



DETERMINATION OF THE HVORSLEV PARAMETERS  
OF VICKSBURG BUCKSHOT CLAY

by

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ABSTRACT

While Coulomb's criterion of failure is useful for practical purposes, it does not give a true picture of the shear strength of soils.

When a load is applied on a cohesive soil, the strength resisting deformation is generally thought of as being provided by two factors, both of an entirely physical nature: true friction and true cohesion. These factors are considered implicitly in the Coulomb's criterion of failure, but no means are given to find the contribution of each one to the shear resistance of the soil.

Hvorslev, in 1937 worked out an experimental procedure to separate the two shear strength parameters. He found that true friction is constant for a given soil, and that true cohesion is a function of the water content at failure.

Remolded backswamp clay was consolidated in large oedometers and from small specimens cut from the chunk of consolidated clay fourteen consolidated undrained triaxial tests with pore pressures measured were run to determine the Hvorslev parameters.

It was found that the true friction is related to the Plasticity Index of the clay, and the cohesion varies with the water content at failure as shown by Hvorslev.

From the analysis of results given in Soil Mechanics literature and those of this investigation, it is concluded that the shear strength of saturated cohesive soils is a function of several factors, but that the main contributing factors are:

- 1) The effective stress at failure.
- 2) The void ratio at failure.
- 3) Time to failure.

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## I. INTRODUCTION

### a) Strength and Strength Theories

"Strength" may be defined as the ability of a material to resist excessive deformation or rupture when loaded.

Many theories of strength have been proposed especially for metals and other cohesive materials (see for example, Reiner 1960, p. 98), but due to the nature of the materials and problems found in Soil Mechanics only strength theories that consider frictional and cohesion resistance are of practical importance in this Science.

### b) Coulomb's Theory of Failure

Coulomb, in 1776, formulated the first strength theory for soils, which is summarized in the following equation:

$$s = c + \sigma_n \tan \phi \quad (1)$$

where  $s$  = shear strength

$c$  = cohesion

$\sigma_n$  = normal stress

$\phi$  = angle of internal friction

This equation represents the equation of a straight line (fig. 1) in which the "y" intercept is the cohesion and the friction increases linearly with the normal pressure. The frictional and cohesion parameters are considered constant for the same soil.

c) The Krey-Tiedemann Theory of Failure

This theory, which is an improvement over the Coulomb's concept of soil strength, is summarized by Hvorslev (1938) and expressed by the following equation:

$$s = \sigma_m \tan \phi_c + \bar{\sigma}_{ff} \tan \phi \quad (2)$$

where  $s$  = shear strength

$\sigma_m$  = maximum preconsolidation pressure

$\tan \phi_c$  = coefficient of cohesion

$\bar{\sigma}_{ff}$  = effective stress in the plane of failure  
at failure

$\tan \phi$  = coefficient of friction

The graphical representation of this condition of failure is given in fig. 2.

In the Krey-Tiedemann failure theory the angle of internal friction is considered constant for the same soil and the cohesion directly proportional to the preconsolidation pressure. This theory does not give an accurate picture of the shear strength behavior of soils because the strength line of a preconsolidated soil is considered a straight line which is not true in most cases.

d) The Hvorslev Theory of Failure

Many people who investigated the question were aware of the importance of water content in the strength behavior of clays (Collin, 1846; Jurgenson, 1934).

However, it was not until 1937, when Hvorslev (1938) found experimentally that the cohesion of soils is a function of the water content (or void ratio) at failure in a saturated cohesive soil.

He represented the condition of failure by the following equation:

$$s = \mu_0 \bar{\sigma}_n + \alpha \sigma_e \quad (3)$$

where  $s$  = shear strength

$\mu_0$  = coefficient of effective internal friction

$\bar{\sigma}_n$  = effective normal stress

$\alpha$  = coefficient of effective cohesion

$\sigma_e$  = equivalent consolidation pressure

If  $\alpha \sigma_e$  is made equal to  $c_r$ , and  $\mu_0$  equal to  $\tan \phi_r$  equation (3) becomes:

$$s = c_r + \bar{\sigma}_n \tan \phi_r \quad (4)$$

where  $c_r$  = true cohesion

$\phi_r$  = true angle of internal friction

As can readily be seen equation (4) is similar to Coulomb's equation (1). The graphical representation of equation (4) is given in fig. 3.

## II. THEORETICAL CONSIDERATIONS

### a) Physical Meaning of the Shear Strength Parameters

Before going further into theoretical considerations and experimental techniques to determine  $c_r$  and  $\phi_r$ , an attempt is made here to review and summarize the present knowledge of the physical meaning of true friction and true cohesion.

TRUE FRICTION - True friction may be considered to be formed by two components:

- 1) The resistance to sliding of two grain surfaces put in contact and acted upon by a normal pressure.
- 2) The resistance to sliding of the two "wavy" surfaces of a soil mass, cut by a plane passing through the voids but not through the grains.

A normal pressure is considered applied to the surfaces in contact.

According to Rosenquist (1959), the first component is caused by "microdilateny", which is due to the resistance towards shear when a mineral particle has to be "lifted up" and passed over the roughnesses of the surface of another mineral particle. The second part, is due to "macrodilateny", a factor similar to "interlocking" in dense sands, fully discussed by Taylor (1948). Both terms are proportional to the effective normal stress in the shear plane.

The second factor is easy to visualize. But when considering the first factor, it is difficult to imagine the roughness of the surface of a mineral particle. This is complicated even more if we take into account the fact that the mineral particles may never touch each other because they can be considered surrounded by a very thin layer of adsorbed water as suggested by Terzaghi (1941) and further advanced by Lambe (1953).

This water may be considered highly viscous near the surface of the particle, and with decreasing viscosity away from the surface. At some distance the water is normal again and becomes pore water. It is not difficult to realize that in this case the concept of "friction" has another meaning because surface roughness is no longer available. The resistance to sliding of two particles in contact is due to the shear strength of the layers of adsorbed water and the friction resembles more a cohesive force than a frictional factor as will be seen later.

Assuming that true contact points are developed the number and size of them is almost proportional to the total force between surfaces as shown by Bishop and Eldin (1950) or:

$$\bar{\sigma} = a \cdot \sigma_p$$

where  $a$  = effective contact area between the grains

$\sigma_p$  = average contact pressure

The maximum contact pressure that can be applied is equal to the crushing strength of the grains and is constant for a given soil.

The second factor, or friction due to the geometry of the surfaces in contact, could be modified slightly to take into account the effect of the viscous layers of water.

If the reasoning above depicts accurately the true friction of clays, it may be expected that this parameter is not constant and could vary with the degree of packing of the mineral particles, and also with the arrangement and orientation of the particles as shown by Lambe. (1958)

#### TRUE COHESION

In general, cohesion may be defined as the resistance to tension forces applied on any plane through a soil mass. Cohesion is divided in two parts:

- 1) Apparent cohesion
- 2) True cohesion

According to Taylor (1948), apparent cohesion is the shear strength which is caused by capillary forces. This pressure is lost in short time if the soil is placed under water.

True cohesion is a factor the nature of which is not completely understood at present because of its complexity. This factor was suggested by Casagrande

(1932) as being a bond between the soil grains forming a honeycomb structure. This theory was later advanced by Terzaghi (1941) when he introduced the concept of the adsorbed layer of water carrying bond stresses. He also suggests, Terzaghi (1948), that the cohesion is due to the shearing strength of the adsorbed layers that separate the grains at the "points of contact".

In a more recent study, Lambe (1958) implies that the true cohesion (colloidal type strength) is due entirely to the electrical forces acting between particles. The kind and nature of these interparticle forces is fully discussed in a previous paper by the same author (Lambe, 1953). Based on the concepts developed in this paper, Aitchison (1957) developed a mathematical expression for the true cohesion and found that this parameter is a function of the ambient effective stress.

Rosenquist (1959), considers the cohesion as due to the bonding of the atoms of mineral grains in contact, and says that (quoted): "There seems to be no fundamental difference between friction and cohesion."

The summary of the concepts discussed above and the feelings of the author on the factors entering the true friction and the true cohesion, is given in Tables I and II.

b) Methods to Separate the Strength Parameters

As was shown by Hvorslev (1938), true friction is constant and true cohesion depends on the water content at failure. Theoretically, at least, it is possible to obtain samples with the same water content at failure but with different effective stress. The variation in shear strength should be due to differences in effective stresses because  $c_r$  and  $\phi_r$  are constants. If the results of the shear tests are plotted as shown in figure 3, the parameters  $c_r$  and  $\phi_r$  may be determined directly from the plot.

In actual practice, a procedure such as this is too laborious and time consuming. Below are presented simpler methods to determine  $c_r$  and  $\phi_r$ :

c) Terzaghi Construction

A graphical procedure was given by Terzaghi (1938) through which may be obtained different effective stresses at the same water content.

This method is shown graphically in figure 4. When a sample of clay is consolidated to  $\sigma_m$  and then rebounded to a lower pressure  $\sigma_1$  (over consolidated) the shear strength diagram also shows a hysteresis loop, as depicted in the lower part of the same figure (4).

Therefore, points A and B with the same water content  $W$  have different shear strengths  $S_A$  and  $S_B$ , respectively. The equation of the straight line involves the

strength parameters  $c_r$  and  $\phi_r$  (Eq. 4).

This method can be used with drained direct and triaxial shear tests. An energy correction has been given by Gibson (1953) to take into account the effect of the volume change at failure in drained tests.

d) Bjerrum Method

For soils of low plasticity, the difference in shear strength for normally consolidated and over-consolidated conditions is rather small, therefore, it is not possible to get sufficient separation between the Mohr circles to obtain reliable values of  $c_r$  and  $\phi_r$ .

Bjerrum (1954), has proposed a method by which different effective stresses can be obtained at the same water content (or void ratio), without using over-consolidated samples.

The method, in short, consists in the consolidation of two samples with quite different initial water content. The consolidation curves are different, the line of the sample at lower initial water content plotting below that for higher initial water content.

If one curve is taken as the normally consolidated line and the other as over-consolidated, the parameters  $\phi_r$  and  $c_r$  may be determined as described before.

e) Undrained Triaxial Tests to Determine  $c_r$  and  $\phi_r$

Because of the long time required to run drained tests on cohesive soils, a modification to the Hvorslev

method has been proposed by Bishop and Henkel (1957) by using the results from undrained tests with pore pressure measurements.

Equation (4) can be written in the following form:

$$s = c_r + (\sigma_n - u) \tan \phi_r \quad (5)$$

where  $\sigma_n$  = total stress in the plane of failure

$u$  = pore pressure

If failure is defined as the maximum deviator stress in the triaxial test, the shear strength is taken as one half of that value, or:

$$s = \frac{1}{2} (\sigma_1 - \sigma_3)_f = \frac{1}{2} (\bar{\sigma}_1 - \bar{\sigma}_3)_f \quad (6)$$

where  $\sigma_1$  = total major principal stress

$\sigma_3$  = total minor principal stress

$\bar{\sigma}_1$  = effective major principal stress

$\bar{\sigma}_3$  = effective minor principal stress

Therefore, Equation (5) becomes, in terms of the principal stresses:

$$\frac{1}{2} (\sigma_1 - \sigma_3)_f = \frac{c_r \cos \phi_r + \bar{\sigma}_3 \sin \phi_r}{1 - \sin \phi_r} \quad (7)$$

#### f) Equivalent Consolidation Pressure

As mentioned before, the dependency of cohesion and water content was found by Hvorslev. He also proved that the cohesion is proportional to the "equivalent consolidation pressure" or:

$$c_r = \alpha \sigma_e \quad (8)$$

where  $\sigma_e$  is the equivalent consolidation pressure which

is defined as the all around pressure which produces the actual water content in a normally consolidated clay.  $\alpha$  is a coefficient of proportionality and is constant for a given clay.

The equivalent consolidation pressure may be determined directly from the  $e$  vs.  $\log \sigma$  curve drawn for a series of tests. An approximate expression for  $\sigma_e$  is found as follows:

Referring to figure 5, the equation of the normally consolidated line may be expressed as:

$$e - e_m = c_c \log \frac{\sigma_m}{\sigma_e} \quad (9)$$

Similarly, the equation of the swelling or rebound curve is expressed as:

$$e - e_m = c_s \log \frac{\sigma_m}{\sigma_c} \quad (10)$$

where  $e$  = void ratio at pressure

$e_m$  = void ratio at the maximum consolidation pressure in the test ( $\sigma_m$ )

$c_c$  = compression index

$c_s$  = swelling index

$\sigma_c$  = consolidation pressure to which the sample was rebounded

Combining Equations (9) and (10) and letting:

$$\frac{c_s}{c_c} = R$$

$$Z = (\sigma_m)^{1-R}$$

The following expression is derived for  $\sigma_e$ :

$$\sigma_e = z \sigma_c^R \quad (11)$$

The validity of Equation (11) is based on the linearity of the rebound curve in the consolidation test. This linear relationship is true for most practical purposes, as considered by Terzaghi and Peck (1948) and Roscoe (1958).

g) Dimensionless Plots

If both sides of Equation (3) are divided by  $\sigma_e$ , the following expression is obtained:

$$\frac{s}{\sigma_e} = \mu_o \frac{\sigma_n}{\sigma_e} + \mathcal{H} = \frac{\bar{\sigma}}{\sigma_e} \tan \phi + \mathcal{H} \quad (12)$$

The ratio  $\frac{s}{\sigma_e}$  is plotted against  $\frac{\bar{\sigma}}{\sigma_e}$  and the straight line drawn through the points gives an average value of  $\phi$  and  $\mathcal{H}$ , as shown in figure 6. Above procedure is only applicable to direct shear tests and was used by Hvorslev in his investigations.

An expression was developed by Bishop and Henkel (1957) to take advantage of dimensionless plot for tri-axial tests. The equation is derived as follows:

Both terms of Equation (7) are divided by  $\sigma_e$  and after rearranging the terms, the following expression is obtained:

$$\frac{(\sigma_1 - \sigma_3)_f}{2\sigma_e} = \frac{c_r \cos \phi_r}{\sigma_e (1 - \sin \phi_r)} + \frac{\bar{\sigma}_3 \sin \phi_r}{\sigma_e (1 - \sin \phi_r)} \quad (13)$$

Equation (13) is the equation of a straight line where the "Y" intercept is:

$$b = \frac{c_r \cos \phi_r}{\sigma_e (1 - \sin \phi_r)} \quad (14)$$

and the slope is:

$$\tan \alpha = \frac{\sin \phi_r}{1 - \sin \phi_r} \quad (15)$$

When the ratio  $\frac{(\sigma_1 - \sigma_3)_f}{2 \sigma_e}$  is plotted against  $\bar{\sigma}_3 / \sigma_e$

the best fitting line drawn through the points gives directly the values of  $b$  and  $\alpha$ , and by applying Equations (14) and (15) an average value of  $\phi_r$  and  $c_r$  is obtained, as shown in figure 7. The equivalent consolidation pressure,  $\sigma_e$ , is obtained from the  $e - \log \sigma$  curve or by applying Equation (11).

#### h) Three-dimensional Plot

By combining Equations (3) and (9) and introducing the coefficient:

$$v = \alpha \sigma_m \exp. (c_c e) \quad (16)$$

the following expression is obtained:

$$s = \mu_o \bar{\sigma}_n + v \exp. (-c_c e) \quad (17)$$

This equation (17) when plotted tri-dimensionally defines a unique surface in space, which Roscoe (1958) called the "Hvorslev surface". Because this plotting is only of an academical interest no more consideration is given here.

### III. EXPERIMENTAL PROCEDURE

#### a) Description of Soil

The soil used in this work was a river backswamp clay from Long Lake  $\pm$  4 miles North of Vicksburg. Its origin and geological formation is fully described by Kolb and Shockley (1957).

The Index Properties of this soil are as follows:

L.L. 69%

P.L. 29%

I.P. 40%

% less than 0.02 mm. 50%  $\pm$

Activity 0.6

Specific gravity 2.74

Classification (U.S.C.S.) CH

#### b) Sample Preparation

The remolded soil was thoroughly mixed with distilled water to form a rather thin slurry, which was deposited in a large oedometer ( $9\frac{1}{2}$ " in diameter). A vacuum was applied to avoid the formation of air bubbles.

The slurry was consolidated to a maximum pressure of 1 K/cm<sup>2</sup> by loads applied in increments. The total time for consolidation was approximately one month. Small blocks were cut from the chunks of consolidated clay, and were stored under moist conditions. Specimens of 1.4" diameter and 8 centimeters long were trimmed out of these blocks to be tested in the triaxial machine.

c) Description of Apparatus

The apparatus used in the testing program was designed and constructed by the Norwegian Geotechnical Institute (see plate I). A detailed description and operational procedures of the equipment have been given by A. Andresen and others (1957).

The principal features of this equipment are described briefly:

- 1) The triaxial cell in which the sample is subjected to an all-round pressure which is controlled by a screw type piston. Pressures less than  $1 \text{ K/cm}^2$  are measured by a mercury manometer. Larger pressures are measured with a Bourdon type manometer.
- 2) The constant pressure cell which keeps the chamber pressure constant.
- 3) The pore pressure device, which permits the measurement of pore pressures in the sample, at constant volume.
- 4) The loading frame, which consists of a screw type loading press, operated by a gear drive unit. The rate of strain can be changed by selecting the proper gear ratio.

Before starting the testing program, all the Bourdon gages were calibrated by means of a "constant load apparatus".

d) Testing Procedure

Techniques for trimming, setting-up and shearing the specimens used in this work are those described by Bishop and Henkel (1957).

To accelerate the time of consolidation, vertical paper filters were placed all-round the sample. The initial water content was the average of three samples taken from the trimmings.

- 1) Consolidation - Immediately after the samples were in place in the triaxial cell, an all-round pressure of  $1 \text{ K/cm}^2$  was applied. Drainage was allowed through a porous stone at the bottom of the sample and the water drained out was measured in a burette. Readings were taken at  $\frac{1}{4}$ ,  $\frac{1}{2}$ , 1, 2, 4, etc. minutes and a time-consolidation curve was plotted as shown in figure 8. Although the primary consolidation was completed in less than one day, a total time of 48 hours was allowed for consolidation.

The chamber pressure was increased in a step-wise method, each increment being twice the previous one. A total time of 48 hours was given to each increment for complete consolidation.

To prepare over-consolidated specimens, a maximum consolidation pressure of  $8 \text{ K/cm}^2$

was obtained by the step-wise procedure, and then the chamber pressure was reduced in the same step-wise manner. Swelling was allowed for two days for each increment in the rebound period.

- 2) Shear tests - When consolidation at the desired effective stress was completed, the triaxial cell was placed in the loading frame. The burette was removed and the pore pressure device connected. The pore pressure apparatus had been previously prepared by removing air bubbles out of the tubing system and saturating it with deaired water.

A gear ratio was selected as to give a rate of strain of approximately 1.25% per hour, To dissolve entrapped air in the specimen, the chamber pressure was raised about 0.3 to 1.0 K/cm<sup>2</sup>, with an average value of 0.5 K/cm<sup>2</sup> for most of the tests. The pore pressure response to increase in chamber pressure was of the order of 10 to 20 minutes.

The undrained tests were run in two different ways:

- 1) For the "consolidated undrained" tests (VBC 2-5 and VBC 2-6) the chamber pressure was kept constant throughout the test and the pore pressures developed were recorded.

- 2) In the "constant volume" tests, the chamber pressure was varied throughout the test and the initial pore pressure was kept constant. The advantages and disadvantages of this type of test have been discussed by Taylor (1948) and Bishop and Henkel (1957).

In both types of tests the deviator stress was measured with a calibrated proving ring with an extensometer to read deflections attached to it. Readings of pore pressure (or chamber pressure) and deviator stress were taken every five divisions of the strain gage (each div. = 0.01 m.m.) at the beginning of the tests, and afterwards were conveniently spaced.

The test was ended a short time after the shear stress started to decrease. The maximum point was determined either by plotting the stress-strain curve, as the test progressed, or by numerical computation of the deviator stress. Time to failure ranged from 4 to 9 hours. The water content at failure was the average for three slices cut at the top, middle, and bottom of the sample (see Table IV). The weight of the entire specimen was also determined at the end of test.

#### IV. RESULTS

##### a) Triaxial Consolidation

A total of 14 triaxial tests were run in this investigation. The specimens were cut from three different batches of soil, prepared as described in section III, b. Batches 2 and 3 had been previously prepared in the Soil Mechanics Laboratory and batch 4 was prepared during the time of this work.

Each test is designed by three letters VBC (Vicksburg Buckshot Clay) and two numbers: the first one refers to the batch number and the second one to the order of test.

From the results of triaxial consolidation, two average curves of water content vs. consolidation were plotted (see figure 9): one curve for specimens of batches 2 and 3, and the other for those of batch 4.

##### b) Triaxial Shearing

From data of the triaxial tests a series of plots was prepared as follows:

- 1) A stress vs. strain curve for each test as shown in figure 10 to 23. In each figure were plotted the equivalent pore pressure, and the effective stress path (Casagrande and Wilson, 1953).

- 2) A plot of the shear strength vs. consolidation pressure is shown in figure 25b. The envelope to the Mohr circles for effective stresses is given in figure 26.
- 3) The water content at failure vs. consolidation pressure is shown in figure 25a (arithmetical scale) and figure 27 (semilogarithmic paper).

From this last figure (27), the equivalent consolidation pressures,  $\sigma_e$ , were determined taking the normally consolidated line of batch 4 as the virgin line for all three batches.

- 4) The relationship between shear strength and void ratio at failure is given in figure 28.
- 5) The effect of over-consolidation on the pore pressure parameter is shown in figure 29.
- 6) In Table III are summarized the most important data obtained from the triaxial test run in this work.

c) Hvorslev Parameters

From figure 30, which was plotted according to the method described in section II,g, the shear strength parameters were found to be:

$$\phi = 16^\circ$$

$$\delta e = \frac{c_r}{e} = 0.08$$

## V. DISCUSSION OF RESULTS

Considerations concerning the physical meaning of true cohesion and true friction were given in Section II a, and summarized in Tables I and II.

As the strength characteristics of clays are due only to the action of these parameters, it may be said that any factor which by its direct action on those physical components changes their magnitude affects consequently the shear strength of cohesive soils.

Taylor (1948) lists eight fundamental and nine inter-related factors affecting the shearing resistance of clays. Below an attempt is made to summarize these factors by studying their effects on the fundamental parameters.

### A. Factors Affecting the Strength Parameters

#### a) Effective Stress

The effective stress is defined by the following equation:

$$\bar{\sigma} = \sigma - u \quad (18)$$

where  $\bar{\sigma}$  = effective stress

$\sigma$  = total stress

$u$  = pore pressure

The effect of capillary forces should be included in the above equation when dealing with partially saturated soils. (For example, Aitchison, 1957)

Effective stress is thought to be the most important factor of shear strength. Its direct

effect on the fundamental parameters,  $\phi_r$  and  $c_r$ , is not really known, but if some consideration is given to the factors summarized in Tables I and II its important role on the magnitude of the angles of microfriction  $\phi_m$  and macrofriction  $\phi_M$  can readily be seen. It also affects true cohesion by changing the number and strength of bonds formed between particles in direct contact. Due to this effect, a direct relationship between effective stresses and shear strength should be expected, as it is shown mathematically by Equations (1) and (4) and experimentally by results plotted in figure 26.

b) Void Ratio

The void ratio is an indirect measure of the separation between particles which affects the true cohesion by varying the magnitude of the electrical forces between them and also the degree of interaction of viscous water layers.

