MEASUREMENT OF ULTRASONIC PROPAGATION VELOCITY IN GASES

by

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ABSTRACT

The propagation of ultrasonic waves is studied in view of temperature determination in gases. On the theoretical side, influence of viscosity, heat conduction, non-perfect-gas behaviour, thermal relaxation and amplitude, is reviewed ; a method is outlined for temperature determination in non-reactive gas mixtures by sound propagation velocity measurement.

The problem historical background is given for correlated sound velocity and gas temperature measurements.

On the practical side, possible application is discussed for instantaneous and synchronized temperature determination in the end-gas region of internal-combustion engines.

Electronic time-measuring devices are studied and orders of magnitude for comparative ratings and accuracies are given.

An apparatus has been designed, operating on pulse-transmission principle ; the apparatus is now in use in the Sloan Laboratories for Aircraft and Automotive Engines at the Massachusetts Institute of Technology, design data and numerical values are given, together with signal wave-form estimates.

Typical operations and results are described and followed by discussion and suggestion for further investigation.

The sound-velocity method for temperature determination is considered as acceptable for research in internal-combustion engines.

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I N D E X

SYMBOLS

- A -	Attenuation Area
- B -	Second virial coefficient
- E -	Young modulus Electromotive force
- G - G_a	Gibbs function Gain
- H -	Hysteresis constant
- K -	Thermal conductivity Dielectric constant
- L -	Path lenght Laplace transform
- M -	Molecular mass
- Q -	Heat energy
- R -	Perfect-Gas constant
- S -	Surface Strain
- T -	Stress Period Power transmission coefficient
- U -	Total internal energy Voltage
- V -	Voltage
- W -	Mechanical work

a	Attenuation constant
c	Velocity of sound
c_v	Specific heat at constant volume
c_p	" " " pressure
c_{ijkl}	Generalized stiffness
f	Frequency
e_{ik}	Piezo-electric constant
h	Ratio of thermal to viscous attenuation
k	Boltzmann constant stiffness constant
m	mass
p	pressure differential pressure
t	time
v	volume
x	abscissa
α	Cubical expansion coefficient
β	Relative concentration Angular frequencies
γ	Ratio of specific heats
Γ	Propagation constant
δ	pulse duration
η_{ijkl}	generalised viscosity coefficient
θ	absolute temperature
λ	wavelenght temperature coefficient of stresses
μ	amplifier gain
ρ	specific mass
φ	angle
Φ	electro-mechanical transformation factor
ω	angular frequency
ξ	particle displacement
σ	entropy
χ	compressional viscosity
τ	relaxation time constant

N.B. Non-numerical subscript and/or exponent is used to indiqate invariants in a specified evolution. Example: k^{σ} is the stiffness at constant entropy.

LIST OF FIGURES.

1. INTRODUCTION

The interdependence of temperature and sound velocity is known since the earliest days of Thermodynamics ; in 1873, J. Mayer suggested the measurement of absolute temperatures by sound propagation velocity determination.

Owing to experimental difficulties, practically no acoustical "thermometer" was constructed before the end of World War II when the availability of ultrasonic transducers made the use of short acoustical waves practical and when the velocity measurements could take advantage of the precision-built electronic circuits developed for Radar purposes.

Even so, acoustical temperature measurements are comparatively expensive and somewhat cumbersome so a few words of justification are in order when this method is suggested.

Our problem is to measure velocity of sound in a gas and deduce the absolute temperature from this result. It will be seen that the "thermometer" to be constructed should resist to extreme variations of temperature (20 to 825°C), to high pressures (1 to 47 atm.), should indicate the temperature <u>of the gas</u> within better than 390 μsec (this means complete thermal equilibrium between the gas and thermometer).

It should be reminded further that the gas volume available for temperature measurement is excessively small (# .125 cu.inches) and still no influence of the "thermometer" on the gas temperature can be accepted.

The target on accuracy temperature measurement is set at 1 per cent.

Dimensional-change thermometers have to be rejected as they are unable to "follow" temperature gradients of the order of 96000° R/sec.

Resistance-change and thermo-couple type thermometers may be designed for the above operating conditions but it is doubtful whether they would resist to repeated measurements since surrounding they have to be placed in is as objectionable as it can be for wires and metallic deposits of 1μ in diameter or thickness : the combustion chamber of an internal-combustion engine where flame temperatures of over 2000° F are produced at the rate of 10 per second. Even with the rather optimistic assumption that the thermo-sensitive element will resist to this treatment, it is questionable whether its sensitivity will not suffer from molecular changes produced by these extreme temperatures.

The idea occured to investigators to use as thermo-sensitive element the gas itself and measure the temperature by one of its properties accessible to external detection ; namely the coefficient $\frac{\partial p}{\partial v}$ or the velocity of sound, closely related to this coefficient.

Advantages of any method of that kind would be :

- Elimination of all "external" body (thermometer bulb, thermo-couple, resistance-pickup) which could create temperature differences and equilibrium lags.
- No possible deterioration of the "thermometric element" as long as the chemical nature of the gas is unchanged.
- Instantaneous and precisely timed measurements are possible with reasonable accuracy.

Investigations of practical possibilities has been decided and temperature determination by means of sound velocity is the problem exposed in the next chapters.

2.) SURVEY OF THE PROBLEM

2.1) Objective

The work presented in this text is an attempt to investigate the instantaneous gas temperature determinations by means of ultrasonic propagational velocity measurements.

One possible use is considered as important: The study of temperature variations in cylinders of internal combustion engines.

Any investigation of this kind must set at an early stage the limits and the depth of the work to be performed; in the present case the scope of the study can be outlined as follows:

- Clarification of the meaning and possible interpretation of the expected experimental results,
- Comparison of methods for ultrasonic energy generation and detection inside of an internal combustion engine cylinder,
- Comparison of available methods for accurate determination of propagational velocity,
- Design and construction of an electro-mechanical apparatus performing the measurement of propagational velocity in accordance with the most promising method available,

- Experimental performance tests of the apparatus,
- Interpretation of the results (expressed as values of the sound velocity in the gas under test) in terms of the temperature of the medium.

As the present work is intended to give a basis for future and more detailed study it seemed important to give, whenever desirable, the suitable theoretical principles governing the operations involved.

No attempt will be made in the limits of the analysis to discriminate the usefulness of this temperature-determination method in the study of internal combustion engines.

2.2) Method

Two classical methods are available for measurement of propagation velocity of sound vibrations. The first makes use of the relationship between the propagation velocity c, the wavelength and the frequency of vibration f. The frequency can be determined quite accurately, especially if electro-mechanical transducers are employed for the sonic power generation.

Unfortunately, the situation is quite different for wavelength measurements. In accessible media, interferometric methods can be used with satisfactory accuracy, bu the special requirements imposed by our problem ("instantaneous" temperature measurements) eliminate automatically any procedure of that kind.

The second method is based on the very definition of propagation velocity which is known to be the ratio of the distance traveled by the sonic disturbance to the time required to span that distance. The distance can be measured, calculated or standardized with an accuracy considerably better than the one we may expect of the overall measurement results, the problem of measuring propagation velocity is then essentially the determination with convenient precision the time of travel.

Although excellent timing devices are readily available as standard laboratory tools, it will be seen that the measurement of the travel time is still quite a serious problem. It seems to be solved at present, but it is by no means unreasonable to believe that some more work can be usefully performed to increase the precision and reduce the cost of the apparatus.

The second method has been chosen in this project, for reason of simplicity.

2.3) Facts and phenomena involved in the proposed measurement process

2.31) Special requirements imposed by the investigation.

The purpose of this study is the measurement of gas temperatures with special regard to application in internal combustion engine analysis. This sets some physical limitations on the measurement methods, on the apparatus design and on the expected consistency of results.

A typical medium under test will be the end-gas portion of an internal combustion engine cylinder where following physical conditions prevail:

Volume: In order to not disturb the "normal" operation of the engine, the space available within the end-gas for the measurement is approximately .125 cu. in. (=2.05 cm3), represented as a cube of 1/2 inch in all dimensions. All surfaces around the test chamber must be:

- Mechanically resistant to the same extent as any other part of the combustion chamber,
- Temperature resistant,
- Dimensionally stable; the sound path-length inside the test chamber must be known with an accuracy better than .1%.

Pressure: Analysis of the engine operation requires to perform temperature measurements at any point of the engine cycle; extreme conditions of pressure will be met at the start of compression (14.8 psia = 1.03 atm) and in the end-gas, during the combustion (675 psia = 47 atm). Even more severe conditions are set by the rapid variation of the above values during the normal engine cycle; representative rates of pressure change are

540 psi/sec at the start of compression
177000 psi/sec in the end gas.

It is clear from these figures that any workable measurement method and possible apparatus have to withstand pressures of this order and give consistent results independently of the pressure.

Temperature: The temperature inside the combustion chamber is influenced by three important factors:

(a) Physical evolution of the gas (inlet, compression)
(b) Chemical phenomena (combustion)
(c) Heat exchange with the surrounding.

The combined action of these three effects makes the behaviour of temperatures in a given point of the combustion chamber a somewhat unpredictable function of time, despite the considerable theoretical and experimental efforts made in this sense.

A considerable number of approximations have been worked out and our limiting conditions will be based on values yielded by these approximations.

Representative values at extreme conditions are 813° R (= 353° F = 180° C) at the start of compression and 1890° R (= 1520° F = 825° C) in the end-gas region.

Rate of change in temperature is typically 7200° R/sec (start of compression) to 96000° R/sec (end-gas).

<u>Time</u>: One of the main limitations imposed on the measurement technique is the over-all duration of the measuring process. It will be shown that the velocity of sound propagation can be approximated by

$$c^2 = \frac{\gamma R \Theta}{M}$$

and therefore the relative change in velocity will be one-half of the relative change in temperature. If 1% is the limit set on variations permissible during the measurement, the maximum time delay compatible with this requirement will be the one yielding 2% in temperature change. It is readily seen that this time is $\frac{.02 \times 813}{7190} = 2250\ \mu$sec at compression start, and $\frac{.02 \times 1890}{96000} = 390\ \mu$sec in the end-gas.

These extraordinary short durations available to perform high-precision temperature measurement determine practically the whole method and experimental equipment.

Construction: In addition to the above "primary" requirements determined by the physical nature of the medium in which sound velocity measurements have to be performed a number of "secondary" conditions have to be complied with, even if these are imposed only for simplicity, economic and safety considerations:

The following factors must receive attention:

- Compatibility with actual operation of firing internal combustion engine, i.e. resistance to vibration, noise and handling,
- Absence of essential modification in the engine behaviour, although special design of the combustion chamber is permissible,
- Reasonable cost and space requirements.

Both primary and secondary considerations restrict considerably the possible measurement methods; at this point it will be in order to review shortly the main physical phenomena occuring in the expected measurement procedure.

2.32) Sound Propagation

2.321) Linear Acoustic plane waves

Linear acoustic waves are generated in any continuous physical medium if following conditions are satisfied:

(a) Particle displacement amplitude ξ small compared to the wavelength λ ,

(b) Physical average properties of the medium unchanged in presence of all new conditions generated by the acoustical disturbance,

(c) No energy exchange between possible motion modes of particles, and

(d) Static (average) temperature along the propagation path unchanged in the time.

The governing principles of the gas behaviour under these conditions are

- The ideal gas Law:

$$\frac{P_0 V_0}{\Theta_0} = R \qquad (1)$$

- The condition of adiabatic evolution (= without heat exchange)

$$dQ = \frac{\partial Q}{\partial p} dp + \frac{\partial Q}{\partial v} dv = 0 \qquad (2)$$

- Newton's inertia Law applied to any elementary section of the wave propagation line:

$$-S dp = \rho S dx \cdot \frac{\partial^2 \xi}{\partial t^2} \qquad (3)$$

Net force — Elementary mass — Acceleration

An elementary derivation of the wave propagation velocity can be summarized as follows:

Differentiating (1)

$$p_0\, dv_p + v_0\, dp_v = R\, d\Theta = \frac{p_0 v_0}{\Theta} d\Theta \qquad (4)$$

(Subscripts dv_p and dp_v standing respectively for volume variations at constant pressure and pressure variations at constant volume)

Then the infinitesimal increments contained in (2) can be expressed in terms of (4)

$$dv_p = v_0 \frac{d\Theta}{\Theta_0} - \frac{v_0}{p_0} dp_v = v_0 \frac{d\Theta}{\Theta_0}$$ since in dv_p the pressure variation is identically 0,

$$dp_v = p_0 \frac{d\Theta}{\Theta_0} - \frac{p_0}{v_0} dv_p = p_0 \frac{d\Theta}{\Theta_0}$$ since in dp_v the volume variation is identically 0.

Then (2) can be written as

$$dQ = \left(\frac{\partial Q}{\partial p}\, p_0 \frac{d\Theta}{\Theta_0} \right) + \left(\frac{dQ}{\partial v}\, v_0 \frac{d\Theta}{\Theta_0} \right) \qquad (5)$$

Introducing the usual notations

c_v for specific heat at constant volume $\left(\frac{\partial Q}{\partial \Theta} \right)_v$

c_p for specific heat at constant pressure $\left(\frac{\partial Q}{\partial \Theta} \right)_p$

$$c_v = \frac{\partial Q}{\partial p} \frac{p_0}{\Theta_0} \quad \text{and} \quad c_p = \frac{\partial Q}{\partial v} \frac{v_0}{\Theta_0}$$

or, in an equivalent form

$$\left(\frac{\partial Q}{\partial p}\right)_v = C_v \frac{\theta_0}{p_0}$$

$$\left(\frac{\partial Q}{\partial v}\right)_p = C_p \frac{\theta_0}{v_0}$$

and (2) will be obtained in the form

$$dQ = C_v \, \theta_0 \frac{dp_v}{p_0} + C_p \, \theta_0 \frac{dv_p}{v_0} = 0$$

or better

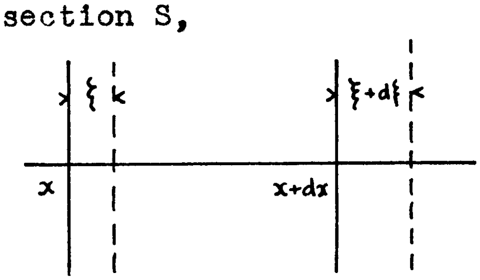
$$C_v \frac{\partial p_v}{p_0} = - C_p \frac{\partial v_p}{v_0}$$

and introducing

$$\frac{C_p}{C_v} = \gamma$$

$$\frac{dp_v}{p_0} = - \gamma \frac{dv_p}{v_0} \qquad (6)$$

The meaning of this equation can be clarified by considering a plane wave propagation path x, with a given section S,



then $v = S \, dx$

$dv = S \, (\xi + d\xi - \xi)$

$= S \, d\xi$

therefore $\frac{dv}{v} = \frac{d\xi}{dx}$

and (6) can be written as

$$\frac{dp_v}{p_0} = - \gamma \frac{d\xi}{dx}$$

or

$$dp_v = -\gamma p_0 \frac{d\xi}{dx} \qquad (7)$$

Equation (7) shows that for any displacement of elementary particle ξ a restoring force dp_v exists, unless $\frac{d\xi}{dx} = 0$, i.e. the whole gas mass is displaced by the same amount.

At this point, eq. (7) can be combined with eq. (3) to introduce the acting inertia force according to Newton's Law.

$$-S\,dp = \rho S\,dx \cdot \frac{\partial^2 \xi}{\partial t^2} \qquad (3)$$

As in this expression dp is a change of pressure along propagation path

$$dp_v = \frac{\partial p}{\partial x}\,dx \qquad \text{and}$$

$$-S\frac{\partial p}{\partial x}\,dx = \rho S\,dx \frac{\partial^2 \xi}{\partial t^2}$$

$$-\frac{\partial p_v}{\partial x} = \rho \frac{\partial^2 \xi}{\partial t^2} \qquad (3')$$

In eq. (7) let us make the change of variables

$dp_v = p$, and call hereafter p,

the differential pressure or the "sound pressure";

Then $$\frac{\partial p}{\partial x} = -\gamma p_0 \frac{\partial^2 \xi}{\partial x^2}$$

and combining with (3')

$$\frac{\partial^2 \xi}{\partial x^2} = \frac{\rho}{\gamma p_0} \frac{\partial^2 \xi}{\partial t^2} \qquad (8)$$

This equation is known to represent a plane displacement wave propagation, the velocity being c defined by

$$c^2 = \frac{\gamma p_o}{\rho} \qquad (9)$$

This value of c^2 is only function of the absolute tempera- ture θ (within restrictions specified in further discuss- ions)

$$c^2 = \frac{\gamma p_o}{\rho} \quad \begin{cases} \rho = \dfrac{m}{V_o} \\ \dfrac{p_o}{\rho} = \dfrac{p_o V_o}{m} = \dfrac{R\theta}{M} \end{cases}$$

$$c^2 = \frac{\gamma R \theta}{M} \qquad (9')$$

If eq. (7) is first derived with respect to t,

then $\dfrac{\partial p}{\partial t} = -\gamma p_o \dfrac{\partial^2 \xi}{\partial x \partial t}$

and $\dfrac{\partial^2 p}{\partial t^2} = -\gamma p_o \dfrac{\partial}{\partial x}\left(\dfrac{\partial^2 \xi}{\partial t^2}\right)$

and by (3')

$$\frac{\partial^2 p}{\partial t^2} = \frac{\gamma p_o}{\rho} \frac{\partial}{\partial x}\left(\frac{\partial p}{\partial x}\right) = \frac{\gamma p_o}{\rho} \frac{\partial^2 p}{\partial x^2}$$

or

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \qquad (10)$$

Sound pressure and particle displacement waves are seen to be propagated with the same velocity c.

With the assumption of adiabatic and linear wave propagation, the sound velocity is essentially independent of the intensity of the disturbance (as long as conditions for linear wave propagation are satisfied) and independent at the same time of the frequency or component frequencies of the pressure "wave". These facts are characteristic of linear sound wave propagation and their effect is the transmission without distorsion of any pressure pulse shape generated in one point of the propagation path.

The linear plane wave propagation concept legitimates the introduction of the specific acoustical impedance, of constant and easily determined transmission and reflexion coefficients at the separation boundary of two media.

Although elementary calculations can be used for rough estimate of propagation velocity, if other than quasi-ideal gases are under experiment, if high frequencies are considered, corrections have to be introduced in order to take into account

- the non-adiabatic evolution of elementary volumes within the gas,
- the viscosity forces between adjacent particles,
- the deviation of the gas from the perfect-gas Law,
- the relaxation phenomena occuring between the macroscopic sound waves and the molecular structure of the gas.

The net effect of these deviations from the elementary case considered above is :

(a) Attenuation and phase distorsion occuring along the propagation path,

(b) Abnormal attenuation at discrete sound frequencies,

(c) Dependence of sound velocity on the frequency,

(d) Apparition of non-linear effects with consecutive errors when applying the superposition principle.

A brief summary of the differences in behaviour between real gases and gaseous mixtures is given below.

2.322) Determination of adiabatic elastic constants and stress relations

The total stress T in an elementary volume within an anisotropic medium is function of the strain S, the rate of change of the strains $\dot{S}$, and the temperature Θ ; the function has to include provision for different values of the respective elastic, viscous and thermal coefficients.

Expressed in form of a tensor equation:

$$T_{kl} = T_{kl}\left(S_{ij} , \dot{S}_{ij} , \Theta \right) \quad (11)$$

then

$$dT_{kl} = \frac{\partial T_{kl}}{\partial S_{ij}} dS_{ij} + \frac{\partial T_{kl}}{\partial \dot{S}_{ij}} d\dot{S}_{ij} + \frac{\partial T_{kl}}{\partial \Theta} d\Theta \quad (12)$$

The entropy σ of the element of volume can be expressed in a similar form

$$d\sigma = \frac{\partial \sigma}{\partial S_{ij}} dS_{ij} + \frac{\partial \sigma}{\partial \dot{S}_{ij}} d\dot{S}_{ij} + \frac{\partial \sigma}{\partial \Theta} d\Theta \quad (13)$$

T_{kl} = Components of total stress,

S_{ij} = Components of strain (relative change in dimension)

$\dot{S}_{ij}$ = Components of time derivatives of strain (relative velocities)

Θ = Absolute temperature

σ = entropy

The partial derivatives are constants for a given material, for a given state of strains, stresses and temperature* and can be written as

$$\frac{\partial T_{kl}}{\partial S_{ij}} = C^{\theta}_{ijkl} + j\, H^{\theta}_{ijkl}$$

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*Piezo-electric, magneto-strictive and electro-strictive effects are not considered at this place.

$C^{\theta}_{ijkl} + j\,H^{\theta}_{ijkl}$ = Complex isothermal elastic stiffness, accounting for internal elastic and hysteresis effects.

$\dfrac{\partial T_{kl}}{\partial \dot{S}_{ij}} = \eta^{\theta}_{ijkl}$ = generalized viscosity coefficient.

$\lambda^{\theta}_{ij} = -\dfrac{\partial T_{kl}}{\partial \theta} = \dfrac{\partial \sigma}{\partial S_{ij}}$ = temperature coefficient of stresses,

at constant strain,*

$\dfrac{\partial \sigma}{\partial \theta} = \dfrac{\rho C_v}{\theta}$ since $d\sigma = \rho\, C_v\, \dfrac{d\theta}{\theta} = \dfrac{dQ}{\theta}$

at constant strain (volume).

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*This relation can be derived from the fundamental thermodynamic potential by defining the Gibbs function as

$G = U - T_{kl}\, S_{ij} - \theta\, \sigma$ (where U is the total internal energy) then $dG = 0$,

expresses the fact that the change in internal energy is equal to the sum of changes in potential energy, $(T_{kl} dS_{ij})$, and the heat energy

$$dQ = d\sigma \cdot \theta$$

G, is known to be a perfect differential, further

$S_{ij} = -\dfrac{\partial G}{\partial T_{kl}}$ and $\sigma = -\dfrac{\partial G}{\partial \theta}$;

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therefore

$$\frac{\partial S_{ij}}{\partial \theta} = -\frac{\partial^2 G}{\partial T_{kl}\,\partial \theta} = -\frac{\partial^2 G}{\partial \theta\,\partial T_{kl}} = -\frac{\partial \sigma}{\partial T_{kl}} = \alpha_{ij}$$

(α_{ij} = linear expansion coefficient.)

This latter equation yields further

$$-\frac{\partial T_{kl}}{\partial \theta} = \frac{\partial \sigma}{\partial S_{ij}} = \alpha_{ij}\,\frac{\partial T_{kl}}{\partial S_{ij}} = \alpha_{ij}\,E_{ijkl}$$

E is the generalized Young modulus of the medium.

The infinitesimal coefficient

$$\frac{\partial \sigma}{\partial \dot{S}_{ij}} = 0 \text{ since at constant temperature}$$

$$d\sigma = \frac{dQ}{\theta} \quad ; \qquad dQ = \Theta \frac{\partial \sigma}{\partial \dot{S}_{ij}} \, d\dot{S}_{ij}$$

must be 0, even if $d\dot{S}_{ij} \neq 0$ as there can be no stored energy in the element of volume at constant rate of change in strain.

Eq. (12) and (13) can then be written in finite form for T and S

$$T_{kl} = \left(c^{\theta}_{ijkl} + j\, H^{\theta}_{ijkl} \right) S_{ij} + \eta^{\theta}_{ijkl} \dot{S}_{ij} - \lambda^{S}_{kl} \, d\theta$$

$$d\sigma = \lambda^{\theta}_{ij} S_{ij} + \rho \frac{C_v}{\theta} \, d\theta \tag{14}$$

Assuming first adiabatic evolution (i.e. no heat exchange between particles),

$dQ = 0$ and therefore

$d\sigma = \frac{dQ}{\theta} = 0.$

The second equation in (14) can be written as

$$d\theta = - \frac{\theta \, \lambda^{\theta}_{ij} S_{ij}}{\rho \, C_v}$$

and from the first equation in (14)

$$T_{k\ell} = \left(c^{\theta}_{ijk\ell} + j\,H^{\theta}_{ijk\ell}\right) S_{ij} + \eta^{\theta}_{ijk\ell}\,\dot{S}_{ij} + \frac{\theta\,\lambda^{\theta}_{ij}\,\lambda^{\theta}_{k\ell}}{\rho\,c_v}\,S_{ij} \qquad (15)$$

or, since this equation (15) holds for σ = Cte = 0, we can write

$$c^{\theta}_{ijk\ell} + \frac{\theta\,\lambda^{\theta}_{ij}\,\lambda^{\theta}_{k\ell}}{\rho\,c_v} = c^{\sigma}_{ijk\ell} \qquad (16)$$

$$\eta^{\theta}_{ijk\ell} = \eta^{\sigma}_{ijkl} \qquad (17)$$

the second members represent the <u>adiabatic elastic and viscosity constants</u>; the elastic constants are quite different for adiabatic and isothermal conditions.

$$T_{k\ell} = \left(c^{\sigma}_{ijk\ell} + j\,H^{\theta}_{ijk\ell}\right) S_{ij} + \eta^{\sigma}_{ijk\ell}\,\dot{S}_{ij} \qquad (15')$$

In case of gases and liquids mechanical constants are essentially the same in all directions, so $C_{ijkl} = C_{11} = C_{12} = C_{13} = k$ (constant of cubical elasticity), and we can express λ^{s}_{ij} in terms of this constant and the coefficient of thermal cubic expansion α

$$\lambda^{s}_{ij} = \left(\frac{\partial T_{k\ell}}{\partial \theta}\right)_s = c_{ijk\ell}\,\alpha_{ij}\,, \qquad \left(\alpha_{ij} = \frac{\partial S_{ij}}{\partial \theta}\right)$$

in the case of liquids and gases,

$$\lambda_{11} = c_{11}\alpha_1 + c_{12}\alpha_2 + c_{13}\alpha_3$$

and since,

$$\alpha_1 = \alpha_2 = \alpha_3 = \frac{\alpha}{3} , \qquad \lambda_{11} = k\alpha$$

Then eq. (16) can be written as

$$\left.\begin{aligned} k^{\sigma} &= k^{\theta} + \frac{\theta (k^{\theta})^2 \alpha^2}{\rho c_v} \\ k^{\sigma} &= k^{\theta} \left[1 + \alpha^2 \theta k^{\theta} / \rho c_v \right] \end{aligned}\right\} \qquad (18)$$

Consider now the heat energy absorbed by an evolution at constant pressure:

$$dQ = dW + dq \quad \left\{ \begin{array}{l} W = \text{mechanical work accomplished} \\ q = \text{heat gained by the elementary volume.} \end{array} \right.$$

$$dW = p\ dV = T\ dS \qquad\qquad dS = d\theta \cdot \alpha$$

$$dW = T\alpha\, d\theta$$

$$dq = \rho\, C_p d\theta$$

At constant strain (volume) the heat energy received would have been $\rho C_v\, d\theta$, therefore the difference represents the mechanical energy received in the evolution at constant pressure (stress).

$$\rho (C_p - C_v)\, d\theta = T\alpha\ d\theta$$

and then

$$\rho \left(C_p - C_v \right) d\theta = K^{\theta} \alpha^2 \theta\, d\theta \qquad (19)$$

which, introduced in (18) gives

$$\frac{k^{\sigma}}{k^{\theta}} = \left[1 + \frac{\rho (c_p - c_v)}{\rho c_v} \right] = \frac{c_p}{c_v} = \gamma \qquad (20)$$

2.323) <u>Application to plane adiabatic wave propagation in viscous medium</u>

Let us take again eq. (15')

$$T_{kl} = \left(c^{\sigma}_{ijkl} + j H^{\theta}_{ijkl} \right) S_{ij} + \eta^{\theta}_{ijkl} \dot{S}_{ij}$$

and make now following assumptions (legitimate in case of gas or liquid), hysteresis being neglected

- No shearing stiffness, therefore $C^{\sigma}_{ijkl} = k^{\sigma}$
- Existence of two viscosity coefficients, shearing viscosity η and compressional viscosity χ such that the component

$$\eta_{11} = \frac{\partial T_{11}}{\partial \dot{S}_{11}} = \chi + 2\eta$$

Then, eq. (15') will be developed in the 6 following equations:

$$T_{11} = k^{\sigma} \left[S_{11} + S_{22} + S_{33} \right] + (\chi + 2\eta) \dot{S}_{11} + \chi \left[\dot{S}_{22} + \dot{S}_{33} \right]$$

$$T_{22} = k^{\sigma} \left[S_{11} + S_{22} + S_{33} \right] + (\chi + 2\eta) \dot{S}_{22} + \chi \left[\dot{S}_{11} + \dot{S}_{33} \right]$$

$$T_{33} = k^{\sigma}\left[S_{11} + S_{22} + S_{33}\right] + (\chi + 2\eta)\dot{S}_{33} + \chi\left[\dot{S}_{11} + S_{22}\right]$$

$$T_{12} = \eta\,\dot{S}_{12} \qquad (21)$$

$$T_{13} = \eta\,\dot{S}_{13}$$

$$T_{23} = \eta\,\dot{S}_{23}$$

This system of equations is fit to handle the particular case of plane wave propagation considering as the only acting force directed along the X axis (component T_{11})*

$T_{12} = T_{13} = 0$

Then, Newton's inertia Law can be written as

$$\rho\, dx\, \frac{\partial^2 \xi}{\partial t^2} = \frac{\partial T_{11}}{\partial x}\, dx$$

or

$$\rho \frac{\partial^2 \xi}{\partial t^2} = \frac{\partial T_{11}}{\partial x} = k^{\sigma} \frac{\partial}{\partial x}\left[S_{11} + S_{22} + S_{33}\right] + (\chi + 2\eta)\frac{\partial^2 S_{11}}{\partial x\, \partial t} \quad (22)$$

We have seen (pp 11) that the strain $\frac{dV}{V} = S$ is equal to $\frac{\partial \xi}{\partial x}$ if ξ is the particle displacement.

---

* T_{22} and T_{33} are not implied to be equal to zero, but their influence on plane wave propagation can be neglected.

On the other hand, $\frac{\partial S_{22}}{\partial x} = \frac{\partial S_{33}}{\partial x} = 0$

Then, eq. (22) can be written as

$$\rho \frac{\partial^2 \xi}{\partial t^2} = k^\sigma \frac{\partial^2 \xi}{\partial x^2} + (\chi + 2\eta) \frac{\partial}{\partial x} \frac{\partial^2 \xi}{\partial x \, \partial t} \tag{23}$$

This is the governing law of plane wave motion, calculated under adiabatic condition taking into account compressional and shearing viscosity.

Assuming harmonic function for

$$\xi = X e^{j\omega t}$$

eq. (23) can be written as

$$-\rho\omega^2 X = k^\sigma \frac{\partial^2 X}{\partial x^2} + j\omega(\chi + 2\eta) \frac{\partial^2 X}{\partial x^2}$$

$$= \left(k^\sigma + j\omega(\chi + 2\eta)\right) \frac{\partial^2 X}{\partial x^2}$$

Comparing this expression to (8) and (9) it appears that it represents a plane wave propagation with the complex velocity

$$c^2 = \frac{k^\sigma + j\omega(\chi + 2\eta)}{\rho} \tag{24}$$

It is easy to observe that, if the viscosities are neglected, χ and $\eta = 0$, and $c^2 = \frac{k^\sigma}{\rho}$

Or,

$$k^\sigma = \left(\frac{\partial T}{\partial S}\right)_\sigma = -\frac{\partial p}{\frac{\partial v}{v}}$$

and the adiabatic evolution being defined as

$$p_o v_o^{\gamma} = C$$

$$\gamma p_o v_o^{\gamma-1} dv + v_o^{\gamma} dp = 0 \; ; \qquad \gamma p_o = -\frac{\partial p}{\partial v} v_o = k^{\sigma}$$

and $\frac{k^{\sigma}}{\rho} = \frac{\gamma p_o}{\rho}$ value found previously for the elementary theory.

Eq. (24) can be interpreted as the expression of the "velocity" in case of viscous fluid submitted to adiabatic acoustical disturbance.

It is useful to compare the so-called "attenuation" with the theoretical case where this attenuation is zero.

If no viscosity exists, the wave equation was found being

$$\frac{\partial^2 \xi}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \xi}{\partial t^2} \; ; \qquad \text{if} \quad \xi = X e^{j\omega t}$$

$$\frac{\partial^2 X}{\partial x^2} = -\frac{\omega^2}{c^2} X$$

and

$$X = X_1 e^{j\frac{\omega}{c}x} + X_2 e^{-j\frac{\omega}{c}x}$$

$\Gamma = \frac{\omega}{c}$ is called the propagation constant.

In case of the free propagation $X_2 \doteq 0$, and

the modulus of $X_x = X_1 e^{j\omega/c\, x}$

$$|X_x| = X_1 \sqrt{\cos^2 \frac{\omega}{c} x + \sin^2 \frac{\omega}{c} x}$$

$$= X_1$$

and the attenuation, as expected, is 0.

(By definition, we will call the attenuation

$$A_x = \text{Log.} \frac{X_x}{X_1} \text{ nepers.})$$

If viscosity occurs, the wave equation will be

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{k^\sigma + j\omega(\chi + 2\eta)}{\rho} \frac{\partial^2 \xi}{\partial x^2}$$

and, with the same assumption of harmonic function for ξ

$$\frac{\partial^2 X}{\partial x^2} = \frac{-\rho\omega^2}{k^\sigma + j\omega(\chi + 2\eta)} X$$

Make $\Gamma^2 = \rho\omega^2 / k^\sigma + j\omega(\chi + 2\eta)$ the square of propagation constant,

Then,

$$\frac{\partial^2 X}{\partial x^2} = -\Gamma^2 X$$

and

$$X_x = X_1 e^{j\Gamma x} + X_2 e^{-j\Gamma x}$$

In case of infinite propagation ($X_2 = 0$), the wave equation will reduce to

$$X_x = X_1 e^{j\Gamma x}$$

if $j\Gamma = a + jb$ $X_x = X_1 e^{ax} e^{jbx}$

The ratio of moduli of two successive maxima of same sign will be

$$\frac{X_2}{X_1} = e^{a\lambda}$$

$a = \text{Log} \frac{X_2}{X_1}$ is the attenuation per unit lenght,

$a\lambda$ is the attenuation for I spatial "wavelenght".

To find actual values of the attenuation and of the propagation velocity, the complex propagation constant has to be analysed:

$$\Gamma^2 = \frac{\rho\omega^2}{k^\sigma + j\omega(\chi + 2\eta)} = \frac{\rho\omega^2\left(k^\sigma - j\omega(\chi+2\eta)\right)}{(k^\sigma)^2 + \omega^2(\chi+2\eta)^2} *$$

then

$$\Gamma^\# = \sqrt{\frac{\rho\omega^2\left(k^\sigma - j\omega(\chi+2\eta)\right)}{(k^\sigma)^2 + \omega^2(\chi+2\eta)^2}} \# \omega\sqrt{\frac{\rho k^\sigma}{(k^\sigma)^2 + \omega^2(\chi+2\eta)^2}}\left(1 - \frac{j\omega(\chi+2\eta)}{2k^\sigma}\right)$$

$$\Gamma = \omega\sqrt{\frac{\rho}{k^\sigma}}\sqrt{\frac{1}{1 + \frac{\omega^2(\chi+2\eta)^2}{(k^\sigma)^2}}}\left(1 - \frac{j\omega(\chi+2\eta)}{2k^\sigma}\right)$$

---

* Order of magnitude for air at normal conditions excited at 2 Mcps:

$k^\sigma = 1{,}46 \cdot 10^6$	dynes/cm^2	dimensions	$M L^{-1} T^{-2}$
$\rho = 1{,}3 \cdot 10^{-3}$	g/cm^3	"	$M L^{-3}$
$2\eta + \chi = 2. \cdot 10^{-4}$	g/cm-sec	"	$M L^{-1} T^{-1}$
$\omega = 4\pi \cdot 10^6$	rad/sec	"	T^{-1}

The velocity is inverse of the imaginary part of $j\Gamma$ multiplied by ω, therefore the propagation velocity square will be

$$c^2 = \frac{\omega^2}{\left[\mathcal{J}m(j\Gamma) \right]^2} = \frac{k^\sigma}{\rho} \left(1 + \frac{\omega^2 (\chi + 2\eta)^2}{(k^\sigma)^2} \right)$$

and the absorbtion will be given by

$$a = \omega^2 \sqrt{\frac{\rho}{k^\sigma}} \; \frac{\chi + 2\eta}{2 k^\sigma}$$

Conclusion

At adiabatic evolution of viscous gas the velocity is modified by a factor

$$\left(\frac{c'}{c} \right)^2 = 1 + \frac{\omega^2 (\chi + 2\eta)^2}{(k^\sigma)^2}$$

the net effect on the velocity being much smaller than probable experimental errors. Orders of magnitude will be given in Section 2.3291.

The attenuation caused by viscosity, expressed in nepers per centimeter will be

$$a = \omega^2 \sqrt{\frac{\rho}{k^\sigma}} \; \frac{\chi + 2\eta}{2 k^\sigma}$$

We will see in the next paragraphs that this cause of attenuation is small compared to thermal losses and relaxation effects.

