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DIRECTED TECHNICAL CHANGE AND WAGE INEQUALITY

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Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality*

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Abstract

I consider an economy where skilled and unskilled workers use different technologies. The rate of improvement of each technology is determined by a profit-maximizing R&D sector. When there is a high proportion of skilled workers in the labor force, the market for skill-complementary technologies is larger and more effort will be spent in upgrading the productivity of skilled workers. An implication of this theory is that when the relative supply of skilled workers increases exogenously, the skill premium decreases in the short-run, but then increases, possibly *even above its initial value*, because the larger market for skill-complementary technologies has changed the direction of technical change. This suggests that the rapid increase in the proportion of college graduates in the U.S. labor force may have been causal in both the decline in the college premium during the 1970s and the large increase in inequality during the 1980s. The paper also derives implications of directed technical change for residual wage inequality and shows that calculations of the impact of international trade on inequality that ignore the change in the direction of technical progress may be misleading.

Keywords: Endogenous Technical Change, Relative Supply of Skill, Returns to Education, Skill-Biased Technological Change, Skill-Technology Complementarity, Wage Inequality.

JEL Classification: O14, O33, J31.

I. Introduction

Between 1811 and 1816 in Britain, the Luddites destroyed machines believing that they would make their skills obsolete. In 1826 in Lancashire, hand-loom weavers attacked weaving machines. In 1830 during the Captain Swing riots, agricultural workers destroyed threshing machines. But technical progress could not be halted and these skilled workers were quite soon replaced by the machines they tried to fight (see Mokyr, 1990). In contrast to the skill replacing technological advances of the nineteenth century, new technologies today appear to be complementary to skilled and educated workers rather than the unskilled.[1] Why is this? As suggested by the historical examples like the spinning jenny and the assembly line which increased the productivity of unskilled labor, the answer that new technologies are by their very nature "skill-biased" is not satisfactory. Even computers which are seen as the prototype example of "skill-biased" innovations can be and are sometimes used as complementary to unskilled labor. For example, most workers making deliveries and most employees at fast food restaurants and supermarkets use computers and scanners. Motivated by this reasoning, this paper starts from the premise that new technologies are not complementary to skilled labor by nature, but *by design*. This premise then raises the question of why over this century, and especially over the past two decades, the productivity of skilled and educated labor has been upgraded more than that of the unskilled.[2] This is the question addressed in this paper.

Most technologies, once invented, are largely *nonrival* goods: they can be used by many firms and workers at low marginal cost. When there are more skilled workers, the market for skill-complementary technologies is larger and the inventor will be able to obtain higher returns. As a result, when the proportion of skilled workers is high, more effort will be devoted to the invention of skill-complementary technologies rather than those complementary to unskilled labor. This implies that the impact of an increase in the supply of skilled workers on the skill premium will be determined by

[1]See Griliches (1956) and Bartel and Lichtenberg (1987) for micro estimates. See, among others, Berman, Bound and Griliches, (1994), Goldin and Katz (1996), Autor, Katz and Krueger (1997) for studies that document the relation between relative wages and technology.

[2]In practice, skill is multi-dimensional. The skills replaced by nineteenth century inventions are different than those complemented by computers today. For most of the paper I focus on education which is the most easily observable component of skills. I will then return to other dimensions of skill in an attempt to explain the rise in residual (within group) wage inequality.

two competing forces: the first is the conventional *substitution* effect which makes the economy move along a downward sloping relative demand curve. The second is the *directed technology effect* which can be thought of as shifting the "short-run" relative demand curve for skills. This effect is driven by the fact that an increase in the supply of skilled workers increases the profitability of technologies complementarity to skilled labor. Two implications of this approach are: (1) if the directed technology effect is sufficiently pronounced, in the long-run an exogenous increase in the supply of skilled workers can *increase* the skill premium; (2) irrespective of whether it dominates the substitution effect, the directed technology effect implies that as an economy accumulates more skills, the fraction of new technologies complementary to skilled workers should increase, which is consistent with the fact that technological change appears to have become more skill-complementary over time.

This theory suggests an alternative explanation for the changes in the structure of wages in the U.S. over the past two decades. The conventional wisdom is that these changes are due to exogenous technological developments. In particular, many believe that there was an *exogenous acceleration* in the skill-bias of technical change starting in the 1970s.[3] The most serious problem for the conventional view is the lack of any reason why exogenous skill-biased technical change should have accelerated in the 1970s, coinciding with the unprecedented increase in the relative supply of skilled workers. The puzzle is in fact more striking. In Katz and Murphy's words; "for the 1963-87 period as a whole and most strongly for the 1980s, the groups with the largest increases in relative supplies tended to have the largest increases in relative wages" (1992, p. 52). In contrast to the conventional approach, suppose we are in a world of directed technical change and consider a large and exogenous increase in the supply of college graduates (as in the U.S. during the late 1960s and 1970s, see the discussion in the text). This increase in the proportion of college graduates in the labor force will first move the economy along a short-run (constant technology) relative demand curve as in Figure 1, reducing the college premium. Then, the induced change in the direction of technical progress will increase the marginal product of college graduates and shift the relative demand curve to the right. Suppose first that the substitution effect dominates the directed technology effect. The improvement in skill-complementary technologies will then increase the

[3]Observe following Autor, Katz and Krueger (1997) that from 1940 to 1970 relative supply of college graduates increased by 7.7 percentage points and the college premium fell slightly. In contrast, from 1970 to 1990, the supply increased by 11.6 percentage points and the skill premium increased. A constant rate of skill-biased technical change cannot account for this pattern.

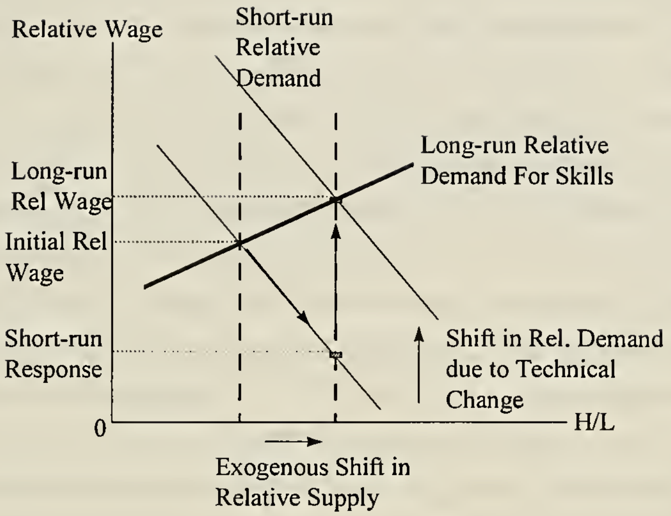

Figure 1: Dynamics of the skill premium when the directed technology effect dominates the substitution effect.

college premium from its short-term low but never above its initial level. In this case, the model can explain why the college premium increased during the 1980s, but not why it increased *above* its level before the supply shock. In contrast, if the directed technology effect is sufficiently strong, the model predicts that in the long-run, the college premium should *increase*. This is the case drawn in Figure 1 and offers a more complete explanation for the changes in the U.S. college premium over the past twenty five years. I will also show how this mechanism may account for the increase in residual wage inequality during the 1970s while the college premium was falling. Furthermore, the theoretical results also apply to any group using technologies different from the rest of the labor force and experiencing an increase in relative supply. So the model is consistent with a positive association between changes in the relative supplies of more disaggregated groups, and their relative wages over the medium run as found by Katz and Murphy (1992) and Bound and Johnson (1992). Therefore, the analysis in this paper suggests that the unprecedented increase in the supply of college graduates during the 1970s may have been causal both for the technological developments and the changes in the structure of wages during the 1980s as well as the decline of the college premium during the 1970s.

There are other episodes in which a large increase in the supply of skills appears to have affected the direction of technical change. The enrollment and graduation rates for high school doubled in the 1910s, mostly due to changes in the location and curricula of schools and the decline in transport costs (Goldin and Katz, 1995). The high school premium fell sharply in the 1910s. Yet, despite the even faster increase in the supply of high school skills during the 1920s, the relative wages of high skill graduates levelled off and started a mild increase. Goldin and Katz (1995) conclude that the demand for high school graduates *must have expanded sharply* starting in the 1920s, presumably due to changes in the office technologies and increased demand from new industries such as electrical machinery, transport and chemicals.[4]

An analysis of the implications of international trade on wage inequality provides an interesting application of this theory. The key observation is that trade will affect the direction of technical change. If the U.S. starts trading with the LDCs and sells technologies to LDC firms, the size of the market for technologies complementary to unskilled labor will increases and wage inequality will decline, or at most increase by only a small amount. However, if due to lack of international property rights protection, it is not possible to sell new technologies to LDC firms, trade will simply increase the relative price of the skill intensive good. This relative price change will induce further effort in upgrading skill-complementary technologies. I show that in this case conventional calculations underestimate the impact of trade on wage inequality because they ignore the change in the *direction* of technical change.

This paper is related to some older literature on induced innovations, including the theoretical work by Fellner (1961) and Kennedy (1964), the empirical studies by Schmookler (1966) and Hayami and Ruttan (1970), and the historical work by Habakkuk (1962). These studies (with the exception of Schmookler) discuss the impact of factor prices on induced innovations. I treat factor prices as endogenous, point out the importance of the fixed costs of innovation and of market size. These features yield the crucial result that a larger relative supply of a factor can lead to faster upgrading of technologies complementary to this factor, which is the opposite of the results that these papers discuss but in line with the evidence provided by Schmookler. This paper also builds on and extends the work of Aghion and Howitt (1992) and Grossman and Helpman (1991) by allowing technological change to be

[4]The high school premium never reached its 1900 level, partly because of the continued rapid increase in the supply of high school graduates. In any case, note that the theory offered in this paper does *not* predict that every increase in the relative supply of skills should be associated with increased skill premium, especially when the system is away from the balanced growth path as U.S. is likely to have been in the 1900s. The model however predicts that every increase in supply should have an impact on the direction of technical change.

directed towards different groups. Finally, a number of recent papers also suggest that changes in the supply of skills may change the demand for skills. This point is first made in Acemoglu (1996) using a search model: when there is a sufficient fraction of workers who are skilled, firms find it profitable to create jobs specifically targeted for this group. The result is a fall in the level of unskilled wages and an increase in skilled wages. Krugman (1997) has recently constructed a signalling model with some common features. A recent paper by Kiley (1997) considers an expanding varieties model and shows that an increase in the proportion of skilled workers can increase wage inequality. Walde (1997) compares the technology choice of economies differing with regards to the skill level of their high school graduates. He shows that an economy with less skilled high school graduates may choose a technology which makes little use of high school skills and have a high skill premium.

The plan of the paper is as follows. Section II analyzes the basic model and contains the most important results of the paper. Section III endogenizes the relative supply of skills which is treated as exogenous in Section II. Section IV discusses the impact of international trade on the direction of technical change, and on wage inequality through this channel. Section V concludes.

