A MATHEMATICAL STUDY OF THE CONTROLLED MOTION OF AIRPLANES

By

SHIH-NGE LIN

B.S. Chiao-Tung University July 1935

M.S. Massachusetts Institute of Technology, 1937

Submitted in Partial Fullfillment of the

Requirements for the Degree of

DOCTOR OF SCIENCE

From the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

1939

Sigmature of Author..

Department of Aeronautical Engineering, May, 11, 1939

Professor in Charge of Researdh.

Chairman of Departmental

Committee on Graduate Students ..

Head of Department...

May,11,1939

Professor George W.Swett,

Secretary of the Faculty

Massachusetts Institute of Technology

Cambridge,Mass.

Dear Sir,

I hereby submit a thesis entitled A MATHEMATICAL STUDY OF THE CONTROLLED MOTIONS OF AIRPLANES in partial fulfillment of the requirements for the degree of Doctor of Sdience.

Very truely yours,

Shih-Nge Lin

Massachusetts Institute of Technology

Cambridge,Mass.

ACKNOWLEDGEMENT

The author wishes to take this opportunity to express his gratitude and deep appreciation for the in-valuable suggestions rendered by Prof.O.C.Koppen, Prof.H.L.Hazen,Prof.S.H.Caldwell and Prof.C.S.Draper.

To Mr.John L.C.Lŏf,the author feels indebted for his constant assistance and help in solving the differential equations and in planning the set-up for the Differential Analyzer.

To Mr.Weiss,the author wishes to express his appreciation for advice and suggestions on the treatment of the fundamental theory of automatic control.

Table of Contents

Summary and Abstract

The controlled stability of the longitudinal and lateral motions of the N.A,C.A. average airplane is investigated mathematically by finding the roots of the stability equations and by solving the disturbed motion due to atmospheric gusts. The non-dimensional derivatives of the average airplane are used in the equations of motion so that the results are believed to be applicable to all kinds of airplans of conventional design.

Chapter I treats the controlled longitudinal motion with the control system giving a stabilizing moment as a function of the angle of inclination in pitch. The effect of control lag enters into the equation of motion as due to the inertia and damping of the control system, making the ~~final form of the~~ stability equation ~~become~~ a sextic. The motions of the control and the controlled member are solved and thoroughly discussed. The effect on the ~~the~~ stability due to a constant time lag is investigated and compared with that due to inertia lag.

Chapter II deals with the controlled longitudinal motion in general. The effects of varying the static stability of the airplane and of varying the tail size on the controlled stability as well as the disturbed

motion.

Chapter III treats the controlled lateral motion in general. Effects due to constant time lag are examined. The stability and ~~controlled~~ the disturbed motion due to rolling gust are solved under nearly all possible variations of the airplane characteristics. The motions produced by rolling and yawing moments due to control operation are solved for various airplane characteristics.

Finally, the possibility of two-control operation is examined in Chapter IV. Both the controllability and the controlled stability are investigated. Comparisons based on ~~between~~ the required airplane characters and the relative controllability between the rudder-elevator and the aileron-elevator control systems are made.

A method of solving the roots of the Quartic, Quintic and Sextic equations with good accuracy and ~~xxxxxx~~ considerable ~~xxx~~ saving of labor is ~~are~~ presented.

The use of M.I.T. Differential Analyzer in solving the disturbed motion due to atmospheric gusts is described. The following points are the main results and conclusions of the research.

Results and conclusions on the mathematical part of the stability research

a. The M.I.T. Differential Analyzer is found to be the most efficient method in investigating disturbed motions due to gusts when a great number of solutions under various airplane characteristics are desired.

b. The method of successive approximation in solving the roots of the Cubic, Quartic, Quintic, and Sextic equations involved in the longitudinal and lateral stability problems is so efficient in saving of labor that the examination of Routh's Discriminant is no longer worth doing. It is applicable to all possible cases involved in longitudinal and lateral stability problems.

c. The correlation between the stability criteria and the magnitude of disturbed motions is seen to be unique despite the difference in the nature of the problem. For all kinds of motions, in order that the disturbed motion be satisfactorily small, each mode of motion must be properly damped. The degree of damping should be measured in terms of the number of oscillations damped to half rather than the time to damp to half amplitude.

The number of oscillations damped to half should be smaller than 0.6 for satisfactory controlled stability.

d. The effect of control lag is seen to decrease the damping for the short oscillations. The effect of inertia lag is essentially equal to that of the constant time lag, except in its possibility of overshooting.

e. The logic of simple vibration system is still applicable to the complicated stability problems. In longitudinal motion, the sum of the damping in the long and short period modes of motion is always equal to the sum of , m_q, x_u, and z_w despite the magnitude of the other derivatives. The control derivatives, such as m_θ derived from displacement control can only equalize the damping among the two modes of motion, but can not introduce extra damping to the controlled system. In the lateral motion, the devivatives due to aileron and rudder control as well as l_v ~~and~~ n_v and n_p and l_r can only influence the relative distribution of the damping among various modes of motion. Only l_p, n_r and y_v are responsible for sum of the damping of the system.

f. The effectiveness of the error-control derivative lies in ~~its~~ their ability to ~~equalize the~~ provide for proper amount of damping for all modes of motion.

Results and conclusions for the purpose of airplane designers' reference.

a. The check of satisfactory stability should be carried out on the controlled stability equations.

b. The control derivatives due to elevator, rudder, and aileron operation can estimated by the following expressions,

(1) $m_\theta = 1/b_1(\partial C_L/\partial\alpha')(\partial\alpha'/\partial\beta)(\partial\beta/\partial\theta)(S'/S)$

(2) $l_\phi = (2/a_1)(\partial\gamma/\partial\phi)(\partial C_l/\partial\gamma)$

(3) $n_\phi = (2/c_1)(\partial\gamma/\partial\phi)(\partial C_n/\partial\gamma)$

(4) $n_\psi = (1/c_1)(\partial C_L'/\partial\alpha')(\partial\alpha'/\partial\eta)(\partial\eta/\partial\psi)(S'/S)$.

c. For proper damping of short oscillations due to control lag,

l_p must be greater than $\mu l_\phi t_a$

n_r must be greater than $\mu n_\psi t_r$

m_q must be greater than $\mu m_\theta t_g$

d. For a control system having a natural undamped period, T_n, the equivalent constant time lag, t_g can be estimated as $t_g = .25\ T_n$.

e. For comfort flight, $m_w = 0$ for cruising speed, horizontal flight should be sought. m_q should be as large as possible (limited only by controllability).

f. Control devices giving $m_q, m_{\dot{q}}$ can be used together with m_θ control to achieve optimum performance.

g. Due to the extremely large control lag in using throttle control for longitudinal stabilization, it is not advisable to ~~xxxx~~ do so.

h. Neither adverse nor favorable yawing moment due to aileron ~~xxx~~ is desirable for conventional airplane. For satisfactory aileron design,

$$C_n/C_l \text{ must be smaller than } (c_l/a_l)\cancel{(a_l/a_l)}(y_v n_r + \mu n_v)/(y_v l_r + \mu l_v)$$

at high angle of attack. If y_v is small, and $a_l = c_l$ then, C_n/C_l must be smaller than n_v/l_v .

i. For an average airplane, $n_v = .5$ to 1 gives good stability and controllability. Too large n_v (such as larger than +2) make the rudder control ineffective, and too small n_v such as smaller than .25, the long oscillation is unstable.

j. The smallest l_v should give $(\mu C_L l_v)(\mu n_\psi) = 100$ at low angle of attack.

k. For two-control operation using rudder-elevator system, a large dihedral approximately ten to twenty times larger than that of the average airplane should be used. The tail size should be from two to four times ~~the~~ larger than the tail of the average airplane.

l. The dihedral for an airplane using aileron-elevator control should be very small or even negative. The tail size should be very large, approximately twice as large to four times as large as that of the average airplane. The aileron must be operated according to the angle of yaw as well as the angle of roll.

m. Due to the fact that the yawing motion due to secondary effect of the applied rolling moment lags behind that produced by the direct effect of yawing moment even for an airplane having large n_v or static stability, it is believed that the controllability for a rudder-elevator control system is superior to that for an aileron-elevator system.

Introduction

The present status of stability research has been a matter of controversial nature. It is frequently stated that there is uncertainty as to what is desired in the stability characteristics. Aeronautical engineers are used to judge the characteristics of the airplane in terms of its inherent stability,namely,its ability to return to the original course after being disturbed. They try to measure the stability characters in terms of the period and damping of the uncontrolled airplane. The judgement would be true if the airplane is always flying in a calm and smooth air so that no gusts to subject the airplane to disturbances in pitch,roll and yaw. However,the airplane if uncontrolled,flying in rough air will be disturbed so violently that it is objectionable to its occupants. Furthermore, an airplane, will not,by itself,return to any given course in azimuth after a disturbance. It lacks the sense of direction. In order to bring about course stability, it is necessary to provide a control operated by a device,having a sense of direction.,such as a human pilot or compass-controlled automatic pilot. Due to the control operation, the stability characteristics are modified so that the original criteria for uncontroled airplane may not apply at all to the controlled one.

On the other hand,no control system is ideal itself. It is subject to oscillation and lag.in addition to certain parasite defects such as the adverse yawing moment introduced by aileron. It is believed,therefore, the stability of the controlled airplane should be taken as the criteria for good control charaters. As it is quite possible that the uncontrolled airplane as well as the control system alone may have less desirable stabilty characters,but their combination may give the best compromise.

It is the purpose of this thesis to investigate the controlled characteristics of the airplane and derive the stability criteria from the most desirable controlled motion.

S. N. Kin.
2/13/39.

Chapter I.

Longitudinal Stabilization As A Function Of The Angle Of Inclination In Pitch

1. Introduction.

Automatic stabilization as a function of the inclination of the airplane to the horizontal forms the basic principle of the Sperry and Smith Automatic Pilots which are now most widely used in commercial aviation. The Sperry Automatic pilot has a gyroscope hung on gimbals in a fixed position relative to the horizontal. This gyroscope, being capable of changing its position with respect to the airplane without encountering appreciable resistance, produces a deflection of the elevator control proportional to the disturbed inclination in pitch θ and thus opposes further increase in disturbances.

Jones[1] pointed out that the manual pilot is most sensitive to the disturbance of the angle of inclination θ, Therefore, it is believed that the study of the automatc stabilization as a function of inclination leads in certain degree to better understanding of the controlled longitudinal stability of an airplane controlled by manual pilot.

In the following mathematical analysis, particular attention is paid to the method of representing the control lag and to the study of the effect of control lag on the control and the controlled airplane in relation to their performances. As few theoretical studies of the automatic piloting[2] in published literature to date presented satisfactory representation of the control lag, the following analysis which involves complicated mathmatical manipulation, is believed to be worth doing.

2. Equations of Motion.

From the description of the Sperry Automatic Pilot[3] it is seen that in the control system, the elevators are connected to a servo-piston which is moved by hydraulic pressures. The variation of the hydraulic pressure is a function of the disturbed inclination in pitch, θ. The elevators are so connected that a negative pitching moment is produced whenever a positive θ is present.

Let, s=Movement of the servo-piston from its neutral position.

m_c= Equivalent mass inertia of the control system including the elevator surfaces.

$\frac{\partial F}{\partial \dot{s}}$= Equivalent viscous damping force per unit rate of servo-piston displacement, $\dot{s}$.

$\frac{\partial F}{\partial \theta}$= Equivalent driving force acting on the piston per unit disturbance in θ.

$\frac{\partial F}{\partial s}$= Equivalent restoring force of the follow-up system per unit piston displacement, s.

$\frac{\partial M}{\partial s}$= The generated pitching moment, M per unit piston displacement, s.

Then, the simultaneous equations of motion of a controlled airplane can be written as follows,

$$m(\dot{u}+Wq)=u\partial X/\partial u+w\partial X/\partial w+q\partial X/\partial q+mg\theta\cos\theta_o \qquad (2.1)$$

$$m(\dot{w}-Uq)=u\partial Z/\partial u+w\partial Z/\partial w+q\partial Z/\partial q+mg\theta\sin\theta_o \qquad (2.2)$$

$$B\ddot{\theta} =u\partial M/\partial u+w\partial M/\partial w+q\partial M/\partial q+s\partial M/\partial s \qquad (2.3)$$

$$m_c\ddot{s}=\theta\partial F/\partial\theta+(-)\dot{s}\partial F/\partial\dot{s} - s\partial F/\partial s \qquad (2.4)$$

The first two equations (2.1) and (2.2) are same as the stability equations for the uncontrolled airplane in the X and Z directions because no additional forces due to the control system are involved. In the third equation (2.3), the term, $s\partial M/\partial s$, in addition to the uncontrolled stability equation in rotary direction about Y-axis, represents the pitching moment generated by the control operation.
The fourth equation, (2.4) is the equation of motion of an equivalent control considered as a simple vibration system of one degree of freedom.

Following the method of treating the uncontrolled stability equations, the determinental equations which characterize the motion of the airplane can be written as follows,

$$\begin{vmatrix} D - X_u & -X_w & -g & 0 \\ -Z_u & D - Z_w & -g\theta_o - DU_o & 0 \\ -M_u & -M_w & D^2 - DM_q & -M_s \\ 0 & 0 & -F_\theta & D^2 + DF_{\dot{s}} + F_s \end{vmatrix} = 0 \qquad (2.5)$$

Where, $D=d/dt$,

$X_u = \frac{1}{m}\,\partial X/\partial u$, $M_u = \frac{1}{B}\,\partial M/\partial u$,etc. as defined in the uncontrolled stability derivatives.

Similarly, the following control derivatives are defined,

$$M_s = \frac{1}{B}\,\partial M/\partial s$$
$$F_\theta = \frac{1}{m_c}\,\partial F/\partial \theta$$
$$F_s = \frac{1}{m_c}\,\partial F/\partial s$$
$$F_{\dot{s}} = \frac{1}{m_c}\,\partial F/\partial \dot{s}$$

The dimensions of the above determinant are found to be as follows,

$$\begin{vmatrix} 1/T & 1/T & L/T^2 & 0 \\ 1/T & 1/T & L/T^2 & 0 \\ 1/TL & 1/TL & 1/T^2 & 1/LT^2 \\ 0 & 0 & L/T^2 & 1/T^2 \end{vmatrix}$$

Following the same procedure as in non-dimensionalizing the stability derivatives of the uncontrolled airplane, we multiply the derivatives of

the first row by T

the second row by T

the third row by LT

the fourth row by T

the third column by T/L

the fourth column by T

As in uncontrolled stability derivatives, we take

L=the distance between the c.g. of the airplane and the c.p. of the horizontal tail area as the unit of length.

m=the mass of the airplane as the unit of mass.

$T=m/\frac{\rho}{2}SU$ = the unit of time.

and define $\mu = m/\frac{\rho}{2}SL$,

The non-dimensional determinant is then as follows,

$$(2.6)\quad \begin{vmatrix} d - x_u & -x_w & -\mu C_L\theta_o - d(x_q - \mu\sin\alpha_o) & 0 \\ -z_u & d - z_w & \mu C_L\sin\theta_o - d(z_q + \mu\cos\alpha_o) & 0 \\ -m_u & -m_w & d^2 - dm_q & -\mu m_s \\ 0 & 0 & -f_\theta & d^2 + df_{\dot{s}} + f_s \end{vmatrix}$$

The four new non-dimensional derivatives due to the introduction of control operation are evidently defined as,

$$\mu m_s = M_s \times LT^2 = \frac{1}{B}\,\partial M/\partial s \times (L) \times (m/\tfrac{\rho}{2}SU)^2 \ldots\ldots (2.7)$$

$$f_\theta = F_\theta \times T^2/L = \frac{1}{m_c}\,\partial F/\partial\theta \times (m/\tfrac{\rho}{2}SU)^2/L \text{ ....(2.8)}$$

$$f_s = F_s \times T^2 = \frac{1}{m_c}\,\partial F/\partial s \times (m/\tfrac{\rho}{2}SU)^2 \text{..............(2.9)}$$

$$f_{\dot{s}} = F_{\dot{s}} \times T = 1/m_c \times \partial F/\partial\dot{s}(m/\tfrac{\rho}{2}SU) \text{.............(2.10)}$$

3. The 'Ideal' or no lag control.

If the control system possesses neither mass inertia nor damping effect, the quantities , m_c and $f_{\dot{s}}$ are zero. Then the variable s can be eliminated from the equattions of motion by the relation $s = (f_\theta/f_s) \times \theta$ so that the control term $\mu m_s s$ becomes $\mu m_s\ (f_\theta/f_s)\ \theta = \mu m_\theta \theta$ where m_θ is by definition, equal to $m_s(f_\theta/f_s)$...(2.11)

It is seen that for 'Ideal' control, the stability equations characterizing the motion of the airplane, reduces from the sixth order to the fourth one.

4. Expressions for the control derivatives

Derivation of m_s and m_θ,

Let, C_L' = Lift coefficient for the horizontal tail.

S' = Horizontal tail area.

L = Tail length, = the distance between c.g. of airplane and c.p. of the tail area.

β = Elevator angle

α' = Tail incidence angle.

$\partial C_L'/\partial\alpha'$ = Slope of the lift curve for the tail plane.

Then, the pitching moment generated by the horizontal tail plane is,

$$M = (\partial C_L'/\partial\alpha')\alpha' \, S'\rho/2 \, U^2 \, L \quad \text{by neglecting slip stream effect} \ldots\ldots(4.1)$$

Therefore, $\partial M/\partial s = (\partial C_L'/\partial\alpha')S'\frac{\rho}{2}U^2L(\partial\alpha'/\partial s)$.........(4.2)

Let, $B = b_1 m L^2$ ....(4.3)

Substituting equations (4.2) and (4.3) into (2.7), and write $\partial\alpha'/\partial s = (\partial\alpha'/\partial\beta) \times (\partial\beta/\partial s)$, we get the expression

$$m_s = 1/b_1(\partial C_L'/\partial\alpha')(\partial\alpha'/\partial\beta)(\partial\beta/\partial s)L(S'/S). \quad (4.4)$$

and $$m_\theta = m_s(f_\theta/f_s) \ldots\ldots(4.5)$$

It is pointed out by Weiss[4] that the nondimensional derivatives f_θ, f_s and $f_{\dot{s}}$ can be most conveniently expressed in terms of the damping ratio ζ and the undamped natural period T_n of the equivalent control system defined as follows,

Consider the control as a vibration system of one degree of freedom. Then the equation of motion of the equivalent control system can be written as,

$$m_c\frac{d^2s}{dt^2} + (\partial F/\partial\dot{s})\frac{ds}{dt} + (\partial F/\partial s)s = 0 \text{ if the external}$$

force applied to the contral system is kept zero.

It is found convenient in studying the vibration equation to define[5] $T_n' = 2\pi/\sqrt{F_s}$ as the undamped natural period of the equivalent control system.

and $$\zeta = F_{\dot{s}}/(F_{\dot{s}})_{critical} = F_{\dot{s}}/2(2\pi/T_n')$$

=damping ratio.

as it is found out that when the vibration system is critically damped, the derivative $(F_{\dot{s}})_{critical}$ is equal to $2(2\pi/T_n')$. Thus, we obtain, by definition,

$$F_s = (2\pi/T_n')^2 \qquad (4.6)$$

and, $$F_{\dot{s}} = 2\zeta(2\pi/T_n') \qquad (4.7)$$

and $$F_\theta = F_s(\partial s/\partial\theta) = (2\pi/T_n')^2(\partial s/\partial\theta) \qquad (4.8)$$

Substituting (4.6),(4.7) and (4.8) into equations (2.9), (2.10) and (2.8) respectively, we get,

$f_s = (2\pi/T_n')^2(T)^2 = (2\pi/T_n)^2$ if $T_n = T_n'/T$ by expressing the natural period of equivalent contral system in terms of the time unit used in the nondimensionalizing system instead of in seconds. ..(4.10)

Similarly we obtain,

$$f_{\dot{s}} = 2\zeta(2\pi/T_n) \qquad (4.11)$$

and $$f_\theta = (2\pi/T_n)^2(\partial s/\partial\theta)/L \qquad (4.12)$$

And since $f_\theta/f_s = (\partial s/\partial\theta)/L$, the expression for m_θ can be simplified from (4.4) and (4.5) to

$$m_\theta = 1/b_1(\partial C_L'/\partial\alpha')(\partial\alpha'/\partial\beta)(\partial\beta/\partial\theta)(S'/S) \qquad (4.13)$$

5. Qualitative Discussion of the non-dimensional determinant.

The non-dimensional determinant (2.6), thought complicated in its expression, can be arranged into a convenient form for discussion purpose, being first pointed out by Weiss[4]. It is noted that the minor of (2.6) formed by the first three rows and columns is the determinant for the uncontrolled airplane. Denoting this minor by Δ_0 and the expression, $\begin{vmatrix} d - x_u & -x_w \\ -z_u & d - z_w \end{vmatrix}$ by Δ_1, (2.6) can be written as,

$$\Delta_0 + (\Delta_1)x(-\mu m_s f_\theta)/(d^2+df_s+f_s) = 0 \ldots\ldots(5.1)$$

Substituting (4.5), (4.10) and (4.11) into (5.1), we get

$$(\Delta_0) - \mu m_\theta(\Delta_1) + (\Delta_0)x\left[(dT_n/2\pi)^2 + 2\zeta(dT_n/2\pi)\right] = 0.(5.2)$$

Denote $k = T_n/2\pi$ ....(5.3)

(5.2) can be written as,

$$(\Delta_0) - \mu m_\theta(\Delta_1) + (\Delta_0)x(k^2d^2 + 2\zeta kd) = 0 \ldots\ldots\ldots(5.4)$$

The division into three separate terms in (5.4) makes it clearer to see the physical significances.

For uncontrolled airplane, the terms m_θ, k, and ζ are zero, so that (5.4) reduces to $\Delta_0 = 0$ ....(5.5)

For 'Ideal' or no lag control, k and ζ are zero, so that (5.4) reduces to $(\Delta_0) - \mu m_\theta(\Delta_1) = 0$ ......(5.6)

Let the coefficients of the quartic equation in d obtained from (5.5) for the uncontrolled airplane be written as $A_o d^4 + B_o d^3 + C_o d^2 + D_o d + E_o = 0$ ..(5.7)

Let the quartic equation in d obtained from (5.6) for the 'ideally' controlled airplane be written as

$$A_1 d^4 + B_1 d^3 + C_1 d^2 + D_1 d + E_1 = 0 \quad \text{...(5.8)}$$

It is found out by expanding the determinant, that

$$\begin{aligned} A_1 &= A_o = 1 \\ B_1 &= B_o \mp \\ C_1 &= C_o - \mu m_\theta \\ D_1 &= D_o + \mu m_\theta (x_u + z_w) \\ E_1 &= E_o - \mu m_\theta (x_u z_w - z_u x_w) \end{aligned} \quad \text{..................(5.9)}$$

Let the sextic equation in d obtained from (5.4) for the real control be written as

$$a'd^6 + b'd^5 + c'd^4 + d'd^3 + e'd^2 + f'd + g' = 0 \quad \text{...(5.10)}$$

It is found that ,

$$\begin{aligned} a' &= k^2 \\ b' &= 2\zeta k + k^2 B_o \\ c' &= 1 + 2\zeta k B_o + k^2 C_o \\ d' &= B_1 + 2\zeta k B_o + k^2 D_o \\ e' &= C_1 + 2\zeta k D_o + k^2 E_o \\ f' &= D_1 + 2\zeta k E_o \\ g' &= E_1 \end{aligned} \quad \text{..................(5.11)}$$

The expressions shown in (5.11) are very convenient, as it is seen that when k and ζ are zero, a' and b' are then zero, leaving

$c' = 1$

$d' = B_1$

$e' = C1$

$f' = D_1$

$g' = E_1$

which are the coefficients of the 'Ideal' control.

Dividing a',b',c' etc by a' throughout, (5.10) becomes $d^6 + B'd^5 + C'd^4 + D'd^3 + E'd^2 + F'd + G' = 0$...(5.12)

It is seen that when k and ζ are very large and approadh infinity, such as when the elevator is locked, we get,

$A' = 1$

$B' = B_o$

$C' = C_o$

$D' = D_o$

$E' = E_o$

$F' = G' = 0$

Which give coefficients for the uncontrolled airplane automatically. It should be noticed that each of the above special cases, results a quartic equation from the original sextic one so that the additional degree of freedom introduced by the control operation disappears.

6.Numerical Investigation

In order to answer the following questions that,

a. What is the effect on the longitudinal dynamic stability of an airplane of m_θ introduced by the control ?

b. What is the modification due to the presence of the control inertia,represented by k and the control damping ,represented by ζ on the controlled longitudinal stability ?

c. What is the relation between the third oscillation introduced by the additional degree of freedom of the control system,(hereafter called control oscillation) and the inertia and damping and spring effect of the control system ?

d. How is the disturbed motion of a controlled airplane due to certain gust affected by the control derivative m_θ and the natural period and damping of the control system ?

it is necessary to carry out numerical investigation based on the numerical values of certain airplane derivatives which are well known to us. In the following investigation the aerodynamic derivatives of the Fairchild 22 airplane are used. The aerodynamic character of the F-22 airplane, has been thoroughly investigated in the N.A.C.A. stability research program. Furthermore, as the non-dimensional derifatives are used, the results obtained are believed to be very close representatives for all modern average monoplanes, of the similar type.

7. The aerodynamic derivatives and the characteristics of the F-22 high wing mono-plane[6]

a. Characteristics

Wing area ,S ....171 sq.ft.

Span,b ....32.83 ft.

Stabilizer area ....15.8 sq.ft.

Elevator area ....10.4 sq.ft.

Tail length,L ....14.69 ft.

Weigth,W ....1600 lbs.

Radius of gyration in pitch ....4.41 ft.

Wing setting ....1°.

b. Assumed flight condition

Horizontal flight at air speed ....133 f.p.s.

Power -off flight at about 3000 ft.altitute where the density of air is ρ..................0.00218 slug/ft^3.

Based on the above flight conditions,the fundamental units in the non-dimensional system are

Unit of mass,m...........................50 slugs

Unit of time,T...........................2 seconds

Unit of length,L.........................15 ft.

The parameter, μ........................20

c. The aerodynamic derivatives used[7].

x_u .... $-.15$

x_w .... $+.40$

z_u .... -1.0

z_w .... -4.5

m_w .... -3.0

m_q .... -6.0

$\theta_o, \alpha_o, x_q, z_q$ and m_u are all assumed negligible compared with the other terms in the stability equations.

C_L .... $-.45$

It should be mentioned here, that the numbers given here are obtained from the calculated non-dimensional derivatives based on measured results rouned off to the nearest significant figures because in the first place, the measured quantities are accurate within 10% or so and in the second place, it is found out that in solving the disturbed motions due to gusts by means of the M.I.T. Differential Analyzer, the above rounded figures simplify the procedure considerably.

d. Estimation of the control Characteristics

Based on the airplane characteristics, it is found that

$b_1 = B/mL^2$ 0.10

$\partial C_L'/\partial\alpha'$ for tail aspect ratio of 3...... -3.5

S'/S 15%

$\partial\alpha'/\partial\beta$ 0.5

Substituting into (4.13) and (4.4), it is found that

$$m_\theta = -2.55(\partial\beta/\partial\theta) \qquad (7.1)$$

$$m_s = -2.55(\partial\beta/\partial s)L \qquad (7.2)$$

From the data furnished by the Sperry Co.[8], it is found out by mormal operation of the Gyro-pilot,

$\partial\beta/\partial\theta$ = 0.25° to 0.8° elevator movement per degree change of attitude................0.25 to 0.8

Thus, m_θ = 0.5 to 2.0 are the range for normal operation.

Again, it is reported by Sperry Co. that 'A linear motion of the servo-piston of 1 inch varies angular control motion from 4° to 15° ' Thus

$\partial\beta/\partial s$ =4° to 15°/inch = .84 to 3.1 radians per ft.

With L = 15 ft., m_s= 32.2 to 110 are the range for normal operation.

It is interested to ~~pe~~ point out here that while m_θ; being a function of the gearing ratio of control,is physically possible to vary from 0 to infinity,is limited to certain range for satisfactory control operation due to the limitation ~~for~~ due to the control lagging effect. It is later found that the m_θ given by Sperry Co. checks beautifully with the theorectically found ranges for satisfactory operation. It is also noted that Weiss[4] investigated the longitudinal stability for 'Ideal' control by using m_θ from zero to -0.5 which is on the small side normal operation range. Klemin[9] on the other hand, estimated m_θ after converted into the non-dimensional system, ranges from -2 to -20 which are on the larger side of the normal operation range. It is believed by the writer that Klemin's method of estimating m_θ by allowing the angle of attack α to vary as well as the independent variable θ is subject to criticism because it is contrary to the basic principle of partial differentiation.

The control derivatives f_s, $f_{\dot{s}}$ and f_θ are known by equations (4.10), (4.11) and (4.12) if proper values of ζ and T_n are assumed. The exact values of ζ and T_n depends on the characteristics of the control system used. For a given control system including the elevator surfaces, the values for T_n and ζ can be easily measured in a vibration testing laboratory by recording the response of the control system due to an arbitrary initial disturbance in pitch.. The approximate values of T_n and ζ for such an equivalent control system can be easily computed from the response record based on the elementary principle of vibration mechanics.[10]

For the purpose of numerical investigation for a hypothetical control system, it is recommended that ζ ranges from 0 to a value not much greater than unity be used as ζ equal to unity is the case when the control system is critically damped. T_n should have a range not much greater than the short period of the short oscillation determined from derivatives of the uncontrolled airplane or rather the ideally controlled airplane as it is logical to believe that when the natural period of the control system is large compared with the period of the controlled member, excessive control lag will result unsatisfactory operation.

8. Tabulation of numerical results.

In the following tabulation of results, the symbols used are,

P_L = Period of long oscillations in T unit.

P_S = Period of short oscillation in T unit.

P_C = Period of control oscillation in T unit.

$T_{1/2}$ = Time to damp to half amplitude in T unit.

$N_{1/2}$ = Number of oscillations damped to half amplitute.

a. Uncontrolled Airplane

$A_o = 1$

$B_o = -(x_u + z_w + m_q) = 10.65$

$C_o = x_u z_w + (x_u + z_w) m_q - \mu m_w - z_u x_w = 89.0$

$D_o = m_q (z_u x_w - x_u z_w) + \mu m_w x_u = 15.5$

$E_o = -C_L \mu m_w z_u = 27.0$

The Quartic can be exactly factered into[11]

$$(d^2 + 10.51d + 87.1)(d^2 + 0.14d + 0.311) = 0$$

Roots	Long Oscillation			Short Oscillation		
	P_L	$T_{1/2}$	$N_{1/2}$	P_S	$T_{1/2}$	$N_{1/2}$
$-5.25 \pm 7.71i$	11.3	9.9	.878	.816	.132	.162
$-0.07 \pm .553i$						

Table 8.1.

b.' Ideal ' Control

$A_1 = 1$

$B_1 = 10.65$

$C_1 = 89 - 20\, m_\theta$

$D_1 = 15.5 - 93\, m_\theta$

$E_1 = 27 - 21.5\, m_\theta$

Table 8.2

m_θ	A_1	B_1	C_1	D_1	E_1
0	1	10.65	89.0	15.5	27
-.25	1	10.65	94.0	38.7	32.4
-.50	1	10.65	99.0	62.0	37.8
-1.0	1	10.65	109	108.5	48.5
-1.5	1	10.65	119	155	60
-2	1	10.65	129	203.5	70
-5	1	10.65	189	480.5	134.5
-10	1	10.65	289	945.5	242.0

m_θ	Quartic in d factered into
0	$(d^2 + 10.51d + 87.1)(d^2 + 0.14d + 0.311) = 0$
-.25	$(d^2 + 10.26\, d + 89.6)(d^2 + .391d + 0.362) = 0$
-.50	$(d^2 + 10.0\, d + 92.3)(d^2 + .63\, d + 0.41) = 0$
-1.0	$(d^2 + 10.6d + 97.2)(d^2 + 1.06d + .50) = 0$
-1.5	$(d^2 + 10.3\, d + 104)(d^2 + 1.44\, d + .58) = 0$
-2.0	$(d^2 + 8.91\, d + 113)(d^2 + 1.74\, d + .62) = 0$
-5	$(d^2 + 7.8\, d + 166.0)(d^2 + 2.85\, d + .81) = 0$
-10	$(d^2 + 7.15\, d + 263)(d^2 + 3.57\, d + .92) = 0$

Table 8.3

m_θ	Roots	periods	Time to half	$N_{1/2}$
0	$-5.25 \pm 7.71i$	.816	.132	.162
	$-.07 \pm .553\,i$	11.3	9.9	.878
-.25	$-5.13 \pm 7.95\,i$	.79	.135	.171
	$-.196 \pm .57\,i$	11.0	3.53	.321
-.50	$-5.0 \pm 8.2\,i$	.765	.139	.180
	$-.315 \pm .556\,i$	11.25	2.2	.196
-1.0	$-5.3 \pm 8.3\,i$	.756	.13	.172
	$-.53 \pm .47\,i$	13.35	1.31	.098
-1.5	$-5.15 \pm 8.8\,i$	.715	.134	.187
	$-.72 \pm .245\,i$	25.6	.963	.038
-2.0	$-4.96 \pm 9.4\,i$	.669	.14	.21
	-1.24	infinity	.560	0
	-.50	infinity	1.39	0
-5.0	$-3.9 \pm 12.3\,i$	.51	.178	.350
	-2.54	infinity	.273	0
	- .32	infinity	2.16	0
-10.0	$-3.56 \pm 15.81i$	.397	.194	.490
	-3.31	infinity	.209	0
	- .27	infinity	2.56	0

c. Varrying m_θ for a control system having $k = 0.1$, $\zeta = 0.5$

$a' = .01$	$A' = 1$
$b' = .2065$	$B' = 20.65$
$c' = 2.955$	$C' = 295.5$
$d' = 19.7$	$D' = 1970$
$e' = 90.8 - 20\, m_\theta$	$E' = 9080 - 2000\, m_\theta$
$f' = 18.2 - 93 m_\theta$	$F' = 1820 - 9300\, m_\theta$
$g' = 15.5 - 21.5\, m_\theta$	$G' = 1550 - 2150\, m_\theta$

m_θ	E'	F'	G'
0	9080	1820	1550
-.5	10080	6470	2625
-1.0	11080	11120	3700
-2.0	13100	20400	7000
-3.0	15100	29700	8000

m_θ	Sextic Equations in d^{11}
0	$(d^2 + .17d + .18)(d^2 + 10.3\,d + 90)^2 = 00$
-.5	$(d^2 + .67d + .30)(d^2 + 13.3d + 111)(d^2 + 6.7d + 80) = 0$
-1.0	$(d^2 + 1.15d + .43)(d^2 + 15.3d + 137)(d^2 + 4.65d + 66.5) = 0$
-2.0	$(d^2 + 1.95d + .70)(d^2 + 16.3d + 153)(d^2 + 2.4\,d + 65.5) = 0$
-3.0	$(d^2 + 2.55d + .70)(d^2 + 21.0d + 230)(d^2 + -3.0d + 83.0) = 0$

Table 8.4

m_θ	Roots	Periods	$T_{1/2}$	$N_{1/2}$
0	$-5.2\pm7.9\,i$	.795	.133	.17
	$-.085\pm.416i$	15.1	8.16	.54
	$-5.2\pm7.9i$	.795	.133	.17
-.5	$-6.7\pm8.15i$	.772	.103	.134
	$-.335\pm.442i$	14.2	2.05	.145
	$-3.35\pm8.35i$	.751	.206	.275
-1.0	$-7.71\pm8.83i$	.712	.090	.127
	$-.575\pm.28i$	22.4	1.20	.054
	$-2.33\pm7.82i$	.80	.297	.370
-2.0	$-8.15\pm9.36i$	.672	.085	.172
	-1.475,-.475	Infinity	.47,1.46	0
	$-1.2\pm7.82i$	.80	.578	.725
-3.0	$-10.5\pm11.0i$	.571	.066	.115
	-2.24,-.315	Infinity	.31,2.2	0
	$+1.5\pm8.6i$	.73	....	....

d. Varying the natural period of the control system at constant $m_\theta = -2.0$ and damping ratio $\zeta = 0.5$

$$a' = k^2$$

$$b' = k + 10.65\,k^2$$

$$c' = 1 + 10.65k + 89\,k^2$$

$$d' = 10.65 + 89\,k + 15.5\,k^2$$

$$e' = 128.9 + 15.5\,k + 27\,k^2$$

$$f' = 201.5 + 27\,k^2$$

$$g' = 70$$

k	A'	B'	C'	D'	E'	F'	G'
.01	1	206.5	11154	115415	1.29×10^6	2.02×10^6	7×10^5
.05	1	30,65	700	6050	51800	81250	2.8×10^4
.10	1	20.65	295	1970	13100	20400	7000
.20	1	15.65	167.3	728	3330	5190	1750
.40	1	11.60	123.6	305	870	1326	437
.80	1	11.90	121	143.3	250	350	109.3

k	Sextic Equations in d
.01	$(d^2+1.755d+.627)(d^2+195d+9600)(d^2+8d+120)=0$
.00	$(d^2+1.74\,d+.62)(\ldots\ldots\ldots\ldots)(d^2+8.9d+113)=0$
.05	$(d^2+1.84\,d+.66)(d^2+22.9d+405)(d^2+6d+105)=0$
.10	$(d^2+1.95\,d+.70)(d^2+16,3d+153)(d^2+2.4d+66)=0$
.20	$(d^2+2.06\,d+.745)(d^2+12.2d+98.0)(d^2+1.0d+24)=0$
.40	$(d^2+2.04\,d+.705)(d^2+9.3d+94)(d^2+.28d+6.6)=0$
.80	$(d^2+1.58d+.47)(d^2+10.8d+107)(d^2-.44d+2.2)=0$
Infinity	$(d^2+0.14d+.311)(d^2+10.5d+87.1)(\ldots\ldots\ldots\ldots)=0$

Table 85

k	Roots	Periods	$T_{1/2}$	$N_{1/2}$
0	-1.24,-.50	Infinity	.56,1.39	0
	-4.96 ± 9.4i	.669	.14	.21
	-98.5 ±28	Infinity	.008,.014	0
.01	-4.0 ±10.2i	.616	.173	.281
	-1.25,-.50	Infinity	.555,1.39	0
	-11.5 ±16.55i	.38	.06	.16
.05	-1.352,-.49	Infinity	.512,1.41	0
	-3.0 ±9.8i	.641	.23	.36
	-8.15 ± 9.36i	.672	.085	.126
.10	-1.475,-.475	Infinity	.47,1.46	0
	-1.22 ± 7.82i	.80	.567	.71
	-6.08 ± 7.81i	.80	.114	.143
.20	-1.582,-.48	Infinity	.438,1.44	0
	-.50 ± 4.88i	1.30	1.40	1.08
	-4.65 ± 8.52i	.739	.149	.20
.40	-1.60,-.44	Infinity	.432,1.58	0
	-.14 ± 2.57i	2.44	4.95	2.0
	-5.4 ± 8.8i	.72	.128	.178
.80	-1.18,-.40	Infinity	.586,1.73	0
	+0.22 ± 1.466i	4.28	......	,....
	-5.25 ± 7.71i	.816	.132	.162
Infinity	-.07±.553i	11.3	9.9	.878

e. Varying the damping ratio ζ at $m_\theta = -2.0$, and $k = 0.1$

$A' = 1$

$B' = 10.65 + 20\zeta$

$C' = 189 + 213\zeta$

$D' = 1080 + 1780\zeta$

$E' = 12920 + 310\zeta$

$F' = 20150 + 540\zeta$

$G' = 7000$

ζ	A'	B'	C'	D'	E'	F'	G'
.00	1	10.65	189	1080	12920	20150	7000
.25	1	15.65	242	1525	12998	20285	7000
.50	1	20.65	295	1970	13100	20400	7000
1.0	1	30.65	402	2860	13230	20690	7000
1.6	1	42.65	530	3925	13416	21020	7000

Sextic Equations in d

ζ	
0	$(d^2+1.72d+.62)(d^2+11.6d+108)(d^2-2.6d+92) = 0$
.25	$(d^2+1.81d+.65)(d^2+13.75d+134)(d^2+.06d+81) = 0$
.50	$(d^2+1.95d+.70)(d^2+16.3d+153)(d^2+2.44d+65.5)=0$
1.0	$(d^2+2.32d+.855)(d^2+22.0d+160)(d^2+5.65d+51.5)=0$
1.6	$(d^2+2.6d+1.2)(d^2+31d+124)(d^2+9d+48)=0$

Table 8.6

ζ	Roots	Periods	$T_{1/2}$	$N_{1/2}$
0	$-5.8 \pm 8.6i$	.730	.12	.165
	$-1.22, -.50$	Infinity	.57, 1.4	0
	$+1.3 \pm 9.5i$	.66	......	......
.25	$-6.88 \pm 9.3i$	.675	.10	.148
	$-1.27, -.55$	Infinity	.47, 1.44	0
	$-.03 \pm 9.0i$	.70	23.3	33.4
.50	$-8.15 \pm 9.36i$	.67	.085	.127
	$-1.475, -.475$	Infinity	.47, 1.44	0
	$-1.2 \pm 8.0i$	.80	.582	.727
1.0	$-11.5 \pm 5.38i$	1.17	.06	.051
	$-1.57, -.55$	Infinity	.44, 1.26	0
	$-2.82 \pm 6.6i$	.95	.25	.264
1.6	$-26.3, -4.7$	Infinity	.025, .148	0
	$-2.0, -.60$	Infinity	.345, 1.15	0
	$-4.5 \pm 5.3i$	1.19	.155	.130

9. Discussion of the numerical investigation

In order to see the effect of varying the control characteristics, the following curves plotted from the numerical investigation are presented here.

a. Fig.9.1 The Effect of varying m_θ on Long Oscillation 'Ideal Control'

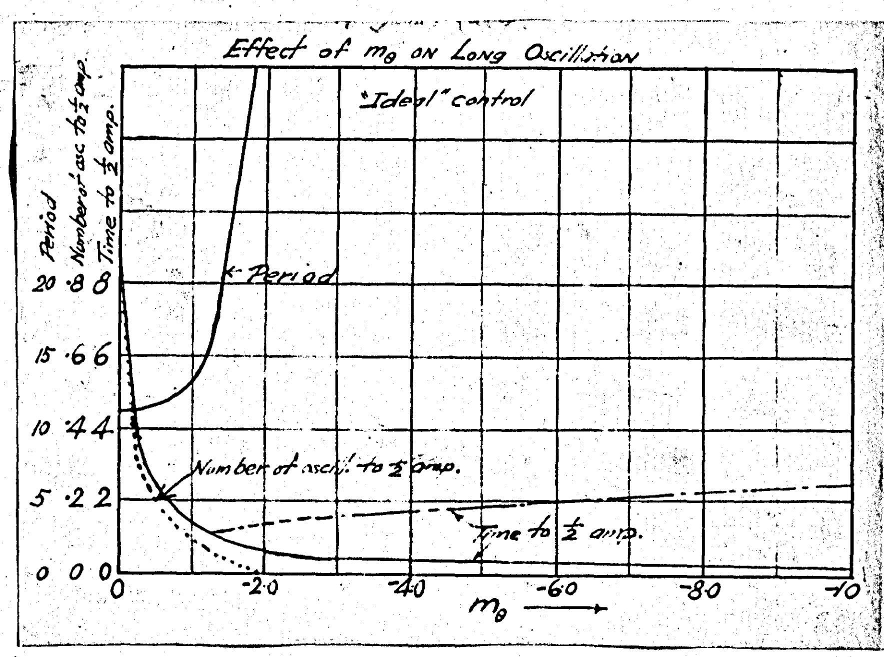


Fig.9.1 shows the effect of m_θ on the long oscillation. As m_θ increases from zero to -2, the period remains almost constant at first and then increases rapidly into longer and longer ones. At m_θ = -2, the period is infinity. On the other hand, the damping increases rapidly at the beginning, and improves slowly as m_θ approaches -2. Further increase of m_θ converts the long oscillation into two rapidly damped aperiodic motions, the damping of one mode of the

9.Discussion of the numerical investigation

In order to see the effect of varying the control characteristics, the following curves plotted from the numerical investigation are presented here.

a. Fig.9.1 The Effect of varying m_θ on Long Oscillation

'Ideal Control'

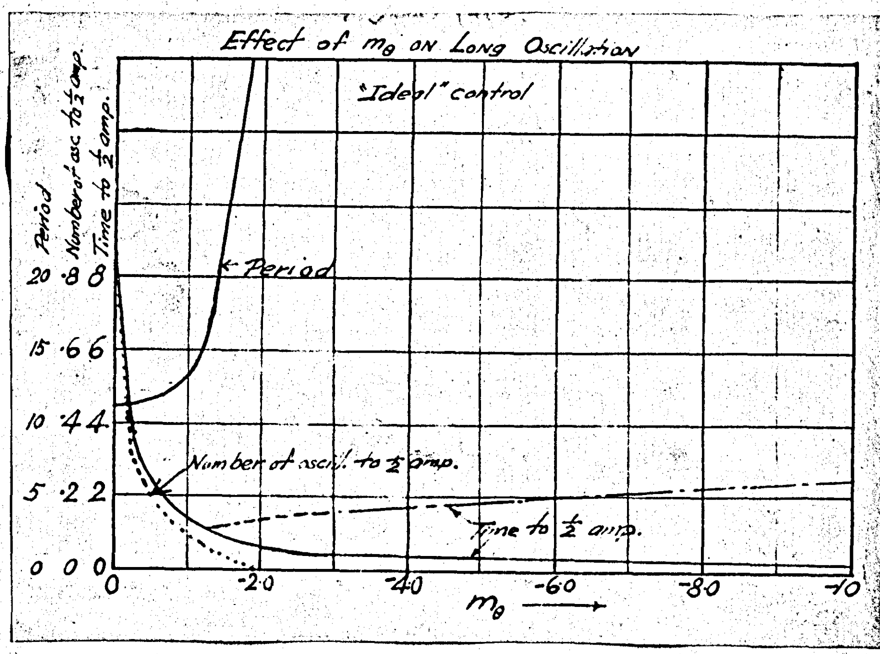


Fig.9.1 shows the effect of m_θ on the long oscillation. As m_θ increases from zero to -2, the period remains almost constant at first and then increases rapidly into longer and longer ones. At m_θ = -2,the period is infinity. On the other hand, the damping increases rapidly at the beginning, and improves slowly as m_θ approaches -2. Further increase of m_θ converts the long oscillation into two rapidly damped aperiodic motions,the damping of one mode of the

aperiodic motion deacreases though slowly, as m_θ increases furthermore. This shows that too large m_θ is not very desirable even for 'Ideal' control.

Fig.9.2

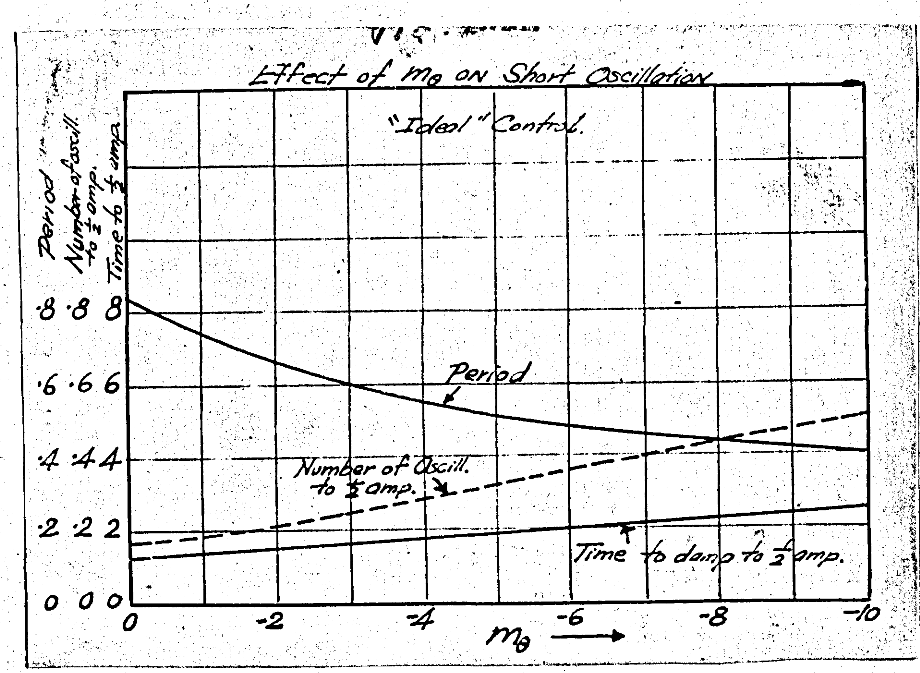


Fig.9.2 shows the effect of m_θ on short oscillation. Both the period and the damping decrease as m_θ increases. The rate of decrease of the damping is so slow that the short oscillation remains to be well damped even at very large values of m_θ. Weiss[12] has shown that in a system of one degree of freedom, such as the automatic control of a naval ship, it is impossible to introduce damping into the controlled member by merely using simple error comtrol such as m_θ here. Damping in simple system can only be obtained through error 'Derivative' control. But the

complex airplane system allows a didplacement error control, m_θ, to influence the damping of the controlled member considerably. It is believable that the m_θ control acts as a sort of equalizing valve to improve the poor damping distribution between the long and short oscillations, instead of introducing additional damping to the system.

b. The Effect of varying m_θ for a control possessing Inertia and Damping.

Fig.9.3

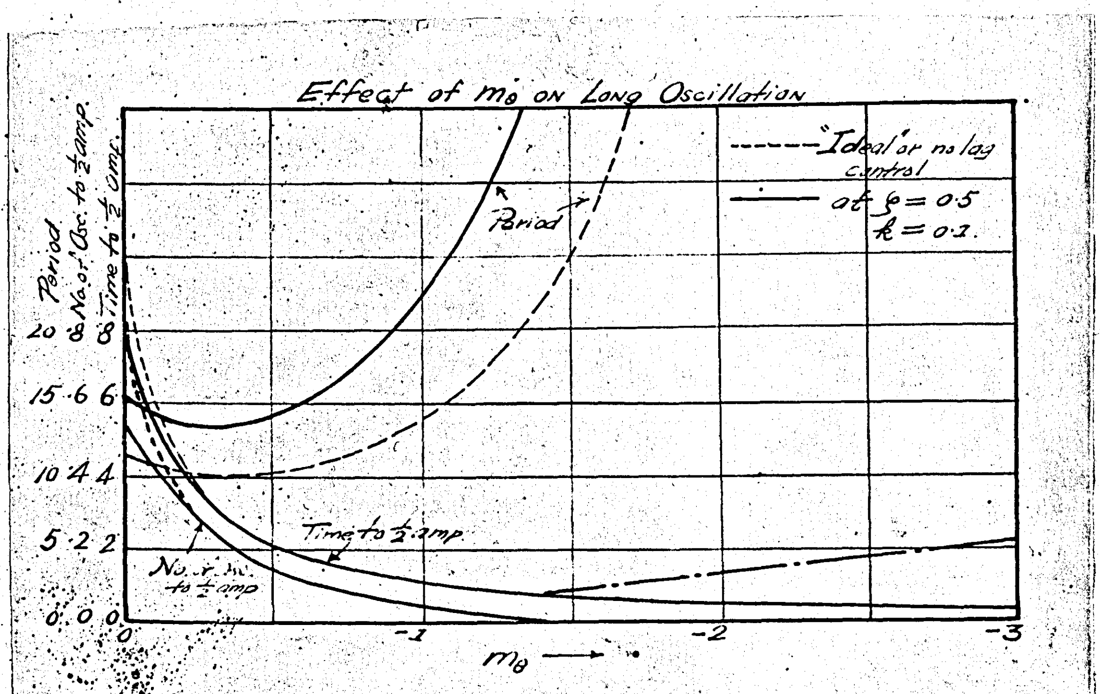


Fig.9.3 shows the effect of varying m_θ of a real control as compared to that of 'Ideal' control, on long oscillations. It is seen that the effect on long oscillation is, in general, not altered much by the presence

of control inertia and damping. It is seen that the period is lengthened slightly and the damping is reduced slightly.. The effect of control inertia and damping on long oscillation can be considered as negligible.

Fig.9.4

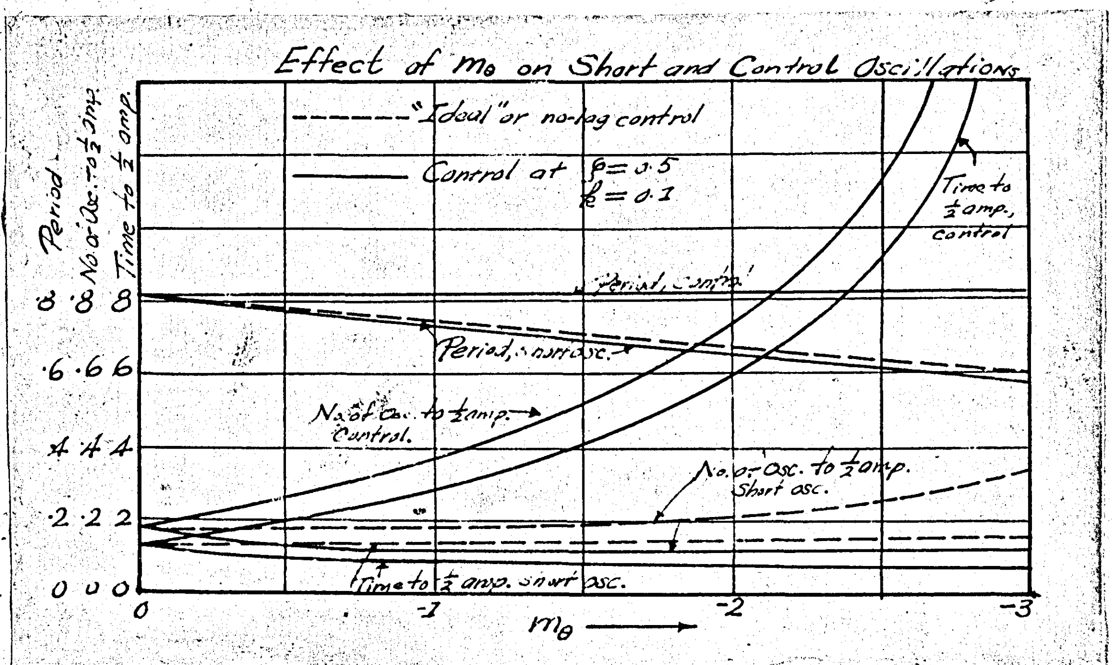


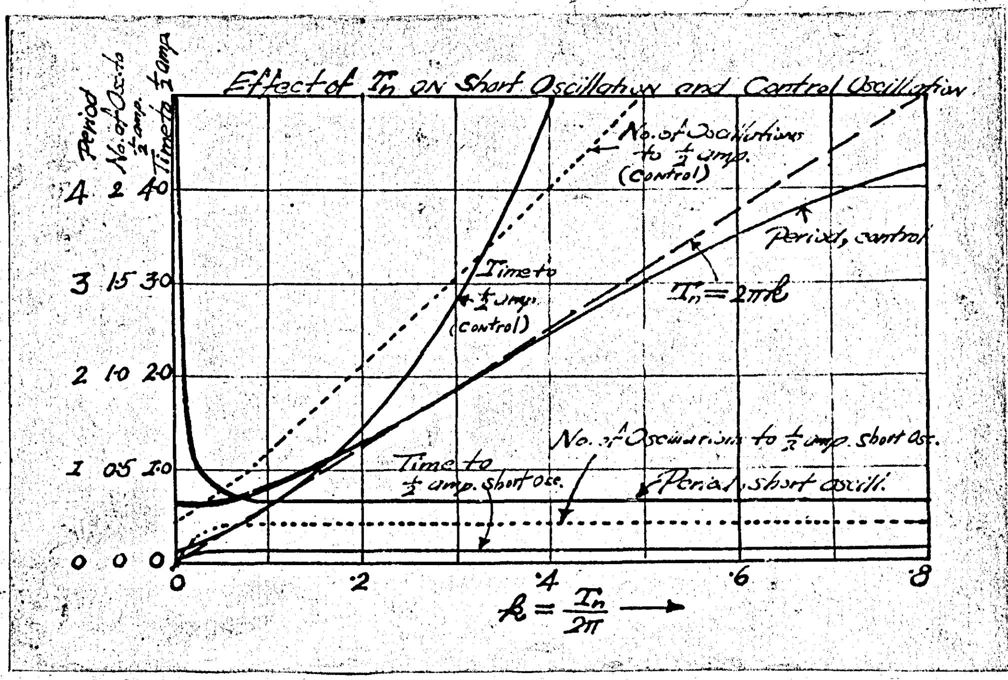
Fig.9.4 shows the effect of varying m_θ on the short oscillation as well as its effect on the control oscillation due to the additional degree of freedom due to the presence of control inertia and damping. The effct on short oscillation due to the presence of the control inertia and damping is seen to be negligible as compared to that of the 'Ideal' control. The short oscillation remains to be well damped except at very large values of m_θ. However, the effect of m_θ on the damping of the

control oscillation is tremendous, though the period of the control oscillation is substantially unaffected by the variation of m_θ. It is interesting to note that as m_θ approaches zero, the differences between the period and damping of the short oscillation and that of the control oscillation become~~s~~ smaller and smaller until at m_θ equals to zero, these two different modes of motion become~~s~~ one, namely the short oscillation of the uncontrolled airplane. As shown in the sextic equation in d for $m_\theta = 0$ in paragraph 8,c, ~~ad~~-a double imaginery roots are obtained. This is a mathematical way of expressing the physical fact that at $m_\theta = 0$, the additional degree of freedom introduced by the control is in coincidence with that of the short oscillation of the airplane. This fact alone, impresses us how important is the relation between the short oscillation and the control operation system.

Fig.9.4 also shows the limitation of m_θ to be used in satisfactory control operation at a given control ~~control~~ characteristics. For a control system having $k = 0.1$, corresponding to a natural period of .63 T sec., and a damping ratio of 0.5, the damping of the control oscillation becomes negative at $m_\theta = -3$.

c. The Effect of varrying the natural period of the Control System.

Fig.9.5



It is noted in the numerical investigation of paragraph 8,d, that the effect of varying k at constant m_θ and damping ratio, on long oscillation is almost negligible and unimportant. The damping of the long oscillation is reduced as k increases but the long oscillation remains to be well damped except at very large k where the control oscillation is already unstable.

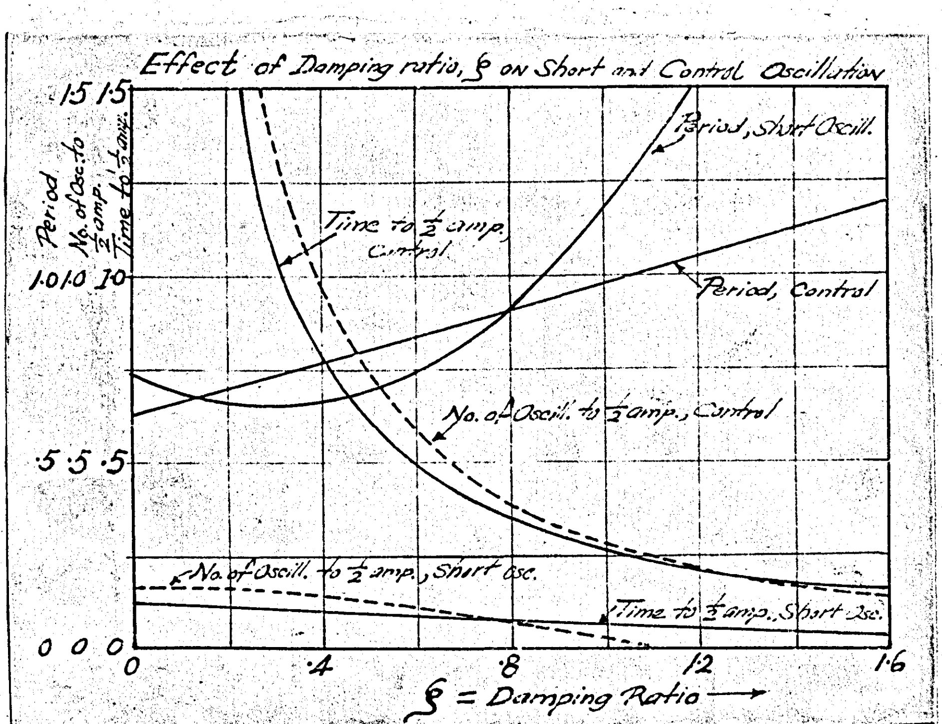
The effect of varyong k on the short oscillation and the control oscillation is shown in fig.9.5. It is seen that the period and the damping of the short oscillation remain unchanged except at very small k, As k approaches zero, the period of the short oscillation increases very rapidly. At k = 0, both the period and the damping of of the short oscillation become infinity. This is again a place to see the mathematical procedure of expressing the physkcal phenomena.It is quite evident to us that when k is zero, the controlled motion reduces to that of'ideally' controlled one which results a stability equation of the fourth order because the additional degree of freedom due to the control system no longer deviates from that of the displacement in pitch. The mathematical process in approaching such a physical fact is seen to be accomplished by adding tremendousl, damping to the short oscillation and at the same time, lenthening its period so that this mode of motion is eventually negligible.

It is also interesting to know that at k= 0, the period and damping of the control oscillation are exactly equal to that of the short oscillation for the ideal control. Thus,it is logical to believe that for 'Ideal' control, the short oscillation actually disappears by having infinite damping and period and the control oscillation takes the place of the short oscillation. Again we see how closely the short oscillation is associated with the controlled motion of the airplane.

Fig.9.5 also shows how closely is the relation between the period of the control oscillation and the natural period of the control system. Except when k approaches zero,the period of the control oscillation is almost identical with the undamped natural period of the control system. At very small k, the period of the control oscillation is very close to the period of the short oscillation for 'Ideal' control instead of approaching zero. The damping of the control oscillation is, however, not constant though the damping ratio of the control system remains unchanged. It decreases rapidly as the the natural period of the control system increases. At k = 0.4, the control oscillation is almost undamped.

d. The Effect of Varying the Damping Ratio of the Control System.

Fig.9.6



It is seen in the numerical investigation of paragraph 8e, that the effect of varying the damping ratio at conatant m_θ and k on long oscillation is comparably unimportant. The damping of the long oscillation improves slightly as the damping ratio of the control system increases. The damping of the long oscillation remains to be satisfactory even when the damping ratio of the control system is zero.

The effect of varying damping ratio on the short oscillation and the control oscillation is shown in fig. 9.6. It is seen that at large damping ratio of the control system, the short oscillation changes into two rapidly damped aperiodic motions, so that for a well damped control system for a given k and m_θ, the short oscillation plays no more important role in determining the stability of the motion. The control oscillation, however, is very much influenced by the variation of damping ratio of the control system. It is seen from fig.9.6, as the damping ratio of the control system is reduced to .25, the control oscillation becomes unstable. As the damping indreases to 1, the damping of the control oscillation improves considerably. Further increse of the damping ratio improves the damping of the control oscillation at a much slower rate. Therefore, it is believed that for a certain combination of m_θ and k, there is a most desirable damping ratio for the control system, as excessive damping in the control system is always undesirable. The period of the control oscillation increases but slightly. For damping ratios greater than unity, the period of the control oscillation remains to be finite despite the fact that the control system is already more than critically damped.

10. The Disturbed Motion in a Sharp Vertical Gust.

a. Introduction

Under the investigation of the previous sections, emphasis was placed on the decay of the oscillatory motion of the airplane after a certain initial disturbance, Nothing was indicated about the magnitude of the disturbed motion, and it was assumed though not stated, that there was only a single disturbance encountered by the airplane so that it is possible for the stable airplane to resume the position of equillibrium.

In actual flight conditions, especially in rough weather, the airplane may be subjected to repeated gusts which occurs sufficiently rapidly so that the motion after one gust only partly decays before the airplane is again displaced by a following gust. The magnitude of the first maximum displacement of the oscillatory motion immediately after the disturbance, which is defined as the 'FIRST SURGE ERROR' is then as important as the stability of the motion.

However, the mathematical complication involved in the analytical method to determine the amplitude and phase of the various modes of motion is tremendous. Even the modern method of operational calculus[13] is too laborious for a systematic investigation of the controlled motion which has three different modes of oscillatory motion. The writer is very fortunate in having the opportunity of using the M.I.T.Differential Analyzer to solve the disturbed motions by mechanical means with high precision and efficiency for such purpose. As the detailed description of the procedure involved in using such a machine is known to most of us[14] through the numerous published investigations carried out by the machine on many problems in Electrical Engineering, and the particular limitations involved in solving the airplane stability problem will be described in a separate article, the disturbed motions due to a vertical sharp gust w_o in plotted curves alone, will be presented here.

b.The Disturbed Pitching Angle, θ due to a sharp vertical gust, w_o for an 'Ideally' controlled Airplane.