The effect of the void ratio on the strength of soils is given mathematically by Equation (17) and corroborated experimentally by results plotted in figure 28 following the procedure suggested by Rutledge (1947).

c) Time

The effect of time on the shear strength of cohesive soils is most difficult to understand.

Taylor (1953) reports an increase of 21% in undrained shear strength from samples consolidated in 40 minutes to those consolidated in 10,000 minutes. Casagrande (1951) reports reductions as large as 80% when soils were sheared undrained over a relatively long period of time.

The increase of shear strength with time of consolidation could be due to the formation of new bonds when particles rearrange themselves into more stable positions, or just changes in effective stresses which occur during shear. The effect of time of consolidation could be worthy of consideration when dealing with geological time-scale (for instance, consolidation of clays into shales), but it is not of much importance for "normal" times of consolidation.

The time dependence of the shear strength of clays due to the rate of loading could be due to the action of the viscous layers of adsorbed water whereby clays behave in a viscous-plastic manner which is characteristic of the Bingham body studied in Rheology (for example, Reiner, 1960). Taylor (1948) suggested this visco-plastic behavior when he introduced the rate of shearing in Equation (15.4).

d) Structure

Structure is considered here as the arrangement and degree of packing of the soil particles. It has a direct effect on the angle of macrofriction  $\phi_M$ .

The effect of arrangement of particles was studied experimentally by Hvorslev (1938) and was theorized by Lambe (1958).

e) Exchangeable Cation

The effect of the exchangeable cation is directly upon the water-cation and particle-cation linkages, changing to a certain extent the true cohesion.

The action of exchangeable actions on the true cohesion of clays has been reported by Michaels (1950) to be of minor importance.

f) Temperature

Temperature has a direct effect on the electrical forces acting between particles (Lambe, 1958) and also on the viscosity of the adsorbed water. The combined action of both effects can change the cohesive parameter and may be the frictional parameter, hence a variation in the shear strength of clays should be expected.

## B. Discussion of the Physical Properties of Strength Parameters

One of the main arguments against the concept of true friction and true cohesion as it has been considered here, seems to be the fact that it is difficult, if not impossible, to differentiate each one from the other. For example, Rosenquist (1959) and Trollope and Chan (1960) suggest that there is no substantial difference between cohesion and friction, because both factors are basically due to electrical forces or bonds formed between grain surfaces in contact. This might be true if all the matter is considered as being only concentrated energy, but when "real" particles (i.e. mineral grains) are formed, this action is not true in all cases.

Shear strength of sand is mostly due to the friction between grains (microfriction) and also to the geometrical interaction between wavy surfaces (macrofriction). If change of volume occurs during shear, a new factor has to be taken into account and this is the energy necessary to cause the change in volume. This factor is called "dilatancy". It is also true that some electrical attraction between sand grains in contact exists and that the formation of bonds (crystallization) at the surfaces in contact can be expected. However, it is thought that the magnitude of these factors is small when compared with the main factors named above.

In soils possessing cohesion, the separation of friction and cohesion would be difficult if tangential forces and displacements are considered only, but it is not difficult to visualize the difference between the two parameters, if cohesion is considered to be the strength resistance to normal components (see Table II), and friction the resistance to tangential or shearing forces (Table I).

It can be argued that in a direct shear test only tangential displacements occur, thereby the mobilized force is due only to the action of frictional forces. This is true if the surface where the shear force is being applied is perfectly smooth, but due to the roughnesses of soil particles at the surface and the irregularity of the plane of failure the surfaces in contact tend to separate, mobilizing in this way the cohesive component of shear strength. If any change of volume occurs during this normal movement of surfaces "dilatancy" occurs and an energy correction must be added to the frictional component (Gibson, 1953). If no change of volume occurs, the tendency to dilate throws negative stresses in the pore water affecting the magnitude of the effective stress and also the cohesive component (see Table II, item 4).

In Powder Metallurgy (for example, Goetzl, 1949), the behavior of two different kinds of powdered metals is studied: hard metals (chromium, steel, etc.) and soft metals (iron,

lead, etc.). The powder of the first group of metals does not acquire coherent properties even when submitted to very high pressures, because contacts are limited to rather few points which do not provide enough bondings to form a coherent material. It is felt that the behavior of sands under pressure is similar to that of hard metals, if proper consideration is given to the magnitude of the applied forces in shear tests and also to the difference in physical properties of the sand grains as compared to those of metal powder grains.

When soft metals are compacted at similar pressures strongly coherent compacts can be obtained. This behavior is similar to the action of plastic soils even if the type and magnitude of the cohesive forces are quite different.

Clays have been subjected to tension forces (for example, Haefeli, 1950) and shown to have strength which can be considered of cohesive nature only.

In summary, it is felt that true cohesion and true friction have different meaning although basically both properties are due to electrical forces only. Their exact contribution to the shear strength of soils is not well known at present, but some idea of their magnitude can be obtained by the method developed by Hvorslev in 1937.

## C. Discussion of Test Results

### 1) Consolidation

Figure 9 shows the equilibrium water content at a given consolidation pressure. Tests of batches 2 and 3 fall on the same curve, while those of batch 4 fell on a curve similar in shape but moved to the right. This behavior could be ascribed to great differences in water content of the slurry prepared for batch 4 and that for batches 2 and 3 (for example, Bjerrum, 1954). However, it is known that the initial water content of all three batches was about the same, therefore, this unusual behavior must be due to another reason. One factor that might be important is the difference in ambient temperature when the batches of soil were under consolidation. The other factors are not known. In Section II f, the equivalent consolidation pressure was defined as the "all around pressure which produces the actual water content in a normally consolidated clay". As mentioned above and also by inspection of figure 9, it is observed that two normally consolidated lines are available: one for tests of batches 2 and 3, the other for tests of batch 4. A problem seems to be presented here, because some one of the two curves must be chosen as the virgin line. The selection of the adequate

line was based on the following reasoning:  
Bjerrum (1954) found that if two samples of the same soil were remolded with different initial water contents and consolidated, two different consolidation curves result. He also showed that the shear strength curves in a Mohr's diagram were different. Therefore, it is possible to get the same water content at different consolidation pressures by using normally consolidated samples. Based on this principle the virgin consolidation line for tests of batch 4 was chosen as the basic line to find the equivalent consolidation pressure.

2) Shearing

a) Lag in pore pressure build-up

The time-lag of pore pressure due to an increase in chamber pressure varied between 10 and 20 minutes (see Section III d). Taylor (1944) reported time lags of the order of 30 to 50 minutes in larger specimens, but no satisfactory explanation was given for this phenomenon. It is felt that this delaying in pore-pressure build-up is due mostly to the inadequacy of the pore pressure measuring technique, but no conclusive results have been obtained at present.

b) Shear strength in terms of total stresses

As shown in figures 25b and 26 the shear strength in terms of effective stresses is higher than the shear strength in terms of total stresses but their ratio varies significantly from the two to one ratio suggested by Taylor (1948).

It is also observed that the consolidated undrained angle,  $\varphi_{cu}$ , is the same as the true friction angle  $\varphi_r = 16^\circ$ . This close similarity between these parameters is also shown for 4 clays in Table I of the paper by Gibson (1953).

If figures 25a and 25b are studied more carefully some interesting conclusions can be obtained:

- i) For the same consolidation pressure the water content at failure is greater for tests of batch 4 than that for batches 2 and 3.
- ii) The shear strength line for samples of batch 4 plots below the line for samples of batches 2 and 3, which shows the importance of the water content at failure on the shear strength of soils. The value of the Hvorslev approach is definitely shown by studying tests 2-6 and 4-12: In figure 24 both tests show

the same normal effective stress in the plane of failure at failure,  $\bar{\sigma}_{ff}$ , but as they have different water content at failure (figure 25a) they should have different shear strength which is also true as depicted in figure 25b (see also Table III). Therefore, the shear strength of clays cannot be expressed only in terms of effective stress but also in terms of the water content (or void ratio) at failure. This example is an experimental proof, qualitative at least, of the validity of equation 17.

iii) There is some scattering in the points for low consolidation pressures for preconsolidated samples. This cannot be explained satisfactorily unless it is assumed that a small amount of air was admitted when the sample was swelling. It is observed also that the shear strength which corresponds to those points shows some scattering but that the points with lower water content consistently show higher shear strength which means that the scattering of points was not due to mechanical or arithmetical errors as might be initially expected.

c) Shear strength in terms of effective stresses

Variation of shear strength with effective stresses is given by the envelopes to the Mohr

circles as depicted in figure 26. Shear strength in terms of the effective stress on the plane of failure at failure is given by the end points of the vector curves shown in figure 24. Two envelopes to the Mohr circles were drawn (figure 26): One for normally consolidated samples and another for overconsolidated samples, but the spread between these two lines seems to be large, because not much difference in shear strength is expected when results of shear tests are plotted in terms of effective stress. (For example, Bishop and Henkel, 1957). The rather large separation of the two lines can be due to the increase in shear strength caused by the use of too high rate of strain. Although normally consolidated and overconsolidated samples were sheared at the same strain-rate, the pore pressure build-up could be lower for overconsolidated specimens because of their lower permeability (Bishop and Henkel, 1957).

The locus of effective stress on the plane of failure in a triaxial test is called "vector curve" (Casagrande and Wilson, 1953) or "effective stress path" (Whitman, 1959). In figure 24 appear the vector curves for all triaxial

tests run in this work. If these curves are analyzed with some detail, several interesting conclusions can be reached. For instance, the curves can be divided in two groups: The first one is formed by those curves heading to the left (decreasing effective stress) which correspond to normally consolidated samples. The second group is formed by those curves heading to the right (increasing effective stress). These correspond to overconsolidated soil.

The reason for this difference is due to the fact that normally consolidated soils tend to decrease in volume when sheared, but as the volume is kept constant, positive pore pressures are developed to counteract this tendency. Positive pore pressures reduce the effective stresses as expressed by Equation 18.

The behavior of highly overconsolidated soils is quite different: they tend to increase in volume when sheared and as the volume is kept constant, negative pore pressures are developed to counteract the tendency to increase in volume. According to Equation 18 negative pore pressures increase effective stresses.

In slightly and medium overconsolidated soils the positive pore pressure change is less than in normally consolidated soils.

Because of the nature of pore pressure build-up described above, normally consolidated samples produce vector curves of the same shape, but overconsolidated samples produce vector curves which show different shapes depending on the overconsolidation ratio.

At this point it is worthy to discuss the concept of the "critical void ratio line" (CVR) as proposed by Roscoe (1958).

The concept of "critical void ratio" for sands has two different meanings as suggested by Taylor (1948). One is associated with volume changes in drained tests and the other with effective stresses in undrained tests.

In undrained tests there is no change in void ratio because the volume is kept constant during shearing, but the change in effective stress due to variations in pore pressures brings the sample to a condition in which the void ratio at failure becomes a "constant-volume critical void ratio".

When dealing with clays Roscoe (1958) proposed the CVR line as one unique line to which

the stress paths of drained and undrained tests converge. In other words, the end points of the vector curves for overconsolidated soils should head back to the normally consolidated line. This theory as proposed by Roscoe (1958) is restricted to soils with an overconsolidation ratio equal or lesser than eight.

As it is shown in figure 24, there is a slight trend of some of the vector curves to head back to the normally consolidated line which makes an angle of  $23^{\circ}$  with the "X" axis.

To test the validity of the concept put forward by Roscoe, the sample of test 3-7 (figure 14) was sheared to a strain beyond failure as depicted in the stress-strain curve. Although the end point did not fall on the normally consolidated line it is close to it and it is possible that this would happen if the test had been carried out to even larger strains. However, the result of this test is not conclusive because the relaxation test described below could have some effect on the pore pressures developed at end of test.

In summary, the existence of a CVR line could simplify considerably the shear strength behavior were it not for the limitations imposed

on the amount of overconsolidation and the large strains required by moderately overconsolidated soils to reach the CVR line. Some authors (for example, Henkel, 1958) believe that the CVR line exists for drained tests on heavily overconsolidated samples but that this behavior does not apply to heavily overconsolidated soils tested undrained. A relaxation test was done when running test 3-7 and it will be described briefly here: The test was carried on in a normal way until peak deviator stress was reached. At this point the motor was stopped and the sample let remain during 13 hours under constant strain. As it is depicted in figure 14 the deviator stress "relaxed" about 10% and the pore pressure became more negative by about 25%. When test was started again, the deviator stress rapidly reached almost its value before relaxation and the water pressure became less negative.

The change in pore pressure when stresses relaxed shows the tendency of the sample to increase in volume. As the volume remained constant the pore pressure became more negative to offset this volume increase tendency.

As relaxation of stresses occur only in

materials that have viscous properties (for example, Reiner, 1960), the above test throws some light on the viscous-plastic behavior of clays, as it was suggested in item A of this section.

d) Shear strength in terms of void ratio at failure

Rutledge (1947) suggested that a unique relationship existed between shear strength and void ratio at failure and that this relationship was independent of the type of test and the minor principal stress. In figure 28 the results of all triaxial tests were plotted in this way and 4 distinct curves could be drawn through the points, depending if the soil was normally consolidated or overconsolidated or if samples were taken from batches 2 and 3, or from batch 4.

It is clearly shown by this example, that shear strength is dependent not only on the void ratio (or water content) at failure, but also on the stress history of the soil before shearing.

It is worthy to mention here that the water content (or void ratio) at failure is the average for the whole sample and that this water content varies along the height of the sample

as shown in Table IV, in which are presented the results of water content determinations at top, middle and bottom of sample. Because of the inconsistency of results no definite conclusions could be reached on the distribution of water in sample and its effect on the undrained shear strength of clays.

e) Pore pressures

A summary of the pore pressures developed at failure in triaxial tests can be shown if use is made of the parameters A and B as defined by Skempton (1954). Because the clay considered here was saturated the parameter B is unity and the value of A at failure is given by:

$$A_f = \frac{U_f}{\sigma_1 - \sigma_3}$$

where  $U_f$  = equivalent pore pressure at failure

$\sigma_1 - \sigma_3$  = deviation stress

In figure 29 values of  $A_f$  were plotted against the logarithm of the overconsolidation ratio  $\frac{\sigma_m}{\sigma_c}$ . This figure shows that for samples with an overconsolidation ratio of one, the pore pressure at failure is approximately equal to the deviator stress and for those samples with overconsolidation ratios larger than 6 negative pore pressures are developed.

f) True friction and mineralogy of clays

To compare the magnitude of the friction parameter  $\phi_r$  of Vicksburg Buckshot Clay with values of  $\phi_r$  for other clays, figure 1 was prepared by plotting values of  $\phi_r$  versus the Plasticity Index for soils of Table I (Gibson, 1953) and Table 7.2 (Bjerrum, 1954). It is readily observed that despite large scattering of points, there is a definite trend by which the true friction decreases with the plasticity of clays. The value of  $\phi_r = 16^\circ$  for VBC plots on the average line drawn through the points.

The relationship of the friction parameter and the Plasticity Index of clays could be explained if proper consideration is given to the role of the angle of microfriction  $\phi_M$  (Table I) and the physical properties of the adsorbed water on the plastic behavior of cohesive soils.

## VI. CONCLUSIONS

From the study of the physical properties of true friction and true cohesion, it is felt that a sharp boundary does not exist between these two parameters, but that it is possible to get some knowledge of their role in shear strength of cohesive soils if proper consideration is given to the way in which each one acts when a soil is subjected to shear stresses.

After the study of results of triaxial tests it can be concluded that when shearing action is taking place at normal strain rates the main factors contributing to shear strength of cohesive soils are the effective stress at failure and the void ratio at failure, but that due to the fact that some problems exist in which shearing action is very fast (atomic blasts, earthquakes), or very slow (long term stability of slopes) time is also an important factor in shear strength.

Therefore, it can be said that in general the principal factors of shear strength are:

- 1) Effective stress at failure
- 2) Void ratio at failure
- 3) Time to failure.

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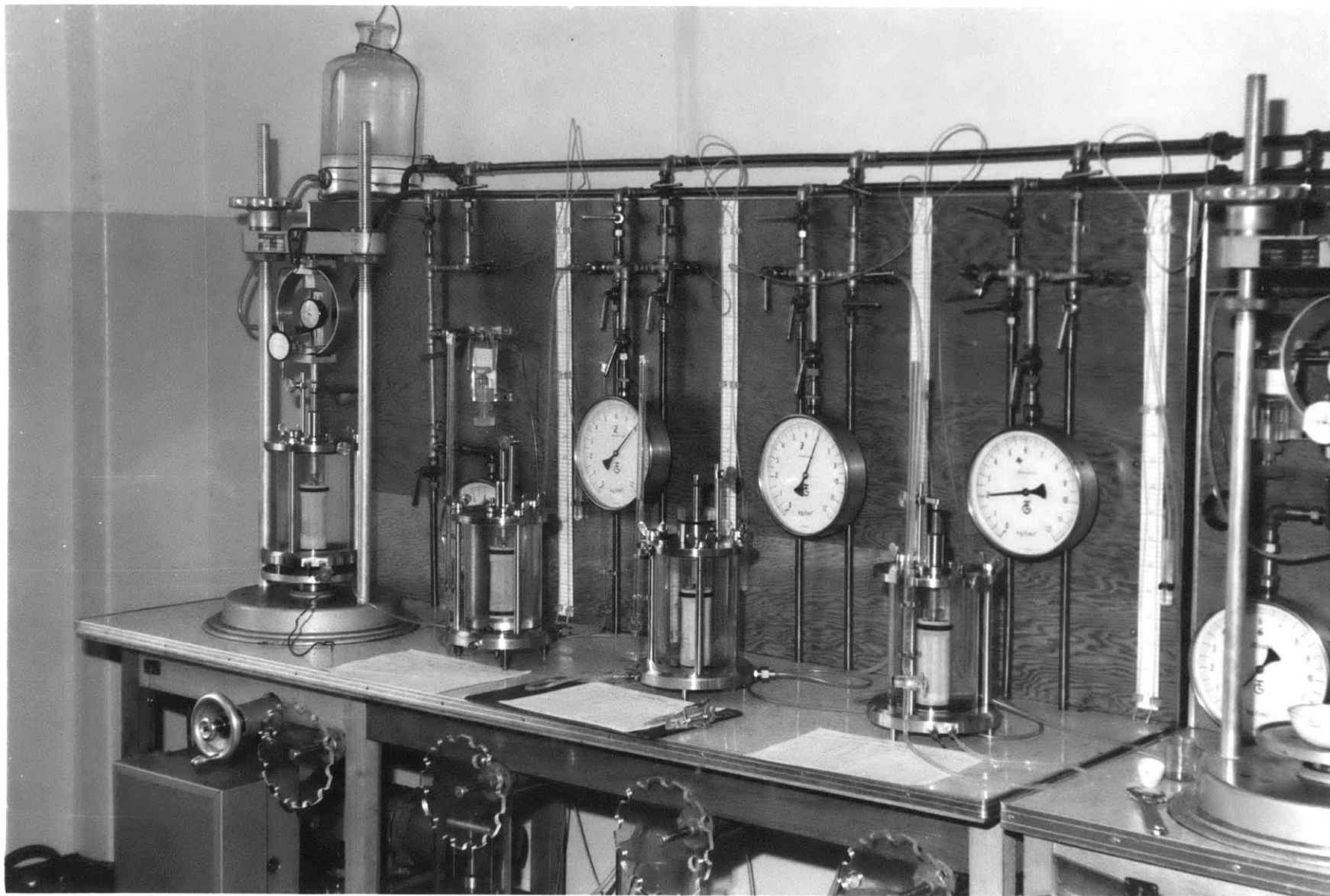
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PLATES, TABLES AND DIAGRAMS



P L A T E I

TABLE I  
SUMMARY OF FACTORS AFFECTING TRUE FRICTION

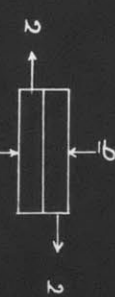

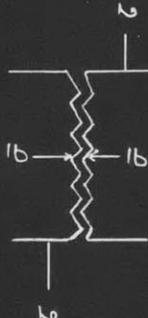
TRUE FRICTION DIVIDED IN:	SCHEMATIC REPRESENTATION:	ACTION DUE TO:
MICROFRICTION	<p data-bbox="535 1386 584 1533"><i>Non-plastic soils</i></p>  <p data-bbox="763 1386 812 1533"><i>Plastic soils</i></p>  <p data-bbox="714 924 860 1155"> <math>\tau = \bar{\sigma} \tan \varphi_m</math>  <math>\varphi_m = \tan^{-1} \frac{\tau}{\bar{\sigma}}</math> </p>	<p data-bbox="633 273 763 819">"Roughness" of particle surfaces in contact, atomic bonds. Net electrical attraction (effect on )</p> <p data-bbox="795 252 860 819">Bonding of highly viscous absorbed water. Net electrical attraction.</p>
MACROFRICTION	 <p data-bbox="1169 924 1299 1113"> <math>\tau = \bar{\sigma} \tan \varphi_M</math>  <math>\varphi_M = \tan^{-1} \frac{\tau}{\bar{\sigma}}</math> </p>	<p data-bbox="1153 273 1299 819">Resistance to shearing forces due to geometric interaction of wavy surfaces. When volume change is permitted the dilatant component must be introduced.</p>

TABLE II  
SUMMARY OF FACTORS AFFECTING TRUE COHESION

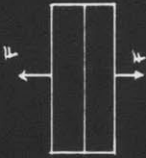
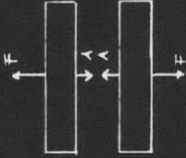
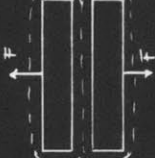

DIVISION	SCHEMATIC REPRESENTATION:	ACTION DUE TO:
1) BOND		Bonds formed between atoms at surface of particles in direct contact.
2) ELECTRICAL ATTRACTION		Net electrical attraction between particles not in contact, but very close together.
3) ADSORBED WATER		Adhesion between layers of highly viscous adsorbed water.
4) PARTICLE-CATION AND WATER-CATION LINKAGE		Link formed by exchangeable cations between mineral grains. Linkage formed by pore water molecules and exchangeable cations (weak link).

