DYNAMICS OF LAND VALUE AND USE IN URBAN GROWTH

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ABSTRACT

It is or should be well known that land value grows faster in areas farther from the center in a growing city. Closely related is the tendency that the gradient of urban population density flattens over time. An analytical explanation is given by a comparative static and a dynamic extension of the simple models of the monocentric city and the capital market.

In a monocentric city whose population, household income, or transportation efficiency is growing continuously, the rate of land value appreciation increases with distance from the city center up to the settlement limit. Beyond the limit, the rate decreases with distance. The opposite will be the case if the city is declining in any one of the aspects of city size. The pattern obtains under a wide range of specifications regarding consumption and production parameters, expectation of market agents, and durability of land use. Existing empirical studies are reviewed and are seen to render strong and abundant support for the analytical conclusion. In addition, the predicted spatial distribution of land value appreciation rate is directly tested by a regression analyis of the land value data of Seoul, Korea.

The dynamic analysis unveils several important properties of land use involving durable and inflexible housing. Two implications are noteworthy. The density of housing structure remains constant over long stretches of time and distance if developers behave with little consideration for future changes in demand (myopic expectations). With full anticipation of future changes, it is shown that land is withheld from development if the growth rate of housing value is higher than a certain fraction of the discount rate. An application of the analytical framework shows that a tax on housing rental or the property value reduces land use density and land value, more severely in areas farther from the city center.

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Chapter 1. INTRODUCTION and SUMMARY

1.1 The Issue

Dynamic evolution is an essential nature of the contemporary city. Its population grows, residents' income rises, and technology develops. Along with these come changes in land value and land use forms. The latter changes do not lend themselves easily to brief yet meaningful summary indications for the city as a whole for they usually are not uniform across areas within a city: in short, the city not only grows or declines in an aggregate sense but its spatial structure undergoes transformation. The same can be said about cross-sectional comparison of cities: they differ not only in size but also in the pattern of land value and population distribution. Part of the variations would be due to purely random factors present at specific times and places. But there also are found unmistakeable systematic patterns of variation linked to aggregate measures of a city.

This thesis is a contribution to the study of the urban spatial structure and its change, an old tradition in urban studies in general and urban economics in particular. This chapter identifies important practical and theoretical issues that this thesis adresses, main points of our arguments, and characteristics of our methodological approaches to them.

Let us first consider the issue of different rates of land value change within a city. This subject of great practical interest will be the focus of our analyses in this thesis though it has not been given comensurate attention by urban economic theorists. It is commonly understood that urban land values rise roughly in step with urban growth. Equally well known is the fact that some parts of the city, urban fringe areas in particular, experience much faster growth of land value than others. How and why does the difference come about? The answer to this question is sought by various segments of the society: investors, developers, home-buyers, and public agencies. A plethora of theories have come from diverse intellectual angles and are vigorously debated for or against on theoretical, empirical, and perhaps ideological grounds. In the following let us briefly comment on a few popular theories.

The cause-effect chain of different land value growth is rather obvious in case the causal factors are well localized: public investments favoring a few selected areas, for example. A spurious application of this line of reasoning has it that, since more new housing is built in outlying areas of a city, that is where the land value will increase more - which in most part seems little more than a confusion of causation and association. A more reasonable version finds the cause of the fast appreciation in suburban land value in the concentration of public infrastructure investments there. But the actual pattern of investment distribution is not so clearly lopsided if one considers a full range of public outlays; and, even so, such effects are found to be responsible for a relatively small part of land value increases. (See, for example, Maisel, 1964.)

Most often, however, the "explosion" of peripheral land values is attributed to "speculation", a term that is employed to designate numerous disparate processes. Perhaps the term is most popularly used to signify various breakdowns of the competitive market: monopoly in the suburban land market, informational inefficiencies leading to speculative bubbles, widespread outright fraud, etc. Many economists have pointed out that these arguments have little logical or empirical validity with possible exceptions for a very limited duration or locality.(See Carr and Smith 1975, for example.) Nevertheless, in boom markets there certainly appears to be a large presence of speculators who are primarily interested in reselling rather than in using land. Therefore, while rejecting the above extreme hypotheses, many still try to define and explain the rapid price rises as effects of speculation, though within the framework of conventional economics.

In one conventional sense, the function of speculators is intertemporal arbitrage, withholding a piece of land from committing it to the immediate highest yield in favor of better long-term benefits, presumably higher capital gains. Provided that the landowner's expectation is rational, however, this action will produce lower, not higher, average rates of land value appreciation. As the actual expectations are more often inaccurate, land value will fluctuate more but will not necessarily grow faster on average. The existence of uncertainty and fluctuations brings into play the second meaning of speculation, risk arbitrage, in which higher rates of return are obtained for higher risk, such as that usually characterizes expectations concerning fringe areas. But, contrary to intuition, the effect of uncertainty on the rate of appreciation is ambiguous (see chapter 2 below.)

In short, under close examination, the intuitive appeals of the above popular theories fail to transtlate into any consistently or generally convincing explanation for the issue. Neither, however, have the critics of these arguments come up with any specific alternatives. Partly for this reason it does not appear fruitful to elaborate on these various arguments. Rather, we begin our analysis by breaking away from the implicit but apparent premise shared by both sides of the above debates: that land value should increase at the same rate across the city if the market is operating efficiently without monopolies, uncertainties, and the like. Instead we simply ask how land value in different parts of a city would grow in just such a market.

Our analysis shows that competitive resource allocation for a growing city would produce an equilibrium spatial pattern such that the land value appreciation rate will be the greatest at the frontier of the city and decreases with distance in either direction, toward the city center or farther into the agricultural area. There is a substantial body of empirical evidence that corroborates this description, but we make the assertion primarily on the basis of a deduction from a simplified equilibrium model of the city and the capital market emphasizing the trade-off mechanism between accessibility, space, and other consumption goods.

As such our analysis cannot be considered a necessary and sufficient explanation for the said pattern. In other words, the fact that the real world behaves as the theory predicts does not prove the existence of a perfect market nor the irrelevance of other possible causes. With this qualification, however, the empirical evidence suggests the relative productivity of our simple analysis in describing the reality. Also, being consistent with established principles, our model is a firmer foundation upon which to examine relevant factors, possibly including the ones mentioned above, than the unquestioned norm of a uniform rate of land value change for a city. We demonstrate this point by considering aspects of speculation employing our own framework of analysis.

Beyond the immediate concern with land values, the analysis can be genaralized to other issues involving related variables. First, due to the duality between price and demand, our prediction about the spatial pattern of land value change can be directly transtlated into that of land use intensity: that density of land use increases more rapidly in areas farther from the city center when the city grows. This phenomenon of flattening density gradient has been recognized and explained by urban economists vastly better than the case of land value dynamics. Still, our analysis represents an improvement in consistency, clarity, and generality.

Another important extension can be made to determine the effects of the property tax, another city-wide parameter that potentially has differential impacts on land value and use of different locations within a jurisdiction. Because of the importance of the property tax in the local economy the subject has not escaped the interests of economists, but mostly the interest has been in aggregate or inter-jurisdictional effect of different rates of the tax and not in the spatial distribution of the impact of a uniform tax rate within a jurisdiction. This latter aspect, however, apparently has significant implications on the matter of equity and efficiency of resource use. Employing the same methods as applied in analyzing growth impacts, we show that an addition to a uniform rate of tax on property income or value would in general produce disproportionately severe impacts on properties in outlying areas, both in terms of loss of income to the owners and reduction of land use intensity.

1.2 Methodological Summary

A general concept underlies all the above predictions: that pieces of urban land take on different characteristics as economic goods, as they become "adapted spaces" and differentiated according to socio-economically defined locational attributes. It is not surprising, then, to find that different locations respond differently to a shock which in itself is not specific to any location, just as demand for and prices of different commodities in an economy react differently to taxation, income growth, etc.

The concept of the city as a set of differentiated spaces is embodied in general equilibrium theories of urban land use, particularly the monocentric city models, in the form of systematic differentiation of land use activities and land rent. These static models have spun off a growing body of theoretical and empirical researches that have substantially advanced our understanding of changes in land use patterns. This thesis belongs to this tradition, adding some substantial and methodological innovations within a unified framework, with more emphasis on dynamics, deductive methods and on the subject of land value change.

As mentioned earlier, the phenomenon of flattening density gradient has received the most attention from urban economists among many aspects of spatially differentiated change. The empirical tendency itself has been well known for over three decades since Colin Clark's study (1951). But a full development of economic studies on the subject owes its beginning to the establishment of monocentric urban models. Especially, Muth's theoretical and empirical studies (1969) offered the first full scale and coherent account of different patterns of population distribution anchored in his equilibrium model of the city, which remains as a standard. Subsequently, numerous studies have been produced, most of them providing impressive amounts of empirical support for the general trend of flattening density gradients, and for Muth's identification of greater income, population, and transportation efficiency as major factors contributing to this trend. All in all, this accumulation of studies has left Muth's original theoretical studies essentially intact.

However, as discussed in our review below, Muth's theoretical analysis for his comparative study of population density gradients is only loosely and partially connected to his formal model of the urban spatial structure. From a formal viewpoint, the efforts by Muth and the subsequent researchers can be characterized more as an estimation of a reduced form equation evaluating likely determinants of density gradients than as an explicit development of a structural model. That approach may not be faulted too much, considering the complexity of actual urban spatial strucutres for which the existing models seem too drastic a simplification. On the other hand, disadvantages of concentrating only on the reduced form equation estimation are well known: opaqueness about interdependencies among variables, susceptibility to weakness of data, inherent limitation on generality, etc. Muth's analysis needs to be re-evaluated also in light of recent estimates of the critical parameter values that he used in his analysis.

There is a growing theoretical literature on comparative static and dynamic analysis based on monocentric models complementing the above empirically oriented research. But, unlike the latter, progress seems far from the point of saturation. On the other hand, except for many intuitive theories some of which were discussed before, there have been few systematic studies, empirical or theoretical, on the subject of land value change. This is surprising in view of the theoretical as well as practical significance of the price variable. The notorious problem of availability and quality of land value data seems to be at least partly responsible for this lack of research, so far as it limits the productivity of efforts toward empirical generalization.

In short, we see a distinct imbalance between theoretical versus empirical works on land use change and between those on land use versus land value. By putting more emphasis on a general and deductive approach that utilizes both comparative static and dynamic analyses, and organizing the study with a focus on land value change, we can not only derive explicit conclusions regarding the neglected issue of land value, but also help clarify some of the analytical ambiguities present in the existing analyses of land use change.

A complete set of basic comparative static properties of Alonso's monocentric model, which is essentially equivalent to Muth's, has been presented by Wheaton (1974). He determined the direction of changes in major variables -- rent, city boundary, utility level, and population density -- resulting from important types of exogenous shocks in two ideal types of the city. He also confirmed the hypothesis of flattening density and rent gradients when income is increasing and when transportation cost is decreasing in a closed city.

We start our spatial analysis in chapter 2 by extending Wheaton's study to complete the enumeration of changes in the rent gradient. We again confirm the tendency that rent and population density gradients flatten with urban growth. In other words (the equivalence of the two expressions is explained in Chapter 2), rent goes up faster in outlying areas, as population, income, and transportation efficiency grow in a closed or an open city. We also show that natural and legal constraints to urban expansion such as waterfront or green-belt have the same effect as the above. But the results depend critically upon the premise of inelastic demand for land with respect to price, which is well supported by recent consensuses. With an opposite assumption, as adopted by Muth, the results would in some cases be reversed or become ambiguous.

The relationship between rent and value changes is derived from conditions of capital market equilibrium. According to this, the above pattern of rent changes implies that land value also increases faster at farther distances from the center in a continually growing city. But the appreciation rate decreases with distance beyond the city limit under rational expectations.

Asset value and hence its change are also dependent upon the discount rate which varies with the market interest rate, risk and tax rate. An increase in the discount rate would decrease land values, particularly the fast growing ones. Its general effect on appreciation rate is ambiguous, but the effect is likely to be negative in areas to be soon urbanized, although the conventional wisdom argues that a higher discount rate always accelerates the appreciation. Our analysis suggests that the risky nature of suburban land markets may not be the cause of high rates of appreciation by itself; rather, the area is intrinsically highly sensitive to small reductions in uncertainty or interest rates brought about by urban or general economic growth.

At this point it seems necessary to examine the rationale of the monocentric city model, since many of its assumptions are obviously unrealistic. The confines of the model are nevertheless accepted by many simply because it is quite useful despite its restrictions, and because there are few effective alternatives for description of the spatial structure and its evolution for a city as a whole. Consider, for example, the most frequently raised question about the assumption of monocentricity, defining household locations solely in terms of the accessibility to a single center of the city, ignoring diffuse travel destinations, amenities, neighborhood externalities, etc. Although the criticism is factually irrefutable, the assumption can be defended as a useful simplification on theoretical and empirical grounds.

With any set of locational attributes, the unique conceptual issues distinguishing the problem of locational choice from other consumption decisions remain basically the same: the socio-economic definition and differentiation of space, as discussed earlier, and the non-convexity of the consumption set due to the constraint that a household must occupy only one site, not a combination of many. Therefore, the properties of trade-off involving accessibility to the center are pertinent to many other dimensions. This sort of generalization has been shown to be possible at the cost of complexity and inability to close a model without added arbitrary specifications. Except for some specialized purposes, however, it may not be worth the trouble since perhaps the most important problem of externalities are left unaccounted for by most extensions of the sort.

Even if relaxing the assumption of monocentricity is possible and may add a few theoretical insights, it is not very useful in an empirical sense because of the difficulty in defining a simple and generally applicable vector of diverse locational dimensions, particularly if one wants a comparison among cities or times. Witness the difficulty in dealing with just the accessibility to the center. Studies have shown that this is not always the dominant determinant of land value but is nevertheless a robust one across cities, thus permitting a simple and clear common dimension.

Another major question concerns the use of comparative static analysis where it is assumed that urban land use can be adjusted instantaneously and costlessly under changing market conditions. In analyzing the supply of housing, this obviously is a very unsatisfactory premise in view of the extreme durability of urban structures. Still, it eventually turns out to be a good approximation in many respects; but this justification is one of our results, and many other dynamic analyses have shown otherwise. Above all, it is impossible for us to sidestep the issue, because we are primarily and explicitly interested in land value, not in the statically defined land rent which is neither observed or otherwise relevant in actual markets of land for durable developments.

Recently there has been a great deal of interest in dynamic analyses of urban land use, mostly within the framework of a monocentric city model. But as we have mentioned above, the issue of land value has received little consistent or explicit treatment. This lack of attention to the price mechanism is perhaps related to difficulties of obtaining clear analytical statements regarding the evolution of land use.

In contrast, our dynamic analysis starts with a careful reexamination of well-known principles of land value determination as well as of optimal land use (chapter 3). This unveils several essential properties of land value that have not been paid due attention in existing studies. It also reveals limitations of popularly adopted analytical compromises in dealing with land value under conditions of durability. We suggest an approach to the study of shadow land rent and land value dynamics that is not encumbered with procrustean simplifications of durable land use. We study information conferred by a well-functioning land market that is analytically as well as practically crucial in determining optimal strategy of land use with durable housing. For example, development or redevelopment of land for urban use is withheld if the value of housing increases faster than a certain limit determined by the discount rate and the factor share of land in construction. The construction input at redevelopment would be about ten times as much as the existing structure.

These formulations define properties of supply of durable housing. The spatio-temporal pattern of housing demand can be easily adapted from the comparative static analysis of the second chapter. Combining these in chapter 4, we obtain a model of dynamic evolution of land use and value that is quite general in its topical and methodological scope.

Patterns of land value distribution and appreciation are shown to be qualitatively the same as the comparative static patterns: land value always declines with distance from the city center, and its rate of appreciation peaks at the urban border in a growing city and declines with distance inward or outward.

As to the evolution of land use, we answer a few theoretical questions left unresolved. For example, we define in what sense it is "normal" for a city to expand outwardly from the center. Conversely we suggest that outside-in or intermittent urban expansion occurs if, for example, the population grows while income is decreasing or if more land becomes available while the urban economy remains stagnant otherwise. Further, if developers' expectations are rational, the structural and population density will generally decline with distance, at least in the initial round of development. In case developers operate under myopic expectations, density of housing would be the same for all locations that have housing vintages belonging to a same generation. That is, the ratio of structural input to land is identical for every housing that is originally built on the converted farm land regardless of the location and the date of the original development; it is another identical value, higher than the original housing density, if the housing is the replacement of the original building; another if the housing is the third inhabitant of the land, etc. Despite this, population density is shown to decrease with distance in general though it can increase or stay constant over distance if the size of an individual house is fixed and the utility level of household decreases in the process of urban growth.

In the last analytical chapter (5), we employ the methods of previous chapters to study the spatial incidence of a property tax on housing in a city. First, under the comparative static framework, we show that an ad valorem tax on housing rental would affect the value and intensity of land use more adversely in outer areas of an open city, in addition to the already known aggregate effect of reduced city size. In the case of a closed city, the same pattern as in the open city is likely to prevail but with more ambiguity.

We also analyze the case of an open city with durable housing. We show that the tax on housing rental will again be felt more severely in locations farther from the center. In dynamic analysis, an important question concerns the timing of development and redevelopment. We obtain the result that a higher tax would delay and reduce the intensity of development or redevelopment of land if factor substitution is inelastic.

Under the framework of dynamic analysis we can analyze the tax assessed on the capital value of a property that is more common in practice than the tax on housing rental dealt with in most conventional analysis. This system of tax in effect raises the discount rate applicable to the rental income stream. This causes differences in growth rates of value to affect the impact of the tax. We show that impacts of this tax are similar to the case with the tax on housing rental income with slight twists. For example, the property tax can facilitate land development in outlying areas in an open or a closed city. Property values will generally decline, and redevelopment in inner areas will be discouraged as in the case with rental income taxation. The gradient of land value steepens under both types of taxation as a result of a higher tax.

In the last chapter, we review existing theoretical and empirical research on the evolution of the urban spatial structure in light of the foregoing anaysis. This is not meant to be exhaustive. Rather, the focus is mainly about how and why some well-known past theoretical studies differ from ours, how far empirical studies support our hypotheses, and what are important remaining ambiguities.

Elsewhere in the present chapter we mentioned the importance and shortcomings of Muth's theory of population distribution. Details of these arguments will be presented first, along with summaries of a few empirical works on flattening density gradient. A substantial departure from Muth's model was made by Harrison and Kain's model of incremental urban growth (1974) formalized by Anas(1978). Their model introduces a very important conceptual element, but cannot be considered a definite improvement over the standard theory in either theoretical refinement or empirical approximation. The conceptual rigidity of these earlier cumulative growth models are being corrected by later developments in dynamics, notably by Brueckner and Wheaton, and we show in some detail how our study complements these analyses.

There are many empirical studies on cross-sectional or historical variations of urban population density gradients, notably by Muth (1969) and Mills (1972, 1980). Studies on land value distribution are less in number and in quality. Although data and estimation methods used in all these studies cannot be termed highly reliable, they as a whole can be seen to provide a consistent support for most observable parts of our theoretical conclusions. To complement the existing indirect or casual evidence we provide a more direct test of the hypothesis about land value appreciation by using the land value series of Seoul, Korea. The result again renders an impressive support for our prediction, particularly considering the degree of abstraction involved.

In recapitulation, our analysis can be characterized as an operationalization and application of the two basic equilibrium concepts essential to understanding urban land use and value: economic differentiation of space in terms of accessibility, and land value as a derived present value. These old concepts and their formalizations have probably been under-utilized. Use of these tools in this study has yielded considerable light on matters of academic and practical interests.

Of course, one should excercise caution not to project the theoretical implications literally onto reality, for our formal model considers only a few elements among the enormous complexity of the urban process in a deterministic framework. Compared with existing studies on similar subjects, however, we sacrifice little scope in order to gain rigor and clarity. In fact, our analytical innovations enable us to consider many important issues that were formerly left unaccounted for in a consistent manner. For example, our dynamic representation of land value permits an explicit comparison between effects of a property value tax on land use intensity and timing in different parts of a city. Overall, our analysis presents a complement to the substantial body of literature with similar aims and approach by offering substantial refinements, elaborations, and sometimes, disagreements.

1.3 Notes on Exposition

Convention allows many alternative terms for each of the important variables considered in this research. Nevertheless, for the sake of consistency, we will employ a unique term for each variable throughout the remainder of this thesis. The choices are made principally in order to avoid confusion, as there are no definite logical grounds in most cases.

We retain the use of the terms "price" and "cost" in the generic meaning with careful qualifications such as "rental price of housing". Although neoclassical economics has all but totally obliterated the distinction between "value" and "price", we shall use the former for a specialized meaning of present or capitalized value of a long-lived asset. In contrast, "rent" and "rental" are used for the hire price of land and housing, respectively, for use for a limited duration. We choose the term "housing rental" in order to distinguish it from land rent and also from the contract rent that includes operating expenses and which is usually expressed as gross rent or simply rent. We use the "discount rate" effective for capitalization as that which includes not only the market "interest" for funds but also other relevant factors such as risk.

Secondly, we need to clarify what physical entities these price terms are for. In urban economics, especially the recent literature on durable housing, the most popular term meaning the component of housing other than land is "(durable) capital". But this could be confusing as to its inclusion of land which is essentially a form of capital, on the one hand, or of labor and other non-material inputs for construction, on the other hand. Therefore, we prefer a more straightforward "(durable) structure." This is a combination of a variety of material and labor inputs. "Housing" again is a composite commodity, a combination of structure and land (and other permanent improvements such as sewage and grading that are ignored in our analysis by the assumption of physical homogeneity of land.) As such, these are not strictly homogeneous physical quantities but economic quantities essentially defined by the market. We assume that such a definition is unique and stable, following the standard practice. We do not see the need to distinguish between occupation of a unit versus consumption of its service.

As we rely heavily on a postulate-deductive mode of analysis, we find it convenient to organize the presentation around a series of dependent but self-contained propositions. But, in order to avoid excessive formalism, they are made to be specific and empirically applicable as far as possible rather than general and empirically non-committal. For example, instead of stating a general theorem, "If 'a' then 'A', If 'b' then 'B', etc.", preceded by "If 'x' and 'y' then 'a', If 'y' and 'z' then 'c', etc."; we will in most cases simply state a proposition "A", having formerly determined that "a" is the most plausible situation. The general case will be discussed in the proof.

For mathematical notation, we will use R'(x;x=b, t=tl) to mean the partial derivative of R with respect to x at b and tl, omitting the designations when they are not particularly significant. R"(x,t) is likewise a second partial derivative. When the ordinary and partial derivatives are one and the same, or when there is no danger of confusion, we will use the notation for an ordinary derivative for convenience. * is used for a logarithmic derivative, or the rate of change. Thus, R*(t) = R'(t)/R is the rate of change in rent, R, over time, t. Frequently used symbols are listed in the following.

List of Symbols

x : distance from the city center

t : time

B : distance of the city boundary from the center

N : total population of the city

n : population per area (population density)

y : income of a household

U : utility of a household

L : land consumed per household

H : housing consumed per household

Z : quantity of composite good other than land (or housing)

a(x) : transportation cost to city center from distance x

Q : quantity of housing per land area

k : quantity of structural input per land area

R : rent per unit area of land

V : present (asset) value of land

r : dicount rate

g : rate of increase in land value

p : rental price of housing net of operating cost

F(t) : present value of housing rental for the life span of a new housing built at time t.

A(t) : value of leasehold of land for the life span of the new housing

c : unit purchase price of structural input

List of Symbols (continued)

E(R) : own price elasticity of demand for land, compensated

E(^R): the same, ordinary or uncompensated

E(p) : own price elasticity of demand for housing, compensated

E(yL): income elasticity of demand for land

s : elasticity of substitution between land and structure in housing production

m : rate of tax on rental income of housing property

w : rate of tax on asset value of housing property

CHAPTER 2. COMPARATIVE STATICS of LAND VALUE and POPULATION DISTRIBUTION

2.1 Equilibrium of a Monocentric City

A household's land use decision as a location decision is different from other consumption choices as it must choose one or few locations foregoing any other. But, like other consumption, it has also to decide how much. Since attractively located lots such as those close to CBD or commanding good vistas are in limited supply, competition forces a trade-off among many locational attributes, amount of space, and other consumption goods through the price mechanism. Alonso's model (1964) describes such a land allocation mechanism with land differentiated solely in terms of distance to the city center. Other authors published models with essentially similar frameworks, though with slightly different practical aims and theoretical structures. For example, the most well-known among them, Muth's (1969) model focuses housing rather than land but otherwise turns out to be formally equivalent to Alonso's in describing urban spatial structure.

These monocentric city models, so called for their common simpifying assumption of a single activity center for a city, have first been used mainly to explain characteristic urban forms in terms of distribution of rent, population densities, and income classes. Muth used the model also to describe variations of the pattern. As discussed in the last chapter, however, his analysis which has been adopted by most subsequent researchers of population distribution, cannot be called a true comparative static analysis. A general and comprehensive comparative static analysis of Alonso's monocentric model has been provided by Wheaton (1974). In this section we review the equilibrium model as developed by Alonso and Wheaton and discuss its isomorphism with Muth's model for the starting point of our comparative static and dynamic extensions.

A household, the only type of land user in this model, makes a location decision as part of the overall consumption decision maximizing utility derived from land, L, and the composite good, Z.

$$\max U = U(Z,L)$$
$$\text{subject to the budget constraint} \qquad (2.1)$$
$$y \geq Z + R(x)L + a(x)$$

where price of the composite good, invariant over location, is taken as the numeraire, R(x) is the rent per land area at location x, and a(x) is transportation cost incurred at the location. Location is here defined solely by the distance from the city center to which every household travel is assumed to be made. Defining the location in such a drastically simple manner, versus defining or simply interpreting x as a vector of many locational attributes, should be viewed as a matter of theoretical and empirical convenience as discussed in Section 1.2.

A landowner maximizes his income by awarding the land to the highest bidder of rent. In a competitive market, these maximization efforts on both sides yield a pair of first order conditions defining a schedule of the rents bid by the household that will allow it a certain level of utility regardless of location.

$$R(x) = U'(L)/U'(Z) = [y - Z - a(x)]/L(x) \qquad (2.2)$$
$$\text{for } U(x) = U \qquad \text{at all } x.$$

This system of relations, equations (2.1) and (2.2), then yields a solution for each of the endogenous decision variables, L, Z, R, in terms of parameters, U, y, a(x).

Wheaton has determined fundamental properties of this household location equilibrium as the following set of theorems under a very general assumption that land and the composite good are both normal goods, that is, both have positive income effects and the utility function is strictly quasi-concave.

Theorem 1.

$\overline{R}'(L) < 0$, and $\overline{R}'(Z) > 0$ where $\overline{R} = U'(L)/U'(Z)$

Theorem 2.

$L'(U) > 0,\ Z'(U) \geq 0;\ L'(y) < 0,\ Z'(y) > 0;$

$L'(x) > 0,\ Z'(x) < 0;\ L'(a) > 0,\ Z'(a) < 0,$ for $a(x)=ax$.