II. The Basic Model

A. Technology and Preferences

Consider the following continuous time infinite horizon economy. There is a continuum H of skilled workers and a continuum L of unskilled workers with identical preferences over the unique consumption good, y. The utility of agent k at time t is:

$$U_{kt} \equiv E_t \int_t^{\infty} \exp(-r(\tau - t)) c_{k\tau} d\tau \tag{1}$$

where c_{kt} is the consumption of agent k at time t, E_t is the expectations operator conditional upon the information set at time t, r is the discount rate and due to linear utility, it will also be the interest rate.[5]

The unique consumption good is produced from two complementary intermediate goods, or *production processes*, one using skilled and the other unskilled labor. The market for intermediate goods is competitive. I denote the total output of these

[5] An equivalent formulation is to assume a general instantaneous utility function $u(c_{k\tau})$ and a perfect international market for lending and borrowing at the rate r.

processes (intermediate goods) by $Y_l(t)$ and $Y_h(t)$, and the aggregate production of the consumption good at time t is:

$$Y(t) = [\gamma_l Y_l^\rho(t) + \gamma_h Y_h^\rho(t)]^{1/\rho} \tag{2}$$

where $\rho \leq 1$. I normalize the price of the final good in each period to 1, and denote the prices of the two intermediate goods by $p_l(t)$ and $p_h(t)$ which, due to competitive pricing, implies:

$$p(t) \equiv \frac{p_h(t)}{p_l(t)} = \frac{\gamma_h}{\gamma_l}\left(\frac{Y_l(t)}{Y_h(t)}\right)^{1-\rho}. \tag{3}$$

(3) is a demand equation, linking the consumption of the intermediate goods to their relative price.

There are m_l and m_h firms in the two intermediate goods sectors, and in the rest of the analysis I will normalize, $m_l = m_h = 1$. The production of Y_h, the skill-intensive good, requires skilled labor while the production of the labor intensive good, Y_l, requires unskilled labor. Namely, firm i in sector s has production function:

$$y_s(i,t) = A_s(i,t)\,[n_s(i,t)]^\beta \tag{4}$$

where $s = l, h$, and $n_s(i,t)$ is the number of workers employed by firm i in sector s at time t, $\beta < 1$ and $A_s(i,t)$ is the productivity of labor in this firm. Since firms in sector l only employ unskilled workers, and those in sector h only hire skilled workers, feasibility requires that $\int n_l(i,t)di \leq L$ and $\int n_h(i,t)di \leq H$ for all t.

The firm level productivity parameter, $A_s(i,t)$, is determined by the quantity and quality of nonlabor inputs (machines) that firm i in sector s purchases at time t. There is a continuum $j_s \in [0,1]$ of *sector-specific* machines for each sector. The fact that each sector uses different machines is the sense in which skilled and unskilled workers use *different technologies* in this model. The quantity of machine j that firm i in sector s uses at time t is denoted by $x_s(i,j,t)$. These machines depreciate fully after use. I denote the highest qualities of these machines available at time t by $\tilde{q}_s(j,t)$ for $j \in [0,1]$ and $s = l, h$. In principle, it is possible that a firm purchases inputs that are not of the highest available quality, but this will not happen in equilibrium (see below). Hence, incorporating the fact that outdated machines will not be used, the productivity parameter of firm i takes the form:

$$A_s(i,t) = \frac{1}{\alpha}\int_0^1 \tilde{q}_s(j,t)\,[x_s(i,j,t)]^{\alpha}\,dj \tag{5}$$

where $\alpha \leq 1-\beta$. If $\alpha < 1-\beta$, intermediate good producers have decreasing returns and will make positive profits (hence their numbers, m_l and m_h are fixed). In contrast, if $\alpha = 1-\beta$, production in both sectors is subject to constant returns, and free-entry will give exactly the same results. Whether $\alpha = 1-\beta$ does not affect the results, and in what follows, I do not take a position on this.

B. Profit Maximization By Firms and Labor Market Equilibrium

Firms (producing y_l and y_h) purchase machines and labor to maximize static profits. Denoting the price of machine $\tilde{q}_s(j,t)$ by $\chi_s(j,t)$ and the wages of skilled and unskilled workers by $w_h(t)$ and $w_l(t)$, firm i solves the following problem at time t:

$$\max_{n_s(i,t),x_s(i,j,t)} \; p_s(t)A_s(i,t)\,[n_s(i,t)]^{\beta} - \int_0^1 \chi_s(j,t)x_s(i,j,t)dj - w_s(t)n_s(i,t)$$

Since this problem is strictly concave, it has a unique solution, implying that all firms in the same sector hire the same amount of labor and inputs (machines). With the normalization, $m_l = m_h = 1$, this implies that $n_l(i,t) = N_l \equiv L$ and $n_h(i,t) = N_h \equiv H$ for all i and t, and the aggregate demands for machine j in sector s at time t, are $X_s(j,s) = \left(p_s(t)\tilde{q}_s(j,t)N_s^{\beta}/\chi_s(j,t)\right)^{\frac{1}{1-\alpha}}$. Given this unique solution to the profit maximization problem, all firms in sector s will have the same productivity parameter (technology), $A_s(t)$. This enables us to determine wages in terms of the final good: $w_s(t) = \beta p_s(t)A_s(t)N_s^{-(1-\beta)}$ for $s = l,h$.

The skill premium (skilled wages relative to unskilled wage) is the main interest for this paper. Using (3), this skill premium, ω, is:[6]

$$\omega(t) \equiv \frac{w_h(t)}{w_l(t)} = \frac{\gamma_h}{\gamma_l}\left(\frac{A_h(t)}{A_l(t)}\right)^{\rho}\left(\frac{H}{L}\right)^{-(1-\beta\rho)}. \tag{6}$$

The skill premium increases when skilled workers become more *scarce*, $\frac{d\omega(t)}{dH/L} < 0$. This is the usual *substitution* effect and shows that, for given technology, the relative

[6] In this section, for some parameter values, skilled workers may have lower wages than the unskilled, i.e. $\omega \leq 1$. One may want to impose $\gamma_h/\gamma_l > (H/L)^{(1-\beta\rho)(1-\rho)-\beta\rho^2}$ to avoid this. When the supply of skills is endogenized in Section III, the skill premium is always positive, and this parameter restriction is not necessary.

demand curve for labor is downward sloping with elasticity $(1-\beta\rho)$. Moreover, when $\rho \in (0,1]$, $\frac{d\omega(t)}{dA_h(t)/A_l(t)} > 0$. This implies that when the skill-complementary technology improves, the skill-premium increases. The converse is obtained when $\rho < 0$. The conventional wisdom is that the skill-premium increases when skilled workers become more productive, not less productive. In support of this, most estimates reveal an elasticity of substitution between skilled and unskilled workers greater than 1 which implies $\rho > 0$.[7] The high degree of substitution between skilled and unskilled workers suggested by the increased share of college educated workers within almost all narrowly defined industries also corroborates this view (e.g. Autor, Katz and Krueger, 1997). Hence, in the remainder of the paper I limit attention to the case $\rho \in (0,1)$ (though the formal analysis does not depend on this parameter restriction).

C. Technological Advances

Technological advances take place as in Aghion and Howitt (1992) and Grossman and Helpman (1991). Namely,

$$\tilde{q}_s(j,t) = \begin{cases} \lambda \times \tilde{q}_s^-(j,t) & \text{if innovation at time } t \\ \tilde{q}_s^-(j,t) & \text{if no innovation at time } t \end{cases}$$

for all $j \in [0,1]$ and $s = l, h$ where $\tilde{q}_s^-(j,t)$ denotes the quality just before time t. $\lambda > 1$ so that innovations improve productivity. Also, I assume that $\lambda > \alpha^{-\frac{\alpha}{1-\alpha}}$, which is a simplifying assumption to be discussed in the next subsection.

Innovations are the result of R&D carried out by research firms using only final output as factor of production. There is free-entry into the R&D sector. If the total amount of R&D activity in technology j for sector s at time t is $z_s(j,t)$, then the probability of innovation is $z_s(j,t)\phi(z_s(j,t))$. The marginal cost of R&D effort (in terms of the final good) for inventing a machine of vintage $\tilde{q}_s(j,t)$ in sector s at time t is $B\tilde{q}_s(j,t)$ where B is a positive constant.[8] I assume that $\phi(.)$ is everywhere smoothly decreasing and $z\phi(z)$ is nondecreasing, that is $\phi(z) + z\phi'(z) \geq 0$. This implies that there are decreasing returns to R&D effort for any particular machine, but more effort increases the probability of discovery. Also, I impose $\lim_{z\to 0}\phi(z) = \infty$ and

[7]See Freeman (1986). Practically, all estimates of the short-run elasticity of substitution between high and low education workers are between $\sigma = 1$ and 2, which implies $1/\sigma = 1 - \beta\rho < 1$, and therefore $\rho > 0$. Since a large part of the substitution between skilled and unskilled workers is within industries, ρ should not be interpreted as the elasticity of substitution between different goods.

[8]Alternatively, the cost of improving vintage $\tilde{q}_s^-(j,t)$ is $B\lambda\tilde{q}_s^-(j,t)$.

$\lim_{z\to\infty}\phi(z) = 0$. These Inada type restrictions on $\phi(.)$ ensure an interior solution and smooth dynamics. In the Appendix, I also discuss the case where $\phi(z) \equiv 1$.

A firm that innovates has a monopoly right over that particular vintage (e.g. it holds a perfectly enforced patent), so it can charge a profit maximizing price and sell as many units of the newly discovered input as it wishes. The marginal cost of producing input $\tilde{q}_s(j,t)$ is $\tilde{q}_s(j,t)$ —i.e. it is linear in quality.

D. Equilibrium R&D Effort

The aggregate demands for technology characterized above are isoelastic, so the profit maximizing price for vintage $\tilde{q}_s(j,t)$ is $\chi_s(j,t) = \frac{\tilde{q}_s(j,t)}{\alpha}$. Hence, the price of each input is a constant markup over the marginal cost of production. This pricing policy follows from the assumption that $\lambda > \alpha^{-\alpha/(1-\alpha)}$ which ensures that even if the next best technology at time t $\hat{q}_s(j,t) = \tilde{q}_s(j,t)/\lambda$ were sold at marginal cost, firms would prefer to buy $\tilde{q}_s(j,t)$ sold at the monopoly price.[9]

Given the monopoly pricing policy every firm in the relevant sector will buy: $x_s(i,j,t) = X_s(j,t) = \left(\alpha p_s(t) N_s^\beta\right)^{1/(1-\alpha)}$. Therefore, the equilibrium productivity in sector s at time t, (5), can be written as: $A_s(t) = \frac{1}{\alpha} Q_s(t) \left(\alpha p_s(t) N_s^\beta\right)^{\alpha/(1-\alpha)}$, where recall that $N_h \equiv H$ and $N_l \equiv L$. Also I have defined $Q_s(t) = \int_0^1 \tilde{q}_s(j,t)dj$ for $s = l,\ h$. These expressions make it clear that the relevant measure of technological know-how in sector s at time t is $Q_s(t)$. I will refer to this as the level of technology in sector s.