TABLE III

## SUMMARY OF TRIAXIAL TESTS ON VICKSBURG BUCKSHOT CLAY

TEST NO.	CONSOL. PRESSURE K/cm <sup>2</sup>	STRENGTH $\frac{1}{2}(\sigma_1 - \sigma_3)$ K/cm <sup>2</sup>	1) STRAIN AT FAILURE %	TIME TO FAILURE (hours)	WATER CONTENT		EQUIVALENT CONSOL. PRESS. Pe	STRESSES AT FAILURE			
					2) INITIAL %	FINAL %		TOTAL			
								$\sigma_1$	$\sigma_3$	$\sigma_1$	$\sigma_3$
2-3	O.C. $\frac{1}{2}$	0.76	8.0	7	46.2	36.1	5.1	2.02	0.50	2.28	0.76
2-4	O.C. 1	1.08	4.5	5	43.3	34.7	6.3	2.12	1.00	2.12	1.00
2-5	O.C. 1	1.16	9.0	7 $\frac{1}{2}$	43.4	34.2	6.0	3.32	1.00	3.36	1.04
2-6	N.C. 8	2.24	7.5	7	47.8	29.1	9.2	12.48	8.00	8.13	3.65
3-7	O.C. $\frac{1}{4}$	0.795	4.0	9	50.5	34.6	6.0	1.84	0.25	2.05	0.46
3-8	N.C. 4	1.26	4.5	4	52.4	36.0	5.2	6.52	4.00	4.42	1.90
3-9	O.C. 2	1.64	5.5	5 $\frac{1}{2}$	51.2	29.5	-	6.30	3.00	5.78	2.48
3-10	N.C. 2	0.68	3.7	8	51.8	42.0	3.2	3.36	2.00	2.24	0.88
3-11	O.C. 4	1.77	5.7	5	50.5	30.5	8.2	7.54	4.00	6.34	2.80
4-12	N.C. 8	2.14	5.5	5	54.2	31.8	8.0	12.28	8.00	7.93	3.65
4-13	O.C. 6	1.96	8.7	7 $\frac{1}{2}$	53.1	32.1	7.7	9.93	6.00	6.86	2.93
4-14	N.C. 6 $\frac{1}{2}$	1.79	5.0	4 $\frac{1}{2}$	52.1	33.8	6.5	9.88	6.50	6.70	3.12
4-15	O.C. 2	1.38	5.0	4	53.2	34.6	6.0	4.76	2.00	4.51	1.75
4-16	N.C. 5	1.47	6.3	5 $\frac{1}{2}$	52.2	36.4	5.0	7.94	5.00	5.16	2.22

1) Failure = Maximum Deviator Stress

2) From Trimmings

3) Air admitted when sample rebounded.

Results not reliable.

N.C. = Normally Consolidated

O.C. = Over Consolidated

TABLE IV

VARIATION OF WATER CONTENT AT FAILURE WITH HEIGHT OF SAMPLE

TEST NO.	TOP		MIDDLE		BOTTOM		AVERAGE WATER CONTENT %
	WATER CONTENT %	% OF WATER CONTENT AT MIDDLE	WATER CONTENT %	% OF WATER CONTENT AT MIDDLE	WATER CONTENT %	% OF WATER CONTENT AT MIDDLE	
2 - 3	36.1	99.2	36.4	100.0	35.8	98.4	36.1
2 - 4	35.1	100.0	35.1	100.0	34.0	96.7	34.7
2 - 5	33.5	94.0	35.7	100.0	33.5	94.0	34.2
2 - 6	29.2	101.0	28.9	100.0	29.1	100.8	29.1
3 - 7	33.9	98.5	34.4	100.0	35.5	103.0	34.6
3 - 8	36.4	102.0	35.6	100.0	36.1	101.2	36.0
3 - 9	29.5	-	-	-	29.5	-	29.5
3 - 10	42.1	-	-	-	41.9	-	42.0
3 - 11	30.6	100.0	30.6	100.0	30.3	99.0	30.5
4 - 12	32.0	102.0	31.4	100.0	31.9	101.5	31.8
4 - 13	32.0	97.2	32.9	100.0	31.4	95.5	32.1
4 - 14	33.8	101.8	33.2	100.0	34.5	104.0	33.8
4 - 15	34.1	98.0	34.8	100.0	35.0	100.5	34.6
4 - 16	36.4	101.2	35.9	100.0	37.0	103.0	36.4

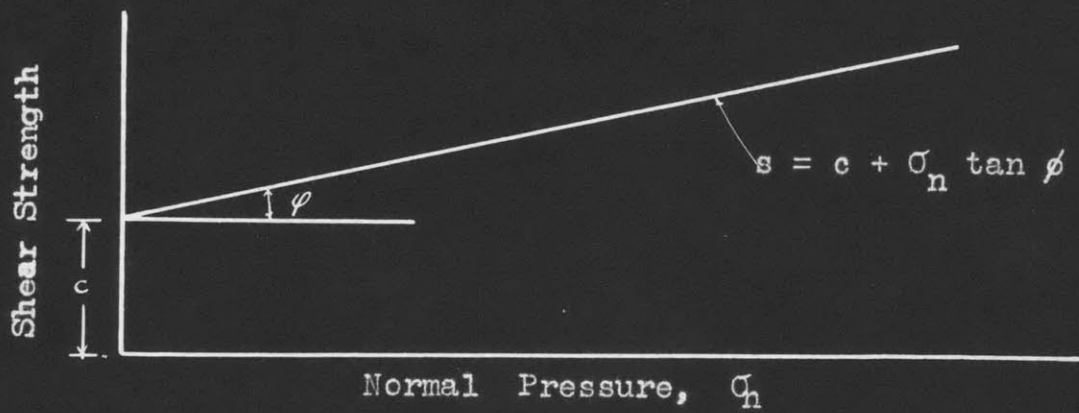


Fig. 1. Coulomb's Condition of failure

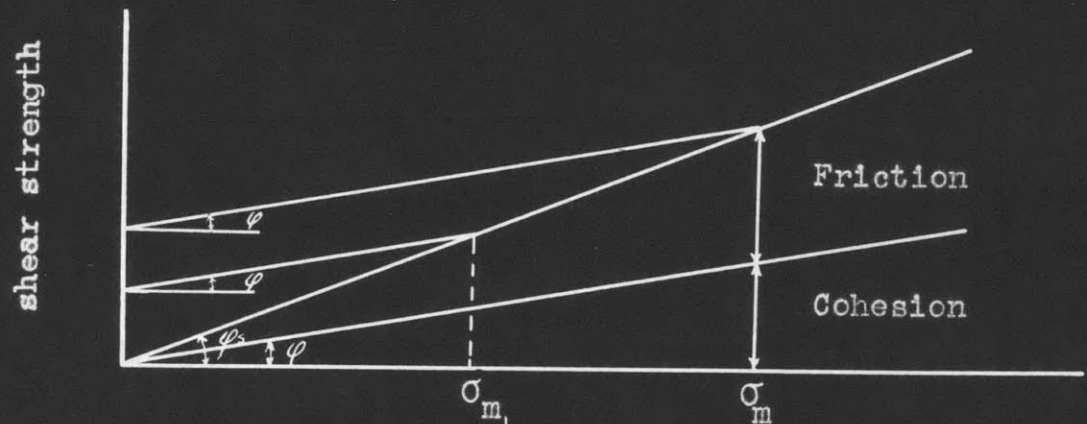


Fig. 2. Krey-Tiedeman failure condition

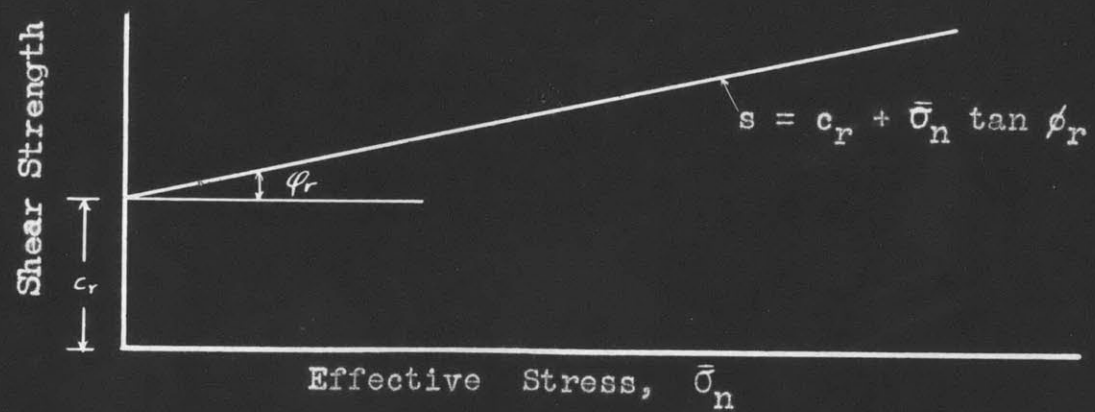


Fig.3. Hvorslev failure condition

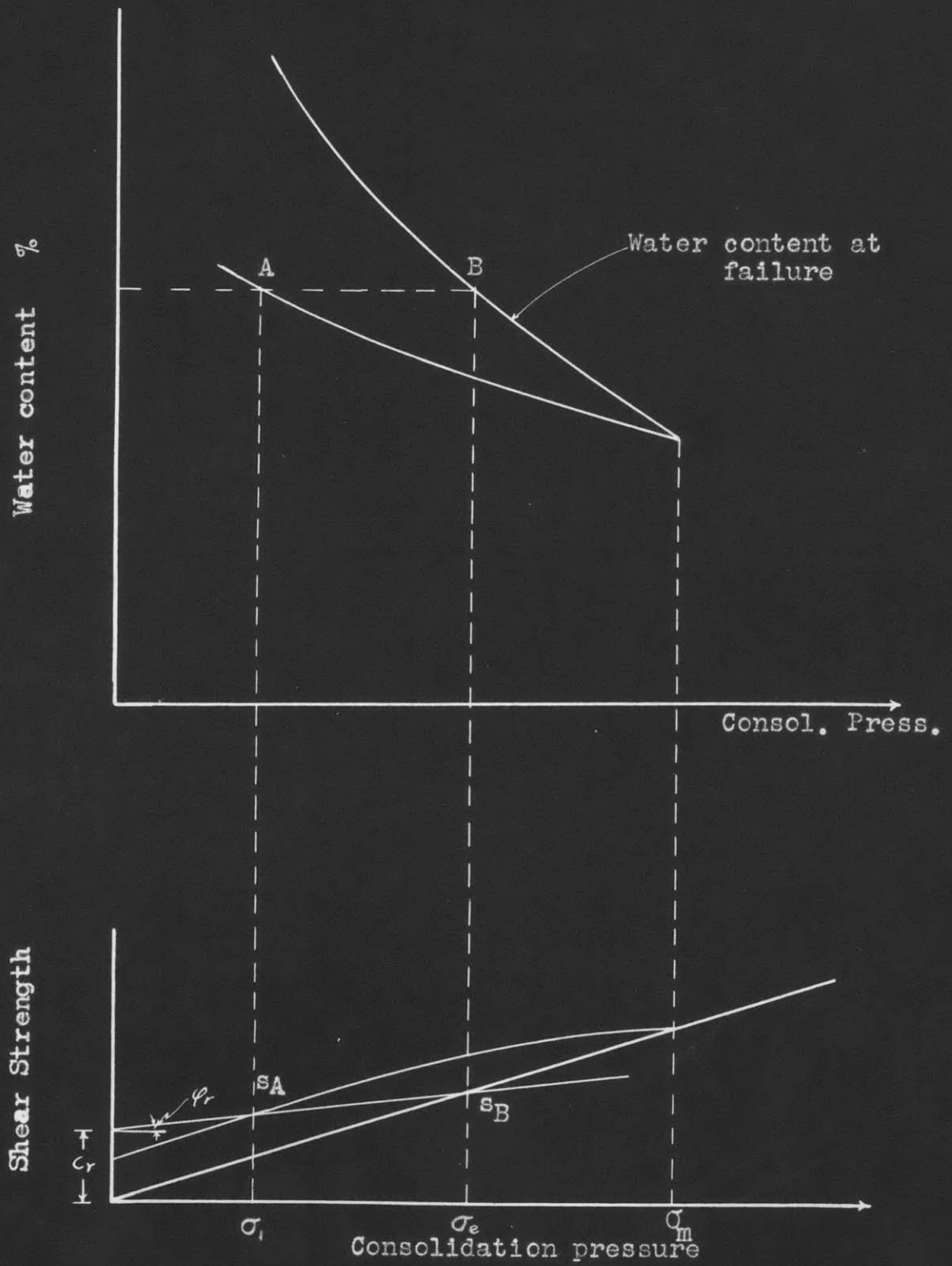
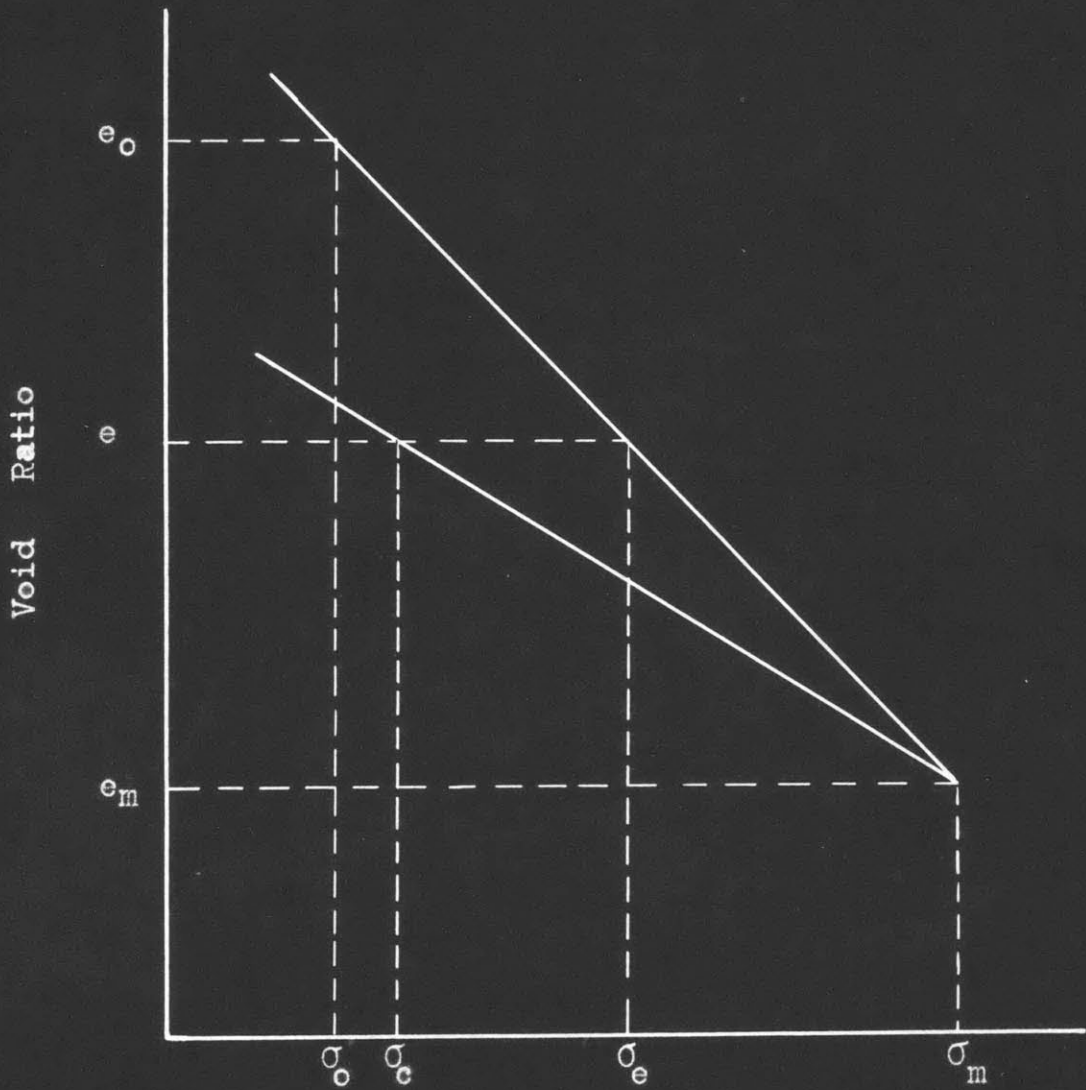


Fig.4. Terzaghi Construction



Log. consolidation pressure  
 Fig. 5. Idealized consolidation curve

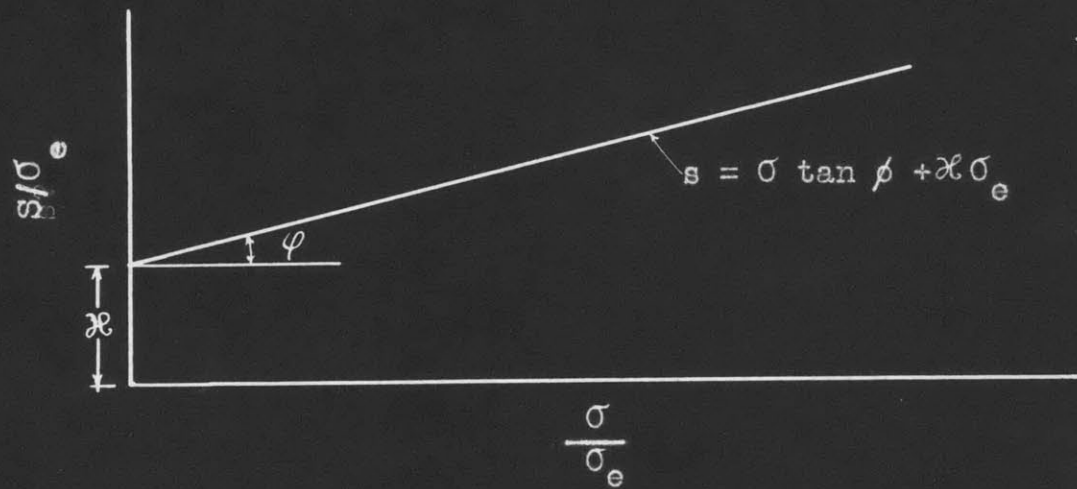


Fig. 6. Dimensionless plot for drained tests

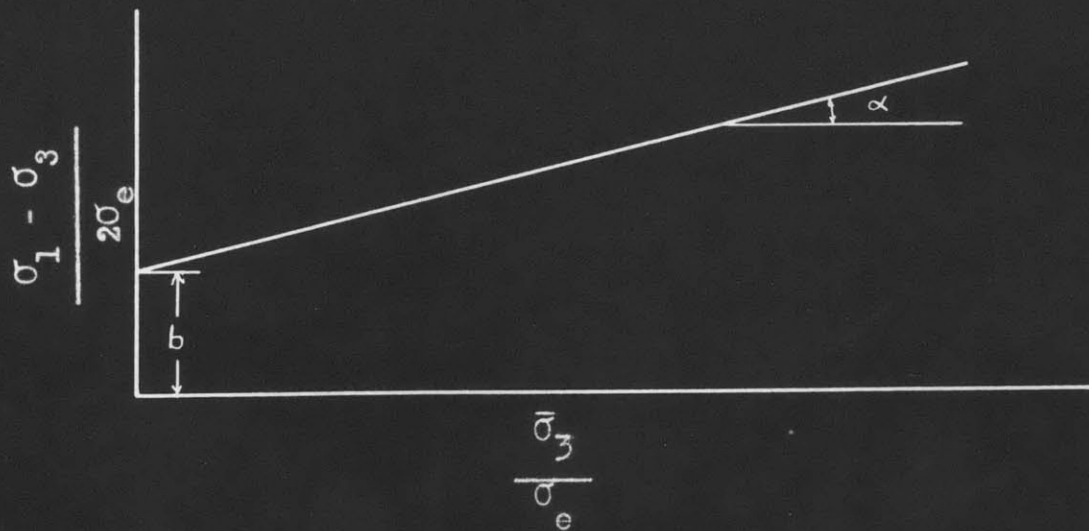
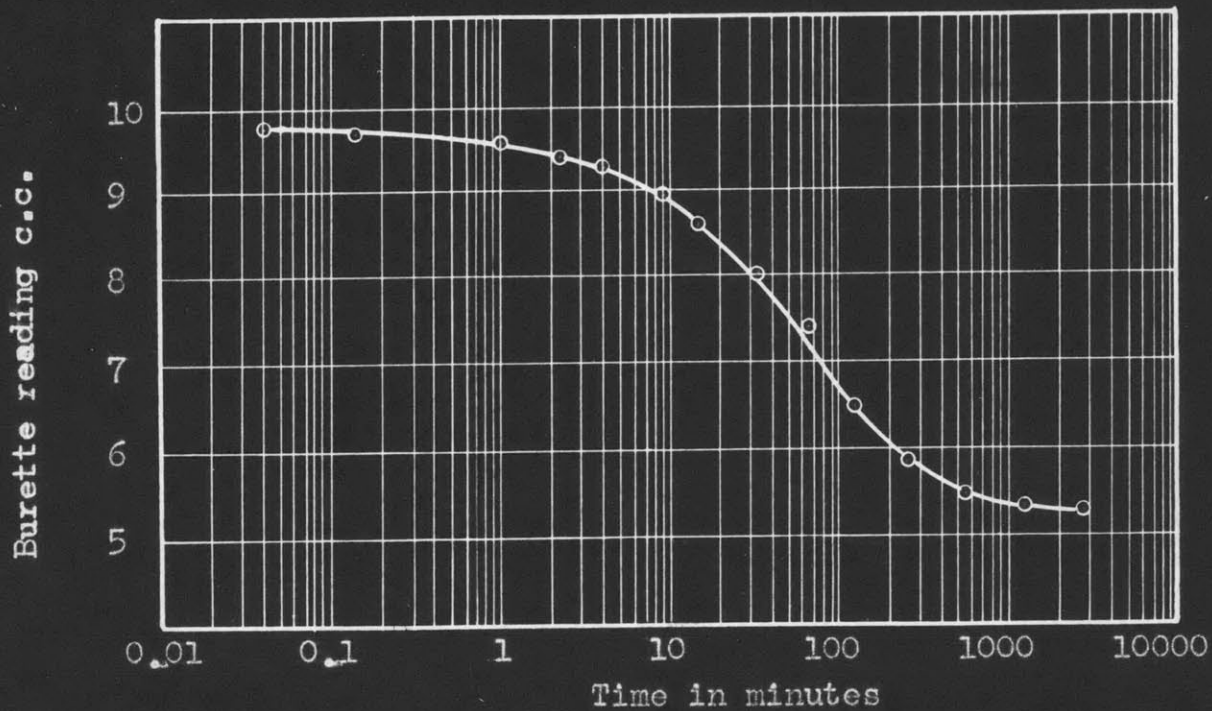
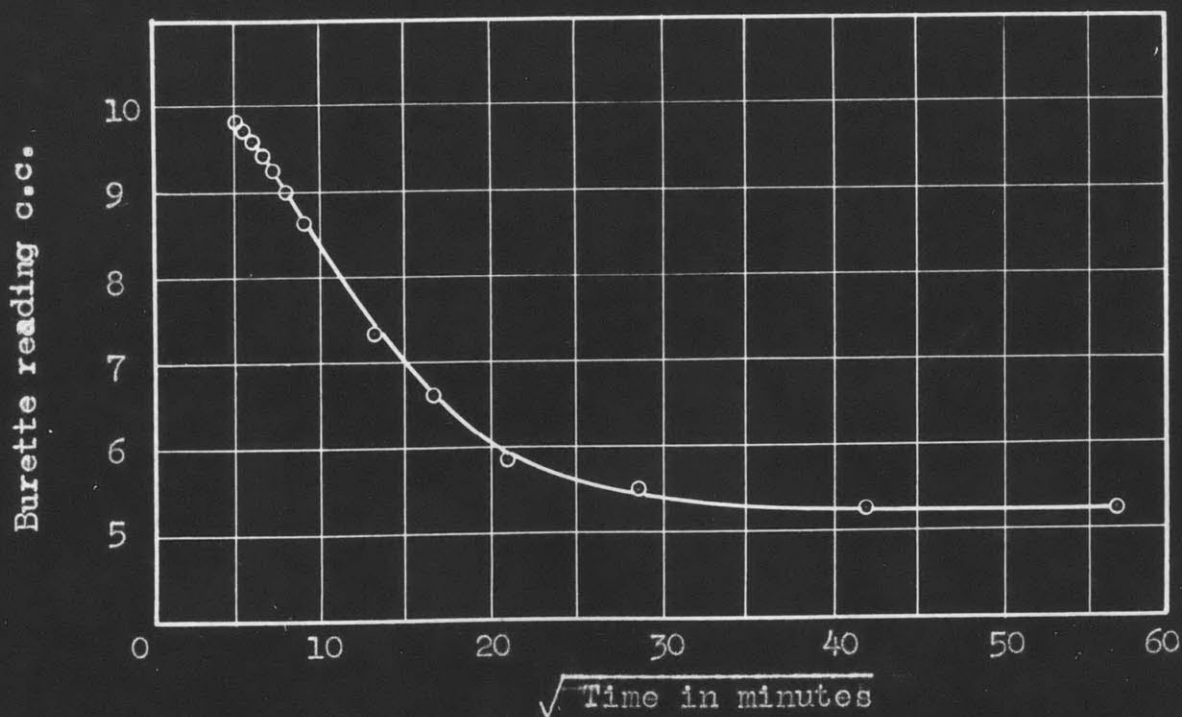