Fig.10.1 shows the effect of varying m_θ on the magnitute of the disturbed pitching inclination due to a sharp vertical gust. It is seen that the magnitude of the surge error as well as the subsequent disturbed oscillations are all improved by increasing m_θ. The rate of reduction in the subsequent oscillation by increasing m_θ is tremendous. The rate of reduction in the first surge error is, however, not quite so rapidly. Taking

DISTURBED ANGLE OF PITCH OF AN AIRPLANE
SUBJECT TO VERTICAL GUST

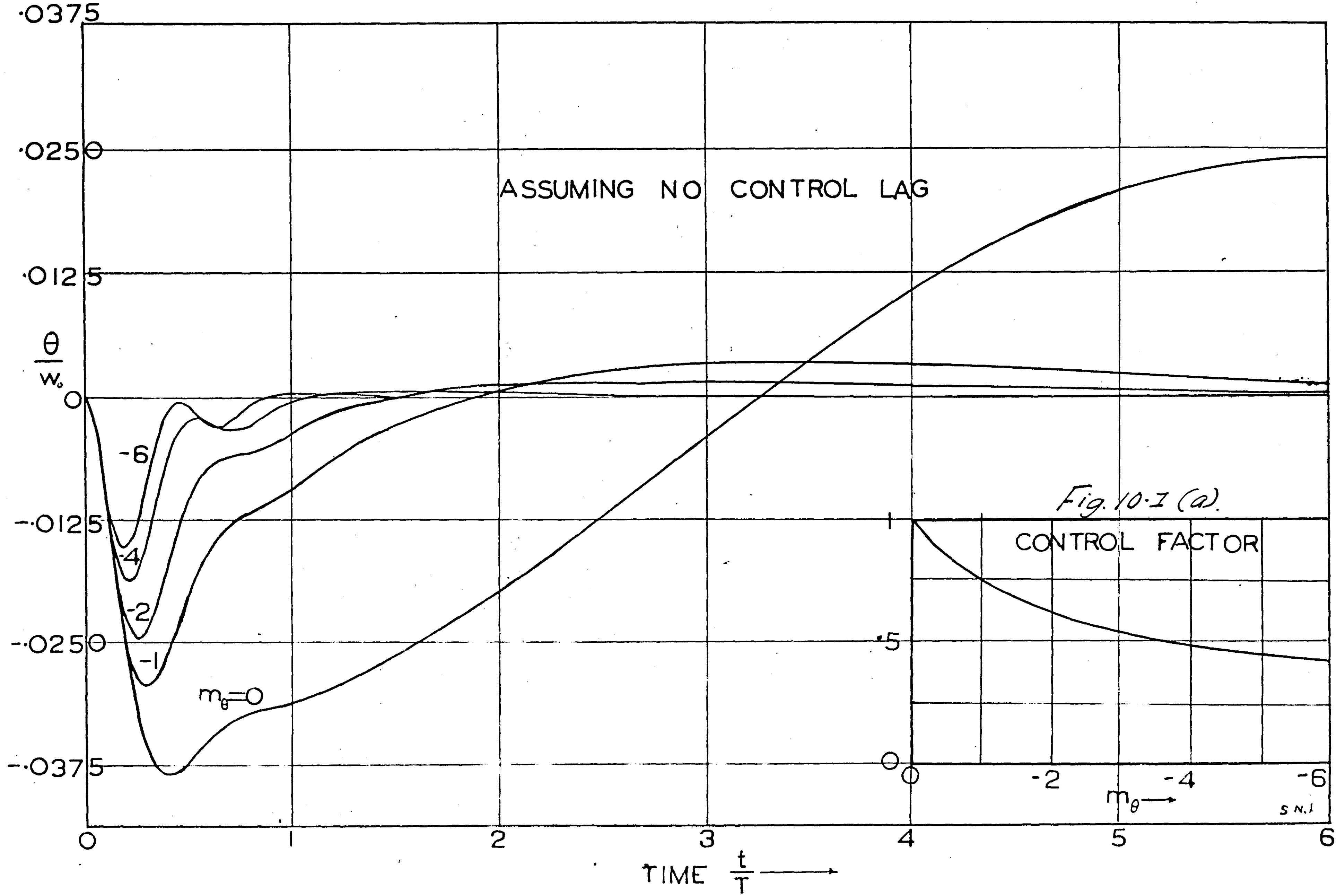


Fig. 10·1 (a).

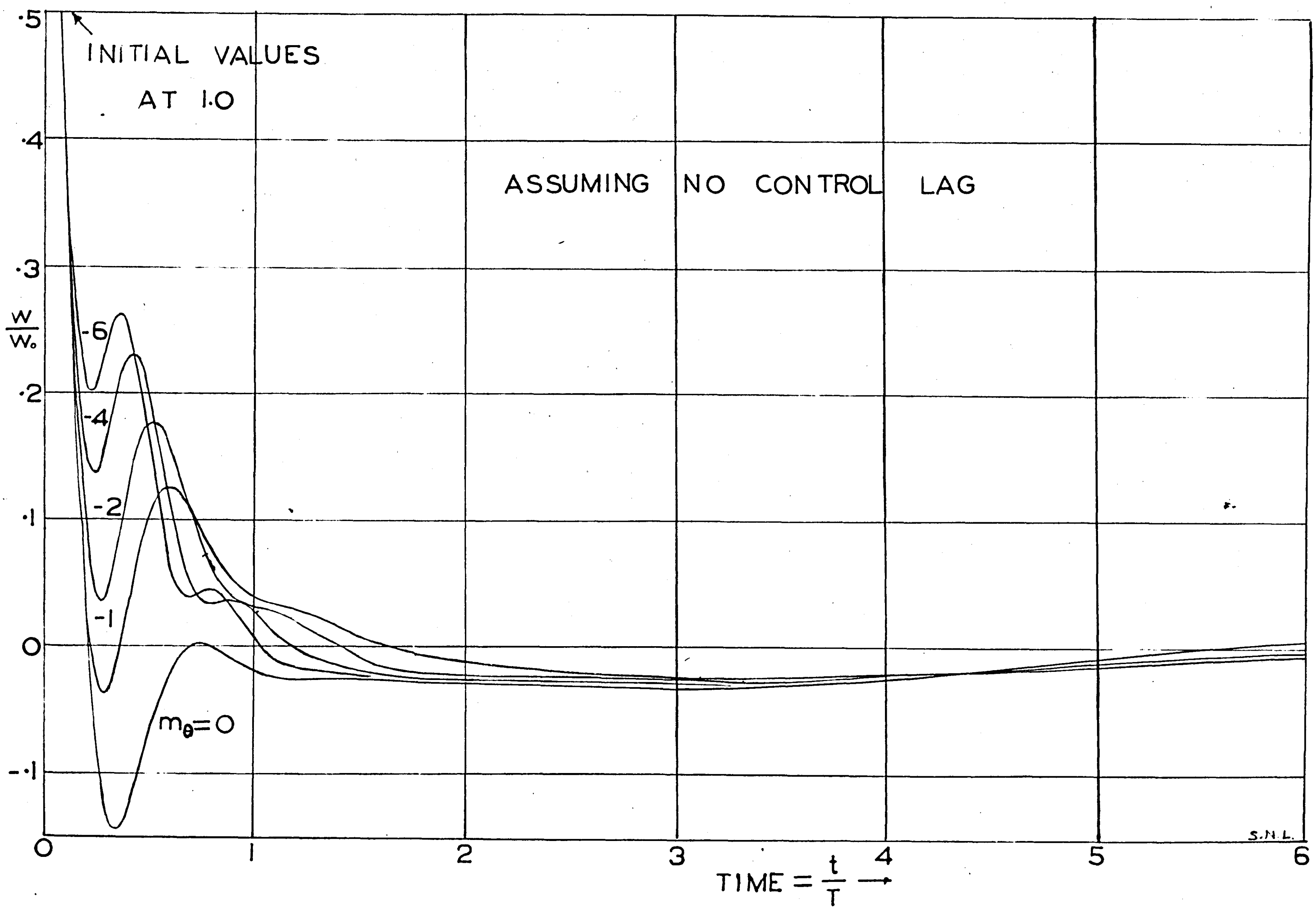
42

the maximum initial surge error for the uncontrolled airplane as unity, the relative magnitudes of the first surge errors at various m_θ indicate the effectiveness of the control derivative m_θ and are plotted in Fig.10.1a as control factor. The control derivative m_θ is seen to be more effective in bringing down the magnitude of the first surge error when m_θ is at smaller values.

c. The Disturbed vertical velocity due to a sharp vertical Gust, for an 'Ideally' controlled airplane.

Fig.10.2 shows the effect of varying m_θ on magnitudes of the vertical velocity. It is seen that the fluctuations in vertical velocity due to a sharp vertical gust are in general improved as m_θ is increased for an 'Ideal' control. However, the improvement in the initial surge error as well as the following oscillations in vertical velocity is seen to be comparatively much less than the inclinations in pitch. This is always true for single control derivative systems, as the influence on the vertical velocity by m_θ is indirect whereby its influence on inclination in pitch is direct. It is also interesting to note that the main influence on the vertical velocity ~~by~~ the m_θ control is seen to be the rapid decrease of the initial over-shooting of the vertical velocity across the zero-axis. Even at m_θ of -2, the vertical velocity

DISTURBED VERTICAL VELOCITY OF AN AIRPLANE SUBJECT TO VERTICAL GUST

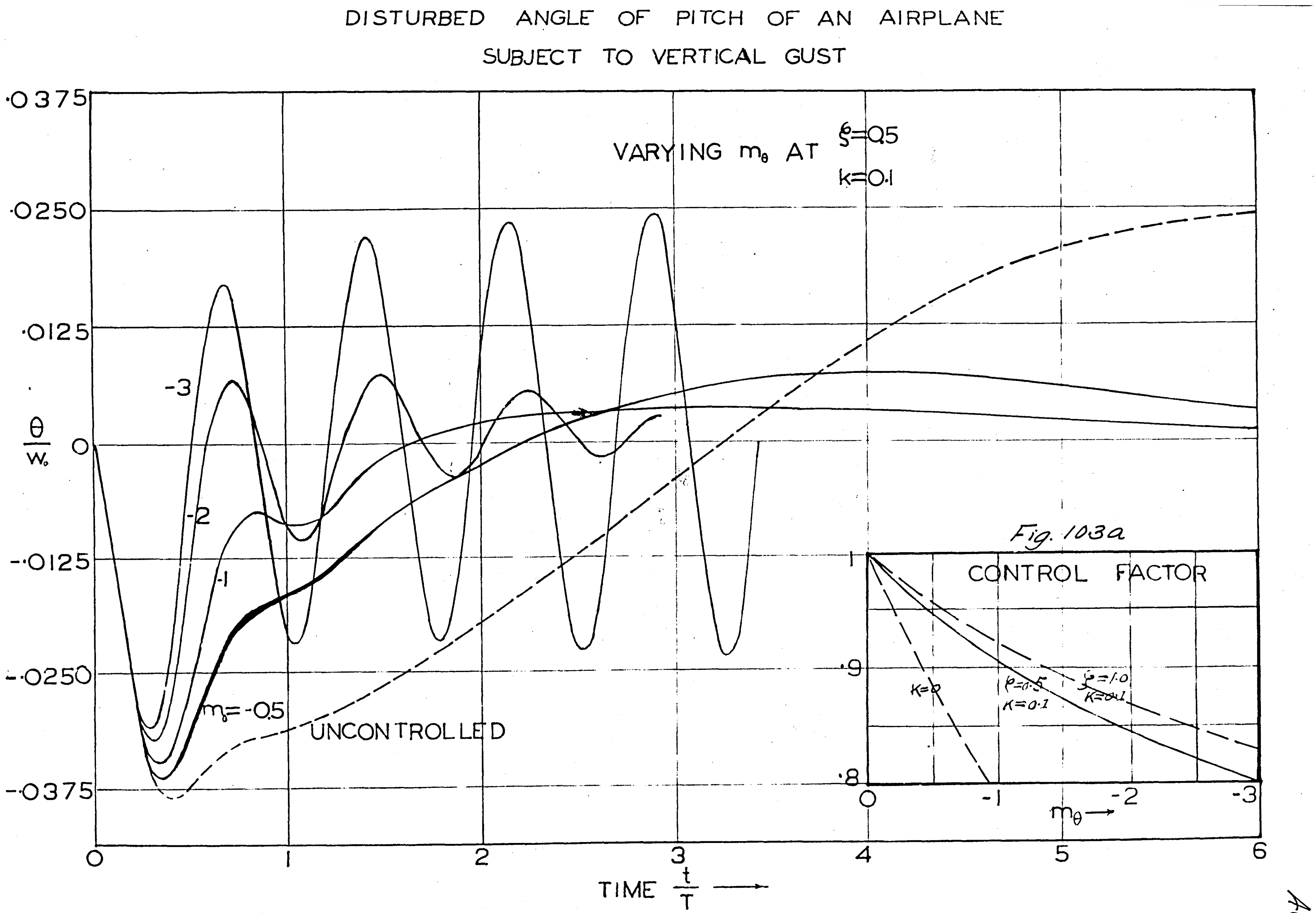


44

no longer over shoots itself across the zero-axis until to the next oscillation. This may improve the pomfort of the passengers considerably by cutting down the duration of high accelertion fluctuation in the z-direction. The initial acceleration is , however,unaffected by variations in m_θ,as it is a purely a function of z_w.

d.Effect of Varrying m_θ for a Control System having Inertia and Damping.

Fig.10.3 shows the effect of varrying m_θ on the disturbed magnitudes of the inclination in pitch due to a vertical sharp gust. It is obvious that the presence of control inertia and damping alters the motion considerably from the motion 'Ideally' controlled.
The ability for the m_θ control to reduce the first surge error is decreased considerably due to the presence of control inertia and damping. The subsequent oscillations in pitch are also increased due to control inertia and damping. At $m_\theta = -3$, the amplitude of the unstable control oscillation is excessive even in the first following oscillation. Fig.10.3a,shows the control factor at different control characteristics. Fig.10.4 shows the effect of varying m_θ on a control system


DISTURBED ANGLE OF PITCH OF AN AIRPLANE
SUBJECT TO VERTICAL GUST
VARYING m_θ AT $\zeta=0.5$ $k=0.1$
$\frac{\theta}{W_0}$
·0375
·0250
·0125
0
-·0125
-·0250
-·0375
-3
-2
-1
$m_\theta=-0.5$
UNCONTROLLED
0
1
2
3
4
5
6
TIME $\frac{t}{T}$
CONTROL FACTOR
1
·9
·8
0
-1
-2
-3
m_θ
K=0
$\zeta=0.5$ K=0.1
$\zeta=1.0$ K=0.1
Fig. 103a

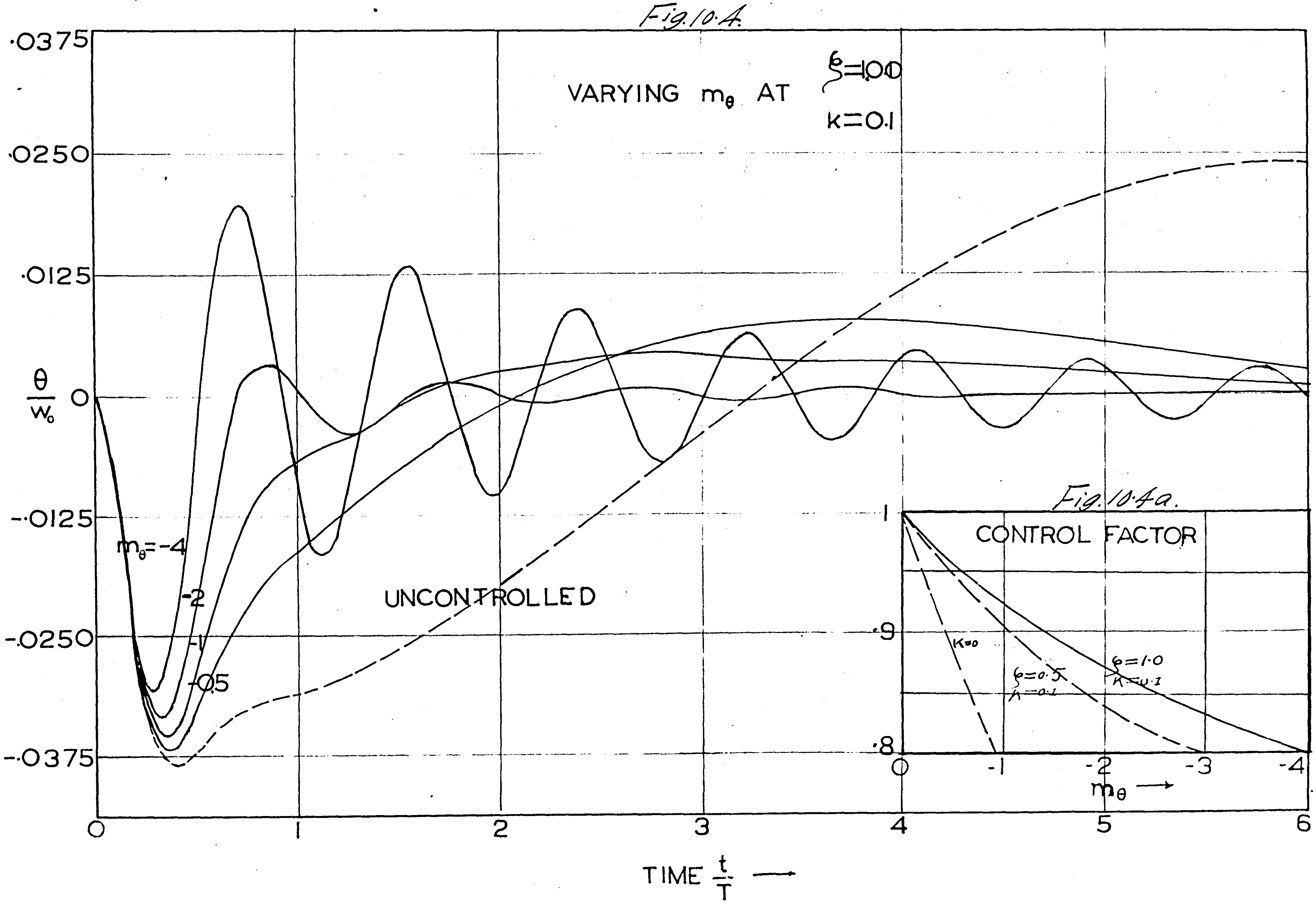
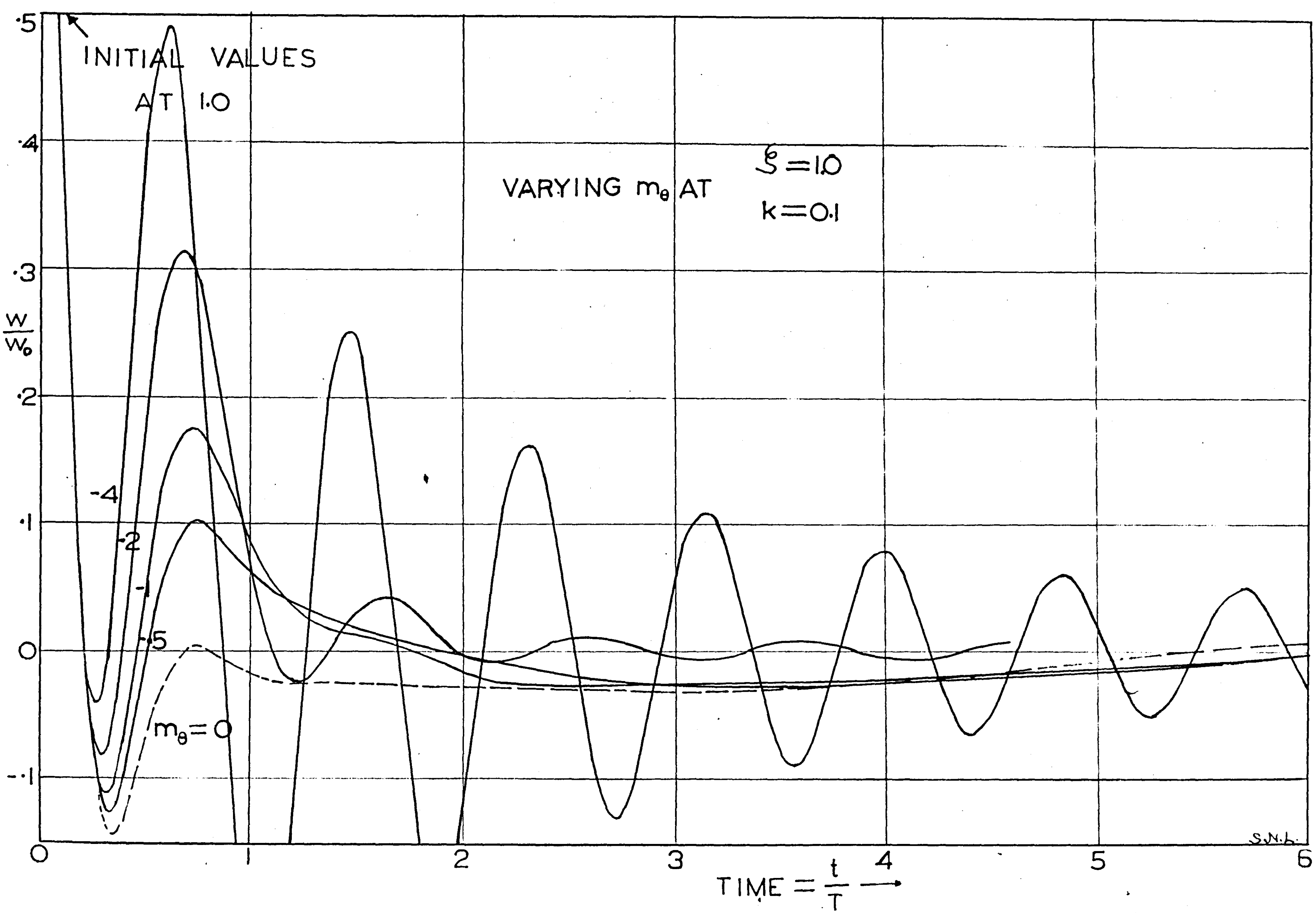


Fig. 10·A.
VARYING m_θ AT
$\zeta = 1.00$
$k = 0.1$
·0375
·0250
·0125
0
-·0125
-·0250
-·0375
$\frac{\theta}{W_0}$
$m_\theta = -4$
-2
-1
-0.5
UNCONTROLLED
0
1
2
3
4
5
6
TIME $\frac{t}{T}$ ⟶
Fig. 10·4a.
CONTROL FACTOR
1
·9
·8
K=0
$\zeta = 0.5$
$K = 0.1$
$\zeta = 1.0$
$K = 0.1$
0
-1
-2
-3
-4
m_θ ⟶
47

DISTURBED VERTICAL VELOCITY OF AN AIRPLANE
SUBJECT TO VERTICAL GUST



with increased damping ratio. It is seen that the effect of increasing the damping ratio of the control system is considerable in reducing the magnitude of the unstable oscillation. It is seen that for a control system having k = 0.1 and damping ratio of 1.0 the controlled motion seems satisfactory for m_θ to be as large as -2.0. Fig.10.5 shows the effect of varying m_θ on the vertical velocity due to vertical gust. It is shown that when the control oscillation is not well damped the magnitude of oscillations in the vertical velocity is very objectional. A well damped control system is therefore,responsible for the comfortness of the passengers flying in rough air. It is interesting to note that when the oscillatory motions are badly damped, the motions in vertical velocity and the inclinations in pitch are nearly in phase with each other.

e.Effect of Varying Control Characteristics.

Fig. 10.6 illustrates the effect of varying k on the magnitudes of the disturbed inclination in pitch due to a vertical gust. The first error surge is increased as k increases. Taking the first surge error for the ideal control as unity, the relative magnitutes of the first

Fig. 10.6

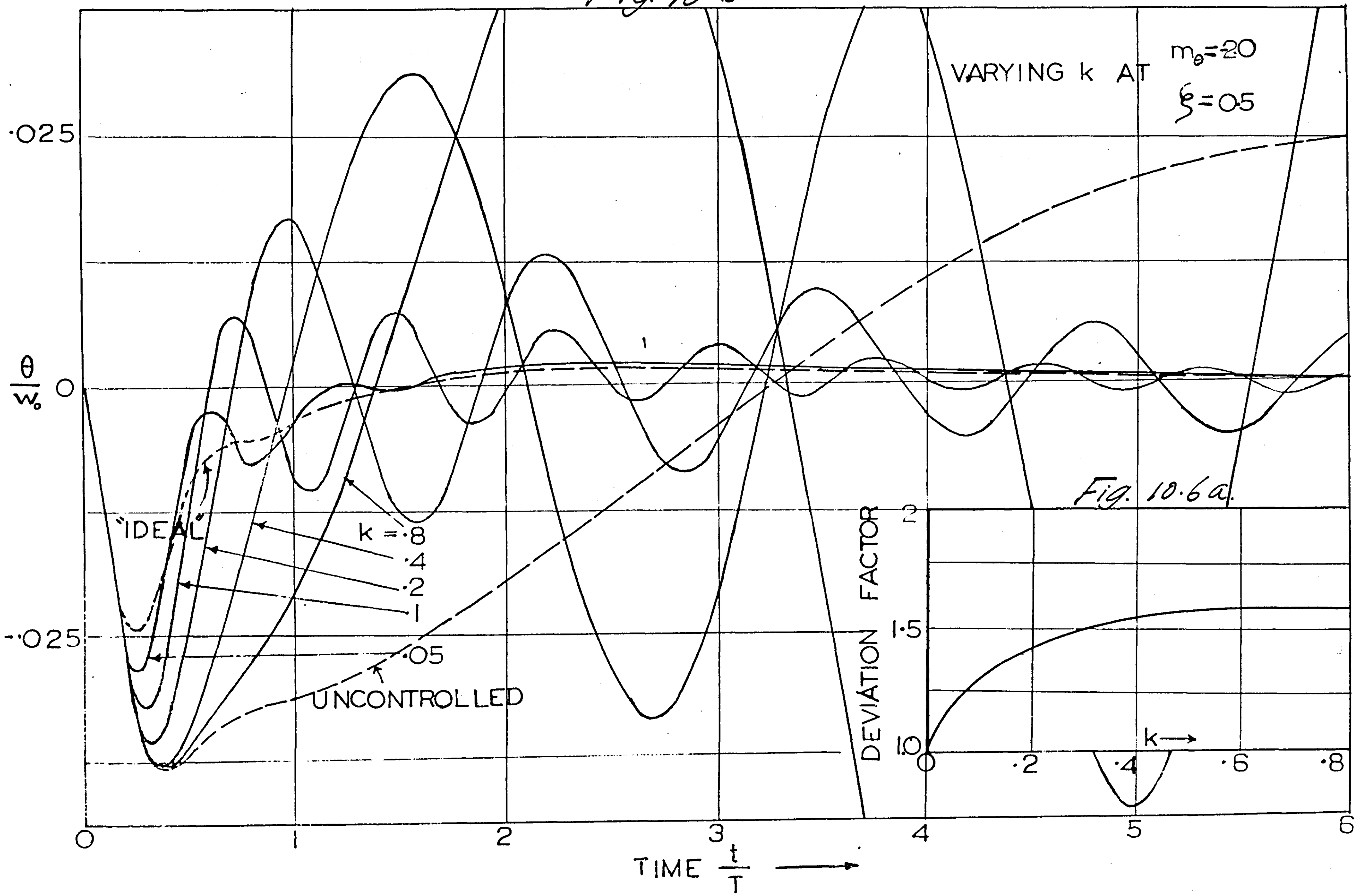


Fig. 10.6 a.

surge error for controls of different k as compared to the ideal one indicate the deviation from the ideal due to the control inertia. This is plotted as Deviation Factor against k in Fig.10.6a. It is seen that the worst first surge error can not exceed that of the uncontrolled airplane. The subsequent oscillations,however, can be tremendous if the control oscillations are not well damped.

The effect of varying the damping ratio on the magnitudes of the disturbed inclination in pitch due to a vertical gust is shown in Fig.10.7. The first surge error is increased by increasing the damping ratio. Taking the maximum amplitude of the first surge error for a control having damping ratio equal to zero as unity, the relative magnitudes of the first surge error at other damping ratios are called the Deviation Factors due to the control damping. They are plotted in Fig.10.7a. Increase of damping ratio though slightly increase the first surge error, is very effective in damping out the subsequent oscillatory motions. Thus, it is seen that a control system which is slightly over-damped, is more satisfactory than under damped one. The disadvantage introduced by adding too much damping in the control system is seem to be much smaller than in other cases of simple systems such as constant speed control.[15]

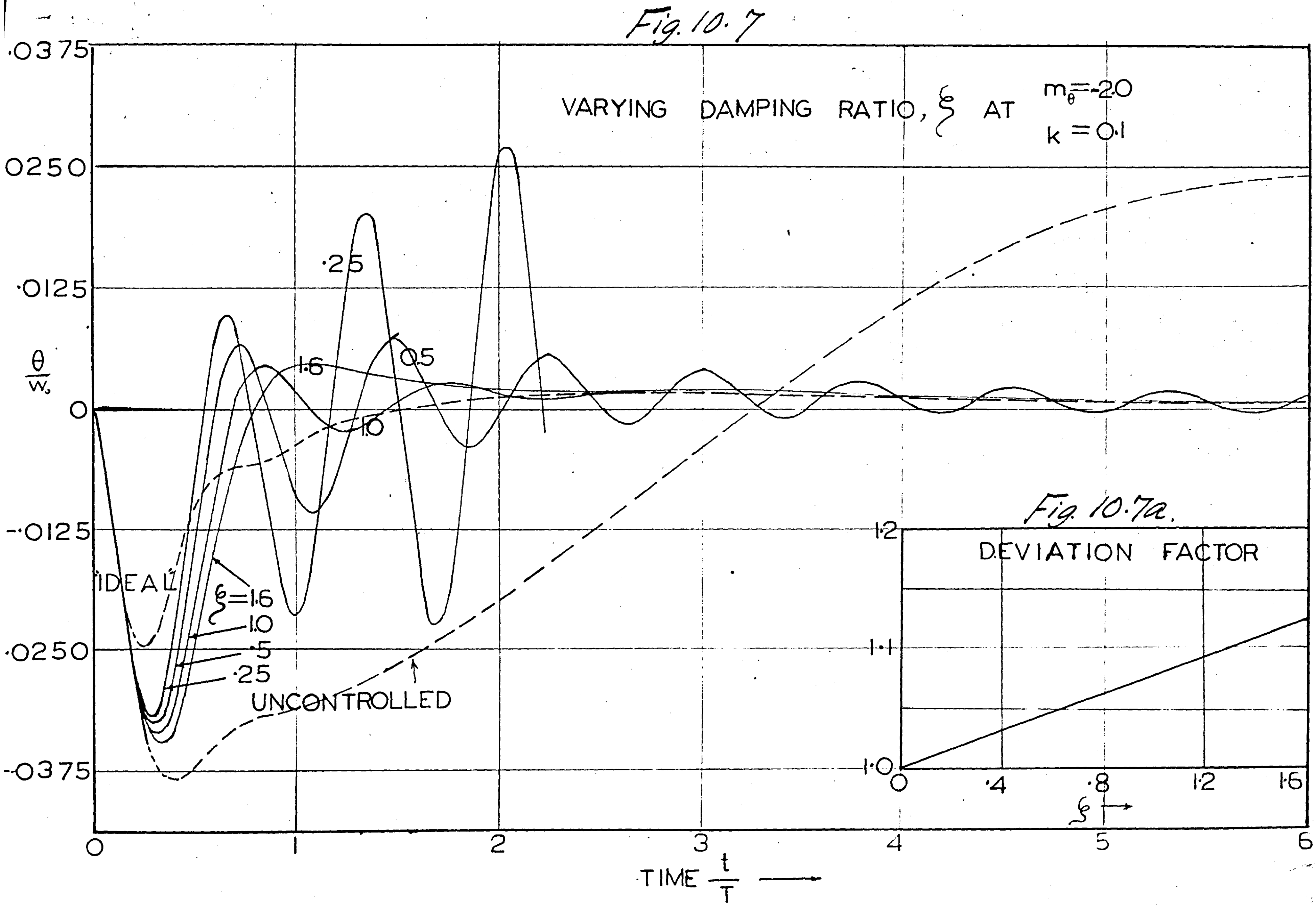


Fig. 10.7
VARYING DAMPING RATIO, ξ AT
mθ = -20
k = 0.1
·0375
0250
·0125
0
-·0125
-·0250
-·0375
θ/w₀
·25
1.6
0.5
1.0
IDEAL
ξ = 1.6
1.0
·5
·25
UNCONTROLLED
0
1
2
3
4
5
6
TIME t/T
Fig. 10.7a.
DEVIATION FACTOR
1.2
1.1
1.0
0
·4
·8
1.2
1.6
ξ

11. The Motions Of The Control

Just as the relation between the vertical velocity w and the pitching inclination θ, the displacement of the servo-piston s is different in its phase relation from the pitching inclination θ. It is pointed out in article 3, that for an 'Ideal' control, the variable, s can be eliminated from the equations of motion, due to the fact that for an 'Ideal' control,

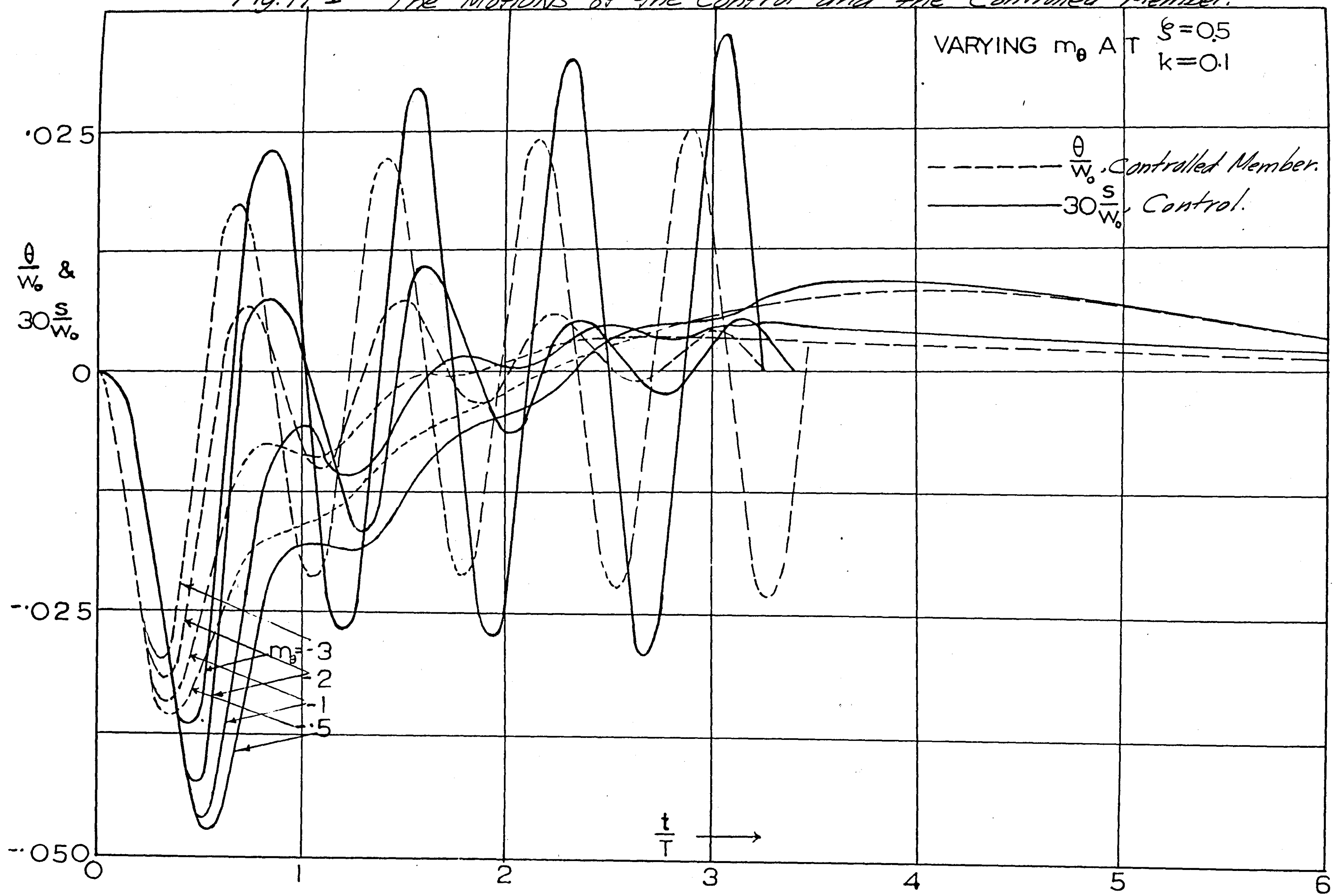
$$s = m_\theta/m_s \times \theta \text{ ....} (11.1)$$

In order to compare the motions of the real control from that of the 'Ideal' the piston displacement s is plotted in a same sheet with the motion for the 'Ideal' control, which is identical with the inclination in pitch, θ except, in a different scale factor M which is represented by the ratio, m_θ/m_s. In the following figures, the piston displacement, $m_s/m_\theta \times s$ is plotted together with θ in a same sheet. From the data in the numerical investigation, m_s/m_θ is found to be $L \times \partial\beta/\partial s$ which is 30 for the airplane F-22.

a. Effect of varying m_θ on the motions of the control.

Fig.11.1 shows the relative motion between the displacement of the control, s and the controlled member θ at various m_θ. It is seen that the time lag for the control behind that of the controlled member, which is called 't_g' is constant for a constant control characteristic

Fig. 11·1 The Motions of the Control and the Controlled Member.



despite the variation of m_θ. However, it is seen that m_θ is responsible for the over-shooting and under-shooting of the control movement. It is seen that the instability of the control oscillation at $m_\theta = -3$, is due to the over-shooting of the control which is lagging behind the controlled member due to the control inertia and damping. When m_θ is small, it under shoots except in the first correcting movement of the control. The effect of the control lag is thus minimized when m_θ is small. The first control movement is larger when m_θ is smaller. This is because the control has to move more to correct a given error when m_θ is smaller. The ratio of the control movement to that of the controlled member is seen to be smaller at smaller m_θ.

b. Effect of varying k on the motions of the control.

Fig.11.2 and Fig.11.3 show the relative motions between the displacement of the control and the controlled member at various natural period of the control system. It is seen that as k increases, the control lag t_g increases as a linear function of k as shown in fig.11.2a. The formula, $t_g = 1.5\ k = .25\ T_n$ found empirically from fig.11.2a seems to be the relation between the control lag and the natural period of the control system, for the airplane F-22. Expressing the control lag in terms of the phase angle $\Theta-\phi-\Phi^{\circ}$ of the control oscillation, and plotting

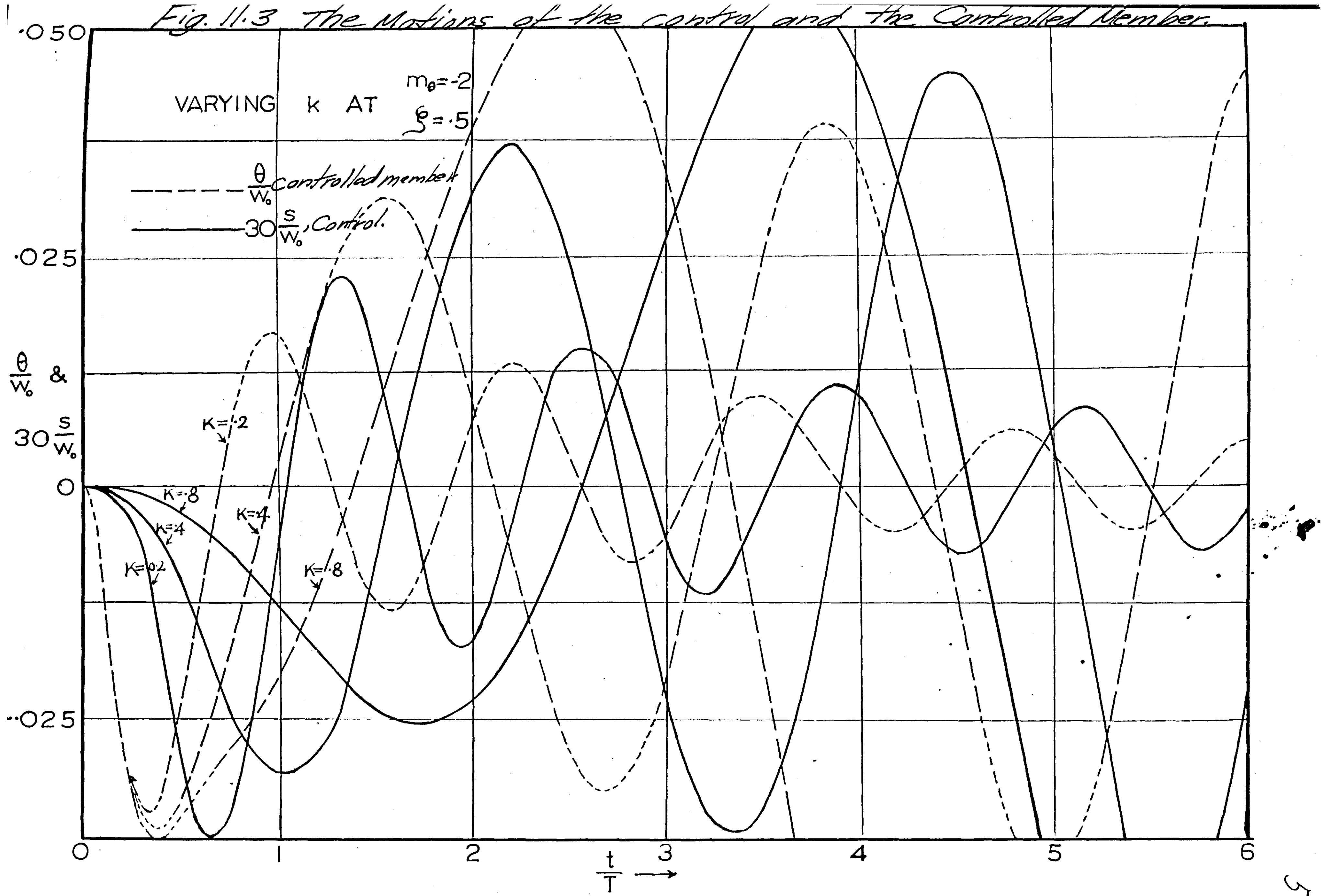
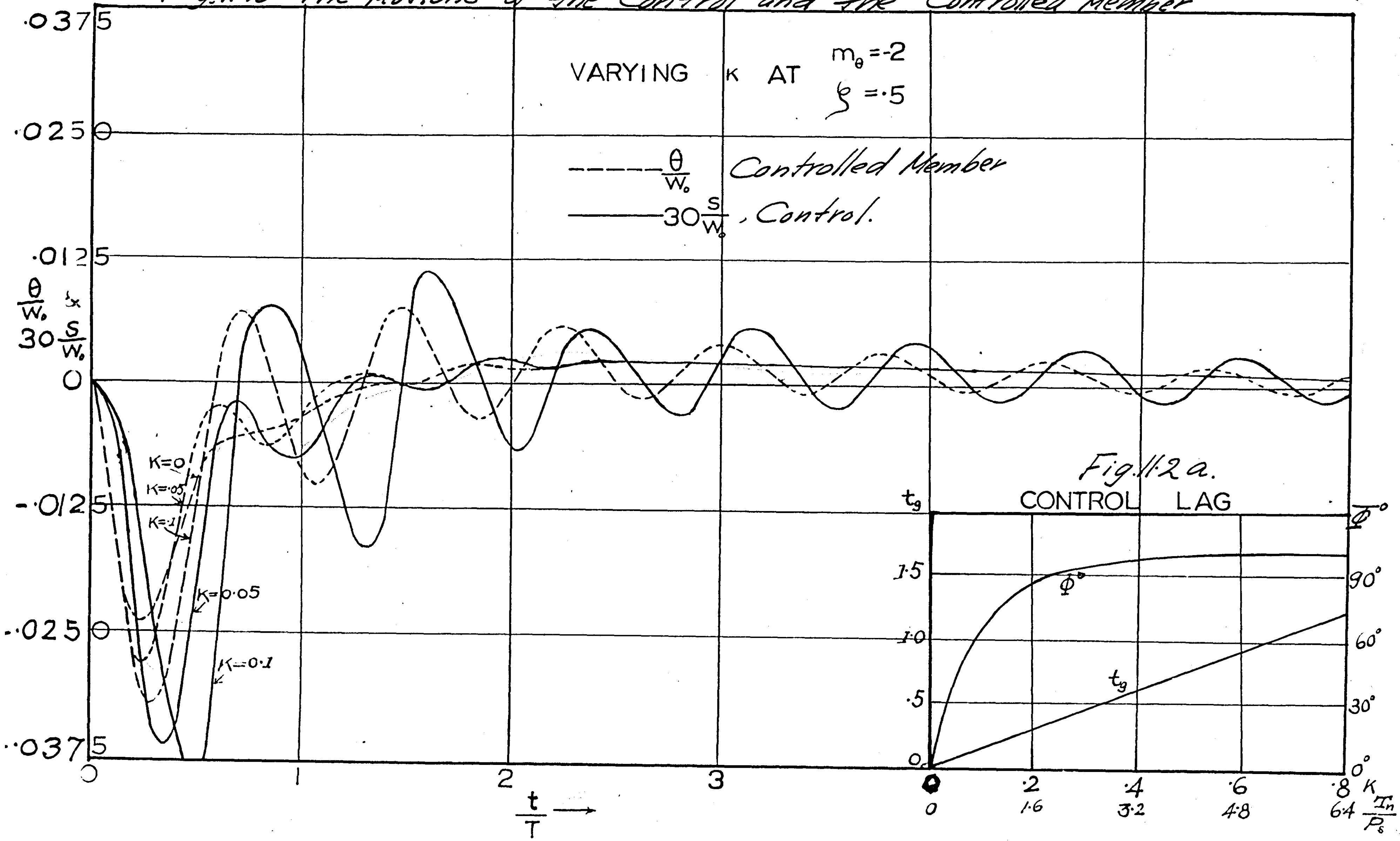


Fig. 11.3 The Motions of the control and the Controlled Member.
VARYING k AT
$m_\theta = -2$
$\zeta = \cdot 5$
$\frac{\theta}{W_0}$ controlled member
$30\frac{s}{W_0}$, Control.
·050
·025
0
-·025
$\frac{\theta}{W_0}$ & $30\frac{s}{W_0}$
K=·2
K=·8
K=·4
K=·2
K=·4
K=·8
0
1
2
3
4
5
6
$\frac{t}{T}$

56

Fig. 11·2 The Motions of the Control and the Controlled Member



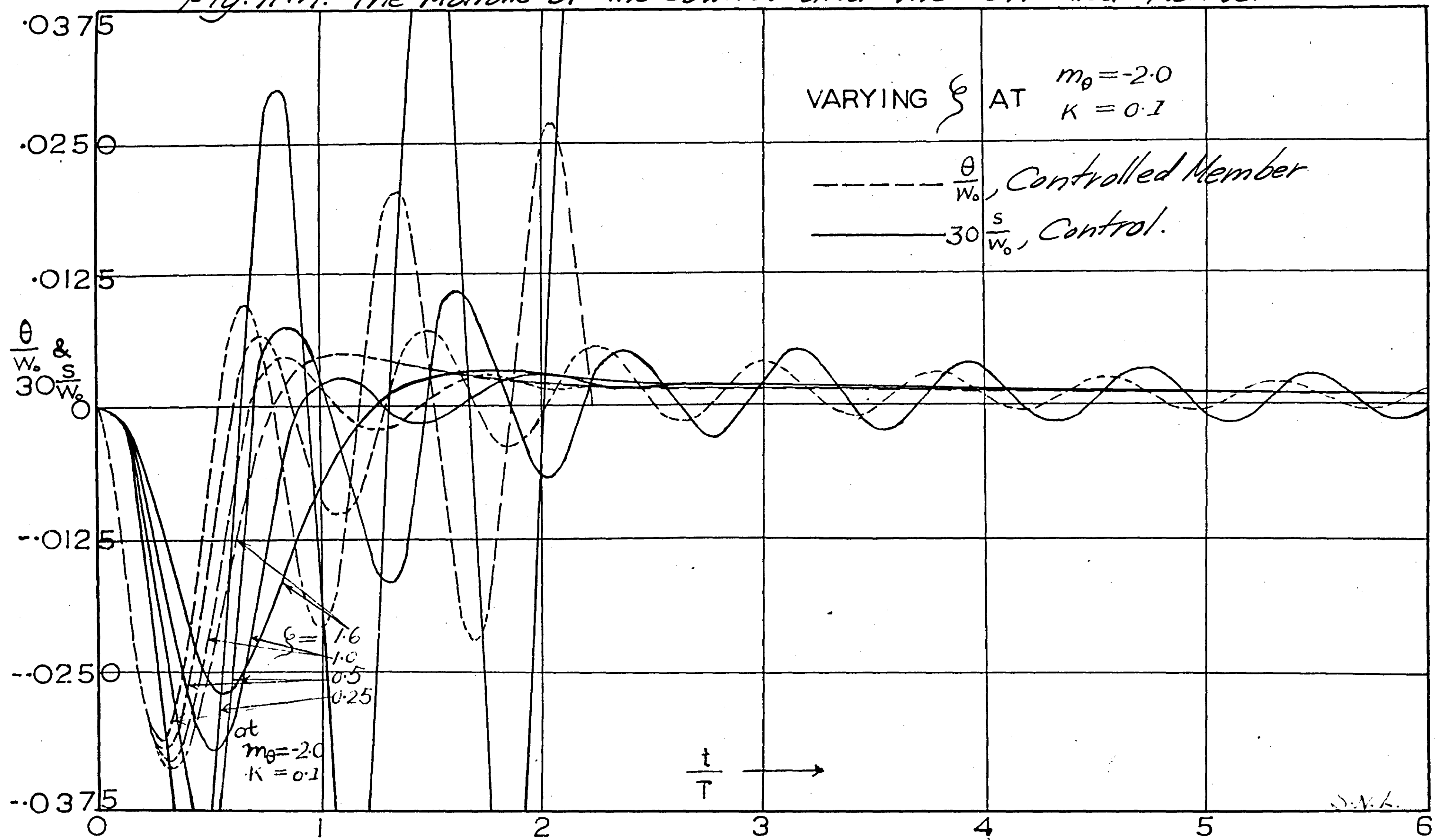
this phase lag against ~~x~~ the ratio of the natural period of the control system to the period of short oscillations of the uncontrolled airplane, T_n/P_s, the curve as shown in Fig.11.2a is very similar to the phase lag between the forced oscillation and the forcing function in a simple vibration system[16]. It is noted that when the control oscillation is unstable due to control lag, the phase angle between the control and the controlled member is nearly 90°.

Again, from figs.11.2 and 11.3, a clear physical picture of the control operation can be obtained. The error of inclination in pitch, θ gets to a quicker start than the control motion. The larger the control inertia is, the slower is the motion at start. Due to the control lag, energy is fed to the disturbance and thus negative damping is introduced into the system as a whole which must be over-balanced by the damping of the airplane itself.

c. Effect of varying the damping ratio on the motions of the control.

Fig.11.4 shows the relative motions of the control and the controlled member at various damping ratio of the control system. It is seen that damping is chiefly responsible for the over shooting of the control. The control lag is hardly affected by the variations of damping ratio. The instability of the control oscillation

Fig. 11. A. The Motions of the Control and the Controlled Member



when the damping ratio is reduced to 0.25, is seen to be entirely due to the excessive over-shooting of the control operation. It is also noticed here, that over-damping is less undesirable than under-damping.

12. Effect of Constant Time Lag on the Stability of the Controlled Motion of Airplanes.

From the study of the motions of the control in article 12, it is seen that due to the inertia of the control system, the motion of the control is lagging behind the controlled member. The time lag of the control is seen to be constant, though not quite exactly, if the control inertia, represented by the natural period T_n is constant. In the airplane, controlled by human pilot, a constant time lag due to the sluggishness of human reaction is introduced into the control. Therefore, it is believed that an investigation of the effect of the constant time lag on the stability of the controlled motion is worthwhile, especially when a short-cut method such as used in the following investigation is available.

In the following investigations, it is assumed that the respomse of the control comes at a constant period of time, t_g, after the error, so that the controlling force or moment at any instant corresponds to the error t_g time units previous. The approximate method used int the following has been mentioned by Minorsky[17], Callender Hartree and Porter[18] and Cowley[19], in connection with problems of automatic control of various simple sys tems.

a. Introduction of the control lag, t_g, into the determinant of an 'Ideally' controlled Airplane.

From (2.6), by neglecting m_u, x_q and z_q and putting θ_o and α_o equal to zero, we get the determinant for an 'Ideal' control

$$(12.1) \qquad \begin{vmatrix} d - x_u & -x_w & -\mu C_L \\ -z_u & d - z_w & d\,\mu \\ 0 & -m_w & d^2 - dm_q - \mu m_\theta \end{vmatrix} = 0$$

For an 'Ideal' control, the moment due to the control at any instant is $\mu m_\theta \theta(t)$, where $\theta(t)$ indicates that the error of inclination in pitch is a function of time. In the derivation of the stability determinant, it is assumed that $\theta(t) = \theta_A e^{dt}$, where θ_A is the amplitude of $\theta(t)$ while d, being found as the root of the determinant, characterizes the period and damping of the motion.

For a control with a constant lag t_g, the moment generated by the control at any instant is $\mu m_\theta \theta(t-t_g)$. Or, it can be written as $\mu m_\theta \theta_o e^{d(t-t_g)} = \mu m_\theta \theta_o e^{dt} \times e^{-dt_g}$ $= \mu m_\theta \theta(t) \times e^{-dt_g}$. Thus the effect of the control lag t_g is seen to be equivalent to multiplying the control derivative μm_θ by the factor e^{-dt}.

Thus, the determinant for the controlled airplane, with a constant control lag, t_g is

$$(12.2)\qquad \begin{vmatrix} d - x_u & -x_w & -\mu C_L \\ -z_u & d - z_w & -d\,\mu \\ 0 & -m_w & d^2 - d m_q - \mu m_\theta (e^{-d t_g}) \end{vmatrix} = 0$$

If the above determinant is to be expanded out to solve for d as in the usual procedure, it would be difficult to do so because d is involved in the exponential form e^{-dt_g} instead of simple expressions.

However, if the constant control lag, t_g, is small compared to the period of the oscillatory motions, which is true for any useful control system, the following approximations can be made.

In terms of series, write

$$e^{-dt_g} = 1 - dt_g + d^2 t_g^2/2 + (\text{ terms of higher powers in } dt_g)$$

As d in general, is found as a complex number, characterizing the damping of the motion by its real part, and the period of the motion by its ~~complex~~ imaginary part. Therefore, the higher power terms in dt_g will be comparable to the first term, unity, only for those modes of motion which are heavily damped and of high frequency ~~and~~ or short period. However, as such modes of motion are found to be of little importance in the criteria of stability and damping of the system as a whole, it is therefore, allowable to neglect the ~~th~~ higher power terms especilly when t_g is only a fractional part of the period of the short oscillations. Thus

$\mu m_\theta e^{-dt_g} = \mu m_\theta(1 - dt_g + d^2 t_g^2/2)$ and the determinant for the controlled airplane can be written as

$$(12.3)\quad 0 = \begin{vmatrix} d - x_u & -x_w & -\mu C_L \\ -z_u & d - z_w & -d\,\mu \\ 0 & -m_w & (1 - \mu m_\theta t_g^2/2)d^2 - (m_q - \mu m_\theta t_g)d - \mu m_\theta \end{vmatrix}$$

Now, dividing the third row of (12.3) by $(1-\mu m_\theta t_g^2/2)$ which is permissible as the determinant is equal to zero.

Let
$$-m'_w = -m_w/(1-\mu m_\theta t_g^2/2)$$
$$-m'_q = -(m_q - \mu m_\theta t_g)/(1 - \mu m_\theta t_g^2/2)$$
$$-m'_\theta = -m_\theta/(1-\mu m_\theta t_g^2/2)$$

then, the quartic equation in d can be expressed in the same form as in (5.8) and (5.9) for 'ideal' control, where

$$A'_1 = 1$$
$$B'_1 = B'_0 = -(x_u + z_w + m'_q)$$
$$(12.4)\quad C'_1 = C'_0 - \mu m'_\theta = x_u z_w + (x_u + z_w)m'_q - \mu m'_w - z_u x_w - \mu m'_\theta =$$
$$D'_1 = D'_0 + \mu m'_\theta(x_u + z_w) = m'_q(z_u x_w - x_u z_w) + \mu m'_w x_u + \mu m'_\theta(x_u + z_w)$$
$$E'_1 = E'_0 - \mu m'_\theta(x_u z_w - z_u x_w) = -C_L\,\mu m'_w z_u - \mu m'_\theta(x_u z_w - z_u x_w)$$

Thus it is seen that the effect of control lag t_g, is to (1) Decrease the inertia in pitch of the airplane from 1 to $(1 - \mu m_\theta t_g^2/2)$

(2) Reduce the m_q of the airplane by $\mu m_\theta t_g$

The equivalent effect on the motion of an airplane due to control lag ~~is~~- can also be interpretated as,

(1)Reduction of the static stability in pitch by the ratio $1/(1 - \mu m_\theta t_g^2/2)$

(2)Reduction of m_q by

(a) subtracting $\mu m_\theta t_g$ from m_q

(b) and then reducing by the ratio $1/(1-\mu m_\theta t_g^2/2)$

(3)Reduction of m_θ by the ratio $1/(1-\mu m_\theta t_g^2/2)$

(Assuming m_u is negligible)

b.Numerical Investigation

In order to see exactly how the control lag affects the stability of the controlled motion,and in order to compare the effects of constant time lag on the controlled motion against the effect due to inertia and damping of the control system,the following numerical investigation has been carried out. Basing on the same derivatives chosen in article 8, we have

$m_\theta = -2.0$, $m_q = -6$, $m_w = -3$ $\mu = 20$

$$A_1' = 1$$
$$B_1' = 4.65 - m_q'$$
$$C_1' = 1.075 - 4.65m_q' - 20m_w' - 20m_\theta'$$
$$D_1' = -1.075m_q' - 3m_w' - 93m_\theta'$$
$$E_1' = -9m_w' - 21.5m_\theta'$$

Table 12·1

t_g	m'_w	m'_θ	m'_q
0	-3	-2	-6
.05	-2.85	-1.91	-3.81
.10	-2.50	-1.67	-1.67
.15	-2.06	-1.38	0
.20	-1.66	-1.11	+1.11
.30	-1.07	-0.72	+2.14

t_g	A'_1	B'_1	C'_1	D'_1	E'_1
0	1	10.65	128.9	201.5	70.0
.05	1	8.46	113.9	190.7	66.8
.10	1	6.32	92.2	164.3	58.4
.15	1	4.65	70.0	134.2	48.2
.20	1	3.54	51.3	106.8	38.9
.30	1	2.51	26.8	62.9	25.1

Table 12·2

t_g	The quartic in d can be factered into
0	$(d^2+1.74d+.62)(d^2+8.91d+112.8) = 0$
.05	$(d^2+1.85d+.662)(d^2+6.61d+101.0) = 0$
.10	$(d^2+1.95d+.705)(d^2+4.37d+82.8) = 0$
.15	$(d^2+2.08d+.755)(d^2+2.57d+63.8) = 0$
.20	$(d^2+2.20d+.810)(d^2+1.34d+47.6) = 0$
.30	$(d^2+2.45d+.976)(d^2+0.06d+25.65) = 0$

Table 12·3

t_g	Roots	Periods	$T_{1/2}$	$N_{1/2}$
0	$-4.96 \pm 9.4i$	.669	.14	.21
	$-1.44, -.50$	Infinity	.48,1.485	0
.05	$-3.31 \pm 9.5i$	.668	.21	.315
	$-1.365, -.485$	Infinity	.508,1.43	0
.10	$-2.185 \pm 8.73i$	.72	.317	.441
	$-1.47, -.48$	Infinity	.472,1.442	0
.15	$-1.285 \pm 7.87i$	.80	.539	.674
	$-1.615, -.465$	Infinity	.428,1.49	0
.20	$-.67 \pm 6.87i$	.917	1.03	1.13
	$-1.732, -.468$	Infinity	.40,1.48	0
.30	$-.03 \pm 5.05i$	1.25	23.1	18.5
	$-1.953, -.497$	Infinity	.355,1.4	0

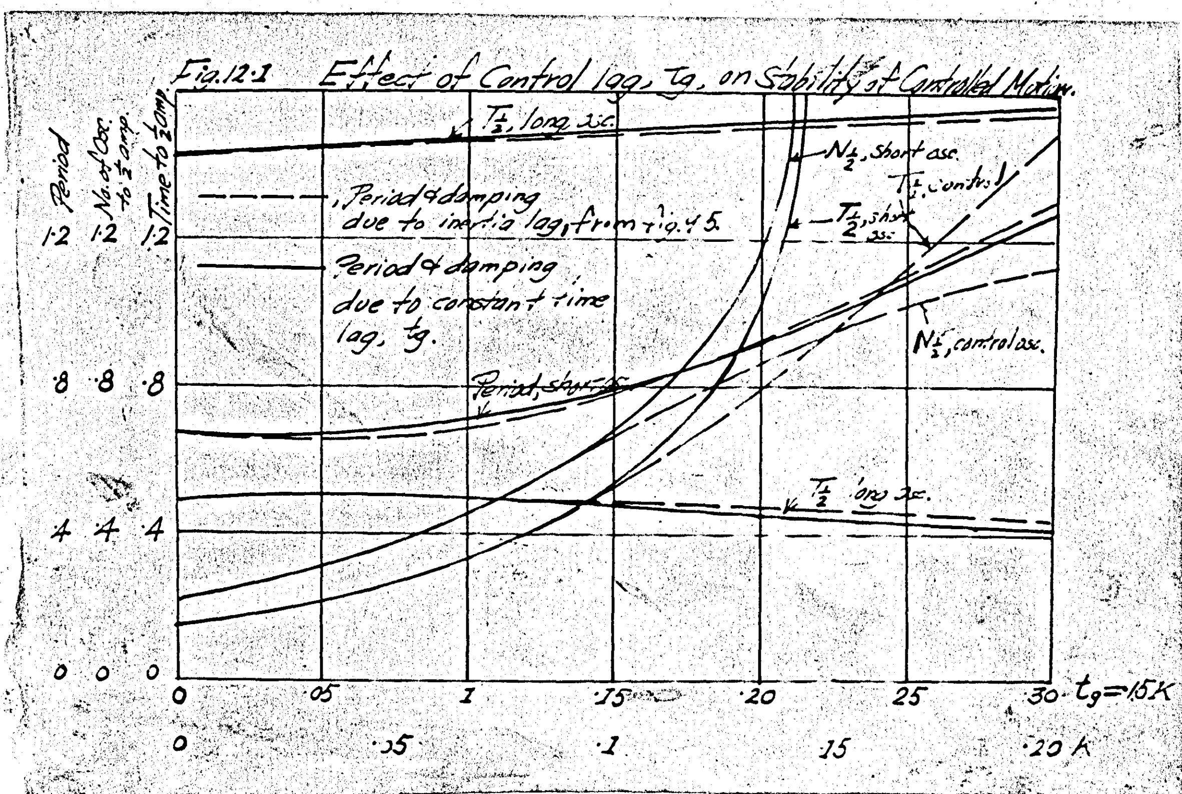


Fig.12.1 compares the the effects on the stability of the controlled motion due to the constant time lag t_g as calculated by the approximate method against the effects on the control oscillation due to the inertia,k of the control system as plotted in Fig.9.5 previously.

It was probably due to ,at least partly, coincidence, that they check almost beautifully,by assuming the relation t_g = 1.5 k, as found in paragragh b,article 11. up to t_g=0.20 The deviation,especially in its effect on damping, from that of the effect due to inertia lag, when t_g is greater than .15 ,may be partly due to the fact that the approximate method used here is only valid when t_g is small compared with the short period of the airplane, and partly due to the different nature of constant time lag and inertia lag.