The equilibrium demands for machines and the monopoly pricing policy imply that the profit level of a leading innovator with vintage $\tilde{q}_s(j,t)$ is: $\pi_s(j,t) = \frac{1-\alpha}{\alpha} X_s(j,t)\tilde{q}_s(j,t)$. Hence, the value of owning the leading vintage of machine j of sector s at time t is:

$$rV_s(j,t) = \pi_s(j,t) - z_s(j,t)\phi(z_s(j,t))V_s(j,t) + \dot{V}_s(j,t) \tag{7}$$

where $z_s(j,t)$ is the aggregate R&D effort to improve machine j in sector s at time t, and I have made use of the fact that the leading monopolist will not perform R&D (Aghion and Howitt, 1992). (7) is a standard value equation; at the flow rate $z_s(j,t)\phi(z_s(j,t))$, the firm loses its monopoly position because there is a new

[9]If this assumption were not satisfied, then the leading vintage of each technique would not be priced at the monopoly price, but at a sufficiently low price to exclude the next best technology (i.e. a limit price). The rest of the results would still remain unchanged, see Barro and Sala-i-Martin (1995) for a discussion of this case.

innovation, and the time derivative of V on the right-hand side of (7) takes care of the fact that $z_s(j,t)$ may be time varying.

Finally, since there is free-entry into R&D activities, additional R&D effort for any machine must not be profitable, thus:

$$\phi(z_s(j,t))V_s(j,t) = B\tilde{q}_s(j,t) \tag{8}$$

where the left-hand side is the marginal return to higher R&D effort directed at machine j in sector s (either by an existing research firm or by a new entrant), and the right-hand side is the marginal cost.[10]

An equilibrium in this economy requires that product, intermediate good and labor markets clear, firms buy the profit maximizing amount of the latest vintage of each technology, innovators follow the profit maximizing pricing policy, and there is no opportunity for any research firm to enter (or exit) and increase its profits. Equations (3), (6), (7) and (8) ensure these conditions and characterize the dynamic equilibrium. The balanced growth path (BGP) is then an equilibrium such that all variables are time-invariant.

E. Characterization of the Equilibrium

Let us start with the balanced growth path and thus drop all time dependence. Using $\dot{V} = 0$, equation (7) implies: $V_s(j) = \left((1-\alpha)\, X_s(j)\tilde{q}_s(j)\right) / \left(\alpha\left(r + z_s(j)\phi(z_s(j))\right)\right)$. Then, the free-entry condition (8) implies that in BGP:

$$\frac{1-\alpha}{\alpha B}\phi(z_s(j))\left(p_s N_s^{\beta}\right)^{1/(1-\alpha)} = r + z_s(j)\phi(z_s(j)) \tag{9}$$

for all $j \in [0,1]$ and $s = l, h$. A number of insights can be obtained from equation (9): research effort to improve machine j in sector s, $z_s(j)$, will be higher when the flow profit of the leading innovator for this machine is high. This flow of profit is increasing in p_s and N_s. Therefore, there is a *price* (p_s) and a *quantity* (N_s^{β}) effect on the incentives to innovate. The price effect is perhaps more familiar: when a certain product becomes more expensive, more effort will be spent to invent the next vintage. This price effect also suggests the reverse intuition to the one discussed in the introduction: when there are more skilled workers, the price of the goods they

[10] This ignores the constraint that total expenditure on R&D should not exceed current output, which will not apply if the economy can borrow from abroad at the rate r. Assuming that ϕ is sufficiently decreasing would also ensure that the expenditure on R&D never exceeds total output.

produce will be low, so there should be more R&D for technologies complementary to unskilled labor. Counteracting this is the quantity effect: when there are more skilled workers, the size of the market for skill-complementary technologies is larger. For $\rho \in (0,1]$, the quantity effect is more powerful, and I argued above why this case makes more sense in the context of the model.[11] Also, note that the quantity effect dominating does not imply that a higher supply of skilled workers will increase the skill premium. This will require further parameter restrictions.

Returning to equation (9), it immediately follows that $z_s(j) = z_s$ for all j and $s = l, h$. In other words, the BGP levels of effort for all skill-intensive (labor intensive) technologies are the same. Next, note that since there is a continuum of skill-intensive inputs, $z_h\phi(z_h)$ is exactly the rate of improvements. Hence $(\lambda - 1)z_h\phi(z_h)$ is the growth rate of $Q_h = \int_0^1 \tilde{q}_s(j)dj$. Similarly $(\lambda - 1)z_l\phi(z_l)$ is the growth rate of Q_l. For BGP, we need Q_h/Q_l to be constant, therefore $z_l = z_h$. Equation (9) then implies that $p = (H/L)^{-\beta}$ along the BGP. Combining this with (3), we obtain:

$$Q \equiv \frac{Q_h}{Q_l} = \left(\frac{\gamma_h}{\gamma_l}\right)^{\frac{1}{1-\rho}} \left(\frac{H}{L}\right)^{\frac{\beta\rho}{1-\rho}} \tag{10}$$

This equation is a crucial result. Q_h/Q_l is the equilibrium technology of the skill-intensive sector relative to the labor intensive sector, and depends on the relative abundance of the two types of labor. The greater the fraction of skilled workers in the economy, the greater their relative productivity as compared to unskilled workers (this is for $\rho > 0$; as noted above, the reverse obtains when $\rho < 0$). The intuition for equation (10) is that equilibrium returns to R&D effort in the two sectors have to be equalized. Since profits to innovation are proportional to the market size, they are effectively proportional to the number of workers using the technology. Therefore, when H increases relative to L, innovation and R&D in the skill-intensive sector become more profitable. This translates into a higher Q_h/Q_l in the BGP. I refer to this as the *directed technology effect*: when the relative supply of skill changes, the *direction* of technical progress changes, leading to different equilibrium technologies for skilled and unskilled labor. Equation (10) immediately implies an important result: as the economy accumulates more skills, technical change will respond to make new technologies more complementary to skilled labor. Therefore, the fact that

[11] For some other situations, the price effect may dominate. For example, Hayami and Ruttan (1970) discusses the different paths of agricultural development in the U.S. and Japan. The scarcity of land in Japan relative to the U.S. appears to have induced a much faster rate of innovation and adoption of fertilizers, increasing output per acre.

new technologies have become more skill-complementary over time is consistent with the approach in this paper.

The BGP R&D effort level can now be determined from (3), (9) and (10) by imposing $z^* = z_l = z_h$, which gives:

$$\frac{r + z^*\phi(z^*)}{\phi(z^*)} = \frac{1-\alpha}{\alpha B}\left[\gamma_h H^{\beta\rho/(1-\rho)} + \gamma_l L^{\beta\rho/(1-\rho)}\right]^{\frac{1-\rho}{\rho(1-\alpha)}} \tag{11}$$

Finally, using (6), we have (proof in the Appendix):

Proposition 1 *There is a unique balanced growth path (BGP) where both sectors and total output grow at the rate $(\lambda - 1)z^*\phi(z^*)$ with z^* given by (11). Along the BGP, Q_h/Q_l is given by (10) and the skill premium is:*

$$\omega = \left(\frac{\gamma_h}{\gamma_l}\right)^{\frac{1}{1-\rho}}\left(\frac{H}{L}\right)^{\eta}, \tag{12}$$

where $\eta \equiv \frac{\beta\rho^2}{1-\rho} - (1-\beta\rho)$.

In the unique BGP there is a 1-to-1 relation between the relative supply of skilled workers and their relative wage. But, this relation can be either increasing or decreasing. The force that tends to make it decreasing is the second term in η, $-(1-\beta\rho)$, which is the elasticity of the relative demand curve for skills for given A_h/A_l (recall equation (6)). This is the usual *substitution effect*, leading to a downward-sloping relative demand curve for skills. If technology were exogenous in this economy, the skill premium would be determined solely by this effect as in equation (6).[12] The counteracting force is the *directed technology effect* which works through changes in relative technologies (Q_h/Q_l). An increase in H/L leads to more R&D activity in the skill-complementary technologies and therefore increases Q_h/Q_l, shifting the "short-run relative demand curve" to the right as in Figure 1. If this effect is sufficiently strong, that is if $\frac{\beta\rho^2}{1-\rho}$ is large enough, the directed technology effect can dominate, and the long-run relative demand curve for skills is *upward sloping.* In this case, an increase in the number of skilled workers relative to the number of unskilled workers will lead to a higher relative price of skill in the long-run. This "perverse" case is more likely to happen when ρ is close to 1 so that the skill-intensive and labor-intensive

[12] A hybrid case is where technology is exogenous, thus there is no R&D, but $A_s(t)$ is still determined by purchases of machines from a monopolist. In this case, the skill premium is again a decreasing function of relative supply of skills, H/L, albeit with the smaller elasticity, $-(1-\beta\rho-\alpha\rho)/(1-\alpha\rho)$, which is the short-run elasticity in the full model as shown in Proposition 3.

production processes (intermediate goods) are close substitutes, and when β is close to 1 so that there are only limited decreasing returns to labor within each sector.

A different intuition for the possibly upward sloping relative demand curve is that there is an important *nonconvexity* in this economy. There is a fixed upfront cost of discovering new technologies,[13] and once discovered, they can be sold to many firms at constant marginal cost. This nonconvexity (nonrivalry of technology use) implies that profit-maximizing R&D firms are more willing to improve the technologies designed for a larger clientele. This is the essence of the *directed technology effect,* and if powerful enough, it ensures an increasing relation between the relative supply of skilled workers and their long-run relative wage. There is an instructive limit case where this effect disappears: $B = 0$ so that there are no upfront fixed costs of discovering new technologies. To make the economy well behaved in this case, also impose $\lim_{z\to\infty} z\phi(z) = \phi_0 < \infty$. Then, we have $z_s \to \infty$ for $s = l, h$, and the growth rate is equal to $(\lambda - 1)\phi_0$. Because the market size for technologies is no longer important, the skill-premium is now decreasing in the relative supply of skills, with an elasticity $-(1 - \beta\rho - \alpha\rho)/(1 - \alpha\rho)$, which is unambiguously negative.

Next, it can be established that the BGP characterized in Proposition 1 is (saddle path) stable. For this proposition, define ϵ_ϕ as the elasticity of the function ϕ. Then (analysis and proof in the Appendix):

Proposition 2 1. *Locally, there exists a unique transition path converging to BGP, so that if $Q \neq Q^*$, then z^h and z^l jump and Q monotonically adjusts to Q^*. If $Q < Q^*$, then $z_h > z_l$ along the transition path and vice versa.*

2. *Suppose $\epsilon_\phi(z)$ is nonincreasing in z. Then, for all $Q \neq Q^*$, there is a globally unique saddle path to BGP along which Q monotonically converges to Q^*. If $Q < Q^*$, then $z_h > z_l$ along the transition path and vice versa.*

The system is always locally stable, and under a fairly weak assumption, it is also globally well-behaved. Figure 2 illustrates the local dynamics with a phase diagram.

F. The Dynamic Response to a Relative Supply Shock

The following result summarizes the dynamic response of the economy to an unanticipated relative supply shock (proof in the Appendix):

[13]The expected discounted cost of discovering vintage $\tilde{q}$ along BGP is $z^* B\tilde{q}/(r + z^*\phi(z^*))$.

