XVII. ELECTRODYNAMICS OF MEDIA*

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RESEARCH OBJECTIVES

During the several years past, work has been in progress on the formulation of the fundamental equations of electrodynamics, with particular attention devoted to obtaining correct expressions for the macroscopic force density distributions for various media of interest. The results of this work, including some results of last year's research will be published as a research monograph by The M. I. T. Press, Cambridge, Massachusetts.[1]

In a self-consistent formulation of electrodynamics, an important question is concerned with the correct identification of momentum density of other than kinetic origin. The feasibility study of a deflection experiment of a polarizable body in a magnetic field to distinguish between a widely accepted expression for the momentum density and the one that we have concluded to be the correct one is now under way.

The study of electrodynamics of moving media led unavoidably into a general treatment for nonlinear media. Such media are of present interest in the optical frequency range and some of their properties are of great engineering importance. In this connection a study is under way of the utilization of nonlinear amplification to obtain from an (optical) amplifier output pulses of duration that is short compared with the inverse (linear) bandwidth of the amplifier.

L. J. Chu, H. A. Haus

References

1. Paul Penfield, Jr. and Hermann A. Haus, Electrodynamics of Moving Media (The M. I. T. Press, Cambridge, Mass., in press); also Special Technical Report Number 14, Research Laboratory of Electronics, M. I. T., Cambridge, Mass.

A. NONLINEAR AMPLIFICATION

1. Introduction

An investigation of the possibility of obtaining and amplifying laser pulses of duration shorter than the inverse bandwidth of the active medium has been initiated. Of particular interest is the CO_2 transition at 10.6 μ. The transitions can be pumped efficiently; however, in the linear region pulse lengths not shorter than ~80 nsec can be obtained, because

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of the inherent bandwidth limitation of any linear amplifier.

In the nonlinear region, the rate at which inversion of a two-level system can be achieved is determined by the field intensity of the applied pulse, and not by the inverse linewidth. Moreover, if the pulse length is shorter than the transverse relaxation time, or becomes so, inertial terms associated with the polarization of such a two-level system become important.

For a linear bandwidth of 50 MHz, and gains and saturation levels typical of a CO_2 system, our results indicate that a peak power intensity ≈1.5 kW for a Gaussian pulse of 20 nsec in width should be sufficient to show some sharpening in 6 m. These power densities are obtainable with a Q-switched pulse. The realizability of the pulse lengths is more questionable.

Even though the model of a homogeneously broadened two-level system is an oversimplification of CO_2 laser operation, it is believed that guidelines for future work on CO_2 systems can be gleaned from these results.

2. Equations of Motion

The equations describing the amplifying medium and the amplifying wave are the density matrix equations for a two-level system and a forward traveling-wave equation for the field. The derivations can be found in several publications.[1-5] The resultant equations are

$$(\dot{\rho}_{22}-\dot{\rho}_{11}) = -\frac{(\rho_{22}-\rho_{11}) - \left(\rho_{22}^{e}-\rho_{11}^{e}\right)}{T_1} + \frac{2}{n\hbar\omega_o}\left(E_o J_p^* + E_o^* J_p\right) \tag{1}$$

$$\dot{J}_p = i\omega_o J_p - \frac{J_p}{T_2} - \frac{\omega_o n |p_{12}|^2}{\hbar}(\rho_{22}-\rho_{11})E_o \tag{2}$$

$$\frac{\partial E}{\partial t} + c\frac{\partial E_o}{\partial t} + \frac{\sigma}{2\epsilon}E_o = -\frac{J_p}{2\epsilon} \tag{3}$$

Here E_o is the field amplitude; J_p, the polarization current; ω_o, the optical frequency, which is assumed to coincide with the atomic frequency; n, the density of particles, which is assumed to be constant; T_1, the lifetime of the excited state; T_2, the polarization relaxation time; $\rho_{22} - \rho_{11}$, the difference between the probability of the upper state being occupied and the probability of the lower state being occupied; and $\rho_{22}^e - \rho_{11}^e$, the equilibrium value of $\rho_{22} - \rho_{11}$. The polarization matrix element connecting the two states is given by p_{12}.