2.324) Attentuation and Velocity change introduced by thermal conductivity

It has been shown (eq. 14) that variations in entropy experienced by unit volume can be written as

$$d\sigma = \overset{\circ}{\lambda}_{ij} S_{ij} + \rho C_v \frac{d\theta}{\theta}$$

The heat exchanged with adjacent particles can be written as

$$\frac{\partial Q}{\partial t} = K \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right)$$

so the equation defining variations in the temperature will be

$$K \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) = \overset{\circ}{\lambda}_{ij}\, \theta \frac{\partial S_{ij}}{\partial t} + \rho C_v \frac{\partial \theta}{\partial t}$$

or

$$\theta \frac{\partial \sigma}{\partial t} = \frac{\partial Q}{\partial t}$$

Let us consider now this equation with assumptions

(a) Plane propagation ($\frac{\partial}{\partial y} = \frac{\partial}{\partial z} \equiv 0$)

(b) Harmonic variations of θ around average value θ_0

$$\theta = \Theta_0 \left(1 + \Theta\, e^{j\omega t} \right) \quad \text{with} \quad \Theta = \Theta_1\, e^{j\Gamma x} \qquad (26)$$

(c) S_{ij} is the result of the wave propagation

$$X = X_1\, e^{j\Gamma x} \qquad \text{with} \quad \xi = X\, e^{j\omega t}$$

$$S_{ij} = S = \frac{\partial X}{\partial x} = j\Gamma X_1 e^{j\Gamma x} = S_1\, e^{j\Gamma x}$$

(amplitude of strain),

with $S = \mathcal{S} e^{j\omega t}$

With these assumptions the space repartition of temperature will be determined by* :

$$k \frac{\partial^2 \theta}{\partial x^2} = \lambda \frac{\partial S_{ij}}{\partial t} \theta + \rho C_v \frac{\partial \theta}{\partial t}$$

$$k \frac{\partial^2 \Theta}{\partial x^2} = j \lambda \omega S + j C_v \rho \omega \Theta$$

$$k \frac{\partial^2 \Theta}{\partial x^2} - j \rho C_v \omega \Theta = j \omega \lambda S_1 e^{j \Gamma x} \quad **$$

or, replacing Θ by its value from (26)

$$\left(- k \Gamma^2 \Theta_1 - j \rho C_v \omega \Theta_1 \right) = j \lambda \omega S$$

$$\Theta_1 = \frac{-j \lambda \omega S}{k \Gamma^2 + j \omega C_v \rho} \quad *** \qquad (27)$$

---

* Assuming temperature deviations to be small, $\frac{\partial \theta}{\partial t} = j \omega \Theta e^{j\omega t}$ and $\frac{\partial^2 \theta}{\partial x^2} = - \Gamma^2 \Theta_1 e^{j \Gamma x} \; \Theta_0$

** The average value Θ_0 vanishes in this expression.

*** Notice in this expression that Γ is complex, as it has been defined in the previous paragraph, owing to the viscous properties of the gas.

This is the value of the relative temperature deviation vs. abscissae along the wave path; x enters implicity in this expression since

$$S = S_1 e^{j\Gamma x}$$

The value given by (27) can be introduced directly in eq. (14) rewritten as valid for plane wave propagation, neglecting hysteresis,

$$T_{11} = k^{\theta} S_{11} + \eta_{11} \dot{S}_{11} - \lambda \Theta_0 \Theta_1$$

$$= k^{\theta} S_{11} + (\chi + 2\eta)\dot{S}_{11} + \frac{j\omega\lambda^2 S_{11}\Theta_0}{K\Gamma^2 + jC_v\rho\omega}$$

$$= k^{\theta} S_{11} + j\omega S_{11}\left[\chi + 2\eta + \Theta_0 \lambda^2/(jC_v\rho\omega + K\Gamma^2)\right]$$

$$= k^{\theta} S_{11} + j\omega S_{11}\left[\chi + 2\eta + \Theta_0 (k^{\theta})^2\alpha^2 \frac{(K\Gamma^2 - jC_v\rho\omega)}{(K\Gamma^2)^2 + C_v^2\rho^2\omega^2}\right.$$

$$T_{11} =^{*}$$

Remembering $k^{\sigma} - k^{\theta} = (\gamma - 1)\dfrac{k^{\sigma}}{\gamma}$

and neglecting $(K\Gamma^2)^2$ with respect to $C_v^2\rho^2\omega^2$

$$T_{11} = k^{\theta} S_{11} + j\omega S_{11}\left[\chi + 2\eta + \frac{K\Gamma^2(\gamma-1)k^{\sigma}}{C_v\rho\omega^2\gamma}\right] + S_{11}\frac{\Theta (k^{\theta})^2\alpha^2}{\rho C_v}$$

$$T_{11} = k^{\theta}\left[1 + \frac{\Theta k^{\theta}\alpha^2}{\rho C_v}\right] S_{11} + j\omega S_{11}\left[\chi + 2\eta + \frac{K\Gamma^2(\gamma-1)k^{\sigma}}{C_v\rho\omega^2\gamma}\right]$$

$$T_{11} = k^{\theta}\left[1 + \frac{\Theta k^{\theta}\alpha^2}{\rho C_v}\right] S_{11} + j\omega S_{11}\left[\chi + 2\eta + \frac{K(\gamma-1)}{C_p}\right] \qquad (28)$$

$$^{*} = k^{\theta} S_{11} + j\omega S_{11}\left[(\chi + 2\eta) + \frac{\rho C_v (k^{\sigma} - k^{\theta})(K\Gamma^2 - jC_v\rho\omega)}{(K\Gamma^2)^2 + C_v^2\rho^2\omega^2}\right]$$

since $\frac{k \Gamma^2 (\gamma-1) k^\sigma}{C_v \rho \omega^2 \gamma} = \frac{K \frac{\omega^2}{c^2} (\gamma-1) k^\sigma}{C_v \rho \omega^2 \gamma} = \frac{K(\gamma-1)}{\gamma C_v} = \frac{K(\gamma-1)}{C_p}$

Eq. (28) shows that the effect of heat conduction is to increase the viscosity and this will obviously react on the absorbtion (= attenuation) and, although to a very small extent, on the propagation velocity.

The proportion in what the viscosity effect will be intensified can be expressed by the factor

$$\frac{\text{Isothermal attenuation}}{\text{Viscous attenuation}} = \frac{K(\gamma-1)}{C_p(\chi+2\eta)}$$

$$= h.$$

order of magnitude of this factor is given below: (Dry air at normal conditions)

K	$= 2.3.10^3$ erg/sec-cm-°C, dimensions	MLT^{-3}
γ	$= 1.4$	0
C_p	$= 9.92 \times 10^6$ erg/ gr-°C	L^2T^{-2}
$\chi+2\eta$	$= 2.10^{-4}$ gr/cm-sec	$ML^{-1}T^{-1}$

$$[h] = \frac{MLT^{-3}}{L^2T^{-2} \cdot ML^{-1}T^{-1}} = 0$$

$$h = \frac{2.3 \times .4 . 10^3}{9.92 . 10^6 \times 2 . 10^{-4}} = .46$$

The important conclusion is that the isothermal attenuation is of the same order of magnitude than the viscous attenuation; in consequence the influence on the propagational velocity can be neglected for the same reason as in 1323.

2.325) Thermal Relaxation*

The thermal relaxation is the modification of energy exchange between external motion of elementary particles in gases, (molecules) namely translation and rotation, and internal motion (vibrations).

The characteristic physical quantities governing relaxation are the "temperatures" θ for the external modes and θ' for the internal vibrations. Accordingly it is convenient to define the respective specific heats at constant strain, C_v and C'_v. *

The macroscopic stresses are known to be independent from the internal motions, so the stress equation for any elementary volume will again be

$$T_{kl} = \frac{\partial T_{kl}}{\partial S_{ij}} S_{ij} + \frac{\partial T_{kl}}{\partial \theta} d\theta \qquad (29)$$

However, entropy of any element of volume is essentially dependent on the external and internal modes and the entropy equation has to be written

$$d\sigma = \frac{\partial \sigma}{\partial S_{ij}} dS_{ij} + \frac{\partial \sigma}{\partial \theta} d\theta + \frac{\partial \sigma}{\partial \theta'} d\theta' \qquad (30)$$

If all temperature differences are small compared to any

---

* This notation will be followed in all expressions related to thermal relaxation.

of the average temperatures, then

$$\frac{\partial \sigma}{\partial \theta} = \frac{\rho c_v}{\theta} \quad ; \qquad \frac{\partial \sigma}{\partial \theta'} = \frac{\rho c'_v}{\theta'} \neq \frac{\rho c'_v}{\theta}$$

Assuming the heat exchange is performed only by conduction, the rate of change in θ' is proportional to the temperature difference with respect to the surroundings:

$$\frac{d\theta'}{dt} = \frac{1}{\tau}\left(\theta - \theta'\right) \qquad (31)$$

τ is a constant having dimension of time and its value is approximatively equal to the <u>relaxation time</u>.

If all variations are assumed to be harmonic then

$$\frac{d\theta'}{dt} = j\omega\theta' \quad ; \qquad d\theta = (1 + j\tau\omega)\, d\theta'$$

Further the partial derivative:

$$\frac{\partial T_{kl}}{\partial S_{ij}} = c^{\theta}_{ijkl}$$

(neglecting hysteresis) and $\quad \dfrac{\partial T_{kl}}{\partial \theta} = -\lambda_{ij}$

as in 2.322.

Eq. (29) and (30) will then be written in the form:

$$T_{kl} = c^{\theta}_{ijkl}\, S_{ij} - \lambda_{ij}\, d\theta$$

$$d\sigma = \lambda_{ij} S_{ij} + \rho c_v \frac{d\theta}{\theta} + \rho c'_v \frac{d\theta}{\theta(1 + j\omega\tau)} \qquad (29')$$

$$\theta\, d\sigma = \theta \lambda_{ij} S_{ij} + \left(\rho c_v + \frac{\rho c'_v}{1 + j\omega\tau}\right) d\theta$$

Adiabatic wave propagation makes $d\sigma = 0$

or

$$d\theta = \frac{-\theta \lambda_{ij} S_{ij}}{\rho c_v + \frac{\rho c'_v}{1 + j\tau\omega}}$$

Replacing in (29'),

$$T_{kl} = c^{\theta}_{ijkl} S_{ij} + \frac{\theta \lambda^2_{ij} S_{ij}}{\rho c_v + \frac{\rho c'_v}{1 + j\tau\omega}}$$

we have found previously that for gases,

$c^{\theta}_{ijkl} = k^{\theta}$ and $\lambda_{ij} = k^{\theta}\alpha$

therefore

$$T_{kl} = k^{\theta}\left[1 + \frac{\theta\alpha^2 k^{\theta}}{\rho c_v + \frac{\rho c'_v}{1 + j\tau\omega}}\right] S_{ij}$$

In plane wave propagation

$$T_{kl} = T_{11} \quad ; \quad S_{ij} = S_{11} = \frac{\partial\xi}{\partial x}$$

and motion will be characterized by the inertia law

$$\rho\frac{\partial^2\xi}{\partial t^2} = \frac{\partial T_{11}}{\partial x} = k^{\theta}\left[1 + \frac{\theta\alpha^2 k^{\theta}}{\rho c_v + \frac{\rho c'_v}{1 + j\omega\tau}}\right]\frac{\partial^2\xi}{\partial x^2} \qquad (32)$$

This equation represents the plane wave propagation, as before the propagation velocity is going to be

<u>Adiabatic</u> wave propagation makes $d\sigma = 0$

or

$$d\theta = \frac{-\theta \lambda_{ij} S_{ij}}{\rho c_v + \frac{\rho c'_v}{1 + j\tau\omega}}$$

Replacing in (29'),

$$T_{kl} = c^{\theta}_{ijkl} S_{ij} + \frac{\theta \lambda^2_{ij} S_{ij}}{\rho c_v + \frac{\rho c'_v}{1 + j\tau\omega}}$$

we have found previously that for gases,

$$c^{\theta}_{ijkl} = k^{\theta} \qquad \text{and} \qquad \lambda_{ij} = k^{\theta}\alpha$$

therefore

$$T_{kl} = k^{\theta}\left[1 + \frac{\theta \alpha^2 k^{\theta}}{\rho c_v + \frac{\rho c'_v}{1 + j\tau\omega}}\right] S_{ij}$$

In plane wave propagation

$$T_{kl} = T_{11} \quad ; \quad S_{ij} = S_{11} = \frac{\partial \xi}{\partial x}$$

and motion will be characterized by the inertia law

$$\rho \frac{\partial^2 \xi}{\partial t^2} = \frac{\partial T_{11}}{\partial x} = k^{\theta}\left[1 + \frac{\theta \alpha^2 k^{\theta}}{\rho c_v + \frac{\rho c'_v}{1 + j\omega\tau}}\right] \frac{\partial^2 \xi}{\partial x^2} \qquad (32)$$

This equation represents the plane wave propagation, as before the propagation velocity is going to be

$$c^2 = \frac{k^\theta}{\rho}\left[1 + \frac{\theta \alpha^2 k^\theta}{\rho c_v + \frac{\rho c'_v}{1 + j\omega\tau}}\right] \tag{33}$$

Remembering relations (shown in 2.322)

$$\frac{k^\theta \theta \alpha^2}{\rho c_{v_0}} = \gamma - 1 = \frac{k^\sigma}{k^\theta} - 1 \qquad \left\{ \begin{array}{l} c_{v_0} = c_v + c'_v \\ \text{total specific heat at} \\ \text{constant strain.} \end{array} \right.$$

Taking again $j\Gamma$ for roots of the characteristic equation of (32), the velocity will be as usually

$c = \frac{\omega}{\mathcal{Im}(j\Gamma)}$ and after convenient simplifications

$$c^2 = \frac{c_0^2 + \omega^2 \tau^2 c_\infty^2}{1 + \omega^2 \tau^2} \tag{34}$$

with $c_0^2 = \frac{k^\sigma}{\rho}$ and $c_\infty = c_0 \sqrt{1 + \frac{\gamma - 1}{\gamma} \frac{c'_v}{c_{v_0}}}$

The attenuation can be calculated as before found to be*

$$A = \mathrm{Re}\,(j\Gamma)$$

or

$$A = \pi \frac{(\gamma - 1)}{\gamma} \frac{c'_v}{c_{v_0}} \omega\tau \Big/ 1 + \omega^2 \tau^2 \frac{c_\infty^2}{c_0^2}$$

Orders of magnitude are given below. (2.3293)

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*W.P. Mason, Piezo-Electric Crystals and their application to Ultrasonic, A.8.

2.326) <u>Variation of Sound velocity with Pressure</u>

It was shown (2.321) that the elementary expression for sound velocity is

$$c^2 = \frac{\gamma R \Theta}{M} \qquad (9')$$

The constancy of c vs. pressure supposes constant values of $\gamma = \frac{c_p}{c_v}$, further, the derivation of (9') is based on the perfect-gas Law:

$$pv = R\Theta$$

For real gases however corrective terms must be introduced to represent experimental results; the usual form is given as

$$pv = R\Theta \left(1 + \frac{B}{v} + \frac{C}{v^2} + \ldots \right) \qquad (9'')$$

As the sound velocity (in absence of viscosity, heat conduction and thermal relaxation) can be written as

$$c^2 = \frac{k^\sigma}{\rho} = \frac{\gamma k^\theta}{\rho}$$

and since $k^\theta = -\left(\frac{\partial p}{\partial v}\right)_\theta V$

$$c^2 = -\gamma \left(\frac{\partial p}{\partial v}\right)_\theta \frac{V^2}{M} \qquad (36)$$

Supposing 1 molecule of the gas in evolution, eq. (36) can be transformed, taking value of from (9') and neglecting higher order terms:

$$\frac{\partial p}{\partial v} \cong \frac{\partial}{\partial v}\left\{\frac{R\Theta}{M}\left(1 + \frac{B}{V}\right)\right\} = -\frac{1}{V^2}\left(R\Theta + \frac{2R\Theta B}{V}\right)$$

then

$$c^2 \cong \frac{\gamma}{M}\left(R\Theta + 2R\Theta\frac{B}{V}\right)$$

$$c^2 \cong \frac{\gamma}{M}\left[R\Theta + 2pB\right] \qquad (37)$$

Besides this correction, the variations of γ have to be taken into account, the specific heat at constant strain (volume) is

$$C_v = (C_v)_0 - p\left(2\frac{dB}{d\theta} + \theta\frac{\partial^2 B}{\partial\theta^2}\right) \qquad (38)$$

where C_v and C°_v are respectively the specific heats at constant volume, measured at both the temperatures Θ and $0^{\circ}C$

Further

$$C_p - C_v = R + 2p\frac{\partial B}{\partial\theta} \qquad (39)^*$$

Relations (37), (38) and (39) account for deviations of real gases from the perfect-gas law and enable one to transform velocity measurements into temperature measurements

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* Bergman, Ultrasonics 5th German Ed. pp. 355

provided

$B = \varphi(\Theta)$ is a perfectly known function.

It will be seen (2.329) that this assumption is made with little respect to experimental circumstances.

2.327) Non-Linear Propagation Effects*

The purpose of this section is to show how the amplitude of the sonic wave influences on the propagation velocity.

In 2.321, we have admitted implicitly that the specific density ρ is constant. This is close to the physical truth if amplitude of pressure waves is very small; in ultrasonics, especially a practical limit of sound pressure amplitude is found of the order of 10^{-3} atm., the density changes are thus of the order of .1% at normal pressures.

It can be safely assumed that influence of any effect of this nature is going to be without perceptible e r r o r on measurements of velocity this research intends to perform.

As before, let be x the abscissa along the propagation path:

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* R.D. Fay, "Plane Sound Waves of Finite Amplitude" J.A.S.A. Vol. 3, No. 2, Oct. 1931.

ξ = particle displacement,

ρ specific mass $\frac{\partial m}{\partial v}$ when $v \rightarrow 0$,

p pressure amplitude (differential pressure)

μ viscosity coefficient,

θ temperature (abs. ° K)

t time.

static (undisturbed) quantities will be denoted by suscript ($_o$).

The specific mass of any portion of gas is initially: $\rho_o = \frac{dm}{dv}$; therefore $dm = \rho_o\, dv = \rho_o S\, dx$ (hereafter unit area of the wave front will be supposed).

Once perturbed, the displacement of plane (x) is ξ , the displacement of plane (x + dx) is $\xi + d\xi$ the new volume will be

$$S\left(dx + d\xi\right) = dv'$$

and

$$\rho = \frac{\partial m}{\partial v'} = \rho_o \frac{dx}{dx + d\xi}$$

$$\frac{\rho}{\rho_o} = 1 \Big/ 1 + \frac{\partial \xi}{\partial x} \qquad (40)$$

Newton's inertia Law can then be written as:

$$-\frac{\partial p}{\partial x}\, dx = \rho_o \frac{\partial^2 \xi}{\partial t^2}\, dx - \frac{4}{3}\mu \frac{\partial^3 \xi}{\partial^2 x\, \partial t}\, dx \qquad (41)$$

(It can be noted that this equation is identical to (23) derived from the general tensor equation) when

$$-\frac{\partial p}{\partial x} = -\frac{dp}{dx} \quad * \quad \text{and} \quad -\frac{dp}{dx} = k^{\sigma}\frac{\partial^2 \xi}{\partial x^2}, \qquad \chi + 2\eta = \frac{4}{3}\eta$$

(Stokes assumption on compressional and shearing viscosity)

Then

$$k^{\sigma}\frac{\partial^2 \xi}{\partial t^2} = \left(1 + \frac{\partial \xi}{\partial x}\right)\rho\left[\frac{\partial^2 \xi}{\partial t^2} - \frac{4}{3}\frac{\mu}{\rho_0}\frac{\partial}{\partial t}\cdot\frac{\partial^2 \xi}{\partial x^2}\right] \tag{42}$$

The adiabatic behaviour of the gas can be expressed as

$$P\,V^{\gamma} = P_0\,V_0^{\gamma}$$

$$\frac{\partial P}{\partial V} = -\frac{\gamma P_0 V_0^{\gamma}}{V^{\gamma+1}} \quad \text{and} \quad k^{\sigma} = \gamma P_0\left(\frac{\rho}{\rho_0}\right)^{\gamma}$$

Thus (42) will become

$$\frac{\gamma P_0}{\rho_0}\frac{\partial^2 \xi}{\partial x^2} = \left(1 + \frac{\partial \xi}{\partial x}\right)^{\gamma+1}\left(\frac{\partial^2 \xi}{\partial t^2} - \frac{4}{3}\frac{\mu}{\rho_0}\frac{\partial}{\partial t}\cdot\frac{\partial^2 \xi}{\partial x^2}\right) \tag{43}$$

---

*By definition

$$dp = \frac{\partial p}{\partial \xi}d\xi = \frac{\frac{\partial p}{\partial \xi}}{\frac{\partial \xi}{\partial x}}\;\frac{\partial \xi}{\partial x} = -k^{\sigma}\frac{\partial \xi}{\partial x}$$

therefore

$$-\frac{\partial p}{\partial x} = -k^{\sigma}\frac{\partial^2 \xi}{\partial x^2}$$

Complete study of solution of this equation is given in the remarkable paper of R.D. Fay (ft. note p. 41) as far as effects on actual measurement results are concerned, following summary will be suitable:

(a) The waveforms in time and space are periodic but no more sinusoidal.

(b) The net effect of interaction of component frequencies in pressure and displacement waves is to transfer energy from lower to higher frequencies.

(c) Effect (b) is compensated by the fact that viscous attenuation affects more the high frequency components than the low frequency components.

(d) The "stable" waveform is the one which yields balance of the two effects (b) and (c).

(e) Any given wave tends to assume a stable form; the shape depends on the fundamental frequency and on the intensity of disturbance.

(f) The limit of the stable form, when the sound intensity tends toward 0 is the sinusoidal waveform.

(g) At peak amplitudes smaller than 10^{-3} atm., the propagation velocity for any component of frequency f is given by

$$c^2 = \frac{\gamma P_0}{\rho_0} \sqrt{\sec \varphi} \quad \sec \frac{\varphi}{2} \qquad (44)*$$

---

*Orders of magnitude $\varphi = 2.5 \times 10^{-3}$, and $c^2 = \frac{\gamma P_0}{\rho_0}$ within 10^{-3}.

where $\lg\varphi = \frac{4}{3}\frac{\mu}{\gamma p_0} 2\pi f$

The effect on wave propagation is entirely negligible as long as intensities are lower than 10^{-3} atm.

2.328) Propagation in Gas Mixtures of Known Composition

The problem of sound propagation in gas mixtures is not entirely solved in the general case; the validity of statements contained in this section is restricted to

- Pressure and temperature region where neither of the components are subjected to abnormal propagation or absorbtion at the fundamental excitation frequency,
- The respective values of k^σ and γ for components are constant and known in the conditions of the experiment.

According to L. Bergmann,* the influence of each gas component is proportional to its concentration and sound velocity can be computed by the formula (45)

$$c^2 = \frac{R\,\Theta}{\sum_{i=1}^{i=n} \beta_i M_i} \cdot \frac{\sum_{i=1}^{i=n} \beta_i c_{p_i}}{\sum_{i=1}^{i=n} \beta_i c_{v_i}} \qquad (45)$$

*Der Ultraschall, Ed. 5; Zurich, 1949

M_i molecular mass of component i

C_{pi} specific heat at constant pressure in component i

C_{vi} " " at constant volume " " "

βi - concentration of component i

The next paragraph will show how practically the use of this formula is suited for purposes of the measurements to be performed.

2.329) Some Experimental Results and Influence on Interpretation of Measurements

2.3291) Effects of Viscosity

In section 2.323 the effect of viscosity on velocity was shown to be a relative modification given by

$$\frac{c'}{c} = \frac{\sqrt{(k\sigma)^2 + \omega^2(\chi+2\eta)^2}}{k\sigma} = 1 + \varepsilon$$

A few representative results are given in Table I.

This tabulation of representative values is by no means complete, nor can it pretend to any degree of accuracy in description of actual influence of viscosity on sound velocity in the previsible conditions of experiments.

TABLE I

VELOCITY CORRECTION DUE TO VISCOSITY

GAS	Θ °C	p_o atm.	$k\sigma$ dynes/cm²	$(\chi + 2\eta)$ g/cm-sec	ε for 100kcps	ε for 2Mcps	ε for 20Mcps
AIR	0	1	$1.377.10^{6}$	$1.70.10^{-4}$	$3.05.10^{-9}$	$1.22.10^{-6}$	$1.22.10^{-4}$
AIR	750	.	1.350 -	4.26 -	19.70 -	7.90 -	7.90 -
NITROGEN	0	.	1.379 -	1.62 -	2.74 -	1.09 -	1.09 -
NITROGEN	750	.	1.360 -	3.93 -	16.40 -	6.55 -	6.55 -
HYDROGEN	0	.	1.382 -	.83 -	.725 -	.29 -	.29 -
OXYGEN	0	.	1.370 -	1.89 -	3.76 -	1.51 -	1.51 -
OXYGEN	750	.	1.355 -	4.72 -	24.10 -	9.60 -	9.60 -
CH_4	0	.	1.382 -	1.03 -	1.10 -	.44 -	.44 -
C_2H_4	0	.	1.161 -	.91 -	1.21 -	.472 -	.472 -
CO_2	0	.	1.274 -	1.39 -	2.34 -	.93 -	.93 -

It appears however that even for high harmonics of the expected sound wave*, the influence of viscosity on sound velocity is comparatively small and errors introduced by neglecting viscosity terms in the velocity-temperature relationship will be less than .1%.

The effect of viscosity on absorbtion (attenuation) is important; although other, more important causes of absorbtion are present, the viscosity term is by no means negligible.

It is in order to observe here that in most gases viscosity increases with temperature.

2.3292) Effects of Heat Conduction

In section 2.324 the effect of heat conduction was shown as being an increase in viscosity such that an "equivalent" viscosity coefficient can be defined as:

$$\eta' = \lambda + 2\eta + \frac{K(\gamma - 1)}{c_p}$$

A few representative values are given in Table II.

* The fundamental frequency in the realized apparatus is 2 mc/sec for undamped transducers.

TABLE II

VELOCITY CORRECTION DUE TO THERMAL CONDUCTIVITY

GAS	θ °C	p_o atm	K erg/sec-cm	γ	C_p erg/gr-°C	$\frac{K(\gamma - 1)}{C_p}$ gr/cm-sec	h
AIR	50	20	$2.32.10^3$	1.401	$1.01.10^7$	$.92.10^{-4}$	.29
AIR	750	1	2.70. -	1.375	1.05 -	.965 -	.051
NITROGEN	15	"	2.18 -	1.404	1.03 -	.870	.288
NITROGEN	750	"	2.27 -	1.375(?)	1.01 -	.843	.046
HYDROGEN	15	"	13.7 -	1.410	14.2 -	.400 -	.230
OXYGEN	15	"	2.3 -	1.401	.912 -	1.01 -	.265
OXYGEN	750	"	2.38 -	1.375	.950	.940 -	.0392
CH_4	15	"	2.71 -	1.310	2.21 -	.382 -	.146
C_2H_4	15 to 100	"	1.65 -	1.255	1.50 -	.281 -	.098

$$h = \frac{\text{Velocity correction due to heat conduction alone}}{\text{Velocity correction due to viscosity alone}}$$

It appears that the effect of thermal conductivity is in all previsible cases small compared to the one of viscosity; the corrections in velocity are of the order of 5 to 29% of the viscosity corrections.*

Influence of heat conductivity on sound propagation velocity will be therefore neglected a fortiori in forthcoming discussions.**

As before, the fact of having no measurable effect on sound velocity does not imply that effect on sound absorbtion might not be important in some cases. For temperature measurements however this will be of no consequence as long as the sound absorbtion will be compatible with the means of sound generation and detection employed.

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* Sources of physical data for this section are:

- L. Bergmann, Ultraschall, Ed. 5, pp. 384
- Handbook of Chemistry and Physics, Charles D. Hodgman, Editor, 33rd edition, Cleveland, Ohio

** L. Bergman indicates for h the value of $\frac{3}{4} \frac{\gamma-1}{\gamma} K'$ with K' being of the order of .8 to 1.94 for usual gases. This would yield 11% for $K' = 1.5$ and in good agreement with values of Table II.

2.3293) Effects of Thermal Relaxation*

The first detection of thermal relaxation effect was reported by Pierce** on Carbon Dioxide. Values for sound velocity were found as follows:

f	42 KC/s	98 KC/s	206 KC/s
C	258.8 m/s	258.9m/s	260.2m/s

The interferometer method was employed and such "dispersion" in velocity could not have been accounted for with inaccuracies in the experimental technique.

Considerable theoretical work has been carried out mainly by H.O. Kneser in the early 30's; and the theory derived in (2.325) is chiefly based on Kneser's original work, extended, interpreted and experimentally tested by many others.

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* References: H.O. Kneser:

Uber die Schallgeschwindigkeit in Kohlensaure, Phys. 2, Vol. 32

Zur dispersiontheorie des Schalles, Ann Phys. Vol. 11

The Interpretation of the Anomalous Sound Absorbtion in Air and Oxygen in Terms of Molecular Collisions. J.A.S.A., Vol. 5, 1933

L. Bergmann, Ultraschall, Ed. 5, pp. 369 and seq.

** G.W. Pierce, Proc. Amer. Acad., Boston, Vol. 60, pp. 271

As has been shown (Eq. 34) the actual sound velocity is determined by values of three constants:

- c_0 low-frequency sound velocity
- c_∞ high-frequency sound velocity
- τ relaxation time.*

Representative case is the one for CO_2, given in Fig. 1

Measurements of this kind have been performed for a good number of gases and operating conditions, results are of considerable importance in field of theoretical physics especially in determination of the specific heat related to the internal vibration modes of molecules and also for determination of the relaxation time τ.

In the results concerning CO_2** the value of τ was found 5.7 μsec at 18 °C and normal atmospheric pressure.

---

* τ can be given also following signification:

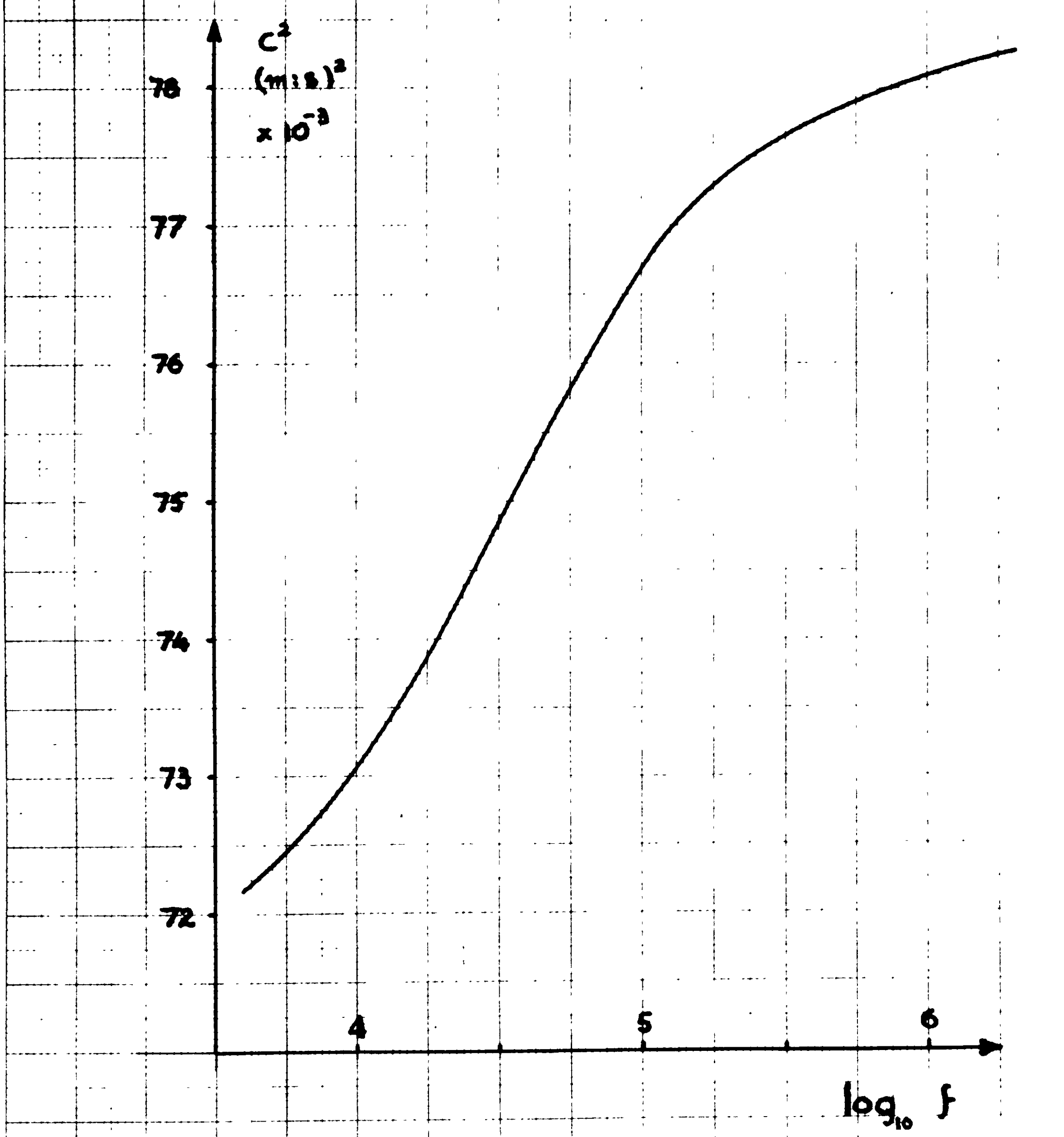
$$(c'_v)_t = (c'_v)_\infty \left(1 - e^{-t/\tau}\right)$$

where $(c'_v)_t$ the "instantaneous" specific heat corresponding to internal modes after square wave excitation at $t = 0$.

$(c'_v)_\infty$ final specific heat characterizing internal modes in the same conditions.

** M. H. Wallmann, Ann. Phys. Vol. 21, pp. 671

Velocity Dispersion in Carbon Dioxyde



F I G. 1

Further investigations* were conducted on CO_2 in view to precise the influence of temperature and pressure on relaxation time and velocities at low and high frequency ranges.

Typical result is shown in Fig. 2 for temperature dependency at normal pressure.

The influence of pressure seems to be reasonably uniform for most of the gases studied, the relaxation time τ appears to be inversely proportional to the pressure.

Any increase of the frequency would have thus the same effect as a corresponding decrease of the pressure.

In most of the experimental works results are given in function of the parameter f/p which seems to be the main governing factor in velocity dispersion.

Fig. 3 gives values for H_2 (para) as found by Stewart**

Mention must be made of the important effects on relaxation (and therefore on velocity dispersion) of presence of so-called "contaminants".

