

The Structural Behavior of A Counter Weighted Cable Stayed Bridge

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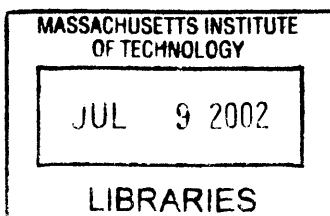
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ABSTRACT

Santiago Calatrava - when he introduced the counter weight cable stayed bridge typology - opened the door to the possibility of designing bridges for pursuits other than the purely utilitarian. This document explores the potential for variation within the typology by reviewing the historical development of the cable stayed bridge, the elements of the structural system, and the variation possible within these elements. A design strategy is then established for the counter weight cable stayed bridge. Using a proposal for Boston, Massachusetts (Charles River Crossing), a counter weight cable stayed bridge is systematically assessed on a component by component basis. Once assembled, the system is analyzed to determine the structural behavior of the system under static and dynamic load.

The results of the analysis revealed that the counter weight cable stayed bridge exhibits complex structural behavior. Bending and torsion are often coupled under both static and dynamic loading. The flexibility of the structural system presents some concern, particularly in the dynamic case. The frequencies of the structure must be correctly established and design measures taken so as to avoid the possibility of exciting multiple modes under loading, resulting in excessive displacements. While challenging, the design of the counter weight cable stay bridge is well within the reach of the modern engineer using current available methods to assess structural behavior.

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Section 1.0 Introduction

In recent times, most if not all, structural design enterprises have chosen to focus their attention on the balancing of strength and weight considerations for the express purpose of achieving a desired level of economy. Nowhere has this become more of a driving force than in the design of bridge structures. The blinding nature of this pursuit has in general demoted these structures to the realm of the banal. The opportunity to create a work of character and beauty has been foregone in order to build “cheap”. While this strategy is by necessity valid for a good number of bridges built today, it precludes the possibility for the design of bridges in character with the likes of the Eads Bridge, Brooklyn Bridge, and Golden Gate Bridge, to name just a few. Santiago Calatrava took a stance against such an agenda and chose to build a bridge of cultural and aesthetic significance when he designed the Alamillo Bridge in Spain. In so doing, he placed a wedge under the weight of tradition and established a precedent for engineers to consider intentions outside of those normally reserved for the modern engineer. Cable supported bridge engineering in particular benefited the most by the introduction of a new bridge typology, the counter weight cable stayed bridge.

This document is intended to investigate the realm of possibilities that the Alamillo Bridge as a typology made available to the structural engineer. It uses at the core of the discussion a proposal for a counter weight cable stayed bridge for Boston’s Charles River. The investigation is intended to illustrate how one might determine the structural behavior of such a bridge, and from such an analysis how one might configure such a bridge for a prescribed set of load cases to achieve a desired structural behavior. **Section 3.0** gives a brief historical perspective on the development of the cable stayed bridge, from a subsystem to provide additional stiffness to an elegant self-sufficient bridge system. Details of the complexity of the required analysis, along with the various cable stayed innovations made along the way of its development are discussed. **Section 4.0** discusses the primary structural elements of a typical cable stayed bridge, and identifies how the counter weight cable stayed bridge varies from this. **Section 5.0** provides an overview of the Boston proposal and its structural system. This proposal will function as a tool for exploring a design and analysis method. **Section 6.0** discusses the primary

applied to bridge structures and their applicability to the Boston example. **Section 7.0** details the analytic method used to generate a preliminary design. This includes the cable analysis, the stiffening girder model (beam on elastic supports), and the element matching required for equilibrium. Additionally, the non-linear analysis of the bridge structure using computer methods will be discussed. **Section 8.0** details the observations made during the analysis and its application to similar problems.

Section 2.0 Brief Historical Perspective

The notion to suspend a cable (rope or chain) across an unsurpassable obstacle as a method of crossing – and ultimately to carry loads – had its beginning in ancient times. From this notion, the idea of cable-supported bridges emerged. Numerous early examples have been noted by scholars and may be found in historical texts on the subject. These bridges were designed based upon empirical observations rather than actual engineering principles and thus are of limited interest here. The development of analytical methods to understand and predict the structural behavior of cable-supported bridges has taken several hundred years to develop. As a result, the design and construction of modern cable-supported bridges has been roughly restricted to the last one hundred twenty years. The lack of accuracy in analytical methods certainly proved to be a hindrance to the design and construction of cable stayed bridge structures. While the notion of using cables to stabilize beams is hardly a new concept – ancient civilizations were using booms, rigging and masts on their sea-going vessels hundreds of years earlier – the pure cable stay bridge still to this day remains relatively novel.

The evolution of bridge engineering was greatly enhanced by necessity after the ravages of World War II. The Germans were instrumental in the pioneering of new bridge design methods and systems to minimize material requirements for a given span. One such conceptual approach was that of the pure cable stayed system. The concept of a cable stayed bridge dates back to 1784. Credit for its first use goes to C. J. Locscher, a German carpenter, who successfully built a timber bridge that used timber inclined stays, fixed to a tower. Several attempts were made to build similar bridges using chain stays. These attempts mostly ended in disaster and fatality, due primarily to the inability to accurately analyze the structural behavior, particularly under dynamic loading. Credit goes to Franz Dischinger, a German professor, who in 1938, proposed a railway suspension bridge that used cable stays as a method to limit deflections, for re-introducing the cable stay concept. Several other cable stay proposals soon followed, but none were ever built. It is important to note that John Roebling used cable stays on the Brooklyn Bridge, roughly seventy years earlier. Nevertheless, it is more probable that Dischinger had a far greater impact in the revival of the cable stay concept due purely to his timing: accurate and

efficient engineering methods were becoming widely available and newer methods, using the power of the computer, were just twenty years away – about the same time the first pure cable stay bridge was built. **Section 2.1** to **Section 2.6** provide a brief overview of the evolution of the cable stayed bridge, from a rather simple device to provide additional stiffness to the highly sophisticated and elegant counter weight cable stayed bridge.

Section 2.1 The Brooklyn Bridge – The First Application of Bridge Cable Stays

It is widely accepted that the first modern cable supported bridge is the Brooklyn Bridge, designed by John A. Roebling. Opening in 1883 and linking Manhattan, New York with Brooklyn, New York, the Brooklyn Bridge was the first suspension bridge to use steel cables. The design of the bridge was influenced greatly by the then recent bridge failures (under dynamic loading) of recently constructed suspension bridges, and is evident by the addition of cable stays and use of a stiffening truss to improve the overall bridge stiffness [Gimsing, 1983]. It is ironic that the first modern suspension bridge was also the first modern bridge to use steel cable stays. While Roebling certainly applied engineering principles to design the Brooklyn Bridge, it is also obvious that he used his engineering intuition to work out the interaction of the overall cable system by some rules of thumb he developed. The availability of analytical methods capable of analyzing the system and in particular the cable stay interaction, were still very much out of reach [Gimsing, 1983]. Nevertheless, the use of cables stays proved to be quiet effective, and their impact upon the suspension system can be seen by the reduced curvature of the main suspension cables (see **Fig. 2.1**).

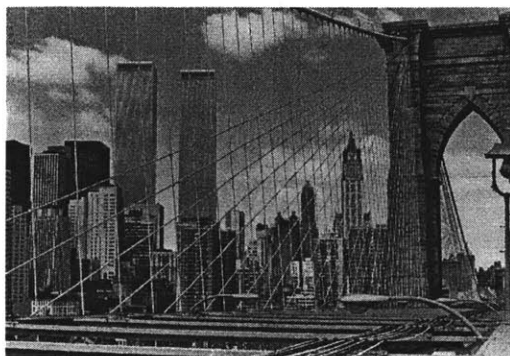


Figure 2.1 The Brooklyn Bridge [Gimsing, 1983]

Section 2.2 Strömsund Bridge – The First Pure Cable Stayed Bridge

The first modern cable stayed bridge was the Strömsund Bridge in Sweden, built in 1956 (see **Fig. 2.2**). The structural system clearly reveals the limitations of the analytical methods available at the time [Gimsing, 1983]. The stiffening girder by configuration provides no torsional rigidity, effectively uncoupling the bending component of the bridge from the torsional component. The cable system is left to handle all torsional effects via the moment arm principle [Gimsing, 1983]. The system uses a limited number of cables to stiffen the deck girder – four points of support at mid-span and two at the back span per side. The pylon is a portal frame that is pinned at the pier. Conceptually, the configuration effectively removes the bending rigidity of the pylon from the system and its overall global effect. The cables radiate out from the top of the frame in a fan configuration. Such a structural system made it reasonable to obtain reliable results from analysis by investigating only two-dimensionally, the force system in the cable planes.



Figure 2.2 Strömsund Bridge [Gimsing, 1983]

Section 2.3 The German Cable Stayed Bridges

As noted previously, the Germans were instrumental in the development of the pure cable stay bridge. The first of the German bridges was the Theodor Heuss Bridge, completed in 1957 (see **Fig 2.3**). The cable system, with the cables in a harp configuration (a design first), was placed into two vertical cable planes along each edge of the deck. The structural system utilized made the analysis of the bridge come at a considerable price, particularly the fixing of the pylons at the piers and the complexity of the stiffening girder scheme – two box girders over the entire span supporting the transverse girders,

which in turn, tied into stiffening girders at the perimeter. The number of indeterminacies was of such a magnitude the designers were forced to make assumptions to reduce the analysis to the two, two-dimensional planes [Gimsing, 1983]. The complexity of the deck system, given today's standard, suggests a great deal of redundancy was present in the final design, with the design assumptions forcing a conservative approach to design. The Theodor Heuss Bridge puts forth strong evidence of the exponential growth in the complexity of analysis that results by blind design choice. Whether the designers knew or not is not the point, the accuracy of their analysis was compromised by their choice.

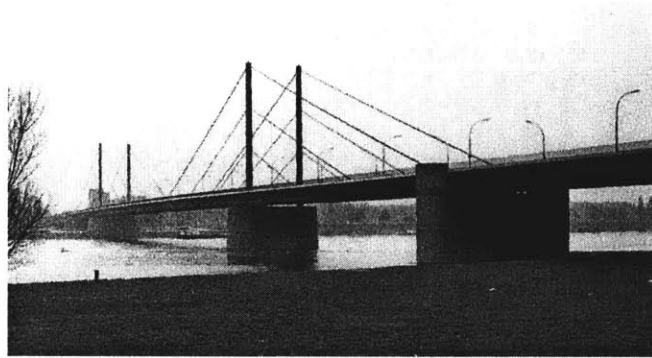


Figure 2.3 Theodor Heuss Bridge [Gimsing, 1983]

The next German cable stayed bridge to be completed was the Severin Bridge, which opened in 1959 (see **Fig. 2.4**). It featured an A-frame pylon with a fan cable configuration, in two non-vertical planes. The cable system and box girders are coupled in both bending and torsion, as a result. Due to this feature, the analysis of the system was made quite difficult [Gimsing, 1983]. The Severin Bridge can boast as being the first asymmetrical cable stay bridge – only one pylon, positioned to one side. In this particular bridge the deck carries a significant compressive force through the stiffening girder and deck due to the asymmetry of the system – one more nuance.

Two similar bridges were built in 1963 and 1964, the Norderelbe Bridge and Leverkusen Bridge (see **Fig. 2.5** and **Fig. 2.6**). Each of these bridges featured a single cable plane supporting a centralized stiffening girder supporting cantilevered roadway decks. The bending and torsional stiffness for both bridges were uncoupled as a result. After the

completion of these bridges the central cable plane became the preferred system [Gimsing, 1983]. The introduction of the computer greatly enhanced the analysis of these systems. Using the computer to solve the beam on elastic supports problem, the designer was left only to analyze the forces within a single cable plane – a one-dimensional problem. Nevertheless, the difficulty in analyzing cable systems is obvious by the apparent minimized number of cables utilized.

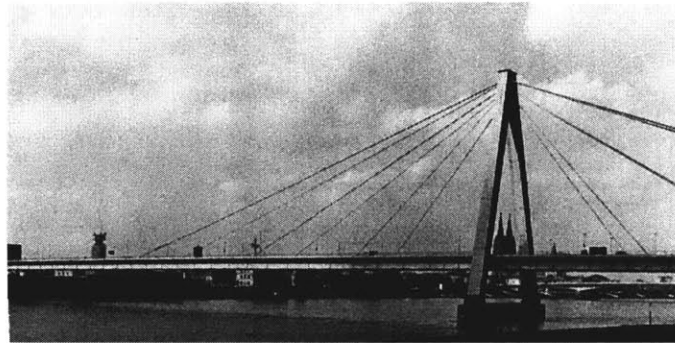


Figure 2.4 Severin Birdge [Gimsing, 1983]

The first multi-cable bridge was the Friedrich Ebert Bridge, completed in 1967 (see **Fig. 2.7**). The bridge featured a central cable plane with 20 cable stays each side of two pylons. The introduction of the computer proved highly beneficial. Multi-cable systems offer a more continuous support of the deck structure, and thus, result in reduced girder cross sections. The virtues of computer methods are clearly obvious; multi-cable systems are highly indeterminate and therefore are nearly impossible to analyze by hand-calculation methods. Computers changed that, and made the analysis of these systems possible. Today, nearly all cable stayed bridges are multi-cable by configuration.