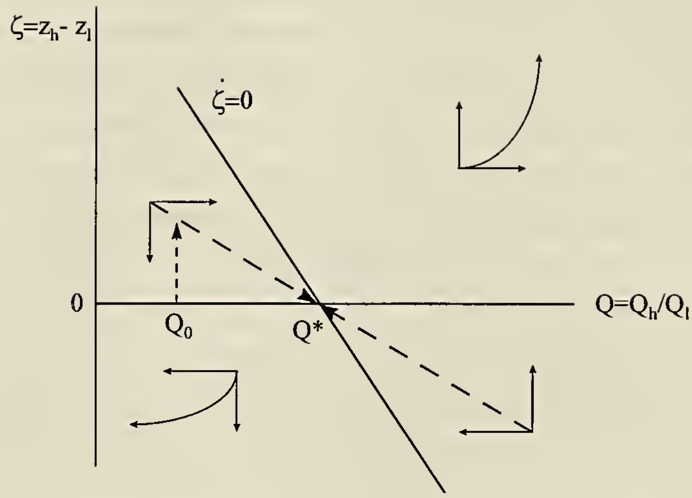

Figure 2: Transition Dynamics Around the Balanced Growth Path.

Proposition 3 *Consider an unanticipated increase from* $\frac{H}{L}$ *to* $\delta\frac{H}{L}$ *at time t starting from a BGP with skill premium* $\omega = \bar{\omega}$. *Immediately after the shift, the skill premium falls to* $\hat{\omega}$ *where* $\log\hat{\omega} - \log\bar{\omega} = -\theta\log\delta$, *and* $\theta \equiv \frac{1-\beta\rho-\alpha\rho}{1-\alpha\rho}$, *and* z_{ht}/z_{lt} *jumps up. The new BGP skill premium* $\tilde{\omega}$ *is such that* $\log\tilde{\omega} - \log\bar{\omega} = \eta\log\delta$.

Therefore, immediately after the relative supply shock, the skill premium falls by $-\theta\log\delta$.[14] The short-run response is for given technological know-how; in other words, Q_h/Q_l is a stock variable and only changes slowly. In terms of Figure 1 in the introduction, this is a move along the short-run (constant technology) relative demand curve. As the economy adjusts to its new BGP, Q_h/Q_l increases (as long as $\rho > 0$), and the skill premium starts increasing from its short-term low. In terms of Figure 1, the constant technology relative demand curve shifts to the right. Therefore, an increase in the relative supply of skills will create a period of rising skill premium in response to the induced shift in the relative demand for skills. This result is not very surprising: many believe that the short-run elasticity of substitution between skilled and unskilled workers is less than the long-run elasticity. So, when $\eta < 0$, the analysis in this paper formalizes this claim. In contrast, when $\eta > 0$, the model's predictions are more surprising and original. Because with the increase in H/L the

[14]Recall that $\delta > 1$ and $-\theta$ is negative because $\beta \leq 1-\alpha$. Also observe that the elasticity is not simply $-(1-\beta\rho)$ because for given Q_h/Q_l, the equilibrium productivities A_h/A_l change in response to a change in H/L.

size of the market for skill-complementary technologies increases, there is more R&D to upgrade these technologies and the skill premium increases above its initial value.

Proposition 3, especially in the case where $\eta > 0$, offers an alternative explanation for the behavior of the U.S. economy during the past twenty five years. There was a large increase in the supply of skills in the 1970s. To put it into perspective, observe that the employment share of college graduates increased by 1.7 percentage points from 1940 to 1950, by 2.8 percentage points from 1950 to 1960, and by 2.8 percentage points from 1960 to 1970. In contrast to this relatively slow increase before 1970, it increased by 6.6 percentage points from 1970 to 1980. Also during this decade, the share of workers with some college but no college degree increased by 7.2 percentage points.[15] These are very large changes in the relative supply of skills preceding the rise in the college premium.

Furthermore, these large supply changes were at least partly exogenous rather than a simple response to anticipated higher returns to education in the future. Enrollment rates had been increasing since the mid 1950s and this trend continued in the 1960s. Two other factors contributed to the sharper increase during the 1960s: (1) until almost the end of the war, the Vietnam era draft laws exempted males enrolled in college from military service. This induced many more young males to stay in college during the late 1960s in order to avoid the draft (see Baskir and Strauss, 1978); (2) government financial aid for college increased by a large amount during this era. The programs of guaranteed student loans, supplemental loans for students and parental loans for undergraduate students began in the 1960s and provided credit to college students, and the Pell Grant program which subsidizes college tuition costs began in 1973, and created yet another rise in college enrollments from 1974 onwards, especially for the group of students with low income parents (see McPherson and Schapiro, 1991, and Kane, 1994). For example, the total federal aid to college students that stood at approximately 2 million dollars in 1963 increased to $14 million in 1970-71 and then to $24 million in 1975-76 (all numbers in 1989-90 dollars). McPherson and Schapiro (1991) estimate that without government support, college enrollments would have been 20% lower during this period (see also Leslie and Brinkman, 1987). Even though 20% may be an overestimate of the direct effect of government support, it is probably in the right range when the effects of Vietnam era draft are taken in to

[15]See Autor, Katz and Krueger (1997). These figures refer to the shares of full-time equivalent (hours worked) of college graduates divided by the full-time equivalent of all workers. The changes in the share of these workers in the labor force are very similar.

account. Using this number, a back-of-the-envelope calculation suggests that without these exogenous factors, the share of college graduates which increased from 13.8% to 20.4% from 1970 to 1980 would have only increased to 17%. Therefore, in terms of the model of this section, there was a substantial and exogenous increase in H/L. The theory predicts that in response to this large increase in H/L, the skill premium should fall first, and then start increasing due to the directed technical change effect. This pattern matches the broad behavior of the U.S. college premium from 1970 to the present. If $\eta < 0$, the increase in the skill premium after the supply shock would not compensate for the initial fall. In contrast, with $\eta > 0$, the model predicts that the technical change should take the skill premium *above* its initial level as was the case in the U.S.

It is also interesting to do some back-of-the-envelope calculations to see whether for reasonable parameter values, the model makes realistic predictions. The three important parameters are β, α and ρ. Recall that ρ is the degree of substitution between the skilled and unskilled production processes, and β is a measure of differential skill intensity of the two production processes[16], and α is the return to machines in the production process. These parameter are not easily pinned down. I choose $\alpha = 0.3$ as a usually accepted value of the returns to machines (capital). I used two different pieces of information to determine β. First, Dunne, Doms and Torske (1997) report the share of workers with college degrees across plants using different technologies. Approximately 33% of employees at establishments using the most advanced technologies are college graduates, whereas the same ratio is less than 10% for those using the least amount of advanced techniques. Second, I calculated the difference between industries at the 90th and10th percentiles in terms of the wage bill share of workers with at least some college education among the 142 industries used in Autor, Katz and Krueger (1997) in the census years 1970, 1980 and 1990.[17] In all three years, this differential is approximately 0.4 (0.40 in 1990, 0.41 in 1980 and 0.38 in 1970). If the number of (full-time equivalent) workers with some college is used instead of their wage bill share, the 90-10 differentials across industries are quite similar. Based on these numbers, I choose values for β between 0.3 and 0.45. Finally, there is a range

[16]In the model, β is also related to the share of labor in total output, but this is due to the simple structure of the model. A more realistic setup would involve both production processes using skilled and unskilled labor, for example, $Y_h = A_h (H_h)^{\beta_1} (L_h)^{\beta_2}$. In this case, β corresponds to $\beta_1 - \beta_2$. Unfortunately, this more general model does not yield analytical solutions.

[17]I thank David Autor for providing me with these industry data calculated from the censuses by Autor, Katz and Krueger (1997).

of estimates for the short-run elasticity of substitution between college graduate and non-college workers. Katz and Murphy (1992) estimate this short-run elasticity as $\sigma = 1.41$. Bound and Johnson (1992) using a slightly different technique, estimate it as $\sigma = 1.70$. Freeman (1986) surveys a number of estimates of this elasticity and concludes that it should be in the range between 1 and 2. In fact, most studies using U.S. data put in between 1.2 and 1.8. In the model, if we consider the short-run as a situation where A_h/A_l is constant, we would have $1 - \beta\rho = 1/\sigma$. Or if we consider the short-run to involve Q_h/Q_l constant but A_h/A_l variable, then we would have $\theta \equiv (1 - \beta\rho - \alpha\rho)/(1 - \alpha\rho) = 1/\sigma$. Therefore, for given α and β, I choose ρ to place both of these numbers between 0.55 and 0.85, which implies σ between1.2 and 1.8, approximately equal to the estimates of Katz and Murphy and Bound and Johnson.

I take the relative supply shock to be the increase in the ratio of college graduates to high school graduates from 1971 to 79, because 1971 was the starting year for the large change in the skill composition of the labor force.[18] This gives the relative supply shock, $\Delta \log(H/L) = \log \delta$, as 0.4 (Katz and Murphy, 1992, Table 8). In addition to the implied (inverse) elasticities of substitution between skilled and unskilled workers and the implied value of η, I report three numbers. The first is the impact effect, $-\theta \log \delta$, which is the immediate effect of the increase in supply as given by Proposition 3. This has no counterpart in reality. The second is the "short-run" response. In the data, I take this to be the proportional change in the skill premium as measured by the average weekly wages of college graduates divided by the average weekly wages of high school graduates from 1971 to 1979, which is reported as -0.10 by Katz and Murphy. This number cannot be directly compared to the impact effect of one time shock given in Proposition 3 because in reality the supply shock took place over a number of years, so technology must have adjusted during this period. Therefore, I compare this number to $\frac{\log \hat{\omega} + \log \tilde{\omega}}{2} - \log \bar{\omega}$, which is the simple average of the change immediately after the shock and the long-run response.[19] Finally, I take the long-run response to be the change in the skill premium from 1971 to 1987 which is reported as 0.024 by Katz and Murphy (1992, Table 8), and compare this to $\eta \log \delta$ $(\log \tilde{\omega} - \log \bar{\omega})$ which is the long-run response implied by the model. Table 1 (at the back) summarizes

[18]Those with exactly 12 years of schooling in the CPS are considered high school graduates and those with at least 16 years of schooling are college graduates. The other categories are ignored, but including them and constructing other measures of supply shifts gives very similar results.

[19]More appropriately, one should use a weighted average depending on the speed of convergence reported in the Appendix. This depends on the exact time path of changes in relative supply and on the function ϕ. However, I see no simple way of mapping the function ϕ to data at this point.

these data and the predictions of the model.