In short, it can be concluded that the effect of the constant time lag on the controlled motion is

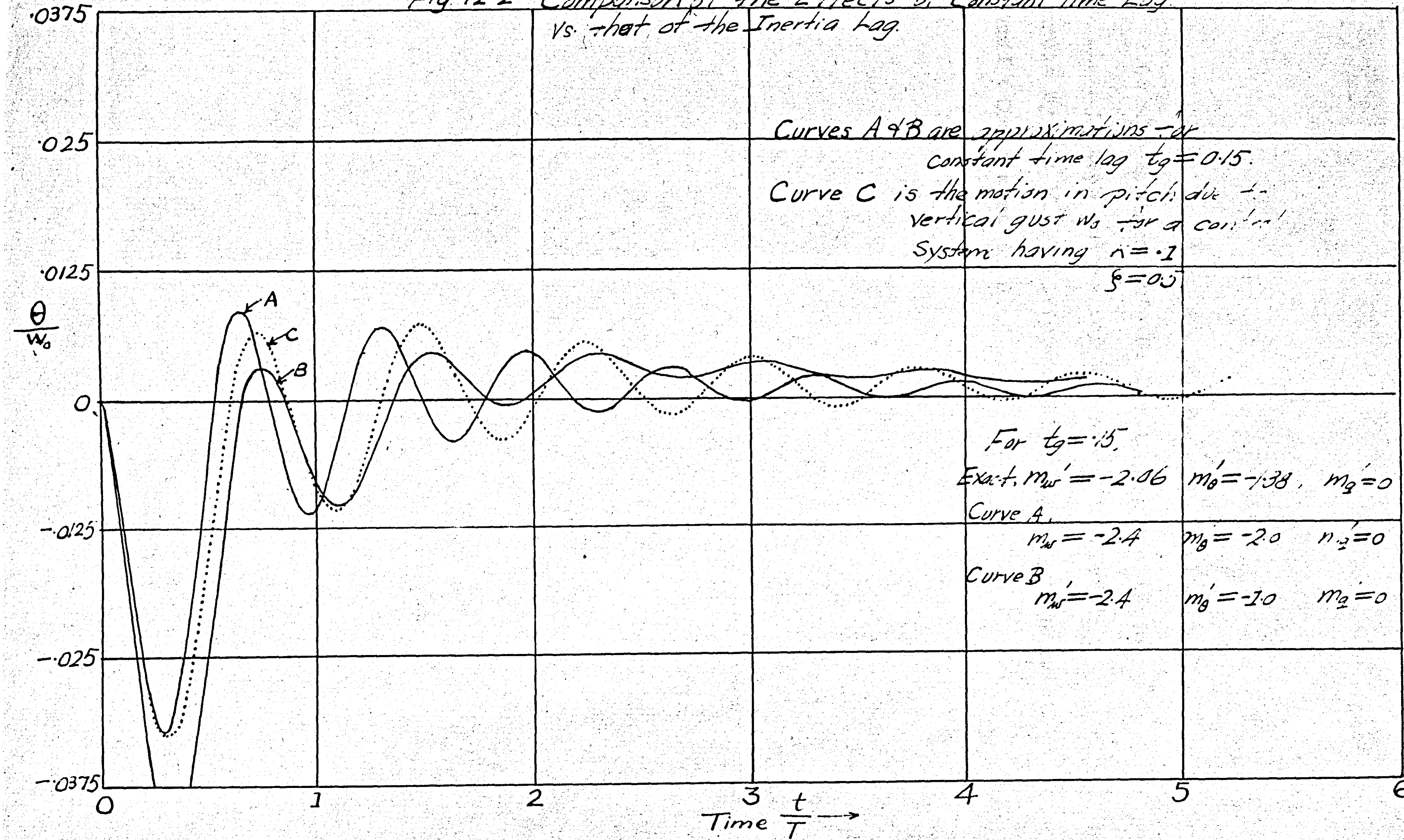
(a) to decrease the damping of the short oscillation just as the inertia lag does on the 'control oscillation',

(b) to increase the period of the short oscilation just as does the inertia lag on the 'control' oscillation'.

(c) to affect the period and damping of the long oscillation only slightly.

It is found that even the magnitude of the disturbed motion as calculated by the above approximate method of introducing the control lag, t_g, instead of providing additional degree of freedom for the control system checks pretty well with the disturbed motion found by solving the equations (2.1),(2.2),(2.3) and (2.4). Fig.12.2 compares the disturbed motion of the inclination in pitch due to a vertical gust as calculated by the M.I.T.Differential Analyzer, for a control derivative $m_\theta = -2$. The dotted curve is found out from equations (2.1,2.2,2.3,2.4) by assuming the inertia factor, $k = 0.1$, and the damping ratio equal to 0.5. The solid curve is found by assuming

Fig. 12·2 Comparison of the Effects of Constant Time Lag vs. that of the Inertia Lag.



constant time lag $t_g = .15$. As shown in table 12.1, $m'_w=2.06$, $m'_\theta=-1.38$, and $m'_q= 0$. Due to difficulty in obtaining exact values for m'_w and m'_θ in plotting curves by the M.I.T. Differential Analyzer, the nearest round values were used.

13. Summary and Conclusions on the study of the longitudinal stabilization as a function of the angle of inclination in pitch.

a. The operation of the elevator control in stabilizing the longitudinal~~me~~ motion of the airplane introduces a control derivative m_θ into the equations of motion of the airplane.

b. The control derivative m_θ in its non-dimensional form can be expressed as

$$m_\theta=1/b_1(\partial C_L/\partial\alpha')(\partial\alpha'/\partial\beta)(\partial\beta/\partial\theta)(S'/S)$$

and is a function of inertia in pitch, tail aspect ratio, tail efficiency, tail size and the gearing ratio of the elevator control of the airplane. The normal operation range for the Sperry Automatic Pilot gives m_θ ranging from -0.5 to -2.0.

c. The introduction of m_θ improves the poor distribution of damping between the long and the short oscillations considerably. The damping of the long oscillation improves so ~~quickly~~ well that for m_θ greater than -2, the long oscillation is transformed into two well damped aperiodic motions. The damping of the short oscillation though decreases slightly, remains to be well damped for all normal values of m_θ.

d. The lag in the control system can be mathematically treated in two convenient ways,

(1) by expressing the inertia, damping and restoring effort of the control system in terms of the non-dimensional parameters k and ζ such that

$k = T_n/2\pi$ where T_n is the undamped natural period of the control system alone expressed in terms of the ~~non~~ time unit of the non-dimensional system.

ζ = damping ratio = ratio of the damping coefficient of the control system relative to that for critically damped one.

(2) By expressing the control lag in terms of ~~as~~ constant time lag t_g, in time unit of the non-dimensional system, as the constant time phase difference between the motion of the control and that of the controlled member.

e. The first method of considering the control lag has the advantage that the parameters k and ζ can be estimated by measuring the free vibration characteristics of the control system. It has, however, the disadvantage of adding one more degree of freedom to the equations of motion of the airplane which are already quite complicated themselves.

f. The second method of considering constsnt time lag is more suitable in approximating the lag due to human reation for case of manually controlled airplane. It also has the advantage that the constant time lag can be very conveniently exproximated with good accurracy without raising the order of the equation characterizing the stability of the motion. The disadvantage of this method is that it lacks means of expressing the degree of overshooting of the control motion.

g. The effect of control lag is to limit the magnitude of m_θ allowable for satisfactory control operation without 'hunting'. It is found that for a control system having a natural period $T_n=0.2\pi$ time unit and a damping ratio equal to 0.5 will 'hunt' as m_θ increases to -2. The control hunting can be cured by increasing the damping ratio to unity.

h. The general effect of the inertia and damping in the control system on the motion of the controlled airplane is found to be quite similar to the simpler system such as the constant speed control investigated by Weiss[20], namely, the inertia of the control system is chiefly responsible for the lag of the control behind the controlled motion while the degree of damping of the control system is responsible for the overshooting of the control motion. A desirable control system is one which has a natural period which is very short compared to the short period of the 'ideally' controlled airplane. It is noticed that an over-damped control system for an airplane is far less undesirable than an under-damped one.

i. The inertia lag is found to be essentially equivalent to constant time lag by the approximate relation

$$t_g = 1.5\ k \mp .25\ T_n$$

It is found that the above relation holds approximately true for the nondimensional airplane derivatives of the airplane F-22 for t_g from 0 to 0.15 time unit.

j. The effect of constant time lag on the motion of the controlled airplane is found to be equivalent to

(1) increase the inertia in pitch. slightly.

(2) decrease the damping in pitch m_q considerably.

k. In general, the method of representing control lag by inertia and damping characteristics of the control system gives satisfactory means of evaluating the controlled motion of the airplane controlled by automatic pilot. The numerical calulations of the motion of the airplane F-22 seem to agree very closely with the information furnished by Sperry Co.[21] The constant time lag method is believed to be a convenient way for the airplane designer to provide sufficient tail size to take care the maximum possible control lag.

l. The period of the third oscillation or called as control oscillation in this thesis is found to be nearly equal to the undamped natural period of the control system except when T_n is very small. As T_n approaches zero, the period of the control oscillation approaches that of the short period of the ideally controlled airplane.

m. From the study of the magnitudes of the disturbed motions under various combinations of the control characteristics, and the numerical calculations of the roots of the corresponding stability equations, it is noticed that the excessively large magnitude of the disturbed motion is always due to that mode of motion which is badly damped. The number of oscillations damped to half amplitude, $N_{1/2}$ seems to be a better criteria for the degree of damping than the time to dampe to half amplitude $T_{1/2}$. It seems that the magnitude of the disturbed motion can always be kept satisfactorily small compared to the uncontrolled airplane by adjusting the control characteristics so as to give $N_{1/2}$ for each mode of motion smaller than 0.6. Any mode of motion with $N_{1/2}$ smaller than 0.2 will contribute very little to the magnitude of the disturbed motion, especially when the period is short at the same time.

n. It is found that although the control derivative m_θ is very effective in giving stability for the longitudinal motion, it is noted quite so effective in reducing the 'first surge error' even for ideal control. At $m_\theta = -2$, the first surge error is approximately 60% as large as that for the uncontrolled airplane when the control system is ideal. For a real control system having the inertia factor $k=0.1$ and damping ratio= 1.0, the first surge error is approximately 85% of the uncontrolled one.

Chapter II

CONTROLLED LONGITUDINAL MOTION IN GENERAL

1. Introduction

From the study of the longitudinal stabilization as a function of the angle of inclination in pitch, it was found that the effect of control derivative has greatly altered the motion of the airplane as compared to the uncontrolled one. As the pitch control, though effective in stabilizing the motion, is ~~by~~ far from being ideal and completely satisfactory. Its effectiveness in reducing the initial surge error due to gust disturbances is found to be unsatisfactory if optimum flight comfort is desired. The control lag, if not properly installed, would cause hunting of the controlled motion. It was also pointed out that the equivalent effect of the control lag is to alter the derivatives of the airplane. Therefore, it is highly desirable to investigate,

a. What combination of the airplane derivatives would give a best result of the controlled motion?

b. What other control devices using disturbance detectors other than the pitching angle detector, could be used ?

c. Is it possible to use certain additional auxiliary control device to cancel the control lag due to the pitch control ?

It is the purpose of this chapter to investigate mathematically the effects of varying different control derivatives on the stability of the controlled motion of the airplane. It is found that the use of the M.I.T.Differential Analyzer is of great value in simplifying the process in solving the disturbed motion due to atmospheric gusts.

2.Possible control devices.

In the longitudinal motion, the pilot can influence the airplane by movement of either the elevator which produces a pitching moment to oppose the disturbance such as the angle of inclination in pitch treated in Chapter I, or the throttle of the power plant which produces a longitudinal force to oppose any disturbance. In either case, the control derivatives thus produced depend on the disturbance detector used in the control device. In the Sperry and Smith gyro pilot, the disturbance detector used is the Sperry gyro which detects the angle of inclination in pitch, θ, thus, if connected to move the elevator, a control derivative m_θ is produced. Haus[1] gives the names and nature of various instruments which can be used as disturbance detectors. The following table shows the corresponding control derivatives that could be produced.

Table 2.1

Disturbance detector	Physical quantity detected	Control derivatives produced by, Elevator movement.	Throttle movement.
1. Airspeed indicator	airspeed, u	m_u	x_u
2. Wind vane	incidence, -w/U	m_w	x_w
3. Free gyro suspended at its c.g.	Absolute inclination in pitch, θ	m_θ	x_θ
4. Motor-driven gyro with precessional moment	Angular velocity, q	m_q	x_q
5. Pendulum to detect acceleration along OX	Direction of apparent gravity du/dt or $\dot{u}$	$m_{\dot{u}}$	$x_{\dot{u}}$
6. Accelerometer along OZ	Apparent gravity, dw/dt or $\dot{w}$	$m_{\dot{w}}$	$x_{\dot{w}}$
7. Lift indicator	Magnitude of lift, wU	m_w	x_w
8. Rate of climb meter	Vertical speed, w	m_w	x_w
9. Torsional accelerometer about OY	Angular acceleration $d^2\theta/dt^2$ or $\dot{q}$	$m_{\dot{q}}$	$x_{\dot{q}}$

It is seen in Table2.1 that many of the control derivatives such as m_w, m_q and so on are identical with the aerodynamic derivatives of the airplane. Therefore they enter into the equations of motion by ~~merely~~ simply adding or subtracting from them. It is evident that while the aerodynamic derivatives have their magnitudes almost fixed, the control derivatives can be varied through a wide range with their maximum values limitted only by the seriousness of the control lag. The sign of the control derivative can also be whatever we want.

3.Equations of motion.

The equations of motion which include every possible control devices can be written as follows,

$$(1+x_{\dot{u}})du/dt - x_u u + x_{\dot{w}} dw/dt - x_w w + (x_{\dot{q}})d^2\theta/dt^2 - x_q(d\theta/dt) - x_\theta \theta = 0$$

$$dw/dt - z_w w - z_u u - \mu d\theta/dt = 0 \qquad (3.1)$$

$$m_{\dot{u}}(du/dt) - m_u u + m_{\dot{w}} dw/dt - m_w w + (1+m_{\dot{q}})d^2\theta/dt^2 - m_q d\theta/dt - m_\theta \theta = 0$$

The non-dimensional determinant can be written as,

$$\begin{vmatrix} d(1+x_{\dot{u}})-x_u & dx_{\dot{w}}-x_w & x_{\dot{q}}d^2-dx_q-x_\theta \\ -z_u & d-z_w & -d\mu \\ dm_{\dot{u}}-m_u & dm_{\dot{w}}-m_w & (1+m_{\dot{q}})d^2-dm_q-m_\theta \end{vmatrix} = 0 \;\ldots\ldots(3.2)$$

Thus it is seen that the effect of $x_{\dot{u}}$ is to increase or decrease the inertia of the airplane in the X-axis,and the effect of $m_{\dot{q}}$ is to alter the inertia of the airplane in pitch. Comparing with the determinant (12.3) of Chapter I, it is seen that the $m_{\dot{q}}$ and m_q control can be used to minimize the effect of constant time lag which is present in the m_θ control. The x_u and m_q control are seen to add merely more damping into the system. It is therefore,necessary to have the m_θ control in addition to them in order to have a good distribution of damping. Variation of x_θ is equivalent to changes of μC_L in the uncontrolled airplane. B.M.Jones[2] has shown that the effect of $m_{\dot{w}}$ is to add some more damping to the short oscillation for unco ntrolled airplane

Although it may be necessary to go through mathematical investigation, in order to have a quantitative idea about the relative merits of each control device, it is quite reasonable to believe that any control system without including the m_θ control will be very unwise as it is the easiest means to apply the control influence and probably the best way of stabilizing both the long period and short period oscillations.

The following numerical investigation will limit to the effects of varying m_w, m_q and x_θ only. As it is known to us that m_w and m_q can be varied not only through control devices shown above, but also can be proportioned at the disposal of the airplane designer by shifting the c.g. position and tail size of the airplane. The effect of varying x_θ is interesting as it is equivalent to changes of flight condition.

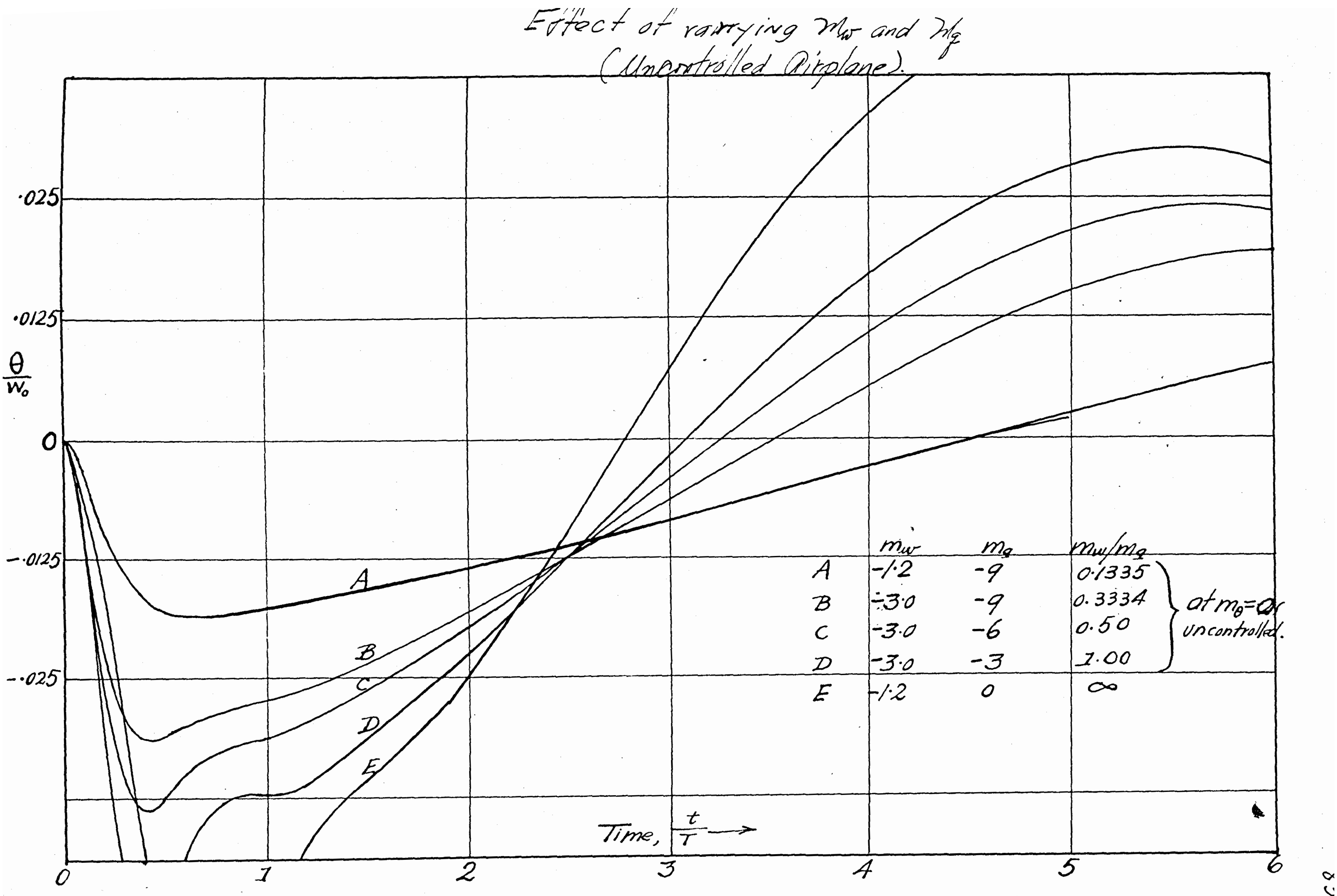
4. Effect of varying m_w and m_q

a. Uncontrolled airplane

The effect of varying m_w for the uncontrolled airplane has been investigated by Jones[3], Haus[4], and other investigators. Using the derivatives for the airplane F-22, Fig. 4.1 shows the disturbed motion in pitch due to a vertical gust as solved by the M.I.T. Differential Analyzer. It is seen that increase of the ratio m_w/m_q merely shortens the period of the long oscillation and increases the magnitude of the disturbed motion. It is also noted that the damping of the long period oscillation is but improved very little through the variation of m_w or m_q. Therefore, study of the uncontrolled stability alone gives the designer little ~~idea~~ information for a quantitative idea about the proper magnitudes of m_w and m_q.

b. With m_θ control having no lag.

Fig. 4.2(A) and (B) show the effect of varying m_q on the disturbed motion due to vertical gust. It is obvious that the principal effect of large m_q is to provide adequate damping for the short period oscillation. If the control has no lag, $m_q = -3$ would be satisfactory. It is seen also that increase of m_q is very effective in cutting down the initial surge error. It is also interesting to note that the variation in the vertical



Effect of varying m_w and m_q
(Uncontrolled Airplane).
·025
·0125
$\frac{\theta}{w_0}$
0
-·0125
-·025
A
B
C
D
E
Time, $\frac{t}{T}$
0
1
2
3
4
5
6
	m_w	m_q	m_w/m_q
A	-1·2	-9	0·1335
B	-3·0	-9	0.3334
C	-3·0	-6	0·50
D	-3·0	-3	1.00
E	-1·2	0	∞
at m_θ=0 or uncontrolled.

Fig. 4.2 (A).
Effect of varying M_q
(Ideal control).

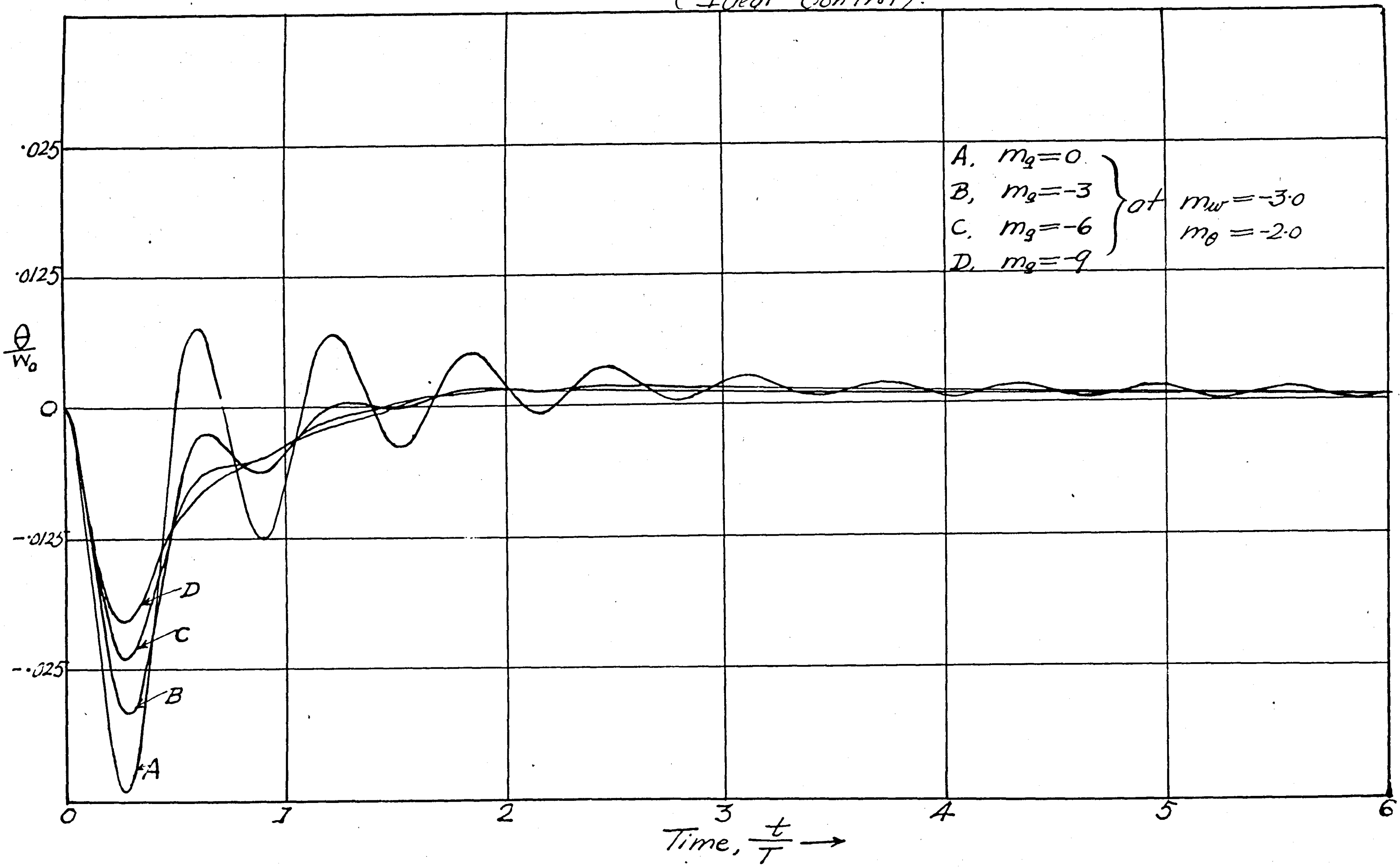
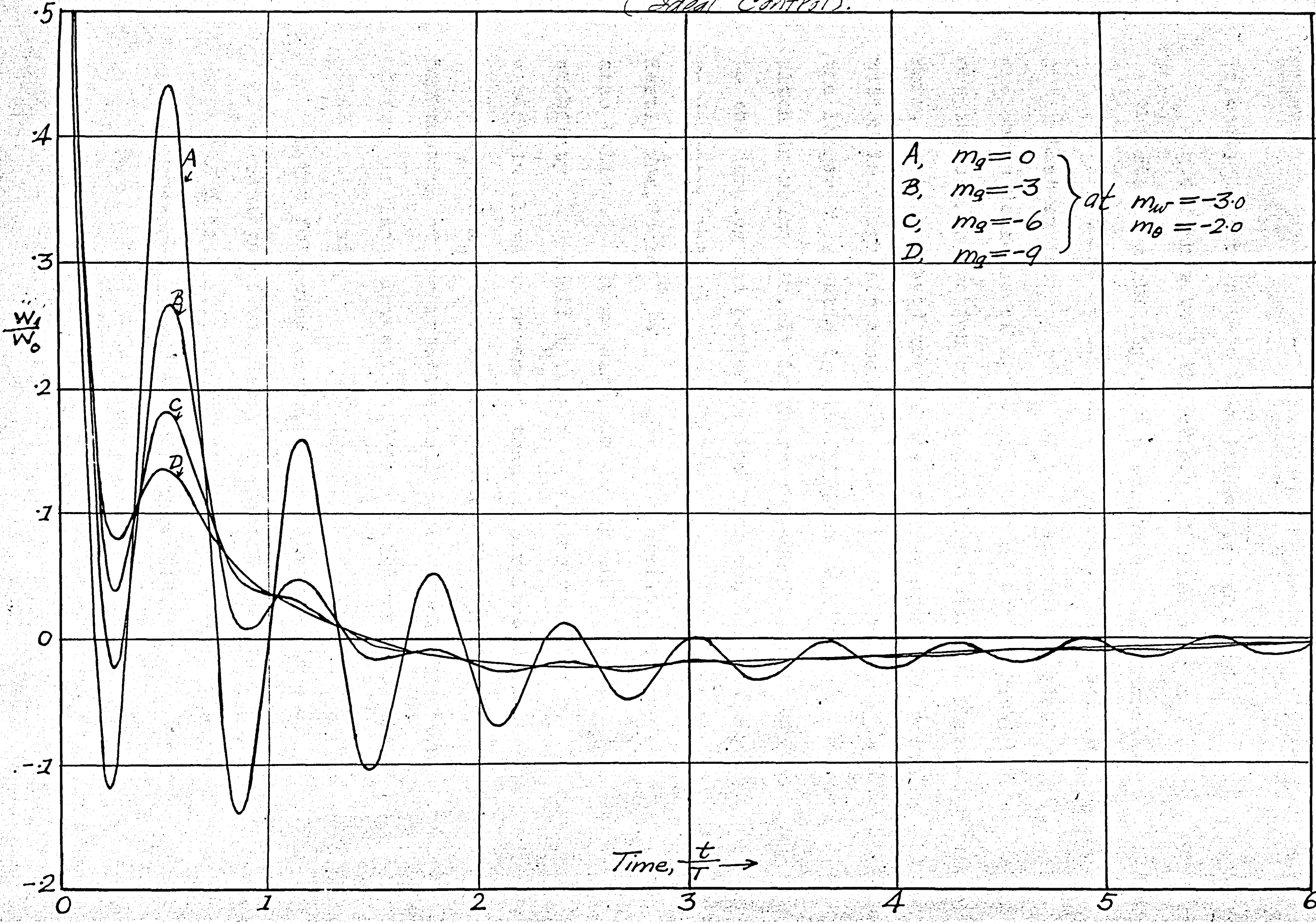


Fig. 4.2 (B).
Effect of varying M_q.
(Ideal Control).



velocity is more severe than the motion in pitch.

Fig.4.3 show that the effect of increasing m_w, or the static stability of the airplane is to increase the initial surge error. Curve C of Fig.4.3(A) and (B) shows that an increse of m_q of 50% over the original one and a decrease of m_w to only 40% of the original one improve the motion due to gust tremwndously. Therefore, it is believed that a correct rating of the pitching motion in rough air should be based on the ratio m_w/m_q, the larger this ratio is ,the airplane will do more pitch in rough air. It is therefore,not directly related to the damping and period of the long oscillation for an uncontrolled airplane,which has been concludded by Hartly ~~and~~ A,Soule'[5] in a flight observation. It is also believed that an airplane having small static stability ,and small m_q would be easier to pitch by a certain elevator movement than an airplane having large m_w and m_q[6] (see fig.56a,Durand,Vol.V). A rough examination of the relative tail size and c.g.location of the airplanes investigated in T.R.578 confirms the above statements.

In the following table, the tail size expressed as a percentage of wing area is a rough measure of m_q while the % of c.g.location relative to M.A.C. is a relative measure of m_w.(all data from T.R.578).

Fig. A.3 (A).
Effect of varying M_w & M_q
(Ideal Control).

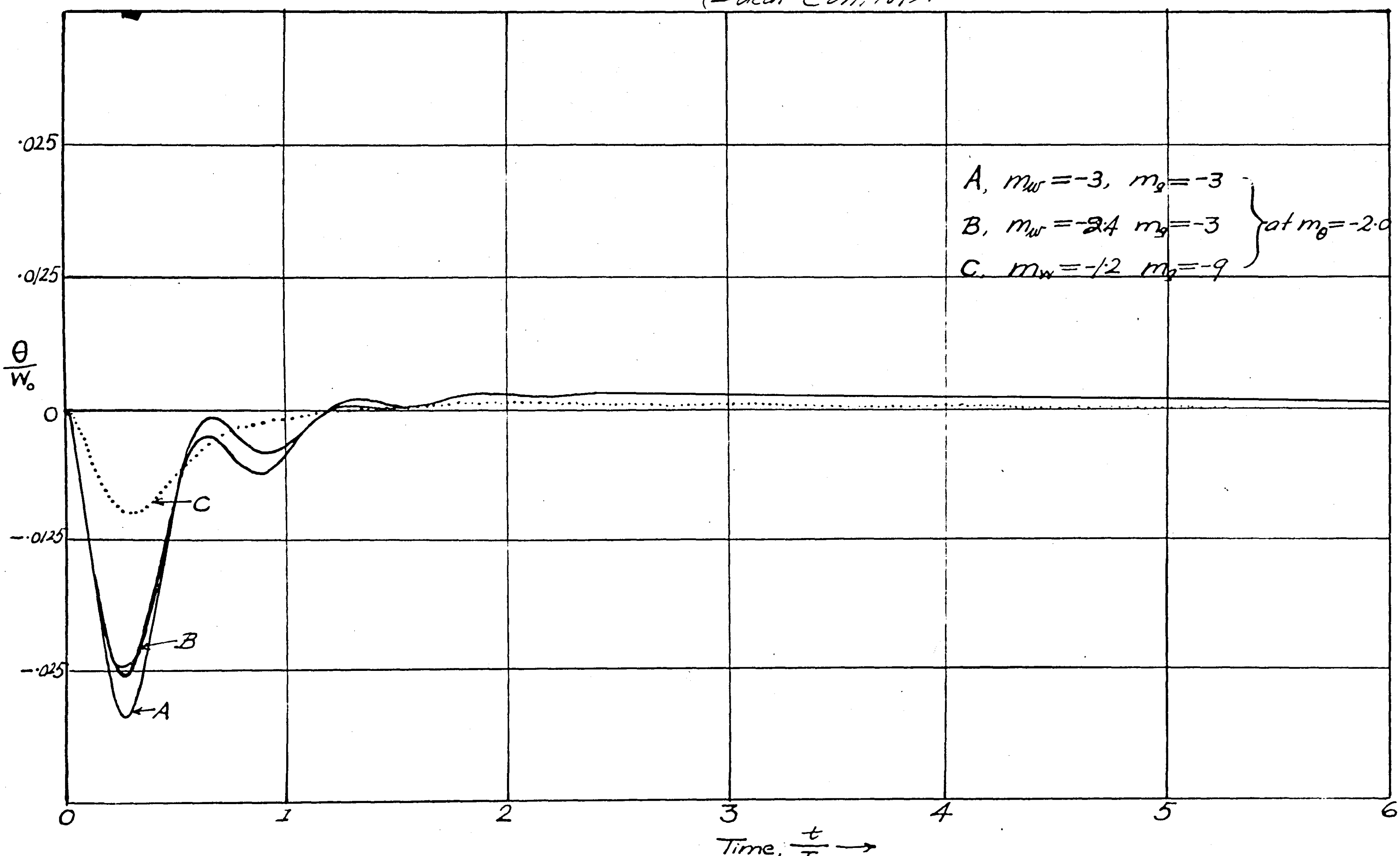


Fig. 4.3 (B).
Effect of varying M_w & M_q
(Ideal control).

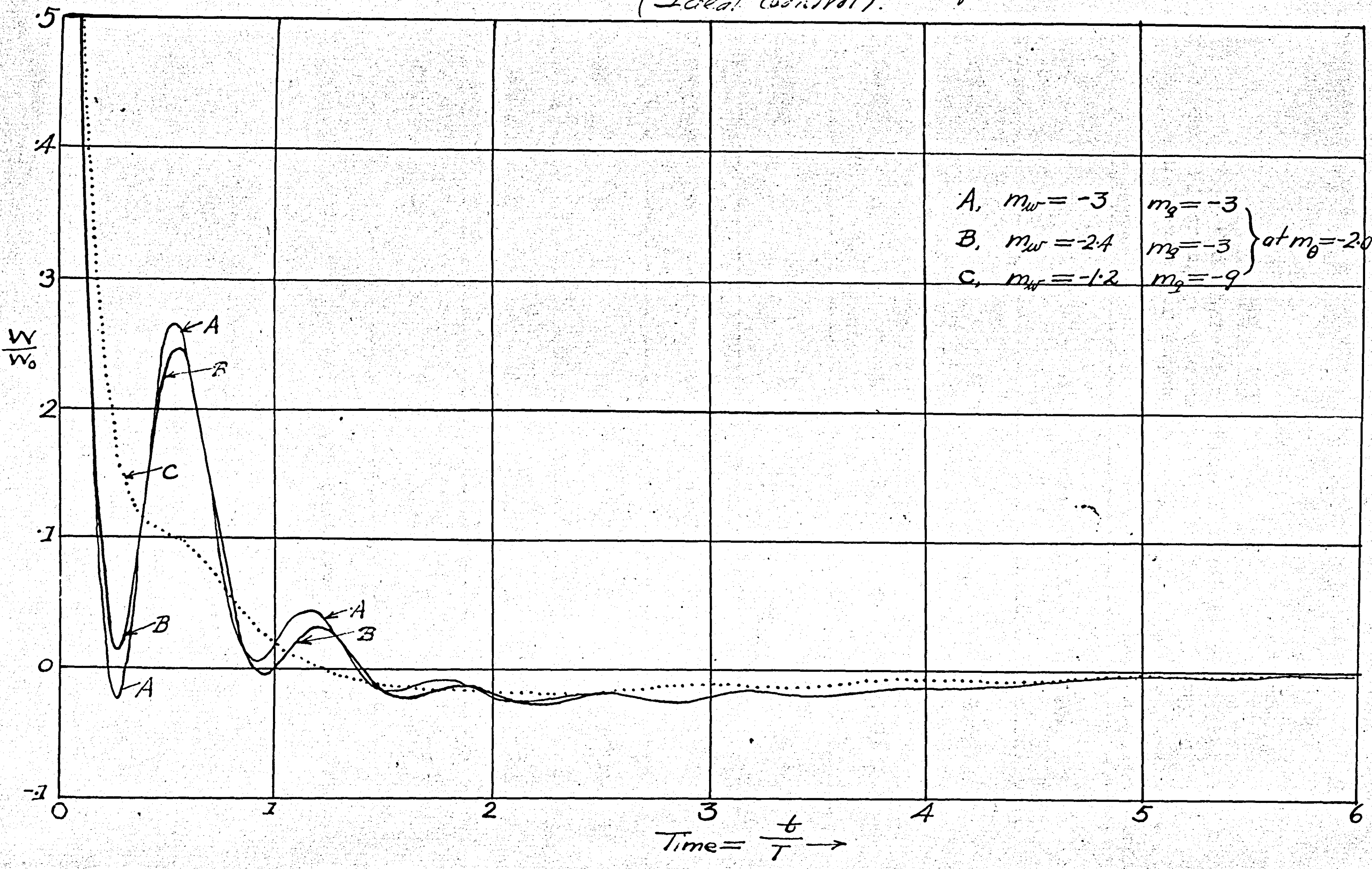


Table 4.1

Airplane	Observed Characteristics Pitching in rough air	Elevator movement	Airplane characters m_q as a function of % of tail size		m_w as a function of c.g.location	
1.F-22	A	B	15.4%,	A	28%,	A
2,NY-2	A	B	9.5%	D	28.6%	A
3,0-2H	A	A	12.5%	C	36,6%	D
4,T4M-1	B	A	12.9%	C	30.9%	C
5,FC2-W2	B	B	14.1%	B	30.6%	C
6,AT	B	B	12.4%	C	33.1%	B
7,F4B-2	C	C	16%	A	38.5%	D
8,XBM-1	D	C	13.1%	B	29.7%	CC

A is used to designate airplanes that are stiffest,require greatest elevator movement,do most pitching in rough air, having largest percentage of tail area, and foremost c.g. lacation.

Owing to the different wing sections used, the c.g. location is not necessary a direct indication of m_w.But in general, airplanes having large tail and rearmost c.g, pitch less in rough air, and airplanes having smallest tail, rearmost c.g.location are easiest to pitch by elevator movement. From the above discussion it is therefore, believed that airplane designers should have the static stability as nearer to zero as possible for a good all round stability and controlability. M_q can neither be too small,lest that the airplane might be unstable, nor too large to avoid the undesirable stiffness in elevator control.

Fig.4.4(A) and (B) show the effect of negative static stability,i,e, m_w being positive. It is seen that a large negative static stability is unbearable as the airplane will be unstable even with m_θ control. A slight negative static stability is ,however,not so disastrous as it may seem to us. In any case, a zero static stability is most desirable.

c.Effect of varying m_w and m_q with control lag.

Fig.4.5 shows the effect of varying m_w and m_q when the inertia lag is present. It is seen that m_q is primarily responsible for minimizing the lagging effect, while m_w only changes the initial surge error greatly. It is believed that m_q should be such that its magnitude

Fig. 4.4 (A).
Effect of varying M_w
(Ideal Control)

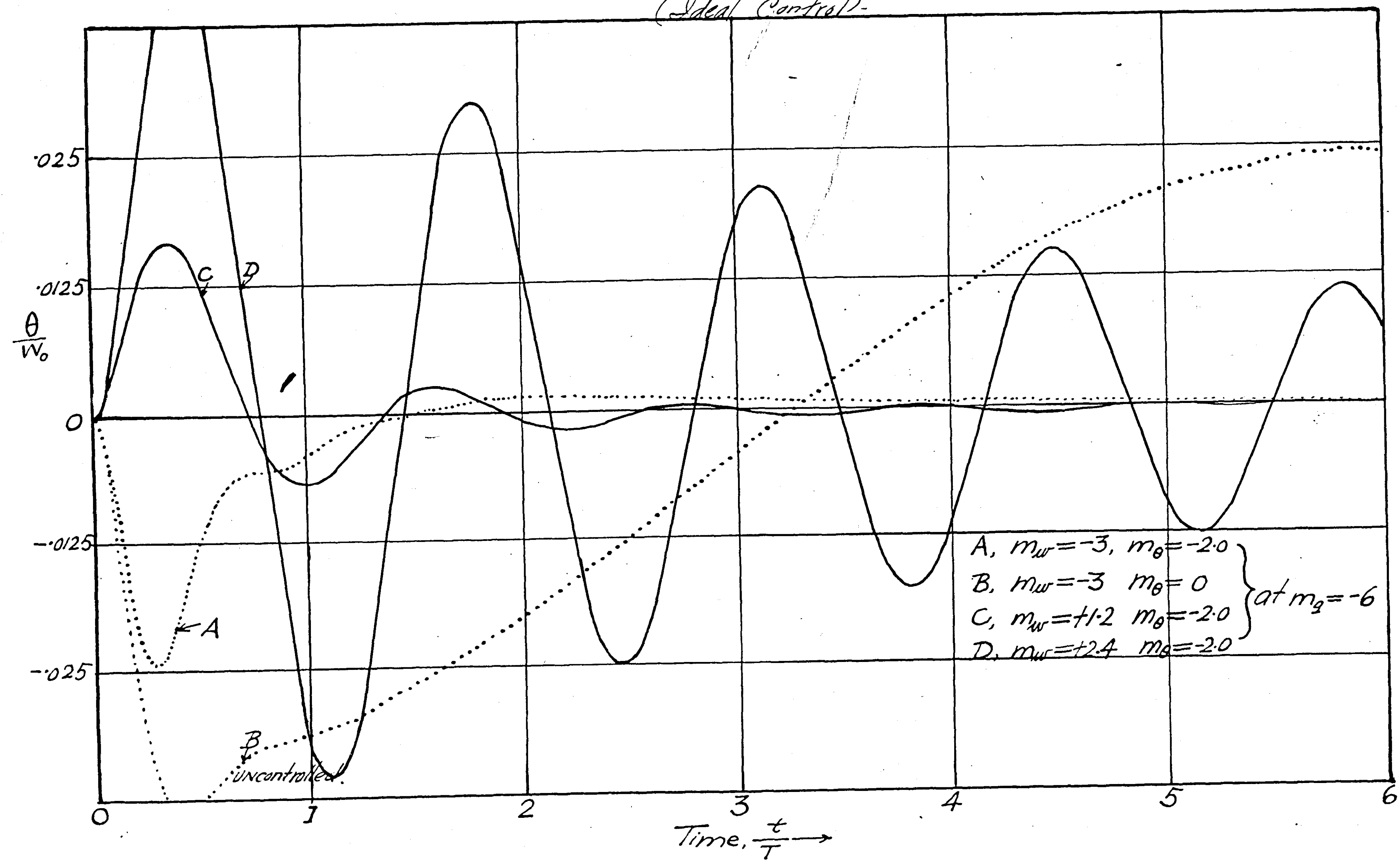


Fig. 4.4 (B).
Effect of varying m_w
(Ideal Control).

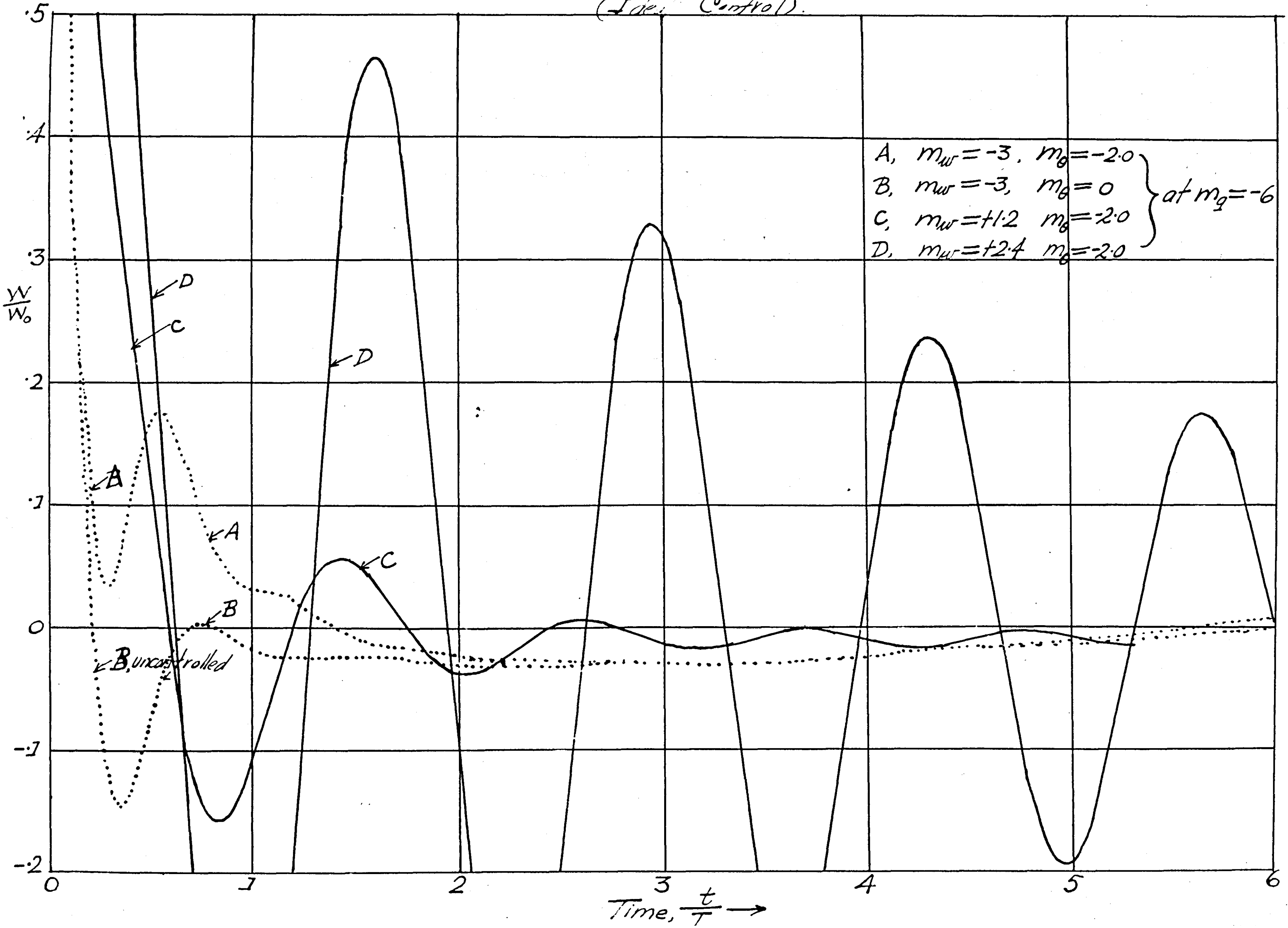
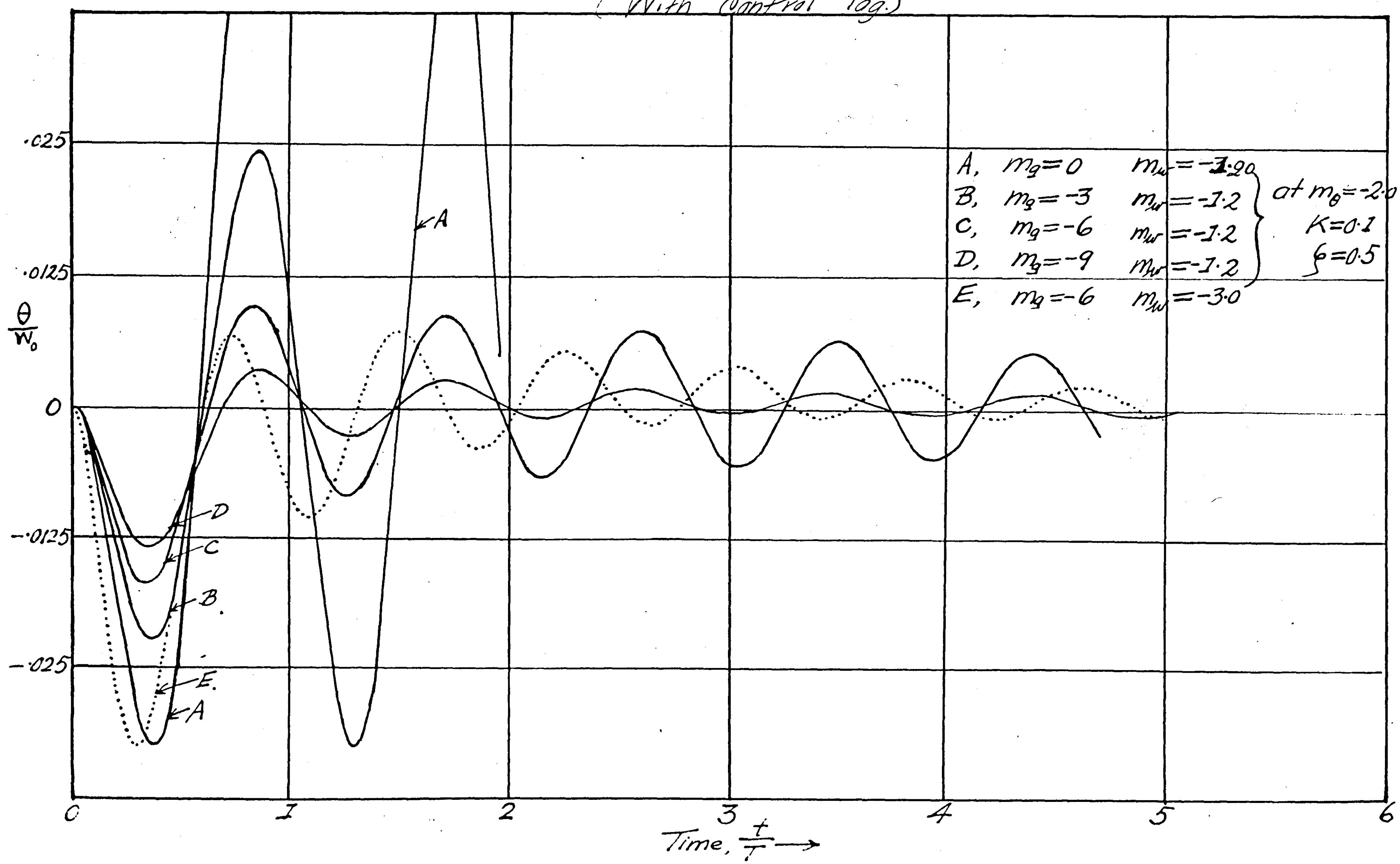


Fig. A.5.

Effect of varying m_w and m_q.

(With control lag.)



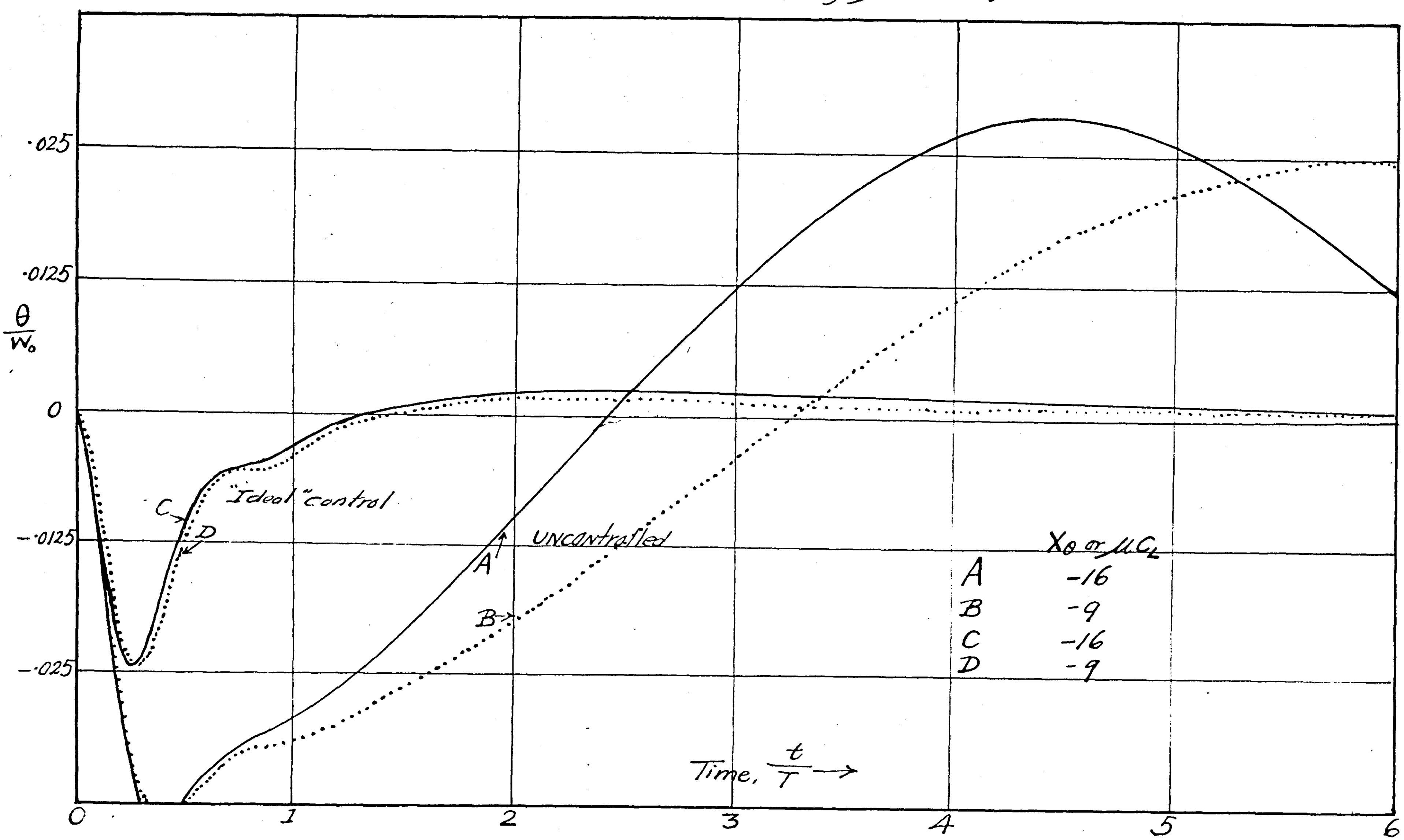
be larger than the equivalent negative m_q due to control lag, t_g, namely, $-\mu m_\theta t_g$, by certain amount such as -3, to provide for satisfactory stability.

5. Effect of varying μC_L or x_θ.

Fig. 5.1 shows the effect of varying μC_L for an airplane controlled and uncontrolled. For an uncontrolled airplane, it is seen that changes of μC_L merely change the period of the long oscillation slightly. The damping of the long oscillation at high angle of attack is less than at low angle. However, for a controlled airplane, variation of μC_L makes little difference in the final criteria of stability.

Fig. 5.1.

Effect of varying μC_L or X_θ



6.Summary and conclusion of the controlled longitudinal motion.

a. From the study of various control devices,it is seen that most of them merely produce devivatives similar to that of the airplane. Therefore,the use of those control devices is equivalent to modify the characteristics of the uncontrolled airplane.

b.The principal advantage of the m_θ control is to equalize the damping of the long and short oscillations,which is a character the other control derivatives lack. Therefore,it is believed that the use of m_θ control is essential for good stability. Other kinds of control should be used in combination with the m_θ control to achieve optimum performance and comfort,such as to reduce the surge error,as well as the effect of control lag introduced by m_θ control.

c.Reduction of m_w to the vicinity of zero and increase of m_q is desirable for both small surge error and comfort of flying. As the initial acceleration due to vertical gust depends on the derivative z_w which can hardly be modified, the only course toward better flight comfort is to reduce the oscillatory motion after disturbance. This is effectively done by having small m_w and large m_q.

d. For good controllability, a small m_q is desirable.

Therefore, it is believed that tail size should not be too large. An instrument detecting the quantity q and thus giving m_q in addition to m_θ should give optimum stability and controllability.

e. The smallest m_q to stabilize the controlled airplane is approximately equal to -3. Additional m_q to provide for control lag should be calculated by the expression, $\mu m_\theta t_g$.

f. $m_w = 0$ is most desirable. A slight negative static stability is allowable if the airplane is properly controlled.

g. The effect of varying μC_L is unimportant for a controlled airplane.

Chapter III

CONTROLLED LATERAL MOTION

1. Introduction

Airplane designers have been confused in interpreting the connection between the results found from the study of the uncontrolled lateral stability equations and the observed stability and controllability of airplanes in flight. The lateral stability equation of an uncontrolled airplane is found to be an equation of the fifth degree. One of the roots of the stability equation is always zero signifying that an uncontrolled airplane is insensitive to direction in azimuth. In the remaining quartic, two of the roots are real with the other two, ~~imaginary~~ complex. One of the real roots is found to be always large and negative, characterizing the rapid subsidence in roll. The remaining real root is small and can be either positive or negative depending on the relative amount of 'lateral static stability' derivative n_v and the 'dihedral derivative', l_v l_v, which are all at the disposal of the airplane designer. It is found that the larger is the dihedral derivative, l_v the wider is the range for n_v without causing spiral divergence (i.e the small real root becomes negative) and osillatory divergence (i.e. the real part of the complex roots to become negative). Only when n_v is very negative, (negative static stability), the oscillatory divergence will separate into two rapidly increasing exponential mode[1]. Thus the study of the uncontrolled stability equation seems to suggest the idea that the dihedral should be as large as possible while the vertical tail size which gives positive static stability n_v is unimportant in the criteria for lateral stability so long as it is not too small to cause excessive negative n_v due to the instability of the fuselage[2]. However, the observed stability and controllability requirements seem to be so closely related to the aileron and rudder control as well as the dihedral and tail size derivativs[3] that any stability criteria without considering the stabilizing effects due to control operations will lead the designer to provide improper proportions of dihedral and tail sizes.

It is, therefore, the purpose of this chapter to study the equations of motions of a controlled airplane in its lateral motion in order to derive a quantitative criteria for satisfactory lateral stability of a controlled airplane.

2.Review of previous work on the controlled lateral stability.

The study of controlled stability was done by Garner[4] as early as 1926. Cowley[5] ~~an~~ examined the mathematical way of expressing control lag. Koppen[6] studied the controlled stability by aileron at high angle of attack. Meredith and Cooke[7] described their physical conception with regard to the effect of control lag on the controlled motion. In general, it is found that the effect of rudder is to give the airplane sensitivity in azimuth. From Garner's mathematical investigation, it is found that for an airplane controlled by rudder alone, the slow spiral motion present in the uncontrolled airplane is changed into a long period bank and yawing oscillation very poorly damped. The rolling subsidence remains as heavily damped as for uncontrolled airplane. The short period oscillation is sensibly unchanged by rudder stabilizing factor. The aileron control adds damping to the long oscillation and is greatly affected by tail size and dihedral of the airplane. Koppen pointed out that at high angle of attack, the damping for the short period oscillation with aileron control alone is very poor and can only be improved by increasing tail sizes.

However, again the difficult mathematical manipulations involved in investigating the roots for the stability equations and particularly in finding the disturbed motions due to atmospheric gusts or control manipulation have limited the previous investigations to very few special cases so that few conclusions toward better airplane and control design could be reached. Again, the methods of representing control lag in previous investigations have been inadequate and confusing so that the effect of control lag can not be conveniently expressed to give the airplane designer a clear conception. Weiss[8] derived the expressions for the equations of motions of an airplane controlled by both aileron and rudder and expressed the control lag in the aileron and rudder operations by adding two more degrees of freedom to the uncontrolled equations of motion. The final controlled stability equation is of the ninth order which is extremely complicated even to solve for the roots of the equation.

3. Procedure of mathematical investigation of the controlled lateral stability.

From the study of the longitudinal stabilization as a function of inclination in pitch in this thesis, it was found that the effect of inertia lag and that due to constant time lag are sensibly identical if the lag is reasonably small compared to the period of the short oscillations of the 'ideally' controlled airplane. It was also pointed out that the effect of constant time lag can be closely approximated without introducing additional degrees of freedom into the equations of motion and thus the numerical investigation of the controlled stability can be greatly simplified especially in the lateral motion. The disturbed motion due to atmospheric gusts is as important as, if not more than the stability criteria and is to be solved by the M.I.T. Differential Analyzer as did for the longitudinal motion.[9] Due to lack of integrators in the present machine in use, the constant time lag equations will be used instead of the inertia lag expressions which requires four more integrators.

4. Various laws of operating lateral controls

Haus[10] gave the following tabulation of disturbance detectors according to the indications of which the ailerons or rudder can be operated.

Disturbance detector	Detected physical quantity	Stability derivative produced by control	
		aileron	rudder
1. Vane with vertical axis	Angle of side-slip, v/U	l_v	n_v
2. Free Gyroscope	Yaw with respect to axes fixed in space, ψ	l_ψ	n_ψ
3. Gyroscope producing precessional couple	Angular velocity of roll, p	l_p	n_p
4. Free Gyroscope	Roll with respect to axes fixed in space, ϕ	l_ϕ	n_ϕ
5. Gyroscope or speedmeter to measure difference in linear speed of wing tips	Angular velocity of yaw, r	l_r	n_r
6. Direction of apparent gravity recorder, or pendulum on ZOY plane or accelerometer along OY	Direction of apparent gravity $g\sin\phi+dv/dt+Vr$	?	?
7 Torsional accelerometer about X-axis	Angular accelation about OX	$l_{\frac{d^2\phi}{dt^2}}$, or $l_{\dot{p}}$	$n_{\frac{d^2\phi}{dt^2}}$ or $n_{\dot{p}}$
8. Torsional accelometer about Z-axis	Angular accela- about OZ	$l_{\frac{d^2\psi}{dt^2}} \sim l_{\dot{r}}$	$n_{\frac{d^2\psi}{dt^2}} \sim n_{\dot{r}}$

Table 4.1.

The stability derivatives due to control operation by the human pilot are probably very complicated. It is believed that human pilots are most sensitive to the angular displacements in roll and yaw so that the effect of human piloting can be considered as producing control derivatives, l_ϕ and n_ϕ due to aileron operation and n_ψ due to rudder control. n_ϕ will be zero if the aileron has no adverse yaw.[11] Garner[12] mentioned that a skillful pilot can produce a control derivative n_r in addition to the derivative n_ϕ through rudder operation.

5. Equations of motion of a controlled airplane

It is seen from table 4.1 that control derivatives l_v, n_v, l_p, n_p, l_r and n_r are already present in the non-dimensinnal equations for uncontrolled airplane. The control derivatives $l_{\frac{d^2\phi}{dt^2}}$ and $n_{\frac{d^2\psi}{dt^2}}$ are merely the equivalent inertias in roll and in yaw, respectively. They are unity for uncontrolled airplane. It was shown in chapter I, article 12, equations (12.3) that increase of inertia in pitch is equivalent to reductions in stability derivatives in~~t~~- the pitching motion such as m_w, m_q and m_θ. Same thing is true here. It is not quite so convenienent to put the control derivatives $l_{\frac{d^2\psi}{dt^2}}$ and $n_{\frac{d^2\phi}{dt^2}}$ into the equations of stability study, ~~so that~~ without raising ~~the order~~ too complicated forms of the equations, so that this type of control will not be considered here. The control derivatives l_ϕ, n_ϕ, l_ψ, and n_ψ are not present in the equations of motion for uncontrolled airplane and they are by far the most important ones because in the controlled motion either by human piloting or by the most succesful Sperry and Smith Automatic Piloting, the control derivatives l_ϕ, n_ϕ, and n_ψ are always present, l_ϕ and n_ψ are due to direct effect of aileron and rudder control, while n_ϕ

is introduced due to the adverse yawing moment present in ordinary aileron arrangement. The control derivative l_ψ has not been investigated before as its presence requires that the operation of the aileron be coupled according to the displacement in azimuth which is not yet tried by any successful automatic control device known to most of us. However, it is pointed out later that this kind of control manipulation is probably important in the aileron-elevator two control operation.

Using similar non-dimensional system, with

m= mass of the airplane as the unit of mass as in the longitudinal motion,

T= $m/\frac{\rho}{2}SV$ = unit of time, where ρ is the density of the air corresponding to the altitude at which the airplane is flying, S is the wing area and V is the air speed of the airplane.

b/2=half the length of the span of the airplane which is quite nearly equal to the tail length, L in the longitudinal motion for normal airplanes, as the unit of length.

the equations of motion for uncontrolled airplane in nondimensional form can be written as,

$$dv/dt - y_v v + \mu C_L \phi + \mu\, d\psi/dt = 0$$
$$d^2\phi/dt^2 - l_v v - l_p p - l_r r = 0$$
$$d^2\psi/dt^2 - n_v v - n_p p - n_r r = 0 \quad \ldots\ldots\ldots\ldots (5.1)$$

the aerodynamic derivative y_p is neglected and the aerodynamic derivative y_r is small compared with μ and is also neglected.

For controls moving ailerons or rudder or both according to the disturbances p,q r or v,some of the derivatives l_p, n_p, l_r, n_r, l_v and n_v can be added or subtracted from the correspomding derivatives and thus altering the stability of the motion.

For motions controlled by human pilot or by Automatic Pilot such as Sperry or Smith's design,the equations of motion for controls without lag,can be written as follaws,

$$dv/dt - y_v v + \mu C_L \phi + \mu r = 0$$
$$d^2\phi/dt^2 - l_v v - l_p p - \mu l_\phi \phi - l_r r = 0$$
$$d^2\psi/dt^2 - n_v v - n_p p - \mu n_\phi \phi - n_r r - \mu n_\psi \psi = 0 \quad \ldots\ldots(5.2)$$

The expressions for μl_ϕ, μn_ϕ and μn_ψ are quite analogous to μm_θ which was derived in Chapter I. They will be derived in later sections.

