

## XII. MICROWAVE AND MILLIMETER WAVE TECHNIQUES

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### 1. NOISE IN NONLINEAR DEVICES

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The spectrum of thermal fluctuations in a physical variable, present in a linear system maintained at a given temperature in equilibrium, is given by Nyquist's theorem. An analogous, but more general, theorem has been derived for the fluctuation spectrum in a driven (i. e., nonequilibrium) nonlinear dissipative one-port system in contact with a heat reservoir at temperature  $T$ . We make the following assumptions:

The system does not store any energy so that the force and the flow variables  $\beta$  and  $\dot{a}$  are instantaneously related.

The system is in a stationary state, driven by a steady force  $\beta = \beta_0$ , which is small so that the system is not very far from equilibrium.

The system is terminated in such a way that the fluctuations in the force variable  $\beta$  are zero.

The transient behavior of  $a$ , as the system relaxes toward a stationary state from a nonstationary one, is the same regardless of whether the nonequilibrium state is reached as a result of spontaneous fluctuations or because of a temporarily applied excitation.

The fluctuations are small enough to be treated under the small-signal approximation.

These assumptions allow the autocorrelation function of the random component  $\dot{a}_s$  of the flow variable  $\dot{a}$  in steady state to be expressed as

$$\langle \dot{a}_s(t) \dot{a}_s(t+\tau) \rangle = 2kT \frac{\overline{\langle \dot{a}_r(t_1) \rangle \beta(t_1)}}{b(t_1-\tau) \beta(t_1)} \delta(\tau), \quad (1)$$

where  $\langle \dot{a}_r(t_1) \rangle$  is the incremental part of the system response due to an incremental zero average periodic force  $b(t_1)$  superimposed on the steady-state force, and the ensemble and time averages are represented by angular brackets and overbar,

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respectively. The fluctuation spectrum for positive frequencies is therefore given by

$$S_{\dot{a}\dot{a}}(f) = 4kT P_{ex} / b^2, \quad (2)$$

where  $P_{ex}$  is the average incremental power dissipation in the system due to  $b(t_1)$ .

This theorem can also be expressed in terms of the input-output characteristics of the nonlinear system. For example, identifying the variables  $\alpha$  and  $\beta$  with the terminal dc voltage  $V$  and current  $I$  of a nonlinear two-terminal device biased at a dc current  $I_0$  leads to the following expression for the mean-square noise voltage per unit bandwidth

$$\overline{v_n^2} = 4kT \left[ \frac{dV}{dI} + \frac{I}{2} \cdot \frac{d^2V}{dI^2} \right]_{I_0}. \quad (3)$$

This result yields the low-frequency noise spectra for Schottky-barrier, p-n junction, and tunnel diodes which are in agreement with those obtained from the first principles. A more detailed paper, entitled "Thermal Noise in Driven Nonlinear Resistive Systems," is being prepared for publication.

## 2. ATMOSPHERIC REFRACTION AT MILLIMETER WAVELENGTHS

Joint Services Electronics Program (Contract DAAB07-76-C-1400)

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The M.I.T. interferometer is now complete, and routine data taking is now in progress. During a late summer shutdown, the stepping motor drive systems were modified to provide position verification to the computer. This modification was required to insure reliable pointing in windy winter weather. The first measurements' refraction effects concentrated on high elevation angles, in order to understand the basic properties of the system.

The first observing program was designed to measure the instrumental and atmosphere stability over a period of weeks. Several quasars and the bright maser source W49N were observed. The data indicate that the long-term phase stability of the instrument is very good, that typically the observed phase on a given object and at a given sidereal time would not vary more than  $30^\circ$  from day to day. The data were taken with a short ( $813 \lambda$ ) and two long ( $\sim 12,000 \lambda$ ) baselines, chosen so that on the strong maser source the observed phase noise will be due to the instrumental local-oscillator system on the short baseline and to the atmosphere on the long baseline, respectively. The