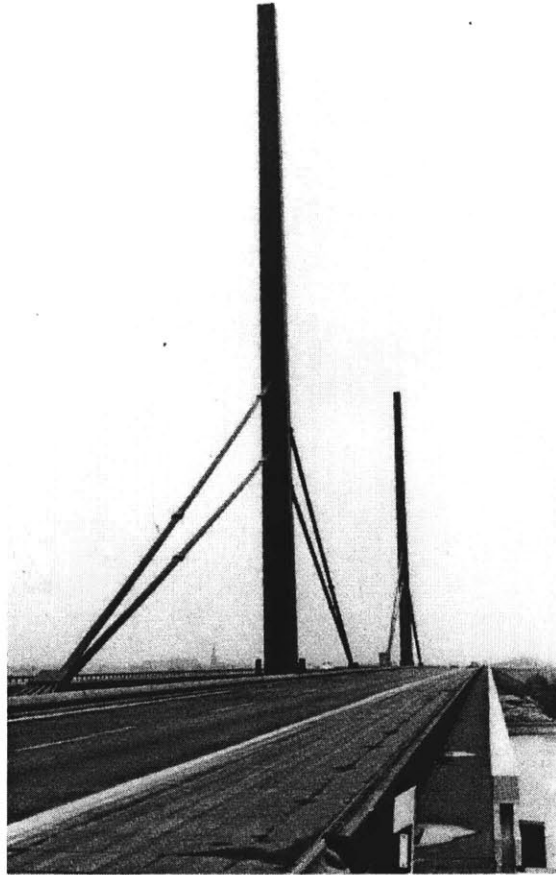


Figure 2.5 Norderelbe Bridge [Gimsing, 1983]

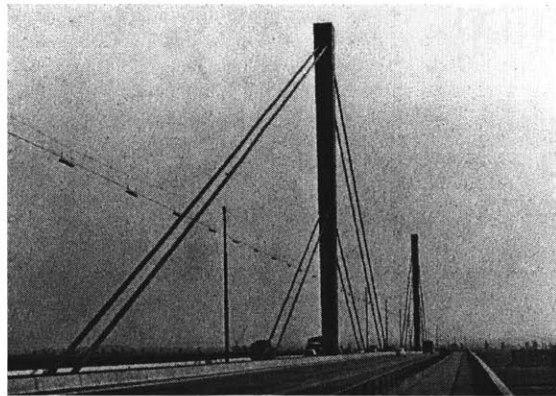


Figure 2.6 Leverkusen Bridge [Gimsing, 1983]

Section 2.4 Knie Bridge – The Effect of Anchorage

In 1969, the Knie Bridge opened to traffic. The bridge has a harp configuration for the cable stays, in two vertical planes. On the side span, at points of anchorage (of the cable stays), intermediate supports were placed (see Fig. 2.8). The efficiency of this anchorage

strategy is clearly obvious by significant reduction in the stiffening girders overall dimensions in comparison to other cable stay bridges [Gimsing, 1983]. Additionally, the pylons are freestanding. As a result, overall torsional rigidity is supplied solely by the cable system. The Knie Bridge is an excellent example for illustrating how variation in the anchorage condition has on the overall structural efficiency. Clearly by anchoring each cable at the back span, the structural system becomes quite efficient both in bending and in torsion.

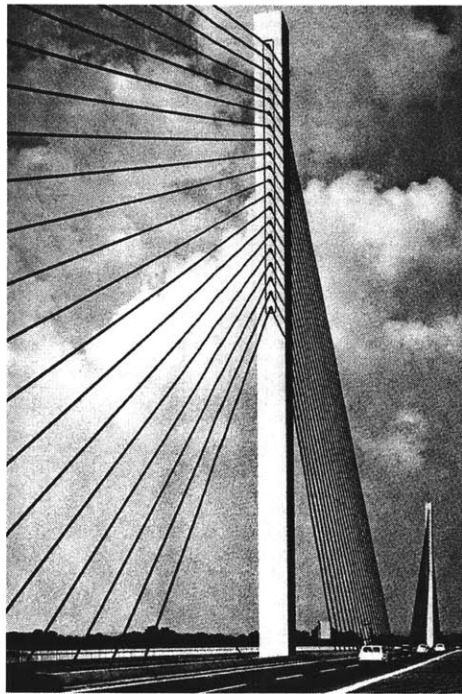


Figure 2.7 Friedrich Ebert Bridge [Taly, 1998]

Section 2.5 Indiano Bridge

The Indiano Bridge was built in 1977 to cross the Arno River near Firenze, Italy (see **Fig. 2.9**). When it opened to traffic it was the first cable stayed bridge to use an earth anchored cable system. In addition to the anchoring system, it was also the first to use inclined pylons. The earth anchor system works by transmitting the vertical and horizontal force components to the ground via an anchor block and anchor stays. The introduction of the inclined pylon was used to effectively decrease the backstay cable force (anchor cables) transmitted to the earth.

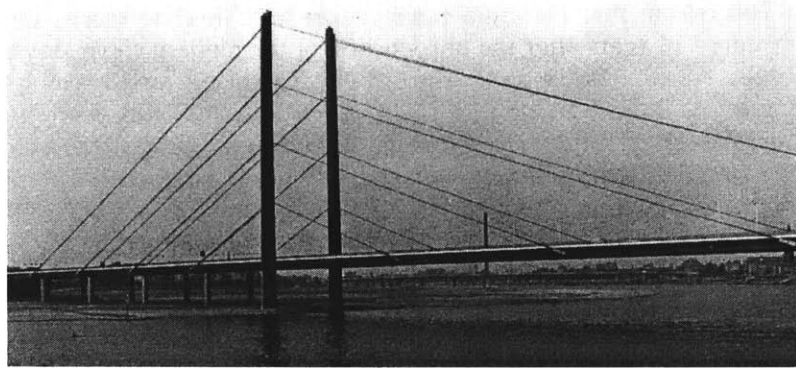


Figure 2.8 Knie Bridge

Section 2.6 Alamillo Bridge – Counter Weight Concept

The Alamillo Bridge (designed by Santiago Calatrava) was completed in 1992 for the Expo' 92 Seville (see **Fig. 2.10**). The uniqueness of the bridge is obvious by the lack of cable backstays. The structural system relies upon the weight of the pylon to achieve equilibrium. The center of mass of the incline pylon sits to the side (in the horizontal direction) of its base. The force action of the pylon mass causes a rotation about the base of the pylon. This rotational force is used to counter-act the rotation induced by the cable stays that support the deck, ultimately bringing the system into equilibrium. The use of the inclined pylon is obvious: to mobilize enough mass to cause a counter-acting rotation to that induced by the cable system. Two vertical cable planes were placed at the center of the transverse span, effectively a single cable plane. This fact considerably simplified the structural analysis of the bridge by uncoupling the bending and torsional rigidities of the system. The structural system works by defining a desired point of equilibrium (a equilibrium loading) and then determining the desired pylon mass to be distributed for equilibrium. When the bridge is loaded over or under the equilibrium case, then the foundation must mobilize a resistance to the overturning moment that these loadings induce [Pollalis, 1999].

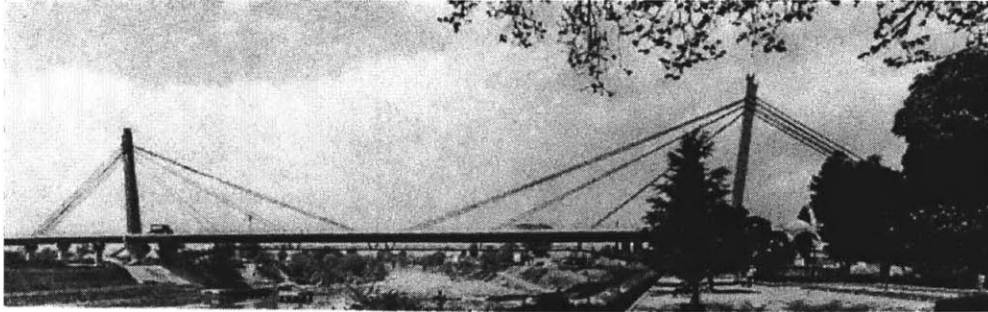


Figure 2.9 Arno Bridge [Gimsing, 1983]

The aesthetic motives for the bridge clearly overshadowed other considerations. The structural system makes this apparent. The required need of the foundation to resist rather large overturning moments when the system is loaded beyond of its equilibrium case, and the high level of structural redundancy to cope with flexibility issues make the bridge design come at considerable cost. And in comparison to other bridges of similar span, the Alamillo Bridge is not competitive in any respect. Nevertheless, the beauty and balance showcased in the concept is quite exceptional. In this particular case, Santiago Calatrava moved outside the domain of purely engineered economy to the domain of engineered aesthetics. Given the history of the cable stay bridge, and the quest to minimize material requirements, this was quite a paradigm shift for the use of the cable stay concept, opening a door to a realm of new possibility.

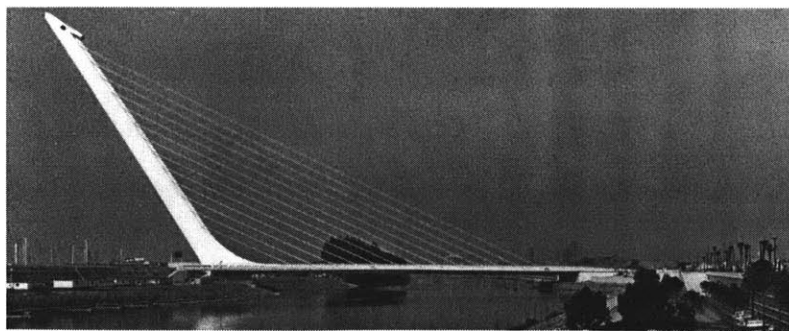


Figure 2.10 Alamillo Bridge [Pollalis, 1999]

Section 3.0 The Structural System, The Functional Components

Cable stayed bridges may be classified by the number of spans they possess. The three-span bridge is by far the most common configuration. The two-span bridge is also quite common. The two-span configuration can be either symmetric or asymmetric. Multi-span bridges have also been built. Each span may be classified as (1) a main span or (2) a back span (also called a side-span). In general, the main span is greater in length than the side spans. **Figure 3.1** illustrates the above discussion.

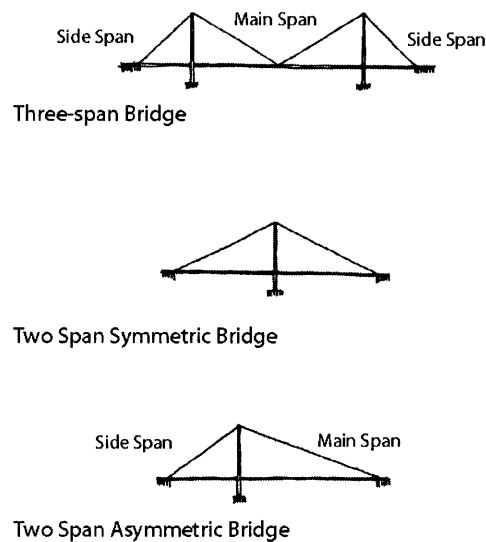


Figure 3.1 Bridge Span Configurations

Cable stayed bridges share a similar structural system that can be divided into four main components. These components are (1) the stiffening girder (sometimes referred to as the bridge deck), (2) the cable system, (3) the pylon(s) (sometimes referred to as the tower(s)), and (4) the foundations (including the anchor blocks) (see **Fig. 3.2**). Cable stayed bridges are further characterized by the configuration of these components. In general, the greatest variation is available in the cable configuration and anchorage support condition used. **Section 3.1** to **Section 3.4** discuss the variation in these components. **Section 3.5** to **Section 3.8** discuss the variation of the counter weight cable stayed bridge from the typical configuration of the cable stayed bridge.

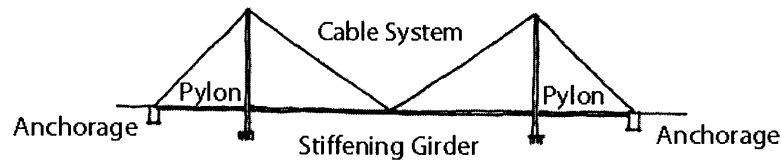


Figure 3.2 Components of Cable Stayed Bridge

Section 3.1 The Stiffening Girder

The stiffening girder performs a primarily utilitarian function. The girder is required to support the roadway deck under funicular loading, while remaining resistant to lateral displacements due to wind effects. In general, the stiffening girder relies upon the cable system for its bending stiffness. In some cases the stiffening girder's torsional rigidity is supplied by the cable system. More frequently though, the stiffening girder provides the torsional rigidity required. The introduction of a box girder is the typical method used to implement torsional rigidity. The bridge designer may use any combination of cable system and stiffening girder to achieve the prerequisite bending and torsional stiffness required for funicular use.