The results in Table 1 suggest that the model's quantitative predictions are not out of line with the data. For a range of parameter choices, the model gives numbers very close to the data. In particular, for $\beta = 0.35$, $\alpha = 0.3$ and $\rho = 0.75$, the model gives short-run elasticities in the right range and implies $\eta = 0.05$. This predicts a short-term fall in the skill premium comparable to the one in the data and then a very large increase in the college premium after this initial fall that takes this premium about 3% above its initial value, which is quite close to the actual behavior of the college premium.[20] Similar results are obtained when $\beta = 0.4$ and $\rho = 0.73$, or when $\beta = 0.45$ and $\rho = 0.7$. Naturally, these numbers should not be overinterpreted because the model is very abstract, and many other changes took place during this time period (including further increases in H/L during the 1980s). Also, if β is reduced below 0.3 or increased above $\beta = 0.5$, the results are no longer in line with the observed patterns. A more careful calibration exercise, taking into account the slow adjustment of technology, the gradual nature of the supply shock and education responses to the changing returns is left for future work.

G. Technical Change and Residual Wage Inequality

The paper so far only analyzed the evolution of the skill premium in response to changes in relative supplies. Based on these results and interpreting the skilled workers as those with a college degree, I suggested that a model incorporating the directed technical change can match the evolution of the college premium in the U.S. However, there are other aspects of the changes in the structure of wages. First, male-female wage differentials have narrowed, and the returns to experience for low education workers increased. I will discuss these in the concluding section. Second, residual (within group) wage inequality began increasing during the 1970s while the

[20] There is a way of obtaining an alternative estimate of η: use equation (12) and regress $\log \omega$ on $\log (H/L)$. With a sufficiently long time series, the slope coefficient gives η. To do this, I took the 25 years of data used by Katz and Murphy and added the data reported by Autor, Katz and Krueger (1997) from the censuses of 1940, 1950, 1960, 1990 and CPS 1995. To obtain $1/\sigma$, Katz and Murphy regress $\log \omega$ on $\log (H/L)$ and a linear time trend. Repeating their exercise with these data gives a similar (but somewhat larger) elasticity $\sigma = 1.66$. I then regressed $\log \omega$ on $\log (H/L)$, *without a linear time trend,* using robust regression (*rreg* in STATA). This gave $\eta = 0.045$ with a t-statistic of 2.3. Since there are only 30 data points, this result should be interpreted with caution. Nevertheless, it is quite encouraging for the theory that this regression gives a value of η very close to the one obtained from the simple calibration exercise, which is also the value necessary to match the behavior of the skill premium from 1970 to the present.

college premium fell (but see also DiNardo, Fortin and Lemieux, 1996). Is this pattern consistent with the approach in this paper? In this subsection, I suggest a simple extension of the model which suggests that the answer is yes.

Suppose that skills are two dimensional: education and ability (for lack of a better term). A fraction $\mu_h > 1/2$ of college graduates are high ability, and the remainder have low ability. The same fraction is $\mu_l < 1/2$ for non-college graduates. For example, ability could be related to the curriculum of college, but not perfectly, so that some of the college graduates do not acquire the necessary ability, while some other workers do in spite of not having attended college. Therefore, ability in this world is not innate, but acquired partly through education. There are H college graduates and L low education workers. Suppose also that the aggregate production function of the economy is (dropping time arguments):

$$Y = \left[\gamma_{hh}\left(A_h\left(\mu_h H\right)^{\beta}\right)^{\rho} + \gamma_{hl}\left(A_h\left(\mu_l L\right)^{\beta}\right)^{\rho} + \gamma_{ll}\left(A_l\left((1-\mu_l)L\right)^{\beta}\right)^{\rho} + \gamma_{lh}\left(A_l\left((1-\mu_h)H\right)^{\beta}\right)^{\rho}\right]^{1/\rho}$$

which basically combines the equivalent equations to (2) and (4), and also imposes that all firms use the same technology and employ all workers, which will be true in equilibrium as shown above. The important assumption embodied in this expression is that technologies are not complementary to education but to *ability* (see Bartel and Sicherman, 1997, for some evidence in support of this). Therefore, both able college graduates and able high school graduates use the same technologies. An analysis similar to the one used previously implies that in BGP, the state of relative technology has to be (see the Appendix for details):

$$\frac{Q_h}{Q_l} = \left[\frac{\gamma_{hh}^{\nu}\left(\mu_h H\right)^{\beta\rho\nu} + \gamma_{hl}^{\nu}\left(\mu_l L\right)^{\beta\rho\nu}}{\gamma_{lh}^{\nu}\left((1-\mu_h)H\right)^{\beta\rho\nu} + \gamma_{ll}^{\nu}\left((1-\mu_l)L\right)^{\beta\rho\nu}}\right]^{\frac{1}{(1-\rho)\nu}} \tag{13}$$

where $\nu \equiv (1-\alpha\rho)^{-1}$. When the number of high ability workers is larger, the BGP ratio of technologies complementary to ability to other technologies has to be larger. This has exactly the same intuition as the previous results.

Now consider an exogenous increase in H/L. Because $\mu_h > 1/2 > \mu_l$, this increases the proportion of high ability workers in the labor force and induces a rise in Q_h/Q_l. The college premium behaves as in the previous section: first it declines because there are more H workers, and then increases with Q_h/Q_l because there is a large fraction of high ability workers among the college graduates. In contrast to the college premium which falls first, *residual wage inequality starts increasing immediately after the shock.* To see this, consider the two measures of residual wage inequality in this model; $\omega^h = w_{hh}/w_{hl}$ and $\omega^l = w_{lh}/w_{ll}$, that is the ratio of the wages of high to low ability workers within their education groups. Similar arguments to those developed before imply that (see the Appendix):

$$\omega^h(t) = c_l \left(\frac{Q_h(t)}{Q_l(t)}\right)^{\kappa} \left(\frac{\mu_h}{1-\mu_h}\right)^{-\theta} \text{ and } \omega^l(t) = c_l \left(\frac{Q_h(t)}{Q_l(t)}\right)^{\kappa} \left(\frac{\mu_l}{1-\mu_l}\right)^{-\theta}$$

where $\kappa \equiv \rho(1+\beta\rho\alpha - \beta\alpha - \alpha\rho)/(1-\alpha\rho) > 0$, and recall that $\theta \equiv \frac{1-\beta\rho-\alpha\rho}{1-\alpha\rho} > 0$. Also, c_l and c_h are suitably defined constants. Since after the relative supply shock Q_h/Q_l begins to increase, residual wage inequality both among the college graduates and among the non-college workers increases immediately.

Therefore, if technologies are more complementary to *ability* rather than to schooling, the approach developed in this paper predicts that in response to an exogenous increase in the relative supply of educated workers, there should first be a drop in the college premium followed by a subsequent increase and throughout this process, residual wage inequality should increase.[21]

H. Discussion

There are a number of issues I wish to discuss informally.

1. The above model can be extended to many groups without affecting the basic results. For example, there could be S groups, each with their own technology. In this case, the model would predict, as Katz and Murphy (1992) and Bound and Johnson (1992) found in the data, that the groups experiencing increasing relative supply may have higher wages. Whether this is a plausible explanation will depend on whether these groups appear to use different technologies in practice. College graduates clearly use different technologies than high school graduates. For other groups, however, this may be less so. I will return to this issue in the concluding section.

2. For simplicity, R&D for sectors l and h were completely separated. If R&D in sector h leads to a discovery of better sector h machines with probability $\psi > 1/2$ and to the discovery of sector l machines with probability $1-\psi$, our results would remain unchanged, only the speed of convergence would be affected. Also, for the results of this paper to apply, it is not necessary that economic motives determine all innovations. Clearly some innovations are exogenous and stem from the advances in

[21]The only other paper that I am aware of which accounts for the increase in residual wage inequality at the same time as the return to education was falling is Galor and Tsiddon (1997), which is again based on exogenous (skill-biased) technical change.

basic science, and we may call these "macroinventions" following Mokyr (1990). It is sufficient that economic motives determine the direction in which these macroinventions are developed. For example, the discovery of the microchip may be largely exogenous, but using this chip to develop personal computers and Windows 95 would be due to the profit opportunities offered by the use of these products.

3. The model purposefully made R&D neutral in its demand for skills (i.e. it uses final output). If the R&D sector is more skill intensive than the rest of the economy, there will be another reason for the returns to skill to increase after a relative supply shock because total R&D effort goes up during periods of transition.

4. Also, observe that in this economy there is no difference between "sector-specific" and "skill-specific" technologies. To draw a distinction between these two cases, suppose $Y_h = B_h F_h(A_h H_h, A_l L_h)$ and $Y_l = B_l F_l(A_h H_l, A_l L_l)$ where F_h is more skill intensive than F_l. Now, B_h and B_l are sector-specific technologies and A_h and A_l are skill-specific. It is straightforward to see that if $B_h = B_l = B$ and R&D firms perform research to improve A_h and A_l, all the results would carry over. What is less clear, but I conjecture to be true, is that if $A_h = A_l = A$, and R&D is directed towards either B_h or B_l, the same results would also obtain. Unfortunately, such a model is very difficult to solve.

5. In this model, an increase in total population leads to a higher growth rate as in the models of Aghion and Howitt (1992) and Grossman and Helpman (1991). This is not important for the focus of the paper. All the results of interest continue to hold if we impose $z_h + z_l = \bar{z}$, but this scale effect is removed. Also, it can be observed from (11) that an increase in H/L leaving total population unchanged may increase or decrease the BGP growth rate. This is because the economy reaches maximum growth when the two sectors are balanced. The restriction $z_h + z_l = \bar{z}$ would also remove this dependence of the growth rate on H/L.

6. More interestingly, the fact that $\phi(.)$ is decreasing implies that the growth rate of the economy declines during transition to a new BGP. This is because during adjustment, z_h increases and z_l falls, and with $\phi(.)$ decreasing, the faster technological improvements in skill-complementary technologies do not compensate for the slowdown in the productivity growth of unskilled workers. Therefore, this approach can account for the growth slowdown during the process of "skill-biased technical change". This result has some similarity to Greenwood and Yorukoglu (1996) and Caselli (1997) who also obtain slower growth during the process of adjustment to new technologies because of costs of adoption and learning. In contrast to these papers,

technological change is endogenous here, and the slower growth during the process of adjustment is because the economy invests mainly in improving skill-complementary technologies at the expense of technologies complementary to unskilled labor.

III. Endogenizing The Choice of Skills

Section II treated the relative supply of skills as exogenous, and mapped it to the supply of college graduates in the data. The increase in the supply of college graduates during the late 1960s and 1970s was argued to be largely exogenous rather than a simple response to anticipated higher returns in the future. Nevertheless, education choices are to some degree forward-looking and respond to the returns. Therefore, it is important to endogenize the choice of skills and ensure that the main results are robust. There are two other issues for which endogenizing the choice of skills is important. First, as noted above, when H/L is treated as exogenous, ω can be less than 1, that is skilled workers may be paid less than the unskilled. Once the choice of skills is endogenized, ω will always be greater than 1. Second, the above theory explained the changes in the structure of wages by using a large increase in H/L. It is important to know whether for the approach to work, exogenous factors need to be responsible for all of the increase in relative supplies, or a relatively small impulse to the cost of education, coupled with the general equilibrium changes in the skill premium, could lead to a large increase in H/L.

