

THE EFFECT PROPERTY TAXES HAVE ON PROPERTY
VALUES : A STUDY OF A SPECIFIC SITUATION

by

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ABSTRACT

Title: The Effect Property Taxes Have On Property Values: A Study Of A Specific Situation

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An important contribution to the revenues of local government is the property tax. This thesis explores the effect property taxes have on property values in a city. To demonstrate this effect a change in the current situation of property taxation is made and the results observed. The current situation in many cities (e.g. Boston) is inequitable taxation—some properties are taxed more heavily than others in the same city. A change in this situation is called revaluation—where each property is reassessed at a value which is then taxed on an equal basis with all other property in the city. Revaluation causes a change in property taxation and this change in the tax causes a change in the market value and it is this process which is examined in this thesis.

Theoretical analysis shows that the percent change in the property taxes from revaluation is equal to the ratio of the current assessment-sales ratio of the city (or total area which is being revalued) over the current assessment-sales ratio of the area which will be changed as a result of

revaluation, minus one. For full revaluation of the city the percent change is

$$\% \Delta \text{taxes}_r = A/S_{\text{city}} / A/S_r - 1$$

where r is any region of the city. This proves to be true for any property and any revaluation. The change in taxes causes a change in the market value of the property which, in turn, causes a reassessment of the property and a new tax once again which changes the value once again. This process resembles the swing of a pendulum and is called an iteration process. The way in which the property tax affects the market value is through the capitalization rate, while the market value affects the property tax through the effective tax rate. The iteration process depends upon the ratio

$$\text{Effective Tax Rate/Capitalization Rate}$$

for any property to reach a point of stabilization.

Empirical analysis proceeds to prove the above theory using a model based on a hypothetical city and performing different revaluations upon it. The results match with the theoretical results and the expected iteration process occurs. Through this method of analysis it is hoped that the reader would come to the conclusion that the effect property taxes have on market value is a specific formula which is true for any city using property taxation as a source of revenue.

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The three people mentioned above were of great help to me, enabling my task to be less difficult than it could have been. However, I wish to state my acknowledgement that this thesis, to the best of my knowledge, is an original work based on known facts. The research, unless other sources mentioned, was primary in character. The formulas and programs presented in this thesis were of my own creation as were the theoretical and empirical results.

Last, I would like to acknowledge this page—the only part of this thesis which is written in the first person.

D.I.D.

May 5, 1974

Cambridge,
Massachusetts

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CHAPTER ONE
INTRODUCTION

Survival of a government is important to those who benefit from that government. The reason for its existence is to give services to those people. In order for an entity to produce it must first consume—the government must receive some revenue or monetary compensation for its services. Since the people use the services, the people should give the monetary compensation. Hence, taxation of the people in exchange for governmental services.¹

One form of taxation is the property tax. The property tax plays an important role in government revenues. On the national scale property taxes are the most important source of city government revenues in fiscal year 1970-71. Figure one shows that property taxes make up 32.8% of total government revenues, while total taxation make up only 49.4%, which means that property taxes make up 66% of total city taxes. In the state of Massachusetts the property tax also plays an important role in local government revenues.

In many urban environments it is found that the property taxes are usually high and that single family houses exist in the parts of the city which are taxed at lower rates than other parts of the city—which usually contain income

¹Some economists argue that taxation is also a measure of the taxpayers' ability to pay. The author wishes not to argue that point and only wanted to introduce the idea of taxation and how it applies to property taxes.

NATIONAL SUMMARY OF CITY FINANCIAL
TRANSACTIONS FOR FISCAL YEAR 1970-71*

Item	Amount**	Percent
General Revenues, total	\$30,575	100.0
Taxes, total	15,090	49.4
Property	10,041	32.8
Sales and gross	2,780	9.1
Other	2,270	7.4
Intergovernmental	9,697	31.7
From state gov.	7,401	24.2
From federal	1,861	6.1
From local	435	1.4
Current Charges	3,579	11.7
Miscellaneous	2,208	7.2

****in millions of dollars**

FIGURE ONE

*Source:

U.S. Bureau of the Census report,
City Government Finances in 1970-71,
Table 1, Summary of City Government
Finances: 1970-71 and Prior Periods.

producing property—i.e. multi-family dwelling unit structures, office buildings, hotels, etc. The way in which different property is taxed at different rates will be discussed later. It should be clear that the property tax, because of its economic characteristics as well as its importance as a major revenue source, has a significant impact upon the property it taxes.

The relationship of the property tax to the value of the property being taxed is not clear. The property tax is directly derived from the city's assessment of the property value, but this assessment of value may not at all be related to the real value of the property. The property value is not the only important matter concerning the impact of the property tax. For owner-occupied single family houses the property tax has a significant impact upon the income of the family. It is not clear that there is a direct correlation between the family income and the value of the family's house, however it is safe to assume that a decrease in value due to an increase in taxes would also mean a decrease in family income. Therefore, there is a correlation between the change in value and the change in family income based on a change in the property tax. This paper will deal with the change in the property tax and its effect on the change in the property value.

Before discussing any relationships between property taxes and property values the terms being used must first be defined. Chapter two deals with the value of property while chapter three deals with property taxation. In order

to best illustrate the effect property taxes have on property values a specific situation outlining a change in property taxes will be made. This situation is the case of revaluation. Revaluation is the process of changing property assessments to a marked "equalization" point where all property is assessed on an "equal" basis.

To be more specific: the constitution of the State of Massachusetts directs that all property be assessed at full market value, which is not the case in many cities and towns in Massachusetts. Boston is such a case where the assessments of different properties range from 30% to 80% of full value. Revaluation has already occurred in many places in Massachusetts. The threat of revaluation of all cities and towns in Massachusetts leads to the question of the impact of this revaluation upon the urban environment. The purpose of this thesis is to show the effect this revaluation has upon the property tax and what effect the change in the property tax will have upon the property value. The result should be a specific effect which is true for any city using the property tax as a source of revenue.

In order to do this, a real situation will be examined both theoretically and empirically. The process of revaluation will be first examined for any city on a theoretical level. Then a model based on a hypothetical city² will go through the same process in order to test the theory just

²The author has had experience with an actual revaluation study of a real city. The reason for not using a model of a real city is to prove the last statement of the second paragraph on this page.

presented. The outcome of this experiment should agree with the theoretical outcome. The results of the experiment will be discussed afterwards.

The situation as it stands is a city where the property tax is a major source of revenue. The property within the city limits is taxed, thus people who work in the city and do not live in the city do not get so taxed (however, they do get taxed by the property taxed in their own community). The people who own their homes get taxed on their houses. The people who live in city apartment buildings also get taxed, indirectly through their rents. The burden of the property tax is passed on through the landlord to the tenant—a fact which will be proved in detail in chapter two on capitalization of income. Because it is difficult to stop the landlord from passing on the property tax to the tenants, the city makes a decision regarding the type of tenant being taxed. Because businesses can afford the property tax more easily than residents, the city purposefully taxes commercial property more heavily than residential property. This causes a certain inequity between types of property, for if all property were assessed at 100% of full value commercial property would not and could not be taxed more heavily than residential property. Along the same lines, because values change from year to year but assessments do not and because of the nature of the city

assessing practices, property of the same type, such as residential, are taxed differently depending upon the location of the property. Residential property in one part of the city is taxed more heavily than residential property in another part of the city. This brings in a different taxation inequity according to the region of the city which would also disappear if 100% valuation was enforced throughout the city.

These inequities described above are measured by one special variable: the effective tax rate, which is the tax rate of the city as applied to the real value of property rather than to the city's official assessment of the value of property (to which the regular tax is applied). Since there are inequities present between properties with regards to a constant city tax rate (regular rate), it is obvious that the effective tax rate is different for different properties and its measure is the inequity between those different properties. The effective tax rate can also measure the inequities present in the property tax between different cities and towns and states. Once there is a common data base (i.e. real market value) then a comparison can be made.

The result of revaluation is the change in the effective tax rate. One form of revaluation is where all properties of the same type receive the same effective tax rate. In this case the tax burden will be redistributed within each property type so that the inequities will stay

between the property types but will be erased within each type along regional lines.

The other significant case of revaluation is full revaluation—all property in the city receives the same effective tax rate. In this case the tax burden is redistributed throughout the city —between property types as well as areas. Assuming that the total revenues needed remain constant then the tax rate will change depending upon what percent of real value the city (new) assessments are based on. Law calls for assessments to be 100% of full value, but this paper will examine other percentages in order to show that the effect upon market value is the same no matter what percent is chosen.

Although the present situation of taxation is unfair for different properties it will be seen that revaluation would bring in other inequities not present beforehand. Under full revaluation residential property will receive an increase in tax burden while non-residential property will enjoy a decrease of the burden.

The examination of this problem will be on both a theoretical and empirical level. The empirical level will utilize a computer model with which a controlled experiment can be conducted. The results of this experiment will be discussed and the meaning of these results will be further examined.

CHAPTER TWO
THE VALUE OF PROPERTY

In dealing with property one must consider that there are two basic types in connection with property taxation: real property and personal property. Personal property is that which is "movable", non-permanent structures not for any sort of habitation by people. Real property is that which is meant to be a permanent structure upon land. Land is a real property. A factory is real property. A machine or oil tank is personal property. This paper will be concerned only with the value of real estate—real property only.

There are many ways to value one's property. The value to the owner can be considerably different than the value to the tenant. The value to the seller is at many times different than the value to the buyer. This paper will be concerned with two types of value and the relationship that exists between them. A discussion of the two values: assessed value and market value; and their relationship: the assessment-sales ratio.

Assessed Value

Property is taxed based on its value. The people who tax the property—the city officials—place values on all property in the city. The part of city government which performs this function is the assessing department. It is the job of the assessing department to assess the value of city property, giving each property an assessment of its true value.

The assessment has the major function of being the foundation for an individual property's tax. The tax rate, which will be discussed in detail in chapter three, is directly applied to the official assessment given by the city assessors in order to determine the amount of taxes that specific property owner must pay for that specific property. The assessment is only the value which the city assessors give to property. It may or may not be equal to the market value of the property and it does have a specific relationship to the market value. In any case, a property owner may appeal to the city for a reassessment, claiming the present assessment of the property is unfair.

In most cases of assessments, land is first assessed as if there were no structure upon it. Then the structure which is there is assessed. Depending upon the city the land may be assessed equally or may be assessed at a value which reflects the land's location or "potential".

Market Value

The definition of market value not only plays an important part in the taxation of property but also finds itself as a main factor on the marketplace of property sales, exchanges and mortgages. The "fair market value" of a certain property usually determines its selling price or exchange value on the "free market" and it is a basis for obtaining a mortgage on the property. A free market is where a property is placed for sale and anyone has the chance to buy it. This is when its sales price is determined, although its real value may not equal that. However, the term "real value" is somewhat debatable. If the sale of a property occurs on a free market where the owner is not forced to sell and the buyer is not forced to buy, then the sales price which is agreed upon by both parties—the buyer and the seller—based only on the condition of the property in question, is called the fair market value of the property. This — the fair market value — is the value which assessors allegedly base their assessments of property on. However, the "fairest" market value for property taxation is a question that will probably never be answered to everyone's satisfaction.

The type of value which assessors (as well as appraisers for other purposes—i.e. insurance, loans, real estate marketing, etc.) seek is the "normal" value

rather than the price which the property obtains from its sale at the particular moment of appraisal. The fair market value of a property as defined previously is the legal value of assessment for taxation purposes. This definition calls for a hypothetical, and most probably an impossible, sale, since many factors which do not have direct connections to the property in question usually affect a sale of a piece of property. Therefore, the assessor (or appraiser) must come up with the means to approximate the fair market value for all property. There are three basic and most commonly used methods for the appraisal of real estate and the approach to the normal and "correct" fair market value.

Comparable Sales Method

This method of property appraisal is mainly what its name is: comparable sales. To approach the value of a piece of property using this method there must have been recent sales of Comparable property. The term comparable encompasses a large range in the science of appraisal. In order for properties to be "comparable" for appraisal purposes they must be nearly identical, plus the sale used must be of the same type as the sale cited in the definition of fair market value. This is normally not the case, but if the buyer is willing to buy and able to and not under any duress and the seller the same then the sale can be used as a comparable sale for a similar property.

There are many drawbacks to the comparable sales method. One drawback is obtaining the comparable sales. For commercial and industrial properties that is nearly impossible, since office buildings are so different and so seldomly sold as that of industries as well. This is why the comparable sales method is mainly used for residential property and vacant land. Yet, there are homes in residential areas that are passed down through family lines and the sales, if any, are between family members. This is an area where the comparable sales method would be unable to approach the fair market value of the residential

property. The homes themselves may be somewhat comparable to homes in other areas, but the uniqueness of this area also plays a part in the value of its property. In an area where few sales occur a sale may be questionable for use in the comparable sales method. In this area where few sales occur those properties that are up for sale may differ in some way from the other properties, which might be the reason they are up for sale. Such a sale would not be useful for the comparable sales method.

Other drawbacks in the comparable sales method is that the buyer may be obtaining a favorable rate of interest for a mortgage which would be assumed from the seller and so buying the property at an inflated price. Also, the buyer may make a deal with the seller to state the price at a deflated level so as to lower the tax assessment upon it. The housing market has little defense against fluctuations: such as interest rates, inflation, new construction, etc.

With vacant land the comparable sales method is not infallible as well. Land values change between and within different areas of the city. And yet, the comparable sales method is usually the best method to use for appraising the value of single family houses and vacant land.

Replacement and Reproduction Cost Method

This method utilizes two separate concepts—thus two different methods actually emerge from this one—the replacement cost and the reproduction cost. The cost of reproduction is the cost that it will take to remake a building today out of the same materials it was originally made of. The cost of replacement is the cost of remaking the same building today out of existing materials. Because of the rising prices of construction materials, especially those used years ago—i.e. brick and stone—and the availability of cheaper materials, it would be less expensive to replace a building than to reproduce a building. For example, it would be cheaper to replace a stone building using concrete than to reproduce in stone.

The second part of this method is computing the depreciation of the property based on the replacement or reproduction cost. This part is the most difficult of the process since depreciation is due to economic and physical factors affecting the property in question. Even if the property does not physically deteriorate it may be worth less because a newer architecture or newer facility (e.g. with air-conditioning) is wanted. Location is also a factor as well as wanting an older structure (such as an older house). This is where the replacement-reproduction method is hard to defend and its treatment of this one term—depreciation—makes it a somewhat crude method.

Capitalization of Income Method

A method most commonly used by real estate firms, insurance companies, banks, and city assessors for determining the market value of property is capitalization of income. This uses the approach of first determining the income the property in question produces. Since single-family houses and vacant land not used as commercial parking lots do not produce income, this method is usually not for that type of property. However, there is a way to use the capitalization of income method on non-income producing property, but that is not important to this paper and therefore shall not be discussed here. The capitalization of income method for determining market value is for income-producing property—i.e. apartment houses, office buildings, stores, etc.

The first part of this method is the operating statement of the property. The net income the property produces is directly related to its value. This relationship is the capitalization rate. The determination of the capitalization is the second and most difficult part of the process. Applying the rate to the net income of the property results in that property's market value. Since most income-producing property is bought on the basis of the income it will bring to the owner and since most mortgages are also given on that same basis, the capitalization of income method is used and regarded as the best method of determining market value by most real estate investors and lenders.