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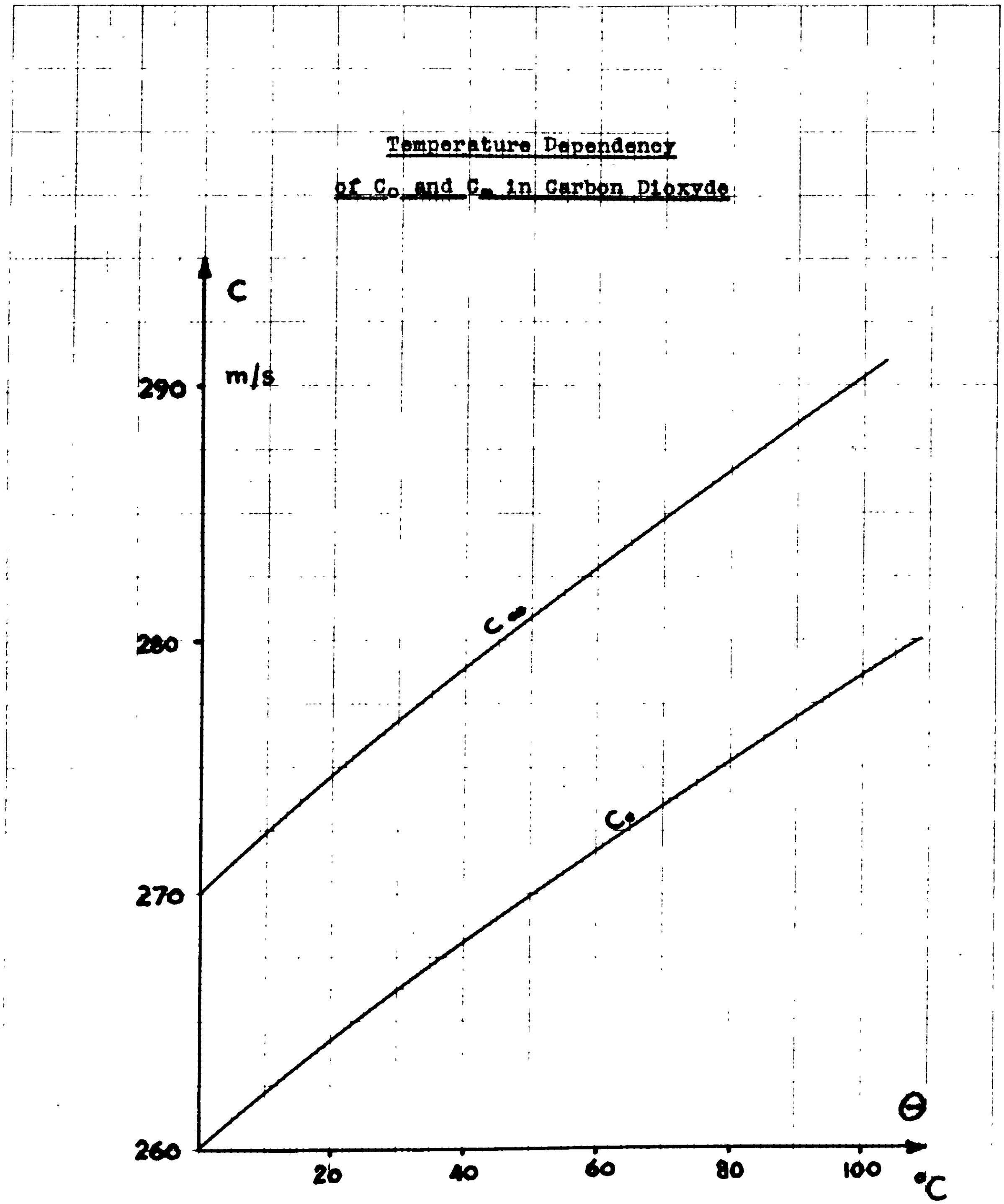
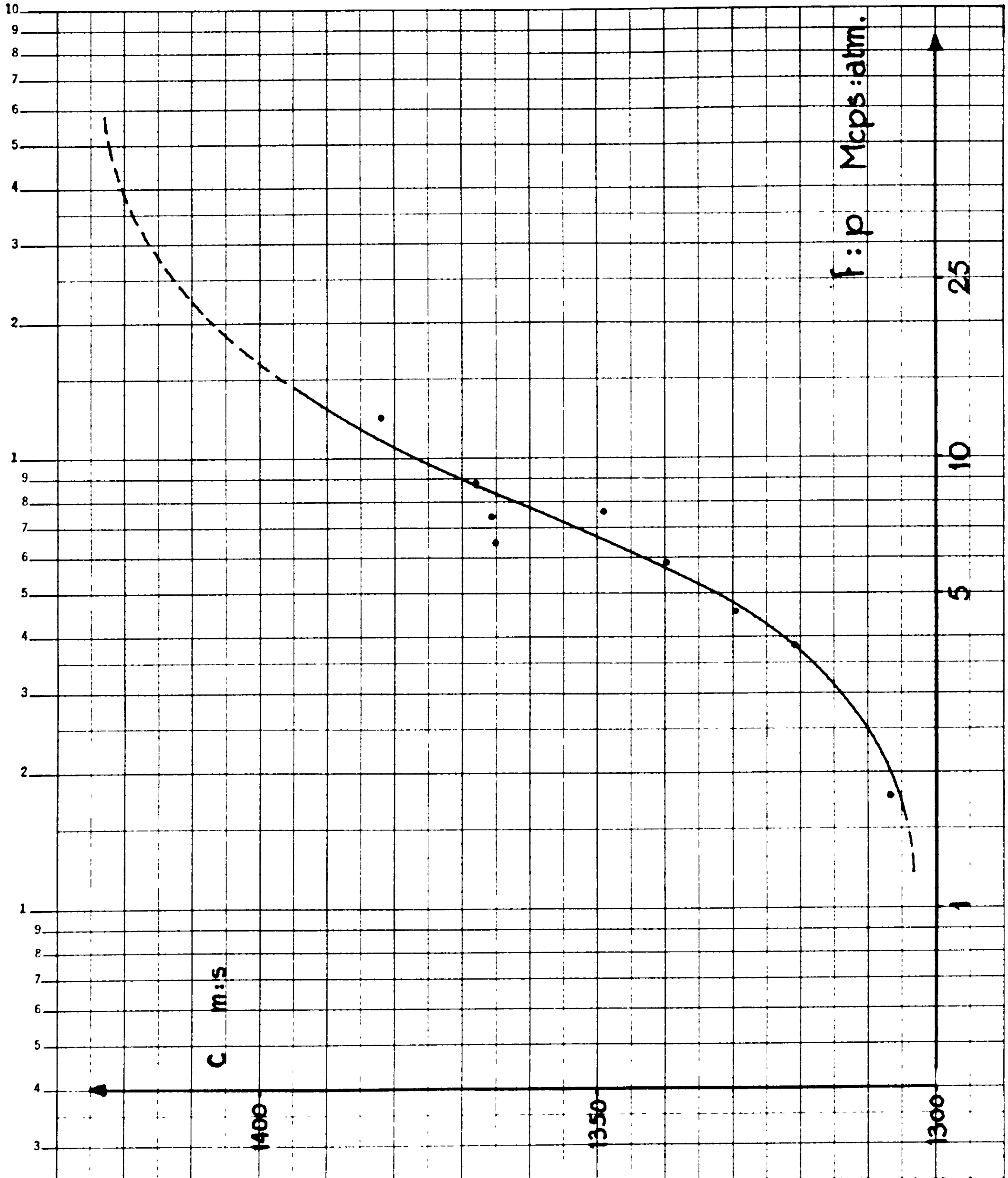
* W. H. Pielemeier, J.A.S.A., Vol. 15, 1943, pp. 22

** E. S. Stewart, Phys. Rev., Vol. 69 - 632, See Annexed Bibliography.


Temperature Dependency
of C_0 and C_∞ in Carbon Dioxyde
C
m/s
290
280
270
260
C_∞
C_0
θ
20
40
60
80
100
°C

F I G. 2



F I G. 3

Velocity Dispersion in Hydrogen

Although present in small amounts (2% to 12%) certain gases such as He, CH_4, H_2, HCl and especially H_2O can modify the log of the relaxation frequency* from 4.7 to 6.25, i.e. the frequency from 50.000 c/s to 1.8 Mc/sec at 1 atm.

The cause of this phenomenon is not yet thoroughly explained; it would appear that presence of contaminants tend to decrease the number of collisions necessary to exchange one energy quantum between molecules; the time lag for complete energy equilibrium would be thus smaller in presence of "contaminants" than in absence of these latter; therefore the total specific heat C_{v^o} prevails for much higher frequencies as it would do so in the pure gas.

Few representative values are given hereafter Table III according to Eucken and Kuhler (see Reference Pages).

Although considerable experimental effort was spent in this field, a number of essential factors are still unavailable in the current literature. In particular, if tendency of displacement of relaxation frequency vs. temperature is indicated, no experimental confirmation has been published up to date.

---

* Relaxation frequency is the one yielding the velocity

$$\frac{C_o + C_\infty}{2}$$

TABLE III

NUMBER OF MOLECULAR COLLISIONS TO RELEASE ONE ENERGY-QUANTUM FROM "PRINCIPAL-GAS"

	Contaminant	
Principal gas	N_2O	CO_2
H_2	650	300
CO	3600	-
CH_4	840	2400
H_2O	105	105
Pure	7500	50000

Nor was investigation made of the influence of rapidly changing temperatures and steep temperature gradients within the propagation medium.

The conclusion from the above facts could be summarized as follows:

- The relationship between sound velocity and temperature at any given frequency between 10^4 and 10^7 cps can not be accurately predicted for the probable medium under investigation.*
- The errors that can be introduced by relaxation in any of the components can be of the order of 5 to 10% of the velocity.
- It is recommended to choose the measurement frequency in such a way as to keep out of the relaxation domain of any of the components.
- If precise measurements are to be made, explore a priori the given gas mixture in the given temperature pressure-frequency region ascertain the absence of relaxation.**

---

* Mixture of N, hydro-carbons, steep time and space gradients of temperature and pressure.

** As it might be cumbersome to change the sound frequency, one can make use of the remark on p. 53, i.e. replace frequency changes by pressure changes.

2.3294) Effects of Pressure

The causes of variation in sound velocity with pressure were given in 2.326.

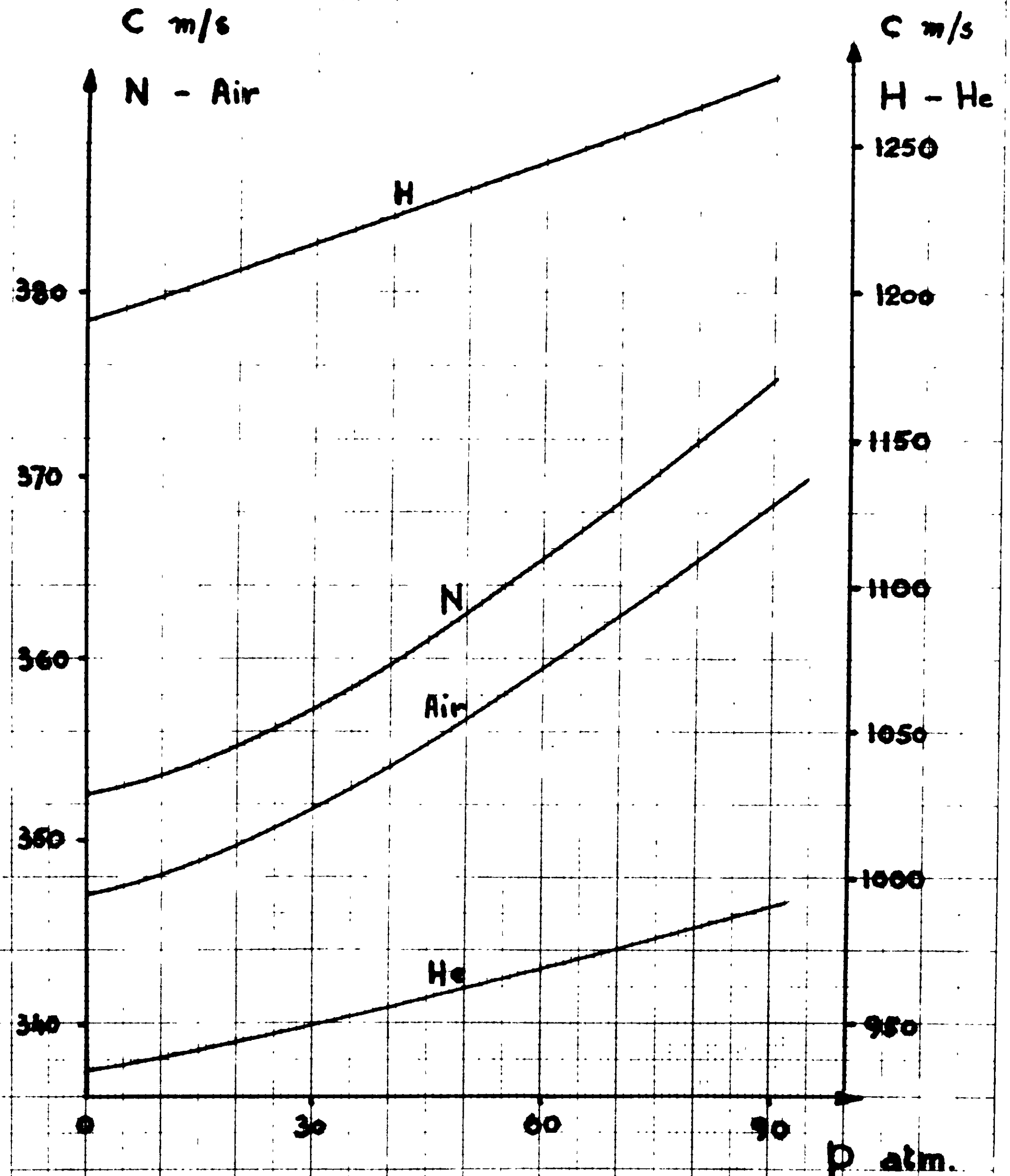
The sound velocity at any given pressure can be calculated if the ratio of specific heats γ and the compressibility k are known. Both of these coefficients, together with the specific mass have been throroughly studied and available for all the components of the gas mixtures to be studied. As an example, Fig. 4 gives (according to Hodge) representative examples for N_2, Air Hydrogen and Helium.

2.3295) Outline for Temperature Determination Procedure from Sound Velocity Measurements

The whole section 2.32 has been devoted to the relationship which is supposed to hold between sound velocity and temperature in gases, taking into account such parameters as chemical composition of gas, pressure, sound frequency, sound pressure amplitude and some others.

It will be in order, now in conclusion to show, by a concrete example how the temperature can be determined when sound velocity is known in presence of the other necessary parameters.

Sound Velocity as Function of Pressure in N, H, He and Air



F I G. 4

Data will be available usually in following form:

(a) Composition of the gas, say components 1, 2, ...etc. with respective concentrations β_1, β_2 ....etc.

(b) Approximative pressure, measured either in static operation or by so-called "engine indicator"*; it can be assumed that pressure is known at least at $\pm$ 2% of accuracy

(c) Frequency and amplitude of the sound wave used to measure the sound velocity; as a rule these quantities will be inherent to the construction of the measuring apparatus and will not be subjected to any change in the measurement process.

(d) Thermodynamical characteristics of the components; gas tables give usually respective values of

Isothermal stiffness	K_1^θ	K_2^θ	K_3^θ ....
Ratio of specific heats	γ_1	γ_2	γ_3 ....
Molecular masses	M_1	M_2	M_3 ....

(Values of K^θ and γ are usually, but not always known as functions of p and θ)

(e) Data on relaxation characteristics of the gas composition <u>at the given pressure, frequency and temperature</u>.

---

* M.I.T. High-speed Engine Indicator, see Sloan Lab. No. 8

It is evident that this latter information is rather unprobable to be found in the current literature; it is therefore suggested to neglect for the first approximation relaxation effects and correct results if any discontinuity in velocity measurements is observed.

(f) Sound velocity as determined by measurement

Then, knowing the composition, the partial pressures p_i will be determined by

$$p_i = \beta_i \frac{\sum m_i}{\sum M_i \beta_i} \frac{R\,\theta}{V} \qquad (46)$$

p_i = partial pressure of component (i)

β_i = concentration of component (i), (ratio $\frac{\text{Mass of } C_i}{\text{Total Mass}}$

m_i = mass of component (i)

M_i = Molecular mass of (i)

R = Universal gas constant, = 8.3136 . 10^7 ergs

θ = Abs. temperature*

V = Volume of the gas mixture studied.

This value has to be compared to the experimental value givenby data (b)

$$P_i = \beta_i \, p_t \qquad \left\{ p_t = \text{total indicated pressure} \right. \qquad (47)$$

---

* Approx. value of θ, as computed by $\theta = \frac{M\,c^2}{R\gamma}$ can be used for determination.

In case of divergence between the two values of p_i the experimental value is to be used in subsequent calculations.

For each component we have, since p_i C_{pi} C_{vi} and K_i^θ are known

$$k_i^\sigma = \frac{C_{pi}}{C_{vi}} k_i^\theta$$

The total stiffness of the mixture is

$$K^\sigma = \sum k_i^\sigma \tag{48}*$$

The sound velocity in the gas mixture will be

$$c^2 = \frac{K^\sigma}{\rho} = V \frac{\sum \frac{C_{pi}}{C_{vi}} k_i^\theta}{\sum m_i} \tag{49}$$

and thus

$$\sum \frac{C_{pi}}{C_{vi}} k_i^\theta = \frac{c^2}{V} \sum m_i$$

The "equivalent" isothermal stiffness K^θ is then given by

$$K^\theta = \frac{\sum \frac{C_{pi}}{C_{vi}} k_i^\theta}{\frac{\sum \beta_i C_{pi}}{\sum \beta_i C_{vi}}} \tag{50}$$

---

* This is consequence of the definition of $K^\sigma = V \left(\frac{\partial p}{\partial v}\right)_\sigma$

and $\partial p = \sum dp_i$, $V \frac{\partial p}{\partial v} = V \sum \frac{\partial p_i}{\partial v} = \sum k_i^\sigma$

therefore

$$K^{\theta} = \frac{c^2}{V} \sum m_i \frac{\sum \beta_i C_{vi}}{\sum \beta_i C_{pi}} \tag{51}$$

then

$$\left(\frac{\partial p}{\partial v}\right)_{\theta} = \frac{K^{\theta}}{V} = \frac{c^2}{V^2} \sum m_i \frac{\sum \beta_i C_{vi}}{\sum \beta_i C_{pi}} \tag{52}$$

For real gases we can write

$$\left(\frac{\partial p}{\partial v}\right)_{\theta} = \varphi(p, \theta) \frac{m}{M} \frac{R\theta}{V^2} \qquad (53)^*$$

and since $K^{\theta} = \sum K_i^{\theta}$

$$\left(\frac{\partial p}{\partial v}\right)_{\theta} = \sum \left(\frac{\partial p_i}{\partial v}\right)_{\theta} \tag{54}$$

The determination of θ is then straightforward, the curves

$$\left(\frac{\partial p_i}{\partial v}\right)_{\theta} = \varphi_i(p, \theta) \frac{m_i}{M} \frac{R\theta}{V^2}$$

are plotted with θ as abscissae, (Fig. 4bis)

It is useful to notice that in the general case for engine study, for points far away from critical points, usually ideal-gas conditions are sufficiently approached to take $\varphi = 1$,

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* For ideal gas $\varphi(p, \theta) \equiv 1$ for the whole domain of p, and θ

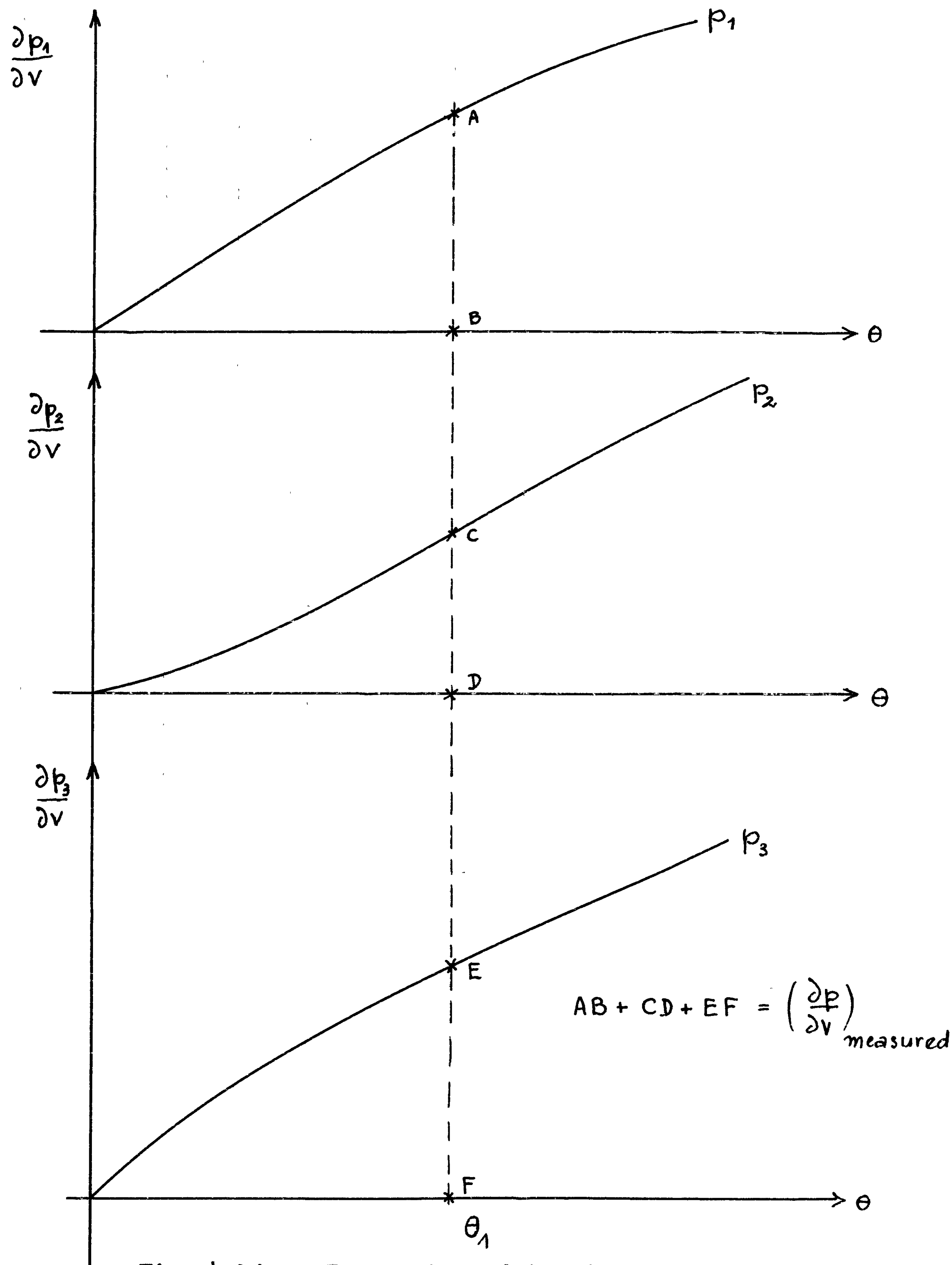


Fig. 4 bis Temperature determination from partial compressibilities and

compute then θ from (52) and (53) modified as

$$\theta = \frac{(\sum \beta_i M_i)(\sum \beta_i c_{vi})}{R \sum \beta_i c_{pi}} C^2 \qquad (55)$$

This formula is obviously applicable to pure and ideal gases.

The above analysis gives all the necessary characteristics of the components which have to be known if satisfactory results are to be obtained.

Curves of temperature vs. sound velocity can be plotted in advance for gas mixtures of known composition.

It is necessary at present to see how the influence of the thermal relaxation can be accounted for.

In general, i.e. for any given composition of gaseous mixture no data will be available in the literature on relaxation characteristics. It is possible, however, that for one of the most important components (usually N_2) the dispersion curve is found in previous experiments.

If at any point of the engine's evolution cycle, the partial pressure of this component becomes such that f/p reaches the dispersion region, special attention should be paid to the results translating C^2 in temperature θ.

If unexpected results occur (such as "irregular" computed values of Θ) it must be made sure by steady-state measurements that relaxation is not the cause of this irregularity.

Steady-state measurements can be performed in controlled-atmosphere chamber whose physical arrangement should reproduce the essential features of the actual combustion chamber. The description of such experiments and apparatus is beyond the scope of the present work.

2.3296) Partial Conclusion on Temperature Measurements by Sound Velocity Method

It has been shown that, except in cases of thermal relaxation, there is bi-unique correspondence between the sound velocity c and the absolute temperature Θ. Effects of viscosity, of heat conduction and of non-linear propagation can be neglected with the physical arrangement proposed in this paper.

Effects of deviation from perfect-gas law can be taken into account by graphical methods, however, in most cases a good approximation will be obtained with simple calculations based on the perfect-gas Law.

In all cases (except for those where noticeable relaxation is present) the temperature can be determined from the sound velocity with systematic errors less than .3%.

2.33 ULTRASONIC POWER GENERATORS AND DETECTORS

2.331 Characteristics and operation of electro-mechanical transducers.

Considering the special problem of temperature measurements in engine cylinders the only generators and detectors described here will be those suitable to this purpose.

The facts exposed in Sect.(2.31) define practically the type of sound generator to be used ; the limitations are even more severe for detectors of acoustical power.

Table IV gives a summary of presently known and used ultrasonic power generators (power ratings are indicative and valid in the ultrasonic spectrum over 20 KC).

Some of the generators acting as transformers of electrical energy into sound energy are at present known as a special case of power transducers. The term "transducer" will be employed henceforth to designate devices intended to perform the electro-acoustical power conversion.

The discussion in Sect.(4) s h o w s that the frequency will be of the order of 1.0 MCps. Only piezo-electric transducers are suitable for practical power

T A B L E IV

Characteristics of Ultrasonic Power Generators

Type	Frequency Range	Power Range	Observations
	KCps		
Galton Whistle	40	10W	Operates in open atmosphere ; frequency and amplitude change with pressure.
Gas-Jet Pulsator	120* 500*	1W/cm2	(*) Air (*) H
Liquid Whistle	32 (H_2O)		Power limited by cavitation and rupture of liquid vein.
Siren	22	6W/cm2	
Holtzmann Generator	33	10W	
Arc Generator	2000		Frequency generated contains high proportion of harmonics. Operates at high temperature.
Electro-dynamic	200	.02W	
Electro-Static	30		
Piezo-Electric	150000	6W/cm2	In pulsed operation only
Magneto Strictive	175	17W/cm2	Dependent on temperature

generation in this frequency range, and the forthcoming analysis will be restricted to piezo-electric transducers in the 1000 KCps domain.

Design of piezo-electric transducers has to consider following characteristics :

- Piezo-electric constant of the transducer material; variation of this constant with temperature, orientation, treatment and time,
- Mechanical properties of the transducer material such as elastic constants, specific mass, and, eventually Poisson's and Lame's Constants,
- Electrical properties such as dielectric constant K and dielectric strenght;
- Associated electrical driving circuit,
- Associated mechanical load; more specifically the "useful load" which is the medium receiving the generated sound and the parasitic or artificial loads resulting from mechanical mounting and damper loads.

The operation of piezo-electric transducers is based on the deformation of crystal lattices by external electric fields.

The lattice deformation is apparent at macroscopic scale by changes in the crystal dimensions. The force acting in presence of dimensional changes will be governed by the elastic properties of the crystal ; if the superimposed electric field is variable, the crystal will follow these variations, introducing however its own "response"* defined by specific mass, elastic constant and dimensions.

Usually piezo-electric crystals are surrounded by solids (mountings and loads) or by fluids ; in either case the force acting on the crystal boundary is transmitted to some extent at the surrounding medium ; the proportion of mechanical power actually delivered to the surroundings depends mainly on the ratio of acoustic impedances of the crystal and the "load".

Acoustic impedance ratios define the transmission coefficient and this latter factor appears to be the measure of <u>power efficiency</u> of the crystal for the given load.

As this efficiency is one of the main limitations on transducer design, it is important to materialize by investigation of the orders of magnitude of transmission factors from usual transducer crystals to some typical loads.

Values in Table V give clear indication that irrespectively of the engineering solutions adopted, the first and most critical condition to be satisfied is

---

* The term "response" is employed here with the meaning "Amplitude and phase variations vs. amplitude and frequency of external sollicitations".

TABLE V

Power Transmission Coefficients

(Crystal to Load.)

*Crystal	Zc	Air	Water	Mercury	Steel	Brass
Quartz	15.1	$11.2.10^{-5}$	.312	.971	.792	.840
A D P	5.9	$29.2.10^{-5}$	.635	.730	.452	.502
Rochelle Salt	6.04	$28.5.10^{-5}$	.621	.743	.460	.520
Barium Titanate	27.5	$6.26.10^{-5}$	.195	.972	.970	.975
	$\times 10^5$	Z'c = 43	$1.45.10^5$	$18.6.10^5$	40.10^5	$34.4.10^5$

(*)A D P : Current abbreviation for $NH_4\ H_2\ PO_4$.

that of obtaining detectable sound energy in the gas under investigation.

The situation is even worse than it would appear from the transmitter consideration only ; as the detecting method is similar in principle to the one used in the transmitter, the same "mismatch" (low factor of transmission) will make it difficult to detect the sound energy once transmitted to the gas.

It can be stated that whenever crystal transducers are used to generate and to detect sound power in gas (at normal pressure) the detected signal will be an infinitesimal fraction of the generated power ; the order of magnitude of the signal level is

70 to 85 below the transmitter output power.

It will be seen in the section dealing with design of the electro-mechanical apparatus that these powers, although close to the receiver noise levels, are possible to use and give satisfactory solution to the problem.

The previsible frequency range (around 1 MCps) indicates imperatively the same type of transducer for the production as for the detection of sound energy.

It is concluded from this summary discussion that, subject to engineering design and check of orders of magnitude, piezo-electric transducers are to be used in the temperature measurement apparatus, as well for the transmitter as for the detector .

2.332 Discussion of possible application of piezoelectric transducers to temperature measurement apparatus.

2.3321 Frequency.

In section (2.31) the lenght of the sound path was fixed at ½" # 1.2 cm ; the maximum velocity of sound will be approximatively at 825° C # 1100° K, 650 m/s the minimum duration of sound travel will be

$$\frac{1.2}{650.10^{2}} = 18.5\mu\,\text{sec}$$

If the figure of 1 per cent is accepted as desirable accuracy on propagation velocity and assuming the lenght of the sound path to be known by far better accuracy, then any time-measuring device incorporated in the sound-velocity apparatus will have to determine time intervals with accuracies better than .18 μ sec.

Whatever the time-measuring procedure may be, it will have to deal with the signal (let us suppose for the present discussion that this signal is sinusoidal) transmitted through the sound path.

The transmitted signal can act by one or several of its characteristics, frequency, amplitude, waveform or phase, but the best <u>resolution in time</u> obtained by any timing method will be proportional to the fundamental period of the signal.

If the detection method employed discriminates time to better than 2/10 of the period, then the condition of accuracy stated in this section calls for a fundamental frequency of at least :

$$f = \frac{2/10}{.18 \cdot 10^{-6}} = 1.1 \text{ Mcps}$$

Although the gas mixtures to be investigated have not yet been thoroughly explored for possible thermal relaxation, preliminary experiments seem to indicate that the range from 1 MCps to 2 MCps is reasonably free from abnormal velocity or absorbtion.

The frequency of 2 MCps is adopted in the design, further reasons for this choice will be given when discussing the sound radiation inside the combustion chamber.

2.3322 Power.

It is usefull to find, at this early stage of design, the order of magnitude of the power involved.

As a first approximation the receiver noise level will set the lower limit of signal strenght.

The signal to noise ratio at the output of the receiver has to be at least 20 dB. The total thermal noise power at the input will be in the 0 - 10 MCps band

$$P_{NT} = K \Theta \Delta f = 1.38 . 10^{-16} . 300 . 10^{7}$$

$$= 415 . 10^{-9} \text{ ergs : sec}$$

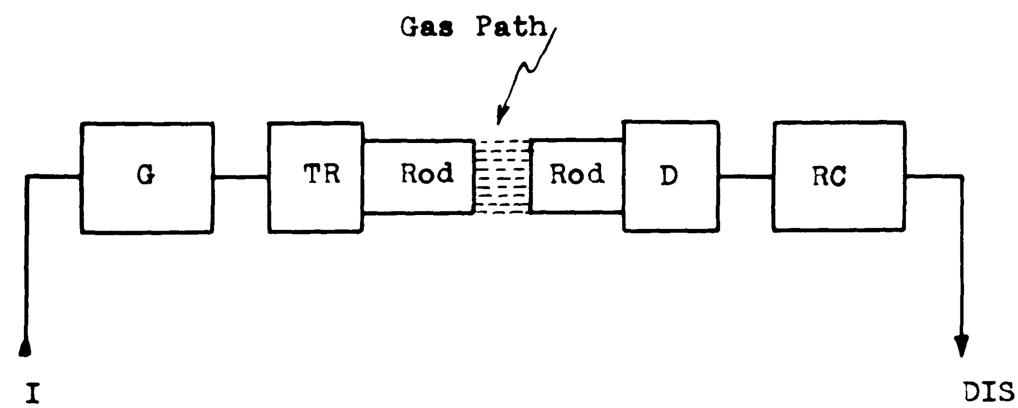
$$= 4.15 . 10^{-14} \text{ w}$$

Let us adopt as a basis for discussion the schematical arrangement given by figure (5), then it is possible to evaluate roughly the power level at each point of the sound path.

The values summarizing results of this evaluation are given in Table VI ; it has to be made clear that they are only intended to serve as indications (or better upper or lower limits) to determine what amount of power the generator has to handle.

Certain assumptions are obviously based on experimental values and have to be considered as being

Schematical Arrangement of Sound-Velocity Apparatus



I Initiation
G Generator
TR Transmitter
D Detector
RC Receiver
DIS Display

F I G. 5

T A B L E VI

Power levels in the Electro-Acoustical Path.

Element	Insertion loss		Minimum level	
	Cause	dB	Watts	dB
Thermal noise input to receiver	Molecular agitation in input circuit	0	$4.15.10^{-14}$	0
Receiver noise	Receiver circuit noise	4		4
Receiving transducer	Electro-mechanical coupling factor	5		9
Coupling Rod (Receiver side)	Matching and attenuation	15		24
Rod-Gas Contact	Brass-Gas Acoustical mismatch	39		63
Gas Attenuation	Viscosity and heat conduction	5		68
Gas-Rod Contact	Gas-Brass Acoustical mismatch	39		1o7
Coupling Rod (Transmitter side)	Matching and attenuation	15		122

Transmitting Transducer	Electro Mechanical coupling factor	5		127
Transmitter	Output and line match	4		131
Transmitter Output			# .5 Watt	131dB

deduced from preliminary vereficatIons.

A considerable safety factor has to affect the value found by approximations of the type used in table VI ; practically it will be in order to design the transmitter to deliver 5 to 7 watt R.F. power to the transducer, in order to reach the receiver noise level.

2.3323 Temperature and pressure.

The conditions prevailing in the gas are up to 825° C and 45 atmospheres ; the transducer materials are, in general, sensitive to both temperature and pressure.

Further, if good transient characteristics of the overall vibrating system are to be obtained, high harmonics of the engine pressure wave would fall in the reach the amplifier band and their effect would be considerably greater than the sound pressure effect.

For all these reasons, solid (metallic) coupling rods are to be provided, together with suitable cooling arrangement. Design should prevent engine pressure to be transmitted onto the crystal. These construction features enable the crystals to operate at room temperature and under atmospheric pressure.

2.3324 Bandwidth.

The last important condition of proper operation is the necessary bandwidth for the desired accuracy.

The accuracy of reading was defined in Sect. 2.3321 ; details of .18 μs were to be observed. This would indicate at least a 5 MCps band ; amplifiers certainly can be constructed according to this specification ; the difficulty will reside chiefly in the transducers where the choice of mechanical circuit parameters is more restricted than in the usual electric network design.

2.3325 Partial Conclusion of section 2.33.

The main operating features of crystal transducers being reviewed and orders of magnitude for operating frequency and power estimated, nothing is found to be prohibitive for use of crystal transmitter and receiver in the apparatus to be designed.

Pending on actual design data, the schematical arrangement of Fig. 5 will be used in the subsequent discussion.

3. HISTORY OF THE PROBLEM UP TO THE PRESENT

3.1 INTRODUCTION TO THE PROBLEM BACKGROUND

Temperature determination of a medium by means of sound velocity measurement requires, as it appears from Section (1.) homogeneity of the medium across the whole propagation path ; its complete isotropy constant and identical temperature in all points and known and uniquely determined relationship between sound velocity and temperature.

As a complementary condition, no interference with temperature of the body under investigation has to occur when sound power generators and detectors are introduced.

These conditions are impossible to realise in a great number of measurement problems ; even where some of them are satisfied with good engineering approximation, the last condition stated eliminates the sound-velocity method from practically all cases.

The only experiments where sound-velocity determinations can be considered are :

- Measurement in very large volumes of liquids, such as lakes or seas ; when average temperature are to be obtained, and variations in specific mass are negligible.

- Measurements in gases atvery high temperature, when radiation pyrometers are not practical ;
- Temperature measurements in closed unaccessible volumes, subjected to rapidly changing physical conditions.

The very limited nature of possible applications seems to have somewhat discouraged the zeal of experimenters and specific examples of sound velocity measurements as a means for temperature determination are rather scarce in the current technical literature. Some of them will be mentionned in Sect.(3.3.)

The second part of our problem, the sound velocity measurement process itself, seems in return to suffer somewhat from plethora of authors, methods and descriptions. There is difficult to find a single issue of specialised papers since 1946 without being introduced to some new application of ultrasonic velocity and absorbtion measurement.

The objectives of these measurements are selected from a very wide range of physical problems ; determination of properties of matter, investigation of molecular structures and experimental proof for theoretical physics are only a few of them.

Some of the most typical examples are mentioned in the next paragraph. The detailed study of comparative ratings of the described methods is beyond the scope of this work ; our attention will be mainly directed to objectives of the various authors and, whenever applicable, to the possible application to temperature determination.

This section will be closed by a brief reference to the importance of temperature determinations by means of sound velocity.

The primary objective of this research was the study of sound-velocity method in view of utilisation in analysis of internal-combustion engines. It is assumed that precise knowledge of "end-gas" temperatures may help to compare and improve fuel mixtures and engine designs.

As far as temperature determinations in internal-combustion engines are concerned, the sound-velocity method appears to be the only one which can be properly synchronized with the engine operation in order to yield precise and instantaneous measurements.

3.2 EXAMPLES OF SOUND VELOCITY MEASUREMENTS

3.21 EXAMPLES OF SOUND VELOCITY MEASUREMENTS IN SOLIDS

(a) F.A. Firestone and J,R. Frederic give a typical method to determine characteristics of metallic bodies by reflectoscopy. (Ref. Nr 8)

A Thyratron-pulsed Quartz crystal is coupled by oil layer to the metal surface and reflection of the sound wave is displayed on ordinary CR scope.

The frequency is of the order of 5 MCps and the crystal oscillations are "killed" by gas damper tube in order to avoid interference with the received pulse. As the main objective is detection of flaws and gas occlusions, the calibration of the CR sweep and the pulse waveform are not of primary importance. The authors do not mention the expected accuracy on sound velocity, as this quantity is not specifically the purpose of their apparatus. (1946)

(b) F.W. Struthers and H.M. Trent give account of another apparatus, used equally for flaw detection but operating on another principle. (It was chiefly designed for Diesel cylinder testing.) (Ref, Nr 25)

The transmitting transducer is a frequency-modulated crystal coupled to the material under test which is usually constant in dimensions and physical characteristics.

As thickness resonance of the load occurs sharp peak in impedance is observed ; this impedancepeak is detected by plate voltage maximum in the driving output tube and displayed on CRO screen swept by the modulation voltage.