Suppose now that there is a continuum 1 of unskilled infinitely lived agents in the economy at date $t = 0$. Each unskilled worker chooses whether and when to acquire education to become a skilled worker. For agent x it takes K_x periods to become skilled, and during this time, he earns no wages. The distribution of K_x is given by the function $\Gamma(K)$ which is the only source of heterogeneity in this economy. This can be interpreted as due to credit market imperfections or differences in innate ability.[22] The rest of the setup is unchanged. To simplify the exposition, I assume that $\Gamma(K)$ has no mass points other than at $K = 0$.

I now define a BGP as a situation in which H/L and relative wages remain constant. It is straightforward to see that in BGP, there will be a single-crossing type property. That is, if an individual with cost of education K_x chooses schooling, another with $K_{x'} < K_x$ must also acquire skills. Therefore, there will exist a cutoff

[22] This formulation is general enough to nest the case in which the economy starts out with a combination of skilled and unskilled workers, and then the unskilled decide whether to acquire skills. This is because $\Gamma(K)$ can have a mass point at 0 who will immediately become skilled.

level of talent, $\bar{K}$, such that all $K_x > \bar{K}$ do not get education. Then the first equation that must hold along BGP is *the relative skill condition*:

$$\frac{H}{L} = \frac{\Gamma(\bar{K})}{1 - \Gamma(\bar{K})} \tag{14}$$

The second condition for a BGP imposes that all workers choose the privately optimal skill level. Once again, we can simply look at an agent with talent $\bar{K}$ and make sure that he is indifferent between acquiring skills and not. Suppose $\bar{K}$ does not acquire any skills. Then, his return at time t, R^{ne}, can be written as:
$R_t^{ne} = \int_t^{\infty} \exp(-r(\tau - t)) w_{l\tau} d\tau = w_{lt} \int_0^{\infty} \exp(-(r-g)\tau) d\tau = w_{lt}(r-g)$ where I have used of the fact that along the BGP wages grow at the constant rate $g = (\lambda - 1) z^* \phi(z^*)$ which is itself a function of H/L. In contrast, if $\bar{K}$ decides to acquire education, he receives nothing for a segment of time of length $\bar{K}$, and receives w_{ht} from then on. Therefore, the return to agent $\bar{K}$ from acquiring education, $R^e(\bar{K})$, can be written as: $R_t^e(\bar{K}) = \int_{t+\bar{K}}^{\infty} \exp(-r(\tau - t)) w_{h\tau} d\tau = \exp(-(r-g)\bar{K}) w_{ht} / (r-g)$.

In BGP, $R_t^e(\bar{K}) = R_t^{ne}$ for all t. Hence, the second equation that must hold in BGP, is *the indifference condition for $\bar{K}$*:

$$\left(\frac{\gamma_h}{\gamma_l}\right)^{\frac{1}{1-\rho}} \left(\frac{H}{L}\right)^{\eta} = \exp((r - g(H/L))\bar{K}) \tag{15}$$

where g the BGP growth rate as a function of H/L and the left-hand side of (15) is the skill premium from (12). A BGP equilibrium with endogenous skill formation is given by the intersection of (14) and (15). The relative skill condition, (14), is everywhere increasing as drawn in Figure 3. The indifference condition, (15), can be decreasing or increasing. In particular, when $\eta > 0$, (15) is likely to be upward sloping, and multiple BGP equilibria, as drawn in Figure 3, are possible. Intuitively, when $\eta > 0$, a higher H/L increases ω, encouraging workers with high K to obtain education and increasing H/L further.

We can think of government policy (e.g. the grant programs in the U.S. or the Vietnam era draft laws) as reducing the cost of education, and shifting $\Gamma(K)$ to the left. For given $\bar{K}$ such a left-ward shift of $\Gamma(K)$ would increase H/L. If $\eta > 0$, the return to education ω would also rise, thus raising $\bar{K}$ and H/L further. Therefore, the prediction of the model in this case is that subsidies to education lead to an increased tendency to acquire education and also to a larger education premium due to the *directed technology effect.*

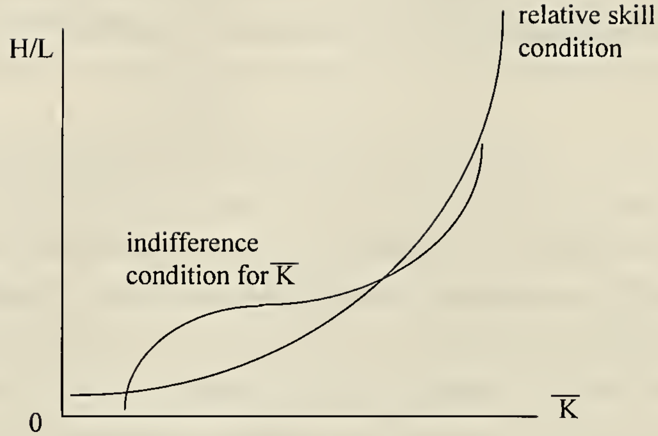

Figure 3: Balanced Growth Path Equilibrium With Endogenous Skills.

Unfortunately, it is quite difficult to work out the full dynamics of education, technology and wages. This is due to two reasons: (i) away from the BGP, even if an agent with K_x finds it beneficial to get education at date t, some agents with $K < K_x$ may prefer to wait; (ii) agents acquiring education will not take part in the production process and this will influence relative wages and education incentives. Nevertheless, without carrying out the full analysis, two key properties can be determined: (a) the equilibrium with the highest H/L is locally stable starting with H/L smaller than the BGP level; (b) even when $\eta > 0$, the economy will never jump to a new equilibrium, instead it will travel there slowly by building more skills and on the way creating more skill-complementary technologies. For the first property, note that since by construction, H/L can never decrease, the system cannot cycle, and retains the same properties as before, thus is locally stable. The reason why it does not jump to the new BGP but adjusts slowly is the same as in Section II: technologies only adapt slowly (Q_h/Q_l is a stock variable). When a large fraction of agents decide to acquire education, for a long while wages will actually be lower for the skilled workers. Moreover, with $\eta > 0$, the relative wage of unskilled workers is lowest when H/L is at its highest level. So, an ideal time to make human capital investments is when all other workers have already completed their investments. Therefore, some of the workers will have an incentive to wait a long while before starting to invest in skills, creating a type of war of attrition and leading to a slow rise in H/L. This pattern of slowly increasing H/L and a gradual shift in technologies towards more skill-complementarity is similar to the experience of the U.S. economy over the past century.

Perhaps the most interesting exercise to perform is to see how much of the increase in H/L in the 1970s can be attributed to the endogenous propagation of the government impulse. For example, if the model does not offer any propagation, all of the change in relative supplies must be due to exogenous factors (i.e. none of these agents would have gone to college without the government's subsidy and Vietnam era draft laws). For a simple calibration, I take $r = 0.05$, and treat the growth rate $g(H/L)$ as constant and equal to 0.02. I take the distribution of education costs, $\Gamma(K)$, to be triangular with one unknown parameter, which is the simplest approximation to the normal distribution. In particular $\Gamma(K) = \frac{1}{2a^2}K^2$ for $K \leq a$ and $\Gamma(K) = -1 + \frac{2}{a}K - \frac{K^2}{2a^2}$ for $K > a$. I start the economy with $\Gamma(\bar{K}) = 0.14$ (i.e. $H/L = 0.16$) which is approximately the relative share of college graduates in the labor force in 1970 (Autor, Katz and Krueger, 1997), and $\omega = 1.54$ which is the college high-school wage differential in 1970. Solving equations (14) and (15) gives $a \approx 27$ and $\bar{K} = 14.4$. Next, I perform the following exercise. Suppose that ω rose to 1.65 (approximately its value in 1990). How much would H/L have increased without any change in the distribution Γ? With $w = 1.65$, the new cutoff level would have been $\hat{K} = 16.7$ and H/L would have increased to 0.23. This is a 44% increase in H/L implied by the dynamics of the model. So, if we limit ourselves to the original 40% increase in H/L between 1971 and 1979 or to the 54% increase between 1970 and 1980, it would have been sufficient for the government to induce only a small fraction of the poor (constrained) agents, who would have otherwise remained unskilled, to go to college. The rest of the agents would have acquired skills in response to the changing returns, creating the large increase in the supply of college graduates and the resulting changes in equilibrium technologies. However, this calculation has to be interpreted with care. H/L continued to increase after 1980. For example, the fraction of full time equivalents with a college degree increased from 21% in 1980 to 26% in 1990. So, if we compare the predicted increase in the supply skills to the change from 1970 to 1990, approximately half of the increase in skills must be directly caused by the government, rather than about 10% of it. Nonetheless, this simple exercise suggests that small changes in education costs can cause large increases in the relative supply of skills and important changes in the structure of wages.

IV. The Impact of Trade on Technology

Increased trade with the LDCs (the South) where skilled labor is scarce is often suggested as a potential cause of increased wage inequality and contrasted to the explanations based on technology. Since technology has been treated as exogenous in the wage inequality literature, there has been little effort in uncovering the links between these two explanations. This section will show that the *direction* of technical change is influenced by trade, thus modifying or qualifying many of the conclusions reached in the previous literature regarding the impact of trade on inequality.

Suppose that an economy in BGP, the North, with the ratio of skilled to unskilled workers equal to H^N/L^N, begins trading with an economy, the South, which has a skill ratio $H^S/L^S < H^N/L^N$. There is no endogenous skill accumulation in either economy. What happens to wage inequality in the North? I will answer this question under three different scenarios: (a) no directed technical change; (b) directed technical change and new technologies sold to firms in the South on the same terms as firms in the North; (c) directed technical change and no property rights enforcement in the South.

The first scenario is for benchmark, and the truth presumably lies somewhere between 2 and 3, so that there is some sale of technology to the firms in the South, but the enforcement of intellectual property rights is less than perfect. Throughout this section, there will be no endogenous skill formation and I will simply compare BGP's. Also, in this model, factor price equalization is guaranteed without further restrictions because each sector employs only one of the non-traded factors (see Ventura, 1997, for a similar structure).

A. No Technical Change

Let A_l and A_h be exogenously given. Denote the steady state (BGP) skill premium in the North before trade opening by ω^N, and the skill premium after trade opening by ω^W, and let $\Delta \log \omega = \log \omega^W - \log \omega^N$. Also define: $H^W = H^N + H^S$, $L^W = L^N + L^S$, $H^W/L^W = \hat{\delta} H^N/L^N$ where $\hat{\delta} < 1$ by the fact that the North is more skill intensive than the South. Then, equation (6) from Section II implies:

$$\Delta \log \omega_{NTC} = -(1 - \beta\rho) \log \hat{\delta} > 0 \tag{16}$$

where the subscript NTC denotes the fact that this expression refers to the case with no technical change. This is the standard effect of increased trade: since the South is less skill-intensive, trade with the South increases the relative price of skills. In this exercise I took A_h/A_l as given, and as discussed above, one can also take Q_h/Q_l as given. In this case, the result would be $\Delta \log \omega_{NTC} = -\theta \log \hat{\delta} > 0$, which is very similar, and does not affect any of the comparisons below.