In determining the net income of a property all the incomes brought to the property must first be tallied. These incomes must be associated with the building itself and not with the activities that go on within it. The earnings of the tenants that occupy the building do not have any association with the value of the building itself. The rents those tenants pay, however, do. Also, special services given by the owner of the building for the tenant's use (i.e. laundry, custodial work, etc.) also connect with the building's value. The total of all rents and all incomes from special services is the gross income of the property. The gross income minus all operating expenses equals the net income. Operating expenses include heat, water, electricity, management, repairs, cleaning and any other expense that have to do with the upkeep of the building or is essential to the building's operation. Real estate taxes (property taxes) and depreciation of the building³ are also operating expenses while all mortgage payments, interest and amortization, are not operating expenses, for they are due to the owner's availability of funds and ability to take out a mortgage and not due to the building's operation. Real estate taxes and depreciation are directly related to the existance of the building itself.⁴ An example of an operating statement is in figure two.

³Land cannot be depreciated in any case.

⁴Some realtors do not include depreciation in the operating statement and add a factor to the capitalization rate instead.

OPERATING STATEMENT
FOR AN INCOME-PRODUCING PROPERTY

+Gross Rent	
-Vacancies	
<hr/>	
= Net Rent	
+Services	
<hr/>	
=Gross Income	
+Repairs	
+Maintainance	
+Management	
+Insurance	
+Miscellaneous	
<hr/>	
= Operating Expenses	
+Real Estate Taxes	
+Depreciation	
<hr/>	
=Total Expenses	
+Gross Income	
-Total Expenses	
<hr/>	
=Net Income	

FIGURE TWO

The capitalization rate (cap rate) is actually the ratio of net income to market value. It is the per cent return an investor receives on a certain property. If a building is worth 1,000,000 dollars, then a 10% return would be 100,000 dollars. If in fact that building does give the owner a return of 100,000 dollars then the cap rate of that building is 10. However, the process of determining the cap rate is not so simple. The location, age and looks of the building are factors very important to the derivation of the cap rate. The actual net income may not be good enough for the derivation, since a low income might be due to mismanagement, not the building itself. Also, net income might be the only figure which is given and one must first determine the cap rate in order to arrive at the value. In other words, although the cap rate is the ratio of the net income to market value, this definition cannot be used for finding the cap rate, since value is also an unknown. There are other methods for finding the cap rate.

The cap rate is simply the return (the rate of return) an investor will agree to take for a certain property. Since real estate has risk involved, the rate of return must be significantly greater than those rates given for less risky investments. It stands to reason that the cap rate cannot be lower than the dividends given to savings accounts and stock investors. Basically, and most importantly, the cap rate is not based on the quantity of

the return, but rather, it is based on the quality of the return. In other words, if the net income of property doubles, then the value doubles—the cap rate stays the same. On the other hand, if the neighborhood surrounding the property becomes "better", then the risk involved is lessened and so the cap rate is lessened and the value goes up although the net income stayed the same. If uncertainty is changed (i.e. the lease of the major tenant is changed from 20 years to 40 years), then the cap rate is changed—thus the value of the property is changed. To illustrate:

$$\text{cap rate} = \frac{\text{net income}}{\text{value}}$$

$$\therefore \text{value} = \frac{\text{net income}}{\text{cap rate}}$$

net income= \$10,000

cap rate= 10% (.10)

$$\begin{aligned} \text{value} &= \frac{\$10,000}{.10} \\ &= \$100,000 \end{aligned}$$

next year, net income=12,000 dollars

$$\begin{aligned} \text{then now} \\ \text{value} &= \frac{\$12,000}{.10} \\ &= \$120,000 \end{aligned}$$

the next year, major tenant moves out,

now the cap rate=12% (.12)

$$\begin{aligned} \text{so} \\ \text{value} &= \frac{\$12,000}{.12} \\ &= \$100,000 \end{aligned}$$

This method of selecting the proper capitalization rate is highly subjective, based on past appraisal experience and knowledge of all influencing factors. Another selection process, which this author invented, is by means of the mortgage received for the property. Mortgage lenders give money for property based on property value. The method used for determining the property's value is the capitalization of income method.

A mortgage is given on a certain property. The ratio of this mortgage to the property's value is called the Loan to Value Ratio.⁵ The mortgagor (the owner of the building taking out the loan) pays back the mortgage in two parts: the principal (amortization) and the interest of the loan. The payments are in monthly installments for a certain number of years (the term of the mortgage). The payment made is always the same—thus it is called the Constant Payment. The ratio of the constant payment to the loan (the mortgage) is called the Constant. The constant is also found on a mortgage table—if the interest rate and the term of the mortgage is known, then the constant can be found. This also brings up another relationship: the ratio of net income to the constant payment is called the Debt Coverage Ratio. This ratio is important to the lender, for the debt coverage ratio should never be less than one, since if it is then the property is not bringing in enough revenue to pay the constant payment. Therefore, lenders usually want a debt coverage

⁵In Mass. the loan to value ratio has a legal maximum of 75%.

ratio higher than one to insure the payments and to lessen their (the lender's) risk.

If a mortgage has been taken out on a property, but the mortgage itself is not known, three significant factors can be known: the loan to value ratio, the constant, and the debt coverage ratio. The capitalization rate is equal to their product.

To prove:

$$1. \text{Cap Rate} = \text{Loan/Value} \times \text{Constant} \times \text{Debt Coverage Ratio}$$

$$\text{Debt Coverage Ratio} = \text{Net Income/Constant Payment}$$

$$a/b \times c = (a \times c)/b$$

$$2. \text{Cap Rate} = (\text{Loan} \times \text{Constant})/\text{Value} \times \text{Net Income/Constant Payment}$$

$$\text{Constant} = \text{Constant Payment/Loan}$$

$$\text{thus, Loan} \times \text{Constant} = \text{Constant Payment}$$

$$3. \text{Cap Rate} = \text{Constant Payment/Value} \times \text{Net Income/Constant Payment}$$

$$a/b \times c/a = c/b$$

$$4. \text{Cap Rate} = \text{Net Income/Value}$$

--which is the definition of the capitalization rate, thus proving the above equation 1.

The capitalization of income method is a very detailed and complex approach to the determination of market value. That is the reason for using the method and the same reason for not using the method. In comparison, the comparable sales and the replacement-reproduction cost methods are crude approaches. And yet, if one does not

go through a detailed analysis for the net income and capitalization rate, then this method would be equally crude.

For the purpose of this paper, the model presented in chapter four uses the capitalization of income method for determining the market value of all property except single family houses and vacant land where the comparable sales method is used.

The Assessment-Sales Ratio

The relationship between assessed value and market value is extremely important when dealing with the inequities of the property tax. In Massachusetts the state law calls for the assessment to be equal to the market value. If this were true then the assessment-sales ratio would be equal to one. The assessment-sales ratio measures the amount of deviation the assessment is from the market value—not a true measure of deviation, but rather the percent of market value. When the assessment is valued at half the value then the assessment-sales ratio is .50.

The inequities present in property taxation become apparent when studying the assessment-sales ratio between property types and between regions of the city. In many cities and in the model examined later in this paper the assessing practice causes different assessment-sales ratios to exist between property types and city regions. An example of this would be where an office building is assessed at 120,000 dollars, its value is 150,000 dollars—thus the assessment-sales ratio is .80; while an apartment building nearby is assessed at 15,000 dollars, value of 45,000 dollars—assessment-sales ratio of .33; and another apartment building with the same assessment of 15,000 dollars, but being in a different neighborhood of much less

quality its value is 30,000 dollars—its assessment-sales ratio is .50. One of the reasons for this difference is the negligence of the assessing department—properties would not be reassessed regularly, so when the market changes and the assessment stays the same, the assessment-sales ratios change causing differences in tax burden between different properties in different areas of the city.

A confusing issue with the assessment-sales ratio is what it actually measures. Since it was shown in the previous section on market value that the sales price is not necessarily equal to the market value, then the assessment to sales ratio would not be a true measure of the percentage of market value the assessment is. In order to deal with this problem another observation should first be made. There are two commonly used, yet different, assessment-sales ratios for property. For an example, examining the residential property of a certain region in a city it is found that each property has its own assessment-sales ratio. Totaling up these ratios and taking the average over the total number of properties gives the average assessment-sales ratio for residential property in that region. Taking the average of these ratios over the total amount of dollars of market value gives a different assessment-sales ratio. This second ratio can also be computed by merely dividing the total assessment of the residential property in that region by its total market

value. This is shown by:

For any region r and any property type p :

a_i = assessment of property i

s_i = market value of property i

n = total number of properties

The first assessment-sales ratio mentioned (averaged over the number of properties) is defined as:

$$\text{ASR\#1} = \frac{\sum_{i=1}^n (a_i/s_i)}{n}$$

which can also be written as:

$$\text{ASR\#1} = \frac{1}{n} \sum_{i=1}^n (a_i/s_i)$$

The second assessment-sales ratio (averaged over the amount of market value (total) dollars) is:

$$\text{ASR\#2} = \frac{\sum_{i=1}^n ((a_i/s_i)s_i)}{\sum_{i=1}^n s_i}$$

which is equal to:

$$\begin{aligned} \text{ASR\#2} &= \frac{\sum_{i=1}^n (a_i(s_i/s_i))}{\sum_{i=1}^n s_i} \\ &= \frac{\sum_{i=1}^n a_i}{\sum_{i=1}^n s_i} \end{aligned}$$

which is the total assessment divided by total market value.

The first ratio has each assessment-sales ratio weighted by a factor of one, totaled, then divided by the sum of the one's—which is n . This is also equal to weighting each a/s ratio (assessment-sales ratio) by a factor of $1/n$ and summing the results. The second ratio has each a/s ratio weighted by its own factor of s (market value), totaled, then divided by the sum of the market values—which is $\sum_{i=1}^n s_i$. This can also be computed by weighting each ratio by a factor of $s_i / \sum_{i=1}^n s_i$ —each individual s divided by the total s , which results in a simplification as shown above with the total assessment divided by the total market value.

The reason for discussing these two ratios is that they measure different problems with the assessment. The first ratio - ASR#1 - measures the percentage that the assessment is of market value for an average property in region r of property type p , while the second ratio - ASR#2 - measures the percentage the assessment is of market value for that total region r of property type p — the percentage ratio for an average dollar of true market value. Although the first ratio measures the true assessment inequities between properties, the second ratio is the significant ratio to be used for a study of revaluation. In other words, ASR#1 can be used to describe differences between different individual properties, but ASR#2 can be used to describe differences between property

types and regions within the city. The next chapter will discuss the direct connection of the property tax to the second assessment-sales ratio - ASR#2.

The assessment-sales ratio that this paper will be concerned with is the total assessment over the total market value for each property type within each region of the city. Individual assessments and values will not be in question, market value need not be computed on an individual basis, thus the individual sales prices will not have to be used at all. The model in chapter four gives the total assessments and total market values for all property types and regions in the city. From this data the assessment-sales ratio (ASR#2) can be computed and used for revaluation.

CHAPTER THREE

THE TAXATION OF PROPERTY

In order for the city to receive its revenues for survival it must be like any other government and tax the people. One source of revenue—a major source of revenue for cities—is the property tax. The property tax places the burden of supplying revenues to the city upon all property owners within the city's boundaries. Apartment owners pass down their property tax payments to the tenants as shown in figure two of the preceding chapter. Thus, the tenants who live in the city, but do not own land, must pay property taxes to the city.

The operation of the property tax has three major parts: the revenues needed and to be collected through property taxation, the assessments of all property in the city, and the tax rate which when applied to the assessments bring in the revenues. In order to determine the tax rate, first the assessments of all property must be established. Then, knowing what expenditures will be made, the total revenues needed can be determined. Dividing the total revenues by the total assessment results in the tax rate. An example of how the tax rate is determined for a city is in figure three. Consequently, to determine how much the tax for a certain property is one must multiply the assessment by the tax rate.

TAX RATE CALCULATION*

<u>Item</u>	<u>(Fictitious) Amount (\$)</u>
Appropriations	
City Budget	200,000,000
County Budget	30,000,000
School Budget	99,000,000
State Assessments	5,000,000
Overlay	15,000,000
Deficits	1,000,000
Total A	350,000,000
Revenues (excluding property tax)	
State Aid	41,000,000
State Tax Revenues	3,000,000
Local Taxes	43,000,000
Available Funds	13,000,000
Total R	100,000,000
Property Tax Revenues (Total A minus Total R)	250,000,000
Total Taxable Valuation	1,197,390,000
TAX RATE	\$208.79

FIGURE THREE

*Source:

Annual Report of the Assessing Department
for the Year 1971, Assessors' Office, City
Hall, Boston Mass., January 1, 1972.

Tax Rate = Total Revenue/Total Assessment

∴ Total Revenue = Tax Rate X Total Assessment

or, Total Taxes Paid = Tax Rate X Total Assessment

so, Individual Taxes Paid = Tax Rate X Individual Assessment

To compare the taxes one property pays to the taxes another property pays an equal base must be constructed. One base to compare property taxation is the market value of all property. The effective tax rate is the total revenue divided by the total market value (rather than divided by the total assessment to get the regular tax rate). This is the tax rate based on the real (market) value of the property. In this way the "fairness" of property taxation can be compared between cities and between types of properties and regions in the city.

For a certain property type in a certain region of a city:

P = Taxes paid by the property type in that region (tot. rev.).

T = Tax rate for the entire city.

E = Effective Tax Rate for the property type in region.

a_i = assessment of property i

s_i = market value of property i

$$1. P = \sum_{i=1}^n (a_i)(T) \quad \text{by definition}$$

$$2. P = T \times \sum_{i=1}^n a_i \quad \text{distributive law}$$

$$3. E = P / \sum_{i=1}^n s_i \quad \text{by definition}$$

But from 2, P can be replaced

$$4. E = (T \times \sum_{i=1}^n a_i) / \sum_{i=1}^n s_i$$

therefore,

E =

$$5. T \times \left(\sum_{i=1}^n a_i / \sum_{i=1}^n s_i \right) \quad \text{associative law}$$

and the quantity $\sum_{i=1}^n a_i / \sum_{i=1}^n s_i$ is the assessment-sales ratio (ASR#2 from chapter two). Since the tax rate is a constant for the entire city, a change in the assessment-sales ratio means a change in the effective tax rate. The assessment-sales ratio has other qualities relating to taxation:

$$6. P = T \times \sum_{i=1}^n a_i \quad \text{from 2}$$

$$\text{since } a = a/s \times s, \text{ then } \sum_{i=1}^n a_i = \frac{\sum_{i=1}^n a_i}{\sum_{i=1}^n s_i} \times \sum_{i=1}^n s_i$$

and so,

$$7. P = T \times \sum_{i=1}^n s_i \times \frac{\sum_{i=1}^n a_i}{\sum_{i=1}^n s_i}$$

which proves that the taxes paid is equal to the product of the tax rate, the market value, and the assessment-sales ratio. So,

$$\text{Assessed Value} = \frac{\text{Tax Rate}}{\text{Taxes Paid}}$$

and

$$\text{Market Value} = \frac{\text{Tax Rate}}{\text{Taxes Paid}} \times \frac{\text{Assessment}}{\text{Sales}} \text{ Ratio}$$

and this: the relationship of the property tax (taxes paid)

to the property value (both its real - market - and assessed).

The effects of revaluation upon this relationship will be examined in the next chapter.

CHAPTER FOUR
REVALUATION: A CONTROLLED EXPERIMENT

Hopefully, by showing the effects that occur from revaluation an understanding of the true relationship between the value and the tax of a property will be made. The significance of this relationship should also become apparent as this experiment proceeds. This experiment will be in two parts. First the entire revaluation process will be done theoretically and the mathematics of the theoretical results will be discussed. The second part will be several runs of a computer program (listed in the appendix) which will perform revaluation upon a model of a city which is presented in this chapter.