Theorem 3.

$R'(U) = -1/LU'(Z);\quad R'(y) = 1/L\ ;$

$R'(x) = -a'(x)/L\ ;\quad R'(a) = -x/L\ .$

Theorem 4.

$dU'(Z)/dx = L'(x)\ [U''(L,Z) - U''(Z)R] > 0\ .$

The first Theorem confirms the convexity of the indifference curve. The second defines the directions of change in consumption of land and the composite commodity as a result of change in basic parameters. The envelope theorem is employed in deriving the third Theorem that specifies the changes in bid rent following changes in the parameters. The fourth states that the marginal utility of the composite good and hence the marginal utility of nominal income increases with distance, because the total consumption outlay decreases by the amount of transportation cost.

In Muth's model, households' utility is expressed as a function of the composite good and housing, a combination of land and structure. In other words, the household's utility maximization problem is now stated just like the maximization in terms of land and the composite good (equation (2.1)) except that housing is substituted for land, and housing rental for land rent,

$$\max U = U(Z, H(h,L)) \tag{2.1.H}$$

subject to $y \geq Z + p(x)H + a(x)$

where H is quantity of housing consumed

h is the amount of structural input for housing

p(x) is the marginal rental price of housing

Strictly speaking, the composite good, Z, in the above does not include the housing structure and thus is a smaller subset of the composite good in Alonso's model. However, we will maintain the same symbol to denote the "other" goods than the quantity in focus, land or housing.

Implicit in this formulation is the weak separability of the utility function in housing and the composite goods, that the marginal rate of substitution between land and structure is independent of the other consumption. This assumption is no stronger than that implied in Alonso's framework where the marginal substitution between housing structure and other goods is independent of the amount of land consumed. If housing preference is separable from other consumption, the production of housing using land and structure can be uniquely defined, and the household's consumption can be specified only in terms of quantity of housing regardless of the proportion of land and structure put in housing.

Once we adopt this framework, spatial equilibrium of households involving land as a distinct consumption good can be exactly duplicated to the one involving housing. Distribution of the rental price for housing equalizes the utility obtainable in different locations, and the land rent schedule equalizes housing producers' profit at zero. Deferring discussioin of relationships between the production and consumption, we obtain the first order conditions of household maximization of exactly the same form as the above, with housing (H) and its rental price (p) substituted for L and R. Thus it is apparent that all the above analyses and Theorems would hold exactly for Muth's model with appropriate substitution of variables of consumption. That is, the marginal rental price of housing is written

$$p = U'(H)/U'(Z) = (y - a(x) - Z)/H \quad (2.2.H)$$

And the third Theorem of Wheaton's, for example, can be restated as Theorem 3H:

$$p'(U) = -1/HU'(Z) \ ; \quad p'(y) = 1/H \ ;$$
$$p'(x) = -a'(x)/H \ ; \quad p'(a) = -x/H \ .$$

Solutions for land consumption and rent can then be obtained through the housing production function which we will examine in detail in the next chapter. This will be a more involved process, but the final solutions should not be different in the two versions because under the assumption of malleable structure and a competitive market the land demand and rent should be the same whether land is considered as a separate and direct utility argument or an indirect one as a factor of production. In case structure and land are considered inseparable because of durability of the combination, this equivalence would not hold exactly, as will be discussed in detail in Chapter 3.

If the two theoretical models are equivalent for the comparative static analysis, the simplicity and generality of Alonso's model dictate that we utilize this as far as possible. In this framework, the specification of the model is completed by considering citywide equilibrium conditions which will constrain the local equilibrium defined above. The first (border) condition is that land can be used for residence only if it is bid away from alternative uses. In this model the alternative use is agriculture whose bid rent is assumed to be constant over time and location. In other words, the urban bid rent at the city border, B, must be as large as the farm rent, R(f).

$$R(B) = R(f) \tag{2.3}$$

The second condition specifies total population of the city, N, that must be housed within the boundary. Letting ϕ the available radian of land at distance x,

$$N = \int_0^B \frac{\phi(x)x}{L(x)}dx \tag{2.4}$$

A solution of the complete system requires specifying all but two of the six parameters, y, N, U, R(f),(scalars, assuming homogeneous population) a(x), and ϕ(x). Of particular interest are the interdependencies among income level, total population, and utility level. Wheaton has proposed two types of the adjustment mechanism. In an open city, an exogenous shock causes in- or out-migration so that the utility of households are maintained at the national level wherever they live. In a closed city, utility is endogenously determined following changes in other parameters by the equilibrium land allocation process. While the open city model is more in line with the classical assumption of interregional equilibrium, Wheaton argues that the closed city model may better represent the actual cities of mature economies. But, in general, real cities would belong to some points on a continuum between these two ideal types, subject to two different yet parallel adjustment processes that they represent.

For each type, Wheaton determined comparative static effects of the above parameter changes on land consumption and rent, as well as other endogenous parameters. The results can be summarized as Table 1. It shows whether an endogenous variable increases (+) or decreases (-) or remains unaffected (0) with an increase in an exogenous variable. When both signs are shown, it means the variable increases in some places but decreases in others.The first line of the Table, for example, should read: increasing income in a closed city increases utility level and expands the city area, increases land consumption everywhere, and decreases rent at central areas but increases it in outer areas.

Table 1. Comparative Static Adjustments

	endogenous changes in							
	a closed city				an open city			
Increase in	U	L	R	B	N	L	R	B
y	+	+	-,+	+	+	-	+	+
a'(x)	-	-	+,-	+	-	+	-	-
R(f)	-	-	+	-	-	0	0	-
N	-	-	+	+				
U					-	+	-	-

2.2 Relationships between Various Gradients

Using the above framework, we are going to investigate time-series or cross-sectional variations in the distribution of rent and related variables among locations. Since location is represented by the distance from the center in the monocentric city model, these differences are expressed by respective derivatives with respect to distance. We will be working most of the time with logarithmic derivatives, referred as gradients, mainly because of their mathematical convenience.

We have seen that bid rent decreases and land consumption per household increases with distance from the city center (Wheaton's Theorems 2 and 3). It also means that the population density (n), number of households per land area, decreases with distance as the density gradient is simply the negative inverse of that of per household land consumption

$$n^*(x) = \frac{dn}{dx}/n = \frac{1}{dx}\left(\frac{1}{L}\right)/\left(\frac{1}{L}\right)$$

$$= - L'(x)/L = -L^*(x) < 0 \qquad (2.5)$$

Also, flattening of the rent or density gradient (that is, increase in its numerical value) over time is equivalent to the statement that the rate of rent or density growth is higher at farther distances, as

$$d(R'(t)/R)/dx = R''(x,t)/R - R'(x)R'(t)/R^2$$

$$= d(R'(x)/R)/dt \qquad (2.6)$$

It is important to note that our use of the logarithmic gradients does not presuppose the hypothesis contained in the so-called exponential density function popularized by Clark and Muth, that the density gradient is constant over all distances in a city. Our gradients are local. Nevertheless, we can define a city-wide gradient as the average of local gradients, and if local rent gradients flatten at all distances, the former also flattens; and more importantly, it means a consistent increase of the rate of rent growth with distance.

Since households adjust their bid rent and land consumption to compensate for the difference in commuting cost, it follows from the definition of the compensated price elasticity of demand

$$L^*(x) = - E(R)\ R^*(x) \qquad (2.7.a)$$

where $E(R) = - L'(R;\ U \text{ constant})\ R/L$

is the (absolute value of) compensated elasticity of demand for land with respect to rent.

Likewise, when the composite good, housing, is the variable of interest as in Muth's system,

$$H^*(x) = - E(p)\ p^*(x) \qquad (2.7.b)$$

where $E(p)$ is the compensated elasticity of demand for housing with respect to rental price, p.

Therefore, the variation in a household's expenditure on land in the distance is given as

$$\frac{d[R(x)L(x)]}{dx} = R(x)L(x)\ [\ R*(x) + L*(x)\ ]$$

$$= R'(x)L(x)\ [\ 1 - E(R)]$$

Then it follows:

[LEMMA 2.1] (a) A household spends less (same/more) on land at farther distances from the city center if demand for land is inelastic (unit-elastic/elastic) with respect to rent, i.e.,

$$d(RL)/dx \gtreqless 0 \qquad \text{iff} \quad E(R) \gtreqless 1$$

(b) The same with housing expenditure.i.e.,

$$d(Hp)/dx \gtreqless 0 \qquad \text{iff} \quad E(p) \gtreqless 1$$

The following Lemma is equivalent to the above except that it deals with variation in housing expenditure in terms of real purchasing power adjusted for the difference of income spendable on consumption goods, and therefore of marginal utility of income, rather than in nominal dollars. Note that

$$LRU'(Z)=L[U'(L)/U'(Z)]U'(Z)=LU'(L); \text{ and } \quad HpU'(Z)=HU'(H) .$$

[LEMMA 2.2]

(a) $$d(LU'(L))/dx \gtreqless 0 \qquad \text{iff} \quad E(\hat{R}) \gtreqless 1$$

(b) $d(HU'(H))/dx \overset{>}{\underset{<}{=}} 0$ iff $E(\hat{p}) \overset{>}{\underset{<}{=}} 1$

Proof.

On differentiating real term expenditures,

$$d[LRU'(Z)]/dx = d[LU'(L)]/dx$$
$$= U'(L)L\,[\,R^*(x) + L^*(x) + L^*(x)\{U''(Z,L) - U''(Z)\}/U'(Z)] \qquad (2.8)$$

Constructing a bordered Hessian of the household maximization problem of equation (2.1) and denoting its determinant as D, and cofactors as D(i,j),

$$D = \begin{vmatrix} U''(L,L) & U''(L,Z) & -R \\ U''(Z,L) & U''(Z,Z) & -1 \\ -R & -1 & 0 \end{vmatrix}$$

we can now define elasticities in terms of these determinants

$$L'(R;\ U\ \text{constant}) = U'(Z)\,D(1,1)/D = U'(Z)/D$$
$$L'(y) = -D(1,3)/D = [\,U''(Z,L) - U''(Z,Z)R\,]/D$$

or,

$$U''(Z,L) - U''(Z,Z)R = DL'(y) = -\,U'(Z)L'(y)/L'(R)$$
$$= U'(Z)E(y)R/E(R)y \qquad (2.9)$$

Substituting this in equation (2.8)

$$d[LU'(L)]/dx = U'(L)LR^*(x)[\,1 - E(R) - E(y)RL/y\,]$$
$$= U'(Z)LR'(x)\,[\,1 - E(\hat{R})\,]$$

by the Slutsky equation.

Noting $R'(x)<0$ completes the proof. Likewise for (b).

Q.E.D.

How we should interpret these Lemmas to bear upon the empirical world depends on values of the relevant price elasticities. Fortunately, price elasticity of housing demand has been extensively studied by many, and the wide difference in earlier estimates has been reduced to a small range in recent studies. Polinsky and Ellwood (1979) obtained the elasticity estimate between 0.67 and 0.72 using a carefully constructed estimation method and set of data on the U.S. housing market. They also showed that earlier estimates are brought close to the range once corrected for various biases. Another study with a set of controlled experimental data produced an estimate of the long-term elasticity a little below this range, though with a wider confidence interval, and the short-term elasticity much smaller. (Quigley and Hanushek, 1980) These and other recent estimates appear to leave little doubt about the inelasticity of housing demand with respect to price.

The question remains, though, whether the estimate is of the compensated (Hicksian) elasticity or the ordinary (Marshallian) one. If income is taken account of using a data set for one city and time, utility can be considered controlled for and the measured elasticity would be a compensated one. But the study by Polinsky and Ellwood used a national pooled sample, and the second study used longitudinal data for individual cities estimation. Therefore, Polinsky and Ellwood's estimate may probably fall between true values of compensated and ordinary elasticities. Even if the estimated value of 0.7 is close to the true compensated elasticity, the small income elasticity (around 0.5 in their estimation and less than 1 according to most studies) suggests that the ordinary elasticity would not exceed 1 (by the Slutsky equation. See the proof of Lemma 2.2.) Also, based on available information on housing consumption in countries other than the U.S. (for example, Lluch and Powell, 1977; and Follain, et al., 1980), the inelasticity of housing demand with respect to rental price appears to be a general phenomenon.

A strong case for inelasticity of demand for land can be derived from these evidences through the following Hicks-Allen formulation for derived demand, assuming a competitive factor market and elastic supply of structural inputs.

$$E(R) = (1 - K)E(p) + sK$$

where K is the factor income share of structural input in housing, and

s is elasticity of substitution between land and structure in housing production.

The pair of compensated elasticities in this equation can be substituted by the ordinary ones. The most widely used value for the substitution elasticity is 0.5, as estimated by Muth (1972), and no other study puts the value over one (McDonald 1980). These inelasticities of both housing demand and substitution imply, as apparent from the above equation, that the demand for land must also be inelastic regardless of the factor shares. Using probable values -- $E(p) = .5$, $s = .5$, and $K = .7$ -- yields an estimate of .5 for $E(R)$. This agrees well with more direct estimates by several researchers (0.35 to 0.7 according to Witte, 1977, and Sirmans and Redman, 1979)

On the strength of these evidences, we can restate the Lemmas into the following propositions which will be used repeatedly throughout this thesis. Note that the Cobb-Douglas utility or production function assumes the ordinary demand or substitution elasticity of 1 which is larger than the empirical studies suggest, but is still compatible with the two Propositions except that $LU'(L)$ would also stay constant for all distances.

[PROPOSITION 2.1] In a monocentric city, (a) land or (b) housing expenditure per identical household decreases with distance from the city center, i.e.,

$$\text{(a)} \quad dRL/dx < 0 \text{ ; and (b) } dHp/dx < 0 .$$

[PROPOSITION 2.2] In a monocentric city, (a) real term expenditure on land per household decreases with distance; and (b) real term housing expenditure does not increase with distance, i.e.,

$$\text{(a) } d[LU'(L)]/dx < 0 \text{ ; and (b) } d[HU'(H)]/dx \leq 0 .$$

Lastly, we can infer a simple relationship between gradients of rent and density by combining equations (2.5) and (2.7.a)

$$n^*(x) = E(R)R^*(x) \tag{2.10}$$

Inelasticity of demand for land will make the density gradient flatter than the rent gradient. The relationship between changes in the two gradients can also be inferred directly by differentiating the above

$$dn^*(x) = E(R)\, dR^*(x) + R^*(x)dE(R)$$

Below we shall see that $R^*(x)$ increases when the level of rent, R, increases generally. Then, by the Le Chatelier principle, the price elasticity decreases. In other words, the two terms in the above equation move in the same direction. Thus,

[PROPOSITION 2.3] The population density gradient increases or decreases as the rent gradient increases or decreases.i.e.

$$\text{sign}(dn^*(x)) = \text{sign}(dR^*(x))$$

2.3 Spatial Pattern of Land Rent and Population Density Changes

The foregoing provides us with the basis for determining the change in the rent gradient and hence the density gradient resulting from a change in some aggregate parameter of a city. Particularly, we are interested in differences of land rent growth rates among locations in a growing city. As discussed above, a flattening rent gradient would mean that rent grows faster at a farther location.

Urban growth means in the foremost sense an increase in population size or in the residents' income level. In an open city, the two go together, as population is increased in response to growing income so that the utility level is equalized. As Wheaton has shown, the growth expands total area of the city and increases land rent and land use density everywhere. Since

$dR/dy = R'(y) = 1/L$, by Theorem 3 of Wheaton.

Changes in the gradient are obtained by differentiating the above:

$$\frac{d}{dx}\left(\frac{dR}{Rdy}\right) = \frac{d}{dx}\left(\frac{1}{RL}\right) = -R^*(x)[1 - E(R)]/RL > 0$$

by Proposition 2.1. Thus,

[PROPOSITION 2.4] An increase in population and household income in an open city flattens rent and density gradients .

Another obvious measure of urban size is the physical expanse of the city. For a given terrain, population increase will result in physical expansion. This can also result from an improved transportation system. Let us first restate the relevant Theorems of Wheaton's which are limited to transportatin cost that is linear in distance with a general cost function

$$a(x) = \int_0^x a'(x')dx' \qquad a'(x') > 0 \text{ for all } x'. \qquad (2.9.a)$$

A reduction of the total transport cost due to some improvement (I) will increase rent and density as

$$R'(I;x) = -\frac{da(x)}{L\ dI} > 0\ ;\quad L'(I;x) = R'(I;x)L'(R) < 0 \tag{2.9.b}$$

But reduction in the marginal transport cost would generally be uneven. It is quite common in growing cities that congestion increases transportation cost in inner parts of the commuting route while expanding car ownership and highways reduce the cost in outlying areas. The impact of transportation investments on rent gradient must be evaluated considering these conflicting factors.

$$\begin{aligned} \frac{dR^*(x)}{dI} &= -\frac{da'(x)}{RL\ dI} - \frac{da(x)}{dI}\, a'(x)\cdot[\ 1 - E(R)\ ]/(RL)^2 \\ &= R^*(x)\{\ \frac{da'(x)}{a'(x)dI} + [1 - E(R)]\ \frac{da(x)}{a(x)dI}\ \frac{a(x)}{RL}\ \} \end{aligned} \tag{2.10}$$

Since the expenditure on land would at least be comparable to that on transportation and hence $RL/a(x)[1-E(R)]$ would be large, this equation tells us that

[PROPOSITION 2.5] (a) When either the total or marginal transportation cost is reduced and neither is increased, the rent and density would increase and their gradients would flatten.
(b) The local gradient would flatten as long as the marginal cost is reduced, provided that the total cost is not increased at a much higher rate than the rate of reduction in marginal cost.

The effects of income increase and transportation improvement in a closed city have been determined by Wheaton and can be summarized as follows.

[PROPOSITION 2.6] An increase in income or transportation efficiency in a closed city would lower rents in central areas, raise them in outer areas, and hence flatten the gradient.

The logic behind this proposition can be informally explained as the following. First, from Wheaton's Theorem 4, we know that the marginal utility of income, $U'(Z)$, increases with distance. Since the dollar amount of income increases are the same over all distances, this will shift the demand for locations in favor of outer areas, thus flattening gradients. In central parts of the city, even with the same demand for land, the least one can do with the additional income is to spend it all on the composite good so that $dy = dZ$. Actually, a better use of the money is possible, so $dU > U'(Z)dy$. Therefore,

$$dR(0)/dy = R'(y) + R'(U)\, dU/dy$$
$$= \frac{1}{L} - \frac{1}{LU'(Z)} \frac{dU}{dy} < 0 \quad .$$

The effects of transportation improvement can be evaluated in the same manner, for a reduction in transportation cost effectively increases income spendable on goods. The flattening of rent gradient will be even more obvious because the cost saving also increases with distance.

Sometimes the utility level of residents is said to increase with urban growth. But this will be the result of some other growth impetus in a closed city as we have seen just above, and the utility increase cannot be considered an independent cause of urban growth. In fact, for an open city, an increase in an exogenous level of utility can be seen as a cause of decline in urban size, as outmigration must occur in order that the city residents' consumption and utility be increased. By Wheaton's Theorem 3,

$$R'(U) = -1/LU'(Z) < 0 \; ; \; L'(U) = R'(U)L'(R) > 0 \, .$$

As the rent at the border rent is reduced as is everywhere, the border is pushed in; increased land consumption and reduced city size combine to decrease the total population. Its effect on rent gradient follows directly from Proposition 2.2.

$$\frac{d}{dx}\frac{R'(U)}{R} = \frac{d}{dx}\left[-\frac{1}{LU'(L)}\right] < 0 \qquad \text{as } R = U'(L)/U'(Z)$$

This effect for an open city can be considered a partial effect of induced utility change in the case of a closed city:

[PROPOSITION 2.7] Increasing utility, exogenous for an open city and endogenous for a closed city, will reduce rent and steepen its gradient everywhere and will reduce the city area and population.

For a closed city, growth in population alone causes a drop in utility level as Wheaton has shown. No other parameter of consumption changes, so the net effect is exactly opposite to the above Proposition. Namely,

[PROPOSITION 2.8] Population increase in a closed city would expand the city, raise rent and flatten its gradient everywhere.

The effect of utility difference on urban spatial structure enables us to consider the impacts of various direct constraints on the physical growth of cities. These include economic ones such as higher farm rent, natural ones such as water and steep hills, and legal ones such as the greenbelt and other prohibitive regulations.

It has already been shown by Wheaton that higher farm rent would decrease utility. Then, Proposition 2.6 makes clear that it will result in higher rent and flattened rent gradient everywhere.

Some growth control measures like the greenbelt can be seen as a legal limit to the city border superseding the market condition of border (equation (2.3)). By differentiating the condition that the size of population does not change in a closed city (equation (2.4)), and rearranging,

$$dU/dB = - N'(B)/ N'(U) > 0$$

as $N'(B) = l(B)B/L(B) > 0$; and

$N'(U) < 0$ by Proposition 2.6.

Therefore, an extended legal border, if it supersedes the border determined by market competition between urban and rural sectors, will make the rent gradient steeper. Conversely, a regulatory or physical limitation on urban area would flatten the gradient.

Sometimes the limitations do not present themselves at the outer edge encircling the city but start right from the city center located alongside a limiting feature such as waterfront or steep hills. This case can be simplified as a reduced arch (ϕ) of a semicircle and can be analyzed in a similar manner. It is fairly obvious that utility would decrease as a result, but the proof is provided for the sake of completeness. Differentiating the border condition,

$$\frac{dR(B)}{d\phi} = R'(B)\,\frac{dB}{d\phi} + R'(U;B)\,\frac{dU}{d\phi} = 0$$

or

$$\frac{dB}{d\phi} = -\,\frac{R'(U;B)}{R'(x;B)}\,\frac{dU}{d\phi}$$

Substituting this in the condition $dN = 0$ and rearranging,

$$\frac{dU}{d\phi} = N/\phi^2 \left\{ \int_0^B [xL'(U)/L^2]dx + \frac{B}{L(B)}\,\frac{R'(U;B)}{R'(x;B)} \right\} > 0$$

as $L'(U) > 0$, $R'(U) < 0$, and $R'(x) < 0$.

Therefore, $dB/d\phi > 0$, and by Proposition 2.5, $dR^*(x)/d\phi > 0$.

Note that all these constraints would change the aggregate size of an open city but would not change any of the consumption parameters - namely, utility, income, and transportation cost - and hence leave the rent and land consumption unchanged. In sum,

[PROPOSITION 2.9] Restraints on urban physical growth by zoning, terrain, or high farm rent would raise rent and density and flatten the gradients everywhere in a closed city. They will decrease the physical size and population but will not change the spatial structure of an open city.

Apparently this Proposition would be useful mainly for cross-sectional comparisons, and would have little to do with growth impacts in themselves. It is interesting to note that the smaller in area a closed city is forced to become by these limitations, the flatter the rent gradient is; whereas the larger the city becomes by growth in population and the like, the flatter the gradient becomes.

In conclusion, we can say very generally that urban growth flattens the rent gradient and hence the population density gradient, as every important aspect of growth - in population, income, and transportation system - results in the flattening of the gradients. However, urban growth is sometimes ambiguous. It is not very rare that a city grows in one aspect but declines in another as in some metropolitan areas of the U.S. where population declined while income level continued to grow. But in most of these cities income growth clearly dominated the population decrease which was neither consistent or substantial. The last Proposition also introduces uncertainty to our conclusion. Some aspects of urban expansion can be likened to a removal of limiting factors: for example, advances in civil engineering equipments such as tractors and bulldozers can open up areas previously left in unbuildable condition for new urban use. This kind of expansion would in the first sense be a result of growth in other basic measures but it would partially offset the trend toward flattening gradient.

2.4 Spatial Pattern of Land Value Changes

The last section makes it clear that urban rent increases at a faster rate in areas more distant from the city center when the monocentric city grows in any basic measure of city size — population, income level, transportation efficiency, and hence physical area. If the city is reduced in size, the opposite happens: rent at a more distant location declines faster. In order to transtlate this pattern into that of land value, the price for a permanent or other long-term title to the rent stream, we need to consider conditions of equilibrium in the capital asset market.

In a competitive capital market, a prospective landowner would be indifferent between owning a piece of land or other asset if the return from the land, rent plus the capital gain, is expected to equal the opportunity cost of capital

$$r(t)V(t) = R(t) + V'(t) \qquad (2.11)$$

where $r(t)$ is the rate of return for alternative investments

$V(t)$ is the land value at the start of period t

$V'(t) = V(t+dt) - V(t)$ is the increase in land value

R(t) is the present period rent receivable at the end of the period

Rewriting this equation we obtain the following expression for the land value appreciation rate, g, as the difference between discount rate and rent-value ratio.

$$g = \dot{V}^{*}(t) = r - R/V \qquad (2.12)$$

This two-period equilibrium implies the following multi-period equation for capitalization of rent stream, which is considered the definition of value.

$$V(t) = \int_{t}^{T} R(t') \exp[-\int_{t}^{t'} r(u)du] \, dt' \qquad (2.13.a)$$

The end period, T, is infinity for the ordinary value of land. Some finite future time, independent of the starting period, can be assigned to it without invalidating the analyses below. The usual capitalization formula is a simplification of the above, with the assumption of a constant discount rate, i.e.,

$$V(t) = \int_{t}^{\infty} R(t') \exp[-r(t'- t)] \, dt' \qquad (2.13.b)$$

Land value declines with distance as long as the present or the future rent does:

$$V'(x;t) = \int_{t}^{\infty} R'(x;t') \exp[-r(t'-t)] \, dt' \qquad (2.14)$$

A similar expression can be given to the land value change over time. The capitalization equation (2.13.b) can be integrated by parts to yield

$$V(t) = \frac{R(t)}{r} + \frac{1}{r}\int_t^T R'(t') \exp[- r(t'- t)]dt'$$

Comparing this with equation (2.11), we obtain the expression of change in value as the present value of the stream of rent changes.

$$V'(t) = \int_t^T R'(t') \exp[-r(t'- t)]\, dt \qquad (2.15)$$

Note that we can define derivatives of rent, though it may not be a continuous function in time or distance, as long as the variations are bounded. (By Lebesgue's Theorem: See Riesz and Sz.-Nagy, 1954.)

Juxtaposing definitions of land value and its change, we have:

[LEMMA 3] If the discount rate remains constant throughout time, the rate of land value appreciation is an average of future rent growth rates weighted by discounted rents of respective periods. i.e.

$$\begin{aligned} g(t) &= V'(t)/V(t) \\ &= \int_t^T R^*(t')R(t') \exp[\ .\ ]\, dt' \Big/ \int_t^T R(t') \exp[\ .\ ]\, dt' \end{aligned} \qquad (2.16)$$

A critical problem with the above analysis of land value change lies in the fact that land value is based on expected future rents and discount rates, although the value itself is a present quantity as are the present rent and discount rate. The speculative element of it is all contained in the capital gain, and the relationships above hold only with the expected gains which could well be different from actual gains. How the real events correspond to expectations and how expectations are formed and work in people's mind are fundamental questions that are far from any satisfactory resolution and that distinguish different models of capital market behaviour. Any elaborate study of this subject is beyond the scope of this thesis and we will only consider two simplest models.

