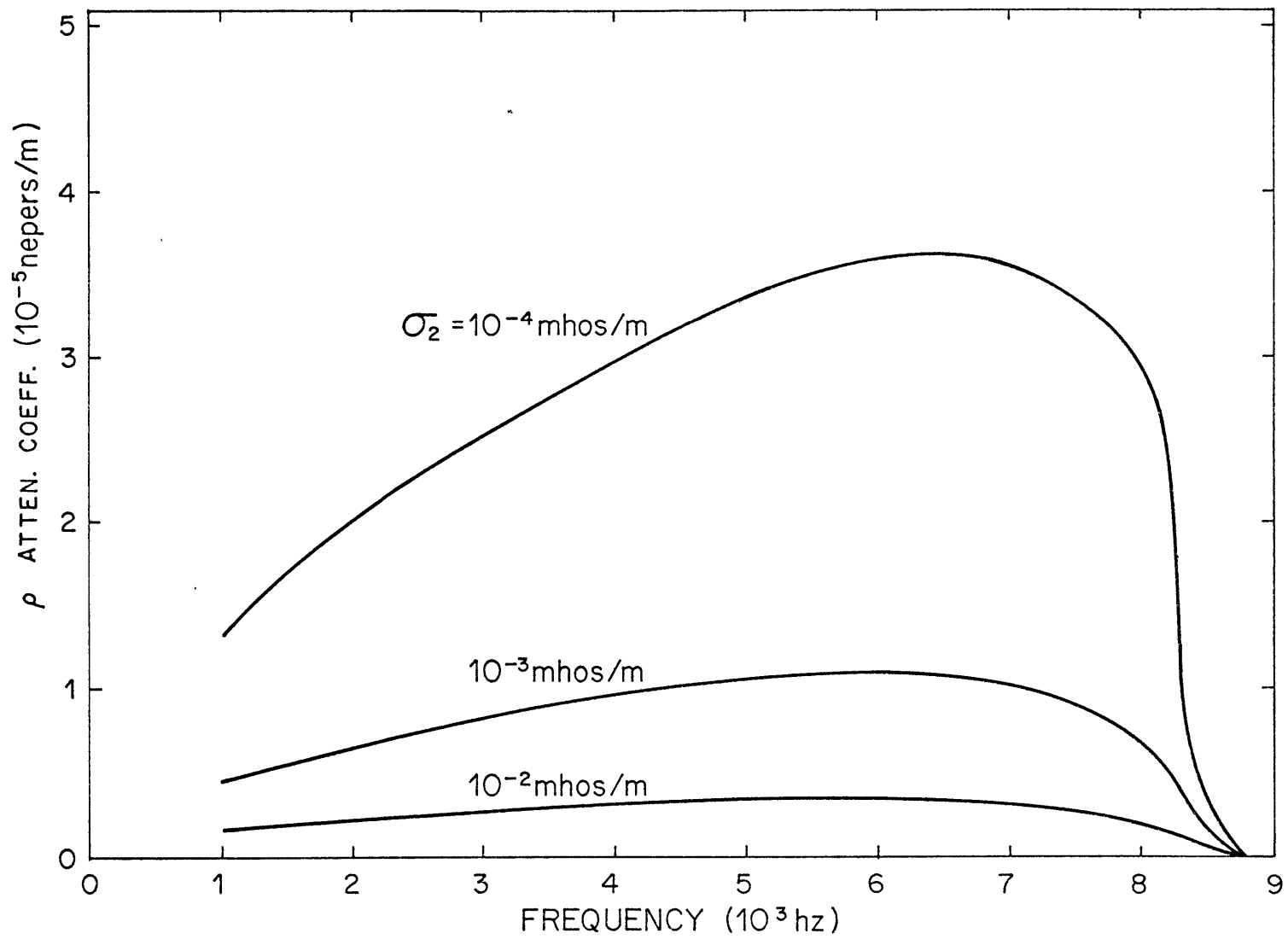


Figure 10. Attenuation coefficient for a conducting basement at h_1 =one kilometer. $K_1=9$

