

TOPOGRAPHIC INFLUENCES ON THE PATH
OF THE GULF STREAM

by

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B.A., Amherst College
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Submitted to the Department of Geology and Geophysics on August 22, 1962, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

ABSTRACT

Meander patterns observed in the Gulf Stream east of Cape Hatteras exhibit correlations with bottom topography. A vorticity balance analysis reveals that if the current extends to the bottom of the ocean without reversing direction, as implied by some recent deep velocity measurements, its path must indeed be controlled principally by the topography of the Continental Rise, with a smaller constraint imposed by the meridional variation of the Coriolis parameter. Approximate current paths calculated according to these mechanisms agree sufficiently well with observed paths to confirm the topographic explanation of Gulf Stream meanders.

The observed variety in meander patterns is attributed to fluctuations in current direction and path curvature near Cape Hatteras. It is conjectured that the New England Seamounts may so deflect the Stream as to effect significant modifications in its path.

These conclusions make very doubtful any direct relation between large-scale meanders and possible instabilities of the Gulf Stream east of Cape Hatteras, and between the distribution of wind stress and the separation of Stream from coast.

Thesis Supervisor: William S. von Arx

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PREFACE

The bulk of this thesis consists of an article, intended for publication, concerning the role played by bottom topography in determining the existence and gross character of Gulf Stream meanders in the open ocean east of Cape Hatteras. It is complete in itself, including abstract, text, figures, and references. Preceding the article is a brief review of observational studies east of Cape Hatteras which are considered by the author to have been important in the development of present-day conceptions of meanders in the open-ocean Gulf Stream; and of the significant ideas which have been proposed to explain these phenomena. Also included is a separate set of references pertinent to this review alone.

I should like to express my thanks to the following:

Mr. Frederick Fuglister, for very considerable instruction concerning the character of the Gulf Stream, for permission to use unpublished results of his Gulf Stream survey of 1960, and for a critical appraisal of this thesis;

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HISTORICAL REVIEW

Systematic studies undertaken in the last thirty years have revealed that in the open ocean east of Cape Hatteras, the Gulf Stream traces tortuous courses which differ significantly from one time to another. Since the Stream between Cape Hatteras and the Grand Banks forms the boundary between the slope water and the Sargasso Sea (Iselin, 1936), its position has generally been identified by the contrast in properties - particularly temperature - between these two water masses. Thus during the thirties, Church (1932, 1937) and Hachey (1939) analyzed thermograph records of surface temperature obtained from commercial vessels traveling over regular ship lanes in the north-western quadrant of the western North Atlantic. They found migrations of the position of the surface Gulf Stream on their lines of observation of roughly one to two hundred miles. They noted also the occasional presence of masses of warm water north of the Stream, which they interpreted as incursions of Gulf Stream water which had broken away from the Stream proper. Thus they inferred the existence of meandering current paths, but stressed rather, apparent northward and southward migrations of the Stream axis as a whole.

In the same period, Iselin (op. cit., 1940) made numerous sections across the Gulf Stream, which revealed much of its vertical temperature and salinity structure.

Like Church and Hachey, he emphasized migrations of the entire Stream axis, but associated them with seasonal variations in volume transport. North of the Stream, he occasionally observed large depressions in the main thermocline, corresponding to the surface warm masses of Church and Hachey; in addition, sometimes south of the Stream, he found elevations in the thermocline. He referred loosely to both features as "eddies", since sloping isopycnal surfaces imply geostrophic currents, but pointed out that from single sections there was no possibility of identifying them positively as such; a single section, after all, cannot distinguish between an eddy adjacent to a current, two currents separated by a counter-current, and a single, extravagantly meandering current. (Fuglister (1955) has discussed such alternative analyses of oceanographic data in great detail.) Since Church and Hachey were also studying, in effect, single or widely spaced sections, the configurations of their warm masses must be regarded as equally uncertain.

Thus it became desirable to make extensive, closely spaced measurements in the Gulf Stream region on a quasi-synoptic basis, in order to reduce these ambiguities. The invention of loran during the Second World War made possible the frequent navigational fixes required for such programs, and the concurrent invention of the bathythermograph permitted the necessary rapid measurements of temperature in the surface layers of the ocean. Therefore in May of 1946, the Atlantis made a new type of survey of the

Gulf Stream: it ran a zig-zag pattern of many short bathy-thermograph sections across the Stream in order to define rather tightly the current path over a fairly long distance (Fuglister and Worthington, 1947). These sections clearly showed meanders in the path, as did other data taken later in 1946 and in 1947. Concerning these observations, Iselin and Fuglister (1948) pointed out that while one portion of the stream might be moving north at the rate of a mile or two a day, another might be moving south just as fast. Hence the inferences of Church, Hachey, and Iselin of general northward and southward migrations of the entire Stream, based on temperature variations along only a few observation lines, became rather questionable.

In late fall, 1948, the New Liskeard made another survey of the sort just described. Ford and Miller (1952) reported considerable meandering of the surface Stream on this occasion, and the existence of a very large, anticyclonic meander centered at longitude 64°W .

To obtain a detailed, synoptic picture of a portion of the Stream more extensive than any previously studied, a six-ship survey was carried out in June, 1950, between Cape Hatteras and longitude 50°W , in the manner of the post-war cruises cited above (Fuglister and Worthington, 1951). The observations again showed meandering, which increased in amplitude downstream from Cape Hatteras, and which was of a pattern different from any seen before.

Particularly engrossing was a very long, narrow, cyclonic meander near longitude 60°W, which was observed to pinch off and turn into a great eddy south of the Stream proper. The entire pattern showed changes during the two weeks of study, the most rapid by far occurring where the cyclonic eddy was forming.

Another method of determining the path of the Stream was tried by Stommel, et al. (1953), who mounted a radiation thermometer in an airplane, and searched for the sharp surface temperature gradient characteristic of the Stream. Their data were fragmentary, but indicated small meanders just east of Cape Hatteras.

In an attempt to eliminate ambiguities in interpretations of current structure, due to the necessity of interpolating between ship sections, and to investigate possible branching or formation of multiple currents in the Stream, Malkus and Johnson (1954) let a ship simply drift with the current. On both their cruises they found that the ship would in fact drift out of the current, so that they had to move repeatedly to the left in order to stay with the main Stream. Consequently, the relation of their drift tracks to the actual paths of the Stream was somewhat uncertain, particularly near the down-stream ends of the tracks, where the structure of the environment became especially confusing. Although it was impossible to draw any conclusions about branching or multiple currents, the track of the first drift

cruise, made in June 1954, is rather interesting in that it closely resembles the Gulf Stream path found by the New Liskeard, and hence perhaps describes an instance in which the Stream nearly repeated a course taken several years earlier.

The extensive programs of the I. G. Y. inhibited further study of the Gulf Stream east of Cape Hatteras until 1960, when Fuglister (in press) organized another multiple ship survey, designed to continue for ten weeks, in contrast to the two-week program of 1950. Furthermore, whereas almost all the 1950 observations were made in the near-surface water of the Stream proper, it was planned in 1960 to obtain a three-dimensional picture of the Stream and its environment between longitudes $68^{\circ}30'W$ and $48^{\circ}30'W$ by intensive use of deep hydrographic stations and neutrally-buoyant Swallow floats. The spacing of the sections led inevitably to uncertainties in the interpretation of the density field, but the station data combined with electromagnetic observations of "lines of zero set" (von Arx, 1960) showed plainly that the Stream was meandering elaborately. The station data also indicated a high degree of coherence among the current paths at all depths. A most important discovery made with Swallow floats and hydrographic stations was that at that time, and at least in the small areas examined, the Stream extended all the way to the bottom of the ocean, with deep velocities in the same direction as those at the

surface. The meander pattern was seen to change during the period of observation, although at rates smaller than those measured in 1950.

Observational data are still far from adequate to form a complete picture of the behaviour of the Gulf Stream between Cape Hatteras and the Grand Banks, but the features seen to date have been sufficiently intriguing to inspire attempts at dynamical interpretation. By far the most popular point of view adopted toward Gulf Stream meanders has been to regard them as derived from small disturbances to some initially uniform unstable flow; such disturbances would amplify into full-blown meanders by feeding on an accessible energy source implicit in the structure of the initial flow. Formidable mathematical difficulties, however, have generally restricted stability analyses to very simple fluid models; thus the instabilities considered have, until very recently, been of two kinds: (1) barotropic instability, associated with the shapes of velocity profiles in barotropic geostrophic currents, and (2) baroclinic instability, associated with the horizontal density gradients in baroclinic geostrophic currents without lateral shear. Most applications of these studies have been to meteorological phenomena, but they are not inherently limited to air as the fluid medium.

Thus Kuo (1949) investigated the character of small, non-divergent, horizontal wave motions superimposed

on zonal geostrophic currents in a barotropic fluid - specifically, an atmosphere - and found as a necessary condition for amplifying waves, and hence barotropic instability, the existence of critical points on the cross-stream velocity profile at which the absolute vorticity assumed extreme values. Haurwitz and Panofsky (1950) examined instabilities of this kind in several models intended to have particular application to the Gulf Stream; they considered currents composed of vertical layers in each of which the initial velocity either was constant or varied linearly in the cross-stream direction. They placed lateral boundaries near the left-hand sides of some of their currents, and found that small perturbations in these models were considerably more stable than those in models without close boundaries. Accordingly, they suggested that one might expect to find large meanders in the Gulf Stream only in the open ocean, rather than south of Cape Hatteras, where the Stream is constrained by the Continental Shelf.

Both Charney (1947) and Kuo (1952) have studied baroclinic instabilities in horizontally uniform zonal currents with constant vertical shear. They found that these flows were unstable with respect to disturbances having wave lengths shorter than some critical value, and that there existed a "most unstable wave length" which would presumably dominate the motion.

Effects of density differences have also been

studied in a simple rotating model consisting of two fluid layers of different densities, the lower, denser layer being motionless (Stommel, 1958, pp. 129-131). Stern (1961) recently considered the lateral boundary conditions to be satisfied by disturbances to currents in this system, and deduced that small perturbations can feed on the gravitational potential energy associated with the initial geostrophic flow only if they feed as well on the kinetic energy of the initial flow. Thus the release of potential energy, the essential feature of baroclinic instability, is intimately related in this model to the release of kinetic energy, the essential feature of barotropic instability. Stern studied this situation in more detail by examining the stability of a laminar jet in a homogeneous fluid layer which overlaid a denser layer at rest. He found that "large-scale" instabilities could occur only for velocity profiles with extreme values of potential vorticity, and he suggested that an effect of such instabilities might be to limit the depth of the main thermocline in the interior of the ocean. His expectation that this stability criterion had validity in other flows more complex than the one he considered was confirmed by Charney and Stern (1962) who discovered that "zonal flow in a stratified rotating atmosphere which (a) is bounded by rigid horizontal boundaries, or (b) extends to infinity, is stable with respect to axially asymmetric disturbances if the gradient of potential vorticity in isen-

tropic surfaces does not vanish, and (a) the potential temperature is constant at the rigid boundaries, or (b) the perturbation energy is reflected at infinity". This latter study revealed the important destabilizing effect in baroclinic instability of the horizontal density gradients at horizontal boundaries.

Unfortunately, the mathematical difficulties noted above have prevented, to date, a realistic analysis of the stability of the Gulf Stream. Consequently, it has been very difficult to assess the significance for the Stream of the physical processes described in these studies.

In contrast to American emphasis on unstable waves in attempts to understand Gulf Stream meanders, Japanese oceanographers, in corresponding efforts to interpret meanders in the Kuroshio, have largely confined their attention to stable waves. The similarities between the two currents make these ideas employed by the Japanese worth consideration in dynamical studies of the Gulf Stream. Thus in analogy to the treatments of atmospheric flow by Rossby (1940) and Bjerknæs and Holmboe (1944), Ichiye (1955) and Fukuoka (1958) have regarded the Kuroshio as characterized by constant transport of absolute vorticity, i.e. as a current which forms stable, stationary Rossby waves. Their calculations of amplitudes and wave lengths compare favorably with those observed in certain Kuroshio meander patterns. The analysis made by Bolin (1950), moreover, of

the generation of such waves by ridges was used by Fukuoka (1957) to explain the existence of meanders in the Tsushima current.

Saint-Guily (1957) applied this idea of the stationary Rossby wave to the Gulf Stream itself, essentially by calculating constant absolute vorticity trajectories. He noted good qualitative agreement between the shapes of such trajectories and the current paths described by Fuglister and Worthington (1951), but found serious discrepancies between calculated and observed meander dimensions.

This thesis proposes a new explanation of meanders in the open-ocean Gulf Stream, based principally upon the constraint exerted by sloping bottom topography on quasi-geostrophic currents which extend to the bottom of the ocean. The idea is sufficiently simple to permit empirical confirmation by comparing with observed current paths the solutions to an approximate equation isomorphic to that considered by Rossby (op. cit.). Not only is this mechanism indeed able to produce the gross observed features of Gulf Stream paths, but estimates of the forces and accelerations actually involved in meandering flow imply that it is the only possible explanation of these features.

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Abstract

Meander patterns observed in the Gulf Stream east of Cape Hatteras exhibit correlations with bottom topography. A vorticity balance analysis reveals that if the current extends to the bottom of the ocean without reversing direction, as implied by some recent deep velocity measurements, its path must indeed be controlled principally by the topography of the Continental Rise, with a smaller constraint imposed by the meridional variation of the Coriolis parameter. Approximate current paths calculated according to these mechanisms agree sufficiently well with observed paths to confirm the topographic explanation of Gulf Stream meanders.

The observed variety in meander patterns is attributed to fluctuations in current direction and path curvature near Cape Hatteras. It is conjectured that the New England Seamounts may so deflect the Stream as to effect significant modifications in its path.

These conclusions make very doubtful any direct relation between large-scale meanders and possible instabilities of the Gulf Stream east of Cape Hatteras, and between the distribution of wind stress and the separation of Stream from coast.

1. INTRODUCTION

It has been generally assumed in recent years that meanders in the Gulf Stream represent amplified perturbations to some fairly uniform, unstable current (Haurwitz and Panofsky, 1950; Stommel, 1958, pp. 129-131; Stern, 1961). Although the difficulties in integrating equations with variable coefficients have prohibited stability analyses of current models having sufficient complexity to resemble the actual Gulf Stream, the wave-like character of observed meander patterns has made explanations based on instability hypotheses very appealing.

The few long, fairly well-determined current paths actually observed east of Cape Hatteras, however, exhibit certain correlations with the trend of isobaths on the Continental Rise. This circumstance, while not conclusive, suggests a topographic influence on the path of the Stream, and makes pertinent a dynamical inquiry into the possibility and nature of such an effect. A study is therefore undertaken here of the vorticity balance typically satisfied in these paths. The analysis reveals that a convincing description of their gross features can be given in terms of the constraint exerted by sloping bottom topography on a quasi-geostrophic current which extends to the bottom of the ocean; with generally minor modifications imposed by the meridional variation in Coriolis parameter. The analysis

implies, moreover, that only this mechanism is quantitatively sufficient to account for these features. Thus it appears that the meanders in the Gulf Stream east of Cape Hatteras are not associated with unstable waves after all, but are segments of topographically-controlled stable waves, analogous to stationary Rossby waves (Rossby, 1940).