Revaluation is the process where the property tax base is changed. There are two cases of revaluation : one in which the tax base remains the same between the total property types for the city but is equalized at a certain percentage of market value for all areas within each property type. The second revaluation changes the tax base of the entire city so all property is assessed at a certain percentage of market value. This, in fact, redistributes the taxable property by increasing the "taxability" of some properties while decreasing the taxability of other properties.

In both revaluations a certain property will receive a new assessment-sales ratio. Since its market value is its

true value the property must change its assessment in order to meet the new ratio. After this occurs for the entire city the tax rate is computed and applied to the new assessments of all individual properties in order for the city to receive its property tax revenues. Therefore, that certain property will now have to pay increased or decreased taxes—depending upon whether its old assessment-sales ratio was lower or higher than its new assessment-sales ratio. A lower-old than higher-new assessment-sales ratio means increased property taxes for that property. This is because before revaluation the properties with the lower-old assessment-sales ratios were paying lower than their "fair" share of property taxes—"fairness" is based on market value. And so, because that certain property now has to pay increased (or decreased) property taxes then there is an added (or subtracted) expense. This will affect the market value in such a way that when the property taxes go up the market value will go down. Because of this new market value the revalued assessment does not match the assessment-sales ratio as it is supposed to do. So, if the market value decreases then the assessment must decrease as much in order to keep the revalued "fair" assessment-sales ratio constant. Because of this new decreased assessment the property tax which is based on it will also decrease. The decrease in taxes mean an increase in market value, thus an increase in assessment and another (increased)

property tax. This process will continue and hopefully reach an equilibrium point. What this describes is an iteration process where each full change (including the change in the market value, the assessment, the property tax) , in order to meet the requirements of the assessment-sales ratio, is called an iteration. The first iteration is the basic revaluation and more iterations result from it. A revaluation occurring over a number of years could have each year as an iteration, but this might have bad effects on the city as it will be discussed later. What follows is an examination of this process of revaluation.

The Theory

This section will discuss what exactly should happen for the revaluation of a city. The determination of the change in property tax payments for both types of revaluation will be made. The results will show that there is a significant difference between the two types of revaluation. Afterwards, the long-range effects of these changes on the market value will be determined as well. It will be shown that the procedure for determining this effect is the same for both revaluations and the difference between the two revaluations with regards to the effect the property tax changes have on market value will be zero.

In order to explain this properly the use of mathematical equations is in order. By using symbols instead of names or real numbers simplifies the explanation and hopefully simplifies the comprehension. The symbols used for this exercise and their meanings are:

n or m - number of properties in question

a_i - assessment of individual property i

A_{pr} - total assessment of property type p in region $r = \sum_{i=1}^n a_i$

A_T - total assessment of city = $\sum_{p=1}^n \sum_{r=1}^n A_{pr}$

s_i - market value of individual property i

S_{pr} - total market value of property type p in region $r = \sum_{i=1}^n s_i$

S_T - total market value of city = $\sum_{p=1}^n \sum_{r=1}^n S_{pr}$

- a/s_i - assessment-sales ratio of property i
 A/S_{pr} - assessment-sales ratio of property type p in region r (ASR#2)
 A/S_T - assessment-sales ratio of city = A_T/S_T (ASR#2)
 net_i - net income of property i
 cap_i - capitalization rate of property i = $C = net/s$
 tax_i - taxes paid on property i (Ntax is new taxes paid after revaluation)
 Tax_{pr} - taxes paid for property type p in region r (Ntax_{pr})
 Tax_T - taxes paid for city= total revenues (remains constant)
 R - tax rate for city (NR is new tax rate after revaluation)
 e_i -effective tax rate for property i
 E_{pr} - effective tax rate for property type p in region r
 E_T - effective tax rate for city ($E_T = R \times A/S_T$)

Assuming that the city receives special monies to perform a revaluation on its property and all other expenditures and revenues remain the same:

Total revenues from property taxes= Tax_T , a constant

The situation before any revaluation is:

1. $Tax = R \times A$
2. $Tax_T = R \times A_T$
3. $\therefore R = Tax_T/A_T$
 since $A_T = S_T \times A/S_T$, then
4. $Tax_T = R \times S_T \times A/S_T$
5. and $R = Tax_T/(S_T \times A/S_T)$
 $= Tax_T/S_T \times 1/(A/S_T)$
6. $E_T = Tax_T/S_T$

so from 4,

$$7. E_T = R \times A/S_T$$

$$\text{since Tax}_{pr} = R \times A_{pr}$$

$$\text{and tax}_i = R \times a_i \text{ from 1, then}$$

$$8. E_{pr} = R \times A/S_{pr} \text{ and}$$

$$9. e_i = R \times a/s_i$$

The first type of revaluation is within property types among all the regions of the city. In this case the effective tax rates of each property type for the entire city (E_{pr} for r =city total) stay the same from before revaluation to after revaluation. What this revaluation calls for is that within a single property type all individual properties in every region possess equal effective tax rates—equal to the effective tax rate of that total property type. Consequently the amount of taxes paid by the total property type also stays the same from before revaluation to after revaluation.

when $r=T$ the region is the city total (i.e. E_{pT})

Revaluation:

$$\text{make } E_{pr} \text{ for all } r \text{ in } p = E_{pT},$$

$$10. A/S_{pT} = A/S_{pr} \text{ for } r=1 \text{ to } n \text{ regions for property type } p$$

and it is known that

$$11. A_{pT} = \sum_{r=1}^n A_{pr}$$

so from 10 and 11 the new assessments are,

after revaluation,

$$12. \text{new } A_{pr} = A/S_{pT} \times S_{pr}$$

therefore, after revaluation

$$13. A_{pT} = A/S_{pT} \times \sum_{r=1}^n S_{pr}$$

and since

$$14. \text{new } A/S_{pT} = \text{old } A/S_{pT}$$

and new $S = \text{old } S$, then

$$15. \text{new } A_{pT} = \text{old } A_{pT}$$

$$16. A_T = \sum_{p=1}^m A_{pT} \text{ for all property types } p=1 \text{ to } m, \text{ then}$$

$$17. \text{new } A_T = \text{old } A_T$$

and since by definition of this exercise Tax_T

is constant, from 2 it is obvious that

$$18. NR = OR \text{ (new } R = \text{old } R)$$

therefore, using the new (after revaluation) figures

$$19. \text{Tax}_{pT} = A_{pT} \times R$$

$$20. \text{NTax}_{pT} = \text{OTax}_{pT}$$

$$21. \text{NTax}_{pr} = \text{new } A_{pr} \times R$$

$$\text{since old } A_{pr} = A/S_{pr} \times S_{pr}$$

$$\text{and new } A_{pr} = A/S_{pT} \times S_{pr}, \text{ then}$$

$$22. \text{new } A_{pr} = \text{old } A_{pr} \times (A/S_{pT}) / (A/S_{pr})$$

where A/S_{pr} is the old a/s ratio of property type

p and region r and since $\text{OTax}_{pr} = \text{old } A_{pr} \times R$, then

$$23. \text{NTax}_{pr} = \text{OTax}_{pr} \times (A/S_{pT}) / (A/S_{pr})$$

and, through equivalent means,

$$24. E_{pT} = \text{new } E_{pr} = \text{old } E_{pr} \times (A/S_{pT}) / (A/S_{pr})$$

This shows that by revaluing property within the property types there will be a change in the property

taxes for each area within each property type. The new total property tax for each area (within each property type and for total property types) is equal to the old property tax times the ratio of the total property type assessment-sales ratio over the regional property type assessment-sales ratio (the new a/s ratio divided by the old a/s ratio). This is what should be expected, since the assessment-sales ratio is extremely significant in determining property tax practices and instrumental in the process of revaluation. A most interesting figure is the change in the tax payments and the relationship of that to the change in the assessments.

$$25. \Delta \text{Tax}_{pr} = \text{NTax}_{pr} - \text{OTax}_{pr}$$

from 23 it is now

$$26. \Delta \text{Tax}_{pr} = \text{OTax}_{pr} \times (A/S_{pT}) / (A/S_{pr}) - \text{OTax}_{pr} \\ = \text{OTax}_{pr} \times ((A/S_{pT}) / (A/S_{pr}) - 1)$$

and from 21 and 22,

$$27. \Delta \text{Tax}_{pr} = R \times \text{old } A_{pr} ((A/S_{pT}) / (A/S_{pr}) - 1) \\ = R \times \Delta A_{pr}$$

$$28. \Delta A_{pr} = \text{new } A_{pr} - \text{old } A_{pr} = \text{old } A_{pr} ((A/S_{pT}) / (A/S_{pr}) - 1)$$

From this the percentage change of the taxes paid and of the assessments can be calculated.

$$29. \% \Delta \text{Tax}_{pr} = (\text{NTax}_{pr} - \text{OTax}_{pr}) / \text{OTax}_{pr} \\ = \Delta \text{Tax}_{pr} / \text{OTax}_{pr}$$

from 26 it can be written as

$$30. \% \Delta \text{Tax}_{pr} = (\text{OTax}_{pr}((A/S_{pT})/(A/S_{pr}) - 1))/\text{OTax}_{pr}$$

this can be easily simplified to get

$$31. \% \Delta \text{Tax}_{pr} = (A/S_{pT})/(A/S_{pr}) - 1$$

and for the assessments

$$32. \% \Delta A_{pr} = \Delta A_{pr}/\text{old } A_{pr}$$

and 28 gives a similar substitution

$$33. \% \Delta A_{pr} = (\text{old } A_{pr}((A/S_{pT})/(A/S_{pr}) - 1))/\text{old } A_{pr}$$

which gives a similar simplification

$$34. \% \Delta A_{pr} = (A/S_{pT})/(A/S_{pr}) - 1$$

Two results come from this. The first, which is not too surprising is that the percent change in the taxes paid is equal to the percent change in the assessments. It will be shown that this is true for any revaluation. The second result tells exactly what the percent change will be, regardless of the actual data and, as it will be shown next, regardless of the type of revaluation made. This percent change for revaluation is

$$35. \% \text{change} = (A/S_{\text{new}})/(A/S_{\text{old}}) - 1$$

The reason for the percentage change from old valuation to new (revalued) valuation is the above result given in equation 35 should be obvious. The percent change from old to new or from a to b has the definition, where $f(x)$ is any function of x and a is the "old" value x takes and b is the "new" value,

$$\% \Delta f(x) \frac{b}{a} = \frac{\Delta f(x) \frac{b}{a}}{f(a)} = \frac{f(b) - f(a)}{f(a)}$$

Easy simplification arrives at

$$\% \Delta f(x) \frac{b}{a} = \frac{f(b) - f(a)}{f(a)} = \frac{f(b)}{f(a)} - \frac{f(a)}{f(a)} = \frac{f(b)}{f(a)} - 1$$

which means that for any function the percent change from one point to another is the ratio of the functions at those points minus one. This percent change is the percent that $f(b)$ is greater (or less) than $f(a)$.

In the case where the functions are the taxes paid, assessed valuation, and assessment-sales ratio it has been shown that they are proportional to each other by the factors of the city tax rate and the market value, both of which remain constant during this revaluation. It is obvious from equations 3, 4, and 18 that

$$\frac{NTax}{OTax} = \frac{\text{new } A}{\text{old } A} = \frac{\text{new } a/s}{\text{old } a/s}$$

but this is the case where the tax rate is constant. The next type of revaluation has a change in the city tax rate.

The second type of revaluation is complete reassessment of all property so that the city has one assessment-sales ratio. Massachusetts law wants 100% valuation—assessment-sales ratio of one for the entire city. That is a special

case of full revaluation and this section will deal with the general case where the assessment-sales ratio for the entire city can take on any percentage.

Let Q = the new A/S_T for revaluation, then

$$A/S_T = A/S_{pr} = a/s_i = Q \text{ for all property.}$$

Since

$$36. A = S \times A/S$$

$$37. A_T = S_T \times A/S_T = S_T \times Q \text{ and } A_{pr} = S_{pr} \times Q ; a_i = s_i \times Q$$

therefore,

$$38. \text{Tax}_T = NR \times S_T \times Q$$

since Tax_T and S_T remain constant, combining 2 and 38,

$$39. NR \times Q = R \times A/S_T$$

therefore,

$$40. NR/R = \text{old } A/S_T / \text{new } A/S_T$$

which means that in the case where the tax rate is not constant and the total city taxes are constant then

$$\frac{\text{new } A_T}{\text{old } A_T} = \frac{\text{new } A/S_T}{\text{old } A/S_T} = \frac{R}{NR}$$

the tax rates are inversely proportional to the assessment-sales ratios. Now the examination will investigate the changes in the taxes paid by the property types and areas.

$$42. NR = R \times A/S_T / Q$$

$$43. N\text{Tax}_{pr} = NR \times \text{new } A_{pr}$$

$$44. \text{new } A_{pr} = Q \times S_{pr}$$

$$45. N\text{Tax}_{pr} = NR \times Q \times S_{pr}$$

$$= R \times A/S_T \times S_{pr}$$

which places the amount of taxes paid by a property type in terms of the tax rate, the assessment-sales ratio, and the market value of that property type p in region r .

$$46. E_{pr} = \text{NTax}_{pr} / S_{pr}$$

$$47. E_{pr} = NR \times Q = R \times A / S_T$$

Since NR , R , Q , and A/S_T are constant for all property types and regions (after revaluation) the effective tax rates (after revaluation) for all property are the same.

$$48. E_T = E_{pT} = E_{pr} = e_i$$

which is what should happen for revaluation for the entire city. If all property has the same assessment-sales ratio, then

$$49. E = R \times A/S : E_T = E_{pr} = e_i = NR \times Q$$

and since all the effective tax rates (which were different before revaluation) are now equal ($=E$), then

$$50. \text{NTax}_{pr} = E \times S_{pr}$$

What is E ? From 6 it is known that

$$51. E_T = \text{Tax}_T / S_T$$

which remains constant from before revaluation to after revaluation since Tax_T and S_T do not change. This means that

$$52. \text{NTax}_{pr} = \text{Tax}_T \times S_{pr} / S_T$$

The new taxes paid by a property type after revaluation will be equal to the total city taxes times the ratio of the market value of that type and region over the market value of the city. This is even more interesting when one notes that before revaluation

$$53. OTax_{pr} = Tax_T \times A_{pr}/A_T$$

the taxes paid were equal to the total city taxes times the ratio of the assessments. Total revaluation causes the property taxes to be based upon the market value rather than the assessed value regardless of the assessment-sales ratio chosen for the city. Complete city revaluation results in all properties having the same assessment-sales ratio and the same effective tax rate. The effects that follow are the same no matter what that assessment-sales ratio or effective tax rate is. Because the total revenues from property taxes remain constant the main tax rate will change as a proportion of the effective tax rate by the factor of the new assessment-sales ratio. In fact, the city's effective tax rate does not change.

$$54. E_T = Tax_T/S_T \\ = R \times A/S_T$$

$$56. E = NR \times Q = R \times A/S_T \\ = Tax_T/S_T = E_T$$

as was shown before in equations 48 - 51. But as shown in

equation 47,

$$57. E_{pr} = NR \times Q = R \times A/S_T$$

for revalued property and

$$58. E_{pr} = R \times A/S_{pr}$$

for property before revaluation. Each effective tax rate will equal the city's effective tax rate no matter what percentage revaluation occurs. Because of this, the impact and redistribution of tax burden will be the same if the revaluation is 100%, 75%, or even 33% across-the-board. Note that

$$\frac{\text{old } E_{pr}}{\text{new } E_{pr}} = \frac{\text{old } A/S_{pr}}{\text{new } A/S_{pr}} = \frac{\text{old } A_{pr}}{\text{new } A_{pr}} = \frac{\text{new } R (NR)}{\text{old } R (r)}$$

since new $E_{pr} = E$ and new $A/S_{pr} = Q$, which shows the ratio of the old assessment to the new assessment and the relationships therein.