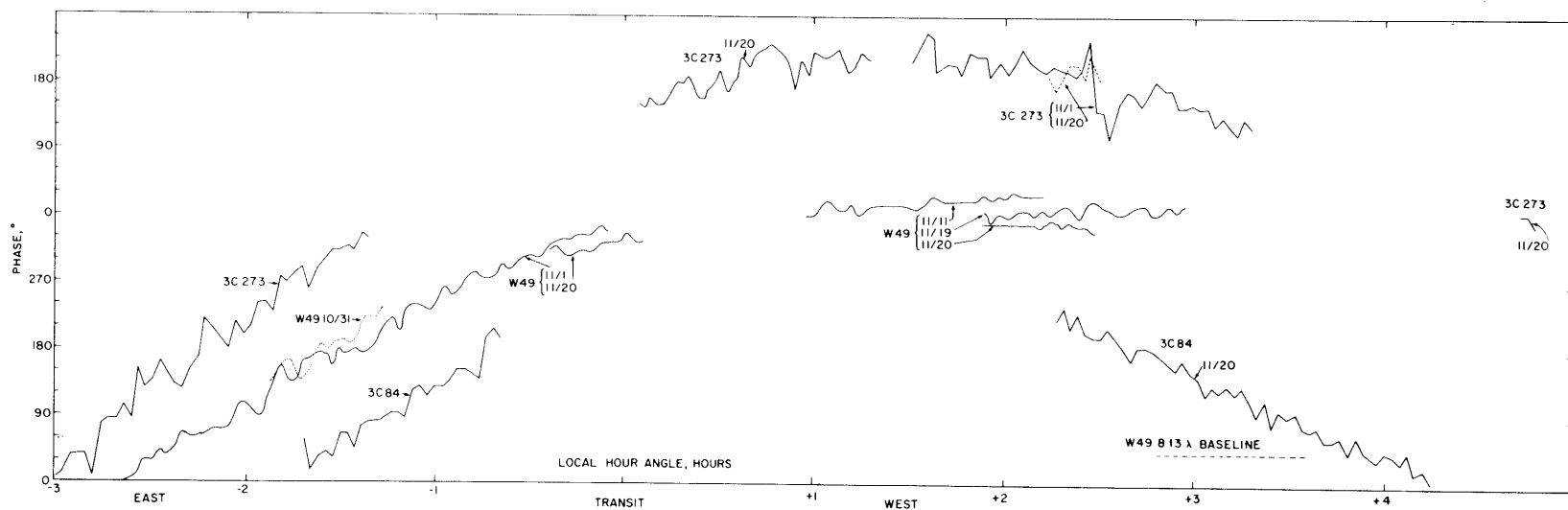


Fig. XII-1. The interferometer phase output as a function of local sidereal time. The solid and dotted lines represent the data from the long ( $\sim 12,000 \lambda$ ) baseline, while the dashed line represents data from the short ( $813 \lambda$ ) baseline. The W49 source dominates the receiver noise, so any phase fluctuation on the short baseline would be instrumental, and the fluctuation on the long baseline W49 data is due to the atmosphere.

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phase stability of the instrument over periods of hours appears to be of the order of  $1^\circ$  rms. Observations of the long baseline indicate that the atmosphere stability can be as good as  $3^\circ$  rms at antenna elevations as low as  $30^\circ$ . The repeatability of the instrument as shown in Fig. XII-1 is a display of phase data vs sidereal time taken with the long baseline averaged over 2.5 minutes. The curvature of the data is due to an error of approximately 1 cm in the positions assumed for the antennas when the data were taken, as is the phase offset associated with the differing declinations of the various sources. The instrumental phase stability is shown by the record marked "W49 813  $\lambda$  Baseline" in the figure.

An overview of the on-line computer program, SUPERSYS, has been prepared with an emphasis placed on the program's architecture and rationale. A similar explanation is given for the 75-MHz bandwidth digital correlation and delay line system. The system auto load is also described as an example of the programming necessary to build a user-oriented facility.