2. TOPOGRAPHIC CORRELATIONS WITH OBSERVED CURRENT PATHS

Several surveys of the Gulf Stream have served to chart nearly synoptic segments of its path east of Cape Hatteras. These studies have been greatly facilitated through the role played by the Stream as the boundary between the Sargasso Sea and the slope water (Iselin, 1936), for the Stream position can be conveniently located according to the pronounced horizontal difference in properties between the two water masses. Five path segments have been defined with sufficient precision in this manner, over sufficiently long distances, to make inter-comparisons potentially informative. Each one is indicated in Figure 1 by the positions at some depth of an isotherm lying in the center of the sharp horizontal temperature gradient characteristic of the Stream. These curves are based on figures included in articles to be cited below. The variety in path indicators is unfortunate, but was imposed by the

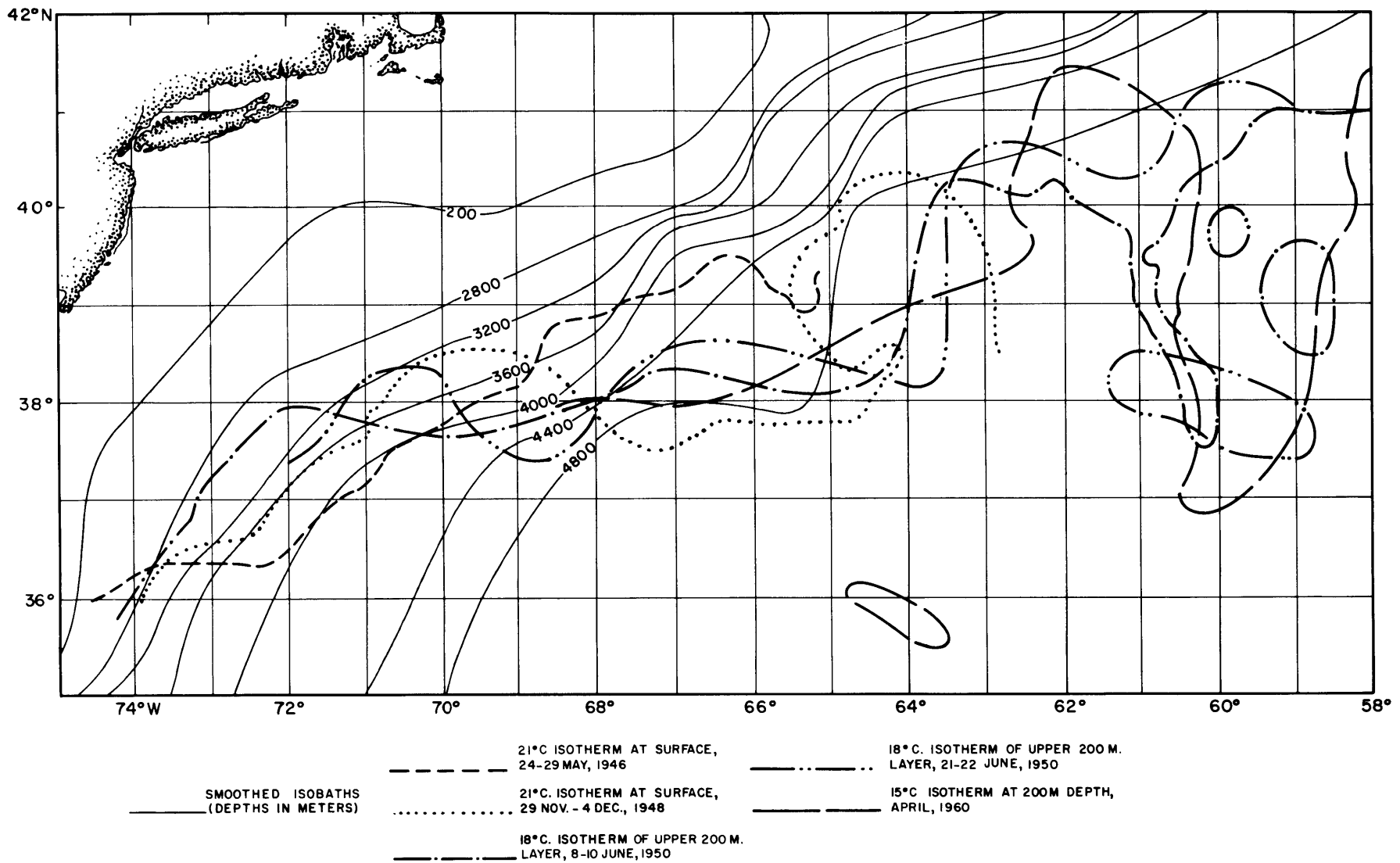


Figure 1. Observed paths of the Gulf Stream overlying smoothed bottom topography. Isotherms from articles cited in the text are used as path indicators.

variety in nature of the source material.

The two earliest current paths are both represented in Figure 1 by surface positions of the 21°C. isotherm. The relevant observational data consist of many short, closely-spaced, bathythermograph sections made in late May, 1946, on board the Atlantis (Fuglister and Worthington, 1947; Iselin and Fuglister, 1948), and in late November and early December, 1948, on the New Liskeard (Ford and Miller, 1952).

Bathythermograph sections were again used to locate the Stream during a multiple ship survey in June, 1950 (Fuglister and Worthington, 1951). The paths taken by the Stream at the beginning (8-10 June) and at the end (21-22 June) of the survey are shown by the positions of the 18°C. isotherm of the upper 200 meter layer. As may be seen in the figure, a long, narrow, cyclonic meander, which was discovered early in the survey, broke away from the Stream to form an eddy, while an eddy east of the meander moved to the northwest, and diminished in size.

On a much longer multiple ship survey, made in the spring of 1960 (Fuglister, in press), sections were run along meridians spaced two degrees (about 180 km) apart. The temperature data obtained are the basis for the current path indicated in Figure 1 by the positions of the 15°C. isotherm at the 200 meter level. Although observations were separated by distances considerably greater than in the

previous surveys mentioned, the interpolation was generally confirmed by electromagnetic "lines of zero-set" (von Arx, 1960). Ambiguity remains concerning possible connections between the great cyclonic meander and the eddy southwest of it; that portion of the current path, however, has no particular importance in the present study.

A span of fourteen years thus separates the earliest from the most recent path observation. One should remember, however, that the two paths of 1950 are separated by only two weeks.

Figure 1 includes a very rough representation of bottom topography in the general area of these surveys. The isobaths are based on U. S. Coast and Geodetic Survey chart No. 1000 (14th Ed., revised as of 20 March 1961) and U. S. Navy Hydrographic Office chart No. 6610-L (1st Ed., revised as of 24 October 1960); they are shown in highly smoothed form, because it was felt that to have superposed detailed current paths on detailed isobaths would have obscured the general trend of the deep topography, the feature of significance to this discussion. The coastline and 200 meter curve roughly define the limits of the Continental Shelf; seaward to a depth of about 2000 meters lies the Continental Slope; the deep isobaths, increasing in depth at 400 meter intervals from 2800 to 4800 meters, depict the general shape of the Continental Rise. The depth of the Abyssal Plain is about 5000-5200 meters. For

clarity, the New England Seamounts, located east of longitude 65°W , have been omitted from the figure. Other topographic features seaward of the 4800 meter curve are comparatively insignificant.

Clearly, although the Gulf Stream no longer presses against the Continental Slope after it passes Cape Hatteras, it by no means enters directly onto the Abyssal Plain, but approaches it only after flowing for more than a thousand kilometers over the Continental Rise. Probably the most striking characteristic common to the current paths in this region is one pointed out by Fuglister (in press): that the downstream amplification of their meanders does not occur gradually, but abruptly at longitudes 65° - 62°W , where the four current paths which extend that far east turn abruptly northward and subsequently develop great anti-cyclonic meanders. It is noteworthy that in these same longitudes there exists a similar bend in the deep bottom contours. A curious feature of these four meanders, moreover, is that the envelope of their crests coincides to a remarkable degree with the 4600 meter isobath. Far upstream, furthermore, the general trend of current paths is northeastward, similar to that of the isobaths, and where the bottom contours bend rather sharply eastward, near longitudes 71° - 69°W , so also does the trend of current paths.

Although these correlations are somewhat tenuous, they suggest that bottom topography may provide a control on

the path of the Gulf Stream which is involved fundamentally in the dynamics of meanders. Such a notion would certainly be frivolous were it in fact true, as was for so long assumed, that the deep water of the ocean is virtually motionless. Recent temperature and salinity profiles, however, have revealed that the horizontal density gradient associated with the Stream, and indicative of vertical shear of horizontal motion, persists down to the bottom of the ocean, although with considerable reduction in magnitude from its thermocline value (see, for instance: Fuglister, 1960). On the other hand, direct observations of deep motions beneath the surface Gulf Stream are rather sparse, and those relevant to the Stream east of Cape Hatteras are nearly limited to Fuglister's survey of 1960 (Fuglister, in press). On that occasion, neutrally-buoyant Swallow floats were set out in deep water directly beneath the surface Stream, and followed for several days. At the same time the float tracks were bracketed with pairs of hydrographic stations. By integrating vertically the horizontal density gradient obtained from the station data, and using the measured float velocity as a constant of integration, curves of geostrophic velocity with depth could then be calculated. These showed a Gulf Stream which extended to the bottom of the ocean without reversing direction, and with bottom velocities averaging about eight to ten centimeters per second. Hence present evidence, though sparse,

indicates the existence of deep flow in the Gulf Stream, through which topographic features could conceivably affect the current path at all depths.

3. THE VORTICITY BALANCE OF THE MEANDERING STREAM

The topographic correlations thus prompt a detailed, systematic inquiry into the possibility that bottom topography is affecting the Stream, and into the nature of the mechanism responsible. Perhaps the most straightforward manner of attack is simply to write down the full equation of motion, and estimate the order of magnitude of each term by inserting values of relevant quantities typical of the meandering Stream. Such a procedure, if practical, should readily identify the dominant forces acting on the Stream, and reveal the existence of any topographic control.

To a first approximation, Gulf Stream velocities are related in constant proportionality to the cross-stream pressure gradients. Since, however, it is the small departures from this proportionality which are crucial to meandering motions, it is desirable for present purposes to eliminate the first-order balance by studying not the momentum equation itself, but the vertical component of its curl, the vorticity equation. Furthermore, in order to remove terms pertaining only to the internal current

structure, and not to the position of the Stream as a whole, it proves useful to integrate the equation over a volume \mathcal{R} fixed in space, which spans the entire width of the current, and extends from the sea surface to the top of any bottom frictional boundary layer.

The velocity field in the Gulf Stream and its environment has as yet been only partially described. It will therefore be possible to use only a crude representation of it in the analysis. (Explicit designation of units will generally be omitted, and the c.g.s. system understood throughout.) We shall consider a jet-like current having a total volume transport V of the order 10^{14} (equal to one hundred million cubic meters per second); a total transport of momentum per unit mass M of the order 5×10^{15} ; and a volume transport per unit depth near the ocean bottom T of the order 10^8 . That these are reasonable numbers by which to characterize the Stream is seen in noting that they describe approximately a jet 125 km. wide, with a triangular cross-stream velocity profile, whose maximum decreases linearly with depth from a surface value of 160 cm/sec to a 1200 m value of 16 cm/sec; and subsequently remains constant down to an ocean bottom at a depth of 4500 m. This current has a cross-stream averaged bottom velocity of 8 cm/sec, consonant with the calculations pertaining to portions of the Gulf Stream in 1960.

The Stream exhibits downstream variations in

volume transport connected with fluxes across its lateral boundaries, but since, east of Cape Hatteras, fluxes over a meander quarter-wave length (about 80-100 km) do not seem to amount to much more than 10-15% of the total transport (Fuglister, in press), we shall neglect them and treat the current transport as constant.

Since we are to investigate an integrated vorticity balance, we shall be concerned with certain lateral boundary effects. These involve velocities or their space derivatives in the flanking waters outside the region of concentrated flow; such quantities apparently are small compared with those in the Stream itself. Thus were we to consider vertically-averaged motions in the flanking water, for instance, of the order 5-10 cm/sec, with horizontal scale of the order 50 km, we would find all associated lateral boundary effects to be negligible (the demonstration is omitted for brevity). Such effects are therefore disregarded in the analysis following.

Furthermore, motions of the sea surface with time scales long enough for them to affect the Stream seem much too small to make any contribution to the integrated vorticity balance. Vertical displacements accompanying typical lateral movements of meander patterns, for instance, are entirely inconsequential (this demonstration is also omitted). Therefore we shall treat the sea surface as effectively stationary.

We shall occasionally refer quantities to a local Cartesian coordinate system (x, y, z) , with corresponding unit vectors $(\underline{i}, \underline{j}, \underline{k})$ pointing eastward, northward, and upward.

In a reference frame fixed with respect to the rotating earth, the momentum equation is:

$$\frac{\partial \underline{V}}{\partial t} + \nabla \left(\frac{1}{2} \underline{V} \cdot \underline{V} \right) - \underline{V} \times \nabla \times \underline{V} + 2 \underline{\Omega} \times \underline{V} = \alpha \nabla p - \nabla \varphi + \underline{F} \quad (1)$$

The symbols are defined as:

- \underline{V} - water velocity relative to the earth, with horizontal and vertical components, \underline{V}_H and w
- ∇ - gradient operator, with horizontal and vertical components, ∇_H and $\partial/\partial z$
- $2 \underline{\Omega}$ - twice the angular velocity of the earth, with vertical and meridional components, f (the Coriolis parameter) and e
- t - time
- α - specific volume
- p - pressure
- φ - gravity potential
- \underline{F} - frictional force per unit mass

It will prove useful occasionally to write \underline{V}_H as $v \underline{l}$, where v means the magnitude of \underline{V}_H and \underline{l} denotes a unit vector in the direction of \underline{V}_H .

Since the compressibility of sea water is very slight, we may approximate the continuity equation as

$$\nabla \cdot \underline{V} = 0 \quad (2)$$

Operating on (1) with the curl, and extracting the vertical component of the result, yields the vorticity equation:

$$\frac{\partial \zeta}{\partial t} + \nabla \cdot (\zeta + f) \underline{V} - \nabla \cdot w (2\Omega + \nabla \times \underline{V}) + \underline{k} \cdot \nabla \alpha \times \nabla p - \underline{k} \cdot \nabla \times \underline{F} = 0 \quad (3)$$

where $\zeta \equiv \underline{k} \cdot \nabla \times \underline{V}$

We shall make the customary β -plane approximation in which we retain only the first-order term in the Taylor series expansion of $f(y)$. Thus we represent the Coriolis parameter as: $f = f_0 + \beta y$, where f_0 is the value of the parameter at some latitude in the area under study, and β , the corresponding value of df/dy .

We shall integrate (3) over a volume \mathcal{R} , as described above. The bounding surface Σ of the volume is to be composed of six sub-surfaces: two nearly vertical surfaces respectively paralleling each side of the current, but lying outside the flow; two nearly vertical surfaces crossing the current at arbitrary points to connect the first pair, but so oriented as to be normal to the flow; a nearly horizontal surface, lying along the sea surface; and another nearly horizontal surface, lying just above the bottom frictional boundary layer. Let the bottom surface be designated by \mathcal{E} ; the top surface, by \mathcal{I} ; the pair of vertical surfaces which cross the current, by \mathcal{S} ; and the pair which parallel the

current, by L . Let it be understood, moreover, that an integral over "surface" S or L is to imply the sum of two integrals, one over each member of S or L .

We proceed then, to examine the relative magnitudes of the various terms in the volume-integrated vorticity equation. We shall consider each term individually. The integral representing the net flux of relative vorticity out of R is easy to evaluate, and turns out to be a dominant member of the equation; hence we shall first estimate this term, then neglect or retain the others according to their magnitudes in comparison.

Let volume and surface elements be denoted by $d\tau$ and $d\sigma$; let approximate equality be indicated by \cong , and equality in order of magnitude, by \sim .

A. Relative Vorticity Flux

$$\int_R \nabla \cdot \zeta \underline{v} d\tau = \int_{\Sigma} \zeta \underline{v} \cdot \underline{n} d\sigma$$

by Gauss's Theorem, where \underline{n} denotes a unit normal to Σ , directed out of R . Since the integral above over L involves only the small velocities outside the current, and the integral over I depends on motions of the sea-surface, neither makes a significant contribution. Therefore,

$$\int_{\Sigma} \zeta \underline{v} \cdot \underline{n} d\sigma \cong \int_S \zeta \underline{v} \cdot \underline{n} d\sigma + \int_E \zeta \underline{v} \cdot \underline{n} d\sigma$$

In natural coordinates, $\zeta = K v - \partial v / \partial a$, where

K is the curvature of the horizontal streamlines, and a is a local cross-stream coordinate, counted positive to the left (for an observer looking downstream). Then since $\underline{\eta}$ is either parallel or anti-parallel to \underline{V}_H on S ,

$$\int_S \underline{V} \cdot \underline{\eta} \, d\sigma = \int_S K v^2 \underline{l} \cdot \underline{\eta} \, d\sigma - \int_S v \frac{\partial v}{\partial a} \underline{l} \cdot \underline{\eta} \, d\sigma$$

but,

$$\int_S v \frac{\partial v}{\partial a} \underline{l} \cdot \underline{\eta} \, d\sigma = \int_S \underline{l} \cdot \underline{\eta} \frac{\partial}{\partial a} \left(\frac{1}{2} v^2 \right) da \, dz$$

and since this integral consequently involves only velocities outside the current, we may disregard it. Gulf Stream meanders are typically characterized by $K \sim 10^{-7}$, so that:

$$\int_S \underline{V} \cdot \underline{\eta} \, d\sigma \cong \int_S K v^2 \underline{l} \cdot \underline{\eta} \, d\sigma \sim KM \sim 5 \times 10^8$$

where M as defined above, is the total transport of momentum per unit mass.

Next we estimate the integral over surface E . Since the thickness of the bottom frictional boundary layer must be very much less than the depth of the open ocean and the lateral dimensions of the current, surface E must lie closely parallel to the ocean bottom, and the component of velocity normal to E in the integral must be that imposed by the boundary layer dynamics. Consequently, we may estimate the order of magnitude of $\underline{V} \cdot \underline{\eta}$ satisfactorily by the negative of w_e , the vertical velocity induced at the top

of an Ekman layer lying between a level bottom and a deep geostrophic current of Gulf Stream characteristics.

Within such a layer,

$$\frac{f}{\alpha} \underline{k} \times \underline{V}_H = -\nabla_H \rho + \frac{\partial \underline{\tau}}{\partial z}$$

where $\underline{\tau}$ represents the horizontal frictional stress acting across horizontal planes. If, in order to study only direct frictional effects, we consider α and f constant, and apply equation (2), the vertical component of the curl of the above equation is:

$$\frac{f}{\alpha} \frac{\partial w}{\partial z} = -\frac{\partial}{\partial z} (\underline{k} \cdot \nabla_H \times \underline{\tau})$$

We integrate over the thickness of the Ekman layer. If subscript b denotes a bottom value,

$$\frac{f}{\alpha} w_e = \underline{k} \cdot \nabla_H \times \underline{\tau}_b$$

since w must vanish at the bottom, and $\underline{\tau}$ at the top of the layer.