Satisfactory stability and dependable operation is reported. (1947)

(c) A.W. Nolle and S.C. Mowry of the Massachusetts Institute of Technology, describe a pulse reflection method, used to study bulk-wave propagation in high polymers. (Ref Nr 18)

The transmitter crystal is pulse-modulated with 10 to 30 MCps carrier and 2 μsec pulse duration. The signal is reflected by metallic plane reflector and compared after amplification to the transmitted signal electrically delayed and attenuated.

The amplitude of the reflected signal is characteristic of the absorbtion of materials tested.

Calibrated-sweep, AR test scope is used to measure the travel time of the acoustic signal. No account is given of the waveform, but it would appear that the errors on the velocity due to scope reading are small compared to other sources of systematic errors.(1948)

(d) Another interesting principle is used in apparatus described first by R.D. Holbrook of the Brown University and later by N.P. Cedrone and D.R. Curran. The objective of the experiments is to detect small changes in sound velocity. (Ref. Nr 5 and 12)

Two-crystal transceiver is retriggered by the transmitted pulse after travel through the test path of constant lenght. The pulse repetition frequency is then function of the sound velocity ; the P R F can be measured by C R O display or by synchronising an oscillator and measure the beat frequency with a crystal-controlled standard oscillator.

The transmitted pulse operates at 10 to 30 MCps carrier frequency with P R F from 50 to 100 KCps. (1948)

The common feature of all these modern methods of sound velocity measurement is the relative freedom from difficulties related to waveform and/or amplitude considerations.

All solid materials tested have acoustic impedances of the same order of magnitude than the usual transducer crystals ; it is by no means exceptional to receive in the detecting device up to 3 per cent of the transmitted pulse power ; therefore far above noise and crosstalk level.

Owing to the comparatively small attenuation of solid materials, reasonable lenghts of sound path can be used ; they range from a few inches to several feet ; then transit times are large enough to reduce the importance of wave leading-edge errors to less than other systematic errors inherent to methods and materials.

3.22 EXAMPLES OF SOUND VELOCITY MEASUREMENTS IN LIQUIDS

(a) J.R. Pellam and J.K. Galt describe an apparatus for sound velocity measurements in liquids, with application to a large number of organic liquids.

15 MCps is used with pulses of 1µsec ; transit time and attenuation is studied on AR scope display. The accuracy claimed is .1 per cent on velocity and 5 per cent on absorbtion. (Ref Nr 19)

The setup is working on the reflectoscope principle and, according to the authors, dependable and simple operation is achieved. (1946)

(b) Brother John Quinn relates studies of ultrasonic absorbtion in Benzene. The method used is the liquid interferometry ; the acoustic impedance of the liquid is modified by micrometric adjustment of the reflecting plate facing the crystal. (Ref. Nr 21)

The impedance variations are transmitted in the driving circuit and detected by thermo-couple. Temperature regulation of the liquid is achieved with fluctuations less than .1° C. (1946)

(c) C.E. Teeter Jr. studies absorbtion in liquids by refelction method using crystal transducer, metal reflector amd AR synchroscope display. The wave-form of transmitted pulses is given in author's description ; it appears to be rather poor with a rise time of the order of several cycles.

The apparatus however gives satisfactory results for absorbtion ; the stated accuracy is 10 per cent on the attenuation constant in average conditions. (1946). (Ref. Nr 27)

(d) A typical example of velocity measurement in liquids by optical means is described by C.J. Burton in connexion with study of ultrasonic propagation and absorbtion in liquid mixtures. (Ref. Nr 2)

Monochromatic light source (Hg-arc) is used to illuminate the liquid excited by Quartz transducer. The operating frequency is 4 to 40 MCps with approximatively 75 Watts electrical input power on the crystal.

Photomultiplier or photographic plate is used to record the diffraction pattern produced by the ultrasonic beam.

Much of what has been said on velocity measurements in solids could be repeated for studies on liquids. In general matching between transducer and liquid is acceptable (although not as good as in many solids) ; on the other hand, precautions are easier to be taken in order to insure homogeneity and constant temperature throughout the whole liquid. Sometimes special construction has to be considered (when acoustic power levels are high) to avoid stirring and/or cavitation in the liquid.

As in case of solids, considerable improvement in the quality of measurement has been obtained with the newly developed AR scopes, designed during World War II for radar purposes. The commercial availability of this kind of equipment puts at the disposal of otherwise not electronically equipped experimenters convenient means to measure delays down to 1μ sec with accuracies better than 1 per cent or .1μ sec, whichever is larger.

3.23 ULTRASONIC VELOCITY MEASUREMENTS IN GASES

(a) H.C. Rothenburg and W.H. Pielmeier of the Pennsylvania State College have designed apparatus for velocity and absorbtion studies in air. Transducers are of the ribbon type, frequencies used are from 10 to 100 KCps. (Ref. Nr 22)

The propagation chamber is about 15 feet long; serious difficulties are related owing to insufficient level at the receiver, measurement results are however acceptable. The apparatus uses one transducer at each end ; pulses of about 10 to 12 cycles are fed in the transmitter.

The received pulse is displayed on ordinary CRO tube with calibrated sweeps, or with superimposed markers.

(b) I.F. Zartmann gives account of measurements of sound velocity in gases by means of interferometer method. (Ref Nr 31)

Continuous-wave transmitter is operated from 500 to 1500 KCps. Metal reflecting plate is used with highly accurate micrometric adjustment. Distances of sound travel are very small (½ to 3/4 of an inch) and thermocouple is used as impedance maxima and minima detector.

No statement is made on accuracy and levels but it can be assumed that the coupling between gas and transducer was extremely small, and so the detection of extrema of impedance must have been difficult. (1949)

(c) More promising seems to be the method given by M. Greenspan and M.C. Thomson Jr. in 1952.(Ref Nr 10)

Interferometer is constructed with two

crystals ; pressure and temperature of gas being constant, the receiver input is function of the distance between the two transducers, or to be more specific, between the internal faces of the long coupling rods, introduced to increase the distance between transmitter and receiver. This was to be done in order to have a constant electric cross-talk signal which can be neutralized in this case by electrical means. Detection is made by recording transmitted amplitude versus distance between radiating and receiving surfaces.

Indications are in the above methods that all ultrasonic investigations in gases suffer from the same difficulty : how to get appreciable amount of power in and out of the gas.

As at high frequencies (b) and (c) C W methods are used, crystals can be driven at their best-power level and the signal strenght is not considered as being the limiting factor in these experiments.

Wavelenght is gases are exceedingly small in gases driven at frequencies in the MCps range ; interferometric measurements are therefore increasingly difficult to perform with a reasonable degree of accuracy.

Pulse methods are not subject to this difficulty but short propagation paths make the overall band-width of great importance as precise detection of the leading edge is imperative. Bandwidth and gain of vibrating systems are known to be opposite requirements and this consideration is definitely the limiting factor when ultrasonic pulse excitation is employed

3.3 MEASUREMENT OF SOUND VELOCITIES IN VIEW OF TEMPERATURE DETERMINATIONS

3.31 TEMPERATURE DETERMINATION IN LIQUIDS

At very origin of transducer development the main objective was ultrasonic underwater detection for military purposes. P. Langevin gives in his relation* the method for underwater temperature determination by sound velocity measurement, but states that density changes, unsuspected underwater currents and presence of planctonic interference, make practical measures of this kind impossible. This experiment was only mentionned for its historical interest.

(*) P. Langevin, Communication a l'Academie des Sciences, Paris, 1919.

3.32 TEMPERATURE DETERMINATION IN GASES.

(a) The first typical measurement of this kind is described by C.G. Suits, in 1935. His method was aimed at the determination of arc temperatures, as radiation pyrometers were not suitable owing to the selective radiation of the arc column. (Ref Nr 26)

A condenser discharge is produced in a gap located inside the arc. The intense ultrasonic wave (about 75 KCps) is propagated through the arc and picked up by a spark microphone* located outside of the arc.

The "transmitter" operated under 14 KV at the P R F of about 15 to 20 per second.

Remarkable accuracy was obtained in the display method : a C R O sweep was initiated by the primary spark acting as a radio transmitter.

The C R O displayed at the same time a timing frequency and the received signal.

Temperature measurements of the order of 3000° C are reported ; although no claim of accuracy is made.

(b) Another experiment is described in the Proceedings of the Royal Society of London by G .G.Sherratt and E. Griffiths in 1934. (Ref.Nr23)

(*) As it is known, this kind of "transducer" uses the voltage variation across a spark gap when pressure around the spark is modified.

Although the objective was the specific heat determination in gases at temperatures between normal and 2000° C, the equipment and procedure is perfectly suitable for temperature determinations ; data given by the authors are of importance in interpretation of our measurement results.

The method used is gas interferometry, with 7907 cps and 27422 cps quartz plate, vibrating on the longitudinal modes.

Temperature of the interferometric chamber is measured by disappearing-filament type optical pyrometer aimed at the reflecting piston. No provision seems to have been made for the non-blackbody radiation characteristics of this reflecting piston.

(c) Another acoustical interferometer method, operating at 304.4 KCps is proposed by A. Van Itterbeek and P. Mariens, both connected with a number of precise and complete velocity and absorbtion measurements in the ultrasonic field. (Ref. Nr 28, 29, 30)

The interferometric chamber is 3.9 cm in diameter and 12 cm in lenght ; acoustic impedance maxima and minima are detected by tube voltmeter across the driving crystal.

Good measurement results are given for O_2 and H_2, between -100° C and 100° C.

(d) As an example of pulsed method in high temperature gases, the preliminary work performed by M. James C.Livengood of the Sloan Laboratories at the Massachusetts Institute of Technology has to be mentionned.

The theoretical survey of the problem was made by Professor Jordan J. Baruch of the Acoustic Laboratory at the Massachusetts Institute of Technology.

The method is a two-crystal pulse transmission procedure at 2 Mcps carrier frequency, synchronized with the engine cycle and displayed on AR scope.

Criticized features are the pulse shape, the overall signal level and the influence of electric "leakage" from the transmitter.

The apparatus was constructed and motoring tests on actual engine gave good experimental results as compared to other methods of temperature determination.

The process of measurement was judged to be promising and the possible simplification and improvement of this apparatus is the primary objective of our work.

Much of the mechanical construction of the Sloan Laboratory experiment was available and had to be used for the present investigation ; the combustion chamber and the engine characteristics are imposed as "nec variatur" data of our problem.

Extensive use was also made of part of the electronic equipment available in the Sloan Laboratories for Automotive and Aircraft Engines, such as synchroscope and video-receiver circuitry, together with M.I.T. Engine Indicators.

4. METHODS FOR DIRECT SOUND VELOCITY MEASUREMENTS

4.1 INTRODUCTION

In Sect. (2) and (3) correlation between temperature and sound velocity has been explicited and some historical indications were given.

Presently the use of this sound-velocity measurement technique in internal-combustion engine cylinders will be discussed.

The frequency of the sound will be considered as given (Sect. 2.33) in the 1 to 2 MCps range, the power generated(*) of the order of 5 to 10 W ; and methods of measuring the sound velocity across gas path of approximatively ½" has to be examined.

Crystal transducers are considered as being the only practical ultrasonic sources in this frequency range.

All the possible measurement methods will have in common following elements :

a- Power source for transmitter and receiver

b- Acoustic path from transmitter to receiver

(*) At the output of the electrical generator.

c- Transmitting and receiving transducers

(In some cases the same transducer can perform both functions)

d- Driver (transmitter)

e- Receiver

f- Time measurement circuits

g- Synchronisation with engine cycle.

Parts (a) and (g) can be determined at once the power will be supplied for the whole apparatus from the 115^{V} + or - 10 per cent 60 cps voltage source ; suitable transformers and rectifiers are provided.

The synchronisation of the measurement process with the engine cycle will be accomplished by means of a breaker on the engine distribution shaft arranged to make one contact at each cycle (1 per 2 engine shaft revolutions) at adjustable crank angle by continuous manual control with accuracy better than + or - $.5^{\circ}$. As far as the sound-velocity apparatus is concerned, the synchronisation will be represented as a closing contact with a timing precision better than + or - $.1^{\circ}$ of crank-angle.

4.2 SOUND PATH ARRANGEMENT

The physical lenght of the sound path has to be .5 ± .1 inch in order to not introduce a too large "end gas" volume in the engine cylinder.

The question of the transmissive or reflective sound propagation arrangement has now to be discussed in light of the probable absorbtion, homogeneity and circuit considerations.

Fig.(6) gives the two possible arrangements. In the transmission (or two-crystal) method, the transmitter T generates the sound "signal" propagated by the coupling rod C_1 into the gas path G, transmitted in C_2 and picked up by the receiving transducer R.

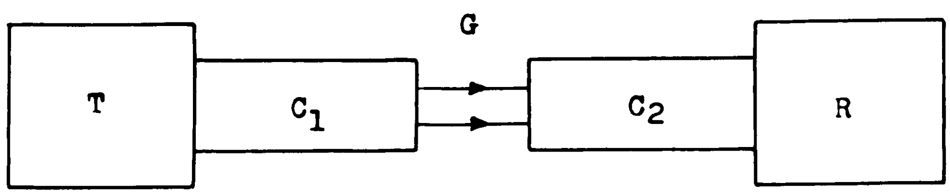
In the reflection arrangement (Fig.6b), the transducer TR acts as transmitter to excite the coupling rod C ; the sound wave in G is reflected by the solid reflector R, reintroduced in the bar C and received by the transducer acting as pick-up. The respective merits and inconvenients are summarized as follows:(see Table VII)

In reflection process, the influence of lack of homogeneity in the gas is important in view of the comparatively high absorbtion in the gas path.

If at the boundary between two layers of different acoustic impedance the sound wave is reflected, Principle of Transmission and Reflection Method

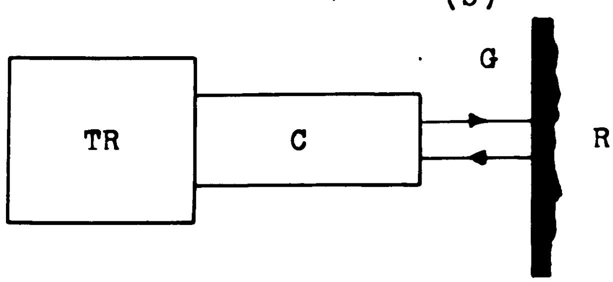
Transmission Method

(a)



Reflection Method

(b)



F I G. 6

TABLE VII

	Transmission	Reflection
Gas path absorbtion at normal pressure	Approx. 5 dB in absence of relaxation	Approx. 5 dB in absence of relaxation
Signal to noise ratio	Acceptable but critical	# 5 dB below transmitted signal
Boundary transmission losses	Approx. 75 to 80 dB	Approx. 75 to 80 dB
Parasitic reflections in coupling Rods	Small in amplitude, can be eliminated	Very large in amplitude, cannot be eliminated unless sound velocity is known quite accurately.
Reflection and scattering at boundaries of different gas layers	Unimportant	Comparatively important.
Electric circuit	Reasonable in complexity and easy to adjust	Comparatively complex and adjustments are critical

the signal reaching the transducer will be of the same order of magnitude as the main signal, since the smaller power propagated by this "parasitic" signal can be compensated by the much smaller distance it has to travel in the highly absorbing gas.

If the reflecting layer is in the vicinity of the reflector, then the parasitic signal amplitude may be much smaller than the main signal, but the respective phases are almost the same, so ambiguity on the main signal leading-edge will occur and make interpretation impossible.

The only real advantage of reflection procedure may be the smaller interference of the presence of sound equipment with the engine construction. It would eliminate the obligation of special combustion chamber with acoustical insulation and reduce the number of special orifices from two to one.

In the transmission method (often called two-crystal method) the influence of gas homogeneity is much less important.

If the sound beam is reasonably narrow, it can be assumed that all points of a given wavefront are at the same temperature ; even if the temperature is not constant along the propagation path, the compound velocity will be very closely the same for all points of the wave.

Partial reflection will attenuate the signal but will not produce any parasitic sound effect on the receiver and will therefore not interfere with measurements of propagation velocity.

Another important consideration is the transmitter circuitry. If the same transducer is electrically switched to the receiver shortly after initiation of the transmitted signal, oscillations of the transmitter have to be suppressed electrically and mechanically before the received signal is expected. This sould set a limit of 10 to 15 μ sec on the transmitted signal duration ; which is by no means easy to obtain, especially when parasitic reflections in coupling rods are present.

For all these reasons, transmission method has been chosen for the first series of experiments.

The acoustic path will be then defined with following components :

- Transmitter crystal (including electrodes)
- Coupling bar (usually brass in contact with the crystal and acoustically insulated from the engine cylinder walls and provided with separate water cooling)
- Gas
- Coupling bar (same as for the transmitter)
- Receiving crystal (including electrodes).

The direct measurement of sound propagation velocity is based on simultaneous determination of the gap lenght L and the time of travel across the gap.

L being known with a satisfactory degree of accuracy the main problem is to measure a time interval T, this is usually done by determining the delay between two "corresponding" points of the transmitted and received electrical signal.

Often the determination of "corresponding" points involves some real difficulties ; the leading edge is often used if acceptable waveforms are transmitted.

The problem of direct sound velocity measurement is then reduced to the (electronic) measurement of the time interval between two electrical "events" hereafter called "initiation" or "trigger" and "signal" respectively.

It will now be necessary to investigate the electronic timing devices at our disposal.

4.3 ELECTRICAL MEANS FOR TIME INTERVAL MEASUREMENTS

4.31 FEEDBACK CIRCUIT ELEMENT METHOD

4.311 Definition and properties.

A great number of circuits can be arranged to operate according to a well defined function of a time interval introduced in the circuit as a so-called "phase difference", Then, one or more of the circuit operating parameters, voltage, current or frequency will be characteristic of the time interval introduced and can be used to "measure" this time interval.

A convenient way to make circuit operation dependent on time interval (hereafter called "phase" or "delay") is to use a feedback link containing this delay; then the feedback factor will be complex and its argument will directly depend on the time delay. Since both the overall gain and the generalized impedance function are dependent on the feedback factor, the natural oscillatory modes of any similar system are essentially determined by the time interval to be measured.

Frequency (and amplitude) in such circuit can be easily measured to yield values of time delay by comparison.

The accuracy of this sort of time measurement is determined by three factors :

(a) The possibility to measure the quantity (frequency or amplitude) governed by the delay-controlled feedback factor ; this usually is well within the necessary accuracy limits ; frequency determinations by beat methods are not exceptional to ± .ol per cent while amplitudes can be measured, although with some precautions, to ± 1.5 per cent even in the radio-frequency range.

(b) The inherent stability of the circuit itself which has to be at least as good as the order of magnitude of the accuracy on the intended measurements. This consideration is, in our opinion, the limiting factor in the use of feedback-controlled time measuring devices ; as the stability of operation is in great proportions governed by the circuit selectivity, any modification in the feedback factor will react on the stability as well as on the frequency (and/or amplitude.)

(c) Further limitation is imposed in continuous-wave sinusoidal operation by the fact that the phase angle (argument) of the feedback factor cannot be over 2π radians, or ambiguous indications will be given by frequency determinations. This sets a limit to the maximum time-interval range that can be measured, as it will be seen in the next paragraph.

4.312 Continuous Wave Self-Oscillatory Link.

Fig. 7 represents schematically a time delay measuring device operating on the CW self-oscillatory link principle. The main amplifier has a gain μ_1 and feeds its output to the acoustic link having a "gain" G_A ; the receiver of the acoustic path will be connected to the feedback amplifier with the gain μ_2. (All gains are assumed to be complex.) The equivalent gain, $-\frac{E_{11}}{E_{10}}$ of the main amplifier is equal to

$$\frac{E_{11}}{E_{10}} = \frac{1}{1 - \mu_1 \mu_2 G_A} \tag{56}$$

or E_{11} will have finite value for $E_{10} = 0$ if $\mu_1 \mu_2 G_A = 1$ (56')

If we suppose for sake of simplicity that

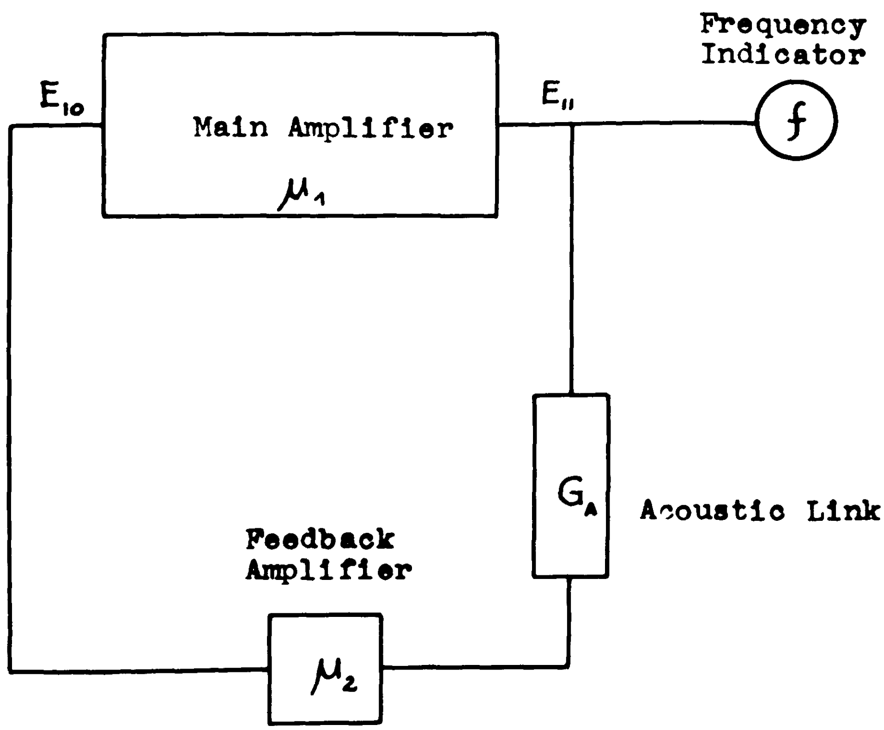
$$\operatorname{Arg}(\mu_1 \mu_2) = 0$$

(this can always be satisfied with good approximation, but is not necessary for the discussion), then Eq.(56') is equivalent to

$$\begin{aligned} [\mu_1 \mu_2 G_A] &= 1 \\ \operatorname{Arg} G_A &= 2K\pi \end{aligned} \tag{57}$$

and $K = 1, 2, 3 \ldots$

Typical Feed-back Circuit



F I G. 7

If L is the lenght of the acoustic path, c the sound velocity, the travel time of the sound wave front is $\frac{L}{c}$ and the phase angle, compared to a sinusoidal oscillation of angular frequency ω will be $\frac{L \cdot 2\pi f}{C} \times \frac{1}{2\pi}$ $\frac{2\pi Lf}{C}$.

If the phase angle Arg G_A is to be equal to $2K\pi$, then

$$\frac{2\pi L f}{c} = 2k\pi$$

and the self-oscillation frequency is found to be

$$f = k\frac{c}{L} \qquad (58)$$

which can be interpreted as defining the self-oscillatory period as being an <u>integer submultiple</u> of the acoustic transit time.

It is readily seen that a given sound velocity c will sustain the same oscillation frequency with $K = 1$ than the sound velocity $\frac{C}{2}$ with $K = 2$, or $\frac{C}{n}$ with $K = n$,

and no unique value can be assigned to c unless values of

$$\left[\mu_1 \mu_2 G_A\right] < 1 \quad \text{for} \quad f > \frac{c_{max}}{L}$$

The measurement will then be interpretable down to $\frac{Cmax}{2}$.

It is important to examine the order of magnitude of the maximum frequency to be employed :

Cmax # 1960 ft/sec = 600 m/sec

L # .5" = .0127 m

f # 47 KC

This frequency is far below the practical limits for the measurements discussed in this paper.

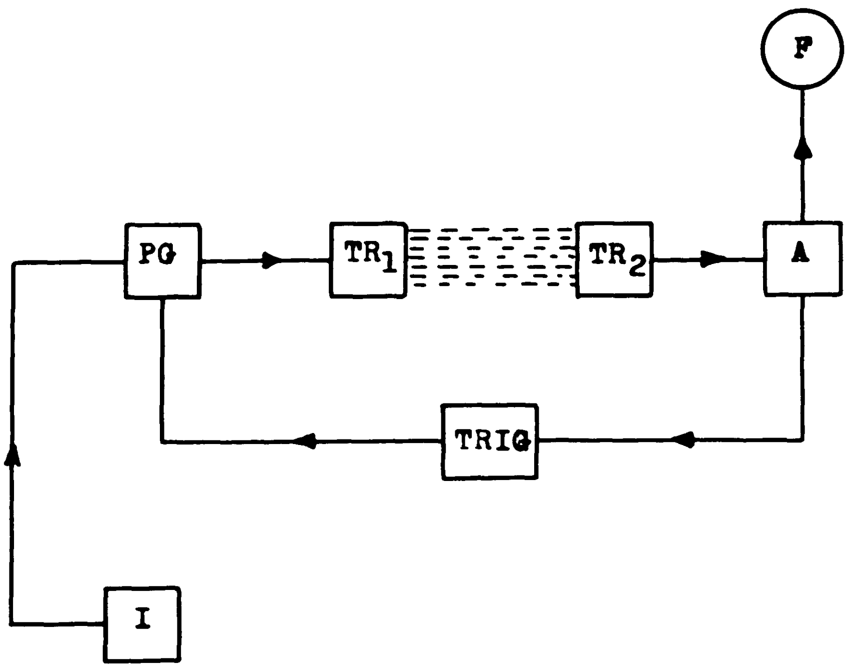
4.313 Self-triggered pulse Method.

Another way to introduce the acoustic path in a feedback loop is described by R.D. Holbrook.(Ref.12)

Schematically, the apparatus consists of a pulse generator supplying radio-frequency short pulses into the acoustic system which includes the two crystal transducers, coupling rods and gas path. The output of the receiving transducer is then amplified and shaped to triggering voltage and connected to the input of the pulse generator.

Once initiated by the internal one-shot trigger J the system will continue to operate as periodic pulse transmitter-receiver ; the pulse repetition frequency will be direct function of the sound velocity in the acoustic path if the gap lenght and electric delays in

Schematical Self-Triggered Circuit



I Initiator

PG Pulse Generator

TR_1 Transmitter

TR_2 Receiver

A Amplifier

F Frequency Indicator

TRIG Trigger Circuit

F I G. 8

other parts of the circuit are constant ; measurement of the P R F will give the sound velocity across the gas.

Following conditions have to be fullfilled for proper operation :

a- The electric delay has to be independent of the signal strenght and pulse repetition frequency,

b- The pulse generator must be designed to accept rather high pulse repetition rates,

c- The time allowed for the measurement has to be long enough for proper P R F determination.

These last two conditions are somewhat impractical for the sound-velocity measurement in engines as the P R F would be of the order of 1/18 μsec # 55.000 pps, while the time available for measurement is very short (Sect.1.31) and can be down to 390 μ sec.

Measurement of pulse repetition frequencies at such rate in such a short time require special electronic apparatus such as high-speed calibrated CR sweep circuits or pulse counters ; the cost and operation of such devices would more than annihilate the advantages of the relative simplicity of the measurement process itself.

The accuracy on velocity measurement is determined partly by the ratio of

$$\frac{\text{Electrical delay time in Amplifier}}{\text{Acoustical Delay time.}}$$

and the electric delay time is known to be of the order of .3μsec for a six or seven stage amplifier.[*]

This would indicate that in the particular case of our measurements, determination errors or variations in the electrical delay are unimportant.

4.32 INTERNAL COMPARISON METHODS.

4.321 Definition and Characteristics.

The name of "Internal comparison method" will be given to timing procedures wherein the time delay is measured by comparison of the "received" signal subjected to the delay to the undelayed "transmitted" signal itself.

Although a large number of comparison methods can be imagined, only two are in current practice and both use continuous wave transmission.

The accuracy will depend on two features :

a) Stability of the transmitted signal,

b) Accuracy of comparison between transmitted and received signal.

(*) Britton, Chance and others, Electronic Time Measurements, Sect. 3.6.

4.322 Continuous Wave Phase detection.

This method uses a sinusoidal power source driving the transmitting transducer ; another transducer receives the signal which is amplified and its phase compared to the transmitted wave.

As before the sound propagated with the velocity C over a gap of lenght L introduces a time lag $\frac{L}{C} = T$ which corresponds in the transmitted signal to a phase angle lag of ωT if ω is the angular sound frequency. Then c can be determined by measuring the phase lag ωT.

A number of methods are available for phase difference measurements but all are characterised by the fact that no phase difference multiple of 2π can be detected ; i.e. any phase difference $\varphi + 2\pi$ will be interpreted by the available measuring devices as a phase difference of φ .

This amounts to accept either an ambiguity on the sound travel time (which can be φ/ω or $(\varphi+2\pi)/\omega$ or $(\varphi+4\pi)/\omega$ etc) yielding as many values of the sound velocity, or to limit the operating angular frequency to

$$\omega_{max} < \frac{2\pi \Delta C}{L}$$

ΔC being the range of velocities to be measured ; it can be readily seen that in the actual problem this condition would limit ω with $\Delta C = 300$ ft/sec , $L = .5''$

$$f_{max} = \frac{\omega_{max}}{2\pi} = 7200 \text{ cps}$$

Another possibility is to determine the approximative sound velocity by another method (for example by sending pulsed signals through the same transducers) and get accuracy from the CW phase.

Serious objections are present however to limit the possibility of CW phase detecting method in the internal-combustion engine problem.

The use of the low frequencies is out of question for practical reasons, first, the power transmitted by any given radiator with a given particle displacement is proportional to the frequency, second, in order to avoid parasitic reflections arising from large values of the ratio $\frac{\lambda}{R}$(*) dimensional restrictions are to be considered for the piston area and for the lenght of the gas path.

Further, the measurement of phase relationship between two sinusoidal functions is a problem of considerable delicacy even at industrial frequencies, when accuracies better than $\pm 1^{o}$ are sought.

Let us consider for sake of argument that $\pm .5^{o}$ of accuracy can be realised in phase measurement and see what error this might introduce on sound velocity.

$$\Delta\varphi = 1^{\circ} \qquad \Delta T = \Delta\varphi/\omega$$

$\omega = 2\pi . 7 . 200$ rad/sec (assuming the limit

(*) λ wavelenght, R radius of radiating piston.

mentioned above), then $\frac{\Delta T}{T} = \frac{\Delta \varphi}{\varphi}$ if $\frac{\Delta \omega}{\omega} \ll \frac{\Delta \varphi}{\varphi}$

$\Delta C/C = \Delta L/L + \Delta T/T$

Assume $\frac{\Delta L}{L}$ negligible, then

$$\frac{\Delta C}{C} = \frac{\Delta T}{T} = \frac{\Delta \varphi}{\varphi}$$

Or $\frac{\Delta \varphi}{\varphi}$ can reach very high values ; suppose that φ is determined by addition or multiplication of angles where $\cos \varphi$ is the determining factor, Then the quantity accessible to measurement, say resulting amplitude A, is given by

$$A = k \cos \varphi \qquad \{ k = \text{constant}$$

and

$$\frac{\Delta A}{A} = - \frac{k \sin \varphi \, d\varphi}{k \cos \varphi} = - tg \varphi \, \Delta \varphi$$

or

$$\frac{\Delta \varphi}{\varphi} = \frac{-1}{\varphi . tg \varphi} \; \frac{\Delta A}{A}$$

If a constant relative error affects the measurement of A, the relative error on φ goes to ∞ as $\frac{-1}{\varphi^2}$ when $\varphi \longrightarrow 0$

It is in order to mention at this place that special precautions have to be taken in order to eliminate any amplitude variation in any device acting as adder or multiplier (mixer) as amplitude variations affect phase measurements in a very important proportion.

Assume the two added components are such that the resulting voltage is V

$$V(t) = E_1 \sin \omega t + E_2 \sin (\omega t + \varphi)$$

then

$$V^2 = E_1^2 + E_2^2 + 2 E_1 E_2 \cos \varphi$$

and $$\cos\varphi = \frac{V^2 - E_1^2 - E_2^2}{2 E_1 E_2}$$

Assuming same relative errors on determination of V, E_1, E_2, the error on cos φ is

$$\frac{\Delta\cos\varphi}{\cos\varphi} \cong 2\frac{\Delta V}{V}\left[1 + \frac{V^2 + E_1^2 + E_2^2}{V^2 - E_1^2 - E_2^2}\right]$$

and the second member is again seen to be very large when $V^2 \cong E_1{}^2 + E_2{}^2$ or $\varphi \cong \frac{\pi}{2}$.

All these considerations eliminate from our discussion commercial two-coil type phasemeters and are as well defavorable for direct Lissajons-pattern phase measurements on CR screens.

One apparatus deserves however special mention for accurate phase-measurements*; it operates by clipping the two incoming signals and generating sharp "pips" at the positive-going zero-crossing of each of them ; the pips are fed respectively in the "on" and "off" grids of a bistable circuit (Eccles-Jordan or sometimes called flip-flop) the average value of the resulting square wave on any of the flip-flop plates is then function of the phase difference between the incident signals and can be used to measure this phase difference.

(*) Technology Instrument Corporation.

With the best commercial instruments, the absolute error on phase angle measurements is of the order of 3^o ; it as been shown that when phase angles are small (say 25^o) errors of that kind are absolutely prohibitive.

Readings on any type of phase meter take an appreciable amount of time ; usually several orders of magnitude over the 390 μsec, found for maximum permissible duration of one single measurement.

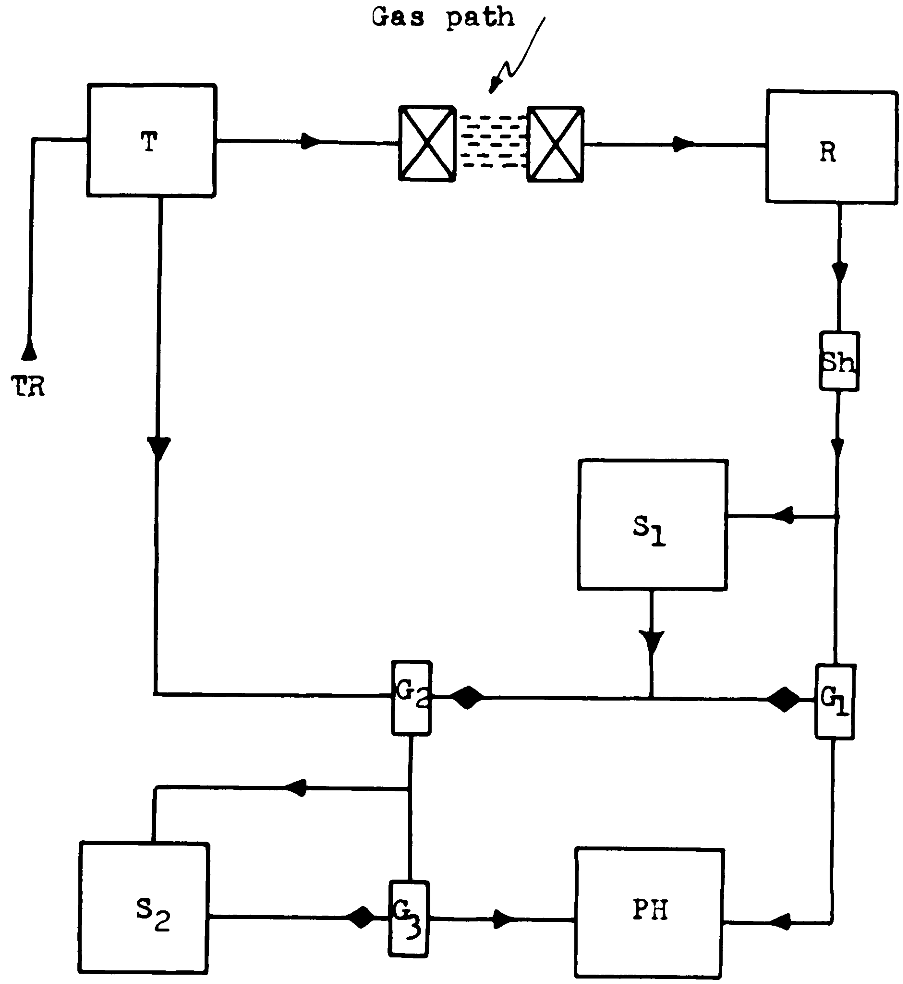
The CW phase detection, although interesting in other applications has definitely to be rejected for the investigation described herein.

4.323 Pulse Wave Phase Detectors.

The above mentioned phasemeter operating by transformation of sinusoidal signals into "pips" suggests immediately to the reader the possible use of this phasemeter in pulsed-operated sound velocity measurement devices.

Fig. 9 gives one of the possible arrangememts ; T and R are ultrasonic transmitter and receiver sending pulses through the gas path ; electronic scaler S_1 opens gate tubes G_1 and G_2 for each second transmitted pulse ; the phasemeter is "switched" once for 2 transmitted pulses and its operation stops after 10 double cycles by operation of scaler S_2.

Principle of PW Phase Detector



TR Trigger

T Transmitter

R Receiver

Sh Shaper

S_1 Scaler 1 : 2

G_1 Gate tubes operated
G_2 by S_1

S_2 Scaler 1 : 10

G_3 Gas tube operated by S_2

PH Phasemeter

F I G. (9)

The output of the phasemeter flip-flop can be used to charge a condenser through a constant current device and give thus, by measurement of the condenser voltage, direct reading of the time delay or sound velocity.