The change in the taxes paid is

$$59. \Delta \text{Tax}_{pr} = N\text{Tax}_{pr} - O\text{Tax}_{pr}$$

by using 52 and 53

$$60. \Delta \text{Tax}_{pr} = \text{Tax}_T \times S_{pr}/S_T - \text{Tax}_T \times A_{pr}/A_T$$

$$= \text{Tax}_T \times (S_{pr}/S_T - A_{pr}/A_T)$$

but combining 52 and 53 to get

$$61. \text{Tax}_T = N\text{Tax}_{pr}/(S_{pr}/S_T) = O\text{Tax}_{pr}/(A_{pr}/A_T)$$

and

taking from equation 61, it is shown that

$$\begin{aligned}
 62. \text{NTax}_{\text{pr}} &= \text{OTax}_{\text{pr}} / (A_{\text{pr}}/A_{\text{T}}) \times S_{\text{pr}}/S_{\text{T}} \\
 &= \text{OTax}_{\text{pr}} \times (A_{\text{T}}/S_{\text{T}} \times S_{\text{pr}}/A_{\text{pr}}) \\
 &= \text{OTax}_{\text{pr}} \times ((A_{\text{T}}/S_{\text{T}})/(A_{\text{pr}}/S_{\text{pr}}))
 \end{aligned}$$

which gives a familiar result

$$\begin{aligned}
 63. \Delta \text{Tax}_{\text{pr}} &= \text{OTax}_{\text{pr}} \times ((A_{\text{T}}/S_{\text{T}})/(A_{\text{pr}}/S_{\text{pr}})) - \text{Otax}_{\text{pr}} \\
 &= \text{OTax}_{\text{pr}} \times ((A_{\text{T}}/S_{\text{T}})/(A_{\text{pr}}/S_{\text{pr}}) - 1)
 \end{aligned}$$

and the percent change is obviously

$$\begin{aligned}
 64. \% \Delta \text{Tax}_{\text{pr}} &= (A_{\text{T}}/S_{\text{T}})/(A_{\text{pr}}/S_{\text{pr}}) - 1 \\
 &= (A/S_{\text{T}})/(A/S_{\text{pr}}) - 1
 \end{aligned}$$

Interestingly enough, this does not say that the effects of the two types of revaluation will be the same. In both revaluations the a/s ratio A/S_{prT} is not the same as A/S_{T} . This is because in the first revaluation the base, the unchanging a/s ratio, is the a/s ratio for each property type - A/S_{prT} - while in the second revaluation the base is the city a/s ratio - A/S_{T} - and the property type a/s ratios - A/S_{prT} and A/S_{pr} - change to become A/S_{T} . So in each case the percent change is (as expected) symbollically equal.

$$65. \% \Delta \text{Tax}_{\text{pr}} = A/S_{\text{prT}}/A/S_{\text{pr}} - 1$$

where for revaluation one it is for each property type and for revaluation two the quantity A/S_{prT} is really A/S_{T} .

$$66. \% \Delta A_{\text{pr}} = \% \Delta \text{Tax}_{\text{pr}}$$

for both types of revaluations. So, equation 65 works for the assessments as well.

It has been shown how the payment of property taxes will change for property types within regions of a city. This is also true for the property tax payment by an individual property.

$$67. \Delta \text{tax}_i = \text{Ntax}_i - \text{Otax}_i$$

$$= \text{Otax}_i (\text{new } A/S_{pr} / \text{old } a/s_i - 1)$$

where for revaluation one: $\text{new } A/S_{pr} = A/S_{PT}$
 and for revaluation two $A/S_{pr} = A/S_T = Q$

and

$$68. \% \Delta \text{tax}_i = \text{new } A/S_{pr} / \text{old } a/s_i - 1$$

or since $\text{new } a/s_i = \text{new } A/S_{pr}$ then

$$69. \% \Delta \text{tax}_i = \text{new } a/s_i / \text{old } a/s_i - 1$$

Since the change in taxes are equivalent, using the term Δtax for the change in the tax payment is for individual properties and total property types and regions. The effect of revaluing the property tax upon the market value of property is easily illustrated by the capitalization of income method of determining market value. Since property taxes is an expense item on a property's operating statement (see figure two in chapter two) the change in property taxes (a net increase or a net decrease) would be the change in the net income assuming all other operations equal. The capitalization rate is based on quality, not quantity, as discussed in chapter two. Therefore, from the definition of

market value the change in market value resulting from the change in property taxes would be directly related.

70. market value = net income/cap rate

from the above definition

71. Δ tax = Δ net income

therefore,

72. Δ market value = Δ net income/cap rate
= Δ tax/cap rate

This means that after revaluation there would be a change in property taxes which in turn cause a change in market value and so, as a result of revaluation, property takes on a new market value.

73. new market value = old market value + Δ market value
(Δ market value can take on a positive or negative value)

Since there is a new market value, a new revaluation must be made based on this new valuation—since the assessment-sales ratio was changed and must be returned to "normal" to return the effective tax rates to "normal", because Tax = E X S. This new revaluation changes the assessments. The change in the assessments causes a change in payment of taxes again which will cause yet another change in the market value. This routine is what is commonly called an iteration process. Because property taxes have a direct effect on market value, revaluation will also have a direct effect

on market value. Since revaluation results in property taxes to be directly derived from market value (through a common assessment-sales ratio), a change in market value will cause a change in property taxes. Therefore, this iteration process is somewhat circular. First, property taxes are changed so that they are based on market value. This change in property taxes cause a change in the market value. Since property taxes cause this change, the change in market value will cause another change in taxes and so on. The short-term effects of this iteration process can be bad and the long-term effect could be some equilibrium, but that may also be bad if the short-term bad effects had an influence or permanent effect on the long-term process.

The question now to be asked is: will there be an equilibrium point? Does this iteration process converge? From equation 72 it is known that

$$74. \Delta s = \Delta t / c$$

where s is the market value, t is the taxes paid, and c is the capitalization rate of the property.

First, when revaluation begins,

$$75. \Delta t = N_t - O_t$$

$$76. N_t = NR \times a_{\text{new}} \\ = NR \times s \times a/s_{\text{new}}$$

$$O_t = R \times a_{\text{old}} \\ = R \times s \times a/s_{\text{old}}$$

and from 56 it is known that

$$77. E = NR \times a/s_{\text{new}} = R \times a/s_{\text{old}}$$

for the city totals and it is also known that the point of revaluation is to keep all properties at effective tax rate E , therefore since E and a/s and NR is constant after revaluation it is obvious that after the market value changes

$$78. Nt = E \times s_{\text{new}}$$

$$79. Ot = E \times s_{\text{old}}$$

and so

$$80. \Delta t = E \times \Delta s$$

$$(\text{remember equation 74. } \Delta s = \Delta t/c)$$

This does not necessarily mean that $E=c$ because equations 74 and 80 are not immediately compatible since Δt and Δs are different for both equations. In fact, if the capitalization rate for a property did equal the effective tax rate then the process would not converge. To prove this and the process itself assume that the revaluation (first iteration) occurred and the change in the property tax is Δt_1 , then

$$81. \Delta s_1 = \Delta t_1/c$$

$$82. \Delta t_2 = \Delta s_1 \times E \quad \text{or} \quad \Delta t_2 = \Delta t_1 \times E/c$$

$$83. \Delta s_2 = \Delta t_2/c \quad \text{or} \quad \Delta s_2 = \Delta t_2/c$$

$$= \Delta s_1 \times E/c \quad = \Delta t_1 \times E/c^2$$

$$84. \Delta t_3 = \Delta s_2 \times E \quad \text{or} \quad \Delta t_3 = \Delta s_2 \times E$$

$$= \Delta s_1 \times E^2/c \quad = \Delta t_1 \times E^2/c^2$$

$$85. \Delta s_3 = \Delta s_1 \times E^2/c^2 \quad \text{or} \quad \Delta s_3 = \Delta t_1 \times E^2/c^3$$

which means that, continuing to n,

$$86. \Delta s_n = \Delta s_1 \times E^{n-1}/c^{n-1}$$

and

$$87. \Delta t_n = \Delta t_1 \times E^{n-1}/c^{n-1}$$

where $\Delta s_1, \Delta s_2, \Delta s_3, \dots, \Delta s_n$ are the n changes in the market value

and $\Delta t_1, \Delta t_2, \Delta t_3, \dots, \Delta t_n$ are the n changes in the property tax.⁶

A. If $E=c$ then $\Delta s_n = \Delta s_1$ and $\Delta t_n = \Delta t_1$ and the process would never converge, the change would always stay the same.

B. If $E > c$ then the process would diverge to the point where $\Delta s_n \rightarrow \infty$ and $\Delta t_n \rightarrow \infty$ as $(E/c)^{n-1}$ becomes larger.

C. If $E < c$ then the process would converge to the point where $\Delta s_n \rightarrow 0$ and $\Delta t_n \rightarrow 0$ as $(E/c)^{n-1}$ becomes smaller.

Assuming no other revenues are needed from property taxes the effective tax rate must be smaller than the capitalization rate in order for the entire revaluation iteration process to converge to an equilibrium point. This is usually the case since the effective tax rate is, as the cap rate is, a measure of return. It is probable that the return the city expects to receive should be at a lower rate than the return the investor expects to receive on a certain property.

⁶The changes in these equations are magnitudes only.

Other questions about this process are now relevant considering the above result. How large does n have to be — how many iterations must go on — to reach convergence, the equilibrium point? That depends upon the value of the fraction E/c which would be then multiplied by itself $n-1$ times. It is true that n must be infinity for $(E/c)^{n-1}$ to be zero. For the purposes of a city doing the revaluation process there is no need to go beyond the point where the change (Δt and Δs) is less than one cent ($<.01$). For this, the magnitude of n , depends upon the magnitude of e/c , but it is doubtful that n will be greater than 460, since that is what n is for $E/c = .99$, for $E/c = .75$, $n= 17$; $E/c=.25$, $n=4$. This shows that equilibrium should be arrived at quickly, especially if the process is done on a computer. This will be examined further in the model.

Another question which arises from this is: why not use the values at the equilibrium point immediately as the solution? The answer to that is: this equilibrium point has values which, when placed immediately at revaluation, call for assessing property at different percentages of market value, some at percentages higher than 100%, which is usually illegal.

This section dealt with revaluation on a theoretical level. The results presented here should prove to be true when tested with actual data.

The Model

Before performing the actual experiments the basic laboratory model must be presented. This section will describe the model to be used and the current situation before any revaluation occurs. In order to see the actual effects of revaluation a comparison with the current situation will be done for each revaluation.

All of the tables dealing with this model and revaluation are in one section at the back of this chapter starting on page 74 for easy access and manipulation. The tables are grouped as follows: One - City Totals by property types; Two - Residential Property by wards; Three - Non-Residential Property by wards; Four - Iteration Process for residential and non-residential. Each group has three different tables (except for group four which has only the pro-revaluation tables): C - Current; R1 - Property Type Revaluation; R2 - Full Revaluation.

The part of the city which will be examined in this model is the taxable real property and the revenues received from that. From here on, the model will be referred to as the city, with the note that only taxable real property is considered. The total revenue needs will not change—this is a basic control in the experiment itself—and the tax rate is determined by dividing the total revenues by the total assessment of all real taxable property in the city. Table 1-C describes the city in its current

situation where the revenues received from property taxes is 250 million dollars. Although the regular tax rate is .209, the effective tax rate is .065. Residential property has less of a tax burden than non-residential property. New property is property which has been constructed after 1960. It is interesting to see that this new property has a lighter tax burden than old property—property existing prior to 1960. This is mainly due to tax benefits the city gives in order to promote new development. Although residential property has a total market value which is 55% of the city total market value, it only consists of 50% of the city tax base. Tables 2-C and 3-C describe the distribution of these figures for residential and non-residential property over different regions of the city. Boston has twenty-two political regions used for property taxation which are called wards. This model also has 22 wards. It is interesting to see the differences between wards. Note that ward 15 has the lowest assessment-sales ratio for residential property. This means it will be the hardest hit by revaluation—regardless of the type of revaluation occurring, since this ward has the lowest tax burden of the entire city. For the revaluation within property types all the wards in the residential property type will be equalized with the residential total. This means that those wards with assessment-sales ratios lower than .28 -

the assessment-sales ratio for residential property, will receive an increase in taxation, while those with higher assessment-sales ratios could expect a decrease due to revaluation. For full revaluation the changes will be quite different. All properties with a lower assessment-sales ratio than that for the city total should expect an increase in property taxes.

In order to go through this in a detailed fashion, describing the expected changes, revaluation must now be examined.

The Experiment

Two separate revaluations will occur upon the model presented above. What should be observed is a change in tax burden between property types and areas of the city. The results from revaluation will be directly compared to the respective values of the basic pre-valuation (current) model. A process which should also occur because of this revaluation is the iteration of the impact of the change in taxes upon the change in market value.

The first type of revaluation is best described as property type revaluation where the differences that lie between property types for the total city remain the same. Since the relative tax burdens remain the same, then the effective tax rates of the property types also remain the same. The effective tax rates could be changed but the relative differences between them must stay the same for this revaluation. The assessment-sales ratios stay the same between the property types (or the relationships stay the same if one wishes to change the a/s ratios).

Table 1-R1 shows the city under property type revaluation. There is no difference between this table and table 1-C. The revenue need remains constant, as part of the control. However, the tax rate also does not change since the total assessment did not change. The basic property tax burdens between total property types remained the same.

The change which occurs through property type revaluation is within each property type. Tables 2-R1 and 3-R1 show the change in the distribution of the tax burden between the wards for residential and non-residential property.

It is ward 12 which exhibits the greatest decrease in tax burden for residential property—a decrease of 41.5%, while ward 15 has the greatest increase in tax burden—40.1%. It is interesting to examine the relative changes for new and old residential property. Only eight wards have a total increase in property taxes and six of those wards show new property having to carry a greater increase than old. Two wards, 7 and 16, have a total decrease in tax burden with the new property increasing in the tax burden, while ward 1 also decreases in taxation, but it is the old property which increases in tax burden. Ward 15, which has the greatest increase in tax burden, also has its new property decreasing in tax burden.

The change for non-residential property is quite different. Ward 16 has the greatest increase in taxes—87.5%, more than twice the percent increase for the greatest ward in residential property. Ward 22 has the greatest increase in tax burden for new property—108.6%, but new property makes up only twenty percent of the total assessment of that ward. The increases in tax burden seem to be much greater than those for residential property.

On the average, non-residential property exhibits either a tax increase of 42.3% or a tax decrease of 26.1%, while residential property has an average increase of 19.9% and an average decrease of 19.0% for property tax burden. It is clear that property type revaluation affects the non-residential property more significantly than residential property within a ward.