A frequently employed simple model, the myopic foresight model, assumes that people behave as if the future would be just like today. Rents are expected to be constant throughout the future, and capital gains are not expected. Then, from (2.11)

$$V(t) = R(t)/r \quad \text{or} \quad R/V = r \qquad (2.17.a)$$

And the gradients of land value and rent are the same

$$V^*(x) = \frac{R'(x)}{r} \Big/ \frac{R}{r} = R^*(x) \qquad (2.17.b)$$

Although people expect no capital gains, the next day may prove them wrong. An increase in rent already taken place has to be accomodated, and the land value will rise to bring the rent-value ratio back to r. Therefore, <u>ex post</u>,

$$V^*(t) = R^*(t) \qquad (2.17.c)$$

The value appreciation rate would increase with distance but fall to zero beyond the border just as the rate of rent growth.

Under the usual circumstances of continued economic changes, however, this model is clearly implausible for it implies that landowners reap excess return on their land everytime the rent increases or suffer loss everytime it decreases, no matter how repeatedly it happens. Alternative models of expectations are based on the fact that substantial if noisy information is available for prediction of the future, and people use it. The hypothesis of rational expectations states that the resulting prediction is on average about as good as the prediction of relevant economic theories (John Muth, 1961). At a further extreme is the perfect foresight model which simply assumes that people know all relevant future events exactly and certainly. But many rational expectation models are in effect equivalent to this extreme model as the market processes are modeled in such a way that real events turn out to match people's prior expectations in probabilistic terms. In other words, real events seldom surprise people so seriously as to force a change in their behaviour while surprise always occurs but never induces people to change their behaviour under the myopic expectations model.

Both are of course extreme and unrealistic. Nevertheless, it appears to be the better strategy to concentrate on the perfect a model of perfect foresight or simple rational expectations for two reasons. First, substantial rationality of people's expectations is probably indicated by the healthy existence of active capital markets, past studies of firms' expectations and dynamic adjustments, etc. Second, as far as the expectations are concerned, the myopic foresight model can be regarded as a special or degenerate case of the perfect foresight model in the sense that the unchanging future is one of many possible course of the future. So the former can be easily derived as a special case if we analyze the case of the latter, more general pattern of expectations.

In the land market of the growing city, what would rational expectations would entail? We suppose that people are aware of the trend of flattening rent gradient and the expectations are stable. Further, the expected path of urban growth could be characterized as monotonic i.e., people would expect fluctuations but the long-term projection would be a continuation of the trend in one direction. Then the actual rate of growth of land value is the same as the mean of expected ones given by the equation (2.12). Its variation among locations is obtained by differentiating the same equation.

$$\frac{dV^*(t;x)}{dx} = -\frac{\quad}{V}\frac{dR(x,t)}{dx} + \frac{dV}{dx} R/V^2$$

$$= \{ - R^*(x;t) \int_t^\infty R(x,t') \exp[-r(t'- t)]\, dt' + \int_t^\infty R'(x;t') \exp[-r(t'- t)]\, dt' \} R/V^2 \qquad (2.19)$$

Inside the border, the present rent gradient, R*(x:t), is negative. Further, since the rent gradient is expected to flatten over time,

$$R^*(x;t)\ R(x,t') < R^*(x;t')\ R(x,t') = R'(x;t')$$
$$\text{for } t' > t$$

Therefore, in equation (2.19) the first term in the brackets is larger than the absolute value of the second. Hence

$$dV^*(t;x)/dx > 0 \qquad \text{for } x \leq B$$

That is, the rate of appreciation is higher as the distance from the center is increased toward the border.

Beyond the border, the farm rent does not vary over distance but land value gradient is negative as the land is expected to begin earning urban rent at some future time (equation (2.14)). Then the first term in equation (2.19) is zero and the second term is negative, hence

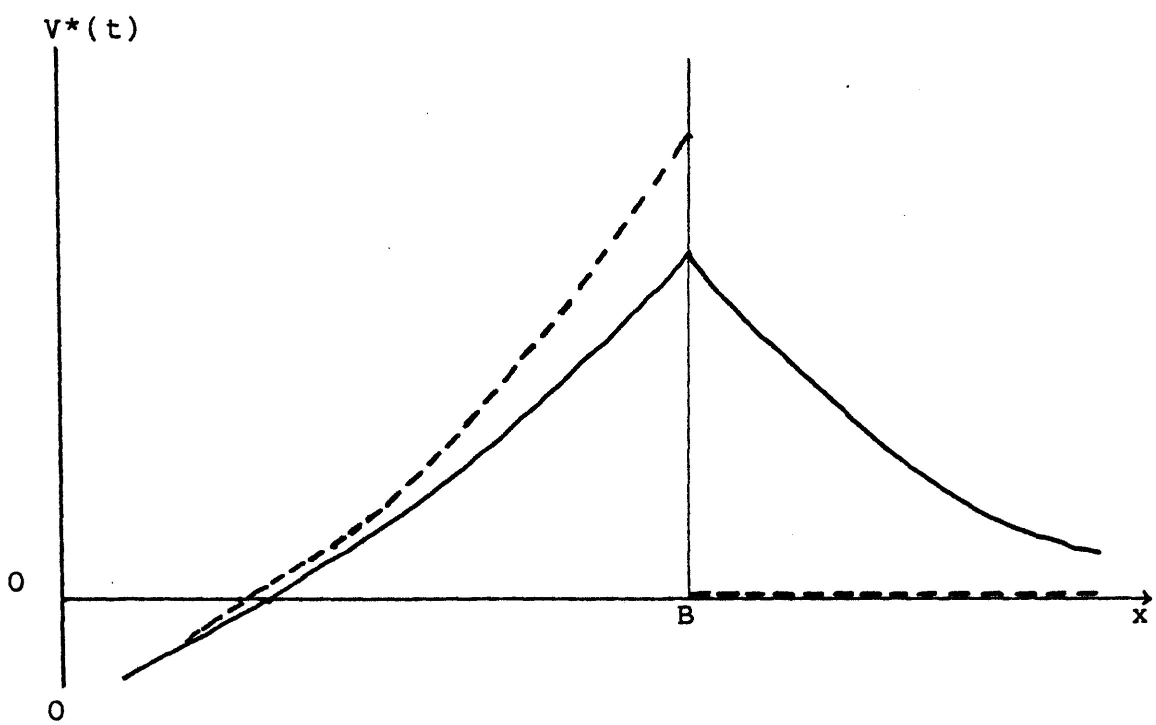
$$dV^*(t;x) < 0 \qquad \text{for } x > B \ .$$

i.e., the rate of appreciation decreases with distance from the boundary away from the city center. The slope of the rate of appreciation in distance would exhibit a sharp edge at the border because the rent gradient is negative just inside the border, and zero just outside it. (Of course, there would be a certain point far outside the city limit beyond which urban use is not expected for a foreseeable future and the gradients of rent, value and the value appreciation rate remain flat.)

The following Proposition and Figure 1 summarize the above analysis of spatial distribution of the land value appreciation rate.

[PROPOSITION 2.10] The rate of land value appreciation increases with distance from the center in a growing monocentric city up to the border. Beyond the border, it falls sharply to zero under myopic expectations but falls gradually to zero under rational expectations.

Figure 1. Distribution of Land Value Appreciation Rates



- - - - under myopic expectations

——— under rational expectations

So far we have assumed that the discount rate remains unchanged over time or location. But the market interest rate changes with economic fluctuations, especially in developing countries, and risk and the property tax rate which make up the remainder of the total effective discount rate change substantially with time and location.

But the inflation rate does not constitute a separate influence as long as it is accounted for in a consistent way. That is, if rent is expressed in current market dollars, it should be discounted by the nominal rate; if rent is in constant dollars, the real rate should be used. The two methods yield equivalent capitalization since the real rate is by definition the nominal rate less the inflation rate.

It is now firmly established that uncertainty is compensated by a higher rate of return. It is axiomatic that, other things being equal, people feel more uncertain about events of a distant future than an immediate future. Where the rent and hence value is growing fast, the future rents carry more weight (in calculating the harmonic mean of the discount rate) relative to the present rent than is the case when the rent and value are growing slowly. Consequently, the faster growing land value is affected more by periods over which uncertainty is relatively high, and hence it incorporates higher overall uncertainty. Uncertainty would affect the whole city as well in times of economic flux, but again it will be more pronounced for lands whose value depends heavily upon expectations far into the future.

A tax levied as a percentage of the value of a property in effect raises the discount factor, as the land should yield not only the market interest but also the ad valorem tax on property value. With a tax rate, w, the equilibrium relation between land value, rent and capital gain can be expressed,

$$(i + w)V(t) = R(t) + V'(t)$$

where i is the market interest rate.

This translates into a capitalization formula with (i+w) the applicable discount rate.

Let us now evaluate how the total effective discount rate affects the land value by differentiating the capitalization equation, assuming rent is independent of the discount rate

$$\frac{dV(t)}{dr} = \int_t^{\infty} (t - t')\, R(t')\, e^{-r(t'-t)}\, dt'$$

$$= -\frac{V(t)}{r} - \frac{1}{r}\int_t^{\infty} (t' - t)\, R'(t')\, e^{-r(t'-t)}\, dt'$$

$$= -\left[\, V(t)/r + V'(t)/r^2 + V''(t)/r^3 + \ldots.\right]$$

$$= -\int_t^{\infty} V(t')\, e^{-r(t'-t)}\, dt'$$

$$= -V(t)/\ (r - G) \qquad (2.20)$$

$$\text{where} \quad G = \int_t^{\infty} V'(t')\, e^{-}\, dt' \Big/ \int_t^{\infty} V(t')e^{-}\, dt'$$

is the weighted average growth rate of land value.

In other words, an increase of the discount rate reduces the level of the land value, more severely in faster growing areas. Also, the growth rate itself would be affected by the discount rate change as apparent from equation (2.12), which says that the appreciation rate is the difference between the discount rate and the rent-value ratio. Some draw an inference that a higher discount rate causes faster appreciation, neglecting the rent-value ratio which should change with the discount rate (for example, Vining and Hiraguchi, 1977). Some others consider the latter effect but reach the same conclusion on the basis of a few specific time paths of rent (see Walters, 1978). But it seems an haste to make a generalization from such a partial analysis. Differentiating equation (2.12),

$$\frac{dg}{dr} = \frac{dV^*(t)}{dr} = \frac{d}{dr}(r - R/V)$$

$$= 1 + R\frac{dV}{dr}/V^2$$

$$= 1 - (r - g)/(r - G) \qquad (2.21)$$

from (2.12) and (2.20).

Therefore, the effects of the change in the discount rate on the level and appreciation of land value can be summarized as

[LEMMA 4] (a) An increase in the discount rate will reduce land value proportionately more in areas where future land value is expected to appreciate faster:

$$V^*(r) = - 1/(r - G)$$

(b) An increase in the discount rate will raise the appreciation rate if the present appreciation rate (g) is higher than the average future appreciation rate (G), reduce it otherwise:

$$dV^*(t)/dr = (g - G)/(r - G)$$

To evaluate this Lemma adequately, we need to know the average of future growth ratea of value, G, particularly in comparison with the present growth rate of value, g, which is the average of future growth rates of rent (see equation 2.16). From equations (2.15) and (2.20.a), we can write

$$G(0) = \int_0^\infty t'R^*(t')\, R(t')\, e^{-rt'} dt' / \int_0^\infty t'R(t')\, e^{-rt'} dt'$$

That is, G can also be called an average of future rent growth rates just as g is, but with heavier weights assigned to more distant futures than in the case of g. Conversely, rent growth in the immediate future becomes less important in evaluating G. Unfortunately, we cannot derive from this many explicit insights about the relationship between the two average rates without assuming a certain time path of rent. However, several points are easily read from the above expression.

First, when rent is expected to grow at a constant (or zero) rate, value is also expected to grow at the same rate and hence the discount rate does not affect the appreciation rate (as g = G). Second, the two average rates would move together among locations within the monotonically growing city as in general the future growth will be faster in areas growing faster at the present. Therefore, land value in outer areas would be more sensitive to discount rate changes. Third, however, difference in G would be proportionately smaller than that in g because the rent gradient grows flatter and hence differences in the rent growth rate would in general become smaller as time goes by.

The second and the third points leave it difficult to define the relative impact of the discount rate on the appreciation rate. However, it is clear that G is likely to be larger than g in areas awaiting urban development in the near future because the zero growth of farm rent before urbanization would be given less weight in calcualting G than in g. Therefore, higher uncertainty and hence a higher discount rate applicable for those areas would make the pace of appreciation slower than under a risk-free rate.

These implications, particularly the last one, contradict the conventional wisdom that argues for appreciation rates increasing with the discount rate, particularly in the suburban area. This argument seems to be an empirical generalization of the frequently observed association between high rates of discount and appreciation but analytically it neglects the effect of the discount rate on the rent-value ratio. But we have shown that fringe area land values would appreciate faster without or despite the high uncertainty and discount rate. This is not to deny the influence of the discount rate which obviously varies over time and locations, but its impact should probably be understood in ways opposite to the conventional reasoning.

Changes in market interest rates are usually exogenous to a city. Locally, uncertainty would be reduced and land values raised in general as the city matures and the simple passage of time improves information. Capital gains resulting from these general influence would be higher in outer areas because the inherently faster rate of rent growth would make the value more sensitive to changes in the discount rate.

An abnormally high risk or interest rate should not be regarded as a permanent feature that causes rapid appreciation in itself but as a transient feature of a particular area or period which is liable to relatively quick abatement. If it is abated, high capital gains result, that is apparently an abnormally fast appreciatiation but is essentially an one-time occurrence. The reduction in uncertainty is also likely to cause acceleration of appreciation in suburban land markets but this shift is not general, and some other areas may experience a slowing down of appreciation. At any rate, this is probably not as important an aspect of land value changes resulting from variations of the discount rate when compared with the one-time change in land value level itself also resulting from variations in the discount rate.

Chapter 3. LAND VALUE and USE with DURABLE HOUSING

3.1 Defining Value of Land for Durable Use

In the last chapter we have assumed that land use is adjustable without cost or delay and that land rent is uniquely determined for a given location and time. But, in reality, urban land is used in combination with structures that are extremely durable and can be changed only at very high costs. In this situation, although the rental price of a land use property as a whole (in our case, housing) is determined by the market as usual, land rent is not observed as a market variable. Nor can rent be derived from the former through the marginal value product rule because the input combination of land and strucuture at a particular moment cannnot be and usually is not adjusted to be optimal as the marginal value rules would prescribe under presently prevailing market parameters.

This apparently causes a lot of problems in defining and analyzing urban land value and use. In this section we will examine essential elements of the concept of land value under conditions of durable land use. It is in a large measure redundant since rules of land valuation with or without durable inputs have been known all along under the name of "the best and highest use principle". Nevertheless, it is felt necessary to clarify the basic conceptual issues because understanding of market pricing is always a crucial step in analyzing any resource allocation problem and because the known principles are, or became, vague as persitent confusion in literature attests.

Let us define what is the quality that we call "durability" that calls for special analytical treatment. First of all, it is important to distinguish between technical durability meaning that a good does not wear off easily, and economic durability meaning that a particular good or a part of it is not easily disposed of. Capital goods are durable in the sense that it is uneconomical to disengage any factor in use and transfer it to another situation even when the existing combination of the factors is not optimal in view of the current market conditions.

It is uneconomical in the sense that the factor loses most of its value if transferred. Staying put, its value may be less than what it would command as the original resource but more than its value in any other use. It is this economic inflexibility that constrains the optimization problem and renders the comparative static analysis inadequate. Physical durability is usually a necessary condition but is not a sufficient one. An irrigation pump, for example, is very durable, and it is just as valuable even if it is transferred to another place or to another purpose. Thus the one-period cost of the pump is well defined as the hire price covering the interest and depreciation on its one-time purchase price. This type of flexible combinations of physically durable factors can be handled adequately within a comparative static framework, although the production of the durable factor itself may call for a different framework.

On the other hand, most urban land use structures once committed to a property are of little use for any other purposes. Structural members of a new single story house are practically of no value for any other building and only marginally valuable even for on-site redevelopment such as building a two story house at the same place. Marginal adaptations of existing structures can usually be made without great loss of efficiency, but only within a narrow limit.

What is the consequence of introducing this quality, the economic durability or inflexibility of a particular land use, in the analysis of urban housing and land market ?

From the consumer's point of view, durability means that an available house comes with predetermined proportions of component factors, land and structure, probably different from those that he would have liked if he were free to assemble his own or to make any marginal modifications, adding or deleting portions of land and structure. However, he is not allowed such choices, but only the choice of this or that prepackage called a house. As a consequence, the price for each factor is not relevant to his choice of residence, only the rental price of the whole house is. Under certain restrictions implicit (hedonic) prices can be assigned to attributes of the house, but such prices in general have only tenuous relationships with the overt market.

From the housing producer's point of view, separate and implicit price for each input factor is again irrelevant for his supply decisions. Further, rental income from a house for each period cannot dictate his production decision in isolation from revenues and costs incurred in other periods. Breaking up the problem into that of multi-factor production and that of multi-period production, let us first consider the derivation of static land rent and value when land is considered an input to housing production instead of a separate consumption good as in the last chapter.

When most input factors for land use are essentially perishable as in agriculture, one-period land rent is defined as the maximum surplus of revenue from the land use over the cost of non-land inputs so that the zero profit condition is maintained. That some inputs are physically durable does not necessarily cause any problem in defining the rent as we have seen in the example of an irrigation pump. This is the essence of the assumption of malleability in comparative static analysis of urban land use. It is assumed that parts of a structure can be rearranged and moved to another place as to befit whatever the prevailing market condition dictates without cost of such adjustments or loss of their value as fresh input factors. The housing producer can essentially hire these inputs period by period at the cost of depreciation and interest. On the other side of the ledger, his revenue is also well defined by the market demand for housing. Thus the housing producer tries to optimize the input combination to maximize the surplus of the current revenue over current cost, which is defined as the land rent. Formally, following the discussion in section 2.1 on Muth's framework of urban housing market analysis,

$$\max R = \max_{h,L} (pH(h,L) - rch)/L$$

where h is the housing structure used by a household, and land (L) and housing (H) are also expressed in amounts per household. c is the marginal one-time purchase price of the structural input; r is the discount rate applicable to the cost of structure, including interest and depreciation rates.

If the returns to scale is not constant, the producer would have also to decide how many of these houses to build. In case the housing is produced by a constant returns to scale technology, we can write the production function in the density form, with the structural density as the sole argument:

$$Q = Q(k) \quad \text{with } Q = nH \text{ and } k = nh$$

where Q is the quantity of housing per land area, or housing density; k is the amount of the structural input per land area, or structural density; and n is the number of households per area.

The assumption of constant returns to scale in housing production is of course a very common one and also has proven to be a good empirical approximation. This in combination with the assumption of weak separability of preference for housing (see section 2.1) implies homotheticity of subutility function in housing input factors. Though certain homothetic utility functons are found to be empirically untrue, a wide variety of utility functions can be specified within the restrictions of homotheticity, closely approximating acutal consumption behaviour.

However, in order for this implication of homotheticity not to degenerate into homogeneity of utility function, which is too restrictive, we should be careful to specify that marginal rental of housing is dependent on the size of a household's dwelling unit, i.e.

$$p = p(H, y, U)$$

where p is the marginal rental price of housing.

With this precaution, we will employ the assumption of the constant returns to scale production function expressed in density form throught the remainder of this thesis. Then the housing producer's maximization problem and the first order condition can be written

$$R = \max_{H,k}\{pQ - r\acute{c}k\} \qquad (3.1)$$

$$pQ'(k) = rc \qquad (3.2.a)$$

In case the producer wants to make an adjustment to existing housing, the standing structure can be rearranged appropriately without cost, as long as the cost of additional input is justified by the additional revenue, i.e.

$$p[Q - q] = rck' \qquad (3.2.b)$$

where q is the existing quantity of housing per area, and k' is the additional structural input.

The last discussion provides a point of departure for the complications involved in situations where land use is durable and the costless adjustment is ruled out. Under durability, if a housing is built so as to maximize the current profit, or rent as defined in equation (3.1), it will turn out to be an inefficient land use when housing rental or input cost changes. This type of inefficiency cannot be totally avoided as long as adjustment is costly, but the developer must seek to maximize the efficiency of resource use for the entire life of the housing by making a long-term plan based on the expectation of the future. Since a short-term maximization problem is no longer relevant, short-term rent is not the primary concern to the developer. Land value is the objective variable to maximize, and it is defined as the surplus of the stream of revenue over cost from the best possible programmme of land use over all future periods:

$$V(t(0)) = \text{supra} \sum_{i} \int_{t(i)}^{T(i)} [P(t',i) - c(t')k(t',i)] e^{-r(t'-t(0))} dt' \qquad (3.3.a)$$

$$i = 0, 1, 2, \ldots ,$$

where $P(t',i)$ is the net rental income per land area at time t', from land use i that is established at time $t(i)$ and replaced at time $T(i)$; $k(t',i)$ is the durable input added at time t' to the land use i ; $c(t')$ is the unit purchase price of the durable input; and $T(i)$ is the end period when the housing built at time $t(i)$ will be replaced.

Alternatively, denoting the profit for the i th phase of development or the leasehold value of land for the life of a housing built at time t(i) as A(i),

$$V(t(0)) = \sum_{i} A(i)e^{-r(t(i)-t(0))}$$

$$= A(0) + V(t(1))e^{-r(t(1)-t(0))} \qquad (3.3.b)$$

This definition of land value is not easy to apply in practice because the future is difficult to foresee and the market for land is usually very thin. But at least conceptually it is straightforward enough as long as land is not being occupied for a durable use. A good deal of confusion persists, however, about correct evaluation of land already being used for housing or other urban purposes. Land is often mistakenly evaluated on the basis of existing use. But the critical point is that land value is based on the optimal land use programme beginning from the current period and the above definition should be applied regardless of the existing use.

In general, every input factor used to produce a durable real asset loses not only some of its original value as a resource but also its separate economic identity, so that it is irrelevant to evaluate individual inputs. But real estate is an exception, for land can always be used as any open land after the previous building is swept away: it makes no difference to a new development whether or not there used be a building on the land. On the other hand, material and labor inputs for construction become completely sunk in the particular land use and cannot be transferred, as illustrated in the above example. This does not mean they have no economic value. Obviously, a property is usually more valuable with a building on the site than without it. But, in principle, the value of a real estate is arrived at by summing the discounted future incomes from the property as a whole, not separate values of structure and land, although the latter approach is often taken in practice. Value of the structure is determined as the remainder of this value of property as a whole after subtracting land value which is independently definable at the opportunity cost as an unused resource.

It appears inconsistent that different evaluation rules are applied to the two factors of the same final product. Indeed, values of both factors should be derived by the same principle from the value of the property to be built, but only at the time of its construction. After the construction, the rental price of the particular property is the only relevant and determinate price in the market for the final product, housing. The production is already completed and the factor costs are no longer relevant in determining the availability and consumption of the existing house. But the owner should decide whether to keep operating it as a housing, to renovate it, or to sell it to a producer (possibly himself) as a factor of production for a new housing or another kind of real estate.

Since the durability renders the structural members unusable as materials for new development, the value of the latter option consists solely of the value of land which is reusable. In other words, land value enters the land use decision problem as a reference to the value of complete redevelopment. This role of the land market in resource allocation requires that land value not be dependent upon the existing use but solely upon the best possible future use. Therefore at any time land value should be defined by the optimal programme of land use starting from that moment as specified in equation (3.3). When this value of land exceeds that of the existing house as a whole, it is time to replace it with a new building. In other words, the value of existing structure defined as the remainder expresses the value of keeping the structure over that of razing it and building anew in its stead.

Correct evaluation itself is not inherently more or less difficult with durable land use than with malleable use. But durability poses a serious problem in analyzing variations of land value with respect to time or location because the discrete series of land use successions contained in the definition of land value resists classical techniques of analysis. The preferred solution by economists to this problem is simply to assume it away in various ways. Let us briefly examine implications of a few widely adopted simplifications.

In most empirical studies of housing production, land value at the time of new housing development is considered the true resource cost of land for housing. But the leasehold value is the true measure of the latter, and it would be smaller than the full market value of land which includes the land value retrievable when the housing is eventually replaced (Refer to equation (3.3.b)). The simplification would be harmless for comparing different land values if the life-span of the projected land use and the discount rate are large enough so that the end-period value would be negligible or if the project life-span and the land value appreciation rate are the same for different developments. But these assumptions contain substantial errors about urban housing and land markets as we shall see in the next chapter.

In modeling dynamics of land use, assuming myopic expectations on the part of developers eliminates the analytical problem. If developers expect no change in market parameters, they will try to maximize (perceived) land value on the basis of the present rental income which is expected to be received in perpetuity. Hence,

$$V(t{:}myopic) = \max_{k}\{p(t)Q(t)/r - c(t)k(t)\} \qquad (3.4.a)$$

or

$$R(myopic) = \max_{k}\{pQ - rck\} \qquad (3.4.b)$$

where $V(myopic) = R(myopic)/r$

Comparing these with equations (2.17) and (3.1) we can see that the analytic situation reverts to the static one under myopic expectations, with the difference that land use change cannot happen as easily as with the malleable structure. We have argued in section 2.4 that market agents' expectations are probably a lot more sophisticated than that, especially if market conditions change recurrently and substantially. Besides, we shall see later that analytical simplicity gained by the restrictive assumption is largely illusionary.

Sometimes it is assumed that land use never changes once it is put in place. This may or may not be a reasonable empirical approximation of urban land use. At any rate, however, we cannot discuss the land value of built-up areas under the no-change assumption, as we have seen that possibility of land use change is the only reason why land value is relevant and determinate for already urbanized areas.

Obviously these are attempts to get around the unwieldiness of the full and correct definition of land value. Analysis is made difficult because the land use decision for every discrete stage and period is dependent not only on current parameters but also upon those of every period of the relevant future. The difficulties would be avoided without resorting to one of the above unsatisfactory simplifications if we can define opportunity cost, or rent, of land for each short period. But it is even more unacceptable to define it, as often done, by reference to the existing use. We will find in the following section an appropriate definition of the shadow rent for each period unencumbered by the above compromises. But before we get such a definition it is necessary to spell out implications of the land market on optimal urban land use.

3.2 Rules of Optimal Land Use and Shadow Rent

We have seen above that land value is determined and used chiefly to decide whether to keep or replace the existing land use. The decision for a property owner is choosing the right time, t*, to replace the existing use to maximize his income, the sum of discounted rental income from the present use and land value at time t*:

$$\max_{t^*} J(0) = \int_0^{t^*} P(t',-)\, e^{-rt'}\, dt' + V(t^*)\, e^{-rt^*} \qquad (3.5)$$

where $P(t',-)$ is rental income derived from the existing land use, net of current cost at time t'.