Fig. 7. Dimensionless plot for Triaxial tests



a) Logarithm of time plot



b) Square root of time plot

Fig. 8. Time-consolidation curves

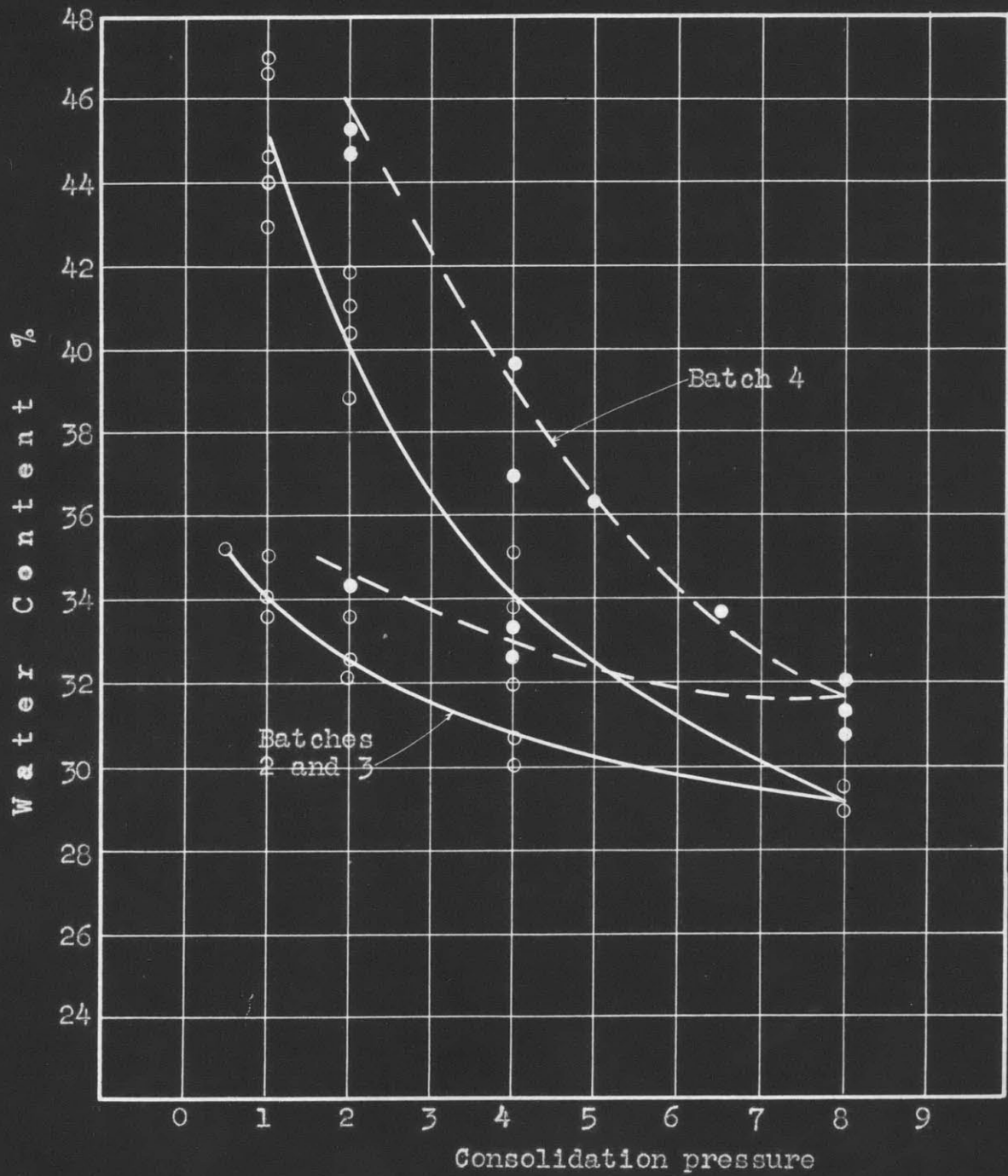


Fig. 9. Consolidation curves

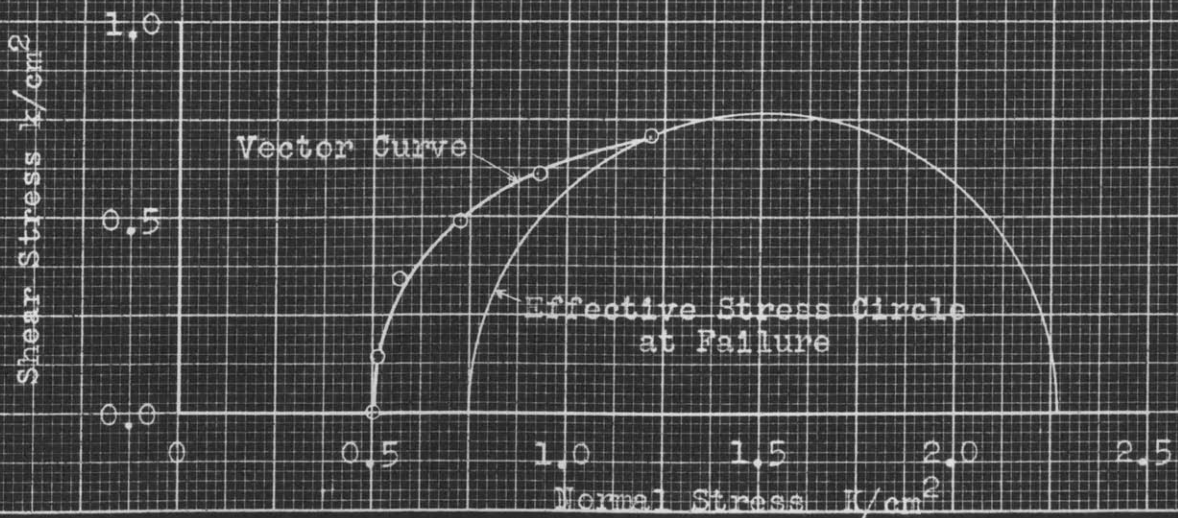
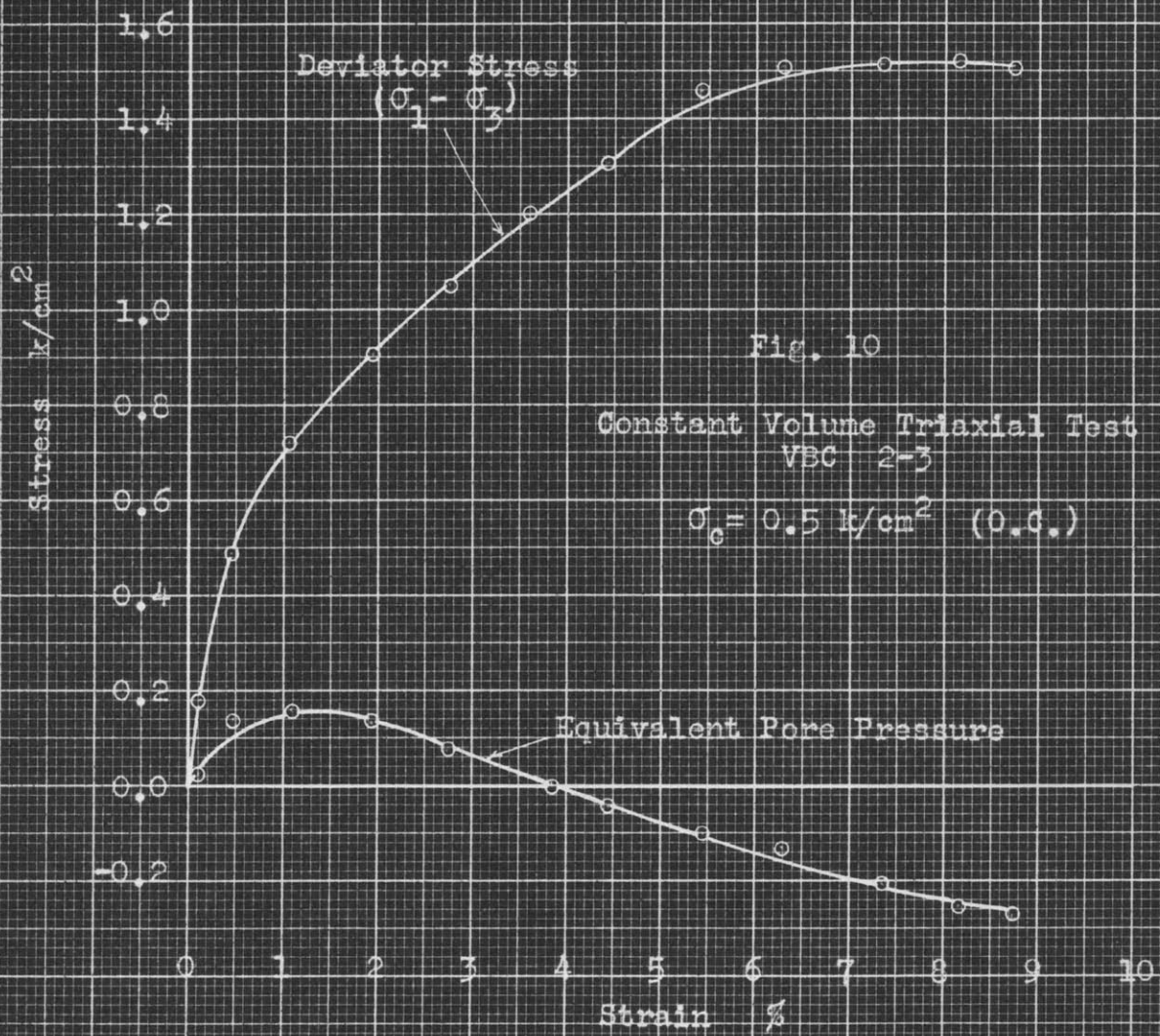
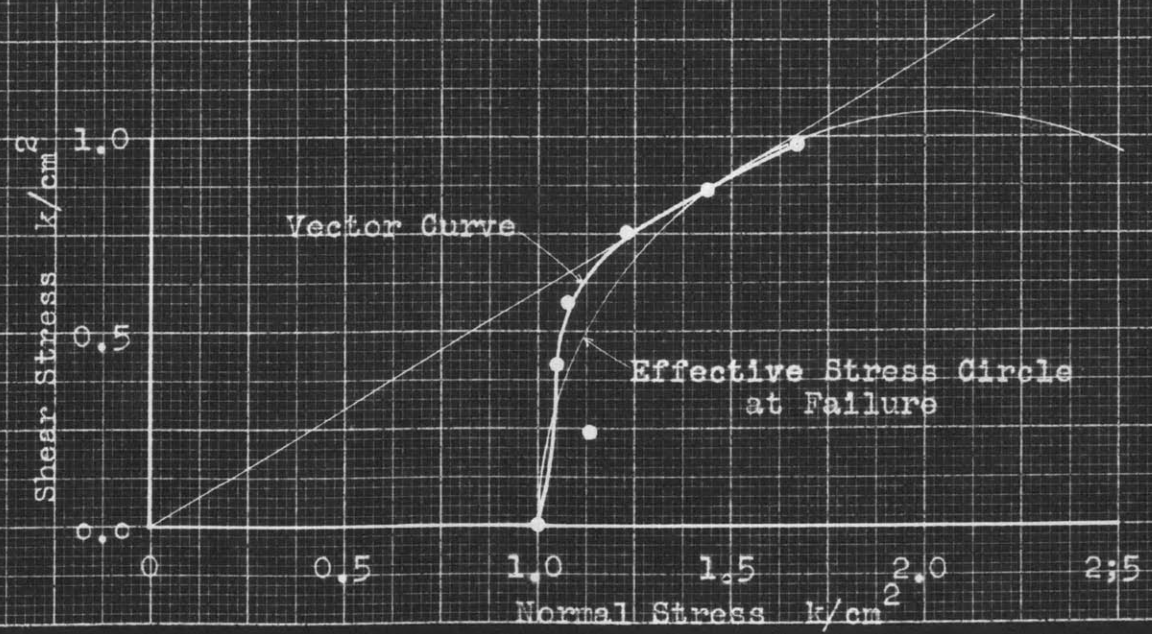
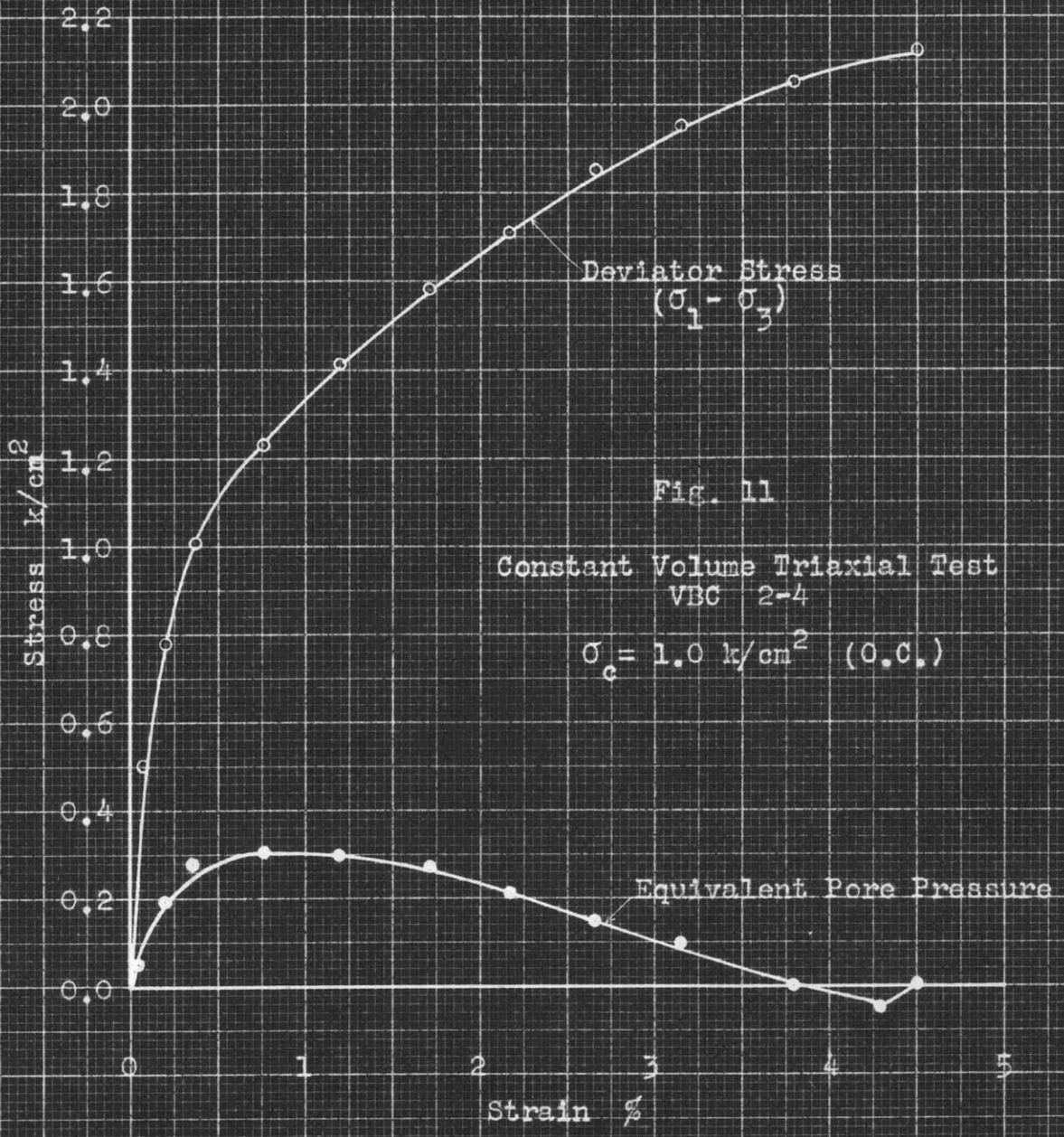
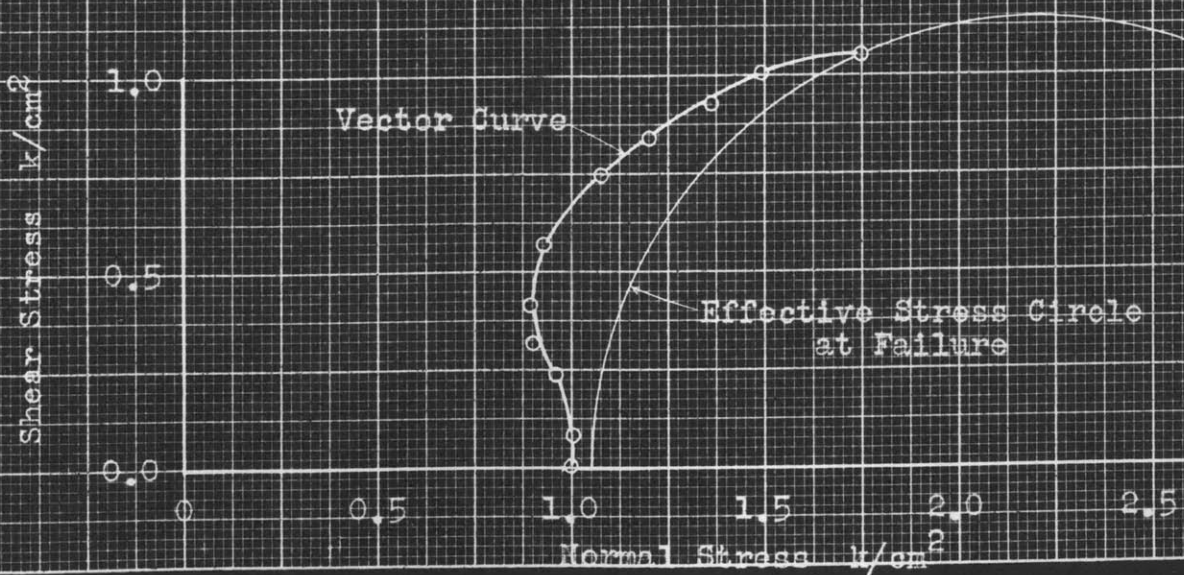
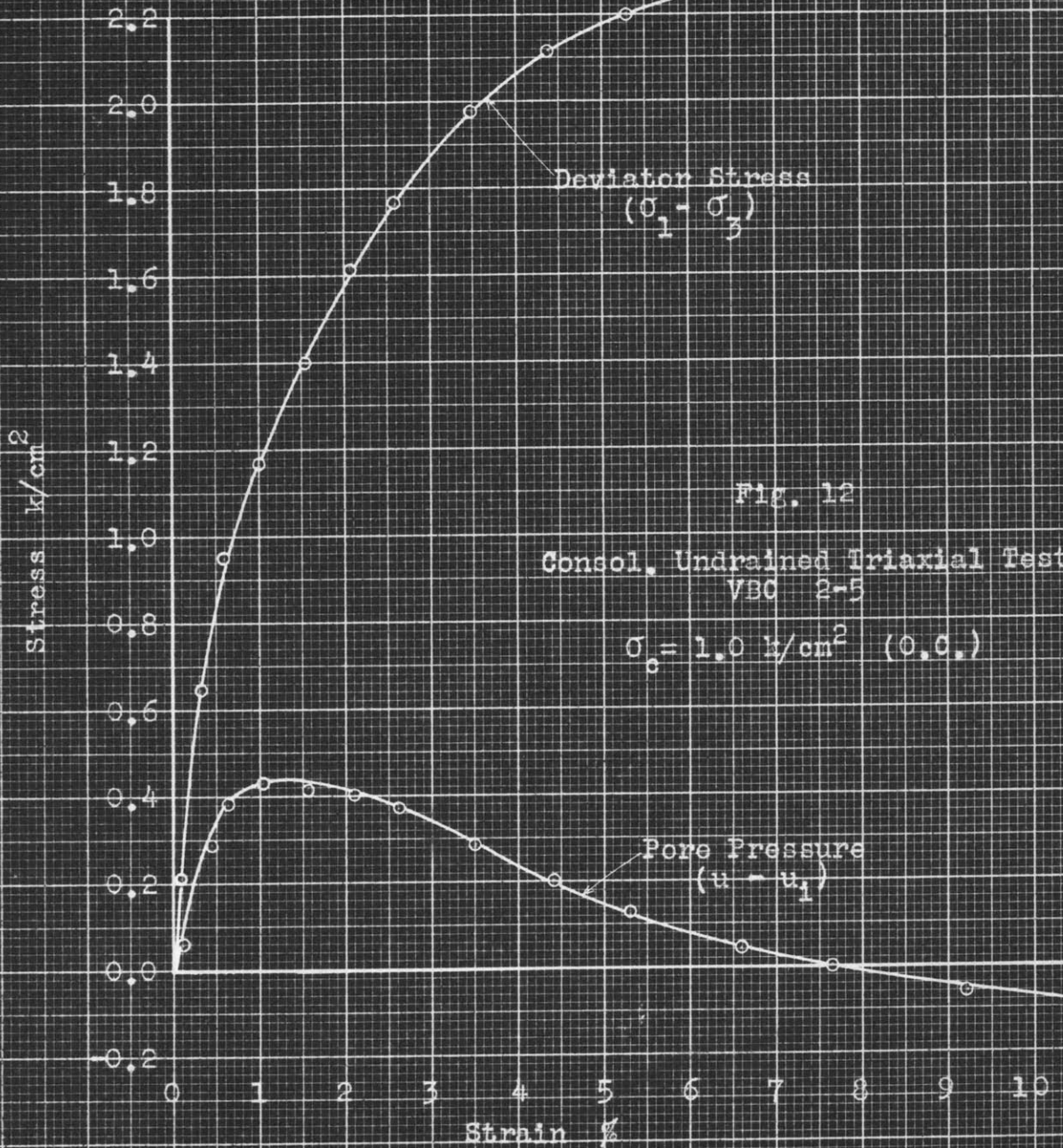
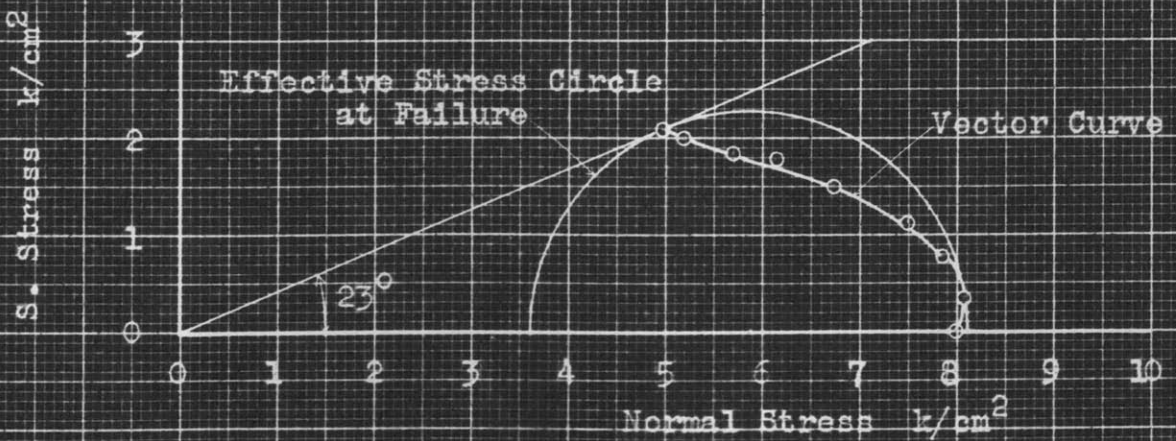
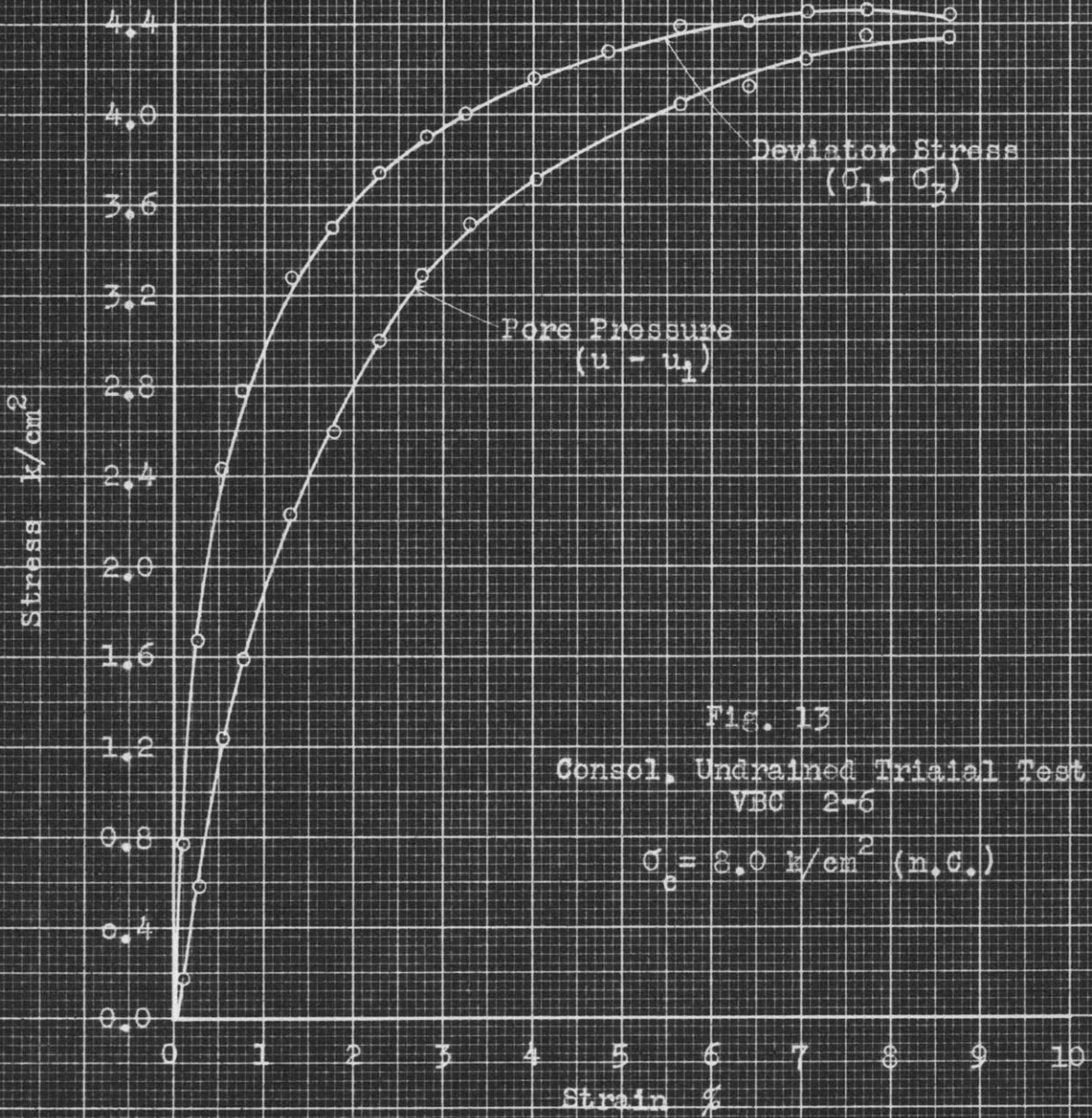