Full revaluation changes the city's taxation distribution much differently. Table 1-R2 shows the change in the city after undergoing full revaluation. The table displayed has the new assessment-sales ratio at 100% of market value. At any other assessment-sales ratio the tax changes are exactly the same. This was examined by changing the new assessment-sales ratio for full revaluation. With a different ratio the tax rate and the assessments change, but the amount of taxes paid and the change in the tax burden remain the same. Table 1-R2 shows that with 100% valuation the tax rate is 6.5%, but figure four shows that with 33% valuation the tax rate is 19.8%. As the tax rate increases the total assessment decreases. The effective tax rate of the entire city remains the same - 6.5%.

The total city tax burden did not change, since the total revenues collected remained constant. New property received an increased tax burden of 45% while old property enjoyed a decrease of 10%. The largest increase in the property taxes to be paid belongs to new office space. Oddly enough, the greatest decrease in the tax burden belongs

Tax Rates Under Full Revaluation

Total Revenues= \$250,000,000

Total Market Value= \$3,820,180,000

Effective Tax Rate= 6.5%

Assessment-Sales Ratio	Tax Rate
1.00	.0654419
.75	.0872559
.50	.1308838
.33	.1983089
.25	.2617677

FIGURE FOUR

to old office space, thus giving total office space a decrease of 10%—equal to the average change of total non-residential property. Total residential property will increase by 10% while non-residential property enjoy a decrease of 10%. However, new residential property increases less than new non-residential property and it is the old non-residential property which receives the decrease while old residential property still has an increase in tax burden.

Full revaluation also causes changes to occur in the wards within property types. Tables 2-R2 and 3-R2 illustrate this case. At first glance one would think that the changes in the wards are quite different from that for property type revaluation. Careful examination will show that the shifts in the tax burden for each ward is relatively the same for both property type revaluation and full revaluation. In other words, the shift of the residential wards in full revaluation is 10% greater because of the total 10% shift of residential property due to full revaluation, while property type revaluation had zero total shifts. This is also true for non-residential property in reverse.

The effective tax rate, which also remained constant with the total revenues and total market value, is .065 - equal to the regular tax rate under 100% valuation. This means that the city is asking for a 6.5% return on its value in the market. This is a very important consideration

when examining the iteration process which occurs after revaluation. Since the changes are the same no matter what the percentage of market value is used for full revaluation the iteration process will also be the same for any ratio used in full revaluation. Table four shows a series of iterations for residential and non-residential property.

In the case of residential property the process took 35 iterations before the total value would stabilize completely (to the .01 precision). Non-residential property reflects this approximately. New property took longer to stabilize since it had the lowest capitalization rate and the process is dependent upon the fraction

$$\frac{\text{Effective tax rate}}{\text{Capitalization rate}}$$

while old property had less iterations.

The end result of the process is the total market value staying the same, but a shift between property types and between new and old property occurring. Residential property ends with a market value 3.5% less than its original market value, while non-residential property increased its market value by 3.5%. New residential property decreased its market value by 10% and new non-residential property had a decrease of 14% in market value. Old property increased in market value. This change in market value as a result of the iteration process is the exact change in tax burden, since taxes are based directly on market value from revaluation.

The end values are called the equilibrium values. While it is possible to assess residential property at 96% of market value it would be quite impossible to assess non-residential property at 104% of its market value. A duplication of this that might be more possible is a "freeze" of the relationships between residential and non-residential property, which is what is done with property type revaluation. With property types revaluation there is no iteration for property types, but there are changes in the wards and consequently iteration processes. It is clear that whenever there is a change in tax burden a change in market value will occur and the process of iteration will also occur. The only way to avoid this is to assess property at percentages below and above 100% market value or to keep the situation as it stands before any revaluation.

The Results

Shifting of the tax burden is shown on an aggregate scale. Although a ward may show a total decrease in tax burden it is very likely that there are properties in that same ward which receive tax increases as a result of revaluation. This is quite easily understood when viewing the property-type revaluation on a different level. While the property type does not change its tax burden the wards within it does, so as a ward changes one way the individual properties within that ward may change differently and give an aggregate result for the ward.

The greatest increase in tax burden, along property types, falls upon residential property under full revaluation. Along different wards, those wards which received an increase from property-type revaluation received an additional increase of 10% if residential and enjoyed a 10% decrease if the property was non-residential, when full revaluation occurred.

Another tax burden increase came upon new property. New office space, which received tax concessions usually found itself with the greatest increase of property taxes. The increased costs due to higher taxes may cause a slowing of new development.

If the landlord is allowed to pass the tax increases on to the tenant, then it is the tenant, residential and non-residential, who will be affected greatly by either

revaluation. Usually the tax is an expense item on the operating statement of a property, so it is passed on to the tenant—especially the residential tenant who has short-term leases.

Equation 65 in this chapter said that

$$\% \Delta \text{taxes}_{pr} = (A/S_{pT}) / (A/S_{pr}) - 1$$

is the expected change in tax burden (or assessments) for any property type p and ward (region) r . From table 1-C it is found that residential property has an assessment-sales ratio of .284 and full revaluation would bring it to an equilibrium point with the assessment-sales ratio for all property - .313. So it can be expected that

$$\% \Delta \text{taxes} = (.313) / (.284) - 1 = 1.10 - 1 = .10$$

which is 10%. Checking table 1-R2 it is found that the percent shift in taxes for residential property is indeed 10%, agreeing with the theoretical result. It will be found that the theoretical results agree with the empirical results for both revaluations.

The iteration results also agree with the theoretical results. There are two ways to avoid the iteration process. The first way is to revalue the property at the equilibrium values of the iteration process which would be very unlikely to do and the second being the situation as it stands keeping the inequities that caused the want of revaluation in the first place. One method which might ease the impact

of revaluation is a gradual revaluation, not going to a complete finish immediately. The changes might not be as significant this way.

Through this experiment it has been shown that there is a connection between the property tax and the property value which has great impact upon policy decisions affecting the urban environment. This connection is true no matter what the situation is and it is a single formula which proves true for any model.

Table 1-C

the status of the city before revaluation

total revenues= 2.5 E+8

tax rate= 0.208787

proptype	mkt val	assedval	a/sratio	taxes pd
single res	7.0116 E+8	1.72898 E+8	0.246589	36098900
new	59790000	12555900	0.21	2621500
old	6.4137 E+8	1.60343 E+8	0.25	33477400
multi res	1.39479 E+9	4.22493 E+8	0.302908	88210928
new	1.6486 E+8	41215000	0.25	8605140
old	1.22993 E+9	3.81278 E+8	0.31	79605789
total res	2.09595 E+9	5.95392 E+8	0.284068	124309793
new	2.2465 E+8	53770900	0.239354	11226642
old	1.8713 E+9	5.41621 E+8	0.289436	113083150
total non-re	1.72423 E+9	6.02003 E+8	0.349143	125690206
new	7.7436 E+8	1.62218 E+8	0.209487	33868982
old	9.4987 E+8	4.39785 E+8	0.462995	91821223
office	5.7787 E+8	2.01668 E+8	0.348985	42105500
new	3.1698 E+8	63396000	0.2	13236200
old	2.6089 E+8	1.38272 E+8	0.53	28869300
retail	5.8404 E+8	1.73204 E+8	0.296563	36162753
new	3.7339 E+8	78411900	0.21	16371352
old	2.1065 E+8	94792499	0.45	19791401
hotel-motel	97060000	31408600	0.3236	6557694
new	41930000	8805300	0.21	1838430
old	55130000	22603300	0.41	4719270
industrial	2.396 E+8	107785000	0.449854	22504100
new	42060000	10515000	0.25	2195390
old	1.9454 E+8	97270000	0.5	20308670
utilities	21600000	7128000	0.33	1488230
new	1000000	330000	0.33	68899.6
old	20600000	6798000	0.33	1419330
vacant land	1.567 E+8	62680000	0.4	13086700
new	1000000	400000	0.4	83514.6
old	1.557 E+8	62280000	0.4	13003200
other	50360000	18129600	0.36	3785220
new	1000000	360000	0.36	75163.2
old	49360000	17769600	0.36	3710050
all property	3.82018 E+9	1.19739 E+9	0.313439	2.5 E+8
new	9.9901 E+8	2.15989 E+8	0.216203	45095624
old	2.82117 E+9	9.81406 E+8	0.347872	2.04904 E+8

Table 1-R1

the status of the city after proptype revaluation

total revenues= 2.5 E+8

tax rate= 0.208787

proptype	assedval	a/sratio	taxes pd	%shift tp
single res	1.72898 E+8	0.246589	36098900	0
new	12555900	0.21	2621500	0
old	1.60343 E+8	0.25	33477400	0
multi res	4.22493 E+8	0.302908	88210929	1.13365 E-6
new	41215000	0.25	8605140	0
old	3.81278 E+8	0.31	79605789	0
total res	5.95392 E+8	0.284068	124309793	0
new	53770900	0.239354	11226642	0
old	5.41621 E+8	0.289436	113083150	0
total non-re	6.02003 E+8	0.349143	125690206	0
new	1.62218 E+8	0.209487	33868982	0
old	4.39785 E+8	0.462995	91821223	0
office	2.01668 E+8	0.348985	42105500	0
new	63396000	0.2	13236200	0
old	1.38272 E+8	0.53	28869300	0
retail	1.73204 E+8	0.296563	36162753	0
new	78411900	0.21	16371352	0
old	94792499	0.45	19791401	0
hotel-motel	31408600	0.3236	6557694	0
new	8805300	0.21	1838430	0
old	22603300	0.41	4719270	0
industrial	107785000	0.449854	22504100	0
new	10515000	0.25	2195390	0
old	97270000	0.5	20308670	0
utilities	7128000	0.33	1488230	0
new	330000	0.33	68899.6	0
old	6798000	0.33	1419330	0
vacant land	62680000	0.4	13086700	0
new	400000	0.4	83514.6	0
old	62280000	0.4	13003200	0
other	18129600	0.36	3785220	0
new	360000	0.36	75163.2	0
old	17769600	0.36	3710050	0
all property	1.19739 E+9	0.313439	2.5 E+8	0
new	2.15989 E+8	0.216203	45095624	0
old	9.81406 E+8	0.347872	2.04904 E+8	0

the status of the city after full revaluation **Table 1-R2**

total revenues= 2.5 E+8

tax rate= 6.54419 E-2

proptype	assedval	a/sratio	taxes pd	%shift tp
single res	7.0116 E+8	1	45885300	27.11
new	59790000	1	3912770	49.2569
old	6.4137 E+8	1	41972500	25.3758
multi res	1.39479 E+9	1	91277766	3.47671
new	1.6486 E+8	1	10788800	25.3758
old	1.22993 E+9	1	80489007	1.10949
total res	2.09595 E+9	1	1.37163 E+8	10.3397
new	2.2465 E+8	1	14701500	30.9522
old	1.8713 E+9	1	122461506	8.29333
total non-re	1.72423 E+9	1	112836959	-10.2261
new	7.7436 E+8	1	50675623	49.6225
old	9.4987 E+8	1	62161300	-32.3018
office	5.7787 E+8	1	37816935	-10.1853
new	3.1698 E+8	1	20743800	56.7197
old	2.6089 E+8	1	17073100	-40.8605
retail	5.8404 E+8	1	38220712	5.69083
new	3.7339 E+8	1	24435400	49.2569
old	2.1065 E+8	1	13785345	-30.3468
hotel-motel	97060000	1	6351790	-3.13981
new	41930000	1	2743980	49.2569
old	55130000	1	3607810	-23.5514
industrial	2.396 E+8	1	15679900	-30.3242
new	42060000	1	2752490	25.3758
old	1.9454 E+8	1	12731100	-37.3121
utilities	21600000	1	1413550	-5.01836
new	1000000	1	65441.9	-5.01836
old	20600000	1	1348104	-5.01836
vacant land	1.567 E+8	1	10254800	-21.6401
new	1000000	1	65441.9	-21.6401
old	1.557 E+8	1	10189300	-21.6401
other	50360000	1	3295660	-12.9335
new	1000000	1	65441.9	-12.9335
old	49360000	1	3230210	-12.9335
all property	3.82018 E+9	1	2.5 E+8	0
new	9.9901 E+8	1	65377155	44.9745
old	2.82117 E+9	1	1.84623 E+8	-9.89805

Table 2-C

the status of the city before revaluation

total revenues= 2.5 E+8

tax rate= 0.208787

proptype	mkt val	assedval	a/sratio	taxes pd
1				
total res	81529999	23487800	0.288088	4903942
new	700000	175248	0.250354	36589.4
old	80829999	23312600	0.288415	4867350
2				
total res	31990000	7592100	0.237327	1585130
new	180000	34705.8	0.19281	7246.11
old	31810000	7557390	0.237579	1577880
3				
total res	125720000	39330803	0.312844	8211740
new	79720000	22341300	0.280247	4664560
old	46000000	16989500	0.369337	3547180
4				
total res	94310000	36619800	0.388291	7645710
new	1000000	317613	0.317613	66313.4
old	93310000	36302148	0.389049	7579400
5				
total res	2.3045 E+8	51605400	0.223933	10774505
new	7810000	1482500	0.189821	309526
old	2.2264 E+8	50122854	0.22513	10464979
6				
total res	33090000	9953730	0.300808	2078200
new	1160000	280875	0.242134	58642.9
old	31930000	9672850	0.302939	2019560
7				
total res	45210000	14072300	0.311266	2938110
new	1200000	283386	0.236155	59167.2
old	44010000	13788900	0.313314	2878950
8				
total res	20850000	8995250	0.431427	1878090
new	2710000	1109840	0.409534	231719
old	18140000	7885410	0.434697	1646370
9				
total res	18380000	7794171	0.424057	1627320
new	4110000	1591040	0.387115	332188
old	14270000	6203130	0.434697	1295130
10				
total res	30870000	10005600	0.324122	2089040
new	1510000	410760	0.272027	85761.2
old	29360000	9594880	0.326801	2003280

proptype	mkt val	assedval	a/sratio	taxes pd
11				
total res	51330000	18864300	0.367509	3938600
new	3160000	923366	0.292204	192786
old	48170000	17940889	0.372449	3745820
12				
total res	49850000	24222800	0.485913	5057390
new	3690000	1475330	0.399819	308030
old	46160000	22747400	0.492795	4749360
13				
total res	37350000	12496192	0.33457	2609040
new	1640000	492697	0.300425	102868
old	35710000	12003500	0.336138	2506170
14				
total res	75050000	31790690	0.423593	6637470
new	2880000	942706	0.327329	196824
old	72170000	30848000	0.427435	6440640
15				
total res	42530000	8621340	0.202712	1800020
new	3580000	1064820	0.297436	222320
old	38950000	7556520	0.194006	1577700
16				
total res	94329999	28366100	0.300712	5922470
new	4800000	1058210	0.220461	220941
old	89529999	27307900	0.305014	5701530
17				
total res	90449999	31325200	0.346326	6540290
new	3570000	949791	0.266048	198304
old	86880000	30375440	0.349625	6341980
18				
total res	2.9631 E+8	69943527	0.236048	14603300
new	54270000	10179900	0.187579	2125430
old	2.4204 E+8	59763629	0.246916	12477800
19				
total res	105359999	29683800	0.281737	6197580
new	2730000	616140	0.225692	128642
old	102630000	29067600	0.283228	6068930
20				
total res	2.8751 E+8	66942677	0.232836	13976700
new	15750000	2660110	0.168896	555394
old	2.7176 E+8	64282600	0.236542	13421300

proptype	mkt val	assedval	a/sratio	taxes pd
21				
total res	1.3823 E+8	37219400	0.269257	7770910
new	15920000	3212300	0.201778	670686
old	122309999	34007102	0.27804	7100230
22				
total res	115250000	26458700	0.229577	5524230
new	12560000	2168260	0.172632	452704
old	102690000	24290467	0.236542	5071520
23*				
total res	2.09595 E+9	5.95392 E+8	0.284068	124309793
new	2.2465 E+8	53770900	0.239354	11226642
old	1.8713 E+9	5.41621 E+8	0.289436	113083150