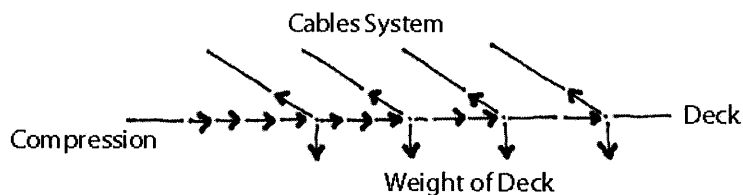


Figure 3.3 Growth of Axial Force in Deck

Beyond the bending and torsional requirements, the stiffening girder must also resist an axial force due to the cable system. Each cable transmits a normal force component to the deck at its anchorage. The deck must resist each force component locally and the

summational effect of these components globally. **Figure 3.3** illustrates the growth in the deck axial force. The weight of the deck equilibrates the system, resisting the vertical force component of each cable.

Section 3.2 The Cable System

In terms of variation, the cable system of a cable supported bridge offers the greatest option, both functionally and aesthetically. When cable stayed bridges are categorized, it is quite typical to identify them primarily by the configuration of the cable system.

Figure 3.4 illustrates the various possibilities for cable stay arrangement. The primary function of the cable system is to provide the stiffening girder with the required bending rigidity for funicular use. Cable stayed bridges may have a limited number of stays, or may – as has become the trend – have a relative large number of stay cables (multi-cable system). The advantage of the multi-cable system is that the stiffening girder becomes nearly continuously supported, and thus, the overall bending stiffness of the girder required to meet funicular loading is reduced. The benefit is materialized in a rather large cost savings that is a result of reduced member cross-sections, and a reduction in the required reinforcement at points of cable anchorage at the deck level [Gimsing, 1983]. Each cable may be composed of a single wire strand (mono-strand cable) or an assembly of several strands (multi-strand cable). The latter has become the more common.

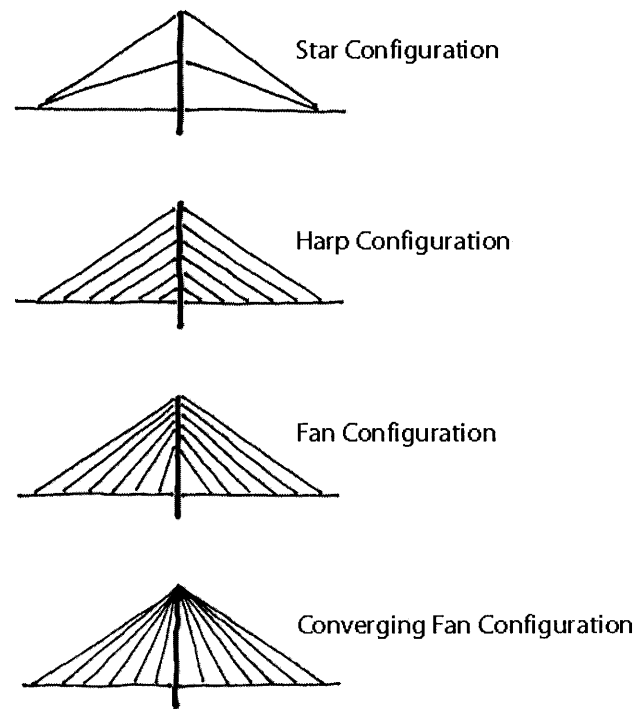


Figure 3.4 Typical Cable Configurations

Within the transverse direction of the cable stayed bridge the cable plane placement offers still more variation. **Figure 3.5** illustrates the realm of possibilities. For most cable stayed bridges the typical solution is to support the deck with two vertical cable planes along the perimeter of the deck. With this configuration, the deck is supported vertically and torsionally. Some cable stayed bridges have been designed using only one vertical cable plane. In this particular case the deck is supported vertically by the cable system, but torsional effects must be resolved within the stiffening girder. This scenario generally requires the introduction of a box girder(s) with a sufficient torsional rigidity to resolve the torsional effects. In some cases, where the bridge is divided into separate traffic areas, vertical cable planes may be placed between these as a means of traffic division and structural optimization. By placing the cables nearer the center, the bending moment of the transverse deck beam is reduced as compared to a deck beam supported

by cables at the deck perimeter. The downside of such a cable placement is that the torsional support of the cable system is reduced. Combinations of the above configurations, i.e. bridges of four or more cable planes, have been used with great success to provide direct transfer of deck loads to the cable system. The cable planes may also be placed in an inclined orientation in the transverse direction to allow for more dramatic bridge geometries, while effectively providing the deck both vertical and torsional support (see **Fig. 3.6**). By using inclined cable planes, a centrally located pylon may be used while successfully avoiding the use of a single vertical plane of cables and the structural requirements that it demands.

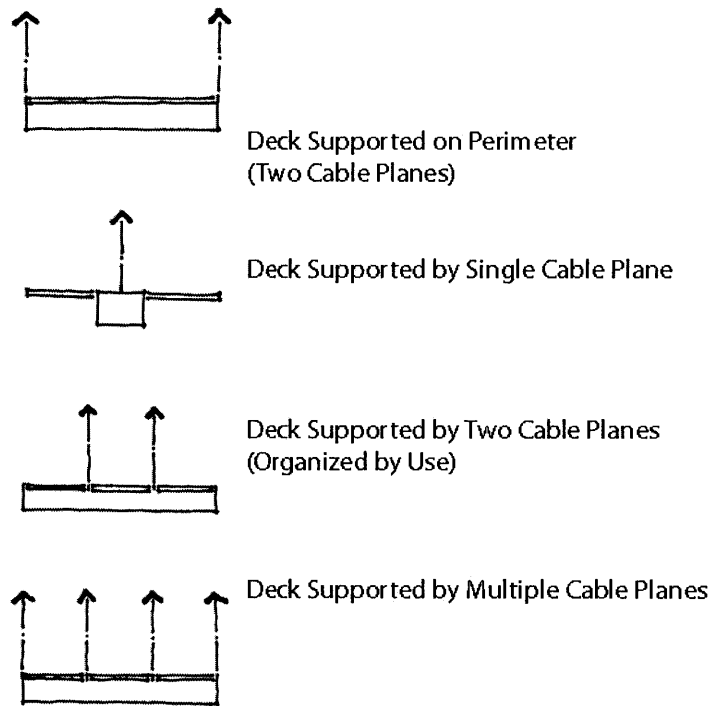


Figure 3.5 Cable Plane Placement (Transverse Direction)

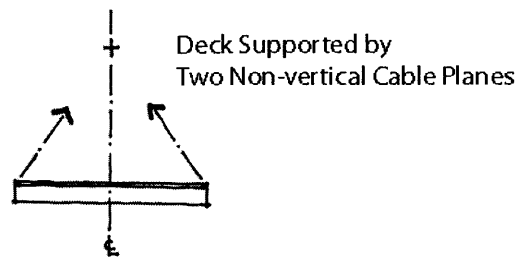


Figure 3.6 Inclined Cable Planes (Transverse Direction)

Section 3.3 The Pylon(s)

As the primary vertical element, the pylon(s) transfers the downward vertical force transmitted by the cable system to the bridge foundation. Each cable stay transmits a downward force component to the pylon (see **Fig. 3.7**). The summation of the downward forces due to the cable system will be significant. In order to deal with the growth of the compressive axial force, the pylon must be made of a material strong in compression – typically concrete. Depending upon how the pylon is fixed at the base, the pylon may experience global bending effects beyond the local bending effects from the cable system. Additionally, the cable system – in the transverse direction – may induce some torsional effect. The determination of the fixity at the base and the cable configuration in the transverse direction, therefore, determine to a large extent the overall sizing of the pylon. **Figure 3.8** illustrates some of the common pylon configurations used.

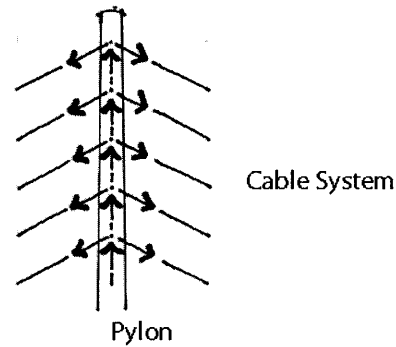


Figure 3.7 Growth of Axial Force in Pylon

Section 3.4 Anchorage

For the cable stayed bridge to function properly, the cable system must be able to calibrate itself so the overall structure is in equilibrium. Within a cable stayed bridge, equilibrium is achieved by transmitting a cable force to the cable anchorage and it being resolved by a boundary reaction. In general, there are two methods to anchor cable stayed bridges, (1) the earth anchor system and (2) the self-anchored system [Gimsing, 1983]. In the earth anchor system, both horizontal and vertical force components are transferred to the earth via a foundation structure. The self-anchor system transfers the vertical cable force component to the earth and the horizontal cable force component to the stiffening girder. The self-anchor system is more commonly employed. **Figure 3.9** illustrates the two anchorage systems.

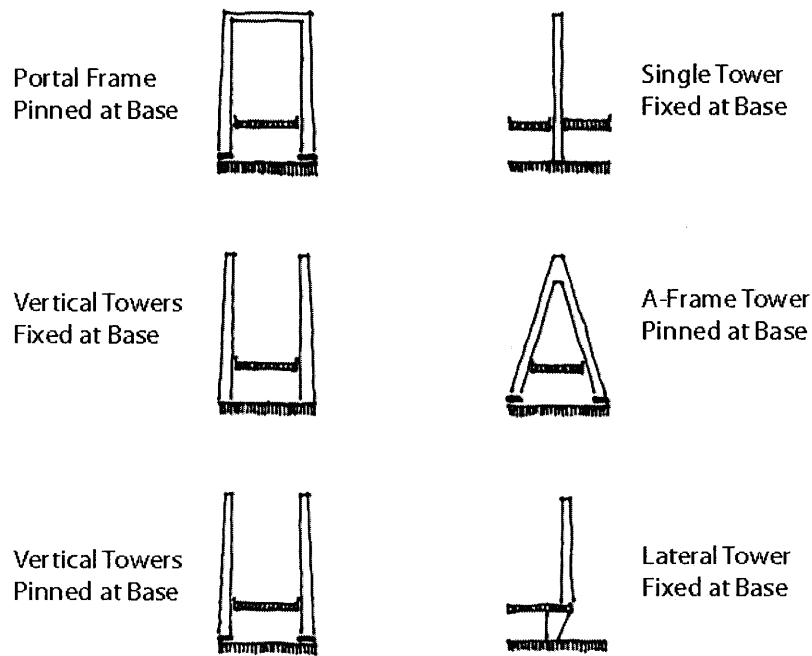


Figure 3.8 Pylon Configurations [Taly, 1998]

The combination of the anchorage system and the chosen cable system will determine the order of stability of the cable stayed bridge. Cable stayed bridges can be classified into one of three categories based upon the degree of stability of the system [Gimsing, 1983]. The three categories are as follow: (1) Stability of the first order. In this case stability is achieve without nodal point displacement. (2) Stability of the second order. In this case stability is achieved only by nodal point displacement under loading. (3) Unstable. In this case stability is impossible to achieve by the cable system alone. **Figure 3.10(a)** illustrates a cable system stable to the first order, and **Figure 3.10(b)** illustrates a cable system in an unstable configuration.

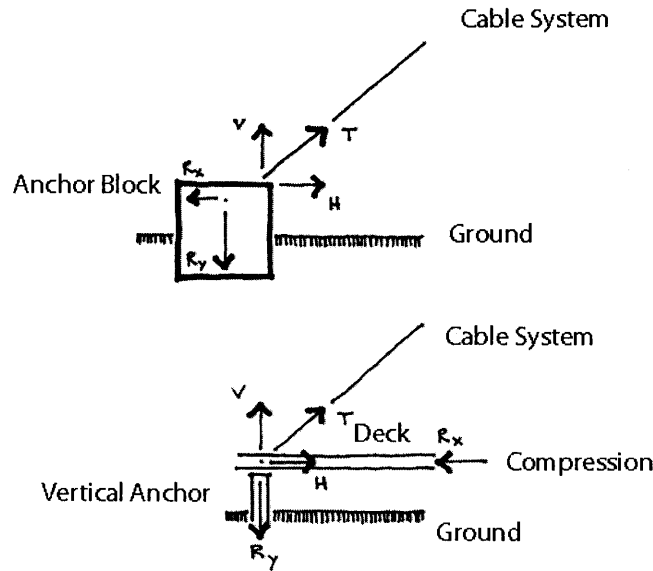
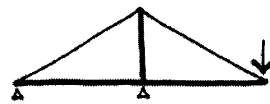


Figure 3.9 Anchorage Methods



(a) Cable System Stable in First Degree



(b) Unstable Cable System
(needs a reactionary moment to counter loading)

Figure 3.10 Stability of Cable System

Section 3.5 Stiffening Girder – Counter Weight Cable Stayed Bridge

In general, the stiffening girder for a counter weight cable stayed bridge functions identically to the stiffening girder of the typical cable stayed bridge. The deck's self-weight is supported by the cable system vertically. Because the cable stays are not exclusively vertical, a horizontal force component must be addressed. The cable

system's horizontal force components sum over the deck and consequently result in a rather large compressive force. Several methods are available to deal with the compressive forces that develop within the deck [Gimsing, 1983]. **Figure 3.11** illustrates the three options available. In case (1) the deck's compressive force is resolved by putting the deck in tension. This solution is obtained by a pinned support at the end opposite the pylon, and allowing the deck to rest on a roller support at the pylon. In case (2) the deck's compressive force is resolved by putting the deck into a combination of compression and tension. This solution is obtained by pinning both ends of the deck, and tensioning the end opposite the pylon. In case (3) the deck is allowed to go into compression completely. This solution is obtained by fixing the deck at the pylon end, and placing a roller support at the opposite end.