This yields the first order condition

$$\exp[-rt^*] = 0 \quad ; \text{ or}$$

$$P(t,-) - rV(t^*) + V'(t^*) = 0 \qquad (3.6)$$

In other words, the owner should keep the property indefinitely, or replace it when current income is no more than the opportunity capital cost of land value gained by waiting one more period. Current rental income $P(t^*,-)$ is farm rent, $R(f)$, for conversion of land from farm to urban use; or housing rental $p(t^*)q(t^*)$ for redevelopment of built-up land, where p is the rental price of housing and q is the quantity of existing housing per land area to be replaced.

On the other side of the ledger, the housing developer assesses the value of land on the basis of the optimal use possible at the time. His maximization problem is expressed by the definition of land value itself, equation (3.3). We can reduce the scope of this maximand to optimization of land use for one immediate stage and of finding an optimal time to replace the land use of the stage. Rewriting equation (3.3.b),

$$V(t) = \max_{\{k\},T} \Big\{ \int_t^T P(t',Q)\, e^{-r(t'-t)}\, dt' - \int_t^T c(t')\, k(t')\, e^{-r(t'-t)}\, dt' + V(T)e^{-r(T-t)} \Big\} \qquad (3.7)$$

where Q is quantity of housing per land area, or density of housing, initially built at time t; and T =T(t,{k}) is the end period of development, endogenously determined.

The optimal series of structural inputs, {k(t')}, is most likely be discontinuous - beginning from large construction cost, and then a few substantial renovation outlays, with small maintenance expenses inbetween. As this is too large a departure from continuity, it is impossible to obtain a analytical solution of the above dynamic optimization problem. It is still possible to solve for individual period inputs, but it is much simpler yet does not affect the essential feature we are looking for if we convert it to a yet simpler optimization problem by ignoring all but the initial construction cost.

This simplification amounts to regarding large scale renovations as practically a redevelopment, other maintenance costs as current expenses, and the depreciation of structure as progressing exponentially and hence being represented by the discount factor. In either dress, this assumption does not seem to be a serious misrepresentation in light of the empirical findings on depreciation and maintenance. For example, Mendelssohn(1977) found that home improvement and maintenance expenditures are extremely small in comparison with construction cost, and further, they are determined more by the economic characteristics of the occupant than by physical condition of housing, supporting the notion of these costs as essentially current expenses not the durable investments that affect the whole revenue generation plan. Margolis (1982) found that net depreciation or addition to existing housing is very small (0.4% annual depreciation), implying that economic obsolescence is the major aspect of housing replacement.

The last point brings us to the question of what determines the economic obsolescence of housing. First, the termination time, T, can be specified as a function of the structural input per land area. This is because too low a density yielding too little rental income would not be a viable form of use to maintain when a project of much higher density and income is feasible. On the other hand, it is reasonable to assume that the marginal rental price of housing and the unit cost of structural inputs are independent of housing density. Therefore they are analytically separable.

But we must add as an important determinant of income and viability of land use the size of a housing unit, that is, how many houses a given combination of land and structutre (quantity of housing) is divided into. In general, there will be an optimal quantity of housing for a household with a particular utility function and budget. The household will bid the highest marginal rental for an optimal sized house but less if it is too small or too large. As the budget, prices and the utility leval change, so does the optimal size of a dwelling unit. The quantity of housing per dwelling unit can be modified by adding to or subtracting from the existing structure, which we assumed away; or by subdivision or merger of the existing quantity of housing per land area, but probably at a substantial cost. In general both propositions are very costly. But at the time of initial construction it would not be significantly more or less costly to package a given quantity of housing in more or less number of units. Again it is not strictly necessary but it simplifies our analysis without substantially affecting the conclusions to ignore the cost of initial packaging.

These simplifications and added considerations can be formally written as

$$P(t',Q) = p(t',H)Q \quad ; \qquad ck = ck(Q) \quad ; \text{ and}$$

$$T = T(t,H(t),Q(t)) \qquad (3.8)$$

where $Q(t)$ is the density of housing built at time t; $p(t')$ the marginal rental price of unit quantity of housing; and $H(t)$ is the size of a dwelling unit determined at the time of construction.

To further simplify the exposition, let us abbreviate as $F(H(t),t,T)$ the marginal capitalized value of the rental stream of the new housing, i.e.

$$F(H,t,T) = \int_t^T p(H(t),t')e^{-r(t'-t)}dt'$$

Then, by definition

$$F'(H;t,T) = \int_t^T p'(H;t')e^{-r(t'-t)}dt' \tag{3.9}$$

and

$$F'(t;T,H(t)) = - p(H,t) + rF \tag{3.10}$$

With this notation we can write the maximization of the housing developer as the following.

$$V(t) = \max_{H,k,T} \{F(t,T,H)Q(t,T,k) - ck(t) + V(T)\, e^{-r(T-t)}\} \tag{3.11}$$

The life-span of the housing, $(T-t)$, is endogenously determined, but the condition of its optimality is the same as equation (3.6), and need not be repeated here. Then the first order condition for optimal structural density is reduced to

$$Q'(k;t)F(t) - c = 0 \quad (3.12)$$

where Q(t) and F(t) are now defined as optimum quntatities as of time t. The second order condition is

$$Q''(k;t)F(t,T) < 0 \quad (3.13)$$

which is satisfied as long as the production function is concave.

The optimal size of a dwelling unit must satisfy

$$\begin{aligned} F'(H) &= 0 \\ F''(H) &< 0 \end{aligned} \quad (3.14)$$

For a strictly quasi-concave utility function that will satisfy the above second order condition, a household's consumption of housing and other goods is optimized with the maximum marginal rental of housing at the point where the budget curve is tangent to the indifference curve. If the household must occupy a house with less quantity of housing than the optimal size, it would bid less per quantity of housing. At the same time the houshold would be willing to increase not only the quantity consumed but also the marginal payment if a larger house is available. In case the given house is too big, the bid rental would increase as the dwelling unit becomes smaller, i.e., closer to the optimal size. (For a detailed discussion of the issue, see Muth, 1973.) Equation (3.14) states that the initially determined size should be an intermediate one, too big for times of lean demand but too small for times when larger houses are in demand, so that on average the effects of these price deviations from the contemporary optimal level should even out during the lifespan of the building. This very commomsensical condition prevents the question of optimal dwelling size from unduly complicating our subsequent analyses.

These conditions enable us to express the opportunity cost of capital invested in land ownership, $rV - V'(t)$, in a form that does not explicitly depend upon land value but upon present land use quantities. Differentiating land value as defined in equation (3.11)

$$dV(t)/dt = V'(t) \qquad \text{by the envelope theorem}$$

$$= F'(t)Q(t) + F(t)Q'(t) - ck'(t) + rV(T)e^{-r(T-t)}$$

Substituting

$$F(t)Q'(t) - ck'(t) = 0 \qquad \text{again by the envelope theorem}$$

$$F'(t) = rF(t) - p(t) \qquad \text{from (3.10)}$$

in the above,

$$V'(t) = Q(t)[rF(t) - p(t)] + V(T)\, e^{-r(T-t)}$$

$$= r[F(t)Q(t) - ck(t) + V(T)\, e^{-r(T-t)}] + rck(t) - p(t)Q(t)$$

$$= rV(t) + rck(t) - p(t)Q(t) \qquad \text{by eq.(3.11)}$$

i.e.,

$$rV(t) - V'(t) = p(t)Q(t) - rck(t) \tag{3.16}$$

by the definition (equation (3.5))

Combining this with the condition for optimal time for development (equation (3.6)), we obtain the following important Lemma.

[LEMMA 5] At the optimal time of development t*,

(a) $J'(t^*) = P(t^*,-) - R(t^*) = 0$

(b) $J''(t^*) = P'(t^*;-) - R'(t^*) \leq 0$

where $P(t^*,-) = R(f)$ for farm-to-urban conversion

$= p(t^*,H-)q(t^*)$ for redevelopment of built-up land; and

$R(t^*) = p(t^*,H^*)Q(t^*) - rck(t^*)$

where $q(t^*)$ is density of existing housing to be replaced at time t*;

$Q(t^*)$ is optimal density of new housing to be built at time t*;

$k(t^*)$ is optimal structural density;

H^* is the size of a new dwelling unit; and

$H-$ is the size of the old dwelling unit.

Here (a) is a mere reexpresion of the combination of equations (3.6) and (3.16). (b) is the second order condition for optimal timing of land use change: In order for the time satisfying the first order condition to be the maximizing, not minimizing, point the difference between the property rental income over the opportunity cost of capital should be decreasing over time.

Readers would have noticed that we used the symbol for rent, R, to stand for the opportunity cost of capital in land ownership ($rV-V'(t)$). Indeed there is a logical basis for such definition. Though durability makes land value, not rent, the primary land use parameter, it does not mean that land rent is altogether irrelevant or indeterminate. With whatever form of land use, land rent for a short period can be conceptualized as the potential profit lost when land is left idle. In terms of durable land use, it can be thought of as the profit foregone by delaying for a period the best and highest land use program as specified in the definition (equation (3.3) or (3.11)) of land value. In terms of capital market equilibrium, it should equal the opportunity cost of the capital used to buy the land (rV) minus the gain from the delay that allows more profitable land use ($V'(t)$). In short,

$$R(t) = rV(t) - V'(t)$$

It is easy to verify mathematically (by the basic properties of the homogeneous differential equation) that this term is consistent with the concept of land value for permanent ownership or for leasehold as the present value of the discounted stream of rents. i.e.,

$$\int_{t(i)}^{T(i)} [rV(t') - V'(t')]e^{-r(t'-t)} dt'$$

$$= V(T(i))e^{-r(T(i)-t)} - V(t(i))e^{-r(t(i)-t)}$$

$$= A(i)e^{-r(t(i)-t)}$$

Or, for integration from the present to the infinity,

$$\int_{t}^{\infty} R(t')e^{-r(t'-t)} dt' = V(t)$$

These relationships among rent, land value, and leasehold value are exactly the same as those in the static case as they should be. As a consequence, we can take advantage of the relationships between changes of land rent and value established in the last chapter to analyze variations of land value for durable housing. But it is important to remember that land rent for durable land use is only a shadow variable derived from land value, not vice versa; therefore it cannot contain any more fundamental information than is already contained in land value. For much of our spatial analysis below, it is sufficient for us to know that shadow rent as defined above exists and that it is determined uniquely as long as land value is well defined. Still, the particular subset of information extracted as the shadow rent constitutes all the essential information that we need for much of the analysis of land use change.

The significant aspect of the derivation of rent as in equation (3.16) is the absence of any direct reference to the future land use in the final definition of rent. This reexpression in quantities observable for each period only, eliminating the burden of considering a long string of future land use, offers us a few significant insight into the process of land use change as we shall elaborate in the next section.

Since land value is defined in reference to the optimal development possible at the moment even as other housing has already been built and is preempting the land, equation (3.16) defines the rent of urban land for all periods, not only for the time of actual development. Hence land value can be obtained on integration of the rent defined as above according to the capitalization formula.

An important qualification here is that this is valid only for land being taken for the kind of urban use as defined in the foregoing maximization problem. The land value (given by equation (3.9) or (3.11)) upon which we base this definition of rent is the shadow value of land for urban use, housing. Maximization of this land value starting from today may be dominated by another option of keeping the land for agriculture for now and building housing later. In this case, the value of the latter option is the realized land value and the former, the surplus generatable by developing urban housing immediately, remains as a mere shadow value. Only if the option of immediate housing development is superior to any other option of using the land (as raw land), the shadow value becomes realized as the market value.

Similarly, the shadow urban rent would be less than farm rent before conversion to urban use actually happens. Having considered this, we can define land value and rent for any period since farm rent is easy to define.

3.3 Further Properties of Optimal Land Use

The most basic properties of optimal land use are given by the last Lemma which is actually a slightly different, a more concise expression of the same information contained in equations (3.7) and (3.11) through (3.14). We will explore further implications of these properties in this section.

First, we note in Lemma 5 that our definition of urban land rent is of exactly the same form as that under the assumption of malleability or of durable development under the assumption of myopic expectations despite the fact that we have not imposed such strong assumptions in deriving the definition (equations (3.1)). More to the point, it implies that the relationship between optimal land use investment and present rental income is identical under any expectations of the future (See equations (3.2) and (3.4)). The first equivalence is only a formal resemblance, but it would be important to elaborate on the latter point. Let us briefly recapitulate the analysis of the last section concerning land use under the assumption of housing malleability or myopic expectations.

In case land use is malleable, land rent for housing production is derived from the present housing rental as the surplus of the latter over the cost of the structural inputs which are malleable and hence which incur only the one-period opportunity cost of the purchase price. Since by assumption the existing structure can always be modified, land use expenses for adding more structural inputs to the existing housing should not exceed, and in equilibrium should be equal to, the additional revenue, i.e.

$$rck'(\text{static}) = p[Q - q] \qquad (3.2.b)$$

where k' represents the additional structural inputs

In case land use is durable but developers are assumed to behave as if the present rental would remain the same in the future as in the present, changing market conditions will render the building inefficient but the land use cannnot be changed bit by bit. It will be changed only if the existing building is rendered completely obsolete by the standard of a newly possible development, which again is determined under myopic expectations so as that the expected profit from a new development is at least equal to the present value of the rental income stream from the existing housing (the right-hand side). In this case the expected profit is the surplus of the present value of the new housing over construction cost (the left-hand side of the immediately following equation) - in short, land value.

$$pQ(\text{myopic})/r - ck(\text{myopic}) = pq(\text{myopic})/r \quad \text{, or}$$

$$p[Q - q] = rck \tag{3.17}$$

Note that the above equation is different from equation (3.2.b) which defines the condition for an optimal level of additional malleable input only in that additional input k' is substituted by the completely new structure, k(t). It reflects the restriction imposed by the strict durabilty of structure that if a modification is to be made to the land use it cannot be an addition to the existing structure but it should be a completely new construction. But this optimality condition under the restriction is identical to Lemma 5 that is valid with any form of expectations for the future.

In other words, as soon as the optimal time to replace an existing land use is determined, the problem of finding an optimal investment and housing density facing a developer is not different from the one under the simple myopic expectations even if the developer knows the rental will change over time. The correct amount of the structural input can be decided using only the knowledge of the present housing rental and the density of the housing that is being replaced.

Of course, this does not mean that land use will change in the same way under the three different regimes. Lemma 5 implies that the density of a new housing will be the same under all regimes if the density of the existing housing are identical at a given time of development. However, the timing of replacing an existing land use will be different depending upon the pattern of expectations. With durable land use it is land value and its change that determine the timing, and it is eventually developers that should determine the land value, with far more complications under the general expectations than under myopic ones. Thus, the identical form of the decision rules does not imply any magic formula to simplify the decision of durable land use, except in the sense that land value can be considered an exogenous variable for a single developer if there is an active land market.

However, the formal equivalence of the optimality condition for land use change does have a significant implication for choice of analytical strategy. In the comparative static world land use change will occur every time the housing demand changes, but specification of the timing of change in durable land use requires specification of land use history of a city, under any form of expectations. Once the timing is specified for locations it is not a problem to define the optimal density even with a general form of expectations. Therefore, we do not gain much simplicity in describing the dynamics of land use in return for restricting the pattern of expectations to that of myopic ones.

If the preceding reveals the similarity among land use decisions under different regimes, we can in contrast find from the same Lemma an essential difference that durability and expected growth of housing demand would make. We have seen in the last section that land value should be increasing at the time of land use change if the income from the existing property is not decreasing over time. This would normally, though not necessarily, imply that the value of new housing would also be increasing. But an even more basic condition can be seen to limit the rate of increase of the latter. From Lemma 5, it is obvious that urban rent is positive at the time of development as long as the rental income of the existing use is positive.

$$R(t^*) = p(t^*)Q(t^*) - rck(t^*)$$

$$= P(t^*,-) > 0 \qquad (3.18)$$

Substituting

$$c = FQ'(k) \qquad \text{from (3.12)}$$

$$K = Q'(k)k/Q$$

$$F'(t) = rF(t) - p(t) \qquad \text{from (3.15)}$$

and rearranging,

$$R(t^*) = Q\,[p - rFK]$$

$$= Q\,[\,-F'(t^*) + rF - rFK\,]$$

$$= FQ\,[\,r(1 - K) - F^*(t^*)] > 0$$

i.e.,

$$F^*(t^*)/r < 1 - K \qquad (3.19)$$

In other words,

[PROPOSITION 3.1] A high growth rate of housing value relative to the discount rate or a large share of construction cost in the value would deter land use change.

This reveals a very important characteristic of durable land use: that durability imposes an efficiency cost in terms of suboptimality due to the inflexibility in the face of changing demand conditions, and the cost is higher if the changes are faster. The cost of inefficiency would be especially serious if the developer has to extract a larger share of cost from the inefficiently employed structural inputs.

Therefore, it will be more profitable to wait and optimize at a higher density (see Proposition 3.2 below) rather than to capitalize on the high expected rental income, however larger this may be than the present income from the existing use, foregoing a more efficient later development. This has the implication for aggregate housing supply that, when a city is growing fast, over-building (relative to the present demand) in some places in anticipation of larger future demands will be more or less offset by the (we might say, speculative) holding of land in other areas. It also implies that the housing rental and land value at the city border would be much higher than under myopic expectations.

Optimal housing and structural densities can be more explicitly related to the capital value of housing, or the present value of the rental stream, by elaborating on condition (3.12). Differentiating,

$$FdQ'(k) + Q'(k)dF - dc = 0$$

Substituting

$$dQ'(k) = Q''(k)dk = Q''(k)dQ/Q'(k)$$

and rearranging,

$$dQ/Q = E(F)\ [dF/F - dc/c]$$

$$\text{where } E(F) = -\,Q'(k)^2/QQ''(k) > 0$$

by equations (3.12) and (3.13)

We may obviously call the last term the elasticity of housing density with respect to the (unit capital) value of new housing (relative to unit cost of structural inputs). This can be shown to be equivalent to the increase of structural input in response to change in land cost relative to its own unit cost, c — the relevant cost of land here being the leasehold value, A, covering the lifespan of the housing. Since, by equation (3.12),

$$dA = QdF \quad , \text{ and } \quad c = FQ'(k) \quad ;$$

ignoring the variation of the structural input cost,

$$d(ck)/dA = c\ dk/QdF = FQ'(k)dk/QdF = (dQ/Q)/(dF/F) = E(F)$$

This then can be related to the elasticity of substitution between the two factors in housing production, s.

$$s = \frac{dk}{d(A/c)}\ \frac{A/c}{k} \qquad \text{by definition}$$

$$= \frac{dk}{dA/c}\frac{A}{ck} = \frac{cdk}{dA}\frac{A}{ck} \qquad \text{since} \quad dc = 0$$

$$= E(F)\ (1-K)/K \qquad \text{from the above equation.}$$

where $K = ck/FQ = Q'(k)k/Q$

is factor income share of structure.

$A/ck = (1-K)/K$ by definition

Summarizing the above relationships,

[LEMMA 6]

$$E(F) = -\ Q'(k)^2/QQ''(k)$$

$$= Q^*(.)/F^*(.) = ck'(.)/A'(.)$$

where "." stands for a dimension x, or t.

$$= sK/(1-K) \qquad .$$

It is easy to show that variation in unit input cost can be accommodated by substituting F/c for F in the above, or $[F^*(.)-c^*(.)]$ instead of $F^*(.)$.

This Lemma makes it apparent that the elasticity of housing production (per land area) would not be constant as the factor income shares are known to vary among locations and as a city grows. For example, with $s = 0.5$, $E(F) = 1.2$ for $K=0.7$, and $E(F)=2.0$ for $K=0.8$. We have seen in section 2.2 that empirical evidence strongly suggests that the elasticity of substitution between land and structure is smaller than one, which implies that the factor share of structure would decline as cost of land increases. On the other hand, it is usually assumed that the elasticity of substitution itself does not vary substantially, and if it does at all, it increases with the factor share of structural input (see Sirmans and Redman, 1979). Also, it is apparent that the leasehold value of land, or the surplus from housing development, increases with housing value and density. Therefore,

[PROPOSITION 3.2] As the capital value of new housing and hence optimal housing density increases, the factor income share of structure and the elasticity of optimal density with respect to housing value decrease. i.e.

$$dK/dF < 0 \quad ; \text{ and}$$
$$dE(F)/dK < 0$$

Using the preceeding analysis, we can find the ratio of rental income from a new development to that from the existing property to be replaced, a matter of practical interest. Rearranging and substituting as in deriving equation (3.19), Lemma 5 can be rewritten

$$P(t^*,-)/p(t^*)Q(t^*) = 1 - KrF/p \qquad (3.20)$$

For example, assuming that the average rate of expected future growth in rentals is negligible so that rF(t*) is approximately equal to $p(t^*)$, the ratio would simply be equal to the factor share of land, $1-K$. That is, if $K = 0.8$, the new revenue would be five times as much as before. Though large as this ratio is, it probably is an underestimation because we have ignored two very important deterrents of (re-) development, the increase of rental and the cost of demolition or site-preparation.

The ratio of structural densities in redevelopment, $k(Q)$ versus $k(q)$, can be approximated by the following:

$$Q - q = k'Q'(\bar{k}: \bar{k}=k(Q)-bk') \qquad (3.21)$$

by the mean value theorem

where $k' = k(Q) - k(q)$

$0 < b < 1$

By Taylor expansion, the last term is

$$Q'(\bar{k}) = Q'(k(Q)) - bk'Q''(k(Q)) + b^2 k'^2 Q'''/2 - ....$$

Ignoring the third and higher derivatives and substituting in equation (3.21) and assuming that marginal rental of the new and the old housing differs by about the same amount (in opposite directions) from the maximum level, i.e., $p(H-) = p(H^*)$,

$$k'[Q'(k) - bk'Q''(k)] = Q - q$$

$$= rFQ'(k)k/p \qquad \text{from (3.20)}$$

or,

$$rF/p = \frac{k'}{k} - \frac{bQ''(k)k'^2}{Q'(k)k}$$

$$= \frac{k'}{k} - \frac{1 - K}{s}(k'/k)^2 \qquad \text{from Lemma 6}$$

The positive solution of this equation is

$$k'/k = [-1 + \sqrt{1 + 4b(1-K)rF/ps}\,]\; s/2b(1 - K)$$

$$= 1 - k(q)/k(Q) \qquad (3.22)$$

For example, assuming $s = 0.5$, $K = 0.8$, $rF/p = 1$, and $b = 0.5$:

$$k(Q)/k(q) = 8.4$$

We have mentioned above why this high ratio would still be an underestimation. Therefore, this example implies that redevelopment is justified only if the new building is close to or even over ten times as capital intensive as the existing structure, emphatically confirming the extreme economic durability of urban structures.

CHAPTER 4. SPATIAL DYNAMICS OF A GROWING CITY

In this chapter we combine the analytical results of chapter 2 and the tools developed in chapter 3 to examine spatio-temporal patterns of land use and value in a dynamically growing city. Durable housing is expected to cause departures from the comparative static patterns, and a number of substantial departures have been suggested by many recent dynamic models. We evaluate the significance of the suggested departures. Detailed review of the previous models is not attempted here but deferred until the next chapter. We first consider spatial differences in land value and its appreciation as in the second chapter.

4.1 Dynamics of Land Value

Some authors argued that land value can increase with distance from the city center when durabiltiy is introduced, a very important departure from the comparative statics of the monocentric city model and one that is infrequently but not rarely observed empirically. While acknowledging this theoretical and empirical possibiltiy, we can nevertheless show as a matter of simple logic that it is not possible as long as we maintain the basic premise of the simple monocentric city model, namely that cost of transportation to the city center is the only intrinsic distinction among locations.

Suppose an optimal land use program is defined for a certain location (as in equation (3.3)), and suppose further that it is exactly copied at another location closer to the center. Since residents always prefer and thus are willing to pay more for a closer location, all other things being equal, the same housing will yield more revenue at the closer location while the construction cost, by premise, is the same. Hence, at the closer location there will be more surplus and higher land value. In fact, the closer site must be taken better advantage of so that it will yield even more surplus. Because land value depends only on the possible optimal use and does not depend on the existing use, the preceeding is always true. The pairwise comparison can be generalized for any location that is or will be affected by the access to the center. Therefore, land value decreases with distance everywhere in the city and also around it if the city is expected to expand. If land is evaluated solely on the basis of presently feasible land use, the slope of agricultural land value would of course be zero. Thus,

[PROPOSITION 4.1] In a monocentric city, land value decreases with distance from the city center. Further, with expectations of future urban expansion, the same is true for farm land near the city.

Though at times a positive land value gradient has been argued to be possible with durable housing even under the assumption of the monocentric city (Anas,1978, for example), this seems to be only the result of an incorrect definition of land value.

As long as the correct definition of land value is used, the above Proposition is valid without depending upon any special assumptions. It is true, to consider one important element that we have ignored in the analysis of the last chapter, that demolition cost of a badly run-down existing structure subtracts from the property value and can even make it negative. But the critical point to remember here is that the negative value should be attributed to the existing structure and hence to the property as a whole; but land value should always be defined as the value of raw land, whether the existing structure is highly valuable or highly detrimental.

The point of the foregoing is not to maintain that land value must always be dèclining in any circumstances but merely that a positive land value gradient is impossible almost by definiton within the simple monocentric model. An explanation for a positive land value gradient cannot be provided by conditions of durability within this framework but must be sought from entirely different perspectives. Factors other than the accessibility that affect land value are too numerous to list here.

To analyze the pattern of secular change in the land value distribution, we have to determine beforehand the change of the housing rental that is the underlying price from which the former is derived. To this end, let us assume that consumption of housing by households can be adjusted instantaneously, given the housing stock of a city which is durable. The assumption means first that households can relocate quickly and costlessly as demand condition changes. This may not be a gross simplification as the adjustment on average takes a reasonably short time in comparison with supply adjustments. Secondly it means that housing suppliers can reassemble the housing so that each house would be an optimum size (quantity of housing per dwelling unit) for the prevailing demand conditions, although they cannot add to the existing housing density without redevelopment. This assumption of durable yet divisible housing is not incorporated in our main body of analysis as it is hardly acceptable as we have discussed in section 2.2. For now, however, we want to specify the distribution of the maximum rental among locations, and shortly thereafter we show that the final analysis does not depend on the suboptimality of dwelling unit size in an essential way.

Under the assumptions we can describe the changes in housing consumption and pricing essentially as a comparative static process, although the housing stock and hence land consumption must be determined under the constraints of durability. In Chapter 2, we have shown that the comparative static patterns of urban land rent is equally valid for distribution of housing rental prices.

For an open city, local comparative static changes can be specified by reference only to locally applicable parameters (income and transportation cost) without concern for the adjustment of the city as a whole (in utility level and total population). Total availability of housing and the ratio of land and structure cannot be adjusted fully when housing is durable, and hence the total city population and the amount of land used for housing are largely influenced by the existing housing stock. But this does not affect income of the household, utility level, and transportation cost. Therefore the rental price and quantity of housing consumed by a household at a given location are not affected by durability of the existing stock, if households can adjust their consumption as assumed. Consequently, the comparative static properties of housing rental, that are exactly the same as those of land rent with proper substitutions as we have seen in section 2.1, can be transferred to the case with durable housing without qualitative change.