The familiar Ekman spiral is obtained by setting $\underline{\tau} = A_v \partial \underline{V}_H / \partial z$ where A_v is a constant coefficient of vertical eddy viscosity. Then

$$\underline{k} \cdot \nabla_H \times \underline{\tau}_b = A_v \frac{\partial \zeta_b}{\partial z}$$

In the spiral, $\partial \zeta_b / \partial z = \zeta_e \pi / d$, where subscript e indicates a value at the top of the layer, and the Ekman depth d is defined as $\pi (2A_v \alpha / f)^{\frac{1}{2}}$. Thus we

obtain the convenient representation

$$w_e = \frac{d}{2\pi} \zeta_e$$

We take $A_v \sim 100$ as a reasonable value near the bottom of the ocean (Sverdrup, Johnson, and Fleming; p. 482); then since $\alpha \cong 1$, and $f \cong 10^{-4}$ in mid-latitudes,

$$d \sim \pi \sqrt{2} \times 10^3, \text{ or about 40 meters. Furthermore,}$$

$\zeta_e \sim 2 v_{em}/W$, where W is the current width, and v_{em} the maximum horizontal velocity at the top of the layer.

Since for a triangular cross-stream velocity profile,

$v_{em} = 2T/W$, we shall take $\zeta_e \sim 4T/W^2$. For the Gulf Stream $W \sim 10^7$, so

$$w_e \sim \frac{4Td}{2\pi W^2} \sim 3 \times 10^{-3}$$

Now we are able to estimate the integral over E .

We note first that the appropriate dimensions to be used in estimating the area of E are the current width W , and the meander quarter-wave length q , which for Gulf Stream meanders is of the order 10^7 . Then,

$$\int_E \zeta_e \underline{v} \cdot \underline{\eta} d\sigma \sim \frac{4T w_e q}{W} \sim 1 \times 10^6$$

This integral is less than 0.3% of that over S ; hence we disregard it, and conclude that:

$$\int_R \nabla \cdot \zeta_e \underline{v} d\tau \cong \int_S K v \underline{v} \cdot \underline{\eta} d\sigma$$

B. Planetary Vorticity Flux

$$\int_{\mathcal{R}} \nabla \cdot f \underline{v} d\tau = \int_{\Sigma} (f_0 + \beta y) \underline{v} \cdot \underline{n} d\sigma$$

by Gauss' Theorem again. Since $\nabla \cdot \underline{v} = 0$

$$\int_{\Sigma} f_0 \underline{v} \cdot \underline{n} d\sigma = 0$$

and we examine only the flux of βy out of \mathcal{R} . The integrals over L and I are to be disregarded for the same reasons given in the preceding section. Since the amplitude of Gulf Stream meanders is typically of the order 10^7 , and $\beta \cong 2 \times 10^{-13}$ for the latitudes in question,

$$\int_S \beta y \underline{v} \cdot \underline{n} d\sigma \sim \beta y V \sim 2 \times 10^8$$

where V , as defined above, is the total volume transport of the Stream. The term is comparable to the relative vorticity flux.

On the other hand,

$$\int_E \beta y \underline{v} \cdot \underline{n} d\sigma \sim \beta y w_e W_g \sim 6 \times 10^5$$

and is only about 0.1% of the relative vorticity flux, so is to be neglected. Therefore,

$$\int_{\mathcal{R}} \nabla \cdot f \underline{v} d\tau \cong \int_S \beta y \underline{v} \cdot \underline{n} d\sigma$$

C. Vertical Velocity Term

$$\int_{\mathcal{R}} \nabla \cdot w (2\Omega + \nabla \times \underline{v}) d\tau = \int_{\Sigma} w (2\Omega + \nabla \times \underline{v}) \cdot \underline{n} d\sigma$$

by Gauss's Theorem once more. We require an estimate of the vertical velocity. This must be principally composed of a frictional component as estimated above, and of a "topographic" component. Since the upper surface of the boundary layer parallels the bottom topography closely, the quasi-geostrophic velocity at the top of the layer must also be parallel to the bottom, and hence have a "topographic" vertical component $w_T = -\underline{v}_{HE} \cdot \nabla_H D$, where \underline{v}_{HE} is the value of \underline{v}_H on surface E , and D represents the ocean depth, counted positive. Since $|\nabla_H D| \sim 8 \times 10^{-3}$ on the Continental Rise, $|\underline{v}_{HE}| \sim T/W$, and the cosine of the angle between \underline{v}_{HE} and $\nabla_H D$ is typically of the order 1/2 (as may be seen in Figure 1), $w_T \sim 4 \times 10^{-2}$. The topographic component of w is therefore more than an order of magnitude greater than the frictional component; hence we consistently neglect the latter in comparison with the former.

As before, we disregard the integrals over surfaces I and L .

We examine, then, the contribution made by the integral over E . Because $|\nabla_H D| \ll 1$, $2\Omega \cdot \underline{\eta} \cong f$. Therefore,

$$\int_E w 2\Omega \cdot \underline{\eta} d\sigma \cong \int_E f w_T d\sigma \sim f_0 w_T W q \sim 4 \times 10^8$$

This integral is of the order of the relative vorticity flux, and must be retained in the approximate vorticity balance.

The integral over E involving relative vorticity, however, is much smaller than that involving planetary vorticity. Quite generally,

$$\nabla \times \underline{v} = \zeta \underline{k} + \underline{k} \times \partial \underline{v}_H / \partial z - \underline{k} \times \nabla_H w$$

Since $|\underline{v}_H| \gg w$ and the vertical scale of variation of \underline{v}_H (the ocean depth D) is much smaller than the lateral scale of w (q or W), $|\nabla_H w| \ll |\partial \underline{v}_H / \partial z|$, and we may immediately disregard the contribution of vertical velocity to $\nabla \times \underline{v}$. Since $|\nabla_H D| \ll 1$, $\underline{n} \cdot \underline{k} \cong 1$, but $\underline{n} \cdot \underline{j} \cong \underline{n} \cdot \underline{i} \sim |\nabla_H D|$. Therefore,

$$\frac{\zeta_E \underline{n} \cdot \underline{k}}{2\Omega \cdot \underline{n}} \cong \frac{\zeta_E}{f} \sim \frac{4T}{W^2 f} \sim 4 \times 10^{-2}$$

$$\frac{\underline{n} \cdot \underline{k} \times (\partial \underline{v}_H / \partial z)_E}{2\Omega \cdot \underline{n}} \sim \frac{|\nabla_H D| |\underline{v}_H| / D}{f} \sim \frac{|\nabla_H D| T}{f W D} \sim 2 \times 10^{-3}$$

where the ocean depth $D \sim 5 \times 10^5$. Hence we neglect these two contributions.

Furthermore, E is so nearly a level surface that we may replace it in the integration by its horizontal projection E_H . In addition, since $\beta y / f_0 \sim 2 \times 10^{-2}$, we may replace f by f_0 . Thus

$$\int_E w (2\Omega + \nabla \times \underline{v}) \cdot \underline{n} d\sigma \cong f_0 \int_{E_H} w_T d\sigma$$

Finally we compare the integral over surface S

with that over surface E .

$$\frac{\int_S w z \underline{\Omega} \cdot \underline{n} d\sigma}{\int_E w z \underline{\Omega} \cdot \underline{n} d\sigma} \sim \frac{w_T e W D}{w_T f W a} \sim \frac{D}{a} \sim 5 \times 10^{-2}$$

since $e \sim f$ in mid-latitudes. Furthermore, on S ,

$$\underline{n} \cdot \nabla \times \underline{v} \cong \underline{n} \cdot \underline{k} \times \partial \underline{v}_H / \partial z \sim v_s / D \quad \text{where } v_s \text{ is a typical}$$

surface velocity, of the order 10^2 . Hence

$$\frac{\int_S w (\nabla \times \underline{v}) \cdot \underline{n} d\sigma}{\int_E w z \underline{\Omega} \cdot \underline{n} d\sigma} \sim \frac{w_T v_s W}{w_T f W a} \sim \frac{v_s}{f a} \sim 10^{-2}$$

The entire integral over S is thus to be neglected in comparison with that over E , and we conclude that:

$$\int_R \nabla \cdot w (z \underline{\Omega} + \nabla \times \underline{v}) d\tau \cong f_0 \int_{EK} w_T d\sigma$$

D. Solenoid Term

We next estimate the magnitude of the term,

$$\int_R \underline{k} \cdot \nabla \alpha \times \nabla \rho d\tau = \int_R \underline{k} \cdot \nabla_H \alpha \times \nabla_H \rho d\tau$$

Since velocities in the Gulf Stream are nearly geostrophic,

$|\nabla_H \rho| \cong |v_H| f / \alpha$. Furthermore, in that $\nabla_H \alpha$ is usually very nearly parallel to $\nabla_H \rho$, we should greatly overestimate the component of $\nabla_H \alpha$ normal to $\nabla_H \rho$ by introducing for it the average cross-stream specific volume gradient $\Delta \alpha / W$, where $\Delta \alpha$, the cross-stream specific volume difference, is of the order 10^{-3} in the thermocline.

Thus

$$\int_{\mathcal{R}} \underline{k} \cdot \nabla_H \alpha \times \nabla_H p \, d\tau \ll \int_{\mathcal{R}} |\nabla_H \alpha| |\nabla_H p| \, d\tau \sim \frac{\Delta \alpha f V \rho}{W \alpha} \sim 1 \times 10^7$$

The solenoid term then is much less than 2% of the relative vorticity term and is consequently to be disregarded.

E. Friction Term

We now assess the importance of the friction term,

$$\int_{\mathcal{R}} \underline{k} \cdot \nabla \times \underline{F} \, d\tau = \int_{\mathcal{R}} \underline{k} \cdot \nabla_H \times \underline{F}_H \, d\tau$$

We represent \underline{F}_H in the usual manner as,

$$\underline{F}_H = \alpha \frac{\partial \underline{\tau}}{\partial z} + A_H \nabla_H^2 \underline{V}_H$$

where $\underline{\tau}$ is, as above, the horizontal frictional stress acting across horizontal planes, and A_H is a constant coefficient of lateral eddy viscosity ($\partial \underline{\tau} / \partial z$ is multiplied by α in order to make the term a force per unit mass).

We look first at the term in $\underline{\tau}$.

$$\int_{\mathcal{R}} \underline{k} \cdot \nabla_H \times \alpha \frac{\partial \underline{\tau}}{\partial z} \, d\tau \cong \bar{\alpha} \int_{IH} \underline{k} \cdot \nabla_H \times \underline{\tau} \, d\sigma - \bar{\alpha} \int_{EH} \underline{k} \cdot \nabla_H \times \underline{\tau} \, d\sigma$$

where IH and EH are the horizontal projections of surfaces I and E , and $\bar{\alpha}$ is a vertically and laterally averaged value of α . The integral over surface EH contributes nothing, since, by definition, $\underline{\tau}$ is negligibly small at the top of the frictional boundary layer. The stress at the sea surface is to be identified with the wind stress $\underline{\tau}_w$, whose mean annual curl Munk (1950) gives for mid-latitudes as of the order 10^{-8} . We compare this integral with the vertical velocity term:

$$\frac{\int_{EH} \underline{k} \cdot \nabla_H \times \underline{\tau}_W d\sigma}{\int_{EH} f_0 \omega_T d\sigma} \sim \frac{|\nabla_H \times \underline{\tau}_W|}{f_0 \omega_T} \sim 2.5 \times 10^{-3}$$

Thus even with stresses an order of magnitude greater than the annual means estimated by Munk, the contribution of wind stress to the vorticity balance would be of no importance.

We turn to the lateral friction terms.

$$\int_{\mathcal{R}} \underline{k} \cdot \nabla_H \times A_H \nabla_H^2 \underline{V}_H d\tau = A_H \int_{\mathcal{R}} \nabla_H \cdot \nabla_H \underline{\zeta} d\tau = A_H \int_{\Sigma} \nabla_H \underline{\zeta} \cdot \underline{n} d\sigma$$

by Gauss's Theorem. We require an estimate of A_H . Unfortunately, no very credible measurements of it have been made. Munk's theory of the wind-driven circulation (op. cit.), however, would yield a current of the proper Gulf Stream width, 100 km, for $A_H = 3 \times 10^6$. Presumably this value constitutes a fair estimate of its order of magnitude, at least.

The integral over \mathcal{L} involves cross-stream velocity derivatives. South of Cape Hatteras, where the Stream presses against the Continental Slope, the rigid lateral boundary may very well sustain large derivatives on the left-hand edge of the Stream, and therefore permit lateral diffusion of relative vorticity to balance the net northward transport of planetary vorticity, as suggested in Munk's theory. Where the rigid boundary is replaced by the slow-moving slope water, however, such large derivatives are impossible. Motions in the slope water and Sargasso

Sea, for instance, of the order of those suggested at the beginning of this section would allow only an insignificant diffusion of vorticity across surface L . Therefore the corresponding integral over L is disregarded.

Since $\nabla_H \zeta$ is a horizontal vector, the integral over E is proportional to the area of the vertical projection of E , which is of the order qD' , where D' is the cross-stream change in elevation over E . Since the elevation change is of the order of the product of W , $|\nabla_H D|$, and the cosine of the angle between \underline{V}_{HE} and $\nabla_H D$, $D' \sim 4 \times 10^4$. Furthermore, $|\nabla_H \zeta| \sim \partial^2 v / \partial a^2 \sim 4v_{EM} / W^2$, where v_{EM} is the cross-stream maximum value of $|\underline{V}_{HE}|$, so that with $v_{EM} \sim 2T/W$,

$$A_H \int_E \nabla_H \zeta \cdot \underline{n} d\sigma \sim \frac{8TqD'A_H}{W^3} \sim 1 \times 10^6$$

Thus even were A_H an order of magnitude greater than estimated, the integral over E would still be insignificant in the vorticity balance.

Since the elevation change over surface I amounts only to about one meter for the Gulf Stream, the integral over I is immediately seen to be negligible.

The integral over S may conveniently be written:

$$A_H \int_S \nabla_H \zeta \cdot \underline{n} d\sigma = A_H \int \frac{\partial}{\partial l} (Kv) d\sigma - A_H \int_S \frac{\partial v}{\partial l \partial a} d\sigma$$

where l represents a coordinate in the direction of flow, normal to a . Since the integrand in the second integral

involves a cross-stream derivative, the integral itself depends only on velocities outside the Stream, and is hence to be neglected. The other integral also turns out to be small:

$$A_H \int_S \frac{\partial}{\partial l} (Kv) d\sigma \sim \frac{A_H K V}{g} \sim 3 \times 10^6$$

Again, an order-of-magnitude increase in A_H would leave the term still very much less than the relative vorticity flux. The entire friction term is therefore negligible.

F. Time Derivative Term

We have left now only the term,

$$\int_R \frac{\partial \zeta}{\partial t} d\tau$$

which is a difficult one to evaluate. Since Gulf Stream meanders move and change shape in a virtually unknown fashion, it is not possible to represent $\partial \zeta / \partial t$ in a form suitable for a convenient estimation of the integral. An important component of the time changes observed during the multiple ship surveys of 1950 and 1960, however, was a translation of the meander pattern as a whole. Probably, then, we can derive a fair idea of the relative magnitude of the time derivative term simply by considering a current pattern in which time changes are due solely to the propagation of meanders with some constant speed C chosen to be consonant with observed changes in the Gulf Stream.

We imagine, then, a current referred to Cartesian coordinates (ξ, η, z) - ξ and η are horizontal dimensions - deformed into a wave which travels in the ξ direction with speed c , so that

$$\frac{\partial \zeta}{\partial t} = -c \frac{\partial \zeta}{\partial \xi}$$

Then,

$$\int_{\mathcal{R}} \frac{\partial \zeta}{\partial t} d\tau = -c \int_{\mathcal{R}} \frac{\partial \zeta}{\partial \xi} d\xi d\eta dz = -c \int_{\mathcal{Q}} \frac{\partial}{\partial \xi} \left(\int \zeta dz \right) d\xi d\eta$$

where the vertical integration is carried out over the entire vertical extent of \mathcal{R} , and \mathcal{Q} is that portion of a level surface cut out by the figure composed of surfaces \mathcal{S} and \mathcal{L} . We designate the bounding contour of surface \mathcal{Q} by

\mathcal{G} . Since

$$\frac{\partial}{\partial \xi} \left(\int \zeta dz \right) = \underline{\eta}_0 \cdot \nabla \times \left(\underline{\eta}_0 \int \zeta dz \right)$$

where $\underline{\eta}_0$ is a unit vector in the η direction,

$$c \int_{\mathcal{Q}} \frac{\partial}{\partial \xi} \left(\int \zeta dz \right) d\xi d\eta = \oint_{\mathcal{G}} ds \underline{\eta}_0 \cdot \underline{t} \int (Kv - \partial v / \partial z) dz$$

by Stokes's Theorem, where ds is an element of \mathcal{G} , and \underline{t} , a unit vector tangent to \mathcal{G} , directed in the sense of the integration.

The velocity and its derivatives are negligibly small along those segments of \mathcal{G} which lie in surface \mathcal{L} , so that integration over them contributes nothing. The integrals of $\partial v / \partial z$ over the remaining segments of \mathcal{G} also

involve only velocities outside the current, so we disregard them too. Therefore:

$$\int_R \frac{\partial \xi}{\partial t} d\tau \cong -c \oint ds \eta_0 \cdot \int K v dz \sim cKV$$

During the survey of 1950, the meander pattern moved down-stream at a rate of about 7 cm/sec; during 1960, the speed was rather smaller, about 2-3 cm/sec. A typical value of c , then, would perhaps be 5 cm/sec. Thus we estimate cKV to be of the order 5×10^7 , or 10% of the relative vorticity flux.

It must be emphasized that this simple scheme of a wave moving without change in shape is by no means a satisfactory description of time changes in meander patterns; its utility is limited to making order-of-magnitude estimates. Application of the scheme, however, has suggested that local time changes in the vorticity distribution do not constitute a significant feature of the integrated vorticity balance of the Stream. Hence the term in the equation which describes time variations is to be omitted.