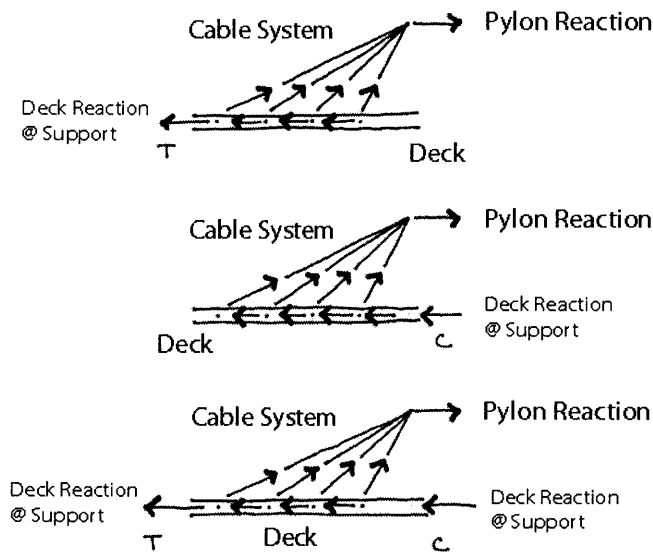


Figure 3.11 Deck Systems [Gimsing, 1983]

In the case of the counter weight cable stayed bridge, the stiffening girder must have sufficient bending and torsional rigidity to remain functional under loading. In the case of the traditional cable stayed bridge (except for the “harp” configuration), the cable system adjusts cable tension at the primary anchorage (stability of the first order). This allows the cable system to adjust the supplied stiffness to the stiffening girder. In the

case of the counter weight cable stayed bridge, the bridge deforms to meet equilibrium requirements. This is done primarily at the pylon and deck through bending. Because equilibrium is achieved by displacement, it is necessary to insure that while equilibrium is possible, the bridge will remain functional in its deformed and equilibrated state. The need to remain operational is assured through provision of a sufficient bending rigidity to the stiffening girder. This must also be the case for torsional effects. While the cable system may supply some torsional rigidity to the overall bridge structure, torsional effects are best managed at the stiffening girder. The introduction of a box girder, like in the case of the traditional cable stayed bridge, is the straightforward solution. A combination of the box girder and a cable placement in the transverse direction may also be employed to meet torsional rigidity demands.

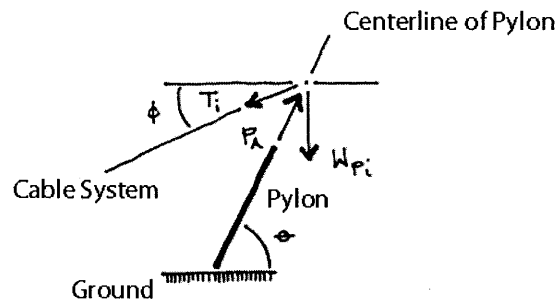


Figure 3.12 Pylon Equilibrium (Counter Weight Scheme)

Section 3.6 The Cable System – Counter Weight Cable Stayed Bridge

In a manner similar to the traditional cable stayed bridge, the cable system of the counter weight cable stayed bridge supports the stiffening girder vertically. To do so, the cables are post-tensioned to a desired level such that an effective stiffness is achieved at each cable anchorage. The equivalent system is then a beam on elastic supports, where the cables behave like springs. The ideal configuration for the cables is the “harp” layout. The need to mobilize enough mass at the pylon to support the deck requires that there be a sufficient length between anchorages along the pylon. The “fan” configuration, while still a possibility, makes the mobilization of the pylon mass difficult. The shortening

length between cable anchorages along the pylon for the “fan” configuration would by necessity require the pylon width and depth to grow rapidly to satisfy equilibrium. The result would be something visually of the nature of an upside-down pyramid. Such a structure for a pylon would not be ideal, as the dynamic behavior would be problematic.

Section 3.7 The Pylon(s) – Counter Weight Cable Stayed Bridge

The pylon for the counter weight cable stayed bridge differs in function to some extent from the traditional cable stayed bridge. The primary function of the counter weight cable stayed bridge pylon is to supply the prerequisite mass needed for equilibrium of the structural system. It is the gravitational effect upon this mass, and the downward resultant that calibrates the system. **Figure 3.12** illustrates the force actions at the pylon, locally, and how they are ultimately resolved.

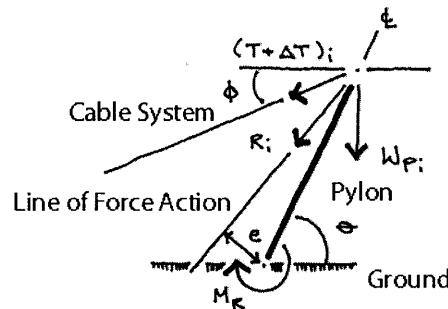


Figure 3.13 Pylon Equilibrium (Counter Weight Scheme - under loading)

As is the case with the traditional cable stayed bridge, the cables transfer a downward force to the pylon. This compressive force grows steadily as the individual effects are summed. For the typical case, this force remains entirely axial (except in cases where the pylon is fixed at the base and experiences some bending effects). Within the counter weight cable stayed pylon, as the bridge deforms and the system seeks an equilibrium state, the compressive force moves out of line from the normal of the cross section. This movement induces a moment within the pylon. As each cable transmits a downward

force component to the pylon, the effect grows and the pylon displaces more. This result leads to a rapid growth of a moment force within the pylon that must be resolved at the foundation. It is this particular matter – the rather large moment developed within the pylon – that makes the counter weight cable stayed bridge unique from the traditional type. Therefore, the bending rigidity of the pylon for the counter weight cable stayed bridge must be significantly large, such that displacements are minimized and the growth in the moment force controlled to a manageable level. **Figure 3.13** illustrates the above discussion.

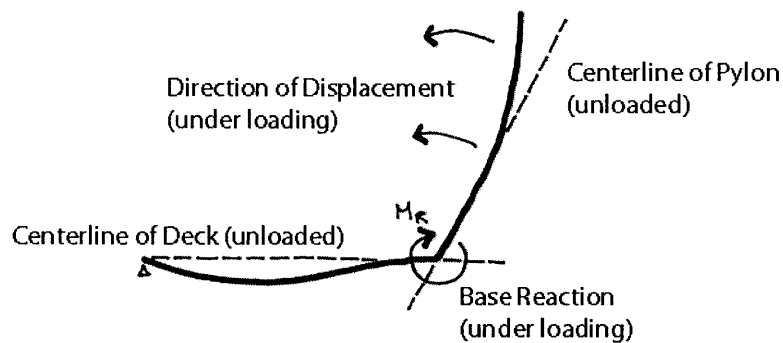


Figure 3.14 Anchorage (Base Reaction - Counter Weight Scheme)

Section 3.8 Anchorage (or Lack Thereof) – Counter Weight Cable Stayed Bridge

Of the differences between traditional cable stayed bridges and the counter weight variation, that of anchorage is the most fundamental. Regardless of the cable configuration, be it the “fan,” “harp” or some modified version, the traditional cable stayed bridge has some degree of anchorage that ultimately limits the extent of the bridge’s displacement. In the case of the counter weight cable stayed bridge, the system has no anchorage device to achieve this. Instead the system will, under increasing load, fold in upon itself and eventually collapse, a consequence from the system being unstable (see **Fig. 3.14**). To overcome this limitation, two conditions are required: (1) There must be a sufficient stiffness in the pylon and deck girder to limit deflections and

displacements. (2) There must be a sufficient capacity for the pylon to resist the rather large moment that it encounters at its end support and transfers to the foundation.

Section 4.0 The Boston Model

Given the options discussed in **Section 3.0**, and in particular **Section 3.5** to **Section 3.8**, a proposal was developed to meet the functional demands for a highway crossing of the Charles River¹. The primary functional requirement of the proposal is to carry nine lanes of traffic between Boston and Cambridge, Massachusetts. The nine lanes may be configured into three combinations of inbound and outbound lanes to meet peak traffic demands. Because the proposal is for a major highway bridge, stringent criteria needed to be met in terms of performance issues. These issues were primarily concerned with the structural behavior of the proposal under loading. The counter weight cable stayed bridge by nature, will tend to be a fairly lively bridge structure under loading. As a result, the configuration of the bridge was driven mainly to satisfy performance criteria. **Section 4.1** to **Section 4.4** discuss the four primary structural components of the proposal. **Appendix C** contains scaled drawings of the proposal. **Figure 4.1** is a rendering of the bridge proposal. **Figure 4.2** illustrates the bridge geometry and how the force system is resolved between elements.

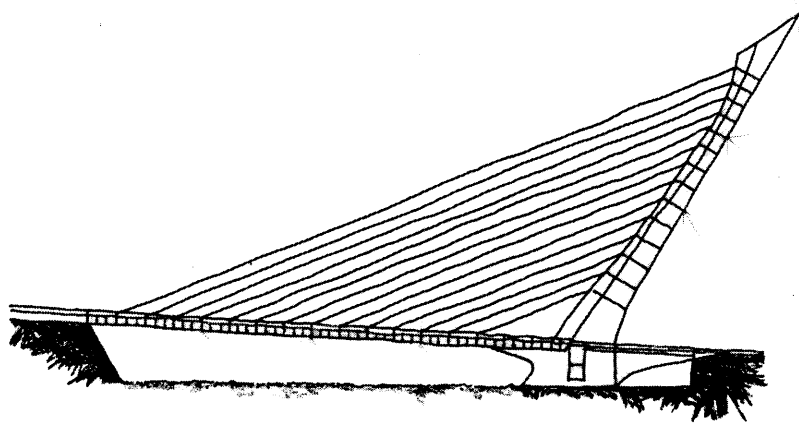


Figure 4.1 Rendering of Boston Proposal (Charles River Crossing)

¹ The Charles River Crossing proposal can further be reviewed in **A Proposal for the Charles River Crossing** by Greg Otto. This document goes into greater detail on the various functional aspects that ultimately determined the configuration of the bridge model.

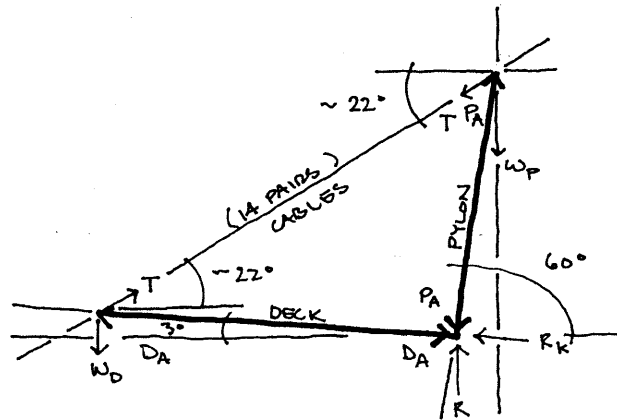


Figure 4.2 Bridge Geometry and Element Equilibrium

Section 4.1 The Stiffening Girder – Boston Model

The stiffening girder must support nine traffic lanes. The design calls for three separate concrete decks (three traffic lanes each), separated by two cable planes. The decks were designed as continuous spans supported every 12 feet. The deck design was ultimately governed by the desire to minimize the total deck weight. The inclusion of the primary structural elements that support the deck is addressed in the effort. **Figure 4.3** illustrates the optimization effort – deck weight is minimized for the 12-foot span between transverse girders. At points of support, steel girders transfer the deck loading to box beams. These beams run parallel to the span, and are supported by the cable system every 36 feet. **Figure 4.4** provides a cross-sectional view of the deck. The design of the box beam is governed by the necessity to meet bending and torsional rigidity requirements globally, as well as stress limits locally. On the local level, areas of concern are points of cable anchorage and transverse girder connection. On the global level, sufficient capacity to resist bending and torsional effects is required to certain that functional operation remains possible under prescribed loading. In the lateral direction, the stiffening girder is further stiffened by diagonal bracing within the central span. This bracing performs a secondary function as well, by providing intermediate support to the transverse girders on the underside, effectively reducing the unsupported length of these

members. Because these members experience a compressive force from the cable system, the inclusion of these members is quite important, well beyond what they provide in terms of stiffening in the lateral direction. **Figure 4.5** illustrates the primary structural elements of the stiffening girder.