No better accuracy is expected from this type of measurement (as it is subject to a great extent to the imperfections of the CW phase detecting method); but a fact of considerable importance is the averaging feature of this sort of apparatus. It will be seen in the discussion of experimental results, that even if errors resulting of the operational technique are excepted, the sound velocity is not constant for the same setting of externally controlled and arbitrary parameters, but is subjected to fluctuations of 1 to 2 per cent around its average value. These fluctuations are tentatively explained in our final discussion but it is necessary to mention here that in our opinion, the average value of a fixed number of experimental determinations (20 in the above example) will be to greater use to the "customer" (i.e. people interested in internal-combustion engine study) than singular determinations of sound velocity and temperature, even if these latter may be performed to .5 or even .2 per cent of accuracy.

4.324 Frequency-modulation Time delay measurement.

The transmitted continuous wave is frequency-modulated and the received signal, delayed by propagation in the gas path, is compared to the transmitter output.

One practical case is when the modulating wave is triangular (Fig.10) such that $\frac{\partial f}{\partial t} = a$ = constant for the whole duration of a "cycle". Then

$$df = a\,dt$$

and

$$dt = L/c = df/a$$

L and a being inherent constants of the apparatus c is only function of df and measurement of the frequency difference yields an unambiguous value of the sound velocity provided the modulation cycle is long enough to be superior to $\frac{L}{C_{min}}$; Cmin being the minimum value of the expected sound velocity.

Unfortunately the beat-frequency measurement (which is the convenient way to determine difference of two frequencies) is not suited to ultra-short time intervals available for measurements.

If however the frequency deviation is chosen low enough to operate an electronic counter for example by the clipped and differentiated beat frequency then the apparatus could be used even in single-shot operation.

The possible arrangement is shown in(Fig.11)

Frequency Modulated Wave

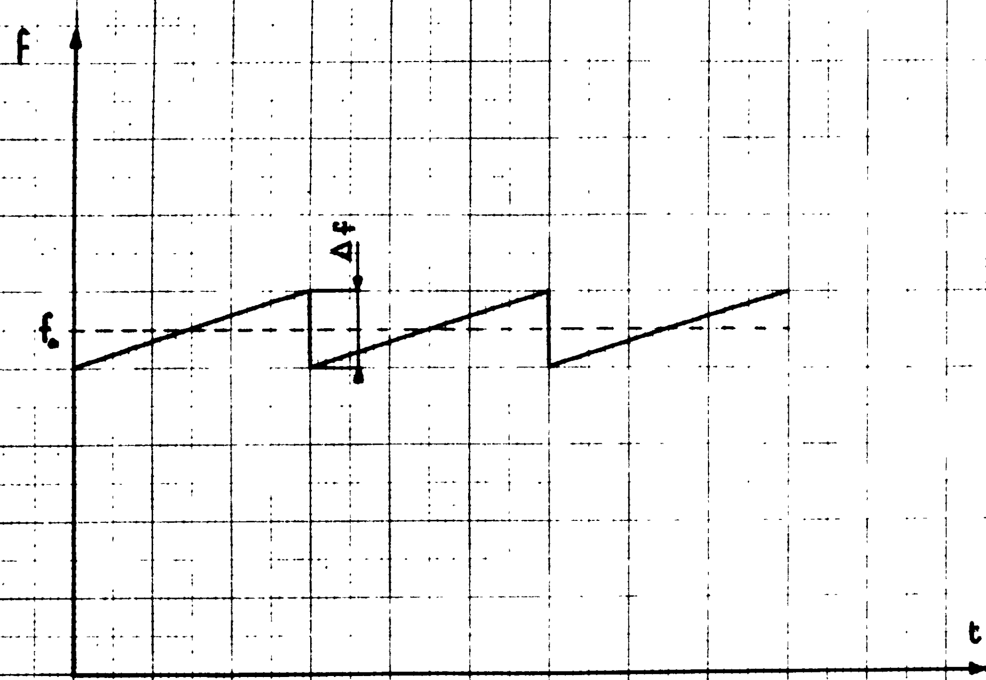
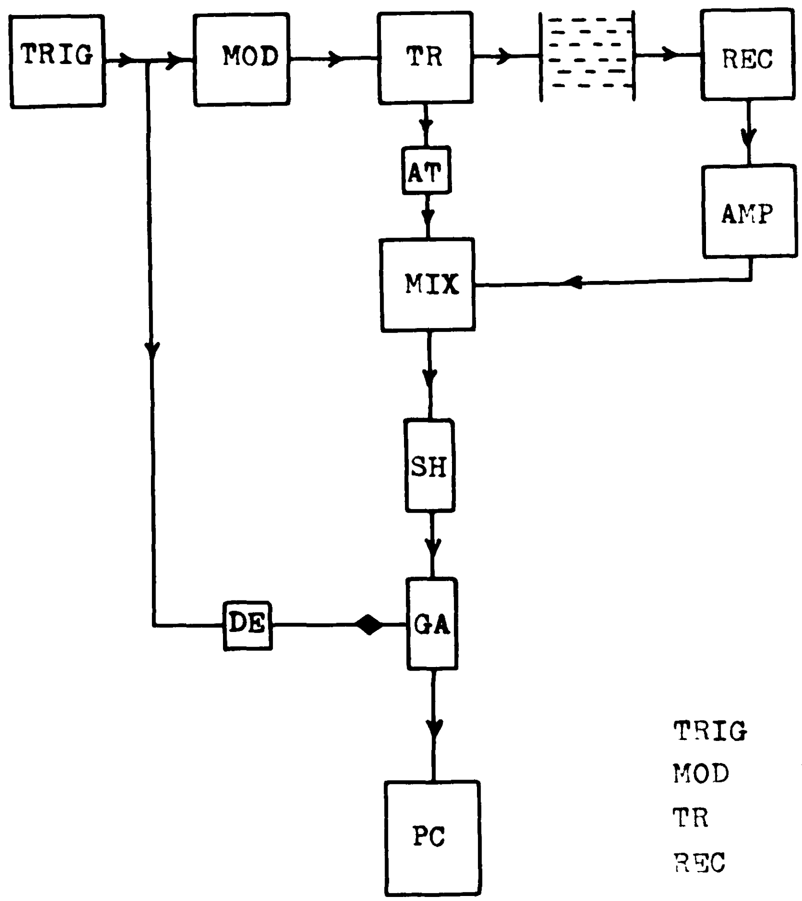


FIG. 10

Princiole of F M Apparatus



TRIG	Trigger
MOD	Modulator
TR	Transmitter
REC	Receiver
AMP	Amplifier
AT	Attenuator
MIX	Mixer
SH	Shaper
GA	Gate Tube
PC	Pulse Counter

F I G. 11

The operation is best explained by concrete example. Trigger TR1G is operated by the crankshaft braker and fires the modulator delivering voltage of say 350 μsec fundamental period. (One modulation cycle is sufficient.)

The transmitter can be pulsed in synchronism with the trigger or may operate in CW ; its waveform will follow in frequency the modulation voltage ; a frequency deviation of $\pm$ 25 per cent ($\pm$ .5 MCps for mean frequency of 2 MCps) can be transmitted over transducers coupled to metal rods ; the beat frequency for 18 μ sec.acoustical delay will be 18/350 $\cdot$ 1.0 MCps = 51.5 KCps (this is the beat frequency for the maximum dound velocity associated with the gage lenght of L = .5" ; the maximum beat frequency is obtained for the minimum sound velocity, will be approximatively 40/350 . 1 MCps = 114 KCps.

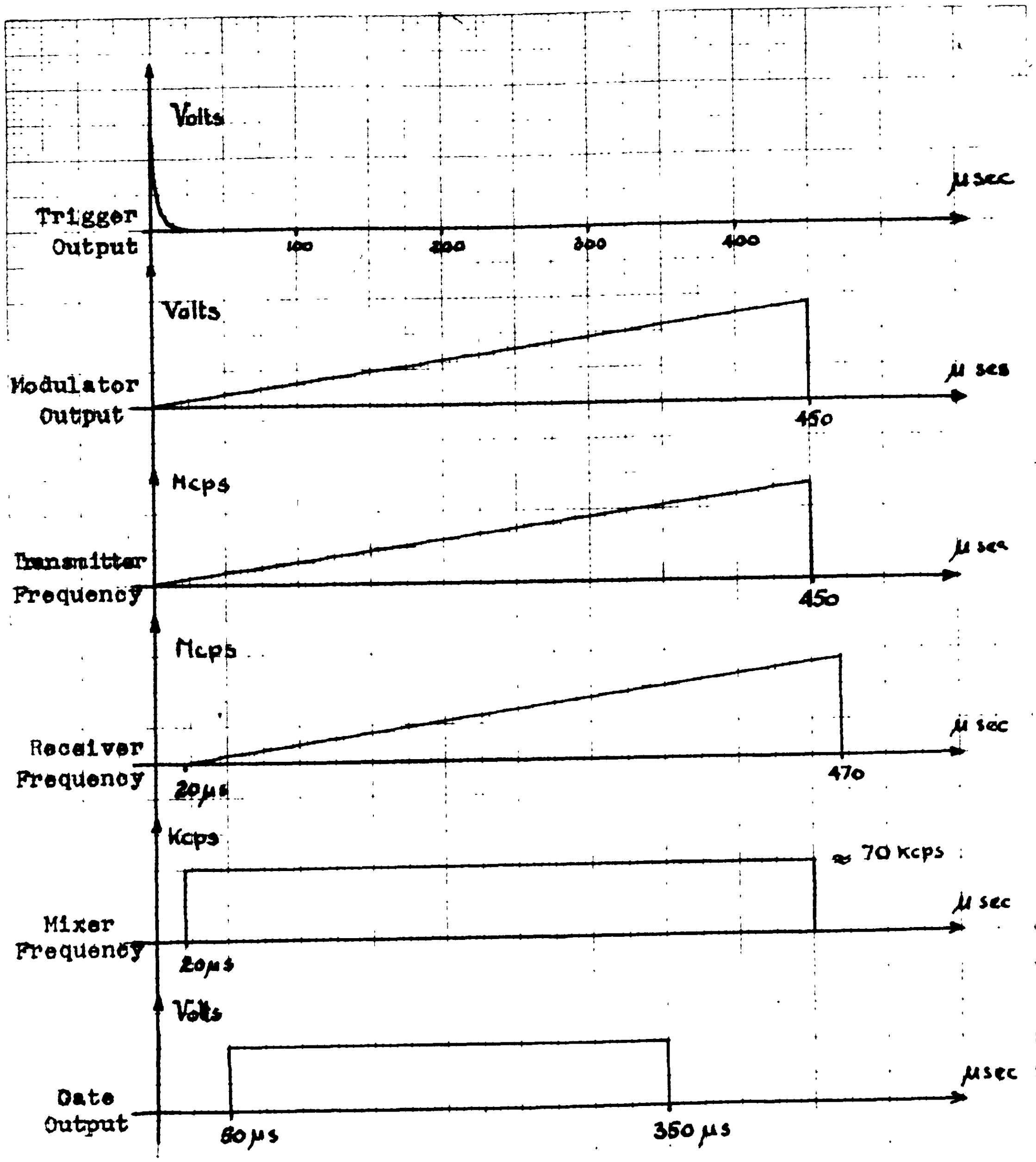
The gating tube, delayed by delay DE of about 50 μsec. opens the counter for accurately 300 μ sec. The number of pulses received by the counter will be at least

$$300 \cdot 60.10^3 \cdot 10^{-6} = 18 \text{ pulses} \pm .5,$$ the error in counting may be up to 2.5 per cent.

This error can be reduced to 1 per cent with

a) higher mean frequency

b) increase of the modulation cycle duration (say 500 μ sec.)



F I G. 12 - F M Apparatus Waveform

c) By adjoining another step counter which would count the number of beat pulses for 25 or 30 engine cycles, then the determination error due to the integer number of counts will become negligible and averaging effect is obtained by use of a great number of cycles.

The advantages of this method are considerable with respect to PW methods described in the next paragraph in as much that errors due to leading edge distorsion and difficulties due to the low signal to noise ratio are eliminated.

Amplitude variations due to the change in average specific mass of the gas from cycle to cycle are also suppressed, this is a very important factor to reduce the operators eye fatigue, if A-R scope display is used instead of counter.

In conclusion, this method is the best theoretically ; however PW method was adopted temporarly in the first experiment uniquely for sake of simplicity as the immediate purpose was to explore the domains of the various previsible parameters.

4.33 EXTERNAL COMPARISON METHODS

4.331 Definition and characteristics

External comparison is used for time delay measurement if determination is made by comparing the quantity to be measured to independently produced and calibrated timing devices.

A good and readily graspable example is the ordinary watch used to "measure time" or better to compare time delays from arbitrarly fixed "origins" such as timing radio signals emitted periodically by primary standard-controlled government agencies.

Two operations are usually preformed (usually (without conscient reflex on behalf of the operator)

a) Sensorial determination of the "event", (to be more specific, let us assume visual perception of the winner's arrival in a car race,)

b) Assuming the spectator is informed of the time of departure as expressed by his own time measuring device ; simultaneous observation of the indication given by this instrument at the time of (a).

This example is somewhat naive but it illuminates difficulties of this kind of time measurement ; errors are likely to be introduced for following reasons :

- Improper interpretation of the "event" (hereafter called perception error), (if the observer is not able for any reason to detect the accurate time of target-line crossing in the hypothetical race.)
- Lack of synchronism between observation of the event and of the timing indicator (clock) ; it is obviously impossible, with the primitive method of a stop watch, for example, to get indication of the arrival time with mathetical accuracy ; reasons for this are finite velocity of light, arbitrarly slow propagation of nervous impulsions in the observer's body, etc. This error may be called "coincidence error".
- Inadequate calibration or stability of the clock itself ; poor construction, magnetic fields, frictional forces are usually the main reasons. This may be called "calibration" error.

All three sources of error can be made extremely small ; when comparison with electrical variations in potential are to be compared the universally employed method is to display them simultaneously on the screen of cathode ray oscillograph where perception and coincidence errors are practically limited only by the focus of the electronic beam, whereas calibration errors can be reduced to arbitrary small value by adequate timing circuitry, and, as supplementary refinement standardisation by means of known time delays.*

Another possible way to "display" time intervals is to compare them with relaxation periods of free-running multistable circuits ; multiple descriptions and commercial designs are currently available; some of them claim accuracies of .4μ sec on time delays down to a few μ sec.

A few examples of external comparison methods are described in sections 4.332 through 4.335.

4.332 Oscillograph Sweep Circuits

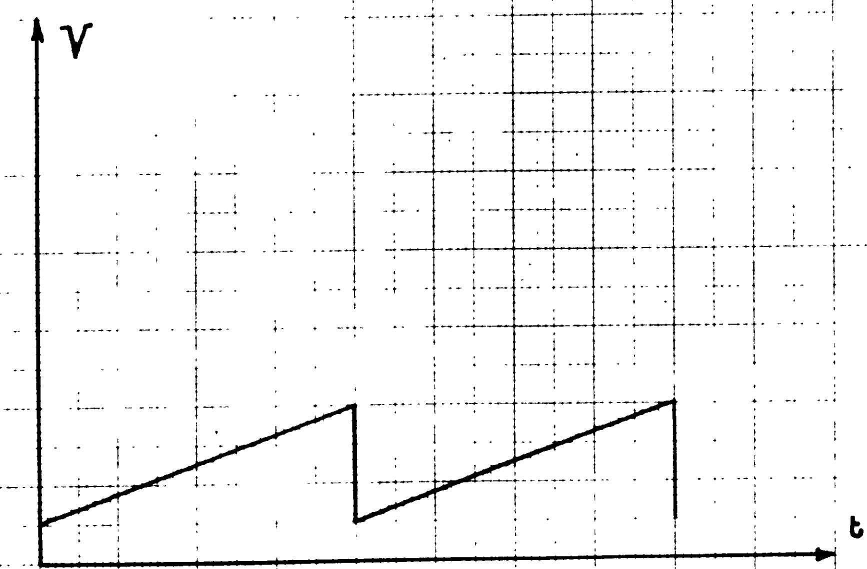
Sweep circuits are designed to supply the so-called time axis for oscillograph displays ; essentially their aim is to provide known and usually periodic voltage functions of time which may be fed in one of the deflection plate pairs of a CR tube.

Linear or "sawtooth" sweep circuits (Fig.13) are preferred for the realistic and readily interpretable display they give of time function voltages connected to

---

(*) The question of "known" time delays has been intensively discussed in this last decade. Although the matter is of no consequence in our problem, it may be stated that crystal-controlled "markers", provided crystals are adequately cut and temperature regulated, can be and actually are used as primary standard of time intervals ; with far better reason than such unknown and inaccurate notions as the earths rotation, astronomical observations,etc.

Sawtooth Voltage



F I G. 13

the signal plates (Usually referred to as "vertical plates" although their position in space is more often horizontal)

A considerable number of practical circuits are available and standard in literature*; they range from the simple constant-current loading and gas-tube dishharge of condensers to the more elaborate Integrator-Comparator, "phantastron" and "sanitron" circuits ; linearities of ± .05 per cent and even ± .02% are reported although accuracies of this order of magnitude presuppose serious constructional stability such as voltage regulation, absence of microphonic effects etc.

Usually the "sweep" is initiated by the start of the time interval to be measured ; then the end of the time interval is "displayed" on the signal plates. The oscillographic pattern is visually observed and time delay is read by superimposed carthesian coordinate systems.

For special displays sweep voltages can be generated to have almost any periodic shape ; sinusoidal sweeps are used sometimes for their simplicity ; "function sweeps" are in use in a great number of fields, in particular in ferromagnetic permeability studies where it is convenient to display the magnetic induction vs. magnetizing current, in reciprocating engine investigation where graphical representation of cylinder pressure vs.

(*) Britton Chance andOthers, Electronic Timing Circuits pp. 115 and Seq.

piston position is basic for performance studies.

Non-periodic phenomena can be photographically or visually observed (and sometimes serious advantage can be taken of the CR scope brightness persistence Characteristics) with triggered sweeps which are initiated and thus automatically synchronized with the "signal" although the leading edge of the signal in this case is not perfectly reproduced unless special precautions are taken.

4.333 Use of Markers or Timers.

Since World War II, considerable effort was spent to increase accuracies of time interval measurement by oscilloscopic means ; a good way to reduce the influence of calibration errors is to superimpose calibrated signals such as crystal-controlled sinusoidal voltages ; crystal-controlled sharp pulses imposed to the modulation grid (markers).

These convenient time calibrators can participate in synchronisation and time scale expansion applied to the sweep voltage ; their phase with respect to the sweep initiation can be controlled and measured.

Calibration errors in scopes equipped with adjustable precision markers are almost negligible in all applications.

4.334 A-R Test Scopes

A-R scopes were designed for testing and servicing radar transmitters and receivers.

Essentially, they feature following characteristics :

- Broad-band receiver, (300 cps to 10 MCps within 6 dB)
- Calibrated linear sweep ranges, representative values are 150, 1500, and 4500 μsec sweep durations, (A-sweeps)
- Expanded sweeps (called R sweeps) representing a fraction (1/5, 1/10, 1/25) of the total sweep on full screen width ; R-sweeps can be delayed by manual adjustment to display "signal" with increased precision and measurable phase ;
- Provision for internal and external triggering ;
- Various observation aids, such as markers, strobes,etc to decrease calibration errors.

It is accepted that A-R scopes can yield a current accuracy on time delay measurements of ± .1 per cent or .1 μsec, whichever is greater.

As an A-R scope is available for the investigation to be performed, this method of time interval measurement was automatically accepted in this paper ; operation thereof is simple and dependable and provided the signal itself is well defined the instrument is ideal for our purposes.

5. SELECTED METHOD AND APPARATUS

5.1 METHOD DESCRIPTION

5.11 PRINCIPLE

From the discussion in Sect.2.2, Sect 2.332 and 4.334 the following principles of operation are deduced :

a) The gas temperature can be determined by sound velocity measurement ;

b) Sound-propagation velocity can be measured by the direct method ; i.e. lenght and transit time determination;

c) Pulse-wave, external comparison method is adequate to transit time determination ;

d) Two-transducer-transmission sound travel has to be arranged.

e) A - R test scope will be used as time-calibrated comparator.

5.12 OUTLINE OF OPERATION

The proposed experimental setup is represented on Fig. 14. The engine distribution shaft S is connected with the braker BR which short-circuits its terminals once per engine cycle ; the contact phase is manually adjustable at $\pm 180^{o}$ distribution shaft angles ; i.e. 720^{o} crank angle.

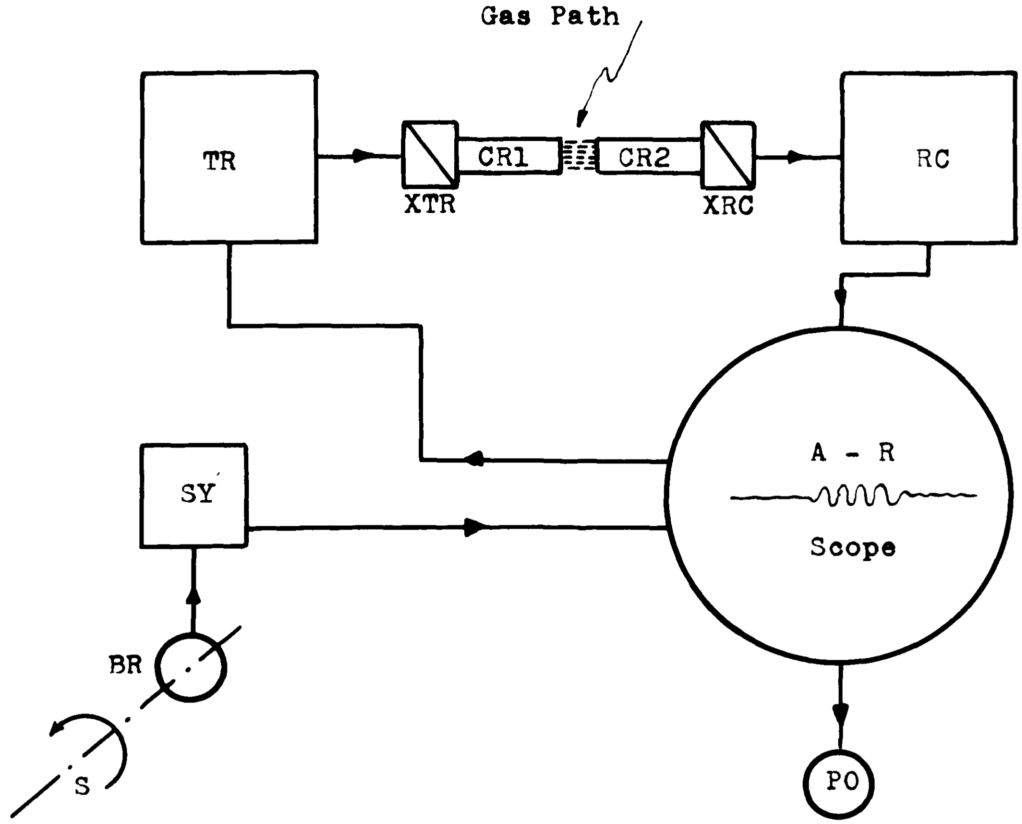
The braker is generating in the synchronizer SY a square pulse ; the positive going rise is used to trigger the A-R scope which is provided with sharp-rising positive or negative trigger voltages available at special output terminals.

This trigger is fed into the transmitter at the time $t_o = 0$; the transmitter TR delivers the driving pulse to the crystal approximately .5 μsec after the trigger.

Transmitter crystal XTR receives the electric impulse and sends out the acoustical "signal" starting at .5 μ sec through the coupling rod CRI to the gas path.

After transmission in the gas and propagation in the coupling rod CR2 the acoustical pressure wave is impressed on the receiving crystal with the delay $.5+2t_{CR} + t_G$;(t_{CR} being the propagation delay in the coupling rods and t_G the gas path delay.)

F I G. 14 - Principle of Proposed Apparatus



S Engine Distribution Shaft
BR Breaker
SY Synchronizer
TR Transmitter
XTR Transmitter Crystal

XRC Receiver Crystal
RC Receiver

CR1
CR2 Coupling Rods

PO Calibrated delay

Receiver RC amplifies (and possibly shapes) the output of XRC and is directly connected to the A-R scope signal input, delayed however by the receiver's own delay t_{RC}.

The A-R scope is equipped with high-precision delay circuits where time interval can be measured by adjustment of a multiturn calibrated potentiometer PO.

The procedure is then straightforward ; first, a known atmosphere is introduced in the gas gap, pressure, temperature and chemical nature of the gas have to be known quite accurately.

The A-R scope is then operated with internal trigger to get a "static" reading(*) ; by means of the time calibrated delay adjustment PO(**) the signal start is placed on an arbitrary fixed screen ordinate.

Reading of PO yields the time interval

$$t_1 = .5\,\mu s + 2t_{CR} + t_{G1} + t_{RC}$$

where $.5\,\mu$ sec is the time interval between trigger and start of transmitted pulse

t_{CR} " " " delay introduced by the coupling rods

t_{G1} " " " delay introduced by the known gas path

t_{RC} " " " " " by the receiver

(*) The PRF in this operation can be of the order of 150 cps in order to get steady image on the screen.

(**) Actually, A-R scopes are calibrated in "yards" i.e. time required by radio waves to perform two-way trip for a given distance. Easy conversion can be made by remembering

164 yds → 1 μs

t_{G1} has to be calculated (from the gap lenght and gas temperature) then the overall time delay due to the electro-acoustic circuit (gas-path excepted) is

$$t_1 - t_{G1} = t_{ea} \quad \text{(Electro-acoustic delay)}$$

The engine is then started ; the breaker is triggering now the A-R scope set on "external trigger" operation, once per cycle with phase adjustable with respect to the top-center of the engine.

If t_2 is the new reading of PO obtained by placing the transmitted signal on the same fixed screen ordinate that was used in calibration, the difference

$t_2 - t_{ea} = t_{G2}$ is the time delay introduced by the gas gap <u>at the time of the sound pulse</u>.

Velocity will be then determined by

$$C = \frac{L}{t_{g2}}$$ and

gas temperature according to considerations in Part.(1).

Repeated determinations of t_{g2} for various settings of the breaker BR yield

t_{G2} gas path delay

C sound velocity

Θ abs. temperature

as function of the crankshaft angle Ψ .

Usual parameters, such as engine speed, inlet and exhaust pressure ; inlet and exhaust temperatures, jacket temperature, fuel-air ratio and speed, have to be recorded in order to complete one "run".

5.2 APPARATUS DESCRIPTION

5.21 ENGINE AND SPECIAL COMBUSTION CHAMBER

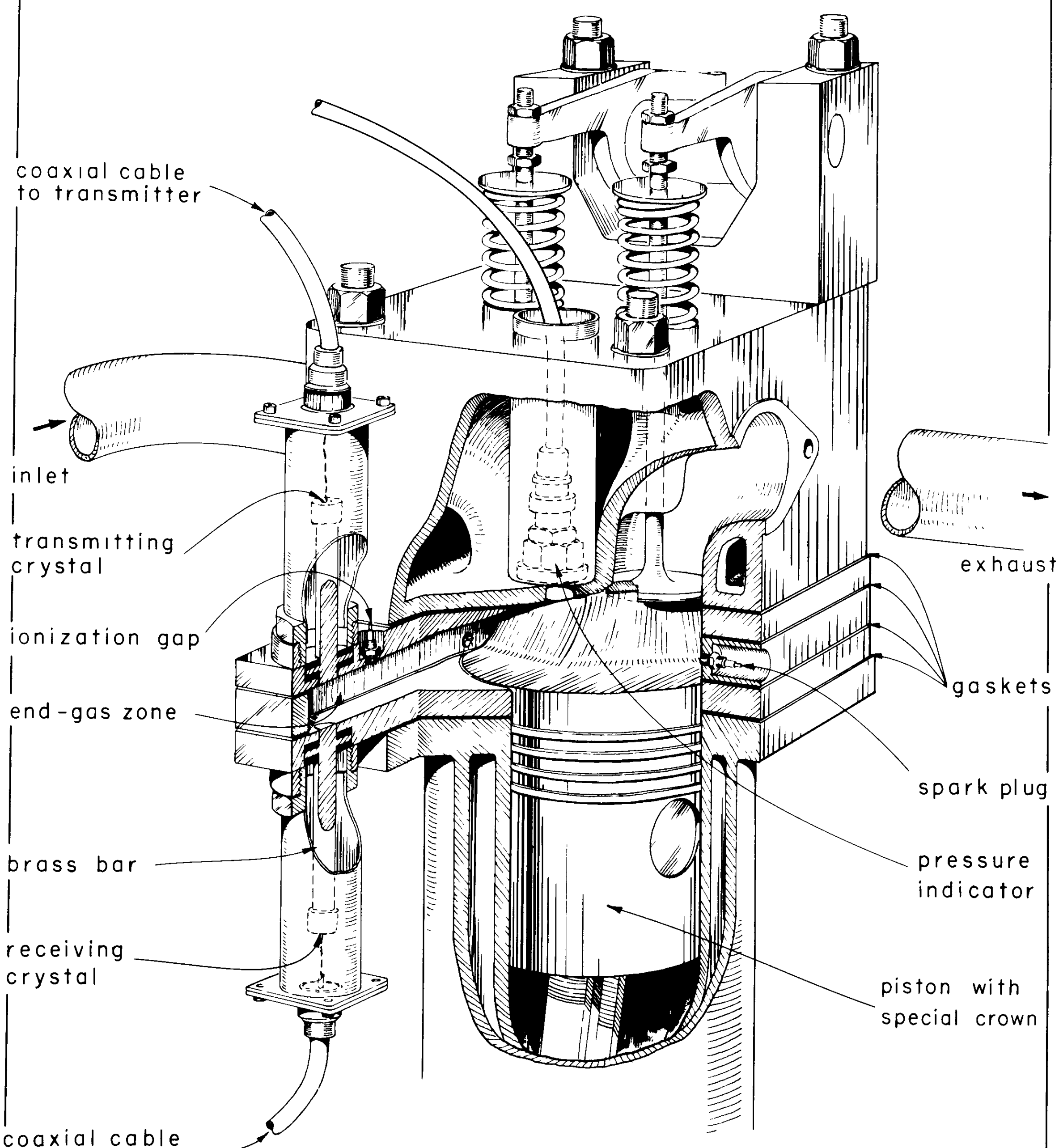
Actualoperation of the proposed apparatus has been performed on a CFR engine* which is standard in the fuel research field.

Speeds of 600 to 1500 rpm are used and variable compression ratio in the range of 4.0 to 9 is available. Best power fuel-air ratio and spark advance can be determined by electric generator driven by the crankshaft. Phase-adjustable breaker is accessible on the distribution shaft.

A special combustion chamber has been constructed (Fig.15) with provision to insert transmitting and receiving transducers**, designed to fit a special CRC 3¼" Bore Cylinder and the standard CRC crankcase.

(*) Described by H.L. Horning, The Co-operative Fuel Research Committee Engine, Trans. S.A.E. 26-1931.

(**) J.C. Livengood, Report Submitted to Coordinating Research Council Inc.,Sloan Laboratories, M.I.T. Feb.1952.



SPECIAL COMBUSTION CHAMBER AND CYLINDER HEAD

FIG. 15

The clearance volume is reduced by special construction of the piston (Fig.15)

The combustion chamber is made of three "sandwich" plates, separated by Teflon gaskets, and cooled by the engine water circulating system.

The bottom and top plate are designed to receive respectively the receiving and transmitting crystal cartridges ; the central plate provides the end-gas region at about 3¼ inches from the ignition points,

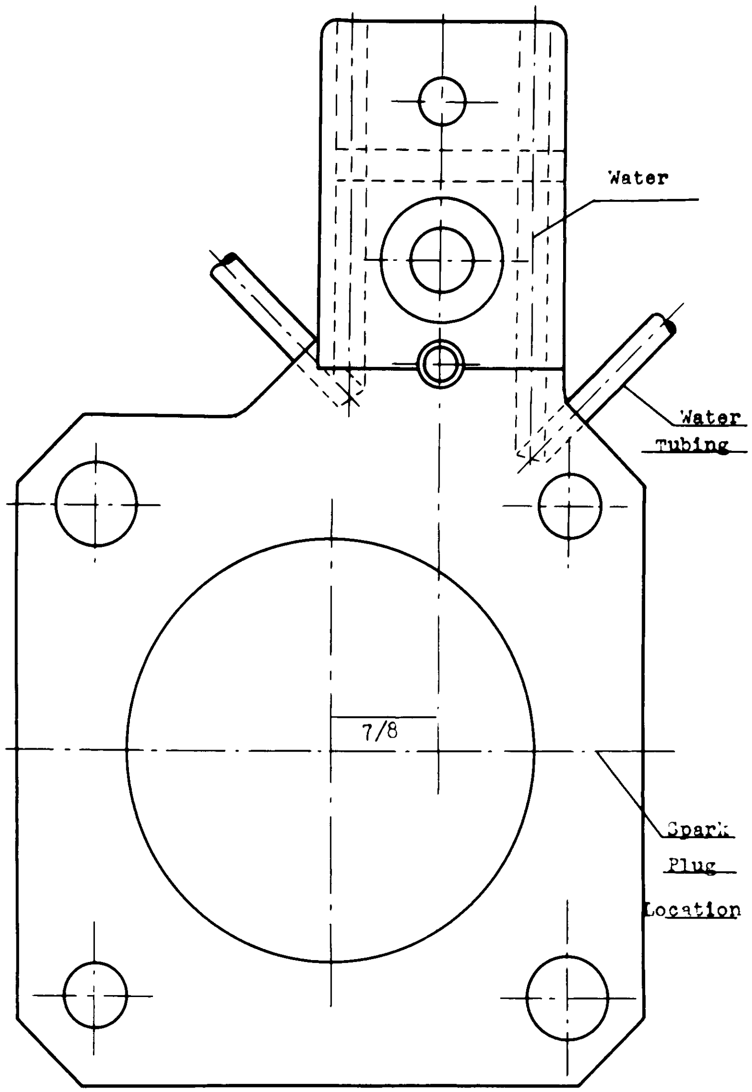
Orifices for the spark plugs are in the center plate.

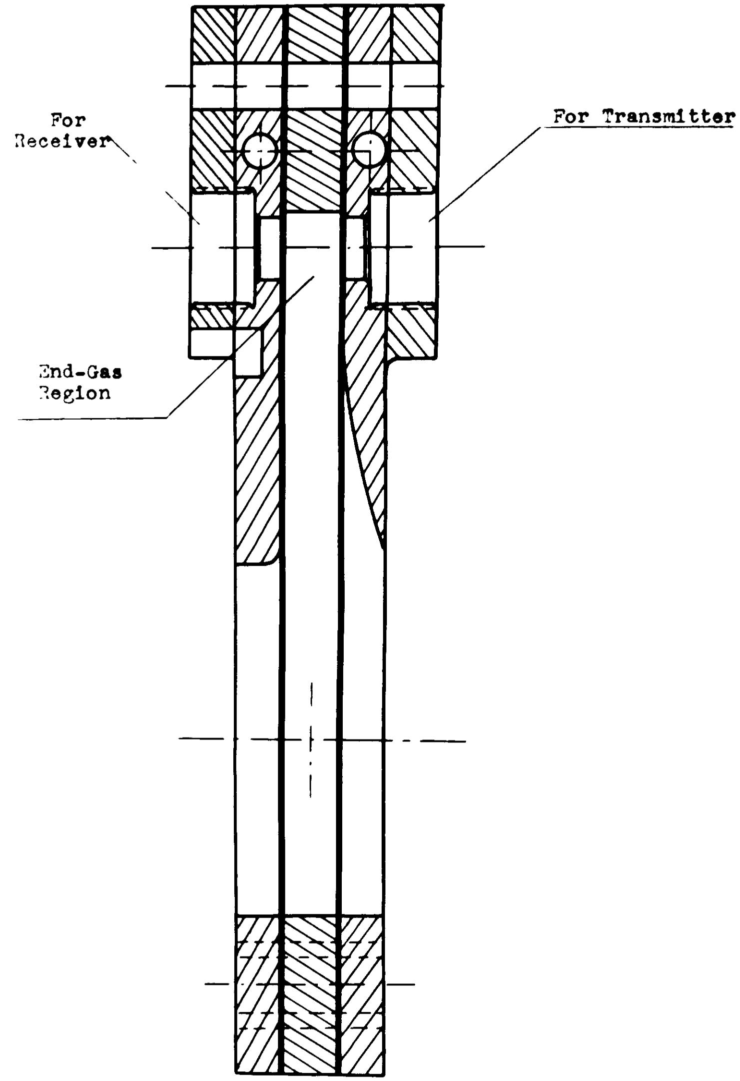
Electrically all three "sandwich" plates are connected together and provide a ground as good as these inhomogeneous ferro-magnetic masses can be at radio-frequencies.

The insulation between sandwich plates is a quite effective sound barrier, no parasitic "acoustical" signal through the metal structure was detected at <u>any</u> stage of the experimental work.