Inasmuch as the bottom frictional boundary layer plays no important role in the vorticity balance, it too will henceforth be ignored. We shall regard the quasi-geostrophic velocities at the top of the layer to be equivalent to bottom velocities, and replace surfaces E and EH by bottom surfaces B and BH .

Thus empirical analysis reveals that only a few

terms in the volume-integrated vorticity equation are of actual significance to the dynamics of the open-ocean Gulf Stream. To a good approximation, the vorticity transformations occurring in that portion of the Stream are described by the equation:

$$\int_S (Kv + \beta y) \underline{v} \cdot \underline{n} \, d\sigma - f_0 \int_{BH} w_T \, d\sigma = 0 \quad (4)$$

Since w_T depends on the slope of the ocean bottom, we find that topography must indeed be affecting the Stream, as was suggested by the correlations between observed current paths and the trend of deep isobaths. (Of course, since w_T depends also on bottom velocities in the Stream, this inference is valid only if they are not typically very much smaller than those calculated on the basis of the 1960 data.)

Equation (4) describes a very simple meander mechanism, which is composed of two dynamical features. The curvature and β -terms by themselves would describe a stationary Rossby wave: a current of constant volume transport, flowing northward over a level bottom, cannot remain in geostrophic balance, for the northward increase in Coriolis parameter, coupled with the condition of constant transport, implies a gradual increase in magnitude of the Coriolis force acting on the current over that of the pressure gradient force. The net force produced imparts an

acceleration normal to the direction of flow, resulting in eastward deflection of the current, and the development of anticyclonic curvature in the streamlines. The deflective force and curvature increase until the direction of flow becomes due east; as the current continues to turn, now to the south, the force diminishes, and finally vanishes when the current returns to its original latitude, where exact geostrophy is again established. The anticyclonic meander so formed is then followed by a cyclonic meander, since continued southward motion leads to a deflective force of opposite sense to that above.

The curvature and topographic terms alone in (4) would describe a "topographic wave", associated with a similar imbalance between Coriolis and pressure gradient forces. The conservation of volume transport in a current which extends to the bottom of the ocean requires a redistribution of velocity as the current flows over a shoaling bottom: if the current maintains its width, the velocity at all levels must increase, and hence - for constant Coriolis parameter - the Coriolis force acting on the current also; if the current broadens, so that velocities need not increase for the current to maintain its transport, the cross-stream pressure gradient must decrease. In any case, just as in the Rossby wave, there must occur a relative increase in Coriolis over pressure gradient force, and an attendant development of anticyclonic curvature. Similarly,

a development of cyclonic curvature must be associated with flow over a deepening bottom, so that again the current executes a sinuous path. Whereas the axes of Rossby waves lie along parallels of latitude, the axes of these topographic waves must be related to the trend of the isobaths. The meandering current described by equation (4), then, represents a certain combination of these two stable waves.

At first glance, it would seem that a topographic effect achieved through such forced variations in the depth of a quasi-geostrophic current ought also to be apparent in a quasi-geostrophic current which does not extend to the bottom of the ocean, but has a sloping surface of no horizontal motion as a lower boundary (Neumann, 1956). That this cannot be so is plainly shown by equation (4), in which the effect due to the slope of the lower boundary is proportional to the horizontal velocity component on it. Therefore, when the boundary is a surface of zero horizontal velocity, the topographic influence must vanish. Closer scrutiny reveals that the two superficially similar situations are in fact fundamentally different. In a quasi-geostrophic current, the cross-stream pressure gradient must vanish on a surface of no horizontal motion; consequently, if that surface deepens in the down-stream direction, the cross-stream pressure gradient must increase down-stream along all levels above the surface. Therefore there must exist a pressure gradient component parallel to the

main current along at least one side of it, and a corresponding geostrophic flow into the current across that lateral boundary. Thus the depth variation of a current flowing above a sloping surface of no motion does not imply a redistribution of velocity tending to destroy geostrophic equilibrium, but simply a gradual change in the volume transport of the main current.

4. APPROXIMATE CURRENT PATHS

Very likely, then, bottom topography influences the Gulf Stream east of Cape Hatteras in the manner described by equation (4). It remains to determine whether this mechanism can in fact be held to account for the observed meander patterns depicted in Figure 1. We desire, therefore, to make computations of current paths based on the combined topographic and β -effects, and compare them with the observed paths.

With a further specification of volume \mathcal{R} , and a set of credible approximations, it is possible to transform (4) from a sum of integrals into a tractable, ordinary differential equation useful for such computations. Integration over \mathcal{S} in (4) involves two surfaces, which we shall designate \mathcal{S}_1 and \mathcal{S}_2 . Let the upstream surface \mathcal{S}_1 be fixed to cross the current at an inflection in its path, so that

$$\int_{S_1} K v \underline{V}_H \cdot \underline{n} \, d\sigma = 0$$

Furthermore, let the origin of the coordinate system (x, y) be set in S_1 at such a position that

$$\int_{S_1} x \underline{V}_H \cdot \underline{n} \, d\sigma = \int_{S_1} y \underline{V}_H \cdot \underline{n} \, d\sigma = 0$$

On S_2 (where \underline{V}_H is normal to \underline{n}) we define weighted average values of x , y , and K - indicated by bars - according to the relations:

$$\int_{S_2} x \underline{V}_H \cdot \underline{n} \, d\sigma \equiv \bar{x} \int_{S_2} v \, d\sigma = \bar{x} V$$

$$\int_{S_2} y \underline{V}_H \cdot \underline{n} \, d\sigma \equiv \bar{y} \int_{S_2} v \, d\sigma = \bar{y} V$$

$$\int_{S_2} K v \underline{V}_H \cdot \underline{n} \, d\sigma \equiv \bar{K} \int_{S_2} v^2 \, d\sigma = \bar{K} M$$

so that by a current path we shall now mean a curve $\bar{y}(\bar{x})$. In addition, we identify the curvature of $\bar{y}(\bar{x})$ with \bar{K} by making the assumption:

$$\bar{K} \cong \frac{d^2 \bar{y}}{d\bar{x}^2} \left[1 + \left(\frac{d\bar{y}}{d\bar{x}} \right)^2 \right]^{-3/2} \quad (5)$$

The other approximations bear on the topographic term.

$$\int_{BH} w_T d\sigma = - \int_{BH} \underline{V}_{HB} \cdot \nabla_H D d\sigma = - \int_{BH} v_B \frac{\partial D}{\partial l} dl da$$

since the natural coordinate l is in the direction of \underline{V}_{HB} . We shall assume that any variations in bottom velocity and current width in the direction of flow are inversely related to a sufficient extent that we may treat the product of v_B and the horizontal element of distance between streamlines da as independent of l . Then

$$\int_{BH} v_B \frac{\partial D}{\partial l} dl da \cong \int v_B \delta D da \quad (6)$$

where the integration over a is between the lateral bounds of the current, and δD denotes the change in depth along a streamline of \underline{V}_B between the cross-stream boundaries of surface BH : B_1 and B_2 , which lie in surfaces S_1 and S_2 . Furthermore, we define a weighted cross-stream average of δD by the relation,

$$\int v_B \delta D da \cong \bar{\delta D} \int v_B da = \bar{\delta D} T$$

For analytic convenience we shall approximate the actual, curved ocean bottom by a system of planes with slopes and orientations appropriate to the local topography. Accordingly, we represent δD as

$$\delta D = D_x (x_2 - x_1) + D_y (y_2 - y_1)$$

where (x_1, y_1) are the coordinates of a streamline of \underline{V}_B on B_1 , and (x_2, y_2) are the corresponding coordinates on B_2 ; the parameters D_x and D_y are the locally averaged

x and y components of $\nabla_H D$. The essence of this plane approximation is that we treat D_x and D_y locally as constants, but also as parameters which change their values discontinuously from one region of the Continental Rise to another.

We relate $\overline{\delta D}$ to $\overline{y}(\overline{x})$ by making the additional assumptions,

$$\int_{B_1} x v_B da \cong \int_{B_1} y v_B da \cong 0$$

$$\int_{B_2} x v_B da \cong \overline{x} \int_{B_2} v_B da$$

$$\int_{B_2} y v_B da \cong \overline{y} \int_{B_2} v_B da$$

where \overline{x} and \overline{y} are defined as above. The basic assumption here is that the bottom current is not very much out of phase with the vertically averaged current, nor laterally displaced very much from it. Thus

$$\overline{\delta D} \cong D_x \overline{x} + D_y \overline{y} \quad (7)$$

If we now insert these representations into equation (4), we obtain a non-linear, ordinary differential equation. For simplicity we shall henceforth omit the bars indicative of averages, but always understand the current path $y(x)$ to mean strictly the curve $\overline{y}(\overline{x})$ defined above. Thus,

$$\frac{dy^2}{dx^2} \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{-3/2} + Qy - Sz = 0 \quad (8)$$

where $Q \equiv (\beta V - f_0 T D_y) / M$ and $S \equiv f_0 T D_x / M$. For definiteness, we took the upstream surface S_1 to cross the current at the path inflection; were we instead to have the downstream surface S_2 cross at the inflection, we should simply change the sign of each term in (4), and hence not alter (8) at all. Thus equation (8) describes a current path both upstream and downstream from the coordinate origin.

We have already taken V and T to be independent of position along the current path; we shall now treat M as a constant also, in order to make (8) an equation with constant coefficients. Actually, the redistributions of velocity attendant on depth changes require variations in T and M of the same sense. These must be roughly proportional to the percentage changes in depth over meander quarter-wave lengths; according to Figure 1, such changes may be as great as 20% in observed meanders, but since, crudely speaking, it is the ratio of T to M which appears in (8), the error made by ignoring such variations will be much smaller than this. The discussion in Section 3 indicates that variations in V are probably small. It is very hard to estimate the validity of the other approximations which led to equation (8), because they involve details of

the Gulf Stream velocity distributions which are not at all well known. Intuitively, however, the approximations (5), (6), and (7) seem credible.

An immediate difficulty with (8) is its inhomogeneity. This is easily removed: it is clearly possible to transform the linear combination $Qy - Sz$ into a single term by an appropriate rotation of axes, and since curvature is an invariant property of a curve, the differentiated term must retain its form under such a transformation. Thus (8) is cast into the homogeneous form

$$\frac{d^2\eta}{d\xi^2} \left[1 + \left(\frac{d\eta}{d\xi} \right)^2 \right]^{-3/2} + P\eta = 0 \quad (9)$$

where $P \equiv \sqrt{Q^2 + S^2}$, by the rotation,

$$\begin{aligned} \xi &= x \cos \theta + y \sin \theta \\ \eta &= -x \sin \theta + y \cos \theta \end{aligned} \quad (10)$$

in which $\theta \equiv \tan^{-1} S/Q$.

The proportionality between curvature and displacement shows clearly the oscillatory character of $\eta(\xi)$, which we of course expect from our interpretation of equation (4). Equation (9) amounts to a generalization of an equation proposed by Rossby (1940) to describe, in effect, stationary Rossby waves: it reduces to Rossby's equation when $D_x = D_y = 0$. On the other hand, when $\beta = 0$, (9) describes a pure topographic wave. The structure of (9) is worth some attention, for it affords insight into the manner in which the Rossby and topographic waves combine to form the meandering

current described by (4). The Rossby wave is characterized by the vector: $\sqrt{M}^{-1} \nabla_M f$, whose normal gives the direction of the wave axis, and whose magnitude measures the scale of the wave; the topographic wave for a plane ocean bottom is characterized in an identical fashion by the vector: $-f_0 T M^{-1} \nabla_M D$. Equation (9) specifies the combination wave simply by the resultant of these two vectors: its normal defines the axis direction (Θ), and its magnitude (P) is the scale parameter.

To obtain a first integral of (8) we make the substitution, $\eta'' = \eta' \frac{d\eta'}{d\eta}$, where primes indicate differentiation with respect to ξ . Then

$$P \eta d\eta + (1 + \eta'^2)^{-3/2} \eta' d\eta' = 0 \quad (11)$$

whereby

$$P \eta^2 - \frac{2}{\sqrt{1 + \eta'^2}} = \text{constant} \quad (12)$$

We prescribe the constant of integration by denoting the wave amplitude as η_0 , i.e. we set $\eta = \pm \eta_0$ at $\eta' = 0$. Hence the constant equals $P \eta_0^2 - 2$, and

$$P(\eta_0^2 - \eta^2) - 2 \left(1 - \frac{1}{\sqrt{1 + \eta'^2}} \right) = 0 \quad (13)$$

For any value of η' then, formula (13) permits calculation of the associated value of η .

To find the corresponding value of ξ , we must perform another integration. We rewrite (13) in the form,

$$\left(\frac{d\xi}{d\eta}\right)^2 = \frac{[1 + P/2(\eta^2 - \eta_0^2)]^2}{1 - [1 + P/2(\eta^2 - \eta_0^2)]^2} \quad (14)$$

from which it follows that

$$\eta = \int_0^\eta \frac{[1 + P/2(\eta_0^2 - \eta^2)] d\eta}{\sqrt{1 - [1 + P/2(\eta_0^2 - \eta^2)]^2}} \quad (15)$$

Any ambiguities in the sign of the integrand associated with extracting the square root of (14) are to be resolved by taking it consistent with the sign of $d\xi/d\eta$ in the range of integration. Since $\xi(\eta)$ is multi-valued, (14) cannot yield values of ξ in excess of a quarter-wave length of $\eta(\xi)$; such distances, therefore, must be calculated by adding an integral number of quarter-wave lengths to a distance given by (15).

Through an elliptic substitution, one can carry out the indicated integration, and express ξ in a form suitable for computations:

$$\xi = \frac{1}{\sqrt{P}} \left\{ 2E(\varphi, k) - F(\varphi, k) - \frac{k^2 \sin 2\varphi}{\sqrt{1 - k^2 \sin^2 \varphi}} \right\} \quad (16)$$

where $F(\varphi, k)$ and $E(\varphi, k)$ are the elliptic integrals of the first and second kind, and

$$k^2 = \frac{P}{4} \eta_0^2$$

$$k^2 \sin^2 \varphi = \frac{P/2 \eta^2}{2 + P/2 (\eta^2 - \eta_0^2)}$$

To evaluate (15), we make the definitions:

$$a^2 \equiv \frac{4}{P} - \eta_0^2, \quad b^2 \equiv \eta_0^2$$

and use them to put (15) in standard form:

$$z = \left(\frac{2}{P} - \eta_0^2\right) \int_0^\eta \frac{d\eta}{\sqrt{(a^2 + \eta^2)(b^2 - \eta^2)}} + \int_0^\eta \frac{\eta^2 d\eta}{\sqrt{(a^2 + \eta^2)(b^2 - \eta^2)}}$$

We introduce Jacobian elliptic functions by making the substitution (Milne-Thomson, 1950),

$$u = \operatorname{sd}^{-1} \left\{ \frac{\sqrt{a^2 + b^2}}{ab} \eta \middle| k^2 \right\}$$

where $k^2 = b^2(a^2 + b^2)^{-1/2} = P \eta_0^2 / 4$. Then

$$\eta = \frac{ab}{\sqrt{a^2 + b^2}} (\operatorname{sd} u) \quad \text{and} \quad d\eta = \frac{ab}{\sqrt{a^2 + b^2}} (cdu)(mdu) du$$

We note that

$$(a^2 + \eta^2)(b^2 - \eta^2) = a^2 b^2 (mdu)^2 (cdu)^2$$

Therefore

$$\int_0^\eta \frac{d\eta}{\sqrt{(a^2 + \eta^2)(b^2 - \eta^2)}} = \frac{1}{\sqrt{a^2 + b^2}} \int_0^u du = \frac{u}{\sqrt{a^2 + b^2}}$$

and

$$\int_0^\eta \frac{\eta^2 d\eta}{\sqrt{(a^2 + \eta^2)(b^2 - \eta^2)}} = \frac{a^2 b^2}{(a^2 + b^2)^{3/2}} \int_0^u (\operatorname{sd} u)^2 du$$

The latter integral can be carried out through use of the elliptic identity,

$$\frac{d}{du} \left[\frac{(snu)(cnu)}{(dnu)} \right] = \frac{1}{k^2} (dnu)^2 - \frac{1-k^2}{k^2} - (1-k^2)(sdu)^2$$

Thus

$$\int_0^\eta \frac{\eta^2 d\eta}{\sqrt{(a^2+\eta^2)(b^2-\eta^2)}} = \sqrt{a^2+b^2} E(u) - \frac{a^2 u}{\sqrt{a^2+b^2}} - \frac{b^2}{\sqrt{a^2+b^2}} \frac{(snu)(cnu)}{(dnu)}$$

and upon combining these integrations we obtain the result that

$$\xi = \frac{1}{\sqrt{P}} \left\{ 2E(u) - u - \frac{2k^2 (snu)(cnu)}{(dnu)} \right\}$$

This expression for ξ is transformed into the computationally more convenient representation (16) by defining an angle φ through the relation,

$$u \equiv \int_0^\varphi \frac{d\psi}{\sqrt{1-k^2 \sin^2 \psi}}$$

from which it follows that

$$\eta = \frac{ab}{\sqrt{a^2+b^2}} \frac{\sin \varphi}{\sqrt{1-k^2 \sin^2 \varphi}}$$

hence

$$k^2 \sin^2 \varphi = \frac{P/2 \eta^2}{2 + P/2 (\eta^2 - \eta_0^2)}$$

and

$$\xi = \frac{1}{\sqrt{P}} \left\{ 2E(\varphi, k) - F(\varphi, k) - \frac{k^2 \sin 2\varphi}{\sqrt{1-k^2 \sin^2 \varphi}} \right\}$$

Although it is thus not possible to write the

solution of (9) explicitly in the form: $\eta = \eta(\xi)$, nevertheless, given the direction of the current and curvature of its path at some point, equations (9), (13), and (16) permit the calculation of any other point on a path described by (9). Then by inversion of formulas (10), one can find the corresponding points of the solution to (8).