***23 is the city total for residential property**

Table 2-R1

the status of the city after proptype revaluation

total revenues= 2.5 E+8

tax rate= 0.208787

proptype	assedval	a/sratio	taxes pd	%shift tp
1				
total res	23160000	0.284068	4835510	-1.39554
new	167548	0.239354	34981.7	-4.39377
old	23395100	0.289436	4884578	0.353897
2				
total res	9087330	0.284068	1897310	19.6945
new	43083.7	0.239354	8995.31	24.1399
old	9206950	0.289436	1922290	21.827
3				
total res	35712992	0.284068	7456390	-9.19842
new	19081300	0.239354	3983920	-14.5918
old	13314000	0.289436	2779790	-21.6338
4				
total res	26790400	0.284068	5593480	-26.8416
new	239354	0.239354	49973.9	-24.6398
old	27007200	0.289436	5638750	-25.6043
5				
total res	65463400	0.284068	13667900	26.8539
new	1869360	0.239354	390296	26.0948
old	64439937	0.289436	13454200	28.564
6				
total res	9399800	0.284068	1962550	-5.56503
new	277651	0.239354	57969.8	-1.14788
old	9241678	0.289436	1929540	-4.45759
7				
total res	12842701	0.284068	2681380	-8.73795
new	287225	0.239354	59968.7	1.35471
old	12738100	0.289436	2659540	-7.62125
8				
total res	5922810	0.284068	1236600	-34.1562
new	648650	0.239354	135429	-41.5546
old	5250360	0.289436	1096200	-33.4167
9				
total res	5221160	0.284068	1090110	-33.0119
new	983745	0.239354	205393	-38.1697
old	4130250	0.289436	862340	-33.4167
10				
total res	8769170	0.284068	1830880	-12.3577
new	361425	0.239354	75460.6	-12.0107
old	8497830	0.289436	1774230	-11.4337

proptype	assedval	a/sratio	taxes pd	%shift tp
11				
total res	14581200	0.284068	3044360	-22.7046
new	756359	0.239354	157918	-18.0867
old	13942100	0.289436	2910930	-22.2886
12				
total res	14160775	0.284068	2956580	-41.5394
new	883217	0.239354	184404	-40.1344
old	13360300	0.289436	2789460	-41.2666
13				
total res	10609900	0.284068	2215210	-15.0947
new	392541	0.239354	81957.2	-20.3281
old	10335700	0.289436	2157960	-13.8939
14				
total res	21319300	0.284068	4451180	-32.9386
new	689340	0.239354	143925	-26.8765
old	20888600	0.289436	4361252	-32.2855
15				
total res	12081400	0.284068	2522434	40.1336
new	856888	0.239354	178907	-19.5274
old	11273500	0.289436	2353760	49.1892
16				
total res	26796100	0.284068	5594670	-5.53489
new	1148900	0.239354	239875	8.56979
old	25913167	0.289436	5410320	-5.10754
17				
total res	25693900	0.284068	5364550	-17.9769
new	854494	0.239354	178407	-10.0335
old	25146163	0.289436	5250180	-17.2155
18				
total res	84172102	0.284068	17574000	20.3429
new	12989700	0.239354	2712080	27.6019
old	70054987	0.289436	14626500	17.2201
19				
total res	29929400	0.284068	6248850	0.827351
new	653437	0.239354	136429	6.05327
old	29704800	0.289436	6201960	2.19188
20				
total res	81672305	0.284068	17052100	22.0033
new	3769830	0.239354	787089	41.7172
old	78657011	0.289436	16422500	22.3613

proptype	assedval	a/sratio	taxes pd	%shift tp
21				
total res	39266700	0.284068	8198360	5.50055
new	3810520	0.239354	795585	18.6225
old	35400865	0.289436	7391230	4.09845
22				
total res	32738803	0.284068	6835420	23.7354
new	3006290	0.239354	627672	38.6497
old	29722100	0.289436	6205580	22.3613
23				
total res	5.95392 E+8	0.284068	124309793	0
new	53770900	0.239354	11226642	0
old	5.41621 E+8	0.289436	113083150	0

Table 2-R2

the status of the city after full revaluation

total revenues= 2.5 E+8

tax rate= 6.54419 E-2

proptype	assedval	a/sratio	taxes pd	%shift tp
1				
total res	81529999	1	5335480	8.79985
new	700000	1	45809.4	25.1984
old	80829999	1	5289670	8.67658
2				
total res	31990000	1	2093490	32.0706
new	180000	1	11779.5	62.5639
old	31810000	1	2081710	31.9305
3				
total res	125720000	1	8227361	0.190183
new	79720000	1	5217030	11.8439
old	46000000	1	3010330	-15.1346
4				
total res	94310000	1	6171830	-19.2773
new	1000000	1	65441.9	-1.31417
old	93310000	1	6106390	-19.4344
5				
total res	2.3045 E+8	1	15081100	39.9702
new	7810000	1	511102	65.1239
old	2.2264 E+8	1	14570000	39.2262
6				
total res	33090000	1	2165470	4.19925
new	1160000	1	75912.7	29.449
old	31930000	1	2089560	3.46606
7				
total res	45210000	1	2958630	0.698256
new	1200000	1	78530.3	32.7262
old	44010000	1	2880100	4.00276 E-2
8				
total res	20850000	1	1364460	-27.3482
new	2710000	1	177348	-23.4645
old	18140000	1	1187120	-27.8948
9				
total res	18380000	1	1202820	-26.0856
new	4110000	1	268966	-19.0319
old	14270000	1	933857	-27.8948
10				
total res	30870000	1	2020190	-3.29578
new	1510000	1	98817.3	15.2238
old	29360000	1	1921380	-4.08861

proptype	assedval	a/sratio	taxes pd	%shift tp
11				
total res	51330000	1	3359130	-14.7125
new	3160000	1	206797	7.26721
old	48170000	1	3152340	-15.8438
12				
total res	49850000	1	3262280	-35.4947
new	3690000	1	241481	-21.6047
old	46160000	1	3020800	-36.3956
13				
total res	37350000	1	2444260	-6.31576
new	1640000	1	107325	4.33204
old	35710000	1	2336930	-6.75281
14				
total res	75050000	1	4911420	-26.0047
new	2880000	1	188473	-4.2432
old	72170000	1	4722945	-26.6697
15				
total res	42530000	1	2783250	54.623
new	3580000	1	234282	5.3806
old	38950000	1	2548960	61.562
16				
total res	94329999	1	6173140	4.2325
new	4800000	1	314121	42.1745
old	89529999	1	5859020	2.76221
17				
total res	90449999	1	5919220	-9.49597
new	3570000	1	233628	17.8131
old	86880000	1	5685600	-10.3499
18				
total res	2.9631 E+8	1	19391102	32.786
new	54270000	1	3551530	67.0975
old	2.4204 E+8	1	15839600	26.9415
19				
total res	105359999	1	6894963	11.2526
new	2730000	1	178657	38.8791
old	102630000	1	6716310	10.667
20				
total res	2.8751 E+8	1	18815200	34.6181
new	15750000	1	1030710	85.5818
old	2.7176 E+8	1	17784502	32.5092

proptype	assedval	a/sratio	taxes pd	%shift tp
21				
total res	1.3823 E+8	1	9046040	16.409
new	15920000	1	1041840	55.3388
old	122309999	1	8004200	12.7317
22				
total res	115250000	1	7542180	36.5292
new	12560000	1	821951	81.5648
old	102690000	1	6720230	32.5092
23				
total res	2.09595 E+9	1	1.37163 E+8	10.3397
new	2.2465 E+8	1	14701500	30.9522
old	1.8713 E+9	1	122461506	8.29333

the status of the city before revaluation

Table 3-C

total revenues= 2.5 E+8

tax rate= 0.208787

proptype	mkt val	assedval	a/sratio	taxes pd
1				
total non-re	76359999	16259000	0.212926	3394670
new	25100000	3982140	0.158651	831417
old	51260000	12276906	0.239503	2563250
2				
total non-re	22830000	9639060	0.42221	2012510
new	2320000	444425	0.191563	92790
old	20510000	9194630	0.4483	1919720
3				
total non-re	2.2353 E+8	65881200	0.294731	13755100
new	1.5131 E+8	34539800	0.228272	7211450
old	72220001	31341400	0.433971	6543650
4				
total non-re	1.6232 E+8	76320650	0.470186	15934727
new	89600000	25708200	0.286922	5367530
old	72719999	50612438	0.695991	10567200
5				
total non-re	2.3316 E+8	64346500	0.275976	13434700
new	1.8296 E+8	41841800	0.228694	8736010
old	50200000	22504700	0.4483	4698670
6				
total non-re	22320000	13204036	0.591579	2756830
new	1080000	225571	0.208862	47096.3
old	21240000	12978500	0.611039	2709730
7				
total non-re	23450000	5455070	0.232625	1138940
new	1620000	204381	0.126161	42672.1
old	21830000	5250690	0.240526	1096270
8				
total non-re	15670000	7013500	0.447575	1464320
new	3910000	1392430	0.356121	290721
old	11760000	5621070	0.477982	1173600
9				
total non-re	16920000	4002980	0.236583	835768
new	4130000	651743	0.157807	136075
old	12790000	3351240	0.26202	699693
10				
total non-re	21130000	13544000	0.640984	2827800
new	1630000	451176	0.276795	94199.6
old	19500000	13092812	0.671426	2733600

proptype	mkt val	assedval	a/sratio	taxes pd
11				
total non-re	31280000	11672800	0.373172	2437129
new	2920000	613574	0.210128	128106
old	28360000	11059250	0.389959	2309020
12				
total non-re	43310000	26861600	0.620217	5608340
new	3110000	898889	0.289032	187676
old	40200000	25962700	0.645838	5420660
13				
total non-re	35420000	13551300	0.38259	2829340
new	8520000	1657280	0.194516	346017
old	26900000	11894100	0.442159	2483320
14				
total non-re	65730000	34368900	0.52288	7175770
new	17630000	4485640	0.254432	936542
old	48099999	29883300	0.621274	6239230
15				
total non-re	39530000	17624200	0.445844	3679700
new	8990000	1870090	0.208018	390449
old	30540000	15754100	0.515852	3289250
16				
total non-re	93420000	17393583	0.186187	3631550
new	40020000	4221550	0.105486	881403
old	53400000	13172000	0.246667	2750140
17				
total non-re	92880000	25804468	0.277826	5387630
new	36360000	5615130	0.154432	1172360
old	56520000	20189337	0.357207	4215260
18				
total non-re	98420000	32040600	0.325549	6689640
new	43410000	8224150	0.189453	1717090
old	55010000	23816400	0.432947	4972550
19				
total non-re	96630000	59703245	0.617854	12465236
new	18980000	5341670	0.281437	1115270
old	77650000	54361576	0.700085	11350000
20				
total non-re	105529999	38788600	0.36756	8098530
new	49210000	9447560	0.191985	1972520
old	56320000	29341000	0.520969	6126010

proptype	mkt val	assedval	a/sratio	taxes pd
21				
total non-re	100449999	26247859	0.261303	5480200
new	41930000	6422240	0.153166	1340880
old	58520000	19825600	0.338784	4139320
22				
total non-re	103939999	22280200	0.214356	4651800
new	39620000	3978750	0.100423	830709
old	64320000	18301400	0.284537	3821100
23*				
total non-re	1.72423 E+9	6.02003 E+8	0.349143	125690206
new	7.7436 E+8	1.62218 E+8	0.209487	33868982
old	9.4987 E+8	4.39785 E+8	0.462995	91821223

***23 is the city ~~total~~ for non-residential property**

Table 3-R1

the status of the city after proptype revaluation

total revenues= 2.5 E+8

tax rate= 0.208787

proptype	assedval	a/sratio	taxes pd	%shift tp
1				
total non-re	26660600	0.349143	5566370	63.9738
new	5258120	0.209487	1097820	32.0425
old	23733126	0.462995	4955160	93.3152
2				
total non-re	7970940	0.349143	1664230	-17.3058
new	486009	0.209487	101472	9.35682
old	9496030	0.462995	1982640	3.27799
3				
total non-re	78043995	0.349143	16294500	18.4618
new	31697400	0.209487	6618000	-8.22921
old	33437500	0.462995	6981300	6.68811
4				
total non-re	56672935	0.349143	11832548	-25.7436
new	18770000	0.209487	3918930	-26.9882
old	33669000	0.462995	7029640	-33.4768
5				
total non-re	81406244	0.349143	16996500	26.5124
new	38327700	0.209487	8002310	-8.39853
old	23242400	0.462995	4852691	3.27799
6				
total non-re	7792880	0.349143	1627050	-40.9811
new	226246	0.209487	47237.1	0.29898
old	9834010	0.462995	2053210	-24.2282
7				
total non-re	8187410	0.349143	1709420	50.0881
new	339369	0.209487	70855.6	66.0468
old	10107200	0.462995	2110240	92.4926
8				
total non-re	5471075	0.349143	1142287	-21.9922
new	819093	0.209487	171016	-41.1754
old	5444820	0.462995	1136810	-3.13542
9				
total non-re	5907500	0.349143	1233410	47.5777
new	865180	0.209487	180638	32.7487
old	5921710	0.462995	1236370	76.7022
10				
total non-re	7377400	0.349143	1540300	-45.5301
new	341463	0.209487	71293	-24.3171
old	9028400	0.462995	1885010	-31.0431

proptype	assedval	a/sratio	taxes pd	%shift tp
11				
total non-re	10921200	0.349143	2280200	-6.43908
new	611701	0.209487	127715	-0.30523
old	13130500	0.462995	2741480	18.729
12				
total non-re	15121400	0.349143	3157140	-43.7063
new	651504	0.209487	136025	-27.5212
old	18612400	0.462995	3886020	-28.311
13				
total non-re	12366700	0.349143	2581990	-8.74226
new	1784830	0.209487	372648	7.6963
old	12454600	0.462995	2600350	4.7124
14				
total non-re	22949187	0.349143	4791480	-33.2269
new	3693250	0.209487	771101	-17.665
old	22270100	0.462995	4649690	-25.4765
15				
total non-re	13801600	0.349143	2881600	-21.6893
new	1883290	0.209487	393205	0.705874
old	14139900	0.462995	2952210	-10.2465
16				
total non-re	32617000	0.349143	6809980	87.523
new	8383660	0.209487	1750400	98.592
old	24723900	0.462995	5162030	87.7002
17				
total non-re	32428427	0.349143	6770620	25.6698
new	7616940	0.209487	1590310	35.6503
old	26168480	0.462995	5463630	29.6154
18				
total non-re	34362700	0.349143	7174470	7.2474
new	9093820	0.209487	1898670	10.5746
old	25469400	0.462995	5317660	6.94033
19				
total non-re	33737714	0.349143	7043980	-43.491
new	3976060	0.209487	830148	-25.5652
old	35951600	0.462995	7506200	-33.8659
20				
total non-re	36845089	0.349143	7692760	-5.01042
new	10308800	0.209487	2152350	9.11648
old	26075881	0.462995	5444290	-11.1282

proptype	assedval	a/sratio	taxes pd	%shift tp
21				
total non-re	35071441	0.349143	7322446	33.6164
new	8783780	0.209487	1833940	36.7713
old	27094470	0.462995	5656960	36.6639
22				
total non-re	36289951	0.349143	7576850	62.8799
new	8299870	0.209487	1732900	108.605
old	29779800	0.462995	6217631	62.7186
23				
total non-re	6.02003 E+8	0.349143	125690206	0
new	1.62218 E+8	0.209487	33868982	0
old	4.39785 E+8	0.462995	91821223	0