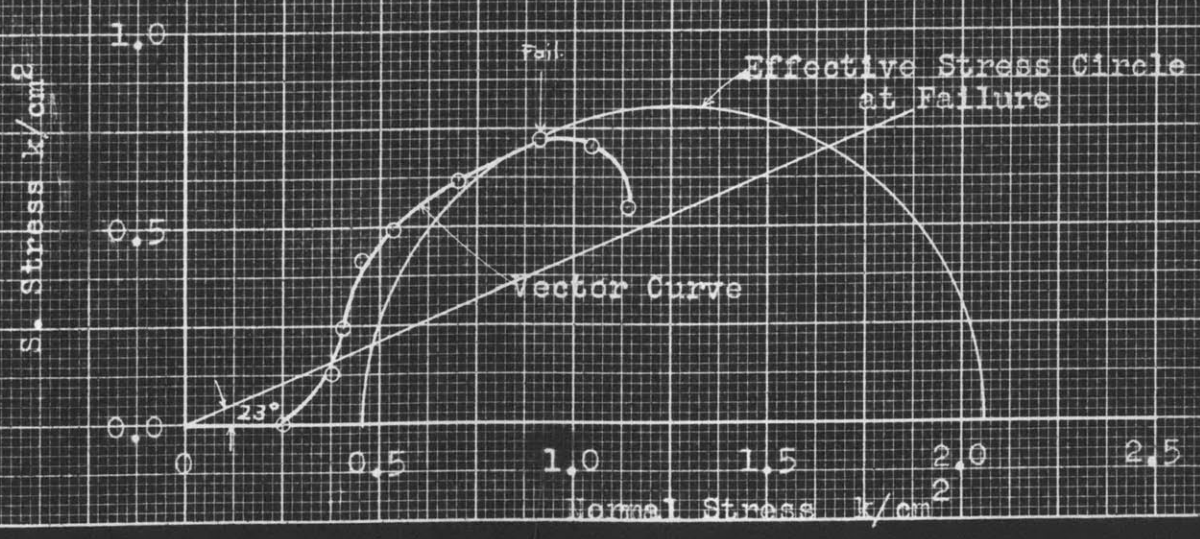
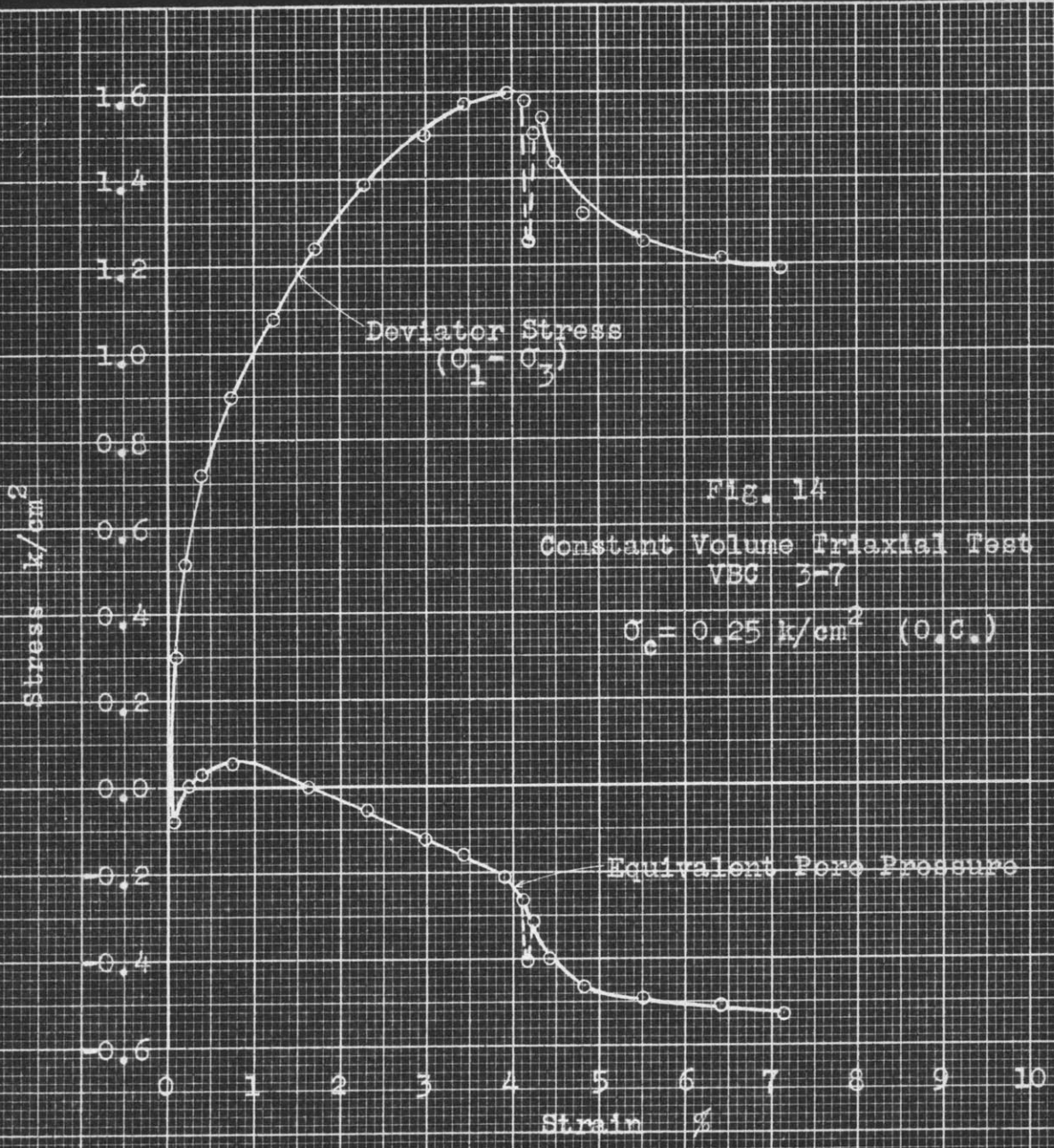
PHOTO BY STATE FROM 55-110  
 KUFFEL & ESSER CO. MADE IN U.S.A.



K&E 10 X 10 TO THE 1/2 INCH 300-310 MADE IN U.S.A. KEUFFEL & ESSER CO.







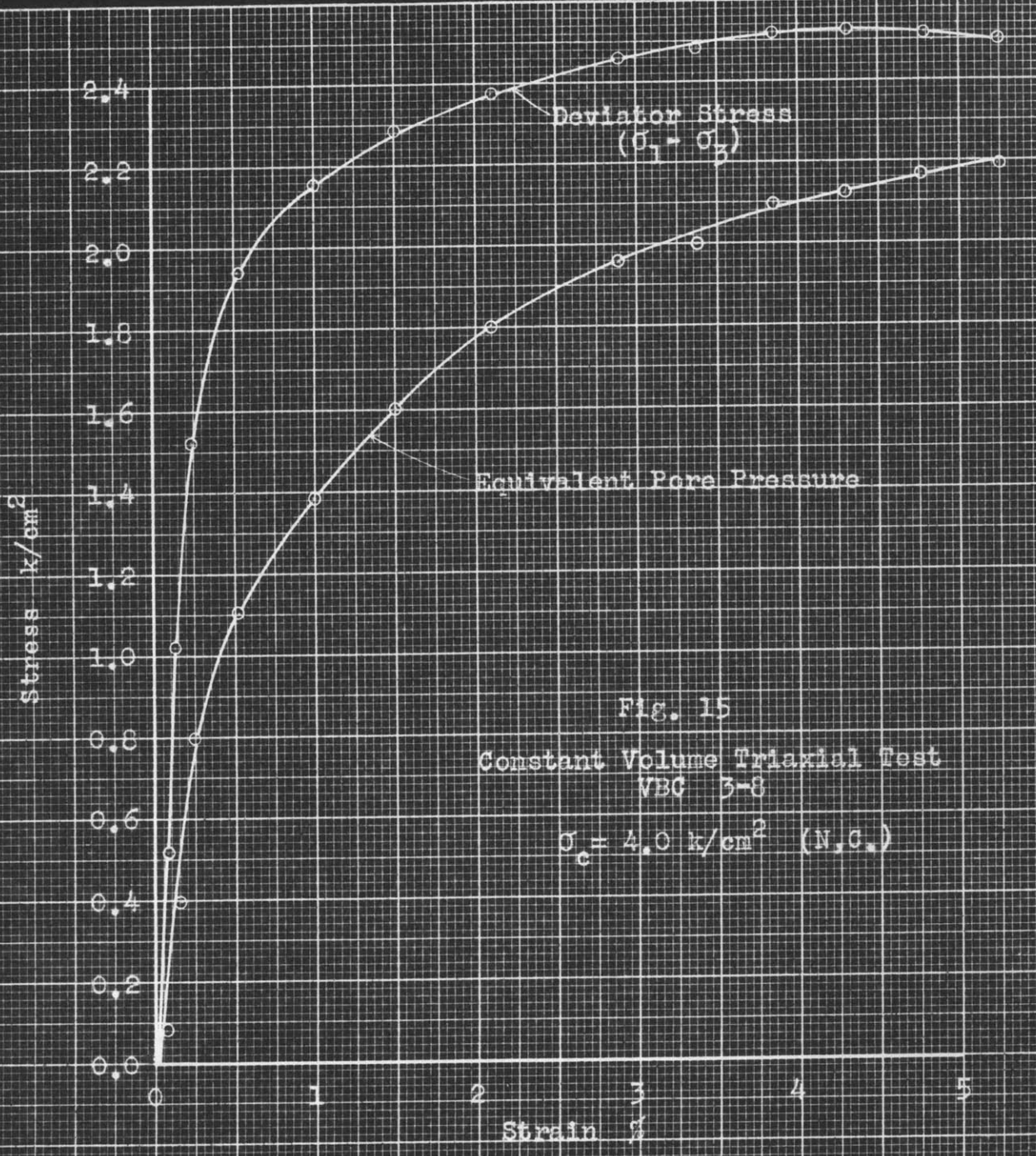
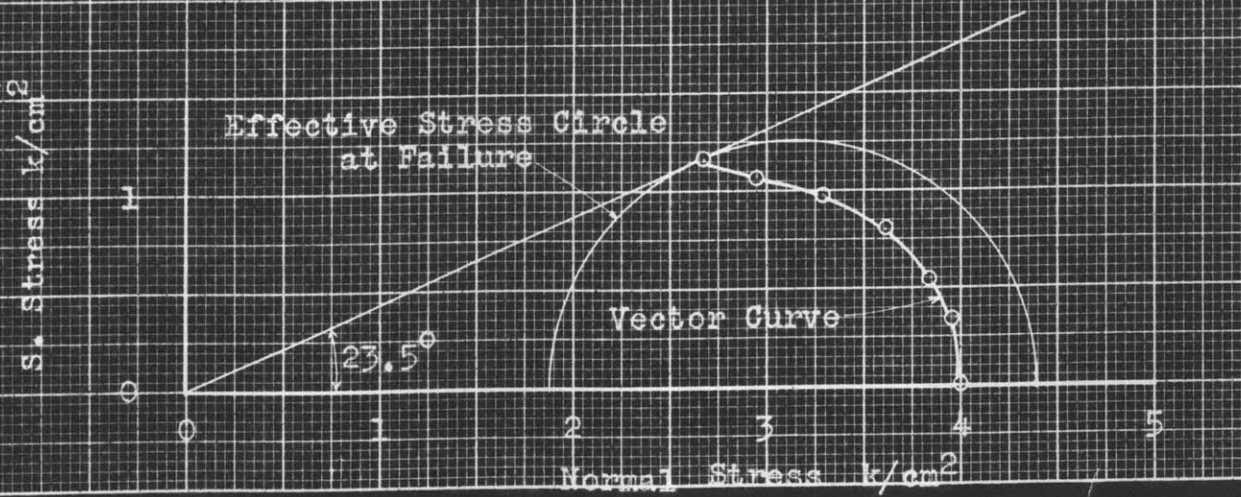
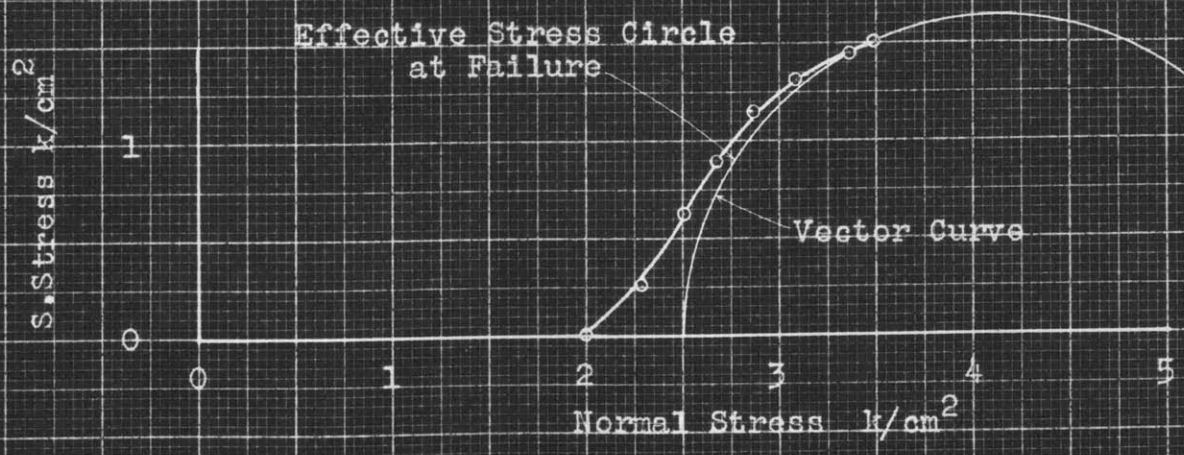
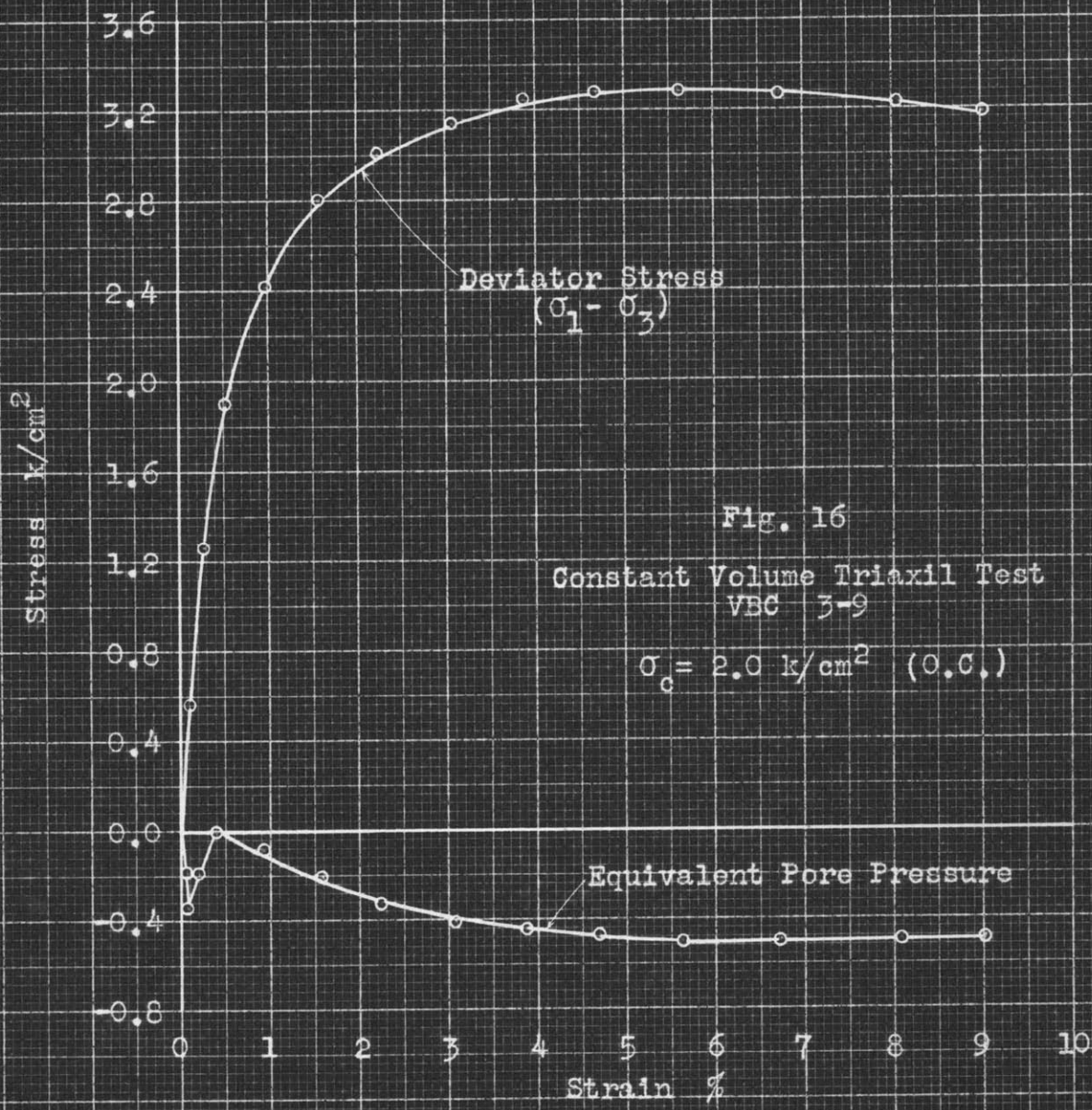
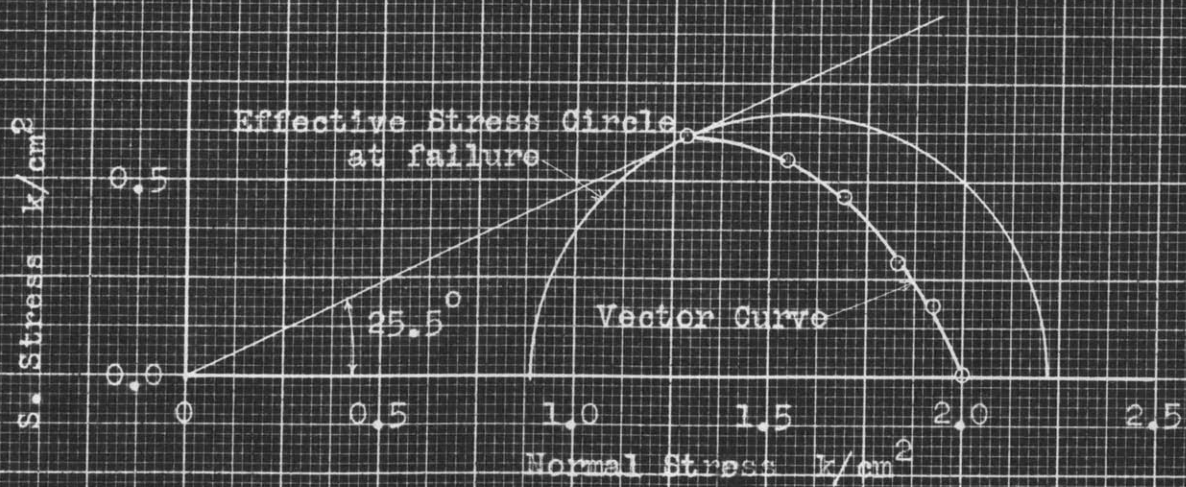
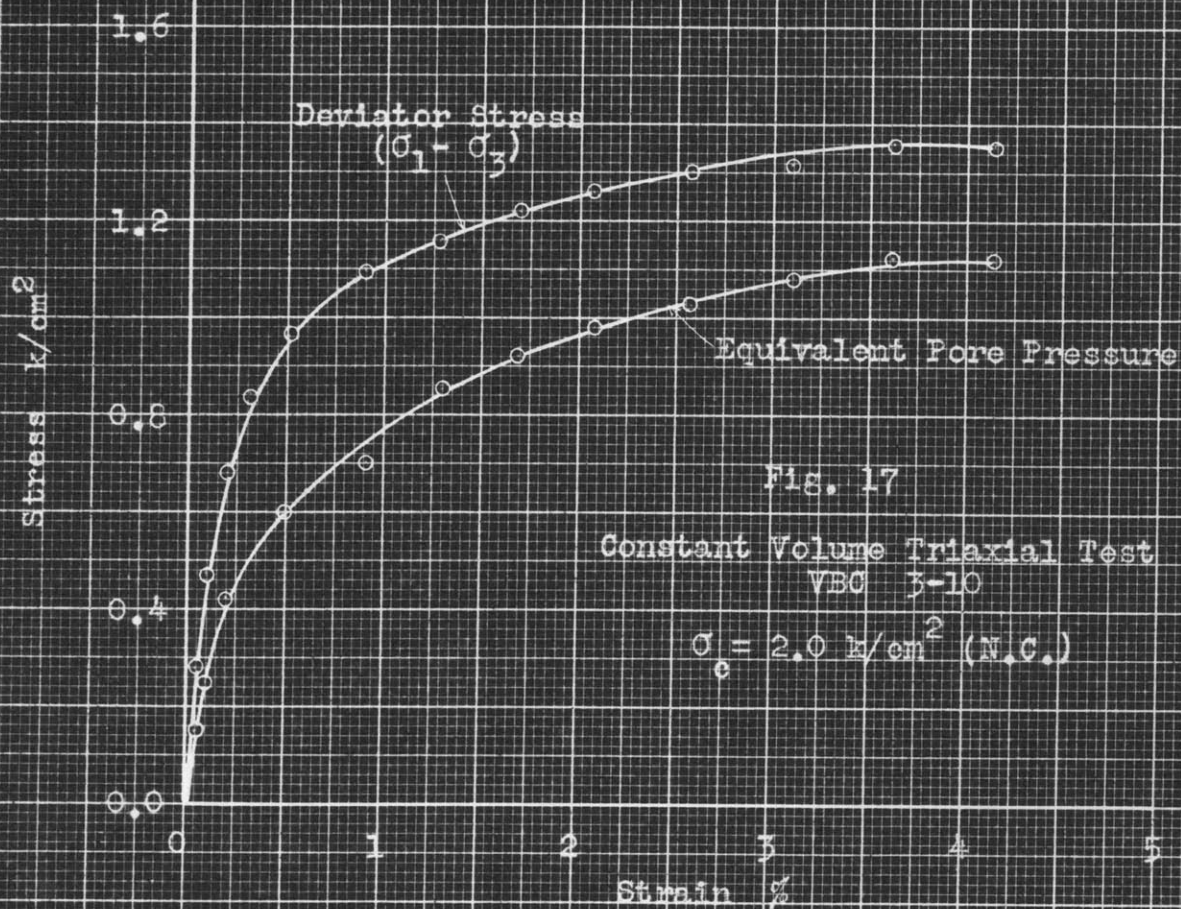
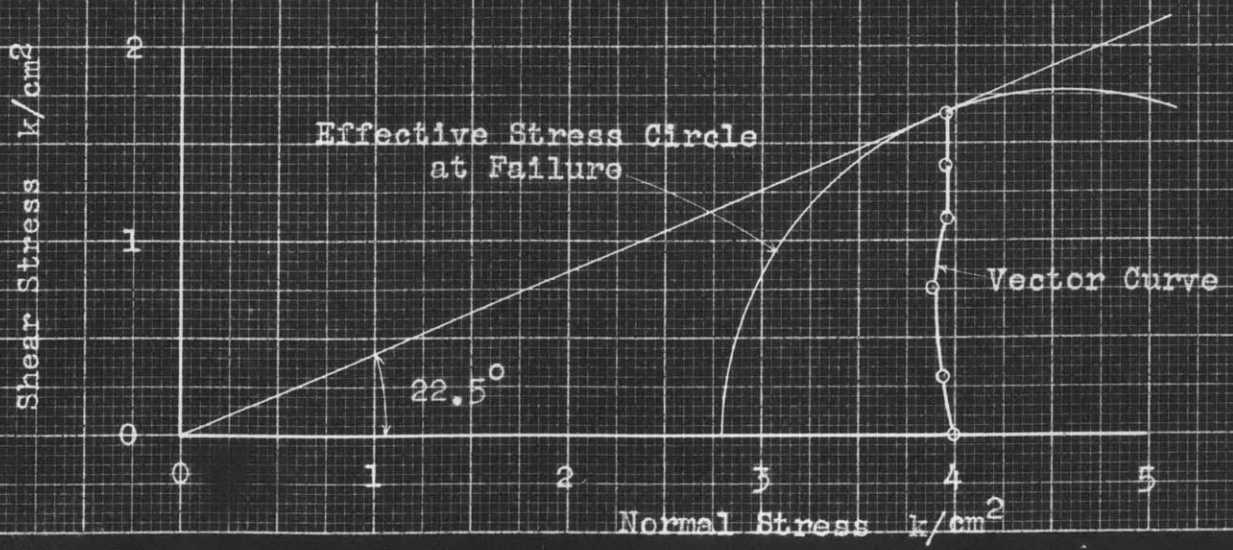
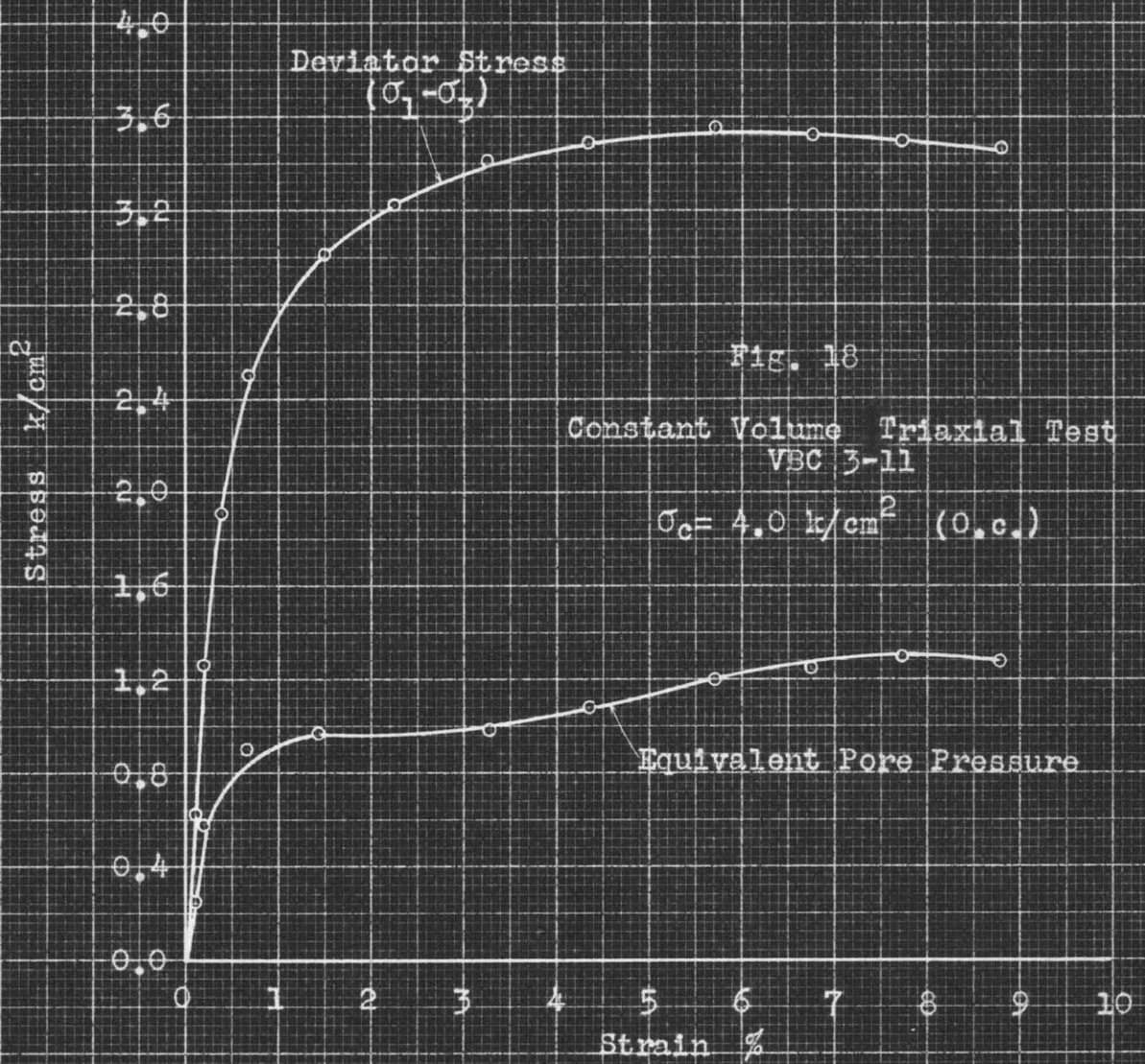