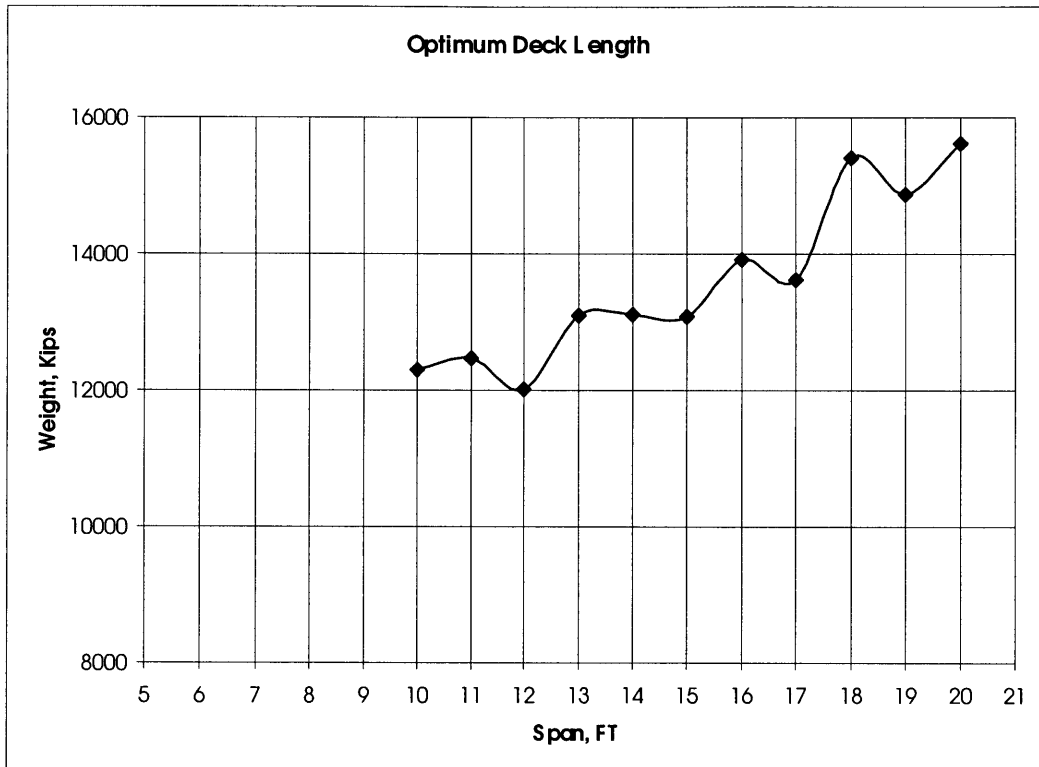


Figure 4.3 Optimization of Deck Length (Weight Consideration)

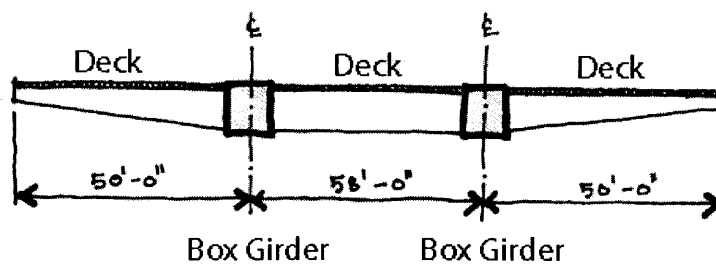


Figure 4.4 Transverse Deck Girder

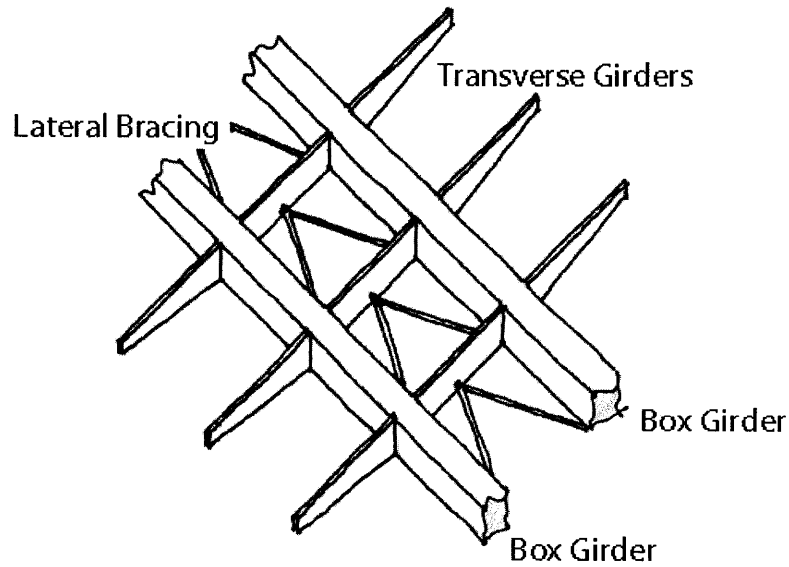


Figure 4.5 Stiffening Girder Axonometric

Section 4.2 The Cable System – Boston Model

The bridge proposal’s cable system has a “harp” configuration in two incline cable planes. The selection of the “harp” configuration was based upon the necessity to mobilize enough mass within the pylon to support the deck section between cables. While height issues do come into play with the “harp” configuration, the desire to avoid the “lollipop” effect with the mass was significantly stronger. The layout of the cable planes in the transverse direction allows the system to participate in providing some torsional rigidity to the overall bridge structure. **Figure 4.6** illustrates the concept.

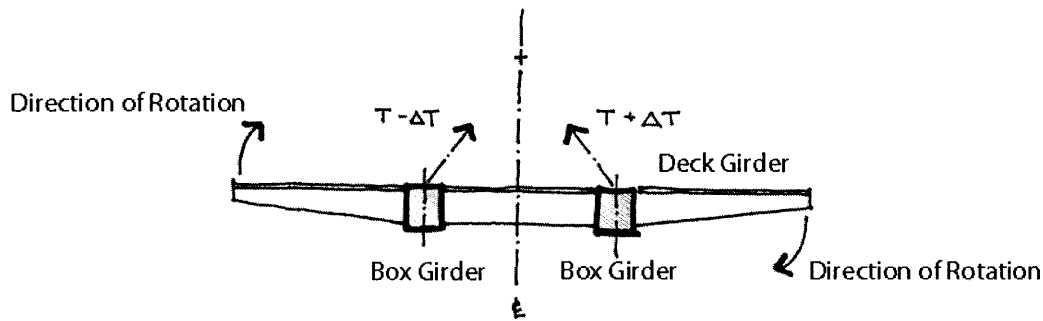


Figure 4.6 Cable System in Torsion

Section 4.3 The Pylon – Boston Model

The overall geometry of the pylon was primarily a consequence of aesthetic intention. Nevertheless, the necessity to provide a force reaction at the pylon to resist the moment induced by the cable system conditioned the geometry to some degree. As noted in **Section 4.1**, the deck weight was minimized; this greatly reduced the mass demands placed upon the pylon. The choice of concrete as the primary material of construction for the pylon was obvious: to provide the prerequisite mass needed for equilibrium. The pylon angle is a direct result of an attempt to minimize the cable quantity, while still providing the required mass effort (equilibrated force from the mass under gravitational effect). The cable system, by configuration, will transfer some torsional effect to the pylon. Therefore, the details of its cross-sectional design must be sufficient to resist the torsional demands placed upon it by the cable system. Because of the great complexity of this particular topic – torsional effects of concrete sections – it has been omitted from further discussion within this report.

Section 4.4 The Foundation – Boston Model

As noted in **Section 3.8**, the counter weight cable stayed bridge is an unstable structure. In order for the bridge system to establish equilibrium the pylon and deck displace. The resulting displacement induces a bending moment within the pylon. As each cable imparts a moment to the pylon the effect grows rapidly. As a result, a rather large

moment must be resolved at the foundation. To meet this demand a rather extensive foundation system is required. **Figure 4.7** illustrates how the loading is resolved at the foundation. For this particular case, a system of 124 drilled shafts (3 foot in diameter) were used, linked by a 15-foot thick pile cap. The shafts are to be set in sound rock, a depth of 10 feet. The average depth is estimated to be between 100 – 130 feet.

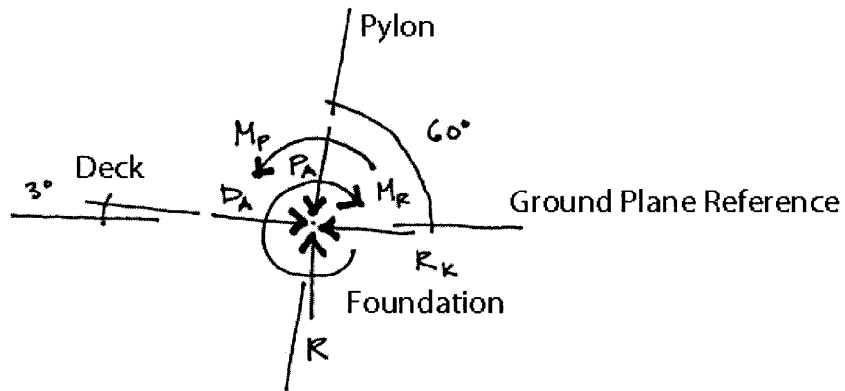


Figure 4.7 Equilibrium at Foundation (under loading)

Section 5.0 Loadings

All bridge structures must be designed to resist a variety of functional and environmental loadings. **Figure 5.1** illustrates the breakdown of loads on bridge structures. The primary guideline for quantifying design loads within the United States is the AASHTO code. **Section 5.1** to **Section 5.4** discuss the primary loads to consider when designing cable stayed bridge structures. Details of their relevance to the Boston example are provided. For a more detailed discussion the references used for this section, and in particular, the **Design of Modern Highway Bridges** by Narendra Taly, should be consulted.

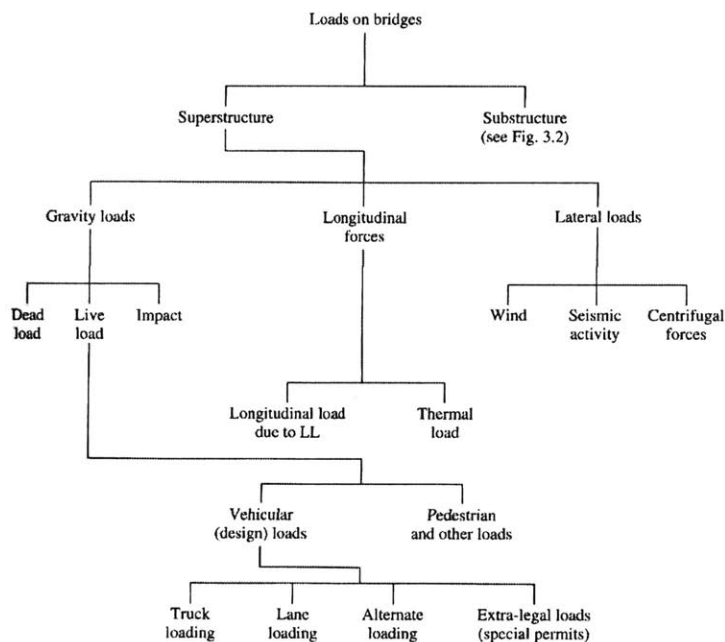


Figure 5.1 Bridge Loads [Taly, 1999]

Section 5.1 Dead Load

The dead load is defined as the weight of the structural system, plus all other fixed loads that remain constant through time. As noted previously, the designer attempts to minimize the weight of the deck for specific reason. This agenda puts less demand upon the system to generate reactionary forces to counter the vertical deflection of the deck. By minimizing the deck weight, the weight requirements at the pylon are reduced, and thus, are within a controllable realm. The primary contributions to the overall dead load

are the concrete (at 150 pcf) of the pylon and deck, and the steel structure (at 490 pcf) of the stiffening girder.

Section 5.2 Live Load

For bridges, the definition of a live load differs considerably from that of buildings. This is due primarily to the fact that for buildings, live loads are primarily static, whereas in bridges these loads are dynamic in behavior, rapidly varying with time. Establishing a working definition then, live loads are those loads, dynamic (transitory) in nature, and are the direct result of vehicular and pedestrian traffic. In general, live loads are defined within the context of a code. The live load effects upon a bridge are a function of several parameters: (1) gross vehicle weight, (2) axle loads, (3) axle configuration, (4) span length, (5) position, (6) multipresence (number of vehicles on the span), (7) vehicular speed, (8) structural stiffness, and (9) bridge configuration [Taly, 1999]. The complexity of the interaction of these parameters is directly proportional to the complexity of analysis that results.