5.22 TRANSMITTER

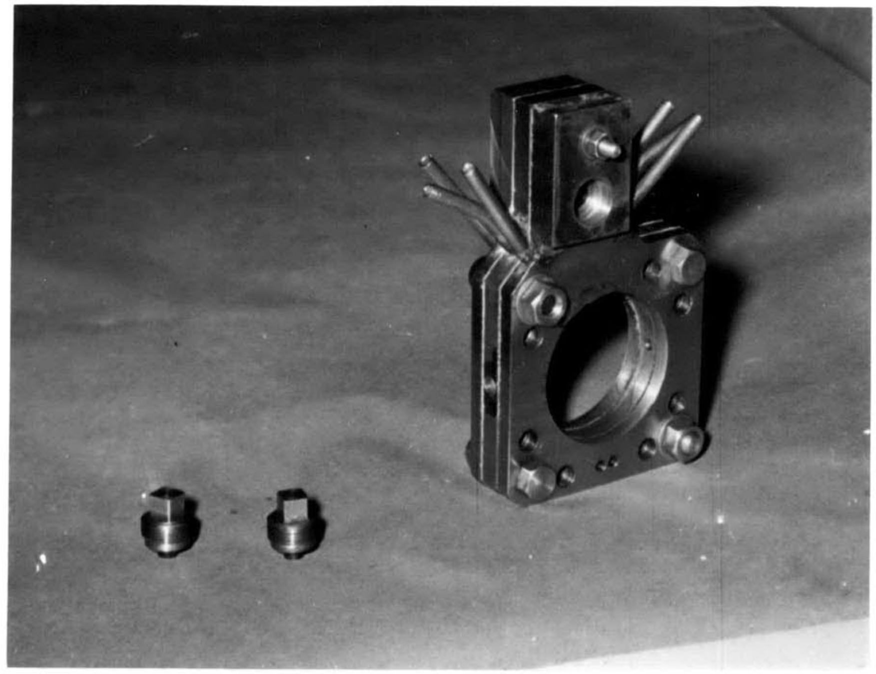
The transmitter is "pulsed" by the input trigger, but has provision to operate directly on the engine braker. The first stage is a 2D21 Thyratron, with





F I G. 15bis

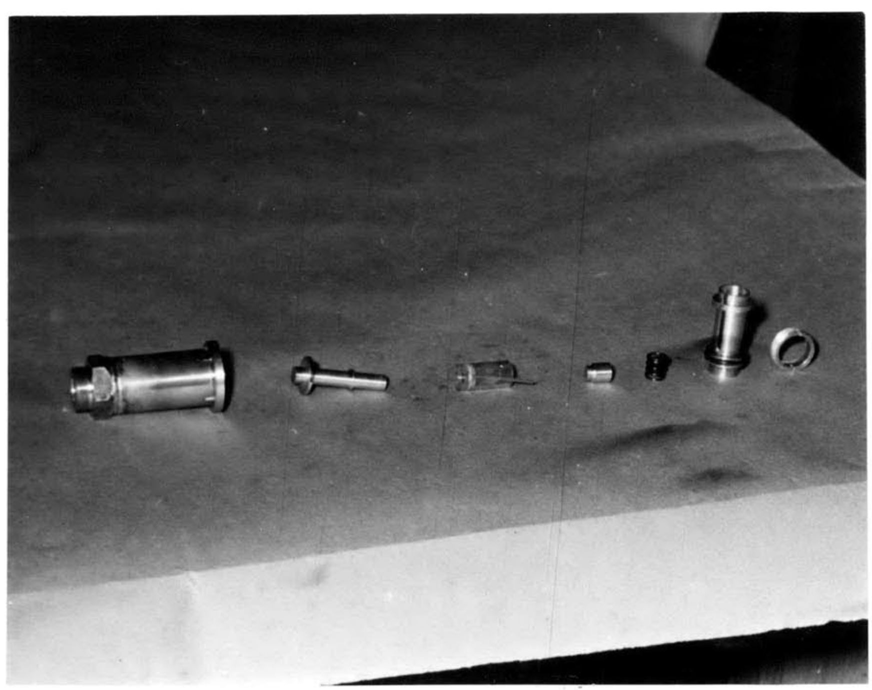
Detail of Combustion Chamber



F I G. 15ter

Sandwich Plate Combustion Chamber

Steel Plugs are in place when engine is used without transducer.



F I G. 17bis

Details of Transducer Construction

Left to right : Cartridge, coupling rod, lucite tube with crystal and electrodes, damping bar, spring, brass case and o-ring, screw.

3.0 msec time constant on the plate and cathode output on 480 Ω. The output pulse is fed in the grid of the final 3D22 Thyratron operating with 650v plate voltage. The transducer is connected to the cathode of this tube and across a tank circuit tuned on the transducer damped natural frequency.

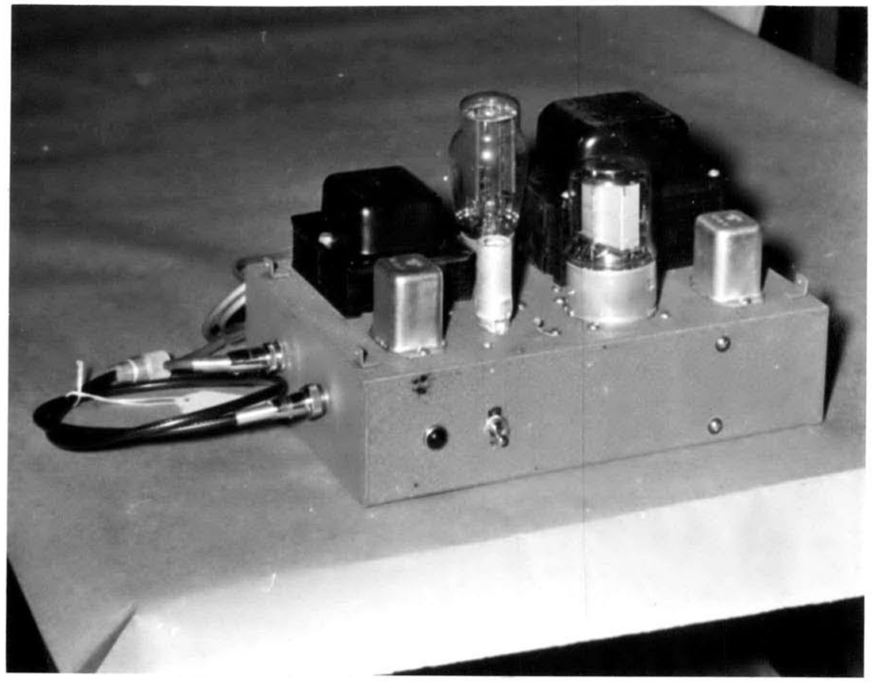
Suitable power supply is contained in the same chassis as the two thyratron tubes. The circuit of the transmitter is given Fig. 16Bis and a photographic view of the transmitter construction in Fig.16

5.23 TRANSDUCERS AND ACOUSTICAL LINKS (Fig.17)

The transmitting and receiving transducers are of identical construction.

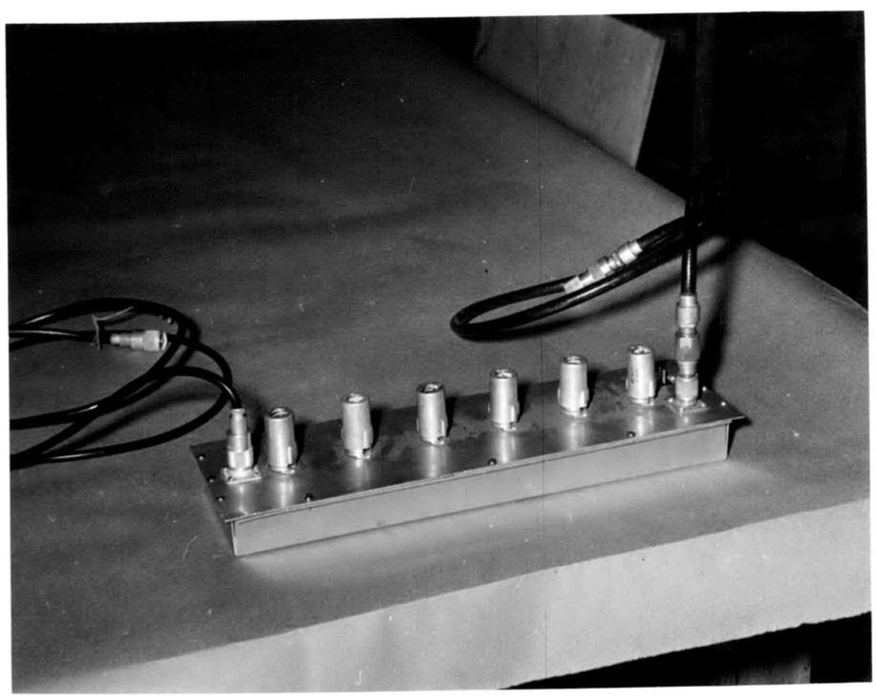
The brass coupling rod 1 is clamped between the combustion chamber plate 2 and the cartridge body 3, gaskets 4 and 5 prevent gas leakage and sound propagation inside the combustion chamber structure. Cold water cooling (independent from the engine jacket cooling) is provided through orifices 6 and 6' and keeps the coupling rod (1) at room temperature.

The Barium Titanate Crystal 7 is hold between two silver-foil electrodes 8 and 8' and bakelite insulators bar 10 and the compression spring 11 which is



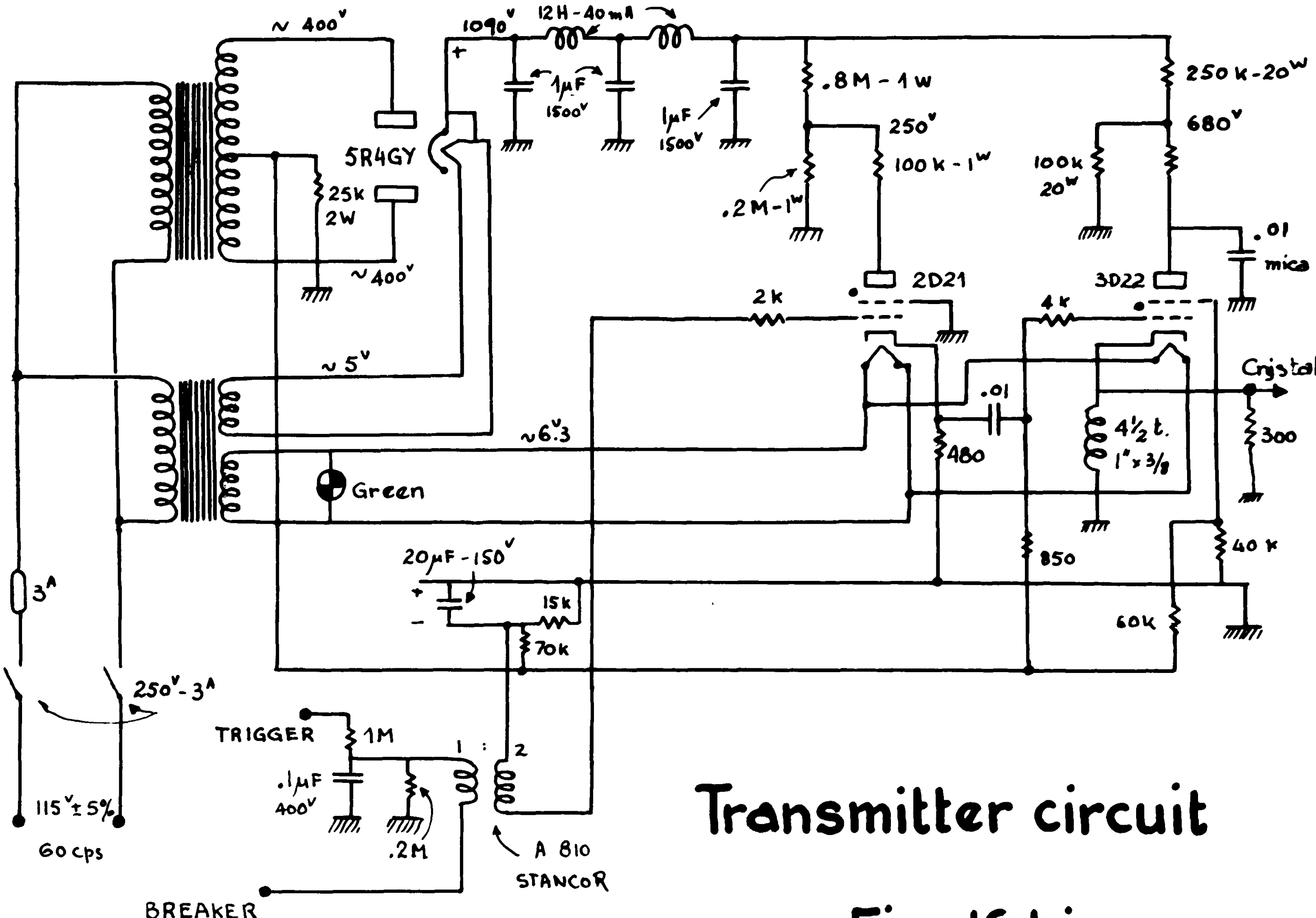
F I G. 16

View of the Transmitter
(Cover removed)



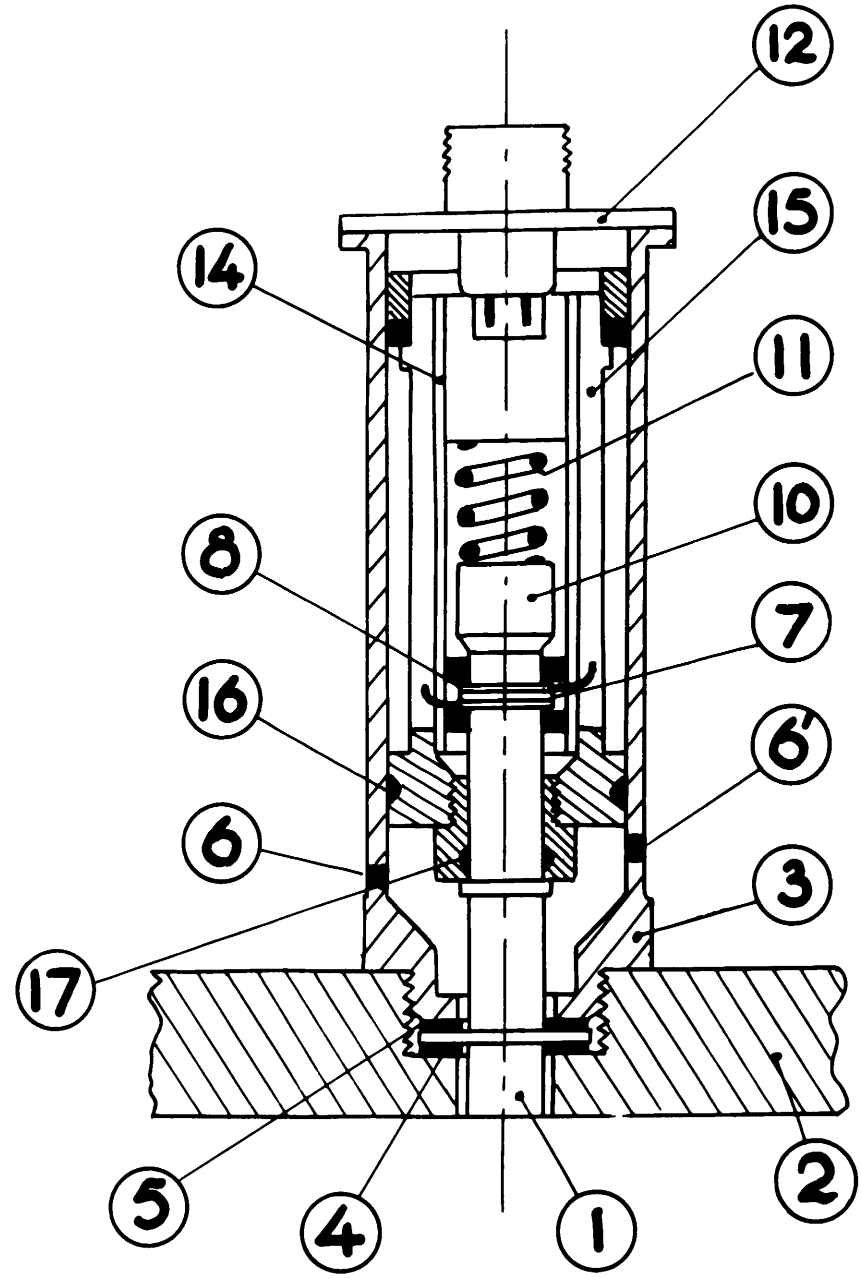
F I G. 18

View of the Receiver



Transmitter circuit

Fig. 16 bis



F I G. 17 - Transducer Construction

pushed towards the crystal by cover-terminal 12. Connexions 13 - 13' are insulated by plastic tubing and hold on the outer surface of the lucite insulator 14.

The whole subassembly is mounted in the brass tube 15 and the O-Rings 16 and 17 prevent water to be in contact with the electric circuit.

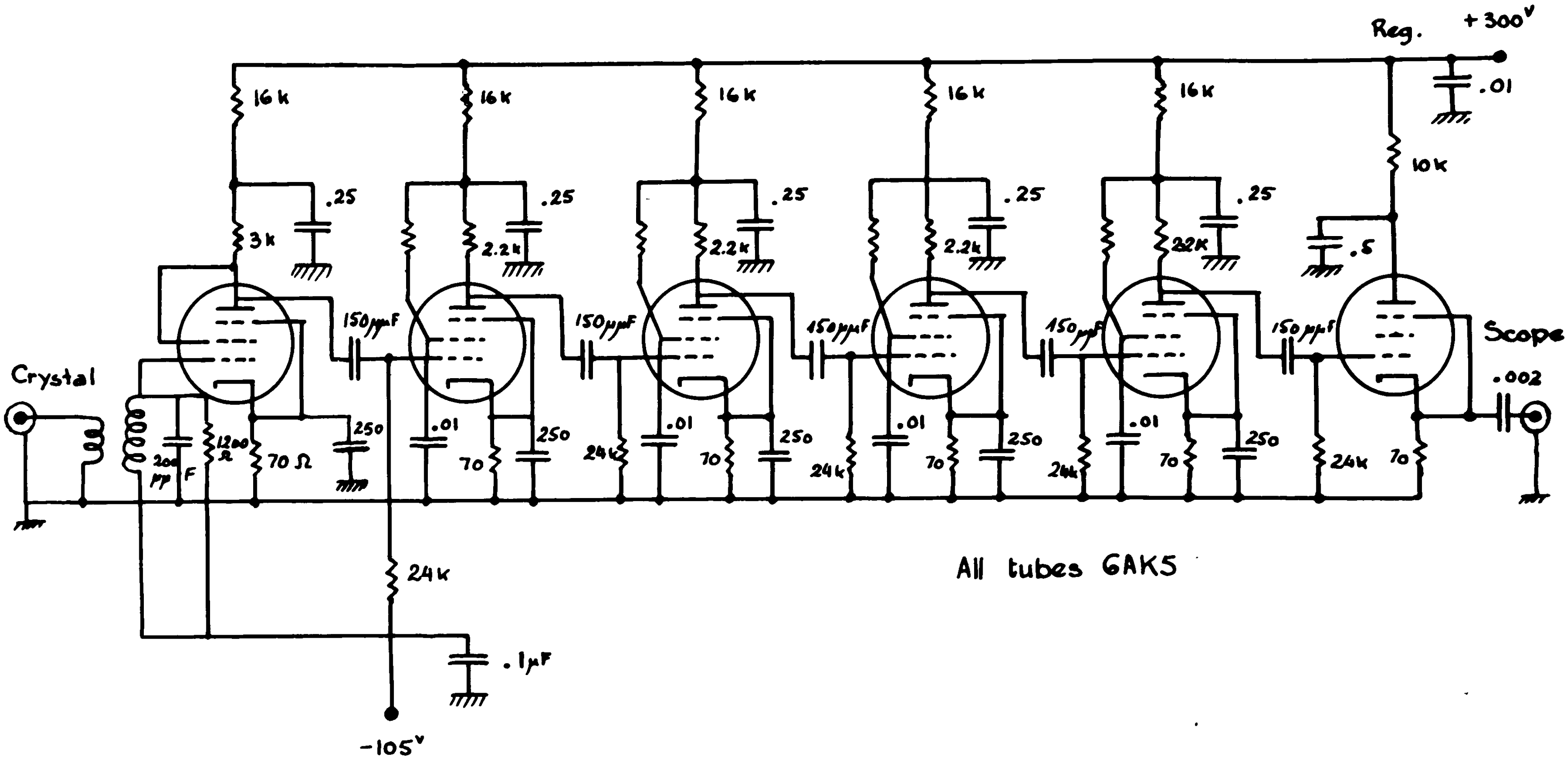
The outer cartridge body provides both mechanical protection and electrical shielding.

It should be pointed out that the described transducer assembly is essentially an experimental modification of the design prepared by Mr. J.C. Livengood (Ref.15) arranged for further investigation of crystal-to-rod transmission and influence of solid damping.

5.24 AUXILIARY EQUIPMENT

5.241 Receiver

The receiver used is a somewhat modified version of a conventional 5 stage video-amplifier, primarly designed for the .5 - 3.5 MCps Band. It uses a transformer input and 5 6AK5 identical stages, with a total maximum gain of the order of 80 dB. A sixth 6AK5 is used as cathode follower to connect the receiver to the AR-scope input (30 $\mu\mu$f - 1 MΩ) through a 72 Ω



Receiver circuit

Fig. 19

coaxial line of about 4 ft in lenght.

Separate power supply is used with adjustable grid bias, to control the amplifier gain from 50 to 80 dB. Receiver circuitry and external view is given in Fig. 18 and Fig.19.

5.242 Synchronizer (Fig. 20)

The engine distribution shaft actuates the braker BR connected to a 67v battery and 30 K resistor. The contact closing produces a sharp voltage rise at the AR-scope trigger input and starts the scope sweep and the scope output trigger. The braker phase is continuously variable and the phase angle (expressed in crank degrees) can be read on a superposed scale.

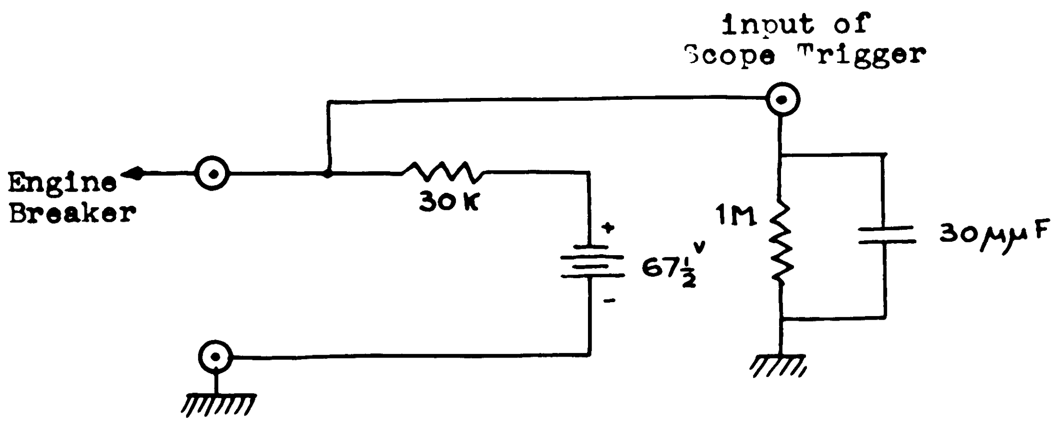
5.243 A - R scope

Type 256-B A-R scope is used in all experiments. Essential features for operation in this experiment are :

(a) Output trigger 100v, rise time less than .05 μsec, used to initiate the transmitter.

(b) Video-amplifier, 300 cps to 10 Mcps to 6 dB with 1, 3, 10, 30 and 100 attenuation ratios at the input and approx. 35 dB gain.

Synchronizer



F I G. 20

Direct sensitivities are 79v/inch on the vertical plates.

(c) Calibrated sweeps (A - sweeps)

<u>20.000 yds Range</u> :

122 μsec or, approximately 30 μsec per inch, for 4" sweep.

<u>200.000 yds Range</u> :

1,220 μsec or approximately 300 μsec per inch, for 4" sweep

<u>Test range</u>

4500 μsec sweep or 1125 μsec per inch.

(d) Expander and calibrated delays. (R - sweep)

The 20.000 yds and 200.000 yds sweeps can be expanded, i.e. the sweep speed increased by factors of 5, 10 and 25 to yield following "writing speeds" :

	20.000 yds		200.000 yds	
X 5	4000 yds -	6 μsec/inch.	40.000 yds -	60 μs/inch
X 10	2000 "	3 "	20.000 "	30 "
X 25	800 "	1.2 "	8.000 "	12 "

Calibrated delays are available on all R sweeps, a 10 turn helicoidal potentiometer can be read from 0 to 20000 with reading accuracies down to ± 5 ; 100 divisions are equivalent to .615 μsec on the 20000 yd range and 6.15 μsec on the 200000 yd range.

The correspondance between yards and sweep time is given by the light velocity, $t = \frac{2L}{C}$ since the scope has been designed for radar operation. For 100 yds

$$t_{100} = \frac{2 \times 100 \times 91.5}{3.10^{10}} = 61.5 \cdot 10^{-8} \text{ sec} = .615 \mu\text{sec}$$

5.3 NUMERICAL JUSTIFICATION OF THE MAIN CHARACTERISTICS

5.31 TRANSMITTER AND TRANSMITTED SIGNAL

For calculation purposes the thyratron 3D22 will be considered as a switch, having a linear voltage variation across its terminals, the rate of variation being defined by the ionisation time.

C_1 Discharge condenser, .ol μf

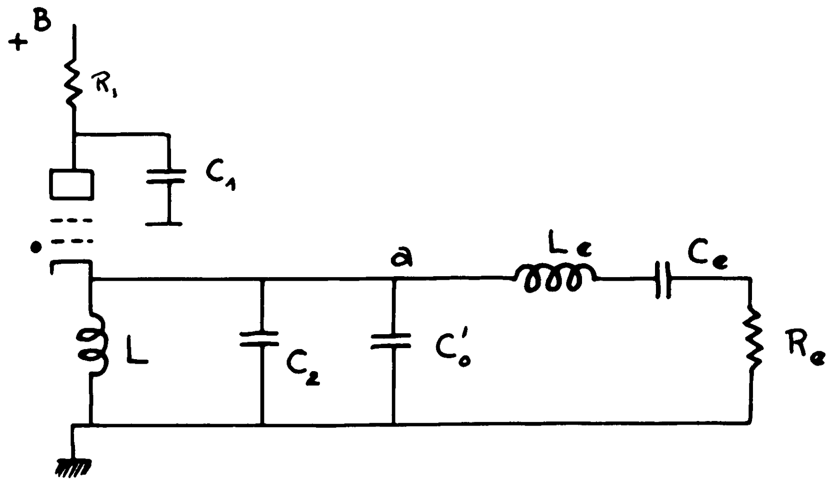
C_2 Wiring and cable capacitance

C_δ Crystal clamped capacitance

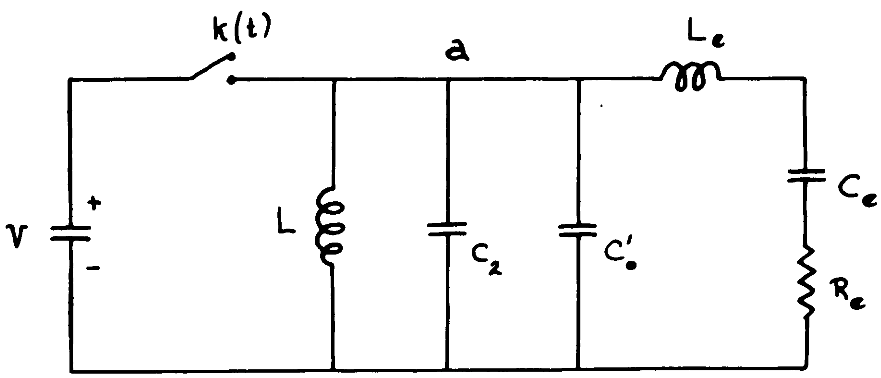
L_e, C_e, R_e : Equivalent electrical parameters of the crystal.

The equivalent circuit will be (Fig.21b), where K(t) represents the switch ; it is known* that if the circuit is in quiescent state before the switching operation, the action of the switch can be replaced by

(*) M.F.Gardner & J.L.Barnes, Transients in linear Systems, II. 14 pp 43.



F I G.21 - Simplified Transmitter Circuit



F I G. 21bis - Modified Transmitter Circuit a passive network with all stored energies set to 0 plus a voltage source equal in magnitude and opposed to the potential difference across the switch and placed in lieu of the switch. Fig.22 represents this second equivalent network, when C_o' and C_2 are replaced by their sum

C_o' C_2 C_o.*

The voltage E is a linearly increasing function at t 0 conventionnally taken at the arrival of the trigger signal on the final grid, its amplitude is the initial voltage across the condenser C_1.

The analysis of the network can be helped by two assumptions :

(a) The branch voltage source E can be replaced by a node current source $C_1 \frac{dE}{dt}$

(b) The natural frequency of the resulting circuit is close to the crystal resonant frequency, then practically the crystal will appear as its radiation resistance and static capacitance in parallel.

Fig. 23 gives the final version of the circuit, where a supplementary damping resistance is introduced in parallel (R'). On node analysis, the voltage transform will be given by

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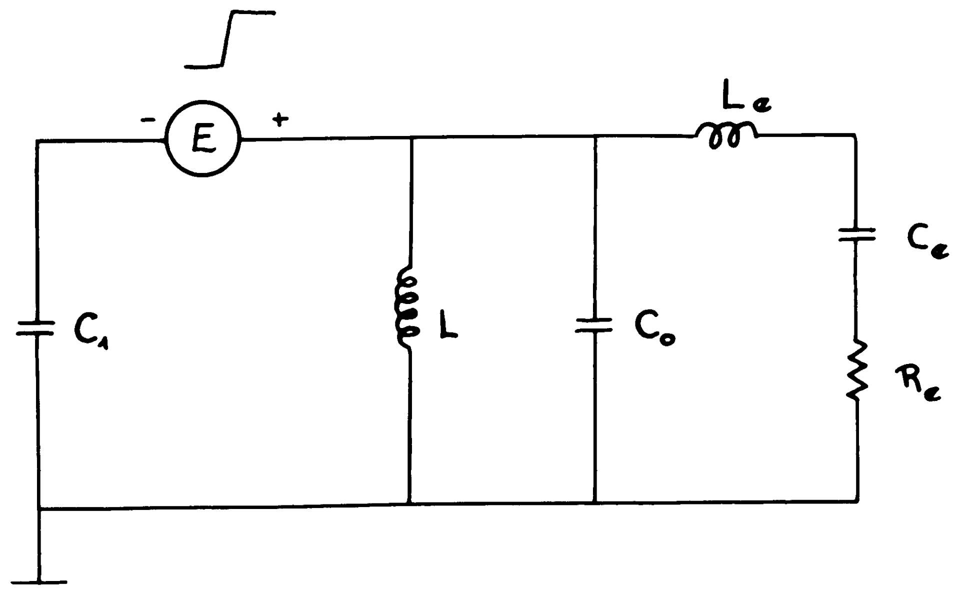
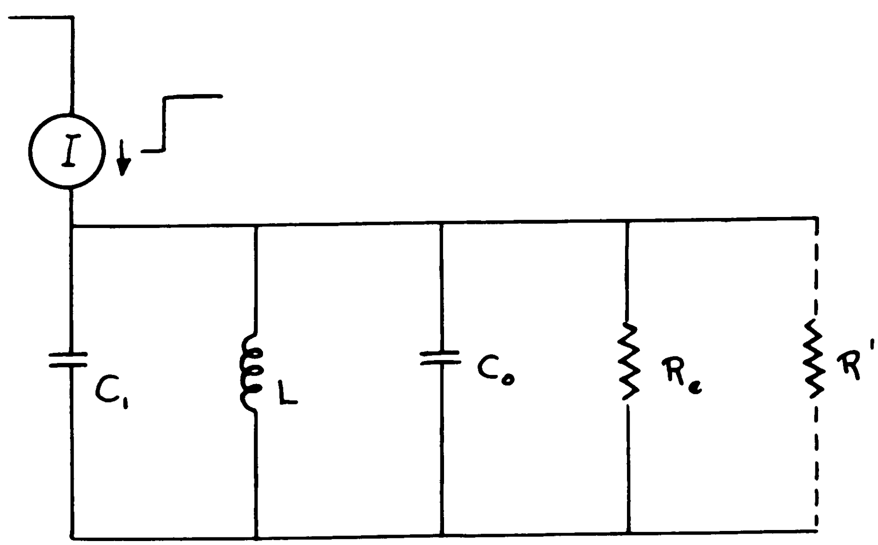
(*) The resistance R_1 has such value (Fig.21) that current from the voltage source B does not affect the circuit operation in a.



F I G. 22 - Equivalent Transmitter Circuit



F I G. 23 - Node Basis Transmitter Circuit

$$U(s) = I(s)\left[(C_1 + C_0)s + \frac{1}{Ls} + \frac{1}{R}\right]^{-1}$$

$$R = \frac{R_e R'}{R_e + R'} \quad , \quad C = C_1 + C_0$$

$$U(s) = I(s)\,\frac{\frac{1}{C}\,s}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

$$U(s) = \frac{I(s)}{C}\,\frac{s}{(s+\alpha)^2 + \beta^2} \qquad \begin{cases} \frac{1}{RC} = 2\alpha \\ \frac{1}{LC} = \beta_0^2 \end{cases} \qquad \beta_0^2 - \alpha^2 = \beta^2$$

$$I(s) = \frac{I_0}{s}$$

$$U(s) = \frac{I_0}{C}\,\frac{1}{(s+\alpha)^2 + \beta^2} \qquad (59)$$

(*) S Complex angular frequency used in Laplace transforms frequency domain. (Ref. Ft note p.132)

The time function transformed in eq. 59 is

$$v(t) = \frac{I_0}{C\beta} e^{-\alpha t} \sin\beta t \qquad (60)$$

The numerical values are : $C_1 = .01 \mu F$

$C_2 = 300 \ \mu\mu F$

$C_0' = 1900 \ \mu\mu F$

$C \cong C_0' + C_1 = .012 \ \mu F$

$L \cong .94 \ \mu H$

$R = 200 \ \Omega$

$f = \frac{\beta}{2\pi} = 1.6$ MCps

Ionisation time approx. $.1 \ \mu sec$

Battery voltage $E = 650v$

Then $I_0 = C_1 \frac{dE}{dt} = .01 \cdot 10^{-6} \frac{650}{10^{-7}} = 65^{A}$

$$\alpha = \frac{1}{2} \frac{1}{200 \times .012 \cdot 10^{-6}} \cong .2 \cdot 10^{6} \ sec^{-1}$$

$$\beta = 2\pi \cdot 1{,}6 \cdot 10^{6} = 10^{7} \ rad\text{-}sec^{-1}$$

$$\beta_0 = \beta^2 + \alpha^2 \neq \beta^2$$

The voltage amplitude will be

$$U_0 = \frac{I_0}{C\beta} = \frac{65}{.012 \cdot 10^{-6} \times 10^{7}} \cong 546^{v}$$

The shape of the transmitted voltage is recorded in Fig.24

This supposes a perfect tuning between the pseudo-frequencies of the electrical and mechanical circuit.

The decay of oscillations is important ; the ratio of two consecutive peaks of same sign is

$$\frac{V_1}{V_0} = e^{-\frac{2\pi\alpha}{\beta}} = e^{-2\pi \cdot \frac{.2 \cdot 10^6}{10^7}} = e^{-.126} \cong .85$$

The time to reach 1/100 of the first peak is given by

αt = Log 100 = 4.6 and

$$t \# \frac{4.6}{\alpha} \quad \frac{4.6}{.5 \cdot .41} 10^{-6} \quad \frac{11.2}{.5} \mu\text{sec} = 22.4 \mu\text{sec}$$

Fig. 24 confirms experimentally the above values.

5.32 TRANSDUCER CHARACTERISTICS*

Barium Titanate disk transducers are used in the experiments covered by this work. Reasons for this choice are :

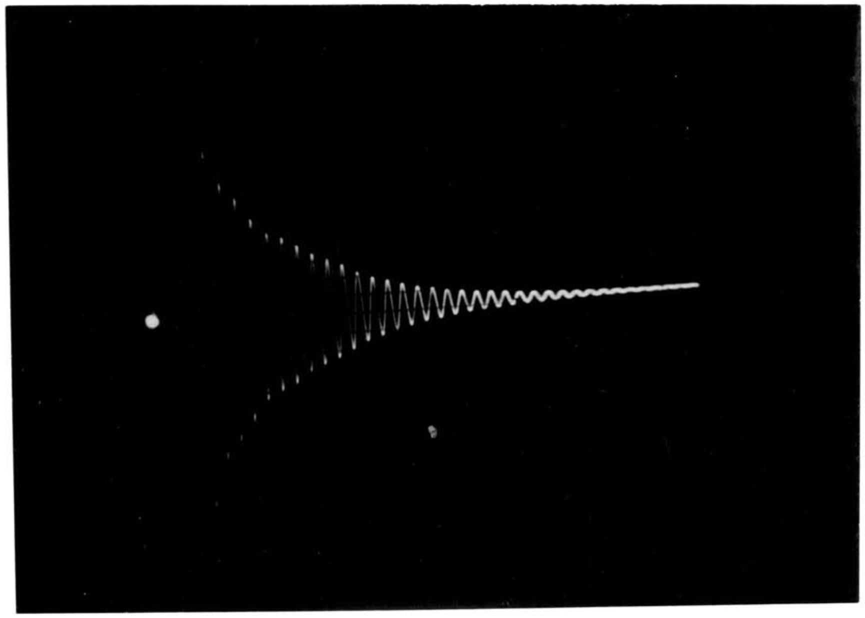
(a) Availability as stock items

(b) Comparatively high output

(c) Ruggedness

As the whole transducer setup is water cooled, it is important to have a crystal resisting to moisture. Temperature coefficient is no serious objection, since cold-water cooling (independent from jacket coolant) is provided.