The equations of motion expressing all possible control derivatives given in Table 4.1 can be written as follows, $dv/dt - y_v v + \mu C_L \phi + \mu r = 0$ as before, because no control forse is possible in the Y-direction.

$$(1 - l_{\dot p}) d^2\phi/dt^2 - l_v v - l_p p \cancel{xk} - \mu l_\phi \phi - l_{\dot r} dr/dt - l_r r - l_\psi \psi = 0$$
$$(1 - n_{\dot r}) d^2\psi/dt^2 - n_v v - n_{\dot p} d^2\phi/dt^2 - n_p p - \mu n_\phi \phi - n_r r - \mu n_\psi \psi = 0 \quad \ldots\ldots(5.3)$$

6. Non-dimensional determinant of controlled airplane

Let the non-dimensional determinant for the uncontrolled airplane be written as,

$$\begin{vmatrix} d-y_v & \mu C_L & d\,\mu \\ -l_v & d^2-dl_p & -d\,l_r \\ -n_v & -dn_p & d^2-dn_r \end{vmatrix} = \quad = 0 \ldots (6.1)$$

Let the stability equation of the quintic in d obtained by expanding (6.1) be written as,

$$A_o d^5 + B_o d^4 + C_o d^3 + D_o d^2 + E_o d + F_o = 0 \ldots\ldots\ldots\ldots (6.2)$$

where, $A_o = 1$

$B_o = -l_p - n_r - y_v$

$C_o = l_p(y_v + n_r) + y_v n_r + \mu n_v - n_p l_r$

$D_o = \mu l_v(n_p + C_L) - l_p(y_v n_r + \mu n_v) + n_p l_r y_v$

$E_o = -\mu C_L(l_v n_r - n_v l_r)$

$F_o = 0$

Let the determinant with the rudder and aileron control be written as,

$$\begin{vmatrix} d-y_v & \mu C_L & d\,\mu \\ -l_v & d^2-dl_p-\mu l_\phi & -dl_r \\ -n_v & -dn_p-\mu n_\phi & d^2-dn_r-\mu n_\psi \end{vmatrix} = 0. (6.3)$$

Let the stability equation of the quintic in d obtained by expanding (6.3) be written as,

$$A\,d^5 + B\,d^4 + C\,d^3 + D\,d^2 + E\,d + F = 0 \ldots\ldots (6.4)$$

where,

$$A = 1$$

$$B = B_o$$

$$C = C_o - \mu n_\psi - \mu l_\phi$$

$$D = D_o + \mu n_\psi(l_p + y_v) + \mu l_\phi(y_v + n_r) - \mu n_\phi l_r$$

$$E = E_o - \mu n_\psi(y_v l_p) - \mu l_\phi(y_v n_r + \mu n_v) + \mu n_\phi(y_v l_r + \mu l_v)$$

$$F = F_o - \mu n_\psi(\mu C_L l_v) - y_v(\mu n_\psi)(\mu l_\phi)$$

The determinant expressing all possible control derivatives given in Table 4.1 is,

$$\begin{vmatrix} d - y_v & \mu C_L & d\mu \\ -l_v & d^2 - l_{\dot p} d^2 - dl_p - \mu l_\phi & -d^2 l_{\dot r} - dl_r - \mu l_\psi \\ -n_v & -d^2 n_{\dot p} - dn_p - \mu n_\phi & (1 - n_{\dot r})d^2 - dn_r - \mu n_\psi \end{vmatrix} = 0 \quad (6.5)$$

The quintic in d can be written as,

$$A'd^5 + B'd^4 + C'd^3 + D'd^2 + E'd + F' = 0, \ldots\ldots\ldots (6.6)$$

where,

$$A' = (1 - l_{\dot p})(1 - n_{\dot r}) - l_{\dot r} n_{\dot p}$$

$$B' = \cancel{B} - y_v(1 - l_{\dot p})(1 - n_{\dot r}) - n_r(1 - l_{\dot p}) - l_p(1 - n_{\dot r}) - n_{\dot p} l_r - \cancel{X}_{\dot p} - l_{\dot r} n_p$$

$$C' = \underline{C} - \mu n_{\dot p} + l_{\dot r} y_v n_p + l_r n_{\dot p} y_v - l_r n_p - \mu l_\psi n_{\dot p}$$

where $\underline{C} = C$ except the term μn_v is replaced by $(1 - l_{\dot p})(\mu n_v)$

and the term μn_ψ is multiplied by $(1 - l_{\dot p})$

the term μl_ϕ is multiplied by $(1 - n_{\dot r})$

$$D' = \underline{D} + \mu C_L n_v L_{\dot{r}} + \mu l_{\dot{\psi}}\, y_v n_{\dot{p}} - n_p \mu l_{\dot{\psi}} + \mu n_{\dot{\phi}} y_v l_{\dot{r}}$$

Where $\underline{D} = D$ with $y_v \mu l_{\dot{\phi}}$ ~~replaced~~ multiplied by $(1-n_{\dot{r}})$

$y_v \mu n_{\dot{\psi}}$ multiplied by $(1-l_{\dot{p}})$

$$E' = E + \mu l_{\dot{\psi}}(y_v n_p - \mu n_{\dot{\phi}}).$$

$$F' = F + \mu^2 C_L l_{\dot{\psi}} n_v + \mu l_{\dot{\psi}}\, \mu n_{\dot{\phi}} y_v$$

It is evident that any attempt to discuss the stability equation in the complete form would lead to confusion. It is therefore, advisable to limit our discussions in the following sections to the equations of motion for simple human piloting, as well as that for the present Sperry and Smith Automatic Pilot, namely, equations (5.2) and determinant (6.3).

7. Control system with constant time lag.

Let t_a = constant time lag in the aileron control system.

t_r = constant time lag in the rudder control system.

For lag due to human reaction, t_a and t_r would be equal. For the equivalent lag due to inertia in the aileron and rudder control system, t_a may be different from t_r. As it is noticed that the aerodynamic rolling moment due to aileron control lags behind the motion of the aileron by certain amount depending on the location, size, shape and so on of the aileron system[13] it is believed that t_a is in general greater than t_r.

Following the principle of Chapter I, article 12, the determinant (6.3) with constant time lag can be written as follows,

$$(7.1). \quad \begin{vmatrix} d-y_v & \mu C_L & d\,\mu \\ -l_v & d^2-dl_p-\mu l_\phi(e^{-dt_a}) & -dl_r \\ -n_v & -dn_p-\mu n_\phi(e^{-dt_a}) & d^2-dn_r-\mu n_\psi(e^{-dt_r}) \end{vmatrix} = 0$$

Write

$$e^{-dt_a} = 1-dt_a + d^2t_a^2/2 + (\text{ terms neglected})$$

$$e^{-dt_r} = 1-dt_r + d^2t_r^2/2 + (\text{ terms neglected})$$

then, (7,1) becomes,

$$
(7.2)\quad \begin{vmatrix} d - y_v & \mu C_L & d\,\mu \\ -l_v & (1 - \mu l_\phi t_a^2/2)d^2 - (l_p - \mu l_\phi t_a)d - \mu l_\phi & -d l_r \\ -n_v & (-\mu n_\phi t_a^2/2)d^2 - (n_p - \mu n_\phi t_a)d - \mu\, n_\phi & (1 - \mu n_\psi t_r^2/2)d^2 - (n_r - \mu n_\psi t_r)d - \mu n_\psi \end{vmatrix} = 0
$$

Comparing the determinant (7.2) with (6.5), it is obvious that those controls listed in table 4.1 which produce the derivatives, $l_{\dot p}, l_p, n_{\dot p}, n_p, n_{\dot r}$ and n_r can be used TOGETHER with the controls producing derivatives l_ϕ, n_ϕ, and n_ψ with their values so ajusted as to cancel out the undesirable lagging effect produced by introducing the controls l_ϕ, n_ϕ and N_ψ.

The question is now to find out,

a, whether the controls l_ϕ, n_ϕ and n_ψ are desirable or not,

b. If so, what are their operational ranges, under various airplane characteristics, flight conditions and so on.

In order to answer the above questions, derivations for the control derivatives, l_ϕ, n_ϕ and n_ψ for certain control system must be estimated and numerical investigations to determine their effects on the stability of the motion and the magnitudes of the disturbed motion due to certain gust must be carried out.

8. Expressions of the control derivatives l_ϕ, n_ϕ and n_ψ

Using the similar notations for uncontrolled airplane derivatives adopted by Koppen[14], the dimensional derivative L_ϕ is defined as ,

$$L_\phi = 1/A_1 (\partial L_a/\partial\phi) \ldots\ldots (8.1)$$

where L_a is the rolling moment due to aileron, and can be written in terms of the non-dimendional rolling moment coefficient, C_1 as follows,

$$L_a = C_1 \tfrac{\rho}{2} SbV^2 \ldots\ldots (8.2)$$

where C_1 is the rolling moment coefficient adopted by N.A.C.A.[15] Write the moment of inertia about X-axis in terms of the non-dimensional coefficient a_1 as in the longitudinal motion,

$$A_1 = a_1 m(b/2)^2 \ldots\ldots (8.3)$$

Substituting (8.2) and (8.3) into (8.1), we have,

$$L_\phi = \partial C_1/\partial\phi \; (\tfrac{\rho}{2}SV^2)/(a_1 mb/4) \ldots\ldots (8.4)$$

By definition of non-dimensionalization, we have

$$\mu l_\phi = L_\phi (m/\tfrac{\rho}{2}SV)^2 \ldots\ldots (8.5)$$

$$\text{As } \mu = m/\tfrac{\rho}{2}S\tfrac{b}{2} \ldots\ldots (8.6)$$

Substituting (8.4) and (8.6) into (8.5), we have,

$$l_\phi = \{2/a_1\} \partial C_1/\partial\phi \ldots\ldots (8.7)$$

Denoting the angular displacement of aileron by γ,

we have, $$l_\phi = (2/a_1)(\partial C_1/\partial\gamma)(\partial\gamma/\partial\phi) \ldots\ldots (8.8)$$

Following the same procedure, it is found that

$$n_{\phi} = (2/c_1)(\partial C_n/\partial\gamma)(\partial\gamma/\partial\phi)\ldots\ldots\ldots\ldots\ldots(8.9)$$

where C_n is the nondimensional yawing moment coefficient due to aileron investigated by N.A.C.A.[15]

The expression for n_{ψ} is exactly analogous to m_{θ} and can be written as,

$$n_{\psi} = (1/c_1)(\partial C_L'/\partial\alpha')(S'/S)(\partial\eta/\partial\psi)(\partial\alpha'/\partial\eta).(8.10)$$

where, c_1 = non-dimensional moment inertia coefficient about Z-axis

$\partial C_L'/\partial\alpha'$ = slope of lift coefficient of vertical tail,

S'/S = % of vertical tail size.as compared to wing area.

η = angular displacement of rudder control.

α' = effective angle of attack of vertical tail.

9.Evaluation of derivatives based on data of the N.A.C.A 'Average Airplane'.

a.Characteristics of the N.A.C.A.Average airplane F-22.[16]

Weight,W....................................1600 lb.

Wing span,b.................................32 ft.

Wing area,S.................................171 ft^2

Wing loading,W/S............................9.4 lbs/ft^2

Area of fin and rudder,S'...................10.8 ft^2

Tail length.................................14.6 ft.

I_x.......................................1,216 slug-ft^2

I_z.......................................1700 slug-ft^2

b.Assumed flight conditibn.

Horizontal flight at air speed..............126 f.p.s.

Altitude....................................3000 ft.

Air density, ρ.........................,0.00218 slug/ft^3

Based on the above flight conditions,the fundamental units in the non-dimensional system are,

Unit of mass,m..............................50 slugs

Unit of time,T..............................2seconds

Unit of length..............................16 ft.

The parameter, μ........................16

c.The aerodynamic derivatives of the average airplane used in the numerical analysis as a basis for comparison[17]

The following derivatives are rounded figures for the convenience in using the M.I.T.Differential Analyzer.

Vertical tail size......................6% of wing area

Dihedral.................................5°

y_v[18]....................................zero or $-1/2$

n_v..................................... $+1$

l_v..................................... -2

l_p.....................................-16

l_r.....................................$+ 4$

n_p.....................................$-1/2$

n_r.....................................-2

C_L.....................................$-1/2$

d.Estimation of the control derivatives

Based on the data of a typical plain aileron of size, $0.40c_w$ by $0.30\ b/2$ given in Fig.6 of T.R.605 it is estimated that,

$$\partial C_l/\partial\gamma = 0.038 \text{ per } 20^\circ \text{ of aileron displacement}$$

$$= 0.11 \text{ per radian of aileron displacement.}$$

$$a_1 = 0.09 \text{ approximately}$$

Substituting into (8.8),

$$l_\phi = 2.50(\partial\gamma/\partial\phi)$$

Assume $\partial\gamma/\partial\phi = 0.8$ as normal operation of aileron control, then, $l_\phi = -2.0$ may be considered as reasonable[19] and l_ϕ varies from -0.5 to -10 .

From Fig.13 of T.R.605, for the typical aileron, on a rectangular wing,

$C_n/C_l = -0.216\ C_L$ ....(9.1)

Thus $C_n/C_l = -0.11$ at $C_L=0.5$

$c_l=0.134$ for F-22 airplane by converting the data $k_z= 0.183$ as given in T.R.638

Substituting into (8.9),

$n_\phi = 0.2\ (\partial\gamma/\partial\phi)$......................(9.2)

Thus, n_ϕ varies from 0.1 to 1 depending on C_L, and $\partial\gamma/\partial\phi$.

To estimate n_ψ, by similar process as in estimating m_θ, assume, $c_l= 0.134$

$\partial C_L'/\partial\alpha' = -2.5$

$\partial\alpha'/\partial\eta = 0.5$

$S'/S = 0.06$ or 6%

Substituting into (8.10),

$n_\psi = -0.6(\partial\eta/\partial\psi)$

and n_ψ varies from -.2 to -2.0 and -0.5 can be taken as normal. For convenience of comparison, the following control derivatives are taken as basis,

μl_ϕ .... -32

μn_ϕ ....-8

μn_ψ ....-8

The adverse yaw due to aileron is approximately twice as large as that for the real airplane F-22 to show its effect on the stability clearer.

10. Numerical Investigation

In the following pages, the stability of the motion defined by the roots of the determinantal equation (6.3) for various control derivatives and various dihedral and tail sizes are investigated. The disturbed motion due to a rolling gust represented by p_o[19] as solved by the M.I.T. Differential Analyzer under various control derivatives as well as airplane characteristics is presented simultaneously. The roots of the quintic equations are solved with good accuracy by the short-cut method found out by the writer[20]. Due to the limitation of the Differential Analyzer, the derivative y_v has to be assumed as zero in most of the investigations. As it is the least important term in the stability equation, it is believed that the absense of that term will not alter the general conclusion greatly.

As in the longitudinal stability investigation,

$T_{1/2}$ = Time to damp to half amplitude in T units of time.

$N_{1/2}$ = Number of oscillations damped to half amplitude.

a. Uncontrolled airplane

y_v	A_o	B_o	C_o	D_o	E_o	F_o
0	1	18.0	50	288.0	0	0
-.5	1	18.5	59.0	305	0	0

....

y_v	Quintic factered into
0	$(d+0)(d+0)(d+16)(d^2+2d+18) = 0$
-.5	$(d+0)(d+0)(d+16)(d^2+2.5d+19.1) = 0$

....

y_v	Roots	Periods	$T_{1/2}$	$N_{1/2}$
	-16	Infinity.	0.0433	0
-0.5	0,0	Infinity	Infinity	....
	$-1.25\pm4.2i$	1.50	0.554	0.370
	-16	Infinity	0.0433	0
0	0, 0	Infinity	Infinity	....
	$-1.0\pm4.12i$	1.525	0.693	0.455

Table 10.1

From the above calculations, it is seen that the uncontrolled airplane F-22 is a spirally neutral airplane, as $E_o = 0$. As it is expected, the airplane without rudder control has no stability in azimuth, being indicated by $F_o = 0$. The effect of y_v is seen to add damping to the oscillatory motion, often called the short period lateral oscillation to dis tinguish it from the comparatively long period oscillatory motion introduced by rudder control. It should be mentioned here that the dihedral angle, 5° assumed here is actually a little bit lagger than the real airplane which has dihedral of approximately 2° to 3°. Therefore, the real airplane would be spirally unstable due to smaller l_v.

The disturbed motion due to a sharp-edged side gust v_o for the same airplane has been investigated by Jones[21] with the conclusion that the fin area and wing dihedral are of primary importance in determing the magnitudes of the disturbed motion due to side gusts, and will not be repeated here. It is seen, however, that investigations of the uncontrolled stability lead to very few definite indications and informations for airplane designers.

b. Airplane with rudder control alone.

Substituting the airplane derivatives into (6.4), with $\mu l_\phi = 0$, $\mu n_\phi = 0$, $y_v = 0$, we have,

$$A = 1$$

$$B = 18$$

$$C = 50 - \mu n_\psi$$

$$D = 288 - 16(\mu n_\psi)$$

$$E = 0$$

$$F = -16(\mu n_\psi)$$

Table 10.2

μn_ψ	A	B	C	D	E	F
0	1	18	50	288	0	0
-4	1	18	54	352	0	64
-8	1	18	58	416	0	128
-16	1	18	66	544	0	256

μn_ψ	Quintic in d factored into		
	long oscillation	subsidence	short oscillation
0	$(d + 0)(d + 0)$	$(d + 16)$	$(d^2 + 2d + 18) = 0$
-4	$(d^2 - .0285d + .182)$	$(d + 16)$	$(d^2 + 2.03d + 21.9) = 0$
-8	$(d^2 - 0.044d + 0.31)$	$(d + 16)$	$(d^2 + 2.04d + 25.8) = 0$
-16	$(d^2 - 0.058d + 0.48)$	$(d + 16.05)$	$(d^2 + 2.01d + 33.65) = 0$

μn_ψ	Roots	Period	$T_{1/2}$	$N_{1/2}$
	-16	Infinity	.0433	0
0	0,0	Infinity	Infinity	......
	$-1 \pm 4.12i$	1.525	.693	.455
	-16	Infinity	.0433	0
-4	$+.0143 \pm .424i$	14.8	unstable	unstable
	$-1.015 \pm 4.57i$	1.375	.685	.498
	-16	Infinity	.0433	0
-8	$+.022 + .553i$	11.35	unstable	unstable
	$-1.02 \pm 5.06i$	1.24	.680	.55

μn_ψ	Roots	Period	$T_{1/2}$	$N_{1/2}$
	-16.05	Infinity	.043	0
-16	$+.029 \pm .686i$	9.16	Unstable	unstable
	$-1.01 \pm 5.72i$	1.10	.690	.626

From the above calculations, the effect of rudder control is seen to give the airplane a restoring capacity in azimuth by having F no longer zero. From (6.4), it is seen that $F = -\mu n_\psi(\mu C_L l_v + \mu l_\phi y_v)$, and F will be zero so long as μn_ψ is zero. Therefore, it is necessary to have rudder control in order to provide for directional stability unless the aileron is operated according to angle of yaw to give the derivative l_ψ (see equation 6.6)

The introduction of directional stability by means of rudder, however, has the disadvantage of combining the original spiral mode of motion into a long period oscillation with inadequate damping. This is seen mathematically by examining (6.4) to be due to the fact that for $y_v = 0$, introduction of the control derivative μn_ψ can not modify the coefficient E which is zero for the present airplane, and is negative for spirally unstable airplane. It is evident that for an airplane, to be controlled by rudder alone must have E_o large and positive, which requires that the dihedral must be unusually large. This will be discussed in detail in the study of two-control operation. It is also seen from the expression for F, that in order to have μn_ψ effective in providing the course stability, l_v must not be zero. This is particularly important when y_v is small and aileron control not powerful. In general, it can be concluded that dihedral must be present in order to have effective rudder control. The effect of y_v is seen as follows,

Table 10.3

At $\mu n_\psi = -8$, $\mu l_\phi = 0$, $\mu n_\phi = 0$

y_v	A	B	C	D	E	F
0	1	18	58	416	0	128
-.5	1	18.5	67	437	32	128

Quintic factored into

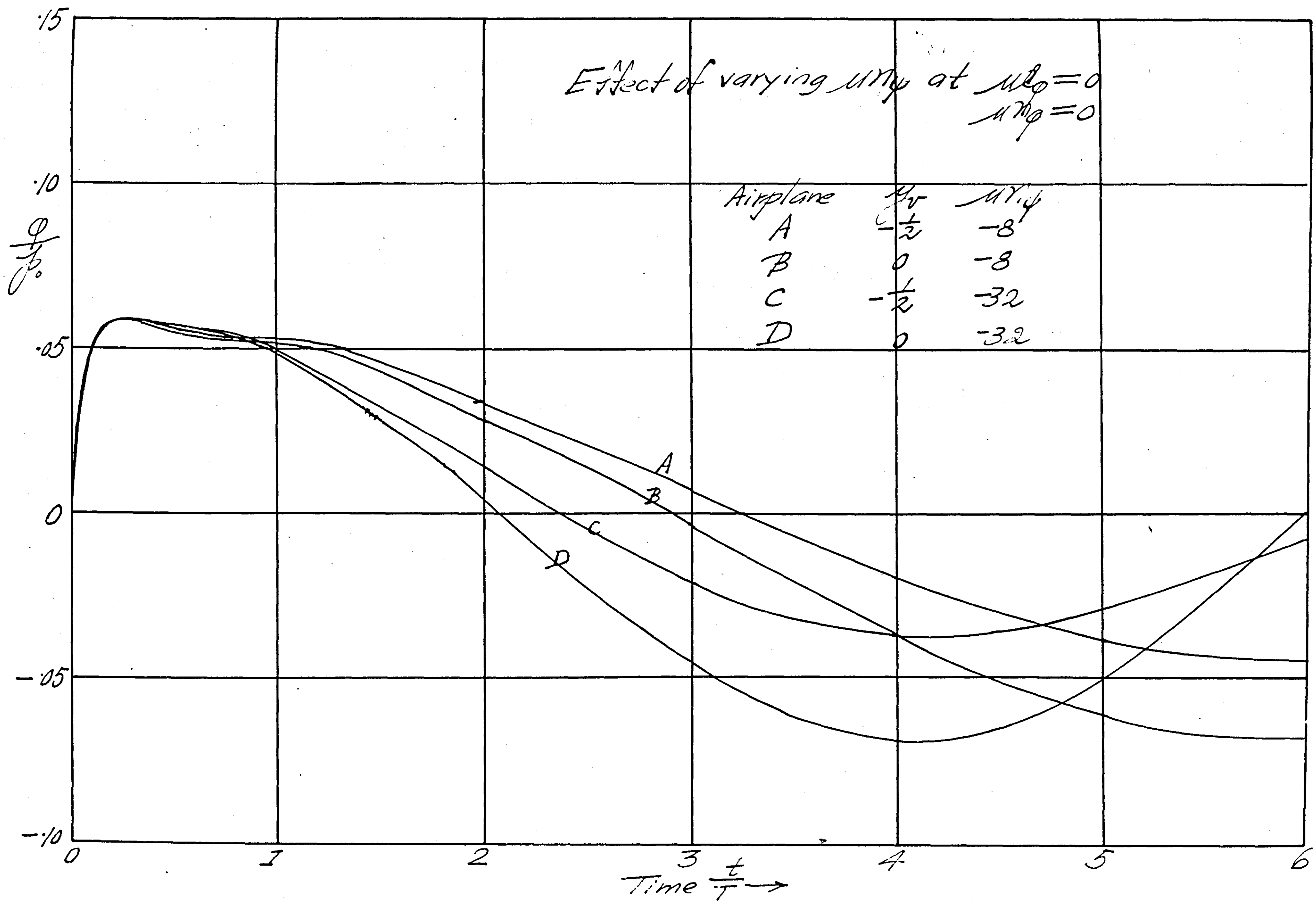
y_v	Long oscillation	Subsidence	Short oscillation
0	$(d^2 - .044d + .31)$	$(d + 16)$	$(d^2 + 2.04d + 25.8) = 0$
-.5	$(d^2 + .082d + .294)$	$(d + 16.05)$	$(d^2 + 2.4d + 27.0) = 0$

y_v	Roots	Period	$T_{1/2}$	$N_{1/2}$
	-16	Infinity	.433	0
0	$+.022 \pm .553i$	11.35	unstable	unstable
	$-1.02 \pm 5.06i$	1.24	.680	.55

y_v	Roots	Period	$T_{1/2}$	$N_{1/2}$
	-16.05	Infinity	.43	0
-.5	$-.041 \pm .54i$	-~~Infinity~~ 11.6	16.9	1.46
	$-1.2 \pm 5.06i$	1.24	.58	.466

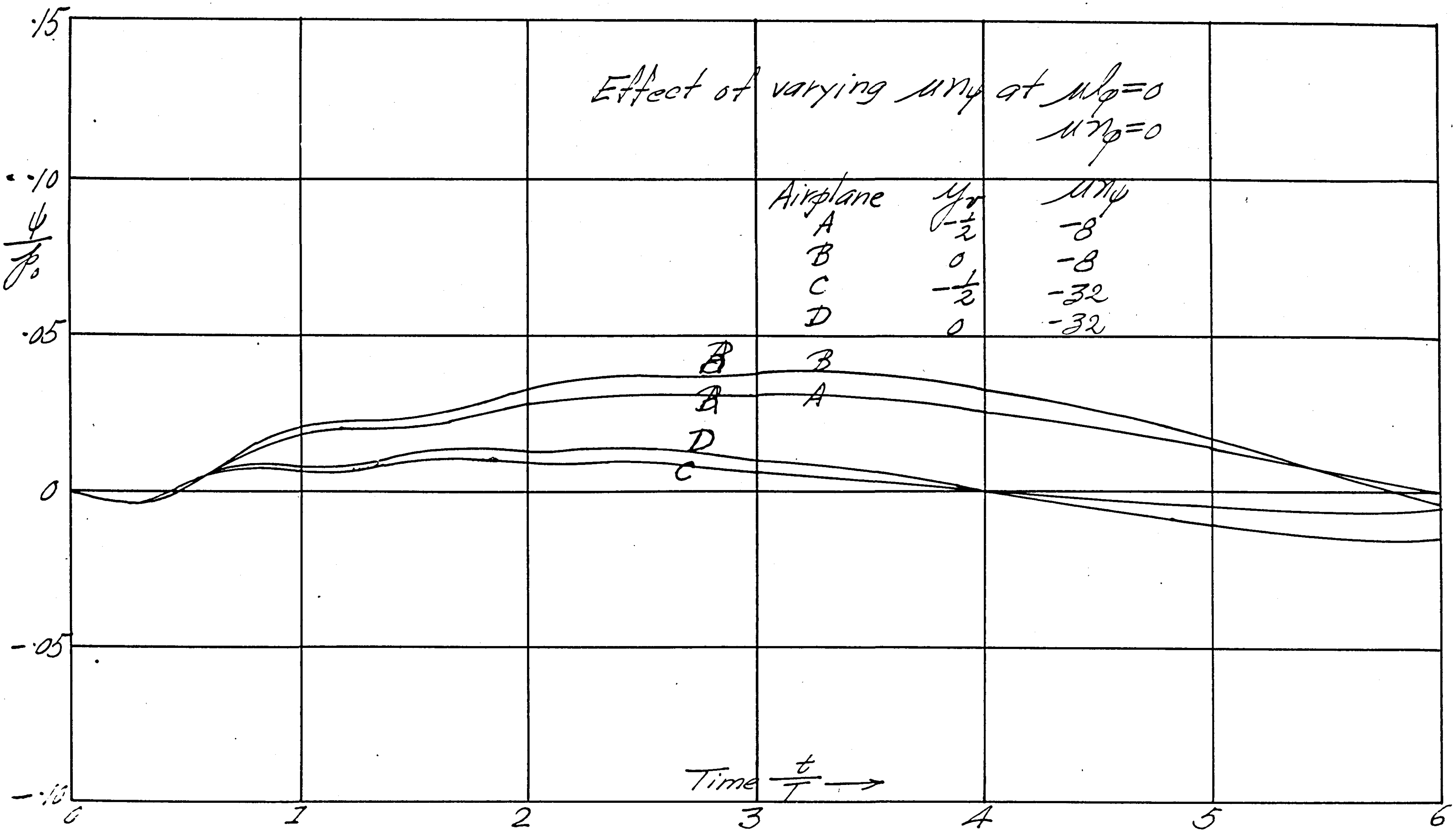
The effect of y_v is to add damping to both the long and short oscillations. However, the long oscillation is still far from being adequately damped, unless y_v is unusually high. Fig.10.1A shows the disturbed motion in roll due to a rolling gust p_o for a~~n~~ y_v ~~un~~controlled rudder airplane. Fig.10.1(B) shows the corresponding motion in yaw with reference to certain azimuth such as that furnished by a direction gyro, or a compass. Increase of rudder control is seen to decrease the magnitude of the disturbance in azimuth without improving the damping of the motion. The disturbance in roll is hardly effected by rudder control except in varying its period of oscillation. A large y_v is seen to be desirable in reducing the magnitude of disturbed motion as well as in improving the damping for both long and short oscillations.

Fig. 10.1 (A).



Effect of varying μn_ψ at μl_φ = 0
μn_φ = 0
Airplane y_v μr_ψ
A −1/2 −8
B 0 −8
C −1/2 −32
D 0 −32
A
B
C
D
·15
·10
·05
0
−·05
−·10
φ/β₀
0
1
2
3
4
5
6
Time t/T →

Fig. 10·I (B).



Effect of varying μn_ψ at $\mu l_\phi = 0$
$\mu n_\phi = 0$
Airplane | y_v | μn_ψ
A | $-\frac{1}{2}$ | -8
B | 0 | -8
C | $-\frac{1}{2}$ | -32
D | 0 | -32
$\frac{\psi}{p_0}$
·15
·10
·05
0
−·05
−·10
Time $\frac{t}{T}$ →
0
1
2
3
4
5
6
B
A
D
C

c. With both rudder and aileron control, the adverse yaw due to aileron being assumed to be zero.

Substituting $y_v=0$, $\mu n_\psi=-8$, $\mu n_\phi=0$, into (6.4), we have,

$A = 1$

$B = 18$

$C = 58 - \mu l_\phi$

$D = 416 - 2(\mu l_\phi)$

$E = -16(\mu l_\phi)$

$F = 128$

Table 10.4

μl_ϕ	A	B	C	D	E	F
0	1	18	58	416	0	128
-16	1	18	74	448	256	128
-32	1	18	90	480	512	128
-48	1	18	106	512	778	128
-64	1	18	122	544	1024	128
-96	1	18	154	608	1536	128
-128	1	18	186	672	2048	128

Quintic in d factored into

μl_ϕ	Long Oscillation	Subsidence	Short oscillation
0	$(d^2-.044d+.31)$	$(d+16)$	$(d^2+2d+18) = 0$
-16	$(d^2+.582d+.314)$	$(d+15)$	$(d^2+2.42d+27.1)=0$
-32	$(d+.322)(d+.988)$	$(d+13.8)$	$(d^2+2.91d+28.3)=0$
-48	$(d+.185)(d+1.965)$	$(d+12.35)$	$(d^2+3.5d+29.0)=0$
-64	$(d+.13)(d+3.33)$	$(d+10.4)$	$(d^2+4.14d+28.3)=0$
-96	$(d+.0885)$ $(d^2+13.6d+73.8)$		$(d^2+4.27d+20.4)=0$
-128	$(d+.0635)$ $(d^2+14.38d+116)$		$(d^2+3.58d+17.4)=0$

μm_ϕ	Roots	Period	$T_{1/2}$	$N_{1/2}$
	-16	Infinity	.0433	0
0	+.022±.553i	11.35	unstable	unstable
	-1.02± 5.06i	1.24	.68	.55
	-15	Infinity	.0462	0
-16	-.291±.479i	13.1	2.38	.182
	-1.21±5.06i	1.24	.572	.462
	-13.8	Infinity	.0502	0
-32	-.322,-.988	Infinity	2.15,.70	0
	-1.455±5.12i	1.23	.395	.322
	-12.35	Infinity	.0562	0
-48	-.185,-1.965	Infinity	3.75,.352	0
	-1.75± 5.1i	1.23	.395	.322
	-10.4	Infinity	.0666	0
-64	-.13, -3.33	Infinity	5.33,.208	0
	-2.07±4.9i	1.28	.335	.205
	-6.8±5.25i	1.20	.102	.085
-96	-.0885	Infinity	7.85	0
	-2.14±3.97i	1.58	.324	.205
	-7.19±8i	.79	.096	.120
-128	-.0632	Infinity	11.0	0
	-1.89±3.71i	1.70	.366	.215

The numerical investigation shows the following interesting points,

(1),The function of the control derivative l_ϕ is to improve both the damping of the long and that of the short osdillation,its effect on long oscillation being so well that at $\mu l_\phi = -32$, the long oscillation splits into two exponentially damped subsidence.

(2),Analogous to the function of m_θ in longitudinal stability equation, the control derivative μl_ϕ merely equalizes the damping by drawing the damping from the rolling subsidence and adding it to both the long and short oscillations. The rolling subsidence remains to be well damped.

(3),For extremely high values of μl_ϕ,such as -96 and -128,the two heavily damped modes of aperiodic motions combine again into a heavily damped short oscillation,indicating that at very large aileron control,the degree of freedom in roll approaches zero in a similar way as the case in longitudinal stability when the inertia of the control-system is very small[22].

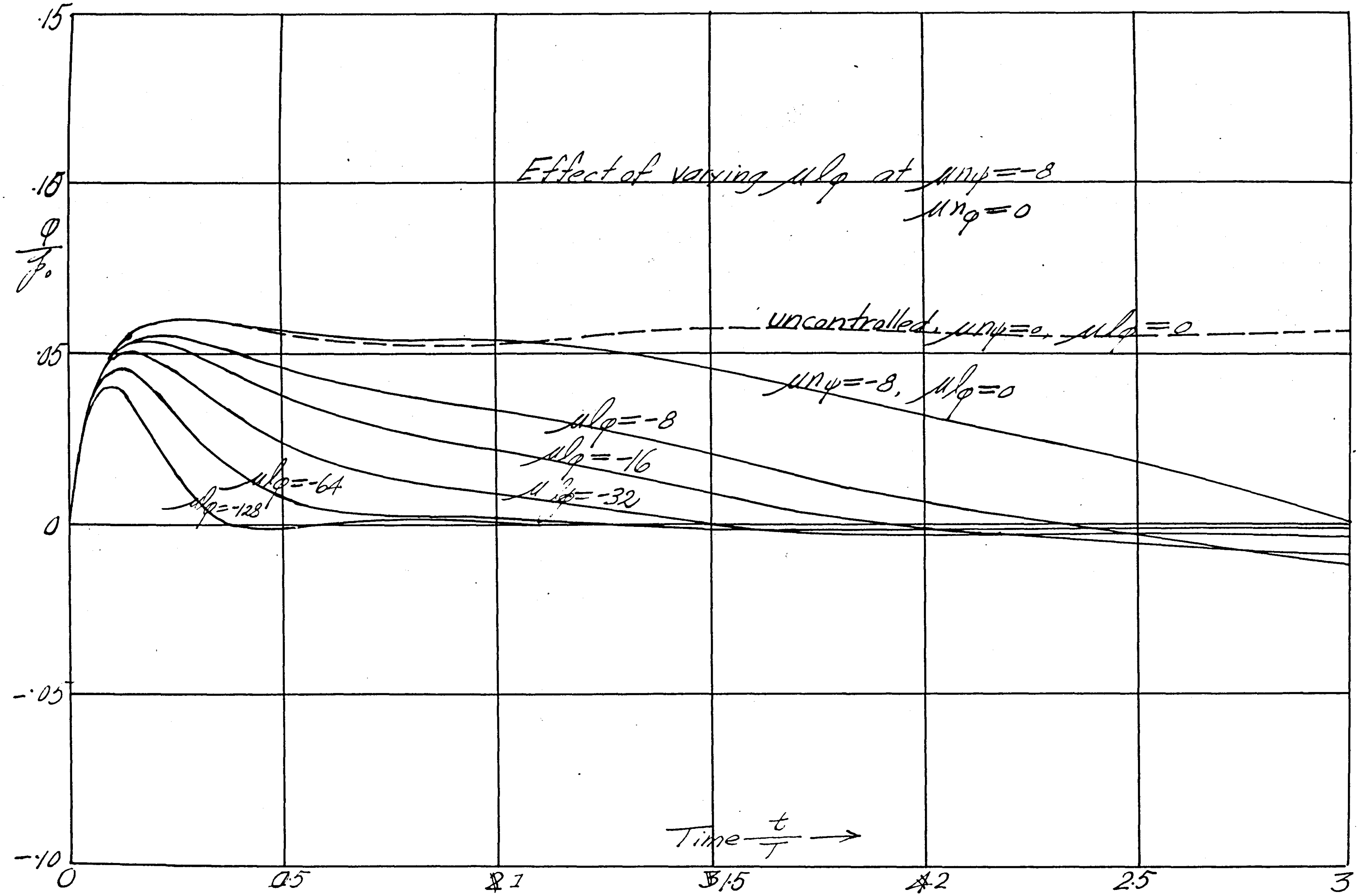
(4) In mathematical expression,It is seen that the most important function of l_ϕ in contributing to the stability of the motion is to add greatly

to the positiveness of E, see (6.4). It is seen also that $E = E_o - (y_v n_r + \mu n_v)\mu l_\phi - \mu n_\psi (y_v l_p)$

As the terms due to y_v are small compared to that due to μn_v, it is necessary that n_v be large enough to render the control due to aileron to be effective. This explains why experienced designers had long suggested a minimum positive n_v be imposed on any airplane.[23] This is especially so for airplanes having small y_v, high μ or effective wing loading. This fact can not be seen through the study of uncontrolled stability, as increase of n_v would make E_o more negative. A numerical investigation showing the effect of varrying tail size on the controlled stability will be presented later.

Fig.10.2(A) shows the disturbed motion due to a rotary gust at various aileron control. The uncontrolled airplane after disturbed by a ratary gust cannot recover itself to the position of zero roll and yaw as indicated by the two zero roots in the stability equation. Increase of aileron control not only stabilizes the motion in roll, but also decrease the errors in azimuth as shown in Fig.10.2(B)., though at a smaller rate. The transient surge error is improved but slightly by increasing aileron control. It is shown later that the lagging effect prevents the aileron control μl_ϕ from increasing beyond -64. It is also interesting to note that the

Fig. 10.2(A).



Effect of varying μl_ϕ at $\mu n_\psi = -8$, $\mu n_\phi = 0$
$\frac{\varphi}{f_0}$
uncontrolled, $\mu n_\psi = 0$, $\mu l_\phi = 0$
$\mu n_\psi = -8$, $\mu l_\phi = 0$
$\mu l_\phi = -8$
$\mu l_\phi = -16$
$\mu l_\phi = -32$
$\mu l_\phi = -64$
$\mu l_\phi = -128$
Time $\frac{t}{T}$
·15
·10
·05
0
-·05
-·10
0
0.5
1
1.5
2
2.5
3

134

exponential mode of motion indicated by the small negative root, -0.0632 as shown in Table 10.4 for μl_q=128, appears in Fig.10.2(B) by the phenomenon that curve returns to zero very slowly. As the magnitude is much smaller than the uncontrolled airplane, the stability will not be unsatisfactory due to the presence of this small negative root.

d. The effect of adverse and favorable yawing moment due to aileron.

Substituting into (6.4),

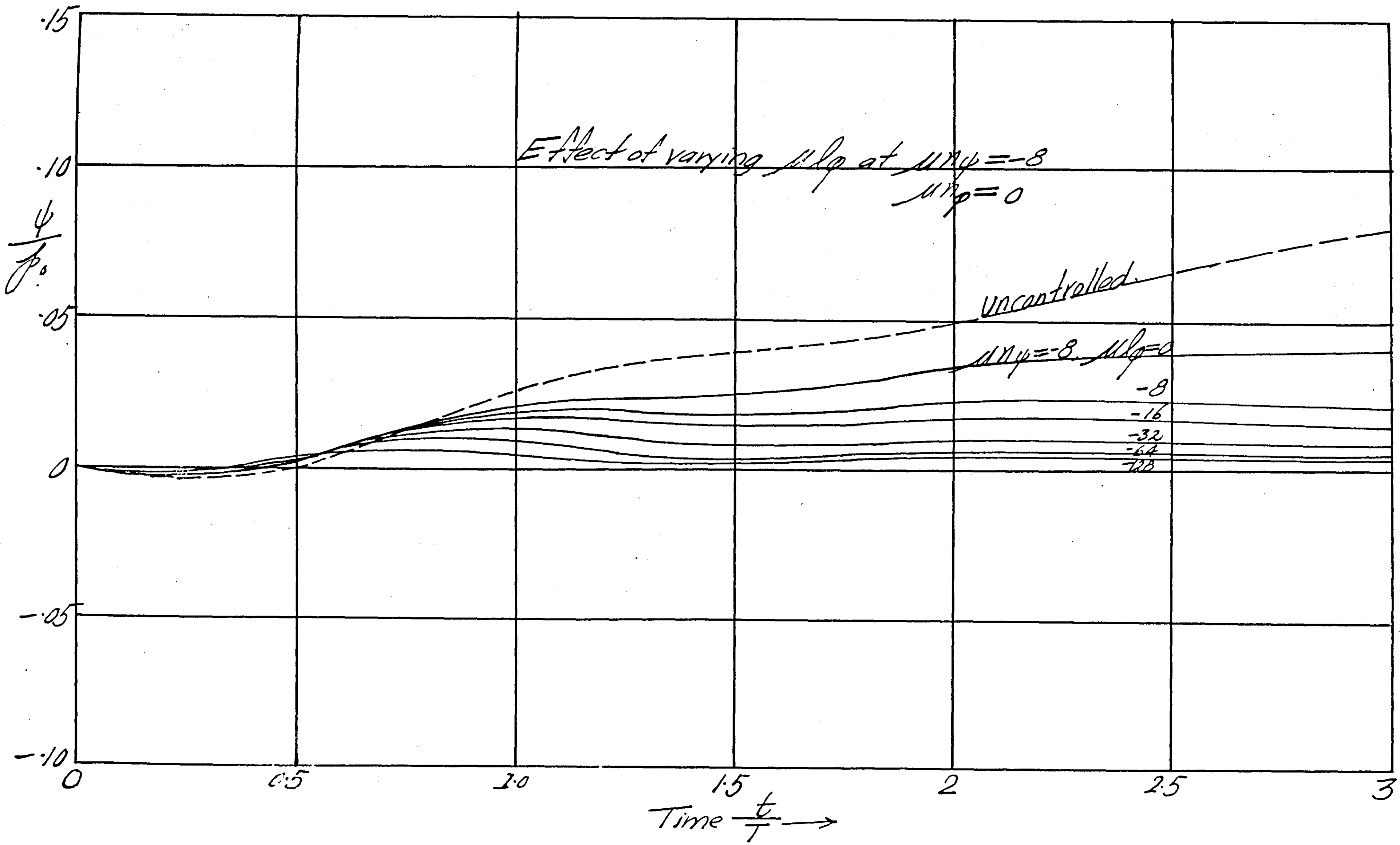
$y_v = 0$, $\mu n_\psi = -8$, $\mu l_\phi = -32$, we have,

$$
\begin{aligned}
A &= 1 \\
B &= 18 \\
C &= 90 \\
D &= 480 - l_r(\mu n_\phi) = 480 - 4(\mu n_\phi) \qquad (10.1) \\
E &= 512 + (\mu l_v + y_v l_r)(\mu n_\phi) = 512 - 32(\mu n_\phi) \\
F &= 128
\end{aligned}
$$

Table 10.5

μn_ϕ	A	B	C	D	E	F
+16	1	18	90	416	0	128
+ 8	1	18	90	448	256	128
+ 4	1	18	90	464	384	128
0	1	18	90	480	512	128
-4	1	18	90	496	640	128
-8	1	18	90	512	768	128
-16	1	18	90	544	1024	128

Fig. 10·2 (B).


Effect of varying μl_φ at $\mu n_\psi = -8$
$\mu n_\varphi = 0$
Uncontrolled.
$\mu n_\psi = -8$, $\mu l_\varphi = 0$
-8
-16
-32
-64
-128
·15
·10
·05
0
-·05
-·10
$\frac{\psi}{p_0}$
0
0·5
1·0
1·5
2
2·5
3
Time $\frac{t}{T}$

μn_ϕ	Quintic in d factored into Long oscillation	Subsidence	Short oscillation
+16	$(d^2-.066d+.309)$	$(d+13.63)$	$(d^2+4.3d+30.5)=0$
+ 8	$(d^2+.583d+.323)$	$(d+13.70)$	$(d^2+3.72d+29.0)=0$
+ 4	$(d^2+.93\ d+.328)$	$(d+13.76)$	$(d^2+3.37d+28.3)=0$
0	$(d^2+.322)(d+.988)$	$(d+13.8\ )$	$(d^2+2.91d+28.3)=0$
-4	$(d\ +.245)(d+1.345)$	$(d+13.85)$	$(d^2+2.57d+28.0)=0$
-8	$(d\ +.228)(d+1.412)$	$(d+13.90)$	$(d^2+2.45d+28.63)=0$
-16	$(d\ +.135)(d+2.295)$	$(d+14.0\ )$	$(d^2+1.57d+29.60)=0$

As the stability of the motion can be well indicated without finding the period and damping as did in the previous table, the table showing the period, $T_{1/2}$, $N_{1/2}$ will not be presented here.

It is seen from the above tabulation that the effect of n_ϕ is to redistribute the damping in different modes of motion. The adverse yawing moment which yields the positive μn_ϕ decrease the damping in the long oscillation whereas the favorable yawing moment corresponding to negative μn_ϕ, decreases damping of the short oscillation. It shifts the damping from one mode of motion at the expense of the other. The heavily damped rolling subsidence remains substantially unaltered, though the adverse yaw seems to shift a slight amount of damping from it to the short oscillation. In general, it can be concluded that the redistribution of damping due to either the adverse or the favorable yawing moment are not desirable.

Fig.10.3(A) and (B) shows the effect of adverse aileron yawing moment on the disturbed motion due to a rolling gust. It is seen that as the damping of the ~~damping of the~~ long oscillation decreases, the magnitude of disturbance increases. Due to difficulty in reversing the sign of μn_q on the differential analyzer, the effect of favorable yaw was not plotted. However, it is believed that the short period oscillation would cause trouble in this case. It is also noticed that the increase in magnitude of the disturbed motion in yaw is comparatively more serious than the motion in roll as what we would expect, due to the direct effect on yawing motion and secondary effect on roll by the adverse yawing moment.

Examining the coefficients in (6.4) or better (10.1), it is seen that the principal effect of μn_ϕ on the stability of the motion is to alter the coefficients D, and E of the stability equation. As shown in (10.1),

$D = 480 - l_r(\mu n_\phi)$, and $E = 512 + (\mu l_v + y_v l_r)(\mu n_\phi)$

it is evident that airplanes of large dihedral will be more sensitive to the undesirable effect of aileron yawing moment. The most critical case would be when l_r is also large, i.e., at high angle of attack because l_r is a function of C_L[24]. A more detailed investigation under the condition of high angle of attack confirms this conclusion.

Fig. 10·3 (A).

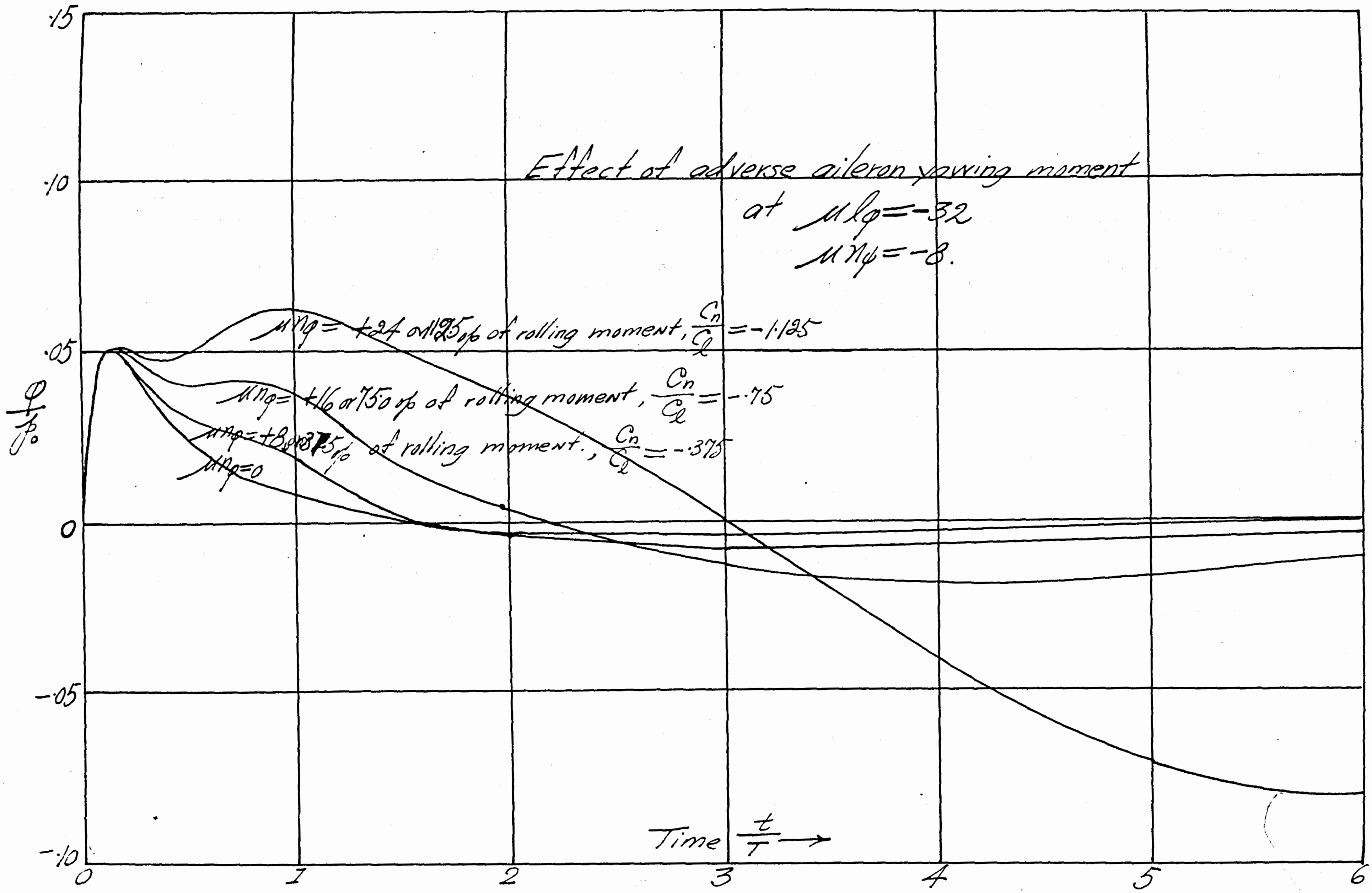
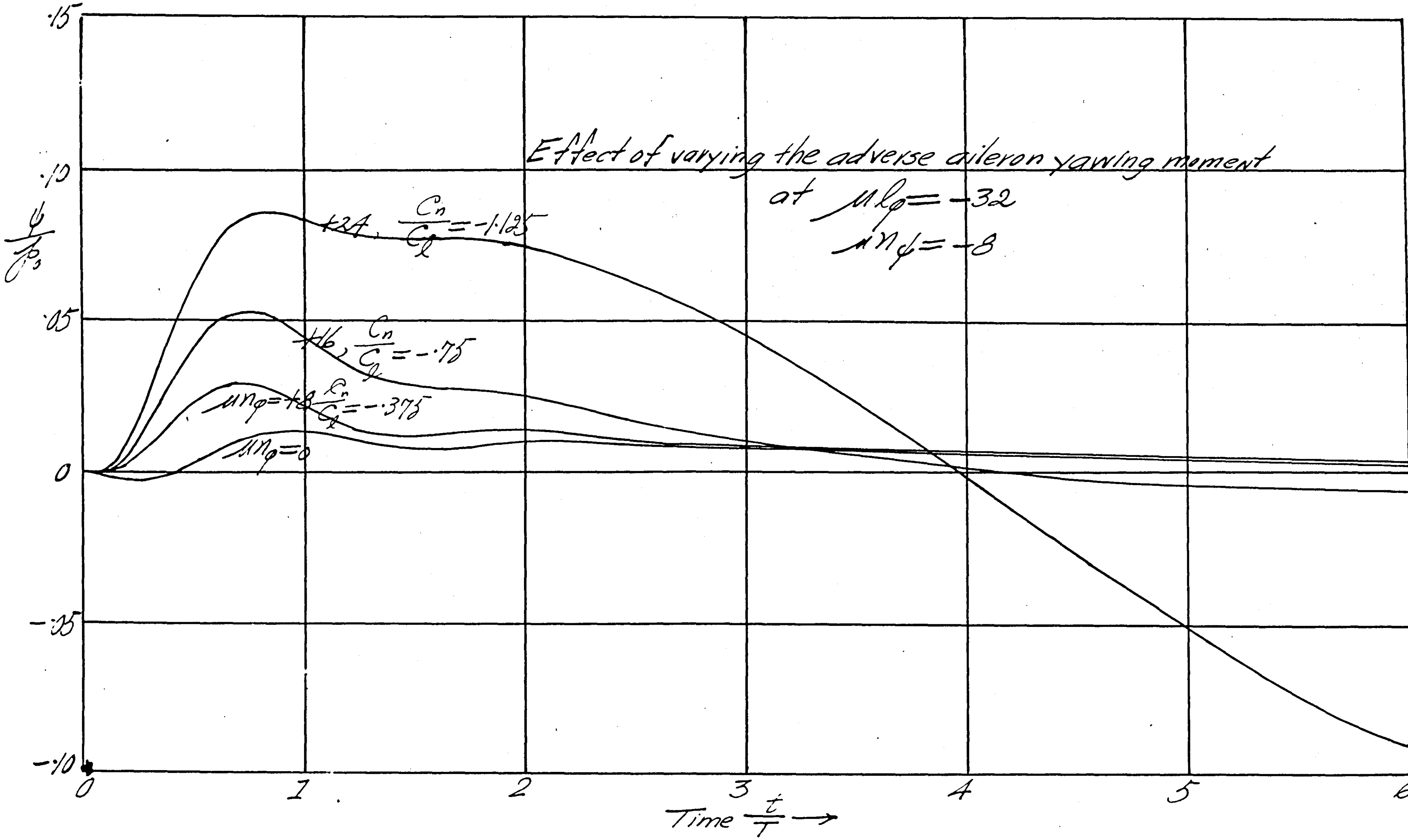

Effect of adverse aileron yawing moment
at $\mu l_\phi = -32$
$\mu n_\psi = -8.$
$\mu n_\phi = +24$ or 125% of rolling moment, $\frac{C_n}{C_l} = -1{\cdot}125$
$\mu n_\phi = +16$ or 75% of rolling moment, $\frac{C_n}{C_l} = -{\cdot}75$
$\mu n_\phi = +8$ or 37·5% of rolling moment, $\frac{C_n}{C_l} = -{\cdot}375$
$\mu n_\phi = 0$
$\frac{\phi}{p_0}$
Time $\frac{t}{T}$ →
·15
·10
·05
0
-·05
-·10
0
1
2
3
4
5
6

Fig. 10·3(B).



Effect of varying the adverse aileron yawing moment
at μl_φ = -32
μn_φ = -8
+24, Cn/Cl = -1·125
+16, Cn/Cl = -·75
μn_φ = +8 Cn/Cl = -·375
μn_φ = 0
·15
·10
·05
0
-·05
-·10
0
1
2
3
4
5
6
Time t/T →

e. Effect of increasing rudder control at $\mu l_\phi = -32$.

It is evident that rudder control is to furnish course sensitivity, as without rudder, the coefficient F would always be zero, except the aileron is operated according to angle of yaw. Fig.10.4 (A) and (B) shows the effect of varying μn_ψ with and without aileron adverse yaw. The principal effect of rudder control is to decrease the error in azimuth due to gust disturbance. Its effect on rolling motion is comparatively less important so long as adequate aileron control is present.

f. Effect of varying tail size on the controlled stability.

The effect of vertical tail size is found to change principally the derivatives y_v, n_v and n_r. In the study of aileron control derivative μl_ϕ, it was pointed out that the ability of aileron control to provide adequate damping for both the long and short oscillations depends on sufficient amount of n_v present.

Assume, $\mu l_\phi = -64$, to show the effect of n_v clearer,

$\mu n_\psi = -8$

$\mu n_\phi = -8$ or zero

$y_v = 0$ to show the effect of n_v on stability clearer.

Compare the stability of a controlled airplane having the following tail sizes,

1. Small tail 2. Normal tail 3. Very Large tail

Fig. 10·A (A).

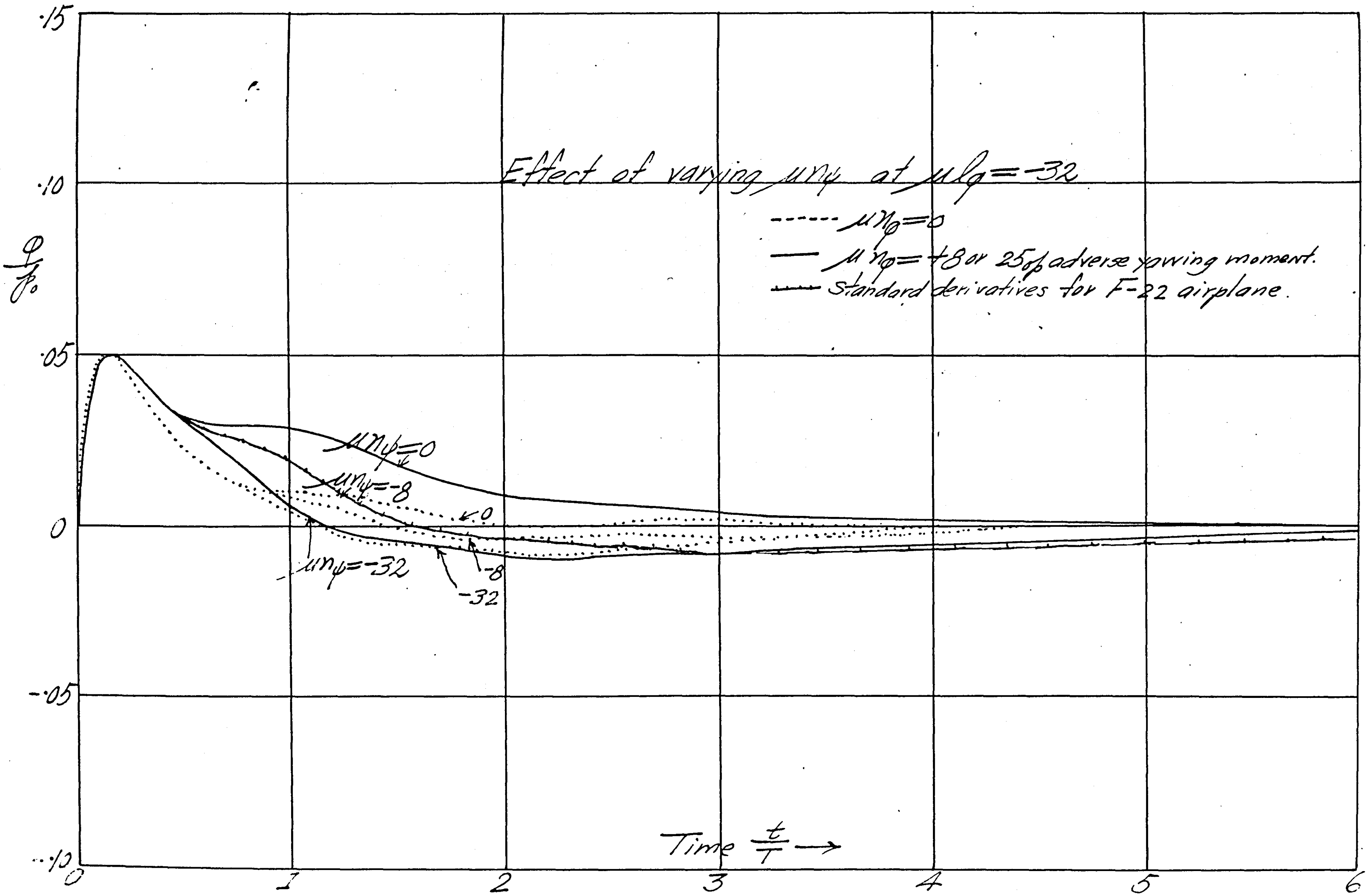
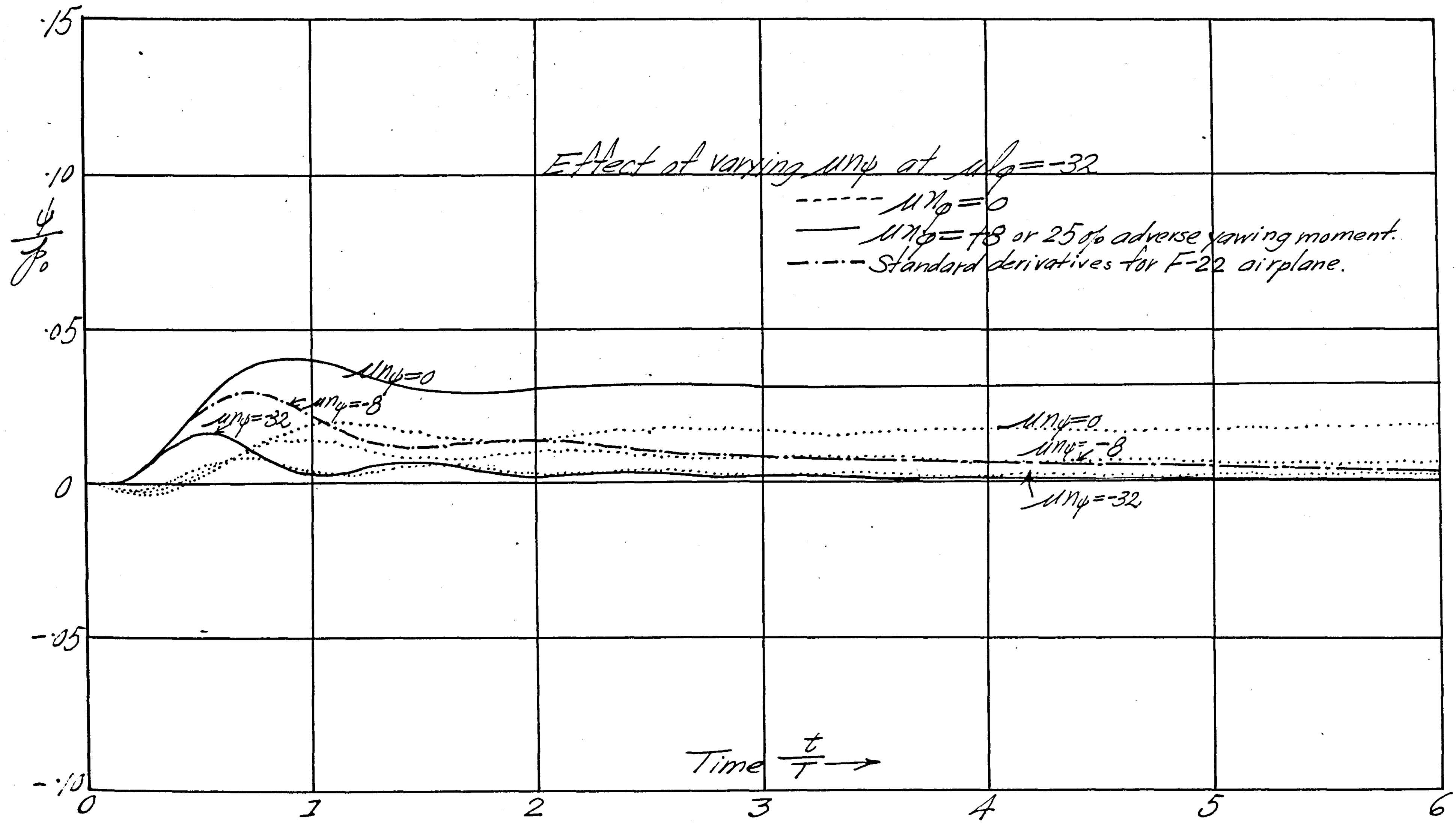

Effect of varying μn_ψ at $\mu l_\phi = -32$
$\mu n_\phi = 0$
$\mu n_\phi = +8$ or 25% adverse yawing moment.
standard derivatives for F-22 airplane.
$\frac{\phi}{p_0}$
·15
·10
·05
0
-·05
-·10
$\mu n_\psi = 0$
$\mu n_\psi = -8$
$\mu n_\psi = -32$
0
-8
-32
Time $\frac{t}{T}$
0
1
2
3
4
5
6

Fig. 10.A (B).


Effect of varying μn_ψ at $\mu l_\phi = -32$
$\mu n_\phi = 0$
$\mu n_\phi = +8$ or 25% adverse yawing moment.
Standard derivatives for F-22 airplane.
$\mu n_\psi = 0$
$\mu n_\psi = -8$
$\mu n_\psi = -32$
$\mu n_\psi = 0$
$\mu n_\psi = -8$
$\mu n_\psi = -32$
$\frac{\psi}{p_0}$
·15
·10
·05
0
-·05
-·10
Time $\frac{t}{T}$
0
1
2
3
4
5
6

Tail Size	n_v	n_r
(1)very small	0	-1
(2)fairly small	+.5	-1
(3)normal(F-22)	+ 1	-2
(4)Very large	+ 2	-4

Substituting into (6.2) and (6.4), we have,

Table 10.6

Tail Size	A_o	B_o	C_o	D_o	E_o	F_o
(1)	1	17	18	32	16	0
(2)	1	17	26	140	0	0
(3)	1	18	50	288	0	0
(4)	1	20	98	272	-32	0

Tail Size	Stability Equation of uncontrolled airplane		
	Long Oscillation	Subsidence	Short oscillation
(1)	$(d+0)\ (d+.602)$	$(d+16)$	$(d^2+.398d+1.66)=0$
(2)	$(d+0)\ (d+0)$	$(d+15.92)$	$(d^2+1.08d+8.8)=0$
(3)	$(d+0)\ (d+0)$	$(d+16)$	$(d^2+2d+18)=0$
(4)	$(d+0)\ (d-.113)$	$(d+14.54)$	$(d^2+5.6d+19.5)=0$

Thus, study of the uncontrolled stability equation show very little on the necessary tail size as it indicates only that the damping of the short oscillation is increased by increasing tail size but at the same time the spiral instability is increased.