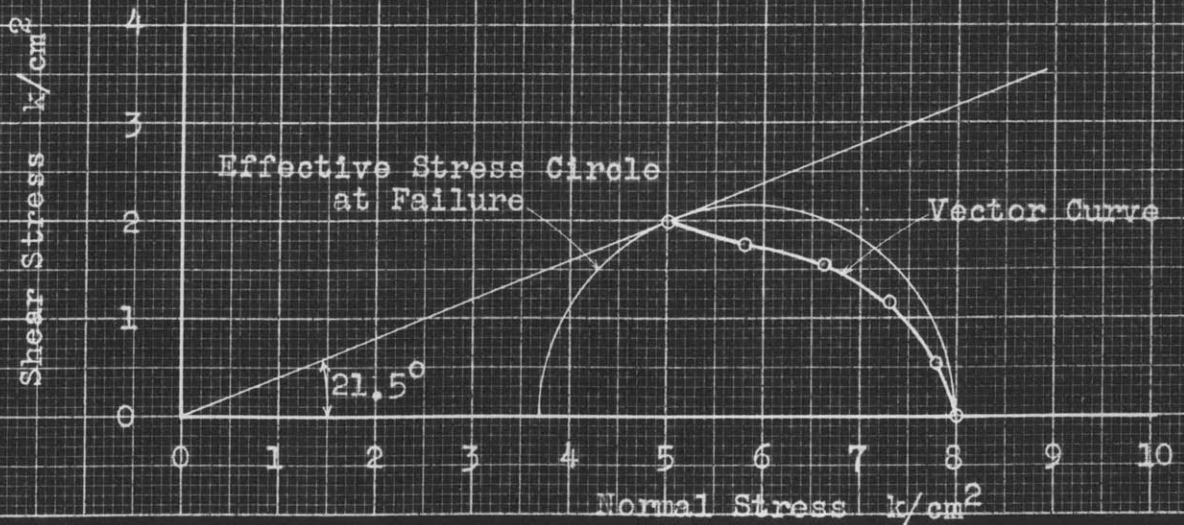
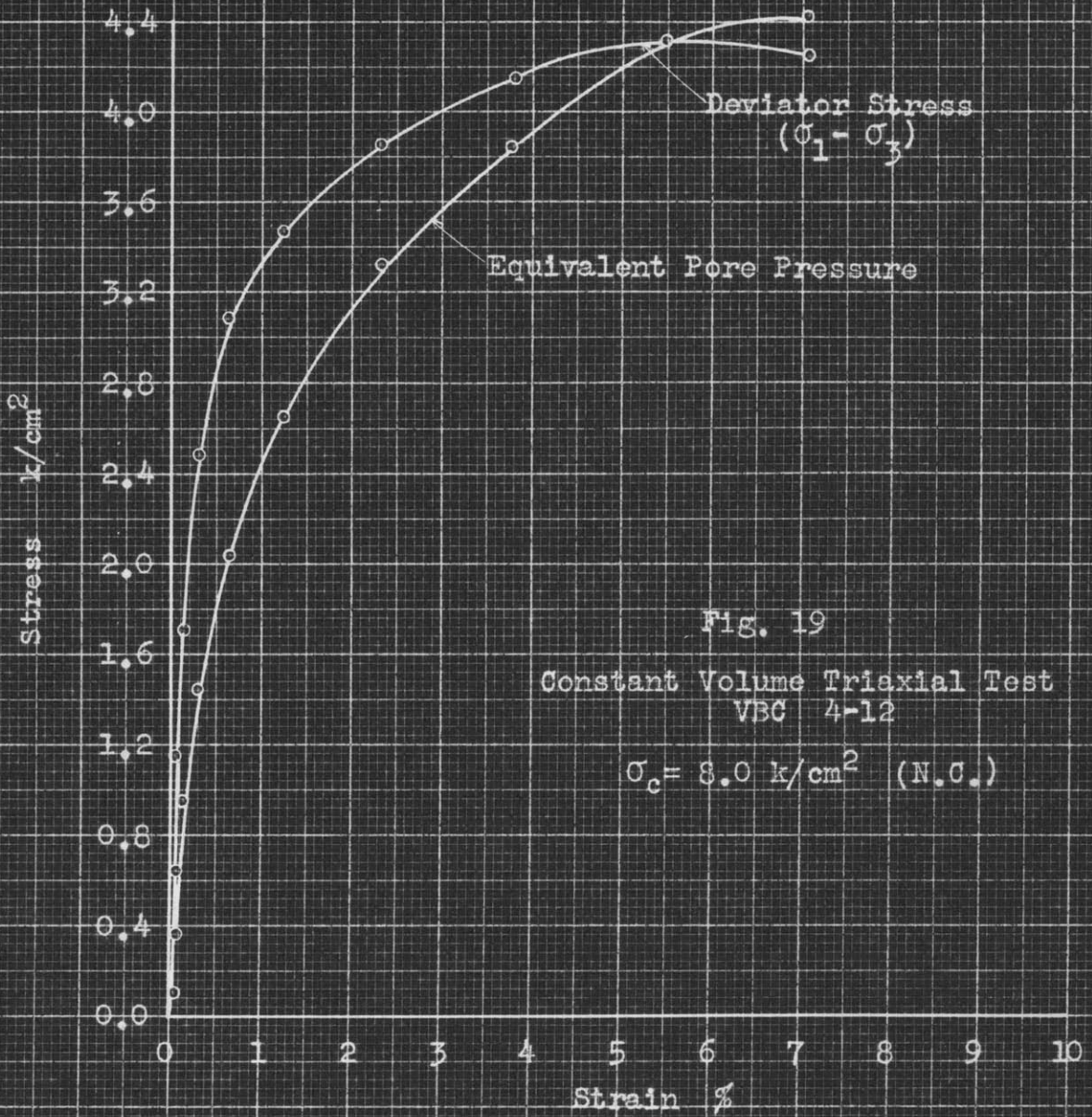
Fig. 15  
 Constant Volume Triaxial Test  
 VBC 3-8  
 $\sigma_c = 4.0 k/cm^2$  (N.C.)

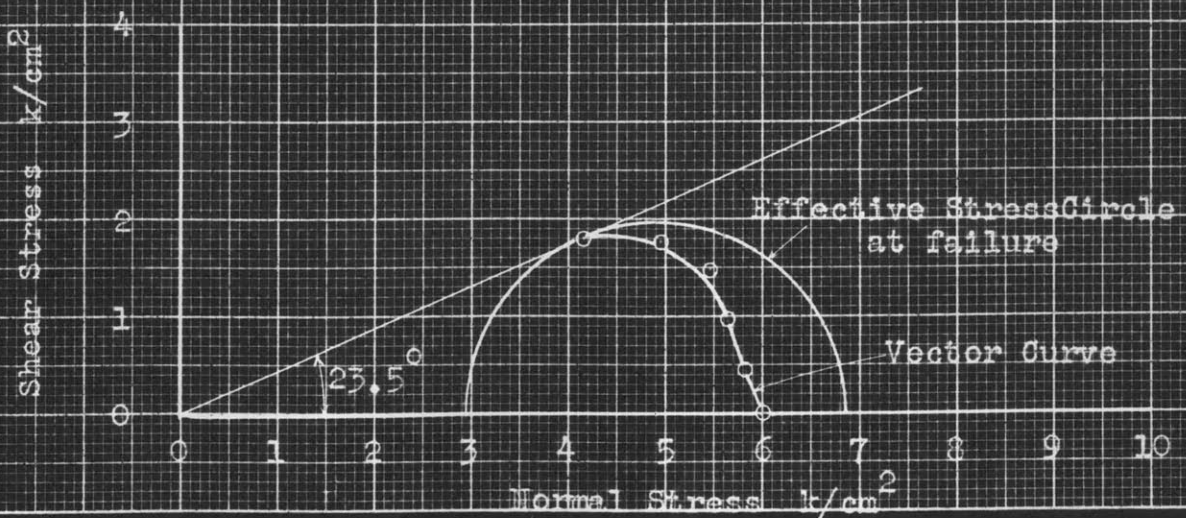
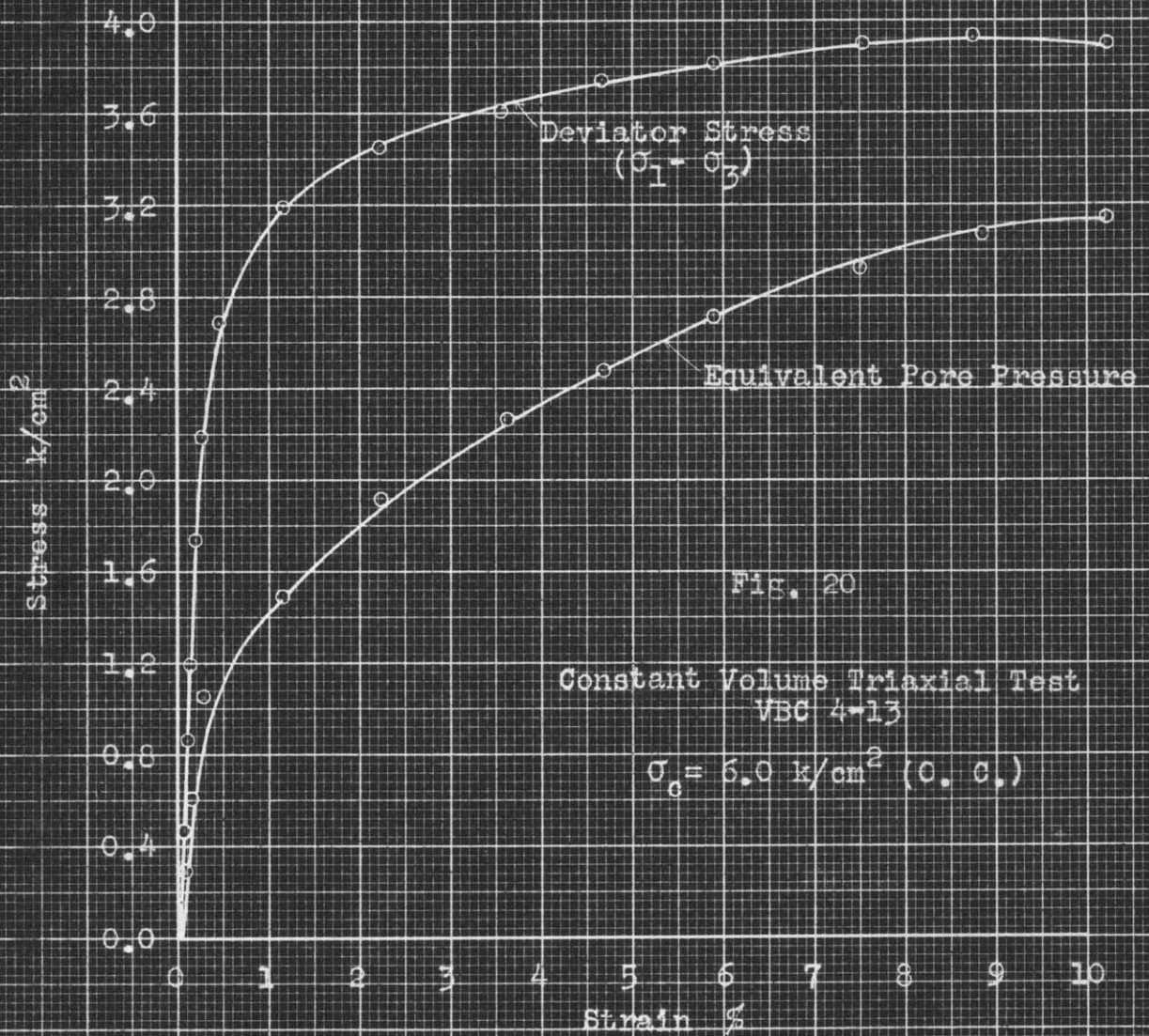