B. Endogenous Technical Change and Full Property Rights

Let us now return to the analysis of Section II where A_l and A_h are endogenous and assume that (i) before trade opening, there were no sales of technology to the firms in the South (and no foreign direct investment in the South by firms in the North); and (ii) after trade opening, firms in the South and the North are symmetric and property rights of R&D producers in the North are fully enforced in the South. We can then use equation (12) from Section II to obtain the BGP skill premia with and without trade, and this immediately gives:

$$\Delta \log \omega_{PR} = \eta \log \hat{\delta} \tag{17}$$

where the subscript PR indicates that in this case there is endogenous technical change and property rights of R&D firms are enforced in the South. The important result is that if $\eta > 0$, contrary to conventional wisdom, trade opening may actually *reduce* the skill premium for exactly the same reasons that a higher supply of skilled workers increased it in Section II.[23] More generally, the directed technology effect implies that when intellectual property rights are fully enforced, it is unlikely that international trade will increase the skill premium by a large amount. Nevertheless, the assumption that intellectual property rights are fully enforced in the South is unrealistic. The next subsection looks at the other extreme case where there is no enforcement of property rights of Northern R&D firms in the South.

C. Endogenous Technical Change and No Intellectual Property Rights in the South

A crude and simple way of modelling the lack of intellectual property rights is to suppose that firms in the South can use the latest machines invented by R&D

[23]However, note that the equivalent of Proposition 3 applies, therefore trade opening first increases wage inequality, and then reduces it back to a lower level.

firms in the North without paying patent fees to Northern firms (but use the same quantities). This specification implies that the market sizes for different machines are unchanged after trade opening. However, there will still be an impact on the direction of technical change because the relative price of skill intensive goods will change.

In this case (9) also has to hold in BGP to equate the return to R&D in skill complementary and labor complementary machines. This implies that in the BGP $p = \left(H^N/L^N\right)^{-\beta}$, therefore, the relative prices will have to adjust back to their original level in order to restore equilibrium.[24] In turn, the price of skill intensive intermediate good relative to the labor intensive intermediate good is also given by (3), which in this case implies: $Q_h/Q_l = (\gamma_h/\gamma_l)^{1/(1-\rho)} \left(H^N/L^N\right)^{\beta\rho/(1-\rho)} \hat{\delta}^{-1}$.

The relative wage is now: $\omega^W = (\gamma_h/\gamma_l)^{1/(1-\rho)} \left(H^N/L^N\right)^{\eta} \hat{\delta}^{-1}$. The change in the skill premium after trade opening is therefore:

$$\Delta \log \omega_{NPR} = -\log \hat{\delta} > 0 \tag{18}$$

where the subscript NPR indicates that there is endogenous technical change but no enforcement of intellectual property rights in the South.

We have: $\Delta \log \omega_{NPR} > \Delta \log \omega_{NTC} > \Delta \log \omega_{PR}$ and in fact if $\eta > 0$, $\Delta \log \omega_{PR} < 0$. That is, if property rights are fully enforced in the South, the decline in the relative supply of skills should not lead to a large increase in the skill premium. In contrast, if intellectual property rights are not enforced in the South, simple calculations that ignore the induced change in the direction of technical progress may be seriously underestimating the impact of the international trade on inequality. Namely, in this case, the market size of different technologies remains unchanged, but trade creates a relative price effect, increasing the profitability of R&D for the skill intensive goods. This magnifies the static impact of trade on factor returns and results in a large long-run elasticity (-1).

To get a sense of how the presence of directed technical change may modify the conventional conclusions, consider the calculations by Krugman (1995). Krugman uses the share of trade with LDCs in the OECD output to calibrate the impact of growing trade on wage inequality. He finds that this can explain a 4.7% increase in the skill premium. Based on this, he concludes that growing trade with LDCs is

[24]Modest (and conflicting) effects of trade on the relative prices of skill intensive goods (e.g. Lawrence and Slaughter, 1993, and Krueger, 1997) are in line with the prediction that the relative price of the skill intensive good staying close to its original level. However, note that a large part of the substitution between skilled and unskilled workers may be taking place within industries.

unlikely to have been the main factor causing the changing structure of wages. Since Krugman is using a model of constant technology, in the context of the current model, this translates into $\log \hat{\delta} \approx 0.05\sigma$ where σ is the short-term elasticity of substitution between skilled and unskilled workers. Using Bound and Johnson (1992)'s estimate of $\sigma = 1.7$, this implies $\log \hat{\delta} \approx 0.085$. Therefore, in the absence of international property rights enforcement, growing trade with the LDCs would imply a 8.5% increase in the skill premium (from (18)). Recall that from 1980 to 1990, the skill premium increased approximately by 11% between 1980 and 1990 and by 13.5% between 1980 and 1995. Hence, with these calculations, trade emerges as a potentially major cause. Although this number should be interpreted with some care since no property rights enforcement may be an extreme assumption, it suggests that recognizing the endogeneity of technical change may modify some of the calculations.

V. Concluding Comments

The wages of college graduates (and of other skilled workers) relative to unskilled labor increased dramatically in the U.S. over the past fifteen years. To many, this is a direct consequence of the complementarity between skill (in its many dimensions) and new technologies. It is not however clear why new technologies should complement skills. History is full of examples of new technologies designed to save on skilled labor. More generally, inventions and technology adoption are the outcome of a process of choice; as a society, we could have chosen to develop (or attempted to develop) many different technologies. Therefore, it is necessary to analyze the *direction of technical change* as well as its magnitude. In its simplest form, this means to pose the question: "why do new technologies complement skills?". This paper gave a preliminary answer. The direction of technical change is determined by the size of the market for different inventions. When there are more skilled workers, the market for technologies that complement skills is larger, hence more will be invented.

I formalized this observation and discussed its implications. I showed that an exogenous increase in the ratio of skilled workers or a reduction in the cost of acquiring skills could increase wage inequality. The likely path is a decrease first, and then a larger increase in the skill premium. These observations fit the U.S. facts where the large increase in the ratio of college graduates during the late 60s and 70s first depressed the college premium and then increased it to higher levels than before.

I conclude with some comments about the implications of this approach and potential future work:

1. The most important area for future work is to develop a test of directed technical change, and its impact on the structure of wages. The testable implication of the model is that after an increase in the supply of college graduates, R&D directed at technologies complementary to college graduates should increase, and these technologies should be upgraded more rapidly. Unfortunately, it is difficult in general to determine which technologies are complementary to skilled workers. Nevertheless, most economists believe that computers are more complementary to skilled and educated workers than the unskilled. For example, Autor, Katz and Krueger (1997) reports that in 1993 only 34.6% of high school graduates use computers in contrast to 70.2% of college graduates. Moreover, Krueger (1993) shows that controlling for education, workers using a computer obtain a wage premium which suggests that they are more skilled. From the R&D expenditure data reported by the NSF, we see that in 1960 company funded R&D expenditure for office computing was 3% of the total company funded R&D expenditure. This ratio has increased to 13% by 1987, suggesting that during this period of rapid increase in the supply of skills, there has been significantly more R&D directed to one of the technologies that complements skills. If other technologies and R&D expenditure can also be classified as complementary to college graduates, the hypothesis of this paper can be tested.

2. Since 1970s, the participation of women in employment has increased substantially and their wages relative to those of male workers increased. Part of this change is likely to be due to reduced discrimination. However, to the extent that male workers use different technologies than female workers, the approach in this paper suggests a new explanation. The degree to which women use different technologies than men within a plant or sector is probably limited. Nevertheless, women tend to work in different sectors and occupations, and these jobs use different technologies than traditionally male jobs (e.g. desk jobs versus construction). Therefore, it is conceivable that the greater participation of women, which is once again not purely a response to higher wages, may have affected the direction of technical change, and via this channel, reduced male-female wage differentials. This hypothesis can be investigated more carefully by studying the relative growth of industries that employ more women, and the relative rate of technical change in these industries. A similar approach can also be developed for the analysis of the return to experience.

3. Throughout the paper, I only discussed the U.S. case. This begs two questions. First, is it appropriate to view the U.S. as an economy rather than the OECD? I

believe the answer to this question to be yes, but this only affects the calibration exercise. Second, and more important, can the approach in this paper account for cross-country differences? To answer this question, one has to distinguish between economies such as the U.K., Australia, Canada and Japan where the college premium increased during the 1980s, and those like France and Germany where it did not change. For the first group of countries, there was also a large increase in the supply of college educated workers during the 1970s, also partly caused by increased government support for education. As predicted by the model in this paper, in the U.K. and Japan, the college premium fell in the 1970s and then increased during the 1980s. In contrast, in France there was a decline in the college premium during the 1970s but no increase during the 1980s (see Katz, Loveman and Blachflower, 1995). It has to be noted that the increase in the relative supply of college educated workers was less pronounced in France. But, it is likely that, as argued by many researchers, labor market institutions also prevented the skill premium from increasing. The interesting point is that this type of labor market rigidities will also affect technical change. In particular, if unskilled labor is priced higher than its market clearing level, there will be two forces affecting the direction of technical change: (i) as there are fewer unskilled workers employed, the market for technologies complementary to unskilled workers is smaller; (ii) because unskilled labor is more expensive, the value of technologies that increase their marginal product is *higher*. These two effects work in opposite directions and can be quantified in future research. But, this simple reasoning already suggests that labor market institutions may have an important effect on the direction of technical change, which requires further study.

4. Finally, a different application of the theoretical framework developed here would involve having capital-complementary and labor-complementary technologies. Such a model would imply different rates of labor productivity growth across economies depending on their initial capital-labor ratios. Another implication would be that a large reduction in employment should be associated with faster upgrading of capital-complementary technologies.

VI. Appendix

Proof of Proposition 1:(10) uniquely defines Q_h/Q_l for $z_h = z_l$. Then, using (3), (9) and (10) and imposing $z^* = z_h = z_l$ gives equation (11), which uniquely defines z^* because the LHS of (11) is strictly increasing in z^* and the RHS is constant. This establishes that the BGP exists and is uniquely defined. Now using (10) to substitute for A_h/A_l in (6), we obtain that in BGP, (12) has to hold, which completes the proof. ■

Transition Dynamics and Proof of Proposition 2: Equation (9) immediately implies $z_s(j,t) = z_s(t)$ out of BGP as well as along it. Thus we only have to determine the time path of z_h, z_l, Q_l and Q_h. The free-entry condition (8) holds at all times. Therefore, differentiating this condition, $\phi(z_s(t))V_s(j,t) = B\tilde{q}_s(j,t)$, with respect to time and using (7) gives:

$$\dot{z}_s(t) = \frac{-\phi(z_s(t))\dot{V}_s(j,t)}{\phi'(z_s(t))V_s(j,t)} \equiv \frac{\dot{V}_s(j,t)z_s(t)}{\epsilon_\phi(z_s(t))V_s(j,t)} \tag{19}$$

for $s = l, h$ and for all j and t and where $\epsilon_\phi > 0$ is the elasticity of the function ϕ. I also normalize $B = (1-\alpha)/\alpha$ in this Appendix to simplify the notation. Combining this with (7) and using (8), we obtain:

$$\dot{z}_s(t) = \frac{r + z_s(t)\phi(z_s(t)) - \phi(z_s(t))\left(p_{st}N_s^\beta\right)^{1/(1-\alpha)}}{\epsilon_\phi(z_s(t))/z_s(t)} \tag{20}$$