Study of the controlled stability, however, shows remarkable difference. Assuming first no adverse aileron yawing moment so that $\mu n_\phi = 0$, from (6.4) we have,

Tail Size	A	B	C	D	E	F
(1)	1	17	90	224	16	128
(2)	1	~~XX~~ 17	~~XXX~~ 98	332	512	128
(3)	1	18	122	544	1024	128
(4)	1	20	170	656	2016	128

Tail Size	Quintic in d factored into		
	Long oscillation	Subsidence	Short oscillation
(1)	$(d^2-.155d+.56)$	$(d+10.43)$	$(d^2+6.73d+21.9)=0$
(2)	$(d^2+.305)(d+2.76)$	$(d+10.0)$	$(d^2+3.95d+15.3)=0$
(3)	$(d+.13)(d+3.33)$	$(d+10.4)$	$(d^2+4.14d+28.3)=0$
(4)	$(d+.065)\underline{\underline{(d-1.9)}}$	$(d+11.44)$	$(d^2+10.4d+91.0)=0$

With adverse aileron yawing moment, $\mu n_\phi = +8$,

Tail Size	A	B	C	D	E	F
(2)	1	17	98	300	256	128
(4)	1	20	170	624	1760	128

Tail Size	Quintic in d factored into		
	Long oscillation	Subsidence	Short oscillation
(2)	$(d^2+.982d+.61)$	$(d+10)$	$(d^2+6.06d+21.0)=0$
(4)	$(d+.075)\underline{\underline{(d\pm1.76)}}$	$(d+11.0)$	$(d^2+10.73d+88.8)=0$

From table 10.6 , it is seen that the selection of proper tail size is very important. Too small tail would cause the long oscillation to be unstable by reducing the stabilizing effect due to aileron control. Too large tail size, or rather too large n_v, as seen in Fig.10.5(B), would have a large error in azimuth with the result that the rudder control seemed to be ineffective. Fig.10.5 and Fig.10.6 show the effects of varrying n_v and n_r separately. It is seen that n_r is responsible for the damping of short oscillation primarily. Increase of n_r increases damping of short oscillation and reduces the error in azimuth, so that a large n_r is always desirable. However, n_v can neither be too large nor too small. $n_v = +.5$ to $+1$ seemed to be best. It is determined by the aileron control derivative μl_ϕ.

g. Effect of varrying dihedral.

From the study of rudder control alone, it was pointed out that when y_v is small or zero, it is necessary to have l_v in order that F may not be zero. It was also pointed out that increase of dihedral would render the adverse yaw due to aileron to be more disastrous. Therefore, l_v can neither be too large nor too small.

The following investigation shows the effect of varrying l_v when y_v is not zero, and no adverse yaw due to aileron is present.

Fig. 10.5 (A).

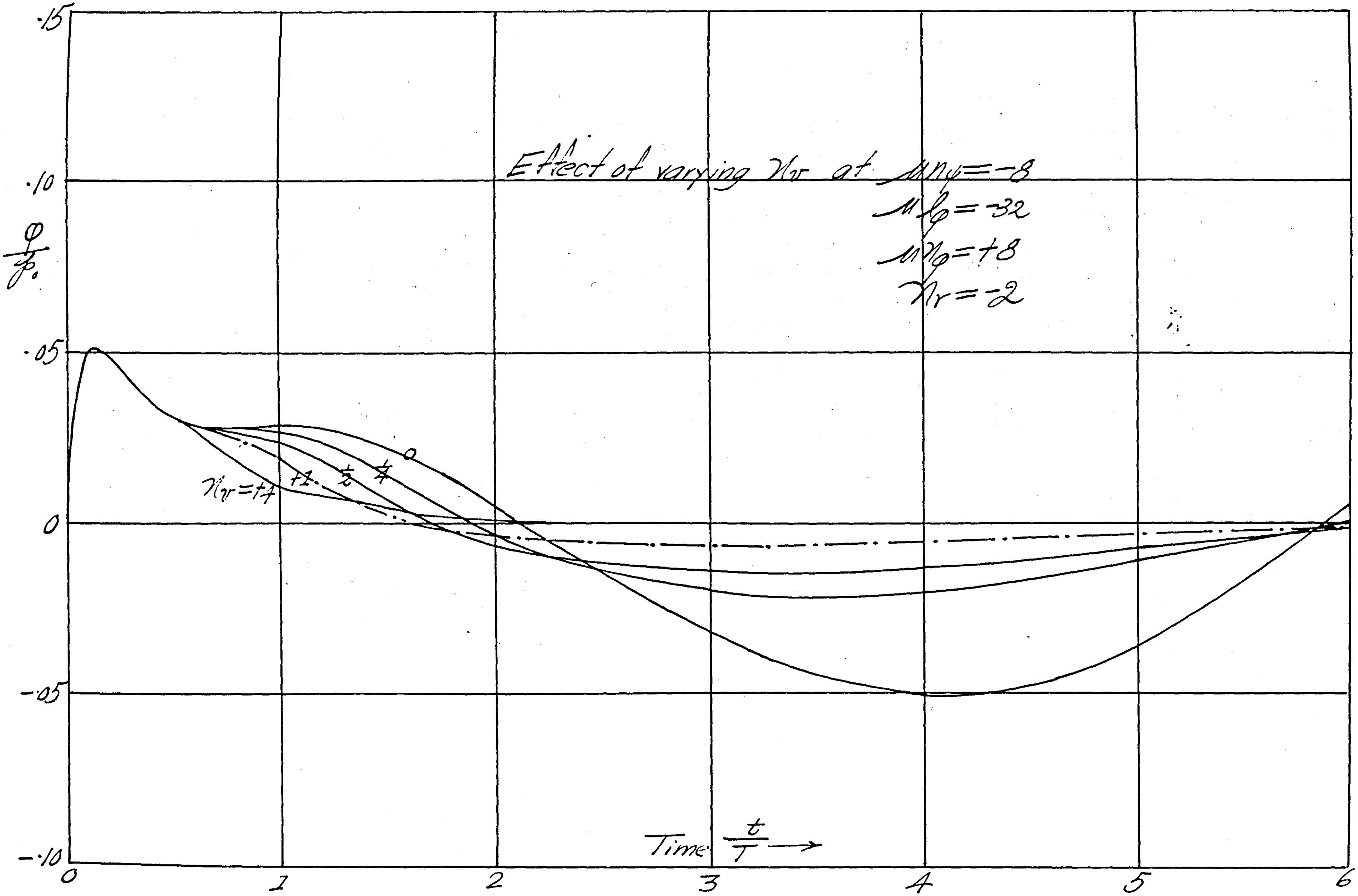

Effect of varying n_v at $\mu n_\psi = -8$
$\mu l_\phi = -32$
$\mu n_\phi = +8$
$n_r = -2$
$\frac{\varphi}{p_0}$
·15
·10
·05
0
-·05
-·10
0
1
2
3
4
5
6
Time $\frac{t}{T}$ →
$n_v = +4$
+1
½
¼
0

Fig. 10·5 (B).

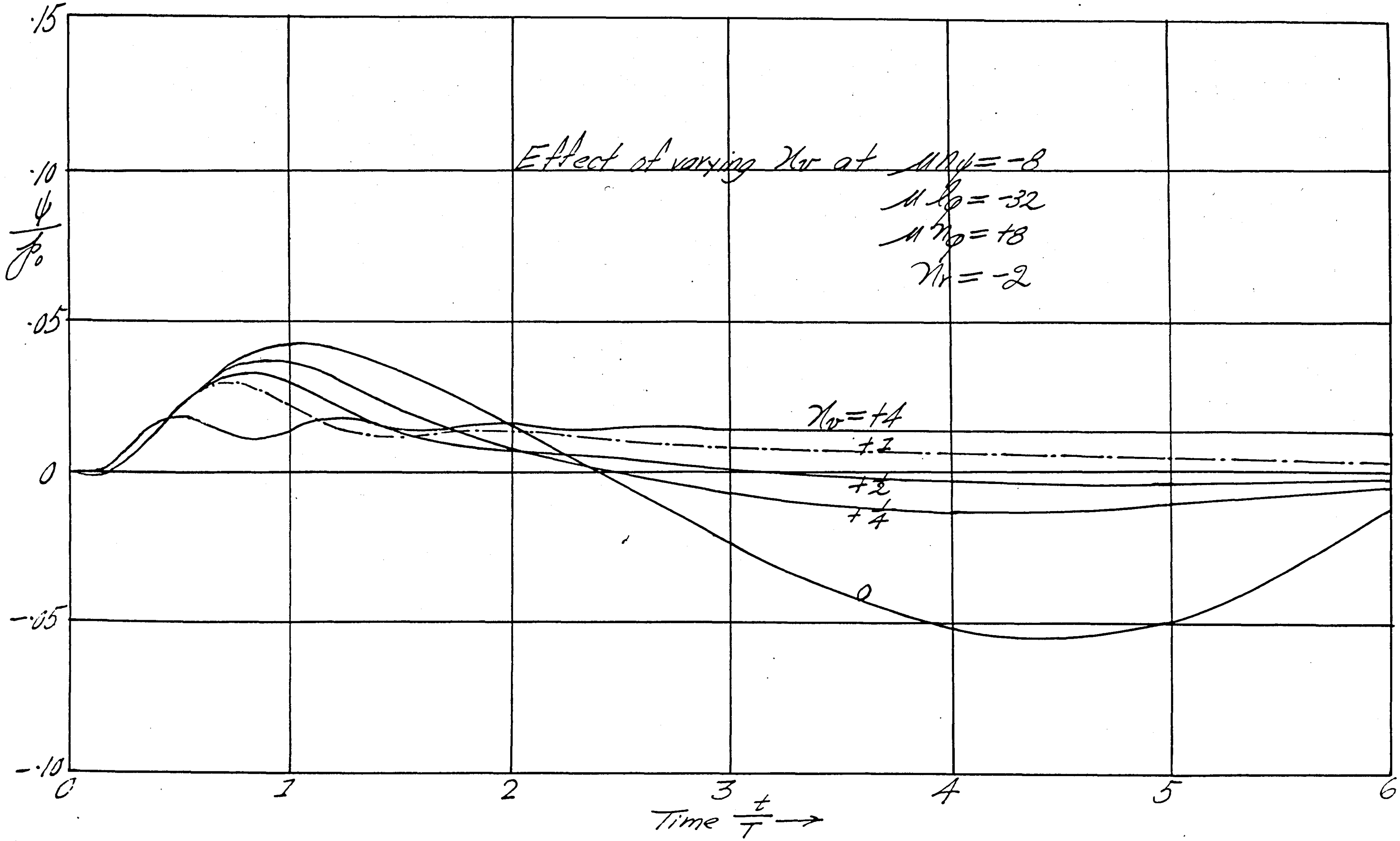
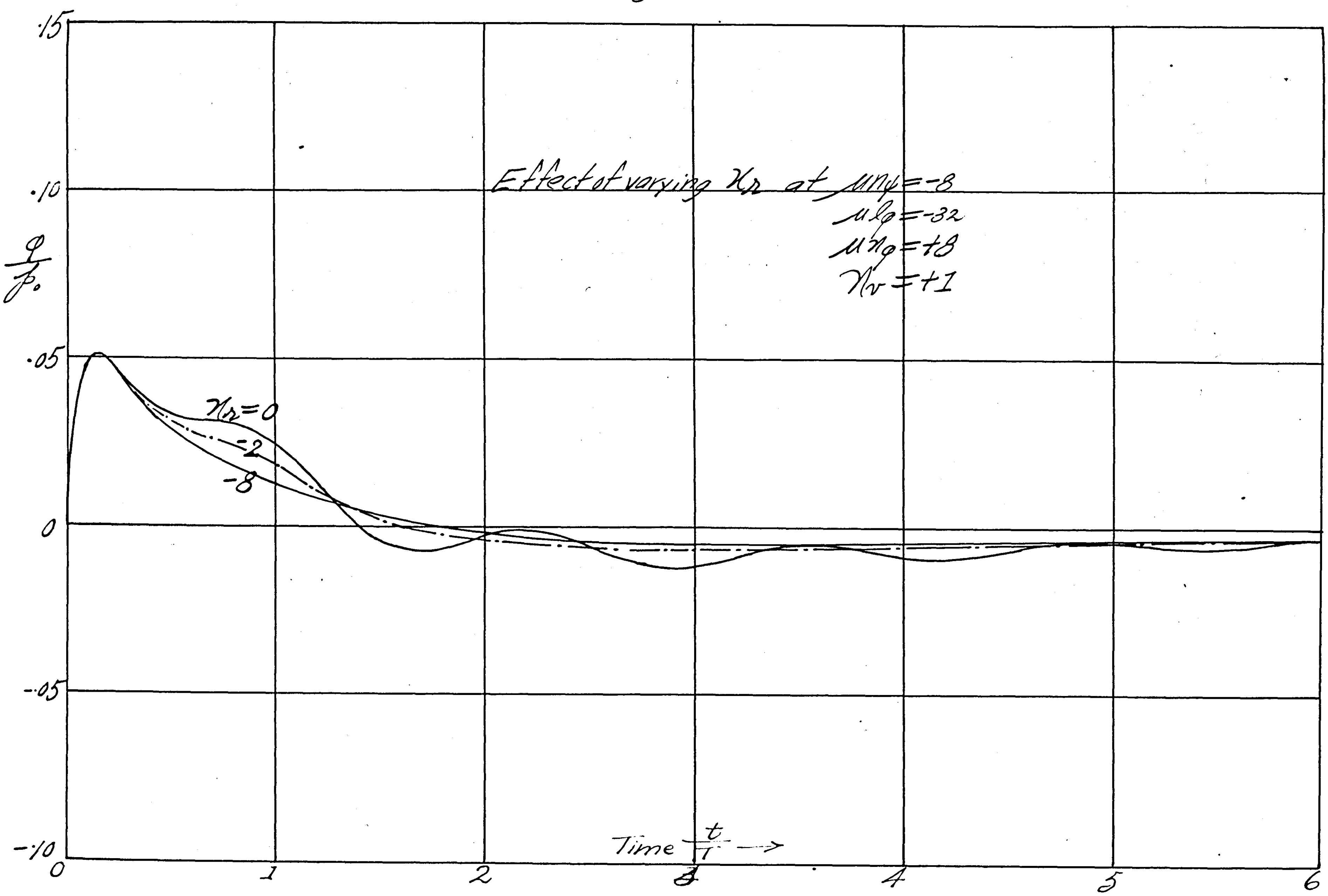

Effect of varying n_v at $\mu n_\psi = -8$
$\mu l_\phi = -32$
$\mu n_\phi = +8$
$n_r = -2$
$n_v = +4$
$+2$
$+\frac{1}{2}$
$+\frac{1}{4}$
0
·15
·10
·05
0
−·05
−·10
$\frac{\psi}{p_0}$
0
1
2
3
4
5
6
Time $\frac{t}{T}$ →

Fig. 10·6 (A).



·15
·10
·05
0
-·05
-·10
$\frac{\varphi}{p_0}$
Effect of varying n_r at $\mu n_\psi = -8$
$\mu l_\varphi = -32$
$\mu n_\varphi = +8$
$n_v = +1$
$n_r = 0$
-2
-8
0
1
2
3
4
5
6
Time $\frac{t}{T}$ →

149

Fig. 10·6 (B).

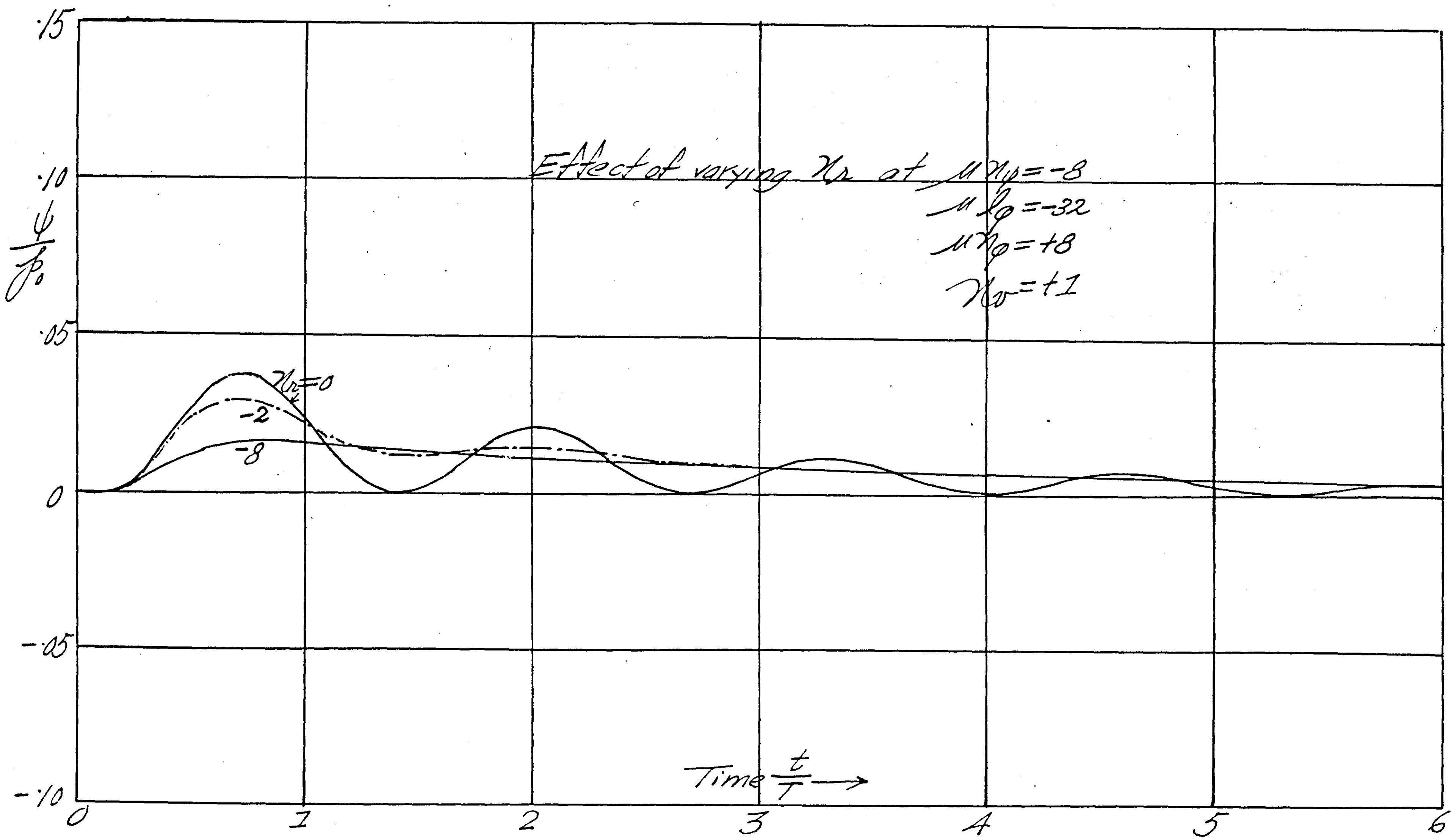

Effect of varying N_r at $\mu N_p = -8$
$\mu l_\phi = -32$
$\mu N_\phi = +8$
$N_v = +1$
$N_r = 0$
-2
-8
$\frac{\psi}{p_0}$
·15
·10
·05
0
-·05
-·10
Time $\frac{t}{T}$
0
1
2
3
4
5
6

Table 10.7

Effect of varrying l_v at $\mu l_\phi = -32$

$\mu n_\psi = -8$

$\mu n_\phi = 0$

$y_v = -0.5$

Substituting into (6.2), we have,
uncontrolled airplane

l_v	A_o	B_o	C_o	D_o	E_o	F_o
0	1	18.5	59	273	-32	0
-2	1	18.5	59	305	0	0
-4	1	18.5	59	337	32	0
-8	1	18.5	59	401	96	0
-16	1	18.5	59	529	224	0

Uncontrolled stability equation

l_v	Long oscillation	Subsidence	Short oscillation
0	$(d+0)(d+-.114)$	$(d+15.87)$	$(d^2+2.84d+17.7)=0$
-2	$(d+0)(d+0)$	$(d+16)$	$(d^2+2.5d+19.1)=0$
-4	$(d+0)(d+.097)$	$(d+16.12)$	$(d^2+2.27d+20.6)=0$
-8	$(d+0)(d+.248)$	$(d+16.37)$	$(d^2+1.88d+23.6)=0$
-16	$(d+0)(d+.442)$	$(d+16.8)$	$(d^2+1.26d+30.14)=0$

Thus it is seen that the effect of varying l_v for an uncontrolled stability equation is to decrease the damping for the short oscillation as well as to have spiral stability, by increasing dihedral. However, as shown by the above investigation, the value for proper l_v is not critical.

For controlled airplane, substituting into (6.4), we have, for no adverse yaw due to aileron,

l_v	A	B	C	D	E	F
0	1	18.5	99	485	544	128
- 8	1	18.5	99	613	672	640
-16	1	18.5	99	741	800	1154

Controlled stability equation

l_v	Long oscillation	Subsidence	Short oscillation
0	$(d^2+1.38d+.34)$	$(d+13.63)$	$(d^2+3.49d+27.6)=0$
-8	$(d^2+1.17d+1.31)$	$(d+14.30)$	$(d^2+3.03d+34)=0$
-16	$(d^2+1.04d+1.84)$	$(d+15.0)$	$(d^2+2.48d+41.83)=0$

Thus it is seen that the effect of increasing dihedral is to,

1. decrease damping and period of short oscillation.
2. Decrease damping and period of long oscillation.
3. increase damping of rolling subsidence.

Therefore, large dihedral is in general undesirable, as the distribution of damping is very unfavorable. However, the limitation for dihedral is still less critical as the damping of both the long and short oscillation remains to be satisfactory even for l_v to be as large as -8.

The disturbed motion showing the effect of varying l_v when the adverse yaw due to aileron is present is plotted in Fig.10.7. It is seen that as l_v increases, the damping for both the long and short oscillation becomes less ,the long oscillation being unstable at $l_v=-16$. When l_v is zero, one real root is zero,as F=0. The error in azimuth remains at certain magnitude, despite the rudder control. However, it is noted that the limitation for l_v is not very strict at low angle of attack. It is the condition at high angle of attack where l_r,C_L,and the aileron adverse yaw all increase rapidly,that the dihedral is required to be kept small.

Fig. 10·7(A).

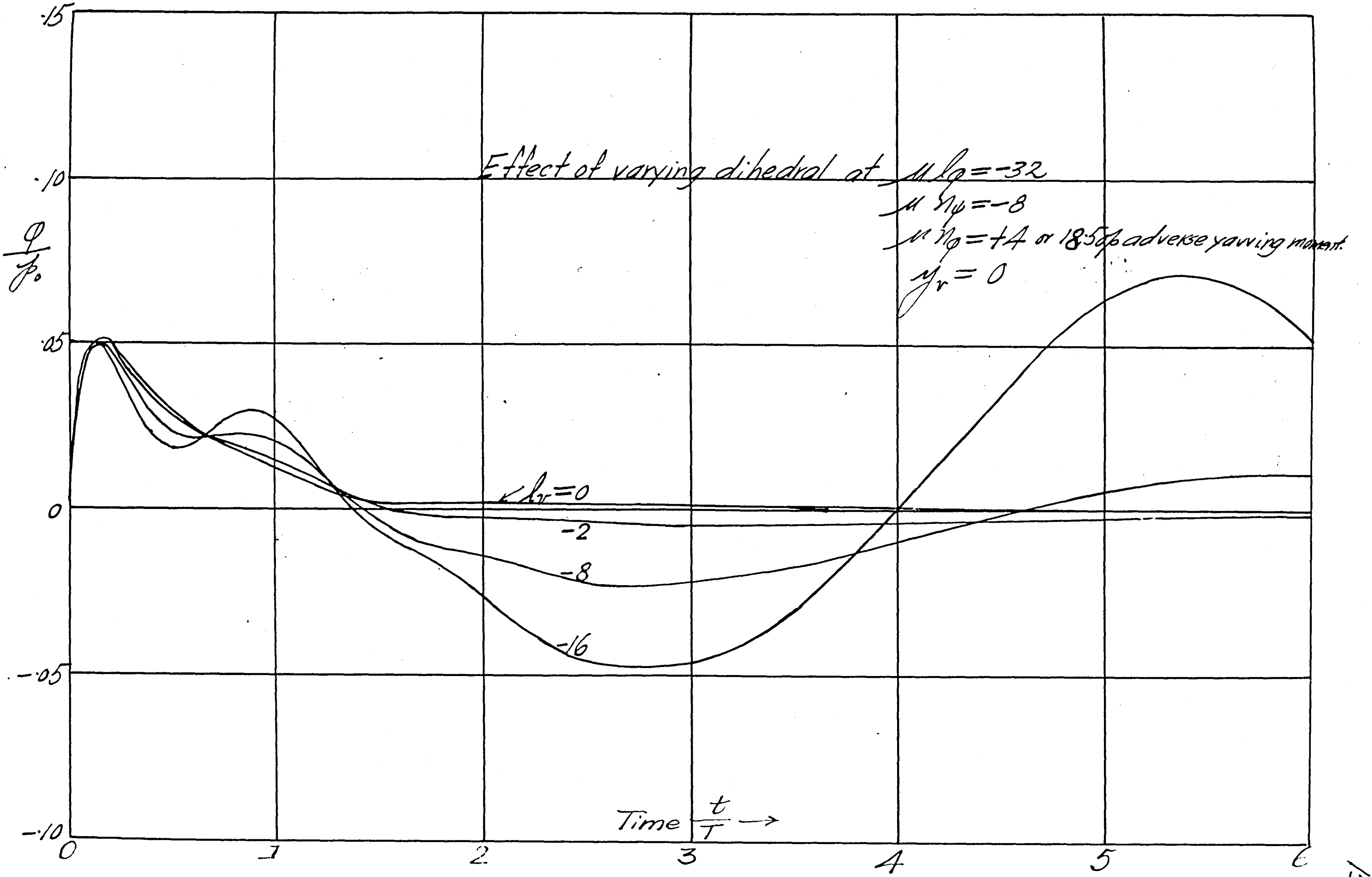
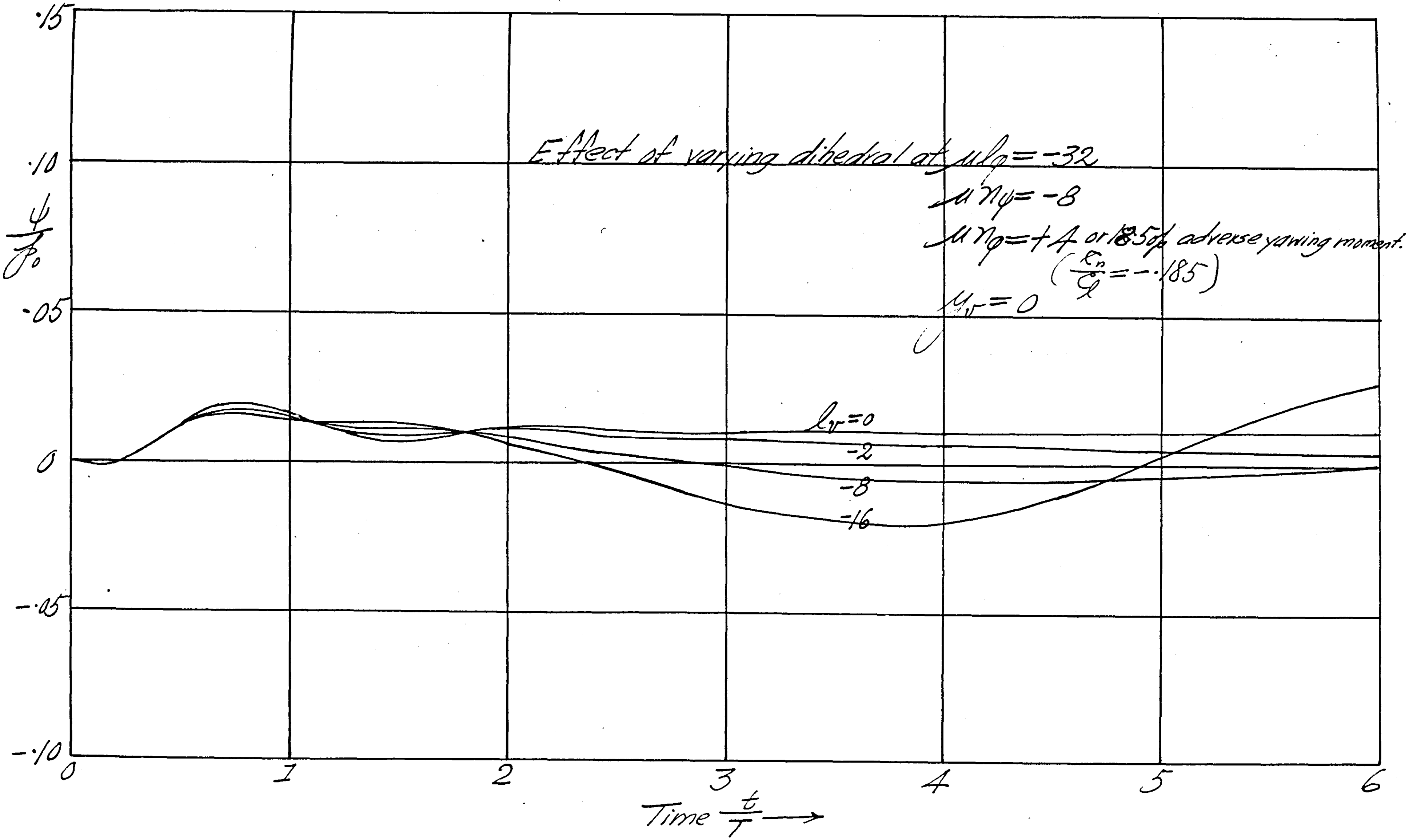

·15
·10
·05
0
-·05
-·10
$\frac{\varphi}{p_0}$
Effect of varying dihedral at $\mu l_\varphi = -32$
$\mu n_\psi = -8$
$\mu n_\varphi = +4$ or 18·5% adverse yawing moment.
$y_r = 0$
$l_v = 0$
-2
-8
-16
Time $\frac{t}{T}$ →
0
1
2
3
4
5
6

Fig. 10.7 (B).



Effect of varying dihedral at $\mu l_\varphi = -32$
$\mu n_\psi = -8$
$\mu n_\varphi = +4$ or 18.5% adverse yawing moment.
$\left(\frac{\varepsilon_n}{\varepsilon_l} = -.185\right)$
$y_v = 0$
$l_v = 0$
-2
-8
-16
$\frac{\psi}{\beta_0}$
·15
·10
·05
0
−·05
−·10
0
1
2
3
4
5
6
Time $\frac{t}{T}$ →

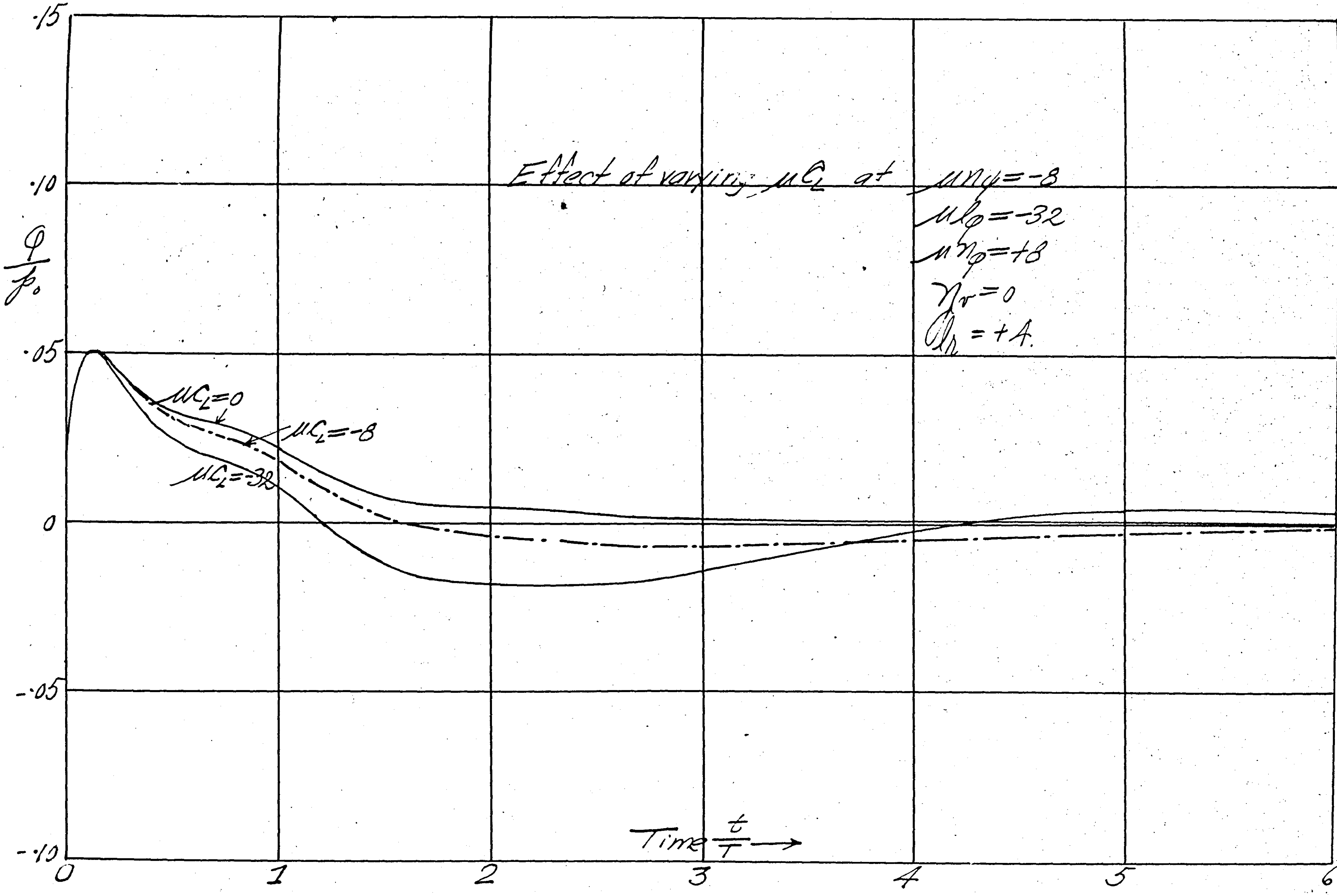
11. Controlled Stability at high angle of attack

The lateral stability at high angle of attack has long been considered to be unsatisfactory for almost every airplane of conventional design[25]. It is the purpose of this article to investigate mathematically the factors principally affecting the controlled lateral stability at high angle of attack and if possible, to find adequate method to improve it.

Examinations of the stability derivatives show that at high angle of attack, the major alterations are the rapid increase of the derivatives μC_L and l_r. The adverse aileron yaw also increase as C_L increases. If the airplane is not stalled, l_p remains substantially constant. y_v, l_v, n_v, n_p and n_r all increase but slightly[26]. there-fore, for the convenience of numerical investigation, it is reasonable to assume that all derivatives remain unchanged except C_L, l_r and aileron yaw.

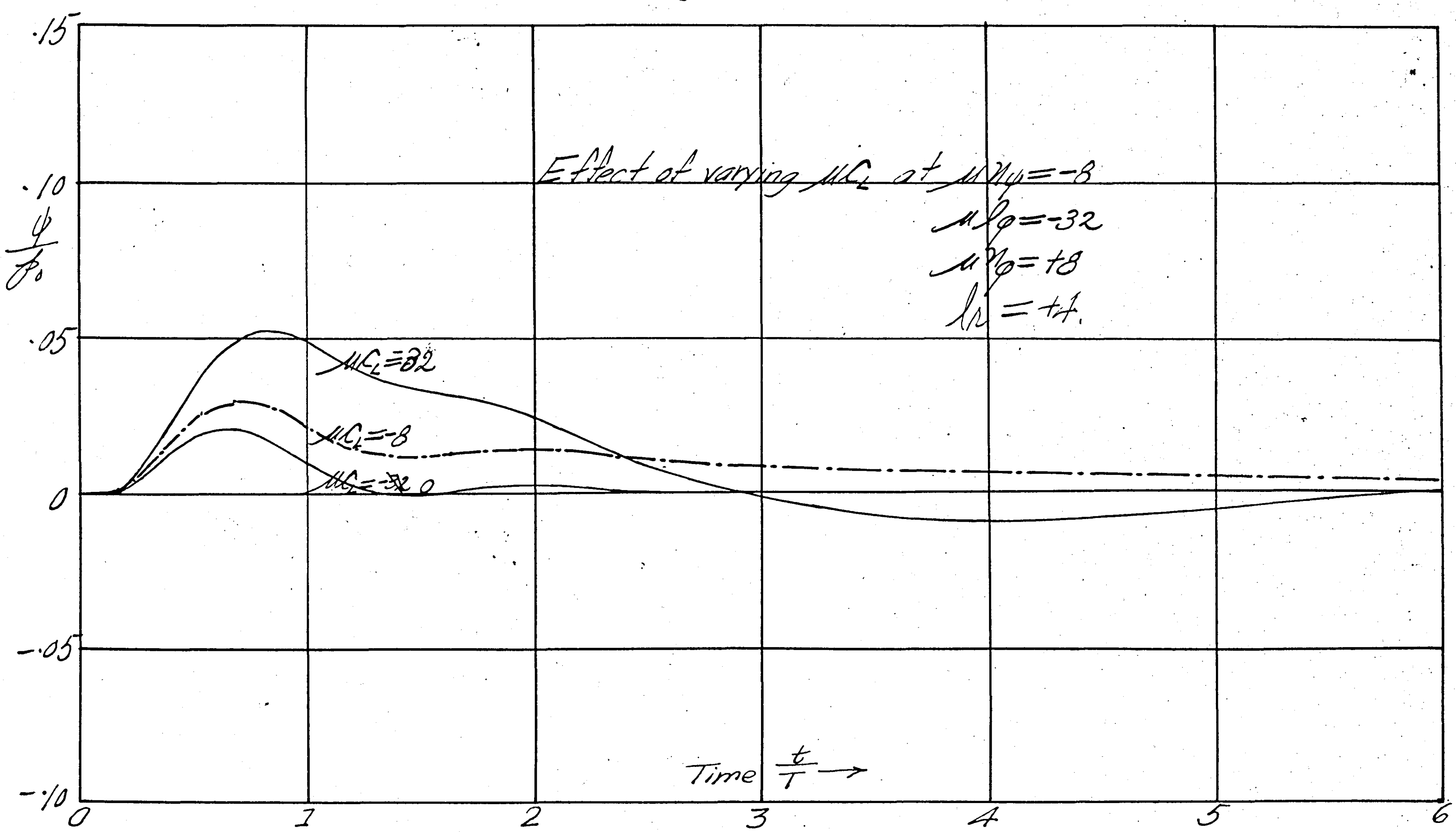
Fig. 11.1 (A) and (B) shows the disturbed motion due to rolling gust at various μC_L assuming constant l_r, it is seen that increase of the term μC_L alone renders the damping of the long oscillation to be less though not very seriously even up to a lift coefficient of -2.0 for the wing loading of F-22. (μ=16). Increase of μC_L is seen to be chiefly responsible for the large surge error in azimuth.

Fig. 11.1 (A).



Effect of varying μC_L at
μn_ψ = -8
μl_φ = -32
μn_φ = +8
η_v = 0
a_h = +4.
μC_L = 0
μC_L = -8
μC_L = -32
φ/p₀
.15
.10
.05
0
-.05
-.10
Time t/T →
0
1
2
3
4
5
6

Fig. 11.1 (B).



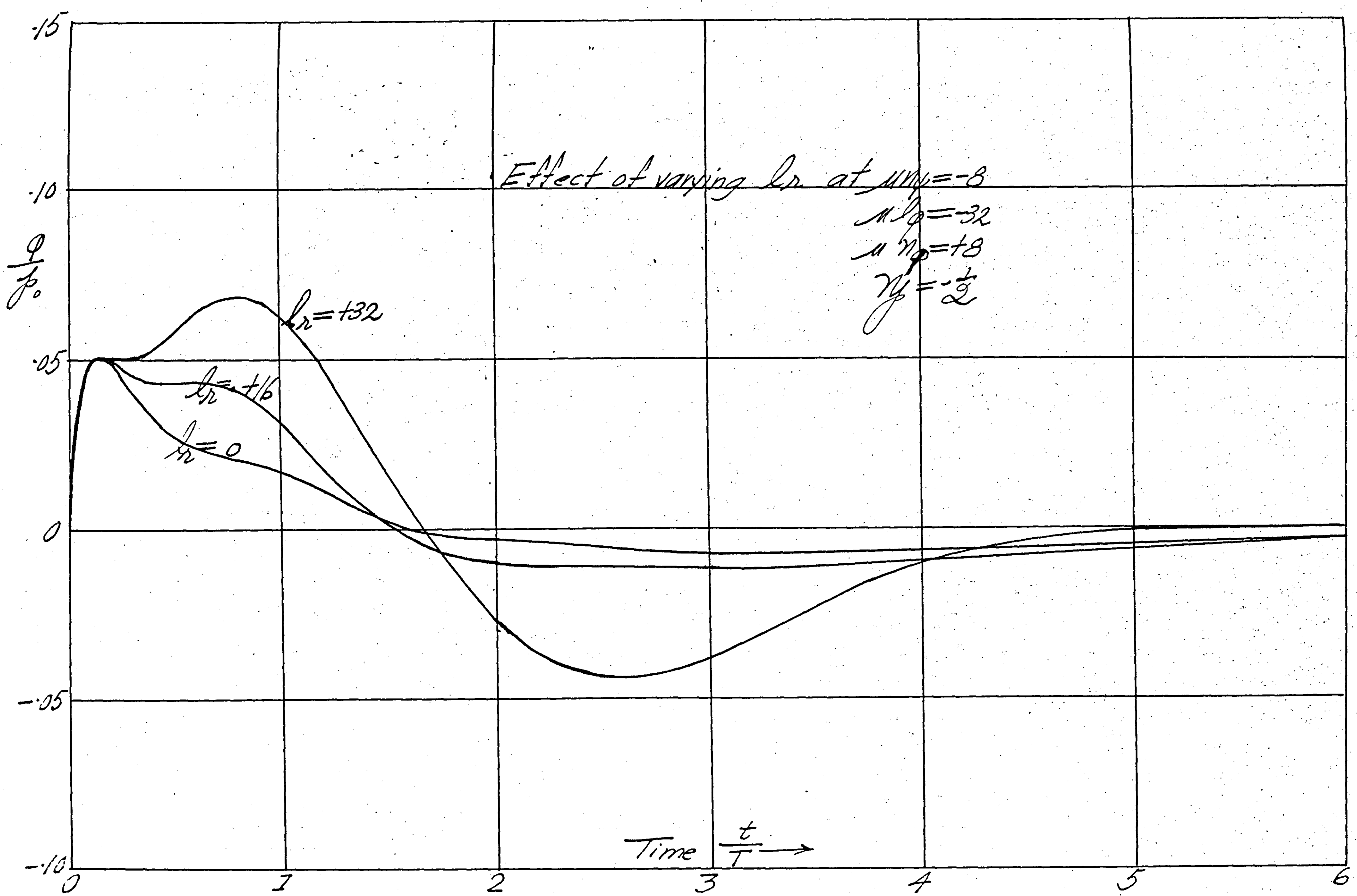
Effect of varying μC_L at $\mu n_\psi = -8$
$\mu l_\phi = -32$
$\mu n_\phi = +8$
$l_r = +4.$
$\mu C_L = 32$
$\mu C_L = 8$
$\mu C_L = 0$
$\frac{\psi}{p_0}$
.15
.10
.05
0
-.05
-.10
Time $\frac{t}{T}$
0
1
2
3
4
5
6

158

Fig.11.2(A) and (B) shows the effect of varying l_r without changing μC_L. It is seen that l_r is responsible for the instability of long period oscillation if increased very much. The magnitude of the disturbed motion in roll and that in yaw are all increased. ~~Opposite to the conclusion pointed out by~~ The effect of changing n_p for a range from zero to -2, makes little difference in the controlled stability of the airplane. This is shown in Fig.11.3(A) and (B). Increase of n_p to -2, seems to decrease the damping of the short oscillation but slightly. The magnitude of disturbance,however, seems to be improved though slightly. Examination of the stability equation in (6.4) justifies this conclusion as the derivative n_p ~~does~~ is not multiplied by any of the control derivatives,such as l_ϕ, n_ϕ or n_ψ. Furthermore, it is only important in determining the value D_o and C_o of the uncontrolled airplane which are over-shadowed by the comparatively much larger influence of the control derivative in determing the value of C and D of the controlled stability equation. Therefore, it is believed quite save to assume $n_p = -.5$ for the calculations on the stability ~~calculation~~ investigation under high angle of attack condition.

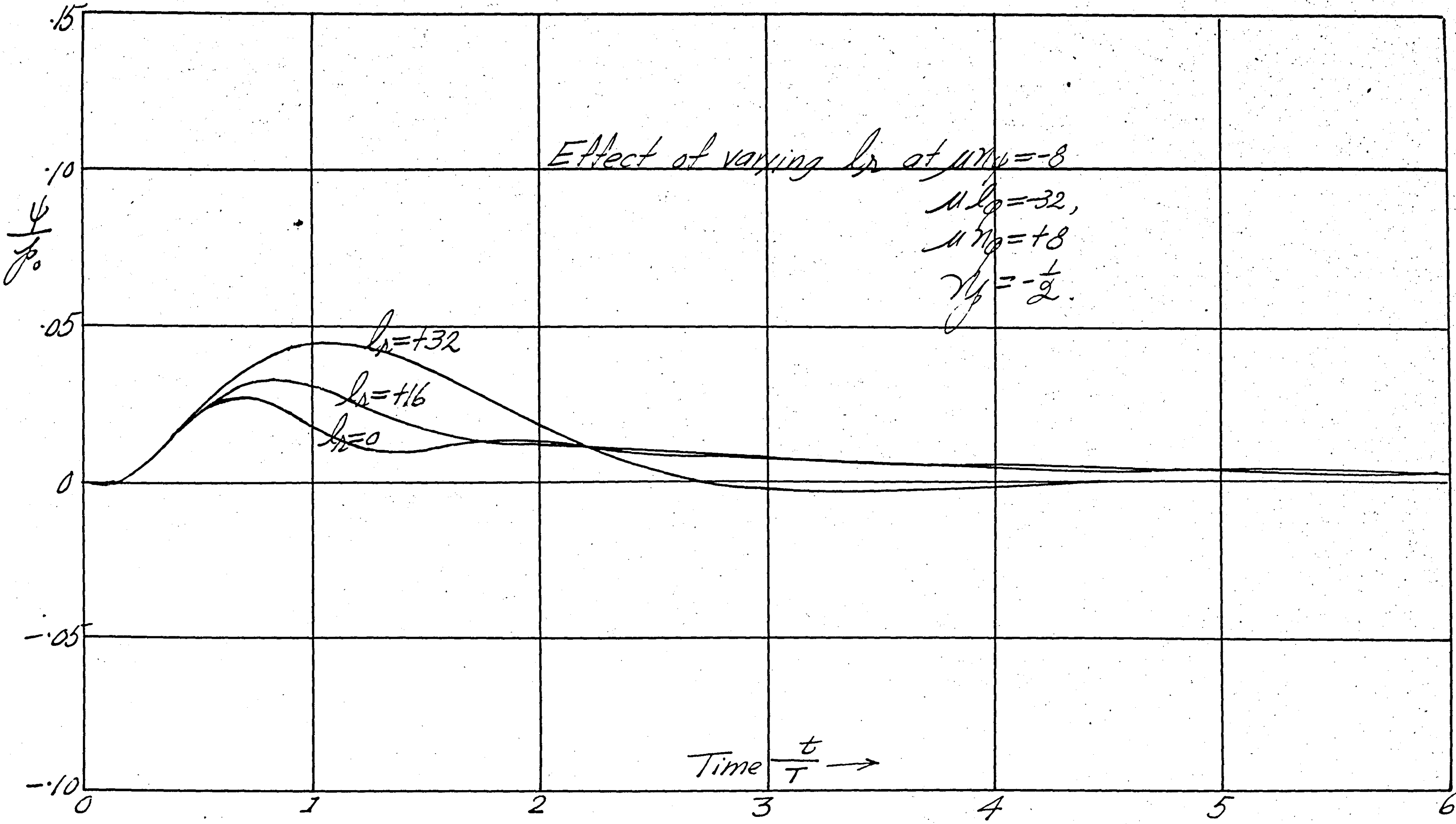
Fig.11.4 shows the effect of varying dihedral, tail size, and aileron adverse yaw at high angle of attack. Fig.11.4(A) shows the disturbance in roll due

Fig. 11·2 (A).



·15
·10
·05
0
-·05
-·10
$\frac{\varphi}{p_0}$
Effect of varying l_r at $\mu n_\psi = -8$
$\mu l_\varphi = -32$
$\mu n_\varphi = +8$
$n_\psi = -\frac{1}{2}$
$l_r = +32$
$l_r = +16$
$l_r = 0$
Time $\frac{t}{T}$ →
0
1
2
3
4
5
6

Fig. 11·2(B).



Effect of varying l_r at $\mu n_\psi = -8$
$\mu l_\phi = -32$,
$\mu n_\phi = +8$
$y_\phi = -\frac{1}{2}$.
$l_r = +32$
$l_r = +16$
$l_r = 0$
$\frac{\psi}{p_0}$
·15
·10
·05
0
−·05
−·10
Time $\frac{t}{T}$ →
0
1
2
3
4
5
6

Fig. 11·3 (A).

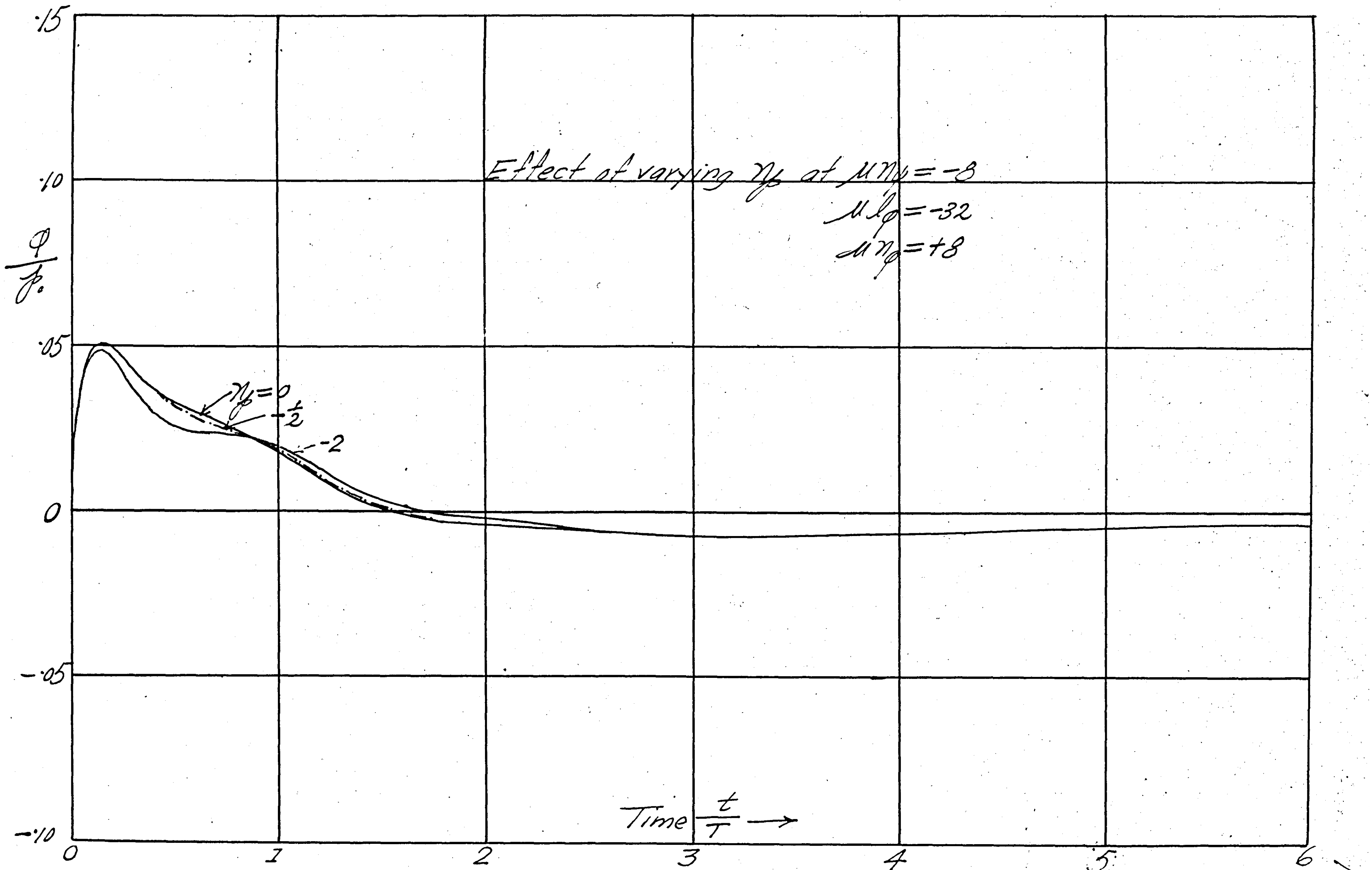

Effect of varying n_β at $\mu n_\psi = -8$
$\mu l_\phi = -32$
$\mu n_\phi = +8$
$n_\beta = 0$
$-\frac{1}{2}$
-2
$\frac{\phi}{\beta_0}$
·15
·10
·05
0
-·05
-·10
0
1
2
3
4
5
6
Time $\frac{t}{T}$ →

Fig. 11·3 (B).

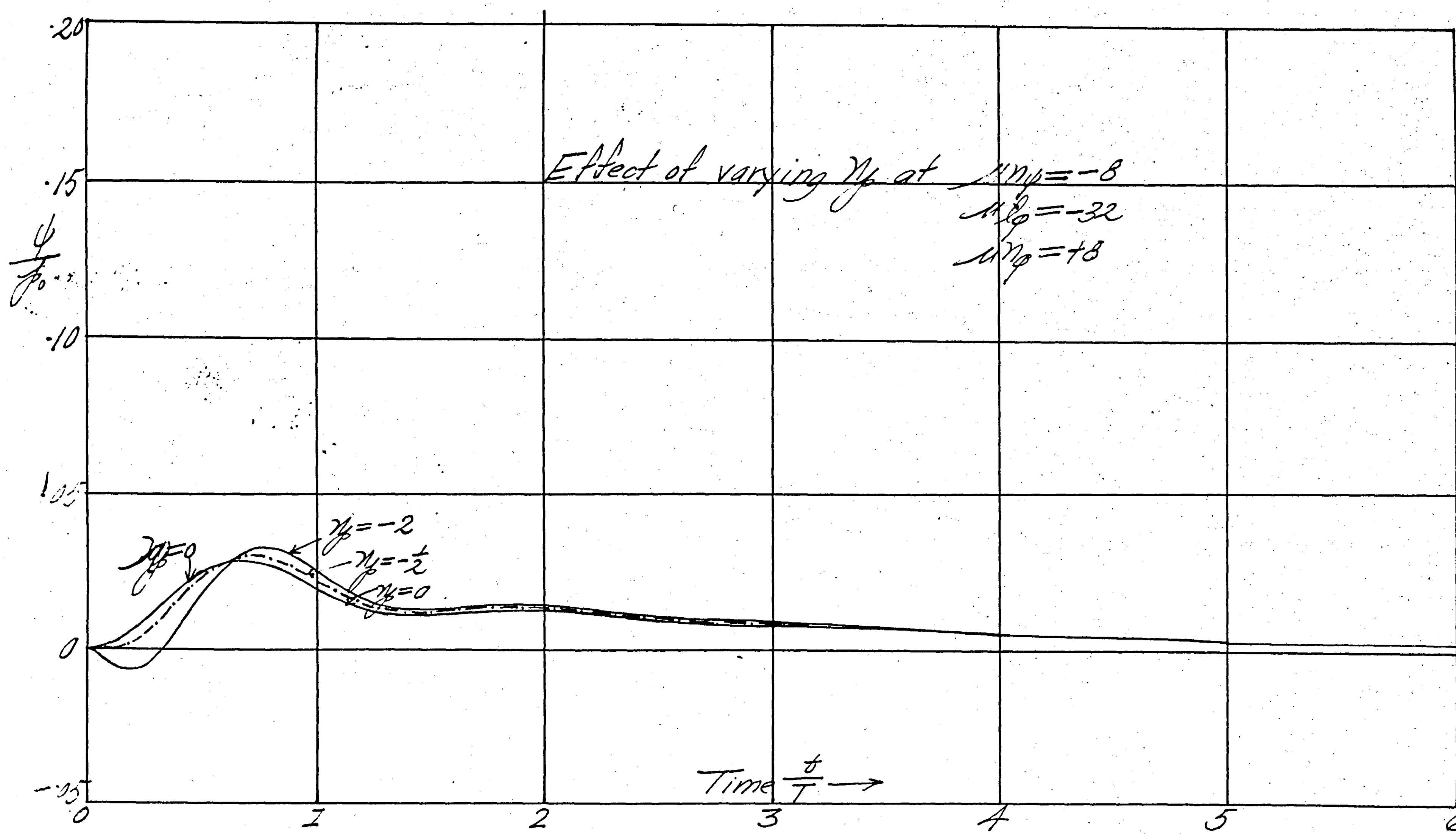
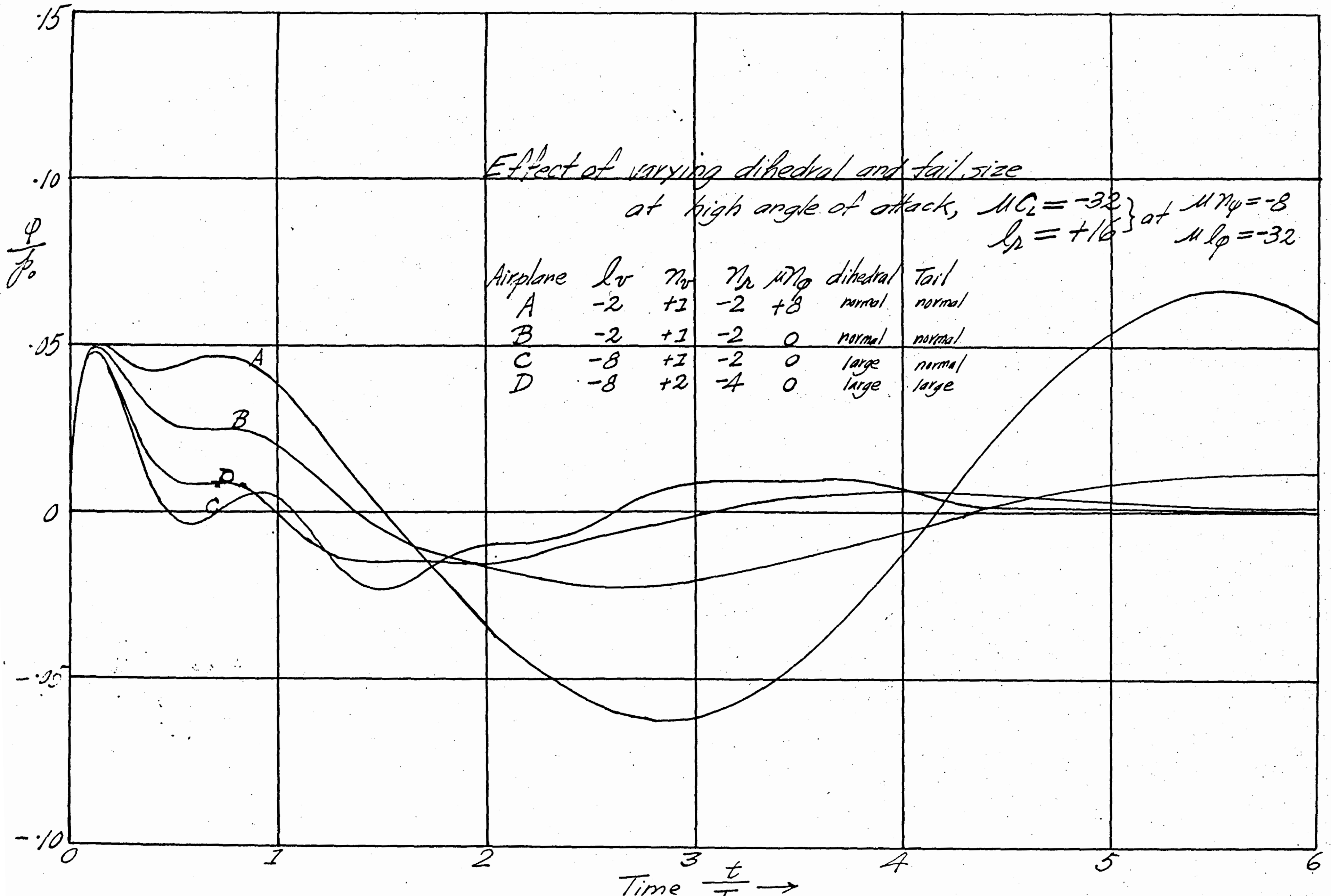
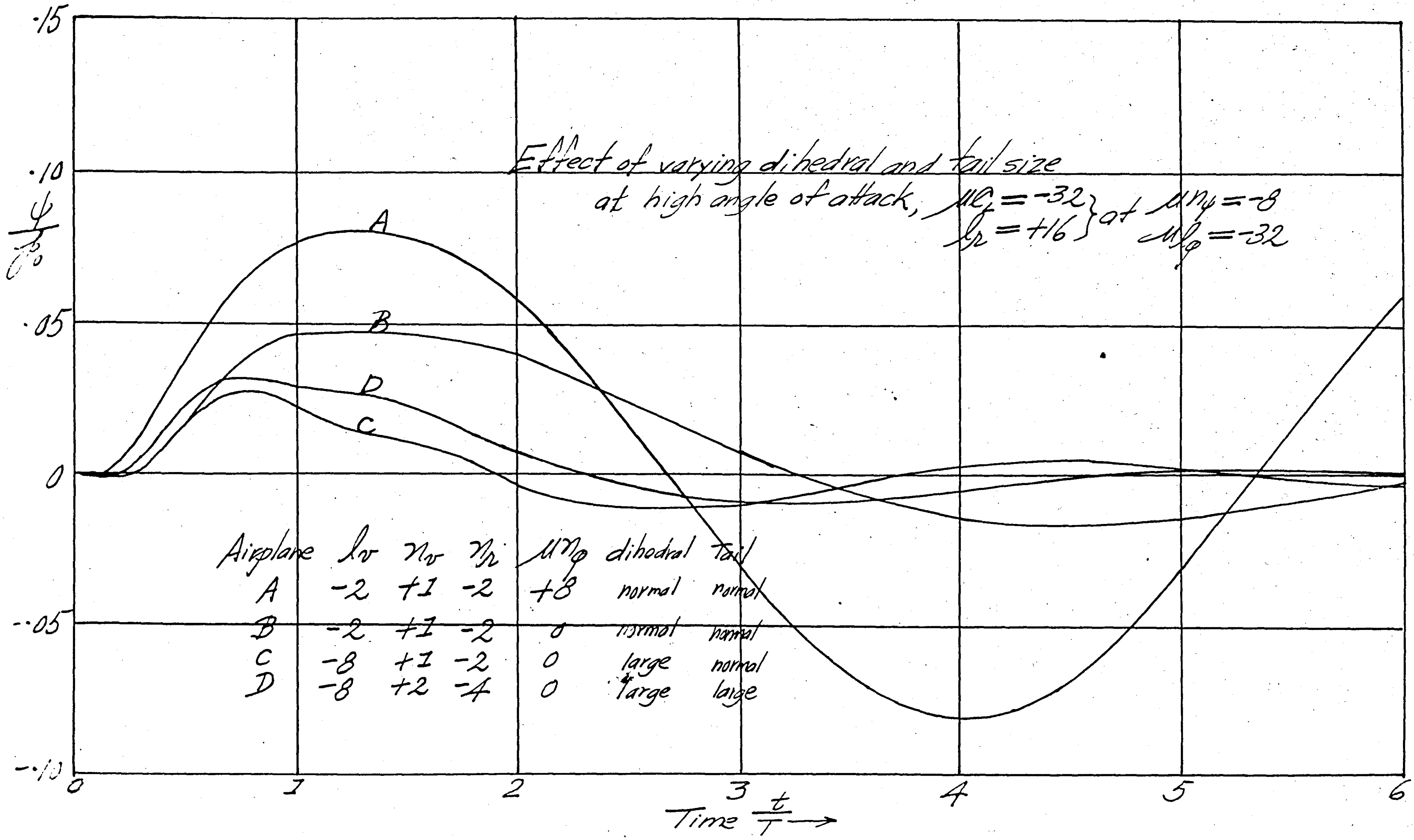

Effect of varying η_ϕ at $\mu n_\psi = -8$
$\mu l_\phi = -32$
$\mu n_\phi = +8$
$\frac{\psi}{p_0}$
·20
·15
·10
·05
0
-·05
$\eta_\phi = 0$
$\eta_\phi = -2$
$\eta_\phi = -\frac{1}{2}$
$\eta_\phi = 0$
Time $\frac{t}{T}$ →
0
1
2
3
4
5
6

Fig. 11·A (A).