For a closed city, however, we must determine the endogenous changes in utility level which we cannot transfer directly from the comparative static to dynamic analysis. In order to determine this, we have to specify long-term planning of households for consumption, savings, and by implication, utility, as well as long-term planning by land and housing suppliers. This does not appear to be a feasible analytical task at present if we are to keep the concept of durability and land value as discussed in the first Section. (This pessimistic view is explained in Malinvaud, 1972.) Intuitively, however, it does not seem an overly dubious proposition to infer as in the comparative static analysis that, other things being equal: population increase would reduce the level of utility, increase housing rental and hence its gradient; and an increase in income and reduction of transportation cost would cause a shift in demand for location in favor of outer locations and hence flatten the rental gradient.

Therefore, we transfer the results of the comparative static analysis of growing cities as the following Proposition with some reservations about the validity of the third part for a closed city.

[PROPOSITION 4.2] In a monocentric city, where housing is homogeneous and divisible,

(a) the marginal rental price of housing decreases with distance from the center, i.e. $p'(x) < 0$;

(b) when the population, income, or transportation efficiency of a city grows, housing rental gradient flattens, i.e.

$dp^*(x)/dx > 0$; and further,

(c) in the case of a growing open city, the marginal rental of housing increases everywhere, i.e. $p'(t) > 0$.

From this we can easily derive the following intermediate Propositions leading to our goal of specifying the pattern of land value appreciation.

[PROPOSITION 4.3] Unit capital value of new housing and the leasehold value of land for new housing decline with distance from the center, i.e,

$F'(x) < 0$; and $A'(x) < 0$

Proof.

$$F'(x) = \int_t^T p'(x;t')\, e^{-r(t'-t)}\, dt' \quad < 0$$

as $p'(x;t') < 0$ for all x and t'

by Proposition 4.2.a.

$A = FQ - ck$ by definition. See equations (3.3) and (3.11).

$$A'(x) = F'(x)Q + FQ'(x) - ck'(x)$$

$$= F'(x)Q < 0 \quad \text{by equation (3.12).}$$

Q.E.D.

The result is the same if we define the rental to vary with the size of the dwelling unit, H(t), because by equation (3.14)

$$F'(H) = 0$$

The termination time, T, is the only relevant endogenous variable that is fixed in the above. But this partial differentiation is adequate for specifying full variation of rent which is of primary interest to us here (see Proposition 4.5 below). This is so because all but the rent of the initial period are unaffected by T(t). Therefore it only equalizes the length of time over which to compare streams of rental without affecting the rentals within the interval. The following is again defined for a given length of housing life.

[PROPOSITION 4.4] (a) In a monocentric city, the gradient of new housing value flattens with monotonic growth of the city's income, population, or transportation efficiency over time. i.e., $dF^*(x)/dt > 0$;

(b) The gradient of the leasehold value of land also flattens with the same growth, unambiguously in an open city and probably in a closed city. i.e.,

$$dA^*(x)/dx > 0 \quad .$$

Proof.

$$\frac{d}{dt}\frac{F'(x;t)}{F(t)} = \frac{-p'(x;t) + rF'(x) + p'(x;T)e^{-r(T-t)}}{F(t)}$$

$$- \quad \frac{F'(x)}{F(t)} \cdot \frac{rF(t) - p(t) + p(T)e^{-r(T-t)}}{F(t)}$$

$$= [\ -p^*(x;t) + F^*(x;t)]p(t)/F(t)$$
$$+ [p^*(x;T) - F^*(x;t)]p(T)e^{-r(T-t)}/F(t) \qquad (4.1)$$

But

$$p^*(x;t) < F^*(x;t) = p^*(x;t'': t<t''<T)$$
$$< p^*(x;T)$$

by the mean value theorem and Proposition 4.2.b.

Therefore,

$$dF^*(x)/dx > 0 \quad .$$

For (b),

$$A^*(x) = F'(x)Q/(1 - K)FQ = F^*(x)/(1-K)$$

$$\frac{dA^*(x)}{dt} = \frac{dF^*(x)}{dt}/(1-K) + F^*(x)\,\frac{dK}{dt}/(1-K)^2$$

The first term is positive by the immediately preceeding. The second will be also positive if the housing value increases over time for then the factor share of structure decreases (see Proposition 3.2), i.e.

$$dK/dt = (dK/dF)(dF/dt) < 0 \quad \text{while} \quad F^*(x) < 0.$$

Housing rental and capital value always and everywhere increase with the growth of an open city and thus the Proposition is unambiguous there. They may decrease in central locations of a growing closed city. In this special instance, the gradient of leasehold value still may flatten because of the positive first term. In most locations the housing rental increases and the Proposition holds.

Q.E.D.

[PROPOSITION 4.5] The gradient of the shadow urban rent of land is negative and flattens in a growing city, i.e.

$$R^*(x) < 0 \qquad \text{and} \qquad dR^*(x)/dt > 0$$

Proof. By analogy with equation (4.1), we can write

$$dA^*(x;t)/dt = [-R^*(x;t) + A^*(x;t)]R(t)/A(t)$$

$$= [R^*(x;T) - A^*(x;t)]R(T)e^{-r(T-t)}/A(t)$$

Reversing the proof of the above Proposition, this would be positive if and only if

$$dR^*(x;t)/dt > 0$$

i.e.,

$$R^*(x;t) < A^*(x;t) = R^*(x;t'') < R^*(x;T) ,$$
$$\text{where} \quad t < t'' < T$$

Since $A^*(x) < 0$ always, it follows that

$$R^*(x) = R'(x)/R < 0 \quad .$$

Q.E.D.

It is noteworthy that we have not been able to prove that the rent gradient is negative in general but only in the case where the city is growing. In other words, in urban decline the rent gradient may be positive. It must be remembered that we have derived this Proposition from the Proposition on leasehold value, not the other way around, just as we derived the definition of rent itself from land value and its change. Since we have already seen that leasehold value and ordinary land value have negative slopes, we can infer that rent may increase with distance but only for a short while.

In proving the conclusion about different land value appreciation rates among locations in the camparative static urban growth (Proposition 2.10) we needed only the equivalent of Propositions 4.1 and 4.5, namely that land value and rent gradients are negative and the latter flattens with urban growth.

Employing exactly the same procedure used in the proof of Proposition (2.10), we obtain its dynamic counterpart that is qualitatively the same.

[PROPOSITION 4.6] In a growing monocentric city, land value appreciates faster in areas farther from the center; outside the city boundary, the appreciation rate is zero under myopic expectations, but positive and decreasing with distance under rational expectations.

4.2 Progression of Urban Expansion

In the comparative static case, land use depends only upon the present parameters and it has a direct correspondence with land rent. With durable housing, land use is more heavily influenced by the past and the future and its relationship with land value is more complicated. Though the last concluding Proposition about land value with durable housing was shown to be qualitatively identical to that with malleable housing, it owes much to the fact that land value is determined only in reference to the future even with durable housing as is the case with land value as a capitalization of future static rents. Durable land use is not so free from the past as the existing structure cannot easily be changed. Despite this, would there be a similar match between static and dynamic land use patterns? Our conclusion in this and the following sections might be interpreted as either a yes or a no, but a much qualified one, either way.

First of all, it is not a foregone conclusion when and where farm land is converted to durable urban use. In a comparative static world, land is used for housing as long as the one-period rent bid by the housing producer (as defined by equation (3.1)) or consumer (equation (2.2)) exceeds farm rent. Since rent is always higher in more central locations, areas within the farthest reach of the urban settlement must be comletely used up by housing. As the city grows, it expands its territory by annexing the immediate neighborhood of the existing development.

However, this may not be the case in a dynamic urban growth. This complication is due to the optimal development rule described in Proposition 3.1. Even if a site can be developed immediately to return a surplus larger than agricultural income, it might be better to wait for a later opportunity for a more efficient use if the housing rental is increasing very fast. Under myopic expectations, of course, there is no such waiting. But in general it is possible that land in more central locations is held in this fashion surrounded by already developed urban areas. In other words, initial city building begins at a point far from the city center and proceed inward toward the center, as suggested by Wheaton(1982a) and Brueckner and von Rabenau(1980).

However, just as the the decline of rent with distance from the center assures that the urban area expands outward from the center in comparative statics, so does the similar pattern of urban shadow rent in dynamics. In a growing city, we have seen that this is the case (Proposition 4.5); thus the city building proceeds in the normal inside-out pattern. To formally elaborate, we first take note that Lemma 5 must hold at any time t* when the conversion occurs. Wholly differentiating $J'(t^*,x) = 0$,

$$\frac{dJ'(t^*,x)}{dx} = J''(x,t^*) + J''(t^*)\frac{dt^*}{dx} = 0$$

or (4.2)

$$dt^*/dx = -J''(x,t^*)/J''(t^*)$$

We know from Lemma 5(b) that $J''(t^*)$ should be negative if the conversion time is to be a maximizing point. Therefore, the sign of dt^*/dx is the same as the sign of the numerator. Partially differentiating Lemma 5(a) with respect to distance,

$$J''(x,t^*) = \partial[R(farm) - R(urban)]/\partial x$$

$$= R'(x:farm) - R'(x:urban)$$

If farm rent is positive and constant, this second derivative is positive as

$$R'(x:urban) = R(farm)R^*(x;t^*)$$

since at time t* farm rent and urban rent are the same.

$$< R(farm)A^*(x;t^*) < 0$$

by Propositions 4.4 and 4.5.

Therefore, dt^*/dx is positive. In other words, the time for conversion arrives later at farther locations and the city expands from the center outward. This conclusion can be reversed only if the gradient of the urban shadow rent is positive. This can happen in urban decline where, as Proposition 4.5 implies by exclusion,

$$R^*(x;t) > A^*(x;t)$$

But note that this implies only the possibility of a positive rent gradient, not a necessity, because the gradient of leasehold value on the right hand side is always negative.

We have expressed a reservation in proving Propositions 4.4 and 4.5 that the rent gradient may not be steeper than the gradient of leasehold value in a growing closed city, since housing rental may decline there even during urban growth. But this problem does not concern us here because land value and hence by implication housing rental should be increasing in order that the conversion actually happens.

Therefore the urban growth as specified in Proposition 4.2 rules out the possibility of outside-in city builiding. To obtain such a pattern, we must have a city that expands its territory while declining in some aspects. In an open city, such a mixed growth is hard to imagine because if any measure of city size increases the others follow suit. However, for a closed city, it can happen as seen in rapid progression of population growth in some Third world cities even while the real income or transportation efficiency decreases.

Another interesting possibility is a case of urban physical expansion without or beyond what can be accounted for by concurrent growth in population, income or transportation efficiency. We can recall Proposition 2.9 that says that if availability of land for the city is increased by removal of some barriers to urban use of land such as legal prohibition of building or physical impediments, utility level of residents increases, the city expands, and the rent gradient flattens.

In order for these changes to have similar effects for a city with durable housing, the following must be true: The availability of land is increasing gradually and this is foreseen by developers, but not before the city is substantially built up and the significant growth of population or income has ceased. This theoretically inelegant scenario is perhaps quite close to real world situations of many cities whose already large urban built-up areas are being extended far and thin into newly opening suburbs. In many of the large cities, essentially insignificant growth of population is dominated by rapid addition of available building land by generous extension of infrastructures and changing life-style that has turned the suburban living as a respectable choice. Then we can expect the additional territory to be filled up from the outside - although what is outside would become the middle in the following period as the boundary of buildable land expands - and the housing rental and land value gradient to steepen. It might be a part of the process of the leap-frogging suburban developments.

In summary of the foregoing:

[PROPOSITION 4.7] Initial conversion of farm land for urban use in a closed city progresses from outside in toward the city center if the expansion is chiefly caused by increasing availability of land; or by growth in population, income, or transportation efficiency coupled with an offsetting decrease in other parameter(s). In all other cases, an open or a closed city expands its territory in the normal manner, outward from the center.

We have so far ignored one other possible course of city building: It may proceed neither outward or inward but simultaneously and continuously over all areas to be eventually built-up fully. For some time, farms and houses would coexist in a neighborhood, housing eventually filling up the area. Hochman and Pines (1982) obtained such a process as the optimal one for a growing open city. They have shown further that areas closer to the center would be developed more heavily than areas far from the center but the difference of land use intensity would be reduced over time. This sounds very similar to the flattening density gradient of a growing city that we have obtained in chapter 2. But the analytical framework of the above study is very dissimilar to ours, and we need to proceed within our framework before evaluating their conclusion.

Formally, the pattern arises when $J''(t^*) = 0$ and the optimal time of development is not uniquely defined. If so, it must also be that

$$J''(x,t^*) = R'(x:\text{farm}) - R'(x:\text{urban}) = 0 \qquad (4.3)$$

so that

$dt^*/dx = 0/0 =$ indefinite

at all locations that are eventually to be fully developed for urban housing. If $J''(t^*)$ is zero but $J''(x,t^*)$ is non-zero, the difference in the optimal development timing between adjacent locations is infinite, i.e., one of the locations is never developed for housing. This should happen only at the border of the city, and condition (4.3) should be true in all locations within the boundary until they are completely filled up with urban housing.

But the condition specified in equation (4.3) is hard to sustain for a long time alongside the condition that urban land value always decreases with distance (Proposition 4.1) and that its slope flattens with urban growth (Proposition 4.6.) We have seen that it is possible for these conditions to coexist for a short while. But it must be only a short while, meaning that the whole city is built up very quickly and goes through redevelopments without expanding the territory. This is clearly implausible.

The condition (4.3) is always true in the trivial sense, if the only opportunity cost of land is the urban rent instead of the farm rent in the equation. This presupposes that there is no competing land use of qualitatively different kind and therefore there is no sharp discontinous break in the use of any site. In other words, Hochman and Pines's model apparently depicts a process of incremental modifications of land use in an already built-up urban area. Though it seems a viable framework for a study of housing production at an aggregate level, it cannot reveal many important properties of discontinuous land development processes and of the competitive market for individual building sites. In short, the simultaneous building of a city is not a plausible pattern in so far as it is difficult to add marginal quantities of housing to agricultural land or to existing housing, as discussed in the last chapter.

Before we conclude this section, it is necessary to point out that the progression of city building discussed above applies only to the conversion of previously undeveloped farm land. After the initial city building has proceeded to a certain extent, some earlier vintages would become ripe for retirement before or at the same time with a still further expansion of the city into the farm land. How soon the original building becomes uneconomical and how the redevelopment progresses would depend upon relative distribution of land use intensity. In the next two sections, therefore, we consider these issues jointly.

4.3 Pattern of Land Use Density and Redevelopment

In comparative statics, densities always decline with distance from the center, as does the housing rental of each period that entirely determines optimal density. Even with durable housing, density of development is entirely dependent upon the present housing rental if developers expect it to remain constant in the future. This is a special case of the maximization discussed in section 3.2, with the present value of expected future rental stream $p(t^*)/r$, where r is the discount rate. Then the relationship between optimum density and housing rental is given by the condition (3.12) with corresponding substitution, and it is exactly the same as the optimality condition for malleable housing given by equation (3.2):

$$p(t^*)Q'(k(t^*)) - rc = 0 \qquad (4.4)$$

Consequently, if the city is built all at once, structural density would decrease with distance and so would the popoulation density because

$$n^*(x) = \frac{d}{dx}\left(\frac{Q}{H}\right)/\frac{Q}{H} = Q^*(x) - H^*(x) < 0 \qquad (4.5)$$

However, as we have discussed in the last section, the city is built up gradually from the center under the myopic expectations with the result that areas developed at different times would reflect parameters effective at respective dates of development. The density of housing older than the current vintage and closer to the center is no longer optimal in view of the current housing rental at the location, but it remains in service as long as it still provides gross revenue larger than the net revenue (gross minus construction cost) of a new development. Therefore it is necessary to specify what has been the optimal density at the time of development of each location. The answer is very simple:

[PROPOSITION 4.8] Under myopic expectations, the density of housing is the same for all initial vintages built on land converted from farm as long as the farm rent, discount rate, and price of structural input are constant over location and time.

It can be easily seen if we reflect on two sets of basic relationships: that housing density depends only on the housing rental prevailing at the time and place of conversion; and the urban shdow rent, for it is determined uniquely for a given set of housing rental and density, should be the same as the farm rent. Nevertheless, since the conclusion appears to contradict earlier research, it may help clarify our meaning if we formalize the reflection.

Lemma 5 specifies the relationship between farm rent, housing rental, and density at any time of conversion, t^*:

$$R(farm) = R(t^*:housing) = p(t^*)Q(t^*) - rck(t^*)$$

From equation (4.4) we know that Q and k are determined completely by p, r, and c. That is, we can write

$$Q(t^*) = Q(k(p(t^*),r,c))$$

Therfore the above Lemma can be restated as

$$R(farm) = R(p(t^*),r,c)$$

This would be valid if and only if $p(t^*)$ and hence $Q(t^*)$ is constant for all t^*, under the assumption of constant farm rent, discount rate, and price of structural input. Note that the first step of the proof is valid only under the assumption of the constant returns to scale in housing production.

The constant structural density does not necessarily mean the same for the population density. Its distribution would depend upon the size of a dwelling unit occupied by each household at different vintages. If housing is divisible so that it can be instantaneously and costlessly repackaged into houses whose size is optimal for the particular location and time, population density decreases with distance, and further, the gradient flattens in urban growth. This is because distribution of population density in this instance is the exact mirror image of optimal housing consumption per household. For, from equation (4.5)

$$n^*(x) = - H^*(x) = E(p)p^*(x) < 0$$

where E(p) is compensated elasticity of housing demand with respect to housing rental.

and, by Proposition 4.2,

$$dn^*(x)/dt = E(p)dp^*(x)/dt > 0$$

However, if the dwelling unit size remains fixed as set at the time of development, the pattern of population distribution is different between a closed and an open city. In the case of an open city, the constancy of the housing rental at the time of development implies that the size of dwelling is always the same. This is because the relationship between the marginal price and the quantity of housing consumed per household stays the same if the utility remains constant, as the following formal derivation shows.

$$\frac{dH(x,t^*)}{dx}\frac{p}{H} = - E(p)p'(x;t^*,H^*) - E(p)p'(t^*;x,H^*)\frac{dt^*}{dx} \tag{4.6}$$

since $p'(H^*) = 0$ for optimum $H^* = H(t^*,x)$

$$= E(p)\frac{dp(t^*)}{dx} = 0$$

Therefore, population density gradient is flat until redevelopment alters the housing density and the dwelling unit size in some places. Adjustment of housing consumption in a closed city is very different from the case of an open city. Cross-sectional variation in housing consumption over location at a certain time is the utility compensating difference in both types of cities. Secular changes in housing rental and consumption are also the same kind in an open city. But, in a closed city, the changes are accompanied with a change in utility. Therefore, we must substitute the ordinary elasticity ($E(\hat{p})$) for the compensated elasticity in equation (4.6), and we must also take into account the income effect. Then,

$$\frac{dH(x,t^*)}{dx}\frac{p}{H} = - E(p)p'(x)$$

$$- [E(\hat{}p)p'(t^*) - \frac{p}{\hat{y}}E(yH)\hat{y}'(t^*)]\frac{dt^*}{dx}$$

where $E(\hat{}p)$ is the ordianry price elasiticity of housing demand;

$E(yH)$ is income elasticity of housing demand; and

$\hat{y}(x)$ is the disposable income after transportation coat at location x, $(y - a(x))$.

$$= - E(p)\frac{dp(x,t^*)}{dx} + \frac{dt^*}{dx}\{p'(t^*)[E(p) - E(\hat{}p)]$$

$$+ E(yH)\frac{p}{\hat{y}}\hat{y}'(t^*)\}$$

$$= \frac{dt^*}{dx}\frac{p}{\hat{y}} E(yH)\{-p'(t^*)H + \hat{y}'(t^*)\} \quad (4.7)$$

since $dt^*/dx = 0$ from the above;

$E(\hat{}p)=E(p) + E(yH)pH/\hat{y}$

by the Slutsky equation.

The terms outside the brackets are all positive. If the housing rental increases solely as a result of an increase in population, the bracketed term would be negative because the housing rental increase is postive and the income change is zero. Then the size of a house decreases and the population density increases over distance. However, if the rental increases either as a result of a rise in the total income or a reduction in the transportation cost, the increase in housing rental cannot absorb all the increase in the spendable income. Thus, the bracketed term becomes positive, the dwelling unit size increases, and the density decreases over time and over distance.

In summary of the foregoing,

[PROPOSITION 4.9] The population density over original housing vintages

(a) is constant over all locations in an open city where housing is indivisible and the size of a dwelling unit is fixed as determined at the time of development;

(b) increases with distance in a closed city with indivisible housing where total population is increasing; but otherwise

(c) decreases with distance from the center, and further, its gradient flattens over time if housing is divisible.

How redevelopment proceeds can be determined by employing the same method used in the last section. Writing the time of redevelopment as t**, and the density of the existing housing as q which is constant over distance,

$$J''(x,t^{**}) = P'(x;t^{**},q) - p'(x;t^{**})Q(t^{**})$$

since $pQ'(x) - rck'(x) = 0$

by equation (4.4)}

$$= p'(x;t^{**})q + qp'(H(q))H'(x;q) - p'(x;t^{**})Q$$

$$= p'(x;t^{**})[q - Q] + qp'(H(q))H'(x;q) \tag{4.8}$$

The first term in the above is positive because the housing rental decreases with distance and because the new housing density must be larger than the old one. However, the second term depends upon specification as in the immediately foregoing analysis. In case dwelling unit size is always adjusted to gain the maximum rental, $p'(H(q)) = 0$. Also in case dwelling unit size is fixed from the time of development but is constant over distance, as we have seen to be the case in an open city, $H'(x) = 0$. Thus the second term vanishes, and $J''(x,t^{**})$ is unambiguously positive.

$$dt^{**}/dx = -J''(x,t^{**})/J''(t^{**}) > 0$$

That is, redevelopment again proceeds outward from the center.

In the case of a closed city where the dwelling unit size remains unchanged over time, we have seen that the size decreases with distance if the city grows purely by force of increasing population. If this pattern of growth continues, the optimum size of housing in later years must be smaller than at the time of earlier developments: therefore, $p'(H(q))<0$. Consequently the second term of equation (4.7) is positive. In case the growth occurs because of continually rising income or improving transportation, the sign of each component of the second term is exactly opposite to the preceeding case, again resulting in the positive second term. In other words, as long as the nature of growth is consistent in time, redevelopment starts from the center and spreads out in a closed city, too. But the situation becomes ambiguous if the optimal size of dwelling unit was decreasing over time and over distance at the time of earlier developments but has since increased, or vice versa.

The density of these second generation housing vintages would be decreasing with distance if the following is negative:

$$dp(t^{**})/dx = p'(x) + p'(t^{**})dt^{**}/dx$$

From equation (4.7) and a similar differentiation with respect to time,

$$\frac{dt^{**}}{dx} = - \frac{J''(x,t^{**})}{J''(t^{**})} = - \frac{p'(x)}{p'(t^{**})}$$

Substituting in the preceeding equation,

$$dp(t^{**})/dx = 0$$

Again, redevelopment occurs when the housing rental reaches a certain constant level. Consequently the density of structure in redevelopment is constant regardless of location. Therefore, except in the case of a closed city where size of dwelling is fixed and its optimal level has increased over time, the pattern of redevelopment is qualitatively the same as that of initial city building. In other words, with "usually" signifying such an exception, [PROPOSITION 4.10] Redevelopment (and re-redevelopment, etc.) usually proceeds outward from the center, with constant density of housing for every redeveloped site. Population density decreases, increases, or stays constant over distance as described in Proposition 4.9.

Combining the results on initial and later developments, we can state the following general cases and exceptions. Here a "generation" of housing means the vintages built as the first housing on a piece of land, the second one, etc.

(i) Structural densities of different vintages of a same generation of housing is constant regardless of location and the time of development.

(ii) Population density at a given moment decreases with distance from the city center if the size of a dwelling unit can be adjusted. If the dwelling size is fixed from the time of development, population density is constant over distance in an open city where utility remains constant through time; increases with distance in a closed city with growing population; and decreases over distance if the income or transportation efficiency has been growing in a closed city.

(iii) There will be a sharp break in population and structural densities at the boundary between succeeding generations of housing. Redevelopment proceeds outward from the center in most instances. Therefore the shift of density at the point of discontinuity is downward over distance.

The crucial element of the conclusion is the first part, that structural densities are the same within the same generation. Its validity depends upon the apparently reasonable assumption that the farm rent, discount rate, and construction input price are constant over time and location and that there is no economies of scale in housing production.

Even with the assumption, the structural density would not be constant over distance if we adopt a discrete time framework. A ring of some width would be developed all at once at the beginning of a discretely marked period. Therefore, the structural density decreases with distance within this area. This framework assumes that land is developed only at certain intervals of time, in other words, that land cannot be developed in the middle of a period even if the condition of development is satisfied.

But the farthest edge of the vintage must exactly satisfy the condition, and the density at the far edge would be the same for each vintage. The inner location of a newer and farther vintage would have a higher density than the edge of the earlier, closer vintage. As a result, the structural and population densities would show a "saw-tooth" pattern with sharp breaks at borders of succeeding vintages. Distribution of the average density of differently aged housing depends entirely upon specification of the time path of urban expansion. If, for example, each succeding vintage were to occupy a ring narrower than the preceding one, its average structural density would be lower than that of the earlier developments and the structural density can be said to decline over distance in a statistical sense.

When developers expect future rentals to be different from the present one, they optimize combination of inputs in reference to the expected present value of the stream of future housing rentals. The relationship between the structural density and the marginal capital value of housing was given by (3.12)

$$F(t^*)Q'(k(t^*)) = c$$

The shadow rent at the time of development must again be equal to the farm rent that is assumed constant:

$$R(farm) = R(t^*) = p(t^*)Q(t^*) - rck(t^*) \qquad \text{Lemma 5}$$

Unlike the case under myopic expectations, the housing rental does not have to be constant for all t^* because different combinations of the values of $p(t^*)$, $F(t^*)$, and hence $Q(t^*)$ can now be compatible with the condition specified in Lemma 5. But the expected change in the future rentals is not enough to cause a fundamental difference in the pattern of development density from that under myopic expectations, the constant structural density. For example, if the housing rental is expected to change at a fixed rate, the present rental can completely define the expected present value, $F(t^*)$, and therefore the condition of Lemma 5 is met with a constant value of $p(t^*)$.

However, We have seen in the first section that the rate of change in housing rental is in general not constant over distance, and in Proposition 3.1 that it is not constant over time, either. If the developers recognize these differences, there will be different densities of development. This flexibility makes analytical determination of density distribution very difficult and we will present only a tentative sketch.