Within the United States as prescribe by AASHTO, design live loads are divided into three categories: (1) design truck loading, (2) design lane loading, and (3) alternative loading [AASHTO, 1992]. For the Boston proposal, the design truck load used was the HS25-44 truck loading. **Figure 5.2** illustrates the application of the HS25-44 truck loading. The design lane loading utilized to perform the analysis is also noted by the HS25-44 distinction. **Figure 5.3** illustrates the use of the HS25-44 lane loading. The alternative loading was neglected from this investigation due to the use of the HS25-44 loadings for the previous two cases. AASHTO permits this within section 3.7.4. In general, the use of these loads is to generate worst case scenarios, and then design accordingly to meet stress and performance criteria.

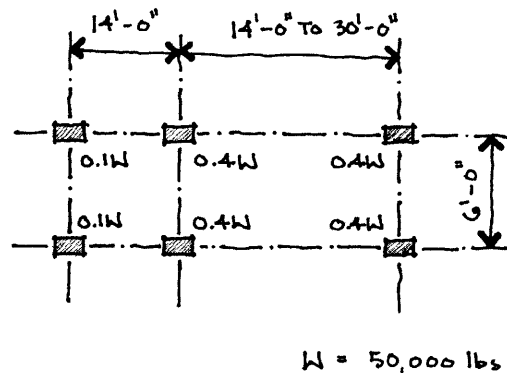


Figure 5.2 HS25-44 Truck Loading [AASHTO, 1992]

A sub-topic in the discussion of live loads is impact loads. Impact loads are those loads typically “live” in nature (live loads), having the additional feature of sudden application. The condition of sudden application is determined by the application time being shorter in duration than the fundamental period of the structure. While the code allows for the investigation of impact loads using fractional values (amplification or impact factors) of the applied live load, the Boston proposal forwent this option and used alternative computer methods to evaluate the dynamic behavior.

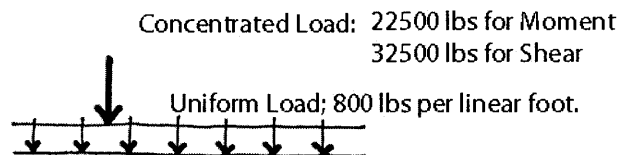


Figure 5.3 HS25-44 Lane Loading [AASHTO, 1992]

Section 5.3 Wind Loads

Wind loads represent the primary source of lateral forces acting on any bridge structure. The effect of wind upon a bridge structure is conditioned by a number of factors: (1)

wind velocity, (2) angle of approach, (3) geometrical configuration of the bridge, (4) surrounding terrain, (5) and gust characteristics. Wind effects may be classified as follows: (1) static wind pressure, (2) dynamic or oscillatory (aerodynamic phenomenon), and (3) buffeting between adjacent structures [Taly, 1999]. Static wind pressure develops as result of a steady wind acting on a structure in the general direction of approach. It results in two force components that act on the structure: (1) lift and (2) drag (see **Fig. 5.4**). Aerodynamic instability (an aerodynamic phenomenon) is defined as “the effect of a steady wind, acting on a flexible structure so as to cause a progressive amplification of these motions to dangerous destructive amplitudes” [Taly, 1999]. The concept of buffeting is defined as a random variation in wind velocity that forces a corresponding random structural vibration. The consequences of wind effect upon bridges cannot be neglected, as evidenced by the collapse of the Tacoma Narrows Bridge at Puget Sound (a result of aerodynamic instability). For cable stayed bridges – and in particular the counter weight cable stayed bridge – aerodynamic instability presents a major concern due to the high degree of flexibility found within the structural system. It is therefore essential to investigate the cumulative dynamic effect of wind-induced loadings. Vortex shedding – a consequence of a steady wind blowing around and across a surface – results in a turbulence that potentially could prove catastrophic (see **Fig. 5.5**). Flutter – an oscillating motion that is a result of two or more modes of vibration being coupled (typically bending and torsion) – also presents itself as a primary concern when investigating wind effects. These latter topics are best investigated within a wind tunnel investigation, and therefore, have been omitted from further discussion (their inclusion within this section is for matters of completeness).

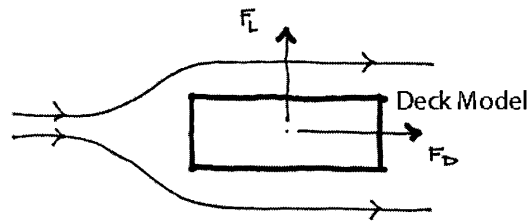


Figure 5.4 Static Wind Pressure

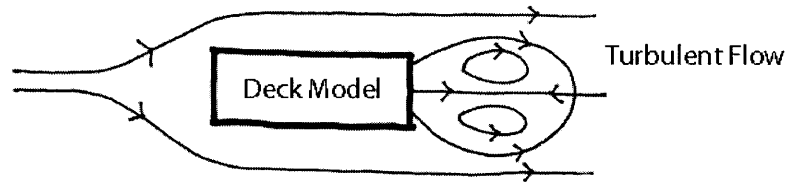


Figure 5.5 Vortex Shedding – von Kármán Vortex Trail

5.4 Seismic Loads

Given the current dependence upon the automobile within the United States, a sufficiently high importance is placed upon the reliability of a bridge structure to withstand a seismic event. Consequently, seismic analytical methods are continuously be evaluated for their effectiveness in predicting seismic effect. Numerous methods are available to investigate seismic phenomenon and their consequence upon bridge structures: (1) response spectrum modal analysis, (2) the time history method, and (3) the equivalent static force method. As like all loads, a number of variables impact the effect: (1) the dynamic response characteristic of the structure, (2) the dynamic response characteristic of the soil, (3) distance relationship to the active fault, and (4) the intensity

of the seismic event [Taly, 1999]. The investigation of seismic events and their effects upon civil structures is a comprehensive topic that requires relevant seismic data for the area. Absent this data, and given the vastness of the topic, the inclusion of seismic considerations is again for matters of completeness.

Section 6.0 Analytical Method

The design of a counter weight cable stayed bridge is an iterative process. The first charge is to establish preliminary sizing of the bridge structural elements. Several methods are available to establish initial cable size, deck girder geometry, and overall system equilibrium. **Section 6.1** to **Section 6.3** detail the methods employed to initially size the structural elements. To begin the design process the range of loadings (the design load combinations) must be established. Graphing these load cases leads to a graph where a minimum load (dead load only) and a maximum load (an extreme design event) are established (see **Fig. 6.1**). Within the range of possible loads as prescribe by code, the designer is charged with selecting an equilibrium load. It will be under this load that the calibration of the bridge's equilibrium will be made. To best select this equilibrium position, a designer should undertake a study to establish a mean loading using a statistical method. The outcome will be a baseline loading, consisting of the entire dead load plus some fraction of the live load based upon current and future usage rates.

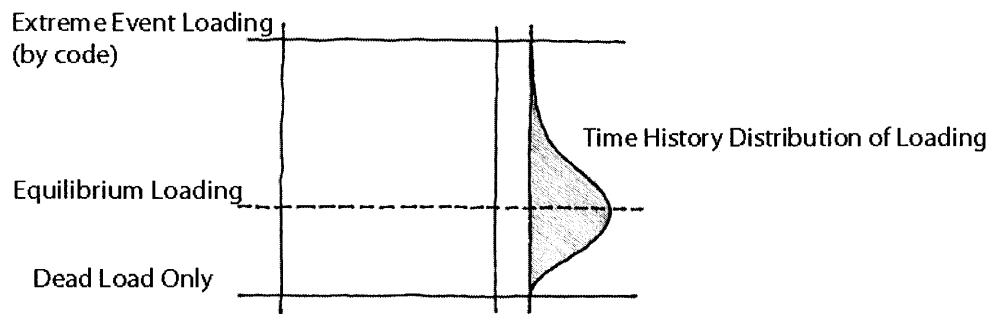


Figure 6.1 Equilibrium Load Case

Upon finalizing the preliminary sizes of the primary structural elements, the next step in the design process is to take these components and incorporate them into a comprehensive model (finite element model) to study the overall structural behavior of the bridge system. **Section 6.4** details the important areas considered when building the bridge model. The FEM software used for this investigation was ADINA.

Section 6.1 Cable Analysis

Given the required bridge span, the overall geometry of the counter weight cable stayed bridge can be established. Many iterations may occur before the final geometry is established. From the geometry, the cable length can be established, as well as, the required tension per cable for bridge equilibrium (see **Section 6.3**). Given the cable length and required tension, the theoretical quantity of cable steel can be established with **Equation 6.1**.

$$Q_i = \frac{\gamma}{\sigma} N_i L_i \quad \text{for each cable, } i. \quad (6.1)$$

where

γ = Density of Cable Material.

σ = Allowable Cable Stress.

N_i = Normal Force, (Cable Tension) Cable, i .

L_i = Cable Length, Cable, i .

The total cable quantity is then given by the summation of all cables, and is given by **Equation 6.2**.

$$Q = \sum_{i=1}^n \frac{\gamma}{\sigma} N_i L_i \quad \text{for the cable system.} \quad (6.2)$$

With the quantity of cable known, the sag of the cable can be determined. The sag perpendicular to the cable is given by **Equation 6.3**.

$$S_p = \frac{\omega_c L^2 \cos\phi}{8T} \quad (6.3)$$

where

S_p = Sag, perpendicular to cable.

ω_c = Unit Weight of Cable.

ϕ = Cable Angle.

T = Cable Tension.

The vertical cable sag is given by **Equation 6.4**. **Figure 6.2** illustrates the system investigated.

$$S_v = \frac{S_p}{\cos \phi} \quad (6.4)$$

where

S_v = Sag, vertical projection.

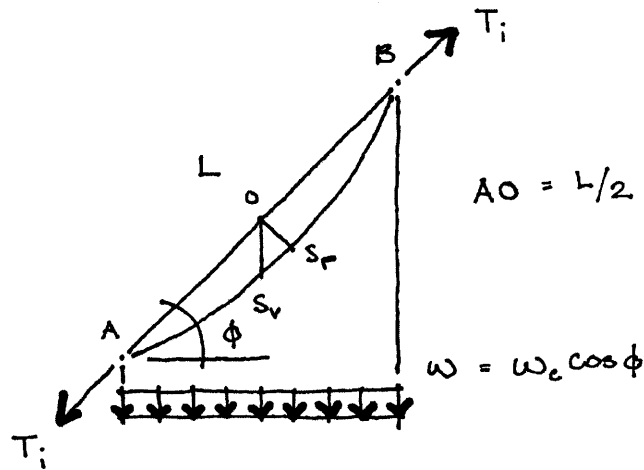


Figure 6.2 Cable Deflection

The analysis of the deflected shape is an important area of concern in the discussion of aesthetics. Should it be omitted, excessive sag within the cable system (particularly the very long cables) could destroy the aesthetic effect.

Having determined the tension in each cable (see **Section 6.3**), the effective tangential stiffness of each cable needs to be determined so the stiffening girder may later be

studied and sized (see **Section 6.2**). **Equation 6.5** establishes the effective stiffness of each cable.

$$k_{T,i} = \frac{A_i E}{L_i} \frac{1}{\left(1 + \frac{1}{12} \left(\frac{A_i E}{T_i}\right) \left(\frac{\omega_i L_i}{T_i}\right)^2\right)} \text{ for each cable, } i. \quad (6.5)$$

where

$k_{T,i}$ = Effective Cable Stiffness, Cable, i .

A_i = Cross-sectional Area, Cable, i .

L_i = Cable Length, Cable, i .

E = Modulus of Elasticity.

ω_i = Unit Weight of Cable, i .

T_i = Cable Tension, Cable, i .

Section 6.2 Stiffening Girder Model

The most effective method to study and effectively size the stiffening girder is the use of the beam on elastic support model. Nevertheless, some very rough approximations need to be made before this model can be employed, to determine initial bending and torsional rigidities of the stiffening girder. Preliminary sizes may then be studied within the beam on elastic support model. To determine the initial bending rigidity, a beam approximation (fixed at one end and simply supported at the other) under uniform loading was used (**Equation 6.6**). The torsional rigidity was determined by the use of **Equation 6.7**. This approximation is for thin-walled hollow shafts, and is relatively effective for the box girder application in the Boston model.

$$\delta = \frac{wl^4}{185EI} \quad (6.6)$$

where

δ = Maximum Deflection, Limited by AASHTO: Span/1000.

w = Uniform Load.

l = Span Length.

E = Young's Modulus.

I = Moment of Inertia of Section, Variable to Determine.

$$\tau_{Allowable} = \frac{T}{2tA} \quad (6.7)$$

where

$\tau_{Allowable}$ = Allowable Shearing Stress, Established by Code.

T = Maximum Torque Applied to Hollow Section.

A = Area Bounded by the Centerline of Wall Cross Section.

From these simple models the box girder dimensions may be established. **Equation 6.6**, establishes the section's moment of inertia, from which the sections width and depth are determined. **Equation 6.7**, establishes the minimum wall thickness of the box girder.

Given the preliminary geometry of the box girder and the effective stiffness of each cable (see **Section 6.1**), the beam on elastic supports may be used to refine the geometry.