It is convenient to normalize the time with respect to T_2,[1] and the field quantities with respect to the energy content associated with the equilibrium population difference, so that

$$F_o = \frac{\sqrt{2\epsilon_o}\, E_o}{\sqrt{n\hbar\omega_o\left(\rho_{22}^e-\rho_{11}^e\right)}} \tag{4}$$

$$K_o = \frac{T_2 J_p}{\sqrt{2\epsilon_o n\hbar\omega_o\left(\rho_{22}^e-\rho_{11}^e\right)}} \tag{5}$$

$$Z = \frac{z}{cT_2}; \qquad T = \frac{t}{T_2} \tag{6}$$

$$N = \frac{\rho_{22} - \rho_{11}}{\rho_{22}^e - \rho_{11}^e} \tag{7}$$

The equations then become

$$\frac{dN}{dT} = -\frac{N-1}{\tau_1} + 4F_oK_o; \qquad \tau_1 = \frac{T_1}{T_2} \to \infty \tag{8}$$

$$\frac{dK_o}{dT} = -K_o - ANF_o \tag{9}$$

$$\frac{\partial F_o}{\partial T} + \frac{\partial F_o}{\partial Z} + \frac{F_o}{\tau_o} = -K_o; \tag{10}$$

with

$$\frac{1}{\tau_o} = \frac{\sigma T_2}{2\epsilon_o}$$

$$A = \frac{|p_{12}|^2\, n\left(\rho_{22}^e-\rho_{11}^e\right) \hbar\omega_o T_2^2}{2\hbar^2\epsilon}$$

3. Integration of the Equations and Results

The set of three equations (8), (9), and (10) can be integrated numerically along the characteristics, two of which are along z = constant, and the other of which is given by V = Z - T = constant. The system is then essentially an infinite set of coupled ordinary differential equations. The Runge-Kutta fourth-order scheme was used. Spatial smoothing was added to the time-smoothed scheme, thereby creating a highly stable predictor-corrector algorithm.

The pulse-sharpening process can be described as follows. The leading edge of the pulse, if intense enough, will cause a depopulation of the upper state, which, if the maximum pulse amplitude is chosen correctly, becomes fully depleted only when the entire pulse has passed. This causes a preferred increase in the amplitude of the leading edge of the pulse. As the maximum amplitude tends to increase, however, the population becomes fully depleted before the entire pulse has passed. The polarization is zero for the fully depleted system; however, inertial effects cause "overshoot" and hence it changes sign. This causes the medium to become an absorber and thus the two-level system tends to re-invert at the expense of the field. Eventually the lagging edge of the field or polarization changes sign once more. This creates another region of population depletion (Eq. 8) trailing the preceding alternating regions of inversion and depletion. The consequence is a pulse which, upon traversing the medium, sharpens, increases in maximum intensity, and develops ringing (Fig. XVII-1).

Several things influence the behavior described above. If the relaxation time T_2 is so small that relaxation overshadows the gain, the polarization and population tend to relax very rapidly and the effect upon the field is diminished. The ringing is highly damped. In the limit when inertial effects are negligibly small, no ringing occurs; and no sharpening occurs except when the front of the pulse is extremely steep (slope of the