The problem is whether the following is negative:

$$\frac{dQ(t^*)}{dx} = Q'(x) + Q'(t^*)\frac{dt^*}{dx}$$

$$= Q(t^*)\ E(F)\ [F^*(x) + F^*(t^*)\frac{dt^*}{dx}] \qquad (4.9)$$

by Lemma 6

We have seen in the first section that the first term in the brackets, the gradient of housing value, is negative and is flattening in urban growth. We have seen in section 3.2 that $F^*(t^*)$ must be positive if the development is to occur on the land whose revenue from the existing use is not decreasing. Therefore, if the urban expansion proceeds from the outside toward the center, the above would be unambiguously negative, i.e. the structural density decreases with distance.

But if the city is built up beginning from the center, as is normally the case, the effect of the secular increase in housing density somewhat offsets the negtive slope of density in distance at one period. A general evaluation of the net effect is not possible but we can limit the range of its value by remembering Proposition 3.1 that says

$$F^*(t^*) < r(1 - K)$$

Rewriting equation (4.8) in a more easily decipherable form,

$dQ/dx < 0$ if and only if

$$F^*(x) < - F^*(t^*)dt^*/dx < - r(1-K)dt^*/dx$$

or (4.10)

$$dx/dt^* > - r(1-K)/F^*(x) = - r/A^*(x)$$

since $A^*(x) = F^*(x)/(1-K)$

(see proof of Proposition 4.4)

The last expression is not difficult to evaluate from common empirical observations. dx/dt^* is simply the width of the ring of urban extension for each period, and the gradient of leasehold value should be steeper than that of ordinary land value in a growing city. (See Proposition 4.5) Let us try and make a rough guess at the numerical values. There are not many reports on empirical values of the land value gradient for cities in the initial growth stage. But one such study by Mills (see the next chapter) on early years of Chicago reports (absolute value of) land value gradient of 0.33 for 1873, and 0.49 for 1857. On a different occasion, he also reports value gradients for Korean cities, and those of medium sized cities were found to be in the range of 0.35 to 0.83. Therefore, it seems safe to assume that the gradient of land leasehold value would be larger than 0.5 in a newly growing city. Then, for a discount rate less than 0.1, the expansion of a city by more than 0.2 miles per year would guarantee the density of structure falling with distance. On average, these values seem fairly conservative estimates, but apparently they constitute a flimsy ground on which to base a definite and general prediction.

Chapter 5. IMPACT of the PROPERTY TAX on LAND USE and VALUE

5.1 Definition of the Issue

A primary policy implication of the foregoing analysis of urban growth impacts is educational: as it is shown that differences in land value appreciation can be explained as an outcome of resource allocation by the competitive (and by standard economic implication, efficient) market, policy makers need be cautious in condemning and trying to suppress every land value appreciation that does not fall into line with the rest of the land market. The conceptual scheme and analytical methods developed in the foregoing context can be applied to a more direct and traditional policy issue, the impact of the property tax on residential real estate.

The property tax is one of the major policy instruments that directly and specifically affect the urban spatial economy, and it has naturally been studied extensively by economists. Almost all existing studies have been concerned with aggregate effects of the tax: for example, the effect of the tax on the level of housing production and housing price in a locality taken as a whole. The implicit and explicit focus of interest is then on the comparative effects of different tax rates between jurisdictions, such as the effect on households' choice of residential location among different communities, or on sectors of production, such as allocation of investment capital between housing versus production equipment. These studies do not say anything about the distribution of the impact among locations within a jurisdiction. We have seen in the foregoing chapters that an ostensibly undifferentiated exogenous shock such as a uniform increase in household income or an increase in aggregate population of a city results in varying effects among locations within the city. It is not difficult to infer that similar differentiation would pertain to the impact of the property tax even though the tax rate is supposed to be uniform for all housing within a city, and that the tax would affect the use of land differently than it does structural factors. We want to determine the pattern of this differentiation.

It is well established, in theory at least, that taxing land value or rent alone does not affect resource allocation as long as the land is taxed without regard to how it is used. But such a pure land tax is hard to find in practice, and normally it is housing rather than land alone that is taxed. Even if housing is subject to taxation, one theoretical conclusion of the existing studies is that it will not affect resource allocation as long as there is no difference in the tax rates across jurisdictions and sectors. However, the uniform tax is again purely hypothetical and normally the tax rates are different across the sectors or across jurisdictions or both. Also, a tax on housing does not affect the use of each input factor in the same manner. The property tax that we deal with in this chapter is the differential tax, a difference in the rate of tax on residential real estate in a city over the rate on other forms of capital in the same city or the rate of real estate tax imposed in other jurisdictions.

There ususally are quite a few municipalities with different tax systems of their own within a typical metropolitan area of the U.S., and this makes the interjurisdictional effect of taxation as one of the most visible aspects of local property tax system. Even then, it is not the only important effect of the property tax. Further, many other countries do not exhibit such a large degree of fiscal segmentation and the tax rate tends to be uniform within a metropolitan area. Then the issue of intra-jurisdictional impact of the tax obviously bears significant implications on the matter of equity and resource allocation.

As mentioned earlier, the subject is generally neglected. But it has not entirely been so, and let us take a brief look at the few studies there are on this subject. Representative of one type of approach to the question is Haurin's analysis (1980). Employing the framework of the monocentric (open) city model, he has found that an increase in the tax rate on housing rental would reduce housing consumption everywhere and steepen the density gradient. We are interested in deriving the same type of result (and in fact reach a similar conclusion for the particular situation). However, we will not adopt his assumption that the tax is reimbursed in monetary goods to taxpayers and that utility and production functions are of the Cobb-Douglas form.

The condition that the local government budget be balanced, which necessiates the former assumption of Haurin's, is adopted in other studies (Polinsky and Rubinfeld, 1978, for example) in the form of local public service expenditure exhausting the gross tax receipt. We feel that this sort of integrated approach is not necessary and even not desirable because few local governments ever balance their budget with property tax revenue alone and because there is a great variation in the way the tax receipt is spent. For these reasons, it is probably more useful to analyze the tax and expenditure questions separately. Carlton (1980) took an approach very similar to ours in analyzing the intrajurisdictional impact of the tax alone in a closed monocentric city setting and with a more general function than the Cobb-Douglas form. We will have more to say about his analysis of the closed city case after we conclude our own analysis of the same.

One serious problem in almost all the analytical studies of the property tax is their definition of the tax itself. A property tax is usually defined as a certain proportion of the rental income from a residential or other real estate, while most often in reality the tax is stipulated as a proportion of the asset value of the property. The two are not equivalent because as we have seen above the value of a property with the same rental income can be quite different depending upon the future growth of the income stream. The neglect is apparently due to the comparative static framework which all the existing analyses adopt because under the assumption of housing malleability the housing rental, land rent, and optimum land use density are not affected by the future growth of rentals which causes the difference between the two taxes. But we shall see below that, if we introduce the durability of housing, the two versions of tax have somewhat different implications as to the land and housing market as well as to the generation of tax revenue. Of course, in some municipalities housing rental is the legal base of tax assessment, and also it is conceivable that in others the actual assessment is based on housing rental even when the statutory stipulation says otherwise. The issue is to be empirically decided, and we will analyze both versions of tax - which we will call housing rental tax and property value tax - in comparative static (section 2) and dynamic settings (section 3), and in an open and a closed city.

5.2 Comparative Static Analysis of a Housing Rental Tax

As we mentioned above, the property value tax is largely irrelevant in the comparative static framework because the base of the value tax, the stream of future rental income, does not affect the present land use. Therefore we analyze only the tax imposed as a certain percentage of the rental income of a housing. With the malleable housing the optimal rule of housing production requires that the cost of an input be equal to its marginal value product. Assuming that owners of capital and structural inputs can peddle their commodity to any market they like and can avoid the differential tax of a particular community, the cost of capital (r) and the cost of structural input (c) do not change under the local property tax. The condition of equilibrium in the housing production under a housing rental tax rate (m) can be stated as the following.

$$(1-m)pQ'(k) - rc = 0 \qquad (5.1)$$

$$R = (1-m)pQ - rck = (1-m)(1-K)pQ \qquad (5.2)$$

where $K = rck/(1-m)pQ = Q'(k)k/Q$
is the factor share of the structural input

where p is the marginal rental of housing, Q is the density of housing, k is the density of the structural input, and R is the land rent. The above is exactly the same as under no taxation except that the after-tax rental income is substituted for the before-tax income. Let us denote the after-tax rental as $\hat{p} = (1-m)p$.

Wholly differentiating the necessary condition for optimality, equation (5.1), with respect to the tax rate m,

$$-pQ'(k) + (1-m)[Q'(k)\frac{dp}{dm} + p\frac{Q''(k)}{Q'(k)}\cdot\frac{dQ}{dm}] = 0 \qquad (5.3.a)$$

or

$$\frac{dQ}{dm}/Q = E(F)\frac{d\hat{p}}{dm}/\hat{p} = E(F)[\frac{dp}{dm}/p - \frac{1}{1-m}] \qquad (5.3.b)$$

from Lemma 6, with E(F) being the elasticity of housing production per land area with respect to the net marginal revenue.

Equation (5.3.a) states that an increase in the housing rental tax rate decreases land use intensity, raises housing rental, or both; and equation (5.3.b) states the relationship among the changes in the tax rate, housing rental, and housing density in an elasticity form. How these effects are realized depends upon the kind of general equilibrium mechanism that distinguishes between a closed and an open city.

In an open city, housing consumption per household and the housing rental should be left unchanged under different taxes as there is no compensating variation in income; otherwise, utility level changes in contradiction to the definition of an open city. Then the after-tax revenue for the housing supplier falls by the amount of the additional tax, and consequently the structural and housing densities decrease everywhere:

$$dQ/dm = Q'(m) = Q'(k)^2/(1-m)Q''(k)$$

from equation (5.3)

$$= - QE(F)/(1-m) < 0$$

One notable feature about this effect is that the negative impact of an additional tax rate is proportionately more severe if the base tax rate is higher.

Land rent will also decrease, and the burden of the tax will fall entirely upon land owners by the assumption of capital mobility. To see it formally,

$$dR/dm = d[(1-m)pQ - rck]/dm$$

$$= -pQ + [(1-m)pQ'(m) - rck'(m)] = -pQ$$

by equation (5.1)

That the land rent is reduced in proportion to the gross revenue from housing, not proportional to land rent or net revenue, implies that the negative impact of the tax is relatively more severe for land which shares a smaller portion of the total rental income or which is already heavily taxed. This can be expressed as steepening of the rent gradient.

$$R'(x) = (1-m)p'(x)Q ,$$

$$R^*(x) = p^*(x)/(1-K) \tag{5.4}$$

from equation (5.2)

Differentiating,

$$dR^*(x)/dm = p^*(x)K'(k)k'(x)/(1-K)^2 < 0$$

since $k'(x) < 0$ and hence
$K'(x) > 0$ by Proposition 3.2

By the same token we can see that the housing density gradient steepens:

$$dQ^*(x)/dm = dQ^*(m)/dx = - [dE(F)/dx]/(1-m) < 0$$

from equation (5.4) and Proposition 3.2

The population density gradient is the difference between gradients of housing density and per household housing consumption (see sections 4.3), i.e.

$$n^*(x) = Q^*(x) - H^*(x)$$

Since the gradient of per household housing consumption does not change while the housing density gradient steepens, the population density gradient moves the same way as the latter.

The effect of the tax on the parameters of the city as a whole can be determined by evaluating the general equilibrium conditions that the land rent at the border (B) be equal to the farm rent and that the population be housed within the border (equations (2.3) and (2.4)). Rewriting the border condition,

$$R(B) = (1-m)p(B)Q(B) - rck(B) \begin{cases} = R(f) & (5.5.a) \\ = (1-m)R(f) & (5.5.b) \end{cases}$$

The first of these applies if the income from farming is not subject to the same tax as housing, and the second is valid when it is. The population identity is

$$N = \int_0^B \phi x \; [Q(x)/H(x)]dx \qquad (5.6)$$

where B is distance from the city center to the border; and ϕ is the radian of the city territory.

If we differentiate the border condition,

$$dR(B)/dm = R'(m;B) + R'(B)dB/dm$$

$$= \begin{cases} 0 & \text{in case farm rent is not subject to the same tax as housing.} \\ -R(f) & \text{in case it is.} \end{cases}$$

Rearranging,

$$dB/dm = \begin{cases} p(B)Q(B)/R'(B) < 0 \\ [p(B)Q(B) - R(f)]/R'(B) < 0 \end{cases}$$

$$\text{since } R(f) = R(B) < p(B)Q(B).$$

Thus we confirm the expectation that the city area shrinks. Total housing available is the increasing function of housing density and available land area and it decreases unambiguously. On the other hand, housing consumption per household does not decrease. Therefore, the population must decrease.

The above analysis of the impact of the rental tax on spatial structure of an open city can be summarized as:

[PROPOSITION 5.1] When there is an increase in the tax rate on housing rental income in an open city, levels of housing density, population density, and land rent fall everywhere; their gradients flatten; the burden of tax is entirely placed on land-owners; but housing rental or consumption per household does not change.

In a closed city, the population is assumed fixed and the level of utility changes instead. We need to determine this aggregate change in utility before we can evaluate local impacts of the rental tax. It is intutitively clear that the level of utility must decrease as a result of an increase in the tax rate as we do not consider the effect of additional public services provided with the added tax revenue. We will nevertheless provide a formal proof because we need a rigorous specification of the utility change in order to evaluate spatial consequences. For this, we first differentiate the border condition with respect to the tax rate:

$$dR(B)/dm = -p(B)Q(B) + (1-m)[p'(U)dU/dm + p'(B)dB/dm] = 0 \quad (5.7)$$

considering only the case where farm rental is not subject to the same tax as housing.

By analogy to Wheaton's Theorem 3, we can write the change in housing rental following a change in utility as

$$p'(U;x) = -1/H(x)U'(Z;x) \qquad 0 \leq x \leq B$$

Substituting in the above,

$$dU/dm = -H(B)U'(Z;B)[p(B)/(1-m) - p'(B)dB/dm] \quad (5.8)$$

Now the desired result can be proved by the method of contradiction. Suppose the utility increases as a result of the tax increase. Then the bracketed term of equation (5.8) should be negative and hence $dB/dm < 0$, i.e. the city shrinks. It can be easily shown that the same must be true even if the farm rent is taxed at the same rate as housing. On the other hand, housing rental decreases everywhere on account of Theorem 3. Then housing consumption per household increases because housing is a normal good, and housing density decreases because the aftertax revenue decreases. From equation (5.6) we can see that these changes should result in a reduction of total population, which contradicts the definition of a closed city.

The spatial distribution of the impact can be specified by evaluating the relative changes in housing rental at different locations. Since the utility change is the same everywhere we can write

$$dp(x)/dm = p'(U;x)dU/dm$$

$$= -\frac{dU}{dm}/H(x)U'(Z;x)$$

$$= \frac{H(B)U'(Z;B)}{H(x)U'(Z;x)}\left[\frac{p(B)}{(1-m)} - p'(B)\frac{dB}{dm}\right]$$

And

$$\frac{dp(x)}{dm}/p(x) = -\frac{dU}{dm}/p(x)H(x)U'(Z;x)$$

We know from Lemma 2 that pHU'(Z), housing expenditure per household multiplied by the marginal utility of income at a particular location, is constant over distance if the ordinary elasticity of housing demand with respect to rental is 1 as in the Cobb-Douglas utility function and decreases with distance if the elasticity is less than 1. Therefore the rate of change in housing rental increases with distance in the case of inelastic demand; in other words, the gradient of housing rental flattens.

It means also that the gradient of after-tax revenue per housing quantity (^p) flattens by the additional tax as the tax rate is uniform over distance by the assumption and

$$\frac{d\hat{p}}{dm}/\hat{p} = \frac{dp}{dm}/p - \frac{1}{1-m}$$

Carlton (1980) analyzed the same (a closed city) case with essentially the same method as ours and reached the similar conclusions as the above, not surprisingly. But we cannot share his conclusion that owners of land at more central locations incur disproportionately heavier burdens of tax. This conclusion of Carlton's was drawn by inferring from the flattening gradient of the after-tax housing rental the same change in the land rent gradient. Let us examine not only the land rent gradient but also housing and population density gradients as their evaluations can be made by the same method. We can write the changes in the gradients resulting from the tax hike as the following.

$$\frac{dQ}{dm}/Q = E(F)\left[\frac{dp}{dm}/p - \frac{1}{1-m}\right] = E(F)\hat{p}^*(m)$$

Hence

$$\frac{d}{dx}\left(\frac{dQ}{dm}/Q\right) = \hat{p}^*(m)\frac{dE(F)}{dx} + E(F)\frac{d\hat{p}^*(m)}{dx} \qquad (5.9.a)$$

Likewise,

$$\frac{d}{dx}\left(\frac{dR}{dm}/R\right) = \frac{d\hat{p}^*(m)}{dx}/(1-K) + \hat{p}^*(m)\frac{dK}{dx}/(1-K)^2 \qquad (5.9.b)$$

Finally,

$$\frac{d}{dx}\left(\frac{dn}{dm}/n\right) = \frac{d}{dx}Q^*(m) + E(p)\frac{d}{dx}p^*(m) \qquad (5.9.c)$$

The elasticity of housing production with respect to the net rental income, the factor share of structure (K) and the rate of increase in housing rental all increase with distance under the assumption that housing demand elasticity and substitution elasticity between structure and land are both less than 1 as we have discussed above. Therefore, we can see from the above that all the gradients flatten if the after-tax rental is increased as a result of a tax hike, in other words, if the housing producer gains in terms of the net revenue as a result of the tax increase. This appears unreasonable and we can prove that this cannot be true in every location. Suppose the after-tax rental increases everywhere. Then housing density increases from the above and so does land rent everywhere. The latter is because, from the definition of land rent,

$$dR/dm = -pQ + (1-m)Q\,dp/dm$$

$$= (1-m)pQ[(dp/dm)/p - 1/(1-m)]$$

But then the city extends its boundary, and with the increased housing density and decreased housing consumption per household the total population of the city must increase, in contradiction to the condition of constant population.

Therefore it must be either that the net revenue, housing density, and land rent decreases everywhere, or that they increase in outer parts and decrease in inner parts of the city. It is not possible to unambiguously decide between the two possibilities. But we can start our evaluation by analyzing a benchmark of sorts, the case when the elasticities of housing demand and factor substitution are both 1, i.e., Cobb-Douglas utility and production functions. In this case, we have already shown above that the before-tax and after-tax rentals of housing increase at the same rate at all locations or that their gradients do not change. Also in this case the factor shares and elasticity of housing production per land area are constant over distance (Proposition 3.2). Therefore, it is easy to see from equations (5.9) that gradients of housing density, land rent, and population density remain constant after the tax change.

Also we know that the utility and housing consumption of each household decrease with the tax rate. Thus the density of housing must also decrease in order that the total population remain constant, implying that the after-tax rental decreases everywhere. It is intutitively clear that this last conclusion should hold for the case where the elasticities of housing demand and factor substitution are in the vicinity of value 1, which could be true empirically. Then the only unambiguous evaluation we can make about the gradients of equation (5.9) is that the gradients of land rent and housing density flatten as a result of the tax hike if the ordinary elasticity of housing demand with respect to housing rental is (slightly) less than 1 and the elasticity of factor substitution is equal to or over 1. Under the more plausible circumstances that both elasticities are less than 1, however, we cannot make any analytical judgement about the direction of change of the gradients.

5.3 Dynamic Analysis of the Property Tax

In this section we examine the impact of a tax on housing rental as well as a tax on the capital value of a residential property, particularly their impact on housing construction and land value. Only the case of an open city will be explicitly analyzed because it is difficult to specify the general equilibrium of a city with durable housing when the utility level is variable under different conditions. The rental price and per household consumption of housing at each location do not change in an open city since the parameters of housing consumption, namely, income, transportation cost and the utility level of a household, remain constant under different taxes.

For a given housing rental stream a raise in the rate of the tax, m, on housing rental income of each period reduces the capitalized net marginal value of unit quantity of new housing by as much as the additional tax rate:

$$d\hat{}F/dm = d(1-m)F/dm = \frac{d}{dm}\int_0^T (1-m)pe^{-rt'}\,dt' = -F(0,T) \qquad (5.10)$$

where T is the time to replace the present housing.

The optimal density of the structure should meet the condition that the capitalized value of the net marginal rental income stream be equal to the marginal cost of construction:

$$(1-m)FQ'(k) - c = 0 \qquad (5.11)$$

Wholly differentiating this condition we find that the rate of reduction in the structural density of new housing following a raise in the tax rate is proportional to the present tax rate and the elasticity of housing production per land area with respect to the marginal value, E(F):

$$\frac{dQ}{dm}/Q = Q^*(m) = -E(F)/(1-m) \qquad (5.12)$$

Since the elasticity of housing production increases with distance if the factor substitution is inelastic (Lemma 6), the density decreases relatively more at locations farther from the center, i.e.

$$\frac{d}{dx}Q^*(m) = -\frac{dE(F)}{dx}/(1-m) < 0$$

In examining the impact of a tax hike on land value we will simplify the analysis by assuming that the new housing will last infinitely. It can be verified by straightforward arithmetic that the conclusion does not change if we consider a series of finite life-spans of successive development, but it will greatly complicate the presentation. Under the assumption,

$$V = (1-m)FQ - ck = (1-m)(1-K)FQ$$

and

$$dV/dm = -FQ \qquad \text{since } (1-m)FQ'(m) = ck'(m) \quad \text{by (5.13)}$$

Thus we can see that the landowner shoulders all the added tax burden as should be expected from the assumption that owners of nonland factors and the city residents can all move their locations to avoid the added burden. It can also be seen easily that the burden relative to the land value is larger at farther distances because the factor share of structure increases with distance under inelastic factor substitution .

$$\frac{d}{dx}\left(\frac{dV}{dm}/V\right) = -\frac{dK}{dx}/(1-m)K^2 < 0$$

An important question in the dynamic analysis is how the timing (t*) of development and redevelopment is affected by a change in the tax rate. Using the same technique as in chapter 4, we can write the effect as

$$dt^*/dm = -J''(m,t^*)/J''(t^*) \qquad (5.15)$$

or

$$\text{sign}[dt^*/dm] = \text{sign}\,[J''(m,t^*)]$$

because $J''(t^*) < 0$ in order for t^* to be the unique optimal time for maximization.

$$\text{where } J'(t^*) = \hat{P}(t^*,-) - R(t^*)$$

$$= \hat{P}(t^*,-) - (1-m)p(t^*)Q(t^*) + rck(t^*)$$

Here the after-tax rental income from the existing land use, $\hat{P}(t^*,-)$, would be $R(f)$, the farm rent, if the site considered for housing development is not subject to the same tax as urban housing; $(1-m)R(f)$ if the farm income is subject to the same rate of tax as housing; and $(1-m)pq$ if the considered site is presently used for housing whose density is q. Let us begin with the first case where the after-tax farm income does not change under different rates of the rental income tax. Then we need only to consider the change in the urban shadow rent specified in the third equation of (5.15).

$$R'(m;t^*) = -\ pQ + (1-m)pQ'(m) - rck'(m)$$

$$= -\ pQ - (1-m)Q'(m)(rF-p) \qquad \text{by equation (5.13)}$$

$$= -\ pQ - (1-m)Q'(m)\dot{F}'(t^*)$$

$$= Q[-p + E(F)F'(t^*)] \qquad \text{by equation (5.14)}$$

If we write the average rate of rental growth as g,

$$p(t^*) = (r-g)F(t^*,\ ) \qquad \text{and} \qquad F^*(t^*) = g$$

by Lemma 3.

Also, by the same token as in Proposition 3.1,

$$g = F^*(t^*) < r(1-K)$$

Substituting in the above,

$$R'(m;t^*) = FQ[g(1+E(F)) - r] \quad (5.16)$$

$$< FQKr(s-1)$$

$$\text{since } E(F) = sK/(1-K)$$

The last term of the above is clearly negative when the elasticity of factor substitution, s, is smaller than 1. Thus,

$$J''(t^*,m) = - R'(m:t^*) > 0$$

Hence, $dt^*/dm > 0$, i.e., the conversion is delayed at the margin of the city. This means that the physical size of the city is smaller under a higher rate of the rental tax. This, in combination with the reduction in housing density everywhere, in turn means that the population of the city would be smaller, too.

On the other hand, if the factors can be substituted more freely, the conversion process can be hastened under a sufficiently fast growth of housing rental. That is, if g in equation (5.16) is large enough campared with the discount rate so as to make the bracketed term positive. This is because under certain circumstances the shadow land rent for housing increases even while the net rental income is decreases. This paradoxical result is possible as the high substitution elasticity makes the housing construction more sensitive to the change in marginal value of the net rental stream and hence the optimal structural input, k, decreases more than the net rental income, (1-m)pQ.

Even if we consider the change in rental income of the existing housing or farm, the basic conclusion remains valid. Since the land use itself is fixed for the existing use, the change in the after-tax rental income can be written as the following.

$$\left.\begin{aligned} d(1-m)R(f)/dm &= -R(f) \\ d(1-m)pq/dm &= -pq \end{aligned}\right\} = -\ [(1-m)pQ - rck]/(1-m)$$

$$\text{since } J'(t^*) = 0$$

$$= -\ pQ + rck/((1-m)$$

$$= -pQ + rFQ'(k)k$$

$$= FQ[-r(r-g) + rK] = FQ[g - r(1-K)]$$

Combining this with equation (5.16),

$$J''(t^*,m) = FQ[g - r(1-K) - g(1+E(F)) +r]$$

$$= FQ[-g + rK] \qquad (5.17)$$

$$> FQKr\ (1-s)$$

The final condition for the sign of dt*/dm again reverts to the case where the existing farm use is not taxed at the same rate as urban housing, contrary to intuition. The only difference is that, with the elasticity of substitution larger than 1, it would take a lesser rate of housing rental growth to make the sign negative and hence development or redevelopment schedule advanced under the even-handed taxation than under a more favorable taxation on existing uses.

Under the ususal scheme of the property tax, the tax bill is supposed to be proportional to the capital value of a property regardless of the present rental income. The equilibrium for the property owner is reached if the rental income and capital gains cover not only the opportunity cost (r) of the fund invested in the property but also the tax bill proportional to its capital value. The condition can be formally stated as the following, with the rate of tax w and the property value Y.

$$(r+w)Y(t) = p(t)q(t) + Y'(t)$$

This is exactly the same as any capital market equilibrium condition except that the effective discount rate in this instance is the market rate plus the tax rate. Therefore the effect of a change in the tax rate can be analyzed just as the case of a change in the discount rate which we have dealt with in section 2.4.

For a new housing development whose life-span is infinite, the change in the capital value of unit quantity of housing can be given by the following which is equivalent to Lemma 4.

$$dF/dw = - F/(r+w-g) \qquad (5.18)$$

It is immediately apparent that the proportionate reduction in the value of housing is larger for locations where the future rental is expected to grow faster (larger g), i.e., at outer locations in a growing monocentric city.

The reduction in the optimal development density follows:

$$\frac{dQ}{dw}/Q = E(F)\ \frac{dF}{dw}/F = -\ E(F)/(r+w-g) \qquad (5.19)$$

One difference in the rate of reduction in density under the value tax from that under the rental tax is that the rate becomes smaller if the present (before change) tax rate is larger, whereas under the rental tax the opposite is true.