5. CURRENT PATH COMPUTATIONS

The formulas developed in the preceding section permit the approximate calculation of current paths governed by the mechanisms which have been inferred to control the Gulf Stream path east of Cape Hatteras. In order to verify that these mechanisms can indeed account for the observed meander patterns of the Stream, an approximate current path has been calculated to correspond to each Stream path shown in Figure 1. Calculated paths were matched to observed paths by using the measured current direction and path curvature at a single point on an observed path as the initial conditions required to determine a solution of equation (8). Some care was necessary in choosing the initial points, because, while the observational data seem generally dense enough to permit estimating current directions with fair precision from the interpolated paths,

they are much too sparse for correspondingly reliable estimates of path curvatures. On the other hand, they do allow rather close definition of the positions of path inflections. Therefore inflection points near the upstream ends of the observed path segments were adopted as initial points for all calculations.

Because considerable labor is involved in the repeated use of formulas (13) and (16), only the positions of selected points of particular interest on a current path were calculated: inflection points, relative maxima and minima in $y(x)$ or $\eta(\xi)$, infinities in dy/dx , or points at distinct boundaries between very different topographic regimes. A path was then drawn to connect these points from knowledge of the general character of solutions to equation (9). The computational procedure was to guess the position of the first point of interest beyond the initial point, estimate the average value of $\nabla_{\mu}D$ over the connecting path segment, insert the value together with the current direction and path curvature at the initial point into (9), (13), and (16), and thus obtain coordinates (ξ, η) for the new point. By inverting (10), these were then translated into coordinates (x, y) . If it happened that the estimated and calculated positions of the point were sufficiently different to make the depth gradient value used in the calculation inappropriate to the topography actually underlying the calculated path segment,

the procedure was repeated with revised values of $\nabla_{\mu}D$ until a self-consistent calculation was achieved. The computed path characteristics at this new point then served as initial conditions for calculating by the same procedure the position of the next desired point.

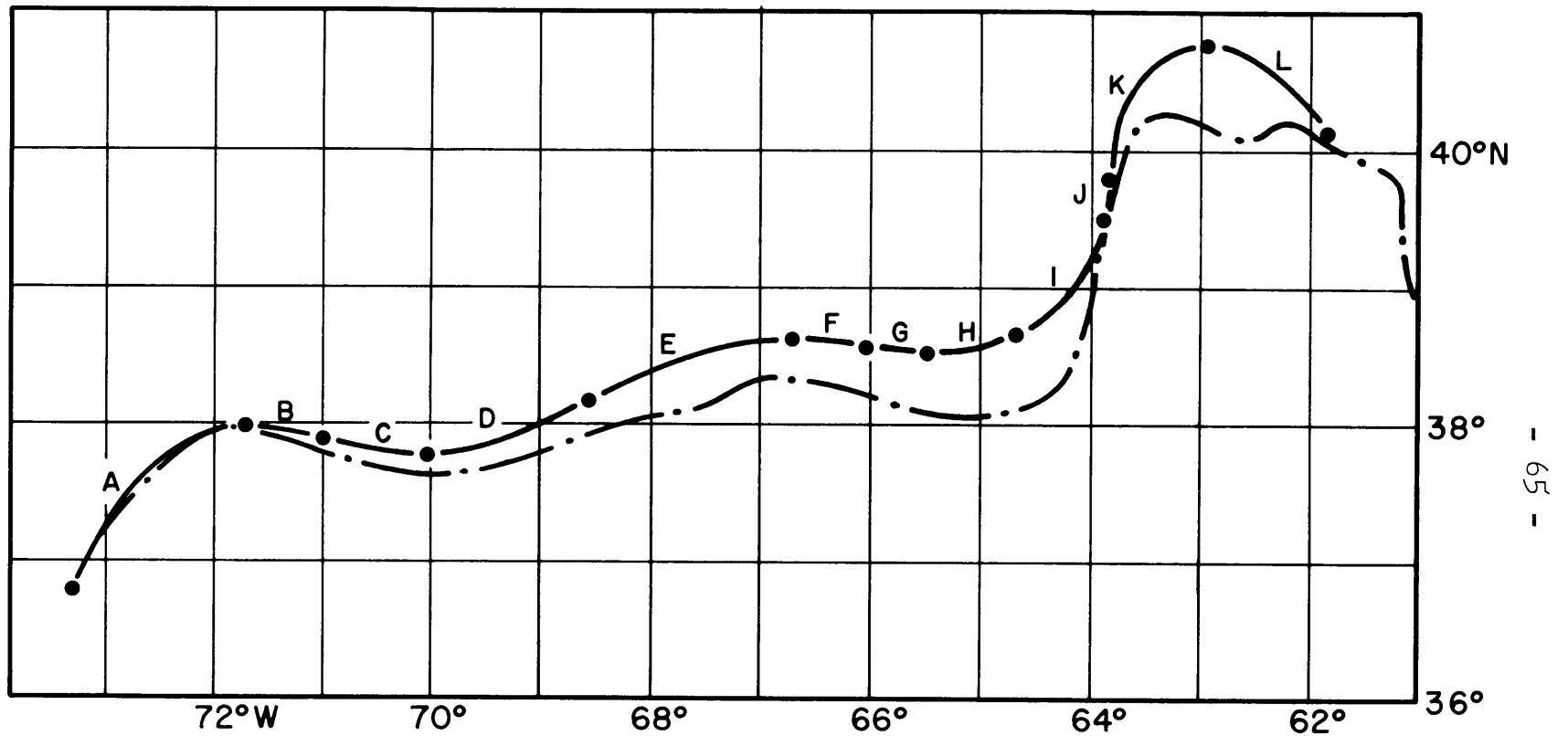
The values adopted for the current parameters V , M , and T were those used for the order-of-magnitude analysis in Section 3, i.e. $V = 10^{14}$, $M = 5 \times 10^{15}$, $T = 10^8$ in c.g.s. units. (The same values were used in all calculations.) Since the latitudes concerned were roughly 36° - $41^{\circ}N$, f_0 was taken as 0.9×10^{-4} , and β , as 1.8×10^{-13} . Values of $\nabla_{\mu}D$ were estimated from the bathymetric data presented on the charts cited in Section 2.

The five pairs of observed and calculated current paths are shown in Figures 2-6. Solid lines denote calculated paths; the dots on them indicate the current positions actually computed. The dashed or dotted lines represent the observed paths as depicted in Figure 1, and described in Section 2. The measured set of the current in degrees true at the initial inflection point (westernmost point of each path) is noted in the legend for every figure. The depth gradients used in the step-wise computations are given on the figures for each path segment by the magnitudes of the gradients D_S and their directions α in degrees true. (The computations were actually made in terms of linear distances between points, which were translated into angular coordinates by the relations appropriate to lati-

tude 38°N: 1° of longitude = 88 km, 1° of latitude = 111 km.)

Figure 2 has to do with the Gulf Stream path observed in early June, 1950. Both it and the calculated path turn eastward near longitude 72°W, and meander very gently until they reach longitude 64°W, where they both turn abruptly northward and form large anticyclonic meanders. Downstream from the initial point the calculated path tends to run a little north of the observed path, but the agreement between the gross features of the two paths is very good.

The path taken by the Stream two weeks later is shown in Figure 3. On 19 June the large cyclonic meander located near longitudes 61°-60°W (see Figure 1) separated from the Stream. Since several assumptions employed in the derivation of equation (8) must be invalid in the process of eddy formation, a calculated path probably ought strictly to be compared to the path observed just prior to separation rather than to that just after. Therefore a segment of the Stream path charted on 18 June (Fuglister and Worthington, 1951) is indicated on the figure by the positions of the 18°C isotherm of the upper 200 m layer. Both the observed and calculated paths in Figure 3 meander much more vigorously upstream of longitude 64°W than those depicted in Figure 2. Like the latter, they both turn abruptly northward at 64°W to form great anticyclonic

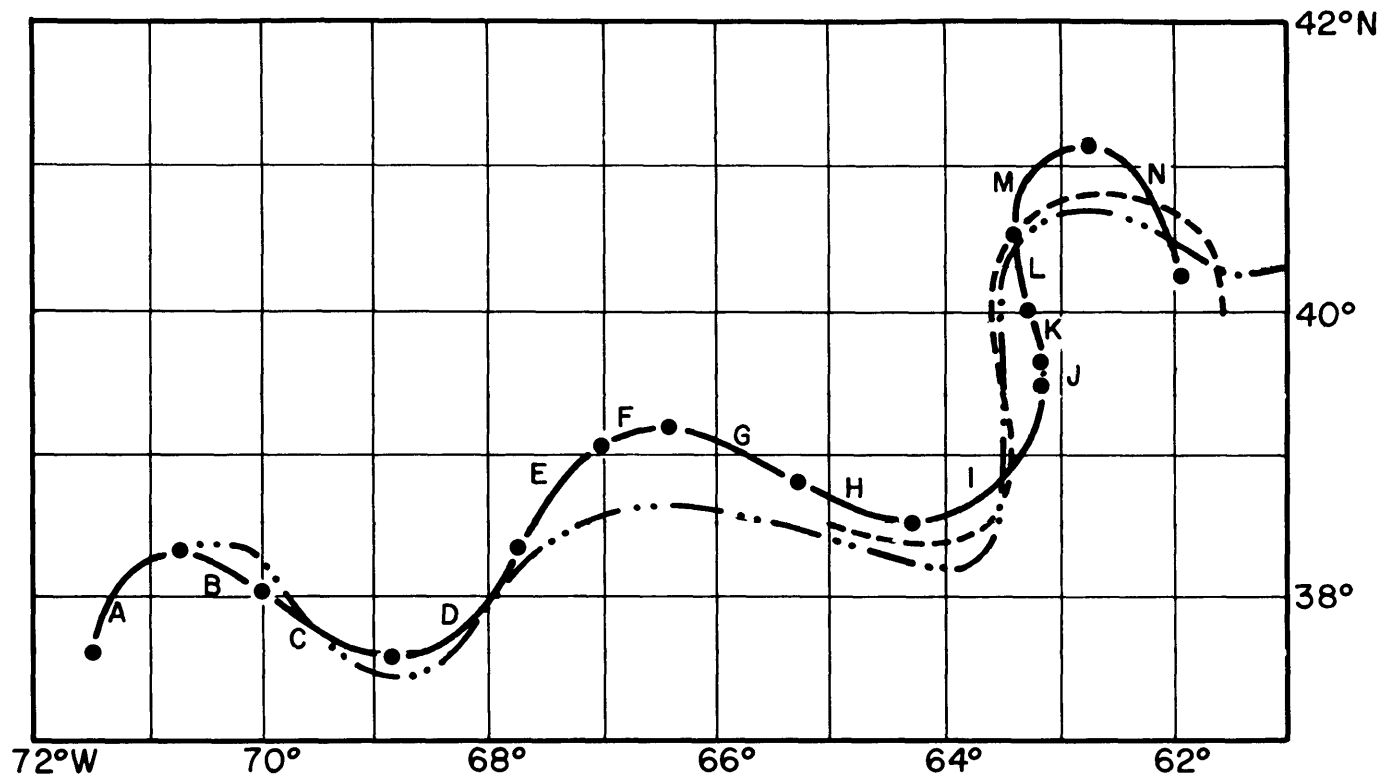


AVERAGE DEPTH GRADIENT FOR EACH PATH SEGMENT:

- · - OBSERVED PATH
 — CALCULATED PATH;
 INITIAL DIRECTION: 015

	D_s	α		D_s	α
A	7×10^{-3}	140	G	5×10^{-3}	120
B	9×10^{-3}	153	H	5×10^{-3}	120
C	9×10^{-3}	170	I	0	—
D	9×10^{-3}	170	J	7×10^{-3}	165
E	10×10^{-3}	165	K	7×10^{-3}	165
F	3×10^{-3}	135	L	7×10^{-3}	165

Figure 2. Comparison of the Gulf Stream path observed on 8-10 June 1950 with a calculated current path matched to the observed path at its upstream end.



AVERAGE DEPTH GRADIENT FOR EACH PATH SEGMENT:

	D_s	α	D_s	α
— · — · — ·	OBSERVED PATH, 21-22 JUNE 1950			
- - - - -	OBSERVED PATH, 18 JUNE 1950			
—	CALCULATED PATH; INITIAL DIRECTION: 011			
A	16×10^{-3}	158	H	6×10^{-3}
B	16×10^{-3}	158	I	0
C	6×10^{-3}	164	J	0
D	9×10^{-3}	158	K	7×10^{-3}
E	12×10^{-3}	146	L	7×10^{-3}
F	12×10^{-3}	166	M	7×10^{-3}
G	4.5×10^{-3}	135	N	7×10^{-3}

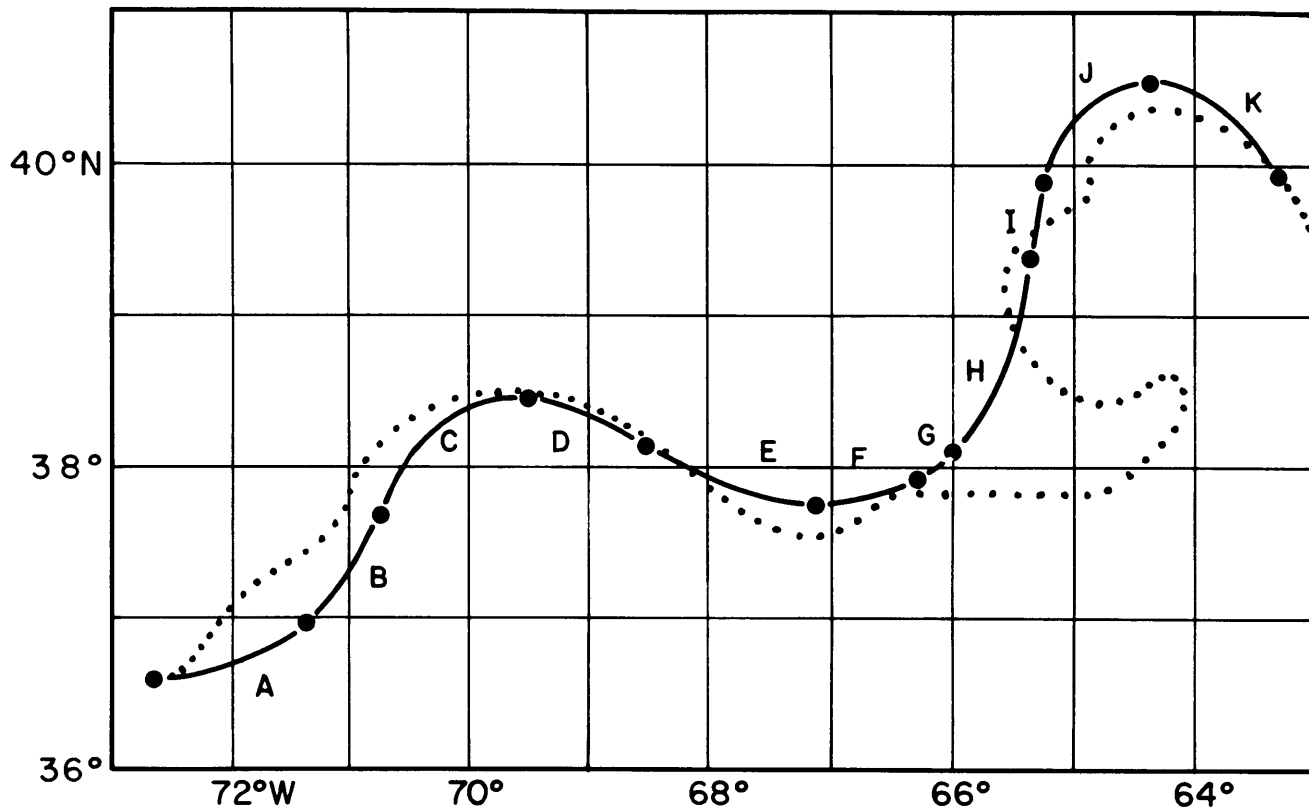
Figure 3. Comparison of the Gulf Stream path observed on 21-22 June 1950 with a calculated current path matched to the observed path at its upstream end. Also depicted is a segment of the path observed on 18 June 1950.

meanders, but meanders of greater amplitude and smaller width than those of Figure 2. The calculated path again tends to run a little north of the observed path, but the correspondence between gross features is as notable as above. The function $\psi(x)$ even becomes multi-valued between longitudes 64° - 63° W, in agreement with the Stream path observed on 18 June.