Effect of varying dihedral and tail size
at high angle of attack, $\mu C_L = -32$, $l_p = +16$ } at $\mu n_\varphi = -8$, $\mu l_\varphi = -32$
Airplane l_v n_v n_r μn_φ dihedral Tail
A -2 +1 -2 +8 normal normal
B -2 +1 -2 0 normal normal
C -8 +1 -2 0 large normal
D -8 +2 -4 0 large large
$\frac{\varphi}{p_0}$
·15
·10
·05
0
-·05
-·10
0 1 2 3 4 5 6
Time $\frac{t}{T}$ →
A
B
C
D

Fig. 11·A (B).


Effect of varying dihedral and tail size
at high angle of attack, $\mu l_\varphi = -32$, $l_r = +16$ } at $\mu n_\psi = -8$, $\mu l_\varphi = -32$
$\frac{\psi}{\beta_0}$
·15
·10
·05
0
-·05
-·10
A
B
C
D
Airplane l_v n_v n_r μn_φ dihedral Tail
A -2 +1 -2 +8 normal normal
B -2 +1 -2 0 normal normal
C -8 +1 -2 0 large normal
D -8 +2 -4 0 large large
0
1
2
3
4
5
6
Time $\frac{t}{T}$ →

to rolling gust. Curve(A) is the combined result of increasing μC_L to -32,(C_L=-2),and l_r from +4 to +16. The motion is evidently unstable with the presence of adverse aileron yaw corresponding to appromimately, C_n/C_l=-0.40. The disturbance in yaw is seen from Fig.11.4(B) to be even more violent. Remove of the adverse aileron yaw improve the conditaon tremendously as seen from the curves B. Without the affect due to aileron adverse yaw,increase of the dihedral makes very little effect, except decrease the damping of the short oscillation but slightly.The magnitude of disturbance,is on the other hand, improved, particularly on the error in azimuth. This is shown by curve C. Increase of tail size adds damping to the short oscillation and improves the disturbance in roll slightly as seen in curve D. However, if the adverse yaw due to aileron is unavoidable,which is true for most of the ordinary aileron design, the presence of large dihedral is disastrous. This is shown in Fig.11.5(A) and (B). For aileron adverse yaw,corresponding to approximately C_n/C_l = 75% at high angle of attack,the large dihedral airplane is almost unmanageable unless the adverse yaw is counterbalanced by rudder movement according to angle of roll in the direction so as to correct this adverse yaw. This is shown by curves A and B in Fig.11.5 As the development into

Fig. 11.5 (A).

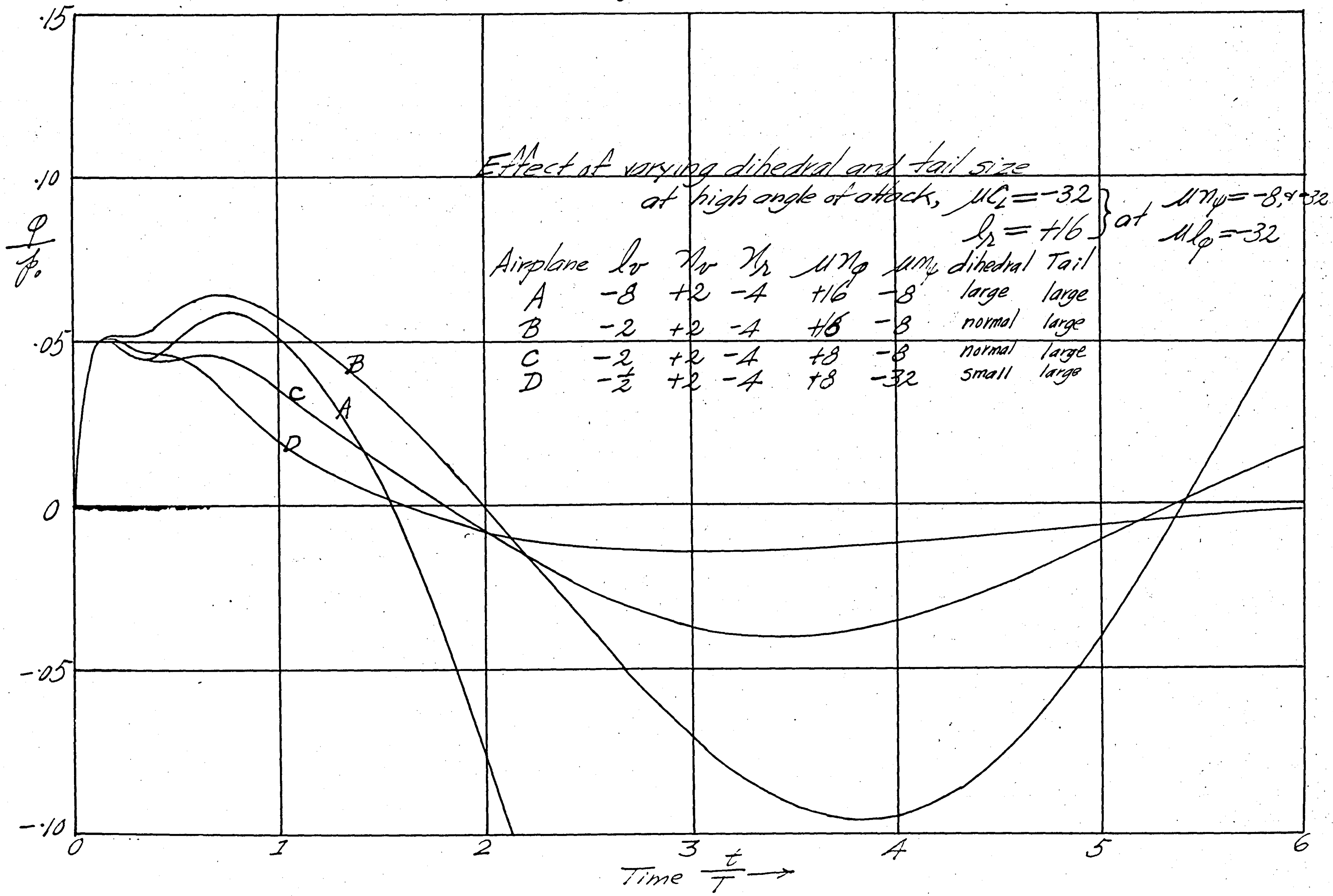
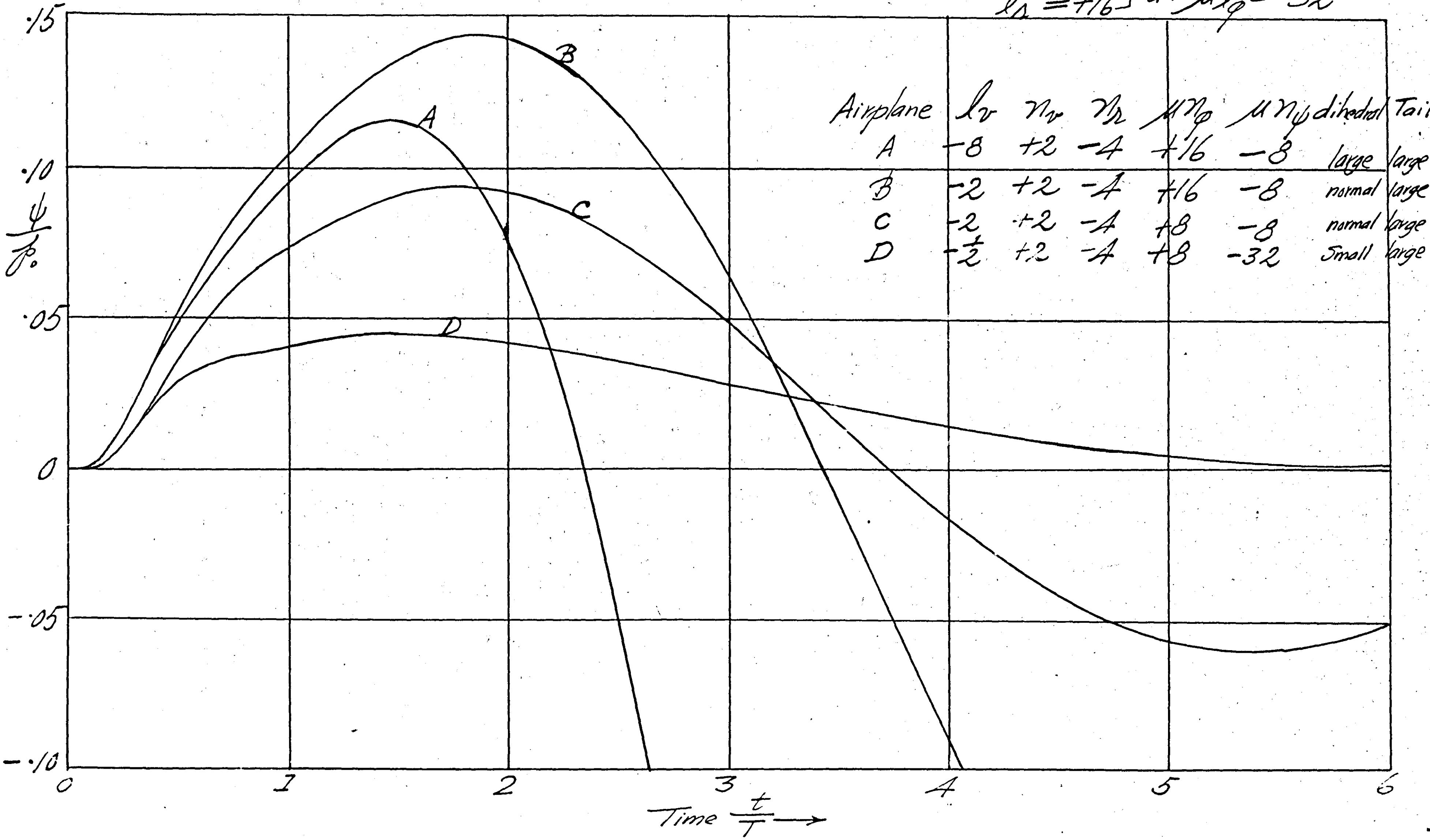

Effect of varying dihedral and tail size
at high angle of attack, μC_L = −32, l_r = +16 } at μn_φ = −8, & −32, μl_φ = −32
Airplane | l_v | n_v | n_r | μn_φ | μn_ψ | dihedral | Tail
A | −8 | +2 | −4 | +16 | −8 | large | large
B | −2 | +2 | −4 | +16 | −8 | normal | large
C | −2 | +2 | −4 | +8 | −8 | normal | large
D | −1/2 | +2 | −4 | +8 | −32 | small | large
φ/p₀
·15
·10
·05
0
−·05
−·10
A
B
C
D
0
1
2
3
4
5
6
Time t/T →

Fig. 11.5 (B).

Effect of varying dihedral & tail size at high angle of attack, $\mu C_L = -32$, $l_r = +16$ } at $\mu l_\varphi = -32$



instability is very rapid, the pilot is required to act on the rudder very quickly in order to avoid the undesirable effect due to control lag. This will be considered later. Decrease of both dihedral and aileron adverse yaw improve the stability considerably, as seen in curve C. A very small dihedral and a stronger rudder control to make the product of μn_ψ by l_v constant so that F of the stability equation (6.4) remains constant, improve the control at high angle of attack to a very satisfactory degree as shown in the curve D. The necessity of large tail is to have large n_r to provide for lag in the rudder control, which can be seen later.

In general, it can be concluded that the limitation of large dihedral is principally due to the presence of aileron adverse yaw. The smallest dihedral is limited by the largest rudder control, μn_ψ without causing control lag to give undesirable short period oscillations. Thus an aileron design giving high adverse yaw at high angle of attack requires a very small dihedral and large tail size. For ailerons giving little adverse yaw, the dihedral can be allowed to be as large as 16° without causing instabilty. As the rudder control is rendered effective by large dihedral, the tail size can be comparatively smaller, to give n_v between +.5 to +1. Too small n_v would decrease the stabilizing effect of aileron control as pointed out before.

A rough criteria for the aileron design can be drawn by examining equation (6.4), and noting the fact that the principal effect of the aileron adverse yaw is to render the coefficients D and E toward negative, and by far the coefficient E is affected most seriously. Therefore, for an ordinary airplane, in order to have the aileron control to be satisfactory, the term $-\mu l_\phi(y_v n_r + \mu n_v)$ must be greater than the term $\mu n_\phi(y_v l_r + \mu l_v)$. In other words, we must have, l_ϕ/n_ϕ ~~maxkxbe~~ larger than $(y_v l_r + \mu l_v)/(y_v n_r + \mu n_v)$. Or C_n/C_l must be smaller than $(c_l/a_l)(y_v n_r + \mu n_v)/(y_v l_r + \mu l_v)$. By neglecting the terms involving y_v, as it is small, and assuming $a_l/c_l = 1$ approximately, we get the criteria for aileron design as C_l/C_n must be greater than n_v/l_v which is known to us before, as an empirical expression.

12. Effect of control lag.

In article 7, it was pointed out that the effect of control lag is to alter the derivatives of the airplane. In the determinant (7.2), it is seen that lag in aileron control is equivalent to,

(1), Increase of the inertia in roll by the ratio from 1 to $1-\mu l_\phi t_a^2/2$.

(2) Reduction of l_p by $\mu l_\phi t_a$

The equivalent effect of lag in rudder control is,

(1), Increase of the inertia in yaw by the ratio from 1 to $1-\mu n_\psi t_r^2/2$

(2) Reduction of n_r by $\mu n_\psi t_r$

If the term $-\mu n_\phi t_a^2/2$ is neglected in considering the aileron lag on the adverse yaw, the effect of aileron lag on the adverse yaw is to increase n_p by $t_a \mu n_\phi$, as μn_ϕ due to adverse yaw is positive.

For the convenience of numerical investigation, it is advisable to divide the second row of (7.2) by $(1-\mu l_\phi t_a^2/2$, and call,

$$l'_v = l_v/(1-t_a^2/2 \times \mu l_\phi)$$
$$l'_p = (l_p-\mu l_\phi t_a)/(1-\mu l_\phi t_a^2/2)$$
$$l'_\phi = l_\phi/(1-\mu l_\phi t_a^2/2)$$
$$l'_r = l_r/(1-\mu l_\phi t_a^2/2)$$

Similarly, the third can be divided by the term, $(1-\mu n_\psi t_r^2/2) = x$ for simplicity, and call,

$\cancel{x t_v}$

$n'_v = n_v/x$

$n'_p = (n_p - \mu n_\phi t_a)/x$

$n'_\phi = n_\phi/x$

$n'_r = n_r/x$

$n'_\psi = n_\psi/x$

For the derivatives of F-22, and assume $t_a = t_r$, At $\mu l_\phi = -32$, $\mu n_\phi = +8$, and $\mu n_\psi = -8$, the table shows the effect of lag in altering the derivatives,

Table 12.1

t_a or t_r	l'_v	l'_r	l'_p	$\mu l'_\phi$	n'_v	n'_p	n'_r	$\mu n'_\psi$	$\mu n'_\phi$
0	-2	4	-16	-32	1	-.5	-2	-8	-8
.15	-1.47	2.94	-8.23	-23.5	.92	-1.56	-.74	-7.4	-7.4
.50	-.40	.80	0	-6.4	.50	-2.25	+1.0	-4	-4

At control lag $t_a = 0.5$, the airplane is evidently very unstable as l_p is zero and n_r is +1 so that B=0 in (6.4) despite the effort of aileron and rudder control. The effect of $t_a = .15$ is investigated as follows,

Table 12.2

t_a or t_r	A_o	B_o	C_o	D_o	E_o	F_o
0	1	18.0	50	288	0	0
.15	1	9	25.4	170	13	0

$t_a = t_r$	uncontrolled stability equation			
	Long oscillation		Subsidence	Short oscillation
0	$(d+0)$	$(d+0)$	$(d+16)$	$(d^2+2d+18)=0$
.15	$(d+0)$	$(d+.0775)$	$(d+8.38)$	$(d^2+.543d+20.16)=0$

Thus the uncontrolled stability equation shows only slight decrease of damping of short oscillation, as the rolling subsidence is still well damped and the airplane is spirally stable.

For controlled stability, substituting into (6.4), we have,

Table 12.3

$t_a = t_r$	A	B	C	D	E	F
0	1	18	90	448	256	128
.15	1	9	56.3	231	185	87

$t_a = t_r$	Controlled stability equation		
	Long oscillation	Subsidence	Short oscillation
0	$(d^2+.583d+.323)$	$(d+13.7)$	$(d^2+3.72d+29)=0$
.15	$(d^2+.88d+.472)$	$(d+5.4)$	$(d^2+2.72d+34)=0$

Thus the lagging effect is seen to be equivalentt to decrease l_p and n_r primarily. The damping in rolling subsidence is decreased very much and the damping of long oscillation is also decreased. The increase of damping of short oscillation is due to the fact that while lagging effect reduces the effectiveness of aileron and rudder control, it minimizes the undesirable effect of the aileron adverse yaw.

As it is difficult to alter the derivatives in the M.I.T. Differential analyzer according to their fractional figures, the disturbed motion for the exact equation with constant time lag is not solved. However, the effect of varying l_p alone is plotted in Fig.12.1.

It is seen that l_p is very important in determining the magnitude of transient surge error as well as the damping for both lonng and short oscillations. When l_p is zero either due to stalling of the wing or due to equivalent effect of excessive control lag, the controlled motion is evidently very objectionable.

The effect of reducing n_r by the lag in rudder control has been plotted in Fig.10.6, showing that the principal effect of n_r is to change the damping of short oscillation.

In general conclusion, it can be said that in any case, the control lag must not cause the damping derivatives l_p and n_r to become zero in order to have satisfactory control. It is interesting to note that while l_p is larger than n_r, the control derivative l_ϕ is also larger than n_ψ so that consideration of lag in rudder control is just as important as that in aileron control.

Fig. 12.3 (A).

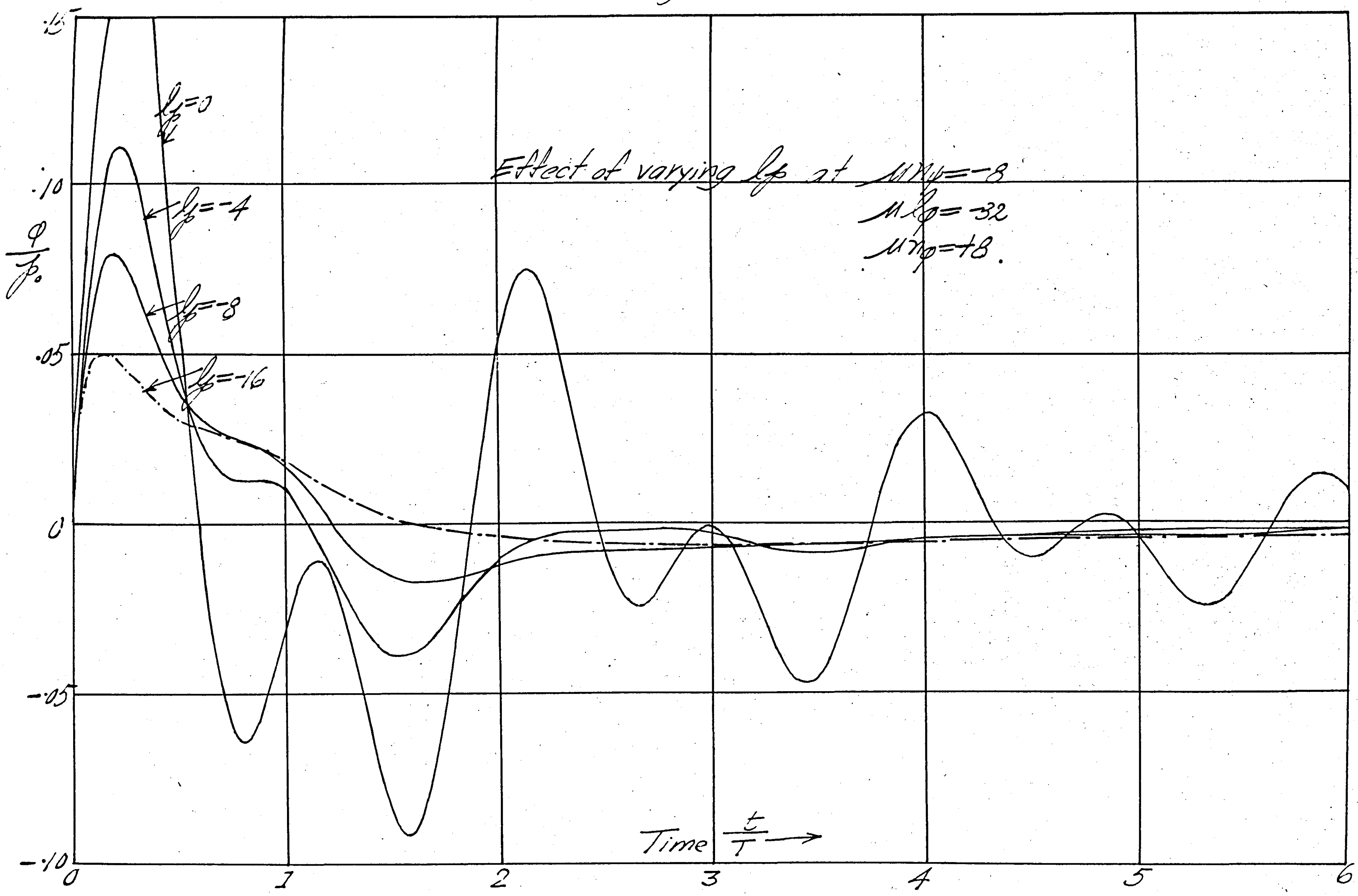
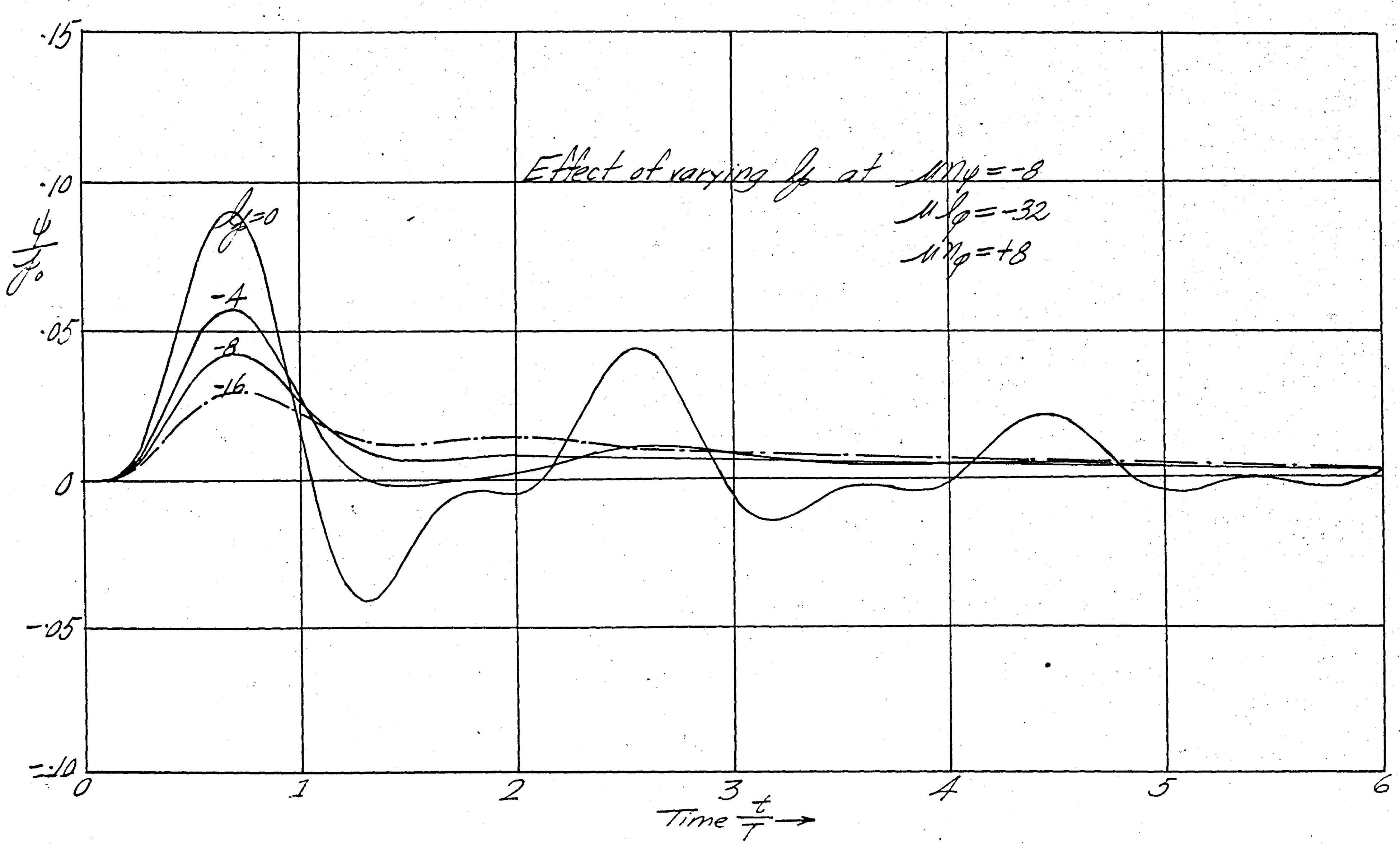

Effect of varying l_p at $\mu n_p = -8$
$\mu l_\varphi = -32$
$\mu n_\varphi = +8$.
$l_p = 0$
$l_p = -4$
$l_p = -8$
$l_p = -16$
$\frac{\varphi}{p_0}$
.15
.10
.05
0
-.05
-.10
Time $\frac{t}{T}$
0
1
2
3
4
5
6

Fig. 12: 1 (B).

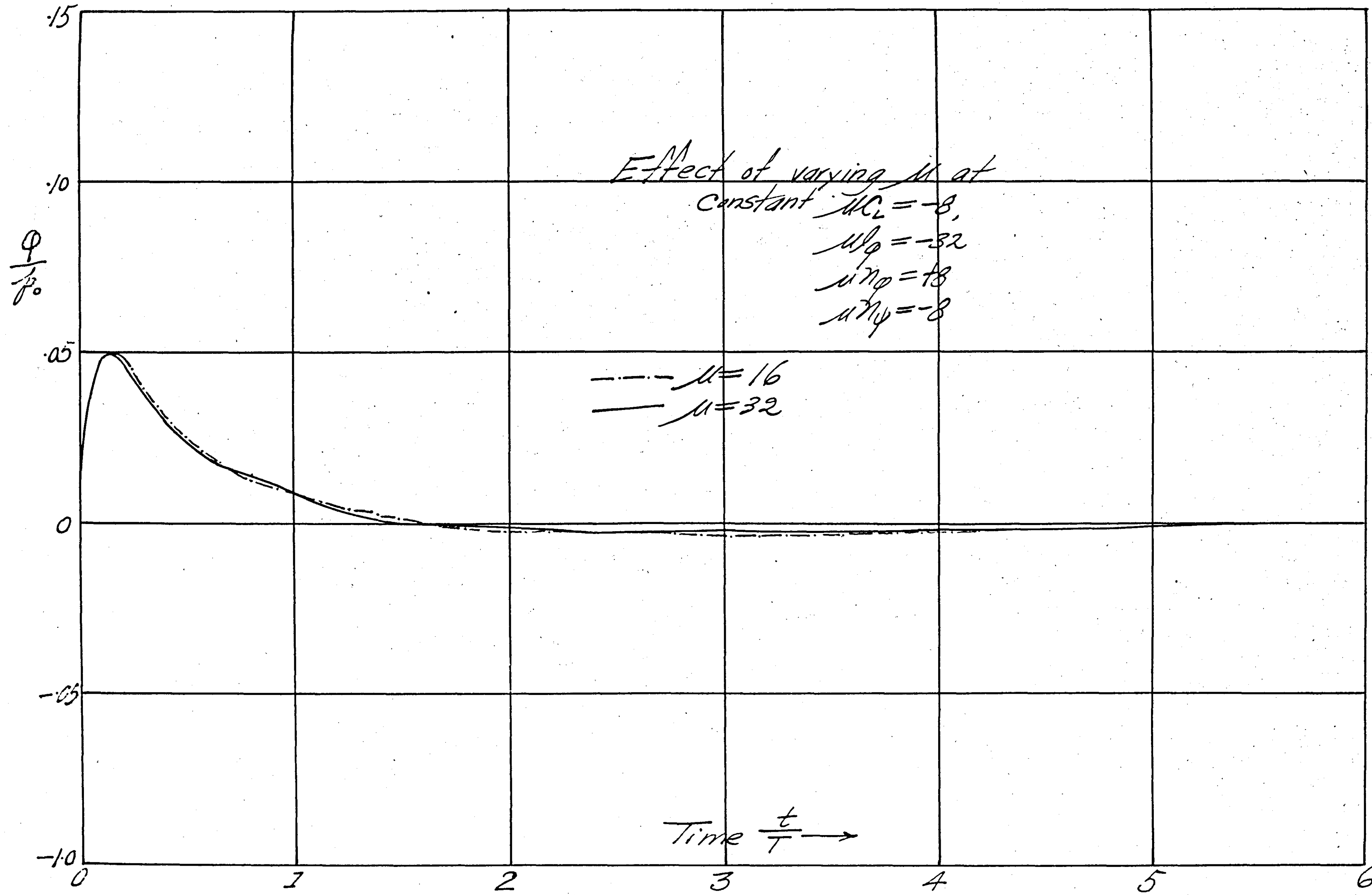

·15
·10
·05
0
-·05
-·10
$\frac{\psi}{f_0}$
Effect of varying l_ϕ at $\mu n_\psi = -8$
$\mu l_\varphi = -32$
$\mu n_\varphi = +8$
$l_\psi = 0$
-4
-8
-16
0
1
2
3
4
5
6
Time $\frac{t}{T}$ →

13. Effect of variation of the parameter μ and the time unit of the non-dimensional system, T.

The variation of μ is due to either change of altitude which changes ρ, or due to different wing loading for different kinds of airplane. Increase of μ would increase μl_{ϕ}, μn_{ϕ}, μn_{ψ} and μC_L the effects of which have all been investigated. Since μ also occurs in the equation of motion as equivalent to y_r, it is interesting to see the effect of varying that term alone. Fig.13.1 shows the effect of varying ~~that term~~ μ alone. It seems variation of that term alone causes little change in the stability of the controlled airplane. Therefore, it is believed that the principal effect of varying wing loading is to modify the control derivatives, making a high μ airplane more sensitive to control lag. It is pointed out by Koppen[28] that ~~that~~ the inertia factors a_1 and c_1 are approximately linear functions of wing loading, being smaller for higher wing loading. The derivatives l_p, l_r, l_v, n_p, n_r and n_v will all be increased for airplanes of higher wing loading, making the criteria for the controlled stability more important and more critical.

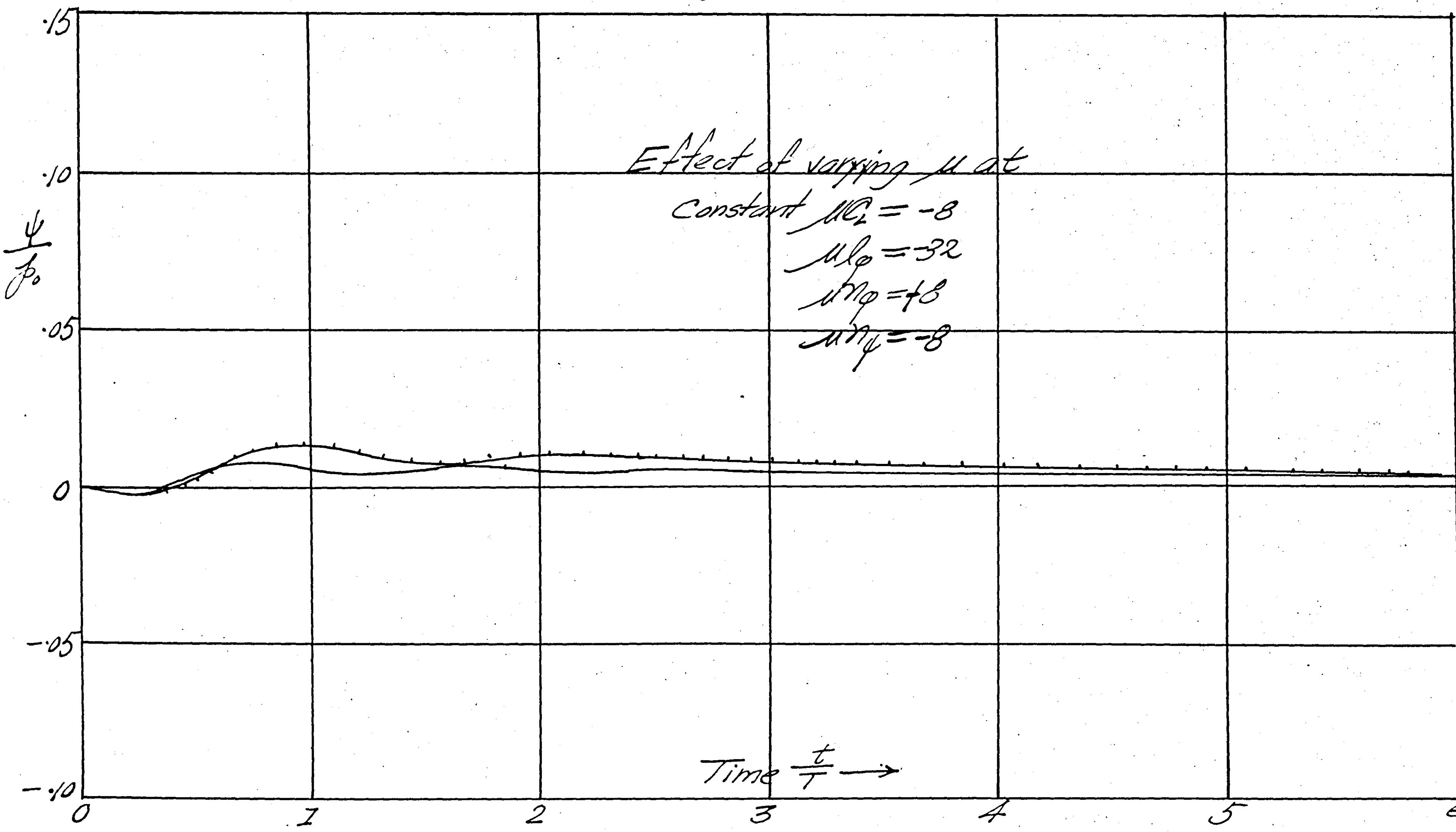
The altitude effect is to change both μ and T. As the lag in the control system is in nondimensional unit for the previous investigations, it would be smaller for a constant lag measured in seconds if T is larger,

Fig. 13.1(A).



Effect of varying μ at constant $\mu C_L = -8$, $\mu l_\varphi = -32$, $\mu n_\varphi = +8$, $\mu n_\psi = -8$
$\mu = 16$
$\mu = 32$
$\frac{\varphi}{p_0}$
·15
·10
·05
0
−·05
−1·0
Time $\frac{t}{T}$ →
0
1
2
3
4
5
6

Fig. 13.1 (B).



Effect of varying μ at
Constant μC_L = -8
μl_φ = -32
μm_φ = +8
μn_ψ = -8
ψ/p_0
·15
·10
·05
0
-·05
-·10
Time t/T →
0
1
2
3
4
5
6

and vice versa. As $T = m/\frac{\rho}{2}SU$, it is seen that airplanes of high speed, low wing loading, flying at low altitude, is most sensitive to control lag. This is approximately the case for racing airplanes, except that wing loading may not be very low.

14. Lateral Controllability

While the definition for the word controllability is different for different writer, it is defined as the relative degree of sensitivity of the airplane in its response to ~~the~~ a certain forcing function introduced by the operation of the control, here. The airplane, is analogous to a vibration system having inertia, damping and spring characteristics of multiple degrees of freedom. The response of the airplane to certain forcing function depends on the ~~eg~~ nature of the forcing function as well as on the nature of the airplane itself. It is the purpose of this article to investigate, under various airplane characteristics, the response of the airplane to a constant applied moment in roll and in yaw, such as that introduced by aileron and rudder control.

Weick and Jones[29] investigated the effect of lateral controls in producing motions of an airplane at various flight conditon and dihedral. Unfortunately, only the equations of motion in roll and yaw were used while the degree of freedom in Y-direction was ignored. Furthermore, due to the complication of mathematics in using the step-by-step method of integration, the investigation was limited to very few cases. Jones[30] B.M., also investigated the response of the airplane to rolling and yawing moment for three different airplane characteristics showing that the airplane characteristics are very important in determing the controlability of the airplane.

It is found that the M.I.T.Differential Analyzer is very convenient in investigations of such purpose. The response of the average airplane F-22 under various design possibilities,to a constant applied rolling and yawing moment is investigated. As the airplane is inherently more sensitive to yawing moment than rolling moment[31],a constant rolling moment causing an initial rolling acceleration,$(d^2\phi/dt^2)_o$ or $\dot{p}_o=5$,or while an initial yawing acceleration, $\dot{r}_o= 2$ were used throughout the investigation. Since the differential equations of motion are linear, the effect on the motion due to any combination of applied rolling and yawing moments can be found by simple addition.of the proper share.

The derivatives for F-22 were used with $\frac{y}{V}=0.5$., except the variable indicated in each case. In each case, the direct roll due to rolling moment as well as the secondary roll due to applied yawing moment are plotted together. Similarly ,the direct yaw due to applied yawing moment and the secondary yaw due to rolling moment are plotted together.

a, Effect of dihedral, derivative, l_v

Fig.14.1(A) and (B) compare the sensitivity of the airplane to roll and to yaw under constant applied moments. It is seen that the dihedral has a remarkable effect on the sensitivity to roll due to direct rolling moment as well as due to secondary effect of the applied yawing moment, as seen from Fig.14.1(A). Increase of dihedral makes the airplane less sensitive to direct rolling moment, but on the other hand, more sensitive to roll as a result of the secondary effect due to applied yawing moment. Basing on the N.A.C.A. criteria of rolling sensitivity, 32, the angle of bank, ϕ, produced in one second, can be taken as a relative measure of the control effectiveness. For the average airplane under the assumed flight condition, the angle of bank at $t/T = 0.5$ should be taken as a standard for comparison, as T=2seconds approximately. It is seen from Fig.14.1(A), that for an airplane having $l_v=-2$, the secondary roll due to applied yawing moment of magnitude 40% as large as the applied rolling moment is always smaller than the direct roll due to rolling moment. For $l_v=-8$, the secondary roll though lags behind the direct roll in the initial stage, it exceeds the direct roll by almost 50% at $t/T=0.5$. At still larger dihedral, $l_v=-16$, the secondary roll exceeds the direct one by almost 300% at $t/T=0.50$. Therefore, on the rolling motion produced by applied moments.

Fig. 14·1 (A).

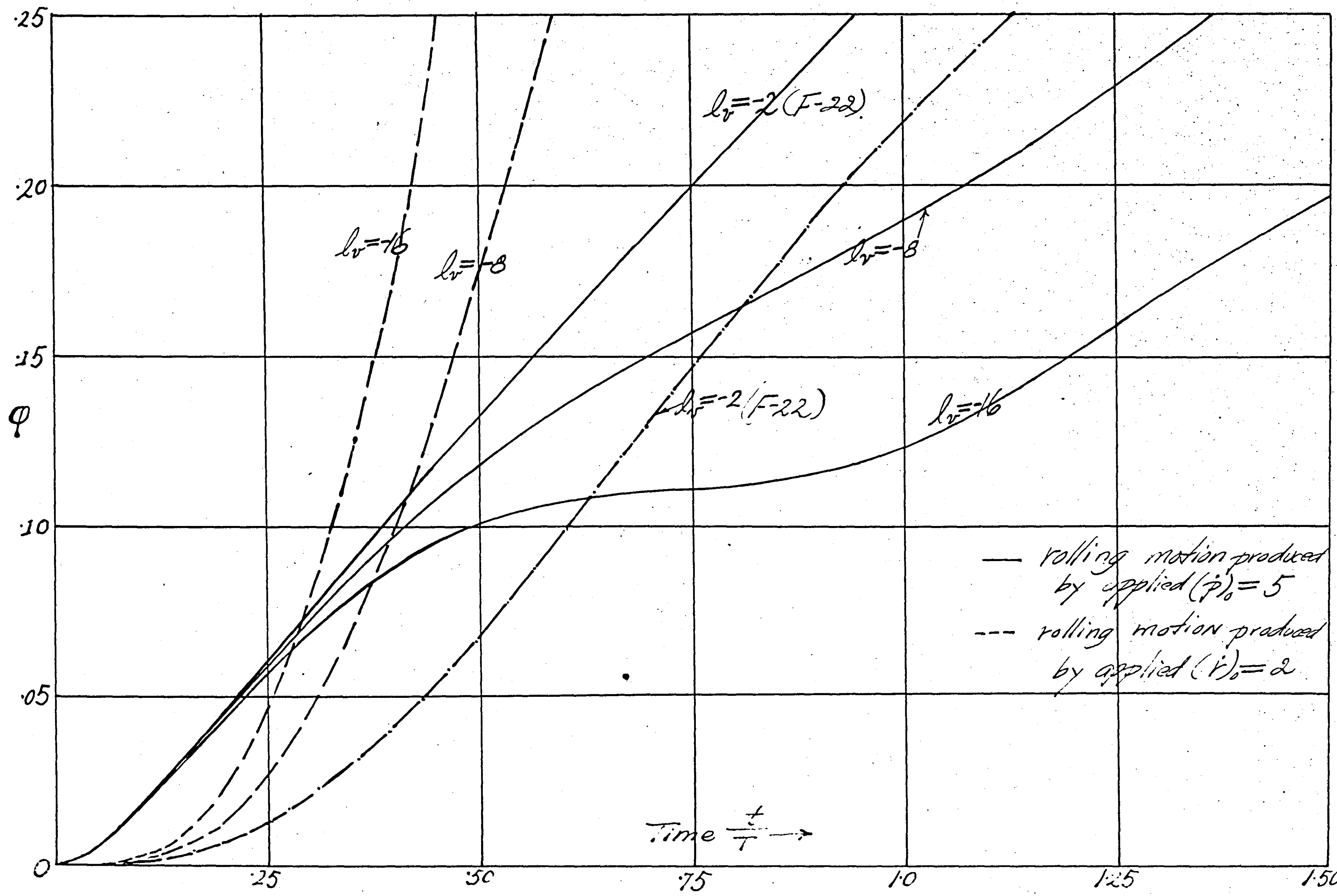
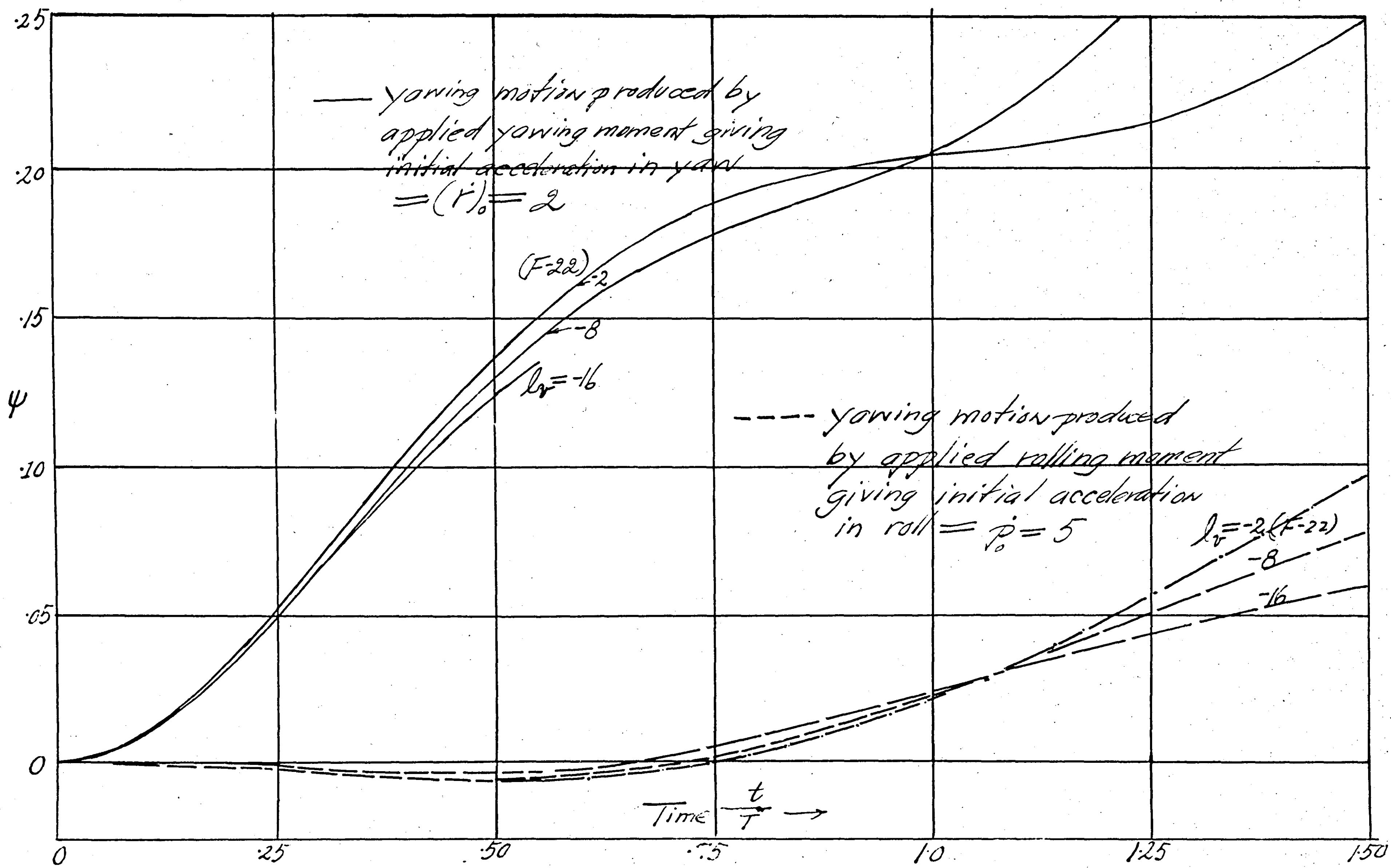


Fig. 14.1 (B).
Effect of varying l_v on the yawing motion produced by applied rolling & yawing moments.



for a same system of aileron design, giving the magnitude of applied rolling moment exactly identical, and also giving a same percentage of aileron adverse yaw, if it is put on an airplane of large dihedral, the pilot would feel that the effectiveness of the aileron control is inferior to the one installed on ~~xxx~~ an airplane of less dihedral, because the negative roll due to secondary effect of the adverse yaw is soon overshadowing the direct roll, for airplanes of large dihedral. At high angle of attack, both dihedral and percentage of aileron adverse yaw increase rapidly, the operation of aileron may give roll in the opposite sense.

Fig.14.1(B) shows that variation of dihedral makes little difference in sensitivity toward either direct yawing moment or secondary effect due to rolling moment.

b. Effect of varying tail sizes, and μ.

Fig.14.2 show the effect of varying n_v, n_r and μ respectively . Curve A is the response curve for the average airplane considered as a basis for comparison. Fig.14.2(A) show the direct and secondary roll due to applied rolling moment and applied yawing moment respectively. It is seen that the effect of large n_v without altering n_r is to increase the direct rolling sensitivity but very slightly. This is shown by comparing B vs A. On the other hand, the secondary rolling sensitivity due

Fig. 14.2 (A).
Effect of varying, n_v, n_r, & μ on the rolling motion, φ produced by applied rolling & yawing moments.

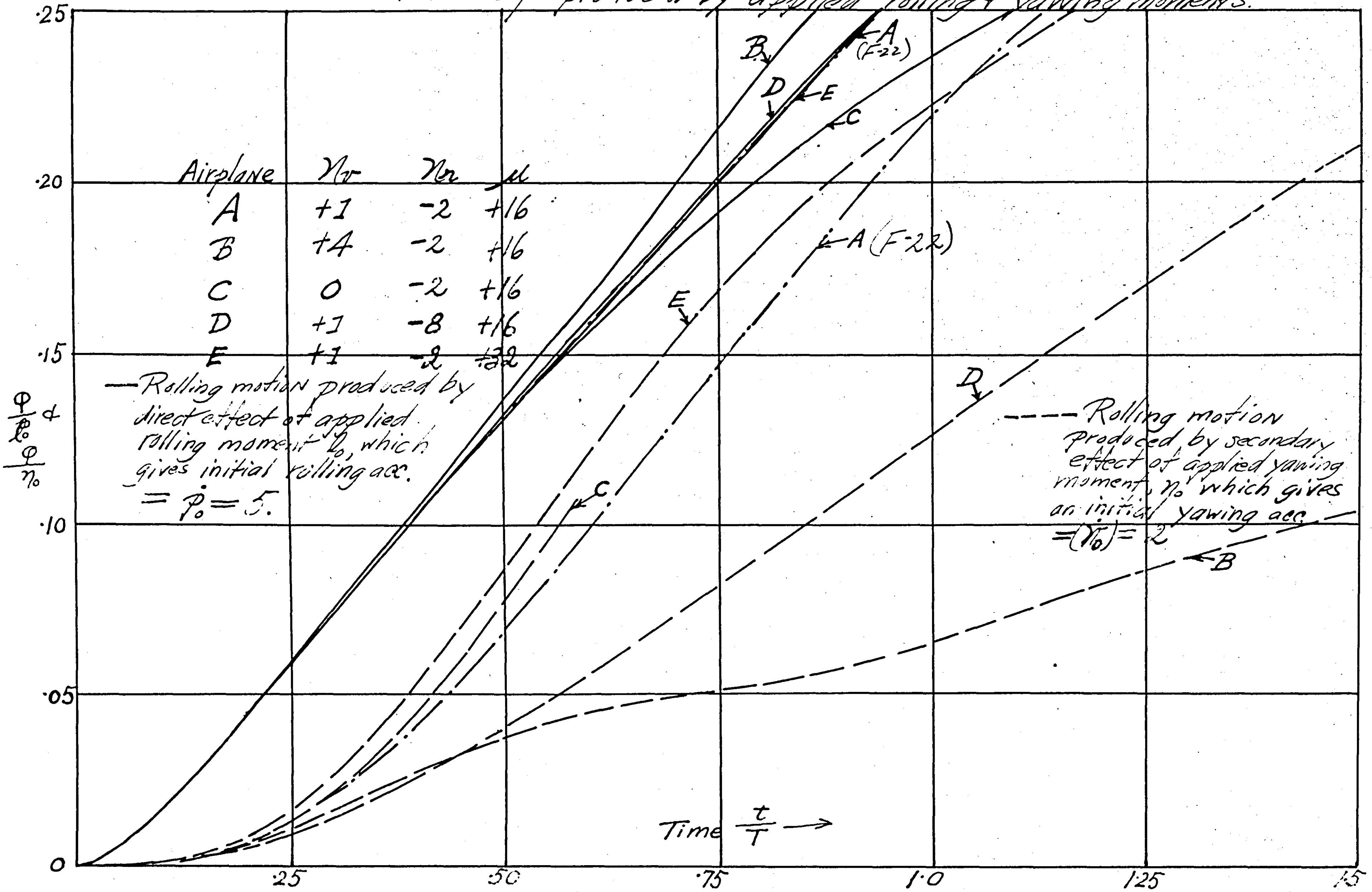
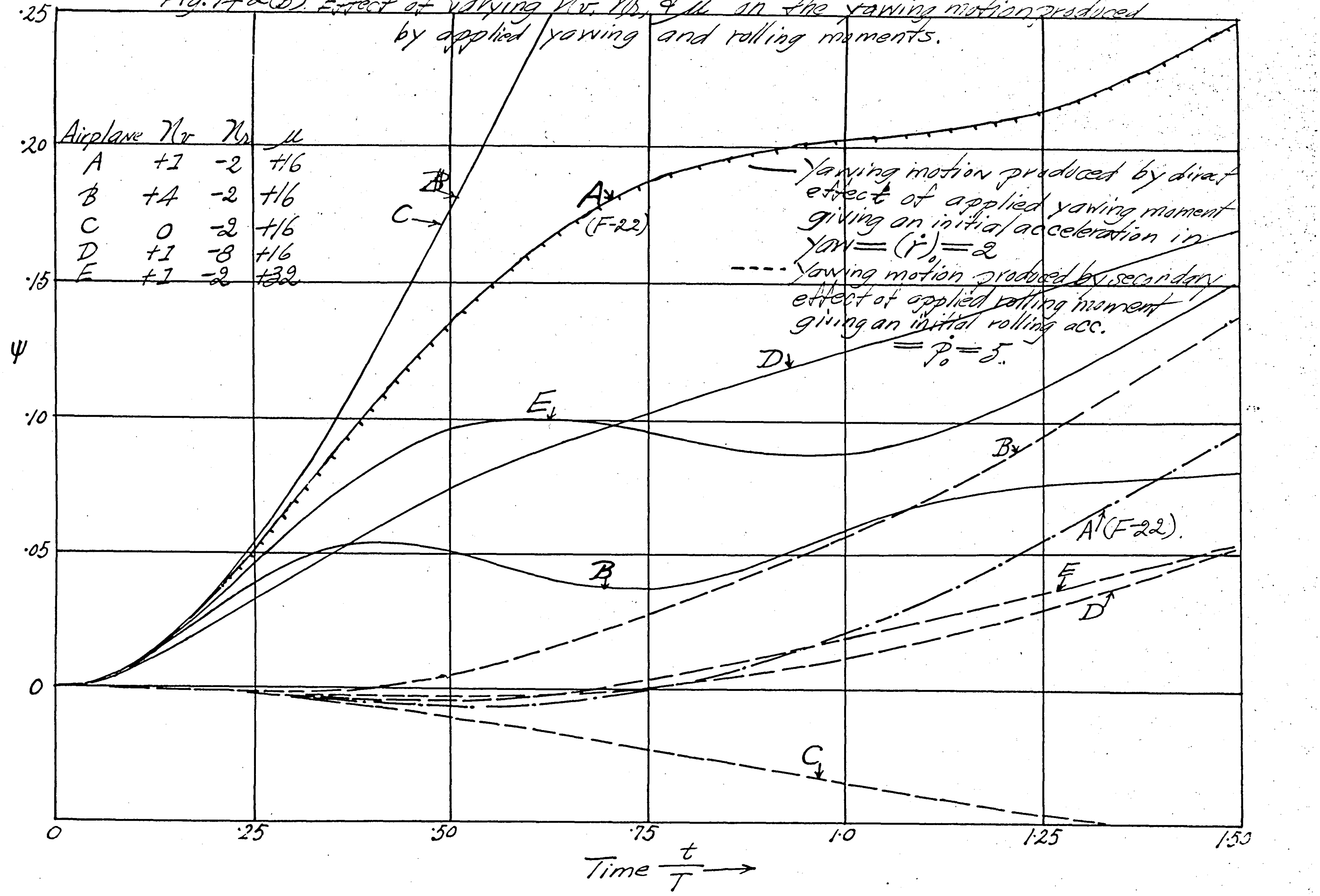


Fig. 14·2(B). Effect of varying n_v, n_r, & μ on the yawing motion produced by applied yawing and rolling moments.



to yawing moment is greatly reduced. Reduction of n_v from +1 to zero shows that the reverse is true. Therefore, airplanes of large n_v is less affected by the disastrous influence of the aileron adverse yaw. Unfortunately, large n_v causes the direct yawing sensitivity due to yawing moment to be greatly reduced as shown by Fig.14.2(B). Therefore, n_v can neither be too large nor too small. With aileron of moderate amount of adverse yaw, $n_v=+1/2$ to +1 is seen to be best compromise. The yawing sensitivity due to secondary effect of rolling moment is increased by increasing n_v and decreased by decreasing n_v, the yaw due to positive rolling moment becomes negative when $n_v=0$, as shown by curve C in Fig.14.2(B).

The effect of increasing n_r alone is to decrease the rolling sensitivity due to the secondary effect of yawing moment. The direct rolling sensitivity is substantially unchanged. Increase of n_r decrease the direct yawing sensitivity just as what would be expected as n_r is the damping coeff. in yaw. The secondary yawing sensitivity is but slightly changed, being smaller for larger n_r.

Variation of μ, only give noticable effect on the direct yawing sensitivity, being decreased ~~in the initial stage but~~ by incresing μ. The secondary rolling sensitivity due to yawing moment is seen to be increased by increasing μ too.

c. Effect of varying μC_L and l_r.

It was pointed out in the study of the controlled stability at high angle of attack that for flight below stall, the important change in the stability derivative can be represented by increase of μC_L and l_r. The sensitivity of the airplane toward the applied moments is shown in Fig.14.3. It is seen that the direct rolling sensitivity is affected only slightly by variations of μC_L and l_r. The increase of rolling sensitivity due to the secondary effect of applied yawing moments is seen to be principally due to increase of l_r. The direct yawing sensitivity is also affected very slightly by changed of μC_L and l_r, but the secondary yawing sensitivity due to rolling moment is increased by increasing μC_L, changes due to l_r being of less importance in this case. It is interesting to note that investigations of the sensitivity give us clear picture why airplanes are poorly affected by the adverse yaw at high angle of attack.

d. Effect of varying l_p, n_p and y_v.

Due to ~~either~~ stalling of the wing, l_p is greatly reduced at high angle of attack. At high angle of attack, the derivatives n_p and y_v are found to increase considerably, so it is interesting to see their effects on the controllability. Fig.14.4 show the effects of varying l_p, n_p, and y_v.

Effect of varying μC_L and l_r on the rolling motion produced by applied rolling and yawing moments.

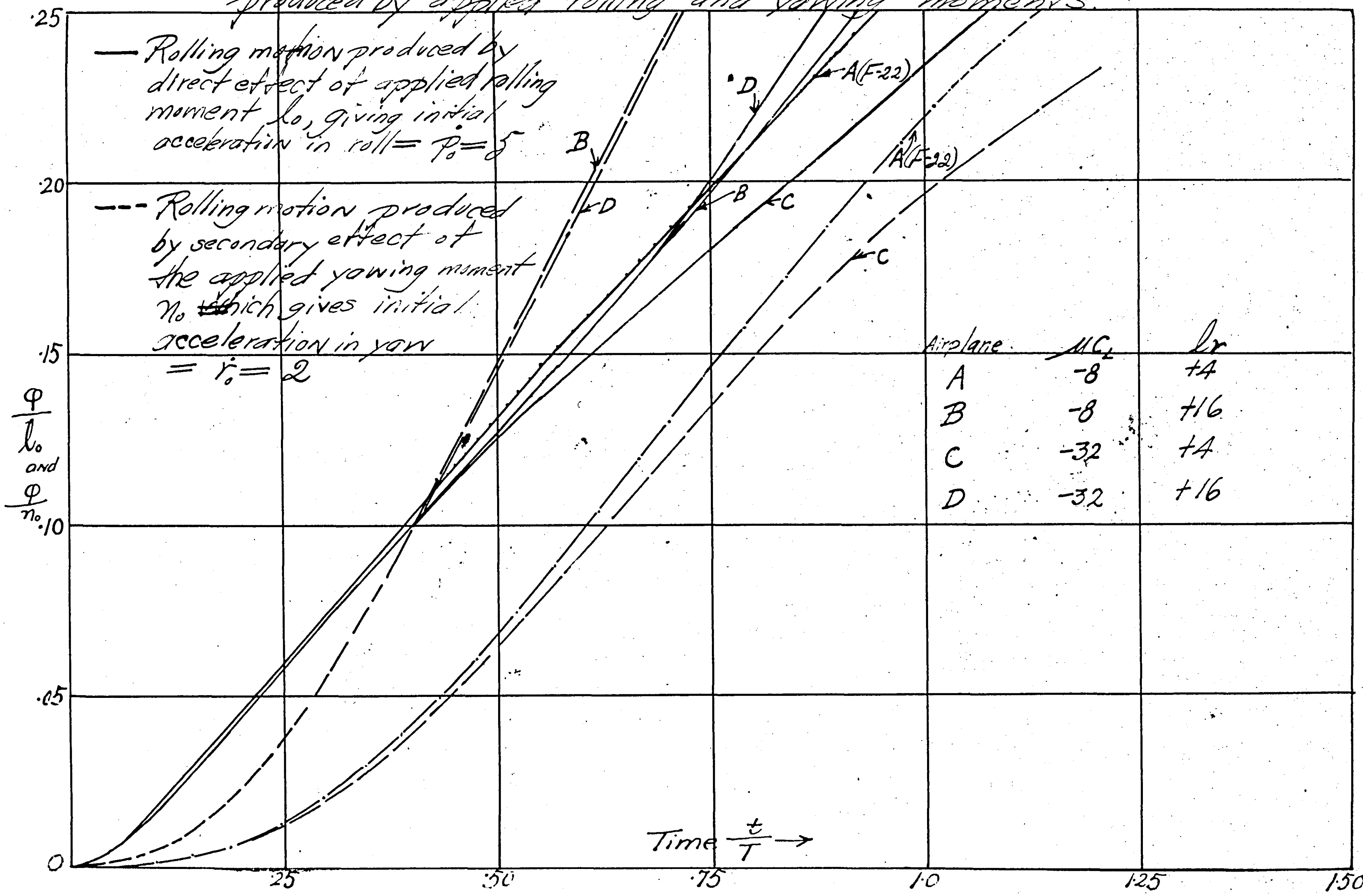
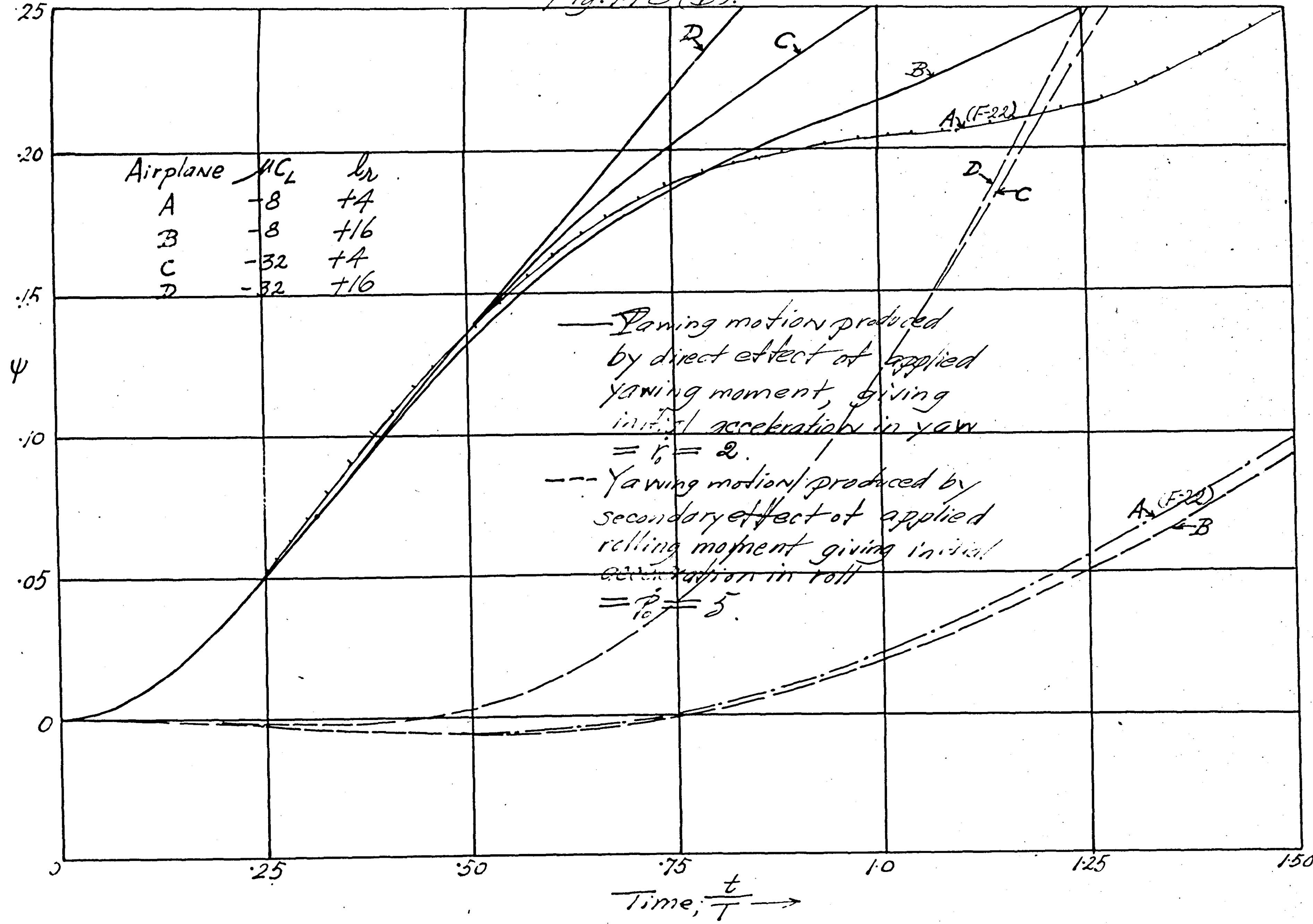


Fig. 143 (B).



Airplane μC_L l_r
A -8 +4
B -8 +16
C -32 +4
D -32 +16
Yawing motion produced by direct effect of applied yawing moment, giving initial acceleration in yaw = ṙ₀ = 2.
Yawing motion produced by secondary effect of applied rolling moment giving initial acceleration in roll = ṗ₀ = 5.
D
C
B
A (F-22)
ψ
Time, t/T
·25
·20
·15
·10
·05
0
·25
·50
·75
1·0
1·25
1·50

Fig. 14·A (A).
Effect of varying l_p, n_p and y_r on the motion in roll produced by applied rolling and yawing moments.