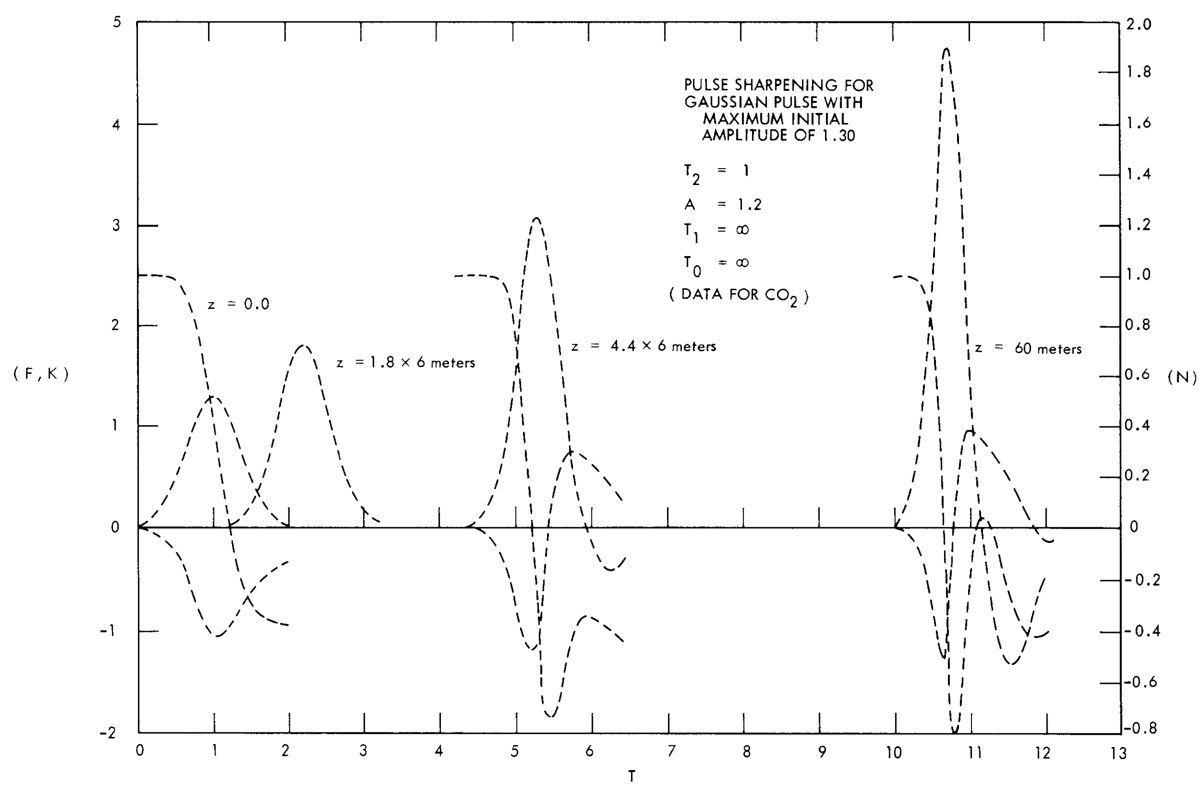

Fig. XVII-1. Pulse sharpening and developing ringing.

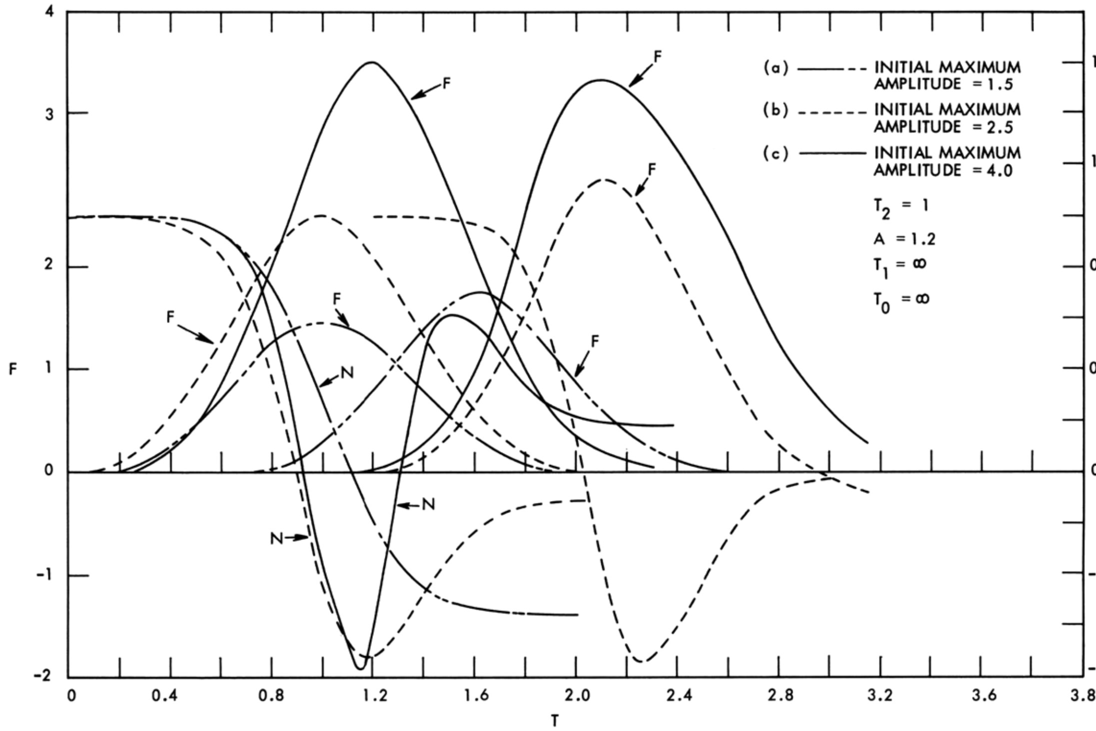

Fig. XVII-2. Initial changes of a Q-switched pulse penetrating an inverted medium.