Figure 6.3 illustrates how the analysis works. The cables each contribute an amount of stiffness to the beam. **Equation 6.8** is the general solution for a beam on elastic supports. The constants of integration must be determined using the boundary conditions as set for the model. The use of this equation is best done within computer programs, such as MATLAB.

$$y = e^{\beta x} (C_1 \sin \beta x + C_2 \cos \beta x) + e^{-\beta x} (C_3 \sin \beta x + C_4 \cos \beta x) \quad (6.8)$$

where

$$\beta = \sqrt[4]{\frac{k}{4EI}}$$

C_1, C_2, C_3, C_4 = Constants of Integration, Determined by Boundary Conditions.

$$k = \frac{k_{eff}}{L}$$

k_{eff} = The Effective Stiffness of Cable Elements, Assume Constant.

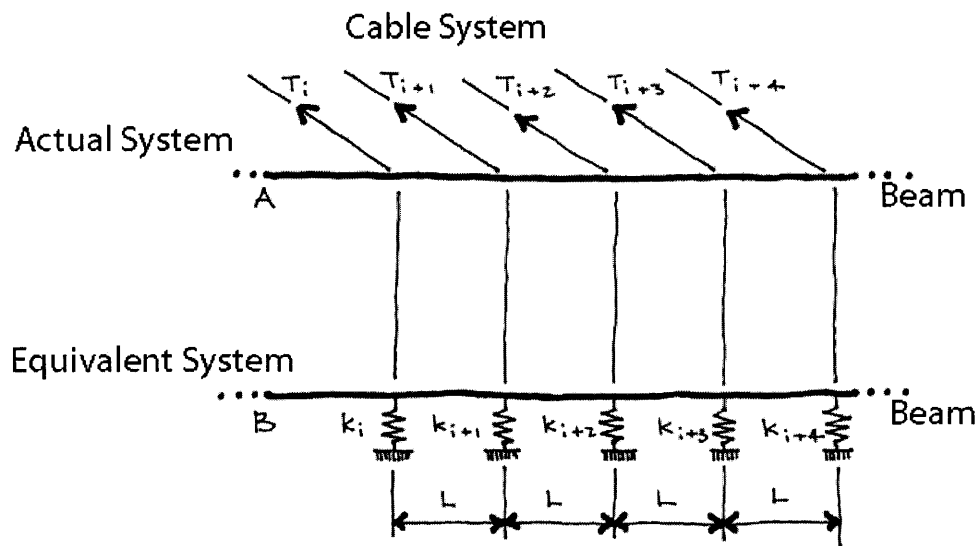


Figure 6.3 Beam on Elastic Supports Model

Section 6.3 Element Matching – System Equilibrium

The counter weight cable stayed bridge works as a system by establishing an equilibrium load. At this load, all forces are axial and the foundation experiences only a vertical and horizontal force component. There are three points of interest in establishing system equilibrium: (1) at the pylon element and cable element node, (2) at the deck element and cable element node, and (3) at the primary foundation at the base of the pylon. For each node along the pylon, each cable force must be matched by an equilibrated pylon weight. Equation 6.9 and Equation 6.10 establish the equilibrium condition. These equations

must be satisfied for the cable-pylon combination at each node. **Figure 6.4** illustrates the equilibrium condition.

$$T_{i,(x-y \text{ plane})} \sin \beta = \omega_{p,i} \sin \varphi \quad (6.9)$$

where

$T_{i,(x-y \text{ plane})}$ = Cable Tension (x - y plane), Cable, i .

β = Angle between Cable and Pylon.

$\omega_{p,i}$ = Weight of Pylon Element, i .

φ = Angle between Pylon and Gravitational Line of Action.

$$P_{A,i} = T_{i,(x-y \text{ plane})} \cos \beta + \omega_{p,i} \cos \varphi \quad (6.10)$$

where

$P_{A,i}$ = Axial Force in Pylon Element, i .

Other variables as previously defined.

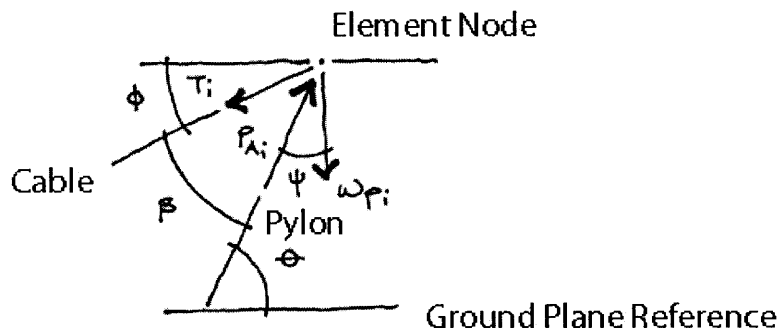


Figure 6.4 Equilibrium at Pylon-Cable Element Node

Equation 6.11 and **Equation 6.12** establish the equilibrium equations at nodes along the deck. Each cable force must be matched by an equilibrated deck weight. **Figure 6.5** illustrates the equilibrium condition.

$$T_{y,i} = \omega_{D,i} - D_{A,i} \sin \alpha \quad (6.11)$$

where

$T_{y,i}$ = Cable Tension in y - Direction, Cable, i .

$\omega_{D,i}$ = Deck Weight, Element, i .

$D_{A,i}$ = Axial Force in Deck, Element, i .

α = Angle between Horizontal and Deck.

$$T_{x,i} = D_{x,i} = D_{A,i} \cos \alpha \quad (6.12)$$

where

$T_{x,i}$ = Horizontal Force Component, Cable Tension, Cable, i .

$D_{x,i}$ = Horizontal Force Component, Deck Axial Force, Element, i .

Other variables as previously defined.

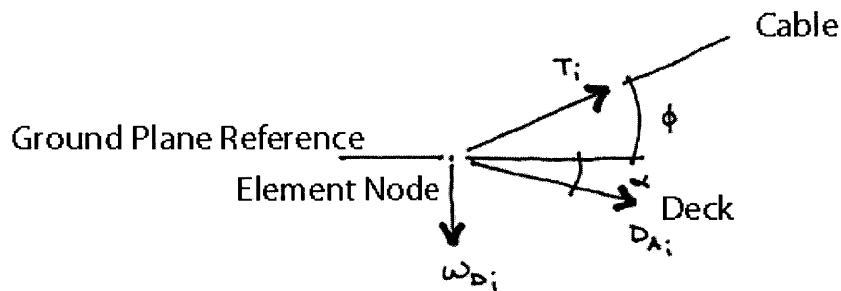


Figure 6.5 Equilibrium at Deck-Cable Element Node

The summation of the axial forces in both the pylon and deck, due to element matching, must be resolved at the pylon base. **Equation 6.13** and **Equation 6.14** establish equilibrium at the foundation. **Figure 6.6** illustrates the equilibrium condition.

$$\sum_{i=1}^n D_{x,i} = \sum_{i=1}^n P_{x,i} \quad (6.13)$$

where

$P_{x,i}$ = Horizontal Force Component, Pylon Axial Force, Element, i .

Other variables as previously defined.

$$R = \sum_{i=1}^n D_{y,i} + \sum_{i=1}^n P_{y,i} \quad (6.14)$$

where

R = Equilibrium Reaction at Foundation.

$D_{y,i}$ = Vertical Force Component, Deck Axial Force, Element i .

$P_{y,i}$ = Vertical Force Component, Pylon Axial Force, Element i .

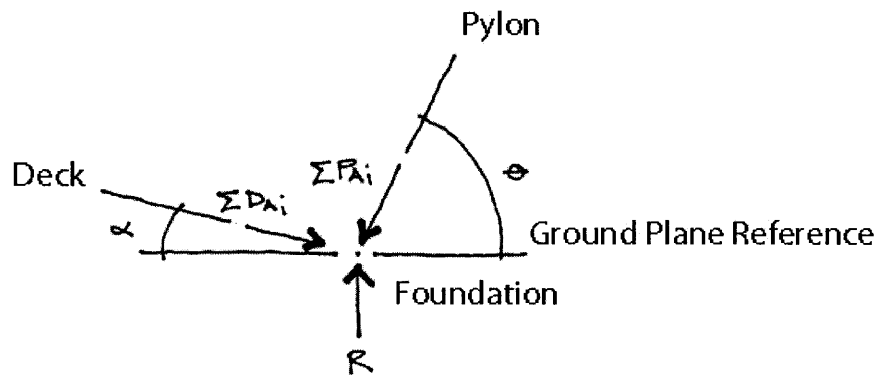


Figure 6.6 Equilibrium at Pylon Base

The results of element matching for the Boston proposal are listed in **Figure 6.8** for the Pylon, **Figure 6.9** for the cable system, and **Figure 6.10** for the deck.

Pylon						
Element No.	P _{iA} kips	P _{ix} kips	P _{iy} kips	V _p ft ³	W _p kips	
1	12280.72	6140.36	10635.42		55016	8252.4
2	10195.61	5097.807	8829.66		45675	6851.25
3	8052.693	4026.347	6973.837		36075	5411.25
4	6933.91	3466.955	6004.942		31063	4659.45
5	6738.592	3369.296	5835.792		30188	4528.2
6	5982.319	2991.16	5180.841		26800	4020
7	5441.009	2720.504	4712.052		24375	3656.25
8	5000.148	2500.074	4330.255		22400	3360
9	4570.447	2285.224	3958.124		20475	3071.25
10	4218.875	2109.437	3653.653		18900	2835
11	4419.773	2209.887	3827.636		19800	2970
12	4620.672	2310.336	4001.619		20700	3105
13	4952.825	2476.412	4289.272		22188	3328.2
14	5368.462	2684.231	4649.224		24050	3607.5

Figure 6.8 Element Matching – Pylon

Cables									
Element No.	T _i , (x-y plane) kips	L _{cx} ft	L _{cy} ft	L _{cz} ft	L _{ci} ft	T _i kips	T _{ix} kips	T _{iy} kips	
1	6515.053	1783	652.6825952	348	1930.333539	6539.805	6040.652	2440.582	
2	5408.882	2433	891.0965606	348	2614.315605	5388.765	5015.028	2026.203	
3	4272.039	3081	1128.761313	348	3299.661665	4242.08	3960.966	1600.334	
4	3678.513	3849	1411.378856	348	4114.352352	3645.791	3410.658	1377.995	
5	3574.895	4508	1653.164281	348	4814.1583	3539.693	3314.585	1339.179	
6	3173.684	5146	1887.082967	348	5492.130928	3140.514	2942.589	1188.883	
7	2886.513	5776	2118.004801	348	6161.917261	2855.144	2676.328	1081.307	
8	2652.632	6399	2346.304388	348	6824.474286	2623.01	2459.477	993.6933	
9	2424.671	7001	2566.737237	348	7464.800402	2397.048	2248.116	908.2978	
10	2238.158	7576	2777.055708	348	8076.442187	2212.263	2075.184	838.4287	
11	2344.737	8138	2982.504293	348	8674.299964	2317.27	2174.002	878.3539	
12	2451.316	8685	3182.33378	348	9256.218304	2422.305	2272.82	918.2791	
13	2627.526	9200	3370.175855	348	9804.039438	2596.152	2436.2984	2887	
14	2848.026	9703	3553.522651	348	10339.09262	2813.755	2640.644	1066.889	

Figure 6.9 Element Matching – Cables

Deck								
Element No.	$W_{D, Typ}$ kips	W_D kips	Required ΔW_D kips	Additional Concrete in Beam, h ft	D_{ix} kips	D_{iy} kips	D_{iA} kips	
1	706.3517	2757.159	1920.247102	35.56013152	6040.652	316.5771	6048.941	
2	706.3517	2289.029	1452.117459	26.89106406	5015.028	262.8265	5021.91	
3	706.3517	1807.92	971.0078806	17.98162742	3960.966	207.5854	3966.402	
4	706.3517	1556.74	719.8285883	13.33015904	3410.658	178.745	3415.339	
5	706.3517	1512.889	675.9774548	12.51810102	3314.585	173.71	3319.133	
6	706.3517	1343.098	506.1858661	9.373812336	2942.589	154.2145	2946.627	
7	706.3517	1221.567	384.655582	7.123251519	2676.328	140.2604	2680.001	
8	706.3517	1122.589	285.6773094	5.290320544	2459.477	128.8957	2462.852	
9	706.3517	1026.117	189.2048158	3.503792885	2248.116	117.8188	2251.201	
10	706.3517	947.1845	110.2727756	2.042088437	2075.184	108.7558	2078.032	
11	706.3517	992.2885	155.3767986	2.877348122	2174.002	113.9346	2176.986	
12	706.3517	1037.393	200.4808215	3.712607806	2272.82	119.1135	2275.94	
13	706.3517	1111.965	275.0528062	5.093570485	2436.2	127.6758	2439.543	
14	706.3517	1205.28	368.3680182	6.821629966	2640.644	138.3903	2644.268	

Figure 6.10 Element Matching - Deck

Section 6.4 FEM Model

The structural analysis of the Boston proposal was finalized using ADINA (a finite element application) to take into account the non-linear behavior of the system. For the sake of simplicity, the structure-soil interaction was neglected from the analysis. The fixity of the model at the supports can be seen in **Figure 6.11**. The deck was constructed of beam elements with diaphragm elements on the top surface (concrete deck). The deck was modeled using 8-node shell elements. The cables were considered to have no stiffness in compression. The cables were modeled with truss elements (thus the necessity for non-linear analysis). An initial strain was applied to provide the necessary pre-stressing (established in the element matching sequence). The pylon was modeled using beam elements (exhibits some non-linear behavior: buckling). A damping ratio of 0.9% was used in the Rayleigh proportional scheme. ADINA was left to optimize the meshing of the elements. Both a static and a dynamic analysis were performed. For the static case, the structure was analyzed under self-weight, equilibrium loading (design equilibrium, see **Fig. 6.1**), and extreme event loading. For the dynamic case, a moving load was employed using the AASHTO specified HS25-44 loading.