The variation in the rate of reduction in the development density, the marginal capital value of housing (old or new), and the land value across locations are similar under the two versions, that is, more severe at outer locations. However, the relative impact of the property value tax can be less severe at farther distances in a declining city where the rate of decline in housing rental is greater at farther locations. It contrasts with the effect under the rental tax that would always be more severe at farther distances as long as the elasticity of substitution is smaller than 1. To wit, by the same procedure as in the analysis of the rental tax, the difference in the proportionate tax burden on the landowner can be stated as the following.

$$\frac{d}{dx}\left(\frac{dV}{dw}/V\right) = -\ \frac{dK}{dx}/(r+w-g)\ -\ (1-K)\frac{dg}{dx}/(r+w-g)^2$$

It can be seen from this expression that the conclusion regarding the impact of the rental tax that the burden of tax grows (or decreases) over distance because of the negative first term is strengthened if the city is growing in general and the second term is negative, but it is mitigated if the city is delining (or the substitution elasticity is high.)

The impact of the tax on timing of housing development or redevelopment can be determined by differentiating the following condition of land use change with respect to the tax rate, as in the previous analysis.

$$J'(t^*) = P(t^*,-) - [pQ - (r+w)ck] = 0$$

In this case we do not need to pay attention to the rental income of the existing land use because the tax does not affect period-by-period rental income itself. Differentiating the bracketed term, which is the urban shadow rent,

$$R'(m;t^*) = pQ'(m) - ck - (r+w)ck'(m)$$

$$= (r+w-g)FQ'(m) - FQK - (r+w)FQ'(m)$$

$$= FQ[-E(F) - K + E(F)(r+w)/(r+w-g)]$$

by equation (5.19)

$$= [g(E(F) + K) - K(r+w)]FQ/(r+w-g)$$

Therefore,

$$J''(t^*,m) = - R'(m;t^*) < 0$$

if and only if

$$g > K(r+w)/(E(F)+K) = (r+w)(1-K)/(1+s-K) \qquad (5.20)$$

However, the growth rate of the housing rental is limited by a condition similar to Proposition 3.2 which for this case is

$$g = \dot{F}^*(t^*) < (r+w)(1-K)$$

Therefore the condition (5.20) can be satisfied only if the elasiticity of substitution is larger than the factor share of structure. If the elasticity of substitution is close to 1 as some authors suggest, this may be satisfied. But in order that the sufficient condition for the inequality (5.20) be satisfied the growth rate must be very large, close to the upper limit given by the last condition. Thus we can conclude that the development would in general be delayed, more so in areas where the housing rental is growing slower, with the possible exception for locations at or close to the city limit where development can be facilitated. Therefore the reduction in the rate of the property value tax would generally facilitate redevelopment of inner city areas and possibly discourage urban encroachment upon the surrounding farm land.

CHAPTER 6. REVIEW OF EVIDENCE AND ALTERNATIVE THEORIES

Chapters 2 and 4 have concluded with a very general prediction: urban land rent and value increase proportionately faster at locations farther from the center in a monocentric city whose population, median income, or transportation efficiency is growing consistently -- be it an open or a closed city, be the process of change comparitive static or dynamic, or be it under myopic or perfect foresight. The pattern is symmetric: the conclusion is reversed if one of the above measures of urban size is decreasing while the others are not increasing, and if the observation is moved beyond the border of the urban area. The exceptions to this pattern are obtained only under conditions infrequently observed in the real world: if the city declines in one of the principal measures of city size but increases in another, or if the city grows through gradual removal of legal or physical barriers to territorial expansion. In these special cases we cannot make an unambiguous prediction. Under the assumption of comparative static change, the conclusion on land value is directly carried over to the pattern of land use intensity or population density. With durable housing, the theoretical correspondence can be confirmed for a more limited number of instances.

In the introductory chapter we claimed that the general conclusion is strongly supported by empirical evidence and that our framework complements previous comparative static or dynamic analyses by clarifying many ambiguities. We will try to substantiate these claims in this chapter. This chapter deals only with the changes in patterns of land value and population distribution in urban growth. There is little empirical or theoretical research on intraurban differentiation of impacts of the property tax beyond what has already been reviewed in chapter 5.

Although we have focused our attention primarily on land rent and value, most existing research has focused on population distribution, the pattern of which is obtained by inference in our analysis. Therefore, we first review empirical evidence of flattening density gradient around the world and over many decades presented by many scholars, and the standard theoretical explanation for it.

6.1 The Standard Model of Urban Population Distribution

It was well known from very early days that inner city areas are more densely populated than outer areas. Colin Clark (1951) studied the phenomenon quantitatively and concluded: "That the falling off of density is an exponential function ... appears to be true for all time and all places studied, from 1801 to the present day, and from Los Angeles to Budapest. This maintenance of the exponential relationship is, however, compatible with very different rates of decline of density, as measured by the coefficient b."
As in the following equation

$$n(x) = n(o)e^{-bx}$$

or (6.1)

$$\log n(x) = \log n(0) - bx$$

where $n(x)$ is the number of people per land area at x miles form the city center;
e is the base of natural logarithm; and
b is the coefficient mentioned in the quotation from Clark, (absolute value of) density gradient unique for a city.

Although it is now used as a standard empirical tool, there has been a substantial controversy about the theoretical validity and empirical usefulness of the negative exponential function. The focal point of contention is the implicit assumption that the density gradient is uniform at all places in a city. This is not a proper context in which to belabor the details of the arguments, but three summary observations appear relevant. First, the function has never been espoused as a general theory by its proponents but only as an empirical hypothesis which can be given a rough theoretical justification. In that spirit, the single gradient parameter should be interpreted as the average gradient not necessarily implying exactly identical values for all locations. Second, despite its simplicity, even many of its critics acknowledge that it explains empirical phenomenon at least as well as many more complicated functional forms. (See McDonald and Bowman, 1976.) Third, and most important, its single parameter of density gradient offers the important advantage of comaparability over time and cities that other complicated functional forms cannot offer.

Table 2. Clark's Density Function Estimates

Region, City and Date	*A*	*b*
Australia—		
Brisbane		
1901	20	·95
1947	40	·75
Melbourne		
1933	100	·55
Sydney		
1947	30	·25
British Isles—		
Dublin		
1936	70	·85
Liverpool		
1921	330	·80
London		
1801	290	1·35
1841	800	1·40
1871	290	·65
1901	210	·45
1921	180	·35
1939	80	·20
Manchester		
1931	40	·25
Ceylon—Colombo		
1946	60	·40
Continental Europe—		
Berlin		
1885	290	1·10
1901	410	·95
Budapest		
1935	280	·90
Continental Europe (*continued*)—		
Oslo		
1938	80	·80
Paris		
1817	450	2·35
1856	240	·95
1896	370	·80
1931	470	·75
Vienna		
1890	170	·80
United States of America—		
Boston		
1900	160	·85
1940	50	·30
Chicago		
1900	110	·45
1940	120	·30
Cleveland		
1940	90	·45
Los Angeles		
1940	30	·25
New York		
1900	250	·55
1910	?	·20
1940	120	·20
Philadelphia		
1900	120	·65
1940	60	·40
St. Louis		
1900	70	·75
1940	40	·45

For legends see equation (6.1)

Source: Clark (1951), Table 1.

In the foregoing theoretical analysis we have been concerned with local gradients whose values may vary among locations. However, we have shown that except for a few specific cases the gradient moves in the same direction at every location of a growing city. Therefore, our prediction of flattening density gradient should show up in the flattening of the average gradient estimated on the basis of the exponential funtion.

Clark noted that there is a tendency for the gradient to flatten over time (see his regression result reproduced in Table 2) and attributed this mainly to improvement of transportation. But a more comprehensive explanation of the tendency as well as the above mentioned justification of the exponential function itself was offered by Muth(1969). Since his work remains the core of the standard theory of the changing pattern of population distribution, we will review his theory and empirical verification in detail.

As we have seen in chapter 2, population density is simply the inverse of land area per capita, and the density gradient is the signed inverse of the gradient of land consumption per capita. Since land consumption is an explicit argument of utility in Alonso's model, per capita land consumption and hence population density are directly obtained from solutions for a consumption optimization problem under given conditions of general equilibrium. However, land enters as a factor in the production of housing in Muth's system where housing is a primary argument of utility. In this case it is necessary to consider utility and housing production functions together in order to figure out per capita land consumption. The population density gradient can now be expressed in terms of demand and production parameters.

$$n^*(x) = - L^*(x) = Q^*(x) - H^*(x)$$

$$= p^*(x) \; [E(F) + E(p)] \qquad (6.2.a)$$

from Lemma 6, with the static housing rental gradient $p^*(x)$ substituting for $F^*(x)$; and equation (2.7.b)

where

$$p^*(x) = - a'(x)/pH \qquad (6.2.b)$$

from Wheaton' Theorem 3.

The approach taken by Muth in determining the comparative statics of the density gradient was direct and partial. He sought to assess the impact of a certain exogenous shock on one or the other variable in the above equations. First, an increase in income will increase the housing expenditure, denominator in equation (6.2.b), and hence the rental price gradient flattens. Reduction of marginal transportation cost has the same effect as it reduces the numerator.

An increase in population would increase the housing rental. If the compensated elasiticity of housing demand with respect to housing rental is larger than 1 as Muth assumed, housing expenditure decreases, and as a result the rental gradient would steepen. On the other hand, the elasticity of production of housing per land area with respect to housing rental, E(F) in equation (6.2.a), decreases with increasing rental if the elasiticity of substitution between land and structure in housing production is less than 1 (see Proposition 3.2.) This will have the effect of flattening the density gradient. The net effect, then, should be determined by comparing magnitudes of these opposite effects by the use of empirically determined parameter estimates. Muth argued the net effect is the flattening of the gradient because the price elasticity of housing demand is only slightly over 1. If housing demand is inelastic with respect to rental, as we suggested in chapter 2, there is no ambiguity in concluding that the rental and population gradients flatten.

By the same token, he observed, elasticity of production decreases if housing rental is raised as a result of an increase in the price of structural inputs or in the property tax. But this is wrong because the housing price increases at a rate less than the input cost, especially if housing demand is elastic as Muth assumed, and the elasticity of production decreases only if the housing rental increases faster than the cost of input (Proposition 3.2). On the other hand, housing expenditure is reduced when the housing price goes up if housing demand is elastic. Therefore, the net effect should be unambiguously a steepening of the density gradient, contrary to Muth's conclusion. The effects of the property tax should be similar: We have seen in chapter 5 that the population density gradient would steepen in a closed or an opencity under inelastic factor substitution and elastic housing demand.

Even if a lower value of housing demand elasticity is used, our framework yields the same conclusion that an increase in the price of the structural input steepens the density gradient. An increase in the price of structural input amounts to a reduction of real income and of farm rent. We have seen in chapter 2 that the former steepens the population gradient for an open or a closed city. Reduced farm rent does not affect the gradient in an open city, but steepens it in a closed city. Therefore, the effect is unambiguously a steepening of the gradient for either type of cities.

The opposite should be true in case the availability of total land increases, and Muth's conclusion that the density gradient steepens as a result should be amended likewise.

As can be noticed in the above discussion, a more serious problem of Muth's approach is the neglect of interdependencies among variables and parameters of the equations (6.2) that define the population density gradient. His separate consideration of each determinant suggests that he dealt with a closed city, while his use of compensated elasticity in determining the effect of increased rental on housing expenditure suggests an open city. These are a minor problem in deriving the theoretical results, but the lack of recognition of the interdependencies hampers interpretation of theoretical and empirical results, as we shall see shortly.

Muth also considerd aspects that do not strictly belong within the comparative statics of a monocentric city. It can be easily seen that the concentration of manufacturing employment in the central city will steepen the gradient, and the existence of multiple subcenters will flatten it. It is also apparent that a large presence of old and substandard housing in the central area would lower the number of inhabitants there, and hence flatten the density gradient.

He also argued that the income elasticity of demand for land is greater than that for structure, and increasing income will therefore shift the demand in favor of a more distant location where households can consume more land. This is the argument frequently repeated by subsequent analysts as the most influential factor. However, it is in fact a spurious explanation.

First of all, available evidence suggests that the income elasticity of demand for land is smaller than that for housing as a whole and hence smaller than that for the structural component alone (see King, 1976, and Wheaton, 1977). If it is very large, the ordinary price elasticity of demand for land can go higher than 1 (see the discussion in section 2.2) and increasing population can cause the gradient to steepen in a closed city. Otherwise, the income elasticity should not have any direct effect on the relative desirability of locations. This is because the market mechanism will ensure that land rent is distributed so that income effects are canceled out and households are indifferent to any location and the specific combination of land and structure that is optimal at the location. We have discussed (while explaining Proposition 2.6) that a shift of demand in favor of a farther location is caused by the difference in the marginal utility of additional income in the case of a closed city, regardless of the income elasticity as long as it is positive.

Despite these errors (by our judgement), little extension or amendment of the basic theory has been offered by others. Muth's empirical test of the theory also remains the most comprehensive in terms of the coverage of determinants of the density gradient. He estimated regresssion coefficients (b) of the log-linear version of the equation (5.1) with population density data for 25 census tracts of each of 46 metropolitan areas of the U.S. in 1950. Then he evaluated the determinants of the cross-sectional variation by means of the ordinary least squares regression with the absolute value of the density gradient for each city (b in equation (6.1)) as a linear function of various factors suggested by his theory. One of the two final equations identify the following as the significant independent variables. In the following list (-) signifies the flattening effect, and (+) the steepening effect.

car registration per capita, as a surrogate for transportation technology	(-)
median family income	(-)
total metropolitan population	(-)
proportion of black population	(-)
proportion of substandard housing in the central city housing stock	(-)
concentration of manufacturing employment	

in the central city (+)

These results are, in short, consistent with Muth's prediction. The last three variables are not dealt in our analysis but the first three are, and Muth and we agree on their expected effects. Though car registration per capita was used to account for transportation technology which usually is extremely hard to quantify, it in fact may be considered a variable reflecting income level and thus may have exacerbated the collinearity problems that we elaborate on below.

In another of the final equations, the following two variables were added:

cost of construction material (-)

dummy variable for a water-front city (-)

The negative coefficient of the construction cost apparently supports Muth's (mistaken) hypothesis and contradicts ours, but its standard error was larger than the coefficient itself. The second, significant at the 15% level, contradicts Muth's hypothesis and supports ours.

More critically, the addition of the latter two variables made the coefficient of the income term statistically insignificant. This term was also found to be very sensitive to different specifications that were tried for intermediate regressions. Muth expressed a puzzlement about this result especially since this contrasts with the consistent significance of the coefficient of the popualtion term whose effect Muth considered to be more ambiguous. The same problem appears in many empirical works by other authors. Mills and Song (1979), for example, found that the coefficient of the income term is statistically insignificant but positive in a regression of density gradients of Korean cities using only population and income as the explanatory variables.

The problem is probably a symptom of high collinearity involving the income term. The median per capita income has a very high correlation with population in theory as well as in reality, especially in developing countries where the urban system is highly open. In Muth's regression, income is perhaps correlated with per capita car registration and construction cost also.

Collinearity is known to reduce the efficiency of estimation and hence make one or the other of the collinear variables appear statistically insignificant. There are numerous examples of econometric studies plagued by the same. For example, in studies of consumption functions, the coefficient of assets or lagged consumption is often found to be insignificant. As is the case in this example, we cannot consider that the statistical insignificance of the income term reflects real insignificance, but that the regression coefficient of one or the other of the highly collinear variables cannot be estimated and interpreted properly without prior restrictions imposed by extraneous data or theory. In this case we should take theory as the guide and consider the susceptibility of the income term to specification as the evidence of collinearity.

Mills's empirical study (1972) of population distribution of the U.S. cities represents a most significant extension of Muth's analysis. This study nicely complements Muth's, as it examines not only cross-sectional but also historical variations in the density gradients, not only population but also employment distributions, not only the final pattern but also the adaptive process. Mills's compilation of historical density gradients shows that the trend of flattening goes as far back as estimation is possible (year 1880, see Table 3) and that it is also true with the distribution of major categories of employment.

For 18 U.S. metropolitan areas Mills obtained gradients of population, and manufacturing, retailing, wholesale, and service sector employments for 1948, 1954, 1958, and 1963 by means of a unique short-cut (see Mills, 1972; for the criticism of the method, see Harrison and Kain, 1974). For some of these cities he could estimate a population density function going back to 1880. To identify important determinants of the density gradients, he estimated regression equations similar to Muth's, including the median family income, total population, a time trend, and the lagged density gradient as the independent variables. The first two are conventional and show the expected effects of flattening density gradient. Here again the coefficient of income term is less significant (17% level) than that of the population term, which is significant at the acceptable 10% level.

Table 3. Mills's Density Function Estimates for U.S. Cities

γ = Density gradient
D = Central city density

(a) Population Density Functions of Six Metropolitan Areas, 1929-63

Metropolitan area		1920	1930	1940	1948	1954	1958	1963
				Population				
Baltimore	γ	.70	.64	.60	.48	.40	.36	.33
	D	69,238	67,630	65,542	51,159	42,693	37,481	34,541
Denver	γ	.87	.83	.76	.59	.45	.38	.33
	D	34,870	36,265	35,334	27,779	22,884	19,678	18,008
Milwaukee	γ	.61	.56	.51	.47	.37	.32	.27
	D	68,304	74,209	65,434	58,318	44,262	37,823	31,123
Philadelphia	γ	.25	.37	.36	.31	.27	.25	.23
	D	67,595	62,034	59,789	53,264	45,714	41,868	38,268
Rochester	γ	1.18	.96	.88	.73	.55	.47	.40
	D	72,729	58,464	50,775	39,682	28,194	24,033	20,527
Toledo	γ	1.43	1.01	.93	.83	.72	.67	.61
	D	85,828	56,260	47,031	41,123	34,661	31,768	28,151

Average Gradients of Population and Employment Density of the Six Metropolitan Areas

Sector	1920	1929	1939	1948	1954	1958	1963
Population*	.84	.73	.67	.57	.46	.41	.36
Manufacturing	.95	.82	.77	.76	.67	.60	.48
Retailing	n.a.	1.02	.90	.76	.73	.58	.41
Services	n.a.	n.a.	1.12	.88	.81	.70	.55
Wholesaling	n.a.	1.43	1.24	1.01	.89	.77	.59

* Figures in columns headed 1929 and 1939 are for 1930 and 1940 respectively.
n.a. = not available.

Table 3. (Continued)

(b) Population Density Functions of Four Metropolitan Areas, 1880-1963

Year		Baltimore	Milwaukee	Philadelphia	Rochester
1880	γ	1.82	.97	.30	1.78
	D	244,730	44,287	39,948	51,400
1890	γ	1.08	.92	.28	1.83
	D	89,300	70,804	45,555	81,600
1900	γ	1.05	.90	.28	1.59
	D	101,200	92,374	56,611	74,500
1910	γ	0.93	.78	.28	1.20
	D	90,100	77,764	64,772	58,400
1920	γ	.70	.61	.25	1.18
	D	69,238	68,304	67,595	72,729
1930	γ	.64	.56	.37	.96
	D	67,630	74,209	62,034	58,464
1940	γ	.60	.51	.36	.88
	D	65,542	65,434	59,787	50,775
1948	γ	.48	.47	.31	.73
	D	51,159	58,318	53,264	39,682
1954	γ	.40	.37	.27	.55
	D	42,693	44,262	45,714	28,194
1958	γ	.36	.32	.25	.47
	D	37,481	37,823	41,868	24,033
1963	γ	.33	.27	.23	.40
	D	34,541	31,123	38,268	20,527

Average of Population Density Gradients of the Four Metropolitan Areas

Year	Average gradient	Year	Average gradient
1880	1.22	1940	.59
1890	1.06	1948	.50
1900	.96	1954	.40
1910	.80	1958	.35
1920	.69	1963	.31
1930	.63		

Source: Mills (1972), pp. 48-49

Table 4. Determinants of Density Function Coefficients of 18 Metropolitan Areas, 1948, 54, 58, 63

Dependent variable		Constant term	Log population	Log SMSA median family income	Log SMSA sector employment	Log time	Lagged dependent variable	R^2
Population	log γ	.9069	$-.2829(10)^{-1}$	$-.1021$		.1415	.9046	.967
		(.9806)	(−1.2017)	(−.9073)		(1.8125)	(19.8500)	
	log D	$.1756(10)^{1}$	$.1679(10)^{-1}$	$.6223(10)^{-2}$		$-.4877(10)^{-1}$	.7939	.919
		(1.1508)	(.6691)	(.0323)		(−.3634)	(20.0988)	
Manufacturing	log γ	−.1380	$-.4706(10)^{-1}$		$.3845(10)^{-1}$	.1020	.8676	.850
		(−.2182)	(−.6185)		(.7928)	(1.0838)	(12.4121)	
	log D	.8332	$-.7051(10)^{-1}$		.1034	$-.6320(10)^{-1}$	.8621	.959
		(1.1807)	(−.7355)		(1.1985)	(−.5330)	(20.5795)	
Retailing	log γ	−.1460	$.7614(10)^{-1}$		$-.8330(10)^{-1}$	−.2608	.8672	.937
		(−.1893)	(.4579)		(−.5464)	(−4.2090)	(12.4597)	
	log D	3.4309	−.4434		.5069	−.3362	.6519	.653
		(1.7690)	(−.9936)		(1.1295)	(−1.8762)	(6.6274)	
Services	log γ	$-.1539(10)^{1}$	.3161		−.2978	−.1240	.8554	.903
		(−2.1154)	(2.4304)		(−2.3830)	(−1.7319)	(12.5529)	
	log D	$-.1245(10)^{1}$	.3343		−.1497	−.3123	.7572	.833
		(−1.0741)	(1.4860)		(−.6753)	(−2.4748)	(12.0124)	
Wholesaling	log γ	−.9544	.1499		−.1226	$-.9226(10)^{-1}$	.9969	.939
		(−2.0377)	(2.1250)		(−2.0460)	(−1.6932)	(18.7097)	
	log D	−.2373	.1518		$-.9531(10)^{-1}$	$-.1583(10)^{-1}$	.8610	.909
		(−.2780)	(1.0848)		(−.6707)	(−1.3432)	(14.6125)	

Regression equation:

$$D_{it} = \alpha_o + \alpha_1 P_{it} + \alpha_2 Y_{it} + \alpha_3 t + (1 - \mu) D_{it-1} + \eta_{it}$$

$$\gamma_{it} = \beta_o + \beta_1 P_{it} + \beta_2 Y_{it} + \beta_3 t + (1 - \lambda) \gamma_{it-1} + \epsilon_{it}$$

where D = density at the center
γ = density gradient
P = total metropolitan population
Y = average family income
t = year
α, β = regression coefficients
μ, λ = adjustment coefficients
i = city

(variables all in logarithm)

t-values are in the parentheses

Source: Mills (1972), pp.54-59.

Dropping many of Muth's variables that are highly correlated with income could have resulted in higher significance of the income coefficient. But the inclusion of the time trend variable must have negated the potential gain in efficiency of estimation, since the income level has gone up consistently over time while populations of many metropolitan areas have not. A time trend is commonly used in many econometric studies to represent myriad influences associated with time and which cannot be otherwise attributed, but it is very hard to interpret. Mills interpreted the significant positive sign of the term's coefficient as meaning the secular increase in the cost of transportation, the net effect of the increase in the cost of travel time due to rising income outweighing cost reduction through improvement in transportaiton technology and road network. But there appears to be no clear ground to support or refute this interpretation.

A lagged dependent variable has a clear enough meaning, used in dynamic analyses to represent the lag in adjustment. In this case the density gradient of the previous period represents the inertia of the urban spatial pattern due to high durability of urban structures. The coefficient was found to be the most significant of all independent variables and close to 1, signifying very slow adjustment of the gradients to changing conditions.

We will not review a host of similar studies of density gradients since they are mostly confirmations of the results obtained by the above two studies without notable methodological improvements. Instead, we reproduce as Table 5 the compilation by Mills and Tan of many research results from around the world, demonstrating that the trend of flattening density gradient is not limited to the U.S. and European cities.

Table 5. Average Urban Density Gradients of Various Countries

Year	*India (12 cities)*	*Year*	*Mexico (3 cities)*
1951	0.675	1950	0.359
1961	0.652	1960	0.335
		1970	0.284
Year	*Brazil (4 cities)*	*Year*	*Korea (12 cities)*
1950	0.182	1966	0.701
1960	0.171	1970	0.670
1970	0.157	1973	0.639
Year	*Japan (22 cities)*	*Year*	*USA (20 cities)*
1965	0.457	1960	0.199
1970	0.391	1970	0.123

Source: Mills and Tan (1980), Table 11

6.2 Dynamic Analyses of Urban Spatial Structure

Muth's theory of population distribution is clearly a comparative static theory with some informal embellishments of dynamic elements. While Mills identifies the durability and the "disequilibrium" nature of land use as an important part of urban spatial process, his description of the process is a version of the standard model of aggregate stock adjustment that does not acknowledge the difference in adjustability among locations and among different vintages of urban land use structures. This essentially comparative static framework has been challenged by theorists who conceptualize the change in land use as radically different from the succession of static spatial patterns. We review in this section a few important contributions in this direction, their main themes and limitations, with particular reference to how they concur with or differ from each other and from our analysis.

Durability is explicitly incorporated as an essential element of urban spatial structure in several large scale econometric models of urban housing and employment distribution (by NBER and by Rothenberg, for example). But a clear statement of the concept within the framework of a simple monocentric city model was first made by Harrison and Kain (1974). Their model "depicts urban growth as a layering process" on the theory that "current levels of population, transportation costs, and other factor prices determine the density of development during this period but the density of past and future development depends on the level of these variables during those time periods." (Ibid.)

Empirically, they define development density as the ratio of single family housing to total new housing built in each period. This ratio is specified as a function increasing with time and decreasing with total existing housing stock of a city. This function is estimated by a regression with data for 83 U.S. metropolitan areas for 17 time periods from pre-1879 to 1960. They assume that each new development is added at the ring immediately next to the existing urban area (the layering process).

Then, for easy comparison with other studies of population distribution, the negative exponential function is fitted to the density profile that consists of computed densities of successive rings. Unfortunately, density functions thus obtained show a disappointing inconsistency with those obtained by Muth (above cited study) and Barr (1970, unpublished, quoted by Harrison and Kain). Harrison and Kain's density gradients of 11 cites are correlated with Muth's with the R-square value of only 0.03, though the correlation improves if the range of comparison is enlarged to 32 cities. Nor do their density gradients show any trend of flattening over time. It is important to remember that Muth's and Barr's density functions are empirical estimates from data of actual popualtion distribution, as are the trend of flattening density gradients; Harrison and Kain's are the outcome of a simulation based on their particular model of urban spatial growth.