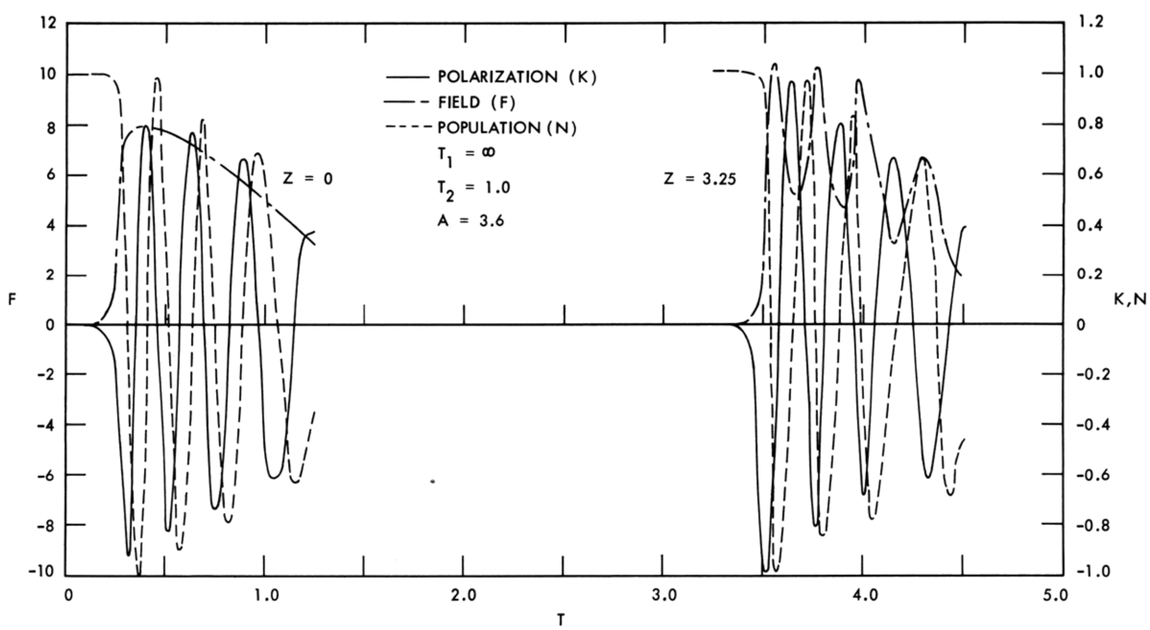

Fig. XVII-3. Development of a pulse with a short initial rise time.

order of $1/T_2$) and the amplitude very large. This case has been discussed by Basov and co-workers.[4] They also consider the effect of a saturable absorber on the sharpening of the characteristics when inertial effects are small. It then becomes a process of sharpening by preferred absorption, rather than by preferred amplification.

If the field amplitude is large, the population can become fully depleted before the peak value of the field has arrived. This causes a pronounced ringing to set in, and can cause the maximum amplitude to initially decrease as the pulse penetrates the medium. This effect is illustrated in Fig. XVII-2 (curve c), in which case the peak energy density associated with the field is much greater than that stored in the medium. The greatest gain in the pulse occurs well ahead of the maximum, which is itself contributing to re-inversion. Hence the maximum is decreasing and the pulse is broadening.

One means of achieving a very narrow pulse would be to chop the initial Q-switched pulse in such a way that its rise time would be very short. Then a much larger amplitude pulse could be used because, for maximum sharpening, the population should be fully depleted in a time comparable to the rise time and the energy required to accomplish this should almost be a constant. Figure XVII-3 shows the development of such a very short pulse. A strong ringing develops because of the large trailing pulse energy. This is essentially the step response obtained by Tang.[2]

The sharpening process can be enhanced by using saturable dyes in conjunction with the amplifying medium. This influence is now being studied with the same program.