Table 3-R2

the status of the city after full revaluation

total revenues= 2.5 E+8

tax rate= 6.54419 E-2

proptype	assedval	a/sratio	taxes pd	%shift tp
1				
total non-re	76359999	1	4997150	47.2056
new	25100000	1	1642590	97.5654
old	51260000	1	3354550	30.871
2				
total non-re	22830000	1	1494040	-25.7622
new	2320000	1	151825	63.6224
old	20510000	1	1342210	-30.0826
3				
total non-re	2.2353 E+8	1	14628200	6.34772
new	1.5131 E+8	1	9902020	37.3098
old	72220001	1	4726220	-27.774
4				
total non-re	1.6232 E+8	1	10622536	-33.3372
new	89600000	1	5863600	9.24203
old	72719999	1	4758938	-54.965
5				
total non-re	2.3316 E+8	1	15258400	13.5751
new	1.8296 E+8	1	11973300	37.0564
old	50200000	1	3285190	-30.0826
6				
total non-re	22320000	1	1460660	-47.0164
new	1080000	1	70677.3	50.0699
old	21240000	1	1389990	-48.7038
7				
total non-re	23450000	1	1534610	34.7399
new	1620000	1	106016	148.443
old	21830000	1	1428600	30.3141
8				
total non-re	15670000	1	1025480	-29.9694
new	3910000	1	255878	-11.9851
old	11760000	1	769597	-34.4244
9				
total non-re	16920000	1	1107280	32.4862
new	4130000	1	270275	98.6219
old	12790000	1	837002	19.6242
10				
total non-re	21130000	1	1382790	-51.1003
new	1630000	1	106670	13.2387
old	19500000	1	1276120	-53.3174

proptype	assedval	a/sratio	taxes pd	%shift tp
11				
total non-re	31280000	1	2047020	-16.0067
new	2920000	1	191090	49.1658
old	28360000	1	1855930	-19.6226
12				
total non-re	43310000	1	2834290	-49.4629
new	3110000	1	203524	8.44464
old	40200000	1	2630770	-51.4678
13				
total non-re	35420000	1	2317950	-18.0744
new	8520000	1	557565	61.1379
old	26900000	1	1760390	-29.1116
14				
total non-re	65730000	1	4301500	-40.0552
new	17630000	1	1153740	23.1917
old	48099999	1	3147760	-49.5489
15				
total non-re	39530000	1	2586920	-29.6975
new	8990000	1	588323	50.6787
old	30540000	1	1998600	-39.2385
16				
total non-re	93420000	1	6113590	68.3466
new	40020000	1	2618990	197.138
old	53400000	1	3494600	27.0697
17				
total non-re	92880000	1	6078250	12.8187
new	36360000	1	2379470	102.963
old	56520000	1	3698780	-12.2527
18				
total non-re	98420000	1	6440796	-3.71986
new	43410000	1	2840830	65.4445
old	55010000	1	3599960	-27.6033
19				
total non-re	96630000	1	6323650	-49.2697
new	18980000	1	1242090	11.3712
old	77650000	1	5081570	-55.2284
20				
total non-re	105529999	1	6906090	-14.7242
new	49210000	1	3220398	63.2628
old	56320000	1	3685690	-39.8354

proptype	assedval	a/sratio	taxes pd	%shift tp
21				
total non-re	100449999	1	6573640	19.9526
new	41930000	1	2743980	104.641
old	58520000	1	3829660	-7.48095
22				
total non-re	103939999	1	6802040	46.2236
new	39620000	1	2592810	212.12
old	64320000	1	4209230	10.1576
23				
total non-re	1.72423 E+9	1	112836959	-10.2261
new	7.7436 E+8	1	50675623	49.6225
old	9.4987 E+8	1	62161300	-32.3018

Table 4

iteration process
total non-re

n	total value	-tax chng	new value	-tax chng	old value	-tax chng
1	1.72423 E+9	-12853247	7.7436 E+8	16806600	9.4987 E+8	-29659900
2	1.84108 E+9	7646741	5.8762 E+8	-12220658	1.19704 E+9	16175000
3	1.77156 E+9	-4549252	7.23405 E+8	8886040	1.06224 E+9	-8821031
4	1.81292 E+9	2706471	6.24671 E+8	-6461331	1.13575 E+9	4810545
5	1.78831 E+9	-1610152	6.96463 E+8	4698245	1.09566 E+9	-2623430
6	1.80295 E+9	957922	6.44261 E+8	-3416248	1.11753 E+9	1430686
7	1.79424 E+9	-569893	6.82219 E+8	2484065	1.1056 E+9	-780224
8	1.79942 E+9	339044	6.54618 E+8	-1806250	1.11211 E+9	425494
9	1.79634 E+9	-201707	6.74688 E+8	1313380	1.10856 E+9	-232044
10	1.79818 E+9	120001	6.60095 E+8	-955002	1.11049 E+9	126545
11	1.79709 E+9	-71393	6.70706 E+8	694413	1.10944 E+9	-69011
12	1.79773 E+9	42473	6.6299 E+8	-504930	1.11001 E+9	37635
13	1.79735 E+9	-25268	6.686 E+8	367151	1.1097 E+9	-20525
14	1.79758 E+9	15031	6.64521 E+8	-266968	1.10987 E+9	11193
15	1.79744 E+9	-8943	6.67487 E+8	194121	1.10978 E+9	-6104
16	1.79752 E+9	5320	6.6533 E+8	-141151	1.10983 E+9	3328
17	1.79747 E+9	-3165	6.66899 E+8	102635	1.1098 E+9	-1815
18	1.7975 E+9	1883	6.65758 E+8	-74629	1.10982 E+9	989
19	1.79749 E+9	-1121	6.66587 E+8	54265	1.10981 E+9	-540
20	1.7975 E+9	666	6.65984 E+8	-39458.5	1.10981 E+9	294
21	1.79749 E+9	-397	6.66423 E+8	28691	1.10981 E+9	-161
22	1.79749 E+9	236	6.66104 E+8	-20862	1.10981 E+9	87
23	1.79749 E+9	-141	6.66336 E+8	15169.5	1.10981 E+9	-48
24	1.79749 E+9	83	6.66167 E+8	-11030.5	1.10981 E+9	26
25	1.79749 E+9	-50	6.6629 E+8	8020.5	1.10981 E+9	-15
26	1.79749 E+9	30	6.66201 E+8	-5832	1.10981 E+9	8
27	1.79749 E+9	-19	6.66266 E+8	4240.5	1.10981 E+9	-6
28	1.79749 E+9	10	6.66218 E+8	-3083.5	1.10981 E+9	3
29	1.79749 E+9	-6	6.66253 E+8	2241.5	1.10981 E+9	-2
30	1.79749 E+9	3	6.66228 E+8	-1630	1.10981 E+9	1
31	1.79749 E+9	-2	6.66246 E+8	1184.5	1.10981 E+9	-1
32	1.79749 E+9	1	6.66233 E+8	-861.5	1.10981 E+9	0
33	1.79749 E+9	-1	6.66242 E+8	626	1.10981 E+9	0
34	1.79749 E+9	0	6.66235 E+8	-455.5	1.10981 E+9	0
35	1.79749 E+9	0	6.6624 E+8	331	1.10981 E+9	0
36	1.79749 E+9	0	6.66237 E+8	-241	1.10981 E+9	0
37	1.79749 E+9	0	6.66239 E+8	175	1.10981 E+9	0
38	1.79749 E+9	0	6.66237 E+8	-128	1.10981 E+9	0
39	1.79749 E+9	0	6.66239 E+8	93	1.10981 E+9	0
40	1.79749 E+9	0	6.66238 E+8	-68	1.10981 E+9	0
41	1.79749 E+9	0	6.66239 E+8	49	1.10981 E+9	0
42	1.79749 E+9	0	6.66238 E+8	-36	1.10981 E+9	0
43	1.79749 E+9	0	6.66238 E+8	26	1.10981 E+9	0
44	1.79749 E+9	0	6.66238 E+8	-19.5	1.10981 E+9	0
45	1.79749 E+9	0	6.66238 E+8	14.5	1.10981 E+9	0
46	1.79749 E+9	0	6.66238 E+8	-11	1.10981 E+9	0
47	1.79749 E+9	0	6.66238 E+8	7.5	1.10981 E+9	0
48	1.79749 E+9	0	6.66238 E+8	-5.5	1.10981 E+9	0
49	1.79749 E+9	0	6.66238 E+8	3.5	1.10981 E+9	0
50	1.79749 E+9	0	6.66238 E+8	-2.5	1.10981 E+9	0

n total value -tax chng new value -tax chng old value -tax chng

51	1.79749	E+9	0	6.66238	E+8	1.5	1.10981	E+9	0
52	1.79749	E+9	0	6.66238	E+8	-1.5	1.10981	E+9	0
53	1.79749	E+9	0	6.66238	E+8	1	1.10981	E+9	0
54	1.79749	E+9	0	6.66238	E+8	-1	1.10981	E+9	0
55	1.79749	E+9	0	6.66238	E+8	0.5	1.10981	E+9	0
56	1.79749	E+9	0	6.66238	E+8	-0.5	1.10981	E+9	0
57	1.79749	E+9	0	6.66238	E+8	0	1.10981	E+9	0

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iteration process
total res

n	total value	-tax chng	new value	-tax chng	old value	-tax chng
1	2.09595 E+9	12853243	2.2465 E+8	3474890	1.8713 E+9	9378356
2	1.9791 E+9	-7646737	1.8604 E+8	-2526710	1.79916 E+9	-4721060
3	2.04862 E+9	4549248	2.14115 E+8	1837250	1.83547 E+9	2376579
4	2.00726 E+9	-2706470	1.93701 E+8	-1335930	1.81719 E+9	-1196370
5	2.03187 E+9	1610152	2.08544 E+8	971395	1.8264 E+9	602251
6	2.01723 E+9	-957924	1.97751 E+8	-706333	1.82176 E+9	-303173
7	2.02594 E+9	569895	2.05599 E+8	513598	1.8241 E+9	152617
8	2.02076 E+9	-339046	1.99893 E+8	-373454	1.82292 E+9	-76828
9	2.02384 E+9	201707	2.04042 E+8	271551	1.82351 E+9	38675
10	2.022 E+9	-120002	2.01025 E+8	-197453	1.82321 E+9	-19469
11	2.02309 E+9	71391	2.03219 E+8	143575	1.82336 E+9	9800
12	2.02245 E+9	-42473	2.01623 E+8	-104398	1.82329 E+9	-4934
13	2.02283 E+9	25268	2.02783 E+8	75911.1	1.82333 E+9	2484
14	2.0226 E+9	-15033	2.0194 E+8	-55197.5	1.82331 E+9	-1251
15	2.02274 E+9	8943	2.02553 E+8	40135.8	1.82332 E+9	629
16	2.02266 E+9	-5321	2.02107 E+8	-29184.1	1.82331 E+9	-317
17	2.02271 E+9	3165	2.02432 E+8	21220.8	1.82332 E+9	159
18	2.02268 E+9	-1883	2.02196 E+8	-15430.4	1.82331 E+9	-81
19	2.02269 E+9	1119	2.02367 E+8	11219.9	1.82331 E+9	40
20	2.02268 E+9	-666	2.02243 E+8	-8158.38	1.82331 E+9	-21
21	2.02269 E+9	396	2.02333 E+8	5932.13	1.82331 E+9	11
22	2.02269 E+9	-236	2.02267 E+8	-4313.5	1.82331 E+9	-7
23	2.02269 E+9	141	2.02315 E+8	3136.38	1.82331 E+9	3
24	2.02269 E+9	-85	2.0228 E+8	-2280.63	1.82331 E+9	-2
25	2.02269 E+9	50	2.02306 E+8	1658.25	1.82331 E+9	0
26	2.02269 E+9	-30	2.02287 E+8	-1205.88	1.82331 E+9	0
27	2.02269 E+9	17	2.02301 E+8	876.875	1.82331 E+9	0
28	2.02269 E+9	-10	2.02291 E+8	-637.75	1.82331 E+9	0
29	2.02269 E+9	5	2.02298 E+8	463.75	1.82331 E+9	0
30	2.02269 E+9	-3	2.02293 E+8	-337.25	1.82331 E+9	0
31	2.02269 E+9	1	2.02297 E+8	245.125	1.82331 E+9	0
32	2.02269 E+9	-1	2.02294 E+8	-178.25	1.82331 E+9	0
33	2.02269 E+9	0	2.02296 E+8	129.5	1.82331 E+9	0
34	2.02269 E+9	0	2.02294 E+8	-94.25	1.82331 E+9	0
35	2.02269 E+9	0	2.02296 E+8	68.5	1.82331 E+9	0
36	2.02269 E+9	0	2.02295 E+8	-49.875	1.82331 E+9	0
37	2.02269 E+9	0	2.02295 E+8	36.25	1.82331 E+9	0
38	2.02269 E+9	0	2.02295 E+8	-26.375	1.82331 E+9	0
39	2.02269 E+9	0	2.02295 E+8	19.125	1.82331 E+9	0
40	2.02269 E+9	0	2.02295 E+8	-14	1.82331 E+9	0
41	2.02269 E+9	0	2.02295 E+8	10	1.82331 E+9	0
42	2.02269 E+9	0	2.02295 E+8	-7.25	1.82331 E+9	0
43	2.02269 E+9	0	2.02295 E+8	5.125	1.82331 E+9	0
44	2.02269 E+9	0	2.02295 E+8	-3.75	1.82331 E+9	0
45	2.02269 E+9	0	2.02295 E+8	2.625	1.82331 E+9	0
46	2.02269 E+9	0	2.02295 E+8	-2	1.82331 E+9	0
47	2.02269 E+9	0	2.02295 E+8	1.5	1.82331 E+9	0
48	2.02269 E+9	0	2.02295 E+8	-1.25	1.82331 E+9	0
49	2.02269 E+9	0	2.02295 E+8	0.875	1.82331 E+9	0
50	2.02269 E+9	0	2.02295 E+8	-0.75	1.82331 E+9	0

n total value -tax chng new value -tax chng old value -tax chng

51	2.02269	E+9	0	2.02295	E+8	0.625	1.82331	E+9	0
52	2.02269	E+9	0	2.02295	E+8	-0.625	1.82331	E+9	0
53	2.02269	E+9	0	2.02295	E+8	0.5	1.82331	E+9	0
54	2.02269	E+9	0	2.02295	E+8	-0.5	1.82331	E+9	0
55	2.02269	E+9	0	2.02295	E+8	0.375	1.82331	E+9	0
56	2.02269	E+9	0	2.02295	E+8	-0.5	1.82331	E+9	0
57	2.02269	E+9	0	2.02295	E+8	0.25	1.82331	E+9	0
58	2.02269	E+9	0	2.02295	E+8	-0.25	1.82331	E+9	0
59	2.02269	E+9	0	2.02295	E+8	0.125	1.82331	E+9	0
60	2.02269	E+9	0	2.02295	E+8	-0.125	1.82331	E+9	0
61	2.02269	E+9	0	2.02295	E+8	0	1.82331	E+9	0

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CHAPTER FIVE

CONCLUSION

The value of property takes a significant role in property taxation. When the term "valuation" is used it means the assessment of property for taxation purposes where property is valued based upon (in some relationship) market value. The question which arises is: how should property be valued? The different ways of valuing property will have a direct effect upon the different tax burdens which will fall on property.