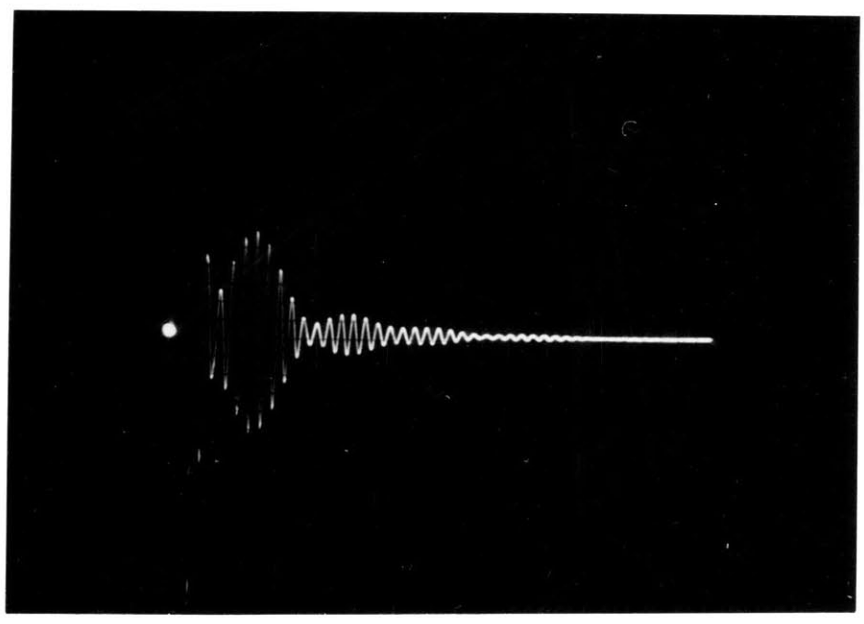
(*) This section is largely based on Dr. Th. F. Hueter's course, "Physical Ultrasonics" delivered at M.I.T. in Spring 1953.



F I G. 24

Transmitted Pulse in direct view on CRO Screen

Vertical axis : 1" = 300v approx.
Horizontal axis : 1" = 8 μ sec



F I G. 24bis

Transmitted Pulse with incorrect tuning

Vertical axis : 1" = 300v approx.
Horizontal axis : 1" = 8 μsec
Note presence of beat frequency of period approx. $7\frac{1}{2}$ μsec.

$$e_{ik} = \frac{K_o}{4\pi} K \, d_{ik} = \frac{\text{Polarisation}}{\text{Relative Strain}}$$

with $K_o = 1$ Statfarad/cm

$$e_{ik} = \frac{1}{4\pi} \cdot 1700 \cdot .46.10^5 = 6.24.10^6 \text{ Statcoulomb/cm2}$$

$$= 20.8 \text{ Coulomb/m2}$$

This value is somewhat higher than usual piezo-electric constants for this type of material ; it is safe to depend on approx. 16 Coulomb/m2, or

$$e_{ik} = 4.8.10^6 \text{ Statcoulomb/cm2}$$

Transformation ratio for symetrical load :

$$\phi = \frac{2A \, e_{ik}}{l} = \frac{2 \cdot 1.26 \cdot 4.8.10^6}{.1} = 1.21.10^8 \quad \text{cgs}$$

(Dimensions of ϕ are statcoulomb/cm-1)

Motional Mass :

$$M = \tfrac{1}{2} \rho A l$$

$$= \tfrac{1}{2} \cdot 5.7 \cdot 1.26 \cdot .1 = .358 \text{ gr}$$

Motional Stiffness :

$$k = \frac{\pi^2}{2} c_{hk} \frac{A}{l} =$$

$$k = \left(\frac{9.86}{2} \cdot 16.10^{11} \cdot \frac{1.26}{.1}\right)$$

$$= 9.95.10^{13} \text{ dynes/cm}$$

Equivalent inductance :

$$L_e = \frac{M}{\Phi^2} = \frac{.358}{(1.21)^2 10^{16}} = .244.10^{-16} \quad \text{CGS}$$

$$L_e = .244.10^{-16} \;.\; 9.10^{11} \;.\; 10^6 \;\mu H = 21.9 \;\mu H$$

Equivalent capacitance :

$$C_e = \frac{\Phi^2}{k} = \frac{(1.21)^2 10^{16}}{9.95.10^{13}} = 147 \text{ Statfarads} = 164 \;\mu\mu F$$

The undamped angular frequency of the crystal is :

$$\beta_o^{\;2} = \frac{1}{L_e \; C_e} = \frac{1}{21.9.10^{-6} \;.\; 164.10^{-12}}$$

$$= 2.84.10^{14} \; \text{rad}^2\text{-sec}^{-2}$$

This angular frequency corresponds to

$$f = 2.68 \quad \text{MCps}$$

The radiation impedance is for symetrical brass load

$$Z_R = 2 \; A \; \rho_B \; C_B$$

$$= 2 \;.\; 1.26 \;.\; 38.10^5 = 9.60.10^6 \quad \text{cgs}$$

The equivalent electrical resistance is then

$$R_e = \frac{Z_R}{\Phi^2} = \frac{9.60.10^6}{1.46.10^{16}} = 6.55.10^{-10}$$

$$= 6.55.10^{-10} \;.\; 9.10^{11} = 590 \;\Omega$$

This yields for the damper resistance R' (p.) 304 Ω if we are to obtain R = 200 Ω in the electrical circuit of Fig.23.

The crystal Q can be calculated by

$$Q = \frac{\pi}{2} \quad \frac{\rho C}{2\rho_B \, C_B}$$ for the fundamental frequency

and where ρC is the acoustical impedance of Barium Titanate and $2\rho_B \, C_B$ the acoustical impedance for the symetrical brass load.

$$Q \;\#\; \frac{\pi}{2} \cdot \frac{27.5 \cdot 10^5}{2 \cdot 38 \cdot 10^5} = .57$$

The clamped capacitance C_0' can be determined by

$$C_0' = \frac{K_0}{4\pi} \quad K \quad \frac{A}{I}$$

$$\frac{1700 \cdot 1.26 \cdot 1}{.1 \quad 4\pi} = 1700 \text{ Statfarad}$$

$$1900 \ \mu\mu F$$

5.33 WAVEFORM IN THE TRANSMITTER

The pressure and/or particle waveform has now to be considered.

The transducer is represented by an equivalent series circuit L_e C_e R_e (Subscripts will be dropped in this section)

$$L = 21.9 \ \mu H$$

$$C = 164 \ \mu\mu F$$

$$R = 590 \ \Omega$$

The applied voltage has been found in Sect. 5.31

$$v = v_0 \, e^{-\alpha t} \sin \beta t \quad \begin{cases} U_0 = 546 \text{ v} \\ 2\alpha = .41.10^6 \\ \beta = 1.0.10^7 \end{cases}$$

The angular damped frequency $\beta = 1.0.10^7$ rad/sec of the electrical circuit is chosen to match the damped crystal frequency β_m.

$$\beta_m^2 = \beta_0^2 - \alpha_m^2 = \frac{1}{LC} - \left(\frac{R}{2L}\right)^2$$

$$= \frac{10^{18}}{21.9 \ . \ 164} - \left(\frac{596}{2 \ . \ 21.9}\right)^2 .10^{12}$$

$$= 2.78.10^{14} - 1.81.10^{14} \ \# \ 1.10^{14}$$

$\beta_m \# 1.10^7$ rad : sec.

The damping factor α_m of the crystal is then

$\alpha_m = 1.34.10^7$ much larger than the damping factor of the electrical circuit ($\frac{1}{2}.41.10^6$). It will be <u>assumed for the first few cycles of the transmitted wave that the electrical impulse is undamped</u>.

Then $v = v_0 \sin \beta t$

$$U(s) = U_0 \frac{\beta}{s^2 + \beta^2}$$

The transformed series impedance of the crystal is

$$Z(s) = \frac{\frac{1}{L} \ s}{(s + \alpha_m)^2 + \beta_m^2}$$

and the expression of the motional current in the crystal will be

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V_0 \beta}{L} \frac{s}{(s^2+\beta^2)\left[(s+\alpha_0)^2+\beta_m^2)\right]} \tag{61}$$

Inverse transform of the second member can be found* as being

$$\mathcal{L}^{-1}\left\{\frac{s+a_0}{(s^2+\lambda^2)\left[(\alpha_0+s)^2+\beta_m^2\right]}\right\} =$$

$$= \frac{1}{\lambda}\left[\frac{a_0^2+\lambda^2}{(\beta_0^2-\lambda^2)^2+4\alpha^2\lambda^2}\right]^{1/2} \sin(\lambda t+\varphi_1) + \tag{62}$$

$$+ \frac{1}{\beta}\left[\frac{(\alpha-a_0)^2+\beta^2}{(\beta_0^2-\lambda^2)^2+4\alpha^2\lambda^2}\right]^{1/2} e^{-\alpha t} \sin(\beta t+\varphi_2)$$

with

$$\varphi_1 = tg^{-1}\frac{\lambda}{a_0} - tg^{-1}\frac{2\alpha\lambda}{\beta_0^2-\lambda^2}$$

$$\varphi_2 = tg^{-1}\frac{\beta}{a_0-\alpha} - tg^{-1}\frac{-2\alpha\beta}{\alpha^2-\beta^2+\lambda^2} \tag{63}$$

$$\beta_0^2 = \alpha^2+\beta^2$$

The general discussion of this equation (62) can be restricted, for sake of simplicity to three important features :

(a) <u>General Form</u> :

If $\lambda=\beta$,i.e. the damped angular frequencies of the electrical and mechanical circuits are the same,

(*) M.F. Gardner & J.L.Barnes, Transients in linear Systems, pp 344 n.1359.

the current is the superposition of two sinusoidal functions, the first undamped(*), the second strongly damped, since the mechanical damping α_m is high compared with the angular frequency β_m. After decay of the second term (within a few cycles) the waveform is practically imposed by the electrical pulse.

If $\lambda \# \beta$, i.e. the electrical circuit is unperfectly tuned on the crystal, eq.62 shows addition of the two components and presence of beat frequency $(\lambda - \beta)$ is to be expected. Fig.24 and 24bis give experimental confirmation of these facts. The amplitude modulation should not appear after a few cycles in the received wave, owing to the rapid decay of the second component in eq.62.

(b) Signal Peak :

The first maximum of the current wave will be practically determined by the first term of (62) ; it can be readily seen that the second term is negligible after short time compared to the oscillation period. Then

$$I_{max} \# \frac{U_0 \beta}{L} \frac{1}{\beta} \left[\frac{\beta^2}{\alpha^4 + 4\alpha^2\beta^2} \right]^{1/2}$$

(*) In fact slightly damped, owing to the presence of damping in the electrical circuit.

with $V_0 = 546v$ $\quad$ $L \quad 21.9\,\mu H$

$\beta_m = 10^7$ rad/sec $\quad$ $\alpha_m = \frac{R}{2L} = \frac{590}{2.21.9} 10^6$

$= 1.34.10^7 \ sec^{-1}$

the value of I_{max} is found .79 Amp.

The crystal particle velocity can be found at the radiating surface by

$$\dot{\xi} = \frac{I}{\phi} = \frac{.78 \ . \ 3.10^9}{1.21 \ . \ 10^8} = 19.6 \quad cm/sec^{-1}$$

The instantaneous peak power radiated in the brass rod is then

$$P = \frac{Z_c \dot{\xi}^2}{2} = \frac{38.10^5}{2} \ . \ (19.6)^2 = 73.10^7 \quad ergs/sec\text{-}cm2$$

$$= 73 \quad watts/cm2$$

This number is obviously an instantaneous value ; the average power transmitted during the pulse is much smaller (less than 10 per cent of the peak) and the duty cycle $= \frac{\text{Pulse duration}}{\text{Repetition period}}$ is of the order of

$\frac{25 \ sec}{10^5 \ sec} = 2.5.10^{-4}$ so the average power during the cycle is absolutely negligible and no heating of the crystal has to be expected.

(c) Leading edge of the transmitted signal

The simplest way to study the first $\frac{1}{4}$ cycle of the transmitted wave is to assume that at the very start of the pulse the "current does not know what is going to happen inside the crystal", which is a simplified statement of the fact that the mechanical damping has no influence on the start of the oscillation as long as critical damping is not approached. The leading edge can thus be examined by means of the simplified transform (with perfect tuning assumed)

$$\left\{ \mathcal{L}^{-1} \right\} = \frac{V_0}{L} \beta \frac{s}{(s^2 + \beta^2)^2}$$

whose inverse transform is(*):

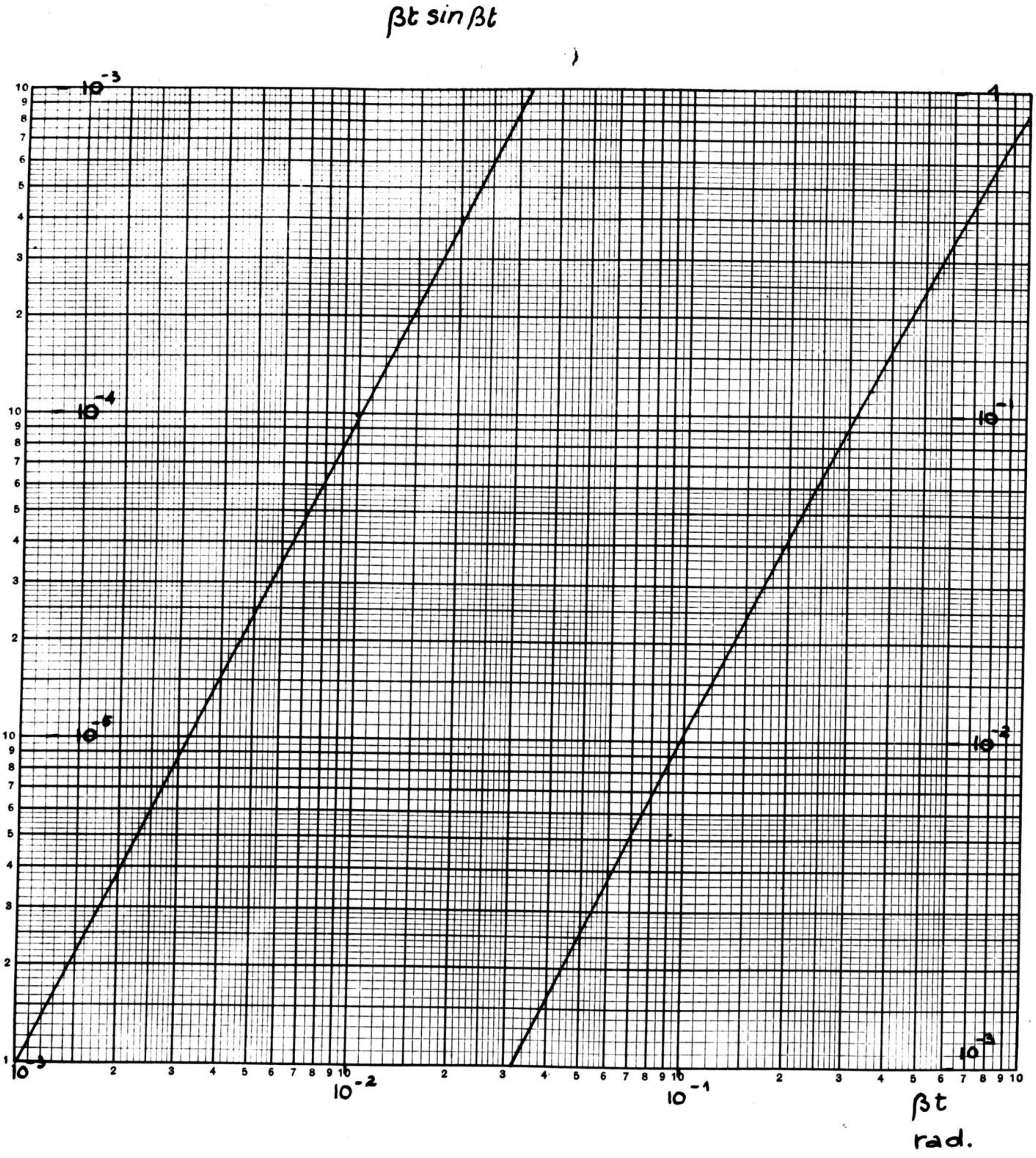
$$I(t) = \frac{V_0 \beta}{L} \frac{1}{2\beta} t \sin \beta t \qquad (64)$$

The function $t \sin \beta t \rightarrow t^2$ when $t \rightarrow 0$ and has therefore a 0 derivative at the origin. (Fig. 25)

This point is of considerable importance, it shows that unless α_m (mechanical damping) is large enough to invalidate the assumption made in 5.33 c ; no matter how steep the voltage function will be, the current pulse will have a zero derivative at the origin.

If on the other hand α_m is great, it is

(*) See reference p. 142



F I G. 25 - **Leading edge Function**

N.B. The zero time derivative at the origin can not be observed, as curves are plotted in logarithmic scales for greater precision.

known that the circuit loses its oscillatory character and the current waveform follows closely the voltage waveform.

It can be seen (by investigation of a non-resonant circuit) that with $\alpha_m > \beta$

$$I \# \frac{V}{R} = \frac{V_0}{R} e^{-\alpha t} \sin \beta t \quad \text{and} \tag{65}$$

$$\left(\frac{\partial I}{\partial t}\right)_0 = \frac{\beta V_0}{R} \tag{65bis}$$

In conclusion, the form of the leading edge of the transmitted signal is closely related to the mechanical damping of the crystal ; horizontal leading edge would be obtained with no damping ; finite derivative of the leading edge can be obtained (eq.65bis) at the expense of the current amplitude (eq.65).

A compromise is practically imposed by the physical nature of damping ; it is not possible to go too far in mechanical damping owing to the limited mechanical impedance of available materials and to the fact that coupling between crystal and load is increasingly difficult when great differences in impedance are present.

5.34 ACOUSTICAL PATH

In Sect.5.33 the sound signal was found to be (at least for the few first cycles) of approximately sinusoidal form, with a "first peak" power of the order of 80 watts. This signal will have to pass through the acoustical path made up with following elements :

a) Boundary Crystal - Silver-foil electrode

b) " Electrode - Bakelite insulator

c) " Bakelite - Brass Coupling-rod

d) Coupling rod

e) Boundary Brass - Combustion-Chamber gas

f) Gas path

The same elements are found in reverse order at the receiving crystal.

In the present stage of investigation, propagation characteristics in (a), (b), (c) and (d) are not taken into account ; interfaces have been thoroughly cleaned and then covered by pure vaseline ; experiments showed the preparation of contact surfaces to be of considerable importance.

Transmission and waveform distorsion will be studied for elements (e) and (f) only ; a power attenuation of 15 dB at each dide are assumed for a, b, c and d, irrespective of the wave frequency. This is obviously a very rough approximation of the facts (although verified by experiments to a certain extent); all these acoustical elements a, b, and c are thin layers and they can act as perfect reflectors if thickness is of the same order of magnitude as the wavelenght. The approximation is legitimated only by the presence of the gas path, where total transmission losses of the order of 90 dB occur, together with attenuation characteristics strongly dependent on the frequency.

The radiating "piston area" is circular, and of approx. same diameter as the crystal. Its surface is polished and should be frequently cleaned in actual operation. The wavelenght in the gas region is very small (# .2 mm) and asperities, such as engine deposits, can cause considerable attenuation and scattering.

The ratio of piston radius to wavelenght is

$$\frac{R}{\lambda} = \frac{6.25}{.2} \# 31$$

and the ratio $\frac{\text{Path lenght}}{\lambda} = \frac{12.5}{.2} \# 62$

Both are satisfactory for directive radiation and assumption of plane-wave-propagation in the test cavity.

At the radiating and receiving brass surfaces, the power transmission coefficient can be calculated

$$T = \frac{4r}{(r+1)^2}, \qquad r = \frac{Z_{gas}}{Z_{brass}} = \frac{41}{38.10^5} = 1.08 \cdot 10^{-5}$$

$$T = \frac{41}{38 \cdot 10^5} = 1.08.10^{-5}$$

$T = 4.32.10^{-5}$ at the transmitting rod, and is the same at the receiving rod.

In sinusoidal operation, the total loss occuring atthe two piston surfaces is

$$T^2 = 18.6.10^{-10} = 1.86.10^{-9} \quad \# \quad 93.5 \text{ dB}$$

The average absorbtion in the gas path at atmospheric pressure can be estimated by elements given in Sect. 2.323 and 2.324. Assuming no thermal relaxation to be present at the operating pressure and frequency, and neglecting in first approximation absorbtion by elements other than atmospheric air,

$$a = \omega^2 \sqrt{\frac{\rho}{k\sigma}} \quad \frac{\chi + 2\eta}{2k\sigma}$$

$$a = (10^7)^2 \sqrt{\frac{1.3.10^{-3}}{1.46.10^6}} \quad X \quad \frac{1.7.10^{-4}}{2.1,46.10^6} \quad .176 \text{ Nepers/cm}$$

The gas path lenght being 1.27 cm ;

$$A = 1.27 \cdot .176 = .224 \text{ Nepers}$$

The influence of heat conductivity can be taken into account by the factor h calculated in 2.324. $h = .46$

Total attenuation in the gas path :

$A(1 + .46) = .326$ Nepers $= 2.84$ dB

It should be remembered that A is pressure (or particle velocity) attenuation, the power attenuation is twice this value, i.e. # 5.5 dB

The total attenuation at fundamental frequency will be (for elements (e) and (f)),

$93.5 + 5.5 = 99$ dB (Power level)

For elements (a), (b), (c) and (d) a total attenuation of 30 dB has to be added ; the acoustical signal reaching the receiver crystal is then 129 dB below the transmitted signal. (This is roughly equivalent to 10^{-13})

The influence of frequency on attenuation is pointed out in 2.324 ; even if elements (a), (b), (c) and (d) are supposed frequency-independent, the gas path acts as low-pass filter and its effect will contribute to round the leading edge of the transmitted signal.

Let us assume, in order to illuminate the action of the gas attenuation that the acoustic signal is a square pulse modulated sinuoid. (Fig.26)

The frequency spectrum of a square pulse is known* if δ is the pulse duration

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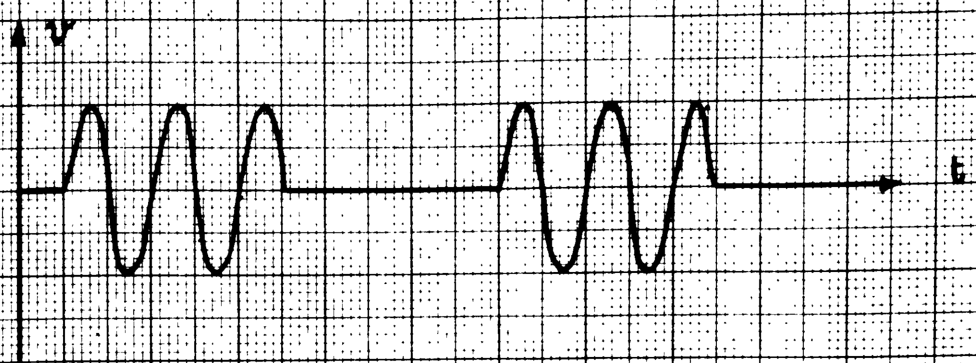
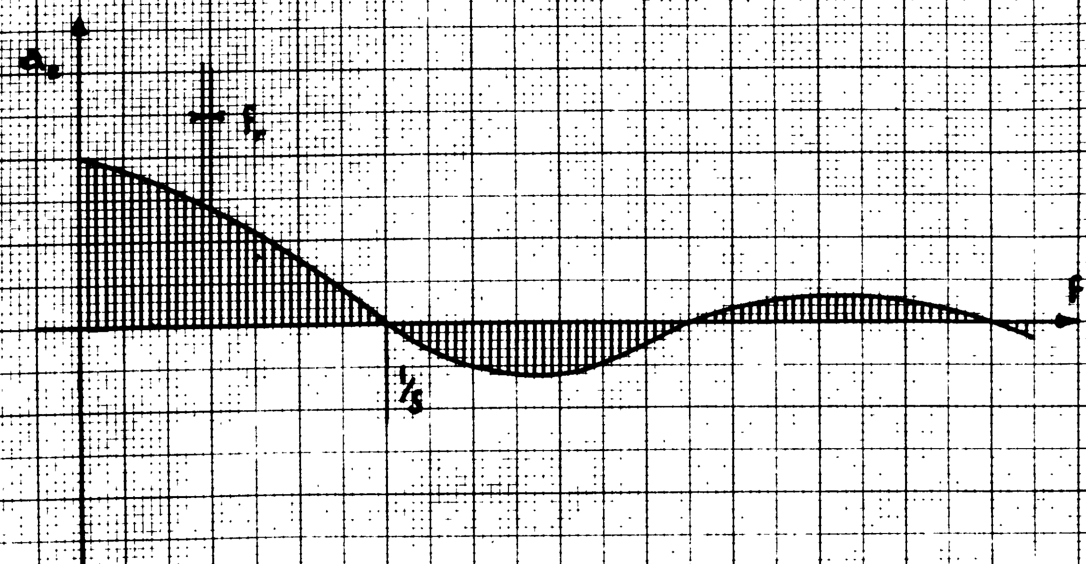
(*) Members of the M.I.T. Radar School Staff, Principles of Radar, 1946. pp 4 - 14.



F I G. 26 - Square Pulse Modulated Sinusoid



F I G. 27 - Frequency Spectrum of Square Pulses

τ_r the pulse repetition period

$f_r = \frac{1}{\tau_r}$ the pulse repetition frequency

the harmonic of the K^{th} order with respect to f_r will be (in relative amplitude)

$$a_k = \frac{2\delta}{\tau_r} \quad \frac{\sin \pi k f_r \delta}{\pi k f_r \delta} \quad \text{(frequency } K f_r\text{)}$$

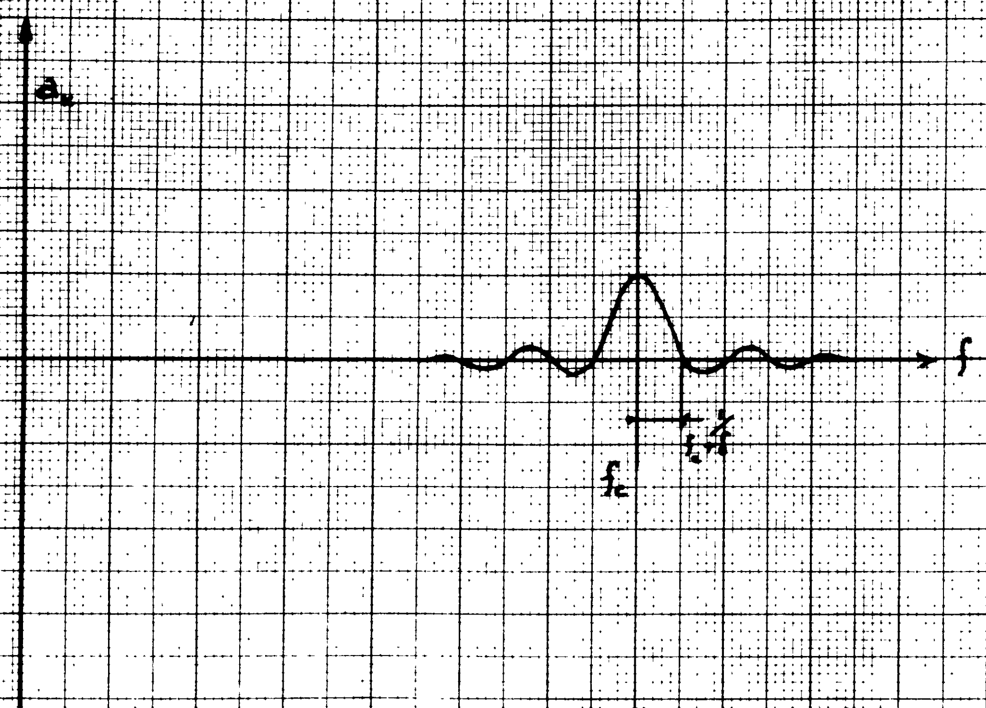
The spectrum is represented in Fig.27. If a "carrier" frequency of f_c is modulated with the pulse, the resulting spectrum is the superposition of all the frequencies $f_c \pm k f_r$ and can be represented as in Fig.28.

Reasonable reproduction fidelity is obtained(*), if frequencies up to $\frac{1}{\delta}$ fall within the half-power points of the filter ; in our problem however, if a sinusoidal pulse-modulated wave has to be quasi-perfectly reproduced much higher frequencies must fall within these limits. Actually, power attenuation of 6 dB defines the half-power point ; the gas attenuation has to be a_0 + 6 dB (if a_0 is the power attenuation at the fundamental frequency)

$$a_0 + 6 \text{ dB} = 11.5 \text{ dB}, \quad \text{so}$$

$$\frac{\omega_1^2}{\omega_0^2} = \frac{11.5}{5.5} = 2.08$$

(*) ibid. 4-20 and seq.



F I G. 28 - Spectrum of Square Pulse Modulated Sinusoid.

$$f_1 = f_o \sqrt{2.08} = 1.6.10^6 \;.\; 1.44 = 2.31 \text{ MCps}$$

The half-frequency band of the "gas filter" is only 2.31 - 1.6 = .71 MCps ; so the expected rise-time of the transmitted signal-envelope is approx. $\frac{1}{.71.10^6}$ i.e. 1.4 μsec, or with 1.6 MCps signal, 2.3 cycles.

The findings of this section can be summarized as follows :

(a) The power attenuation in the acoustical path will be of 129 dB, assuming a total loss of 30 dB in all elements, gas path and gas-metal boundaries excepted ;

(b) The gas acts as low-pass filter increasing the rise-time of the transmitted signal ; order of magnitude of the leading-edge rounding is of 2 to 3 cycles irrespectively of frequency characteristics of other acoustical elements, such as crystal electrodes, insulators, brass coupling rods, etc... the electronic amplifier (receiver) bandwidth will be certainly easy to make better than the acoustical filter itself.

5.35 RECEIVED SIGNAL

The receiving crystal will be mechanically excited by the incident signal, at abt. 129 dB below the transmitted power and with a waveform defined by Sect. 5.33 and 5.34.

The point of interest is the voltage input in the receiver RC.

Tuned circuit input is used, so that the crystal operation can be approximated by the equation

$$\frac{V_o}{p_o A} = \frac{2 \phi R_e}{Z_R + \phi^2 R_e} \qquad (66)^*$$

where best signal is obtained with $R_e = \frac{Z_R}{\phi^2}$ **

then

$$\frac{V}{p_o A} = \frac{1}{\phi} \qquad (67)$$

and $V_o = \frac{p_o A}{\phi}$

If sinusoidal operation is assumed,

$$W = \frac{p_o^2}{Z}$$

(p_o is the effective value of the sound pressure on crystal surface)

and $p_o = \sqrt{W Z}$

$$= \sqrt{80.10^{-13} \cdot 10^7 \cdot 38.10^5}$$

$$= \sqrt{8 \cdot 3.8 \cdot 10} = 17.4 \text{ dynes/cm2}$$

then $V = \frac{17.4 \cdot 1.26}{1.21.10^{-8}} = 18.1.10^{-8}$ Statvolts

$54.3.10^{-6}$ Volts $= 54\ \mu$V.

(*) T.F. Hueter, Physical Ultrasonics, pp.4.65

(**) For symetrically loaded transducers.

The input resistance R_e has to be

$$\frac{2\ Z_R A}{\phi^2} = \frac{38.10^5}{1.47.10^{16}} \text{ Statohms}$$

$$R_e \ \# \ 620 \ \Omega$$

The actual thermal noise voltage is then

$$V_N = \sqrt{4k\ \theta\ R_e\ \Delta f}$$

k $1.38.10^{-23}$ joules/°K

Θ Abs. Temperature °K

R_e Ohms

Δf bandwidth

with

$R = 620\ \Omega$

$\theta = 300^\circ$ K

$\Delta f = 3.10^6$ cps

$V_N = 2.7\ \mu V$

The receiver noise figure was tentatively set at 4 dB (power) or 2 dB in voltage, so the total noise f.e.m. at the receiver input will be $3.5\ \mu V$

Then the signal-to-noise ratio is $\frac{54}{3.5} = 15.5$ or 23.5 dB

It must be however stated that in actual operation somewhat smaller signal-to-noise ratio is observed.

5.36 MEASUREMENT ACCURACY

The merit of the sound velocity method for temperature determinations is dependent on three factors :

(a) Accuracy of relationship between sound velocity and gas temperature ;

(b) Accuracy on gage lenght measurement ;

(c) Accuracy on travel time determination.

Factors (a) and (b) have been discussed previously, if c is known, θ can be determined with better than .3 per cent accuracy. The value of the apparatus is shown by the accuracy obtained on travel-time measurement.

The A-R scope is time-calibrated to better than .1 per cent ; the error on time-measurement is the superposition of

- A-R scope calibration error (.1 per cent maximum)
- Reading error on the yard-scale ; the average reading is 10000 and the smallest division of the scale is 20 ; so .5 divisions represent .1 per cent.
- Leading edge error ; taking in consideration what has been said on leading edge forms, it is safe to assume $\frac{1}{2}$ cycle of possible error on the delay determination,

this amounts to $\frac{1}{2} \cdot \frac{1}{1.6} \cdot 10^{-6} = .3\ \mu$sec. The average time delays being of 25 to 30 μsec this error is of the order of 1 per cent.

The relative error on C can be up to 1.2 per cent (2.4 per cent on temperature). In part 7. possible improvement of accuracy will be discussed.

6. TYPICAL MEASUREMENT RESULTS

6.1 MOTORING TEST

The CFR engine (5.21) is driven by electric motor at 1000 RPM.

Observed engine parameters

Inlet pressure exhaust pressure	7.7" Hg gage
Inlet orifice depression Δp	8.9" H_2O
Jacket temperature	100^o F
Compression ratio	4.2
Barometric pressure	764.8 mm

Gage lenght .0527 ft

Calibration lecture (Air at 100 psi and 73^o F)

T = 12020 yds

In motoring run, values of Table VIII for crank-angle φ and time-delay T_2 (expressed in yards) are observed.

T A B L E VIII

φ	T	t_2	t_{g2}	C $\cdot 10^{-4}$	C^2 $\cdot 10^{-8}$	Abs Θ
o	yards	μsec	μsec	cm/sec^{-1}	cm2/sec^{-2}	oK
-80	11155	68.0	41.7	3.85	14.8	366
-70	11030	67.4	41.1	3.90	15.2	376
-60	10875	66.3	40.0	4.02	16.2	400
-50	10670	65.0	38.7	4.15	17.2	425
-40	10520	64.2	37.9	4.24	17.9	443
-30	10400	63.5	37.2	4.32	18.7	462
-20	10340	63.1	36.8	4.37	19.1	472
-10	10210	62.3	36.0	4.46	19.9	492
0 TC	10250	62.5	36.2	4.44	19.7	486
10	10300	62.8	36.5	4.40	19.4	480
20	10480	63.9	37.3	4.30	18.5	457
30	10620	64.8	38.5	4.17	17.4	430
40	10800	65.9	39.6	4.06	16.5	408
50	10970	66.8	40.5	3.97	15.8	390

<u>Calculation</u> :

Calibration temperature $\theta_c = 73^\circ$ F =	296° K
M for atmospheric air	28.8
R (perfect-gas constant)	$8.31.10^7$ cgs
γ (at 73° F for air)	1.401

$$c^2 = \frac{\gamma RT}{M} = \frac{1.401 \cdot 8.31.10^7 \cdot 296}{28.8} = 11.6.10^8$$

$c = 3.405.10^4$ cm/sec^{-1}

The total delay measured is

$$t_1 = \frac{12020}{164} = 73.4\ \mu\text{sec}$$

The gas path lenght is = 30.48 x .0527 = 1.605 cm

Delay in the gas path

$$t_{g1} = \frac{1.605}{3.405}\, 10^{-4}\ \text{sec} = 47.1\ \mu\text{sec}$$

The electro-acoustical delay is

$$t_{ea} = t_1 - t_{g1} = 26.3\ \mu\text{sec}$$

Then, the sound velocity is

$$C = \frac{L}{t_2 - t_{ea}} \quad \text{with } L = 1.605$$

$t_2 - t_{ea} = t_{g2}$ = gas path transit time is tabulated in Column 4 of Table VIII ; c, c^2 and θ respectively in col. 5, 6 and 7.