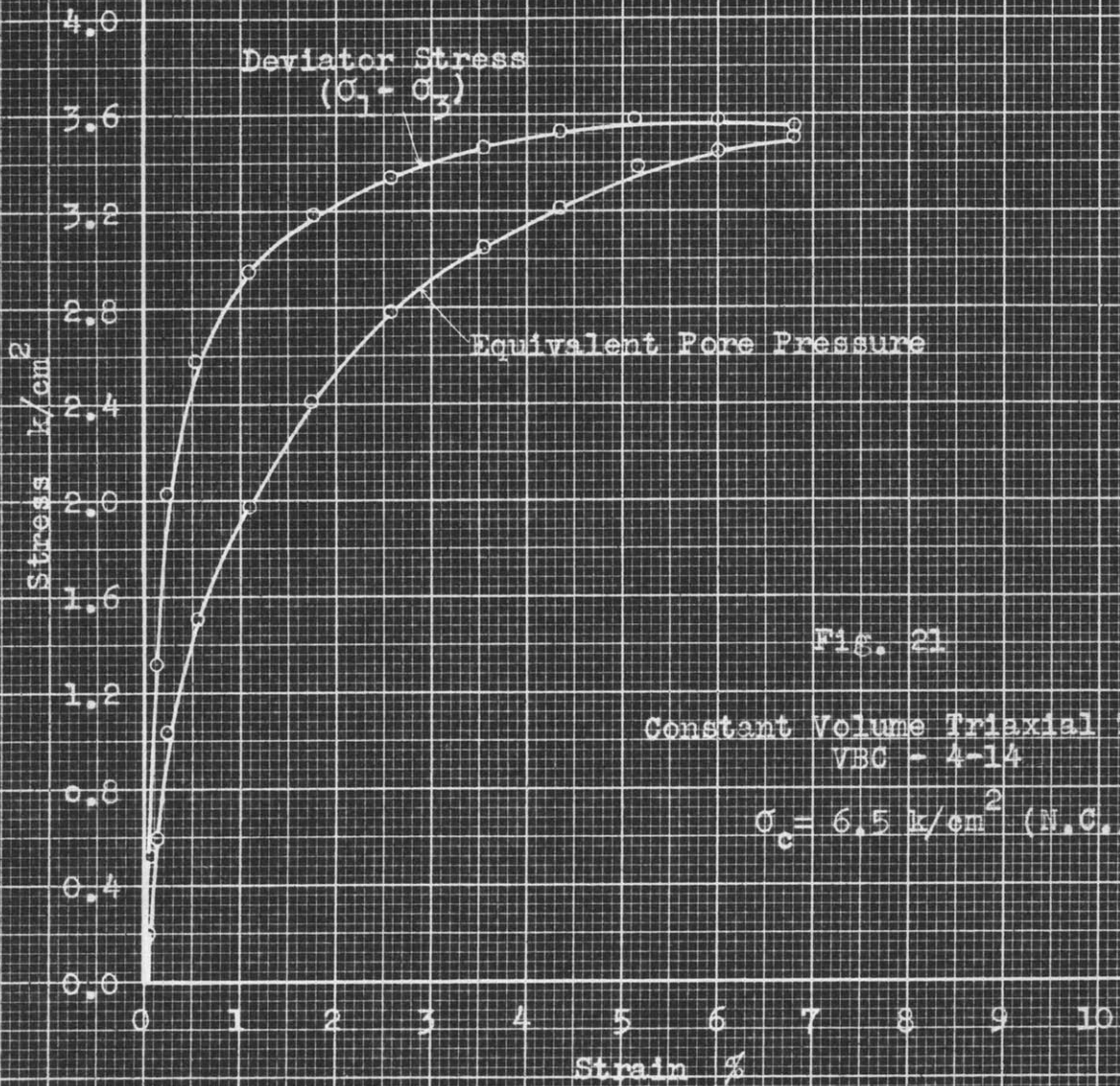
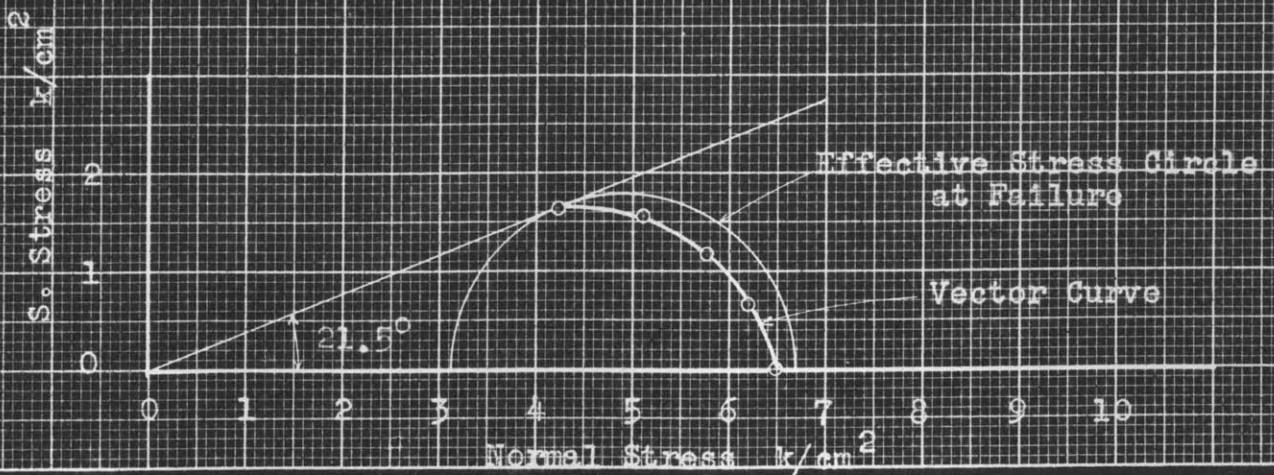
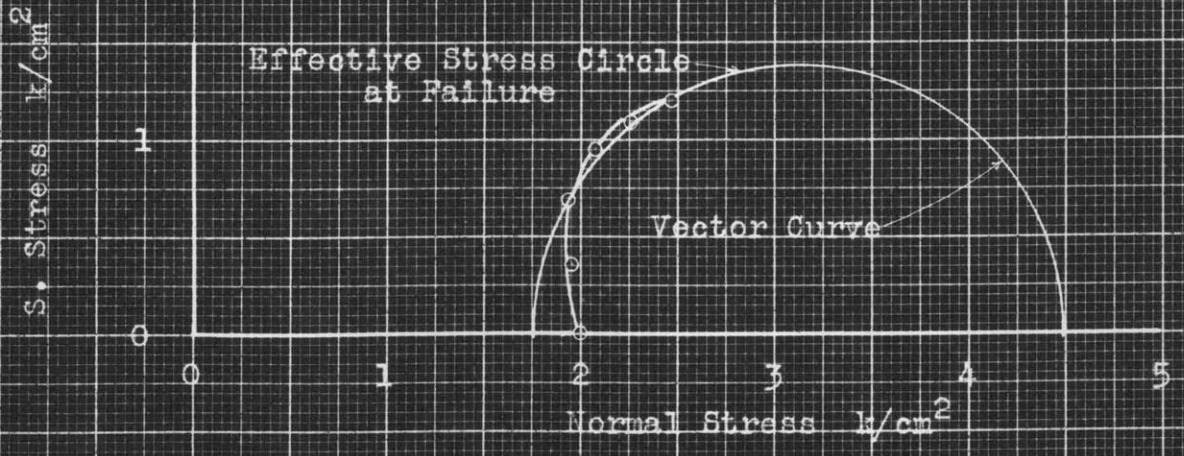
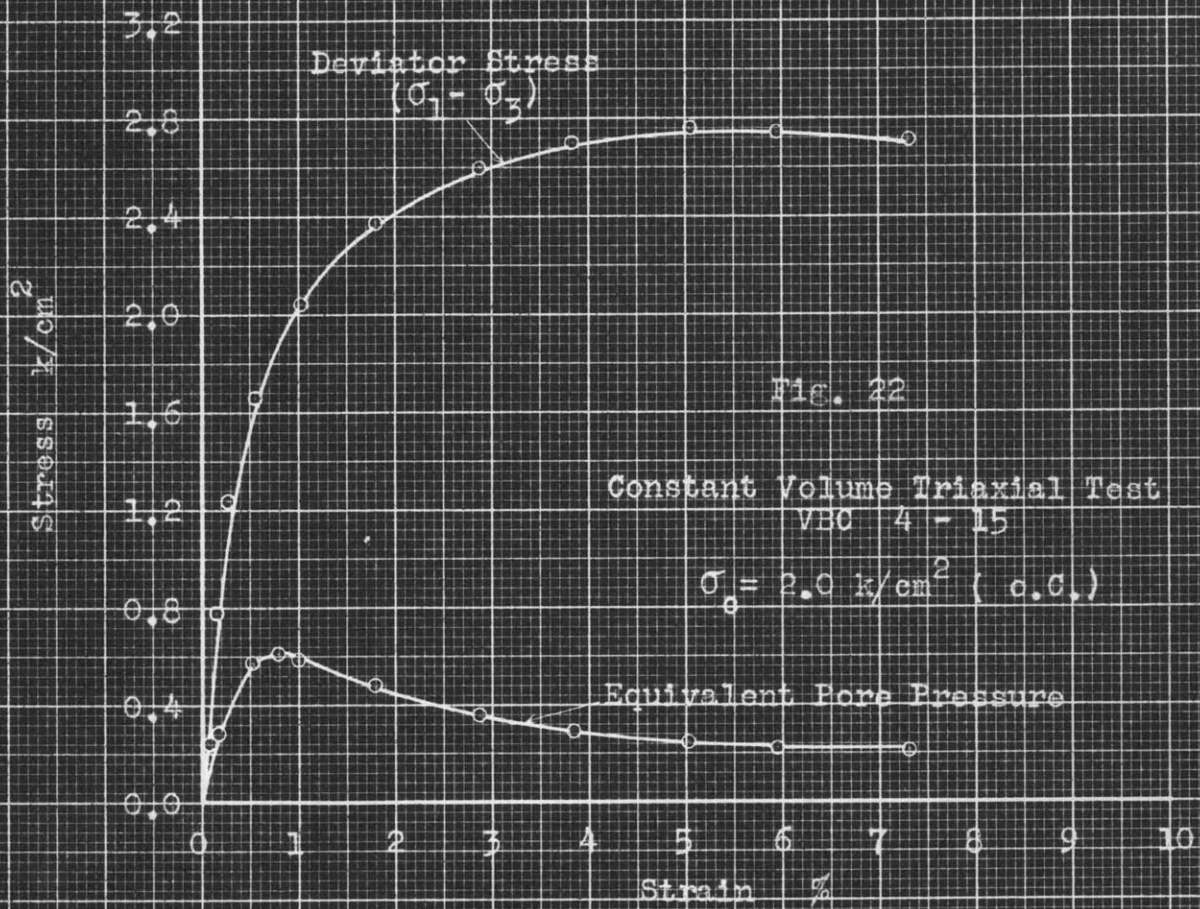


Fig. 21

Constant Volume Triaxial Test  
VBC - 4-14

$$\sigma_c = 6.5 k/cm^2 \text{ (N.C.)}$$





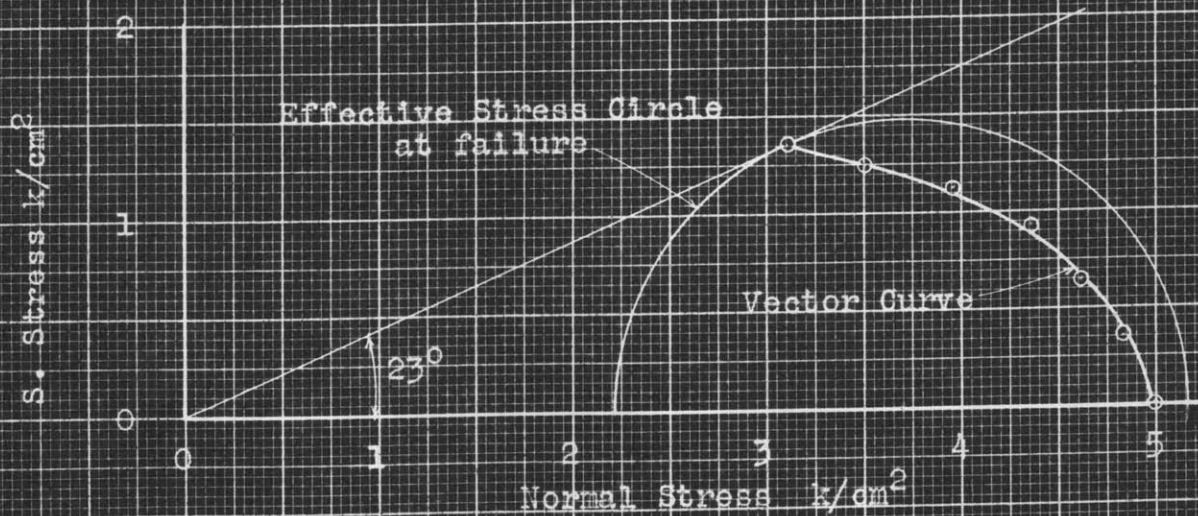
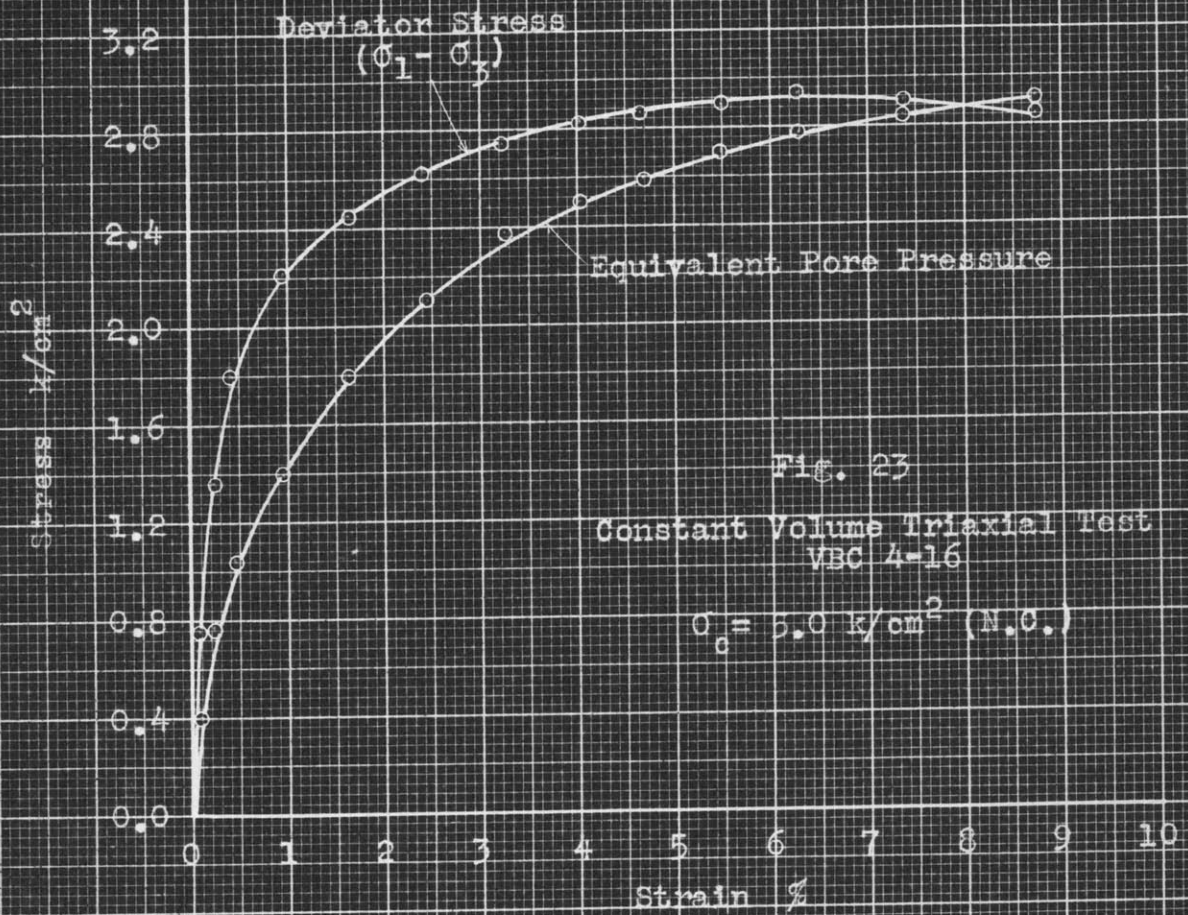
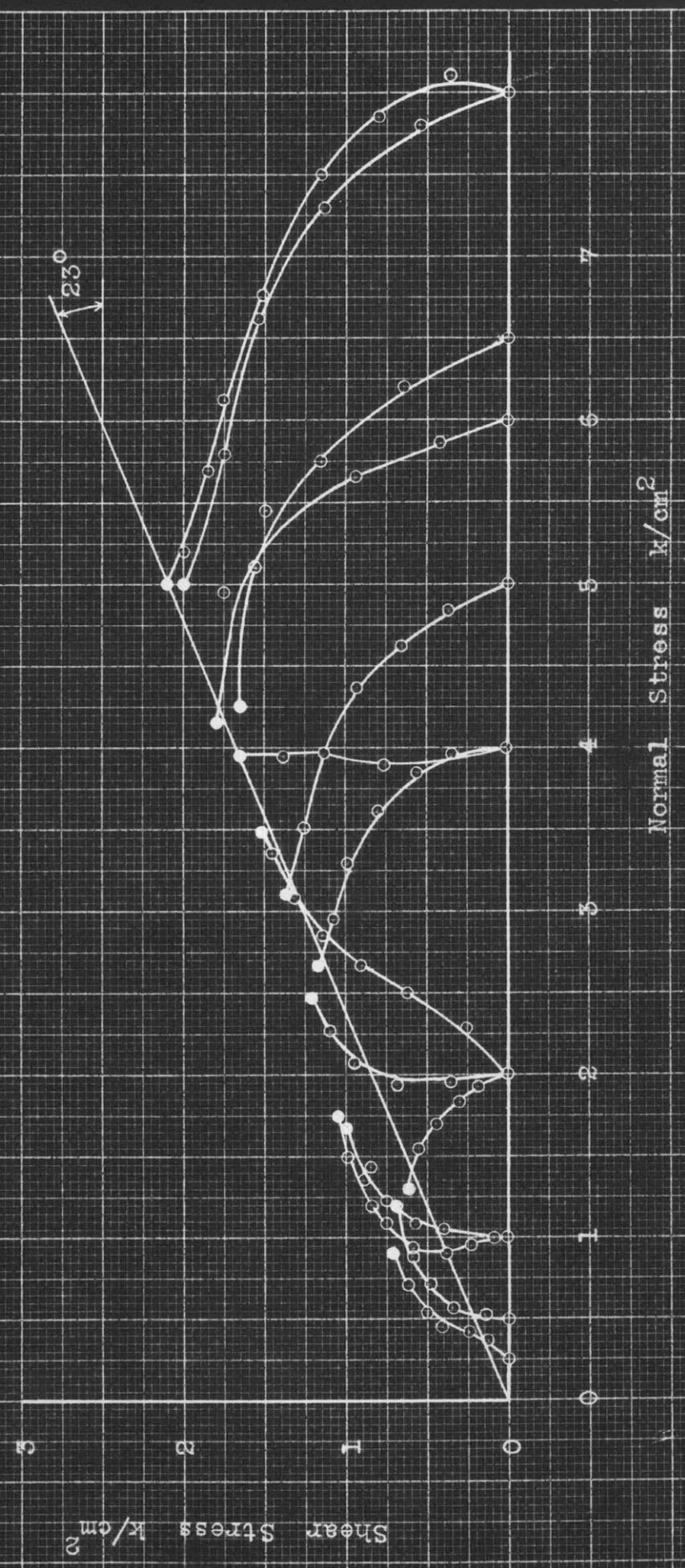