These calculated patterns are easily understood qualitatively in terms of the distribution of isobaths. On the Continental Rise, the contribution of the depth gradient term to the scale parameter P is three to six times greater than that of the β -term. Hence, upstream of longitude 64° W, the paths are guided by the topography in the manner described in Section 3: they meander about axes closely paralleling the isobaths. The amplitudes in late June exceed those in early June simply because the angle between the initial current direction and the trend of isobaths was greater for the former path than for the latter. Both paths inflect somewhat west of the abrupt northward turn in the deep bottom contours, and hence develop large cyclonic curvatures just as they leave the Continental Rise and pass onto the Abyssal Plain. The excess of pressure gradient over Coriolis force acting on the current is therefore relatively large at this point, and deflects the current to the north. Since the subsequent depth gradient is zero, the path becomes a

segment of a stationary Rossby wave, subject only to the restoring effect of the variation in Coriolis parameter. Inasmuch as the β -effect is considerably weaker than the topographic effect experienced upstream, a correspondingly greater northward displacement is required to destroy the deflective tendency than was necessary before. In fact, the calculated currents do not actually inflect and turn eastward again until they are once more constrained by the Continental Rise, now found north of latitude $39^{\circ}30'N$. Thus it is the abrupt bend in isobaths, permitting the currents to run onto the Abyssal Plain with relatively intense cyclonic curvature, which explains the development of the great anticyclonic meanders in the calculated paths.

On both occasions in 1950 the Gulf Stream displayed triple-crested meander patterns; in contrast, the 1948 pattern, shown in Figure 4, is double-crested. Nevertheless the calculation reproduces the observed path with about the same fidelity that it did the 1950 paths. It does, however, miss the sharp indentation in the large anticyclonic meander. This discrepancy probably cannot be explained by weaknesses in the approximations used to transform (4) into (8), because the curvature in the indentation is much too great to be linked to any local topographic feature, and hence to be described by (4) at all. On the other hand, the shape of the observed, indented



AVERAGE DEPTH GRADIENT FOR EACH PATH SEGMENT

..... OBSERVED PATH					
———— CALCULATED PATH; INITIAL DIRECTION, 080°					
		D_s	α	D_s	α
	A	8×10^{-3}	130	G	4.5×10^{-3} 120
	B	8×10^{-3}	142	H	4.5×10^{-3} 120
	C	11×10^{-3}	153	I	7.5×10^{-3} 120
	D	7×10^{-3}	165	J	7×10^{-3} 165
	E	3×10^{-3}	155	K	7×10^{-3} 165
	F	3×10^{-3}	155		

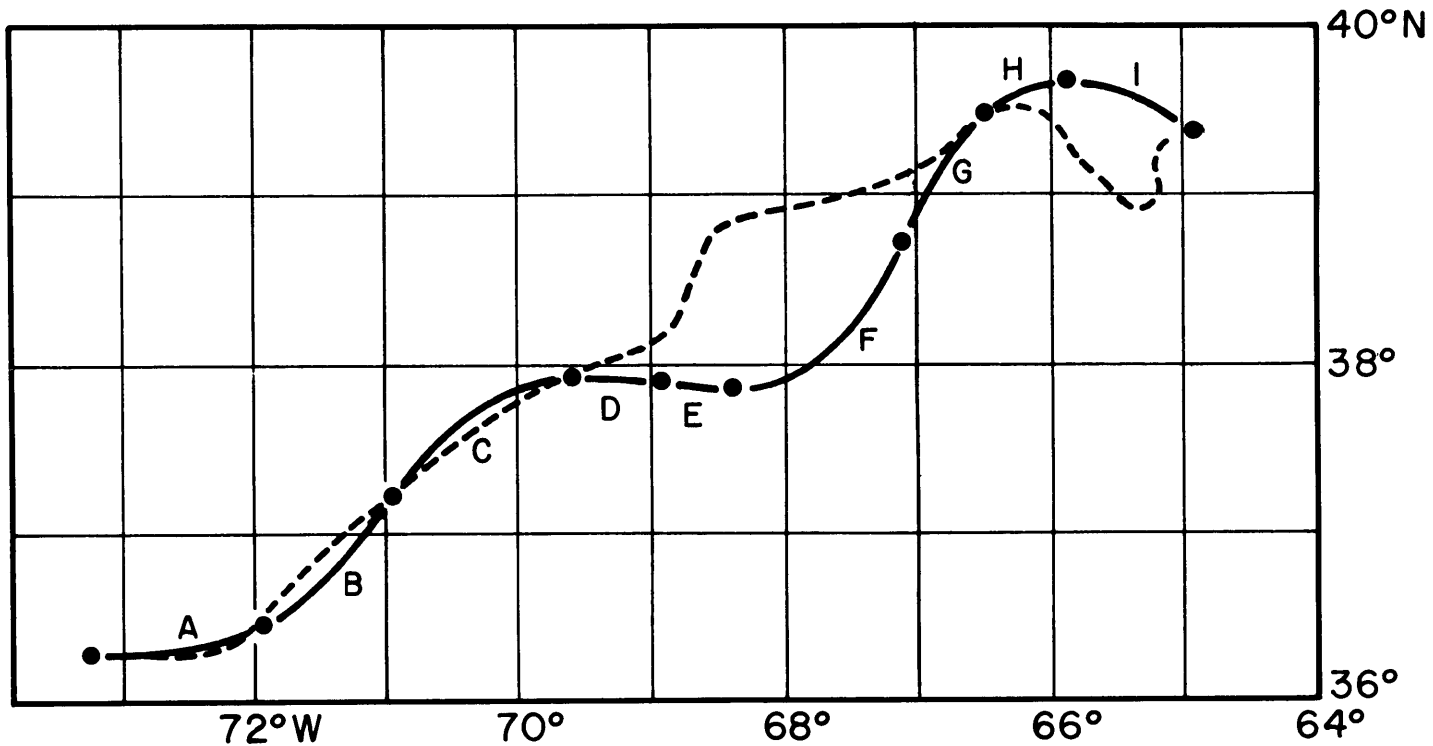
Figure 4. Comparison of the Gulf Stream path observed 29 November-4 December 1948 with a calculated current path matched to the observed path at its upstream end.

path is based on surface temperature, a less reliable indicator of the position of the Stream as a whole than deeper temperatures; the calculated path may actually follow more closely the vertically averaged Stream path than does the "observed" path.

Unlike the other two paths considered, this one never overlies the Abyssal Plain; hence it is not a vanishing of the depth gradient which produces the large anticyclonic meander. Its existence is to be attributed instead to the guiding effect of the northward turning isobaths between longitudes 66° - 65° W. In both the 1948 and 1950 situations, therefore, the sudden, extensive bend in bottom contours is intimately associated with the development of extreme meanders.

Figure 5 shows the Gulf Stream path found in May, 1946, and the corresponding calculated path. The correlation is not so striking as in the previous cases, but is persuasive nevertheless. Since the "observed" path is again actually a surface isotherm, the reservation noted above for the 1948 path should bear on the interpretation of this one also.

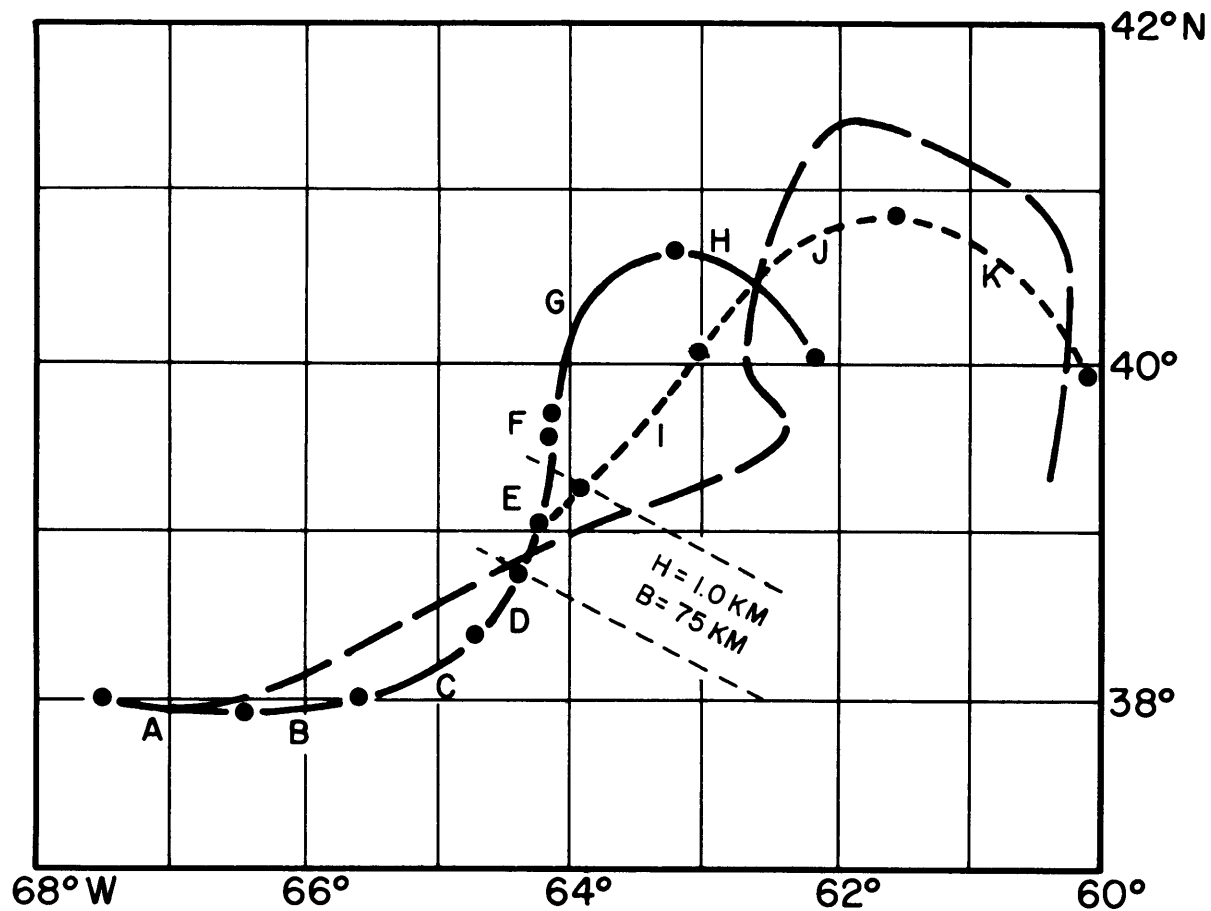
The Stream path charted in April 1960, is shown in Figure 6. For the same reasons given in the discussion of the 1950 paths, the calculated current turns sharply northward just beyond the bend in isobaths, and its path develops another great anticyclonic meander. Since the



AVERAGE DEPTH GRADIENT FOR EACH PATH SEGMENT:

	D_s	α		D_s	α	
----- OBSERVED PATH			F	12×10^{-3}	146	
———— CALCULATED PATH; INITIAL DIRECTION: 090	A	8×10^{-3}	146	G	12×10^{-3}	135
	B	8×10^{-3}	146	H	13×10^{-3}	170
	C	11×10^{-3}	153	I	7×10^{-3}	135
	D	11×10^{-3}	153			
	E	12×10^{-3}	146			

Figure 5. Comparison of the Gulf Stream path observed on 24-29 May 1946 with a calculated current path matched to the observed path at its upstream end.



**AVERAGE DEPTH GRADIENT
FOR EACH PATH SEGMENT:**

	D_s	α
A	3×10^{-3}	155
B	3×10^{-3}	155
C	6.5×10^{-3}	122
D	0	—
E	0	—
F	7×10^{-3}	165
G	7×10^{-3}	165
H	7×10^{-3}	165
I	0	—
J	4×10^{-3}	180
K	3×10^{-3}	180

- — — OBSERVED PATH
- CURRENT PATH CALCULATED FOR LEVEL ABYSSAL PLAIN;
INITIAL DIRECTION: 098
- - - - - CURRENT PATH CALCULATED FOR ABYSSAL PLAIN CROSSED
BY RIDGE AT POSITION OF KELVIN SEAMOUNT

Figure 6. Comparison of the Gulf Stream path observed in April 1960 with a calculated current path matched to the observed path at its upstream end. The branching path is deflected by a fictitious ridge at the position of Kelvin Seamount.

form of the observed path is based on meridional crossings of the Stream at two degree intervals between longitudes $68^{\circ}30'W$ and $60^{\circ}30'W$, the apparent discrepancy between observed and calculated paths on the interval between longitude $66^{\circ}30'W$ and $64^{\circ}30'W$ need not be real. On the other hand, the calculated central longitude of the anticyclonic meander is much more appropriate to the 1950 meanders than to the one actually seen in 1960, which is located nearly two degrees east of it. This is a real discrepancy, greatly in excess of any encountered in the other comparisons. The previous successes of the method in reproducing observed paths, however, suggest that the fault here may lie not in the method itself, but in its application. Some speculation concerning this discrepancy, involving the branching path of Figure 6, is given in the next section.

It became apparent early in these computations that the calculated current paths were rather sensitive to the directions of the estimated depth gradients; directional deviations, it was found could easily steer the calculated currents into topographic regimes different from those underlying the corresponding segments of the observed paths. Since the calculated currents would then be subject to topographic effects unlike those which presumably influenced the observed currents, small errors introduced to the computations in this fashion could readily amplify.

Unfortunately, the available bathymetric data were sufficiently sparse to permit occasional ambiguities of $10-15^\circ$ in the directions of estimated depth gradients. Therefore, since the aim in these computations was not to test a method for forecasting positions of the Gulf Stream, but simply to examine the credibility of a particular explanation for observed meander patterns, the error amplification was reduced by adopting that interpretation of the bathymetric data which yielded the best agreement between observed and calculated paths. (It was of course verified that the topographic interpretations made for the five path calculations were consistent with one another.)

Since so few measurements were available for making an estimate of the bottom transport per unit depth, \bar{T} , the generally close agreement between observed and calculated paths seems at first sight remarkable: one might readily ask whether use of a more accurate, somewhat different value of \bar{T} would destroy it. The coordinates (ξ, η) , however, depend on the bottom transport only through $\sqrt{\bar{P}}$; that this number is a fraction whose numerator and denominator both depend on deep velocities suggests that it may in fact not be strongly sensitive to the value assigned to \bar{T} . An expression for \bar{P} was therefore written in terms of the simple velocity distribution described at the beginning of Section 3. This profile is characterized by two velocities: the cross-stream maximum

bottom velocity v_{BM} , and the difference Δv between v_{BM} and the cross-stream maximum surface velocity. The sensitivity of \sqrt{P} to changes in v_{BM} was then calculated as $(v_{BM}/\sqrt{P})(\partial \sqrt{P}/\partial v_{BM})$, where the derivative was taken holding Δv , the depth gradient, and the distance parameters of the profile constant. With the values cited in Section 3 for the relevant variables, and with $D_x = 4 \times 10^{-3}$, $D_y = -7 \times 10^{-3}$, the sensitivity turned out to be 0.2. This result may be interpreted by noting that if the sensitivity were independent of v_{BM} - as it is not - a 50% change in v_{BM} would induce a change in \sqrt{P} , and hence in calculated values of ϵ and η , of only 10%. For this velocity distribution, \sqrt{P} is thus not very sensitive to variations in bottom velocity, at least in the neighborhood of the particular numbers descriptive of the profile. One is encouraged to believe, therefore, that the close agreement between observed and calculated current paths is not some lucky fluke, but instead, convincing evidence for topographic control as the explanation for meanders in the Gulf Stream.

The amplification by varying topography of deviations between calculated current paths suggests a partial explanation for the great diversity in observed meander patterns. Since bottom topography is fixed, time changes in current paths described by (8) must be associated with changes in initial conditions. Since the Gulf Stream leaves

the vicinity of the Continental Slope and enters the open ocean just as it passes Cape Hatteras, it should be governed by equation (4) at least that far upstream, so that the appropriate "initial" conditions for the topographically controlled Stream must be its direction and path curvature near Hatteras. Changes produced somehow in these characteristics would not only change the shape of the wave described by (8), but also direct the Stream into topographic regimes different from those underlying it at an earlier time; it would then be subject to different deflective tendencies, and hence follow a course perhaps quite different from that taken earlier. Thus, although the topography at a fixed point cannot change with time, variations in current direction and path curvature would cause time changes in the effective topography influencing the Stream; these in turn would transform the initial variations into major alterations in meander patterns downstream. That directional variations of some sort do in fact occur at Hatteras is amply indicated by the well-established fluctuations in current direction and position off Onslow Bay (von Arx, et al., 1955; Webster, 1961).

To test the premise of this explanation, we may ascertain how well the calculated paths of Figures 2-6, if extended upstream according to the solution of (8), converge on Cape Hatteras, the "source" of the open-ocean

Gulf Stream: poor convergence would make doubtful the upstream existence of topographic control. Even a cursory inspection of the distribution of isobaths shows that topographically-controlled currents passing through the initial points of the previous calculations must stem from the general Hatteras area. Upstream path extensions have been calculated, moreover, and are displayed in Figure 7. The points farthest downstream on each segment are the initial points of the corresponding calculated paths discussed above; the computations were carried out upstream from these points and terminated at inflection points near Hatteras. The relevant topographic data are presented in the table following. Clearly the convergence of the several path extensions is very close, and sustains the interpretation of the observed Gulf Stream paths as segments of a topographically-controlled current "originating" in proximity to the Continental Slope near Cape Hatteras.