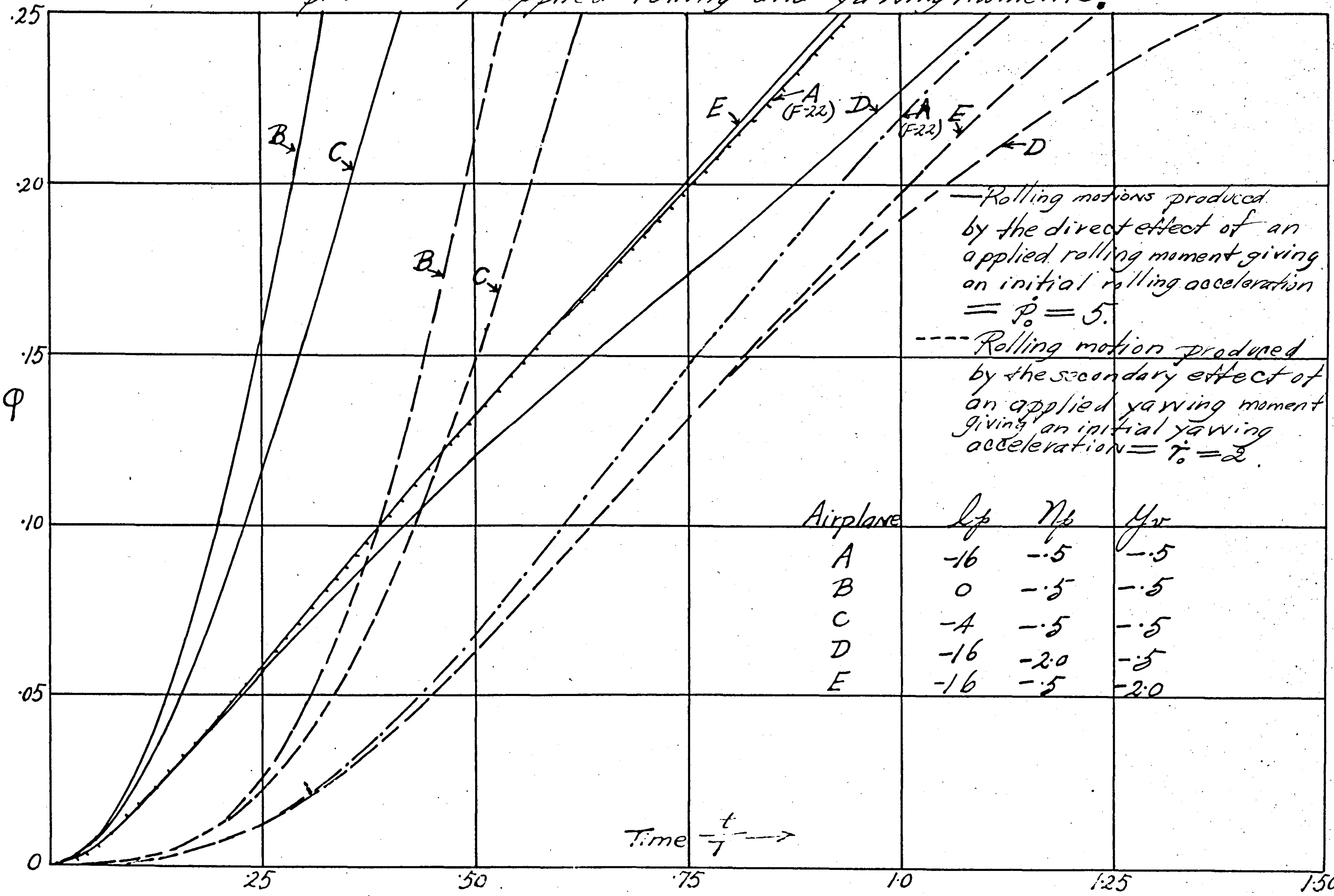
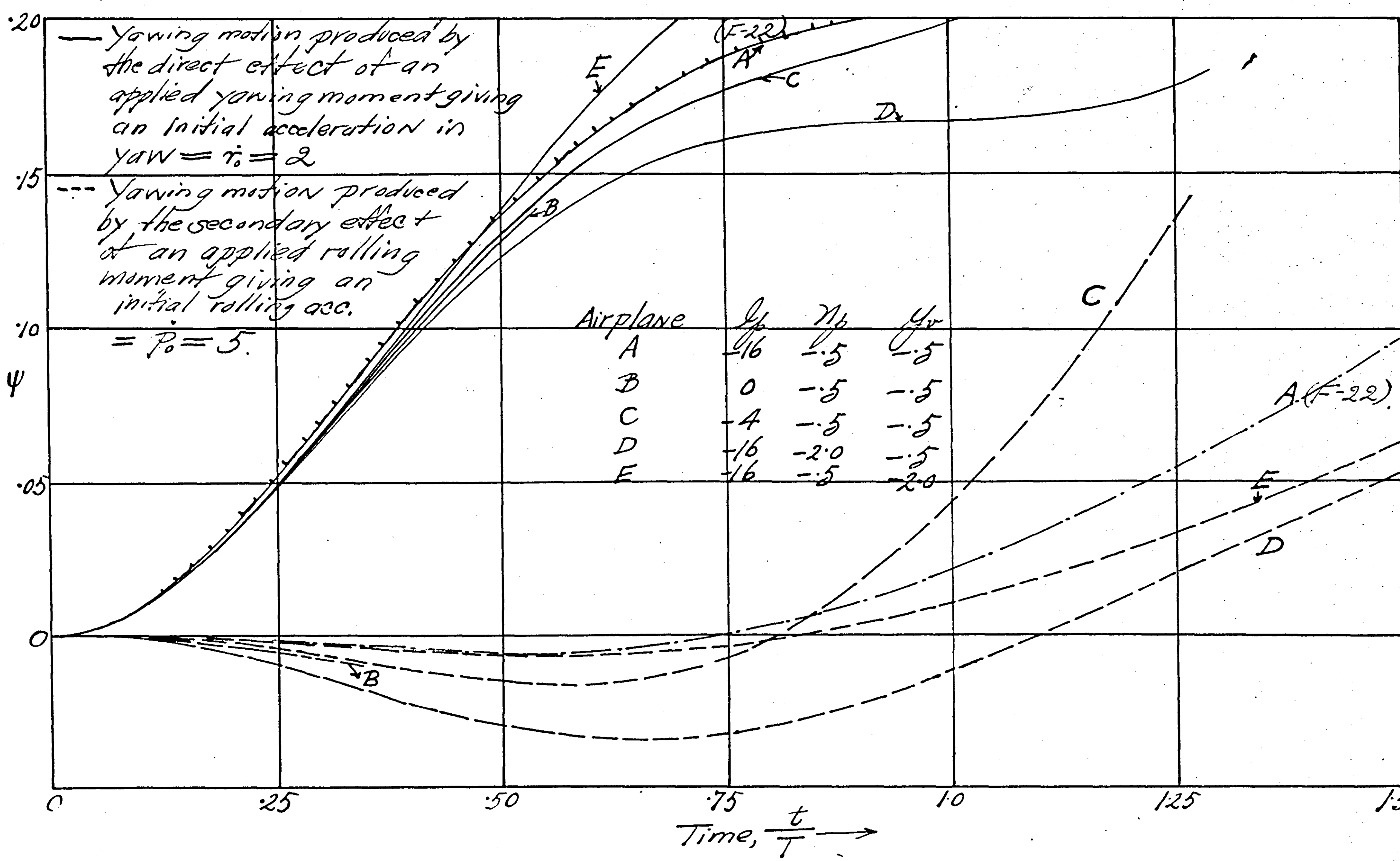


Fig. 14·A (B)

Effect of varying l_p, n_p & y_v on the motions produced by applied rolling and yawing moments.



Decrease of l_p makes the airplane extremely sensitive to both direct and secondary effects of the applied moments in rolling motion.This is because of the fact that l_p is the damping coefficient in roll. The sensitivity to yaw through the secondary effect of the applied rolling moment is also increased. The direct yawing sensitivity is,however,unchanged. Increase of n_p decreases both the motion in roll and in yaw due to either direct or secondary effects of the corresponding applied moments. The negative yawing motion due to the secondary effect of the applied rolling moment is comparatively much larger in the initial stage of t/T from zero to 1 as n_p increases from -.5 to -2.0.

The effect of y_v is in general,less important than the effect due to other derivatives. It is seen that increase of y_v decreases the motion produced by either the direct effect or the secondary effect of the applied moments.

e. Summary on the effects of varying each of the derivatives on the motion of the airplane produced by applied yawing and rolling moments.

Table 14.1

Increase of	Motion in Roll, ϕ produced by direct rolling mom.	Produced by yawing moment	Motion in Yaw, ψ Produced by direct yawing mom.	Produced by rolling moment.
l_v	decreased	Inc.	Dec. slightly	Almost unchanged
n_v	Increased slightly	Decreased very much	Decreased very much	Increased
n_r	Unchanged	Decreased very much	Decreased considerably	Decresed slightly
μC_L	Decreased slightly	Decreased slightly	Increased slightly	Increased considerably
l_r	Unchanged	Increased considerably	Unchanged	Decreased slightly
l_p	Decreased considerably	Decreased considerably	Unchanged	Decreased
n_p	Decreased	Decreased	Decreased	Decreased considerably
y_v	Unchanged	Decreased	Unchanged	Decreased
μ	Unchanged	Increased	Decreased considerably	Unchanged

15. Summary and conclusions on the controlled lateral motion.

a. From the study of various laws of lateral control, It is seen that most of the control devices merely modifye the derivatives of the airplane, without providing control derivatives that ~~is~~ are essential to give good damping distribution among various modes of motion as well as the course stability that the uncontrolled airplane lacks.

b. The advantage of human pilotting and that of the succesfull Sperry and Smith automatic pilot is to introduce the control derivatives l_ϕ and n_ψ. which the uncontrolled airplane lacks. Introduction of n_ψ provide the airplane sensitivity in azimuth while l_ϕ gives good damping distribution among each mode of motion.

c. The adverse yawing moment ~~is~~ of the aileron is present in the controlled stability equation as giving the control derivative n_ϕ. ~~Either the~~ It is found that the derivative due to adverse yawing moment tends to decrease the damping of the long period mode of motion while that due to favorable yaw tends to decrease the damping of the short period mode of motion. They are, therefore, both undesirable for ordinary airplane.

d. In order that the rudder control be effective in providing for the course stability, the dihedral derivative l_v must not be zero. In order that the aileron control be effective in equalizing the damping among various modes of motion, the static stability derivative, n_v must not be zero.

e. The derivatives l_p, n_r and y_v provide the damping for all modes of motion. n_r is chiefly responsible for the damping of short oscillation. Decrease of l_p to zero, the damping for both the long and short oscillation is decreased.

f. The largest l_v allowable is limited by the percentage of aileron adverse yaw present at high angle of attack. For a rough criteria,

$$C_n/C_l \text{ must be smaller than } (a_l/a_l)(y_v n_r + \mu n_v)/(y_v l_r + \mu l_v)$$

at high angle of attack.

A still simpler criteria is,

$$C_n/C_l \text{ must be smaller than } n_v/l_v$$

g. The principal effect of control lag is to decrease l_p by $\mu l_{\dot\phi} t_a$ and n_r by $\mu n_{\dot\psi} t_r$. The lagging effect in aileron and rudder is of the same order of importance.

h. The effect of varying airplane derivatives on the motion produced by applied rolling and yawing moments can be found in Table14.1.

i. The most interesting point about the motion produced by applied rolling and yawing moments is that an airplane of extremely large dihedral can be rolled by the secondary effect of rudder control even quicker than the direct aileron control, and also the fact that an airplane of large static stability, n_v is very sluggish to the direct effect of rudder control.

j. An airplane having large l_p will have less surge error in roll due to rolling gust despite the larger initial rolling acceleration due to the gust.

k. For an average airplane, $n_v = +.5$ to 1 is a practicable range for both good aileron and rudder control.

l. The adequate value for l_v should be such that its smallest value should give $(\mu C_L l_v)(\mu n_\psi)$ greater than 100 at low angle of attack. And its largest value should give l_v/n_v smaller than ~~xk~~ C_l/C_n at high angle of attack.

m. n_r for a given tail size should be greater than $\mu n_\psi t_r$. And l_p should be greater than $\mu l_\phi t_a$.

Chapter IV

STUDY OF TWO-CONTROL OPERATION

1. Introduction.

The possibility of flying airplanes with two-control operation instead of three was long observed to be feasible, if the airplane characters are adequately modified[1]. From the study of lateral controllability, in Chapter III, it is noted that by varying certain ~~af~~ derivatives of the airplane, such as l_v and n_v, the rolling motion due to the secondary effect of applied yawing moment, as well as the yawing motion due to the secondary effect of applied rolling moment can be increased so greatly that they may even exceed the motion produced by the direct effect of the corresponding applied moments. It is therefore, quite possible, to provide an adequately designed airplane, control system having either aileron and elevator alone or rudder and elevator without aileron.

In order to have adequate controlability as well as stability, a combination of all desirable airplane characteristics calculated through adequate mathematical analysis under all flight conditions must be investigated.

Jones[2] and Klemin[3] have attempted to analyze the controlled motion mathematically in order to study the possibility of two-control operation, Unfortunately, inadequate mathematical expressions together with the complicated maniputations involved prevented the previous study to throw much light on the understanding of the possibility of two-control operation.

It is the purpose of this chapter to investigate the desirable conditions of airplane design adopting two-control operation, and to compare the relative merit between the rudder-elevator and aileron-elevator systems. The M.I.T. Differential analyzer is here, again found to be of great value in simplifying the complicated mathematical problems.

2.Conditions from point of view of controlability.

For acceptable two-control operation system, the control must provide adequate lateral controllability,i,e to bank the airplane by rudder control,if the rudder-elevator system is used, as satisfactory as does the original aileron. On the other hand, if the aileron-elevator system is adopted,the aileron must be able to yaw the airplane as satisfactory as the original rudder.

As the lateral controllability under various conditions has been thoroughly studied in Chapter III, It is therefore only a matter of tabulating the desirable conditions under the two different two-control systems. Basing on the N.A.C.A. controllability criteria,the rolling and yawing motion produced in one second,corresponding to $t/T = 0.5$,for the assumed flight condition and the data of the average airplane,are taken as the basis of comparison. The sensitivity to rolling and yawing moment of the average airplane F-22,is taken as basis for judging the degree of satisfaction in controllability,since it is known to us that the airplane F-22 is condidered by pilots to be of good quality in control.

a.Requirements for rudder-elevator control

(1) Rolling motion produced in one second by the secondary effect of applied yawing moment must be as large as the direct rolling motion produced

the original aileron. Since the airplane is inherently more sensitive to yaw than to roll[4], it would be more reasonable to compare the rolling motion produced by 40% of yawing moment against that produced by a 100% rolling moment such as plotted in Figs.14.1,2,3 and 4.

(2) The yawing motion produced by the effect of the applied rudder moment must be as satisfactory as the original rudder control.

b. Requirements for aileron-elevator control.

The requirements for aileron-elevator control system are complicated by the presence of adverse yaw due to aileron.

(1) The resultant yawing motion ~~produced by~~ as a result of the algebraic sum of the positive yaw due to the secondary effect of the aileron rolling moment and the negative yawing motion produced by the aileron adverse yaw should be of satisfactory magnitude compared to the yawing motion due to the original rudder control.

(2) The rolling motion produced by the aileron control fitted on the new airplane characteristics must be as good as the original one.

From the above requirements, it is seen that for an airplane designed for rudder-elevator control must be such that,

(a), it is highly sensitive to roll due to secondary effect of the applied yawing moment.

(b), it is moderately sensitive to yaw due to applied yawing moment.

(c) the sensitivity to roll by applied rolling moment and the sensitivity to yaw by secondary effect of applied rolling moment are less important.

For an airplane designed to adopt the aileron-elevator control system, it must be such that,

(a) it is sensitive to roll due to rolling moment.

(b) it is highly sensitive to yaw due to secondary effect of the applied rolling moment.

(c) it is less sensitive to yaw and to roll due to applied yawing moment, in order to minimize the negative rolling and yawing motion due to the aileron adverse yaw.

The following table lists the desirable magnitudes of each derivative as compared to the original magnitudes of the corresponding derivatives of the average airplane, F-22, in their relative merit for good controllability in two-control operation.

Table 2.1

Desirable magnitudes of the airplane derivatives as compared to the derivatives of the airplane F-22, for good controllability in a two-control system using rudder and elevator.

Derivatives	For large sensitivity to roll due to second-effect of yawing mom.	For good sensitivity to yaw due to direct effect of yawing mom.
l_v	larger	smaller°
n_v	smaller	smaller
n_r	smaller	smaller
μC_L	smaller°	larger°
l_r	larger	unimportant
l_p	smaller	unimportant
n_p	smaller	smaller
y_v	smaller°	unimportant
μ or y_r	larger	smaller

Remark, smaller° means it is unimportant, though smaller is desirable.

Table 2.2

Desirable magnitudes of the airplane derivatives as compared to the derivatives of the average airplane F-22, for good controllability in a two control system using aileron-elevator.

Derivatives	For good sensitivity to yaw due to rolling moment.	For good sensitivity to roll due to rolling moment.	For small sensitivity to roll and yaw, produced adverse yaw.mom.
l_v	unimportant	smaller	smaller
n_v	larger	larger°	~~smaller~~ ~~and~~ larger.
n_r	smaller°	unimportant	larger
μC_L	larger°	smaller°	unimportant
l_r	smaller°	unimportant	smaller
l_p	smaller	smaller	larger
n_p	smaller	smaller	larger
y_v	smaller	unimportant	larger
μ	unimportant	unimportant	smaller

From Table2.1, it is evident that for an airplane to be controlled by rudder-elevator system, the dihedral must be large, the tail size small so that n_v and n_r are small, for good controllability. It is noted that the controllability will be even better at high angle of attack as increase of l_r and decrease of l_p will all increase the controllability. However, in order to meet stability requirement, l_p and n_r may not be small so that adequate damping could be provided.

From Table 2.2, it is seen that for a two-control system using aileron-elevator, the airplane must have very small dihedral, and very large tail so that n_v and n_r can both be large. Large l_p is desirable in providing adequate damping for all modes of motion. The controllability will be less at high angle of attack due to rapid increase of l_r.

Examinations of Figs.14.1,2,3 and 4 show that ~~although~~ the rolling and yawing motions due to secondary effects of the applied moments, all lag behind the corresponding one, which is produced by the direct effects of the control. The rolling motion due to the secondary effect of applied yawing moment can be made to respond the rudder control by increasing the dihedral so quickly that in a time interval of one second, ~~the~~ it can exceed the rolling motion produced by the direct effect of the original aileron control. Even for an airplane having a large tail as large as to give $n_v = +4$, and $n_r = -8$, the rolling motion produced by the secondary effect of the applied yawing moment can still be made as large as that produced by the direct aileron control if l_v is increased to -16. As a matter of fact, at high angle of attack, when l_r is also large, (see Fig.2.1) the pilot may have to move the rudder so gently that the airplane of large dihedral may not be over-banked. However, the yawing motion due to the secondary effect of the aileron control is seen to lag behind the motion due to direct yawing moment so severely that even for a very large n_v airplane, it seems still quite far from being as satisfactory as the direct effect due to rudder(see Fig.14.2(B) curve B)

It is therefore,believed that a two-control system using rudder and elevator will be more desirable than the aileron-elevator system from the point of view of good controllability. The unavoidable aileron adverse yaw as well as the necessity to have rudder for control beyond stall and to get the airplane out of spin further justify the conclusion.

Fig. 2·1 (A).

Effect of varying l_v on the rolling motion produced by the applied rolling and yawing moments.

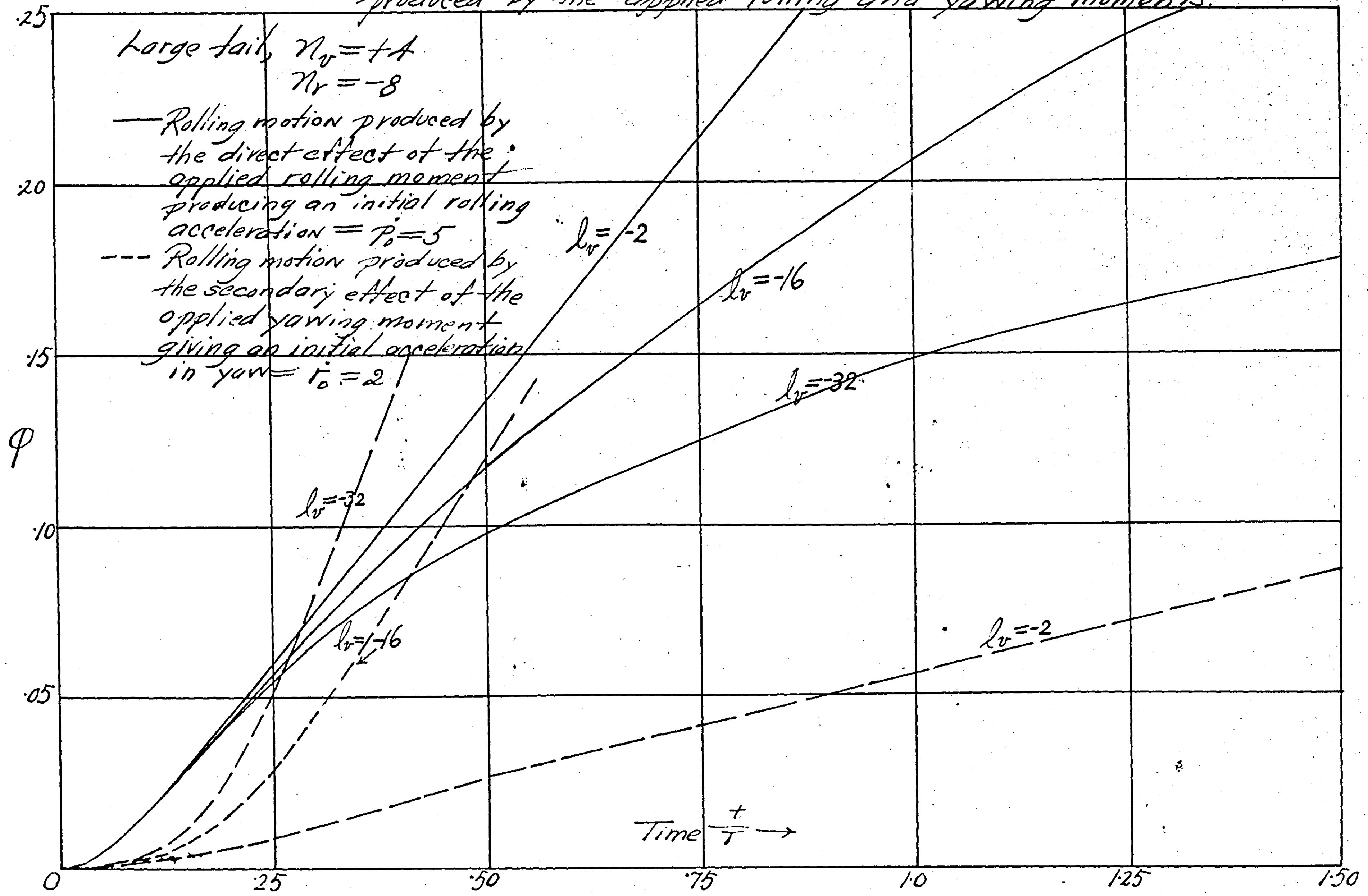
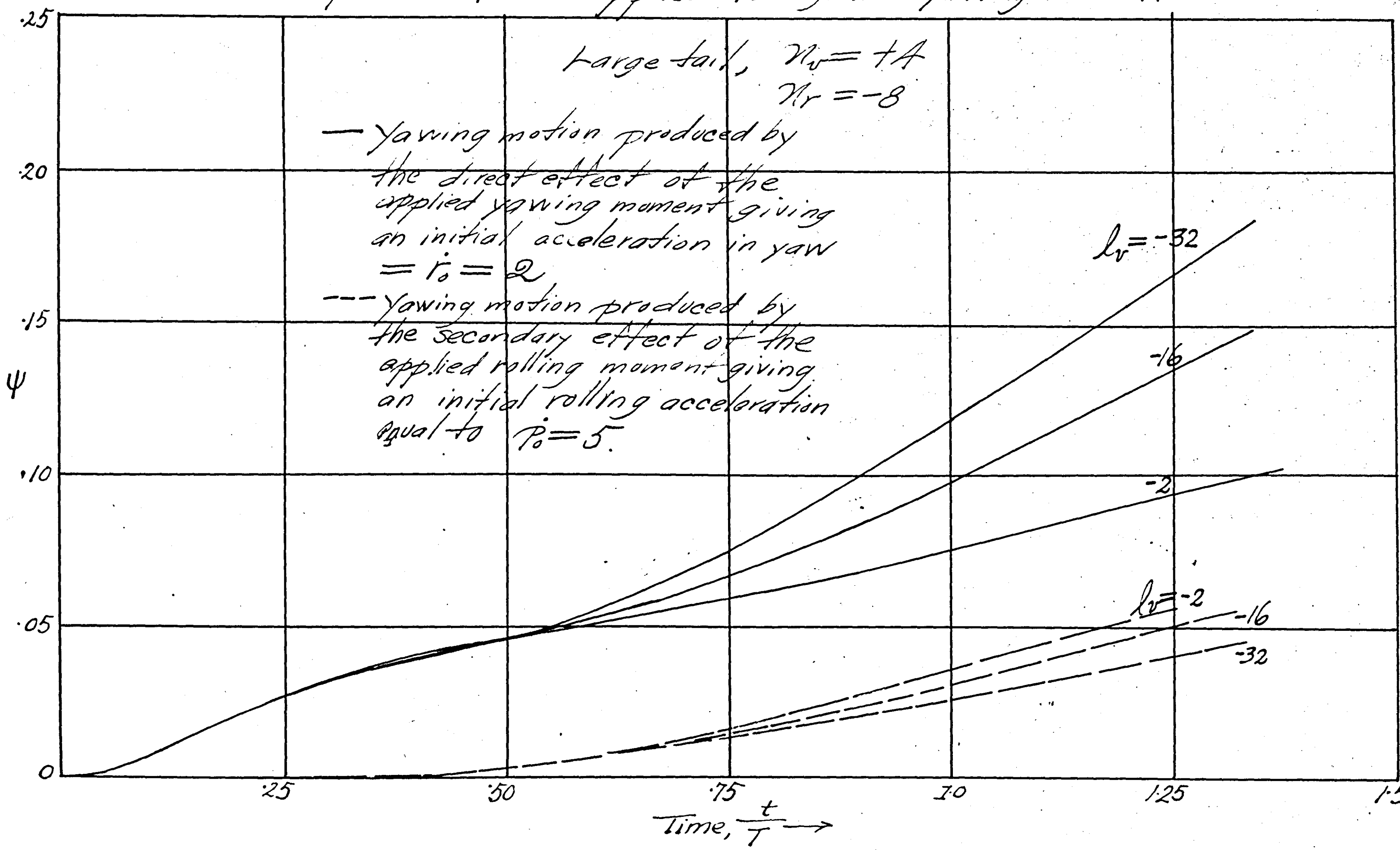


Fig. 2·1 (B).

Effect of varying l_v on the motion in yaw produced by the applied rolling and yawing moment.



3. The stability of an airplane controlled by rudder alone, for two-control operation.

Having the controllability of an airplane using two control operation system investigated, the question is to study the ability of the rudder in stabilizing the disturbances caused by atmospheric gusts. It was noted in the numerical invesgigations of the controlled stability of the average airplane in article 10, Chapter III, that the rudder control derivative μn_{ψ}, while providing for the airplane a restoring moment in azimuth, gives little damping to the long period oscillation. (see Fig. 10.1) It was pointed out that for an ordinary airplane, it is necessary to have the aileron control to give adequate damping for the long period oscillation. Examination of equation (6.4) showa that the rudder control contributes very little to the positiveness of the coefficient, E which is equal to $E_o - \mu n_{\psi}(y_v l_p)$. From numerical manipulation of the roots of the stability equation, it was found that in order to have the long oscillation adequately damped, the coefficient E must be large and positive. The only alternative is therefore to have E_o large and positive. From (6.2), this is seen at once to require an airplane having large $l_v n_r$ but small $n_v l_r$. As l_r cannot be altered by airplane designer, while increase of tail size would increase n_r and n_v simultaneously, the most effective way is seen to increase l_v.

Numerical investigation.

Assuming $\mu n_\psi = -8$, $l_v = -16$, $y_v = -0.5$

using the derivatives for the average airplane, at low angle.

Tail size	n_v	n_r
(1) small	0	-1
(2) normal	1	-2
(3) large	2	-4
(4) very large	4	-8

Tail size	A_0	B_0	C_0	D_0	E_0	F_0
(1)	1	17.5	26.5	265	128	0
(2)	1	20.5	~~102~~ 59	529	224	0
(3)	1	20.5	108	801	448	0

Tail size	Uncontrolled stability equation		
	Long oscillation	Subsidence	Short oscillation
(1)	$(d+0)$ ~~(d+16.835)~~ $(d+0.5)$	$(d+16.835)$	$(d^2+.165d+15.22)=0$
(2)	$(d+0)(d+.442)$	$(d+16.80)$	$(d^2+1.26d+30.44)=0$
(3)	$(d+0)(d+.605)$	$(d+16.81)$	$(d^2+3.09d+44.14)=0$

(Table 3.1)

Thus from table 3.1, it is seem that increase of l_v makes the short period oscillation to be badly damped, by decreasing the damping and period simultaneously.

This requires a large n_r to improve it. Increase of n_r by increasing tail size would increase n_v also, which tends to decrease the damping of the short oscillation too. An ideal case would be to increase n_r either by control device or by increase the tail length[5].

Table 3.2

With rudder control derivative $\mu n_\psi = -8$, substituting into (6.4), we get,

Tail size	A	B	C	D	E	F
(1)	1	17.5	34.5	397	192	1024
(2)	1	18.5	67	661	256	1024
(3)	1	20.5	116	933	512	1024

Tail size	Stability equation for rudder control		
	Long oscillation	Subsidence	Short oscillation
(1)	$(d^2+.34d+3.03)$	$(d+16.825)$	$(d^2+.336d+20.0)=0$
(2)	$(d^2+.252d+1.68)$	$(d+16.82)$	$(d^2+1.43d+36.5)=0$
(3)	$(d^2+.45d+1.2)$	$(d+16.8)$	$(d^2+3.35d+51.1)=0$

Thus it is seen that for a large tail size, the long oscillation will be damped to half in 0.52 oscillations and the short period oscillation will be damped to half in .445 oscillations so that the stability will be considered as satisfactory.

A numerical investigation showing the stability at high angle of attack by increasing μC_L to -32, l_r=16, n_p=-1, y_v=-1, l_p=-16, with large dihedral, l_v=-16, and large tail giving n_v=2, n_r=-4, we get for uncontrolled airplane, $d^5+21\,d^4+132d^3+\cancel{720d^2} +1360d^2+1024d + 0=0$ which is factored into,

$$(d+0)(d+.81)(d+17.7)(d^2+2.49d+71.5)=0$$

With rudder control derivative μn_ψ= -8, we get,

$$d^5+21d^4+140d^3+1496d^2+1142d+4096=0,$$

which is factored into,

$$(d^2+.546d+3.0)(d+17.7)(d^2+2.75d+77)=0$$

showing that the motion is stable at high angle of attack, except that the damping of the short oscillation is seen to be a little bit too small.

Fig.3.1(A) and (B) show the disturbed motion due to a rolling gust p_o at low angle of attack. It is seen that the motion in yaw is so small that it is almost constrained to that mode of motion. The motion in roll is seen to be comparatively large. Increase of rudder control alone cannot improve the damping of long period oscillation. Small n_v but large n_r is seen to be desirable. A large tail is more desirable than small one. Curve E shows a good compromise, though further increase of n_r and decrease of n_v would be favored. In general, it can be concluded that for rudder control alone, the condition

Fig. 3.1 (A)

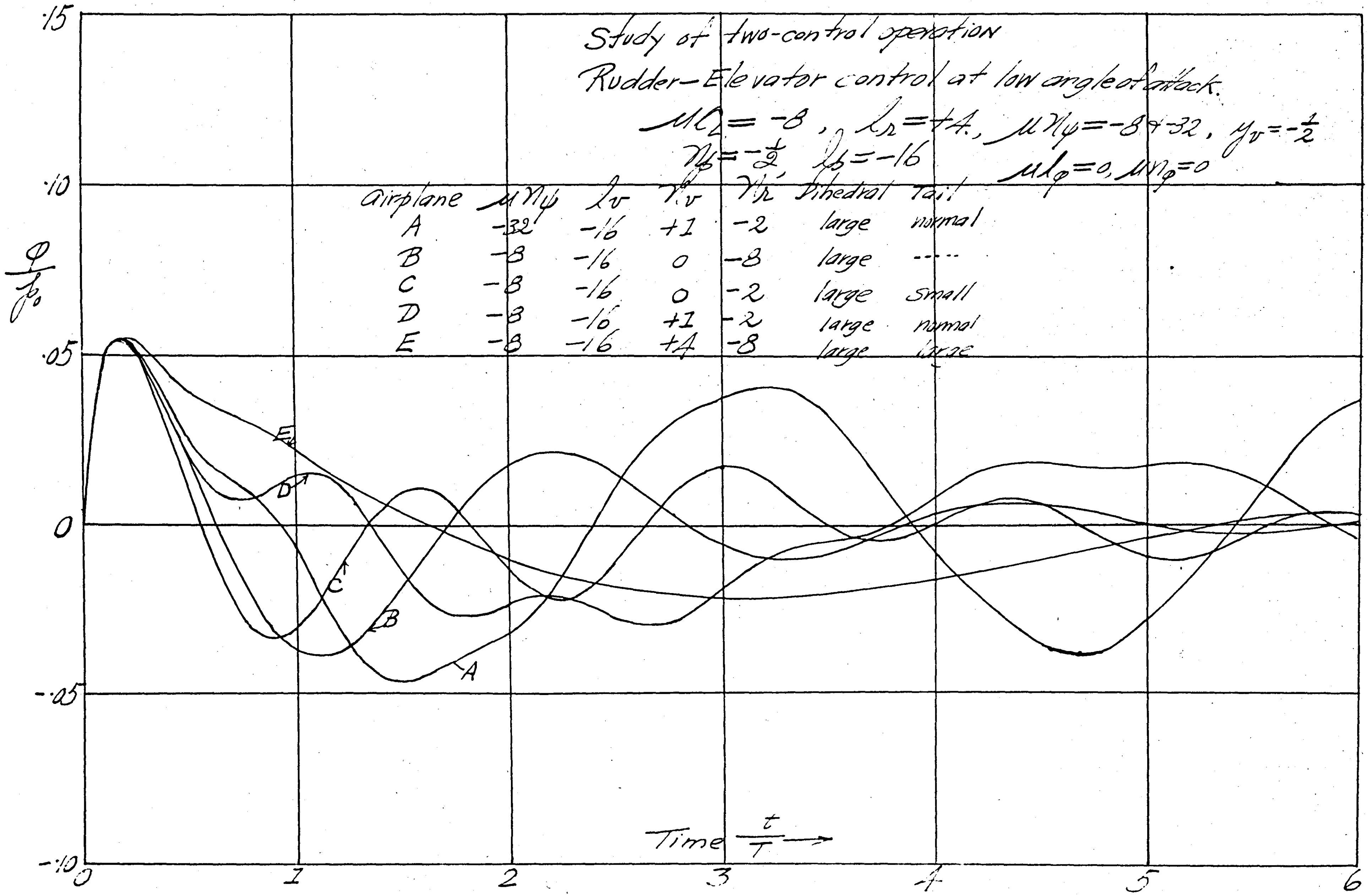
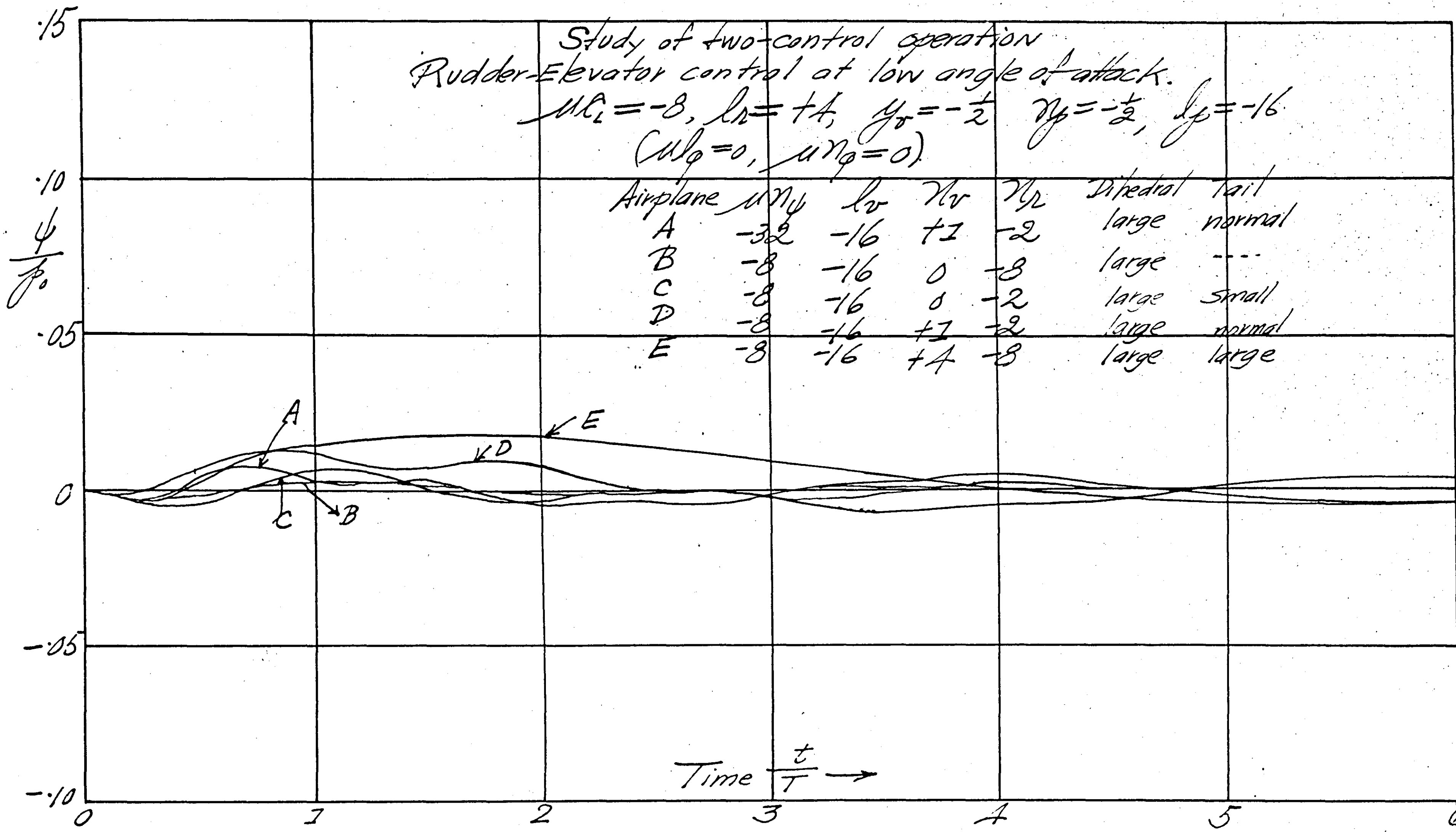

Study of two-control operation
Rudder-Elevator control at low angle of attack.
$\mu C_L = -8$, $l_r = +4$, $\mu n_\psi = -8 \ \& -32$, $y_v = -\frac{1}{2}$
$n_\delta = -\frac{1}{2}$, $l_\delta = -16$
$\mu l_\varphi = 0$, $\mu n_\varphi = 0$
airplane μn_ψ l_v n_v n_r Dihedral Tail
A -32 -16 +1 -2 large normal
B -8 -16 0 -8 large -----
C -8 -16 0 -2 large small
D -8 -16 +1 -2 large normal
E -8 -16 +4 -8 large large
$\frac{\varphi}{p_0}$
·15
·10
·05
0
-·05
-·10
0
1
2
3
4
5
6
Time $\frac{t}{T}$
A
B
C
D
E

Fig. (3.1) (B)



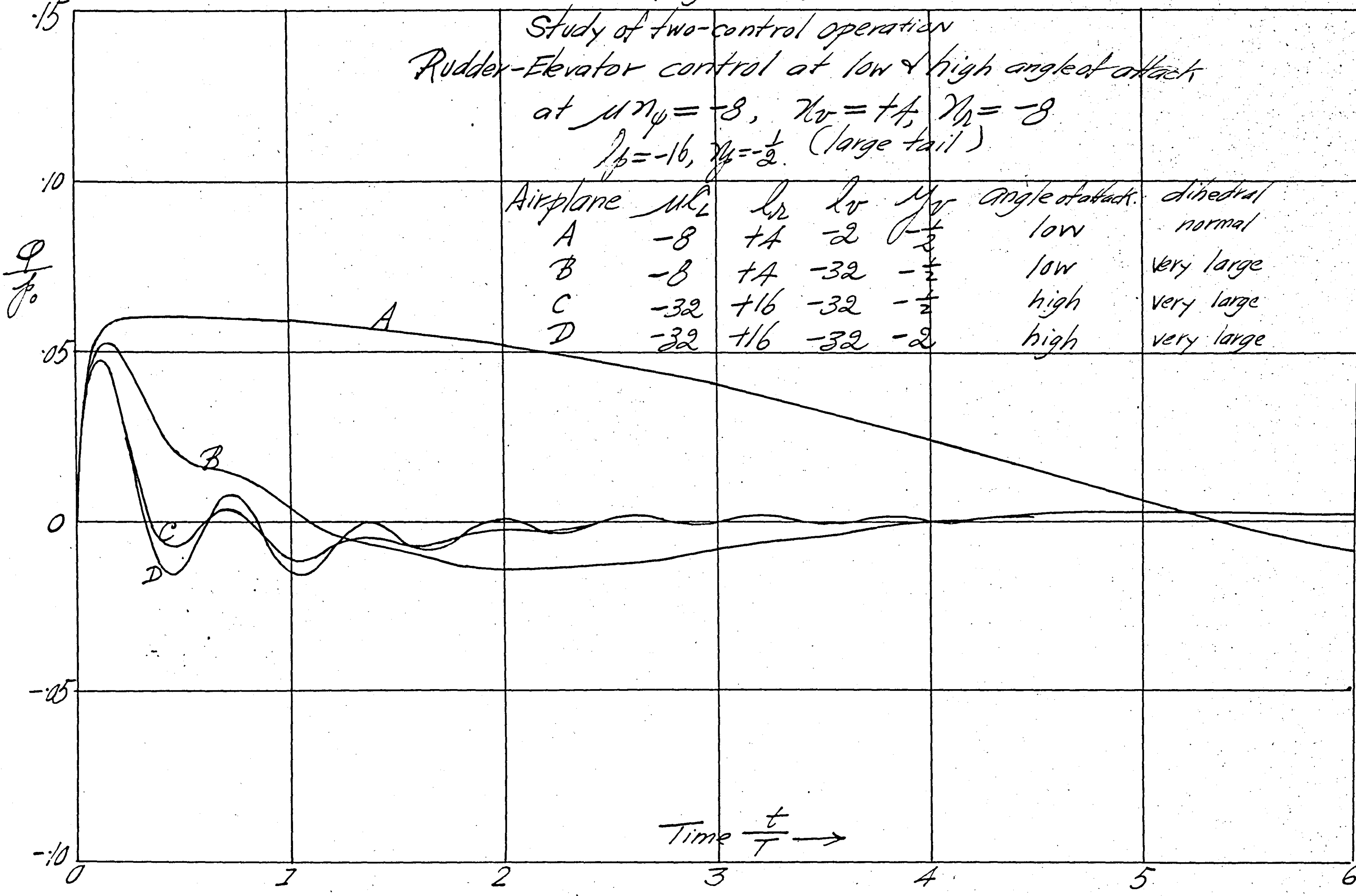
Study of two-control operation
Rudder-Elevator control at low angle of attack.
$\mu \mathcal{L}_\ell = -8,\ l_r = +4,\ y_v = -\frac{1}{2}\ \ n_p = -\frac{1}{2},\ l_p = -16$
$(\mu l_\varphi = 0,\ \mu n_\varphi = 0)$
Airplane μn_ψ l_v n_v n_r Dihedral Tail
A -32 -16 +1 -2 large normal
B -8 -16 0 -8 large ---
C -8 -16 0 -2 large small
D -8 -16 +1 -2 large normal
E -8 -16 +4 -8 large large
A
E
D
C
B
$\frac{\psi}{\beta_0}$
·15
·10
·05
0
-·05
-·10
Time $\frac{t}{T}$ →
0
1
2
3
4
5
6

that $l_v n_r$ be greater than $n_v l_r$ must be satisfied with large margin of positiveness.

Fig.(3.2) shows the effect of l_v at low angle of attack and the effect of varying y_v at high angle of attack. Curve A of Fig.3.2 has $l_v = -2$, showing that the long oscillation is badly damped. Increasing l_v to -16, (curve E of Fig.3.1A) improves the damping comsiderably. Further increase of l_v to -32, with large tail and low angle of attack as before, the disturbed motion is seen to so good that it is even better than that with aileron control, This is shown as the curve B of Fig.3.2(A).

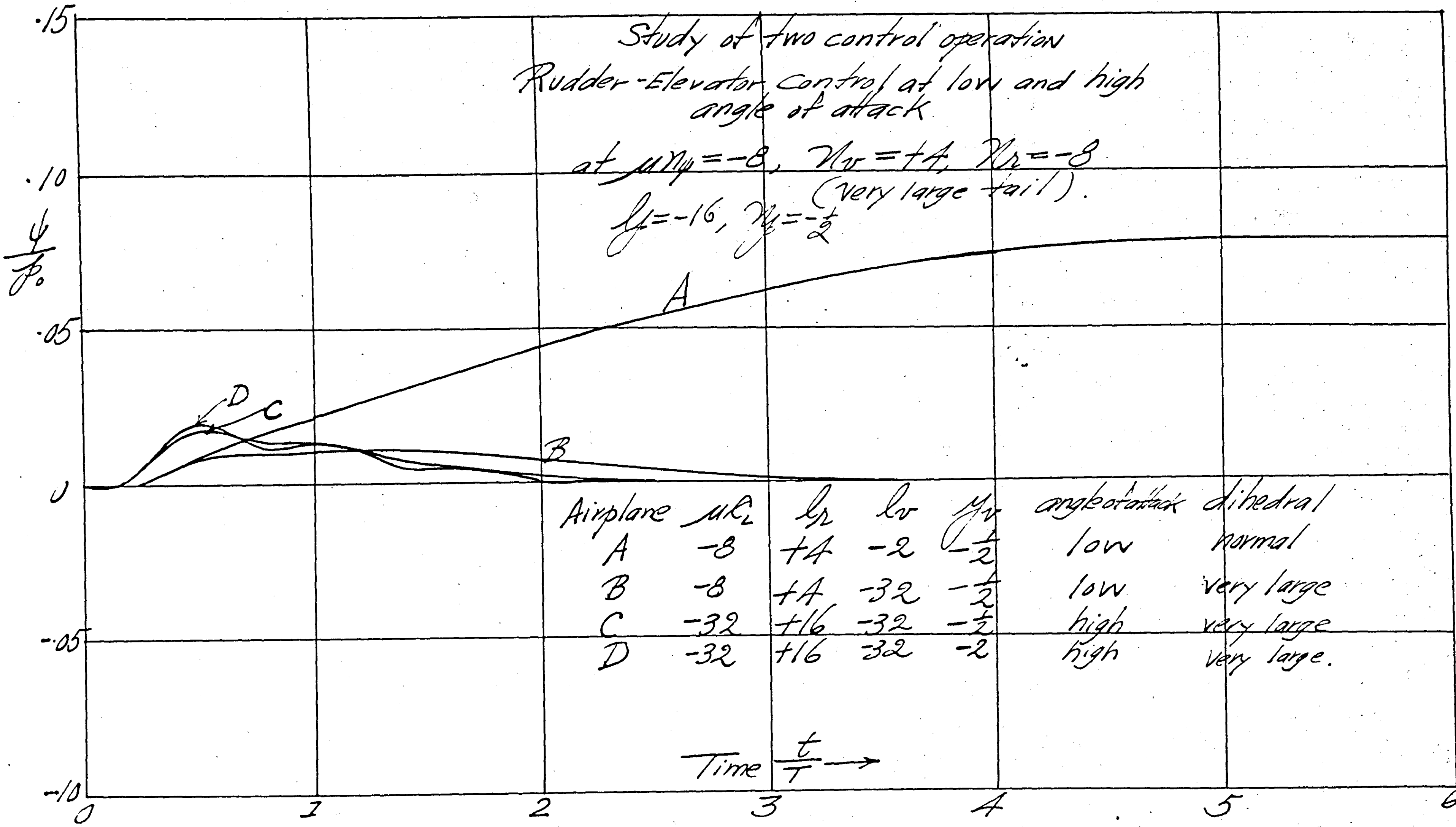
At high angle af attack, the damping of short oscillation is a little bit less. This is shown by curves C and D. of Fig.3.2. The long oscillation is even better damped. In any case, for an airplane having l_v approximately ten times larger than that of the average airplane, and a tail size four times as large as that of F-22, the two control operation with rudder and elevator will be satisfactory from the point of view of both controlled stability and lateral controllability.

Fig. 3:2 (A)



Study of two-control operation
Rudder-Elevator control at low & high angle of attack
at μn_ψ = -8, n_v = +4, n_r = -8
l_p = -16, y_ψ = -1/2 (large tail)
Airplane μC_L l_r l_v y_v angle of attack dihedral
A -8 +4 -2 -1/2 low normal
B -8 +4 -32 -1/2 low very large
C -32 +16 -32 -1/2 high very large
D -32 +16 -32 -2 high very large
φ/p₀
.15
.10
.05
0
-.05
-.10
A
B
C
D
Time t/T →
0
1
2
3
4
5
6

Fig. 3.2 (B)


Study of two control operation
Rudder-Elevator Control at low and high angle of attack
at $\mu n_\psi = -8$, $n_v = +4$, $n_r = -8$ (very large tail).
$l_\psi = -16$, $y_\psi = -\frac{1}{2}$
$\frac{\psi}{p_0}$
.15
.10
.05
0
-.05
-.10
A
B
C
D
Airplane μk_z l_r l_v y_v angle of attack dihedral
A -8 +4 -2 $-\frac{1}{2}$ low normal
B -8 +4 -32 $-\frac{1}{2}$ low very large
C -32 +16 -32 $-\frac{1}{2}$ high very large
D -32 +16 -32 -2 high very large.
Time $\frac{t}{T}$
0
1
2
3
4
5
6

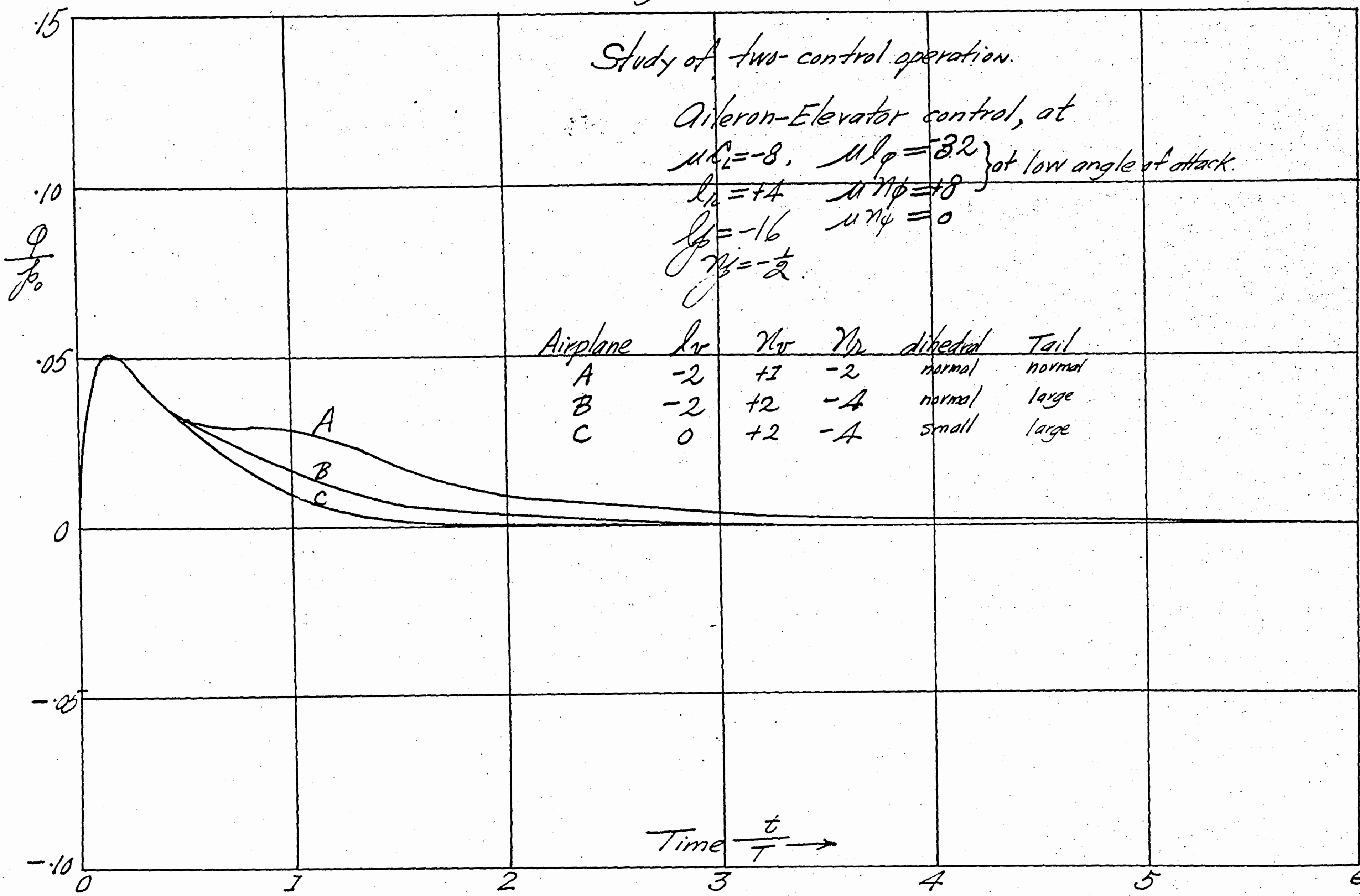
4. The stability of an airplane controlled by aileron-elevator for two-control operation.

From Equation (6.4), it is seen that an airplane without rudder control, the coefficient F will be zero unless the aileron is operated according to angle of yaw in addition to angle of roll.

Fig.4.1(A) and (B) show the disturbed motion for an airplane without rudder. The motion in roll is always satisfactory even with considerable amount of adverse yaw, at low angle of attack. The effect of having the coefficient F=0, is seen to result a constant error in azimuth even ~~after~~ a long time after the disturbance. Without moving the aileron according to angle in azimuth, the only possible way to overcome this difficulty is to increase the tail size and to reduce the aileron adverse yaw so that this constant error in azimuth may have a magnitude so small that its presence will not cause any practical difficulty. This is only good at low angle of attack, as seen from curves B and C of Fig.4.1.

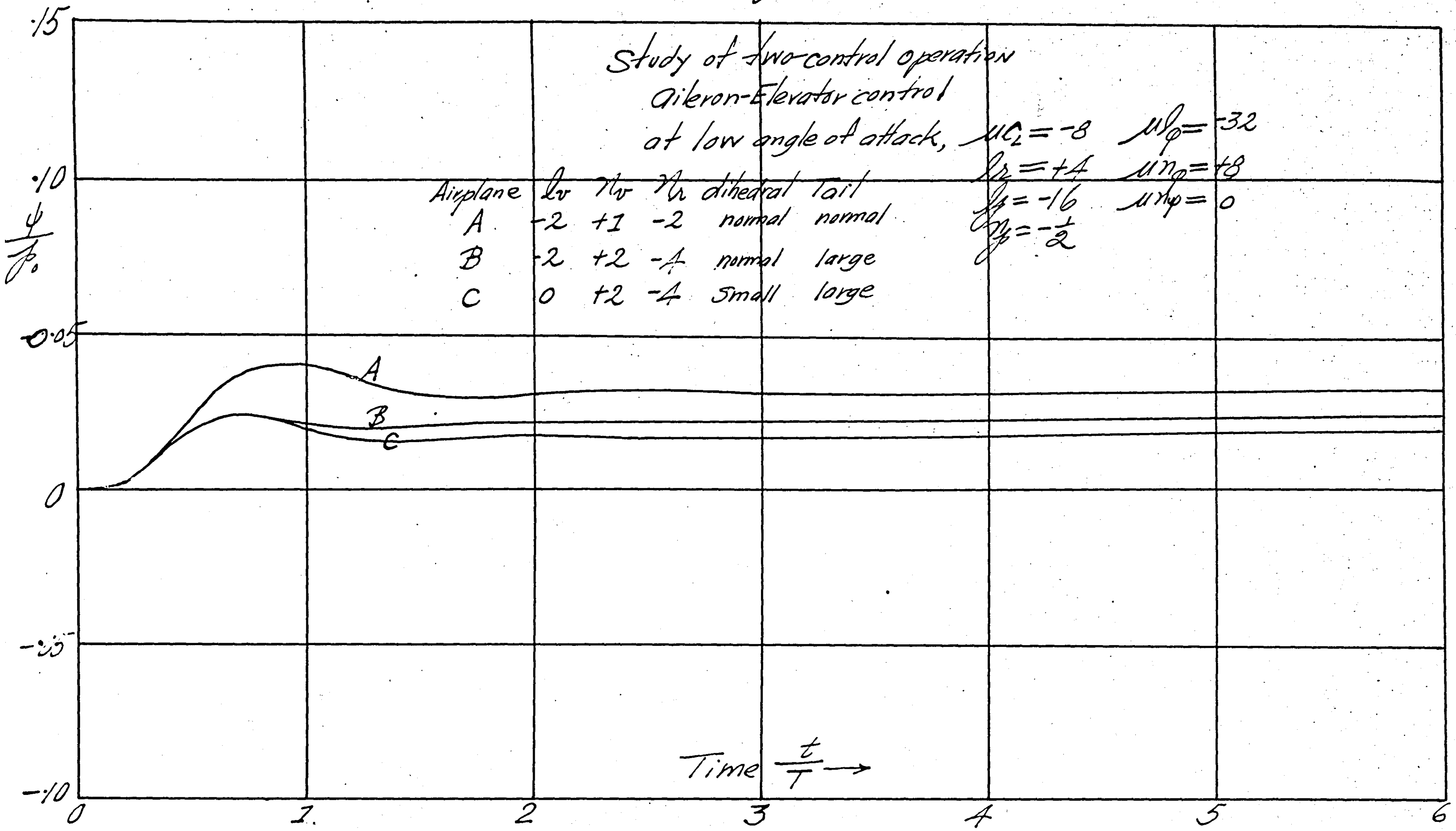
At high angle of attack, it is seen that error in azimuth is so large that even with a large tail, zero adverse yawing moment due to aileron, zero dihedral, could not improve the instability in azimuth. This is shown by curves A and B of Fig.4.2(A) and (B). Even with l_r

Fig. A·1 (A).



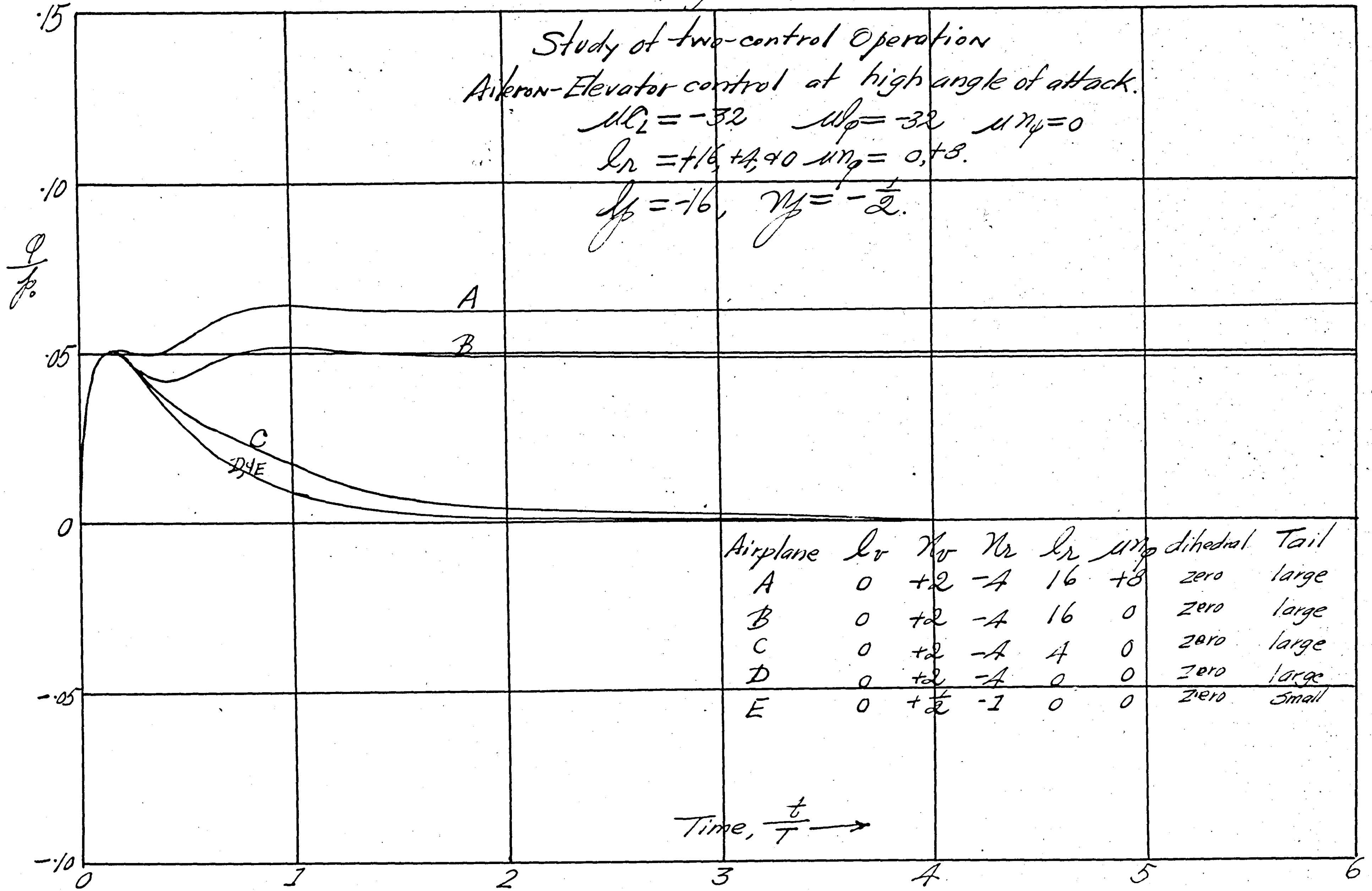
Study of two-control operation.
Aileron-Elevator control, at
$\mu C_L = -8$, $\mu l_\varphi = -32$, $\mu n_\varphi = +8$ } at low angle of attack.
$l_r = +4$
$\mu n_\psi = 0$
$l_\beta = -16$
$n_\beta = -\frac{1}{2}$
Airplane
l_v
n_v
n_r
dihedral
Tail
A -2 +1 -2 normal normal
B -2 +2 -4 normal large
C 0 +2 -4 small large
$\frac{\varphi}{p_0}$
Time $\frac{t}{T}$
·15
·10
·05
0
-·05
-·10
0
1
2
3
4
5
6
A
B
C

Fig. A·1 (B).