FIG. 24. VECTOR CURVES



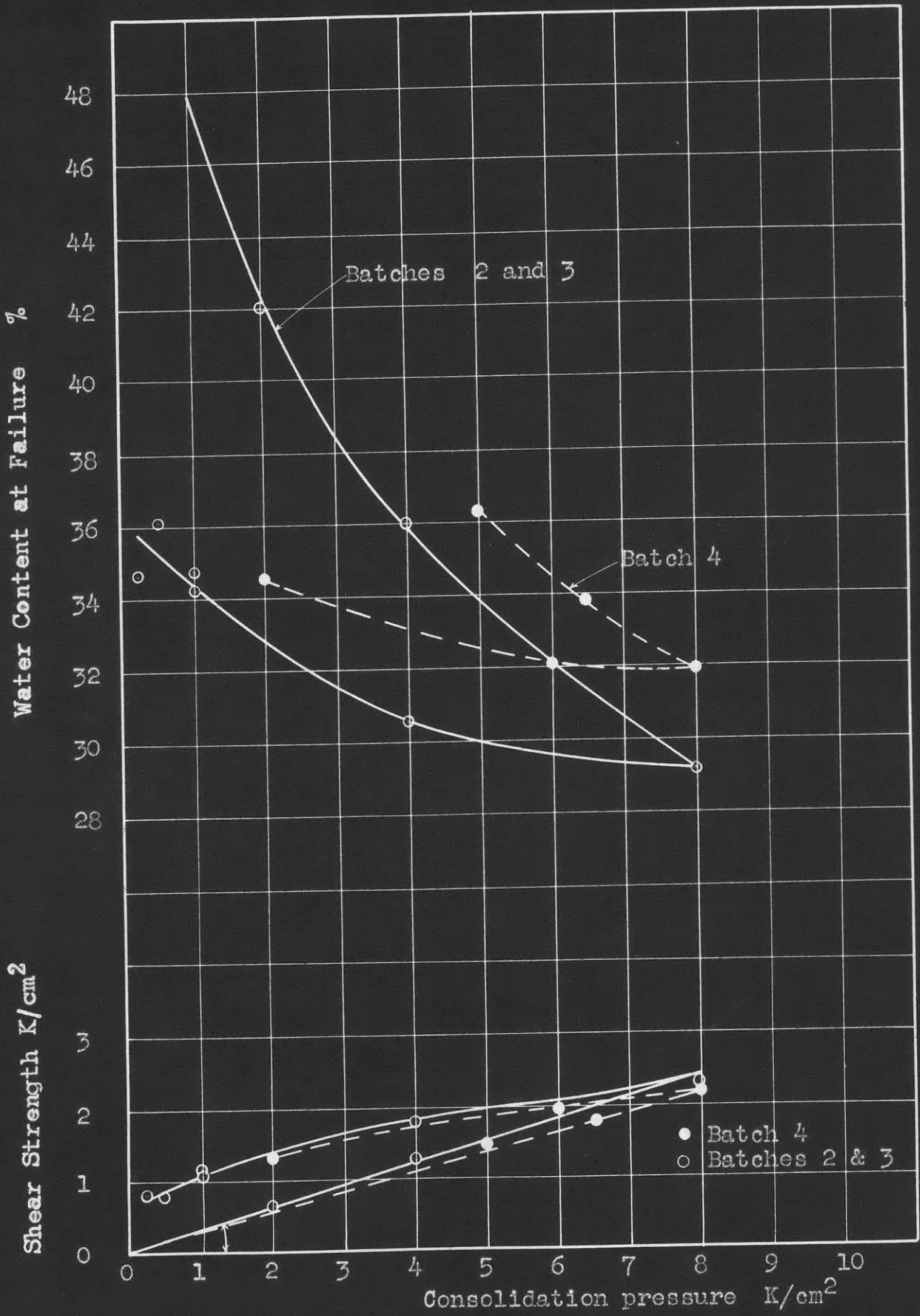


Fig. 25. Water content at failure versus Consol. press. and shear strength

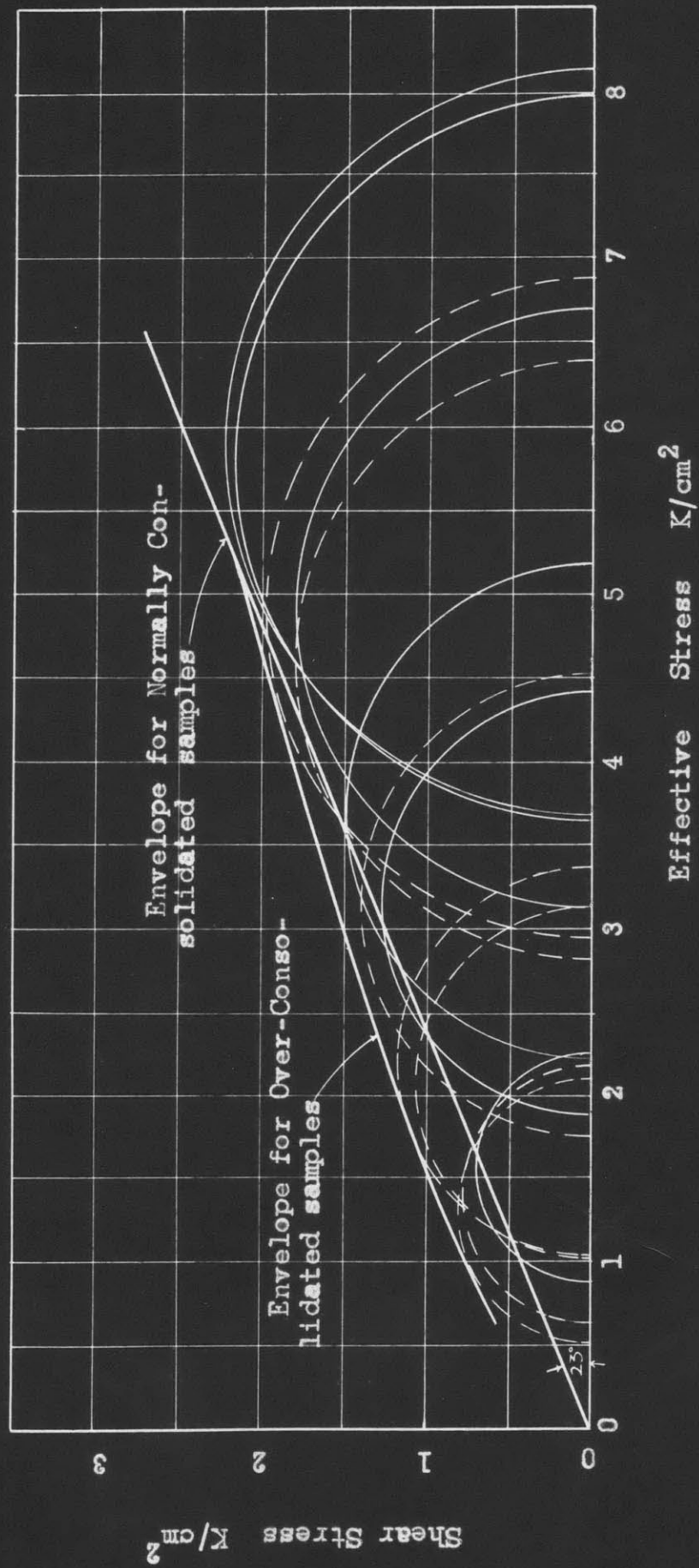


Fig. 26. Mohr circles for effective stresses at failure

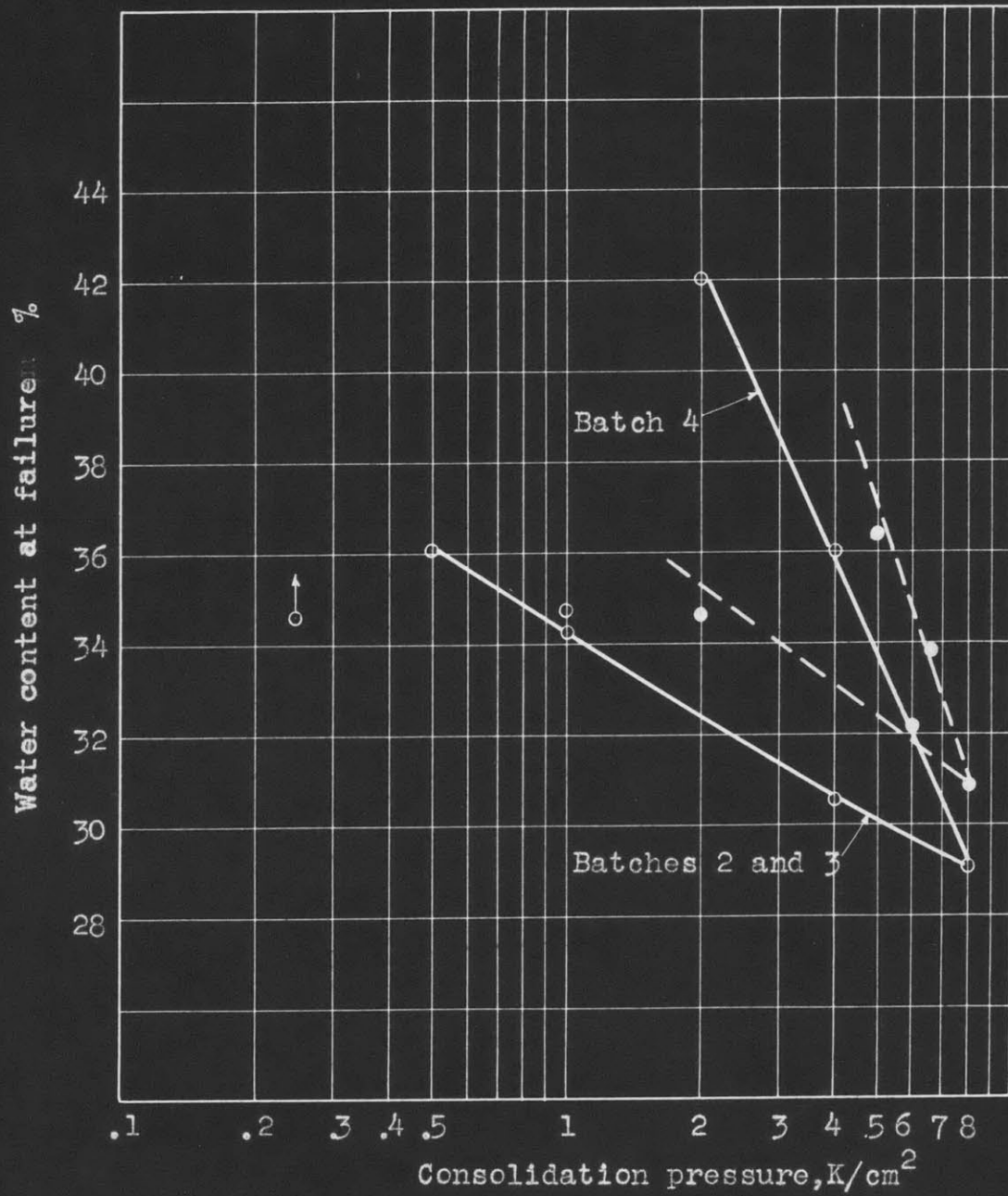


Fig. 27. Consolidation curves  
(Semi-logarithmic plot)

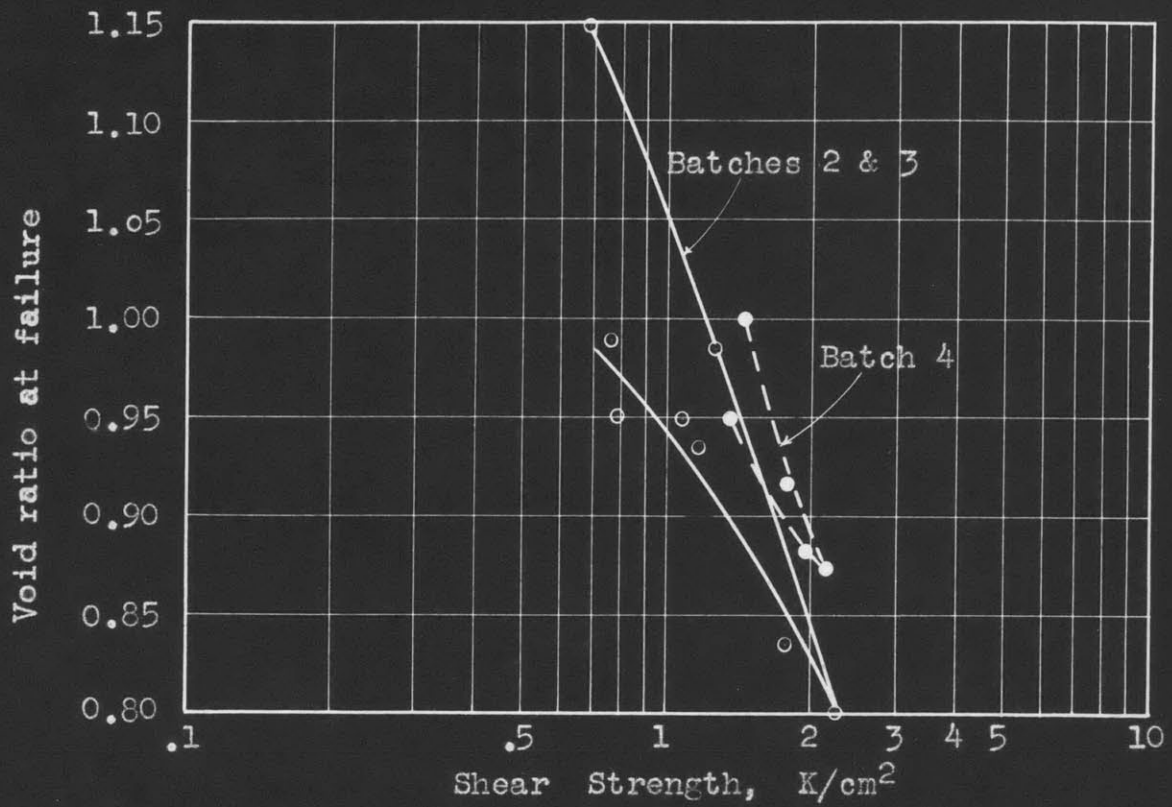


Fig. 28. Shear Strength vs. void ratio

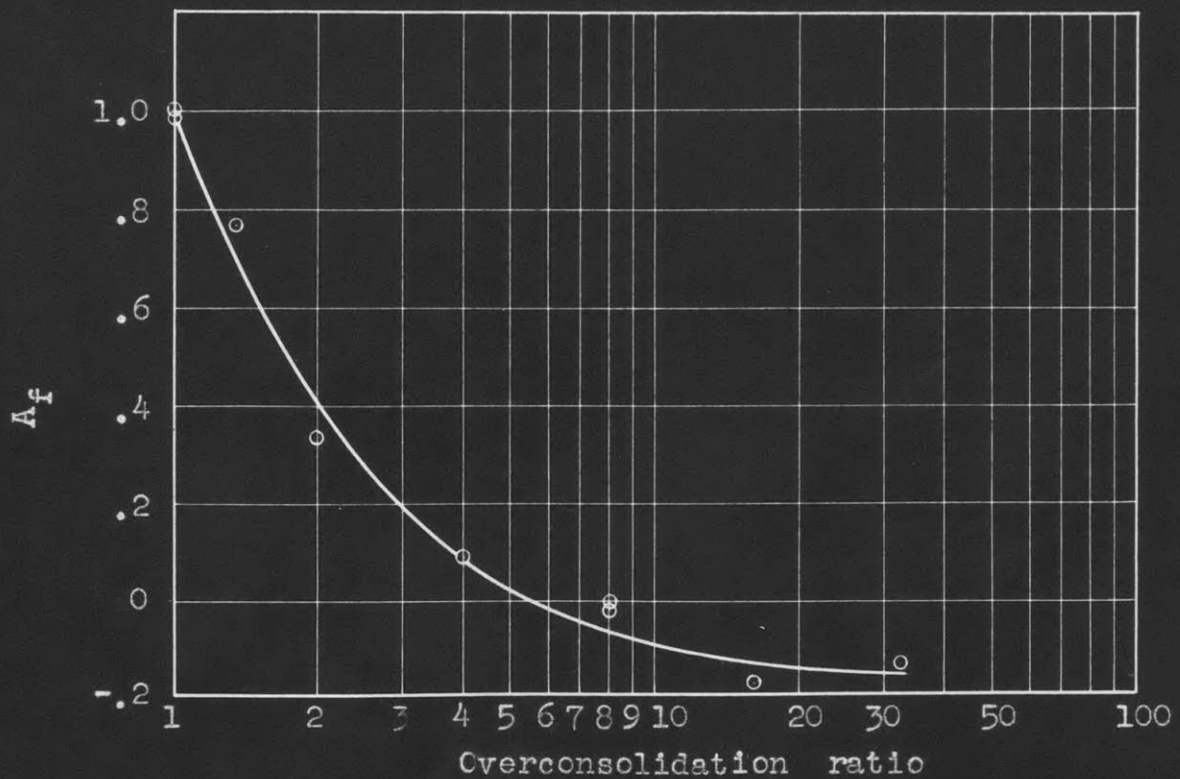


Fig. 29. Effect of overconsolidation ratio on the pore pressure parameter A at failure.

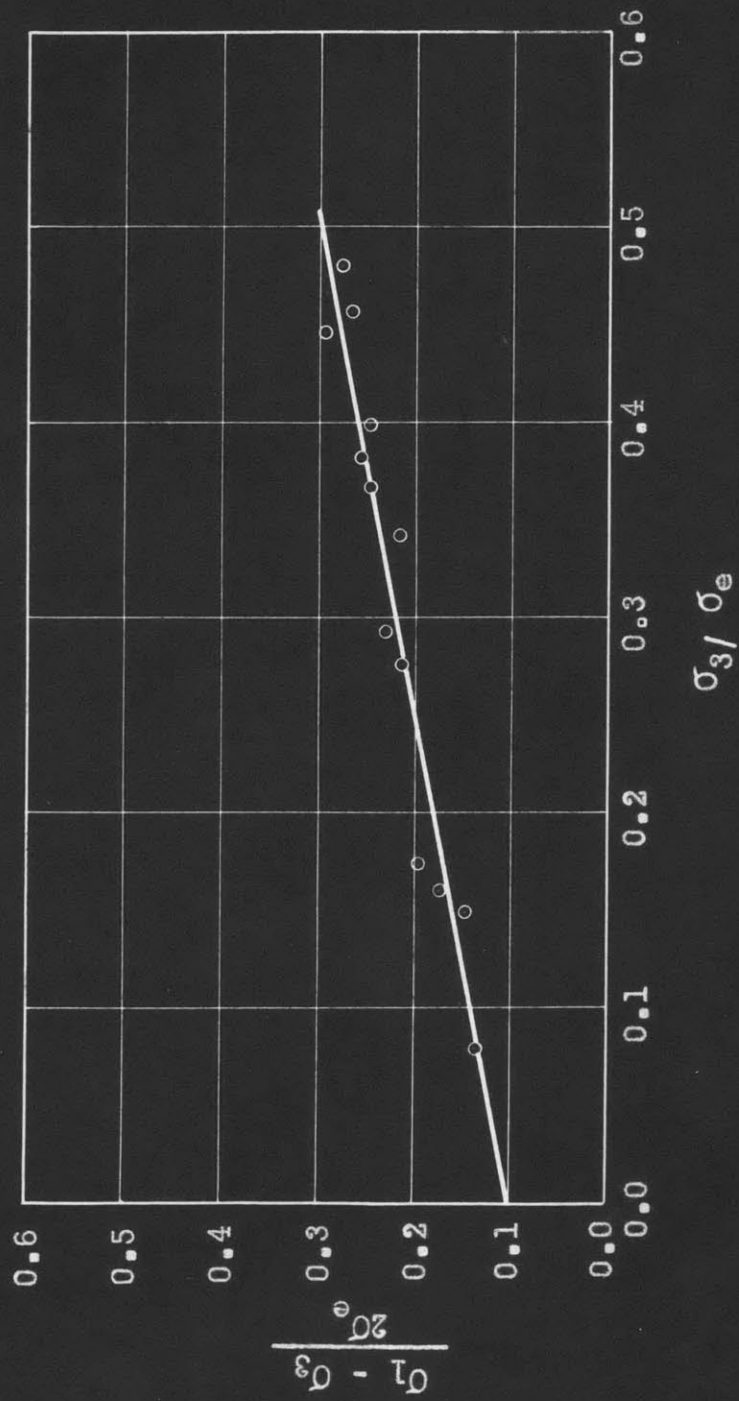


Fig. 30. Dimensionless plot to determine  $\phi_r$  and  $c_r$   
from triaxial tests

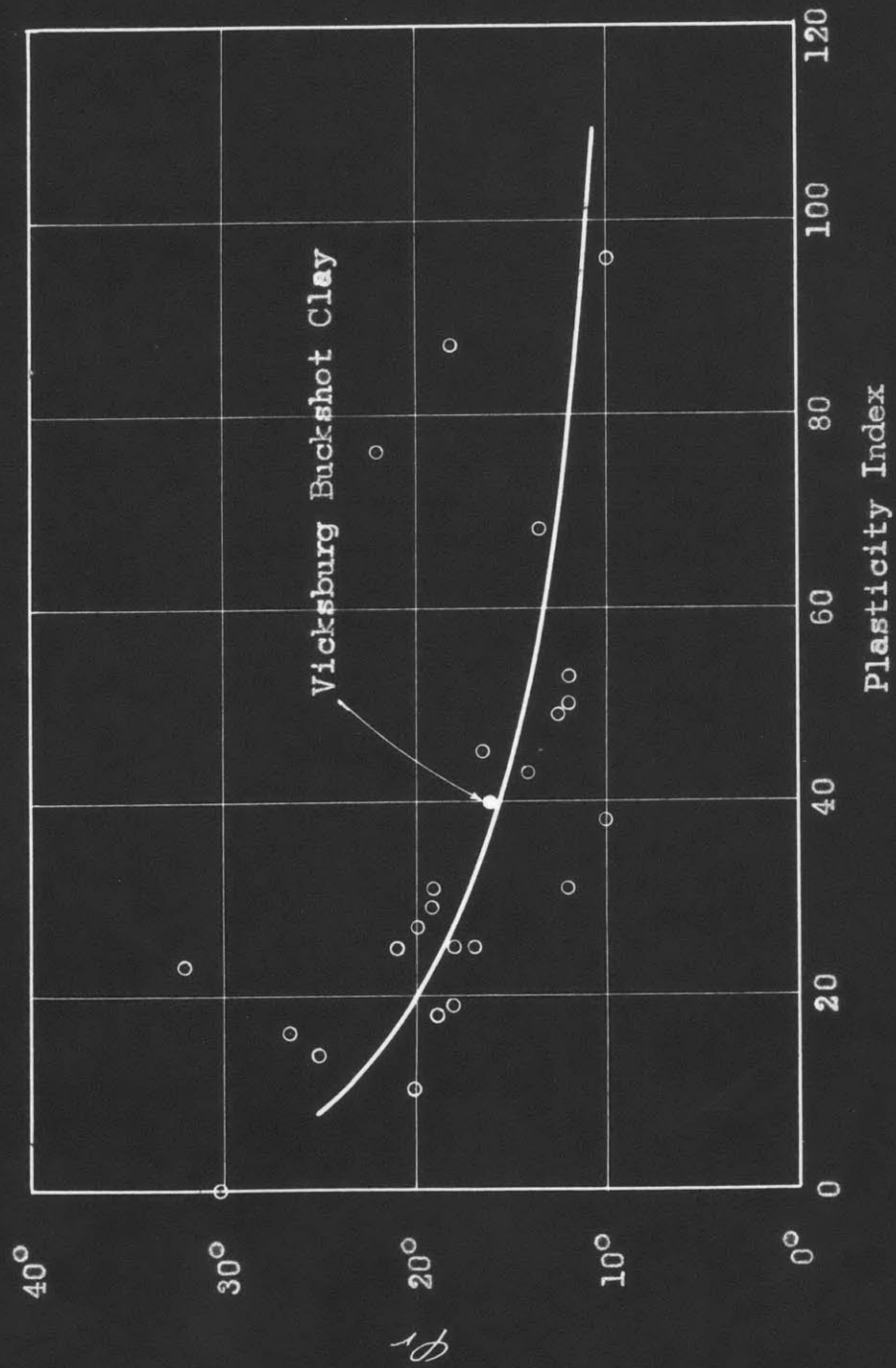


Fig. 31. Relation between  $\phi_r$  and Plasticity Index

APPENDIX

1) Deduction of formula (11):

Referring to figure 5:

$$e_o - e = \log \frac{\sigma_e}{\sigma_o} c_c \quad (1)$$

$$e_m - e_o = - \log \frac{\sigma_m}{\sigma_o} c_c \quad (2)$$

Combining Eq. (1) and (2):

$$e_m - e = \left( \log \frac{\sigma_e}{\sigma_o} - \log \frac{\sigma_m}{\sigma_o} \right) c_c$$

$$e - e_m = c_c \log \frac{\sigma_m}{\sigma_e} \quad (3)$$

Similarly for swelling line:

$$e - e_m = c_s \log \frac{\sigma_m}{\sigma_c} \quad (4)$$

In above equation:

$c_c$  = Compression Index

$c_s$  = Swelling Index

Combining (3) and (4)

$$c_c \log \frac{\sigma_m}{\sigma_e} = c_s \log \frac{\sigma_m}{\sigma_c}$$

$$\text{If } \frac{c_s}{c_c} = R$$

$$\log \sigma_m - \log \sigma_e = R \log \frac{\sigma_m}{\sigma_c}$$

or

$$\log \sigma_e = \log \frac{\sigma_m}{\left(\frac{\sigma_m}{\sigma_c}\right)^R}$$

therefore

$$\sigma_e = \sigma_m^{1-R} \cdot \sigma_c^R$$

if  $\sigma_m^{1-R} = Z$

Equation (11) is obtained:

$$\sigma_e = Z \sigma_c^R$$

2) Determination of cross-sectional area of sample to determine the deviator stress.

a) Drained tests:

$l_0$  = initial length

$v_0$  = initial volume

$a_0$  = initial cross-section area

If  $a$  is the average cross-sectional area after a change  $l$  in length and  $v$  in volume it follows that:

$$a(l_0 + l) = v_0 + v$$

$$a = \frac{l + \frac{v/v_0}{l/l_0}}{1 + \frac{v/v_0}{l/l_0}} \cdot \frac{v_0}{l_0}$$

but  $s = -\frac{l}{l_0}$  = axial strain

$$\text{and } a_0 = \frac{v_0}{l_0}$$

$$\text{therefore } a = a_0 \cdot \frac{1 + \frac{v/v_0}{l - s}}{1 - s} \quad (1)$$

b ) Undrained tests:

In undrained tests there is no change in volume (saturated soil), therefore  $v = 0$  and

$$\frac{v}{v_0} = 0$$

hence Equation (1) becomes:

$$a = a_0 \cdot \frac{1}{1 - s} \quad (2)$$

Equation (2) says that the cross-sectional area in undrained tests is function only of axial strain.