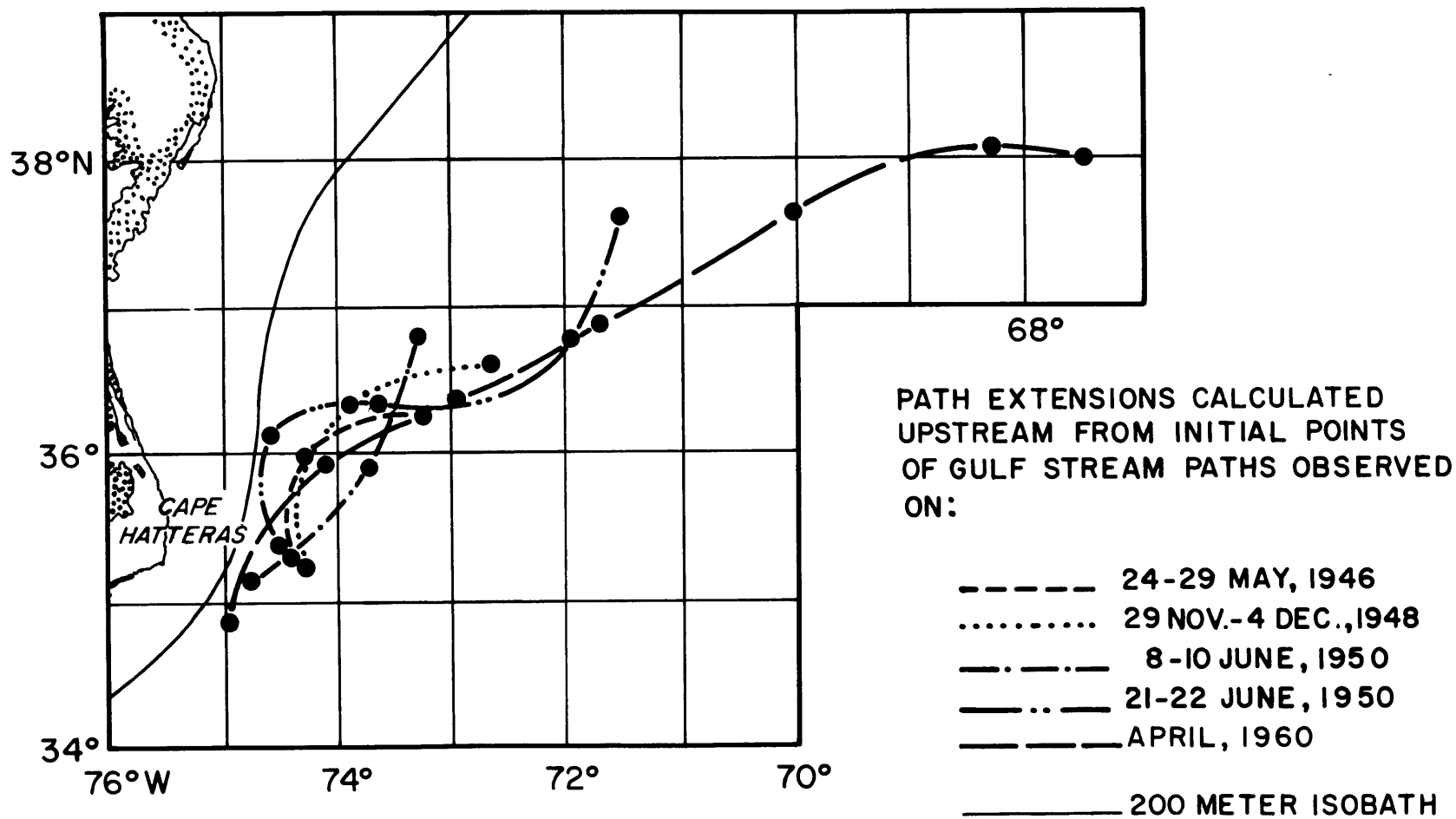


Figure 7. Upstream extensions of calculated current paths.

Depth Gradients Used for Path Extension Calculations

Successive gradients for each interval between adjacent calculated points are given in upstream order. Paths are identified by the dates of the end-point observations.

<u>24-29 May 1946</u>	<u>29 Nov.-4 Dec. 1948</u>	<u>8-10 June 1950</u>	<u>21-22 June 1950</u>	<u>April 1960</u>
$D_s = 14 \times 10^{-3}$ $\alpha = 120$	$D_s = 9 \times 10^{-3}$ $\alpha = 130$	$D_s = 11 \times 10^{-3}$ $\alpha = 112$	$D_s = 12 \times 10^{-3}$ $\alpha = 120$	$D_s = 7 \times 10^{-3}$ $\alpha = 165$
$D_s = 21 \times 10^{-3}$ $\alpha = 122$	$D_s = 21 \times 10^{-3}$ $\alpha = 108$	$D_s = 15 \times 10^{-3}$ $\alpha = 124$	$D_s = 8 \times 10^{-3}$ $\alpha = 140$	$D_s = 10 \times 10^{-3}$ $\alpha = 165$
			$D_s = 19 \times 10^{-3}$ $\alpha = 120$	$D_s = 3 \times 10^{-3}$ $\alpha = 135$
			$D_s = 19 \times 10^{-3}$ $\alpha = 120$	$D_s = 8 \times 10^{-3}$ $\alpha = 146$
				$D_s = 9 \times 10^{-3}$ $\alpha = 130$
				$D_s = 20 \times 10^{-3}$ $\alpha = 120$

6. SOME SPECULATION CONCERNING EFFECTS OF SEAMOUNTS

This section is concerned with two significant discrepancies between observed and calculated current paths which have appeared during the course of computations. While it has not proved possible to account for them, their geographical association with seamounts inspires some speculative discussion concerning possible effects of seamounts on current paths.

One discrepancy was noted in the discussion of Figure 6. It was found that the current path calculated to correspond with the course of the Gulf Stream as charted during April 1960 developed its large anticyclonic meander approximately 150 km west of that observed. Since this error is much larger than any in the other figures, and since the basic data for the computation are of comparable quality to those used elsewhere, it seems improper to ascribe the error either to inaccuracies in the basic data, or to weaknesses in the approximations through which equation (4) was replaced by (8).

The path calculations shown in Figures 2, 3, and 6 were terminated after the large anticyclonic meanders, despite the availability of observational data for downstream comparisons, because of a second discrepancy which would plainly develop. The local meander axes near the downstream ends of these paths are nearly zonal; the

symmetry properties of $\eta(\xi)$, implicit in equation (9), therefore require that the next cyclonic meanders - overlying a level Abyssal Plain - should extend no farther south than the preceding ones. Those actually observed in 1950 and 1960 (shown on Figure 1), however, clearly reach a good deal farther south than their predecessors; path continuations calculated in the above fashion would hence differ significantly from the observed paths.

The previous successes of the method developed for computations suggest that these discrepancies may not be due to any fundamental error in the underlying mechanism, but perhaps, instead, to a faulty application. A striking topographic feature to which no attention has yet been given is the New England Seamount Arc, located on the Abyssal Plain. (This feature is described by Northrup, et al., 1962.) It happens, moreover, that just before developing its great anticyclonic meander, the current path observed in 1960 fairly definitely passes over Kelvin Seamount, centered at $38^{\circ}50'N$, $60^{\circ}00'W$. The currents of 1950, on the other hand, seem just to have grazed it, although the path of 8-10 June is somewhat indefinite in this area, because no crossing of the current was made between latitudes $38^{\circ}20'N$ and $40^{\circ}00'N$ (F. C. Fuglister, private communication). Furthermore, in flowing southward to form the large cyclonic meanders, the currents in both years apparently passed over three additional seamounts in

the Arc: San Pablo Seamount, centered at 39°00'N, 60°40'W; Manning Seamount, centered at 38°10'N, 60°50'W; and Rehoboth Seamount, centered at 37°30'N, 60°00'W. One naturally wonders, therefore, whether seamounts might affect ocean currents in some way which would account for disparities in path calculations, obtained by treating the Abyssal Plain as everywhere level.

The discrepancies found are of different kinds: one has to do with meander amplitudes, the other with a displacement of a meander along the general trend of flow. Apparently, if a seamount were to deflect a current sufficiently to the right, both discrepancies would be explicable: if the calculated current of Figure 6 were so deflected (eastward), its subsequent anticyclonic meander would be displaced downstream from that calculated for the level Abyssal Plain. Downstream continuations of the calculated current paths in Figures 2 and 3, moreover, would pass onto the Abyssal Plain, and turn gradually eastward under the β -effect; if they were subjected in this passage to deflections to the right, they would be forced farther south than if not. If each path were subjected to repeated deflections from the three seamounts underlying the observed paths, the cyclonic meanders would acquire much greater amplitude indeed than those formed in a current flowing over an entirely level plain.

That this single effect could thus account for

both discrepancies suggests that seamount deflections may actually occur. Unfortunately, the method previously developed for path computations cannot be used to verify the effect: since the lateral scale of a seamount is smaller than the width of the Stream, the approximation of plane topography, essential for reduction of (4) to an ordinary differential equation, is invalid. In fact, because of the cross-stream variation in depth gradient, it seems likely that nothing short of a non-linear partial differential equation can even grossly describe currents flowing over seamounts. It has not proved possible, therefore, to deduce the effects of these features on the Gulf Stream, nor, consequently, to determine whether they should deflect the Stream as suggested.

On the other hand, in order to illustrate graphically how deflections could rectify discrepancies, and to demonstrate two very suggestive additional consequences of deflecting tendencies, we may abandon the problem of seamounts as such, and study a much simpler, perhaps analogous one: namely, flow across very long ridges. It is well known (Queney, 1948; Bolin, 1950) that a "large-scale" ridge forces rising and sinking motions in a current, whose net effect is to deflect the current to the right. Furthermore, since the faces of a ridge can be represented as planes, the deflected current path can be calculated by the method developed in the preceding sections. It is not claimed, of

course, that currents actually respond to seamounts as if to ridges; the following inquiry into ridge effects should be regarded, rather, as a geophysical game whose interest for a study of the Gulf Stream is justified by the verisimilitude of calculated meander patterns.

At the positions of the four seamounts noted above, then, we introduce ridges having cross-sectional profiles described by isosceles triangles, with dimensions constant along the ridge axes. It turns out that if the ridge heights H and the basal widths B are modeled crudely on the corresponding seamounts, then the associated deflections are quantitatively sufficient to account for the discrepancies. For the current parameters V , M , and T , we adopt the same values as in Section 5.

The pair of parallel straight lines on Figure 6 indicates the basal bounds and axial direction of the fictitious ridge located where the observed current path passed over Kelvin Seamount; the ridge dimensions are given between the lines. The axial direction was chosen to make the previously calculated path normally incident on the ridge. The dashed curve branching away from the solid curve represents the deflected current path. The small increase in amplitude of the displaced meander over that originally calculated is not to be attributed directly to the deflection, which on the contrary tended to flatten the meander, but rather to the eastward decrease in slope of the Continental

Rise, which led to a smaller topographic constraint on the displaced meander than on the one first calculated. (The bathymetric data on H. O. Chart No. 6610-L were too sparse beneath the displaced meander for any estimate at all of the local depth gradient; the estimate shown is based instead on echo-soundings made along longitudes $60^{\circ}30'W$ and $62^{\circ}30'W$ during the 1960 Gulf Stream survey.) That the amplitude of the displaced meander is perceptibly smaller than that observed, and the half-wavelength, greater, is due to the "flattening" cited, rather than to any deviation from topographic control eastward of Kelvin Seamount. If the calculated path were characterized at its inflection point near $40^{\circ}N$, $63^{\circ}W$ by the more northward direction of the observed current at the same latitude, the calculated meander, in both shape and size, would correspond very closely to the observed meander.

Figure 8 shows two downstream continuations of the current path which was calculated to correspond upstream with that observed on 21-22 June 1950. For comparison, the large cyclonic meander observed on 18 June is also shown, as indicated by the positions of the $18^{\circ}C$ isotherm of the upper 200 m layer (Fuglister and Worthington, 1951). It will be recalled that on 19 June this meander separated from the Stream as an eddy. The solid line represents a current flowing over an entirely level Abyssal Plain; for the reason given above, the meander has an amplitude con-

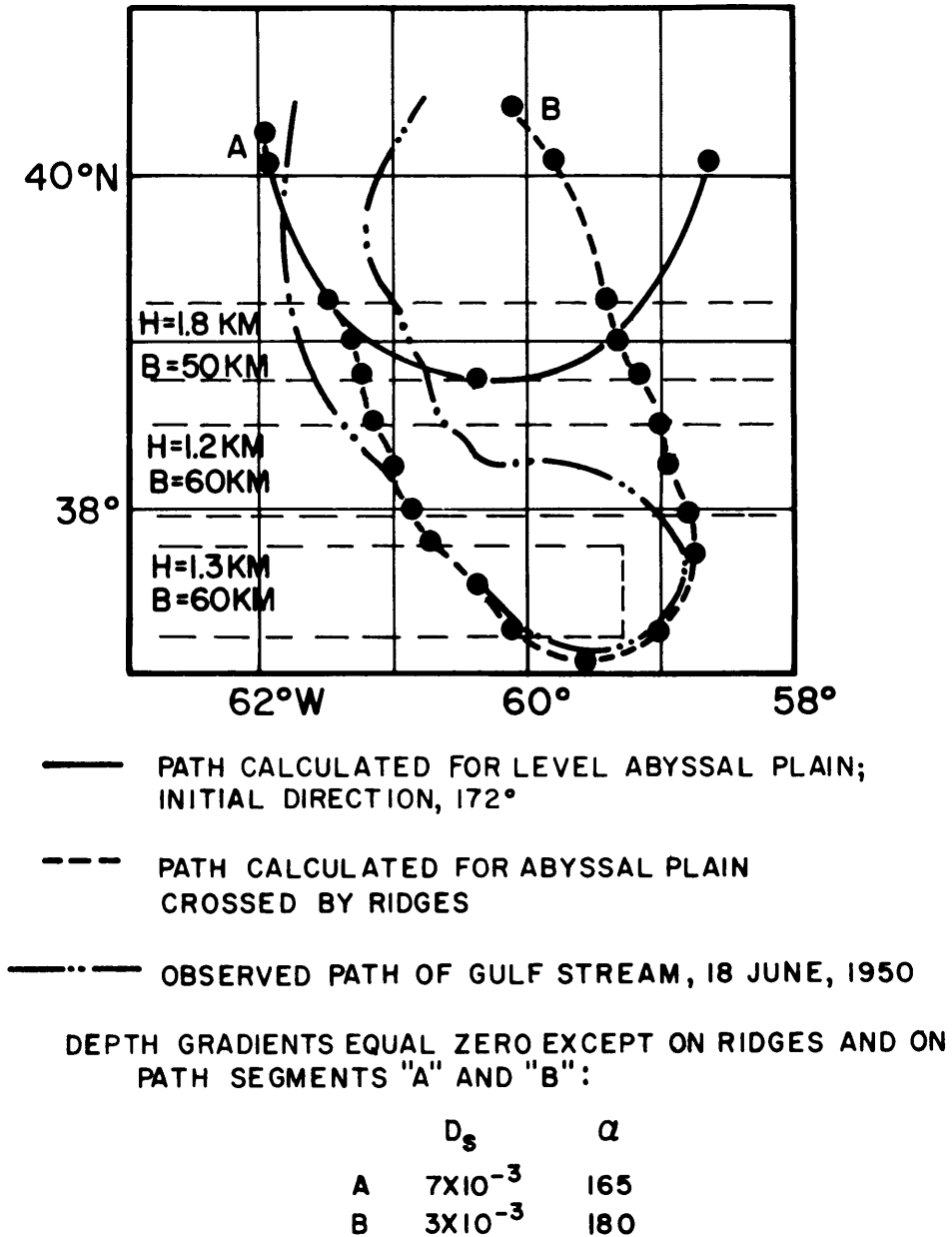


Figure 8. Calculated amplification and tilting of a meander by ridges. Depicted are downstream continuations of the calculated current path of Figure 3. Also shown is the large cyclonic meander observed on 18 June 1950.

siderably smaller than that observed. The dashed continuation describes a path affected by three zonal ridges, whose basal limits are indicated by pairs of parallel lines, and whose dimensions are given between the lines, as in Figure 6. The initial point for both computations is the terminal point of the calculated path shown in Figure 3, located over the Continental Slope a short distance from the Abyssal Plain.

We see that repeated ridge deflections can indeed markedly amplify a meander. Symmetry considerations require that if the deflected current, after turning northward, were to pass over the same ridges that it did while flowing south, then the eastern half of the cyclonic meander should be a mirror image of the western half. On 18 June, however, the Gulf Stream passed over the southernmost seamount only while flowing southward, and missed it completely during its return flow north. It seemed fitting, therefore, to cut off the southernmost ridge between the western and eastern portions of the cyclonic meander, so that the northward-flowing calculated current would not be influenced by it. The eastern half of the dashed continuation represents a computation carried out on that basis. A remarkable effect of this device is to impart an east-west tilt to the calculated meander which corresponds very closely to that observed. In addition, the device keeps the tilted meander rather uniformly narrow, in qualitative agreement

with observation, and in contrast to the meander symmetric about a meridian.

It happens that the ridges force the calculated current so far south that the β -effect produces a curvature in the path segments overlying the Abyssal Plain south of the middle ridge so intense as to make them segments of non-inflecting curves, i.e. of curves resembling plane projections of helices. The particular non-inflecting curves have cyclonic curvature, whose magnitude decreases with increasing latitude according to the β -effect. Since these curves repeatedly intersect themselves, only limited, non-intersecting portions of them can represent current paths. Although equation (4) probably describes such portions, equations (8) and (9) do not, since they presuppose inflections; nor, therefore, do formulas (13) and (16), the tools for all preceding path computations. Hence it was necessary to render (4) in a form descriptive of segments of non-inflectional paths, and then to obtain new formulas for calculating path coordinates. This derivation is quite straightforward, and is appended to the end of this section.