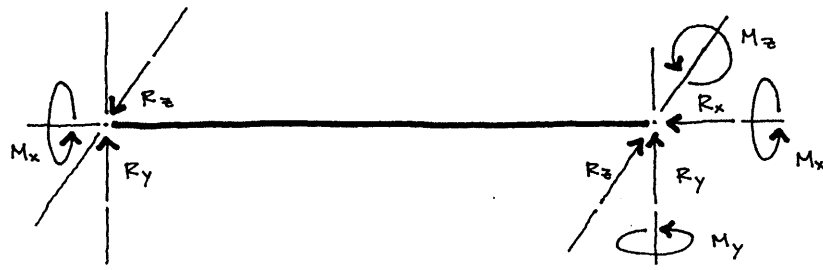


Figure 6.11 Fixity of Model

Section 7.0 Structural Behavior

The structural behavior of the cable stayed counter weight bridge is a rather complex phenomenon. Whether under static (except for the symmetrical case) or dynamic loading, the bridge structure's reactions are governed by the coupling of the torsion and bending characteristics of the structural system. The analysis of the Boston proposal is agreement with this assessment. **Section 7.1** discusses the initial static analysis, and the response of the model under loading (live load and static wind pressure). **Section 7.2** discusses the complexity of the dynamic analysis, and the results of the investigation. In both sections, some design advice is provided, based upon the author's experience in developing the Boston proposal.

Section 7.1 Behavior under Static Loading

In general, the counter weight cable stayed bridge is well behaved under static loading. This of course is dependent upon the selection of an appropriate point of equilibrium (see **Section 6.0**, and in particular **Fig. 6.1**), and assumes that the typical range of loadings is relatively small about equilibrium. If these conditions are met, then the deflection of the primary structural members can be controlled by design. The investigation of the stiffening girder and in particular its deflection under loading is critical. The bending rigidity of the stiffening girder is found in the girder itself, as well as in the bending rigidity of the pylon and its overall effect upon the cable system when loaded (see **Section 6.2**). **Figure 7.1** graphs the deflection of the stiffening girder and compares the FEM analysis (for both the static and dynamic case – first mode) with the beam on elastic supports model. The location of Node 1 is at the pylon. Node 18 is at the opposite embankment.

Attention should be given to the pylon such that sufficient bending rigidity is available to meet demand, and that the foundation can provide the prerequisite reactionary moment. When the bridge is unloaded, such that the load is less than the equilibrium case, the foundation is required to resist a negative overturning moment. The reverse is true when the bridge is loaded beyond its equilibrium case. The degree to which the pylon deflects

is directly proportional to the moment that the foundation system must resist. Therefore, the best available design measure to control the moment at the foundation is to limit the deflection of the pylon.

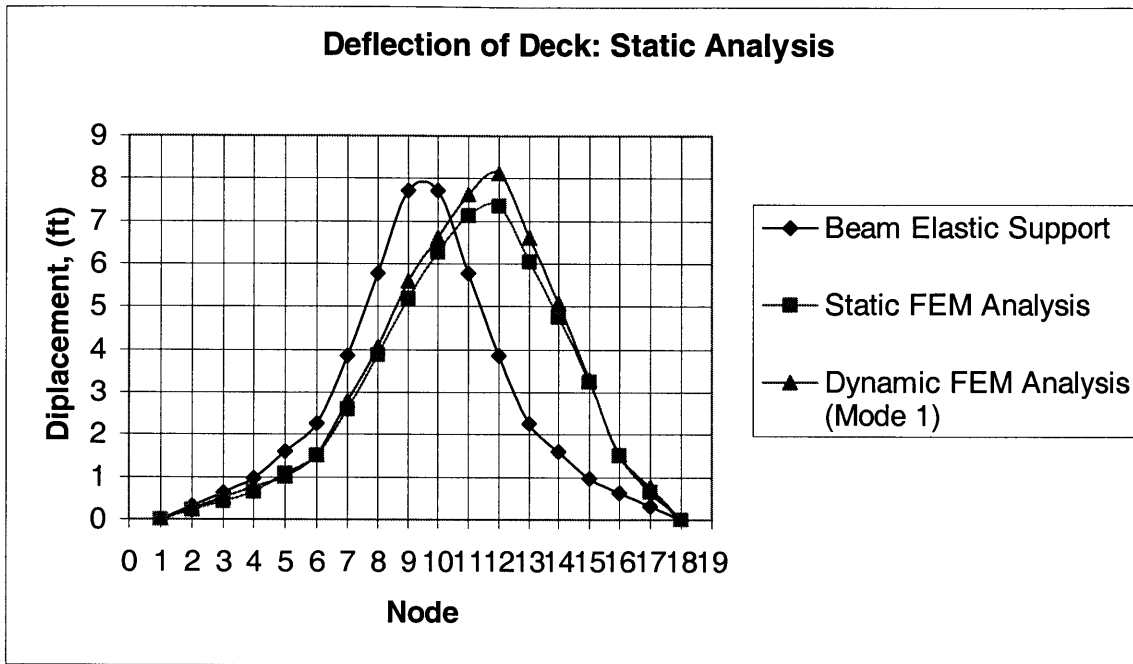


Figure 7.1 Deflection of Stiffening Girder

Santiago Calatrava when designing the Alamillo Bridge chose to make the bridge very stiff. In so doing he assumed that a static analysis of wind effects could be done within reasonable accuracy. The results of a contractor sponsored wind tests (wind tunnel investigations) essentially confirmed what Calatrava assumed, that a sufficiently stiff structure experiences marginal dynamic effect [Pollalis, 1999]. A similar approach was taken when designing the Boston model, faced with a lack of wind testing facilities and the marginal value found in performing wind analysis with FEM methods for dynamic effects. In the lateral direction, because of the width of the deck (9 traffic lanes) and the extensive bracing system below the deck, the bridge experienced marginal lateral displacements (maximum of 3 feet off center) using static analysis. The diaphragm action of the deck coupled with the diagonal bracing provided sufficient stiffness in the lateral direction of the deck to resist the rather large static design wind pressure (75 psf).

Section 7.2 Behavior under Dynamic Loading

The structural behavior of the counter weight cable stayed bridge under dynamic loading can be problematic. The counter weight cable stayed bridge is an extremely flexible structure, and as a result, dynamic effects can have rather severe effects. Of issue is the fact that counter weight cable stayed bridges have a complex structural behavior in which the translational and torsional modes are commonly coupled, and are prone to excitation under dynamic loading. **Figure 7.2** lists the first ten modes and their corresponding periods of the Boston model. For the Boston model, a forcing function of variable magnitude was applied in the vertical and lateral direction in an attempt to simulate the dynamic loading of the traffic. The results of this investigation show that the first several modes are relatively close to each other and potentially could couple when the bridge is loaded.

Mode Number	Frequency (Hz)	Mode Type
1	0.2867	Vertical
2	0.3756	Vertical
3	0.5145	Torsion
4	0.6102	Vertical
5	0.6388	Torsion
6	0.6423	Lateral
7	0.677	Vertical
8	0.7249	Vertical
9	0.8575	Vertical
10	0.8801	Torsion

Figure 7.2 First Ten Modes

The first few modes suggest that wind effects (gusting wind) may be problematic. For Mode 1 and Mode 2, the concern is obvious as their periods (3.488 seconds and 2.662 seconds, respectively) are similar to what could be expected from a gusting wind (3 to 5 seconds) [Taly, 1999]. The excitation of these modes could pose a considerable threat. Consideration of either providing more stiffness or damping (assumed damping ratio: less than 1%) would be obvious corrective measures.

The dynamic effect of traffic also presents some design concern. The relative closeness of each mode, suggest that two or more modes may easily couple and be excited leading

to excessive displacements. Given the functional demands of the counter weight cable stayed bridge system, sufficient structural capacity to resist excessive deformations must be provided to meet demand. Additional attention should be given to how the dynamic effects of traffic couple with wind effects, and the torsional consequence of the coupling. Given the complexity of that inquiry, it is obvious that wind tunnel testing is essential, regardless the stiffness of the system, i.e. the assumption that a static wind analysis is satisfactory more likely incorrect.

Section 8.0 Observations

The structural behavior of the counter weight cable stayed bridge exhibits complex tendencies to couple bending and torsional effects under loading. Considerable attention should be given to the study of the interaction of the dynamic load cases – particularly wind and traffic – in generating a system response. Further, optimization of the structural system is required to a certain that the dynamic load cases do not excite the system's modes of vibration. While this investigation touched upon the dynamic effects, considerable effort should be given to the study of wind effects upon the bridge using wind tunnel testing. Employing the information obtained from such testing would better the accuracy of this investigation. Further, the study of the counter weight cable stayed bridge under seismic load (topic omitted here) is essential, given the flexibility of the system. Failure to do so might ultimately prove catastrophic.

In conclusion, given quality information concerning the loading of the system, an effective design strategy can be employed to control the structural behavior of the system. The ability of the system to meet the structural demands of the design load combinations in a controlled manner is possible provided an effective design strategy is employed and all structural effects are considered. The fine-tuning of the structural parameters based upon a comprehensive structural analysis will result in an optimized structural system to meet the wide spectrum of design loads expected. Should such a strategy be employed, the counter weight cable stayed bridge typology offers exceptional design potential.

Appendix A – References

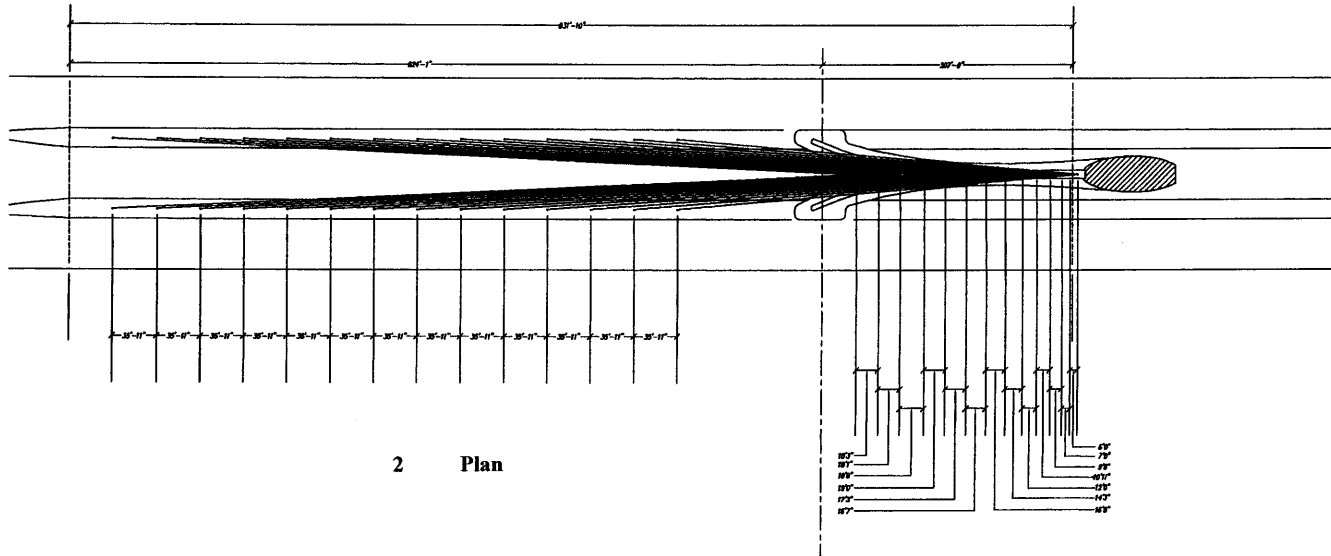
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Appendix B – Software List

The following programs were used to develop the Boston proposal and this thesis:

FEM Analysis	ADINA 7.4
Numeric Methods	MATLAB 5.3
Operating System	MacOS 8.6
	Windows 2000
Document Composition	MS Word 98/2000
Calculations (Spreadsheet)	MS Excel 98/2000
Drafting (CAD)	AutoCAD 2000

Appendix C – CAD Drawings, Boston Proposal



2 Plan