The poor performance of the model was attributed by the authors to the neglect of several factors, most important of them being the alteration of existing housing and variations in the size of a building lot. But even after most neglected aspects except the alterations are corrected for with additional data, the density profile is still quite far from actual figures. This may indicate either a serious drawback in the scheme of the model or the great importance of housing alterations. But it is hard to pinpoint where the critical problem lies. Since their principal explanatory variable of the incremental density is a time trend which only begs another explanation, we do not have any explicitly specified relationship that links present development density to present parameters, let alone past or future.

Harrison and Kain's concept of cumulative urban growth was explicitly and faithfully formalized by Anas (1978). He assumes, following Harrison and Kain, that developers have myopic expectations about market parameters and that housing structure never changes once built. In order to specify patterns of relevant price variables he additionally assumes that utility and production functions are Cobb-Douglas, structural inputs are paid the permanent annuity of interest cost of their purchase price, and land rent is the remainder of housing rental after the payment for structural inputs.

Various scenarios of urban growth are analyzed in this framework, and empirical phenomena are interpreted in light of the analytical results. Because his analysis presents a most clear break from comparative statics of a monocentric city and his interpretation expresses the high hopes initially placed on dynamic modelling, it seems instructive to carefully examine Anas's main conclusions.

He argues that "Harrison and Kain perform an empirical study of urban densities ...their predictions of densities for Boston are at least as good as those of the empirical studies of Mills, Muth, and Barr which are based on long run adjustment assumptions and do not recognize durability." However, we have noted above that Harrison and Kain's result is a simulated one based on particular assumptions and empirical parameters and is generally not as good as the empirical results that were obtained by normal statistical methods independent of any particular model of urban spatial structure.

Anas observes that growth of income and improvement of transportation in typical American cities explain the negative density gradient, and that the growth of city population can be explained by the same factors using the open city model. But his analysis shows that the density gradient is positive if population is growing for a closed city. Then, what should be the model of a city whose population has been increasing but whose density gradient is negative?

Third, he observes that land rent is purely an opportunity rent and land value could be increasing with distance because of durability if the welfare is increasing. He also argues that under the same conditions abandonment of central city housing can be explained as the result of negative rental price of housing due to durability. We have seen in chapters 3 and 4 that if structure is assumed to be permanently maintained there is no opportunity rent or land value for built-up urban land, and that if land value exists it should decrease with distance in a monocentric city. Housing abandonment that is explained by Anas as a result of the negative housing rental cannot be possible if redevelopment is allowed and no externalities are admitted.

The last paragraph points to the most apparent weakness of the above two models, namely the assumption of permanent durabiltiy. As we have emphasized before, durability is primarily an economic condition rather than a technical constraint; if the building is no longer viable there is no reason why it should be kept indefinitely when the land can be redeveloped profitably. Anas claims that the assumption is a good approximation because in the decade of 1950's, for example, only 3.8% of the housing stock was demolished. But the figure must be viewed in light of the fact that this is about 17% of new construction on vacant lots, and that significant additions and alterations of the existing stock add another 20%. (U.S. Census of Housing, 1960)

This slow but eventual adaptation of urban land use is explicitly taken into account in later dynamic models of myopic urban land use by Brueckner (1980) and Wheaton (1982b). Both of these models have theoretical structures very similar to each other and to Anas's except for the above mentioned assumption of no replacement. Both derive conditions for replacement; Wheaton's numerical example is close to our example of section 3.3. They rely on simulation to explicitly describe spatial characteristics of urban housing. Brueckner simulates an open city where income, population, and utility growth are exogenously specified. Wheaton simulates a closed city where the level of utility is endogenously determined. Allowing for redevelopment makes density drop sharply at the border of housing generations as we have described in chapter 4. Structural density generally decreases with distance even among the same generation of housing developments, but this seems to be the result of the discrete time framework and a slightly increasing level of utility (in the case of Brueckner's simulation). Must we then think of the general decline of population density over distance as merely a historical accident as the cumulative growth model implies, or as evidence that the utility level increased over time?

We are inclined to answer "neither" on two grounds. First, we have seen in section 4.3 that population density will decrease with distance even under myopic foresight in the case where the size of a dwelling unit is adjustable during the lifetime of a building. Although alteration of the size is certainly costly, it is probably not so prohibitive as adjusting the housing density itself. This is because splitting up a house usually is not a very costly proposition and because passive adjustment is possible and frequent: even if house size remains fixed, if there are heterogeneous households they choose housing so that the size fits their characteristics.

Secondly, myopic foresight is perhaps not a good description of developers' expectations in the real market place. We have seen in the discussion toward the end of chapter 4 that the housing density would probably decline with distance under rational expectations unlike the case under myopic foresight. There are a number of models that assume perfect foresight on the part of developers. We have seen in chapter 2 that in most instances it is sufficient to assume rational expectations, not necessarily perfect foresight, to obtain results essentially different from myopic foresight models. Of many perfect foresight models we review here two of the recent efforts by the same authors as above: Brueckner and von Rabenau (1980) and Wheaton (1982a).

The structures of the two models are quite different. Brueckner and von Rabenau investigate a setting with two periods and three locations where one land use change is possible; Wheaton studies a setting with continous location and multiple time periods but with no redevelopment. Brueckner and von Rabenau show that declining density is the most plausible outcome of urban growth under perfect foresight. Wheaton provides a set of simulations most of which show general decline of population density. But in one of Wheaton's five simulations, where population continues to grow while income and transportation cost remain constant, density increases with distance over a fairly long stretch of distance and housing vintages. A close examination of the simulation result reveals that this positively sloping density curve occurs during periods of very slow physical expansion, confirming our prediction of section 4.3.

As we have mentioned in section 4.2, these authors also suggested and demonstrated the possibility of reversed, outside-in growth of a city under perfect foresight. In analytical terms, however, their specifications of the condition for such development paths are rather ambiguous. Brueckner and von Rabenau say that the pattern is possible under fast enough growth of housing rental over time, but obviously this necessary condition is not at all informative.

Wheaton clarifies the situation by an ingenious extension of Alonso's bid rent approach. A piece of land held until, and developed at, a certain period generates a certain maximum surplus; a different surplus if developed at another period. These can be called land values bid by different options of developing land at different periods. If an option of development at a particular period outbid all earlier development options, the land will be held until that period. For this waiting to be longer at more central locations, the bid value function for a later development option must have a steeper slope in distance. This is shown to be equivalent to the condition that the rate of increase in optimal population density be larger than the discount rate. Defining the condition in this way, in terms of parameters of development, makes it easier to quantify the situation. Still, it remains to be specified in terms of urban growth characteristics and utility and housing production functions, those that define the optimal population density. For this Wheaton presents simulations of different growth paths employing a log-linear utility function. The reverse growth pattern is shown to result when income is decreasing (3% per year) while population grows (3% per year).

Our analysis provides a clue why this development path is obtained. We have seen in chapter 2 that the housing rental gradient steepens if income decreases in a closed city under very general utility function. On the other hand, the population increase leaves the gradient of the housing rental unchanged because the log-linear utility function assumes that an ordinary price elasticity of housing is 1 (Lemma 2 and Proposition 2.8). Therefore, the net effect is the steepening of the housing rental gradient, and the reversed expansion path would result as we have concluded in Proposition 4.7.

We have reviewed major strands of the evolving literature on urban spatial dynamics: from Harrison and Kain's pathbreaking introduction of the concept of cumulative growth, Anas's formalization of the concept, Brueckner and Wheaton's introduction of replaceable housing and their subsequent consideration of expectations more complex than myopic ones. Our contribution can be assessed clearly in this context. First, we further the development toward generalization of expectation schemes, by adopting rational expectations and replceabilty of housing, and toward generalization of utility and production functions by limiting them only by a range of demand elasticity and substitution easticity. Second, by establishing a set of general yet simple rules of durable land use we have been able to determine some important issues of dynamics without relying on the opaque method of simulation. Third, we have shown that a clearly defined concept of land value can aid both the analysis of durable land use as well as the analysis of the land value dynamics itself which has been largely neglected by analysts despite its practical importance.

6.3 Empirical Patterns of Land Value Appreciation

Empirical patterns of population density change reviewed in the first section should be considered evidence for our prediction of the spatial pattern of land value appreciation thanks to the theoretical dependence between the two variables. More direct evidence seems indispensable, however, because land value is in itself at least as important as land use density and because one cannot take the mechanism relating the two variables as granted.

Our conclusions regarding land value appreciation rates were set out in the comparative static version (Proposition 2.10) and the dynamic version (Proposition 4.6), which are qualitatively identical, and summarized at the beginning of this chapter. We have also argued in the introductory chapter that the predicted pattern, faster growth of land value in outer areas, has been widely observed empirically and almost taken for granted. Nevertheless, this has not been so systematically investigated as the density pattern probably due to the well known problems in quantity and quality of land value data. Land transactions are not frequent. Nor are they straightforward as they usually involve complex credit arrangements and the like. Assessments tend to be unreliable if they exist at all. But, where there is a reasonable set of data on land value changes, analysts made good use of it, and our conclusion is well tested for.

In the U.S., Chicago's land values for a hundred years since the infancy of the city were compiled by Homer Hoyt (1969), and those for the twentieth century have been published in Olcutt's Blue Book series. Mills used the former compilation to analyze the relationship between land value and distance from the city center in different years. His estimation of three different regression equations is reproduced in Table 6. The linear function does not fit the data well, but the log-linear and double-log land value functions clearly confirm the decline of land value over distance in all the years studied, 1836, 1857, 1873, 1910, and 1928.

Of central interest to us is the overall trend of secular growth in (or decline in the absolute value of) the distance coefficients of the two logarithmic functions. We have shown before (equation 2.6) that the increase in the distance coefficient of the log-linear function, which is the logarithmic gradient, means that the land value appreciation rate increases over distance. It can easily be shown that the distance coefficient of the double log function also has the same property. Then Mills's result clearly shows that land value appreciated faster in more

Table 6. Land Value Function of Chicago, 1836-1928

Year	Regression	Constant	Distance	R^2
1836	linear	1016	−101.6 (−3.2782)	.0503
	log	5.799	−0.3986 (−27.1104)	.7836
	log-log	6.272	−1.936 (−31.3073)	.8284
1857	linear	6011	−575.1 (−6.9412)	.1911
	log	8.792	−.4874 (−35.3627)	.8597
	log-log	10.40	−2.873 (−34.1262)	.8509
1873	linear	24920	−2333 (−7.2494)	.2009
	log	10.05	−.3300 (−22.4327)	.7066
	log-log	10.34	−1.543 (−20.3243)	.6640
1910	linear	139800	−19220 (−4.4658)	.1386
	log	10.84	−.3275 (−13.2685)	.5867
	log-log	10.70	−1.300 (−16.3365)	.6828
1928	linear	182400	−15590 (−4.2650)	.1150
	log	11.85	−.2184 (−11.7969)	.4985
	log-log	11.96	−.9886 (−10.8135)	.4551

Regression Equation:

linear : $V(x) = A + bx$

log : $\log V(x) = A + bx$

log-log : $\log V(x) = A + b \log x$

where x is the distance from the city center, miles
$V(x)$ is land value at distance x

t-values are in the parentheses

Source: Mills (1969), pp.245-247

distant locations in every interval since 1857. Thanks to the fact that the two regression equations are single parameter models we can test the significance of difference in the coefficients for adjacent years by use of the reported t-values. Under both specifications the increase (flattening) of the gradient is statistically significant at less than a 1% confidence level. Although the trend is apparently marred by the steepening of land value gradient from 1836 to 1857, this opposite direction of change can be easily reconciled with our theory once we closely examine the nature of the data.

Chicago was but a tiny settlement of eight thousand people occupying an area of barely four square miles in 1836 (Hoyt, 1939). The ensuing booms of the railroad construction and the Civil War brought a forty-fold increase in population to about 320,000 in 1871, who were spread over all over the present legal boundary of the city, which extends about nine miles to the west and twenty five miles to the north and south, although much of the area was not tightly filled in until the early twentieth century. The picture is not clear for the year 1856, but we presume that the extent of the settlement was small compared to the present city area as the population in 1857 was about 90,000, less than one third of that in 1876. Nevertheless, Hoyt reports land values for all locations within the present city limit, and Mills uses this full data for his regression.

Therefore, we can infer that the land value changes that are reported for 1836 to 1857 mainly concern those outside the urban settlement. In those areas, the land value gradient should steepen, i.e., land values would increase faster at locations closer to the city boundary according to our hypothesis for the case of a city growing under rational expectations. The gradient might very well have flattened within the city, but in estimation of the land value function it probably was dominated by the steepening trend in the majority of the study area.

Chicago land values of the twentieth century reported in Olcutt's Blue Books were analysed by Yeates (1965). He estimated a double-log equation including four distance variables (distances from the city center, area shopping center, transit station, and lake Michigan), and two other variables (of population characteristics). The result of his regression is reproduced in Table 7. Despite the added specifications which would have reduced the statistical significance of the parameters, the coefficients of the distance from the center are significant and show the expected negative sign and the trend of flattening over time. In fact, most of the other distance variables show the same consistent behaviour, rendering support to the suggestion that the influence of distance from the city center on land use variables is generalizable to other distance variables. (Refer to the discussion in section 1.2) Here, again, the pattern opposite to our prediction for a growing city is shown to have occured in the decade of the great depression (1930 to 1940).

McDonald and Bowman (1979) report the continuation of the flattening of the land value gradient in their study of Chicago land values for 1970 and 1980, but reviewing this work in detail

Table 7. Chicago Land Value Functions, 1910-60

Yeates' Regression Results

	1910	*1920*	*1930*	*1940*	*1950*	*1960*
Multiple R^2	77	65	37	34	24	18
b_1	−.837*	−.673*	−.268*	−.275*	−.268*	−.173*
b_2	−.038	−.122*	−.156*	−.134*	−.080	−.092*
b_3	−.450*	−.414*	−.367*	−.285*	−.227*	−.146*
b_4	−.248*	−.240*	−.214*	−.140*	−.152*	−.050
b_5	+.105*	−.008	+.039	+.044	−.016*	−.137*
b_6	+.005*	+.001	−.003*	−.002*	−.002*	−.002*
Correlation between log c_i and log p_i	−.63	−.56	−.23	−.18	−.20	+.04

* means significantly different from zero at 0.05 level.

Regression Equation:

$$\log V_i = a + b_1 \log C_i + b_2 \log R_i + b_3 \log M_i + b_4 \log E_i + b_5 \log P_i + b_6 N_i + e$$

where: V_i = front foot land value
C_i = distance to central business district
R_i = distance to nearest regional shopping center
M_i = distance to Lake Michigan
E_i = distance to nearest elevated train or subway station
P_i = population density
N_i = percentage of non-white population
e = error
i = "ith" sampling point

Source: Yeates (1965), pp. 57-64

Table 8. Land Value Functions of Four Korean Cities, 1965-75

	SEOUL						PUSAN					
	A		-b		R^2		A		-b		R^2	
	Res.	*Comm.*	*Res.*	*Comm.*	*Res.*	*Comm.*	*Res.*	*Comm.*	*Res.*	*Comm.*	*Res.*	*Comm.*
1970	183,000	417,000	0.201	0.188	0.724	0.582	83,000	222,000	0.112	0.111	0.661	0.355
	(44.6)	(40.2)	(10.2)	(7.5)			(34.3)	(22.5)	(6.4)	(3.4)		
1973	176,000	469,000	0.163	0.187	0.671	0.611	101,000	314,000	0.094	0.089	0.612	0.288
	(48.0)	(43.7)	(9.2)	(7.9)			(38.3)	(25.5)	(5.8)	(2.9)		
1975	242,000	627,000	0.126	0.143	0.723	0.483	191,000	547,000	0.091	0.072	0.600	0.232
	(74.8)	(46.3)	(10.2)	(6.1)			(43.9)	(30.0)	(5.6)	(2.5)		

	TAEGU						SUWŎN					
	A		-b		R^2		A		-b		R^2	
	Res.	*Comm.*	*Res.*	*Comm.*	*Res.*	*Comm.*	*Res.*	*Comm.*	*Res.*	*Comm.*	*Res.*	*Comm.*
1965									0.733		0.718	
									(10.7)			
1966												
1968							13,000		0.833		0.812	
							(17.0)		(13.9)			
1970	101,000	369,000	0.508	0.574	0.867	0.785						
	(48.2)	(41.0)	(11.1)	(8.3)								
1973	100,000	483,000	0.348	0.526	0.818	0.621	51,000	210,000	0.675	0.819	0.827	0.795
	(58.5)	(31.4)	(19.2)	(5.6)			(33.9)	(38.2)	(14.6)	(10.8)		
1975	139,000	784,000	0.316	0.520			69,000	261,000	0.581	0.766	0.821	0.765
	(65.0)	(36.7)	(8.7)	(6.0)			(41.5)	(38.9)	(14.4)	(9.9)		

t-values are in the parentheses.
Regression Equation: same as the log function of Table 6.
Res. means residential land value; Com. commercial.
Source: Mills and Song (1979), Table 28.

would be redundant.

In Korea, various classes of land values are assessed annually by a quasi-public body. Mills and Song (1979) estimated the log-linear regression equation for four Korean cities covering various years of 1960's and early 1970's. Their result, reproduced in Table 8, shows that the gradients of commercial as well as residential land values flattened even over fairly short intervals (two to three years) in these rapidly growing cities. Population of the four cities grew at the annual rate of six to eleven percent during the studied period. The flattening of residential land value gradients during the short intervals are statistically significant at 7.5% level of confidence, except for Pusan where the significance level is about 15%. Comparing gradients of different cities we note that the gradient of Pusan, whose seaside and hilly terrain severely limit the supply of land for urban use, has the gradient flatter than that of the much larger city of Seoul. This is consistent with our prediction of Proposition 2.9 regarding natural and institutional barriers to urban expansion. Otherwise, the larger the city, the flatter is the gradient, again as expected.

There are a few studies that report the same general pattern of land value change in European cities. One of them, a wide survey of many cities (UN, 1973) concludes as follows.

> The expansion of city functions within a metropolitan area has brought a more rapid price increase in areas far from the

center of the town. Data on the development of land prices at Copenhagen,... Paris, Lyon, Milan, Zurich are significant for many metropolitan areas in different countries...

The data on which this conclusion is based were not processed through standard statistical techniques and consequently are not easily comprehensibe without the accompanying text to which we refer the readers. For yet another study of land value of European cities in support of our argument, readers are referred to Hallett (1979).

The foregoing renders an abundance of empirical evidence to our hypothesis. A rigorous support for it is found in the statistical significance of shifts in the land value gradient reported by Mills(1968) and Mills and Song (1979). However, the skeptic still might argue that the reported trend only provides a joint test of the negative exponential function as well as our own hypothesis, but it does not strictly verify either hypothesis in isolation. A more straightforward approach is to estimate the relationship between the distance from the center and the land value appreciation rate itself directly.

For this we analyze changes in residential land value of districts of Seoul, Korea, for the period from 1963 to 1976. The data consist of annual assessments by the Korea Appraisal Board of Grade A (good) residential land value of each district (but 1966 data are missing). Aerial distance from the city Hall to the municipal administrative office of each district (Dong) is used as the measure of the distance. Availability of published land value series (by Korea Home Builders' Association) and identifiability of districts on the official map limited the usable observations to 55. There are now about 700 districts in the city of Seoul but land value series for only about 80 of them were available presumably because many of the present districts are recent annexations of previously rural areas or subdivisions of old districts. Apparently name and area changes also occured preventing a reliable match between the available land value data and the official adminitrative map for more districts.

Seoul has a constrictive terrain for a city of seven million population (1982 estimate): its expansion is limited by steep hills to the north and the northwest of the city center, a large hill to the immediate south of the city center, and then the river beyond the hill. Thus it grew mainly towards the east and the northeast and a little to the west until the early 1960's when it began to break into areas beyond the hills and the river. These new sections now constitute a larger urban area than does the area along the older east-west axis.

In its most general form, our hypothesis simply posits an (increasing) monotonical relationship between land value appreciation and the distance in a growing city, and another (decreasing) monotonical relationship in a declining city or outside the boundary of a growing city. Such a relationship can be tested by the method of Spearman's rank correlation coefficient which has an advantage of being robust and approaching its asymptotic property for a small number of observations (about

10). We identified eleven to thirteen districts lying along each of the east-northeast, west-northwest, and south corridors of development, and computed the correlation between ranks of distance and of land value appreciation rates for 1963-70 and 1970-76 periods. The result is summarized in Table 9 below.

The magnitudes of most of rank correlation coefficients (Spearman's Rho) indicate that the hypothesis of the monotonic relationship between land value appreciation and the distance cannot be rejected at a 3.0% confidence level. Even the correlation for the southern areas in the 1970's is still significant at the 7.5% level.

Table 9. Rank Correlation between the Distance from the Center and Land Value Appreciation Rate, 1963-70 and 1970-1976

Direction from the city center	Number of observations	Spearman's Rho 1963-70	Spearman's Rho 1970-76
East and Northeast	12	.918 *	.694 *
West and Northwest	13	.804 *	.701 *
South, including South of the River	11	.711 *	.579

note: coefficients marked with * are significant at 3.0% level.

If the observations were to be pooled, we would have a sample of fair size with which an ordinary (parametric) statistical analysis would be viable. But due to the above mentioned difference between the new and old areas of development we felt that all the observations should not be pooled together. We therefore divided them into two groups of a still reasonable number of observations: 27 districts on the east-west axis (sector A) which form the traditional city territory, and 28 districts to the south and north of the City Hall (sector B) many of which began to be built up only since the 1960's. Land value appreciation rates were taken for 1963-67, 1967-70, 1970-73, and 1973-76, as well as the longer intervals, 1963-70 and 1970-76.

Though our hypothesis has not been specified in a parametric form so far, an appropriate one should feature such properties as are found in the quadratic function specified below: The dependent variable, the rate of growth in land value, would increase with the distance from the city center up to the bondary of the city and decreases with distance thereafter. We would expect the inflection point to be visible, i.e., the coefficient of distance to be positive and that of squared distance negative, provided that our data points include districts beyond the full-fledged urban area.

$$V^*(t,i) = a(t) + b(t)x(i) + c(t)x(i)^2 + e(i,t) \qquad (6.3)$$

where $V^*(t,i)$ is the land value appreciation rate of district i for the period t, i.e., difference between logarithm of land value of the beginning of the period and that of the end of the period;

$x(i)$ is the distance from the City Hall to the district i, in km;

a(t,s), b(t,s), c(t,s) are regression coefficients to be estimated for each sector s (sector A, B, or the whole city, T) and period t; and

e(i,t) is the error term.

The point of inflection, or the distance of the border B*(s,t), can be determined by differentiating the equation:

$$B^*(s,t) = -b(s,t)/2c(s,t)$$

We estimated equation (6.3) by the ordinary least squares method assuming as usual that the error term is normally distributed with mean of zero and the variance independent of the variables of regression. However, the result showed a clear presence of heteroscedasticity: the error terms (regression residuals) were larger for farther distances, threatening the viability of the OLS estimation. After a few experimentations with various weights, we found that weighting by the reciprocal of distance restores the distribution of residuals to reasonable approximation of homoscedasticity in most cases. Therefore we simply divided both sides of the regression equation by the distance and estimated it with the OLS method.

But even after the modification the second problem remained: despite large values of R-square, it seldom happened that all the coefficients are statistically significant in estimations for sector A and the whole city. This problem signals the insignificance of the border effect in the traditional sector (A). The sector has been almost fully developed to the sample limit which coincides with the real topographical limits, a large mountain range to the east and the river to the west, though they are far removed from the city center unlike the above mentioned barriers to the immediate north and south. If there is no inflection point, one of the distance terms is superfluous, and its inclusion can only harm the estimation by imparting bias and inefficiency. Suspecting a nonlinear relationship, we chose to drop the linear distance term, bx, from the equation and instead estimate the following.

$$V^*(t) = a' + c'x^2 + e \qquad (6.4)$$

Weighted least squares estimation of this equation produced significant coefficients for almost all estimations as well as reasonably homoscedastic distribution of residuals. The result is reported for sector A and the pooled data in Table 10. The simplified equation performed well enough also for sector B which contains the newly developing areas. Estimation of the equation for each sector and the combined sample showed that the difference between the sectors are not significant enough to invalidate the pooled regression.

But the original equation with bx is retained to be reported for sector B because it produced useful information on the city border with larger R-square values while seriously hurting the significance of coefficients only for one period, 1967-70. The calculated inflection distances are 9.7km for the 1963-67 period, 10.3km for 1970-73, and 10.7km for 1973-76.

There would certainly be many other factors besides the distance from the city center whose inclusion would be advisable for the sake of realism and explanatory power of the regression equation. However, such a practice can be confusing to the theoretical argument involving a simple model. The reported result clearly indicates that the distance variable has a strong enough explanatory power to stand alone in accounting for the difference in land value appreciation. The strong fit of these equations with essentially one independent variable is a pleasant surprise considering the level of abstraction of the theoretical model behind the regression equation, and it confirms the more indirect or casual evidence we have reviewed above.

In sum we believe the empirical evidence reported above supports our theoretical prediction almost to the point of redundancy, though the quality of each piece of evidence might be called into question. The most well documented is the part of the conclusion concerning land value appreciation within the growing city, but other implication regarding those outside the urbanized area in a growing city and those during times of decline is also evidenced at a few instances.

Table 10. Regression of Land Value Appreciation on the Distance from the Center, Seoul, Korea.

year	sector	constant	b	$cx10^2$	R^2
1963 -67	T	1.340 *** (.051)		.64 *** (.20)	.866
	A	1.230 *** (.049)		.55 *** (.27)	.940
	B	.855 *** (.355)	.284 (.127)	-1.46 *** (.94)	.695
1967 -70	T	.736 *** (.028)		.485 *** (.111)	.859
	A	.702 *** (.034)		.638 *** (.192)	.894
	B	.735 *** (.136)	.0504 (.0568)	-.621 * (.422)	.799
1970 -76	T	.441 *** (.038)		.808 *** (.152)	.687
	A	.419 *** (.036)		.797 *** (.202)	.658
	B	.212 (.205)	.325 *** (.086)	-1.553 *** (.634)	.267
1970 -73	T	.020 (.027)		.313 *** (.106)	.503
	A	.036 (.030)		.294 ** (.167)	.436
	B	-.415 *** (.140)	.171 *** (058)	-.832 ** (.432)	.282
1973 -76	T	.414 *** (.026)		.495 *** (.102)	.643
	A	.383 *** (.025)		.503 *** (.139)	.818
	B	.203 * (.135)	.154 *** (.056)	-.720 ** (.421)	.553

REGRESSION EQUATION: Equation (6.3) for Sector B;
Equation (6.4) for Sectors A and T.
SECTORS: T: All 55 districts; A: 27 districts; B: 28 districts
STANDARD ERROR is in the parentheses
SIGNIFICANCE LEVEL: *** significant at 2.5% level;
** significant at 5%;
* significant at 10% level.
UNITS: Land Value - Thousands of Won (current) per pyeong;
Distance - km.
DATA : compiled by the Korea Housing Builders' Assoc.,1982.

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JUE: Journal of Urban Economics
RES: Review of Economics and Statistics
RSUE: Regional Science and Urban Economics

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