In the case of all property being valued at an equal level (i.e. 100% valuation), residential property has a tax burden equal to that of office buildings. Any increase in the tax rate will be equally levied upon all property. It would be interesting to see the results of a study which measures the amount of use of city services each property type and area receives. If the study concludes that there is equal use of city services among all classes and locations of property (it may be true that different services are used by different properties—this is concerned with total usage of all services), then the argument against equal level valuation of all property would be weakened. If, however, the amount of use of and benefit from city services differs according to property type and/or area,

then equal level valuation of all properties for taxation purposes could be seen as unfair, according to the "taxation for payment of city service" doctrine introduced in chapter one.

Valuing property at different levels according to type and area might be a solution, if it is residential property which benefits the most from city services, then residential property will be taxed at the highest level. Another doctrine used in taxation theory is "the ability to pay", which could be used as an argument for the non-residential property to be taxed the heaviest, since most businesses have more ability than most residents to pay the property tax. It would be easy for people to agree with the taxation of property policy that is set with a higher tax burden for non-residential property than for residential if a study would show that (or if one could prove that) non-residential property benefits most from city services. This way both doctrines of taxation are satisfied.

A solution to the problem of what level of taxation should be placed upon different properties is not just satisfying theories of taxation. Property can be taxed according to the ability to pay and the benefits it receives from the city in terms of public service. This is not an answer, for then the reason for residents in one area of the city receiving better city services than residents in

other areas could be that the residents in the first area are paying higher taxes, thus deserving better service. And yet, it was the city, not the residents, which decided upon the policy of taxation. From this, for the same type of property, areas should have equal tax burdens as well as public service benefits.

The question of tax policy does not only rest upon the principle on which it is based. The impact this policy will have upon the existing taxation situation is a real concern—especially to public policy makers, who are elected by the people that are being affected by the policies made. The politician would then try to lessen the tax burden for the residents—the homeowners and tenants—of the city relative to the tax burden held by commercial and industrial property. Therefore, one could see the politician fighting against full revaluation and discouraging property type revaluation from occurring too rapidly. However, the tenants in the city might not view this stand as the politician would want them to. The politician is stating a policy which calls for a relatively less (relative to non-residential property) tax burden for residential property, thus fewer tax dollars paid by homeowners and apartment dwellers. This policy helps the landlord for the tax burden is low. Yet, even if the tax burden was increased, it would be passed on to the tenant. Thus, what the tenant is not happy with is the

obvious lack of effect a property tax change would have on the landlord. A solution, in the mind of the apartment dweller in the city, would not be a change in the magnitude or direction of the property tax, but rather a change in the property tax itself. The tax itself and the policy would want to be changed as well as the administration of the policy being changed to match the "fairness" which the policy dictates.

There are taxes which might be deemed as being more "fair" than the property tax. Some examples of other taxes are the sales tax, the income tax, and the transportation taxes which take several forms, three being highway or bridge tolls, gasoline tax and mass transit fares. There are similarities between the sales tax and the various transportation taxes mentioned, one being that these taxes are considered to be nonprogressive while the income tax (especially the Federal income tax) is considered to be progressive. Progressivity in taxation refers to taxing in steps or different stages. In the case of the progressive income tax every income level or stage (\$10,000-\$15,000 could be such a stage) pays a certain proportion or percentage which increases at every level. Thus, as the income level increases not only does the absolute dollar value of taxes increase but the proportion of income to be paid for taxes also increases. A less progressive tax would be having the proportion the same for all income

levels (i.e. 30% of income). This would still be considered progressive (although not as progressive as the first example) since the absolute value of taxes increase with the increase of income. Even less progressive are those taxes that remain the same dollar amount for all levels of income, as the sales tax and transportation taxes do. The sales tax is progressive, however, when using the sales price rather than income as the base—as the price increases so does the dollar amount of sales taxes.

A question that arises from this discussion is: where does the property tax fit in with the level of progressivity? When comparing the property tax with value a significant progressivity is apparent. As the (assessed) value increases, the tax increases. In the case of equal level valuation the newer and better quality property pays a higher proportion of gross income for property taxes. This is because of the capitalization rate, which is lower for the newer and better quality property than for the poorer quality property. In the case of unequal tax valuation, the tax progressivity follows the "progressivity" of the tax burdens themselves.

Although the property tax may be considered progressive when using value and gross income of the property as the base, it is not as progressive when comparing it with the income of the property's tenants. Since the property tax is passed on to the tenants of the respective properties,

the same property tax falls upon all tenants, with different levels of income, in a building. One could argue that in an apartment building the residents will have similar, if not very close, income levels, which should satisfy the progressivity of the property tax, since the high quality apartments that get taxed the heaviest will contain high income tenants. Assuming that is true, the property tax can still be proved to be nonprogressive. The progressivity that exists when dealing with the property tax on the value of property base disappears when dealing on the level of tenants income base. Assuming equal level valuation, each tenant is taxed on a rate based on the city tax rate and the number of dwelling units in the building. Since each building has a different number of dwelling units, the building rates will be different. The values of the buildings also differ—usually the value increasing as the number of units increase. However, the value per dwelling unit decreases as the number of units in a building increase.⁶ Therefore, the tenants living in the large luxury apartments pay a lower rate than those living in small less-quality buildings for the property tax. This places the property tax at a nonprogressive level—less progressive than the sales tax. This would also be true for unequal valuation

⁶This is true for properties in the Boston-Cambridge area. A 3-family house would be valued at 30,00 dollars while a 200-unit building would have a one million dollar value, a fifty percent difference in the 200-unit building's favor.

within residential property and within each area if areas are also valued differently. This is the case where the tax itself can be called regressive, rather than nonprogressive.

If the progressive tax structure is a solution one seeks for the city, then the property tax is far from the answer. An income tax—regulated along city or state lines, would be the answer to a progressive path. The property tax could still be maintained, but the importance of it for city revenues should be lessened. Doing so would also lessen the impacts of any revaluation of property in the city. The property tax raises the problems of progressivity, administration, and equity.

Equity is the quality or state of being fair and impartial. Whether or not a government can be equitable is a question for another thesis. The equity of taxation and the principles behind it is another question. Fairness is a subjective quality—viewed differently by different people. Instead of taxing people according to one doctrine of equity—such as the progressive tax—different taxations of different "fairness" might be a better solution. It has never been proved to everyone's satisfaction that the progressive tax structure is the most equitable. And yet, it is also clear to many that the sales tax is also inequitable. There is no fine solution. A compromise and a complete understanding must be made. The task is for the planners and policy makers to do. Hopefully, the impact of their decisions upon the urban environment will be favorable.

APPENDIX A

CONSTRUCTION OF THE MODEL

The model presented in this thesis is based on a hypothetical city. All data that exists in the basic model and used for further experimentation is fictitious and should be treated as an academic exercise only, proving the hypotheses presented in the section on revaluation theory, which are true for any city using property taxation.

Market Value

Market Value was constructed for each property type and ward using the capitalization of income method, except for residential and vacant property where the comparable sales method was used.

Assessed Value

An inventory of the assessments of all property was conducted in order to compute Assessed Value.

Assessment-Sales Ratio

Dividing the Assessed Value by the Market Value results in the Assessment-Sales Ratio.

APPENDIX B
CONSTRUCTION OF THE COMPUTER PROCESS

The programs used for this thesis are listed on the pages following.

Model.Basic is the program used for constructing tables One, Two, and Three.

Iter.Basic is the program used for constructing table Four.

The files used for both programs were:

Data - the market values and assessed values of the city

Use - the names of the different property types

Cap - the capitalization rates of the different property types

model.basic 04/27/74

```
No line.
10 file #1:"data"
11 file #2:"use"
20 dim a(23,12),b(23,12),c(23,12),d(23,12),e(23,12),f(23,12)
30 dim g(23,12),h(23,12),i(23,12),j(23,12),k(23,12),l(23,12)
40 dim r(23,12),s(23,12),t(23,12),m(23,12)
45 dim a$(12)
50 mat read #1:a,b,c,d,e,f
55 mat read #2:a$
60 mat m=con
63 let n$="new"
65 let o$="old"
70 let k=1
71 mat a=(1000000)*a
72 mat b=(1000000)*b
73 mat c=(1000000)*c
74 mat d=(1000000)*d
75 mat e=(1000000)*e
76 mat f=(1000000)*f
80 print "input revenues needed"
90 input r
100 print "input situation:current=1,rev1=2,rev2=3"
110 input z
111 for x=1 to 12
112 for w=1 to 23
113 let r(w,x)=a(w,x)/d(w,x)
114 let s(w,x)=b(w,x)/e(w,x)
115 let t(w,x)=c(w,x)/f(w,x)
116 next w
117 next x
120 print "input type, city-1,region-2;and input use"
130 input n,q
140 on z goto 190,190,170
150 print "idiot-you lose"
160 stop
170 print "what % of mkt val-input a decimal-- .xx"
180 input y
190 gosub 500
200 if k=3 then 5000
210 if k=1 then 240
220 print "error-error-error-line 220"
230 stop
240 if z=2 then 280
250 if z=3 then 310
260 print "error line 260"
270 stop
280 gosub 2000
290 let k=2
300 goto 190
310 gosub 1000
```

model.basic 04/27/74

```

310 gosub 1000
320 let k=2
330 goto 190
500 rem this computes the tax rate, asse val, a/sratio,
501 rem taxes paid, mkt val, shifts--new or old.etc
510 let t=r/a(23,12)
511 print
512 print
515 on k goto 660,700
520 print "the status of the city before revaluation"
530 print
535 print
540 print ."total revenues=";r,"tax rate=";t
550 print
555 print
560 print "proptype","mkt val","assedval","a/sratio","taxes pd"
561 if w=11 then 4010
562 if w=21 then 4010
570 on n goto 900,4000
580 print "the status of the city after proptype revaluation"
590 print
595 print
600 print "total revenues=";r,"tax rate=";t
610 print
615 print
620 print "proptype","assedval","a/sratio","taxes pd","%shift tp"
621 if w=11 then 4160
622 if w=21 then 4160
630 on n goto 800,4150
640 print "the status of the city after full revaluation"
650 goto 590
660 mat g=(t)*a
670 mat h=(t)*b
680 mat i=(t)*c
690 on z goto 520,999,999
700 mat j=(t)*a
710 mat k=(t)*b
720 mat l=(t)*c
730 for w=1 to 23
740 for x=1 to 12
750 let g(w,x)=((j(w,x)-g(w,x))/g(w,x))*100
760 let h(w,x)=((k(w,x)-h(w,x))/h(w,x))*100
770 let i(w,x)=((l(w,x)-i(w,x))/i(w,x))*100
780 next x
790 next w
795 on z goto 980,580,640
800 for x=1 to 12
810 print
820 print a$(x),a(23,x),r(23,x),j(23,x),g(23,x)
830 print n$,b(23,x),s(23,x),k(23,x),h(23,x)

```

'Now g h i are
'the percent change
'in taxes paid

model.basic 04/27/74

```
830 print n$,b(23,x),s(23,x),k(23,x),h(23,x)
840 print o$,c(23,x),t(23,x),l(23,x),i(23,x)
850 next x
860 let k=3
870 goto 999
880 rem this line does not belong here
900 for x=1 to 12
905 print
910 print a$(x),d(23,x),a(23,x),r(23,x),g(23,x)
920 print n$,e(23,x),b(23,x),s(23,x),h(23,x)
930 print o$,f(23,x),c(23,x),t(23,x),i(23,x)
940 rem this line does not either
950 next x
960 let k=3
970 goto 999
980 stop
999 return
1000 rem full revaluation at percent y
1010 mat a=(y)*d
1020 mat b=(y)*e
1030 mat c=(y)*f
1040 mat r=(y)*m
1050 mat s=(y)*m
1060 mat t=(y)*m
1070 return
2000 rem property type revaluation
2001 print
2002 print
2010 for x=1 to 12
2020 for w=1 to 23
2030 let a(w,x)=d(w,x)*r(23,x)
2040 let b(w,x)=e(w,x)*s(23,x)
2050 let c(w,x)=f(w,x)*t(23,x)
2060 let r(w,x)=r(23,x)
2070 let s(w,x)=s(23,x)
2080 let t(w,x)=t(23,x)
2090 next w
3000 next x
3100 return
4000 for w=1 to 23
4001 if w=11 then 4004
4002 if w=21 then 4004
4003 goto 4010
4004 print
4005 print
4006 input q
4007 goto 560
4010 print
4020 print w
4040 print a$(q),d(w,q),a(w,q),r(w,q),g(w,q)
```

```
                                model.basic    04/27/74
4040 print a$(q),d(w,q),a(w,q),r(w,q),g(w,q)
4050 print n$,e(w,q),b(w,q),s(w,q),h(w,q)
4060 print o$,f(w,q),c(w,q),t(w,q),i(w,q)
4120 next w
4130 let k=3
4140 goto 999
4150 for w=1 to 23
4151 if w=11 then 4154
4152 if w=21 then 4154
4153 goto 4160
4154 print
4155 print
4156 print
4157 print
4158 input q
4159 goto 620
4160 print
4170 print w
4190 print a$(q),a(w,q),r(w,q),j(w,q),g(w,q)
4200 print n$,b(w,q),s(w,q),k(w,q),h(w,q)
4210 print o$,c(w,q),t(w,q),l(w,q),i(w,q)
4280 next w
4290 let k=3
4300 goto 999
5000 print
5010 print
5020 print
5030 print
5040 print
5050 end
EOF
```

iter.basic 04/27/74

```
No line.
10 file #1: "data"
20 file #2: "cap"
25 file #3:"use"
30 dim a(3,12),d(23,12),e(23,12),f(23,12),m(23,12),n(23,12),o(23,12)
35 dim a$(12),b(3),s(3),q(3),c(3),p(3)
40 mat read #1:d,e,f,m,n,o
50 mat read #2:a
55 mat read #3:a$
60 print "input total revenues, new a/s R,proptype number"
70 input r,w,z.
71 let t2=r/(m(23,12)*w)
72 let t1=r/d(23,12)
80 let e=r/m(23,12)
81 let n=1
83 print "iteration process"
84 print a$(z)
86 print " n";" total value -";"tax chng";" new value -";"tax chng";
   " old value -";"tax chng"
90 let p(1)=t1*d(23,z)
100 let p(2)=t1*e(23,z)
110 let p(3)=t1*f(23,z)
120 let s(1)=m(23,z)
130 let s(2)=n(23,z)
140 let s(3)=o(23,z)
150 mat q=(e)*s
160 mat c=q-p
170 print n;s(1);c(1);s(2);c(2);s(3);c(3)
180 goto 250
185 let b(1)=c(1)/a(1,z)
190 let b(2)=c(2)/a(2,z)
200 let b(3)=c(3)/a(3,z)
210 let n=n+1
220 mat s=s-b
230 mat p=q
240 goto 150
250 if c(1)<0 then 300
260 if c(1)<.01 then 280
270 goto 185
280 if c(2)<0 then 286
281 if c(2)<.01 then 290
285 goto 185
286 if c(2)>-.01 then 290
287 goto 185
290 if c(3)<.01 then 400
295 goto 185
300 if c(1)>-.01 then 320
310 goto 185
320 if c(2)>-.01 then 340
330 goto 185
340 if c(3)>-.01 then 400
350 goto 185
400 end
EOF
```