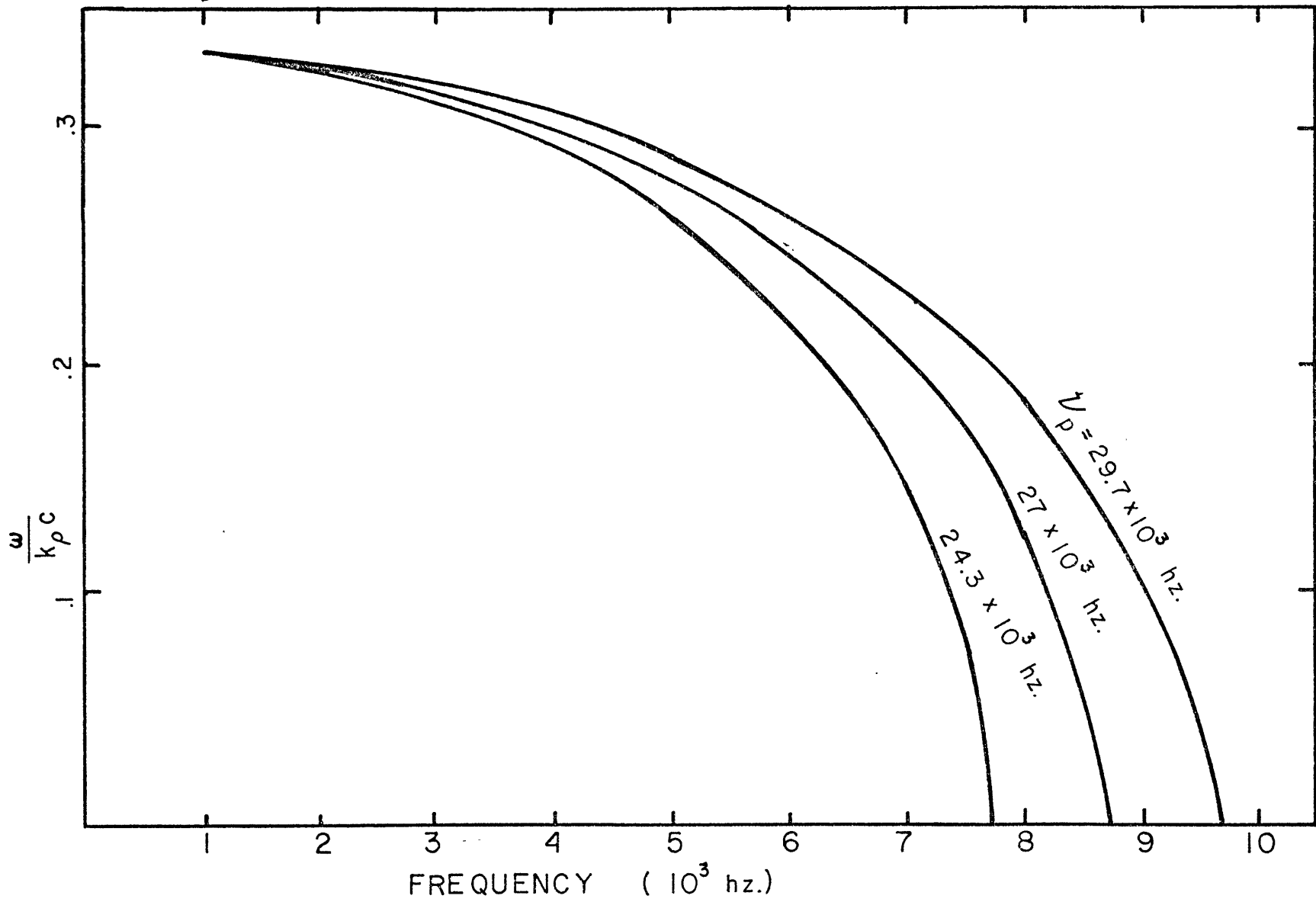


Figure 11. Relative phase velocity of the surface wave for changes of ν_D plus and minus ten percent in the plasma frequency

FIGURE 11

III. APPENDICES

III. A. ELECTROMAGNETIC WAVE PROPAGATION

In rationalized mks units the Maxwell equations for a macroscopic medium are:

$$\nabla \cdot \underline{D} = \rho \quad \text{III.A.1}$$

$$\nabla \cdot \underline{B} = 0 \quad \text{III.A.2}$$

$$\nabla \wedge \underline{E} = - \frac{\partial \underline{B}}{\partial t} \quad \text{III.A.3}$$

$$\nabla \wedge \underline{H} = \underline{J} + \frac{\partial \underline{D}}{\partial t} \quad \text{III.A.4}$$

where the field vectors are related by the constitutive relations:

$$\underline{D} = \epsilon_0 \underline{E} + \underline{P} \quad \text{III.A.5}$$

$$\underline{H} = \underline{B} / \mu_0 - \underline{M} \quad \text{III.A.6}$$

In a linear medium III.A.5 and III.A.6 become:

$$\underline{D} = \epsilon_0 \underline{K} \cdot \underline{E} \quad \text{III.A.7}$$

$$\underline{B} = \mu_0 \underline{K}_{\text{mag}} \cdot \underline{H} \quad \text{III.A.8}$$

and the Maxwell equations are supplemented by Ohm's law:

$$\underline{J} = \underline{\sigma} \cdot \underline{E} \quad \text{III.A.9}$$

If in addition to being linear the medium is isotropic, equations III.A.7, 8, and 9 reduce to:

$$\underline{D} = \epsilon_0 K \underline{E} \quad \text{III.A.10}$$

$$\underline{B} = \mu_0 K_{\text{mag}} \underline{H} \quad \text{III.A.11}$$

$$\underline{J} = \sigma \underline{E} \quad \text{III.A.12}$$

Under the assumption that the fields are oscillatory with $\exp(i\omega t)$ time dependence, Ampere's law then becomes:

$$\nabla \wedge \underline{H} = (\sigma + i\omega\epsilon_0 K) \underline{E} \quad \text{III.A.13}$$

Because of the form of III.A.13 it is convenient to define a complex relative dielectric constant:

$$\check{K} = K - \frac{i\sigma}{\epsilon_0\omega} = K(1 - i \tan \delta) \quad \text{III.A.14}$$

where $\tan \delta$ is the loss tangent of the medium and represents the ratio of conduction current to displacement current. The formalism of a complex dielectric constant absorbs both the explicit charge and the current density in the Maxwell equations, which may now be written:

$$\nabla \cdot \epsilon_0 \check{K} \underline{E} = 0 \quad \text{III.A.15}$$

$$\nabla \cdot \mu_0 K_{\text{mag}} \underline{H} = 0 \quad \text{III.A.16}$$

$$\nabla \wedge \underline{E} = -i\omega\mu_0 K_{\text{mag}} \underline{H} \quad \text{III.A.17}$$

$$\nabla \wedge \underline{H} = i\omega\epsilon_0 \check{K} \underline{E} \quad \text{III.A.18}$$

For a homogeneous medium we may combine the curl of III.A.18 with III.A.17 to obtain an equation for the magnetic field vector,

$$\nabla \wedge (\nabla \wedge \underline{H}) = \frac{\omega^2}{c^2} K_{\text{mag}} \overset{\cup}{K} \underline{H} \quad \text{III.A.19}$$

which is reduced by III.A.16 to the homogeneous Helmholtz equation for the cartesian components of the magnetic field vector:

$$\left(\nabla^2 + \frac{\omega^2}{c^2} K_{\text{mag}} \overset{\cup}{K} \right) \underline{H} = 0 \quad \text{III.A.20}$$

Taking the curl of III.A.17 and combining with III.A.18 and III.A.15 we obtain an identical equation for the cartesian field vector \underline{E} :

$$\left(\nabla^2 + \frac{\omega^2}{c^2} K_{\text{mag}} \overset{\cup}{K} \right) \underline{E} = 0 \quad \text{III.A.21}$$

If the fields are those of a plane wave in the form:

$$\exp(i\omega t - i\mathbf{k} \cdot \mathbf{r}) \quad \text{III.A.22}$$

equations III.A.20 and III.A.21 admit the solution

$$\mathbf{k}^2 = \mathbf{k} \cdot \mathbf{k} = \frac{\omega^2}{c^2} K_{\text{mag}} \overset{\cup}{K} \quad \text{III.A.23}$$

If the linear, isotropic, homogeneous medium under consideration is a temperate plasma in which the collision frequency of momentum transfer for electrons and the ion motion may be neglected, the electron equation of motion may be solved for $\exp(i\omega t)$ fields to yield the complex relative dielectric constant

$$\overset{\cup}{K} = 1 - \frac{\omega_p^2}{\omega^2} \quad \text{III.A.24}$$

where $\omega_p = \left(\frac{n_e q_e^2}{\epsilon_0 m_e} \right)^{1/2}$ is the one dimensional free oscillation frequency of the electron component of the plasma and q_e , m_e , and n_e are respectively the electron charge, mass, and number density.