Finally, noting that $\dot{Q}_s(t)/Q_s(t) = (\lambda-1)\phi(z_s(t))z_s(t)$, we also have:

$$\frac{d(Q_h(t)/Q_l(t))}{dt} = (\lambda-1)(z_h(t)\phi(z_h(t)) - z_l(t)\phi(z_l(t)))\frac{Q_h(t)}{Q_l(t)} \tag{21}$$

Equations (20) and (21) completely describe the dynamics of the system. To analyze the local dynamics and stability in the neighborhood of the BGP, let us linearize these equations, and let $Q = Q_h/Q_l$. Then, around the BGP, $z_l = z_h = z^*, Q = Q^*$, and ignoring constants, we have:

$$\dot{z}_h = \frac{\sigma(z^*)(z_h - z^*)}{\epsilon_\phi(z^*)/z^*} + \psi_1(z^*, Q^*)(Q - Q^*)$$

$$
\begin{aligned}
\dot{z}_l &= \frac{\sigma(z^*)(z_l - z^*)}{\epsilon_\phi(z^*)/z^*} - \psi_2(z^*, Q^*)(Q - Q^*) \\
\dot{Q} &= \sigma(z^*)Q^*(z_h - z_l)
\end{aligned}
$$

where I have defined $\sigma(z^*) \equiv \phi'(z^*)z^* + \phi(z^*) > 0$ to simplify the notation. ψ_1 and ψ_2 are analogously defined, and are both positive. The reason why deviations of Q from Q^* affect z_l and z_h differently is that when $Q > Q^*$, p_h is above its BGP value and p_l is below its BGP value. This linearization enables us to reduce the three variable system to two variables: Q and $\zeta = z_h - z_l$. Specifically:

$$
\begin{aligned}
\dot{Q} &= \sigma(z^*)Q^*\zeta \\
\dot{\zeta} &= \frac{\sigma(z^*)}{\epsilon_\phi(z^*)/z^*}\zeta + \psi(z^*, Q^*)(Q - Q^*)
\end{aligned}
$$

where $\psi(z^*, Q^*) = \psi_1(z^*, Q^*) + \psi_2(z^*, Q^*) > 0$. This linear system has one negative and one positive eigenvalue, and thus a unique saddle path converging to the BGP equilibrium, as is drawn in Figure 2. The rate of convergence to the BGP can be calculated as $\sqrt{\left(\frac{\sigma(z^*)}{2\epsilon_\phi(z^*)/z^*}\right)^2 + \sigma(z^*)\psi(z^*, Q^*)} - \frac{\sigma(z^*)}{2\epsilon_\phi(z^*)/z^*}$. Thus, when $\sigma(z^*) = \phi'(z^*)z^* + \phi(z^*)$ is lower, or when $\phi(.)$ is more steeply decreasing, convergence is slower. In fact, in the extreme case of $\phi(z) \equiv 1$ (where $\sigma(z^*)$ is at its highest), all our BGP results would be unchanged, but ignoring nonnegativity constraints on consumption, there would be no transitory dynamics. That is, when $Q < Q^*$ we would have $z_l = 0$ and $z_h \to \infty$ for an infinitesimally short while. There would be transitory dynamics, however, if nonnegativity constraints on consumption are imposed.

Next, I establish that if $\epsilon_\phi(z)$ is nonincreasing, the system is also globally saddle path stable. Since paths cannot cross and there are no other stationary points of the system, all paths that do not cycle must go to infinity. Therefore, we only have to establish that there are no cycles. Suppose $Q < Q^*$. Note that in this case $p_h H^\beta > p_l L^\beta$. Then consider case *(A)* where $z_l > z_h$. Then using (20) and the fact that $\epsilon_\phi(z)$ is nonincreasing, $\dot{z}_l/z_l > \dot{z}_h/z_h$, therefore, z_l will remain larger than z_h, and $\dot{Q} < 0$, thus, there cannot be any cycles and all paths go to infinity when $z_l > z_h$. Now consider case *(B)* where $z_h > z_l$ and $\dot{z}_l/z_l < \dot{z}_h/z_h$. Now $\dot{Q} > 0$, and also as Q increases p_h falls and p_l increases, therefore from , it will always be the case that $\dot{z}_l/z_l < \dot{z}_h/z_h$. Hence, in this case too, cycles are not possible. Now consider case *(C)* where $z_h > z_l$ and $\dot{z}_l/z_l > \dot{z}_h/z_h$. If as $t \to \infty$, $z_h(t) > z_l(t)$ and $\dot{z}_l(t)/z_l(t) > \dot{z}_h(t)/z_h(t)$, then we converge to $z_h = z_l = z^*$ and $Q = Q^*$, and we know there is a unique saddle path locally and paths cannot cross, therefore, we must be on that path. Instead if $z_h(t) = z_l(t)$ at some point where $Q \neq Q^*$, then once again cycles can be ruled out. We must have either that $z_h(t) > z_l(t)$ and $Q > Q^*$ which puts us in case *(A),* and cycles are not possible and all paths go to infinity. Or, it could be the case that $z_h(t) > z_l(t)$ and $Q > Q^*$, which, by the analogous argument to case *(A),* again rules out cycles. Thus, there must be a unique saddle path from all points $Q < Q^*$. The proof for the case of $Q > Q^*$ is analogous. ■

Proof of Proposition 3: Take $Q_h(t)/Q_l(t)$ as given. Then, given optimal monopoly pricing and profit maximization by firms, we have:

$$\frac{A_h(t)}{A_l(t)} = \frac{Q_h(t)}{Q_l(t)} \left(\frac{p_h(t)H^\beta}{p_l(t)L^\beta}\right)^{\alpha/(1-\alpha)}$$

Now substituting for $p_h(t)$ and $p_l(t)$ and rearranging:

$$\frac{A_h(t)}{A_l(t)} = \left(\left(\frac{\gamma_h}{\gamma_l}\right)^{\alpha/(1-\alpha)} \frac{Q_h(t)}{Q_l(t)} \left(\frac{H}{L}\right)^{\beta\alpha\rho/(1-\alpha)}\right)^{(1-\alpha)/(1-\alpha\rho)}$$

Substituting into (6), we obtain:

$$\omega(t) = \left(\frac{\gamma_h}{\gamma_l}\right)^{1/(1-\alpha\rho)} \left(\frac{Q_h(t)}{Q_l(t)}\right)^{(1-\alpha)\rho/(1-\alpha\rho)} \left(\frac{H}{L}\right)^{-\theta}$$

where $\theta \equiv (1-\beta\rho-\alpha\rho)/(1-\alpha\rho)$ as defined in the text. Therefore, at given technology (Q_h/Q_l), $\Delta \log \omega = -\theta \log \delta$.

Once, the technology adjusts to its new BGP level, then we have the result of Proposition 1, thus $\Delta \log \omega = \eta \log \delta$. ■

Details of the Model with Two-dimensional Heterogeneity: The demands for sector h machines now come from firms employing high ability college graduates and high ability high school graduates, and vice versa for sector l machines. These demand curves are the same as in the text, and have the same elasticity, thus the optimal pricing policy is the same. Therefore, the free-entry condition for sector h machines, analogous to (9), can be written as:

$$\frac{1-\alpha}{\alpha B}\phi(z_h)\left[\left(p_{hh}\left(\mu_h H\right)^\beta\right)^{1/(1-\alpha)} + \left(p_{lh}\left(\mu_l L\right)^\beta\right)^{1/(1-\alpha)}\right] = r + z_h\phi(z_h) \qquad (22)$$

and similarly for z_l, where p_{hh} is the price of the intermediate good produced by high ability college graduates in terms of the final good, p_{lh} is for high ability high school graduates, etc.

Then, from competitive pricing:

$$p_{hh} = (\rho\gamma_{hh})^{(1-\alpha)/(1-\alpha\rho)}\, Q_h^{-(1-\rho)(1-\alpha)/(1-\alpha\rho)}\, (\mu_h H)^{-\beta(1-\rho)/(1-\alpha\rho)}\, Y^{(1-\rho)/\rho}$$

and p_{lh}, p_{hl} and p_{ll} are similarly defined. Substituting these into (22) and simplifying, we obtain (13) in the text.

Finally, consider:

$$\omega^h(t) \equiv \frac{w_{hh}(t)}{w_{hl}(t)} = \frac{\gamma_{hh}}{\gamma_{lh}} \left(\frac{A_{hh}(t)}{A_{hl}(t)}\right)^{\rho} \left(\frac{\mu_h H}{(1-\mu_h)H}\right)^{-(1-\beta\rho)}$$

Substituting for $A_{hh}(t)$ and $A_{lh}(t)$ gives the expression in the text. $\omega^l(t)$ is derived similarly. ■

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Table 1: Some-Back-of-the-Envelope Calculations

	Data	$\beta=0.3$ $\alpha=0.3$ $\rho=0.8$	$\beta=0.35$ $\alpha=0.3$ $\rho=0.8$	$\beta=0.35$ $\alpha=0.3$ $\rho=0.75$	$\beta=0.4$ $\alpha=0.3$ $\rho=0.7$	$\beta=0.4$ $\alpha=0.3$ $\rho=0.73$	$\beta=0.4$ $\alpha=0.3$ $\rho=0.75$	$\beta=0.45$ $\alpha=0.3$ $\rho=0.7$	$\beta=0.45$ $\alpha=0.3$ $\rho=0.65$
$1-\beta\rho$	0.55-0.85	0.82	0.72	0.73	0.72	0.70	0.70	0.68	0.70
$\theta=(1-\beta\rho-\alpha\rho)/(1-\alpha\rho)$	0.55-0.85	0.63	0.63	0.63	0.64	0.62	0.61	0.60	0.63
Long-run elasticity, η	--------	0.68	0.4	0.05	-0.06	0.07	0.20	0.05	-0.16
Impact Effect	--------	-0.24	-0.24	-0.26	-0.25	-0.25	-0.24	-0.24	-0.25
"Short-run", $\Delta\log\omega_{71-79}$	-0.10	0.01	-0.05	-0.12	-0.14	-0.11	-0.08	-0.11	-0.16
"Long-run", $\Delta\log\omega_{71-87}$	0.024	0.27	0.16	0.02	-0.02	0.03	0.08	0.02	-0.06

Note: The first two rows give the inverse of the two alternative short-run elasticities between college and high school graduate workers implied by the model. The range of this parameter is given by the estimated elasticities discussed in the text. The third row gives the long-run elasticity implied by the model. The fourth row gives the impact effect, which is the proportional fall in the skill premium in case of a 40% unanticipated increase in H/L. The sixth row gives the long-run change implied by the model. This is interpreted to correspond to the increase in the college premium from 1971 to 1987 as reported by Katz and Murphy (1992). The model's prediction is given by $\eta\log\delta$ where $\log\delta$ is the proportional change in H/L from 1971 to 1979, which is given as 40% by Katz and Murphy (1992). Finally, the fifth row is the simple average of the impact effect and the long-run effect, in other words: $(\theta+\eta)\log\delta/2$

Date Due