θ is determined by $\theta = \frac{M c^2}{\gamma R} =$

$$= \frac{28.8}{1.401 \cdot 8.31.10^7} c^2$$

6.2 FIRING TEST

Same CFR engine is used as in 6.1, with following parameters :

Inlet and jacket temperature	160° F
Fuel : Octane at F=.078 Fuel-Air ratio	
Orifice depression	5.3" H_2O
Atmospheric pressure	762.7 mm
Engine speed	1000 RPM
Gage lenght	1.605 cm

Calibration is made at 73° F and values for t_{ea} and t_{gl} are found again to be respectively 26.3 and 47.1 μsec

Table IX summarizes results for T yards as function of crank-angle φ.

Sound velocity is calculated as in 6.1 ; then temperature of the gas mixture is graphically determined from characteristics plotted as function of the residual gas proportion.* (Fig.29)

(*) These curves have been calculated and plotted first by Mr. P.C. Wu of the Sloan Laboratories at the Massachusetts Institute of Technology. Perfect-gas relationship has been admitted for both air-gas mixture and residual gas ; ratio of specific heats is found in J.H Keenan & J. Kaye "Gas tables".

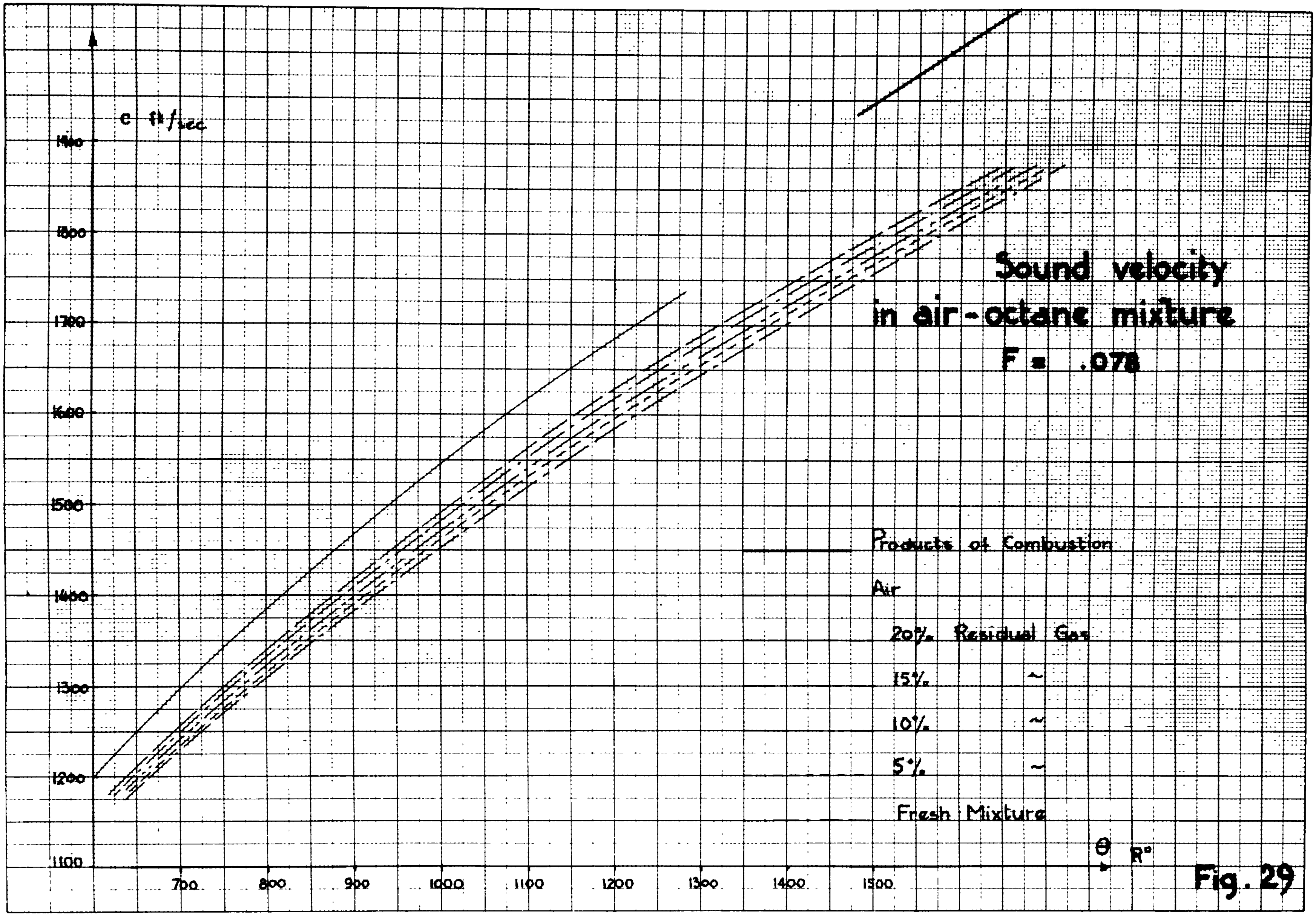

Sound velocity
in air-octane mixture
F = .078
c ft/sec
1900
1800
1700
1600
1500
1400
1300
1200
1100
700
800
900
1000
1100
1200
1300
1400
1500
θ R°
Products of Combustion
Air
20% Residual Gas
15% ~
10% ~
5% ~
Fresh Mixture

Fig. 29

TABLE IX

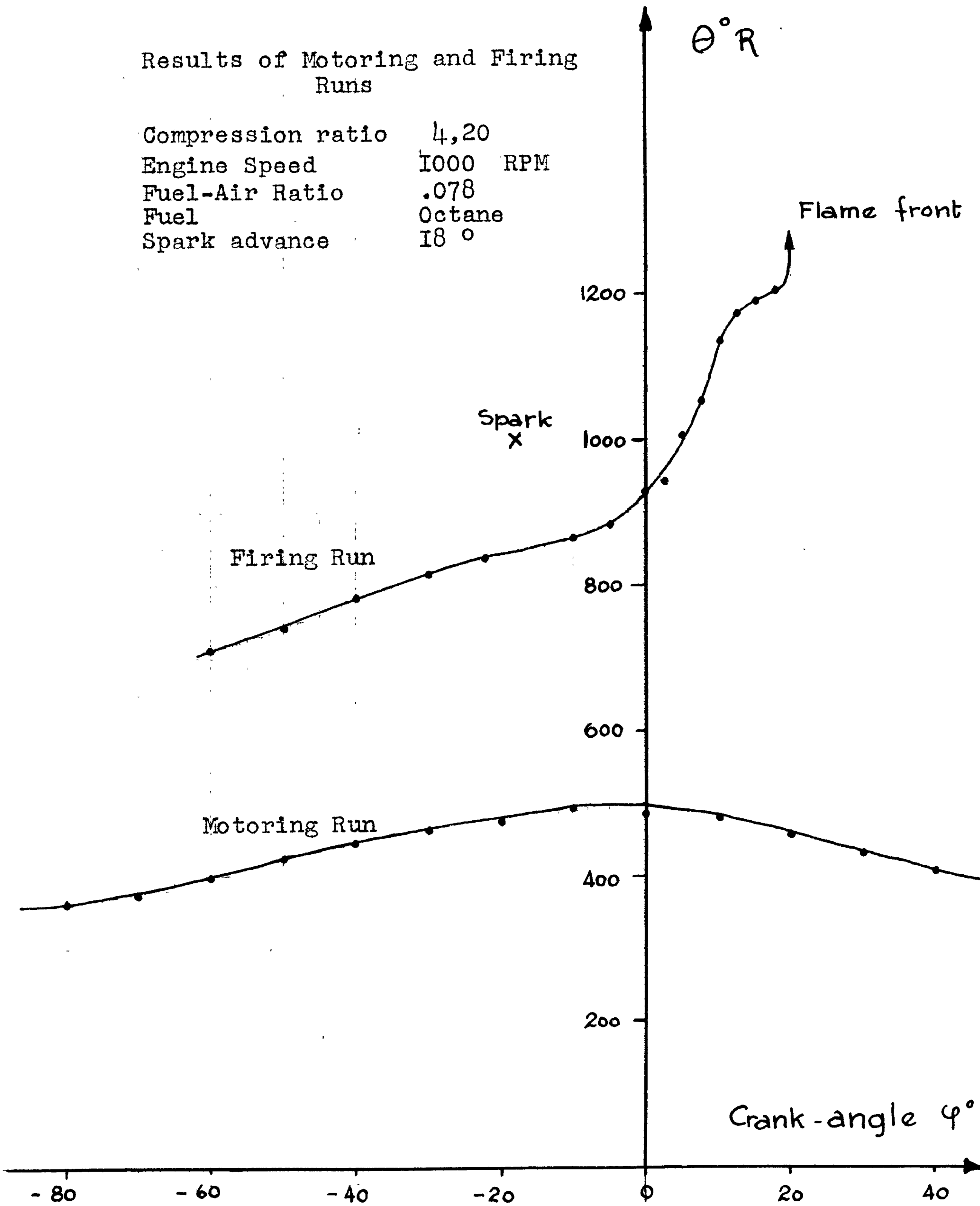
φ	T	t_2	t_{g2}	$\frac{c}{10^{-4}}$	c	θ *
°	yards	μsec	μsec	cm/sec	ft/sec	°R
-60	10762	65.5	39.2	4.10	1250	710
-50	10640	64.9	38.6	4.16	1269	740
-40	10482	63.6	37.3	4.30	1310	785
-30	10380	63.0	36.7	4.37	1332	815
-22	10262	62.5	36.2	4.44	1351	840
-10	10184	62.0	35.7	4.50	1370	865
- 5	10120	61.6	35.3	4.55	1386	885
TC 0	9980	60.8	34.5	4.66	1420	930
2.5	9935	60.5	34.2	4.70	1430	945
5	9770	59.5	33.2	4.84	1472	1005
7.5	9630	58.7	32.4	4.95	1510	1055
10	9452	57.6	31.3	5.14	1565	1137
12.5	9400	57.2	30.9	5.20	1584	1170
15	9330	56.9	30.6	5.25	1600	1190
17.5	9320	56.8	30.5	5.27	1608	1207
20	irregular					
22	"					
25	7100	43.3	17.0	9.45	2880	**
59	7500	45.6	19.3	8.33	2540	**

(*) Assuming 10 per cent residual gas.

(**) These points are in the flame region, therefore the curves of Fig.29 are not applicable.

Points after 17.5° crankangle are irregular and located manifestly inside the flame front. No interpretation is given to sound velocity in this case and values indicated for 25° and 59° should not be considered as functions of temperature only.

Results of Runs 6.1 and 6.2 are represented in Fig. 30 and Fig.30bis.



F I G. 30 and 30 bis

6.3 DISCUSSION

6.31 MEASUREMENT PROCESS AND RESULTS

After setting of initial parameters of the engine run and calibration, an operator familiar with the method can make about 1 determination per 3 minutes. A complete run, including about 20 points, can be performed within an hour.

In absence of any other "rating" method, the value of experimental results can be evaluated by comparison with the perfect-gas law temperatures. In our limited study, there was no time left for extensive investigation of this point ; however the method was in use for about 2 months and a representative superposition of temperature curves obtained by sound-velocity method and perfect-gas law method is given in annex N° I (*)

Differences appearing between the two curves (and deviations from the isentropic curve, also represented) can be accounted for in a satisfactory way by heat losses at the cylinder walls and other secondary phenomena ; it is the opinion of the "users" that temperatures determined by the sound-velocity method can be accepted

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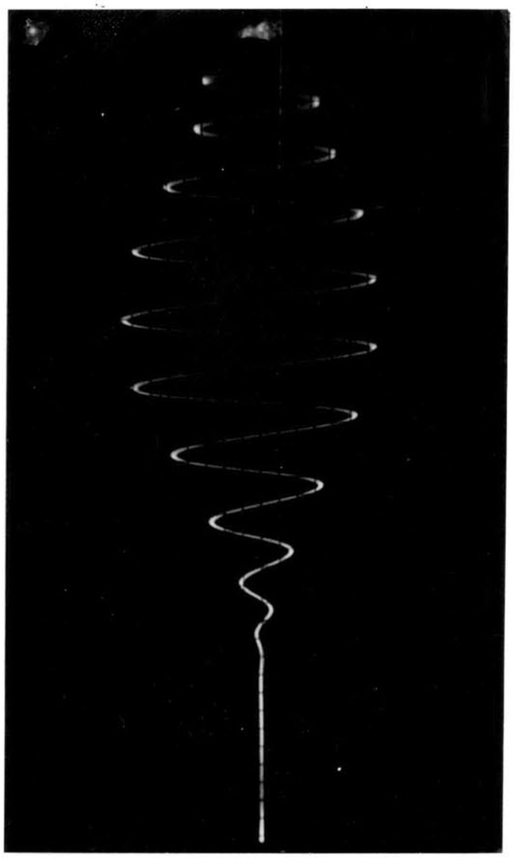
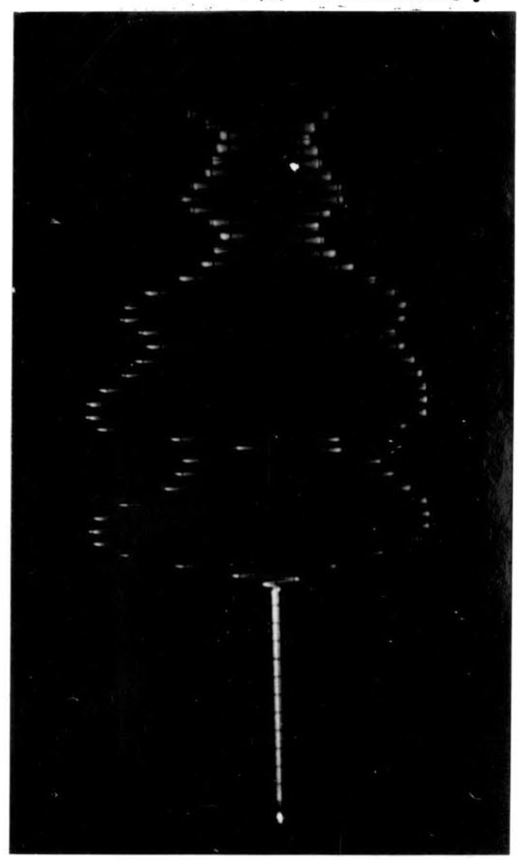
(*) Experiments have been performed by Mr. P.C Wu of the Sloan Laboratories.

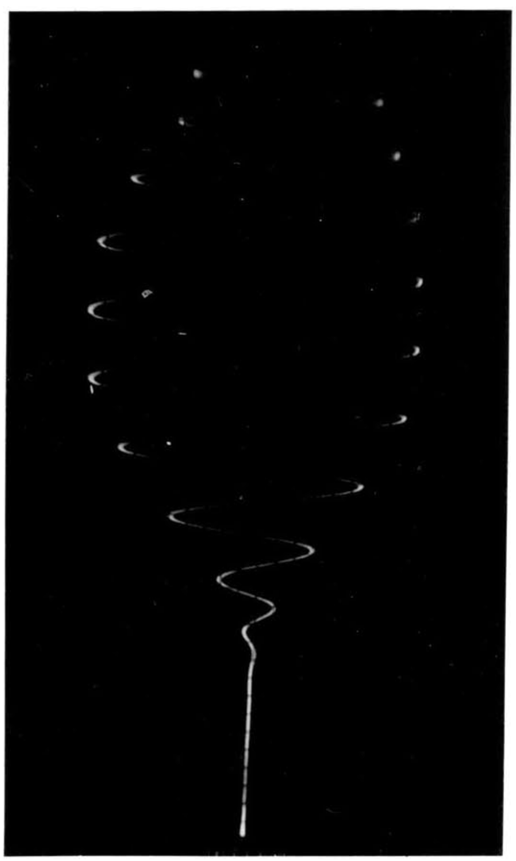
F I G. 3I

5o psia
I" # 2,5 microsec.

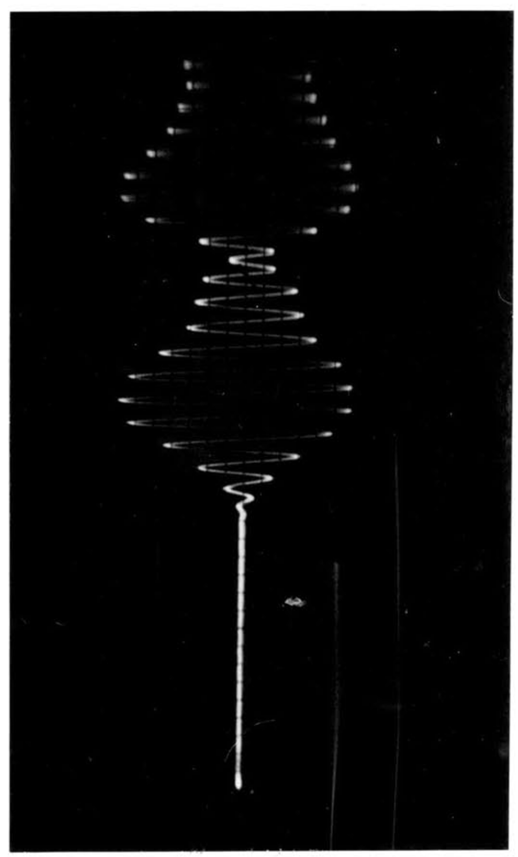


IOO psia
I" # 9,5 microsec.





IOO psia
I" # 2,5 microsecond



50 psia
I" # 5,5 microsecond

Received Signals obtained with Air Backed transducers with 1 per cent maximum error if curves of Fig. 29 are correctly interpreted, i.e. the fuel composition, the fuel-air ratio and the residual gas proportion is accurately estimated.

6.32 ELECTRO-ACOUSTIC SIGNAL

Photographs on Fig.31 and 32 give oscilloscopic picture of the received signals with different pressures and with damped and undamped transducers.

The rounding of the leading edge can be observed in all cases ; the damped waveform appears to be more suitable to experiments than the undamped signal.

The signal-to-noise ratio is acceptable even with atmospheric pressures ; it is however not as good as the theoretical value found in Sect. 5.35. In the most defavourable case, the signal-to-noise ratio is about 12 dB.

The considerable importance of proper shielding has been verified. Critical elements to shield are the transmitter thyratrons and tank circuits ; the receiver input tube and input circuit. Another important factor is the connexion to ground of both coupling rods ; the transmitter rod is at high electrostatic potential by induction and radiates electrically a much stronger signal than the

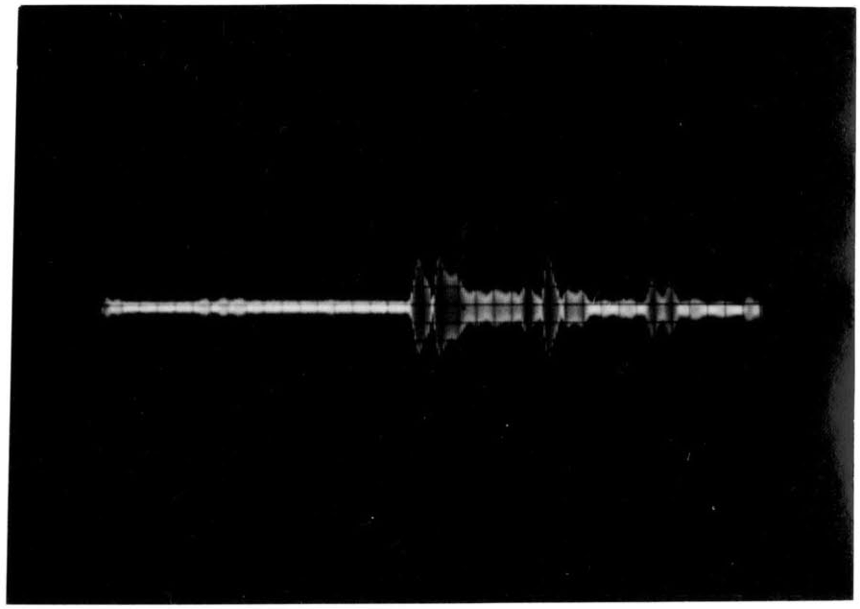


Fig. 33 - A- Display of the received signal at atmospheric pressure and air-backed transducer. Observe left to right : leakage signals, noise level, rounded signal edge and successive reflections in coupling rod. Peak at abt. 50 microvolts, time scale 1" # 35 microsec.

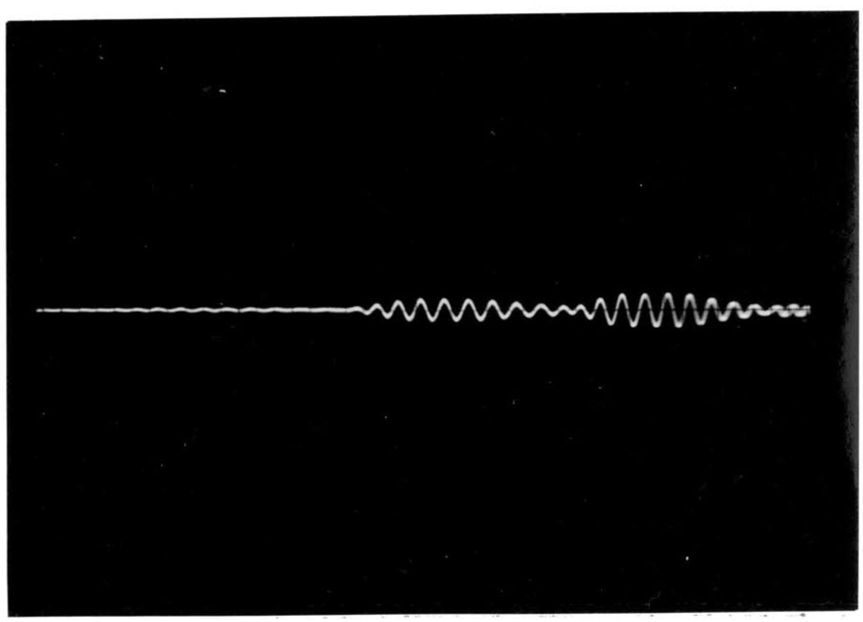


Fig.32 - R-Display of received signal at atmospheric pressure with symetrically loaded transducers. Observe improved leading edge (abt. 2½ cycles) and reduced amplitude (abt. 30 microvolts). 1" # 4,5 microsec.

electro-acoustic wave if the grounding is not satisfactory.

Further difficulty arises in case of improper shielding : the transmission by electro-magnetic radiation of the piezO-electric f.e.m. generated in the transmitter crystal by reflected sound waves in the coupling rod, or, alternatively the reflections in the receiver coupling rod of the sound wave generated in the receiving crystal by the radiation of the main transmitted signal.(See Fig.33) Disturbances of this origin can be completely eliminated by proper shields and ground connexions and no difficulty was experienced in this respect.

The turbulent gas flow in the test cavity causes considerable amplitude differences from cycle to cycle ; the observation of the signal for this reason causes eye fatigue to the observer and can as well cause errors on the leading edge determination.

7. SUMMARY AND CONCLUSION

a) The review of sound propagation theory shows that absolute temperature of non-reactive gas mixtures (in absence of relaxation) is related to the sound velocity and the relationship between the two quantities is known for usual and previsible cases with accuracies considerably better than the expected experimental accuracy.

Thermal relaxation does not appear in experiments, although conditions are such that its presence could have been predicted in some cases. It is assumed that systematic relaxation tests are to be performed in order to get a definite picture of the situation. The transient behaviour of relaxation have not yet been investigated and it is possible that no change in propagation velocity occurs at the relaxation frequency short pulse-excitation.

b) The pulse-transmission technique, associated to precision-sweep test scope, is one of the best methods available ; theoretically its limitations are only the internal calibration accuracy and errors on the leading edge of the received signal.

The waveform of the signal can be improved by use of symetrically loaded transducers, however even the improved leading edge is not sufficiently defined to eliminate scope-reading errors. Signal-to-noise ratio is critical at low pressures, and amplitude modulation by changes in density of the gas is present.

Frequency-modulation type measurement method (Sect. 4.324) appears to be free from leading-edge and amplitude-modulation errors ; the circuit complexity does appear however to be disproportionate with the expected advantages.

The used technique can be improved by :

- Experimental investigation of transducer-coupling-rod transmission.
- Decrease of the frequency, approximately 6 dB could be gained on the signal-to-noise ratio by reducing the frequency to 1 MCps.
- Reduction of the receiver bandwidth, which is presently from .5 to 3.5 MCps and introduces useless noise power ; since the gas path acts as filter with upper half-power frequency at 2.4 MCps.
- Additional refinements in the receiver circuitry, such as clipping (to prevent amplitude-modulation) and one-shot multivibrator or blocking oscillator

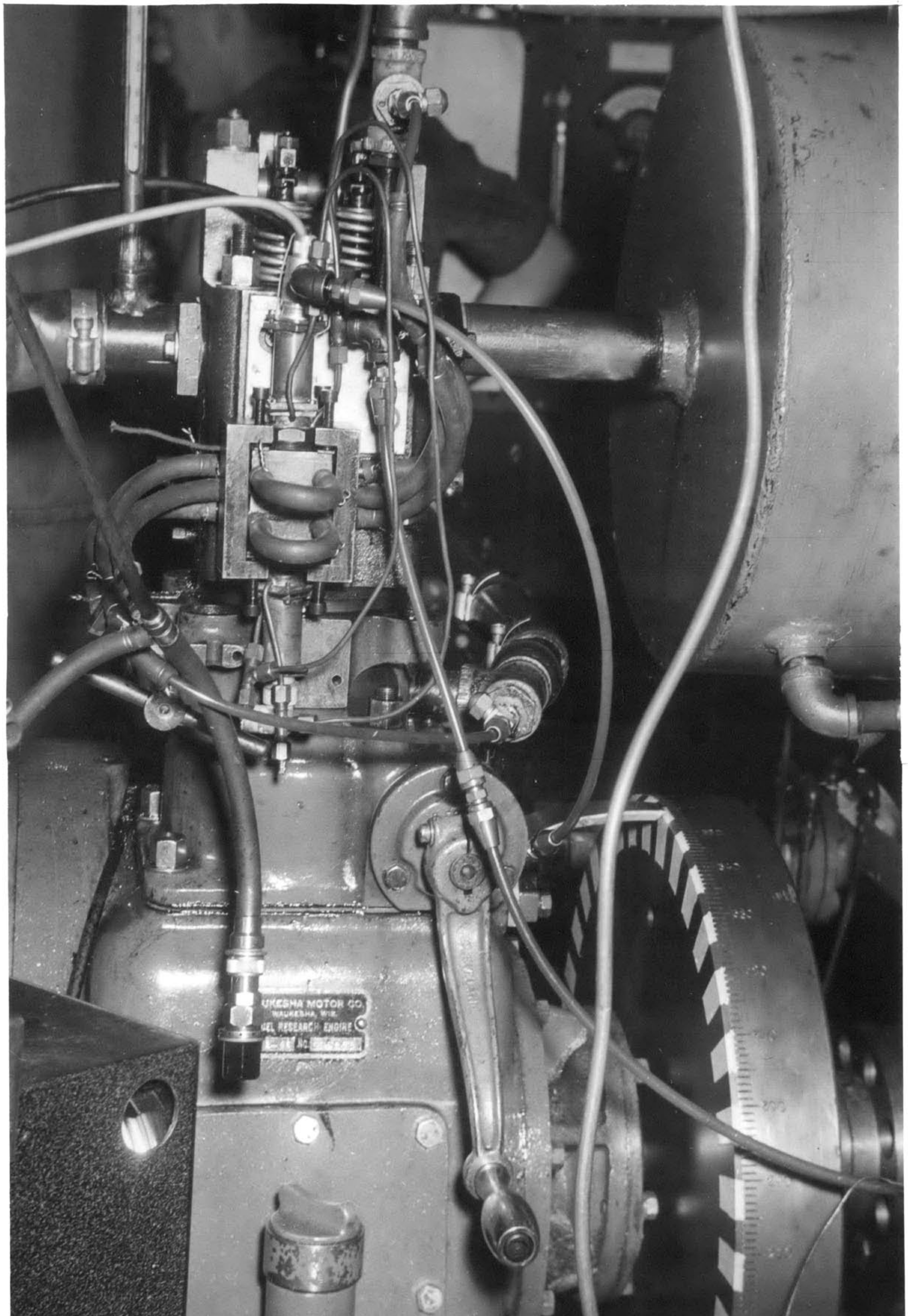


Fig. 34 - View of the Engine Set-Up, showing Engine head, transducers and cooling arrangements.

(to supply perfectly observable "artificial leading edge" at any chosen point of the signal.)

c) The process, as a whole, is acceptable for internal-combustion engine study ; its operation is characterized by quasi-instantaneous and synchronized measurements ; the probable error on temperature is small (2.4 per cent) compared with the expected consistency ; and with the refinements suggested above the accuracy can be much improved.

In actual operation, the total error appears to be around 1 per cent owing to the possibility to determine the leading edge quite accurately at the points of interest, i.e. before the end of the engine compression stroke.

d) The sound velocity method might have other uses in fluid temperature studies. Due attention should be paid however to the fact that gas velocities directed orthogonally to the propagation axis cause considerable attenuation and scattering whereas parallel fluid velocities introduce dispersion in the sound velocity.

The method described and discussed in this paper is suitable to temperature determinations in internal combustion engine cylinders. Minor improvements can and have to be made and it is assumed that this method will become standard in engine temperature analysis.

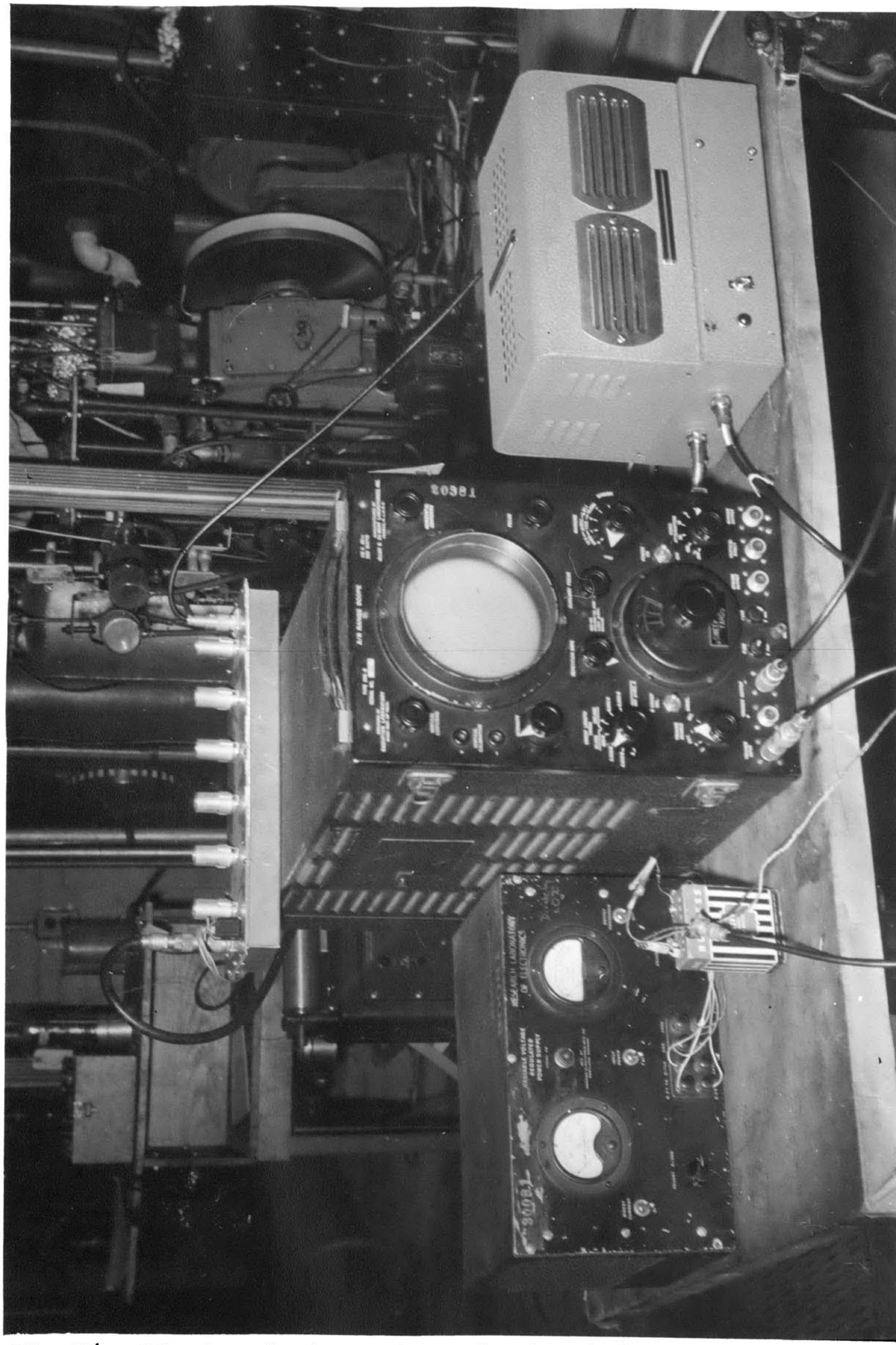


Fig.35 - Electronic Apparatus, showing left to right : Stabilized supply for receiver, synchronizer, receiver, A-R Scope and transmitter.

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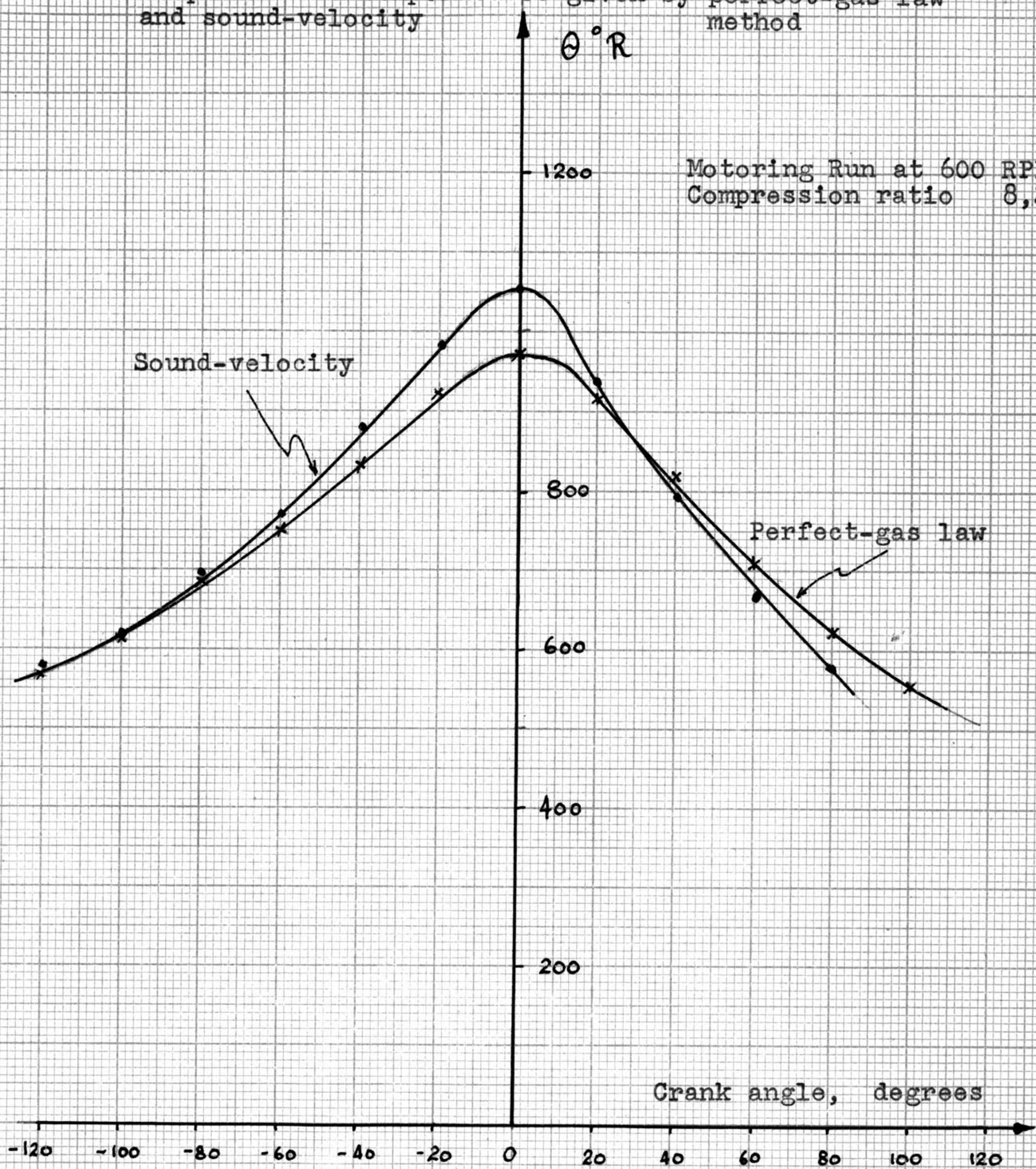
Abbreviations:

J. A. S. A.	Journal of the Acoustic Society of America
J. Chem. Phys.	Journal for Chemical Physics
Z. Phys. Chem.	Zeitschrift fur Physik und Chemic
Phys. Rev.	Physical Review
Proc. Roy. Soc., London	Proceedings of the Royal Society of London

ANNEX I

Comparison of temperatures given by perfect-gas law and sound-velocity method



The sound velocity method yields higher temperatures during the compression stroke and lower temperatures during the expansion stroke.

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