If the plasma medium is in a magnetostatic field the propagation is anisotropic and the tensor forms of the relative dielectric constant and the conductivity are maintained through use of equations III.A.7 and III.A.9. Proceeding as in the isotropic case, the complex relative dielectric constant is defined as:

$$\underline{\underline{K}}^c = \underline{\underline{1}} - \frac{i \underline{\underline{g}}}{\epsilon_0 \omega} \quad \text{III.A.25}$$

where $\underline{\underline{1}}$ is the unity matrix. For plane waves of the form III.A.22, equation III.A.21 is replaced by:

$$\underline{k} \wedge (\underline{k} \wedge \underline{E}) + \frac{\omega^2}{c^2} \underline{\underline{K}}^c \cdot \underline{E} = 0 \quad \text{III.A.26}$$

Again neglecting ion motion and the electron collision frequency, the tensor complex relative dielectric constant becomes in cartesian form:

$$\underline{\underline{K}}^c = \begin{pmatrix} 1 - \frac{\omega_p^2}{\omega^2 (1 - \frac{\omega_c^2}{\omega^2})} & - \frac{i \omega_p^2 (\frac{\omega_c}{\omega})}{\omega^2 (1 - \frac{\omega_c^2}{\omega^2})} & 0 \\ \frac{i \omega_p^2 (\frac{\omega_c}{\omega})}{\omega^2 (1 - \frac{\omega_c^2}{\omega^2})} & 1 - \frac{\omega_p^2}{\omega^2 (1 - \frac{\omega_c^2}{\omega^2})} & 0 \\ 0 & 0 & 1 - \frac{\omega_p^2}{\omega^2} \end{pmatrix} \quad \text{III.A.27}$$

where $\omega_c = \left| \frac{q_e B}{m_e} \right|$ and $\underline{B} = B\hat{z}$.

In the limit $\frac{\omega_c}{\omega} \ll 1$, the tensor III. A. 27 may be replaced by the scalar III.A.24, thereby making the propagation isotropic.

III. B. REFLECTION OF PLANE WAVES FROM A HORIZONTALLY STRATIFIED MEDIUM CONSISTING OF N ELECTROMAGNETICALLY LINEAR, ISOTROPIC, HOMOGENEOUS LAYERS.⁹

As shown in Appendix A, the electromagnetic properties of the n^{th} linear, homogeneous, isotropic layer may be characterized by the independent parameters σ_n , K_n , and $K_{n,\text{mag}}$. In addition, if the fields are those of a plane wave, the cartesian field vectors in the n^{th} layer satisfy the homogeneous Helmholtz equation

$$\left(\nabla^2 + k_n^2 \right) \begin{matrix} \underline{E} \\ \underline{H} \end{matrix} = 0$$

III.B.1

where $k_n^2 = \frac{\omega^2}{c^2} \overset{\vee}{K}_n K_{n,\text{mag}}$ and $\overset{\vee}{K}_n = K_n - \frac{i\sigma_n}{\epsilon_0 \omega}$

Choosing the coordinate system of Figure 1, we consider a transverse magnetic plane wave having only a horizontal magnetic field component to be incident upon the stratified medium. The incident magnetic field may then be written as

$$H_{0,y} \exp(i\omega t - i \underline{k}_0 \cdot \underline{r})$$

III.B.2

From symmetry, the magnetic field in the n^{th} layer will have only a y component which will satisfy the scalar homogeneous Helmholtz equation

$$\left(\nabla^2 + k_n^2\right) H_{n,y} = 0 \quad \text{III.B.3}$$

The general solution to III.B.3 may be written as

$$H_{n,y} = \left[a_n \exp(-i k_{n,z} z) + b_n \exp(i k_{n,z} z) \right] \exp(-i k_x x) \quad \text{III.B.4}$$

where $k_n^2 = k_{n,z}^2 + k_x^2$ and $k_{n,z} = a'_n - i b'_n$ with $a'_n, b'_n \geq 0$. Also, since the incident wave is infinite in extent, k_x is pure real.

The boundary conditions on the fields at the interfaces require the continuity of tangential \underline{E} and \underline{H} . In terms of the magnetic field the conditions at the interface between the n^{th} and $(n-1)^{\text{st}}$ layers may be written as:

$$H_{n,y} = H_{n-1,y} \quad \text{III.B.5}$$

and from III.A.18

$$\frac{\partial H_{n,y}}{\partial z} \left(\frac{1}{i\omega\epsilon_0 K_n} \right) = \frac{\partial H_{n-1,y}}{\partial z} \left(\frac{1}{i\omega\epsilon_0 K_{n-1}} \right) \quad \text{III.B.6}$$