The self-intersecting tendency thus implicit in meanders of very extreme amplitude invites some additional speculation, concerning transformations of meanders into eddies. Were it not that it was deflected northward by the middle ridge, the path segment forming the eastern

portion of the amplified meander would have veered so far eastward on account of its intense curvature as to cut across the western segment. It would be most improper, of course, to consider that curve any longer as a current path, since the notion of a self-intersecting current is meaningless. Even a close approach to intersection, moreover, would invalidate several of the approximations required to justify describing the open-ocean Gulf Stream by equation (4). Nevertheless a self-intersection in such a curve can probably be interpreted physically as a strong tendency for a meander to pinch, detach from the Stream, and turn into a separate eddy. In other words, although the detailed process of eddy-formation must be very complex, its existence, at least, would seem plausibly indicated by self-intersections of the simple curves considered here.

Let us then imagine for the moment that the character of the large cyclonic meander studied in 1950 was in fact determined through seamount deflections of magnitudes similar to those shown in Figure 8. Then the separation of the meander from the Stream could be associated simply with a change in current direction and path curvature upstream, of a sort to shift the eastern segment of the cyclonic meander away from the middle as well as the southern member of the seamount trio. The altered current path would tend then to intersect itself, and hence the

cyclonic meander, to separate from the Stream and form an eddy. This interpretation of the event observed in 1950 has an appealing simplicity, but, of course, an unsubstantiated foundation; it is presented as a corollary to the idea of seamounts deflecting the Gulf Stream.

Thus we conclude that a pattern of deflections correlated with positions of seamounts would account for the eastward displacement of the large anticyclonic meander observed in 1960, for both the amplification and zonal tilt of the large cyclonic meander observed in 1950 (relative to corresponding features in current paths calculated for a level Abyssal Plain), and perhaps, indirectly, even for the transformation of the cyclonic meander into an eddy. Without a clear understanding of the nature of flow in the vicinity of seamounts, however, the existence of such a pattern remains a matter for speculation.

To transform (4) into an ordinary differential equation descriptive of non-inflecting "current paths", we first remove the specification that surface S_1 , cross the path at an inflection. Purely for mathematical convenience, we shall think of "currents" which repeatedly cut across themselves; we ignore the physical nonsense implied, but ultimately, of course, we restrict physical relevance only to non-intersecting segments of the derived curves. The

curvature of the "paths" to be considered is everywhere cyclonic; we reset S_1 , to cross the "current" on a line of minimum cyclonic curvature, K_M , and locate the origin of coordinates (x, y) on S_1 exactly as in Section 4. Then by applying the same approximations to equation (4) as in Section 4, we obtained the equation analogous to (8):

$$\frac{d^2 y}{dx^2} \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{-3/2} + Qy - Sx - K_M = 0 \quad (17)$$

The axial rotation (10) then yields the analogue to (9),

$$\frac{d^2 \eta}{d\xi^2} \left[1 + \left(\frac{d\eta}{d\xi} \right)^2 \right]^{-3/2} + P\eta - K_M = 0 \quad (18)$$

which we simplify by the axial translation,

$$Y \equiv \eta - K_M/P \quad (19)$$

whereby

$$\frac{d^2 Y}{d\xi^2} \left[1 + \left(\frac{dY}{d\xi} \right)^2 \right]^{-3/2} + PY = 0 \quad (20)$$

Since (20) is an exact isomorph of (9), we may write its first integral immediately as,

$$PY^2 - \frac{2}{\sqrt{1+Y'^2}} = C \quad (21)$$

where C is a constant of integration. The square root in (21) is to be assigned both positive and negative values, and, since $Y(\xi)$ does not inflect, $Y' = 0$ when $\sqrt{1+Y'^2} = \pm 1$. When $\sqrt{1+Y'^2} = -1$, the curve $Y(\xi)$ has its minimum cyclonic

curvature, K_M , and $Y(\xi)$ itself assumes its maximum value, $-K_M/p$; therefore $C = K_M^2/p + 2$. Consequently, when $\sqrt{1+Y'^2} = +1$, and $Y(\xi)$ assumes its minimum value, $Y = -\sqrt{K_M^2/p^2 + 4/p}$. Thus if $a \equiv K_M/p$, and $b \equiv \sqrt{K_M^2/p^2 + 4/p}$, then $-b \leq Y \leq -a$.

We next extract an expression for $d\xi/dY$ from (21), with which to write an integral for distances ξ . Since we are concerned with path segments in the neighborhood of a relative minimum in $Y(\xi)$, i.e. near $Y = -b$, we derive specifically an expression for displacements along the ξ -axis, $\Delta\xi$, between the minimum and some arbitrary point on $Y(\xi)$. Thus

$$\Delta\xi = \int_{-b}^Y \frac{\left(\frac{PY^2 - C}{2}\right) dY}{\sqrt{1 - \left(\frac{PY^2 - C}{2}\right)^2}} \quad (22)$$

By making use of the definitions for a and b , (21) is reduced to standard form:

$$\Delta\xi = \int_{-b}^Y \frac{Y^2 dY}{\sqrt{(Y^2 - a^2)(b^2 - Y^2)}} - \frac{a^2 + b^2}{2} \int_{-b}^Y \frac{dY}{\sqrt{(Y^2 - a^2)(b^2 - Y^2)}}$$

We introduce Jacobian elliptic functions by the substitution (Milne-Thomson, 1950):

where $u \equiv \operatorname{dn}^{-1} \left\{ -\frac{Y}{b} \mid k^2 \right\}$
 $k^2 = (b^2 - a^2)/b^2 = (1 + K_M^2/4p)^{-1}$. Thus

$$Y = b(\operatorname{dn} u)$$

$$dY = b k^2 (\operatorname{sn} u)(\operatorname{cn} u) du$$

We note that

$$(Y^2 - a^2)(b^2 - Y^2) = b^2 k^4 (\operatorname{dn} u)^2 (\operatorname{cn} u)^2$$

Therefore

$$\int_{-b}^Y \frac{dY}{\sqrt{(Y^2 - a^2)(b^2 - Y^2)}} = \frac{1}{b} \int_0^u du = \frac{1}{b} u$$

$$\int_{-b}^Y \frac{Y^2 dY}{\sqrt{(Y^2 - a^2)(b^2 - Y^2)}} = b \int_0^u (\operatorname{dn} u)^2 du = b E(u)$$

and, upon combining integrations, we find that

$$\Delta \xi = b \left[E(u) - \left(1 - \frac{k^2}{2}\right) u \right]$$

This formula may be put in a form more convenient for computational purposes by defining an angle φ according to the relation,

$$u \equiv \int_0^\varphi \frac{d\psi}{\sqrt{1 - k^2 \sin^2 \psi}}$$

whereby

$$Y = b(\operatorname{dn} u) = \sqrt{K^2/p^2 + 4/p} \sqrt{1 - k^2 \sin^2 \varphi}$$

and consequently

$$\sin^2 \varphi = \frac{1}{k^2} - \frac{p}{4} Y^2$$

where, as above $k^2 = (1 + K^2/4p)^{-1}$

Then we may rewrite $\Delta \xi$ as,

$$\Delta \xi = \frac{2}{k\sqrt{p}} \left\{ E(\varphi, k) - \left[1 - \frac{k^2}{2}\right] F(\varphi, k) \right\} \quad (23)$$

where $F(\varphi, k)$ and $E(\varphi, k)$ are the elliptic integrals of the first and second kinds, as customarily tabulated.

Equations (21) and (23), then, are the analogues of (13) and (16) for calculating coordinates of non-inflecting "current paths".

7. CONCLUDING REMARKS

It is assumed in this study that the portion of the Gulf Stream overlying the Continental Rise is a flow sufficiently concentrated and intense for it to be assigned lateral boundaries, albeit in a crude and not clearly specified manner; and that the flow is vertically coherent, roughly to the extent that vertical variations in the position of the Stream are smaller than its width. No attempt is made to deduce such a current from physical principles and oceanic boundary conditions; rather, its existence is taken as empirically established. It then makes fair sense to characterize the Stream as a whole by a single curve: a current path. It is further assumed, on the basis of recent observations, that the flow persists to the bottom of the ocean, without reversing direction.

Analysis on this basis of the magnitudes and scales of gross motions in the Gulf Stream, as estimated at present, implies that current paths are determined essentially by a steady-state response principally to variations in ocean depth, and, in a lesser degree, to the meridional variation in Coriolis parameter. Furthermore, approximate descriptions of current paths governed by these mechanisms fit observed path segments so closely as to make very convincing the interpretation of the large-scale meanders as segments of combined topographic and

Rossby waves. This interpretation explains fully the similarity between the trends of deep isobaths and observed current paths, the tendency of the Stream to form large anticyclonic meanders just downstream from the great northward turn in bottom contours, and the correlation between the amplitudes of these meanders and the positions of the underlying isobaths. It is suggested that the apparently slow time-variations in meander patterns are associated with changes in current direction and path curvature near Cape Hatteras, but no attempt is made to describe evolutions of the Stream path from one pattern to another.

No doubt the least firmly established feature of this conception of the Stream is the persistence to the bottom of the ocean of flow in the same direction as the surface current. The only reliable measurements made to date of deep motions beneath the surface Stream consist of the few neutrally-buoyant float observations of 1960. Should these results turn out not to be typical of the Stream, the topographic explanation of meanders would fail. On the other hand, the close agreement between calculated and observed current paths, achieved by assuming bottom velocities as implied by the 1960 measurements, constitutes a fairly compelling argument for such a flow being generally a property of the Stream.

One might ask, then, whether this flow is incon-

sistent with the deep "Gulf Stream countercurrent" hypothesized by Stommel (1957), and perhaps observed south of Cape Hatteras, seaward of the surface Stream, by Swallow and Worthington (1961). Actually, there is no reason to suppose that east of Cape Hatteras, where the Stream is far from the Continental Slope, a deep countercurrent ought to lie directly beneath the tortuous surface Stream. Indeed, if one did, and the system of current-countercurrent were vertically coherent and governed approximately by the vorticity balance inferred in this study, the countercurrent would destabilize the entire Stream: a reversal in direction of bottom flow would change the sign of the topographic component of vertical velocity, and hence change the sense of the topographic deflection; a current path which meandered rather gently for deep flow in the direction of surface flow would therefore be replaced by a self-intersecting current path, implying break-up of the flow, and generally chaotic surface conditions. If a countercurrent is to exist at all, it would seem that east of Cape Hatteras it must be found north of the Gulf Stream, pressed closely against portions of the Continental Slope or Rise sufficiently steep to permit frictional, rather than topographic domination of its vorticity balance. Thus not only is there no conflict with countercurrent notions in supposing that the Stream east of Cape Hatteras does not reverse direction in the deep water, but there is

also reason to believe that it cannot do so.

No attention has been given here to possible instabilities of the Gulf Stream. According to the idea of topographic control, interpretations of the large-scale meanders as self-amplified perturbations to some rectilinear equilibrium flow have proceeded, in a sense, from a fundamental misconception: the topography of the Continental Rise is so irregular that any current which extends to the bottom of the ocean must necessarily meander; a rectilinear equilibrium current is therefore impossible. Apparently the very meanders which have been construed as disturbances superimposed on a basic flow are properly to be regarded as intrinsic features of the basic flow itself. It is conceivable that features of a smaller scale may turn out to be linked to an instability, but they should then be treated as disturbances to an initially meandering basic current. It seems also possible that instabilities in the Stream south of Cape Hatteras might account for the fluctuations in current direction and path curvature near Hatteras to which we have attributed the diversity in observed meander patterns, but instability would then have at most an indirect influence on the meanders downstream.

Since meanders, and hence path-curvature, tend to be averaged out of mean currents, we should expect, on a rough mean basis, to find the net northward advection of planetary vorticity in the Gulf Stream east of Cape Hatteras

to be balanced topographically by a gradual increase in ocean depth beneath the Stream. Thus the mean axis of the Gulf Stream, as estimated on U.S. Navy Hydrographic Office Chart No. 1412 (and reproduced in Iselin and Fuglister, 1948), traces a rectilinear path between Cape Hatteras and longitude 72°W , with an implied current set of 043°T . The underlying isobaths are nearly rectilinear, and are characterized by a depth gradient of magnitude 8×10^{-3} and direction 125°T . According to equations (9) and (10), for values of the current parameters V, M, T , the same as above, the associated current determined entirely by a balance between topographic and β -effects must be rectilinear with bearing 045 . This agreement between directions of the mean and calculated rectilinear currents confirms the expected mean vorticity balance, although the closeness of agreement may be partly fortuitous.

Neumann (1956) suggested the possibility of general balance between "topographic" and β -effects in the Atlantic circulation, but he regarded the flow as bounded below by a sloping surface of no horizontal motion: he assumed that the slope in this surface implied a topographic effect on a current analogous to that imposed on a barotropic current by a sloping, rigid lower boundary. Such an assumption is not justified, however, because, as we have seen, the effect due to the slope of a lower boundary is proportional to the horizontal component of

velocity on it, and hence vanishes when the boundary is a surface of zero horizontal velocity.

Topographic domination of the Gulf Stream vorticity balance must make very doubtful the explanation recently proposed by Carrier and Robinson (1962) for the separation of the Stream from the coast after it passes Cape Hatteras. They considered oceanic circulations forced by a meridionally varying zonal wind stress; these consisted of quasi-geostrophic interior flows coupled to inertial boundary currents; with thin frictional layers added to satisfy realistic boundary conditions, remove discontinuities in certain velocity profiles, and permit diffusion of vorticity out of the system. The qualitative character of the circulations depended on the sense of variation of the wind stress curl as compared with that of the Coriolis parameter: western boundary currents for variation in the same sense; eastern, for variation in the opposite sense. It was considered that the former situation existed in the equatorward half of the North Atlantic, and the latter in the poleward half. An eastward inertial jet was then required at the latitude of maximum wind stress curl, which separated the two gyres. It was suggested that the abrupt change in current direction as the western boundary current entered the mid-latitude jet represented the separation of the Gulf Stream from the coast at Cape Hatteras.

The fundamental conclusion of the present study,

however, is that the path of the Stream, after it passes Cape Hatteras, is controlled for some considerable distance principally by bottom topography, and thus not by the distribution of wind stress. Furthermore, to the extent that separation is conceived as an abrupt change in current direction, the problem is imaginary: as Fuglister has often point out in conversation, and recently in print (in press), the mean axis of the Stream does not change direction at all as it passes Cape Hatteras; the coastline turns instead. It is true that about 300 km downstream from Hatteras, the mean axis, now far distant from the Continental Slope, does bend rather sharply eastward, but this turn occurs where the deep isobaths also bend abruptly eastward, and hence is readily explicable in terms of topographic control. It is difficult, therefore, to see a relation between the zonal wind stress curl and the path of the Gulf Stream between Cape Hatteras and, say, the Grand Banks of Newfoundland.

These remarks should not be taken to imply that there exists no separation problem at all. On the contrary, it is felt that the present study underscores two "Cape Hatteras problems" which are basic to our understanding of the Gulf Stream. From Cape Canaveral north nearly to Cape Hatteras the Stream flows on the Blake Plateau, and its depth is consequently limited to about one kilometer. On this passage the Stream transports water no colder than

about 6°C (Iselin, 1936). On the other hand, if the Stream over the Continental Rise extends to the ocean bottom, then it has a depth just northeast of Cape Hatteras of about four kilometers, and transports water as cold as 3°C (Fuglister, 1960). In what manner, then, does the Stream increase its depth fourfold in the Hatteras area: from what source and by what means does it acquire a large transport of water colder than any carried on the Blake Plateau? Furthermore, if the "Gulf Stream countercurrent" truly exists, and east of Hatteras is to be found shoreward of the Gulf Stream, but south of Hatteras, seaward of the Stream, as suggested by the measurements of Swallow and Worthington (op. cit.) then how is it related to the Stream at Hatteras itself, where apparently it should flow under the Stream?

The other problem is of a dynamical nature, but is probably related to the first. We have spoken of Cape Hatteras as the "source" of the topographically-controlled Gulf Stream. That the Stream on the Blake Plateau should press closely against the Continental Slope strongly suggests that it is controlled by turbulent frictional and inertial boundary current mechanisms, which have been studied in most detail by Carrier and Robinson (op. cit.). They found that the inertial mechanism should be of paramount importance in determining the cross-stream profile of vertically integrated velocity. The effect of the mechanism

on the current as a whole, however, according to the analysis of Section 3, is expressed by cross-stream integrals of the transport of cross-stream velocity shear, and must therefore be "small" for "small" velocities on the lateral boundaries of the current. Without a significant inertial effect, it seems likely, then, that south of Cape Hatteras the net northward transport of planetary vorticity is balanced by frictional diffusion of relative vorticity across the Continental Slope. If this supposition is true, it becomes a matter of fundamental importance to discover how the vorticity balance of the Gulf Stream is transformed in the vicinity of Cape Hatteras from one dominated by frictional diffusion into one dominated by topographically-induced vertical motions.

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BIOGRAPHICAL NOTE

The author, Bruce Alfred Warren, was born in Waltham, Massachusetts, on 14 May 1937. He attended public schools in Arlington, Massachusetts. In September, 1954, he entered Amherst College, where his major field of study was physics. During his senior year he was elected to membership in the Phi Beta Kappa Society, and to associate membership in the Society of the Sigma Xi. He was graduated in June, 1958, with the degree of Bachelor of Arts magna cum laude. During his college summers he was employed at the Woods Hole Oceanographic Institution.

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