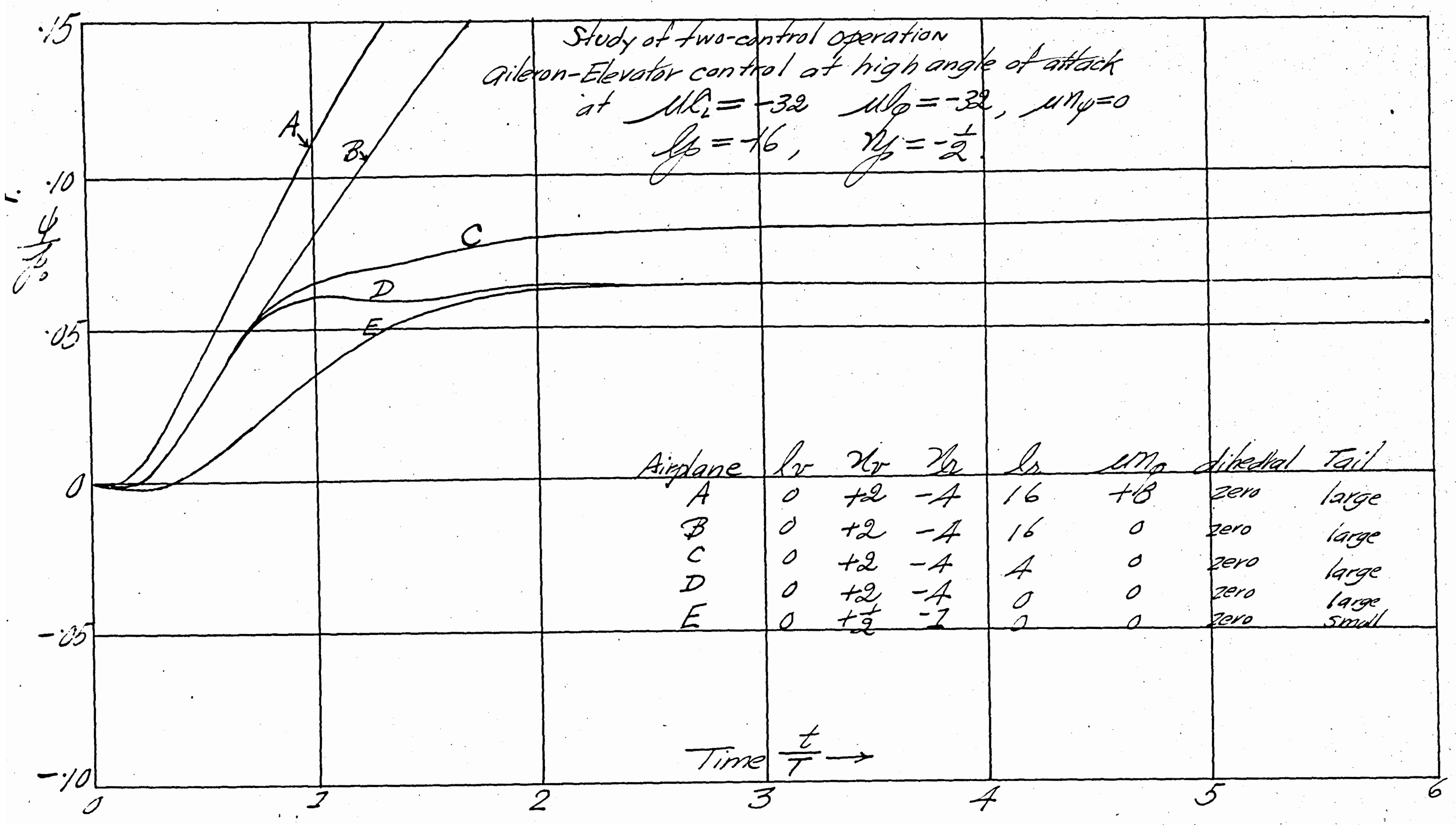
Study of two-control operation
aileron-Elevator control
at low angle of attack, $\mu C_L = -8$ $\mu l_\varphi = -32$
$l_r = +4$ $\mu n_\varphi = +8$
$l_p = -16$ $\mu n_p = 0$
$n_p = -\frac{1}{2}$
Airplane l_v n_v n_r dihedral Tail
A -2 +1 -2 normal normal
B -2 +2 -4 normal large
C 0 +2 -4 small large
$\frac{\psi}{p_0}$
·15
·10
0·05
0
-·05
-·10
Time $\frac{t}{T}$ →
0
1
2
3
4
5
6
A
B
C

Fig. A.2 (A).



Airplane	l_v	n_v	n_r	l_r	μn_p	dihedral	Tail
A	0	+2	-4	16	+8	zero	large
B	0	+2	-4	16	0	zero	large
C	0	+2	-4	4	0	zero	large
D	0	+2	-4	0	0	zero	large
E	0	$+\frac{1}{2}$	-1	0	0	zero	small

Fig. 4·2 (B).



Study of two-control operation
Aileron-Elevator control at high angle of ~~attack~~
at $\mu l_i = -32$ $\mu l_\phi = -32$, $\mu n_\phi = 0$
$l_p = -16$, $n_p = -\frac{1}{2}$.
A
B
C
D
E
Airplane l_v n_v n_r l_r μm_q dihedral Tail
A 0 +2 -4 16 +8 zero large
B 0 +2 -4 16 0 zero large
C 0 +2 -4 4 0 zero large
D 0 +2 -4 0 0 zero large
E 0 $+\frac{1}{2}$ -1 0 0 zero small
ψ / ψ_0
·15
·10
·05
0
-·05
-·10
0
1
2
3
4
5
6
Time $\frac{t}{T}$ →

reduced to zero, the error in azimuth is still too large. Variation of the tail size seems to help the situation very little. This is shown in Fig.4.2 as the curves C, D, and E. Therefore, two-control operation using aileron and elevator will not be satisfactory even without adverse yaw if the the aileron is not operated according to angle of yaw. If the aileron is operated according to the sum of angle of roll and angle of yaw, then, the derivative, l_ψ will also be present in addition to l_ϕ.

Examination of (6.6) of Chapter III., it is seen that if we neglect the terms involing y_v and n_ϕ, the effect of l_ψ is to add a term $\mu C_L(\mu l_\psi n_v)$ instead of the term $-\mu C_L(\mu n_\psi l_v)$ due to rudder control in the coeffécient F'. As μl_ψ is of the order of μl_ϕ which is four times larger than μn_ψ, F' will have a same value if n_v is four times smaller than l_v. Thus, for two control operation, if the aileron is operated so as to give a restoring moment for both angle in roll and in yaw, the controlled stability will be as satisfactory as that of the three control operation. The dihedr al in this case can be very small or zero because the sensitivity in azimuth due to l_ψ depends on n_v instead of l_v as does the rudder control. Accordingly, the disastrous effect due to adverse yaw would be reduced.

5.Summary and conclusions on the study of two-control operation.

a.Two-control operation by using either rudder-elevator or aileron-elevator system would be feasible only when the characteristics of the airplane are properly modified.

b.For good controllability,a rudder-elevator controlled airplane should have large dihedral,small tail size. For good controlled stability, the airplane should have large dihedral as well as tail size. An airplane having dihedral from 10 to 20 times as large as that of the airplane F-22 is required for good stability. The corresponding tail size should be from two to four times larger than that of F-22.

c.An aileron-elevator controlled airplane should have a small dihedral,even zero or negative,and large tail for good controllability and stability, The aileron must be operated according to the sum of the angle of roll and angle of yaw.

d.The motion in roll produced by the secondary effect of the yawing moment can be made as quick as that due to the direct effect due to aileron. But the motion in yaw due to applied rolling moment lags so seriously that two-control operation using rudder is believed more desirable than using aileron.

e, The controllability when the wing is stalled is better for two-control system using rudder instead of aileron. This is another point to favor the rudder-elevator system.

APPENDIX I

METHODS OF SUCCESSIVE APPROXIMATION TO SOLVE THE QUARTIC,QUINTIC?AND SEXTIC STABILITY EQUATIONS.

The numerical solution of the stability equations for the quartic,quintic and the sextic equations have been a difficult problem in the research of airplane stability. Even the semigraphical method of solving the quartic equation usually take 20 to 30 minutes for one set of solution only(see T.R.589). The writer is extremely fortunate in being able to discover the following methods of successive approximations to solve nearly every possible cases which may arise in the airplane stability research. The neccessary number of approximations required to reach the engineering accuracy being so small,usually one or two,that the numerical solution for a quartic equation can be done in one or two minutes.

1.Solution of quartic equations occured in longitudinal stability of airplanes.

It is well known to us that there are always two modes of motion in the longitudinal stability equation for all kinds of airplane,namely the long and the short period oscillations. Writing the quartic equation as $Ad^4+Bd^3+Cd^2+Dd + E = 0$,and it can always be factored into $(d^2+a_1d+b_1)(d^2+a_2d+b_2)= 0$

Denoting the mode of motion which has a long period by the factor $(d^2+a_1 d + b_1) = 0$ and that mode of motion which has a short period by $(d^2+a_2 d+b_2)=0$. Then, it is true for all kinds of airplane that a_1 is much smaller than a_2 and b_1 much smaller than b_2. It is due to this property that the following method of successive approximation applies succesfully. To show the method, it is best by doing two examples, one for an uncontrolled airplane, and one for a controlled one.

a. Uncontrolled airplane, F-22, using data obtained in article 8, Chapter I. We have the stability equation, for uncontrolled airplane,

$$d^4+10.65\,d^3+89.0\,d^2+15.5\,d+27.0=0 \quad \ldots\ldots (A)$$

$$\text{or} \quad A_o d^4 + B_o d^3 + C_o d^2 + D_o d + E_o = 0$$

First approximation,

$$a_1 = D_o/C_o, \qquad b_1 = E_o/C_o = 0.304$$
$$= 0.174$$

then, divide (A) by $d^2+.174d+.304$ as follows,

$$
\begin{array}{r|l|l}
d^2+.174d+.304 & d^4+10.65d^3+89\,d^2+15.5d+27.0 & d^2+10.48d \;+86.9 \\
 & \underline{d^4+\;\;.174d^3+.304d^2} & \\
 & \quad 10.48d^3+88.7d^2+15.5d & \\
 & \quad \underline{10.48d^3+1.18d^2+3.18d} & \\
 & \qquad\qquad 86.9d^2+12.3d+27 & \\
\end{array}
$$

Now, find a_1 for second approximation by $a_1' = 12.3/86.9 = .142$, and $b_1' = 27/86.9 = 0.311$ and repeat the process.

Due to the fact that a_1 is always much smaller than a_2

and b_1 much smaller than b_2, the second approximation gives almost the final solution. Using the brief symbol for sythetic division, the following process can be carried out,

$$
\begin{array}{l|l}
1+.142+.311 & \underline{1+10.65+89+15.5+27.0} \\
 & \underline{1+\ \ .142+\ .311} \\
 & \quad 10.51\ \ +88.7\ +15.5 \\
 & \quad \underline{10.51\ \ +1.49\ +3.27} \\
 & \qquad\qquad 87.2\ +12.23\ +27.0
\end{array}
$$

Then, $a_1 = 12.23/87.2 = .1405$, $b_1 = 27/87.2 = .31$ which is so close to the second approximation that further repeating of the third approximation is not necessary. The final answer can be written as,

$$(d^2+.1405d+.31)(d^2+10.51d+87.2)=0$$

b. Ideally controlled airplane.

The above process applies just as well to the ideally controlled airplane for any value of m_θ. For example, take the case of F-22, with $m_\theta = -2$, we have from Table8,2, $A_1d^4+B_1d^3+C_1d^2+D_1d+E_1 = 0$ which is,

$$d^4+10.65d^3+129d^2+203.5d+70 = 0$$

For first approximation, $a_1 = 203.5/129 = D_1/C_1 = 1.58$

$b_1 = E_1/C_1 = 70/129 = 0.543$

Then, divide as before,

$$
\begin{array}{l|l}
1+1.58\ +.543 & \underline{1\ +\ 10.65\ +129\ +\ 203.5\ +\ 70} \\
 & \quad \underline{1.58\ \ +.543} \\
 & \quad 9.07\ +\ 128.46\ +203.5 \\
 & \quad \underline{9.07\ +\ \ 14.3\ \ +\ 4.92} \\
 & \qquad\qquad 114.16\ \ +198.6\ +70
\end{array}
$$

Then, $a_1 = 198.6/114.16 = 1.74$, $b_1 = 70/114.16 = .613$ for second approximation. Repeating the process,

$$\begin{array}{l|l} 1 + 1.74 + .613 & 1 + 10.65 + 129 + 203.5 + 70 \\ & \underline{1 + 1.74 + .613} \\ & \quad 8.91 + 128.39 + 203.5 \\ & \quad \underline{8.91 + 15.47 + 5.45} \\ & \qquad 112.92 + 198.05 + 70 \end{array}$$

Then, $a_1 = 198.05/112.92 = 1.755$, $b_1 = 70/112.92 = .62$
The value is so close to the second approximation, that the final solution can be written as,

$$(d^2+1.755d+.62)(d^2+8.91d+112.9) = 0$$

2. Solution of the sextic equation for the controlled stability equation with inertia lag.

Similar method of approximation can be used to solve the numerical equation of the sixth order introduced due to the presence of inertia lag. It is known that the lagging effect is principally concerned with the short period oscillation so that the long period mode of motion remains essentially unchanged. Write the sextic as $A'd^6+B'd^5+C'd^4+D'd^3+E'd^2+F'd+G'=0$
It can be factored as $(d^2+a'd+b')(d^4+Bd^3+Cd^2+Dd+E) = 0$
As the factor $d^2+a'd+b'=0$ represents the long period mode of motion, (really, it may be splitted into two exponential mode of motion when a' is larger than b' by a certain margin, namely when $(a'/2)^2$ is larger than b') a' and b' are comparatively much smaller than the

the coefficients B?C,D, and E.

Take, from table 8.4, the case of $m_\theta=-2.0$, $k=0.1$, damping ratio=0.5, in Chapter I,

we have, $d^6+20.65d^5+295.5d^4+1970d^3+13100d^2+20400d+7000=0$

Take for first approximation,

$a'=F'/E'=20400/13100=1.56$, $b'=G'/E'=7000/13100=.535$

and perform the synthetic division as before,

```
1+1.56+.535|1+20.65+295.5+1970+13100+20400+7000
            1+ 1.56+.535
               19.09+294.0+1970
               19.09+29.8 +10.2
                     264.2+1960+13100
                     264.2+412.0+  141
                           1548+12959+20400
                           1548+ 2420+  828
                                 10539+19572+7000
```

Then, $a'=19572/10539=1.86$, $b'=7000/10539=.665$

repeat the process, by still briefer symbols as follows,

```
1+1.86+.665|1+ 20.65 + 295.9 +1970 + 13100 +20400 +7000
               1.86 + .665
               18.79 + 295.2
                     35   +  12.5
                     260.2 +1957.5
                           484    + 173
                           1473.5 +12927
                                  2740+ 980
                                  10187 +19420+7000
```

Then, $a'=19420/10187=$ ~~2.12~~ 1.91 $b'=$ ~~.762~~ .687 for third approximation.

Repeating the above process, we find,

$a'=1.92$, $b'=.692$ as it is so near to the last times, the final solution is,

$$(d^2+1.92d+.692)(d^4+18.74d^3+260d^2+1480d+10100)=0$$

The remaining quartic, however, cannot be sovled by the above successive approximation method as the two modes of motion involved are of nearly same period and damping. It can be easily solved by the semigraphical method described in T.R.589. The complete factoring gives,

$$(d^2+1.92d+.692)(d^2+2.4d+65.5)(d^2+16.3d+153)=0$$

3. Solution of the quintic equation involed in the lateral controlled stability.

The above principle of successive approximation can also be used to solve the quintic equations in lateral stability. The process is however, somewhat different. several illustrations can make the method clear.

a. The cubic and quartic equations of the uncontrolled airplane stability.

For uncontrolled airplane, the coefficient $F=0$, or one of the roots is zero so that the quintic equation reduces to a quartic equation. It is also known to us that for all cases of uncontroled lateral stability equations, one of the roots is allways large and negative, with its magnitude somewhere near to the derivative, l_p, signifying that one mode of the motion is a rapidly damped rolling subsidence.

Take, for example, the coefficients of the uncontrolled stability equation for the airplane F-22 as found in Table 10.1 of Chapter III.

$$d^5 + 18d^4 + 50d^3 + 288d^2 + 0 + 0 = 0$$

Here, due to the fact that E and F = 0, the equation reduces to a cubic. As $1_p = -16$, so one of the real roots is approximately, if not exactly, equal to -16.

Therefore, we divide as follows,

$$
\begin{array}{l}
d + 16 \;\lfloor\; \underline{d^3 + 18\,d^2 + 50\,d + 288} \\
\qquad\qquad \underline{d^3 + 16\,d^2} \\
\qquad\qquad\qquad 2\,d^2 + 50\,d \\
\qquad\qquad\qquad \underline{2\,d^2 + 32\,d} \\
\qquad\qquad\qquad\qquad 18\,d + 288 \\
\qquad\qquad\qquad\qquad \underline{18\,d + 288}
\end{array}
$$

In this case, it happens to be exactly divisible, so that the final result is, $(d + 16)(d^2 + 2d + 18) = 0$

However, in the following case, such as taken from Table 10.2 of Chapter III, for the case of $\mu n_{\psi} = -4$

$$d^5 + 18\,d^4 + 54\,d^3 + 352\,d^2 + 0\,d + 64 = 0$$

Since we know that in this case, $1_p = -16$ as before, so we try to divide by d + 16 . Using the abbreviated form of synthetic division,

$$
\begin{array}{l}
1 + 16 \;\lfloor\; \underline{1 + 18 + 54 + 352 + 0 + 64} \\
\qquad\quad \underline{1 + 16}\,(h) \\
\qquad\qquad (g)\,2 + 54 \\
\qquad\qquad (g)\,\underline{2 + 32}\,(f) \\
\qquad\qquad\quad (e)\,22 + 352 \\
\qquad\qquad\quad (e)\,\underline{22 + 352}\,(d) \\
\qquad\qquad\qquad (c)\,0 + 0 \\
\qquad\qquad\qquad (c)\,\underline{0 + 0}\,(b) \\
\qquad\qquad\qquad\quad (a)\,0 + 64 \\
\qquad\qquad\qquad\quad (a)\,\underline{0 + 0} \\
\qquad\qquad\qquad\qquad\quad 64
\end{array}
$$

In this case, the division is not exact, as there is a remainder of 64. However, we know that the real root is somewhere around 16, as it is seen that had the coefficient 352 been a little bit larger, the remainder would have a chance to be zero instead of 64 as it is now. So, a reverse of the process of division can be carried out as follows, Using the same example, we divide 64 by 16, giving 4, indicating that if the number a, had been 4 instead of zero, then the remainder would be zero. In order to have the number a= 4, the number b should have been -4 instead of zero. In order b to be -4, the number c should have been -4/16 = -.25 This requires that the number d should be 352+.25 = 352.25 instead of 352. Similarly we trace back finding that, e should be 352.25/16 =22.0 or just a little bit larger than 22. And the rest numbers, f, g, and h are so close to their present value that the difference can not be read from the slide rule. So, in this case we say that the real root is -16, though the exact one is really -16.03. And the quintic is factored into,

$$(d +16.0) (d^4+2d^3+22d^2+c\ d + \overset{a}{b}) = 0 \quad \text{or}$$

$$(d +16.0) (d^4+2d^3+22d^2-.25d+ 4) = 0$$

We see that in this way, we neglect the small error involved in a big number such as 16, 2 and 22 . and make good accuracy on the numbers -.25 and 4 which is large compared with zero.

The previous process can be performed with great convenience by the following form suggested by the writer,

Write,

A	B	C	D	E	F
1	18	54	352	0	64
(h)/16 = 1	(g) =(f)/16 =1.999	(e) =(d)/16 =22.0156	(c) =(b)/16 =-.25	(a) =64/16 = 4	
0	(h) =B-(f) =~~2x000~~ =16.00	(f) =C-(e) =31.9844	(d) =D-(c) =352.25	(b) =E-(a) = -4	

The quintic is factored in to,

$$(d+16)(d^4+(g)d^3+(e)d^2+(c)d+a) = 0$$

or in this case, it is,

$$(d+16)(d^4+1.999d^3+22.0156d^2+ -.25d + 4)= 0$$

The quartic equation is then factored by the successive approximation method described for the longitudinal stability equation. The final factoring is,

$$(d+16)(d^2-.0285d+.182)(d^2+2.03d+21.9) = 0$$

The above method of factoring the quintic equation can be carried out successively if the first trial gives the value ~~Rx(k~~), (h)/16 or in general (h)/l_p quite far from being unity.

The following example illustrate the process.

Take, from Table 10.4, the coefficients for the quintic with $\mu l_\phi = -32$,

$$d^5+ 18\ d^4+ 90d^3+ 480\ d^2+ 512d +128 = 0$$

First trial, assume the real root= -16,

1	18	90	480	512	128
	62/16 = 3.88	448.5/16 = 28	504/16 = 31.5	128/16 = 8	
	14.12	62	448.5	504	

As the value 14.12/16 = .882 instead of 1,

we repeat the process by assuming the real root to be -14.12 instead of 16,

Second trial,

1	18	90	480	512	128
	58.5/14.1 = 4.15	444/14.12 = 31.5	502.94/14.1 =35.7	128/14.12 = 9.06	
	13.85	58.5	444.3	502.94	

As the value 13.85/14.12 = 0.98 which is very nearly to be unity

If greater accuracy is desired, the process can be repeated for a third trial, by assuming the real root to be -13.85,

Third trial,

1	18	90	480	512	128
	4.15	32.7	36.3	9.22	
	13.85	57.3	453.7	502.78	

As 13.85/13.85 = 1 so the third trial gives the final answer to be,

$$(d+13.85)(d^4+4.15d^3+ 32.7\ d^2+ 36.3\ d + 9.22)= 0$$

Factoring the quartic by the method of sucessive approximation, we have, finally

$$(d + 13.85)(d^2 + 1.3d + .32)(d^2 + 2.9\,d + 28.3) = 0$$

By the above process and the basic principle, the writer found that nearly all kinds of stability equations involved in aeronautics can be handled with so short a time that any attempt to find the Routh's discriminants to see just whether one of the modes of motion is stable or not is no longer worth doing.

APPENDIX II

DESCRIPTION OF THE USE OF THE M.I.T.DIFFERENTIL ANALYZER TO SOLVE THE DISTURBED MOTION DUE TO GUSTS AND CONTROL MANIPULATIONS

The basic principle as well as the detail process in solving differential equations by using the M.I.T. Differential analyzer will not be described here, as it is done by Bush[1]. Any one who is interested in knowing the detail should consult Bush's paper.

The present description is, therefore, limited to those points which are closely concerned with the solution of the stability problems.

I.Longitudinal stability equations.

a.Equations of motion.

$$du/dt = x_u u + x_w w + \mu C_L \theta$$

$$dw/dt = z_u u + z_w w + \mu \, d\theta/dt$$

$$d^2\theta/dt^2 = m_w w + m_q d\theta/dt + \mu m_s s$$

$$d^2 s/dt^2 = f_\theta \theta - f_{\dot{s}} ds/dt + -f_s s$$

The following coefficients are constant and not to be changed,

$x_u = -.15$

$z_u = -1.0$

$z_w = -4.5$

$\mu = 20$

$x_w = 0.4$

The following coefficients are to be varied as,

$m_w = \underline{-3}, -2.4, -1.2, 0, +1.2, +2.4$.

$m_q = -9, \underline{-6}, -3, 0$

$\mu C_L = -18, \underline{-9}$.

$\mu m_s = -180, -120, -90, \underline{-60}, -45, -30, 0$

The following coefficients are to be functions of, k, damping ratio, and m_θ.

$f_s = (1/k)^2$, $f_{\dot{s}} = (2/k)$(damping ratio) and

$f_\theta = (1/k)^2 (m_\theta / m_s)$

Range of k is to be, .05, $\underline{0.1}$, 0.2, 0.3, 0.4, 0.8

Range of damping ratio is to be, 0, .25, $\underline{.50}$, 1.0, 1.6

Range of m_θ / m_s is to be 12.5, $\underline{25,}$ 50.

Underlined coefficients are normal ones.

1.Equations of motion with normal coefficients,

$$du/dt = -9\theta + .4\,w - .15\,u$$

$$dw/dt = 20d\theta/dt - 4.5w - u$$

$$d^2\theta/dt^2 = -6d\theta/dt - 3w - 1200s$$

$$d^2s/dt^2 = 4\theta - 10ds/dt - 100s$$

2.Equations of motion with maximum coefficients,

$$du/dt = -18\theta + .4w - .15u$$

$$dw/dt = 20d\theta/dt - 4.5w - u$$

$$d^2\theta/dt^2 = -9d\theta/dt - 3w - 1800s$$

$$d^2s/dt^2 = 16\theta - 32ds/dt - 400s$$

3. The estimated maximum ranges for the accelārations and velocitȷes due to a verical velocity $w_o = 1.5$,

$d\theta/dt$ is smaller than 1,

$d^2\theta/dt^2$ is smaller than 5

dw/dt is smaller than 10

du/dt is smaller than 1

d^2s/dt^2 is smaller than 1

ds/dt is smaller than 1/10

t is from zero to 10.

4. Modified form of the equations of motion

(a) With normal coefficients

$$\text{(I)}\ 16du/dt = -144\theta + (32/5)w - (12/5)u$$

$$\text{(II)}\ 4dw/dt = 80d\theta/dt - 18w - 4u$$

$$\text{(III)}\ 5d^2\theta/dt^2 = \underset{(e)}{-30d\theta/dt} \underset{(f)}{-15\,w} \underset{(d)}{-6000s}$$

$$\text{(IV)}\ 32d^2s/dt^2 = \underset{(c,a)}{128\,\theta} \underset{(b)}{-320ds/dt} \underset{(a)}{-3200s}$$

(b) With maximum coefficients

$$\text{(I)}\ 16du/dt = -288\theta + (32/5)w - (12/5)u$$

$$\text{(II)}\ 4dw/dt = 80d\theta/dt - 18w - 4u$$

$$\text{(III)}\ 5d^2\theta/dt^2 = \underset{(e)}{-45d\theta/dt} \underset{(f)}{-15w} \underset{(d)}{-9000s}$$

$$\text{(IV)}\ 32d^2s/dt^2 = \underset{(c,a)}{512\theta} \underset{(b)}{-1024ds/dt} \underset{(a)}{-12800s}$$

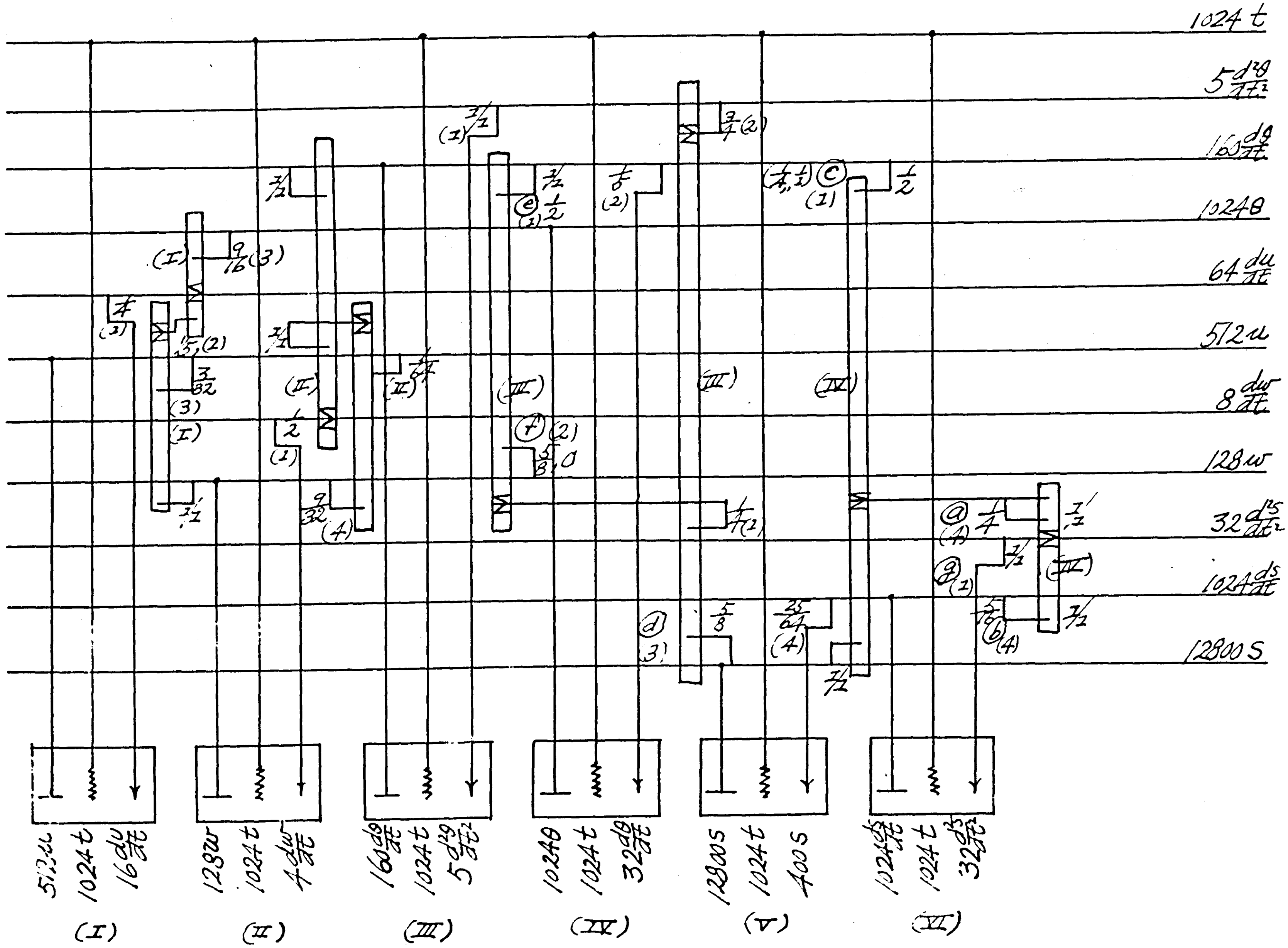
(a),(b)... etc are places where gear ratios can be changed

(c,a) means gear ratio at (c) depends on that at (a).

5, The preliminary set-up diagram for normal coefficients, with provision for maximum and other coefficients are shown in Fig.1 ,(a),(b) etc are places to change the the corresponding gear ratio. Numbers in the brackets indicate the numbers of gears required.

6. The detail set-up of the Differential analyzer is shown in Fig.2.

The symbols used are standard ones with their exact meaning defined by Bush and Cadwell[2].



1024 t
5 d²θ/dt²
160 dθ/dt
1024θ
64 du/dt
512u
8 dw/dt
128w
32 d²s/dt²
1024 ds/dt
12800 S
512u 1024t 16 du/dt
128w 1024t 4 dw/dt
160 dθ/dt 1024t 5 d²θ/dt²
1024θ 1024t 32 dθ/dt
12800S 1024t 400S
1024 ds/dt 1024t 32 d²s/dt²
(I)
(II)
(III)
(IV)
(V)
(VI)

Fig. I.

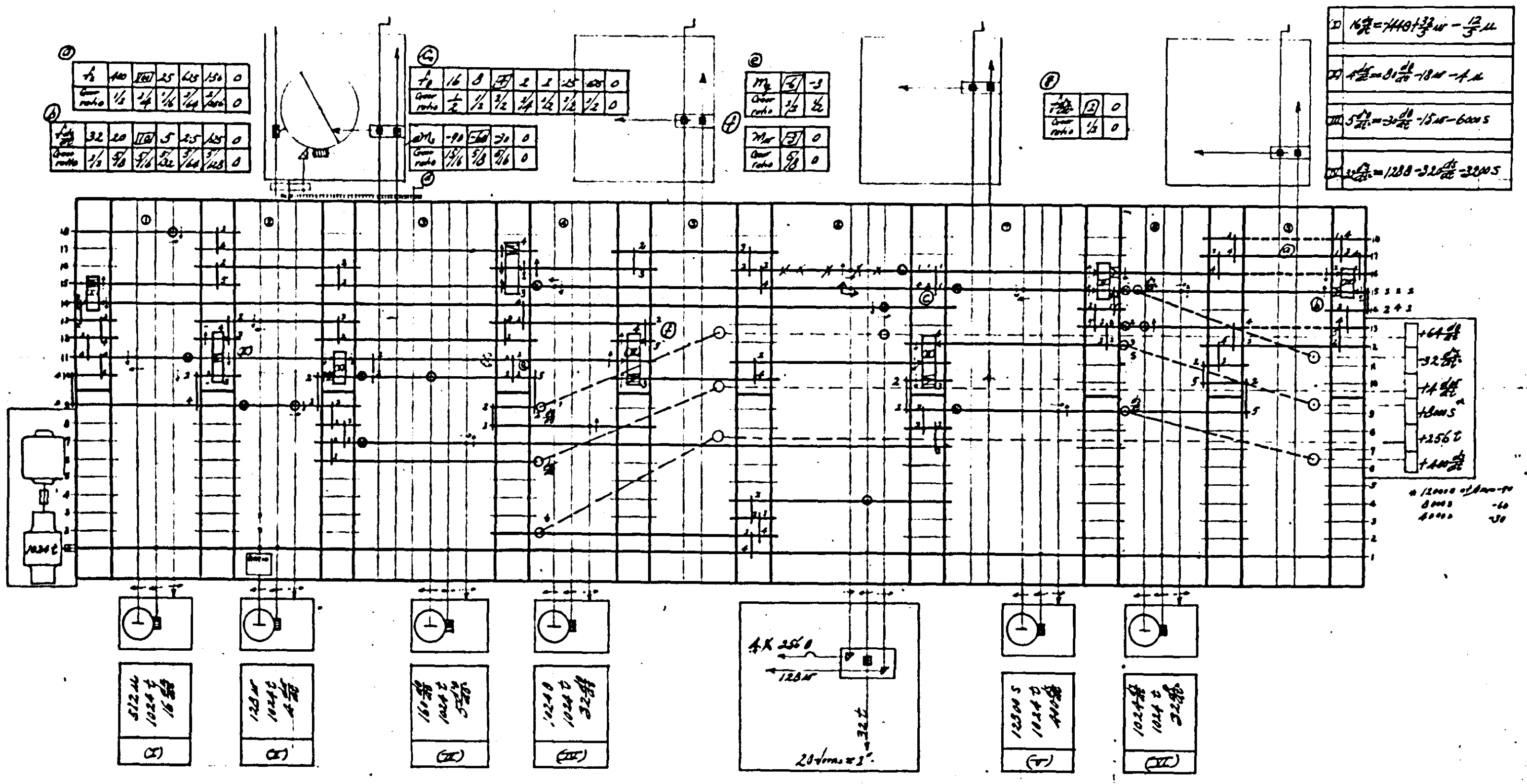


Fig. 2.

244

II. Lateral stability equations.

1. Equations of motion

$$dv/dt = y_v v - \mu C_L \phi - \mu d\psi/dt$$

$$d^2\phi/dt^2 = l_v v + l_p p + \mu l_\phi \phi + l_r d\psi/dt$$

$$d^2\psi/dt^2 = n_v v + n_p d\phi/dt + \mu n_\phi \phi + n_r d\psi/dt + \mu n_\psi \psi$$

2. Ranges of the coefficients,

$y_v = \underline{\underline{0}}, -1/4, -1/2, -1, -2.$

$\mu = \underline{\underline{16}}, 32$

$\mu C_L = \underline{\underline{-8}}, -16, -32$

$l_v = 0, -1, \underline{\underline{-2}}, -4, -8, -16, -32.$

$l_p = 0, -4, -8, \underline{\underline{-16}}.$

$l_r = 0, 2, \underline{\underline{4}}, 8, 16, 32$

$n_v = 0, \underline{\underline{1}}, 1.5, 2, 4, .25, .5$

$n_p = 0, -.25, \underline{\underline{-.5}}, -1, -2.$

$n_r = 0, -1, \underline{\underline{-2}}, -4, -8$

$\mu l_\phi = 0, -16, -8, \underline{\underline{-32}}, -48, -64, -128$

$\mu n_\phi = 0, 4, \underline{\underline{8}}, 12, 16, 32.$

$\mu n_\psi = 0, -4, \underline{\underline{-8}}, -16, -32.$

The underlined coefficients are normal ones.

3. The maximun accelerations and velocities due to an initial rolling gust, $p_o = 1$ are ,

dv/dt is smaller than 10

$d\phi/dt$ is smaller than 1

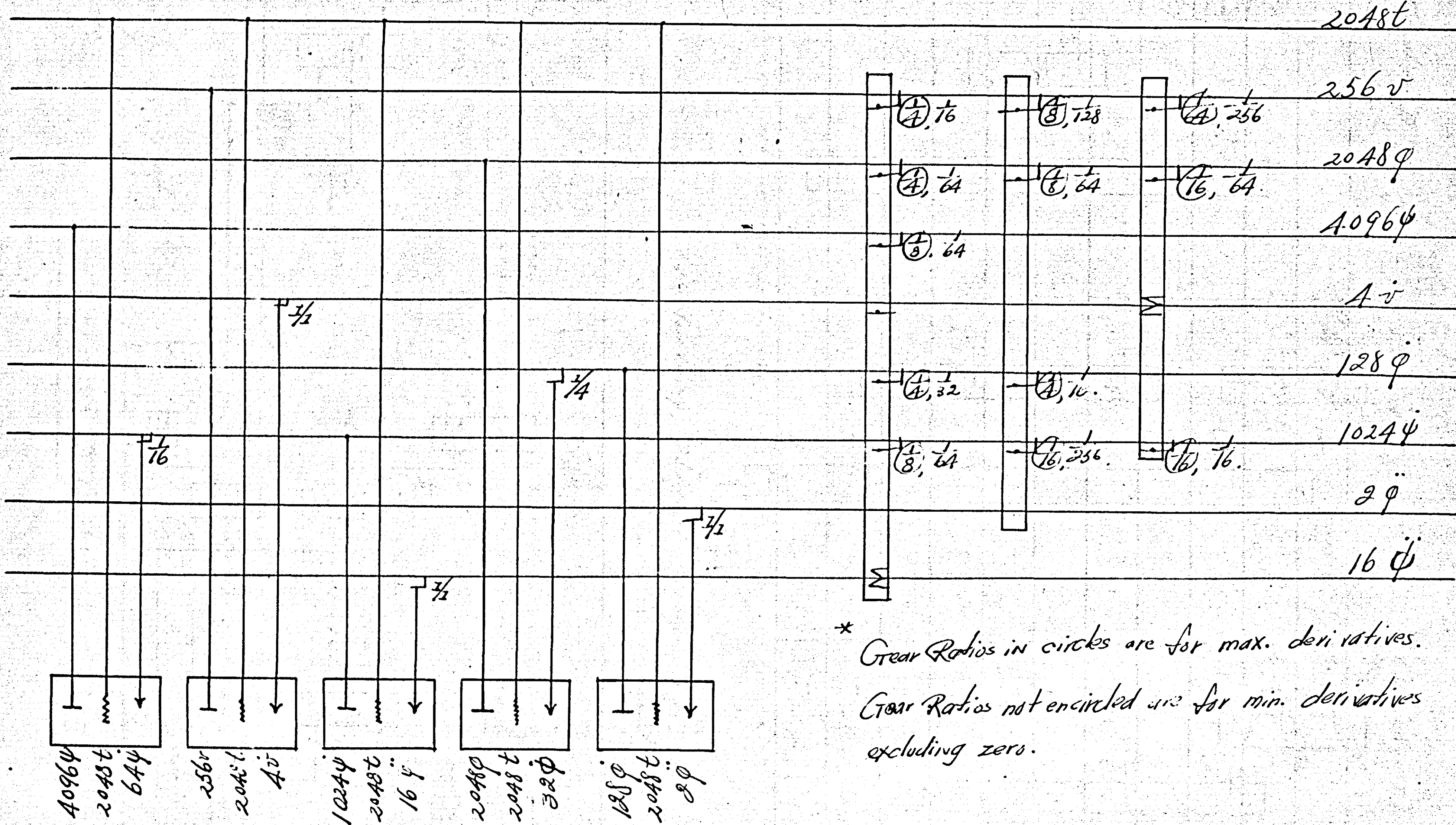
$d^2\phi/dt^2$ is smaller than 16

$d\psi/dt$ is smaller than 1/2

$d^2\psi/dt^2$ is smaller thn 2

v is smaller than 1, ϕ smaller than 10, ψ smaller than 1 and time not longer than 5.

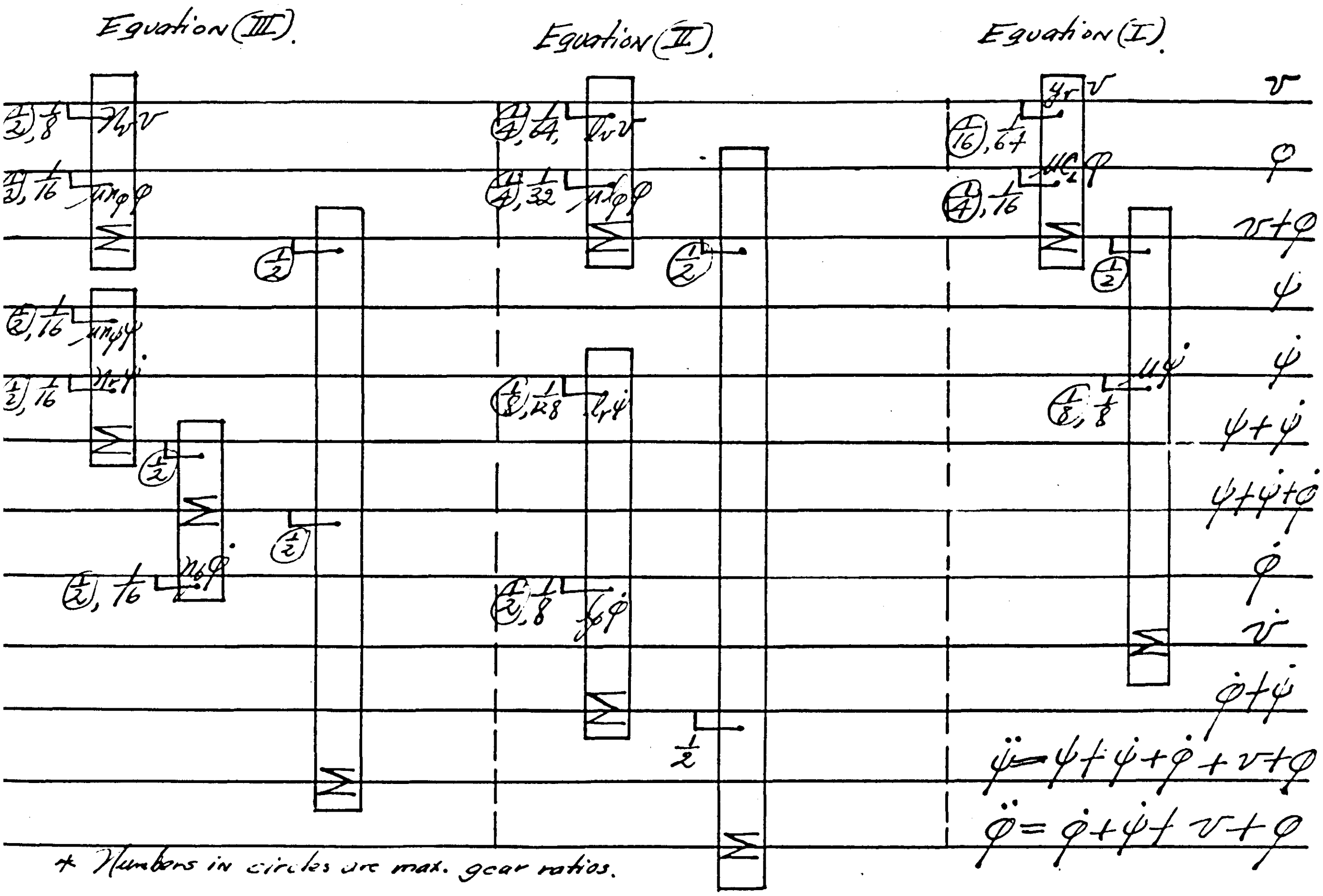
3, Fig.3 shows the connection diagram, Fig.4 shows the preliminary set-up, and Fig.5 shows the detail set-up.



* Gear Ratios in circles are for max. derivatives.
Gear Ratios not encircled are for min. derivatives excluding zero.

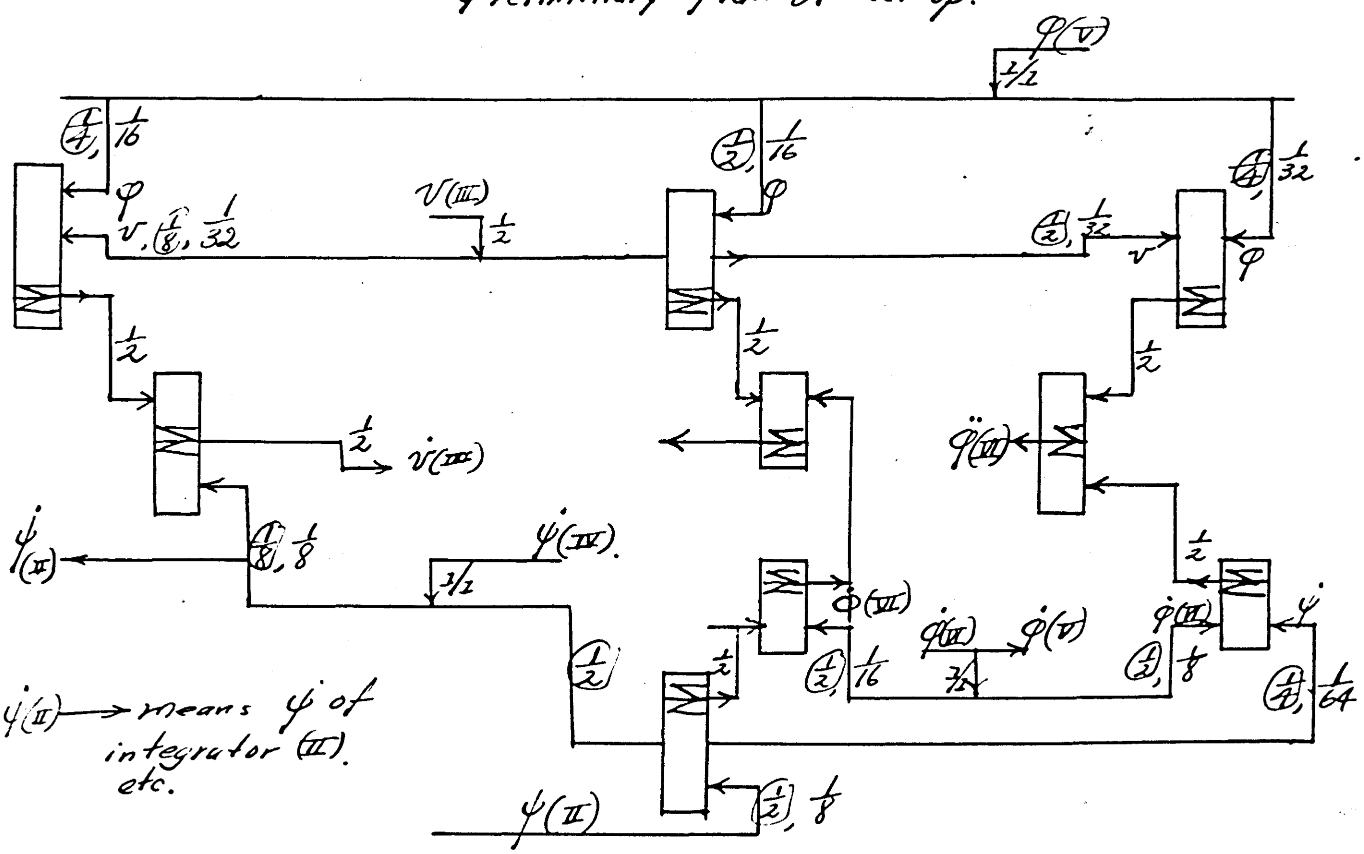
Fig. 3.

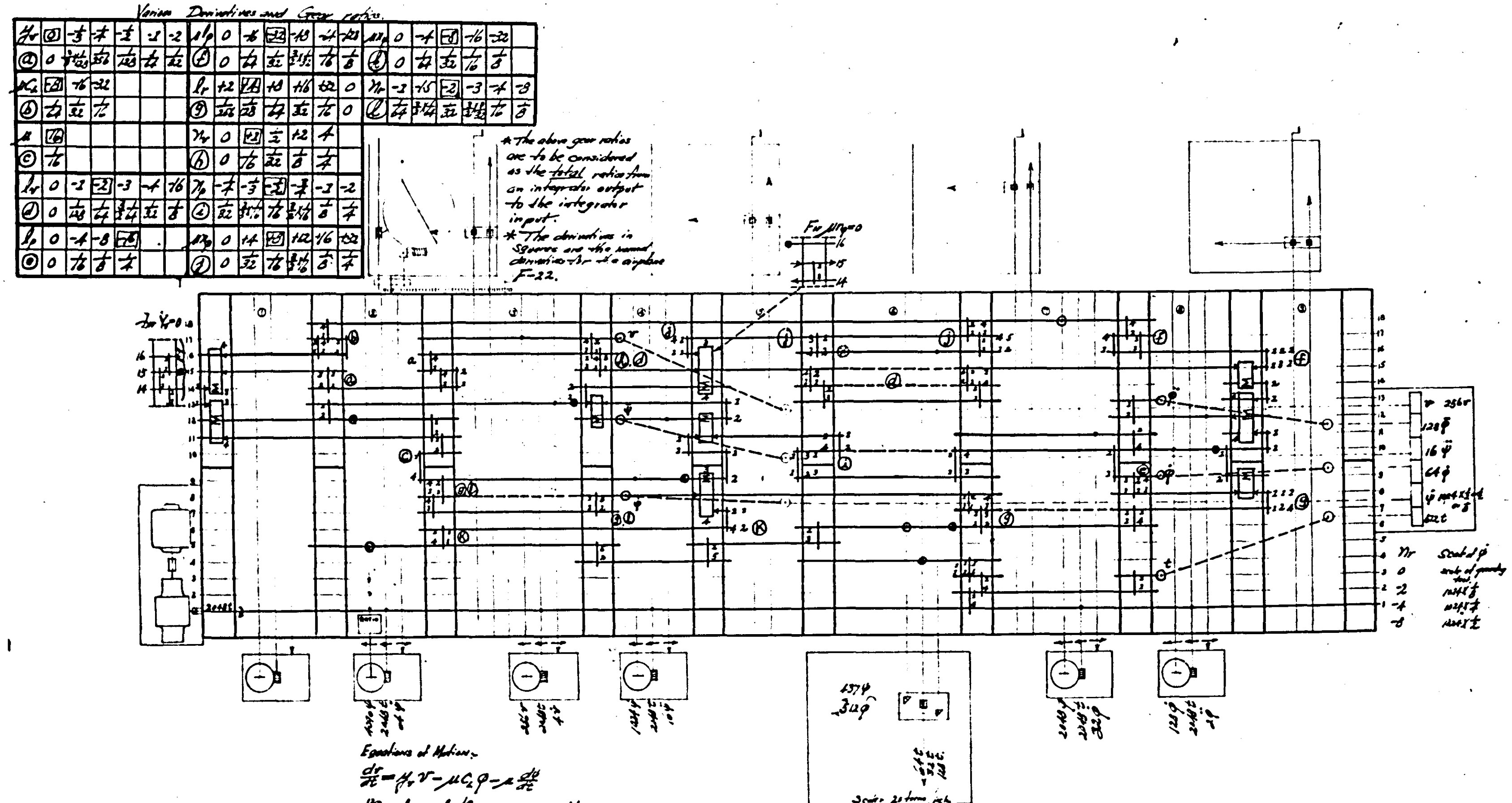
Fig. 4.



* Numbers in circles are max. gear ratios.

Preliminary plan of set-up.





Equations of Motion:

$$\frac{dv}{dt} = y_v v - \mu C_L \varphi - \mu \frac{d\psi}{dt}$$

$$\frac{dp}{dt} = l_v v + l_p \frac{d\varphi}{dt} + \mu l_\varphi \varphi + l_r \frac{d\psi}{dt}$$

$$\frac{dr}{dt} = n_v v + n_p \frac{d\varphi}{dt} + \mu n_\varphi \varphi + n_r \frac{d\psi}{dt} + \mu n_\psi \psi$$

Fig. 5.

III. Comparison of the disturbed motion, solved by analytical method against that plotted by the Differential Analyzer.

The longitudinal motion of the uncontrolled airplane due to a vertical gust, w_o is computed by method of operational calculus, and compare with that solved by the M.I.T. Differential Analyzer.

(a) By Klemin's formula[3],

$$\theta/w_o = e^{-5.255t}(.0385\cos 7.72t+0.0244\sin 7.72t) - e^{-.0702t}(.036\cos .552t+.006\sin .552t)$$

(b) By method of complex number algebra[4],

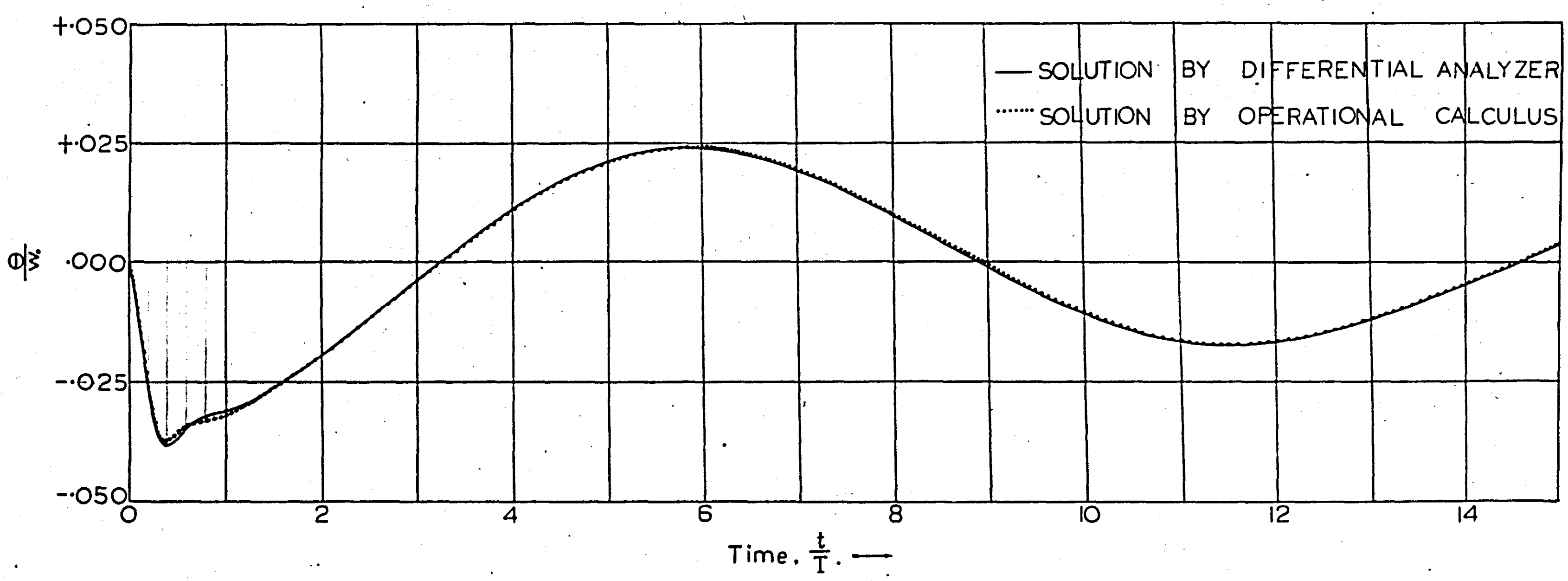
$$\theta/w_o = 0.0475e^{-5.255t}\cos(7.72t-40°) +.037\, e^{-.0702t}\cos(.552t-190.5°)$$

The result found by the complex number algebra is plotted in Fig. 6 side by side with that solved by the Differential analyzer. They check very well except that the damping of the short oscillation as indicated by the curve solved by the machine seems a little bit too low. This is probably due to the effect of back-lash because no front lash unit was used in the set-up. As in this research, no extreme accuracy is required, the result solved by the machine can be considered entirely satisfactory for the purpose.

The most advantageous part of using this machine to solve the stability problem is due to the fact that

Fig. 6

DISTURBED ANGLE OF PITCH OF AN UNCONTROLLED AIRPLANE SUBJECT TO VERTICAL GUST.



corresponding to one set of stability derivatives, seven solutions can be obtained by a single run which lasts only from ten to twenty minutes. In the longitudinal stability, the solutions for the vertical velocity,w and the inclination in pitch,θ are plotted out in the out-put table while the motions, $d\theta/dt$, d^2s/dt^2,ds/dt, dw/dt and s are recorded as a function of time in the recorder. Due to lack of time and space, only part of theeresults are presented in this thesis. IN the lateral stability equations,the variables ϕ, and ψ are plotted on the input table while the variables,v,dp/dt,p, and r and dr/dt are recorded as a function of time.

Suggestions for further development

1. Design and construction of apparatus to determine the the exact nature of control lag, and compare with theory.
2. Study of the nature of human pilotting with regard to the maximum lag, control derivatives produced and the degree of over-shooting.
3. Mathematical investigations of the effects of those control derivatives which have not been carried out, due to lack of time and space, on the stability of the controlled motion.
4. Investigation of the effect of inertia lag on the lateral motion by using the New M.I.T. Differential Analyzer which is not yet available at the time of writing this thesis.
5. Design and contruction of ~~a~~ an airplane using two-control operation system to test the practicability of the performance as predicated by the theorectical investigation.
6. Application of the results obtained from the theorectical study of the controlled stability to aid practical airplane designers by constructing charts, diagrams and so on so as to make the stability estimation an easy task for average designers.

BIBLIOGRAPHY ACCORDING TO NUMBERS OF FOOTNOTES

Chapter I

1.B.Melvill Jones,Dynamics of the Airplane,Vol.V Aerodynamic Theoby,W.F.Durand,Editor in Chief. p.164 Berlin.Julius Springer.1935, Referred to henceforth as Jones,Dynamics.

2.Fr.Haus,Introduction a l'e'tude de la stabilite' dynamique' des avions. Bulletin du Service Technique de L'Aeronautique, No.5,1927.

Fr,Haus Automatic Stability of Airplanes,T.M.695,1932

Fr,Haus,Automatic Stabilitzation,T.M.802,815 1936

Garner,H.M.Lateral stability with special reference to controlled motion,R.and M.1077,1926

W.L.Cowley,On the stability of controlled motion, R. and M. 1235,1928

Alexander Klemin, Longitudinal Stability in relation to the use of an automatic pilot,T.N.666, 1938.

3.Bassett,Preston R. Development and Principles of the Gyropilot. Instruments Vol.9,no.9 Sept.1936 pp.251-254

4.H.K.Weiss, Theoby of automatic control,M.S.Thesis,1938 M.I.T. Part III,Chapter II.

5.C.S.Draper Notes on the Aircraft Instruments,16.55 course,M.I.T.Aero.Eng.Department. or

Den Hartog,Mechanical Vibrations,McGraw Hill Co.,1934

6.Klemin,T.N.666 p.12

7.Klemin, T.N.666 p.12 with conversion into non-dimensional form,with definition of the non-dimensional derivatives as defined in the article by,

A.G.B.Metocalf,Airplane longitudinal stability-A Resume'.Journal of the Aeronautical Science,Vol.4 no.2,Dec.1936,p.61.

8. Klemin,T.N.666,p.29-31

9. Klemin,T.N.666, p.7

10.Den Hartog,J.P. Vibrations.

11.See appendix I of this thesis.

12.H.K.Weiss,M,S.Thesis,M.I.T. 1938, or

H.K.Weiss,Constant speed control theory,Journal of the Aero.Sc. Feb.,1939 pp 147-152

N.Minorsky,Directional Satability of automatically steered bodies. Journ.Am.Soc.of Naval Engrs. May 1922 pp.280-309

13.Klemin, A. Operational solutions in airplane dynamics Jour.of the Aero.Sc. May,1936

Bryant,L.W. and Williams D.H. The application of the method of operators to the calculation of the disturbed motions of an airplane R.and M.1346,1930

Jones,Robert T. A simplified application of the method of operators to the calculation of disturbed motions of an airplane. T.R.560,1936

14. V. Bush, The differential analyzer, a new machine for solving differential equations. M.I.T. E.E. Contribution, serial no. 75, Oct. 1931. or Journal of the Franklin Institute, vol. 212, no. 4, Oct. 1931.

See also Appendix II of this thesis.

15. H.K.Weiss, Theory of constant speed control, Journal of Aero. Sc. Feb. 1939. pp. 147-152, Henceforth, Constant speed control.

16. Den Hartog, Mechanical Vibrations.

17. N. Minorsky, Directional stability.

18. A.Callender, D.R.Hartree, A.Porter, Time lag in a control system philosophical translations of the Royal Society of London, July 21, 1936. vol. 235, pp. 415-444.

19. W.L.Cowley, On the stability of controlled motion, R and M 1235.

20. H.K.Weiss, Constant speed control.

21. Klemin, T.N. 666, pp. 29-31.

Chapter II.

1. Haus Fr., Stability, T.M. 695, Dec. 1932,
2. B.M.Jones, Dynamics p. 153.
3. A.M.Jones, Dynamics, fig. 56,(a-r)
4. Fr. Haus, Theoretical study of various airplane motions after critical disturbance, T.M. 867, June, 1938. A translation from Bulletin du Service Technique de L'Aéronautique, no.17, June, 1937.

   Fr. Haus, Automatic stability of airplane, T.M. 695,1932.
5. H.A.Soulé, Flight measurements of the dynamic longitudinal stability of several airplanes and a correlation of the measurements with pilot's observations of handling characteristics, T.R. 578, 1936.
6. B.M.Jones, Dynamics. p.166, Fig. 56a.

Chapter III.

1. B.M.Jones, Dynamics, p. 203.

2. C.H. Zimmerman, An analysis of lateral stability in power-off flight with charts for use in design. T.R. 589, 1937.

3. Fred E. Weick, H.A.Soulé, and M.N.Gough, A flight investigation of the lateral control characteristics of short wide ailerons and various spoilers with different amounts of wing dihedral. T.R. 494,1934.

Also B.M.Jones, Dynamics.

4. Garner, R and M 1077.

5. Cowley, R and M 1235.

6. O.C.Koppen, Lateral control at high angles of attack. Jour. of Aero. Sc. Jan. 1935. p.22.

7. F.W.Meredith and P.A. Cooke, Aeroplane stability and the automatic pilot, Jour. of the Royal Aero. Society, June 1937, p. 415.

8. H.K.Weiss, M.S.Thesis, M.I.T. 1938.

9. See Appendix II of this thesis.

10. Fr. Haus, T.M. 695.

11. Koppen, Lateral control at high angle of attack. and Garner, R and M 1077.

12. Garner, R and M 1077.

13. Weick, Soulé and Gough, T.R. 494. and F.E. Weick and R.T. Jones Resumé and analysis of N.A.C.A. Lateral control reasearch T.R. 605, 1937.

14. Koppen, Lateral control at high angle of attack.

15. C.H.Zimmerman, T.R. 589, 1937.

16. T.R. 605. and

Soulé,H.A. and McAvoy,W.H., Flight investigation ofl lateral control devices for use with full-span flaps. T.R. 517, 1935.

17. Robert T. Jones, The influence of lateral stability on disturbed motions of an airplane with special reference to the motions(produced by gusts. T.R. 638, 1938, p.2, Table I.

18. Due to the limited mumber of adders for the M.I.T. differential analyzer, the term y_v had to be zero when the term due to aileron adverse yaw i is present. See Appendix II.

19. Koppen, O.C. Lateral controal at high angle of attack. Garner, R and M 1077.

20. See Appendix I.

21. R.T. Jones, T.R. 638.

22. See Fig. 9.5 of chapter I.

23. R.T. Jones, T.R. 638 Fig. 2, p.3. and

Diehl, W.S. Engineering aerodynamics, The Ronald Press, Co. 1936, p. 206.

24. Millikan,Clark B. On the results of aerodynamic research and their application to aircraft construction. Jour. Aero. Sc. Dec. 1936, pp. 43-53.

Korvin-Kroukovsky, B.V. Proportioning the plane forl lateral-stability. Aero. Eng. Supplement to Aviation, Jan. 1929, pp.8-12.

24. Zimmerman, T.R. 589.

25. Weick, Fr., Jones, R.T. T.R. 605.

26. R.T. Jones, T.R. 638, Table I p.2.

27. O.C. Koppen, A simplified study of the lateral motion of an airplane at high angle of attack, Aviation, Sept. 1936. PP.29-31.

28. O.C. Koppen, Notes on the estimation of stability derivatives. Course 16.22, M.I.T.

29. Weick, Fred E. and Jones R.T. The effect of lateral controls in producing motions of an airplane, as computed from wind tunnel data. T.R. 570 1936.

30. B.M. Jones, Dynamics. Fig. 57 a-m, p.194.

31. B.M. Jones, Dynamics. p. 193.

32. Weick and Jones, T.R. 570 and T.R. 605.

Chapter IV.

1. Weick, Soulé and Gough, T.R. 494, 1934.

2. Robert T. Jones, A study of the two control operation of an airplane T.R. 579, 1936.

3. Klemin, A. Motion of the two controal airplane in rectilinear flight after initial distrubances with introduction of controls following an exponential law. T.N. 615, Sept. 1937, N.A.C.A.

4. B.M. Jones. Dynamics. p.193.

5. Zimmerman, T.R. 589.

Appendix II

1.V,Bush,The Differential Anlyzer.A new machine for solving differential equations Serial No.75 E.E.contributions, Oct.1931.Or Journal of the Franklin Institute,Oct.1931.

2.Caldwell,S.H. Sc.D.Thesis,M.I.T. E.E.Department,1933

Autobiography

I was born at Canton,Kwangtung Province,China on July,1,1913 and received my high school education at Peking Academy,Peiping,China. In 1931,I entered Chiao-Tung University,Shanghai,and studied for four years in the Electrical Engineering Department. I received the Degree of Bachelor of Science in July,1935.

After graduation, I won the scholarship offered by the Department of Education,Kwangtung Province, through open competitive examination. I was sent to U.S.A. in September 1935 to enter Massachusetts Institute of Technology to study aeronautical engineering,a branch of technology which was then not yet offered in any engineering schools in China.

I registered as a graduate student in the Aeronautical engineering Department since Oct.1935. I received the Degree of Master of Science in Dec.1937 with my thesis subject on the Design of a Frequency-modulation-type Radio Altimeter,under the supervision of Prof.C.S.Draper. I received a full tuition scholarship for the last two graduate years. I am expecting to get the Degree of Doctor of Science in June,1939.