From III.B.2, $a_0 = H_{0,y}$ and the assumption that the terminating layer N is semi-infinite requires $b_N = 0$. Substitution of III.B.4 into III.B.5 and III.B.6 then yields $2(N-1)$ equations for the $2(N-1)$ unknown coefficients in terms of the known coefficient a_0 . The solution obtained using transmission line theory is:

$$R_{11} = \frac{a_0}{b_0} = \frac{S_0 - Z_1}{S_0 + Z_1} \quad \text{III.B.7}$$

where

$$Z_{n\parallel} = \begin{cases} S_n \frac{Z_{n+1} + S_n \tanh(i k_{n,z} h_n)}{S_n + Z_{n+1} \tanh(i k_{n,z} h_n)} & n = 1, 2, \dots, N-1 \\ S_N & n = N \end{cases}$$

and

$$S_n = \frac{k_{n,z}}{\epsilon_0 \omega K_n} \quad n = 1, 2, \dots, N$$

In the above solution, R_{\parallel} is the reflection coefficient for parallel incidence and Z_1 is the surface impedance.

If we now consider a transverse electric plane wave with only a horizontal electric field to be incident upon the stratified half space, we may proceed by analogy with the previous case of parallel incidence, now writing the electric field in the n^{th} layer as:

$$E_{n,y} = \left[a_n \exp(-i k_{n,z} z) + b_n \exp(i k_{n,z} z) \right] \exp(-i k_x x) \quad \text{III.B.8}$$

The interface boundary conditions are then written in terms of the electric field:

$$E_{n,y} = E_{n-1,y} \quad \text{III.B.9}$$

and using III.A.17:

$$\frac{\partial E_{n,y}}{\partial z} \left(\frac{1}{i\omega\mu_0 K_{n,mag}} \right) = \frac{\partial E_{n-1,y}}{\partial z} \left(\frac{1}{i\omega\mu_0 K_{n-1,mag}} \right)$$

III.B.10

Equations III.B.8, 9, and 10 may be transformed to equations III.B.4,5, and 6 for parallel incidence by making the substitutions:

$$\begin{aligned} E_{n,y} &\rightarrow H_{n,y} \\ \mu_0 K_{n,mag} &\rightarrow \epsilon_0 \tilde{K}_n \end{aligned}$$

III.B.11

Therefore, by analogy with III.B.7, the solution for perpendicular incidence is:

$$R_{\perp} = \frac{b_0}{a_0} = \frac{S'_0 - Y_1}{S'_0 + Y_1}$$

III.B.12

where

$$Y_n = \begin{cases} S'_n \frac{Y_{n+1} + S'_n \tanh(i k_{n,z} h_n)}{S'_n + Y_{n+1} \tanh(i k_{n,z} h_n)} & n=1,2,\dots,N-1 \\ S'_N & n=N \end{cases}$$

and

$$S'_n = \frac{k_{n,z}}{\omega\mu_0 K_{n,mag}} \quad n=1,2,\dots,N$$

In analogy with the surface impedance Z_1 , Y_1 represents the surface admittance.

IV. REFERENCES

1. England, A.W., G. Simmons, and D. Strangway, Electrical conductivity of the moon, J. of Geophys. Res., 73, pp. 3219-3226, 1968.
2. Chung, D.H., W.B. Westphal, and G. Simmons, Dielectric properties of Apollo 11 lunar samples and their comparison with earth materials, J. of Geophys. Res., Nov. 1970.
3. Parkhomenko, E.I., Electrical Properties of Rocks, Plenum Press, New York, 1967.
4. Keller, G.V., Electrical properties of rocks and minerals, Handbook of Physical Constants, edited by S.P. Clark, Jr., pp. 553-571, Geological Society of America, New York, 1966.
5. Chung, D.H., W.B. Westphal, and G. Simmons, Dielectric properties of Apollo 12 lunar samples, Trans. A.G.U., 51, pp. 582, 1970.
6. St. Amant, M., and D.W. Strangway, Dielectric properties of dry geological materials
7. Rossi, B., and S. Olbert, Introduction to the Physics of Space, McGraw Hill, New York, 1970.
8. Johnson, F.S., and J.E. Midgley, Notes on the lunar magnetosphere, J. of Geophys. Res., 73, pp. 1523-1532, 1968.
9. Wait, J.R., Electromagnetic Waves in Stratified Media, Pergamon Press, New York, 1962.
10. Stratton, J.A., Electromagnetic Theory, McGraw Hill, New York, 1941.
11. Allis, W.P., S.J. Buchbaum, and A. Bers, Waves in Anisotropic Plasmas, MIT Press, 1963.