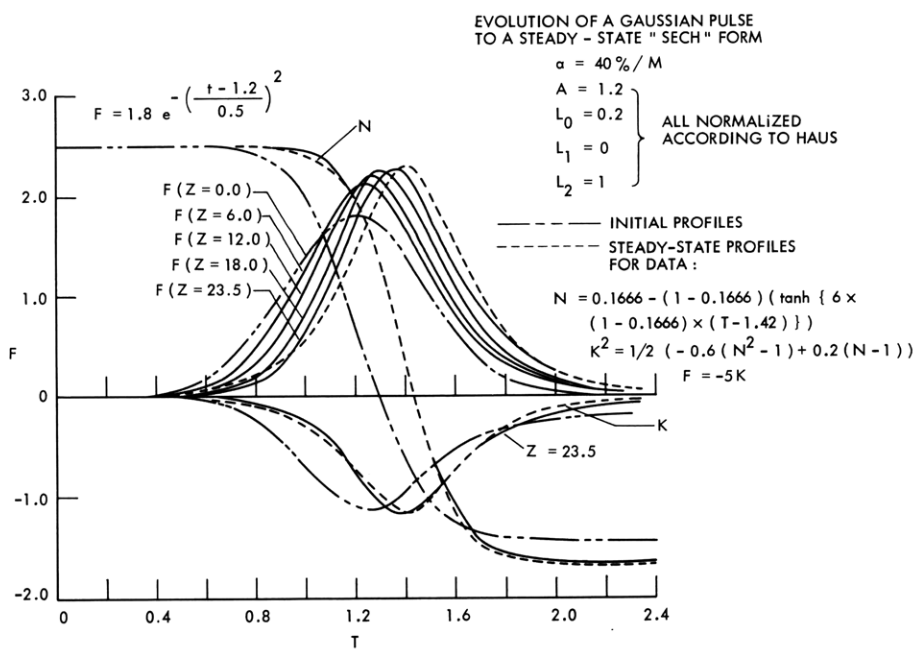

Fig. XVII-4. Evolution of a Gaussian pulse to a steady-state sech profile.

Field loss limits the amount of sharpening and gain which can be obtained from the inverted medium. If the pulse energy is less than a critical value, the pulse at first gains energy and then propagates in a steady-state manner. Similarly, if the pulse energy is greater than the critical value, the energy difference is lost to the medium. A steady-state solution for propagation at the speed of light is given[4, 5] by

$$N = \frac{L_o L_2}{A} - \left(1 - \frac{L_o L_2}{A}\right)\left[\tanh \frac{A}{L_o}\left\{1 - \frac{L_o L_2}{A} \; (T+T_o)\right\}\right]$$

$$K_o^2 = \frac{A}{4}\left(1 - \frac{L_o L_2}{A}\right)^2 \operatorname{sech}^2\left[\frac{A}{L_o}\left(1 - \frac{L_o L_2}{A}\right)(T+T_o)\right]$$

$$F_o = -\frac{K_o}{L_o}$$

$$L_o = \frac{1}{\tau_o}; \qquad L_2 = 1.0.$$

We have shown that an initial waveform of a Gaussian shape does tend to the sech $\{A/L_o(1-L_oL_2/A)(T+T_o\}$ profile (Fig. XVII-4). Thus, for a small L_o ($L_o \ll A/L_2$), the limiting pulsewidth is

$$\frac{1}{c}\left(\frac{L_o}{A}\right).$$

For CO_2, the pertinent data are

$$\lambda = \frac{2\pi c}{\omega_o} = 10.6\ \rho.$$

$$T_2 \approx 20 \times 10^{-9} \text{ sec.}$$

These data give

$$n\left(\rho_{22}^e - \rho_{11}^e\right) = 3.73 \times 10^{12}/\text{cm}^3$$

and

$$\hbar\omega_o\left(\rho_{22}^e - \rho_{11}^e\right) n = 0.69 \times 10^{-7} \text{ J/cm}^3.$$

For a 20-nsec pulse, a maximum amplitude of $F_o \approx 1.3$ is necessary for sharpening. This gives a peak-power intensity of ~1.5 KW/cm^2.

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